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Differensial tenglamalarni yechishda energiya integralining tadbiqi

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KIRISH

Mavzuning dolzarbligi. Xususiy hosilali differensial tenglamalar nazariyasining muhim bo'limlaridan biri, aralash tipdagi tenglamalar o'tgan asrning 50-yillaridan boshlab, tez rivojlanib bordi. Mazkur yo'nalish bo'yicha birinchi fundamental tadqiqotlar F.Trikomi[1] tomonidan olib borildi va muhim natijalarga erishildi. Keyin esa aralash tipdagi tenglamalar nazariyasini rivojlanishida F.I.Frankl [2], I.N.Vekua[3] tomonidan Trikomi masalasining va unga bog'liq boshqa masalalarning muhim tadbiqlarini aniqladi. A.V.Bitsadze birinchi marta aralash tipdagi tenglama uchun Trikomi masalasi va boshqa chegaraviy masalalar uchun ekstremum prinsipini ta'rifladi. Ekstremum prinsipidan bu masalalar yechimining yagonaligi bevosita kelib chiqadi. Yana K.I.Babenko, T.Sh.Kalmenov, A.M.Naxushev, M.S.Salohiddinov va boshqa olimlar tomonidan aralash tipdagi tenglamalar uchun yangi chegaraviy masalalar qo'yildi va tadqiq etildi.

Ma'lumki, M.S.Salohiddinov, A.Xasanov, B.Islomov, S.X.Akbarovalarning ishlarida ikkita buzilish chizig'iga ega va turli tartibli aralash tipdagi tenglamalar uchun, local va nolokal chegaraviy masalalar tadqiq etilgan[4-8].

Magistrlik dissertatsiyasi kirish qismi, ikkita bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan iborat.

Ishning maqsadi . Ikkita buzilish chizig'iga ega va turli tartibli aralash elliptik-giperbolik tipdagi tenglama uchun Trikomi masalalarini qo'yish, bu masalalar yechimini mavjudligini va yagonaligini isbotlash.

Tadqiqot usuli. Trikomi masalalari yechimining yagonaligi energiya integrali usulida hamda A.V.Bitsadze ekstremum prinsipining analogi, yechimning mavjudligi esa integral tenglamalar nazariyasi yordamida isbot etiladi.

Ilmiy yangiliklar. Dissertatsiyada olingan natijalar quyidagicha:

Ushbu

$$signy\,|y|^{m_i}u_{xx} + x^n u_{yy} = 0 \qquad (1)$$

ko'rinishdagi aralash elliptik-giperbolik tipdagi tenglama uchun chekli D sohada Trikomining local va nolocal masalalari qo'yildi, bu yerda $m_i, n = const \quad (i = 1,2)$ va $y > 0$ da $i = 1$, $y < 0$ da $i = 2$ bo'lib, $n < m_1 \le m_2$. Trikomi masalalari uchun A.V. Bitsadzening ekstremum prinsipi analogi keltirildi.

2) Trikomi masalalari yechimining yagonaligi energiya integrali usuli yordamida, bundan tashqari ekstremum prinsipini qo'llagan holda isbotl etildi. Bunda matematik analiz kursida keltirilgan Grin formulasidan keng foydalanildi.

3) Buziluvchan elliptic va giperbolik tipdagi tenglamalar uchun chegaraviy masalalar o'rganildi va bu masalalar yechimlari ko'rinishi aniqlandi.

4) Interal tenglamalar nazariyasini qo'llagan holda, (1) aralash elliptic-giperbolik tipdagi tenglama uchun Trikomi masalalari yechimining mavjudligi isbotlandi.

Amaliy va nazariy ahamiyati. Magistrlik ishi mavzusi aralash tipdagi tenglamalar nazariyasidagi muhim mavzulardan biri bo'lib, unda mazkur nazariya bo'yicha nazariy jihatdan katta ahamiyatga ega bo'lgan yangi natijalar olindi. Bu masalalar fizik, mexanik, texnik, biologic va boshqa jarayonlarni o'rganish bilan uzviy bog'langan bo'lib, ularni aralash tipdagi tenglamalar va bunday tenglamalarga keltiriladigan amaliy masalalarni yechishda qo'llanilishi mumkin.

Ishning aprobatsiyasi. Dissertatsiyada olingan natijalar fizika-matematika fakulteti, Matematika kafedrasining fizika-matematika fanlari doktori G'.Mo'minov rahbarligidagi ilmiy seminarlarda, 2011 yil 8-9 noyabrda da Andijon davlat universitetida o'tkazilgan Respublika ilmiy anjumanda ma'ruza qilingan.

E'lon qilingan ishlari. Dissertatsiyaning asosiy natijalari [21],[22] ishlarda e'lon qilingan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, ikkita bob, xulosa va foydalanilgan adabiyotlar ro'yhatidan tuzilgan.

Ishning umumiy xajmi 64 betdan iborat. Adabiyotlar ro'yhati bo'yicha 22 ta.

Dissertatsiyaning mazmuni. Magistrlik ishining kirish qismi dissertatsiya mavzusiga oid adabiyotlarning qisqacha mazmuni yoritilgan va mavzuning dolzarbligi hamda olingan asosiy natijalarning qisqacha bayoni keltirilgan.

I-bob uchta paragrafdan iborat bo'lib, 1.1-§ da matematik analiz kursidan ma'lum bo'lgan, gamma va beta funksiyalar haqida ma'lumot berilgan.

1.2-§ da esa gipergeometrik funksiya va uning xossalari keltirilgan. Bundan tashqari mazkur paragrafda umumlashgan kasr tartibli integral operatorga ta'rif berilgan. Quyidagi

$$F_{0x}\left[\begin{matrix} a,b \\ c, \dfrac{x^{\kappa}-t^{\kappa}}{x^{\kappa}} \end{matrix}\right] f(x) =$$

$$= \frac{1}{\Gamma(c)} \int_0^x f(t)(x^k - t^k)^{c-1} F\left(a,b,c;\frac{x^{\kappa}-t^{\kappa}}{x^{\kappa}}\right) k t^{k-1} dt$$

$$F_{x1}\left[\begin{matrix} a,b \\ c, \dfrac{x^{\kappa}-t^{\kappa}}{x^{\kappa}} \end{matrix}\right] f(x) =$$

$$= \frac{1}{\Gamma(c)} \int_x^1 f(t)(t^k - x^k)^{c-1} F\left(a,b,c;\frac{x^{\kappa}-t^{\kappa}}{x^{\kappa}}\right) k t^{k-1} dt$$

ko'rinishdagi ifodalar c $(c > 0)$ tartibli, $f(x)$ funksiyadan olingan integral deb ataladi [4], bu yerda $a,b,c - const, k > 0,\ F\left(a,b,c;\frac{x^{\kappa}-t^{\kappa}}{x^{\kappa}}\right)$–Gaussning gipergeometrik funksiyasi.

1.3-§ da giperbo'lik tipdagi tenglamalar uchun qo'yilgan chegaraviy

masalalar yechimining yagonaligini isbotlashda energiya integrali usuli

qo'llanilishi keltirilgan.

1.4-§da esa xususiy hosilali differensial tenglamalar nazariyasining muhim bo'limlaridan, aralash tipdagi differensial tenglamalar haqida ma'lumot berilgan.

II-bob ikkita buzilish chizig'iga ega va turli tartibli aralash tipdagi tenglama uchun chegaraviy masalalarni o'rganishga bag'ishlangan bo'lib, u uchta paragrafdan tuzilgan.

2.1-§.da

$$signy\ |y|^{m_i} u_{xx} + x^n u_{yy} = 0 \quad , \quad m_i, n = const \qquad (i = 1,2) \tag{1}$$

tenglama uchun Trikomi tipidagi TC va T masalalar qo'yilgan, bu yerda $n < m_1 < m_2$ и $i = 1$ da $y > 0$, $i = 2$ da esa $y < 0$.

D-soha, $x > 0, y > 0$ da oxirlari $A(h,0), A_1(0,h_1)$ nuqtalarda bo'lgan, σ Jordan chizig'i bilan, OA_1 kesma va (1) tenglamaning $x > 0, y < 0$ da ushbu

$$OC: \frac{1}{q}x^q - \frac{1}{p_2}(-y)^{p_2} = 0,\ AC: \frac{1}{q}x^q + \frac{1}{p_2}(-y)^{p_2} = 1$$

xarakteristikalari bilan chegaralangan, bu yerda,

$$h = q^{\frac{1}{q}}, h_1 = p_1^{\frac{1}{p_1}}, 2q = n+2,\ 2p_1 = m_1+2,\ 2p_2 = m_2+2\ .$$

D_1 va D_2 - D sohaning elliptic va giperbo'lik qismlari bo'lsin.

Ta'rif. (1) tenglamaning D sohadagi regulyar yechimi deb, quyidagi shartlarni qanoatlantiruvchi $u(x,y)$ funksiyani ataymiz:

1) $u(x,y) \in C(\overline{D}) \bigcap C^1(D) \bigcap C^2(D_1 \bigcup D_2)$ bo'lib, $u_y(x,0)$ $x \to 0$ va $x \to h$ da $(n+2)/(m_2+2)$ dan kichik tartibda cheksizlikka intiladi;

2) $u(x,y)$ (1) tenglamani D_1 va D_2 sohalarda qanoatlantiradi.

TC masala. Ushbu shartlarni qanoatlantiruvchi $u(x,y)$ funksiyani aniqlang:

1) $u(x,y)$ (1) tenglamaning D sohada regulyar yechimi;

2) $u(x,y)$ quyidagi chegaraviy shartlarni qanoatlantiradi

$$u(x,y)/_{\sigma} = \varphi_0(s),\ s \in \overline{\sigma}, \qquad (2)$$

$$u(x,y)/_{OA_1} = \varphi_1(y), 0 \le y \le h_1, \qquad (3)$$

$$\frac{d}{dx^{2q}}(x^{2q})^{\frac{1-\alpha-\beta}{2}} F_{0x}\left[\begin{matrix}\frac{\alpha+\beta-1}{2}, \frac{\alpha+\beta}{2} \\ \beta, \frac{x^{2q}-t^{2q}}{x^{2q}}\end{matrix}\right](x^{2q})^{\frac{2\alpha-1}{2}} u[\theta(x)] =$$

$$= a(x)u_y(x,0) + b(x), 0 < x < h, \qquad (4)$$

bu yerda $\varphi_0(x,y), \varphi_1(y),$ $a(x), b(x)$- berilgan yetarlicha silliq funksiyalar bo'lib,

$$\varphi_0(l) = \varphi_1(h_1), \varphi_1(y) \in C[0,h_1] \cap C^2(0,h_1), \qquad (5)$$

$\varphi_0(s) = \varphi_0(x,y)$ funksiya esa

$$\varphi_0(x,y) = xy\overline{\varphi_0}(x,y), \overline{\varphi_0}(x,y) \in C(\bar{\sigma}), \qquad (6)$$

ko'rinishga ega va

$$(x^{2q})^{\frac{1-\alpha+\beta}{2}} a(x) + \gamma > 0, \qquad (7)$$

yoki

$$(x^{2q})^{\frac{1-\alpha+\beta}{2}} a(x) + \gamma = 0, \qquad (8)$$

γ-berilgan son, $2\alpha = n/(n+2), 2\beta = m_2/(m_2+2), F_{0x}[\quad]$-kasr tartibli umumlashgan integral operator,

$\theta(x) = \left(\frac{x^q}{2}\right)^{1/q} - i\left(\frac{p}{q}\frac{x^q}{2}\right)^{1/p}$ – (1) tenglamani $(x,0)$ dan chiquvchi xarakteristikalarining OC xarakteristika bilan kesishish nuqtasining affiksi.

T masala.(1) tenglamaning D sohada regulyar va (2), (3),

$$u|_{OC} = \varphi_2(x),\ 0 < x < h_3 \qquad (4')$$

chegaraviy shartlarni qanoatlantiruvchi $u(x,y)$ yechimi topilsin, bu yerda $\varphi_2(x)$-berilgan funksiya, $h_3 = \left(\frac{q}{2}\right)^{1/q}$.

Shuni ta'kidlash kerakki, $a(x) = 0$ da TC masaladan T masala kelib chiqadi.

2.2-§.da TC va T masalalar yechimning yagonaligi isbotlangan. T masala yechimining yagonaligi energiya integrali usulida ko'rsatilgan.

Matematik analiz kursidan bizga ma'lum bo'lgan Grin formulasini T masala yechimining yagonaligini isbotlashda qo'llaymiz.

(D) -soha, biror bo'lakli silliq (L) kontur bilan chegaralangan bo'lsin.

Faraz qilamiz, (D) –sohada $P(x,y)$ va $Q(x,y)$ funksiyalar berilgan bo'lib, bu funksiyalar o'zlarining $\frac{\partial P}{\partial y}, \frac{\partial Q}{\partial x}$ hosilalari bilan uzluksiz. U holda ushbu

$$\iint_{(D)}\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dxdy=\int_{(L)}Pdx+Qdy \qquad (9)$$

tenglik o'rinli. Bu ifoda Grin formulasi deyiladi.

Bir jinsli chegaraviy shartli T masalani qaraymiz , ya'ni chegaraviy shartlar

$$u(x,y)/_{\sigma}=0,\quad s\in\overline{\sigma}, \qquad (2')$$

$$u(x,y)/_{OA_1}=0,\quad 0\le y\le h_1, \qquad (3')$$

$$u|_{OC}=0,\quad 0<x<h_3 \qquad (4')$$

ko'rinishda bo'ladi.

Belgilash kiritamiz:

$$u(x,0)=\tau(x),\qquad u_y(x,0)=v(x);$$

D_2 sohaning chegarasini ∂D_2-orqali belgilaymiz: $\partial D_2=OC\cup CA\cup AO.$

D_2 sohada (1) tenglamani qaraymiz:

$$-(-y)^{m_2}u_{xx}+x^n u_{yy}=0. \qquad (10)$$

Bu tenglikni har ikki tomonini u ga ko'paytiramiz va uni D_2 soha bo'yicha integrallaymiz. U holda

$$\iint_{(D_2)}u[-(-y)^{m_2}u_{xx}+x^n u_{xx}]dxdy=0 \qquad (11)$$

hosil bo'ladi.

(10) tenglikning chap tomonida ba'zi almashtirishlarni bajaramiz.

$$u[-(-y)^{m_2}u_{xx}+x^n u_{xx}]=$$

$$=-(-y)^{m_2}uu_{xx}-(-y)^{m_2}u_x^2+(-y)^{m_2}u_x^2+x^n uu_{yy}+x^n u_y^2-x^n u_y^2=$$

$$= -\frac{\partial}{\partial x}[(-y)^{m_2} u u_x] + \frac{\partial}{\partial y}[x^n u u_y] + (-y)^{m_2} u_x^2 - x^n u_y^2.$$

Endi buni (11) ga qo'yamiz

$$0 = \iint\limits_{(D_2)} [-(-y)^{m_2} u_{xx} + x^n u_{yy}] dxdy =$$

$$= \iint\limits_{(D_2)} [(-y)^{m_2} u_x^2 - x^n u_y^2] dxdy -$$

$$- \iint\limits_{(D_2)} \left[\frac{\partial}{\partial x}[(-y)^{m_2} u u_x] - \frac{\partial}{\partial y}[x^n u u_y]\right] dxdy,$$

u holda (9) Grin formulasiga ko'ra, quyidagini olamiz

$$\iint\limits_{(D_2)} [x^n u_y^2 - (-y)^{m_2} u_x^2] dxdy + \int\limits_{\partial D_2} x^n u u_y dx + (-y)^{m_2} u u_x dy = 0,$$

yoki

$$\iint\limits_{(D_2)} [x^n u_y^2 - (-y)^{m_2} u_x^2] dxdy + \int\limits_{O}^{C} u\left(x^n u_y dx + (-y)^{m_2} u_x dy\right) +$$

$$+ \int\limits_{C}^{A} u\left(x^n u_y dx + (-y)^{m_2} u_x dy\right) + \int\limits_{A}^{O} u\left(x^n u_y dx + (-y)^{m_2} u_x dy\right) = 0.$$

(12)

1) 4') shartga asosan, yuqoridagi tenglikdagi OC bo'yicha integral nolga teng bo'ladi:

$$\int\limits_{O}^{C} u\left(x^n u_y dx + (-y)^{m_2} u_x dy\right) = 0;$$

2)

$$\int_A^O u\left(x^n u_y dx + (-y)^{m_2} u_x dy\right) = -\int_0^h x^n u(x,0) u_y(x,0) dx =$$

$$= -\int_0^h x^n \tau(x) v(x) dx.$$

Shunday qilib, (12) dan

$$\iint_{(D_2)} \left[x^n u_y^2 - (-y)^{m_2} u_x^2\right] dxdy + \int_C^A u\left(x^n u_y dx + (-y)^{m_2} u_x dy\right) -$$

$$-\int_0^h x^n \tau(x) v(x) dx = 0,$$

yoki

$$\int_0^h x^n \tau(x) v(x) dx =$$

$$= \iint_{(D_2)} \left[x^n u_y^2 - (-y)^{m_2} u_x^2\right] dxdy + \int_C^A u\left(x^n u_y dx + (-y)^{m_2} u_x dy\right)$$

(13)

Bu yerda isbot qilish mumkinki,

$$\int_C^A u\left(x^n u_y dx + (-y)^{m_2} u_x dy\right) =$$

$$= p_2\, \alpha \int_A^C u^2\, x^{q-2} \left(\frac{q}{p_2}\right)^{\frac{1}{p_2}} (q x^q)^{-\frac{1}{q}} \left(\left(1 + \frac{\beta}{\alpha}\right) x^q - q\right) dx \geq 0$$

(14)

$$\iint\limits_{(D_2)} [x^n u_y^2 - (-y)^{m_2} u_x^2] dxdy = 2^{-2\alpha-2\beta} q^{2\alpha} {p_2}^{2\beta} X$$

$$X \iint\limits_{\Gamma} (\eta + \xi)^{2\alpha} (\eta - \xi)^{2\beta} \frac{\beta(\eta + \xi)^2 - \alpha(\eta - \xi)^2}{\left(\alpha(\eta - \xi) - \beta(\eta + \xi)\right)^2} u_\xi^2 \, d\xi \, d\eta +$$

$$+2^{-2\alpha-2\beta} q^{2\alpha} {p_2}^{2\beta} \int_0^1 \frac{(1+\xi)^{1+2\alpha}(1-\xi)^{1+2\beta}}{\beta(1+\xi)-\alpha(1-\xi)} u_\xi^2 |_{\eta=1} d\xi \geq 0 \qquad (15)$$

tengsizliklar o'rinli.

Endi (13) dan (14) va (18) tengsizliklarga ko'ra, shuni hulosa qilamizki,

$$\int_0^h x^n \tau(x) \nu(x) dx \geq 0. \qquad (16)$$

D_1 elliptik sohada (1) tenglamani qaraymiz va xuddi yuqoridagidek, bu sohada (9) Grin formulasini qo'llab, ushbu munosabatni olamiz

$$\int_0^h x^n \tau(x) \nu(x) dx = \qquad (17)$$

$$= - \iint\limits_{(D_1)} [x^n u_y^2 + y^{m_1} u_x^2] dxdy \leq 0.$$

(16) va (17) tengsizliklardan

$$\iint\limits_{(D_1)} [x^n u_y^2 + y^{m_1} u_x^2] dxdy = 0,$$

bundan esa D_1 elliptik sohada $u_x = 0, u_y = 0 \Rightarrow u(x,y) = const$ ekanini, (2')-(4') bir jinsli chegaraviy shartlarga ko'ra esa $u(x,y) \equiv 0$ deb hulosa qilamiz. D_2 giperbo'lik sohada

(1) tenglama uchun qo'yilgan Koshi masalasi yechimini yagonaligiga ko'ra, D_2 sohada ham $u(x,y) \equiv 0$ ekaniga ishonch hosil qilamiz.

Demak, $D = D_1 \cup OA \cup D_2$ sohada $u(x,y) \equiv 0$, ya'ni (1) tenglama uchun (2')-(4') no'l chegaraviy shartli masala faqat no'l yechimga ega. U holda shuni hulosa qilamizki, (1) tenglama uchun T masala yagona yechimga ega.

TC masala uchun esa A.V.Bitsadzening quyidagi ekstremum prinsipidan

foydalanilgan:

Ekstremum prinsipi. Agar $b(x) = 0$ bo'lsa, u holda $\overline{D_1}$ yopiq sohada TC masalaning $u(x,y)$ yechimi, o'zining ekstremal qiymatlarini faqat $\sigma \cup OA_1$ da qabul qiladi.

Quyidagi teorema o'rinli:

Теорема. Agar (5)-(8) shartlar bajarilsa, u holda TC masala yagona yechimga ega.

2.3-§.TC va T masalalar yechimning mavjudligi integral tenglamalar nazariyasini qo'llagan holda isbot etilgan. Bu masalalarning D_1 sohadagi yechimi, (1) elliptic tipdagi tenglama uchun ND masalaning

$$u(x,y) = -\int_0^h x^n \nu(x) G(x,0;x_0,y_0)\,dx +$$

$$+\int_0^{h_1} y^{m_1} \left.\frac{\partial G}{\partial x}\right|_{x=0} \varphi_1(y)\,dy - \int_\sigma \varphi_0(s) A_s[G]\,ds$$

ko'rinishdagi, bu yerda $\nu(x) = u_y(x,0)$, $G(x,y;x_0,y_0)$

- Grin funksiyasi va D_2 sohadagi yechimi esa, (1) giperbolik tipdagi tenglama uchun ushbu

$$u(x,0) = \tau(x), u_y(x,0) = \nu(x)$$

boshlang'ich shartli Koshi masalasining quyidagi

$$u(x,y)=\frac{\Gamma(2\alpha)}{\Gamma^2(\alpha)}\left(\frac{1}{p_2}(-y)^{p_2}\right)^{-\beta}\int_0^1\left[\frac{1}{q}x^q(2z-1)+\frac{1}{p_2}(-y)^{p_2}\right]^{\beta}\times$$

$$\times\left[z(1-z)\right]^{\alpha-1}\tau\left\{\left[\frac{p_2}{q}x^q(2z-1)+(-y)^{p_2}\right]^{1/p_2}\right\}F(\beta,1-\beta,\alpha;\rho)dz\;-$$

$$-\frac{\Gamma(1-2\alpha)}{\Gamma^2(1-\alpha)}q^{-2\alpha}\left(\frac{1}{p_2}(-y)^{p_2}\right)^{-}\left[\frac{1}{q}x^q\right]^{1-2\alpha}\int_0^1\left[\frac{1}{q}x^q(2z-1)+\frac{1}{p_2}(-y)^{p_2}\right]^{\beta}\times$$

$$\times\left[z(1-z)\right]^{-\alpha}\nu\left\{\left[\frac{p_2}{q}x^q(2z-1)+(-y)^{p_2}\right]^{1/p_2}\right\}F(\beta,1-\beta,1-\alpha;\rho)dz\,.$$

ko’rinishdagi yechimi sifatida aniqlanadi.

I-Bob. Asosiy tushunchalar

Ushbu bobda matematik analiz kursidan ma'lum bo'lgan, gamma va beta funksiyalar haqida ma'lumot berilgan. Maxsus funksiyalar nazariyasidagi gipergeometrik funksiya va uning xossalari, yana umumlashgan kasr tartibli integrodifferensial opertatorlar tushunchasi keltirilgan. Giperbolik tipdagi tenglamalar uchun yagonalik teoremasi energiya integrali yordamida isbotlangan hamda aralash tipdagi tenglamalar tushunchasi yoritilgan.

1.1-§. Gamma va Beta funksiyalar

Quyidagi

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt \quad (a>0,\ b>0)$$

integral Beta funksiya yoki birinchi tur Eyler integrali deyiladi. U ikki ozgaruvchili

a va *b* parametrlarning funksiyasini ifodalaydi.

Beta funksiya quyidagi xossalarga ega:

1^0. Beta funksiya *a* va *b* ga nisbatan simmetrik, ya'ni

$$B(a,b) = B(b,a).$$

2^0. Quyidagi formulalar o'rinli

$$a>1 \text{ da } B(a,b) = \frac{a-1}{a+b-1}B(a-1,b),$$

$$b>1 \text{ da } B(a,b) = \frac{b-1}{a+b-1}B(a,b-1).$$

Agar *a* va *b* lar mos ravishda *m* va *n* natural sonlardan iborat bo'lsa, u holda

$$B(m,n) = \frac{(n-1)!\,(m-1)!}{(m+n-1)!}.$$

3^0. Beta funksiya quyidagicha ham ifodalanadi:

$$B(a,b) = \int_0^{+\infty} \frac{t^{a-1}}{(1+t)^{a+b}}dt.$$

Xususan,

$$B(a,1-a)=\int_0^{+\infty}\frac{t^{a-1}}{1+t}dt=sin\frac{\pi}{a\pi},\quad B\left(\frac{1}{2},\frac{1}{2}\right)=\pi$$

bo'ladi.

Ushbu

$$\Gamma(a)=\int_0^{\infty}e^{-t}t^{a-1}dt \qquad (a>0)$$

integral, Lejandr tomonidan Eylerning ikkinchi tur integrali deb nomlangan

va u Γ ("Gamma") funksiyani aniqlaydi.

Γ funksiyaning sodda xossalarini keltiramiz:

1^0. $\Gamma(a)$ funksiya, barcha $a>0$ qiymatlarda, uzluksiz va barcha tartibdagi hosilalarga ega.

2^0. $\Gamma(a)$ funksiya uchun $\Gamma(a+1)=a\Gamma(a)$ formula o'rinli.

Bu formula yordamida

$$\Gamma(a+n)=(a+n-1)(a+n-2)\dots(a+1)a\Gamma(a)$$

tenglikni olish mumkin. Bundan esa $\Gamma(n+1)=n!$.

3^0. $a\to+0$ da va $a\to+\infty$ da ham $\Gamma(a)=\frac{\Gamma(a+1)}{a}\to+\infty$.

4^0. Beta va Gamma funksiyalar orasida quyidagicha bog'lanish mavjud

$$B(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}.$$

5^0. Quyidagi formula o'rinli

$$\Gamma(a)\Gamma(a-1)=\frac{\pi}{sina\pi}.$$

$a = \frac{1}{2}$ da, bu yerdan

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$$

6^0. Ushbu Lejandr formulasi o'rinli

$$\Gamma(a)\Gamma\left(a + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2a-1}}\Gamma(2a).$$

1.2-§. Gipergeometrik funksiya. Umumlashgan kasr tartibli integral operator

1.Quyidagi[11-13]

$$z(1 - z)\omega''(z) + [c - (a + b + 1)z]\omega'(z) - ab\,\omega(z) = 0$$

ikkinchi tartibli oddiy differensial tenglamaning $z = 0$ maxsus nuqta atrofidagi yechimi, bizga ma'lumki,

$$\omega(z) = F(a, b, c; z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k k!} z^k, \qquad |z| < 1, c \neq 0, -1, -2, \ldots$$

ko'rinishda bo'ladi va u gipergeometrik qator yoki Gaussning gipergeometrik funksiyasi deyiladi. Bu yerda $a, b, c -$ parametrlar, haqiqiy yoki kompleks sonlar.

$F(a, b, c; z) -$ funksiya Eylerning gipergeometrik integrali yordamida quyidagicha ifodalanadi:

$$F(a, b, c; z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c - b)} \int_0^1 t^{b-1}(1 - t)^{c-b-1}(1 - zt)^{-a} dt,$$

$$0 < \operatorname{Re} b < \operatorname{Re} c, \qquad |\arg(1 - z)| < \pi.$$

$F(a, b, c; z) -$ funksiya quyidagi xossalarga ega.

1^0. Gipergeometrik funksiyaning analitik davomi.

$$F(a, b, c; 1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c)\Gamma(c-b)}, \ Re(c - a - b) > 0 \text{ va } c \neq 0, -1, -2, \ldots$$

$$F(a,b,c;z) = (1-z)^{-b}F\left(c-a,b,c;\frac{z}{z-1}\right), |\arg(1-z)| < \pi,$$

$$F(a,b,c;z) = (1-z)^{c-a-b}F(c-a,c-b,c;z), |\arg(1-z)| < \pi,$$

$$F(a,b,c;z) = A_1 z^{-a} F(a, a-c+1, a+b+1-c; 1-z^{-1}) +$$

$$+ A_2 z^{a-c}(1-z)^{c-a-b} F(c-a, 1-a, c+1-a-b; 1-z^{-1})$$

$$A_1 = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \qquad A_1 = \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)},$$

$$c-a-b \neq 0, \pm 1, \pm 2, \ldots; \qquad |\arg z| < \pi, \qquad |\arg(1-z)| < \pi;$$

$$F(a, 1-a+c, c; -z) = (1+z)^{c-1}\left(\sqrt{1+z}+\sqrt{z}\right)^{2-2a-2c} \times$$

$$\times F\left(c+a-1, c-\frac{1}{2}, 2c-1, 4\sqrt{z(1+z)}\left(\sqrt{1+z}+\sqrt{z}\right)^{-2}\right).$$

2^0. Ba’zi elementar munosabatlar.

$$\frac{d}{dz}F(a,b,c;z) = \frac{ab}{c}F(a+1,b+1,c+1;z),$$

$$\frac{d}{dz}[z^a F(a,b,c;z)] = az^{a-1}F(a+1,b,c;z),$$

$$\frac{d}{dz}[z^{c-1}F(a,b,c;z)] = (c-1)z^{c-2}F(a,b,c-1;z),$$

$$\frac{d}{dz}[(1-z)^a F(a,b,c;z)] = -\frac{a(c-b)}{c}(1-z)^{a-1}F(a+1,b,c+1;z).$$

$$F(a,b,b;z) = (1-z)^{-a}, |\arg(1-z)| < \pi.$$

3^0. Gipergeometrik funksiya quyidagi tengsizliklarni qanoatlantiradi[2]:

agar $c-a-b>0, 0\le z\le 1$ bo'lsa, $F(a,b,c;z)\le C_1$;

agar $c-a-b<0, 0<z<1$ bo'lsa, $F(a,b,c;z)\le C_2(1-z)^{c-a-b}$;

agar $c-a-b=0$ bo'lsa , $F(a,b,c;z)\le C_3(1+|\ln(1-z)|)$, bu yerda C_1, C_2, C_3 - ixtiyoriy o'zgarmaslar.

Quyidagi funksiyalar[13]

$$F_2(\alpha,\beta,\beta',\gamma,\gamma';x,y)=\sum_{m,n=0}^{\infty}\frac{(\alpha)_{m+n}(\beta)_m(\beta')_n}{(\gamma)_m(\gamma')_n m!n!}x^m y^n$$

Gorna gipergeometrik funksiyasi,

$$F_3(\alpha,\beta,\beta',\gamma,\gamma';x,y)=\sum_{m,n=0}^{\infty}\frac{(\alpha)_m(\beta)_n(\beta')_m(\gamma)_n}{(\gamma')_{m+n} m!n!}x^m y^n$$

Appel gipergeometrik funksiyasi deyiladi.

Bu funksiyalarni F -Gauss gipergeometrik funksiyasi orqali ifodalash mumkin, masalan

$$F_2(\alpha,\beta,\beta',\gamma,\gamma';x,y)=\sum_{k=0}^{\infty}\frac{(\alpha)_k(\beta')_k}{k!(\gamma')_k}y^k F(\alpha+k,\beta,\gamma;x).$$

2. Quyidagi

$$F_{0x}\left[\begin{matrix} a,b \\ c, \frac{x^\kappa-t^\kappa}{x^\kappa}\end{matrix}\right]f(x)=$$

$$=\frac{1}{\Gamma(c)}\int_0^x f(t)(x^k-t^k)^{c-1}F\left(a,b,c;\frac{x^\kappa-t^\kappa}{x^\kappa}\right)kt^{k-1}dt$$

$$F_{x1}\left[\begin{matrix} a,b \\ c, \frac{x^\kappa-t^\kappa}{x^\kappa}\end{matrix}\right]f(x)=$$

$$=\frac{1}{\Gamma(c)}\int_x^1 f(t)(t^k-x^k)^{c-1}F\left(a,b,c;\frac{x^\kappa-t^\kappa}{x^\kappa}\right)kt^{k-1}dt$$

ko’rinishdagi ifodalar $f(x)$ funksiyadan olingan umumlashgan c $(c>0)$ kasr tartibli integral deb ataladi [4], bu yerda $f(x)$ $x\to x_0$ da

$$f(x)=O\left(\left|x^k-x_0^k\right|^{l_0}\right),\ l_0>\max(0,a+b-c)-1,$$

$a,b,c-const, k>0,\ F\left(a,b,c;\frac{x^{\kappa}-t^{\kappa}}{x^{\kappa}}\right)$ –Gaussning gipergeometrik funksiyasi,

1.3-§.Giperbolik tipdagi tenglamalar uchun yagonalik teoremasi

Teorema. Quyidagi[17],[18]

$$\rho u_{tt}=(ku_x)_x+f(x,t),0<x<1,t>0 \qquad (1)$$

tor tebranish tenglamasining

$$u(x,0)=\varphi(x),\quad u_t(x,0)=\psi(x) \qquad (2)$$

boshlang’ich va

$$u(0,t)=\mu_1(t),\quad u(1,t)=\mu_2(t) \qquad (3)$$

chegaraviy shartlarini qanoatlantiruvchi yagona yechim mavjud. Bu yerda $u(x,t)$ funksiya 2-tartibgacha hosilalari $0\le x\le 1,\ t\ge 0$ sohada uzliksiz va $\rho(x)>0,\ k(x)>0$ uzluksiz funksiyalar.

Bu teoremani isbot qilishdan avval tor tebranishi *energiyasi* $E=K+U$, bu yerda K – *kinetik energiya*, U – *potensial energiya*, ifodasini topamiz. u_t tezlik bilan xarakatlanuvchi dx tor elementi,ushbu

$$\frac{1}{2}mv^2=\frac{1}{2}\rho(x)dx(u_t)^2$$

kinetic energiyaga ega. Butun torning kinetic energiyasi:

$$K=\frac{1}{2}\int_0^l\rho(x)[u_t(x,t)]^2dx.$$

$t=t_0$ da $u(x,t_0)=u_0(x)$ ko’rinishga ega tor tebranishining potensial energiyasi, tor muvozanat holatidan $u_0(x)$holatga o’tishi uchun, bajarilgan ishga teng. $u(x,t)$ funksiya torning t momentdagi holati bo’lib,

$$u(x,0)=0,\ u(x,t_0)=u_0(x).$$

dx element teng ta’sir etuvchi taranglik kuch $Tu_{xx}dx$ ta’sirida dt vaqtda $u_t(x,t)dt$ yo’lni bosib o’tadi. Bunda bajarilgan ish quyidagiga teng

$$\left\{\int_0^l T_0 u_{xx}\,u_t dx\right\}dt=\left\{T_0u_xu_t|_0^l-\int_0^l T_0u_x\,u_{xt}dx\right\}dt=$$

$$=\left\{-\frac{1}{2}\frac{d}{dt}\int_0^l T_0(u_x)^2\,dx+T_0u_xu_t|_0^l\right\}dt.$$

t bo’yicha 0 dan t_0 gacha integrallab,quyidagini olamiz:

$$\int_0^{t_0}\left\{-\frac{1}{2}\frac{d}{dt}\int_0^l T_0(u_x)^2\,dx+T_0u_xu_t|_0^l\right\}dt=$$

$$=-\frac{1}{2}\int_0^l T_0(u_x)^2\,dx|_0^{t_0}+\int_0^{t_0} T_0u_xu_t|_0^l dt=$$

$$=-\frac{1}{2}\int_0^l T_0[u_x(x,t_0)]^2\,dx+\int_0^{t_0} T_0u_xu_t|_0^l dt.$$

Bu tenglikning oxirgi qo’shiluvchilari ma’nosi quyidagicha. $T_0u_x|_{x=0}$ taranglikning torning *x=0* oxiridagi kattaligi;

$$\int_0^{t_0} T_0u_xu_t|_{x=0}dt$$

integral bajarilgan ishni ifodalaydi. *x=l* ga mos qo'shiluvchi xuddi shunday ma'noga ega. Agar torning oxirlari maxkamlangan bo'lsa, u holda torning oxirlarida bajarilgan ish nolga teng (bunda $u(0,t) = 0, u_t(0,t) = 0$). Ravshanki, bu holda $u = 0$ muvozanat holatdan $u_0(x)$ holatga o'tishda bajarilgan ish torni bu holatga o'tish usuliga bog'liq emas va torni $t = t_0$ momentdagi potensial energiyasining teskari ishora bilan olinganiga teng

$$-\frac{1}{2}\int_0^l T_0[u'_0(x)]^2\,dx.$$

Shunday qilib, torning *to'la energiyasi* quyidagiga teng

$$E = \frac{1}{2}\int_0^l [T_0(u_x)^2 + \rho(x)(u_t)^2]\,dx.$$

Teorema isboti: (1)-(3)masalaning 2 ta yechimi $u_1(x,t)$, $u_2(x,t)$, mavjud bo'lsin deb faras qilamiz va quyidagi ayirmani qaraymiz

$$V(x,t) = u_1(x,t) - u_2(x,t),$$

u holda bu funksiya

$$\rho V_{tt}=(kV_x)_x \tag{4}$$

tenglamani va bir jinsli qo'shimcha shartlarni qanoatlantiradi.

$$V(x,0)=0, \qquad V_t(x,0)=0$$

$$V(0,t)=0, \qquad V(1,t)=0\,. \tag{5}$$

$V(x,t)=0$ ekanini isbotlaymiz .

Quyidagi funksiyani qaraymiz :

$$E(t) = \frac{1}{2}\int_0^l \{k\,V_x^2 + \rho\,V_t^2\}\,dx \tag{6}$$

va ko'rsatamizki, u *t* ga bog'liq emas, ya'ni isbot qilamizki,

1) $E(t)= const$; 2)$E(t)=0$.

$E(t)$ funksiyaning fizik ma'nosi ko'rinib turibdiki, u torning t momentdagi to'la energiyasi.

(6) dan

$$\frac{dE(t)}{dt}=\frac{1}{2}\int_0^l(2kV_xV_{xt}+\rho V_tV_{tt})dx$$

tenglikni olamiz, chunki 2-tartibli hosilalar uzliksiz. Bo'laklab integrallasak ,

$$\int_0^l k\ V_xV_{xt}\,dx=[kV_xV_t\,]_0^l-\int_0^l V_t(k\,V_x)_x dx \qquad (7)$$

$V_t(0,t)=0,\ V_t(1,t)=0$ bo'lgani uchun, $\int_0^l k\ V_xV_{xt}dt=-\int_0^l V_t(k\,V_x)_x dx$ bo'ladi.

Bu yerdan

$$\frac{dE(t)}{dt}=\int_0^l V_t[\rho V_{tt}-(kV_x)_x]dx=0,\ E(t)=con\,st. \qquad (8)$$

Boshlang'ich shartlarga asosan ,

$$E(t)=con\,st=E(0)=\frac{1}{2}\int_0^l[k\ V_x^2+\rho V_t^2]_{t=0}dx=0, \qquad (9)$$

chunki

$$V(x,0)=0, \qquad V_t(x,0)=0.$$

(9) formuladan, $k,\rho>0$ bo'lgani uchun, $V_x(x,t)=0$, $V_t(x,t)=0$ ni hosil qilamiz. Bundan $V(x,t)=con\,st$. Boshlangich shartga ko'ra, $V(x,0)=con\,st=0$. Demak , $V(x,t)=0$.

Shunday qilib , agar teorema shartlarini qanoatlantiruchi 2 ta u_1 va u_2 funksiyalar mavjud bo'lsa, u holda

$$u_1(x,t)=u_2(x,t).$$

Ikkinchi chegaraviy masala uchun,

$$V=u_1-u_2$$

funksiya quyidagi

$$V_x(0,t)=0\,,\ V_x(1,t)=0$$

shartni qanoatlantiradi. (9) formuladagi isbotni davomi o'zgarmaydi.

3- chegaraviy masala uchun isbot bir qacha o'zgarishlarga ega.

$V=u_1 - u_2$ funksiya uchun

$$\rho V_{tt}=(kV_x)_x$$

$$V_x\,(0,t) - h_1 V(0,t)=0\,,\ h_1 \geq 0$$

$$V_x\,(l,t) - h_2 V(0,t)=0\,,\ h_2 \geq 0 \qquad (10)$$

(10) dan quyidagini o'rniga qo'yamiz:

$$(kV_x V_t)_0^l=-kh_2 V(l,t)V_t(l,t) - kh_1 V(0,t)V_t(0,t)=$$

$$= -\frac{k}{2}\frac{\partial}{\partial t}\left(h_2 V^2(l,t) + +h_1 V^2(0,t)\right) -$$

$$-\frac{k}{2}\{h_2[V^2(l,t) - V^2(l,0)] + h_1[V^2(0,t) - V^2(0,0)]\},$$

bu yerdan, tenglama va boshlang'ich shartlardan

$$E(t)=-\frac{k}{2}\,|h_2 V^2(1,t) + h_1 V^2(0,t))\leq 0$$

kelib chiqadi.

$E(t)\geq 0$ bo'lgani uchun $E(t)=0$. Ravshanki, $V(x,t)=0$.
Shunday qilib , 3-chegaraviy masala uchun yagonalik teoremasi isbotlandi.

1.4-§. Aralash tipdagi differensial tenglamalar

1. Quyidagi [11],[19]

$$A(x,y)u_{xx} + 2B(x,y)u_{xy} + C(x,y)u_{yy} + F(x,y,u,u_x,u_y) = 0 \qquad (11)$$

ko'rinishdagi ikkinchi tartibli ikki o'zgaruvchili kvazichiziqli differensial tenglamani qaraymiz, bu yerda $A(x,y), B(x,y), C(x,y)$ koeffitsiyentlar biror $\overline{D}$ yopiq sohada aniqlangan, uzluksiz haqiqiy funksiyalar bo'lib, $A^2 + B^2 + C^2 \neq 0$, F -berilgan funksiya, $u = u(x, y)$ - noma'lum funksiya.

Agar D sohada (11) tenglamani elliptik (giperbolik) tipga tegishli bo'lib, ya'ni $B^2 - AC < 0$ $(B^2 - AC > 0)$, D sohaning butun chegarasida yoki bu chegaraning biror qismida $B^2 - AC = 0$ bo'lsa, u holda (11) tenglamani *buziluvchan elliptik (buziluvchan giperbolik) tenglama* deyiladi. D soha chegarasining bu qismi esa (1.1) tenglama uchun *buzilish chizig'i* yoki *parabolik chizig'i* deyiladi.

$\gamma : B^2 - AC = 0$ parabolik chiziq nuqtalarida ikkita hol bo'lishi mumkin:yoki

$$Ady^2 - 2Bdxdy + Cdx^2 \neq 0, \qquad (12)$$

yoki

$$Ady^2 - 2Bdxdy + Cdx^2 = 0. \qquad (13)$$

Ta'rif. Agar (12) shart bajarilsa, u holda (11) tenglama *birinchi tur buziluvchan elliptik (giperbolik) tenglama* deb ataladi. Agar (13) bajarilsa - *ikkinchi tur buziluvchan elliptik (giperbolik) tenglama* deb ataladi.

Endi regulyar yechim tushunchasini keltiramiz.

Ta'rif. (11) tenglamaning D sohadagi *regulyar yechimi* deb, bu tenglamani D sohaning barcha nuqtalarida qanoatlantiruvchi $u(x, y) \in C^2(D)$ funksiyani ataymiz.

Agar tenglama qaralayotgan sohaning bir qismida elliptik, boshqa qismida giperbolik tipga tegishli bo'lsa, u holda u aralash tipdagi tenglama deyiladi. Bu qismlar o'tish chizig'i (yoki sirti) bilan ajratiladi va bunda tenglama parabolik buziladi yoki aniqlanmaydi.

Ta'rif. Agar $B^2 - AC$ ifoda D sohada ishorasini o'zgartirsa, u holda (11) tenglama *aralash tipdagi tenglama* deyiladi.

$B^2 - AC = 0$ tenglama bilan aniqlanadigan γ chiziq (11) tenglamaning parabolik chizig'i yoki bu tenglama tipining buzilish chizig'i deyiladi. Bu yerda ikki hol bo'lishi mumkin:

1) $\gamma: B^2 - AC = 0$ parabolik chiziq (11) tenglamaning xarakteristik yo'nalishlari bilan hech qayerda urinmaydi, ya'ni γ da

$$Ady^2 - 2Bdxdy + Cdx^2 \neq 0;$$

2) $\gamma: B^2 - AC = 0$ parabolik chiziqning har bir nuqtasida urinma (11) tenglamaning xarakteristik yo'nalishlari bilan ustma-ust tushadi, ya'ni γ da

$$Ady^2 - 2Bdxdy + Cdx^2 = 0.$$

Ta'rif. Agar (11) tenglama uchun 1) - shart bajarilsa, u holda u *birinchi tur aralash tipdagi tenglama* deyiladi, 2)- shart bajarilganda esa – *ikkinchi tur aralash tipdagi tenglama* deyiladi.

Misollar. 1) $$yu_{xx} + u_{yy} = 0$$

tenglama birinchi tur aralash elliptik-giperbolik tipdagi tenglamadir. $B^2 - AC = y,\ y = 0$ - bu tenglamaning buzilish chizig'i.

2) Ushbu $u_{xx} + yu_{yy} = 0$ tenglama ikkinchi tur aralash elliptik-giperbolik tipdagi tenglama bo'lib, uning buzilish chizig'i $B^2 - AC = y,\ y = 0$.

3) Quyidagi

$$0 = \begin{cases} y^{m_1}u_{xx} + x^n u_{yy},\ x > 0,\ y > 0 \\ -(-y)^{m_2}u_{xx} + x^n u_{yy},\ x < 0,\ y > 0 \end{cases}$$

tenglama 1-tur aralash elliptik-giperbolik tipdagi tenglama. $x = 0, y = 0$ chiziqlar esa buzilish chiziqlari.

II-Bob. Ikkita buzilish chizig'iga ega va turli tartibli aralash tipdagi tenglama uchun chegaraviy masalalar

Mazkur bobda

$$signy\,|y|^{m_i}U_{xx} + x^n U_{yy} = 0$$

ko'rinishdagi ikkita buzilish chizig'iga ega va turli tartibli aralash tipdagi tenglama uchun chegaraviy masalalar tadqiq etilgan.

2.1-§. TC va T masalalarning qo'yilishi

Quyidagi

$$signy\ |y|^{m_i} u_{xx} + x^n u_{yy} = 0 \quad , \quad m_i, n = const \qquad (i = 1,2) \qquad (1)$$

tenglamani qaraymiz, bu yerda $n < m_1 \le m_2$ и $i = 1$ da $y > 0$, $i = 2$ da esa $y < 0$.

(1) tenglama $y > 0$ *da* $B^2 - AC = -x^n y^{m_1} < 0$ bo'lgani uchun, u elliptik tipga tegishli, $y < 0$ da esa $B^2 - AC = x^n (-y)^{m_2} > 0$ va tenglama giperbo'lik tipga tegishli.

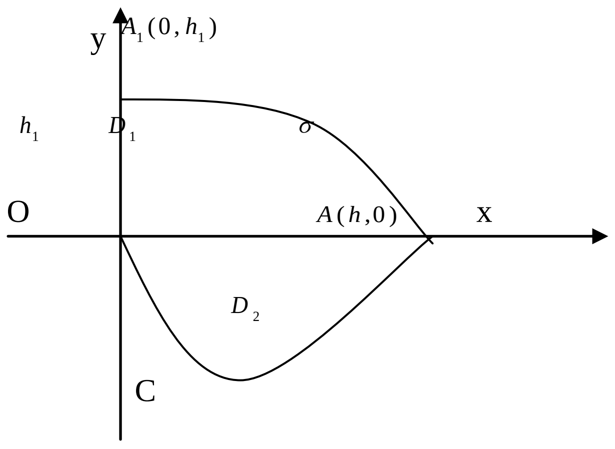
D-soha, $x > 0, y > 0$ da oxirlari $A(h,0)$, $A_1(0,h_1)$ nuqtalarda bo'lgan, σ Jordan chizig'i bilan, OA_1 kesma va (1) tenglamaning $x < 0, y > 0$ da ushbu

$$OC:\ \frac{1}{q}x^q - \frac{1}{p_2}(-y)^{p_2} = 0,\ AC:\ \frac{1}{q}x^q + \frac{1}{p_2}(-y)^{p_2} = 1$$

xarakteristikalari bilan chegaraslangan, bu yerda,

$$h = q^{\frac{1}{q}}, h_1 = p_1^{\frac{1}{p_1}}, 2q = n + 2,\ 2p_1 = m_1 + 2,\ 2p_2 = m_2 + 2\ .$$

D_1 va D_2 - D sohaning elliptic va giperbo'lik qismlari bo'lsin.



Ta'rif. (1) tenglamaning D sohadagi regulyar yechimi deb, quyidagi shartlarni qanoatlantiruvchi $u(x,y)$ funksiyani ataymiz:

1) $u(x,y)\in C(\overline{D})\bigcap C^1(D)\bigcap C^2(D_1\bigcup D_2)$ bo'lib, $u_y(x,0)$ $x\to 0$ va $x\to h$ da $(n+2)/(m_2+2)$ dan kichik tartibda cheksizlikka intiladi;

2) $u(x,y)$ (1) tenglamani D_1 va D_2 sohalarda qanoatlantiradi.

TC masala. Ushbu shartlarni qanoatlantiruvchi $u(x,y)$ funksiyani aniqlang:

1) $u(x,y)$ (1) tenglamaning D sohada regulyar yechimi;

2) $u(x,y)$ quyidagi chegaraviy shartlarni qanoatlantiradi

$$u(x,y)/_{\sigma}=\varphi_0(s),\ s\in\overline{\sigma}, \tag{2}$$

$$u(x,y)/_{OA_1}=\varphi_1(y), 0\le y\le h_1, \tag{3}$$

$$\frac{d}{dx^{2q}}(x^{2q})^{\frac{1-\alpha-\beta}{2}}F_{0x}\begin{bmatrix}\frac{\alpha+\beta-1}{2},\frac{\alpha+\beta}{2}\\ \beta,\frac{x^{2q}-t^{2q}}{x^{2q}}\end{bmatrix}(x^{2q})^{\frac{2\alpha-1}{2}}u[\theta(x)]=$$

$$=a(x)u_y(x,0)+b(x), 0<x<h, \tag{4}$$

bu yerda $\varphi_0(x,y),\varphi_1(y),$ $a(x),b(x)$- berilgan yetarlicha silliq funksiyalar bo'lib,

$$\varphi_0(l)=\varphi_1(h_1),\varphi_1(y)\in C[0,h_1]\cap C^2(0,h_1), \tag{5}$$

$\varphi_0(s)=\varphi_0(x,y)$ funksiya esa

$$\varphi_0(x,y)=xy\overline{\varphi_0}(x,y),\overline{\varphi_0}(x,y)\in C(\overline{\sigma}), \tag{6}$$

ko'rinishga ega va

$$(x^{2q})^{\frac{1-\alpha+\beta}{2}}a(x)+\gamma>0, \tag{7}$$

yoki

$$(x^{2q})^{\frac{1-\alpha+\beta}{2}}a(x)+\gamma=0, \qquad (8)$$

γ-berilgan son, $2\alpha=n/(n+2), 2\beta=m_2/(m_2+2)$, $F_{0x}[\quad]$- umumlashgan kasr tartibli integral operator,

$$\theta(x)=\left(\frac{x^q}{2}\right)^{1/q}-i\left(\frac{p}{q}\frac{x^q}{2}\right)^{1/p}$$

- (1) tenglamaning $(x,0)$ dan chiquvchi xarakteristikalarining OC xarakteristika bilan kesishish nuqtasining affiksi.

T masala.(1) tenglamaning D sohada regulyar va (2), (3),

$$u|_{OC}=\varphi_2(x),\ 0<x<h_3$$

chegaraviy shartlarni qanoatlantiruvchi $u(x,y)$ yechimi topilsin, bu yerda $\varphi_2(x)$-berilgan funksiya, $h_3=\left(\frac{q}{2}\right)^{1/q}$.

Shuni ta'kidlash kerakki,

$$a(x)=0$$

da TC masaladan T masala kelib chiqadi.

2.2-§. Yechimning yagonaligi

TC va T masalalar yechimning yagonaligi isbotlaymiz.

1.T masala yechimining yagonaligi energiya integrali usulida ko'rsatamiz.

Matematik analiz kursidan bizga ma'lum bo'lgan Grin formulasini T masala yechimining yagonaligini isbotlashda qo'llaymiz.

(D) -soha, biror bo'lakli silliq (L) kontur bilan chegaralangan bo'lsin.

Faraz qilamiz, (D) –sohada $P(x,y)$ va $Q(x,y)$ funksiyalar berilgan bo'lib, bu funksiyalar o'zlarining $\frac{\partial P}{\partial y},\frac{\partial Q}{\partial x}$ hosilalari bilan uzluksiz. U holda ushbu

$$\iint_{(D)} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy = \int_{(L)} Pdx + Qdy \qquad (9)$$

tenglik o'rinli. Bu ifoda Grin formulasi deyiladi.

Endi bir jinsli T masalani qaraymiz , ya'ni chegaraviy shartlar

$$u(x,y)/_{\sigma} = 0, \quad s \in \overline{\sigma}, \qquad (2')$$

$$u(x,y)/_{OA_1} = 0, \quad 0 \le y \le h_1, \qquad (3')$$

$$u|_{OC} = 0, \quad 0 < x < h_3 \qquad (4')$$

ko'rinishda bo'ladi.

Belgilash kiritamiz:

$$u(x,0) = \tau(x), \qquad u_y(x,0) = v(x);$$

D_2 sohaning chegarasini ∂D_2-orqali belgilaymiz: $\partial D_2 = OC \cup CA \cup AO.$

D_2 sohada (1) tenglamani qaraymiz:

$$-(-y)^{m_2} u_{xx} + x^n u_{yy} = 0. \qquad (10)$$

Bu tenglikni har ikki tomonini u ga ko'paytiramiz:

$$u[-(-y)^{m_2} u_{xx} + x^n u_{yy}] = 0$$

D_2 soha bo'yicha integrallaymiz, u holda

$$\iint_{(D_2)} u[-(-y)^{m_2} u_{xx} + x^n u_{xx}] dxdy = 0 \qquad (11)$$

hosil bo'ladi.

(10) tenglikning chap tomonida ba'zi almashtirishlarni bajaramiz.

$$u[-(-y)^{m_2} u_{xx} + x^n u_{xx}] =$$

$$= -(-y)^{m_2} u u_{xx} - (-y)^{m_2} u_x^2 + (-y)^{m_2} u_x^2 + x^n u u_{yy} + x^n u_y^2 - x^n u_y^2 =$$

$$= -\frac{\partial}{\partial x}[(-y)^{m_2}uu_x] + \frac{\partial}{\partial y}[x^n uu_y] + (-y)^{m_2}u_x^2 - x^n u_y^2.$$

Endi buni (11) ga qo'yamiz

$$0 = \iint\limits_{(D_2)} [-(-y)^{m_2}u_{xx} + x^n u_{yy}]dxdy =$$

$$= \iint\limits_{(D_2)} [(-y)^{m_2}u_x^2 - x^n u_y^2]dxdy -$$

$$- \iint\limits_{(D_2)} \left[\frac{\partial}{\partial x}[(-y)^{m_2}uu_x] - \frac{\partial}{\partial y}[x^n uu_y]\right]dxdy,$$

u holda (9) Grin formulasiga ko'ra, quyidagini olamiz

$$\iint\limits_{(D_2)} [x^n u_y^2 - (-y)^{m_2}u_x^2]dxdy + \int\limits_{\partial D_2} x^n uu_y dx + (-y)^{m_2}uu_x dy = 0,$$

yoki

$$\iint\limits_{(D_2)} [x^n u_y^2 - (-y)^{m_2}u_x^2]dxdy + \int\limits_O^C u\left(x^n u_y dx + (-y)^{m_2}u_x dy\right) +$$

$$+ \int\limits_C^A u\left(x^n u_y dx + (-y)^{m_2}u_x dy\right) + \int\limits_A^O u\left(x^n u_y dx + (-y)^{m_2}u_x dy\right) = 0.$$

(12)

1) 4') shartga asosan, yuqoridagi tenglikdagi OC bo'yicha integral nolga teng bo'ladi:

$$\int\limits_O^C u\left(x^n u_y dx + (-y)^{m_2}u_x dy\right) = 0;$$

2)

$$\int_A^O u\left(x^n u_y dx + (-y)^{m_2} u_x dy\right) = -\int_0^h x^n u(x,0) u_y(x,0) dx =$$

$$= -\int_0^h x^n \tau(x) v(x) dx.$$

Shunday qilib, (12) dan

$$\iint_{(D_2)} [x^n u_y^2 - (-y)^{m_2} u_x^2] dxdy + \int_C^A u\left(x^n u_y dx + (-y)^{m_2} u_x dy\right) -$$

$$-\int_0^h x^n \tau(x) v(x) dx = 0,$$

yoki

$$\int_0^h x^n \tau(x) v(x) dx =$$

$$= \iint_{(D_2)} [x^n u_y^2 - (-y)^{m_2} u_x^2] dxdy + \int_C^A u\left(x^n u_y dx + (-y)^{m_2} u_x dy\right)$$

(13)

(10) tenglamaning

$$AC : \frac{1}{q} x^q + \frac{1}{p_2} (-y)^{p_2} = 1$$

xarakteristikasi tenglamasidan

$$\frac{1}{q} x^q = 1 - \frac{1}{p_2} (-y)^{p_2} \Rightarrow x^{q-1} dx = (-y)^{p_2 - 1} dy$$

tenglikni olamiz. Bunga asosan, $2q = n+2, n = 2(q-1); 2p_2 = m_2 + 2,$ $m_2 = 2(p_2 - 1)$ ekanini hisobga olib,

$$I_1 = \int_C^A u\left(x^n u_y dx + (-y)^{m_2} u_x dy\right) =$$

$$= \int_C^A x^{q-1}(-y)^{p_2-1} u\left(u_x dx + u_y dy\right) =$$

$$= \frac{1}{2}\int_C^A x^{q-1}(-y)^{p_2-1} d(u^2)$$

yoki bu oxirgi integralni bo'laklab integrallab, quyidagiga ega bo'lamiz

$$I_1 = -\frac{1}{2}\int_C^A u^2\left((q-1)x^{q-2}(-y)^{p_2-1}dx - (p_2-1)x^{q-1}(-y)^{p_2-2}dy\right).$$

Bu yerda

$$2\alpha = \frac{n}{n+2} = \frac{2q-2}{2q} = 1 - \frac{1}{q} \Rightarrow q = \frac{1}{1-2\alpha}; \; 2\beta = \frac{m_2}{m_2+2} = \frac{2p_2-2}{2p_2} = 1 - \frac{1}{p_2}$$

$\Rightarrow p_2 = \frac{1}{1-2\beta}$ ekanini hisobga olsak,

$$I_1 = p_2\,\alpha \int_A^C u^2\, x^{q-2}\left(\frac{q}{p_2}\right)^{\frac{1}{p_2}} (qx^q)^{-\frac{1}{p}}\left(\left(1 + \frac{\beta}{\alpha}\right)x^q - q\right)dx \geq 0 \qquad (14)$$

tengsizlikni hosil qilamiz.

(13) tenglikdagi

$$I_2 = \iint_{(D_2)} \left[x^n u_y^2 - (-y)^{m_2} u_x^2\right] dxdy$$

integralni qaraymiz va bunda ushbu

$$\xi = \frac{1}{q}x^q - \frac{1}{p_2}(-y)^{p_2},\ \eta = \frac{1}{q}x^q + \frac{1}{p_2}(-y)^{p_2}$$

(15)

xarakteristik koordinatalar sistemasiga o'tamiz, u holda D_2 soha Γ uchburchak sohaga o'tadi va

$$I_2 = -2^{1-2\alpha-2\beta}q^{2\alpha}p_2{}^{2\beta}\iint_{\Gamma}(\eta+\xi)^{2\alpha}(\eta-\xi)^{2\beta}u_\xi u_\eta d\xi\, d\eta.$$

(16)

(10) tenglamani (15) xarakteristik koordinatalar sistemasida yozsak,

$$u_{\xi\eta} + \left(\frac{\alpha}{\eta+\xi} + \frac{\beta}{\eta-\xi}\right)u_\xi + \left(\frac{\alpha}{\eta+\xi} - \frac{\beta}{\eta-\xi}\right)u_\eta = 0$$

ko'rinishdagi tenglamaga ega bo'lamiz. Bu tenglikdan, quyidagini olish mumkin

$$(\eta+\xi)^{2\alpha}(\eta-\xi)^{2\beta}u_\xi u_\eta =$$

$$= -\frac{1}{2}(\eta+\xi)^{2\alpha}(\eta-\xi)^{2\beta}\frac{\beta(\eta+\xi)^2 - \alpha(\eta-\xi)^2}{\left(\alpha(\eta-\xi) - \beta(\eta+\xi)\right)^2}u_\xi^2 -$$

$$-\frac{1}{2}\frac{\partial}{\partial\eta}\left(\frac{(\eta+\xi)^{2\alpha}(\eta-\xi)^{2\beta}}{\beta(\eta+\xi)-\alpha(\eta-\xi)}u_\xi^2\right). \quad (17)$$

Endi (17) ni (16) tenglikka qo'yamiz:

$$I_2 = 2^{-2\alpha-2\beta}q^{2\alpha}p_2{}^{2\beta}X$$

$$X\iint_{\Gamma}(\eta+\xi)^{2\alpha}(\eta-\xi)^{2\beta}\frac{\beta(\eta+\xi)^2 - \alpha(\eta-\xi)^2}{\left(\alpha(\eta-\xi) - \beta(\eta+\xi)\right)^2}u_\xi^2 d\xi\, d\eta +$$

$$+2^{-2\alpha-2\beta}q^{2\alpha}p_2{}^{2\beta}\iint_\Gamma \frac{\partial}{\partial\eta}\left(\frac{(\eta+\xi)^{2\alpha}(\eta-\xi)^{2\beta}}{\beta(\eta+\xi)-\alpha(\eta-\xi)}u_\xi^2\right)d\xi\, d\eta$$ yoki

$$I_2 = 2^{-2\alpha-2\beta}q^{2\alpha}p_2{}^{2\beta}X$$

$$X\iint_\Gamma (\eta+\xi)^{2\alpha}(\eta-\xi)^{2\beta}\frac{\beta(\eta+\xi)^2-\alpha(\eta-\xi)^2}{\left(\alpha(\eta-\xi)-\beta(\eta+\xi)\right)^2}u_\xi^2 d\xi\, d\eta +$$

$$+2^{-2\alpha-2\beta}q^{2\alpha}p_2{}^{2\beta}\int_0^1 \frac{(1+\xi)^{1+2\alpha}(1-\xi)^{1+2\beta}}{\beta(1+\xi)-\alpha(1-\xi)}u_\xi^2|_{\eta=1}d\xi \geq 0$$

(18)

Endi (13) dan (14) va (18) tengsizliklarga ko'ra, shuni hulosa qilamizki,

$$\int_0^h x^n\tau(x)\nu(x)dx \geq 0 \qquad (19)$$

D_1 elliptik sohada (1) tenglamani qaraymiz va xuddi yuqoridagidek, bu sohada (9) Grin formulasini qo'llab, ushbu munosabatni olamiz

$$\int_0^h x^n\tau(x)\nu(x)dx = -\iint_{(D_1)}[x^n u_y^2 + y^{m_1}u_x^2]dxdy \leq 0. \qquad (20)$$

(19) va (20) tengsizliklardan

$$\iint_{(D_1)}[x^n u_y^2 + y^{m_1}u_x^2]dxdy = 0,$$

bundan esa D_1 elliptik sohada $u_x = 0, u_y = 0 \Rightarrow u(x,y) = const$ ekanini, (2')-(4') bir jinsli chegaraviy shartlarga ko'ra esa $u(x,y) \equiv 0$ deb hulosa qilamiz. D_2 giperbo'lik sohada (1) tenglama uchun qo'yilgan Koshi masalasi yechimini yagonaligiga ko'ra, D_2 sohada ham $u(x,y) \equiv 0$ ekaniga ishonch hosil qilamiz.

Demak, $D = D_1 \cup OA \cup D_2$ sohada $u(x,y) \equiv 0$, ya'ni (1) tenglama uchun (2')-(4') no'l chegaraviy shartli masala faqat no'lyechimga ega. U holda shuni hulosa qilamizki, (1) tenglama uchun T masala yagona yechimga ega.

2.TC masala yechimining yagonaligini isbotlashda A.V.Bitsadzening ekstremum prinsipidan foydalanamiz. (7) shart bajarilsin, u holda quyidagi ekstremum prinsipi o'rinli.

Ekstremum prinsipi. Agar $b(x) = 0$ bo'lsa, u holda $\overline{D_1}$ yopiq sohada TC masalaning $u(x, y)$ yechimi, o'zining ekstremal qiymatlarini faqat $\sigma \cup OA_1$ da qabul qiladi.

Quyidagi teorema o'rinli:

Теорема. Agar (5)-(8) shartlar bajarilsa, u holda TC masala yagona yechimga ega.

Ekstremum prinsipidan bevosita teoremaning isboti, ya'ni TC masala yechimining yagonaligi kelib chiqadi.

D_2 sohada (1) giperbolik tipdagi tenglama uchun

$$u(x,0) = \tau(x), \qquad u_y(x,0) = v(x);$$

boshlang'ich shartli Koshi masalasining yechimi[15],[16]

$$\begin{aligned} u(x,y) = {} & \frac{\Gamma(2\alpha)}{\Gamma^2(\alpha)} \left(\frac{1}{p_2}(-y)^{p_2} \right)^{-\beta} \int_0^1 \left[\frac{1}{q} x^q (2z-1) + \frac{1}{p_2}(-y)^{p_2} \right]^{\beta} \times \\ & \times [z(1-z)]^{\alpha-1} \tau\left\{ \left[\frac{p_2}{q} x^q (2z-1) + (-y)^{p_2} \right]^{1/p_2} \right\} F(\beta, 1-\beta, \alpha; \rho)\, dz - \\ & - \frac{\Gamma(1-2\alpha)}{\Gamma^2(1-\alpha)} q^{-2\alpha} \left(\frac{1}{p_2}(-y)^{p_2} \right)^{-} \left[\frac{1}{q} x^q \right]^{1-2\alpha} \int_0^1 \left[\frac{1}{q} x^q (2z-1) + \frac{1}{p_2}(-y)^{p_2} \right]^{\beta} \times \\ & \times [z(1-z)]^{-\alpha} v\left\{ \left[\frac{p_2}{q} x^q (2z-1) + (-y)^{p_2} \right]^{1/p_2} \right\} F(\beta, 1-\beta, 1-\alpha; \rho)\, dz. \end{aligned} \tag{21}$$

ko'rinishda bo'ladi.

(21) ni (4) shartga qo'yamiz, u holda D_2 sohada $\tau(x)$ va $\nu(x)$ funksiyalar orasidagi funksional munosabatni olamiz:

$$\left((x^{2q})^{\frac{1-\alpha+\beta}{2}}a(x)+\gamma\right)\nu(x)=$$

$$=k(x^{2q})^{\frac{1-2\alpha}{2}}\frac{d}{dx^{2q}}(x^{2q})^{\frac{1-2\beta}{2}}F_{0x}\left[\begin{matrix}\alpha+\beta,\frac{2\beta-1}{2}\\ 2\beta,\frac{x^{2q}-t^{2q}}{x^{2q}}\end{matrix}\right](x^{2q})^{\frac{2\alpha-1}{2}}\tau(x)-$$

$$-(x^{2q})^{\frac{1-\alpha+\beta}{2}}b(x),\ \ 0<x<h, \qquad (22)$$

bu yerda $k=2^{\alpha-\beta}\frac{\Gamma(2\beta)}{\Gamma(\beta)}$.

(8) shart bajarilganda, (22) munosabatdan

$$\tau(x)=\frac{(x^{2q})^{\beta-\alpha}}{k}F_{0x}\left[\begin{matrix}\alpha-\beta,\frac{1-2\beta}{2}\\ 1-2\beta,\frac{x^{2q}-t^{2q}}{x^{2q}}\end{matrix}\right](x^{2q})^{\frac{\alpha+\beta}{2}}b(x)$$

tenglikni olamiz. Ko'rinib turibdiki, TC masala (8) shart bajarilganda Dirixle masalasiga ekvivalent ravishda keltiriladi.

2.3-§.Yechimning mavjudligi

TC masaladan $a(x)=0$ bo'lganda, T masala kelib chiqadi. Shu sababli, biz mazkur paragrafda faqat TC masalani qaraymiz.

TC yechimning mavjudligini integral tenglamalar nazariyasini qo'llagan holda isbotlaymiz. Bu masalaning D_1 sohadagi yechimi,

$$y^{m_1}u_{xx}+x^n u_{yy}=0 \qquad (1')$$

elliptic tipdagi tenglama uchun (2), (3) va $u_y(x,0)=\nu(x)$, o<x<h, shartli ND masalaning

$$u(x,y)=-\int_0^h x^n\nu(x)G(x,0;x_0,y_0)dx+\int_0^{h_1} y^{m_1}\left.\frac{\partial G}{\partial x}\right|_{x=0}\varphi_1(y)dy-\int_\sigma \varphi_0(s)A_s[G]ds, \qquad (23)$$

ko'rinishdagi yechimi sifatida aniqlanadi, bu yerda $G(x, y; x_0, y_0)$ - (1') tenglama uchun *ND* masalaning Grin funksiyasi [4]:

$$G(x,y;x_0,y_0) = q(x,y;x_0,y_0) - (R_0^2)^{-\alpha-\beta_1} q(x,y;\overline{x_0},\overline{y_0}) \qquad (24)$$

ko'rinishida bo'ladi, bu yerda

$$R_0^2 = \frac{1}{q^2} x_0^{2q} + \frac{1}{p_1^2} y^{2p_1}, \qquad \overline{x_0}^q = \frac{x_0^q}{R_0^2}, \qquad \overline{y_0}^{p_1} = \frac{y_0^{p_1}}{R_0^2}.$$

$q(x, y; x_0, y_0)$ esa (1') tenglamaning

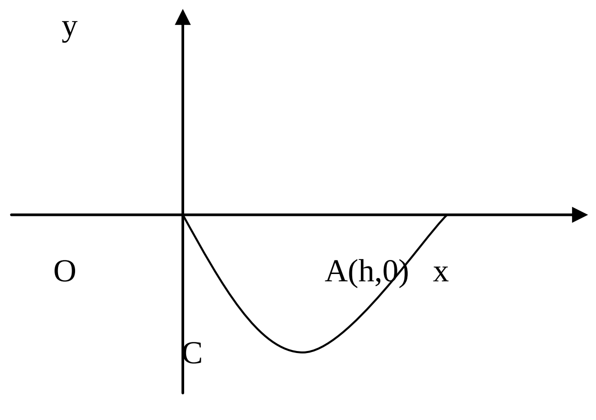
$$q(x,y;x_0,y_0)\big|_{x=0} = 0, \qquad \frac{\partial}{\partial x} q(x,y;x_0,y_0)\big|_{x=0} = 0$$

shartlarni qanoatlantiruvchi, fundamental yechimi bo'lib, u quyidagi ko'rinishga ega:

$$q(x,y;x_0,y_0) = kxx_0 (r^2)^{-\alpha-\beta_1-1} F_2(1-\alpha+\beta_1, 1-\alpha, \beta_1; 2-2\alpha, 2\beta_1; \sigma_1, \sigma_2). \qquad (25)$$

Endi (1) tenglamani D_2 sohada qaraymiz

$$-(-y)^{m_2} u_{xx} + x^n u_{yy} = 0, \quad x > 0, y < 0 \qquad (1'')$$



(1'') giperbolik tipdagi tenglama uchun

$$u(x,0) = \tau(x), u_y(x,0) = \nu(x)$$

boshlang'ich shartli Koshi masalasining yechimi

$$u(x,y)=\frac{\Gamma(2\alpha)}{\Gamma^{2}(\alpha)}\left(\frac{1}{p_2}(-y)^{p_2}\right)^{-\beta}\int_0^1\left[\frac{1}{q}x^{q}(2z-1)+\frac{1}{p_2}(-y)^{p_2}\right]^{\beta}\times$$

$$\times\left[z(1-z)\right]^{\alpha-1}\tau\left\{\left[\frac{p_2}{q}x^{q}(2z-1)+(-y)^{p_2}\right]^{1/p_2}\right\}F(\beta,1-\beta,\alpha;\rho)dz-$$

$$-\frac{\Gamma(1-2\alpha)}{\Gamma^{2}(1-\alpha)}q^{-2\alpha}\left(\frac{1}{p_2}(-y)^{p_2}\right)^{-}\left[\frac{1}{q}x^{q}\right]^{1-2\alpha}\int_0^1\left[\frac{1}{q}x^{q}(2z-1)+\frac{1}{p_2}(-y)^{p_2}\right]^{\beta}\times$$

$$\times\left[z(1-z)\right]^{-\alpha}\nu\left\{\left[\frac{p_2}{q}x^{q}(2z-1)+(-y)^{p_2}\right]^{1/p_2}\right\}F(\beta,1-\beta,1-\alpha;\rho)dz \qquad (26)$$

.

ko'rishdagi yechimi sifatida aniqlandi.

TC masalaning (2),(3) chegaraviy shartlari bir jinsli, ya'ni $\varphi_0(s)=0,\varphi_1(y)=0$ bo'lsin deb faraz qilamiz. (23) da $y=0$ deb olib, $\tau(x)$ va $\nu(x)$ orasidagi I oraliqda, D sohaning elliptik qismidan olingan, funksional munosabatni hosil qilamiz

$$\tau(x)=$$

$$=k\int_0^h\xi^{n}\nu(\xi)\left\{\frac{F\left(\alpha-\beta_1,\alpha,2\alpha;\dfrac{\frac{4}{q^2}\xi^{q}x^{q}}{\left(\frac{1}{q}\xi^{q}+\frac{1}{q}x^{q}\right)^{2}}\right)}{\left|\frac{1}{q}\xi^{q}-\frac{1}{q}x^{q}\right|^{2\beta_1}\left(\frac{1}{q}\xi^{q}+\frac{1}{q}x^{q}\right)^{2\alpha}}-\frac{F\left(\alpha-\beta_1,\alpha,2\alpha;\dfrac{\frac{4}{q^2}\xi^{q}x^{q}}{\left(\frac{1}{q^2}\xi^{q}x^{q}+1\right)^{2}}\right)}{\left(1-\frac{1}{q^2}\xi^{q}x^{q}\right)^{2\beta_1}\left(1+\frac{1}{q^2}\xi^{q}x^{q}\right)^{2\alpha}}\right\}d\xi$$

(27)

Bu yerda quyidagi(1.1-paragraf)

$$F(a,b,c;z)=A_1 z^{-a}F(a,a-c+1,a+b+1-c;1-z^{-1})+$$

$$+A_2 z^{a-c}(1-z)^{c-a-b}F(c-a,1-a,c+1-a-b;1-z^{-1})$$

$$A_1=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)},\qquad A_1=\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)},$$

$$c-a-b\neq 0,\pm1,\pm2,...;\qquad |\arg z|<\pi,\qquad |\arg(1-z)|<\pi;$$

$$F(a,b,c;z)=(1-z)^{c-a-b}F(c-a,c-b,c;z),\quad |\arg(1-z)|<\pi;$$

$$F(a,1-a+c,c;-z)=(1+z)^{c-1}\left(\sqrt{1+z}+\sqrt{z}\right)^{2-2a-2c}\times$$

$$\times F\left(c+a-1,c-\frac{1}{2},2c-1,4\sqrt{z(1+z)}\left(\sqrt{1+z}+\sqrt{z}\right)^{-2}\right)$$

formulalarni ketma-ket qo’llaymiz va (27) ifodani

$$\tilde{\tau}(x)=-\tilde{k}x^{\beta_1-\alpha}F_{ox}\begin{bmatrix}\alpha-\beta_1,\dfrac{1-2\beta_1}{2}\\ 1-2\beta_1;\dfrac{x-t}{x}\end{bmatrix}x^{\frac{2\alpha-1}{2}}\tilde{\nu}(x)-\tilde{k}x^{\beta_1-\alpha}F_{x_1}\begin{bmatrix}\alpha-\beta_1,\dfrac{1-2\beta_1}{2}\\ 1-2\beta_1;\dfrac{x-t}{x}\end{bmatrix}x^{\frac{2\alpha-1}{2}}\tilde{\nu}(x)-$$

$$-\tilde{\tilde{k}}x^{-\alpha-\beta}\int_0^1 t^{\frac{2\alpha-1}{2}}\tilde{\nu}(t)F\left(\alpha+\beta_1,\frac{1+2\beta_1}{2},1+2\beta_1;\frac{x-t}{x}\right)dt+$$

$$+\frac{\tilde{k}}{\Gamma(1-2\beta_1)}\int_0^1 t^{\frac{2\alpha-1}{2}}\tilde{\nu}(t)(1-xt)^{-2\beta_1}F\left(\alpha-\beta_1,\frac{1-2\beta_1}{2},1-2\beta_1;1-xt\right)dt+$$

$$+\tilde{\tilde{k}}\int_0^1 t^{\frac{2\alpha-1}{2}}\tilde{\nu}(t)F\left(\alpha+\beta_1,\frac{1+2\beta_1}{2},1+2\beta_1;1-xt\right)dt,0\le x\le 1$$

(28)

ko’rinishga keltiramiz. Bu yerda

$$\tilde{\tau}(x)=\tau\left[(q^2x)^{1/2q}\right],\quad \tilde{\nu}(x)=\nu\left[(q^2x)^{1/2q}\right],$$

$$\tilde{k}=\frac{2^{2\beta_1-3}}{\pi}\left(\frac{2}{p_1}\right)^{2\alpha}\frac{\Gamma(1-2\beta_1)\Gamma^2(\beta_1)}{\Gamma(2\beta_1)},\qquad \tilde{\tilde{k}}=\frac{2^{-2\beta_1-3}}{\pi}\left(\frac{2}{p_1}\right)^{2\beta_1}\frac{\Gamma(\beta_1)\Gamma(-\beta_1)\Gamma(\alpha+\beta_1)}{\Gamma(2\beta_1)\Gamma(\alpha-\beta_1)}.$$

Endi (1') giperbolik tipdagi tenglama uchun Koshi masalasi yechimining (26) ifodasi asosida,

$$F(a,1-a,c;z)=(1-z)^{c-1}F\left(\frac{c-a}{2},\frac{c+a-1}{2},c;4z(1-z)\right)$$

formulani qo'llab,

$$u[\theta(x)]=\gamma_1(x^{2q})^{\frac{2-\alpha-3\beta_2}{2}}F_{ox}^{-\beta_2}\begin{bmatrix}\frac{\beta_2-\alpha}{2},\frac{\alpha+\beta_2-1}{2}\\ \beta_2,\frac{x^{2q}-t^{2q}}{x^{2q}}\end{bmatrix}(x^{2q})^{\frac{\alpha+\beta_2-2}{2}}\tau(x)-\gamma_2(x^{2q})^{\frac{\beta_2-\alpha}{2}}\times$$

$$\times F_{ox}^{\beta_2-1}\begin{bmatrix}\frac{1-\alpha-\beta_2}{2},\frac{\alpha-\beta_2}{2}\\ 1-\beta_2,\frac{x^{2q}-t^{2q}}{x^{2q}}\end{bmatrix}(x^{2q})^{\frac{\alpha-\beta_2-1}{2}}\nu(x)$$

(29)

tenglikni olamiz, bu yerda

$$\gamma_1=\frac{\Gamma(2\beta_2)}{\Gamma(\beta_2)}2^{\alpha-\beta_2},\qquad \gamma_2=\frac{\Gamma(2-2\beta_2)}{\Gamma(1-\beta_2)}\left(\frac{p_1}{q}\right)^{1-2\beta_2}2^{\alpha+3\beta_2-2}.$$

Endi (29) ni (4) shartga olib borib qo'yamiz, u holda

$$\gamma_1J_1(x)-\gamma_2J_2(x)=a(x)\nu(x)+b(x) \qquad (30)$$

munosabatni hosil qilamiz, bu yerda

$$J_1(x)=\frac{d}{dx^{2q}}(x^{2q})^{\frac{1-\alpha-\beta}{2}}F_{ox}\left[\begin{matrix}\frac{\alpha+\beta-1}{2},\frac{\alpha+\beta}{2}\\ \beta,\frac{x^{2q}-t^{2q}}{x^{2q}}\end{matrix}\right](x^{2q})^{\frac{1+\alpha-3\beta}{2}}\times$$

$$\times F_{ox}\left[\begin{matrix}\frac{\beta-\alpha}{2},\frac{\alpha+\beta-1}{2}\\ \beta;\frac{x^{2q}-t^{2q}}{x^{2q}}\end{matrix}\right](x^{2q})^{\frac{\alpha+\beta-2}{2}}\tau(x) \tag{31}$$

$$J_2(x)=\frac{d}{dx^{2q}}(x^{2q})^{\frac{1-\alpha-\beta}{2}}F_{ox}\left[\begin{matrix}\frac{\alpha+\beta-1}{2},\frac{\alpha+\beta}{2}\\ \beta,\frac{x^{2q}-t^{2q}}{x^{2q}}\end{matrix}\right](x^{2q})^{\frac{\alpha+\beta-1}{2}}\times$$

$$\times F_{ox}\left[\begin{matrix}\frac{1-\alpha-\beta}{2},\frac{\alpha-\beta}{2}\\ 1-\beta;\frac{x^{2q}-t^{2q}}{x^{2q}}\end{matrix}\right](x^{2q})^{\frac{\alpha-\beta-1}{2}}v(x) \tag{32}$$

(31) da x ni $x^{1/2q}$ ga va t ni $t^{1/2q}$ ga almashtiramiz:

$$J_1\left(x^{1/2q}\right)=\frac{1}{\Gamma^2(\beta)}\frac{d}{dx}\int_0^x t^{\frac{\alpha+\beta_2-2}{2}}\tau\left(t^{11/2q}\right)K_1(\sigma)dt\ , \tag{33}$$

bu yerda

$$K_1(\sigma)=\sigma^{\frac{1+\alpha-\beta_2}{2}}\int_0^\infty z^{\alpha-\beta_2}f_1(\sigma z)f_2(z)dz\ , \tag{34}$$

$$f_1(z)=(z-1)_+^{\beta_2-1}F\left(\frac{\alpha+\beta_2}{2},\frac{\alpha+\beta_2-1}{2},\beta_2;1-z\right),$$

$$f_2(z) = (1-z)_+^{\beta_2 - 1} F\left(\frac{\alpha+\beta_2-1}{2}, \frac{\alpha+\beta_2}{2}, \beta_2; 1-z\right),$$

(34) integralni hisoblash uchun Mellin almashtirishini qo'llaymiz. Ya'ni [16]

$$x^{\alpha}\int_0^{\infty} \xi^{\beta} g_1(x\xi) g_2(\xi) d\xi \leftrightarrow g_1^*(s+\alpha) g_2^*(1-\alpha+\beta-s)$$

formulaga ko'ra, (34) ifodadan

$$K_1^*(s) = f_1^*\left(s + \frac{1+\alpha-\beta}{2}\right) f_2^*\left(\frac{1+\alpha-\beta}{2} - s\right) \tag{35}$$

ni hosil qilamiz.

Ushbu [16]

$$(x-1)_+^{c-1} F(a,b,c;1-x) \leftrightarrow \Gamma(c)\Gamma\begin{bmatrix} 1+a-c-s, 1+b-c-s \\ 1-s, 1+a+b-c-s \end{bmatrix},$$

$$\operatorname{Re} c > 0, \qquad \operatorname{Re} s < 1 + \operatorname{Re}(a-c), \qquad 1 + \operatorname{Re}(b-c),$$

$$(1-x)_+^{c-1} F(a,b,c;1-x) \leftrightarrow \Gamma(c)\Gamma\begin{bmatrix} s, s+c-a-b \\ s+c-a, s+c-b \end{bmatrix},$$

$$\operatorname{Re} c > 0, \qquad \operatorname{Re} s > 0 \qquad \operatorname{Re}(a+b-c),$$

bu yerda

$$\Gamma\begin{bmatrix} a_1, a_2, \ldots, a_n \\ b_1, b_2, \ldots, b_n \end{bmatrix} = \frac{\Gamma(a_1)\Gamma(a_2)\ldots\Gamma(a_n)}{\Gamma(b_1)\Gamma(b_2)\ldots\Gamma(b_n)},$$

formulalar asosan,

$$K_1^*(s) = \Gamma^2(\beta)\Gamma\begin{bmatrix} -s, \dfrac{1+\alpha-\beta_2}{2}-s, \dfrac{2-\alpha-\beta_2}{2}-s \\ \dfrac{1-\alpha+\beta_2}{2}-s, \dfrac{\alpha+\beta_2}{2}-s, 1-s \end{bmatrix}, \operatorname{Re} s < 0 \qquad (36)$$

hosil bo’ladi. Buni quyidagi

$$x^{\alpha} g(x) \leftrightarrow g^*(s+\alpha),$$

$$(x-1)_+^{c-1} F_3(a, a', b, b', c; 1-x; 1-\frac{1}{x}) \leftrightarrow \Gamma(c)\Gamma\begin{bmatrix} 1-a'-b'-s, 1+a-c-s, 1+b-c-s \\ 1-a'-s, 1-b'-s, 1+a+b-c-s \end{bmatrix},$$

$$\operatorname{Re} c > 0, \quad \operatorname{Re} s < 1-\operatorname{Re}(a'+b'), \quad 1+\operatorname{Re}(a-c), \quad 1+\operatorname{Re}(b-c)$$

munosabatlar bilan solishtirib,

$$K_1^*(s) \leftrightarrow K_1(\sigma) = \frac{\Gamma^2(\beta_2)}{\Gamma(2\beta_2)}\sigma^{\frac{1-2\beta_2}{2}}(\sigma-1)_+^{2\beta_2-1} \times$$

$$\times F_3\left(\frac{\alpha+\beta_2}{2}, \frac{\alpha+\beta_2}{2}, \frac{1-\alpha+\beta_2}{2}, \frac{1-\alpha+\beta_2}{2}, 2\beta_2; 1-\sigma, 1-\frac{1}{\sigma}\right)$$

munosabatni olamiz.

(33) integralni quyidagicha yozib olamiz:

$$J_1\left(x^{1/2q}\right) = \lim_{\varepsilon\to 0}\frac{1}{\Gamma^2(\beta)}\frac{d}{dx}\int_0^{x-\varepsilon} t^{\frac{\alpha+\beta_2-2}{2}}\tau\left(t^{1/2q}\right)K_1(\sigma)dt =$$

(37)

$$= \frac{1}{\Gamma^2(\beta_2)}\lim_{\varepsilon\to 0}\left[(x-\varepsilon)^{\frac{\alpha+\beta_2-2}{2}}\tau\left[(x-\varepsilon)^{1/2q}\right]K_1\left(\frac{x}{x-\varepsilon}\right)+\int_0^{x-\varepsilon} t^{\frac{\alpha+\beta_2-2}{2}}\tau\left(t^{1/2q}\right)\frac{d}{d\sigma}K_1(\sigma)\sigma_x{}' dt\right].$$

Endi $\frac{d}{d\sigma}K_1(\sigma)$ ni hisoblaymiz. Buning uchun

$$\frac{d}{dx}g(x) \leftrightarrow (1-s)g^*(s-1) \qquad (38)$$

formulani qo'llaymiz, u holda

$$\frac{d}{d\sigma}K_1(\sigma) \leftrightarrow (1-s)K_1^*(s-1) = (1-s)\Gamma^2(\beta_2)\Gamma\begin{bmatrix} 1-s, \frac{1+\alpha-\beta_2}{2}+1-s, \frac{2-\alpha-\beta_2}{2}+1-s \\ \frac{1-\alpha+\beta_2}{2}+1-s, \frac{\alpha+\beta_2}{2}+1-s, 1+1-s \end{bmatrix}$$

Shunday qilib,

$$\frac{d}{d\sigma}K_1(\sigma) = \frac{\Gamma^2(\beta_2)}{\Gamma(2\beta_2-1)}\sigma^{\frac{\alpha-\beta_2-1}{2}}(\sigma-1)_+^{2\beta_2-2}F\left(\alpha+\beta_2-1, \frac{2\beta_2-1}{2}, 2\beta_2-1; 1-\sigma\right).$$

Ushbu

$$\frac{1}{2\beta_2-1}x^{-\frac{\alpha+\beta_2}{2}}\frac{d}{dx}x^{\frac{1-2\beta_2}{2}}\int_0^{x-\varepsilon}t^{\frac{2\alpha-1}{2}}\tau\left(t^{1/2q}\right)(x-t)^{2\beta_2-1}F\left(\alpha+\beta_2,\frac{2\beta_2-1}{2},2\beta_2;\frac{x-t}{x}\right)dt=$$

$$=\frac{1}{2\beta_2-1}x^{\frac{1-\alpha-3\beta_2}{2}}(x-\varepsilon)^{\frac{2\alpha-1}{2}}\tau\left[(x-\varepsilon)^{1/2q}\right]\varepsilon^{2\beta_2-1}F\left(\alpha+\beta_2,\frac{2\beta_2-1}{2},2\beta_2;\frac{\varepsilon}{x}\right)+$$

$$+x^{\frac{1-\alpha-3\beta_2}{2}}\int_0^{x-\varepsilon}t^{\frac{2\alpha-1}{2}}\tau\left(t^{1/2q}\right)(x-t)^{2\beta_2-2}F\left(\alpha+\beta_2-1,\frac{2\beta_2-1}{2},2\beta_2-1;\frac{x-t}{x}\right)dt$$

ayniyatga ko'ra, (37) dan $\varepsilon\to 0$ da limitga o'tib, $J_1(x)$ ni aniqlaymiz:

$$J_1(x)=(x^{2\varepsilon})^{-\frac{\alpha+\beta_2}{2}}\frac{d}{dx^{2q}}(x^{2q})^{\frac{1-2\beta_2}{2}}F_{ox}\begin{bmatrix}\alpha+\beta_2,\frac{2\beta_2-1}{2}\\2\beta_2,\frac{x^{2q}-t^{2q}}{x^{2q}}\end{bmatrix}(x^{2q})^{\frac{2\alpha-1}{2}}\tau(x) \qquad (39)$$

Shunga o'xshash, $J_2(x)$ ni ham hosil qilamiz:

$$J_2(x)=(x^{2q})^{\frac{\alpha-\beta_2-1}{2}}v(x) \qquad (40)$$

Endi (39) va (40) ni (30) ga qo'ysak, $a(x)v(x)+b(x)$

$$\left((x^{2q})^{\frac{1-\alpha+\beta_2}{2}}a(x)+\gamma_2\right)v(x)=$$

$$=\gamma_1(x^{2q})^{\frac{1-2\alpha}{2}}\frac{d}{dx^{2q}}(x^{2q})^{\frac{1-2\beta_2}{2}}F_{ox}\begin{bmatrix}\alpha+\beta_2,\frac{2\beta_2-1}{2}\\2\beta_2,\frac{x^{2q}-t^{2q}}{x^{2q}}\end{bmatrix}(x^{2q})^{\frac{2\alpha-1}{2}}\tau(x)-(x^{2q})^{\frac{1-\alpha+\beta_2}{2}}b(x)$$

$0<x<h_1$.

Bundan

$$\tilde{v}(x) = \frac{\gamma_1}{\gamma_2} q^{2\beta_2 - 1} x^{\frac{1-2\alpha}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} F_{ox} \begin{bmatrix} \alpha + \beta_2, \frac{2\beta_2 - 1}{2} \\ 2\beta_2, \frac{x-t}{x} \end{bmatrix} x^{\frac{2\alpha - 1}{2}} \tilde{\tau}(x) + f(x),$$

$$0 < x < 1, \qquad (41)$$

bu yerda

$$f(x) = -\frac{b(x)}{(x^{2q})^{\frac{1-\alpha+\beta_2}{2}} a(x) + \gamma_2} (q^2 x)^{\frac{1-\alpha+\beta_2}{2}} \qquad (42)$$

I-hol. $m_1 = m_2$ bo'lsin. U holda (28) va (41) munosabatlardan $\tilde{\tau}(x)$ ni yo'qotib, quyidagini olamiz

$$\tilde{v}(x) + \frac{1}{2\sin \pi\beta}\left[R_1(\tilde{v}) + R_2(\tilde{v})\right] + k_1 R_3(\tilde{v}) - k_2 R_4(\tilde{v}) - k_1 R_3(\tilde{v}) = f(x) \qquad (43)$$

bu yerda

$$R_1(\tilde{v}) = x^{\frac{1-2\alpha}{2}} \frac{d}{dx} x^{\frac{1-2\beta}{2}} F_{ox} \begin{bmatrix} \alpha + \beta, \frac{2\beta - 1}{2} \\ 2\beta, \frac{x-t}{x} \end{bmatrix} x^{\frac{2\beta - 1}{2}} F_{ox} \begin{bmatrix} \alpha - \beta, \frac{1 - 2\beta}{2} \\ 1 - 2\beta, \frac{x-t}{x} \end{bmatrix} x^{\frac{2\alpha - 1}{2}} \tilde{v}(x),$$

$$R_2(\tilde{v}) = x^{\frac{1-2\alpha}{2}} \frac{d}{dx} x^{\frac{1-2\beta}{2}} F_{ox} \begin{bmatrix} \alpha + \beta, \frac{2\beta - 1}{2} \\ 2\beta, \frac{x-t}{x} \end{bmatrix} x^{\frac{2\beta - 1}{2}} F_{x_1} \begin{bmatrix} \alpha - \beta, \frac{1 - 2\beta}{2} \\ 1 - 2\beta, \frac{x-t}{x} \end{bmatrix} x^{\frac{2\alpha - 1}{2}} \tilde{v}(x),$$

$$R_3(\tilde{\nu}) = x^{\frac{1-2\alpha}{2}} \frac{d}{dx} x^{\frac{1-2\beta}{2}} F_{ox}\begin{bmatrix} \alpha+\beta, \frac{2\beta-1}{2} \\ 2\beta, \frac{x-t}{x} \end{bmatrix} x^{-\frac{1+2\beta}{2}} \int_0^1 t^{\frac{2\alpha-1}{2}} \tilde{\nu}(t) F\left(\alpha+\beta, \frac{1+2\beta}{2}, 1+2\beta; \frac{x-t}{x}\right) dt,$$

$$R_4(\tilde{\nu}) = x^{\frac{1-2\alpha}{2}} \frac{d}{dx} x^{\frac{1-2\beta}{2}} F_{ox}\begin{bmatrix} \alpha+\beta, \frac{2\beta-1}{2} \\ 2\beta, \frac{x-t}{x} \end{bmatrix} x^{\frac{2\alpha-1}{2}} \int_0^1 t^{\frac{2\alpha-1}{2}} \tilde{\nu}(t)(1-xt)^{-2\beta} F\left(\alpha-\beta, \frac{1-2\beta}{2}, 1-2\beta; 1-xt\right) dt$$

$$R_5(\tilde{\nu}) = x^{\frac{1-2\alpha}{2}} \frac{d}{dx} x^{\frac{1-2\beta}{2}} F_{ox}\begin{bmatrix} \alpha+\beta, \frac{2\beta-1}{2} \\ 2\beta, \frac{x-t}{x} \end{bmatrix} x^{\frac{2\alpha-1}{2}} \int_0^1 t^{\frac{2\alpha-1}{2}} \tilde{\nu}(t) F\left(\alpha+\beta, \frac{1+2\beta}{2}, 1+2\beta; 1-xt\right) dt,$$

$$k_1 = -\frac{2^{1-4\beta}\Gamma^2(1-\beta)}{4\pi\beta\,\Gamma(1-2\beta)}, \qquad k_2 = \frac{1}{2\Gamma(1-2\beta)\sin\pi\beta}.$$

Hisoblash mumkinki [5],

$$R_1(\tilde{\nu}) = \tilde{\nu}(x),$$

$$R_2(\tilde{\nu}) = \cos\pi(1-2\beta)\tilde{\nu}(x) + \frac{k_3}{\pi}\int_0^1 \left(\frac{t}{x}\right)^{\frac{1-2\beta}{2}} \frac{\tilde{\nu}(t)}{t-x} dt + \frac{k_4}{\pi}\int_0^1 \left(\frac{x}{t}\right)^{\beta-\alpha} \frac{\tilde{\nu}(t)}{t-x} dt,$$

$$R_3(\tilde{\nu}) = -k_5\int_0^1 \left(\frac{x}{t}\right)^{\beta-\alpha} \frac{\tilde{\nu}(t)}{t-x} dt + k_5\int_0^1 \left(\frac{t}{x}\right)^{\frac{1-2\beta}{2}} \frac{\tilde{\nu}(t)}{t-x} dt,$$

$$R_4(\tilde{\nu}) = \frac{k_4\Gamma(1-2\beta)}{\pi}\int_0^1 \left(\frac{x}{t}\right)^{\beta-\alpha} \frac{t^{\beta-\alpha}}{1-xt}\tilde{\nu}(t)dt + \frac{k_3\Gamma(1-2\beta)}{\pi}\int_0^1 \left(\frac{t}{x}\right)^{\frac{1-2\beta}{2}} \frac{t^{\alpha+\beta-1}\tilde{\nu}(t)}{1-xt} dt,$$

$$R_5(\tilde{\nu}) = -k_5\int_0^1 \left(\frac{x}{t}\right)^{\beta-\alpha} \frac{t^{\beta-\alpha}}{1-xt}\tilde{\nu}(t)dt + k_5\int_0^1 \left(\frac{t}{x}\right)^{\frac{1-2\beta}{2}} \frac{t^{\alpha+\beta-1}}{1-xt}\tilde{\nu}(t)dt,$$

bu yerda $k_3 = \dfrac{\cos\pi\beta\,\sin\pi(1-\alpha-\beta)}{\cos\pi\alpha}, \qquad k_4 = \dfrac{\cos\pi\beta\,\sin\pi(\beta-\alpha)}{\cos\pi\alpha},$

$$k_5 = \frac{\Gamma(1+2\beta)\Gamma\left(\frac{1}{2}+\alpha\right)\Gamma\left(\frac{1}{2}-\alpha\right)}{\Gamma(\alpha+\beta)\Gamma(1-\alpha+\beta)\Gamma^2\left(\frac{1}{2}+\beta\right)}.$$

R_1, R_2, R_3, R_4, R_5 larning bu ifodalarini (43) ga qo'yib, ba'zi almashtirishlardan so'ng, ushbu singulyar integral tenglamani olamiz

$$\rho(z) + \lambda\int_0^1 \frac{\rho(\eta)}{\eta - z}d\eta + \int_0^1 K(z,\eta)\rho(\eta)d\eta = f_0(z), \quad 0 < z < 1, \tag{44}$$

bu yerda

$$\rho(z) = x^{1-2\beta}(1+x^2)\tilde{v}(x),$$

$$\lambda = \frac{\cos \pi\beta}{\pi(1+\sin \pi\beta)}, \qquad K(z,\eta) = \frac{K_0(z,\eta) - K_0(z,z)}{\eta - z},$$

$$K_0(z,\eta) = \frac{1}{1-t^2}\left[k_6(1 - xt - t^{\alpha+\beta} + xt^{\alpha+\beta-1}) + k_7(1 - xt - t^{\beta-\alpha+1} + xt^{\beta-\alpha}\right],$$

$$f_0(z) = k_8 x^{\frac{1-2\beta}{2}}(1-x^2)f(x),$$

k_6, k_7, k_8-ma'lum o'zgarmas sonlar.

Shunday qilib, biz TC masalaga ekvivalent bo'lgan (44) singulyar integral tenglamani hosil qildik. Bu tenglamaing $h(0)$ sinfda, ya'ni $z \to 0$ da chegaralangan funksiyalarsinfida, indeksi nolga teng [5]. Karleman-Veksa usuli [20] yordamida (44) tenglamani Fredgolm 2-tur integral tenglamasiga olib kelish mumkin, bu oxirgi tenglamaning yechimga ega ekani Trikomi masalasi yechimining yagonaligidan bevosita kelib chiqadi.

II-hol. $m_1 < m_2$ bo'lsin. Bu holda ham (28), (43) dan $\tilde{\tau}(x)$ ni yo'qotamiz, ya'ni

$$\tilde{v}(x) = \lambda_1 P_1(\tilde{v}) + \lambda_2 P_2(\tilde{v}) + \lambda_3 P_3(\tilde{v}) + \lambda_4 P_4(\tilde{v}) + \lambda_5 P_5(\tilde{v}) + f(x) \tag{45}$$

bu yerda

$$P_1(\tilde{v}) = x^{\frac{1-2\alpha}{2}}\frac{d}{dx}x^{\frac{1-2\beta_2}{2}}F_{ox}\begin{bmatrix}\alpha+\beta_2, \frac{2\beta_2-1}{2} \\ 2\beta_2, \frac{x-t}{x}\end{bmatrix}x^{\frac{2\beta_2-1}{2}}F_{ox}\begin{bmatrix}\alpha-\beta_1, \frac{1-2\beta_1}{2} \\ 1-2\beta_1, \frac{x-t}{x}\end{bmatrix}x^{\frac{2\alpha-1}{2}}\tilde{v}(x),$$

$$P_2(\tilde{v}) = x^{\frac{1-2\alpha}{2}}\frac{d}{dx}x^{\frac{1-2\beta_2}{2}}F_{ox}\begin{bmatrix}\alpha+\beta_2, \frac{2\beta_2-1}{2} \\ 2\beta_2, \frac{x-t}{x}\end{bmatrix}x^{\frac{2\beta_2-1}{2}}F_{x_1}\begin{bmatrix}\alpha-\beta_1, \frac{1-2\beta_1}{2} \\ 1-2\beta_1, \frac{x-t}{x}\end{bmatrix}x^{\frac{2\alpha-1}{2}}\tilde{v}(x),$$

$$P_3(\tilde{v}) = x^{\frac{1-2\alpha}{2}}\frac{d}{dx}x^{\frac{1-2\beta_2}{2}}F_{ox}\begin{bmatrix}\alpha+\beta_2, \frac{2\beta_2-1}{2} \\ 2\beta_2, \frac{x-t}{x}\end{bmatrix}x^{-\frac{1+2\beta_2}{2}}\int_0^1 t^{\frac{2\alpha-1}{2}}\tilde{v}(t)F\left(\alpha+\beta_1, \frac{1+2\beta_1}{2}, 1+2\beta_1; \frac{x-t}{x}\right)dt,$$

$$P_4(\tilde{v}) = x^{\frac{1-2\alpha}{2}}\frac{d}{dx}x^{\frac{1-2\beta_2}{2}}F_{ox}\begin{bmatrix}\alpha+\beta_2, \frac{2\beta_2-1}{2} \\ 2\beta_2, \frac{x-t}{x}\end{bmatrix}x^{\frac{2\alpha-1}{2}}\int_0^1 t^{\frac{2\alpha-1}{2}}\tilde{v}(t)(1-xt)^{-2\beta_1}F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; 1-xt\right)dt$$

$$P_5(\tilde{v}) = x^{\frac{1-2\alpha}{2}}\frac{d}{dx}x^{\frac{1-2\beta_2}{2}}F_{ox}\begin{bmatrix}\alpha+\beta_2, \frac{2\beta_2-1}{2} \\ 2\beta_2, \frac{x-t}{x}\end{bmatrix}x^{\frac{2\alpha-1}{2}}\int_0^1 t^{\frac{2\alpha-1}{2}}\tilde{v}(t)F\left(\alpha+\beta_1, \frac{1+2\beta_1}{2}, 1+2\beta_1; 1-xt\right)dt$$

$\lambda_i \quad (i=\overline{1,5})$ - berilgan o'zgarmaslar.

Endi P_1, P_2, P_3, P_4, P_5 ifodalarni hisoblaymiz.

$$P_1(\tilde{v}) = \frac{x^{\frac{1-2\alpha}{2}}}{\Gamma(1-2\beta_2)\Gamma(2\beta_2)} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^s t^{\frac{2\beta_2-1}{2}} (x-t)^{2\beta_2-1} F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) dt \times$$

$$\times \int_0^t s^{\frac{2\alpha-1}{2}} (t-s)^{-2\beta_1} F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; \frac{t-s}{t}\right) \tilde{v}(s) ds =$$

$$= \frac{x^{\frac{1-2\alpha}{2}}}{\Gamma(1-2\beta_2)\Gamma(2\beta_2)} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^x s^{\frac{2\alpha-1}{2}} \tilde{v}(s) ds \int_s^x t^{\frac{2\beta_2-1}{2}} (x-t)^{2\beta_2-1} (t-s)^{-2\beta_1} \times$$

$$\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; \frac{t-s}{t}\right) dt =$$

$$= \frac{1}{\Gamma(1-2\beta_2)\Gamma(2\beta_2)} \int_0^x \tilde{v}(s) \left[\left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_s^x t^{\frac{2\beta_2-1}{2}} (x-t)^{2\beta_2-1} (t-s)^{-2\beta_1} \times\right.$$

$$\left.\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; \frac{t-s}{t}\right) dt \right] ds.$$

$$P_2(\tilde{v}) = \frac{x^{\frac{1-2\alpha}{2}}}{\Gamma(1-2\beta_2)\Gamma(2\beta_2)} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^x t^{\frac{2\beta_2-1}{2}} (x-t)^{2\beta_2-1} F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) dt \times$$

$$\times \int_0^1 s^{\frac{2\alpha-1}{2}} \tilde{v}(s)(s-t)^{-2\beta_1} F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; \frac{t-s}{t}\right) ds$$

O’ng tomonda turgan ichki integralni quyidagicha yozib olamiz:

$$\int_t^1 = \int_t^x + \int_x^1 \quad ,$$

u holda

$$P_2(\tilde{v}) = \frac{x^{\frac{1-2\alpha}{2}}}{\Gamma(1-2\beta_2)\Gamma(2\beta_1)} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \left\{ \int_0^x s^{\frac{2\alpha-1}{2}} \tilde{v}(s) ds \int_0^s t^{\frac{2\beta_2-1}{2}} (x-t)^{2\beta_2-1} (s-t)^{-2\beta_1} \times \right.$$

$$\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; \frac{t-s}{t}\right) dt +$$

$$+ \int_x^1 s^{\frac{2\alpha-1}{2}} \tilde{v}(s) ds \int_0^x t^{\frac{2\beta_2-1}{2}} (x-t)^{2\beta_2-1} (s-t)^{-2\beta_1} \times$$

$$\left. \times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; \frac{t-s}{t}\right) dt \right\}$$

yoki

$$P_2(\tilde{v}) = \frac{1}{\Gamma(1-2\beta_2)\Gamma(2\beta_1)} \left\{ \int_0^x \tilde{v}(s) \left[\left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^s t^{\frac{2\beta_2-1}{2}} (x-t)^{2\beta_2-1} (s-t)^{-2\beta_1} \times \right. \right.$$

$$\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; \frac{t-s}{t}\right) dt +$$

$$+ \int_x^1 \tilde{v}(s) \left[\left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^x t^{\frac{2\beta_2-1}{2}} (x-t)^{2\beta_2-1} (s-t)^{-2\beta_1} \times \right.$$

$$\left. \left. \times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; \frac{t-s}{t}\right) dt \right] ds \right\}.$$

$$P_3(\tilde{v}) = \frac{x^{\frac{1-2\alpha}{2}}}{\Gamma(1-2\beta_2)} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^s t^{-\frac{1+2\beta_2}{2}} (x-t)^{2\beta_2-1} F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) dt \times$$

$$\times \int_0^1 s^{\frac{2\alpha-1}{2}} \tilde{v}(s) F\left(\alpha+\beta_1, \frac{1+2\beta_1}{2}, 1+2\beta_1; \frac{t-s}{t}\right) ds =$$

$$= \frac{1}{\Gamma(1-2\beta_2)} \int_0^1 \tilde{v}(s) \left[\left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^x t^{-\frac{1+2\beta_2}{2}} (x-t)^{2\beta_2-1} \times\right.$$

$$\left.\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha+\beta_1, \frac{1+2\beta_1}{2}, 1+2\beta_1; \frac{t-s}{t}\right) dt \right] ds$$

$$P_4(\tilde{v}) = \frac{1}{\Gamma(1-2\beta_2)} \int_0^1 \tilde{v}(s) \left[\left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^x t^{\frac{2\alpha-1}{2}} (x-t)^{2\beta_2-1} (1-st)^{-2\beta_1} \times\right.$$

$$\left.\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; 1-st\right) dt \right] ds$$

$$P_5(\tilde{v}) = \frac{1}{\Gamma(1-2\beta_2)} \int_0^1 \tilde{v}(s) \left[\left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^x t^{\frac{2\alpha-1}{2}} (x-t)^{2\beta_2-1} \times\right.$$

$$\left.\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha+\beta_1, \frac{1+2\beta_1}{2}, 1+2\beta_1; 1-st\right) dt \right] ds$$

Demak,

$$P_1(\tilde{v}) = \int_0^x \tilde{v}(s) K_{11}(x,s)\, ds, \quad P_2(\tilde{v}) = \int_0^x K_{21}(x,s)\tilde{v}(s)\, ds + \int_x^1 \tilde{v}(s) K_{22}(x,s)\, ds,$$

$$P_3(\tilde{v}) = \int_0^1 \tilde{v}(s) K_{31}(x,s)\, ds, \qquad P_4(\tilde{v}) = \int_0^1 \tilde{v}(s) K_{41}(x,s)\, ds,$$

$$P_5(\tilde{v}) = \int_0^1 \tilde{v}(s) K_{51}(x,s)\,ds \quad ,$$

bu yerda

$$K_{11}(x,s) = \frac{1}{\Gamma(1-2\beta_2)\Gamma(2\beta_2)} \left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_s^x t^{\frac{2\beta_2-1}{2}} (x-t)^{2\beta_2-1} (s-t)^{-2\beta_1} \times$$

$$\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; \frac{t-s}{t}\right) dt \qquad (46)$$

$$K_{21}(x,s) = \frac{1}{\Gamma(1-2\beta_2)\Gamma(2\beta_1)} \left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^s t^{\frac{2\beta_2-1}{2}} (x-t)^{2\beta_2-1} (s-t)^{-2\beta_1} \times$$

$$\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; \frac{t-s}{t}\right) dt \qquad (47)$$

$$K_{22}(x,s) = \frac{1}{\Gamma(1-2\beta_2)\Gamma(2\beta_1)} \left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^s t^{\frac{2\beta_2-1}{2}} (x-t)^{2\beta_2-1} (s-t)^{-2\beta_1} \times$$

$$\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; \frac{t-s}{t}\right) dt \qquad (48)$$

$$K_{31}(x,s) = \frac{1}{\Gamma(1-2\beta_2)} \left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^x t^{-\frac{1+2\beta_2}{2}} (x-t)^{2\beta_2-1} \times$$

$$\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha+\beta_1, \frac{1+2\beta_1}{2}, 1+2\beta_1; \frac{t-s}{t}\right) dt \qquad (49)$$

$$K_{41}(x,s) = \frac{1}{\Gamma(1-2\beta_2)}\left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^x t^{\frac{2\alpha-1}{2}} (x-t)^{2\beta_2-1}(1-st)^{-2\beta_1} \times$$

$$\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha-\beta_1, \frac{1-2\beta_1}{2}, 1-2\beta_1; 1-st\right) dt \quad (50)$$

$$K_{51}(x,s) = \frac{1}{\Gamma(1-2\beta_2)}\left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_2}{2}} \int_0^x t^{\frac{2\alpha-1}{2}} (x-t)^{2\beta_2-1} \times$$

$$\times F\left(\alpha+\beta_2, \frac{2\beta_2-1}{2}, 2\beta_2; \frac{x-t}{x}\right) F\left(\alpha+\beta_1, \frac{1+2\beta_1}{2}, 1+2\beta_1; 1-st\right) dt \quad (51)$$

Shunday qilib, (46)-(51) ga ko’ra , (43) tenglama

$$\tilde{v}(x) + \int_0^1 \tilde{v}(s) K(x,s) ds = f(x) \quad (52)$$

ko’rinishga keladi. Bu yerda

$$K(x,s) = K_1(x,s) + K_2(x,s) \quad (53)$$

$$K_1(x,s) = \begin{cases} K_{11}(x,s) + K_{21}(x,s), \ agar \ \ 0 \le s \le x \\ K_{22}(x,s), \ agar x \ \ \le s \le 1 \end{cases}$$

$$K_2(x,s) = K_{31}(x,s) + K_{41}(x,s) + K_{51}(x,s)$$

$K(x,s)$ yadroni baholaymiz. (46) tenglikni o’ng tomonida turgan integralda $t = s + (x-s)z$ almashtirish bajaramiz, u holda

$$K_{11}(x,s)=k_1\left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}}\frac{d}{dx}x^{\frac{1-2\beta_2}{2}}\int_0^1[s+(x-s)z]^{\frac{2\beta_2-1}{2}}[(x-s)-(x-s)z]^{2\beta_2-1}(x-s)^{1-2\beta_1}z^{-2\beta_1}\times$$

$$\times F\left(\alpha+\beta_2,\frac{2\beta_2-1}{2},2\beta_2;\frac{x-s}{x}(1-z)\right)F\left(\alpha-\beta_1,\frac{1-2\beta_1}{2},1-2\beta_1;\frac{(x-s)z}{s+(x-s)z}\right)dz=$$

$$=k_1\left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}}\frac{d}{dx}x^{\frac{1-2\beta_2}{2}}s^{\frac{2\beta_2-1}{2}}(x-s)^{2\beta_2-2\beta_1}\int_0^1 z^{-2\beta_1}(1+z)^{\frac{2\beta_2-1}{2}}(1-z)^{2\beta_2-1}\times$$

$$\times F\left(\alpha+\beta_2,\frac{2\beta_2-1}{2},2\beta_2;\frac{x-s}{x}(1-z)\right)F\left(\alpha-\beta_1,\frac{1-2\beta_1}{2},1-2\beta_1;\frac{(x-s)z}{s+(x-s)z}\right)dz$$

Bu yerdan ushbu (1.1-paragraf)

$$F(a,b,c,z)\le\begin{cases}C_1, agarc\quad -a-b>0, 0\le z\le 1,\\ C_2(1-z)^{c-a-b}, agarc\quad -a-b<0, 0<z<1,\\ C_3\left(1+\left|\ln(1-z)\right|\right), agarc\quad -a-b=0\end{cases}$$

baholarga asosan,

$$\frac{d}{dz}F(a,b,c,z)=F(a+1,b+1,c+1;z)$$

tenglikni hisobga olib, quyidagini olamiz:

$$\left|K_{11}(x,s)\right|\le\left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}}s^{\frac{2\beta_2-1}{2}}\left[C_1x^{-\frac{1+2\beta_2}{2}}(x-s)^{2\beta_2-2\beta_1}+C_2x^{\frac{1-2\beta_2}{2}}(x-s)^{2\beta_2-2\beta_1-1}+\right.$$

$$\left.+C_3x^{\frac{1-2\beta_2}{2}}(x-s)^{2\beta_2-2\beta_1}\left(\frac{s}{x}\right)^{-\alpha-\frac{1}{2}}+C_4x^{\frac{1-2\beta_2}{2}}(x-s)^{2\beta_2-2\beta_1}\right]=$$

$$=C_1s^{\alpha+\beta_2-1}x^{-\alpha-\beta_2}(x-s)^{2\beta_2-2\beta_1}+C_2\left(\frac{s}{x}\right)^{\alpha+\beta_2-1}(x-s)^{2\beta_2-2\beta_1-1}+$$

$$+C_3\left(\frac{s}{x}\right)^{\beta_2-\frac{3}{2}}(x-s)^{2\beta_2-2\beta_1}+C_4\left(\frac{s}{x}\right)^{\alpha+\beta_2-1}(x-s)^{2\beta_2-2\beta_1}.$$

(54)

Xuddi shunga o'xshash $K_{21}, K_{22}, K_{31}, K_{41}, K_{51}$ larni yuqoridan baholaymiz. Ushbu [11]

$$\frac{d}{dz}\left[z^{c-1}F(a,b,c,z)\right] = (c-1)z^{c-2}F(a,b,c-1;z)$$

formulaga asosan, (54) ga asosan,

$$\left|K_{21}(x,s)\right| \le \left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}}\left[C_1 x^{3\beta_2-\frac{3}{2}} s^{\frac{1}{2}-\beta_2} + C_2 x^{\frac{1-2\beta_2}{2}}\left(\frac{s}{x}\right)^{-\alpha-\frac{1}{2}} s^{\beta_2-2\beta_1+\frac{1}{2}}\right] =$$

$$= C_1 s^{\alpha-\beta_2-2\beta_1} x^{3\beta_2-\alpha-1} + C_2 s^{\beta_2-2\beta_1-\frac{1}{2}} x^{\frac{3}{2}-\beta_2} = \tag{55}$$

$$= C_1\left(\frac{s}{x}\right)^{\alpha-\beta_2-2\beta_1} x^{2\beta_2-2\beta_1-1} + C_2\left(\frac{s}{x}\right)^{2\beta_2-2\beta_2-\frac{1}{2}} x^{-2\beta_1+1}$$

$$\left|K_{22}(x,s)\right| \le \left(\frac{s}{x}\right)^{\frac{2\alpha-1}{2}}\left[C_1\left(\frac{s}{x}\right)^{-2\beta_1} x^{2\beta_2-2\beta_1} + C_2\left(\frac{s}{x}\right)^{-2\beta_1} x^{2\beta_2-2\beta_1-1}\right] \tag{56}$$

Xuddi yuqoridagidek, ko'rsatish mumkinki, $K_2(x,s)$ yadro ham kuchsiz maxsuslikka ega bo'ladi.

Demak, (52) tenglamaning $K(x,s)$ yadrosi kuchsiz maxsuslikka ega, chunki $m_1 < m_2$ yoki $-1 < 2\beta_2 - 2\beta_1 - 1 < 0$, va (52) Fredgolm ikkinchi tur integral tenglamasi $C^2(0,h)$ sinfda yotuvchi yagona yechimga ega bo'lib, bu yechim $x \to 0$ da $\frac{n+2}{m_2+2}$ dan kichik tartibda cheksizlikka intilishi va $x \to 0$ da chegaralangan bo'lishi mumkin.

(52) integral tenglamani yechib, $\nu(x)$ funksiyani topamiz va (27) munosabatdan $\tau(x)$ funksiyani aniqlaymiz, bunda $\tau(x) \in C[0,h] \cap C^2(0,h)$ bo'ladi.

TC masalaning D_1 sohadagi yechimi, (1) elliptic tipdagi tenglama uchun ND masalaning

$$u(x,y) = -\int_0^h x^n \nu(x) G(x,0;x_0,y_0)dx +$$

$$+\int_0^{h_1} y^{m_1} \left.\frac{\partial G}{\partial x}\right|_{x=0} \varphi_1(y)dy - \int_\sigma \varphi_0(s) A_s[G]ds$$

ko'rinishdagi, bu yerda $\nu(x) = u_y(x,0)$, $G(x,y;x_0,y_0)$

- Grin funksiyasi va D_2 sohadagi yechimi esa, (1) giperbolik tipdagi tenglama uchun ushbu

$$u(x,0) = \tau(x), u_y(x,0) = \nu(x)$$

boshlang'ich shartli Koshi masalasining quyidagi

$$u(x,y) = \frac{\Gamma(2\alpha)}{\Gamma^2(\alpha)}\left(\frac{1}{p_2}(-y)^{p_2}\right)^{-\beta} \int_0^1 \left[\frac{1}{q}x^q(2z-1) + \frac{1}{p_2}(-y)^{p_2}\right]^{\beta} \times$$

$$\times [z(1-z)]^{\alpha-1} \tau\left\{\left[\frac{p_2}{q}x^q(2z-1) + (-y)^{p_2}\right]^{1/p_2}\right\} F(\beta, 1-\beta, \alpha; \rho)dz -$$

$$-\frac{\Gamma(1-2\alpha)}{\Gamma^2(1-\alpha)} q^{-2\alpha}\left(\frac{1}{p_2}(-y)^{p_2}\right)^{-}\left[\frac{1}{q}x^q\right]^{1-2\alpha} \int_0^1 \left[\frac{1}{q}x^q(2z-1) + \frac{1}{p_2}(-y)^{p_2}\right]^{\beta} \times$$

$$\times [z(1-z)]^{-\alpha} \nu\left\{\left[\frac{p_2}{q}x^q(2z-1) + (-y)^{p_2}\right]^{1/p_2}\right\} F(\beta, 1-\beta, 1-\alpha; \rho)dz .$$

ko'rinishdagi yechimi sifatida aniqlanadi.

XULOSA

Mazkur ishda aralash tipdagi elliptic-giperbolik tenglama uchun Trikomi tipidagi chegaraviy masalalar o'rganilgan bo'lib, u kirish qismi, ikkita bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan tashkil topgan.

I-bobda matematik analiz kursidan ma'lum bo'lgan, gamma va beta funksiyalar haqida, maxsus funksiyalar nazariyasi bo'yicha ba'zi ma'lumotlar keltirilgan hamda umumlashgan kasr tartibli integral operatorga ta'rif berilgan. Bundan tashqari, matematik fizika tenglamalaridan bizga ma'lum bo'lgan, giperbo'lik tipdagi tenglamalar uchun qo'yilgan chegaraviy masalalar yechimining yagonaligini isbotlashda energiya integrali usuli qo'llanilishi va aralash tipdagi tenglamalar tushunchasi yoritilgan.

II-bob ikkita buzilish chizig'iga ega va turli tartibli aralash tipdagi tenglama uchun chegaraviy masalalarni o'rganishga bag'ishlangan bo'lib, unda Trikomi tipidagi TC va T masalalar yechimning mavjudligi va yagonaligi isbotlangan. Yechimining yagonaligi energiya integrali usulida va A.V.Bitsadzening ekstremum prinsipi yordamida, yechimning mavjudligi esa integral tenglamalar nazariyasini qo'llagan holda ko'rsatilgan.

Magistrlik dissertatsiyasida olingan natijalar, aralash tipdagi tenglamalar nazariyasidagi yangi muhim masalalarni hal etishda va bunday tenglamalarga keltiriladigan amaliy masalalarni yechishda qo'llanilishi mumkin.

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