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TOSHKENT DAVLAT AGRAR UNIVERSITETI

OLIY MATEMATIKA KAFEDRASI

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OLIY MATEMATIKA

(AGRAR UNIVERSITET VA QISHLOQ XO’JALIK OLIYGOHLARINI TALABALARI UCHUN USLUBIY QO’LLANMA).

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Uslubiy qo'llanma qishloq xo'jalik oliygohlarini kunduzgi ta'lim Agroinjeneriya,Iqtisod, Buxgalteriya va barcha Agronomiya yo'nalishi talabalariga «Oliy matematika» fanini yaxshi o'zlashtirishlariga yordam berish maqsadida tayyorlandi. Uslubiy qo'llanmada kunduzgi ta'lim talabalari uchun rejalashtirilgan o'quv soatiga mos mavzular kiritilgan va mavzular bo'yicha qisqacha nazariy tushunchalar bayon yetilib, namunaviy misol va masalalar yechib ko'rsatilgan.

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Taqrizchilar: 1. Sh. Qayumov-ToshDTU«Oliy matematika» kafedrasi dosenti.
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Uslubiy qo'llanma «Oliy matematika» kafedrasining 2006 yil 18 dekabr №5 sonli majlisida muhokama qilinib, chop etishga tavsiya etildi.

Qishloq xo'jaligini mexanizasiyalashtirish fakulteti o'quv-uslubiy komissiyasining 2006 22 dekabr № 3, éèg'ilishi va Universitet Ilmiy Kengashining dekabr № 2006 yil qarorlari bilan chop etishga tavsiya qilindi.

Toshkent-2007 y.

Kirish

Ushbu uslubiy qo'llanmada agrar universitet va qishloq xo'jalik oliygohlarini Agroinjeneriya,Iqtisod, Buxgalteriya va barcha Agronomiya yo'nalishi talabalariga «Oliy matematika» fanini yaxshi o'zlashtirishlariga yordam berish maqsadida tayyorlandi. «Oliy matematika» fanini Agroinjeneriya yo'nalishi talabalariga rejalashtirilgan 234 o'quv soatiga mos mavzular qisqacha bayon etilib, ularga doir namunaviy misollar yechib ko'rsatilgan. Ma'lumki, matematika va tabiiy fanlarni o'qitishdan asosiy maqsad talabalarni mantiqan to'g'ri fikr yuritishga o'rgatish bilan bir qatorda, ularni o'z ish faoliyatlarida uchraydigan amaliy masalalarni echishda matematik usullardan foydalanishlariga yordam berishdir.

Talabalar fanni yanada chuqurroq o'zlashtirishlari uchun tavsiya etilgan adabiyotlardan foydalanishlarini maslahat beramiz.

1. Turli foizlarni hisoblash va ularni qishloq xo'jalik masalalarini yechishga qo'llanilishi.

Oddiy foiz masalalarini yechish.

A haqiqiy sonning yuzdan bir ulushi (bo'lagi) 1 foizi bo'ladi. Masalan 8%=0,08, 45%=0,45, 100%=1, 160%=1,6. Oddiy foizlar bilan bog'liq masalalar quyidagi proporsiya shaklida yechiladi:

A - 100%

B - C %

A,B,C xaqiqiy sonlar noldan farqli bo'lganda, nisbatlari proporsional bo'ladi ya'ni A sonni B ulushiga nisbati, ularni mos foizlari nisbatiga teng bo'ladi

$$A/B=100/C, \quad AC=100B \qquad (1)$$

Oxirgi tenglikdan ixtiyoriy ikkitasi ma'lum bo'lsa, uchinchi miqdorni topish mumkin.

Misol 1. Agar paxtadan 34 % tola chiqsa, 1200 t paxtadan qancha tola chiqadi?

Yechish: A= 1200t, C=34% bo'lganligidan

1200 -100%

B - 34% B=AC/100=1200x34/100=12x34=408t.

Misol 2. Fermer xo'jaligini 5% ni tashkil qiluvchi 8ga erga sholi ekildi. Fermer xo'jaligida qancha er bor?

Yechish: 8 ga - 5%

A - 100% A=8 .100/5=160ga.

Misol 3. Pillaxonaga 76 kg sifatli, 4kg sifatsiz pilla topshirildi. Sifatsiz pilla necha % ni tashkil qiladi?

Yechish: Jami pilla 76+4=80 kg

80 kg - 100%

4 kg - C % C=4.100/80=5%

Misol 4: Ekilgan 800 chigitdan 720 tasi unib chiqdi. Chigitni unuvchanligi necha % ?

Yechish: 800 - 100%

720 - C % C=720x100 /800 =90%.

Oddiy va murakkab foizli jamg'armalarni hisoblash

Oddiy foizli jamg'arma.

A - so'mning ma'lum bir foizi vaqt(oy, yil) o'tishi bilan qo'shilib borsa, jamg'arma hosil bo'ladi. Agar jamg'armada faqat boshlang'ich pul miqdorini foizi qo'shilib borsa, oddiy foizli jamg'arma deyiladi va quyidagi formula bilan hisoblanadi([8] ,[9]):

$$A_n=A\ (1+nr/100) \qquad (2)$$

Bu yerda A boshlang'ich pul miqdori, r- o'sish foizi, n- oylar yoki yillar soni (o'sish muddati), A_n- n muddatdan keyingi oddiy foizli jamg'arma miqdori.

Misol 5: Boshlang'ich A=10000 so'm, oyiga r=20% li oddiy jamg'armaga qo'yilgan bo'lsa, n =5 oydan keyin jamg'arma miqdori A_5 qancha bo'ladi?

Yechish: (2) formulaga asosan 5 oydan keyigi jamg'arma miqdori

A_5=10000 (1+5x20/100)=10000x2=20000 so'm bo'ladi.

Misol 6: To'rt oydan keyin 10 % li oddiy jamg'arma miqdori 14000 so'm bo'lgan bo'lsa,boshlang'ich pul qancha bo'lgan?

Yechish: $n=4$, $r=10\%$, $A_n=14000$ so'm (2) formulaga asosan boshlang'ich pul miqdori

$A=A_p/(1+nr/100)=14000/(1+4x10/100)=14000/1,4=10000$ so'm.

Misol 7. 5000 so'm pul, oyiga 12 % li oddiy jamg'armada necha oydan keyin 8000 so'm bo'ladi?

Yechish: $A_p=8000$ so'm, $A=5000$ so'm, $r=12\%$ (2) formulaga asosan

$N=(A_p-A)x100/(Ar)=(8000-5000))/(50x12)=3000:600=5$ oydan keyin.

Murakkab foizli jamg'arma.

Agar jamg'arma boshlang'ich pul miqdoriga emas, balki har oylik (yillik) o'sish foiziga nisbatan olinsa, murakkab foizli jamg'arma hosil bo'ladi va u quyidagi formula bilan hisoblanadi([8] ,[9]):

$$\mathbf{K_n=K\ (1+p/100)^n=K\ r^n} \qquad \mathbf{(3)}$$

Bu erda K boshlang'ich pul miqdori, p- ma'lum bir muddatda o'sish foizi, n muddat (oy, yil),K_n n -muddatdan keyingi jamg'arma miqdori, $r=1+p/100$.

Misol 8: 10000 so'm ,20%li murakkab jamg'armada 5 oydan keyin qancha bo'ladi?

Yechish: $K=10000$ so'm, $r=20\%$, $p=5$ (3) formulaga asosan

$K_5=K\ (1+r/100)^5=10000\ (1+20/100)^5=10000x1,2^5=10000x2,48832=24883,2$ so'm bo'ladi.Oddiy foizli jamg'arma hisoblangan misol 5 bilan toqqoslasak,murakkab foizli jamg'armada 4883,2 so'm ko'p bo'lishini ko'ramiz.

Misol 9: Uch oydan keyin 10 % li murakkab jamg'arma miqdori 4200 bo'lgan bo'lsa, boshlang'ich pul qancha bo'lgan?

Yechish: $K_3=4200$ so'm, $r=10\%$, $n=3$, $r=1+r/100=1,1$ natijada (3) formulaga asosan:

$$K=(1+R/100)^n=4200/1,1^3=4200:1,331\approx3155,52 \text{ so'm ekan.}$$

O'rtacha o'sish sur'atini aniqlash.

Misol 10: 4 yillik rejani fermerlar uyushmasi bajarishi uchun etishtirayotgan mahsuloti hajmini 90 foiz oshirishi lozim bo'lsa, o'rtacha yillik o'sish sur'ati qanday bo'lishi kerak?

Yechish: $n=4$, $K_4=1,9K$ bundan $K_4=Kr^4$ bo'lganligi uchun, $r^4=1,9$,

$R=(1,9)^{0,25}\approx1,174$, $r/100+1=1,174$, $r/100=0,174$, $r=17,4\%$.

Demak, o'rtacha yillik mahsulot etishtirish hajmini 17,4% oshirgan taqdirda 4 yillik mahsulot etishtirish hajmini 90% oshishiga erishadi.

$\mathbf{K_p=Kr^p}$ formula yordamida iqtisodiy tahlilda keng qo'llaniladigan o'rtacha o'sish sur'atini aniqlash mumkin.

Qimmatbaho qog'ozlarning baho qiytmatini aniqlash.

Ma'lumki har qanday kelajakda beriladigan puldan, qo'ldagi naqd pul qimmatliroq hisoblanadi. Shu sababli ham $M=(1+i)N$ shaklida kelajakda beriladigan pul miqdori, N-hozirgi vaqtidagi pul miqdori orqali aniqlanadi. Bu erda iqr/100, r qo'yilgan pulni boshlang'ich % sharti.

Bir yildan keyingi pul $N=M/(1+i)$ qiytmatga ega bo'ladi. Masalan: Qimmatbaho qog'ozni o'rtacha o'sish foizi 10% bo'lsin. Bir yildan keyingi qimmati M=1000 so'm bo'ladigan bo'lsa, qimmatbaho qog'ozni bugun N=M/(1+i)=1000/1,1=909,09 so'mga sotib olishi mumkin. Agar qimmatbaho qog'oz 2 yildan keyin 1000 so'm qiytmatga ega bo'ladigan bo'lsa, uni bugungi kunda $N=M/(1+i)^2$=1000/1,12 ≈826,45 so'mga sotib olish mumkin.

Xuddi shunday n yildan keyin M so'm bo'ladigan qimmatbaho qog'ozni bugun $N=M/(1+i)^n$ so'mga sotib oladi. Natijada T yildan keyingi qimmatbaho qog'ozni bahosi

$$\mathbf{N=C_1/(1+i)+C_2/(1+i)^2+...+C_T/(1+i)^T} \qquad \mathbf{(4)}$$

Yuqoridagidan farqli ba'zi qimmatbaho, qog'oz egalari ma'lum bir davr oraligida uning r foiz qiymatini olib turadi .Faraz qilaylik n yildan keyin qiymati to'lanadigan qimmatbaho qog'oz egasi, har yili A so'm ya'ni r foiz tayin bir miqdorda pul olib tursa, hozirgi vaqtdagi qog'ozni qiytmati quyidagi formula bilan aniqlanadi:

$$\mathbf{N = A/(1+ i)+ A/(1+ i)^2 +.. + A/(1+ i)^n + M/(1+ i)^n} \qquad \mathbf{(5)}$$

Ìèñîë:11 r=10%, p=3, xar yili A=20 so'm to'lab borilsa, 400 so'mli qimmatbaho qog'ozni bugungi bahosi quyidagicha bo'ladi

$N=20/1,1+20/1,1^2+20/1,1^3+400/1,1^3$=18,8+16,53+15,02+300,53=350,26 so'm.

Xo'jalikning uzoq muddatli qarzini to'lash

Fermer xo'jaligi davlat bankidan ma'lumbir muddatga qarz olganida, qarzini yiliga qancha miqdordan to'lab borishligini bilishi lozim bo'ladi. Olingan qarzni kelishilgan ulushini, har yilni oxirida to'lab boriladi.

Faraz qilaylik fermer xo'jalikgining qarzi K so'm bo'lib, uni n yil muddatga, har yili R so'mdan to'lab borish shartida olgan bo'lsin. Birinchi qarz to'lov RVso'mni yil to'lishi bilan beradi, ikkinchi yil RV^2 so'm, nihoyat n-chi yilga RV^n so'm to'laydi. Bu erda V=1/r=1/(1+ p/100). Qarzini to'liq to'lashi uchun quyidagi tenglik bajarilishi kerak.

$$\mathbf{K =RV+RV^2+...+RV^n=R(V+V^2+...+V^n)} \qquad \mathbf{(6)}$$

(V<1 bo'lganligidan) geometrik progressiyaning hadlar yig'indisini topish formulasiga asosan $K=R((V(V^n-1)/(V-1))$. Bundan

$$\mathbf{R=K(1-V)/(V(1-V^n))} \qquad \mathbf{(7)}$$

Qarzdan qutilish uchun har yili to'lab borilishi lozim bo'lgan badal miqdori (7) bilan topiladi.

Misol 12. Agar K =1000000 so'm qarz, 4 yil muddatga r=25% shartda olingan bo'lsa, qarzdan qutilish uchun yillik to'lov badalini toping.

Yechish. K =1000000 so'm, (r/100) =0.25 , p=4, V =(1/(1+p) =(1/1.25) =0.8

(7)formulaga asosan $R =(K(1-V)/V(1-V^4)$ to'lov badali

$R =(1000000(1-0.8)/(0.85(1-(0.85)^4))$ =(200000)/(0.5904) =338753.38 so'm ekanligi kelib chiqadi.

Chegirib qolinadigan mablag'ni (amartizasiya fondini) hisoblash

Chegirilib qolinadigan mablag'ni hisoblashda chiziqli usul.

Aytaylik ishlab chiqarish vositalarining boshlang'ich bahosi K_0, uni n vaqt (yil yoki oy) o'tishi bilan ishlab chiqarish vositalarining eskirishi natijasida bahosi K_p so'm bo'lsin. B-bir yillik o'rtacha chegirib qolinadigan pul miqdori B=**(K_0-**

$К_р$)/n bo'ladi. Bundan i-chi yilning oxirida ishlab chiqarish vositalarining bahosi quyidagicha bo'ladi: **$К_i$=$К_0$-iB** **(8)**

Natijada chegirib qolinadigan mablag' miqdori quyidagi formula bilan hisoblanadi **S_i=iB** **(9)**

Misol 13. Faraz qilaylik $К_0$=1000000 so'm ,n=4, $К_4$=250000 so'm, r=5 foiz. Topish talab qilingan $К_i$ i-chi yil oxiridagi ishlab chiqarish vositasining bahosi, S_i i-chi yil uchun chegirib qolinadigan mablag' miqdorini hisoblash.

Yechish. Chiziqli usul bilan echamiz, unda yuqorida keltirilgan (8),(9) formulalardan foydalanib, quyidagilarga ega bo'lamiz: B=($К_0$-$К_n$)/n =(1000000-25000)/4=187500 so'm yillik amortizasiya uchun chegirish miqdori.

S_1=1*B=187500 so'm, S_2=2*V=375000 so'm, S_3=3*V=562500so'm, S_4=4*V=750000 so'm.

Xuddi shunday (8) formulaga binoan ishlab chiqarish vositalarining baholari i-chi yil oxirida quyidagicha bo'ladi:

$К_1$=$К_0$-1*B=812500 so'm , $К_2$=$К_0$-2*B=625000 so'm

$К_3$=$К_0$-3*B=437500 so'm $К_4$=$К_0$-4*B=250000 so'm

Bu mavzuda keltirilgan formulalardan foydalanib, qishloq xo'jalik korxonasini uzoq muddatli qarzini yillik to'lash badalini, texnika vositalarining amartizasiya fondlarini bevosita hisoblash mumkin.

2. Qishloq xo'jalik ekinlarining turli ekish sxemalari bo'yicha ko'chatlar sonini va ma'lum bir ko'chatlar uchun zarur bo'lgan yer maydonini aniqlash.

Qishloq xo'jalik ekinlarini ekish sxemalari bo'yicha ko'chat miqdorini yoki ko'chat miqdori ma'lum bo'lganda shu ko'chatlarni ekish uchun zarur bo'ladigan yer maydoni aniqlash masalalarini qisqacha o'rganamiz.

Paxta asosan er sharoitiga qarab turli sxemada ekiladi, shulardan ikki holni qaraymiz: a)90 x 10 b)60 x15 Masalan : a)90 x 10= 900 sm^2 10 ta go'zaning oziqlanish maydoni bo'lsa,

1ga =10000m^2=100000000 sm^2 = 10^8 sm^2 bo'lganligidan 1 ga maydonda o'rtacha **100000000: 900 sm^2 =111111** dona g'o'za bo'ladi ya'ni taxminan ≈ 111100 dona

b) Xuddi shunday usul bilan

a x b sxemada paxta ekilsa 100000000: (аб)dan o'rtacha 1 ga maydonda qancha g'o'za ko'chati bo'lishi aniqlanadi.

s)**Piyoz,sabzi asosan** {(50+10+10)x8}:3 , { (50+10+10)x7}:3 sxemada ekiladi. 1 ga =100 mln sm^2 bo'lganligidan 70 x 8 =560 sm^2 erda 3 dona piyoz bo'lganligidan 1dona piyoz uchun 560:3=186.7 sm^2 oziqlanish maydoni kerak bo'ladi. Demak, 1ga erda o'rtacha **100mln sm^2: 186,7 sm^2 =534759 ≈534700** dona piyoz bo'ladi.

Sabzini ekish sxemasi uchun {(50+10+10)x7}:3=490:3=163,3 sm^2 har bir dona sabzini oziqlanish maydoni bo'ladi. Bu sxemada

100 mln sm^2: 163,3≈612369≈612300 sabzi 1 ga erda o'rtacha bo'ladi.

d)**Bodiring** qo'sh qatorlab ekilganda {(110+70)x40/2}=3600 sm^2 **100 mln sm^2 :3600=27778 dona≈28000** bodiring ko'chati 1 ga maydonda bo'ladi.

{(110+70)x30/2}=90 x30=2700 sm ^{2} ,**100mln sm^2 :2700 sm^2 =37037≈37000** dona bodiring bo'ladi.

e)**Pomidorni** {(110+30)x30/2} ekish sxemasi bo'yicha har bir tupi uchun 2100 sm^2 oziqlanish maydoni kerak bo'ladi, 1 ga maydonda o'rtacha 47620 dona pomidor ko'chati bo'ladi.

3. Mavzuga doir namunaviy misollarni yechimi.

Mavzu nihoyasida er maydoni berilganda shu maydonga ekish mumkin bo'lgan ko'chatlar sonini va ko'chatlar soni ma'lum bo'lganda shu ko'chat uchun zarur bo'lgan er maydonini topishga misollar keltiramiz.

Ma'lumki **10000 m^2=1ga**; Agar juyakni eni 60 sm bo'lsa, 1 ga maydon 10000:60*100 =16666,67 pag.metr bo'ladi. Agar 1 ga maydonda 120000 g'o'za bo'lishi lozim bo'lsa, 1pag.metrda qancha g'o'za bo'lishi kerak? 120000:16667=7 g'o'za bo'lishi kerak, demak 10 pag. metrda o'rtacha 70 ta g'o'za bo'ladi. 1 dona g'o'za uchun qancha pagon santimetr kerak, 100:7,2=13,9 sm≈14 sm

Misol -1. 2400m^2 maydonga necha dona a) pomidor, b)piyoz, s) sabzi, d) kartoshka ekish mumkin?

Yechish: Yuqorida keltirilgan 1 ga maydonga ekish sxemasiga asosan 2400 m^2=0.24 ga ekanligini inobatga olsak

a) 0,24*27777≈6666 dona pomidor. b) 0,24*534700=128328 dona piyoz; s) 0,24*612300=146952 dona sabzi; d) 0,24*47620≈11429 dona kartoshka ekish mumkin.

Misol -2: Har biridan 500 tup bo'lgan a) kartoshka b) pomidor, s) piyoz ko'chatlari uchun qancha er maydoni kerak bo'ladi?

Yechish: 1 dona piyoz uchun(50+10+10)x8 sxemada 70 sm^2 , sabziga (50+10+10)x7 sxemada 70 sm^2, 1 dona kartoshkaga 2100 sm^2 , 1 dona pamidorga 1350 sm^2 bo'lganligidan:

Kartoshka uchun: 0,21 m^{2}x500=105 m^2=1,05 ar(soto'x),

Pamidor uchun: 500x1350=67,5m^2=0,675 ar(soto'x) ,

Piyozga uchun: 500x70=35000sm^2=3,5m2=0,035 ar(soto'x) er maydoni kerak bo'ladi.

Agar bizga bir tupdan olinadigan o'rtacha hosildorlik miqdori berilsa, bevosita 1 ga maydon hosildorligini, daromadini topishimiz mumkin, masalan(taxminiy bahosi):

N	Ekinni turi	1ga maydonda-gi o'rtacha ko'chatlar soni	har bir tupdan olinadigan hosil	1ga maydondan olinadigan hosil (s.)	1kg maxsulotni o'rta-cha baho-si (so'm)	1ga.erdan qilina-digan daromad (so'm)
1	Pomidor	27777	1 kg	277.8	200	5556000
2	Bodiring	27777	0.8 kg	222.2	200	4444000
3	Piyoz	612300	0.08kg	489,8	150	7347000
4	Sabzi	534700	0.04kg	213,8	200	4276000
5	Paxta	111000	0.3 kg	33.0	400	1320000

Albatta har bir ekinni naviga, ekish sxemasiga qarab turli miqdorda hosil olinadi, shu sababli bir tupidan olinadigan hosildorlik ma'lum bo'lsa yuqoridagidek istalgan maydon uchun o'rtacha hosildorlikni, olinadigan daromadni hisoblab topish mumkin.

4. Mustaqil yechish uchun misol va masalalar.

1. Paxtani 90*10 ekish sxemasida 1ga erga o'rtacha 111000 tup g'o'za kuchati bo'lsa, 560 m^2 erda qancha g'o'za ko'chati bo'ladi?
2. Sutni yogliligi 3,8% bo'lsa 2300 kg. sutdan necha kg yog olish mumkin?
3. Uzum quritilganda 25 % mayiz chiqsa, 3 t mayiz olish uchun qancha uzum kerak bo'ladi?
4. Paxtadan 32% tola chiqsa, 140t tola olish uchun qancha tonna paxta kerak bo'ladi?
5. Xo'jalik paxta topshirish rejasidagi 120 t o'rniga, 124 t paxta topshirgan. Xo'jalik rejani necha foiz ortiq bajargan?
6. Yog'liligi 3,5% li 30 litr sut bilan , yog'liligi 4,3% bo'lgan 50 litr sut aralashatirilganida xosil bo'lgan sutni yog'liligi necha foiz bo'ladi?
7. 18000 so'm omonat kassaga yiliga murakkab 60% li o'sish shartida qo'yilgan. 3 yildan keyin jamg'arma miqdori necha so'm bo'lad?
8. 25000 so'm pul omonat kassasiga yiliga oddiy 40% li jamg'armaga qo'yilagan. 3 yildan keyin jamg'arma miqdori necha so'm bo'ladi?
9. 30000 so'm omonat kassasiga yiliga 45% li murakkab jamg'armaga qo'yilgan, 2 yildan keyin qancha bo'ladi?
10. Ombordagi 24 t karamning 30% sotuvga chiqarildi, 35 % tuzlandi. Omborda qancha karam qoldi?
11. Xo'jalik topshirgan 18 t pillaning 90%ni birinchi navga topshirgan bo'lsa, qancha pilla birinchi navga topshirishlgan?
12. Xo'jalik 340 t sabzavot o'rniga 374 t topshirdi. Reja necha foiz oshirib bajarilgan?
13. 2680 ga erni 3.5 % ga beda ekish kerak. Beda necha ga erga ekiladi?
14. 125 kg pomidorni 5 kg mi sifatsiz. Sifatsiz pomidor necha % ?
15. 0.25 ga/ni 125 m^2 necha % ni tashkil qiladi?
16. Agar 1 ga erga (110+30)x30/2 sxemada o'rtacha 28000 dona pomidor ko'chati ekilsa, 2250 m^2 yerga qancha pomidor ko'chati ekiladi?
17. Agar 1 ga erga (110+30)x30/2 sxemada o'rtacha 28000 dona pomidor ko'chati ekilsa 25516 tup pomidor ko'chati uchun necha m^2 er kerak bo'ladi?
18. Paxtani 90*10 ekish sxemasida 1 ga maydonda 111000 tup g'o'za ko'chati bo'lsa 22560 m^2 erda qancha tup g'o'za ko'chati bo'ladi?
19. 58400 m^2 erga (110+70)x30/2 ekish sxemalari bo'yicha qancha kartoshka ko'chati kerak bo'ladi?
20. 40000 tup bodiringga (110+70)x30/2 ekish sxemalari bo'yicha necha ga maydon ajratish kerak?
21. Agar 1 ga erga o'rtacha 512000 sabzi ekilsa, 250 m^2 qancha sabzi ekiladi?
22. {(50+10+10)x8}:3 ekish sxemasida 1 ga maydonda 612300 dona piyoz bo'lsa, 2540 m^2 er maydoniga o'rtacha qancha piyoz ko'chati kerak?

23. Agar 1 ga erga o'rtacha 512000 sabzi ekilsa, 13500 dona sabzi uchun qancha m^2 er kerak?

24.(110+70)x30/2 ekish sxemasi bo'yicha 1ga maydonda 28000 dona bodring bo'lsa, 7400 tup bodiringga qancha ga maydon ajratish kerak?

5. Chiziqli algebra elementlari. Determinantlarni hisoblash qoidalari. Chiziqli tenglamalar sistemasini Кramer, Gauss usullar bilan yechish.

Ikkinchi va uchinchi tartibli determinantlar quyidagicha

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Bu erdagi a_{ij} (i=1,2,3, j=1,2,3) sonlardan iborat bo'lib, determinantning elementlaridir. Uchinchi tartibli determinantda 3 ta satr va 3 ta ustun bo'lib, jami 9 ta elementdan iborat bo'ladi.

Uchinchi tartibli determinantlarni hisoblashda uchburchak, Sarrius, yoyib yozish usullari mavjud.

1) **Uchburchak qoidasi.** (3) uchinchi tartibli determinantlar uchburchak qoidasi bo'yicha quyidagi hisoblanadi.

$$\Delta q \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} =a_1b_2c_3+b_1c_2a_3+a_2b_3c_1-a_3b_2c_1-a_1b_3c_2-a_2b_1c_3$$

2) **Sarrius usuli**

$$\Delta q \left| \begin{array}{ccc|cc} a_1 & b_1 & c_1 & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \\ a_3 & b_3 & c_3 & a_3 & b_3 \end{array} \right| =a_1b_2c_3+b_1c_2a_3+a_2b_3c_1-a_3b_2c_1-a_1b_3c_2- a_2b_1c_3$$

3) **Yoyib yozish usuli.** Determinantni birinchi satr elementlari bo'yicha yoysak quyidagicha bo'ladi.

$$\Delta=a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} -b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} +c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

n ta satr va n ta ustundan tashkil topgan quyidagi determinantga n tartibli determinant deyiladi.

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \end{vmatrix}$$

$a_{n1} \quad a_{n2} \quad \dots \quad a_{nn}$

Determinantlarning asosiy xossalari:

1. Determinantda satrlarini mos ustunlari bilan almashtirib yozilsa, determinantning qiymati o'zgarmaydi.

2. Determinantni istalgan ikki satrini (ustuni) o'zaro almashtirilsa, determinantning faqat ishorasi o'zgaradi.

3. Determinantda biror ustun (satr) elementlari boshqa ustun (satr)ning mos elementlariga proporsional bo'lsa, bunday determinantni qiymati nolga teng bo'ladi.

4 Determinantda biror ustun (yoki satr)ning hamma elementlari m umumiy ko'paytuvchilarga ega bo'lsa, m-ni ko'paytuvchi qilib, determinantning belgisidan tashqariga chiqarish mumkin.

5. Determinantning biror ustun (satr)ni hamma elementlarini $m(m\neq 0)$ soniga ko'paytirib, boshqa ustun (satr)gaga qo'shilsa, determinantning qiymati o'zgarmaydi.

Minorlar va algebraik to'ldiruvchilar. n-chi tartibli determinantning ixtiyoriy r-ta satr va r-ta ustunlaridan (1<r<n)tuzilgan M determinantga r-chi tartibli minor deyiladi. A_{ke}–elementga mos algebraik to'ldiruvchi quyidagicha aniqlanadi $A_{kl}=(-1)^{k+l} M_{kl}$.

Chiziqli tenglamalar sistemasini Kramer usuli bilan echish.

Uchta noma'lumli chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_1x+b_1y+c_1z=d_1 \\ a_2x+b_2y+c_2z=d_2 \\ a_3x+b_3y+c_3z=d_3 \end{cases}$$

Asosiy va yordamchi determinantlarni tuzamiz:

$$\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \Delta_x=\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \quad \Delta_{y=}\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \quad \Delta_{z=}\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Agar $\Delta\neq 0$ *bo'lsa, sistema yagona* $x=\Delta_x/\Delta$, $y=\Delta_y/\Delta$, $z=\Delta_z/\Delta$ *yechimlarga ega bo'ladi.*

Gauss usuli

Chiziqli tenglamalar sistemasini $n \geq 4$ bo'lganda Kramer usuli bilan yechish qiyinlashadi. Gaussni ketma – ket noma'lumni yo'qotish usuli bilan engilroq echish mumkin.

6.Mavzuga doir namunaviy misollarni yechimi.

Misol-1. Ushbu determinantni yoyib yozish usuli bilan hisoblang.

$$\Delta=\begin{vmatrix} 4 & -2 & 4 \\ 10 & 2 & 12 \\ 1 & 2 & 2 \end{vmatrix} =4\begin{vmatrix} 2 & 12 \\ 2 & 2 \end{vmatrix} \quad -2*(-2)\begin{vmatrix} -10 & 12 \\ 1 & 2 \end{vmatrix}$$

$$+4\begin{vmatrix} 10 & 2 \\ 1 & 2 \end{vmatrix} = 4(4-24)+2(20-12)+4(20-2)=8$$

Misol-2. Chiziqli tenglamalar sistemani Kramer usuli bilan yeching:

$$\begin{cases} 2x-3u=5 \\ 3x+2u-4z=-7 \\ x-6y-2z=3 \end{cases}$$

Yechish: Sistemaning asosiy Δ determinantini hisoblaymiz. Masalan, yoyib yozish usuliga asosan (Laplas teoremasi)

$$\Delta = \begin{vmatrix} 2 & -3 & 0 \\ 3 & 2 & -4 \\ 1 & -6 & -2 \end{vmatrix} = 2\begin{vmatrix} 2 & -4 \\ -6 & -2 \end{vmatrix} + 3\begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} + 0\begin{vmatrix} 3 & 2 \\ 1 & -6 \end{vmatrix} = 2(-28) + 3(-2) + 0 = -62 \neq 0$$

Demak berilgan tenglamalar sistemasi yagona echimga ega. Endi Δ_x, Δ_y, Δ_z determinantlarni hisoblamiz:

$$\Delta_x = \begin{vmatrix} 5 & -3 & 0 \\ -7 & 2 & -4 \\ 3 & -6 & -2 \end{vmatrix} = -62 \quad , \Delta_y = \begin{vmatrix} 2 & 5 & 0 \\ 3 & -7 & -4 \\ 1 & 3 & -2 \end{vmatrix} = 62$$

$$\Delta_z = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -7 \\ 1 & -6 & 3 \end{vmatrix} = -124$$

Kramer formulasiga asosan sistemani yechimi $x=1$, $y=-1$, $z=2$ bo'ladi.

Quyidagi tenglamalar sistemasini Gauss usuli bilan yeching.

Misol-3. $$\begin{cases} x_1 + x_2 - 3x_3 - 5x_4 = 0 \\ -x_1 + 2x_2 + x_3 + x_4 = 1 \\ 2x_1 - x_2 - x_3 + x_4 = 3 \\ x_1 + 3x_2 + 4x_3 - x_4 = -1 \end{cases}$$

Birinchi tenglamani mos ravishda 1 ga, -2 ga, -1ga ko'paytirib ikkinchi uchinchi, to'rtinchi tenglamalarga qo'shamiz.

$$\begin{cases} x_1 + x_2 - 3x_3 - 5x_4 = 0 \\ 3x_2 - 2x_3 - 4x_4 = 1 \\ -3x_2 + 5x_3 + 11x_4 = 3 \\ 2x_2 + 7x_3 + 4x_4 = -1 \end{cases}$$

Ikkinchi tenglamani uchunchi teglamaga va ikkinchi tenglamani -2 ga to'rtinchi tenglamani 3 ga ko'paytirib qo'shsak

$$\begin{cases} x_1 + x_2 - 3x_3 - 5x_4 = 0 \\ 3x_2 - 2x_3 - 4x_4 = 1 \\ 3x_3 + 7x_4 = 4 \\ 25x_3 + 20x_4 = -5 \end{cases} \qquad \begin{cases} x_1 + x_2 - 3x_3 - 5x_4 = 0 \\ 3x_2 - 2x_3 - 4x_4 = 1 \\ 3x_3 + 7x_4 = 4 \\ 5x_3 + 4x_4 = -1 \end{cases}$$

Uchinchi tenglamani (-5) ga ko'paytirib, to'rtinchi tenglamani 3ga ko'paytirib qo'shsak

$$\begin{cases} x_1 + x_2 - 3x_3 - 5x_4 = 0 \\ 3x_2 - 2x_3 - 4x_4 = 1 \\ 3x_3 + 7x_4 = 4 \\ -23x_4 = -23 \end{cases}$$

oxirgi tenglamadan $x_4 = 1$, 3 tenglamaga qo'ysak $x_3 = -1$, ikkinchi tenglamadan $x_2 = 1$ va birinchi tenglamadan $x_1 = 1$ echimi kelib chiqadi. Demak, chiziqli tenglamalar sistemasini echimi

(1; 1; -1; 1;).

7. Mustaqil yechish uchun misollar.

Quyidagi determinantlarni hisoblang.

1) $\begin{vmatrix} Sin\ \alpha & Cos\ \alpha \\ -Cos\ \alpha & Sin\ \alpha \end{vmatrix}$, 2) $\begin{vmatrix} 1+\sqrt{2} & 2-\sqrt{3} \\ 2+\sqrt{3} & 1-\sqrt{2} \end{vmatrix}$, 3) $\begin{vmatrix} \sqrt{a} & -1 \\ a & \sqrt{a} \end{vmatrix}$,

4) $\begin{vmatrix} x & 2 \\ x+2 & (x-1) \end{vmatrix} = 0$ tenglamani yeching.

5) $\begin{vmatrix} 13547 & 13647 \\ 28423 & 28523 \end{vmatrix}$, 6) $\begin{vmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{vmatrix}$, 7) $\begin{vmatrix} -x & 1 & x \\ 0 & -x & 1 \\ x & 1 & -x \end{vmatrix}$,

8) $\begin{vmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 2 & 4 & 6 \end{vmatrix}$, 9) $\begin{vmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{vmatrix}$, 10) $\begin{vmatrix} 1 & 2 & 5 \\ 3 & -4 & 7 \\ -3 & 12 & -15 \end{vmatrix}$,

11) $\begin{vmatrix} 1 & 0 & 0 & \dots 0 \\ 0 & 2 & 0 & \dots 0 \\ 0 & 0 & 3 & \dots 0 \\ \dots & \dots & \dots & \dots . \\ 0 & 0 & 0 & \dots n \end{vmatrix}$, 12) $\begin{vmatrix} 1 & 2 & 3 & \dots n \\ -1 & 0 & 3 & \dots n \\ -1 & -2 & 0 & \dots n \\ \dots & \dots & \dots & \dots . \\ -1 & -2 & -3 & \dots 0 \end{vmatrix}$ 13) $\begin{vmatrix} 1 & 2 & 2 & \dots 2 \\ 2 & 2 & 2 & \dots 2 \\ 2 & 2 & 3 & \dots 2 \\ \dots & \dots & \dots & \dots . \\ 2 & 2 & 2 & \dots n \end{vmatrix}$

14) $\begin{vmatrix} 1 & 2 & 2 & \dots & n-1 & n \\ 1 & 3 & 3 & \dots & n-1 & n \\ 1 & 2 & 5 & \dots & n-1 & n \\ . & . & . & . & . & . \\ 1 & 2 & 3 & \dots & 2n-3 & n \\ 1 & 2 & 3 & \dots & n-1 & 2n-1 \end{vmatrix}$ 15) $\begin{vmatrix} 2 & 1 & 0 & \dots. & 0 \\ 1 & 2 & 1 & \dots. & 0 \\ 0 & 1 & 2 & \dots. & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots. & 2 \end{vmatrix}$

№ 1-20 misollarda chiziqli tenglamalar sistemasini Kramer usuli bilan yeching.

1. $\begin{cases} 2x+3y-7z=8 \\ 3x-y+5z=1 \\ x-4y+6z=-7 \end{cases}$ 2. $\begin{cases} 4x-5y+z=3 \\ x-6y+3z=-4 \\ 3x+2y-z=8 \end{cases}$ 3. $\begin{cases} 7x+2y-4z=-6 \\ 4x-2y-z=-4 \\ -x+y+z=3 \end{cases}$

4. $\begin{cases} 5x-6y+5z=-7 \\ 3x-2y+4z=0 \\ -2x+y-7z=-5 \end{cases}$ 5. $\begin{cases} -3x+5y+6z=9 \\ 5x-y-4z=-3 \\ 4x+3y-z=2 \end{cases}$ 6. $\begin{cases} -4x+8y+7z=-1 \\ x+y-5z=-3 \\ -2x-3y+5z=1 \end{cases}$

7. $\begin{cases} 7x+5y-3z=-4 \\ 3x-6y-5z=-1 \\ -2x+3y+4z=3 \end{cases}$ 8. $\begin{cases} 3x+4y+5z=-1 \\ -7x+2y-6z=3 \\ x-y+z=-2 \end{cases}$ 9. $\begin{cases} 3x-5y+z=0 \\ -2x+2y-3z=1 \\ 2x-2y+4z=-2 \end{cases}$

10. $\begin{cases} 2x+7y+z=0 \\ -2x+2y-3z=1 \\ 2x-2y+4z=-2 \end{cases}$ 11. $\begin{cases} 7x-y+z=-3 \\ -4x-2y-z=-3 \\ x+2y+3z=1 \end{cases}$ 12. $\begin{cases} 5x+2y-3z=10 \\ -x-4y+z=-2 \\ - -2x-y-6z=4 \end{cases}$

13. $\begin{cases} 4x+2y-z=0 \\ 2x-y+z=5 \\ -3x+3y+2z=-2 \end{cases}$ 14. $\begin{cases} x-3y+5z=10 \\ -x-y+z=0 \\ 2x+5y-3z=-4 \end{cases}$ 15. 15 $\begin{cases} 3x-2y+5z=3 \\ x+y-3z=4 \\ -x-2y+3z=-3 \end{cases}$

16 $\begin{cases} .\ 7x+5y-3z=0 \\ 8x+y+3z=-3 \\ 2x+4y-4z=2 \end{cases}$ 17. $\begin{cases} 4x-y+5z=3 \\ 7x-3y-4z=5 \\ x-y+5z=-6 \end{cases}$ 18. $\begin{cases} 3x+2y-z=-6 \\ -x-3y+2z=3 \\ 4x+y+z=-5 \end{cases}$

19 $\begin{cases} x+y-2z=-6 \\ 2x+4y-z=-1 \\ 3x+5y-2z=-4 \end{cases}$ 20. $\begin{cases} 2x+3y+4z=0 \\ x+y+5z=-4 \\ -x-2y+3z=-6 \end{cases}$

Quyidagi chiziqli tenglamalar sistemalarini Gauss usuli bilan yeching

1) $\begin{cases} x_1+x_2+2x_3+3x_4=1 \\ 3x_1-x_2-x_3-2x_4=-4 \\ 2x_1+3x_2-x_3-x_4=-6 \\ x_1+2x_2+3x_3-x_4=-4 \end{cases}$ 2) $\begin{cases} x_1+2x_2+3x_3-2x_4=6 \\ 2x_1-x_2-2x_3-3x_4=8 \\ 3x_1+2x_2-x_3+2x_4=4 \\ 2x_1-3x_2+2x_3+x_4=-8 \end{cases}$

3) $\begin{cases} x_1+2x_2+3x_3+4x_4=5 \\ 2x_1+x_2+2x_3+3x_4=1 \\ 3x_1+2x_2+x_3+2x_4=1 \\ 4x_1+3x_2+2x_3+x_4=-5 \end{cases}$ 4) $\begin{cases} x_2-3x_3+4x_4=-5 \\ x_1-2x_3+3x_4=-4 \\ 3x_1+2x_2-5x_4=12 \\ 4x_1+3x_2-5x_3=5 \end{cases}$

8.Matrisa. Matrisalar ustida amallar. Teskari matrisani topish. Matrisaning rangi

Ushbu

$$\begin{Vmatrix} a_{11} & a_{12} & \dots a_{1n} \\ a_{21} & a_{22} & \dots a_{2n} \\ a_{m1} & a_{m2} & \dots a_{mn} \end{Vmatrix}$$

ko'rinishda tuzilgan ifodaga matrisa deyiladi. Màòðèöàäà satrlar soni m, ustunlar soni n . Matrisada satrlar soni ustunlar soniga teng bo'lishi shart emas, a_{ij}(i=1, 2,...,m, j=1,2, ..., n) sonlar matrisaning elementlari deyiladi. Agar $m \neq n$ bo'lsa *to'g'ri burchakli matrisa* deyiladi. Aksincha, mqn bo'lsa, kvadrat matrisa deyiladi. Ushbu ko'rinishdagi 1 satrli n ustunli matrisani A= (a_{11} a_{12} ... a_{1n}) *satr matrisa* deyiladi, m satrli va 1 ustunli matrisa *ustun matrisa* deyiladi

$$B = \begin{Vmatrix} B_{11} \\ B_{21} \\ B_{m1} \end{Vmatrix}, \quad \textit{birlik matrisa}: \quad E = \begin{Vmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ . & . & . & . \\ 0 & 0 & \dots & 1 \end{Vmatrix}$$

A matrisani determinantini │ A │ bilan belgilaymiz.

$$|A| = \begin{vmatrix} a_{11} & a_{12} \dots & a_{1n} \\ a_{21} & a_{22} \dots & a_{2n} \\ a_{n1} & a_{n2} \dots & a_{nn} \end{vmatrix}$$

Agar │A│≠0 bo'lsa, u holda A matrisani *maxsusmas matrisa* deyiladi. Aksincha, │ A│=0 bo'lsa, A *maxsus matrisa* deyiladi.

A matrisadagi satrlarni mos ustunlar qilib yozish natijasida hosil bo'lgan matrisani $A^{ò}$ *transponirlangan matrisa* deyiladi

m satrli va n ustunli ikkita A va B matrisalardan birining hamma elementlari ikkinchisining mos elementlariga teng $a_{ij}=b_{ij}$ bo'lsa, bu matrisalar teng bo'ladi. A=B .

Matrisalar ustida amallar.

a) Matrisani songa ko'paytirish m soniga ko'paytirish uchun uning hamma elementlarini shu m songa ko'paytirish kerak, ya'ni A=(a_{ij}) bo'lsa, mA=(ma_{ij}) bo'ladi.

b) Matrisalarni qo'shish va ayirish. m satrli va n ustunli ikkita (bir xil nxm tartibli) A va B matrisalarni qo'shish (ayirish) uchun ularning mos elementlarini qo'shish (ayirish) kerak A=(a_{ij}) va B=(b_{ij}) bo'lsa, A+B=S S=(s_{ij}), bu erda $s_{ij}=a_{ij}+b_{ij}$ (i= $\overline{1}$,m; j=$\overline{1}$,n).

v) Matrisani matrisaga ko'paytirish. A matrisani B matrisaga ko'paytmasi AB yana matrisa bo'lib, uning elementlarini hosil qilish uchun A matrisaning i-chi satr elementlarini B matrisaning j-chi ustunning mos elementlariga ko'paytirib, natijalarni qo'shish kerak ya'ni $s_{ij}=a_{ij}b_{ij}+a_{i2}b_{2i}+...+a_{in}b_{nj}=\sum a_{ik}b_{kj}$. A matrisani B matrisaga ko'paytirish uchun A matrisadagi ustunlar soni B matrisadagi satrlar soniga teng bo'lishi kerak.

Teskari matrisa tushunchasi. A va X n-tartibli kvadrat matrisalar va E n-tartibli birlik matrisa bo'lsin. Agar Ax=Xa=E tenglik bajarilsa, u holda X matrisani A ga teskari matrisa deyiladi. Teskari matrisa A^{-1} bilan belgilanadi. Har qanday maxsusmas kvadrat matrisa uchun teskari matrisa (yagona) mavjud va u quyidagi formuladan aniqlanadi:

$$A^{-1}=\frac{1}{|A|}\begin{Vmatrix} A_{11} & A_{21} & \dots A_{n1} \\ A_{12} & A_{22} & \dots A_{n2} \\ A_{1n} & A_{2n} & \dots A_{nn} \end{Vmatrix}$$

Bu erda A_{ij} (i,j=1,n) A matrisa elementlarining algebraik to'ldiruvchisi. Xususan, 2-tartibli kvadrat matrisa uchun teskari matrisa ushbu ko'rinishda bo'ladi, ya'ni

$$A^{-1}=\frac{1}{|A|}\begin{Vmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{Vmatrix}$$

3-tartibli kvadrat matrisa uchun esa teskari matrisa ushbu formuladan topiladi.:

$$A^{-1}=\frac{1}{|A|}\begin{Vmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{Vmatrix}$$

bu erda $A_{ij}=(-1)^{i+j}M_{ij}$, a_{ij} elementga mos algebraik to'ldiruvchi.

Matrisaning rangi. A matrisaning noldan farqli minorlaridan

eng yuqorisining tartibiga, matrisaning rangi deyiladi. Matrisaning rangi r(A) bilan belgilanadi.

9. Mavzuga doir namunaviy misollarni yechimi.

Misol-1 Quyidagi matisaga teskari matisani toping

$$A=\begin{Vmatrix} 7 & 2 & 3 \\ 9 & 3 & 4 \\ 5 & 1 & 3 \end{Vmatrix}. \qquad |A|=\begin{vmatrix} 7 & 2 & 3 \\ 9 & 3 & 4 \\ 5 & 1 & 3 \end{vmatrix}=3\neq 0$$ bo'lganligidan maxsusmas matrisa demak, unga teskari matrisa mavjud .

Algebraik to'ldiruvchilarini topamiz:

$A_{11}=(-1)\cdot 5=5$, $A_{12}=(-1)^{1+2}\cdot 7=-7$, $A_{13}=(-1)^{1+3}(-6)=-6$

$A_{21}=(-1)^{2+1}3=-3$, $A_{22}=(-1)^{2+2}6=6$, $A_{23}=(-1)^{2+3}(-3)=3$

$A_{31}=(-1)^{3+1}(-1)=-1$, $A_{32}=(-1)^{3+2}\cdot 1=-1$, $A_{33}=(-1)^{3+3}3=3$

Natijada A matrisaga teskari matrisa quyidagicha bo'ladi:

$$A^{-1}=\frac{1}{3}\begin{Vmatrix} 5 & -3 & -1 \\ -7 & 6 & -1 \\ -6 & 3 & 3 \end{Vmatrix}.$$

Misol-2. Quyidagi matrisani rangini toping.

$$A=\begin{pmatrix}1 & -2 & 1 & 1\\ 1 & -2 & 1 & -1\\ 1 & -2 & 1 & 5\end{pmatrix}$$

A- asosiy matrisani rangini xisoblaymiz buning uchun elementar amallarni bajaramiz

$$A=\begin{pmatrix}1 & -2 & 1 & 1\\ 1 & -2 & 1 & -1\\ 1 & -2 & 1 & 5\end{pmatrix}\to\begin{pmatrix}1 & -2 & 1 & 1\\ 0 & 0 & 0 & 2\\ 0 & 0 & 0 & -4\end{pmatrix}\to\begin{pmatrix}1 & -2 & 1 & 1\\ 0 & 0 & 0 & 2\\ 0 & 0 & 0 & 0\end{pmatrix}.$$

Bu matrisa $M=\begin{vmatrix}1 & 1\\ 0 & 2\end{vmatrix}=2\neq 0$ bo'lganligan $r(A)=2$ kelib chiqadi.

10.Mavzuga doir mustaqil yechish uchun misollar.
Matrisalar ustida amallar. Teskari matrisani topish. Matrisaning rangi.

Matrisalar ustida quyidagi amallarni bajaring:

1) Agar $A=\begin{pmatrix}4 & -3\\ 2 & 1\end{pmatrix}$, $B=\begin{pmatrix}-5 & 4\\ 3 & -4\end{pmatrix}$ bo'lsa $3A+2B$ toping.

2) $A=\begin{pmatrix}5 & 7\\ -1 & 4\end{pmatrix}$, $B=\begin{pmatrix}-1 & 0\\ 3 & 2\end{pmatrix}$, $\begin{matrix}AB=?\\ BA=BA\end{matrix}$ bo'ladimi?

3) $A=\begin{pmatrix}5 & 8 & -4\\ 6 & 9 & -5\\ 4 & 7 & -3\end{pmatrix}$, $B=\begin{pmatrix}3\,2\\ 4\,1\\ 1\,2\end{pmatrix}$, $AB=?$ 4) $A^4=\begin{pmatrix}17 & -6\\ 35 & -12\end{pmatrix}^4=?$

5) Agar $A=\begin{pmatrix}1 & 2\\ 4 & -3\end{pmatrix}$ bo'lsa $4A^2-A=?$ 6) $A^3=\begin{pmatrix}4 & 3 & -3\\ 2 & 3 & -2\\ 4 & 4 & -3\end{pmatrix}^3=?$

7) Agar $A=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$ bo'lsa $4A^n=?$

6) Quyidagi matrisalarni teskari A^{-1} matrisalarini toping:

a) $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ d) $A=\begin{pmatrix}3 & -4 & 5\\ 2 & -3 & 1\\ 3 & -5 & -1\end{pmatrix}$,

b) $A=\begin{pmatrix}Cos\ \alpha & -Sin\ \alpha\\ Sin\ \alpha & Cos\ \alpha\end{pmatrix}$, e) $A=\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{pmatrix}$

s) $A=\begin{pmatrix}2 & 5 & 7\\ 6 & 3 & 4\\ 5 & -2 & -3\end{pmatrix}$, f) $A=\begin{pmatrix}1 & 2 & 1\\ 3 & 1 & 2\\ 5 & -2 & -3\end{pmatrix}$

Quyidagi matrisalarni $r(\mathrm{A})$ rangini toping:

1) $\begin{pmatrix}1 & 2 & 3\\ -1 & 2 & 3\\ 2 & 4 & 6\end{pmatrix}$ 2) $\begin{pmatrix}2 & -2 & 2\\ 1 & 1 & 1\\ 3 & 3 & 3\end{pmatrix}$ 3) $\begin{pmatrix}1 & -2 & 1-1\\ 1 & -2 & 1-1\\ 1 & -2 & 1\ 5\end{pmatrix}$

4) $\begin{pmatrix} 0 & 4 & 10 & 1 \\ 4 & 8 & 18 & 7 \\ 10 & 18 & 40 & 17 \\ 1 & 7 & 17 & 3 \end{pmatrix}$ 5) $\begin{pmatrix} 2 & 1 & 11 & 2 \\ 1 & 0 & 4 & -1 \\ 11 & 4 & 56 & 5 \\ 2 & -1 & 5 & -6 \end{pmatrix}$ 6) $\begin{pmatrix} 2 & 0 & 2 & 0 & 2 \\ 0 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix}$

11. Chiziqli tenglamalar sistemasini matrisaviy usul bilan echish. Кronekker-Кapelli teoremasi.

Bizga uchta noma'lum, uchta chiziqli tenglamalar sistemasini berilgan bo'lsin:

$$\begin{cases} a_{11}x_1+a_{12}x_2+a_{13}x_3=B_1 \\ a_{21}x_1+a_{22}x_2+a_{23}x_3=B_2 \\ a_{31}x_1+a_{32}x_2+a_{33}x_3=B_3 \end{cases} \qquad (1)$$

bu tenglamalar sistemasini matrisaviy tenglama shaklida quyidagicha yozish mumkin: $Ax=B$ (2)

Bu erda

$$A=\begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{3} \end{Vmatrix}, \quad x=\begin{Vmatrix} x_1 \\ x_2 \\ x_3 \end{Vmatrix}, \quad B=\begin{Vmatrix} B_1 \\ B_2 \\ B_3 \end{Vmatrix}$$

agar A maxsusmas matrisa ya'ni uni $\det A\neq 0$ bo'lsa, u holda bu matrisaga teskari A^{-1} matrisa mavjud va u quyidagicha topiladi:

$$A^{-1}=\frac{1}{\det A}\begin{Vmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{Vmatrix}$$

Bu erda $A_{ij}=(-1)^{I+j}M_{ij}$, M_{ij} a_{ij} elementga mos minor u i^{chi} satr j^{chi} ustunni o'chirishdan hosil bo'lgan 2 chi tartibli determinant.

(2) matrisaviy tenglamani ikkala tomonini teskari A^{-1} matrisaga ko'paytirib, tenglamani yechimini topamiz $x=A^{-1}B$

$$X=A^{-1}B=\frac{1}{\det A}\begin{Vmatrix} A_{11}\cdot B_1+A_{21}\cdot B_2+A_{31}B_3 \\ A_{12}B_1+A_{22}B_2+A_{32}B_3 \\ A_{13}B_1+A_{23}B_2+A_{33}B_3 \end{Vmatrix}, \qquad x=\begin{Vmatrix} x_1 \\ x_2 \\ x_3 \end{Vmatrix}$$

tenglikdan matrisaviy tenglamani yechimi kelib chiqadi:

$x_1=1/\det A(A_{11}B_1+A_{21}B_2+A_{31}B_3)$

$x_2=1/\det A(A_{21}B_1+A_{22}B_2+A_{23}B_3)$

$x_3=1/\det A(A_{31}B_1+A_{23}B_2+A_{33}B_3)$

Кronekker-Кapelli teoremasi.

Bizga n noma'lumli m ta tenglamalar sistemasi berilgan bo'lsin

$$\begin{cases} a_{11}x_1+a_{12}x_2+...+a_{1n}x_n=c_1 \\ a_{21}x_1+a_{22}x_2+...a_{2n}x_n=c_2 \qquad (1) \\ a_{m1}x_1+a_{m2}x_2+...a_{mn}x_n=c_m \end{cases}$$

Sistemani asosiy va kengaytirilgan matrisalarini tuzamiz:

$$A = \begin{Vmatrix} a_{11} & a_{12} & \dots a_{1n} \\ a_{21} & a_{22} & \dots a_{2n} \\ a_{m1} & a_{m2} & \dots a_{mn} \end{Vmatrix} \qquad B = \begin{Vmatrix} a_{11} & a_{12} & \dots a_{1n} & c_1 \\ a_{21} & a_{22} & \dots a_{2n} & c_2 \\ a_{m1} & a_{m2} & \dots a_{mn} & c_в \end{Vmatrix}$$

Teorema. (1) sistema echimga ega bo'lishligi (birgalikda bo'lish) uchun A asosiy matrisa rangini B kengaytirilgan matrisa rangiga teng bo'lishligi zarur va etarlidir. Xususan 1) **r(A)=r(B)=n**, ya'ni noma'lumlar soniga teng bo'lsa, u holda (1) sistema yagona yechimga ega bo'ladi.
2) Agar **r(A)=r(B)<n**, ya'ni noma'lumlar sonidan kichik bo'lsa, u holda (1) sistema cheksiz ko'p echimga ega bo'ladi.3)Agar **r (A) ≠ r (B)** bo'lsa (1) sistema yechimiga ega bo'lmaydi.

Agar (1) tenglamalar sistemasida ozod sonlar $в_1 = в_2 = \ldots = в_m = 0$ bo'lsa, bir jinsli tenglamalar sistemasi hosil bo'ladi va u har doim birgalikda va $x_1 = x_2 = x_3 = \dots = x_n = 0$ trivial yechimga ega bo'ladi. Bir jinsli chiziqli tenglamalar sistemasi noldan farqli yechimga ega bo'lishi uchun $r(A) = k < n$ (n noma'lumlar soni) bo'lishi zarur va etarlidir.

12.Mavzuga doir namunaviy misollarni echimi.

Misol- 1. Tenglamalar sistemasini matrisaviy usul bilan yeching.

$$\begin{cases} x_1-2x_2+x3 \\ 2x_1+3x_2-x_3=8 \\ x_1-x_2+2x_3=-1 \end{cases}$$

$$A = \begin{Vmatrix} 1 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{Vmatrix}, \quad x = \begin{Vmatrix} x_1 \\ x_2 \\ x_3 \end{Vmatrix}, \quad B = \begin{Vmatrix} 1 \\ 8 \\ -1 \end{Vmatrix}$$

A matrisani determinantini hisoblaymiz.

$$\Delta = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 10 \neq 0$$

demak, A maxsusmas matrisa ya'ni unga A^{-1} teskari matrisa mavjud. Buning uchun algebraik to'ldiruvchilarni hisoblaymiz:

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} = 5, \quad A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 5, \quad A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -5,$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} -2 & 1 \\ -1 & 2 \end{vmatrix} = 3, \quad A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1, \quad A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} = -1$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -2 & 1 \\ 3 & -1 \end{vmatrix} = -1, \quad A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = 3, \quad A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix} = 7$$

u holda teskari matrisa

$$A^{-1} = \begin{Vmatrix} 5/10 & 3/10 & -1/10 \\ -5/10 & 1/10 & 3/10 \\ -5/10 & -1/10 & 7/10 \end{Vmatrix} = 1/10 \begin{Vmatrix} 5 & 3 & -1 \\ -5 & 1 & 3 \\ -5 & -1 & 7 \end{Vmatrix}$$

2 ga asosan.

$$X = A^{-1}H = 1/10\begin{Vmatrix} 5 & 3 & -1 \\ -5 & 1 & 3 \\ -5 & -1 & 7 \end{Vmatrix}\begin{Vmatrix} 1 \\ 8 \\ -1 \end{Vmatrix} = \frac{1}{10}\begin{Vmatrix} 30 \\ 0 \\ -20 \end{Vmatrix}\begin{Vmatrix} 3 \\ 0 \\ -2 \end{Vmatrix}$$

demak, sistemaning yechimi $x_1=3$, $x_2=0$, $x_3=-2$.

<u>Misol-2:</u>Ushbu $\begin{cases} 7x_1+2x_2+3x_3=13 \\ 9x_1+3x_2+4x_3=15 \\ 5x_1+x_2+3x_3=14 \end{cases}$ sistemani matrisaviy usulda yechilsin.

<u>Yechish</u>: bu sistemani matrisaviy ko'rinishi $Ax=B$ bo'ladi. Bu erda

$A = \begin{Vmatrix} 7 & 2 & 3 \\ 9 & 3 & 4 \\ 5 & 1 & 3 \end{Vmatrix}$, $X = \begin{Vmatrix} x_1 \\ x_2 \\ x_3 \end{Vmatrix}$, $C = \begin{Vmatrix} 13 \\ 15 \\ 14 \end{Vmatrix}$ va $|A| = \begin{vmatrix} 7 & 2 & 3 \\ 9 & 3 & 4 \\ 5 & 1 & 3 \end{vmatrix} = 3 \neq 0$ bo'lganligidan teskari matrisa mavjud. Algebraik to'ldiruvchilarni topamiz:

$A_{11} = (-1)\cdot 5 = 5$, $A_{12} = (-1)^{1+2}\cdot 7 = -7$, $A_{13} = (-1)^{1+3}(-6) = -6$

$A_{21} = (-1)^{2+1}3 = -3$, $A_{22} = (-1)^{2+2}6 = 6$, $A_{23} = (-1)^{2+3}(-3) = 3$

$A_{31} = (-1)^{3+1}(-1) = -1$, $A_{32} = (-1)^{3+2}\cdot 1 = -1$, $A_{33} = (-1)^{3+3}3 = 3$

A matrisaga teskari matrisa

$A^{-1} = \frac{1}{3}\begin{Vmatrix} 5 & -3 & -1 \\ -7 & 6 & -1 \\ -6 & 3 & 3 \end{Vmatrix}$. Natijada sistemani echimi

$$\begin{Vmatrix} x_1 \\ x_2 \\ x_3 \end{Vmatrix} = \frac{1}{3}\begin{Vmatrix} 5 & -3 & -1 \\ -7 & 6 & -1 \\ -6 & 3 & 3 \end{Vmatrix}\begin{Vmatrix} 13 \\ 15 \\ 14 \end{Vmatrix} = \frac{1}{3}\begin{Vmatrix} 6 \\ -15 \\ 9 \end{Vmatrix} = \begin{Vmatrix} 2 \\ -5 \\ 3 \end{Vmatrix}$$

Bundan tenglamalar sistemasini matrisaviy usul bilan yeching $x_1=2$, $x=-5$, $x_3=3$ kelib chiqadi.

Кronekor – Кapeli teoremasiga namunaviy misolni yechimi.

Misol: Ushbu chiziqli tenglamalar sistemasi birgalikdami? Кronekor – Кapeli teoremasidan foydalanib yeching.

$$\begin{cases} x_1 - 2x_2 + x_3 + x_4 = 1 \\ x_1 - 2x_2 + x_3 - x_4 = -1 \\ x_1 - 2x_2 + x_3 + 5x_4 = 5 \end{cases}$$

Yechish: A- asosiy matrisani rangini xisoblaymiz.

$$A = \begin{pmatrix} 1 & -2 & 1 & 1 \\ 1 & -2 & 1 & -1 \\ 1 & -2 & 1 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -4 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Bu matrisa $M = \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = 2 \neq 0$ bo'lganligan $r(A) = 2$ kelib chiqadi. Кengaytirilgan B matrisani rangi

$$B = \begin{pmatrix} 1 & -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & -1 & -1 \\ 1 & -2 & 1 & 5 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -4 & -4 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},\ r(B) = 2$$

Kronekor- Kapelli teoremaga asosan $r(A)=r(B)=2<4$ bo'lganligidan berilgan chiziqli tenglamalar sistemasi birgalikda va cheksiz ko'p echimga ega bo'ladi. No'malumlardan ikkitasi x_1 va x_2 ixtiyoriy tanlanadi (uchinchi tenglama birinchi ikkita tenglamani chiziqli yigindisidan iborat bo'lganligi uchun uni tashlab yubrish mumkin)

$$\begin{cases} x_3+x_4=1-x_1+2x_2 \\ x_3-x_4=-x_1+2x_2 \end{cases}$$

bu erda x_1, x_2 ixtiyoriy tanlanganidan xususan $x_1=0,\quad x_2=0$ bo'lganida sistemani $\begin{cases} x_3+x_4=1 \\ x_3-x_4=-1 \end{cases}$ yechimi $x_3=0,\quad x_4=1$ kelib chiqadi. (0; 0; 0; 1).

13. Mustaqil yechish uchun misollar.

Quyidagi chiziqli tenglamalar sistemasini matrisaviy usul bilan yeching

1. $\begin{cases} 5x+8y-z=-7, \\ x+2y+3z=1, \\ 2x-3y+2z=9. \end{cases}$

2. $\begin{cases} x+2y+z=4, \\ 3x-5y+3z=1, \\ 2x+7y-z=8 \end{cases}$

3. $\begin{cases} 3x+2y+z=5, \\ 2x+3y+z=1, \\ 2x+y+3z=11. \end{cases}$

4. $\begin{cases} x+2y+4z=31, \\ 5x+y+2z=29, \\ 3x-y+z=10. \end{cases}$

5. $\begin{cases} 4x-3y+2z=9, \\ 2x+5y-3z=4, \\ 5x+6y-2z=18. \end{cases}$

6. $\begin{cases} 2x-y-z=4, \\ 3x+4y+2z=11, \\ 3x-2y+4z=11. \end{cases}$

7. $\begin{cases} x+y+2z=-1, \\ 2x-y+2z=-4, \\ 4x+y+4z=-2. \end{cases}$

8. $\begin{cases} 3x-y=5, \\ -2x+y+z=0, \\ 2x-y+4z=15. \end{cases}$

9. $\begin{cases} 3x-y+z=4, \\ 2x-5y-3z=-17, \\ x+y-z=0. \end{cases}$

10. $\begin{cases} x+y+z=2, \\ 2x-y-6z=-1, \\ 3x-2y=8. \end{cases}$

11. $\begin{cases} 2x+y-z=1, \\ x+y+z=6, \\ 3x-y+z=4. \end{cases}$

12. $\begin{cases} 2x-y-3z=3, \\ 3x+4y-5z=8, \\ 2y+7z=17. \end{cases}$

13. $\begin{cases} x+5y+z=-7, \\ 2x-y-z=0, \\ x-2y-z=2. \end{cases}$

14. $\begin{cases} x-2y+3z=6, \\ 2x+3y-4z=16, \\ 3x-2y-5z=12. \end{cases}$

15. $\begin{cases} 3x + 4y + 2z = 8, \\ 2x - y - 3z = -1, \\ x + 5y + z = 0. \end{cases}$

16. $\begin{cases} 2x - y + 3z = 7, \\ x + 3y - 2z = 0, \\ 2y - z = 2. \end{cases}$

17. $\begin{cases} 2x + y + 4z = 20, \\ 2x - y - 3z = 3, \\ 3x + 4y - 5z = -8. \end{cases}$

18. $\begin{cases} x - y = 4, \\ 2x + 3y + z = 1, \\ 2x + y + 3z = 11. \end{cases}$

19. $\begin{cases} x + 5y - z = 7, \\ 2x - y - z = 4, \\ 3x - 2y - 4z = 11. \end{cases}$

20. $\begin{cases} 11x + 3y - z = 2, \\ 2x + 5y - 5z = 0 \\ x + y + z = 2. \end{cases}$

Quyidagi tenglamalar sistemasini birgalikda yoki birgalikda emasligini Kronekor-Kapelli teoremasi bilan tekshirib yechimlarini toping:

1) $\begin{cases} x_1 + x_2 - x_3 = 4 \\ 2x + x_2 + 3x_3 = 8 \end{cases}$

2) $\begin{cases} 3x_1 + x_2 - x_3 - 2x_4 = -4 \\ x_1 + x_2 - x_3 + 2x_4 = 1 \end{cases}$

3) $\begin{cases} x_1 + x_2 - 2x_3 + x_4 = 1 \\ 2x_1 - x_2 + 2x_3 + 3x_4 = 18 \\ -x_2 + 2x_3 - x_4 = 0 \end{cases}$

4) $\begin{cases} x_1 + x_2 - 3x_3 = -1 \\ 2x_1 + x_2 - 2x_3 = 1 \\ x_1 + x_2 + x_3 = 3 \\ x_1 + 2x_2 - 3x_3 = 1 \end{cases}$

5) $\begin{cases} 2x_1 - x_2 + x_3 = -2 \\ x_1 + 2x_2 + 3x_3 = -1 \\ x_1 - 3x_2 - 2x_3 = 3 \end{cases}$

6) $\begin{cases} x_1 + 2x_2 - 4x_3 = 1 \\ 2x_1 + x_2 - 5x_3 = -1 \\ x_1 - x_2 - x_3 = -2 \end{cases}$

Quyidagi chiziqli bir jinsli tenglamalar sistemasini fundamental yechimlarini toping.

1) $\begin{cases} x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + 9x_2 - 3x_3 = 0 \end{cases}$

2) $\begin{cases} x_1 - 2x_2 - 3x_3 = 0 \\ -2x_1 + 4x_2 + 6x_3 = 0 \end{cases}$

3) $\begin{cases} x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + 9x_2 - 3x_3 = 0 \end{cases}$

4) $\begin{cases} 3x_1 + 2x_2 + x_3 = 0 \\ 2x_1 + 5x_2 + 3x_3 = 0 \\ 3x_1 + 4x_2 + 2x_3 = 0 \end{cases}$

5) $\begin{cases} 3x_1 + x_2 + x_3 = 0 \\ 2x_1 + x_2 + 3x_3 = 0 \\ x_1 + 4x_2 + 2x_3 = 0 \end{cases}$

14. Tekislikdagi analitik geometriya elementlari. Ikki nuqta orasidagi masofa, kesmani berilgan nisbatda bo’lish. To’g’ri chiziq tenglamalari.

Bu mavzuga doir misollar yechish uchun quyidagi formula va tushunchalarni bilishlari zarur.

Tekislikda yotuvchi uchta $A(x_1;y_1)$, $B(x_2;y_2)$, $S(x_3;y_3)$ nuqtalar koordinatalari bilan berilgan bo'lsa. 1)$A(x_1;y_1)$, $B(x_2;y_2)$ ikki nuqta orasidagi d masofani topish formulasi

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \qquad (1)$$

2)AB kesmani λ nisbatda bo'luvchi, xususan kesma o'rtasi E nuqtani koordinatasini topish formulasi:

$$x_E=\frac{x_1+x_2}{2},\ y_E=\frac{y_1+y_2}{2} \qquad (2)$$

3)Agar ikkita to'g'ri chiziq $y=k_1x+b_1$ va $y=k_2x+b_2$ burchak koeffisentli tenglamalari bilan berilgan bo'lsa bu to'g'ri chiziqlar orasidagi burchak

$$tg\,\varphi=\frac{k_2-k_1}{1+k_1k_2} \quad (3) \qquad \kappa_1=-\frac{1}{\kappa_2} \quad (4) \qquad k_1=k_2$$

perpendikulyarlik sharti paralellik sharti

4)Berilgan $A(x_1;y_1)$ nuqtadan o'tib, k yo'nalishi ega bo'lgan to'g'ri chiziq tenglamasi: $y-y_1=k(x-x_1)$ (5)

5)Berilgan ikki $A(x_1;y_1)$, $B(x_2:y_2)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1} \qquad (6)$$

6)Uchlarining koordinatalari A,B va S berilgan bo'lsa, uchburchak yuzasini hisoblash formulasi:

$$S_\Delta=\pm\frac{1}{2}\begin{vmatrix}1&1&1\\x_1&x_2&x_3\\y_1&y_2&y_3\end{vmatrix} \qquad (7)$$

7) Berilgan $M(x_0;y_0)$ nuqtadan $Ax+By+s=0$ to'g'ri chiziqqacha bo'lgan masofani topish formulasi:

$$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}} \qquad (8)$$

15. **Mavzuga doir namunaviy masalani echimi.**

Misol 1. Uchlari A(-7;4) , B(5;-5) va S(3;9) nuqtalarda bo'lgan uchburchakni 1) |AB| tomon uzunligini; 2) AB tomon tenglamasini;
3) S uchidan AB tomonga tushirilgan SD balandlik tenglamasini;
4)|SD| balandlik uzunligini; 5) AE mediana 6) S uchburchak yuzini topib, shaklini chizing.

Yechish: 1) |AB| tomon uzunligini (1) formulaga asosan

$$|AB|=d=\sqrt{(5+7)^2+(-5-4)^2}=\sqrt{225}=15$$

2) AB tomon tenglamasini (6) formulasiga ko'ra

$$\frac{y-4}{-5-4}=\frac{x-(-7)}{5-(-7)};\frac{y-4}{-9}=\frac{x+7}{12}; \qquad 3x+4y+35=0$$

oxirgi tenglamani y -ga nisbatan yechib, uchburchakni AB tomonini burchak koeffisentli tenglamasiga ega bo'lamiz

$4y=-3x-35$, $y=-\frac{3}{4}x-\frac{35}{4}$ bundan burchak koeffisienti $\kappa_{AB}=-\frac{3}{4}$ bo'ladi.

3) SD balandlik AB tomonga perpendikulyarligidan (4) ga asosan, uning burchak koeffisienti k=4/3 bo'ladi. Endi (5) formulaga asosan S(3;9) va k=4/3 larga ko'ra SD balandlik tenglamasi $y-9=\frac{4}{3}(x-3)$, $y-9=\frac{4}{3}x-4$; $y=\frac{4}{3}x+5; 4x-3y+15=0$ bo'ladi.

4)(8) formulaga asosan S(3;9) dan $x_0=3$,. $y_0=9$ hamda Ax+By+c=0 tenglamada A=3, B=4, c=35 larga ko'ra SD balandlik uzunligi

$$d=|CD|=\frac{|3*3+4*9+35|}{\sqrt{3^2+4^2}}=\frac{80}{5}=16$$

5) BS tomon o'rtasini koordinatasi E(x;y) (2) formulaga asosan

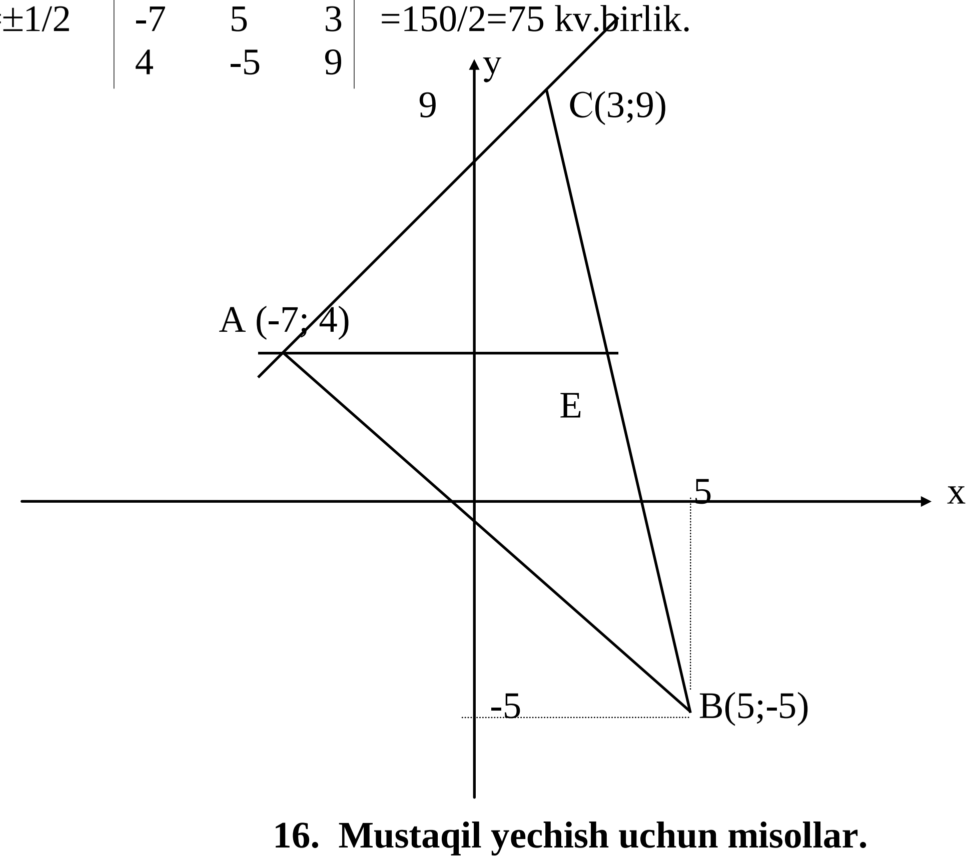
$$x=\frac{x_1+x_2}{2}=\frac{5+3}{2}=4,\ y=\frac{y_1+y_2}{2}=\frac{-5+9}{2}=2$$

E (4;2) kelib chiqadi.

6) Talab qilingan uchburchak yuzasini (7) formulaga asosan topamiz:

$$S_\Delta=\pm 1/2\begin{vmatrix}1 & 1 & 1\\ -7 & 5 & 3\\ 4 & -5 & 9\end{vmatrix}=150/2=75 \text{ kv.birlik.}$$

,



16. Mustaqil yechish uchun misollar.

1-20 .Uchburchakning uchlarini koordinatalari A,B va S berilgan. Topish talab qilingan:

1. BS tomon uzunligini; 2) BS tomon tanlanmasini; 3) A uchidan BS tomoniga tushirilgan AD balandlik tenglamasini; 4) AD balandlik uzunligini; 5)SE medianani $E(x; \quad y)$ koordinatalarini; 6) uchburchakning yuzasi S topilib shakli chizilsin.

1. A (3;6), B) (15;-3), C) (13;11)
2. A (-10;5), B) (2;-4), C) (0;10)
3. A (-4;12), B) (8;3), C) (6;17)
4. A (-3;10), B) (9;1), C) (7;15)
5. A (-7;4), B) (5;-5), C) (3;9)
6. A) (0;3), B) (12;6), C) (10;8)
7. A) (-5;9), B) (7;0), C) (15;14)
8. A) (-8;-3), B) (4;-12), C) (8;10)
9. A) (-5;7), B) (7;-2), C) (11;20)
10. A) (4;1), B) (16;-8), C) (14;6)
11. A) (-7;4), B) (5;-5), C) (3;9)
12. A) (0;3), B) (12;-6), C) (10;8)
33. A) (-5;9), B) (7;0), C) (15;14)
14. A) (4;1), B) (16;-8), C) (14;6)
15. A) (-3;10), B) (9;1), C) (7;15)
16. A) (-4;12), B) (8;3), C) (6;17)
17. A) (10;15), B) (2;-4), C) (0;10)
18. A) (-2;7), B) (10;-2), C) (8;12)
19. A) (-1;4), B) (11;-5), C) (15;17)
20. A) (-6;8), B) (6;-1), C) (4;13)

17. To’g’ri chiziqqa doir mustaqil echish uchun misollar.

1. Kvadratning qarama- qarshi uchlari $B(-2;\ 2)$ va $D(0;-3)$ nuqtalarda. Kvadrat tomonlarining tenglamalarini tuzing.
2. O’tkir burchagi 60^0 bo’lgan ABSD rombda $A(0;\ -1)$ va $C(2;\ 1)$ nuqtalar qarama-qarshi uchlar, AS- kichik diagonal. Romb tomonlarining tenglamalarini yozing.
3. A$(-4;\ 2)$ nuqtalardan o’tib, 1) Ox o’qqa paralel bo’lgan; 2) Ou o’qqa paralel bo’lgan; 3) II va IV koordinat burchaklari bissektrissasiga paralel bo’lgan; 4) $y=\sqrt{3x}-2$ to’gri chiziq bilan 60^0 li burchak hosil qiladigan; 5) $4x+5y-3=0$ to’gri chiziqqa perpendikulyar bo’lgan to’gri chiziqn chizing.
4. $3x-2y+1=0$ va $x+3y-7=0$ to’gri chiziqlarning kesishishi nuqtasidan ularning birinchisiga perpendikulyar to’gri chiziq o’tkazilgan. Hosil qilingan to’gri chiziqdan koordinatalar boshigacha bo’lgan masofa qancha?
5. Uchlari $A(5;\ -2)$, $B(-3;\ -3)$, $C(-1;\ -6)$ bilan berilgan ABS uchburchakning S burchagi bissektrisasi bilan A uchidan chiqqan mediana orasidagi burchakni hisoblang.
6. ABS uchburchakda uning tomonlari o’rtalarining koorinatalari ma’lum: $M_1(-1;\ 5)$, $M_2(3;\ 1)$, $M_3(-5;\ -1)$. Uchburchak tomonlarining va medianalarining tenglamalarini tuzing.
7. To’rtburchak tomonlarining tenglamalari berilgan: $x-2y+2=0$, $x-2y-10=0$, $x-4y-8=0$ va $x-4y+8=0$. Uning yuzini toping.

8. Ushbu $M(4;\ 3)$, $N(5;\ 0)$, $P(-5;\ -6)$ va $Q(-1;\ 0)$ nuqtalar trapesiyaning uchlari ekanligini ko'rsating. Bu trapesiyaning diagonallari orasidagi o'tkir burchakni va trapesiya balangligini toping.

9.Parallelogrammda ikkita tomon tenglamalari $7x-24y+13=0$ va $3x-4y+20=0$ hamda diagonallarning kesishgan nuqtasi $(-7;\ 1)$ ma'lum. Parallelogramm balanuzunliklarini aniqlang.

10. Rombning ikkinta tomoni tenglamasi $x+2y-7=0$ va $x+2y-13=0$ hamda diagonal tenglamsi $x-y+2=0$ berilgan. Romb uchlarining koordinatalarini va romb yuzini toping.

11. A$(-3;\ 1)$ va S$(-7;\ -5)$ nuqtalar kvadratning qarama-qarshi uchlaridir. Uning qolgan ikkita uchining koordinatalarini toping.

IKKINChI TARTIBLI EGRI ChIZIQLAR.

Aylana, ellips, giperbola, parabola va ularning kanonik tenglamalari.

18. Aylana

Ta'rif. Markaz deb ataluvchi M (a; b) nuqtadan bir xil uzoqlikda yotuvchi nuqtalarning geometrik o'rniga *aylana* deyiladi.

A(x;u) nuqta aylanaga tegishli ihtiyoriy nuqta bo'lsa, markazi M (a; b) nuqtada va radiusi R ga teng bo'lgan aylananing normal(kanonik) tenglamasi quyidagicha $(x-a)^2+(y-b)^2=R^2$ bo'ladi. Agar aylananing markazi koordinatalar boshida bo'lsa, a=0, b=0 bo'lib, uning tenglamasi $x^2+y^2=R^2$ bo'ladi. Aylana tenglamasidagi qavslarni ochsak $x^2+y^2-2ax-2вy+a^2+в^2-R^2=0$ tenglama hosil bo'ladi.

19. Aylanaga doir namunaviy misolni yechimi.

Misol 1. Markazi $M(2;\ 3)$ nuqtada, radiusi $R=5$ bo'lgan aylana tenglamasini tuzing va $A(5;\ 1)$ nuqta aylanada yotish, yotmasligini tekshiring.

Yechish: Markazi $M(a;\ в)$ nuqtada, radiusi R bo'lgan aylana tenglamsiga $(x-a)^2+(y-в)^2=R$ asosan $(x-2)^2+(y+3)^2=52$

Bu tenglamani quyidagicha ham yozish mumkin

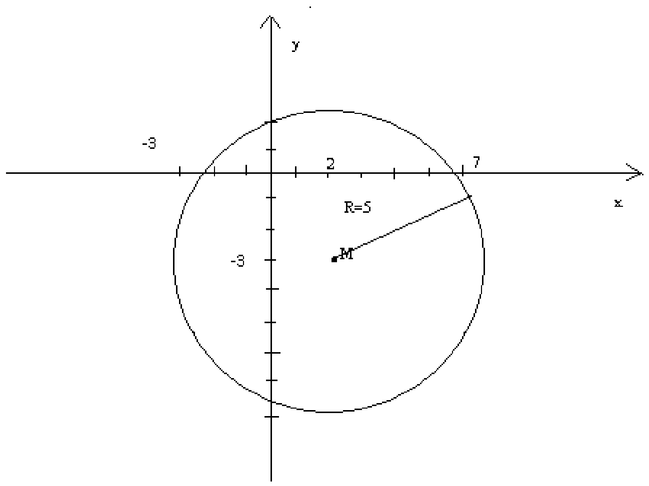
$$x^2-2x+4+y^2+6y+9=25$$

$$x^2-2x+y^2+6y-12=0.$$

$A(5;\ 1)$ nuqtani koordinatalrini aylana tenglamaga qo'yamiz.

$$(5-3)^2+(1+3)^2=3^2+4^2=25$$

Demak, $A(5;\ 1)$ nuqta aylana ustida yotadi.



20. Mustaqil yechish uchun misollar.

1) . Markazi $A(-5;\ 7)$ nuqtada, radiusi 10 ga teng bo'lgan aylana $M(-11;\ 15)$ nuqtadan o'tadimi?

2) . Markazi $B(12;\ -5)$ nuqtada bo'lgan aylana koordinatalar boshidan o'tadi. Bu aylananing tenglamasini tuzing.

3) . Aylana diametrlaridan birining uchlari $M_1(2;\ -7)$ va $M_2(-4;\ 3)$ nuqtalarda yotishi ma'lum. Aylana tenglamasini tuzing.

4) . Diametri $12x+5y+60=0$ to'gri chiziqning koordinata o'qlari orasidagi kesmasidan iborat bo'lgan aylananing tenglamasini tuzing.

5) . Markazi Ox o'qda va $A(6;\ 4\sqrt{2})$ hamda $B(0;\ -2\sqrt{5})$ nuqtalardan o'tuvchi aylananing tenglamasini tuzing.

6) . $A(3;\ -1)$ va $B(-4;\ -8)$ nuqtalardan o'tuvchi aylananing radiusi $r=13$ ga teng. Aylana tenglamasini yozing.

7) . Quyidagi aylanalarning markazi S ning koordinatalarini va radiusi r ni toping:

8) 1) $x^2+y^2-8x+12y-29=0$; 2) $x^2+y^2+16x-20y-5=0$;
3) $x^2+y^2+7y-18=0$; 4) $2x^2+2y^2-12x-7=0$;
5) $4x^2+4y^2+16x-32y-41=0$ 6) $9x^2+9y^2-72x+18y-208=0$

9) 19. $A(3;\ -6)$ nuqtadan o'tuvchi va $x^2+y^2+6x-4y-62=0$ aylana bilan konsentrik bo'lgan aylana tenglamasini tuzing.

10) 20. $x^2+y^2+6x-14y-6=0$ va $x^2+y^2-24x+2y-51=0$ aylanalar markazlari orasidagi masofani toping.

11) 21. $x-y+1=0$ to'gri chiziqning $x^2+y^2-4x+16y-5=0$ aylana bilan kesishish nuqtalarini toping.

12) O'zaro kesishuvchi $x^2+y^2+12x-10y+45=0$ va $x^2+y^2-8x+6y=0$ aylanalarning umumiy vatari tenglamasini tuzing.

21.Ellips.

Ta'rif. Ellips deb, uning ixtiyoriy nuqtasidan F_1 va F_2 fokuslari deb ataluvchi ikki nuqtasigacha bo'lgan masofalar yig'indisi o'zgarmas bo'lgan nuqtalarning to'plamiga aytiladi.

M(x; y) nuqta ellipsga tegishli ixtiyoriy nuqta bo'lsin. F_1 va F_2 fokuslar orasidagi masofa 2s , M(x; y) nuqtadan F_1 va F_2 fokuslargacha bo'lgan masofalar yig'indisi 2a bo'lsa, ellipsning sodda (kanonik) tenglamasi $x^2/a^2+y^2/v^2=1$ bo'ladi, bu erda $b^2=a^2-s^2$, a - ellipsning katta yarim o'qi, b -ellipsning kichik yarim o'qi deyiladi.

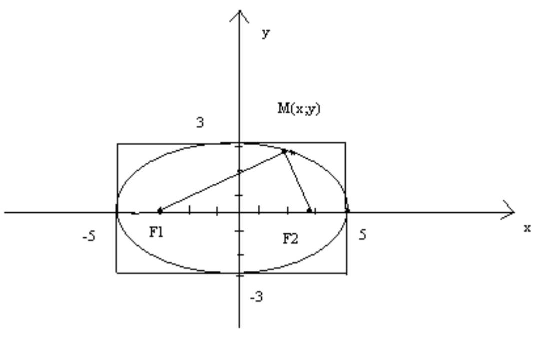
Agar a>b bo'lsa, ellipsning fokuslari Ox o'qida joylashgan bo'lib, uning koordinatalari $F_1(-s; 0)$ va $F_2(s; 0)$ nuqtalarda $s^2 = a^2-b^2$ bo'ladi. Agar a<b bo'lsa, ellipsning fokuslari Ou o'qida joylashgan bo'ladi. Uning koordinatalari $F_1(0; -s)$ va $F_2(0; s)$ nuqtalarda $\mathbf{s^2= b^2-a^2}$ bo'ladi. ε=s/a ellipsning ekssentrisiteti deyiladi. Ellipsning M(x; u) nuqtasidan fokuslargacha bo'lgan masofalar $r=a-Ex$, $r_1=a+Ex$ formulalar bilan aniqlanadi. r va r_1 masofalar fokal radius - vektorlari deyiladi.

22. **Ellipsga doir namunaviy misolni yechimi.**

Misol: $9x^2+25y^2=225$ ellipsni yarim o'qlarini, fokuslarini koordinatalarini, ekssentrisitetini topib shaklini chizing.

Echish. 225ga tenglikni ikkala tomonini bo'lib, ellipsni kanonik tenglamasini hosil qilamiz $\frac{x^2}{25}+\frac{y^2}{9}=1$. Bundan $a=5$, $в=3$, $a>в$ bo'lganligidan ellipsni fokuslari Ox o'qida yotadi, katta yarim o'qi a va $c=\sqrt{a^2-в^2}=\sqrt{25-9}=\sqrt{16}=4$, $F_1(-4;\ \ 0)$, $F_2(4;\ \ 0)$ fokuslarini koordinatalari bo'ladi. Ellipsni ekssentrisiteti $\varepsilon=\frac{2c}{2a}=\frac{4}{5}=0{,}8$,

$|MF_1|+|MF_2|=2a=10$.



23. Ellipsga doir mustaqil echish uchun misollar.

1) . Quyidagi ellipslarning uchlari koordinatalarini, o’qlarini, fokuslarini va ekssentrisitetini toping.

1) $16x^2 + 25y^2 = 400$; 2) $4x^2 + 9y^2 = 36$;

3) $16x^2 + 9y^2 = 144$; 4) $25x^2 + 9y^2 = 900$.

2) . Ellipsning katta yarim o’qi $a = 4$ va ellipsda yotuvchi $M\left(-2;\ \frac{3\sqrt{3}}{2}\right)$ nuqta ma’lum. Ellipsning eng sodda tenglamasini tuzing va M nuqtadan ellips fokusigacha bo’lgan masofani toping.

3) . Kichik yarim o’qi 24 ga teng bo’lgan va fokuslaridan biri $F_2(-5;\ 0)$ koordinatalarga ega bo’lgan ellipsning eng sodda tenglamasini tuzing.

4) . Ellipsning fokuslari orasidagi masofa 30 ga, *Ox* o’qda yotuvchi katta o’qi 34 ga teng. Ellipsning eng sodda tenglamasini tuzing va uning ekssentrisitetini toping.

5) . Yarim o’qlarining yigindisi 36 ga, *Oy* o’qda yotuvchi fokuslari orasidagi masofa 48 ga teng bo’lgan ellisning eng sodda tenglamasini tuzing.

6) . Agar ellipsning fokuslaridan biri $(6;\ 0)$ nuqtada bo’lsa va ekssentrisiteti $e = \frac{2}{3}$ ga teng bo’lsa, uning eng sodda tenglamasini tuzing.

7) . Quyidagi ellipslarning markazini, o’qlarini, uchlarini, fokuslarini va ekssentrisitetini toping.

1) $9x^2 + 18x + 16y^2 - 64y - 71 = 0$

2) $9x^2 + 10y^2 + 40y - 50 = 0$

8) . Katta o’qi 20 ga teng, fokuslari $F_1(-1;\ 0)$ va $F_2(5;\ 0)$ nuqtalarda bo’lgan ellipsning tenglamasini tuzing.

9) . $\frac{x^2}{16} + \frac{y^2}{4} = 1$ ellips va $x^2 - y^2 - 8y = 0$ aylananing umumiy vatari tenglamasini tuzing.

10) 32. $6x^2 + 8y^2 = 120$ ellipsda uning katta o’qidan 3 birlikka teng masofada yotgan nuqtalarni toping.

24.Giperbola.

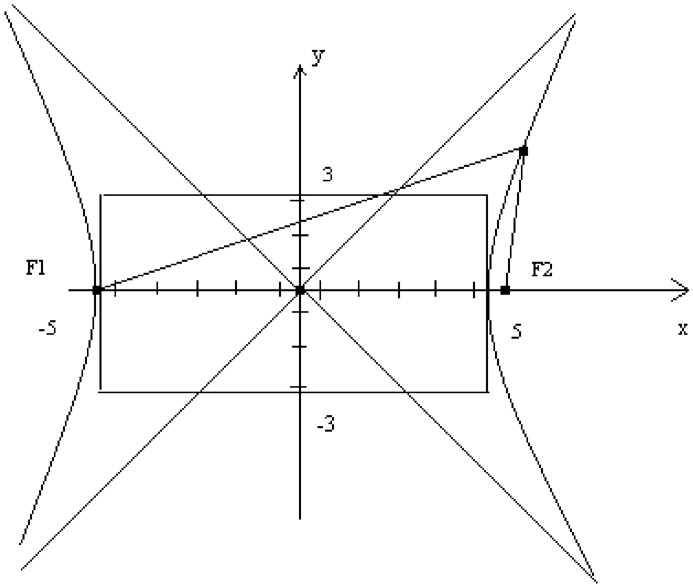
Ta’rif. Giperbola deb uning ixtiyoriy nuqtasidan F_1 va F_2 fokuslari deb ataluvchi nuqtalarigacha bo’lgan masofalar ayirmasi o’zgarmas sondan iborat bo’lgan nuqtalar to’plamiga aytiladi.

Giperbolaning sodda(kanonik) tenglamasi $x^2/a^2-y^2/b^2=1$ bo’ladi, bu erda $b^2=s^2-a^2$, a-ni giperbolaning haqiqiy yarim o’qi; v- giperbolaning mavhum yarim o’qi deyiladi. У=±(b/a)x to’g’ri chiziqlarni giperbolaning asimptotalari deb ataladi. E=2s/2a=s/a>1 nisbatni giperbolaning eksentrisiteti deyiladi.

25.Giperbolaga doir namunaviy misolni echimi.

Misol. $9x^2 - 25y^2 = 225$ giperbolani haqiqiy va mavxum yarim o’qlarini, fokuslarini koordinatalarini, asimptotalarini, ekssentrisitetini topib shaklini chizing.

Yechish: $\frac{x^2}{25}-\frac{y^2}{9}=1$ bo’lganligidan, xaqiqiy yarim o’qi $a=5$, mavxum yarim o’qi $в=3$ bo’ladi. Fokusini koordinatalari $c=\sqrt{a^2+в^2}=\sqrt{25+9}=\sqrt{34}$, $F_1(-\sqrt{34};\ 0)$, $F_2(\sqrt{34};\ 0)$ bo’lib, Ox o’qida yotadi. Asimptotalari $y=\pm\frac{в}{a}x=\pm\frac{3}{5}x$, ekssentrisiteti $\varepsilon=\frac{c}{a}=\frac{\sqrt{34}}{5}\approx\frac{5,83}{5}=1,17>1 \left|MF_2\right|-\left|MF_1\right|=2a=10$.



26. Mustaqil yechish uchun misollar.

1) . Quyidagi giperbolalarning uchlarining koordinatalarini, o’qlarini, fokuslarini va ekssentrisitetini toping.

1) $4x^2-5y^2-100=0$; 2) $9x^2-4y^2-144=0$;

3) $16x^2-9y^2+144=0$; 4) $9x^2-7y^2+252=0$.

2) . Haqiqiy o’qi abssisalar o’qida yotadigan va $M_1(3;\ -2)$, $M_2(-6;\ 2\sqrt{10})$ nuqtalardan o’tadigan giperbolaning eng sodda tenglamasini tuzing. Bu giperbolaning ekssentrisitetini fokuslarining koordinatalarini toping.

3) . Haqiqiy o’qi 6 ga, fokuslari orasidagi masofa 8 ga teng bo’lgan giperbolaning eng sodda tenglamasini tuzing. qo’shma giperbolaning tenglamasini tuzing.

4) . $7x^2-5y^2=35$ giperbola fokuslaridan o’tuvchi va *Ox* o’q bilan 60^0 li burchak tashkil etuvchi to’gri chiziqlarning tenglamalarini tuzing.

5) . $M(-5;\ 2)$ nuqta orqali $9x^2-4y^2=36$ giperbola asimptotalariga parallel bo’lgan to’gri chiziqlar o’tkazing.

6) . Giperbolaning asimptotalari $y=\pm\frac{3}{4}x$ to’gri chiziqlardan iborat, fokuslaridan biri $(-10;\ 0)$ nuqtada joylashgan. Giperbolaning tenglamasini tuzing va uning ekssentrisitetini toping.

7) . $x^2 - 3y = 27$ giperbola asimptotalari va ekssentrisitetlari orasidagi o'tkir burchakni toping.

8) . Ekssentrisiteti $e = 1{,}2$ ga teng degan shartda $\frac{x^2}{64} + \frac{y^2}{28} = 1$ ellips bilan umumiy fokuslarga ega bo'lgan giperbolaning tenglamasini tuzing.

9) . Giperbola $\frac{x^2}{289} + \frac{y^2}{225} = 1$ ellipsning fokuslaridan o'tishi ma'lum, fokuslari esa bu ellipsning uchlarida joylashgan. Giperbolaning tenglamasini tuzing.

10). Giperbola asimptotalarining tenglamalari $y = \pm\frac{3}{4}x$ ko'rinishda, fokuslaridan biri esa $(0;\ 5\sqrt{2})$ nuqtada joylashgan. Giperbolaning tenglamasini tuzing va uning ekssentrisitetini toping.

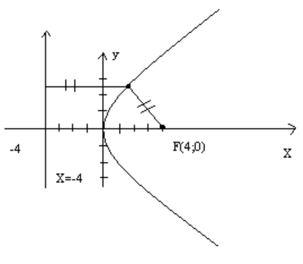
27.**Parabola**

Ta'rif. Parabola deb, uning ixtiyoriy nuqtasidan F fokus deb ataluvchi nuqtasigacha va direktrissa deb ataluvchi to'g'ri chiziqqacha bo'lgan masofalari teng bo'lgan nuqtalar to'plamiga aytiladi. Parabolani sodda tenglamasi $y^2=2rx$ (OX-o'qiga nisbatan simmetrik) , fokusini koordinatasi $F\left(\frac{p}{2};0\right)$. x=-p/2 to'g'ri chiziq direktrissaci.

28. Parabolaga doir namunaviy misolni yechimi.

Misol: $y^2 = 8x$ parabolani fokusini koordinatalarini topib, direktrisasini tenglamasini tuzing.

Yechish: $y^2 = 2px$, $F\left(\frac{1}{2};\quad 0\right)$, $x + \frac{1}{2} = 0$ ga asosan $2p = 8$, $p = 4$, $F(4;\quad 0)$ - fokusini koordinatasi $x + 4 = 0$ diraktrisasini tenglamasini bo'ladi.



29. Mustaqil yechish uchun misollar.

1) . Quyidagi parabolalarning fokusi koordinatalarini toping va direktrisasi tenglamasini yozing.

a) $y^2 = 8x$; v) $x^2 = 10y$;

c) $y^2 = -12x$; d) $x^2 = -16y$.

2) . Uchi koordinatalar boshida va fokusining koordinatalari

a) $F(0,4)$; v) $F(0;\ -3)$; s) $F(6;\ 0)$; d) $F(-2,5;\ 0)$ bo'lgan parabolaning tenglamasini tuzing.

3) . Parabola *Ox* o'qqa nisbatan simmetrik, uning uchi koordinatalar boshida, fokusidan uchigacha bo'lgan masofa 12 ga teng. Parabola tenglamsini yozing.
4) . *Ou* o'qqa nisbatan simmetrik bo'lgan parabolaning uchi koordinatalar boshida bo'lib, $(6;\ -2)$ nuqta orqali o'tadi. Parabola tenglamasini yozing va fokusining koordinatalarini aniqlang.
5) . $y^2 = 36x$ parabolada shunday nuqtalarni topingki, bu nuqtalardan fokusgacha bo'lgan masofa 24 ga teng bo'lsin.
6) . $x^2 = -8y$ parabola fokusi orqali *Ox* o'q bilan 135^o li burchak tashkil etuvchi to'gri chiziq o'tkazilgan. Bu to'gri chiziqning tanglamasini yozing va uning direktrisa bilan kesishish nuqtasini toping.
7) . $y^2 = 48x$ parabolaning fokusi orqali berilgan $y = \sqrt{3}x + 1$ to'gri chiziqqa parallel qilib to'gri chiziq o'tkazilgan. Hosil bo'lgan vatarning uzunligini toping.
8) . $y^2 = 24x$ parabolaning 1) $3x + y - 6 = 0$; 2) $2x - y + 5 = 0$; 3) $y - 6 = 0$ to'gri chiziqlar bilan kesishish nuqtasini toping.

10). $y^2 = 24x$ parabolaning $\frac{x^2}{100} + \frac{y^2}{64} = 1$ ellips bilan kesishish nuqtalarini toping.

11). $y^2 = 36x$ parabola hamda $(x+12)^2 + y^2 = 400$ aylananing umumiy vatari tenglamsini tuzing va zunligini aniqlang.

12). Uchlari koordinatalar boshida, fokuslari, $F_1(-6;\ 0)$ va $F_2(0;\ 6)$ nuqtalarda bo'lgan ikkita parabolaning umumiy vatari uzunligini toping.

30. Ikkinchi tartibli egri chiziqlarga doir aralash masalalar.

1. $x^2 + y^2 + ay = 0$ aylana markazidan $y = 2(a - x)$ to'gri chiziqqacha bo'lgan masofani toping.
2. $9x^2 + 16y^2 = 576$ ellipsga ichki chizilgan kvadrat tomonining uzunligini toping.
3. $x^2 + y^2 + 6x + 2y - 5 = 0$ aylana markazidan $9x^2 + 16y^2 = 144$ giperbola asimptotikasigacha bo'lgan masofani toping.
4. Teng tomoni $x^2 - y^2 = 16$ giperbola ellips fokuslaridan o'tadi. Agar ellips va giperbola ekssentrisitetlarining nisbati $\sqrt{3}$ ga teng bo'lsa, ellipsning eng sodda tenglamasini tuzing.
5. Parabola direktrisasi $9x^2 + 20y^2 = 324$ ellipsni $A(-4;3)$, $B(4;3)$ nuqtalarda kesib o'tadi, bu nuqtalardan parabola fokusgacha bo'lgan masofa $2\sqrt{5}$ teng. Parabola tenglamasini tuzing.
6. Quyidagi tenglamalar qanday egri chiziqlarni ifodalaydi:

a) $x^2 + 2y^2 + 2x - 8y + 5 = 0$, b) $2x^2 + 2y^2 - 3x - 4y - 10 = 0$

s) $3x^2 - 4y - 12x = 0$, d) $3x^2 - 4y^2 - 12x = 0$

e) $3x^2 - 4y^2 - 12x + 24 = 0$

7. To'liq kvadratga ajratib va koordinatalar boshini ko'chirish orqali quyidagi chiziqlarning tenglamalari soddalashtirilsin:

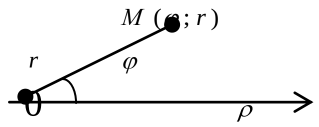
a) $2x^2 + 5y^2 + 12x + 10y + 13 = 0$, b) $x^2 - y^2 + 6x + 4y - 4 = 0$

s) $y^2 + 4y = 2x$ d) $x^2 - 10x = 4y - 13$

8) $9x^2 + 16y^2 = 576$ ellipsga ichki chizilgan kvadrat tomonining uzunligini toping.

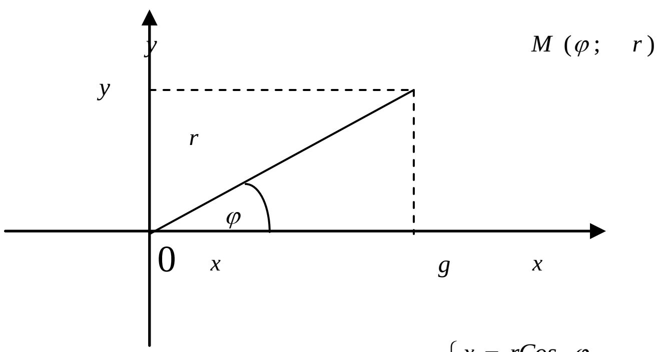
31. Qutb koordinatalar sistemasi.

Tekislikdagi har – bir $M(\varphi; r)$ nuqtani φ - qutb burchagi va qutb radius vektori r orqali bir qiymatli aniqlash mumkin. O nuqta qutb boshi



Agar φ - burchak soat strelkasi yo'nalishi bo'yincha qutb koordinatalar sistemasida olinsa manfiy, soat strelkasiga qarama qarshi yo'nalish bo'yicha olinsa musbat bo'ladi.

Dekart va qutb koordinatalar sistemasi orasidagi boglanishni ko'rsatish uchun ox o'qini qutb o'qi va 0(0;0) qutb boshi qilib olamiz. Natijada trigonometrik funksiyalarni ta'rifiga asosan quyidagi o'rinli bo'ladi:



$Cos\,\varphi = \frac{x}{r}$, $Sin\,\varphi = \frac{y}{r}$ $\begin{cases} x = rCos\,\varphi \\ y = rSin\,\varphi \end{cases}$ yoki $x^2 + y^2 = r^2$

$x^2 + y^2 = r^2$, $r = \sqrt{x^2 + y^2}$ $(r > 0)$, $tg\,\varphi = \frac{y}{x}$.

32. Namunaviy misolni yechimi.

Misol-1. Qutb koordinatalar sistemasida quyidagi chiziqlarni chizing.

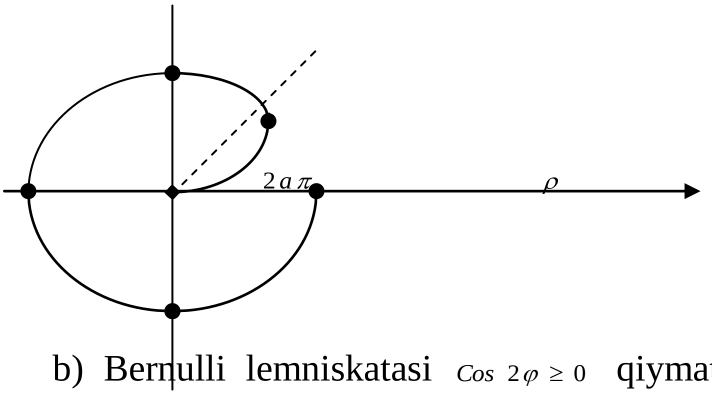
a) $r = a\varphi$ Arximed spirali;

b) $r^2 = a^2 Cos\,2\varphi$ Bernulli lemniskatasi.

Yechish. a) Arximed spiralini yasaymiz

φ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

r	0	$a\frac{\pi}{4}$	$\frac{a\pi}{2}$	$a\pi$	$\frac{3a\pi}{2}$	$2a\pi$



b) Bernulli lemniskatasi $Cos\,2\varphi \ge 0$ qiymatlarida aniqlanganda va $\varphi = 0$, $\varphi = \frac{\pi}{2}$, $\varphi = \pi$, $\varphi = \frac{3\pi}{2}$ eurlarga nisbatan simmetrik bo'ladi, $a = 6$ bo'lsa

φ	0	$\frac{\pi}{12}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$
r	6	$3\sqrt[4]{12}$	$3\sqrt[4]{8}$	3	0

33. Mustaqil yechish uchun misollar

Qutb koordinatalar sistemasida quyidagi chiziqlarni chizing.

1) $r = \frac{a}{\varphi}$, (Giperboloik spiral).
2) $r = a(1 - Cos\,\varphi)$, (Kardoida).
3) $r = a(1 + 2Cos\,\varphi)$, (Paskal chiganogi).
4) $r = aSin\,3\varphi$, (Uch yaproqli gul).
5) $r = aSin\,2\varphi$, (To'rt yaproqli gul).

Ushbu chiziqlarning tenglamalarini Qutb koordinatalar sistemasida yozing ($x = rCos\,\varphi$, $y = rSin\,\varphi$ almashtirish yordamida)

1) $x^2 - y^2 = a^2$, 2) $x^2 + y^2 = a^2$
3) $x^2 + y^2 = ax$, 4) $(x^2 + y^2)^2 = a^2(x^2 - y^2)$
5) $(x^2 + y^2)^2 = 2a^2xy$, 6) $x^4 + y^4 = a^2xy$

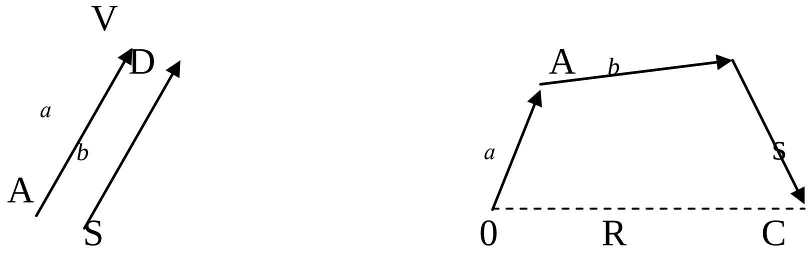
Ushbu chiziqlarni tenglamalarini Dekart koordinatalar sistemasida yozing.($r = \sqrt{x^2 + y^2}$, $tg\,\varphi = \frac{y}{x}$ almashtirish bilan)

7) $rCos\,\varphi = a$, 8) $r = 2aSin\,\varphi$,
9) $r^2 Sin\,2\varphi = 2a^2$, 10) $r = a(1 + Cos\,\varphi)$

34. Vektorlar nazariyasi. Ikki vektorni skalyar, vektorli ko'paytmalari va ularning qo'llanilishi.

Ta'rif. Yo'naltirilgan $\overline{AB}$ kesma vektor deyiladi.

Bunda A nuqta vektorning boshi, B nuqta uning oxiri.Vektor boshi va oxiri ko'rsatilib $\overline{AB}$ ko'rinishda yoki bir xarf bilan $\bar{a}$ belgilanadi. Masalan, tezlik, tezlanish, kuch momenti va boshqalar vektor miqdorlardir. Vektorning koordinatalari deb uning Ox, Ou va Oz koordinata o'qlaridagi proeksiyalari a_x, a_y, a_z ga aytiladi $\vec{a}\{a_x; a_y; a_z\}$ va ular diagonali $\vec{a}$ vektordan iborat bo'lgan paralelpipedning qirralaridir.



Agar $\bar{a}$ vektor *Ox* o'q bilan φ burchak tashkil etsa, vektorning bu o'qdagi proeksiyasi $\text{pr}_x a = |a| \cos \varphi$ bo'ladi. Vektorning uzunligi (moduli) $|\overline{AB}|$ yoki $|a|$ bilan belgilanadi. $|\overline{AB}| = a = \sqrt{a_x^2 + a_y^2 + a_z^2}$.

Bir to'gri chiziqqa paralel vektorlar kollinear vektorlar, bir tekislikka parallel bo'lgan vektorlar komplanar vektorlar deyiladi.

Ikki vektorning yigindisi, ayirmasi va vektorning songa ko'paytmasi uchun quyidagilar o'rinlidir: $(\vec{a} \pm \vec{в})\ \{a_x \pm в_x; a_y \pm в_y; a_z \pm в_z\}$, $\lambda a\{\lambda a_x; \lambda a_y; \lambda a_z\}$ (bu erda λ - skalyar miqdor) .

Ikki vektorning skalyar ko'paytmasi. Ta'rif. Ikki $\vec{a} \cdot \vec{в}$ vektor-ning skalyar ko'paytmasi shu vektorlar modullarining ular orasidagi burchak kosinusi bilan ko'paytamasiga teng

$$\overrightarrow{a \cdot в} = (\vec{a} \cdot \vec{в}) = |\vec{a}| \cdot |\vec{в}| \cos(\vec{a}, \vec{в}) \ , \quad \overrightarrow{a \cdot в} = (\vec{a} \cdot \vec{в}) = |\vec{a}|\, Пр\, |\vec{в}| = |\vec{в}|\, Прв\ \vec{a}$$

Ikki vektorning skalyar ko'paytmasi quyidagicha xossalarga ega.

I $\vec{a} \cdot \vec{в} = \vec{в} \cdot \vec{a}$, **II** $(\vec{a} + \vec{в}) \cdot \vec{c} = \vec{a} \cdot \vec{c} + \vec{в} \cdot \vec{c}$, **III** $(\lambda \vec{a}) \cdot \vec{в} = \lambda(\vec{a} \cdot \vec{в})$

IV Koordinatalari bilan berilgan vektorlarning skalyar ko'paytmasi $\vec{a}\{a_x; a_{y.}; a_z\}$, $\vec{в}\{в_x; в_{y.}; в_z\}$, , $\vec{a} \cdot \vec{в} = a_x в_x + a_y в_y + a_z в_z$, orasidagi burchak va

$$\cos \varphi = \cos(\vec{a}, \vec{в}) = \frac{\vec{a} \cdot \vec{в}}{|a| \cdot |в|} = \frac{a_x в_x + a_y в_y + a_z в_z}{\sqrt{a_x^2 + a_y^2 + a_z^2} \cdot \sqrt{в_x^2 + в_y^2 + в_z^2}}$$. Bundan, paralellik,

perpendikulyarlik shartlari: $\frac{b_x}{a_x} = \frac{b_y}{a_y} = \frac{b_z}{a_z} = m$. $a_x b_x + a_y b_y + a_z b_x = 0$.

Ikki vektorning skalyar ko'paytmasini fizik ma'nosi F kuchni jismga ta'siri natijasida uni S masofaga ko'chirib bajarilgan ishdir. Aq│ $\bar{F}$││ $\bar{S}$│ cosφ.

Ikki vektorning vektor ko'paytmasi. Ta'rif. $\bar{a}$ va $\bar{b}$ vektorlar-ning vektorli $[\bar{a} \times \bar{b}]$ ko'paytmasi deb shunday uchinchi $\bar{c}$ vektorga aytiladiki: 1) Berilgan $\bar{a}$ va $\bar{b}$ vektorlardan yasalgan parallelgramm yuziga teng;2) y parallelogramm

tekisligiga perpendikulyar; Ikki vektorning vektorli ko’paytmasini moduli $[\bar{a} \times \bar{b}] =$
$/\vec{c}/ = |\bar{a}| \cdot |\bar{в}| \sin(\vec{a}, \vec{в})$.

Vektor ko’paytmasining asosiy xossalari. **I** $[\vec{a} \cdot \vec{в}] = -[\vec{в} \cdot \vec{a}]$,
II $[(\vec{a} + \vec{в}) \cdot \vec{c}] = [\vec{a} \cdot \vec{c}] + [\vec{в} \cdot \vec{c}]$. **III** $[(\lambda \vec{a}) \cdot \vec{в}] = \lambda [\vec{a} \cdot \vec{в}]$.

Agar $\vec{a}\{a_x; a_{y.}; a_z\}$, $\vec{в}\{в_x; в_{y.}; в_z\}$, bo’lsa, ularning vektorli ko’paytmasi

$$[\vec{a}\vec{в}] = \vec{a}\{a_{y.}в_z - a_z в_z\}\vec{i} - \{a_x в_{y.} - a_z в_x\}\vec{j} + \{в_x в_{y.} - a_y в_x\}\vec{k} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ в_x & в_y & в_z \end{vmatrix}.$$

Moduli $[\vec{a}\vec{в}] = \sqrt{(a_y в_z - a_z в_y)^2 + (a_x в_z - a_z в_x)^2 + (a_x в_y - a_y в_x)^2}$

Vektorli ko’paytma $a\{a_x, \ a_y, \ a_z\}$ va $b\{b_x, \ b_y, \ b_z\}$ bo’lganda

$$[a \times b] = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}.$$

$\bar{a}$ va $\bar{b}$ vektorlarda yasalgan paralellegrammning yuzi $S = |\bar{a} \times \bar{b}|$, shu vektorlarda yasalgan uchburchakning yuzi: $S_\Delta = \frac{1}{2}|\bar{a} \times \bar{b}|$.

$\vec{i}, \vec{j}, \vec{k}$ ortlari bilan shu vektor orasidagi burchaklari mos ravishda α, β va φ bilan belgilasak $\vec{a}$ vektor-ning yo’naltiruvchi kosinuslari quyidagi formulalar bilan hisoblanadi:

$$\cos\alpha = \frac{x_2 - x_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}, \qquad \cos\beta = \frac{y_2 - y_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

$$\cos\varphi = \frac{z_2 - z_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

35. Mavzuga doir namunaviy misolni yechimi.

Misol. Piramida ucharining koordinatalari berilgan
$A(0; \ 0; \ 1)$, $B(2; \ 3; \ 5)$, $C(6; \ 2; \ 3)$, $D(3; \ 7; \ 2)$. Quyidagilarni topish talab etiladi: 1) $\overline{AB}, \overline{AC}$ vektorlarni va ularni modullarini $|\overline{AB}|, |\overline{AC}|$; 2) $\overline{AB}, \overline{AC}$ vektorlar orasidagi <A burchakni; 3) $\overline{AB}, \overline{AC}$ vektorlarning skalyar ko’paytmasini; 4) $\overline{AB}, \overline{AC}$ vektorlarning vektorli ko’paytmasining modulini. 5) S nuqtadan o’tib, $\overline{AB}$ vektorga perpendikulyar bo’lgan, Q tekislik tenglamasini tuzish.

Yechish. 1) $\overline{AB}$, $\overline{AC}$, $\overline{AD}$ vektorlarni va ularni modellarini topamiz. Vektorlarga doir yuqorida keltirilgan formulalardan foydalanamiz. Ma’lumki, ixtiyoriy vektorni $a_x, \ a_y, \ a_z$ o’qdagi proeksiyalari hamda birlik $\bar{i}, \ \bar{j}, \ \bar{k}$ vektrlari (ortlari) yordamida yagona usul bilan yoyib yozish mumkin.

$$\bar{a} = a_x \bar{i} + a_y \bar{j} + a_z \bar{k}, \quad \overline{AB} = (2-0)\bar{i} + (3-0)\bar{j} + (5-1)\bar{k} = 2\bar{i} + 3\bar{j} + 4\bar{k},$$

$$\overline{AC} = (6-0)\bar{i} + (2-0)\bar{j} + (3-1)\bar{k} = 6\bar{i} + 2\bar{j} + 2\bar{k},$$

$$\overline{AD} = (3-0)\bar{i} + (7-0)\bar{j} + (2-1)\bar{k} = 3\bar{i} + 7\bar{j} + \bar{k}.$$

Bu vektorlarni modullari $|\bar{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$ formulaga asosan

$$\left|\overline{AB}\right| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}, \left|\overline{AC}\right| = \sqrt{6^2 + 2^2 + 2^2} = \sqrt{44} = 2\sqrt{11},$$

$$\left|\overline{AD}\right| = \sqrt{3^2 + 7^2 + 1^2} = \sqrt{59}.$$

2) $\overline{AB}$ va $\overline{AC}$ vektorlar orasidagi φ burchakni hisoblaymiz.

$$Cos\ \varphi = \frac{\overline{AB} \cdot \overline{AC}}{\left|\overline{AB}\right|\left|\overline{AC}\right|} = \frac{2\cdot 6 + 3\cdot 2 + 4\cdot 2}{\sqrt{29}\cdot 2\sqrt{11}} = \frac{13}{\sqrt{319}} \approx 0{,}728$$

Bradis jadvalidan $\varphi \approx 43^\circ$.

3) $\overline{AD}$ vektorni $\overline{AB}$ vektordagi proeksiyasi

$$np_{\overline{AB}} AD = \frac{\overline{AD} \cdot \overline{AB}}{\left|\overline{AB}\right|} = \frac{2\cdot 3 + 3\cdot 7 + 4\cdot 1}{\sqrt{29}} \approx \frac{31}{5{,}39}$$

4) Piramidaning ABC tomoni, uchburchak yuzasini hisoblaymiz

$$S_\Delta = \frac{1}{2}\left|\left[\bar{a}\times\bar{в}\right]\right| = \frac{1}{2}\left|\overline{AB}\times\overline{AC}\right|.$$

Bu erda ikki vektorni ko’paytmasi

$$\left[\overline{AB}\times\overline{AC}\right] = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 3 & 4 \\ 6 & 2 & 2 \end{vmatrix} = -2i + 20\bar{j} - 14k$$

$$\left|\left[\overline{AB}\times AC\right]\right| = \sqrt{(-2)^2 + 20^2 + (-14)^2} = \sqrt{4 + 400 + 196} = \sqrt{600} = 10\sqrt{6},$$

$S_\Delta = \frac{1}{2}\cdot 10\sqrt{6} = 5\sqrt{6}$ kvadrat birlik.

5) Piramidaning xajmini uchta komplanar bo’lmagan $\overline{AB}$, $\overline{AC}$, $\overline{AD}$ vektorlarni aralash ko’paytmasiga asosan hisoblaymiz:

$$V = \frac{1}{6}\left|\left[\bar{a}\times\bar{в}\right]\cdot\bar{c}\right| = \pm\frac{1}{6}\begin{vmatrix} a_x & a_y & a_z \\ в_x & в_y & в_z \\ c_x & c_y & c_z \end{vmatrix} = \pm\frac{1}{6}\begin{vmatrix} 2 & 3 & 4 \\ 6 & 2 & 2 \\ 3 & 7 & 1 \end{vmatrix} = \frac{1}{6}\cdot 120 = 20$$ kub birlik.

36. Mustaqil yechish uchun misollar.

1. Tekislikda $A(0;\ -2)$, $B(4;\ 2)$ va $C(4;\ -2)$ nuqtalar berilgan. Koordinatalar boshidan $\overline{OA}$, $\overline{OB}$ va $\overline{OC}$ kuchlar qo’yilagan. Ularning teng ta’sir etuvchisi $\overline{OM}$ yasalsin va uning o’qlardigi proeksiyalari hamda uzunligi topilsin. $\overline{OA}$, $\overline{OB}$, $\overline{OC}$ va $\overline{OM}$ kuchlar i va j birlik vektorlar orqali ifodalansin.

2. Uchta komplanar $\bar{m}$, $\bar{n}$ va $\bar{p}$ birlik vektor berilgan, $\bar{m},\wedge\bar{n} = 30^0$ va $(n\,p) = 60^0$ $u = m + 2n - 3p$ vektor yasalib uning moduli hisoblansin.

3. Uchta komplanar bo’lmagan $\overline{OA} = a$, $\overline{OB} = b$, $\overline{OC} = c$ vektorlarda parallelepiped yasalgan. Uning mos ravishda $a + b - c$, $a - b + c$, $a - b - c$ va $b - a - c$ larga teng vektor – diagonallari ko’rsatilsin.

4. Tekislikda $A(3;\ 3)$, $B(-3;\ 3)$ va $C(-3;\ 0)$ nuqtalar berilgan, koordinatalar boshidan $\overline{OA}$, $\overline{OB}$ va $\overline{OC}$ kuchlar qo'yilagan. Ularning teng ta'sir etuvchisi $\overline{OM}$ yasalsin va uning o'qlardigi proeksiyalari hamda kattaligi topilsin. $\overline{OA}$, $\overline{OB}$, $\overline{OC}$ va $\overline{OM}$ vektorlar o'qlardagi i va j birlik vektorlar orqali ifodalansin.

5. $M(5;\ -3;\ 4)$ nuqta yasalsin va uning radiusi-vektorining uzunligi hamda yo'nalishi aniqlansin.

6. $r=\overline{OM}=2i+6k$ vektor yasalsin va uning radius- vektorining uzunligi hamda yo'nalishi aniqlansin ($cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ formula bo'yicha tekshirilsin.)

7. $A(1;\ 2;\ 3)$ va $B(3;\ -4;\ 6)$ nuqtalar berilgan. $u=\overline{AB}$ vektor va uning koordinata o'qlaridagi proeksiyalari yasalsin hamda uning uzunligi va yo'nalishi aniqlansin. u vektorning koordinata o'qlari Bilan tashkil etgan burchaklari yasalsin.

8. $A(2;\ 1;\ -1)$ nuqtaga $R=7$ kuch qo'yilgan. Bu kuchning ikki koordinatasi $X=2$ va $Y=-3;$ o'sha kuchni ifodalovchi vektorning yo'nalishi va oxirgi nuqtasi aniqlansin.

9. Paralelgrammning ketma – ket uchta $A(1;\ -2;\ 3)$, $B(3;\ 2;\ 1)$ va $C(6;\ 4;\ 4)$ uchlari berilgan. Uning to'rtinchi uchi D topilsin.

10. $a=-i+j$ va $b=i-2j+2k$ vektorlar orasidagi burchak aniqlansin.

11. Uchlari $A(2;\ -1;\ 3)$, $B(1;\ 1;\ 1)$ va $C(0;\ 0;\ 5)$ nuqtalarda bo'lgan ΔABC ning burchaklari aniqlansin.

12. $a=2i+j$ va $b=-2j+k$ vektorlarda chsalgan parallelgramm diagonallari orasidagi burchak topilsin.

13. $a=i+j+2k$ va $b=i-j+4k$ vktorlar berilgan. $\text{пр}_a b$ aniqlansin.

14. 1) Agar m va n o'zaro 30^o burchak tashkil etuvchi birlik vektorlar bo'lsa, $(m+n)^2$ hisoblansin; 2) agar $a=2\sqrt{2}$ va $b=4$ hamda $(a,\wedge b)=135^o$ bo'lsa, $(a-b)^2$ hisoblansin.

15. Agar m va n - oralaridagi burchagi 60^o ga teng birlik vektorlar bo'lsa, $a=2m+n$ va $b=m-2n$ vektorlarda yasalgan parallelgramm diagonallarining uzunliklari aniqlansin.

16. $a=2m-n$ vektor berilgan bo'lib, bunda m va n oralaridagi burchagi 120^o ga teng birlik vektorlardir.

$cos\ (a,\wedge m)$ va $cos\ (a,\wedge n)$ topilsin.

17. $A(3;\ 3;\ -2)$, $B(0;\ -3;\ 4)$, $C(0;\ -3;\ 0)$ va $D(0;\ 2;\ -4)$ nuqtalr berilgan. $\overrightarrow{AB}=a$ va $\overrightarrow{CD}=b$ vetorlar yasalsin hamda $\text{пр}_a b$ topilsin.

18. Agar 1) $a=3i;\quad b=2k$; 2) $a=i+j;\quad b=i-j;$ 3) $a=2i+3j;\quad b=3j+2k$ bo'lsa, $c=a\times b$ vektor aniqlansin va yasalsin. Har bir hol uchun berilgan vektorlarda yasalgan paralelgramm yuzi hisoblansin.

19. Ushbu 1) $i\times(j+k)-j\times(i+k)+k\times(i+j+k)$;

2) $(a+b+c)\times c+(a+b+c)\times b+(b-c)\times a$; 3) $(2a+b)\times(c-a)+(b+c)\times(a+b)$;

4) $2i\cdot(j\times k)+3j\cdot(j\times k)+4k(i\times j)$

ifodalar qavslarni ochib soddalashtirilsin.

20. $a = 3k - 2j$, $b = 3i - 2j$ va $c = a \times b$ vektorlar yasalsin. c vektorning moduli hamda a va b vektorlarda yasalgan uchburak yuzi hisoblansin.

Fazoda $A(x_1; y_1; z_1)$, $B(x_2; y_2; z_2)$, $C(x_3; y_3; z_3)$ nuqtalar koordinatalari bilan berilan. quyidagilarni topish talab etiladi:

1) $\overline{AB}, \overline{AC}$ vektorlarni va ularni modullarini $|\overline{AB}|, |\overline{AC}|$;

2) $\overline{AB}, \overline{AC}$ vektorlar orasidagi <A burchakni;

3) $\overline{AB}, \overline{AC}$ vektorlarning skalyar ko'paytmasini;

4) $\overline{AB}, \overline{AC}$ vektorlarning vektorli ko'paytmasining modulini.

5) S nuqtadan o'tib, $\overline{AB}$ vektorga perpendikulyar bo'lgan, Q tekislik tenglamasini tuzish.

21) A(1;2;1), B (-1;5;1), C(-1;2;7), D(1;5;9).
22) A(2;3;2), B (0;6;2), C(0;3;8), D(2;6;10).
23) A(0;3;2), B (-2;6;2), C(-2;3;8), D(0;6;10).
24) A(2;1;2), B (0;4;2), C(0;1;8), D(2;4;10).
25) A(2;3;0), B (0;6;0), C(0;3;6), D(2;6;8).
26) A(2;2;1), B (0;5;1), C(0;2;7), D(2;5;9).
27) A(1;3;1), B (-1;6;1), C(-1;3;7), D(1;6;9).
28) A(1;2;2), B (-1;5;2), C(-1;2;8), D(1;5;10).
29) A(2;3;1), B (0;6;1), C(0;3;7), D(2;6;9).
30) A(2;2;2), B (0;5;2), C(0;2;8), D(2;5;10).
31) A(1;3;2), B (-1;6;2), C(-1;3;8), D(1;6;10).
32) A(0;1;2), B (-2;4;2), C(-2;1;8), D(0;4;10).
33) A(0;3;0), B (-2;6;0), C(-2;3;6), D(0;6;8).
34) A(0;2;1), B (-2;5;1), C(-2;2;7), D(0;5;9).
35) A(1;1;1), B (-1;4;1), C(-1;1;7), D(1;4;9).
36) A(1;2;0), B (-1;5;0), C(-1;2;6), D(1;5;8).
37) A(0;1;0), B (-2;4;0), C(-2;1;6), D(0;4;8).
38) A(0;1;1), B (-2;4;1), C(-2;1;7), D(0;4;9).
39) A(0;2;0), B (-2;5;0), C(-2;2;6), D(0;5;8).

FAZODAGI ANALITIK GEOMETRIYA

37. Fazoda tekislik va to'gri chiziq tenglamalari.

Tekislik tenglamalari.

$M(x; \; y; \; z)$ tekislikning ixtiyoriy nuqtasi bo'lsin. $M_1(x_1; \; y_1; \; z_1)$ nuqtadan o'tuvchi va $N\{A; \; B; \; C\}$ vektorga perpendikulyar tekislik tenglamasi $\overline{M_1M} \perp N$ va ikki vektorning perpendikulyarlik shartiga asosan $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$. Tekislikning umumiy tenglamasi quyidagicha $A \times + By + Cz + D = 0$ (1). $N\{A; \; B; \; C\}$ vektor tekislikni normal vektori deyiladi.

Tekislikning koordinata o'qlaridan ajratgan kesmalar bo'yicha tenglamasi:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Ikki tekislik orasidagi burchak

$$cos\ \varphi = \pm \frac{NN_1}{NN_1} = \pm \frac{AA_1 + BB_1 + CC_1}{NN_1} \qquad (2)$$

formuladan topiladi, bunda $\overline{N}$ va $\overline{N_1}$ mos ravishda $A \times + By + Cz + D = 0$ va $A_1 x + B_1 y + C_1 z + D_1 = 0$ tekisliklarni normal vektorlar.

Parallellik sharti: $\frac{A}{A_1} = \frac{B}{B_1} = \frac{C}{C_1}$, Perpendikulyarlik sharti:

$$AA_1 + BB_1 + CC_1 = 0$$

$M_0(x_0; y_0; z_0)$ nuqtadan $Ax + By + Cz + D = 0$ tekisligacha bo'lgan masofa:

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{N} \qquad (3)$$

Berilgan ikki tekislikning kesishgan chizigidan o'tuvchi barcha tekisliklar dastasining tenglamasi quyidagicha yoziladi.

$$\alpha(Ax + By + Cz + D) + \beta(A_1 x + B_1 y + C_1 z + D_1) = 0$$

$\alpha = 1$ deb olish mumkin, u holda dastadan berilgan tekisliklardan ikinchisini chiqarib tashlagan bo'lamiz.

Fazoda to'gri chiziq tenglamalari.

$A(a;\ b;\ c)$ nuqtadan o'tuvchi va $P\{m;\ n;\ p\}$ vektorga parallel bo'lgan to'gri chiziq tenglamalari. $N(x;\ y;\ z)$ - to'gri chiziqning ixtiyoriy nuqtasi bo'lsin. U holda $\overline{AN} \| P$ va ikki vektorning parallellik shartiga ko'ra:

$$\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-c}{p} \qquad (1)$$

(1) tenglama to'gri chiziqning kanonik tenglamasi deyiladi. $P\{m;\ n;\ p\}$ vektor to'gri chiziqning yo'naltiruvchi vektori deyiladi.

(1) tenglamadagi har bir nisbatni t parametrga tenglab, to'gri chiziqning

$$\left.\begin{array}{l} x = mt + a, \\ y = nt + b, \\ z = pt + c \end{array}\right\} \qquad (2)$$

ko'rinishdagi parametrik tenglamalariga ega bo'lamiz.

Ikki nuqtadan o'tuvchi to'gri chiziq tenglamalari:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}. \qquad (3)$$

To'gri chiziqning umumiy tenglamalari:

$$\left.\begin{array}{l} Ax + By + Cz + D = 0 \\ A_1 x + B_1 y + C_1 z + D_1 = 0 \end{array}\right\} \qquad (4)$$

(4) tenglamalardan bir marta y ni, ikkinchi marta x ni yo'qotib, to'gri chiziqning proeksiyalari bo'yicha yozilgan tenglamalariga ega bo'lamiz:

$$\left.\begin{array}{l} x = mz + a, \\ y = nz + b. \end{array}\right\} \qquad (5)$$

tenglamalarni ushbu

$\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-0}{1}$ kanonik ko'rinishda yozish mumkin.

To'gri chiziq va tekislik

$\frac{x-a}{m}=\frac{y-b}{n}=\frac{z-c}{p}$ to'gri chiziq bilan $Ax+By+Cz+D=0$ tekislik orasidagi burchak: $\sin 0=\frac{|N\cdot P|}{NP}=\frac{|Am+Bn+Cp|}{NP}$ (1)

ularning parallellik sharti $(N\|P)$: $Am+Bn+Cp=0$ (2)

ularning perpendikulyarlik sharti $(N\perp P)$: $\frac{A}{m}=\frac{B}{n}=\frac{C}{p}$ (3)

Tekislik bilan to'gri chiziqning kesishgan nuqtasini topish uchun to'gri chiziq tenglamasini $x=mt+a$, $y=nt+b$, $z=pt+c$ parametrik ko'rinishda yozib, tekislikning $Ax+By+Cz+D=0$ tenglamasidagi $x;\ y;\ z$ larning o'rniga ularning t ga nisbatan yozilgan qiymatlarini qo'yamiz. Hosil bo'lgan tenglamadan t_0 ni, so'ngra kesishgan nuqta koordinatalari $x_0;\ y_0;\ z_0$ ni topamiz.

Ikki to'gri chiziqning bir tekislikda yotish sharti:

$$\begin{vmatrix} a-a_1 & b-b_1 & c-c_1 \\ m & n & p \\ m_1 & n_1 & p_1 \end{vmatrix}=0 \qquad (4)$$

38. Mavzuga doir namunaviy masalani yechimi.

Misol-1. $A(1;\ 1;\ 1)$ va $B(0;\ 2;\ 1)$ nuqtalardan o'tib $\overline{a}(2;\ 0;\ 1)$ vektorga parallel tekislik tenglamasini tuzing.

Yechish. $\overline{AB}(-1;\ 1;\ 0)$ va $\overline{a}(2;\ 0;\ 1)$ vektorlar kollinear bo'lmagan vektorlar bo'lganligi sababli bu masala yagona yechimga ega va tekislikni normal tenglamasi

$$\overline{n}=\begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{vmatrix}=\overline{i}+\overline{j}-2\overline{k}=\overline{n}(1;\ 1;\ -2)$$

berilgan nuqtadan o'tib, $\overline{n}$ normal vektorga perpendikulyar $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$, $\overline{n}(A;\ B;\ C)$ tekislik tenglamasiga asosan talab qilingan tekislik tenglmasi

$$(x-1)+(y-1)-2(z-1)=0,\quad x+y-2z=0.$$

Misol -2. To'gri chiziqni umumiy shakldagi tenglamasiga ko'ra uni kanonik tenglamasini tuzing.

$$\begin{cases} x+y-z=0 \\ 2x-y+2=0 \end{cases}$$

Echish. $M(0;\ 2;\ 2)$ nuqta to'gri chiziq ustida yotadi ya'ni uni qanoatlantiradi. To'gri chiziqni yo'naltiruvchi vektori sifatida $\overline{q}(l;\ m;\ n)=[n_1 x n_2]$ olamiz. Bu erda $n_1(1;\ 1;\ -1)$, $n_2(2;\ -1;\ 0)$ har bir tekislikni normal vektorlari, bu tekisliklarni kesishidan to'gri chiziq hosil bo'ladi.

$$\bar{q}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ -1 & 1 & -1\\ 2 & -1 & 0\end{vmatrix}=-\bar{i}-2\bar{j}-3\bar{k}=\bar{q}(-1;\quad 1;\quad -3)$$

Natijada $M(x_0;\quad y_0;\quad z_0)$ nuqtadan o'tuvchi to'gri chiziqni kanonik tenglamasi

$$\frac{x}{-1}=\frac{y-2}{-2}=\frac{z-2}{-3}$$

bu tenglamadan to'gri chiziqni parametrik tenglamasi kelib chiqadi:

$$\begin{cases} x=x_0+\ell t\\ y=y_0+mt\\ z=z_0+nt\end{cases} \Rightarrow \begin{cases} x=-t\\ y=2-2t\\ z=2-3t\end{cases}$$

Misol-3. Berilgan M_0 (5,2,-3) nuqtadan o'tuvchi va $\bar{n}=(2,-1,4)$ vektorga perpendikulyar tekislik tenglamasini tuzing

Yechish. $\bar{n}=(A,B,C)$ demak, A=2, B=-1, S=4, x_0=5, y_0=2, c_0=-3.

A(x-x_0)+B(y-y_0)+C(z-z_0)=0, 2(x-5)-(y-2) + (z+3) =0, 2 x-y+4z+4=0

Yechish. $\overline{M_0M}=(x-2,y-2,z-1)$, $\overline{a_1}$ $\overline{a_2}$, $\overline{M_0M}$ bitta tekislikda yotgani sababli ular kollenear bo'ladi, ularni aralash ko'paytmasi nolga teng bo'ladi,

$$\begin{vmatrix}x-2 & y-2 & z-2\\ 3 & 2 & 5\\ 1 & -1 & 0\end{vmatrix}=0 \quad . \quad (x-2)\begin{vmatrix}2 & 5\\ -1 & 0\end{vmatrix}-(4-2)\begin{vmatrix}3 & 5\\ 1 & 0\end{vmatrix}+(z-1)\begin{vmatrix}3 & 2\\ 1 & -1\end{vmatrix}=0$$

5(x-2)+5(4-2)-5(2-1)=0 soddalashtirishdan so'ng quyidagini hosil qilamiz, x+y-z-3=0

Misol-4. M_0 (1,1,1) nuqtadan 2y+2y-z+3=0 tekislikda bo'lgan masofani toping.

D $=\dfrac{|Ax_0+BY_0+CZ_0+D|}{\sqrt{A^2+B^2+C^2}}$, x_0=1 y_0=1 z_0=1, d=$\dfrac{|2\cdot 1+2\cdot 1-1+3|}{\sqrt{4+4+1}}=\dfrac{6}{\sqrt{9}}=\dfrac{6}{3}=2$

Misol-5. Berilgan tekisliklar orasidagi burchakni toping.

4x-10y+z-3=0 va -11x+8y+7z+5=0

$$\cos\varphi=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}$$ formulaga ko'ra

$$\cos\varphi=\frac{-44-80+7}{\sqrt{16+100+1}\cdot\sqrt{121+64+49}}=\frac{-127+7}{\sqrt{117}\cdot\sqrt{234}}=\frac{-117}{\sqrt{117}\cdot\sqrt{2}\sqrt{117}}=-\frac{1}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$$

$$\cos\varphi=-\frac{\sqrt{2}}{2}\qquad \varphi=\arccos\left(-\frac{\sqrt{2}}{2}\right)=\pi-\arccos\frac{\sqrt{2}}{2}=\pi-\frac{\pi}{4}=\frac{3\pi}{4}=135^0$$

39. Mustaqil yechish uchun misollar.

1. $5x-2y+3z-10=0$; 2) $3x+2y-z=0$; 3) $3x+2z=6$; 4) $2z-7=0$
2. tekisliklar yasalsin.
3. $2x+3y+6z-12=0$ tekislik yasalsin va unga normal vektorning koordinata o'qlari bilan tashkil etgan burchaklari topilsin.
4. $M_1(0;\quad -1;\quad 3)$ va $M_2(1;\quad 3;\quad 5)$ nuqtalar berilgan. M_1 nuqtadan o'tuvchi va $N=\overline{M_1M_2}$ vektorga perpendikulyar tekislik tenglamasi yozilsin.

5. $M_1(0;\ 1;\ 3)$ va $M_2(2;\ 4;\ 5)$ nuqtalardan o'tuvchi va *Ox* o'qqa parallel tekislik tenglamsi yozilsin va tekislik yasalsin.
6. *Ox* o'qdan va $M_1(0;\ -2;\ 3)$ nuqtadan o'tuvchi tekislik tenglamasi yozilsin va tekislik yasalsin.
7. *Oz* o'qdan va $M_1(2;\ -4;\ 3)$ nuqtadan o'tuvchi tekislik tenglamasi yozilsin va tekislik yasalsin.
8. *Oy* o'qqa parallel, *Ox* va *Oz* o'qlardan a va c kesmalar ajratuvchi tekislik tenglamasi yozilsin. Tekislik yasalsin.
9. $M(2;\ -1;\ 3)$ nuqtadan o'tuvchi va koordinata o'qlaridan teng kesmalar ajratuvchi tekislik tenglamasi yozilsin.
10. 1) $2x + y - z + 6 = 0$; 2) $x - y - z = 0$; 3) $y - 2z + 8 = 0$; 4) $2x - 5 = 0$;
11.5) $x + z = 1$; 6) $y + z = 0$ tekisliklar yasalsin.
12. $2x - 2y + z - 6 = 0$ tekislik yasalsin va unga normal vektorning koordinata o'qlari bilan tashkil etgan burchaklari topilsin.
13. 1) $x - 2y + 2z - 8 = 0$ va $x + z - 6 = 0$

   2) $x + 2z - 6 = 0$ va $x + 2y - 4 = 0$
14.tekisliklar orasidagi burchak topilsin.
15. $(2;\ 2;\ -2)$ nuqtadan o'tuvchi va $x - 2y - 3z = 0$ tekislikka parallel tekislik topilsin.
16. $(-1;\ -1;\ 2)$ nuqtadan o'tuvchi va $x - 2y + z - 4 = 0$ hamda $x + 2y - 2z + 4 = 0$ tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin.
17. $M_1(-1;\ -2;\ 0)$ va $M_2(1;\ 1;\ 2)$ nuqtalardan o'tuvchi hamda $x + 2y + 2z - 4 = 0$ tekislikka perpendikulyar tekislikning tenglamasi yozilsin.
18. $M_1(1;\ -1;\ 2)$, $M_2(2;\ 1;\ 2)$ va $M_3(1;\ 1;\ 4)$ nuqtalardan o'tuvchi tekislikning tenglamasi yozilsin.
19. $(4;\ 3;\ 0)$ nuqtadan $M_1(1;\ 3;\ 0)$, $M_2(4;\ -1;\ 2)$ va $M_3(3;\ 0;\ 1)$ nuqtalardan o'tuvchi tekislikkacha bo'lgan masofa topilsin.
20. $4x + 3y - 5z - 8 = 0$ va $4x + 3y - 5z + 12 = 0$ parallel tekisliklar orasidagi masofa topilsin.
21.1) $2x - y + 3z - 6 = 0$ va $x + 2y - z + 3 = 0$ tekisliklarning kesishgan chizigidan va $(1;\ 2;\ 4)$ nuqtadan o'tuvchi tekislik tenglamasi yozilsin.
22. $2x + 2y + z - 8 = 0$ tekislikka parallel va undan $d = 4$ masofa bo'lgan tekisliklarning tenglamalari yozilsin.
23. $4x - y + 3z - 6 = 0$ va $x + 5y - z + 10 = 0$ tekislikning kesishgan chizigidan o'tuvchi va $2x - y + 5z - 5 = 0$ tekislikka perpendikulyar tekislikning tenglamasi yozilsin.
24. 1) $\begin{cases} x = z + 5 \\ y = 4 - 2z \end{cases}$ va 2) $\left\{ \dfrac{x-3}{1} = \dfrac{y-2}{2} = \dfrac{z-3}{1} \right.$
25.to'gri chiziqlarning xOy va xOz tekisliklardagi izlari topilsin va to'gri chiziqlar yasalsin.

26. $\left.\begin{array}{l} x+2y+3z-13=0 \\ 3x+y+4z-14=0 \end{array}\right\}$ to’gri chiziq tenglamalarini: 1) proeksiyalari bo’yicha; 2) kanonik ko’rinishda yozilsin. To’gri chiziqning koordinata tekisliklaridagi izlari topilsin hamda to’gri chiztq va uning proeksiyalari yasalsin.

27. $A(4;\ 3;\ 0)$ nuqtadan o’tuvchi va $P\{-1;\ 1;\ 1\}$ vektorga parallel bo’lgan to’gri chiziqtenglamalari yozilsin. To’gri chiziqning yOz tekislikdagi izi topilsin va to’gri chiziq yasalsin.

28. 1) $\begin{cases} y=3 \\ z=2 \end{cases}$, 2) $\begin{cases} y=2 \\ z=x+1 \end{cases}$, 3) $\begin{cases} x=4 \\ z=y \end{cases}$

29. to’gri chiziqLar yasalsin va ularning yo’naltiruvchi vektorlari Aniqlansin.

30. $A(-1;\ 2;\ 3)$ va $B(2;\ 6;\ -2)$ nuqtalardan o’tuvchi to’gri chiziq tenglamalari yozilsin va uning yo’naltiruvchi kosinuslari topilsin.

31. $\begin{cases} x-y+z-4=0 \\ 2x+y-2z+5=0 \end{cases}$ va $\begin{cases} x+y+z-4=0 \\ 2x+3y-z-6=0 \end{cases}$

to’gri chiziqlar orasidagi burchak topilsin.

32. $\frac{x}{2}=\frac{y}{3}=\frac{z}{1}$ to’gri chiziqning $x=z+1$, $y=1-z$ to’gri chiziqqa parallel bo’lgan to’gri chiziq tenglamalari yozilsin.

33. $N(2;\ -1;\ 3)$ nuqtadan $\frac{x+1}{3}=\frac{y+2}{4}=\frac{z-1}{5}$ to’gri chiziqqacha bo’lgan masofa topilsin.

34. $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z+3}{2}$ va $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z+1}{2}$ parallel to’gri chiziqlar orasidagi masofa topilsin.

35. $\begin{cases} 2x-y-7=0 \\ 2x-z+5=0 \end{cases}$ va $\begin{cases} 3x-2y+8=0 \\ z=3x \end{cases}$

36. $y=3x-1$, $2z=-3x+2$ to’gri chiziq bilan $2x+y+z-4=0$ tekislik orasidagi burchak topilsin.

37. $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z-1}{3}$ to’gri chiziq $2x+y-z=0$ tekislikka parallel ekanligi, $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z+3}{3}$ to’gri chiziq esa shu tekislik ustida yotishi ko’rsatilsin.

38. $\frac{x-2}{1}=\frac{y-2}{2}=\frac{z+1}{3}$ to’gri chiziqdan va $(3;\ 4;\ 0)$ nuqtadan o’tuvchi tekislikning tenglamasi yozilsin.

39. $\frac{x-1}{1}=\frac{y+1}{2}=\frac{z+1}{2}$ to’gri chiziqdan o’tuvchi va $2x+3y-z=4$ tekislikka perpendikulyar tekislikning tenglamasi yozilsin.

40. $\frac{x-3}{2}=\frac{y}{1}=\frac{z-1}{2}$ va $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z}{2}$ parallel to’gri chiziqlardan o’tuvchi tekislikning tenglamasi yozilsin.

41. $x=2t-1$, $y=t+2$, $z=1-t$ to’gri chiziqning $3x-2y+z=3$ tekislik bilan kesishgan nuqtasi topilsin.

42. Ushbu

1) $\frac{x-a}{m}=\frac{y-b}{n}=\frac{z-c}{p}$ va $\frac{x-a_1}{m_1}=\frac{y-b_1}{n_1}=\frac{z-c_1}{p_1}$;

2) $\frac{x+1}{1}=\frac{y}{1}=\frac{z-1}{2}$ va $\frac{x}{1}=\frac{y+1}{3}=\frac{z-2}{4}$

43.parallel bo'lmagan to'gri chiziqlar orasidagi eng qisqa masofa topilsin.

44. $y=z$ tekislik, $\left.\begin{array}{l} x=-z+1 \\ y=2 \end{array}\right\}$ to'gri chiziq yasalsin va: 1) ularning kesishgan nuqtasi; 2) ular orasidagi burchak topilsin.

45. $(1;\ \ 2;\ \ 8)$ nuqtaning $\frac{x-1}{2}=\frac{y}{-1}=z$ to'gri chiziqdagi proeksiyasi topilsin.

46. $\frac{x-1}{1}=\frac{y+1}{-2}=\frac{z-2}{3}$ va $\frac{x}{1}=\frac{y+1}{-2}=\frac{z-1}{3}$ parallel to'gri chiziqlardan o'tuvchi tekislikning tenglamasi yozilsin.

40. Sferik va silindrik sirtlar

1°. Markazi $C(a;\ \ b;\ \ c)$ nuqtada va radiusi R bo'lgan sferik sirtning tenglamasi quyidagicha yoziladi:

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2. \qquad (1)$$

2°. z ishtirok etmagan $F(x;\ \ y)=0$ tenglama, yasovchilari O_z o'qqa parallel silindrik sirtni aniqlaydi. Shunga o'xshash 1) $F(y;\ \ z)=0$ va 2) $F(x;\ \ z)=0$ tenglamalarning har bir yasovchilari 1) Ox, 2) Oy o'qlarga paralel bo'lgan silindrik sirtlarni aniqlaydi.

3°. Yo'naltiruvchi $F(x;\ \ y)=0$, $z=0$ yasovchilari esa $P\{m;\ \ n;\ \ p\}$ vektorga parallel bo'lgan silindrik sirt tenglamasi. Ixtiyoriy yasovchining tenglamasi.

$$\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z}{p}$$

dan iborat, bundagi $(x_0;\ \ y_0;\ \ 0)$ - yo'naltiruvchida yotuvchi nuqta. So'ngi tengliklardan $x_0;\ \ y_0$ ni topib, yo'naltiruvchining tenglamasiga qo'ysak, silindrik sirtning

$$F\left(x-\frac{m}{p}z,\, y-\frac{n}{p}z\right)=0 \qquad (2)$$

tenglamasini hosil qilamiz.

Ellipsoid, giperboloidlar va paraboloidlar

1°. Kanonik tenglamalar. Silindrik sirtlardan boshqa quyidagi kanonik (eng sodda) tenglamalar bilan Aniqlanuvchi oltita asosiy ikkinchi tartibli sirtlar bor:

I. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ - ellipsoid.

II. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ - bir kovakli giperboloid.

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$ - ikki kovakli giperboloid.

III. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$ - ikkinsi tartibli konus.

IV. $\frac{x^2}{p}+\frac{y^2}{q}=2z$ - elliptik paraboloid, ($pq>0$ bo'lganda)

$\frac{x^2}{p}-\frac{y^2}{q}=2z$ - giperbolik paraboloid.

41. Mustaqil yechish uchun misollar.

1. 1) $x^2+y^2+z^2-3x+5y-4z=0$; 2) $x^2+y^2+z^2=2az$ sferalarning markazi va radiusi topilsin hamda ikkinchi sferaning tasviri yasalsin.

2. $\left.\begin{array}{l} x^2+y^2+z^2=a^2 \\ x+y+z=a \end{array}\right\}$ aylanadan va $(a;\ a;\ a)$ nuqtadan o'tuvchi sferik sirtning tenglamasi yozilsin.

3. Koordinatalar chap sistemasida
1) $y^2+z^2=4$; 2) $y^2=ax$; 3) $xz=4$; 4) $x^2+y^2=ax$ sirtlar yasalsin.

4. $x^2+y^2+z^2-2ax=0$ sfera tashqarisida chizilgan, yasovchilari mos ravishda: 1) Ox o'qqa; 2) Oy o'qqa; 3) Oz o'qqa parallel uchta silindrik sirtning tenglamalari yozilsin.

5. $\left.\begin{array}{l} x^2+y^2+z^2=16 \\ x^2+y^2=4x \end{array}\right\}$

Viviani egri chizigining $x=0$; 2 va 4 bo'lgandagi nuqtalarini yasab, koordinatalar chap sistemasining birinchi oktantida uning shakli chizilsin. Egri chiziqning xOz tekislikdagi proeksiyasi parabola ekani ko'rsatilsin.

6. $\left.\begin{array}{l} x^2+y^2+z^2=10y \\ x+2y+2z-19=0 \end{array}\right\}$

aylananing markazi va radiusi topilsin.

7. $x^2+y^2+z^2-2x+y-3z=0$ sirtning markazi C dan o'tuvchi va OC to'gri chiziqqa perpendikulyar tekislikning tenglamasi yozilsin.

8. $\frac{x^2}{a^2}+\frac{y^2}{c^2}=1$, $y=0$ ellipsning Oz o'q atrofida aylanishidan hosil bo'lgan sirtning tenglamasi yozilsin.

9. $\frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{25}=1$ sirt yasalsin va uning: 1) $z=3$; 2) $y=1$ tekisliklar bilan kesimlarining yuzlari topilsin.

10. $\frac{x^2}{a^2}-\frac{z^2}{c^2}=1$, $y=0$ egri chiziqning: a) Oz o'q; b) Ox o'q atrofida aylanishidan hosil bo'lgan sirt tenglamasi yozilsin. Ikala sirt (koordinatalarning chap sistemasida) yasalsin.

11. 1) $x^2+y^2-z^2=4$; 2) $x^2-y^2+z^2+4=0$ sirtlar yasalsin.

12. $\frac{x^2}{16}+\frac{y^2}{4}-\frac{z^2}{36}=1$ giperboloid yasalsin va uning $(4;\quad 1;\quad -3)$ nuqtadan o'tuvchi yasochilari topilsin.

13. $x^2-y^2=4z$ sirt (koordinatalarning chap sistemasida) yasalsin va uning $(3;\quad 1;\quad 2)$ nuqtadan o'tuvchi yasovchilari topilsin.

14. $\frac{x^2}{169}+\frac{y^2}{25}+\frac{z^2}{9}=1$ ellipsoidning eng katta doiraviy kesimi topilsin.

15. Ushbu

1) $x^2+y^2+z^2=2az$;	6) $x^2=2az$;
2) $x^2+y^2=2az$;	7) $x^2=2yz$;
3) $x^2+y^2=2az$;	8) $z=2+x^2+y^2$;
4) $x^2-y^2=2az$;	9) $(z-a)^2=xy$;
5) $x^2-y^2=z^2$;	10) $(z-2x)^2+4(z-2x)=y^2$

sirtlardan har birining nomi aniqlansin va ular yasalsin.

16. $\frac{x^2}{25}+\frac{y^2}{9}-\frac{3z^2}{25}=1$ giperboloidning eng kichik doiraviy kesimlari topilsin.

17. $\frac{x^2}{16}-\frac{y^2}{9}2z$ giperbolik paraboloidning $(4;\quad 3;\quad 0)$ nuqtadan o'tuvchi to'gri chiziqlar yasovchilarining tenglamalari yozilsin.

42. Matematik tahlil. O'zgaruvchi miqdorlar va funksiyalar. Ishlab chiqarish funksiyalariga misollar.

Kundalik hayotimizda o'zgarmas va o'zgaruvchi miqdorlar bilan ish ko'ramiz. Masalan: paxtani hosildorligi, sutdagi yog miqdori va boshqalar o'zgaruvchi miqdorlar bo'lsa, aylana uzunligini diametriga nisbati, uchburchakni ichki burchaklari yig'indisi o'zgarmas miqdorlar bo'ladi.

Intervallar. x o'zgaruvchining $a<x<b$ tengsizliklarni qanoatlantiruvchi qiymatlari to'plami $(a,\quad b)$ oraliq deyiladi. x o'zgaruvchining $a\le x\le b$ tengsizliklarni qanoatlantiruvchi x qiymatlari to'plami $[a,\quad b]$ segment deyiladi

Matematik tahlilda elementlari haqiqiy sonlardan iborat bo'lgan to'plamlar o'rganiladi. X va У to'plamlar berilgan bo'lsin (bu erda interval, segment bo'lishi mumkin).

Sanash natijasida natural sonlar to'plami hosil bo'ladi N={1,2,3,4,…..}. Agar natural sonlar to'plamiga nol va butun manfiy sonlarni qo'shsak u xolda hosil bo'lgan sonlar to'plamini butun sonlar to'plami deyiladi Z={-…-2,-1,0,1,2,…} .

Agar m va n – lar butun sonlar bo'lib, $n\ne 0$ bo'lsin. u holda ushbu $\frac{m}{n}$ ko'rinishdagi sonlarni rasional kasr son deb ataladi. Rasional sonlar to'plamini Q xarfi bilan belgilanadi. Har qanday rasional sonni chekli yoki cheksiz davriy o'nli kasr shaklida ifodalash mumkin: $\frac{1}{2}=0{,}5,\quad \frac{3}{4}=0{,}75,\quad \frac{1}{3}=0{,}333\;....=0,(3)$

Davriy bo'lmagan cheksiz o'nli kasr shaklida yoziladigan sonlar irrasional sonlar deyiladi. Masalan: $\sqrt{2} = 1{,}4145$ ......, $\sqrt{3} 1{,}7328$ .....
$\pi = 3{,}14$ ...., $e = 2{,}7128$ ......

Rasional va irrasional sonlar to'plami haqiqiy sonlar to'plamini tashkil qiladi u R bilan belgilanadi. Haqiqiy sonning absolyut qiymati $|x|$ bilan belgilanadi va u quyidagi tenglik bilan aniqlanadi:

$$|x| = \begin{cases} -x, \text{агар} & x < 0 \\ 0, \text{агар} & x = 0 \\ x, \text{агар} & x > 0 \end{cases}$$

Masalan: $|-5| = -(-5) = 5, \ |3| = 3$

Absolyut qiymatning xossalari:

1^0. $|-x| = |x|$, 2^0. $|x| \le a \Leftrightarrow -a \le x \le a$, 3^0. $|x| \ge a \Leftrightarrow x \le -a$ va $x \ge a$

4^0. $|x+y| \le |x| + |y|$, 5^0. $|x-y| \ge |x| - |y|$, 6^0. $|x \cdot y| = |x| \cdot |y|$, 7^0. $\left|\dfrac{x}{y}\right| = \dfrac{|x|}{|y|} \quad (y \ne 0)$

Masalan ushbu $|x-1| < 2$ tengsizlikni qanoatlantiruvchi oraliq topilsin. Echish $|x-1| < 2 \Rightarrow -2 < x-1 < 2 \Rightarrow -1 < x < 3$.

Ta'rif. Agar X to'plamdagi har bir x songa biror qoida yoki qonun yordamida У to'plamdagi u son mos qo'yilsa, X to'plamda funksiya aniqlangan deyiladi va y=f(x) shaklida yoziladi.

Bunda o'zgaruvchi x ixtiyoriy qiymat qabul qilganligidan erkli o'zgaruvchi ya'ni argument, argumentning qiymatlari to'plami D(y) funksiyaning aniqlanish sohasi deyiladi. Ularga mos u funksiyani qabul qiladigan qiymatlari to'plami E(y) funksiyaning qiymatlar sohasi deyiladi.

Funksiya 1) analitik , 2) jadval, 3) grafik usullarda berilishi mumkin.

Agar u=f(x) funksiya uchun f(-x)=f(x) tenglik bajarilsa juft funksiya, f(-x)= -f(x) tenglik bajarilsa toq funksiya, f(-x)≠±f(x) bo'lsa juft ham, toq ham bo'lmaydi. Juft funksiyani grafigi ordinata (ou) o'qiga simmetrik bo'lsa, toq funksiyani grafigi koordinata boshiga nisbatan simmetrik bo'ladi. Agar T≠0 son uchun f(x+T)=f(x) tenglik bajarilsa, y=f(x) funksiya T davrga ega deyiladi. Masalan, y=sinx, y=cosx 2p davrga ega, y=tgx, y=ctgx p davrga ega bo'lgan funksiyalardir.

43. Mavzuga doir namunaviy misollar.

Asosiy elementar funksiyalar.

1. Chiziqli va kvadratik funksiyalar. Ushbu y=ax+b, y=ax^2+bx+s ko'rinishdagi funksiyalar mos ravishda chiziqli va kvadratik funksiyalar deb ataladi, bunda a,b,s – o'zgarmas haqiqiy sonlar. У=ax+b, D(f)=E(f)=R. y=ax^2+bx+C, D(f)=R .

2. Kasr - rasional funksiya. $y = \dfrac{k}{x} \quad D(f) = (-\infty.0) U (0,+\infty)$

3. Darajali funksiya. $y = x^{\alpha} (x > 0)$. 1) Agar α - butun va α >0 bo'lsa D(f)=R, 2) Agar α - butun va α <0 bo'lsa, $D(f) = (-\infty, 0) U (0, \infty)$

4. Ko'rsatkichli funksiya. y=a^x $(a>0, a\neq 1)$, D(f)=R, E(f)=R_Q
5.Lagorifmik funksiya. $y=\log_a x$ $(a>0, a\neq 1)$ D(f)=R_Q E(f)=R
6. Trigonometrik funksiyalar. $y=\sin x,\ y=\cos x,\ y=tgx\ \ y=ctgx$
1) $y=\sin x,\ y=\cos x,$ funksiyalar uchun D(f)=R, E(f)=[-1,1]

2) $y=tgx$, $x\neq \frac{\pi}{2}+k\pi,\ E(f)=R$. 3) $y=ctgx$, $x\neq k\pi,\ k\in z\ \ E(f)=R$

7. Teskari trigonometrik funksiyalar. $y=\arcsin x,\ \ y=ar\cos x,\ \ y=arctgx$, $y=arcctgx$,.) $y=\arcsin x,$ uchun D(f)=[-1,1]. E(f)=[$-\frac{\pi}{2},\frac{\pi}{2}$].

2) $y=ar\cos x,$ uchun D(f)=[-1,1]. E(f)=[$0,\Pi$]

3) $y=arctgx$, uchun D(f)=$(-\infty,\infty)$. E(f)=($-\frac{\pi}{2},\frac{\pi}{2}$)

4) $y=arcctgx$, uchun D(f)=$(-\infty,\infty)$. E(f)=($0,\pi$)

Ishlab chiqarish funksiyalariga misollar.

Matematik model real ishlab chiqarish jarayonini aks ettiruvchi formal munosabatlar majmuidir. Real hayotda uchraydigan jarayonlarni muqobil matematik modelini yaratish goyatda murakkab vazifa. Model biror jarayon uchun tuzilayotganda qancha ko'p ta'sir etuvchi faktorlar e'tiborga olinsa, shuncha tuzilgan model aniqroq bo'ladi. Ishlab chiqarish funksiyalariga qishloq xo'jaligidan misollar keltiramiz:

1) Tuproqqa solingan o'g'it miqdori bilan ekin hosildorligi orasidagi bog'lanishni chiziqli model shaklida y=y_0+ax karash mumkin. Bu erda y_0 - o'g'itsiz etishtirilgan hosildorlikni miqdori (kontrol), a - tayin bir o'g'itni shu ekin hosildorligiga ta'sirini xarakterlovchi son, x - solingan mineral o'g'it miqdori, y - olingan hosildorlik miqdori.
2) Qoramollarga beriladigan 10% oqsilli makkajo'xori poyasi bilan x (kg), uni vaznini o'sishi u (kg) orasida quyidagi bog'lanish mavjudligi aniqlangan y=-0,0001x^2+0,335x-0,48.
3) Ikki yoshli buzoqni terik vazni quyidagi formula bilan taxminan baholanadi y=578,4-244$e^{-0,00015x}$ bu erda x - beriladigan oziqa miqdori (kg).
4) Birinchi 3 soatda tuproqni suv shimish tezligi y=a/x^α bog'lanish orqali ifodalanadi.

5) Italiyalik iqtisodchi Pareto kapitalistik jamiyatda daromadni taqsimlanishini o'rgangan. Agar u bilan kamida x so'm daromadga ega bo'lgan odamlar sonini belgilasak, $y=\frac{a}{x^m}$ Bu erda a, m-o'zgarmas sonlar.

8). Biror xo'jalikning iqtisodiy rentabelligini aniqlash funksiyasi quyidagicha: $R=\frac{x}{y}-1$

Bu erda R-xo'jalikni rentabelligi, X-har bir maxsulotni narxi,
y - har bir maxsulotni tan narxi. Oxirgi munosabatdan ko'rinib turibdiki, agar: 1) x>y bo'lsa, ya'ni maxsulot narxi tan narxidan katta bo'lsa, R>0 bo'lib, xo'jalik

rentabelli; 2) Agar $x=y$ bo'lsa, xo'jalik o'zini-o'zi qoplaydigan $R=0$; 3) Agar $x<y$ bo'lsa, zarar ko'radigan ($R<0$) xo'jalik bo'ladi.

9).Mitcherlix-Spilman solingan o'g'it bilan, hosildorlik orasidagi bog'lanishni chiziqli modelga qaraganda quyidagi bog'lanish yaxshiroq xarakterlanishini ko'rsatadi

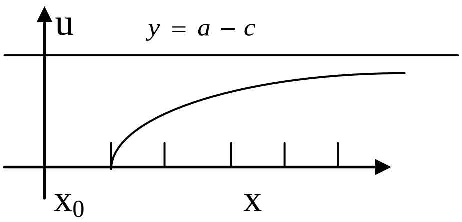
$$y=y_0+A(1-10^{-Kx})$$

Bu erda y_0-o'g'it solinmasdan olingan hosildorlik a-o'simlikning shu o'g'itga bo'lgan eng yuqori (maksimal) talabi, k-ma'lum bir o'g'itni ekin hosildorligiga ta'sirini xarakterlovchi o'zgarmas miqdor. Masalan, bu son - fosfor uchun k=0,8, azot uchun k=1,1, maxalliy o'g'it uchun k=0,4.

<u>Misol-10.</u> Boqilayotgan qoramolga beriladigan oziqa miqdori x kg bilan, uning tirik vazni y (kg) orasida quyidagicha boglanish mavjudligi aniqlangan

$$y = \frac{ax}{в + x} - c$$

bu erda a, b boqilayotgan qoramolni zotiga qarab aniqlanuvchi parametrlar. Agar maksimal oziqa berib borilsa, ma'lum bir muddatdan so'ng, qoramolni tirik vazni (a-s) maksimumga erishadi.



<u>Misol-11.</u> 1 kg yog olish uchun zarur bo'lgan sut miqdori u(kg) bilan, x-sutning yoglilik darajasi orasidagi boglanish funksiyasi $y=\frac{88}{x}$.

<u>Misol-12.</u> Kunlik sut sogib olish u- litr bilan, sigirning yoshi x- orasidagi bog'lanish funksiyasi $y=-0{,}5x^2+6{,}9x-9{,}5$ $(x>2)$ (7)

<u>Misol-13 .</u> Ekilgan bugdoy hosildorligi u(s/ga) bilan, x- 1ga maydonga ekilgan bugdoy donasi orasidagi bog'lanishni ishlab chiqarish funksiyasi. $У=5{,}6+8{,}1x-0{,}7x^2$ (9), x mln dona 1 ga maydonga ekiladigan bug'doy soni u s/ga –hosildorligi, normal bo'yicha 5,8 mln dona bug'doy 1 ga erga ekiladi.

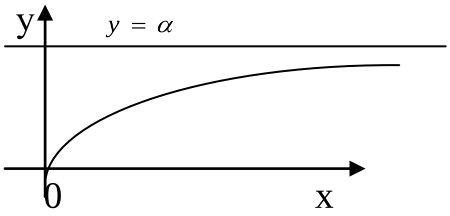
<u>Misol-14.</u> Talab va taklif funksiyasi. Talab-bu ma'lum bir mahsulotga iste'molchilarni ehtiyoji.Bu jarayon mahsulot hajmiga, bahosiga hamda iste'molchilarni sotib olish imkoniyatlariga bog'liq bo'ladi.Taklif-bu ma'lum bir bahoda bozorga sotish uchun chiqarilgan mahsulot miqdori. Agar taklif u, mahsulotni baxosi x so'm bo'lsa, y=f(x) funksiyadan iborat bo'ladi.

Mikroiqtisodiyotda talab va taklif funksiyalarini kesishish nuqtasi muhim hisoblanadi.Bu (x_0, y_0) muvozanat nuqtasi bo'lib, bu nuqtada ishlab chiqarilgan barcha mahsu-lotga talab mavjud, ya'ni zahirada mahsulot turib qolmaydi.

Zaruriy mahsulotlarga bo'lgan talab funksiyasini Tonkvist quyidagicha taklif etgan

$$Y = \frac{\alpha x}{x + \beta}$$

bu erda x-iste'molchining daromad miqdori, u- istemolchining zaruriy mahsulotlarga bo'lgan talabi. Istemolchining daromadi x oshib borsa u ham α ga intiladi.



44. Mustaqil yechish uchun misollar.

Misol. Tegsizliklarni qanoatlantiruvchi x ning o'zgarish intervallarini toping:

1. 1) $|x| < 4$; 2) $x^2 \le 9$; 3) $|x-4| < 1$;

4) $-1 < x-3 \le 2$; 5) $x^2 > 9$; 6) $(x-2)^2 \le 4$

Masalalarda $|x| \le 3$ segmentda berilgan funksiyalarning grafiklari nuqtalar bo'yicha yasalsin:

2. 1) $y = 2x$; 2) $y = 2x+2$; 3) $y = 2x-2$

3. 1) $y = x^2$; 2) $y = x^2+1$;3) $y = x^2-1$

4. 1) $y = \frac{x^3}{3}$; 2) $y = \frac{x^3}{3}+1$;3) $y = \frac{x^3}{3}-1$

5. 1) $y = \frac{6}{x}$; 2) $y = 2^x$; 3) $y = \log_2 x$

funksiyalarning grafiklari yasalsin. Bu egri chiziqlarning koordinata o'qlariga nisbatan joylashish vaziyatlarida qanday xususiyatlarni ko'rish mumkin?

Quyidagi funksiyalarni $D(y)$ aniqlanish va $E(y)$ o'zgarish sohalarini toping .

6. 1) $y = \sqrt{x+2}$; 2) $y = \sqrt{9-x^2}$; 3) $y = \sqrt{4x-x^2}$.

7. 1) $y = \sqrt{-x} + \sqrt{4+x}$; 2) $y = arc \sin \frac{x-1}{2}$.

8. 1) $y = \frac{x(2 \pm \sqrt{x})}{4}$; 2) $y = \pm x\sqrt{4-x}$

9. 1) $y = -\sqrt{2\sin x}$; 2) $y = -\frac{x\sqrt{16-x^2}}{2}$

10. 1) $y = \ln(x+3)$, 2) $y = \sqrt{5-2|x|}$

3) $y = \sqrt{\sin\sqrt{x}}$, 4) $y = arcCos\frac{1-2x}{4}$

5) $y = \ln(1-2Cosx)$, 6) $y = \lg(5x-x^2-6)$

11. Agar $f(x)$ berilgan bo'lsa, $f(f(f(x)))$ ni quyidagi funksiyalar uchun toping:

1) $f(x) = \frac{1}{1-x}$, 2) $f(x) = \frac{x}{\sqrt{1+x^2}}$, 3) $f(x) = a + вx$, 4) $f(x) = Sinx$

12. 1) $f(x) = x^2 - x + 1$ bo'lsa, $f(0)$, $f(1)$, $f(-1)$, $f(2)$, $f(a+1)$ hisoblansin;

2) $\varphi(x)=\frac{2x-3}{x^2+1}$ bo'lsa, $\varphi(0)$, $\varphi(-1)$, $\varphi\left(\frac{3}{2}\right)$, $\varphi\left(\frac{1}{x}\right)$, $\frac{1}{\varphi(x)}$ hisoblansin.

45. Sonlar ketma –ketligi.
Cheksiz kichik va cheksiz katta miqdorlar.
Кetma –ketlik va funsiya limitlari.

Natural sonlar to'plamida aniqlangan f(n), $n\in N$ funksiyaga ketma-ketlik deyiladi. N=1, 2, 3, ..., dagi qiymatlarini ketma-ketlikning hadlari deyiladi. Misol. Barcha toq sonlar ketma-ketligi 1, 3, 5, ..., 2n-1, ... $a_n=2n-1$, $n\in N$.

Ta'rif. Agar biror x_0 son topilsa va ixtiyoriy $\varepsilon>0$ son uchun biror musbat butun $N(\varepsilon)$ topilib, barcha $n>N(\varepsilon)$ larda ushbu $|x_n-x_0|<\varepsilon$ tengsizlik bajarilsa, x_0 sonini $\{x_n\}$ ketma-ketlikni limiti deyiladi va uni lim $x_n=x_0$ ko'rinishida yoziladi.

Masalan: $x_n=n+(-1)^n$ bo'lsin; $n=1, 2, 3, \ldots$ deb olsak, $0, 3, 2, 5, 4, 7, \ldots.$ ketma – ketlik hosil bo'ladi.

1°. Cheksiz kichik miqdorlar. Agar har qanday musbat ε son uchun o'zgaruvchining shunday a_o qiymati mavjud bo'lsaki, a ning undan so'nggi har bir qiymatining absolyut miqdori ε dan kichik bo'lsa, α o'zgaruvchi cheksiz kichik deyiladi. Cheksiz kichik miqdorni α limiti nolga intiladi $\alpha\to 0$.

2°. Cheksiz katta miqdorlar. Agar har qanday musbat c son uchun o'zgaruvchining shunday qiymati mavjud bo'lsaki, x ning undan so'nggi har bir qiymatning absolyut miqdori istalgan c dan katta bo'lsa, x o'zgaruvchi cheksiz katta miqdor deyiladi, u $x\to\infty$ ko'rinishda yoziladi.

Cheksiz katta miqdorlar teskari miqdor cheksiz kichik miqdo rva aksincha (cheksiz kichik o'zgauvchiga teskari miqdor cheksiz katta miqdor) bo'ladi.

3°. Agar o'zgarmas a va o'zgaruvchi x orasidagi ayirma cheksiz kichik miqdor, ya'ni agar $x=a+a$ bo'lsa, o'zgarmas a o'zgaruvchi x ning limiti deyiladi, $\lim x=a$.

Bizga $X\in R$ to'plamda aniqlangan f(x) funksiya va $\delta>0$ son berilgan bo'lsin.

Ta'rif. Agar biror A son topilib, ixtiyoriy $\varepsilon>0$ son uchun biror $\delta>0$ son topilsa va X to'plamning $|x-x_0|<\delta$ tengsizlikni qanoatlantiruvchi barcha x elementlari uchun $|f(x)-A|<\varepsilon$ tengsizlik bajarilsa, A sonini f(x) funksiyaning x argument $x_0$ga intilgandagi limiti deyiladi va uni limf(x)=A ko'rinishida yoziladi.

$\lim\limits_{x\to a-0} f(x)=b$ ni yoki $\lim\limits_{x\to a+0} f(x)=b$ $f(x)$ funksiyaning x ning a ga chapdan yoki o'ngdan intilgandagi limiti deyiladi.

Limitla nazariyasidan foydalanib $x=x_0$ nuqtada aniqlanmagan, ammo uning etarli kichik atrofida aniqlangan y=f(x) funksiya limitini hisoblash mumkin. Umumiy holda limitlar nazariyasi quyidagi aniqmasliklarni ochish uchun hizmat qiladi:

$$\left(\frac{0}{0}\right), \left(\frac{\infty}{\infty}\right), (0\cdot\infty), (\infty-\infty), (1^{\infty})$$

Funksiyaning limitini hisoblashda quyidagi asosiy teoremalardan foydalaniladi. Agar f(x) va g(x) funksiyalar uchun $x\to a$ (yoki $x\to\infty$) da chekli limf(x)=A va limg(x)=B limitlar mavjud bo'lsa, u holda quyidagi teoremalar o'rinlidir:

1) $\lim\limits_{x\to x_0}(f(x)+g(x))=\lim\limits_{x\to x_0}f(x)+\lim\limits_{x\to x_0}g(x)=A+B.$

2). $\lim\limits_{x\to x_0}(f(x)g(x))=\lim\limits_{x\to x_0}f(x)\lim\limits_{x\to x_0}g(x)=AB.$

3). *Agar* $\lim\limits_{x\to x_0}g(x)\neq 0$ *bo'lsa, u holda*

$\lim\limits_{x\to x_0}(f(x)/g(x))=A/B \qquad (B\neq 0).$

4). $\lim\limits_{x\to x_0}[f(x)]^U=[\lim\limits_{x\to x_0}f(x)]^U = A^U$

5). $\lim\limits_{x\to x_0}cf(x)=c\lim\limits_{x\to x_0}f(x)=cA$ ***s-o'zgarmas.***

Limitlar nazariyasiga doir misollar echishda quyidagi birinchi va ikkinchi ajoyib limitlardan foydalaniladi:

1) Birinchi ajoyib limit **$\lim_{x\to 0}(\sin x/x)=1$ yoki $x\to 0$, $\lim(\sin kx/kx)=1$, $k\neq 0$,**

2) . II-chi ajoyib limit $\lim_{n\to\infty}(1+1/n)^n=e$
e=2,7182818284.

46. Mavzuga doir namunaviy misollarni yechimi

Misol 1.

$$\lim_{x\to 4}\frac{x^2-6x+8}{x-4}$$

(0/0) shaklidagi aniqmaslik bo'lib, bu aniqmaslikni ochish uchun kasrni suratidagi kvadrat uchhadni chiziqli ko'paytuvchilarga ajratish $ax^2+bx+s=a(x-x_1)(x-x_2)$ formulasidan foydalanamiz natijada berilgan limit quyidagicha bo'ladi:

$$\lim_{x\to 4}\frac{x^2-6x+8}{x-4}=\lim_{x\to 4}\frac{(x-2)(x-4)}{x-4}=\lim_{x\to 4}(x-2)=\lim_{x\to 4}x-\lim_{x\to 4}2=4-2=2$$

Misol 2.

$$\lim_{x\to\infty}\frac{6x^2+7}{3x^2+2}$$

Bu $\frac{\infty}{\infty}$ aniqmaslik bo'lib, bu aniqmaslikni ochish uchun kasrni surat va maxrajini o'sib boruvchi miqdor x-ni eng katta darajasiga ya'ni x^2 ga bo'lamiz:

$$\lim_{x\to\infty}\frac{6x^2+7}{3x^2+2}=\lim_{x\to\infty}\frac{\frac{6x^2}{x^2}+\frac{7}{x^2}}{\frac{3x^2}{x^2}+\frac{2}{x^2}}=\lim_{x\to\infty}\frac{6+\frac{7}{x^2}}{3+\frac{2}{x^2}}=\frac{\lim\limits_{x\to\infty}(6+\frac{7}{x^2})}{\lim\limits_{x\to\infty}(3+\frac{2}{x^2})}=\frac{\lim\limits_{x\to\infty}6+\lim\limits_{x\to\infty}\frac{7}{x^2}}{\lim\limits_{x\to\infty}3+\lim\limits_{x\to\infty}\frac{2}{x^2}}=\frac{6+0}{3+0}=2$$

Misol 3.

$$\lim_{x\to 0}\frac{tgx}{x}$$

Aniqmaslik, bu aniqmaslikni ochish uchun birinchi ajoyib limitdan foydalanamiz

$$\lim_{x\to 0}\frac{tgx}{x}=\lim_{x\to 0}\left(\frac{\sin x}{\cos x}\cdot\frac{1}{x}\right)=\lim_{x\to 0}\left(\frac{1}{\cos x}\cdot\frac{\sin x}{x}\right)=\lim_{x\to 0}\frac{1}{\cos x}\lim_{x\to 0}\frac{\sin x}{x}=1\cdot 1=1$$

Misol4.

$$\lim_{x\to\infty}\left(\frac{2x-3}{2x+1}\right)^{4x+5}$$

$$\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^{x}=e$$

Bu (1^{∞}) shaklidagi aniqmaslikni ochish uchun ikkinchi ajoyib limitdan foydalanamiz $\lim\limits_{n\to\infty}(1+\frac{1}{n})^n=e\approx 2.7182$

$$\lim_{x\to\infty}\left(\frac{2x-3}{2x+1}\right)^{4x+5}=\lim_{x\to\infty}\left(\frac{2x+1-4}{2x+1}\right)^{4x+5}=\lim_{x\to\infty}\left(1+\frac{-4}{2x+1}\right)^{4x+5}$$

$$2x+1=-4y;4x+5=-8y+3;\ y\to\infty$$

$$\lim_{x\to-\infty}\left(1+\frac{1}{y}\right)^{-8y+3}=\left[\lim_{x\to\infty}\left(1+\frac{1}{y}\right)^{y}\right]^{-8}\cdot\lim_{y\to\infty}\left(1+\frac{1}{y}\right)^{3}=e^{-8}\cdot 1^3=\frac{1}{e^8}$$

47. **Mustaqil yechish uchun misollar.**

1. $n=0,\ 1,\ 2,\ \dots$ deb

$$\alpha=\frac{1}{2^n},\alpha=-\frac{1}{2^n},\alpha=\left(-\frac{1}{2}\right)^n$$

2. $x=1+\frac{(-1)^n}{2n+1}$ o’zgaruvchi qiymatlarining ketma –ketligi yozilsin va uning o’zgarishi grafik usulda tasvirlansin. n ning qaysi qiymatidan boshlab $x-1$ ning moduli 0,001 dan, berilgan musbat ε dan kichik bo’ladi va shunday bo’lib qola beradi?

3. 1) $x=\frac{n}{n+1}$; 2) $x=-\frac{n}{n+1}$; 3) $x=\frac{(-1)^n n}{n+1}$;

4) $x=\frac{8\cos n\frac{\pi}{2}}{n+4}$; 5) $x=\frac{2n+(-1)^n}{n}$; 6) $x=2^{-n}a\cos n\pi$

o'zgaruvchilar qiymatlarining etma – ketliklari yozilsin va ularning o'zgarishi grafik usulda tasvirlansin. Har bir misol uchun $\lim\limits_{n\to\infty} x$ mavjudmi va u nimaga teng?

Ushbu (1-73) limitlarni yeching

1) $\lim\limits_{x\to 2}\dfrac{3x^2-7x+2}{2x^2-5x+2}$

2) $\lim\limits_{x\to 0}\dfrac{\sin 5x}{tg\,2x}$

3) $\lim\limits_{x\to\infty}\left(\dfrac{x+1}{x-2}\right)^{2x+3}$

4) $\lim\limits_{x\to 7}\dfrac{x^2-8x+7}{(x-7)^2}$

5) $\lim\limits_{x\to 2}\dfrac{\sqrt{3x-2}-2}{x-2}$

6) $\lim\limits_{x\to 0}\dfrac{\sin 6x}{3x}$

7) $\lim\limits_{x\to 0}(1+x)^{\frac{2}{x}}$

8) $\lim\limits_{x\to -1}\dfrac{2x^2+x-1}{5x^2+4x-1}$

9) $\lim\limits_{x\to\infty}\dfrac{1+2x-x^2}{4x^2-5x+2}$

10) $\lim\limits_{x\to\infty}\left(\dfrac{2x+3}{2x-1}\right)^{4x}$

11) $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x^2-1}$

12) $\lim\limits_{x\to\infty}\dfrac{2x^2-3}{4x^3+5x}$

13) $\lim\limits_{x\to 0} tg\,2x\cdot ctg\,4x$

14) $\lim\limits_{x\to 0}(1+x)^{\frac{3}{x}}$

15) $\lim\limits_{x\to -1}\dfrac{2x^2+3x+1}{x^3+1}$

16) $\lim\limits_{x\to\infty}(\sqrt{x+1}-\sqrt{x})$

17) $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{x^2}$

18) $\lim\limits_{x\to\infty}\left(\dfrac{x+4}{x+1}\right)^{2x+2}$

19) $\lim\limits_{x\to 2}\dfrac{3x^2-10x+8}{x^2-4}$

20) $\lim\limits_{x\to\infty}\dfrac{\sqrt{x}-2x}{3x+1}$

21) $\lim\limits_{x\to 0}\dfrac{x}{arctg\,2x}$

22) $\lim\limits_{x\to\infty}\left(\dfrac{2x+1}{2x-1}\right)^{3x}$

23) $\lim\limits_{x\to 1}\dfrac{x^2-2x+1}{2x^2-x-1}$

24) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$

25) $\lim\limits_{x\to 0}\dfrac{x\sin x}{1-\cos x}$

26) $\lim\limits_{x\to\infty}\left(\dfrac{3x+2}{3x+1}\right)^{6x-4}$

27) $\lim\limits_{x\to 3}\dfrac{x^2-2x-3}{x^2-9}$

28) $\lim\limits_{x\to 4}\dfrac{\sqrt{2x+1}-3}{\sqrt{x}-2}$

29) $\lim\limits_{x\to\infty}\left(\dfrac{x+3}{x+4}\right)^{2x-1}$

30) $\lim\limits_{x\to-1}\dfrac{x^2-x-2}{x^3+1}$

31) $\lim\limits_{x\to\infty}(\sqrt{x^2+1}-x)$

32) $\lim\limits_{x\to 0}\dfrac{\sin x}{tg\,3x}$

33) $\lim\limits_{x\to 0}(1+5x)^{\frac{3}{x}}$

34) $\lim\limits_{x\to-3}\dfrac{2x^2+5x-3}{x^2+4x+3}$

35) $\lim\limits_{x\to 0}\dfrac{\sqrt{2x+1}-1}{\sqrt{3x+4}-2}$

36) $\lim\limits_{x\to 0}\dfrac{\arcsin 2x}{x}$

37) $\lim\limits_{x\to\infty}\left(\dfrac{4x+2}{4x-1}\right)^{2x+3}$

38) $\lim\limits_{x\to-2}\dfrac{3x^2+5x-2}{x^2+3x+2}$

39) $\lim\limits_{x\to 0}\dfrac{x}{\sqrt[3]{8-x}-\sqrt[3]{8+x}}$

40) $\lim\limits_{x\to 0} tg\,3x\cdot ctg\,5x$

41) $\lim\limits_{x\to\infty}\left(\dfrac{x-3}{x+4}\right)^{x-1}$

42) $\lim\limits_{x\to 3}\dfrac{x^2-4x+3}{x^2-9}$

43) $\lim\limits_{x\to\infty}\dfrac{2x-4}{x+\sqrt[3]{x}}$

44) $\lim\limits_{x\to 0}\dfrac{x}{arctg\,3x}$

45) $\lim\limits_{x\to\infty}\left(\dfrac{3x-2}{3x-1}\right)^{6x+4}$

46) $\lim\limits_{x\to 1}\dfrac{5x^2-4x-1}{x^2-6x+5}$

47) $\lim\limits_{x\to-1}\dfrac{\sqrt{x+2}+x}{x+1}$

48) $\lim\limits_{x\to 0}\dfrac{\sin 5x}{\sin 4x}$

49) $\lim\limits_{x\to 0}(1-2x)^{\frac{4}{x}}$

50) $\lim\limits_{x\to-1}\dfrac{7x^2+8x+1}{2x+2}$

51) $\lim\limits_{x\to\infty}(x-\sqrt{x^2+3x})$

52) $\lim\limits_{x\to 0}\dfrac{tg\,6x}{tg\,3x}$

53) $\lim\limits_{x\to\infty}\left(\dfrac{5x+2}{5x-3}\right)^{2x+1}$

54) $\lim\limits_{x\to-4}\dfrac{2x^2+6x-8}{x^2-16}$

55) $\lim\limits_{x\to 0}\dfrac{x^2}{\sqrt[3]{x^2+1}-1}$

56) $\lim\limits_{x\to 10}\dfrac{5x^2-51x+10}{2x-20}$

57) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{x^2}$

58) $\lim\limits_{x\to 0}\dfrac{3x}{\arcsin 2x}$

59) $\lim\limits_{x\to\infty}\left(\dfrac{4x-3}{4x+2}\right)^{2x+1}$

60) $\lim\limits_{x\to 3}\dfrac{2x^2-7x+3}{x^2-x-6}$

61) $\lim\limits_{x\to 5}\dfrac{x^2-25}{\sqrt{2x-1}-3}$

62) $\lim\limits_{x\to 0} x\cdot ctg\,3x$

63) $\lim\limits_{x\to -1}(2x^3+3)^{\frac{1}{x+1}}$

64) $\lim\limits_{x\to -1}\dfrac{3x^2+2x+1}{2x^2+3x+1}$

65) $\lim\limits_{x\to\infty}\dfrac{x^2-3x+1}{4x^2-5x+2}$

66) $\lim\limits_{x\to 0} x\cdot ctg\,2x$

67) $\lim\limits_{x\to\infty}\left(\dfrac{x+3}{x+1}\right)^{x-2}$

68) $\lim\limits_{x\to 4}\dfrac{8+2x-x^2}{x^2-16}$

69) $\lim\limits_{x\to\infty}\dfrac{x^2-x+3}{x^3+2x^2-1}$

70) $\lim\limits_{x\to 0}\dfrac{tg\,3x}{x}$

71) $\lim\limits_{x\to 4}(x-3)^{\frac{2}{x-4}}$

72) $\lim\limits_{x\to 0}\dfrac{x}{\arcsin\,3x}$

73) $\lim\limits_{x\to 0}\dfrac{tg\,5x}{tg\,3x}$

48. Funksiyaning uzluksizligi va uzilish nuqtalari. Funksiya va argument orttirmalari.

Funksiyaning uzluksizligi -matematikaning muhim tushunchalaridan biridir. Qishloq xo'jaligida mikroorganizmlarni ko'payishi, g'o'zani o'sishi, boqilayotgan qoramol vaznining o'sishi uzluksiz jarayonlarga misol bo'ladi.

X sohada aniqlangan y=f(x) funksiya berilgan bo'lsin. $x_0, x_1 \in X$ nuqtalarni x_1-x_0 ayirmasi $\Delta x=x_1-x_0$ argument orttirmasi , $f(x_0+\Delta x)-f(x_0)$ ayirma f(x) funksiyani $x=x_0$ nuqtadagi orttirmasi $(\Delta f(x_0))$ $\Delta y=f(x_0+\Delta x)-f(x_0)$ deyiladi.

Agar $x_0\in X$ nuqtaning istalgan $\{x;\ x_0-\delta<x<x_0+\delta\}$ $\delta > 0$, atrofida X sohaning cheksiz ko'p elementlari yotsa, x_0 nuqta X - sohaning limitik nuqtasi deyiladi.

Agar $x\to x_0$ da $\lim f(x)=f(x_0)$ (1) tenglik o'rinli bo'lsa, f(x) funksiya $x=x_0$ nuqtada uzluksiz deyiladi.

Funksiyaning $x=x_0$ nuqtadagi uzluksizligiga argument va funksiya orttirmalari yordamida ham ta'rif berish mumkin. f(x) funksiyaning $x=x_0$ nuqtadagi $\Delta f=f(x_0+\Delta x)-f(x_0)$, orttirmasi $\Delta x\to 0$ da, ya'ni $\lim_{\Delta x\neq 0}f(x)=\lim_{\Delta x\to 0}(f(x_0+\Delta x)-f(x_0))=0$ bo'lsa, u holda f(x) funksiyani xqx_0 nuqtada uzluksiz deyiladi.

Agar f(x) funksiya X soxaning har bir nuqtasida uzluksiz bo'lsa, funksiya shu sohada uzluksiz deyiladi.

Ko'rsatish mumkinki, butun rasional, kasr rasional, darajali, ko'rsatkichli, logarifmik va barcha elementar funksiyalar o'zlarining aniqlanish sohasida uzluksizdir.

Funksiyaning uzilishi nuqtalari va uning turlari.

Agar $\lim_{x\to x_0}(x)\neq f(x_0)$ bo'lsa, u holda f(x) funksiya xqx$_0$ nuqtada uzilishga ega deyiladi. Bu tenglik bajarilmaydigan 3 ta hol mavjud: 1) f(x) funksiyaning xqx$_0$ nuqtadagi o'ng va chap limitlari mavjud bo'lib, ular bir-biriga teng emas 2) $x=x_0$ nuqtada f(x) funksiyaning limiti mavjud bo'lsada, $x=x_0$ nuqta f(x) ning aniqlanish sohasiga tegishli emas 3) $x\to x_0$ intilganda f(x) funksiyaning limiti mavjud bo'lmagan hol.

1 va 2 hollarda f(x) funksiya birinchi tur uzilishga ega bo'lib $f(x_0+0)-f(x_0-0)$ ayirma f(x) ni $x=x_0$ nuqtadagi sakrashi deyiladi. 3 holda, ya'ni f(x) ning $x\to x_0$ dagi limiti mavjud bo'lmasa, $\pm\infty$ bo'lsa, f(x) funksiyani $x=x_0$ da ikkinchi tur uzilishga ega deyiladi.

49. Mavzuga doir namunaviy misollarni yechimi.

Misol-1. Uzluksizlikning ta'rifidan foydalanib, $y = x^2$ funksiya son o'qining istalgan nuqtasida uzluksiz ekanligini ko'rsating.

Yechilishi. $x = x_0$ nuqta son to'gri chizigining istalgan nuqtasi bo'lsin; funksiyaning bu nuqtadagi qiymati $y(x_0) = x_0^2$; $x = x_0$ argumentga ixtiyoriy Δx orttirma beramiz. Natijada funksiya

$$\Delta y = (x_0 + \Delta x)^2 - x_0^2 = 2x_0 \cdot \Delta x + \Delta x^2$$

ga teng bo'lgan birorta Δy orttirma oladi. Limitga o'tsak

$$\lim_{\Delta x \to 0} \Delta y = \lim_{\Delta x \to 0} (2x_0 \cdot \Delta x + \Delta x^2) = 0$$

Demak, $y = x^2$ funksich uzluksizlikning birinchi ta'rifiga ko'ra sonlar o'qing istalgan $x = x_0$ nuqtasida uzluksiz ekan.

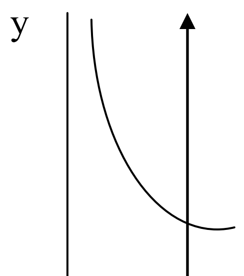
Misol-2. $y = \frac{1}{x+3}$ funksiya berilgan. Uzilish nuqtalarini toping va ularning xarakterini tekshiring. Uzilish nuqtalari atrofida funksiyalarning sxematik grafiklarini yasang.

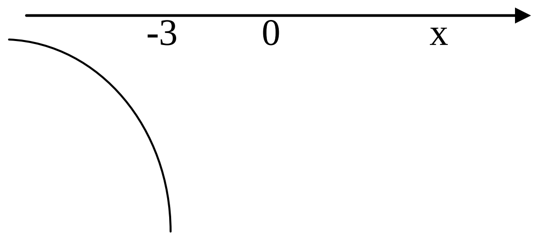
Yechilishi. $y = \frac{1}{x+3}$ funksiya x ning $x = -3$ qiymatidan tashqari barcha qiymatlarida aniqlangan. Bu funksiya elementar bo'lgani uchun, u o'zining $(-\infty,-3)$ va $(-3,+\infty)$ intervallardan iborat bo'lgan aniqlanish sohasining har bir nuqtasida uzluksizdir.

Demak, yagona uzilish nuqtasi $x = -3$ dan iborat ekan (funksiya bu nuqta atrofida aniqlangan, bu nuqtaning o'zida esa uzluksiz-likning sharti buziladi – funksiya bu nuqtada aniqlanmagan). Uzilish xrakterini tekshirish uchun funksiyaning x argumentning $x = -3$ uzulish nuqtasiga intilgandagi chap va o'ng limitlarini topamiz:

$$\lim_{x\to 3+0} \frac{1}{x+3} = +\infty \qquad \lim_{x\to 3-0} \frac{1}{x+3} = -\infty \ .$$

Demak, $x = -3$ nuqtada $y = \frac{1}{x+3}$ funksiya II- tub uzilishga egar.





$x = 0$ nuqtada bu funksiyaning bir tomonlama limitlarini topamiz:

Misol-3. Uzilish nuqtalarini toping va ularning xarakterini tekshiring. Uzilish nuqtalari atrofida funksiyalarning sxematik grafiklarini yasang.

$$y = \begin{cases} x \le 0 & \partial a \quad 2 - x. \\ 0 < x < \dfrac{\pi}{2} & \partial a \quad \cos x, \\ x \ge \dfrac{\pi}{2} & \partial a \quad 0. \end{cases}$$

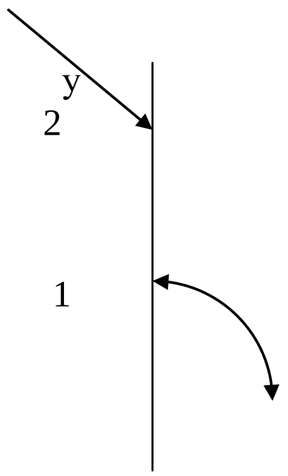
Yechilishi. $y = f(x)$ funksiyaning aniqlanish sohasi bo'lgan son o'qi $(-\infty, \ 0)$, $\left(0, \ \dfrac{\pi}{2}\right)$, $\left(\dfrac{\pi}{2}, \ +\infty\right)$ integravallarga bo'lingan bo'lib, ularning har birida $f(x)$ funksiya mos ravishda $\varphi(x) = 2 - x$, $\varphi(x) = \cos x$, $\eta(x) = 0$ elementar funksiyalar bilan berilgan (3 -rasm). Belgilangan bu intervallarning ichida bu funksiyalar aniqlangan va demak uzuluksiz. Shunday qilib, funksiyani $y = (x)$ funksiyani tashkil etuvchi funksiyalarning aniqlanish sohalari «uchrashadigan» $x = 0$ va $x = \dfrac{\pi}{2}$ nuqtalardigina uzluksizligini tekshirish qoladi.

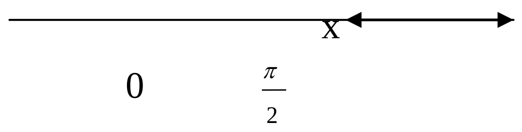
$y = f(x)$ funksiyaning $x = 0$ nuqtadagi bir tomonlama limitlarini hisoblaymiz. $x \le 0$ da $f(x) = 2 - x$ bo'lganligi uchun

$$\lim_{x \to -0} f(x) = \lim_{x \to -0} (2 - x) = 2$$

$0 < x < \dfrac{\pi}{2}$ da $f(x) = \cos x$ bo'lganligi uchun

$$\lim_{x \to +0} f(x) = \lim_{x \to +0} \cos x = 1$$





Demak, $x = 0$ nuqta I tur uzulish nuqtasi ekan; unda $y = f(x)$ funksiya sakrashga ega. $y = f(x)$ funksiyaning $x = \frac{\pi}{2}$ nuqtadagi bir tomonlama limitlari quyidagichadir:

$$\lim_{x \to \frac{\pi}{2} - 0} f(x) = \lim_{x \to \frac{\pi}{2} - 0} \cos x = 0 \qquad \left[0 < x < \frac{\pi}{2} \quad \text{da} \quad f(x) = \cos x \right];$$

$$\lim_{x \to \frac{\pi}{2} + 0} f(x) = \lim_{x \to \frac{\pi}{2} + 0} = 0 \left[x \geq \frac{\pi}{2} \quad \text{da} \quad f(x) = 0 \right].$$

$f(x)$ funksiyaning $x = \frac{\pi}{2}$ nuqtadagi qiymat $f(\frac{\pi}{2}) = 0$ ga teng. Bu erdan $x = \frac{\pi}{2}$ nuqtada $f(x)$ funksiyaning uzluksizligi kelib chiqadi, chunki

$$\lim_{x \to \frac{\pi}{2} - 0} f(x) = \lim_{x \to \frac{\pi}{2} + 0} f(x) = f\left(\frac{\pi}{2}\right) = 0$$

Shunday qilib, tekshiralayotgan $y = f(x)$ funksiya butun son o'qining $x = 0$ nuqtasidan tashqari hamma nuqtalarida uzluksiz. Bu nuqtada u I tur uzulishga – sakrashga ega.

50. Mustaqil yechish uchun misollar.

Masalalarda uzluksizlikning birinchi ta'rifidan foydalanib, berilgan funksiyalar o'zlarining butun aniqlanish sohalarida uzluksiz ekanligini ko'rsating.

1. $y = x^2 - 2x$. 2. $y = 1 - x^3$. 3 $y = \frac{1}{x^2 + 1}$.

4. $y = e^x$. 5. $y = \sin x$. 6. $y = \cos 3x$

Quyidagi masollarda berilgan har qaysi funksiya uchun uzulish nuqtasini toping va ularning xarakterini tekshiring.

7. $y = \frac{1}{2 - x}$. 8. $y = \frac{1}{(x + 5)^2}$. 9. $y = \frac{2}{x^2 - 1}$.

10. $y = \frac{1}{x^2 - 4x + 3}$. 11. $y = \frac{3}{2^x - 1}$. 12. $y = \frac{\sin x}{x}$.

13. $y = tg(x - 3)^2$. 14. $y = \frac{1}{\lg |x|}$. 15. $y = 5^{1/x} + 2$.

16. $y = arcrg \frac{1}{x}$. 17. $y = \lg |\sin x|$. 18. $y = arcctg \frac{1}{x^2}$.

22. $y = \begin{cases} x \leq 1 & \partial a & x^2 \\ x > 1 & \partial a & x + 1 \end{cases}$. 23. $y = \begin{cases} x < -1 & \partial a & -x - 1 \\ -1 \leq x < 0 & \partial a & 0 \\ x \geq 0 & \partial a & \sqrt{x} \end{cases}$.

24. $y = \begin{cases} x \le & \partial a \quad x \\ x > 0 & \partial a \quad \dfrac{1}{x} \end{cases}$. 25. $y = \begin{cases} x < 0 & \partial a \quad \dfrac{1}{x} \\ 0 \le x < \dfrac{\pi}{2} & \partial a \quad \sin x \\ x \ge \dfrac{\pi}{2} & \partial a \quad 0 \end{cases}$.

51. Funksiya hosilasi va differensiali. Hosilani hisoblash qoidalari.

Bizga y=f(x) uzluksiz funksiya berilgan bo'lsin, x va x_0 funksiyaning aniqlanish sohasiga qarashli argumentni ikkita qiytmati bo'lsa, $\Delta x = x - x_0$ argument orttirmasi $x=(x_0+\Delta x)$, $\Delta y = \Delta f(x_0) = f(x_0+\Delta x) - f(x_0)$ funksiyaning x_0 nuqtadagi orttirmasi bo'ladi.

Ma'lumki, funksiya orttirmasining argumenti orttirmasiga nisbati argument orttirmasi nolga $\Delta x \to 0$ intilganda chekli limitga ega bo'lsa, bu funksiyaning x_0 nuqtadagi hosilasi deyiladi va quyidagicha yoziladi.

$\lim \dfrac{\Delta y}{\Delta x} = \lim \dfrac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = y^1$ Hosila y^1, $f^1(x)$ - shaklida yozilishi mumkin.

У=f(x) funksiyani biror nuqtada hosilaga ega bo'lishi uchun, uzluksiz bo'lishidan tashqari nisbatning limiti shu nuqtada $\Delta x \to 0$ mavjud bo'lishi kerak.

Hosilani ta'rifidan foydalanib, barcha elementar funksiyalarni hosilalarini keltirib chiqarish mumkin.

Funksiya hosilalarini hisoblashda quyidagi hosila jadvalidan foydalaniladi:

$1.\ C' = 0,$ $2.\ (x)' = 1,$ $3.\ (x^n)' = nx^{n-1},$

$4.\ (\sqrt{x})' = \dfrac{1}{2\sqrt{x}},$ $5.\ \left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2}$ $6.\ (a^x)' = a^x \ln a,$

$7.\ (e^x)' = e^x,$ $8.\ (\log_a x) = \dfrac{1}{x \ln a},$ $9.\ (\ln x)' = \dfrac{1}{x},$

$10.\ (\sin x)' = \cos x,$ $11.\ (\cos x)' = -\sin x,$ $12.\ (tgx)' = \dfrac{1}{\cos^2 x}$

$13.\ (ctgx)' = -\dfrac{1}{\sin^2 x}$ $14.\ (\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$ $15.\ (\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}}$

$16.\ (arctgx)' = \dfrac{1}{1+x^2}$ $17.\ (arcctgx)' = -\dfrac{1}{1+x^2}$ $18.\ (U+V+W)' = U'+V'+W'$

$19.\ (UV)' = U'V + UV'$ $20.\ (CU)' = CU'$ $21.\ \left(\dfrac{U}{V}\right)' = \dfrac{U'V - UV'}{V^2}$

22) Agarda y=f(t) bo'lib, t=φ(x) bo'lsa, u xolda murakkab funksiyani x argument bo'yicha hosilasi $y_x^1 = y_t^1 * t_x^1$ yoki dy/dx=dy/dt*dt/dx

52. Mavzuga doir namunaviy misollarni yechimi.

Misol-1. $y=5x^4-3x^2+6$ funksiyani hosilasi va differensialini toping.

Yuqorida keltirilgan formulalarga asosan $y^1=(5x^4)^1-(3x^2)^1+(6)^1 =5(x^4)^1-3(x^2)^1+0= 5*4x^3-3*2x=20x^3-6x$ bo'ladi.

Ma'lumki funksiyaning differensiali $dy=f^1(x)dx$ bo'lganligi uchun bizni misolda $dy=(20x^3-6x)dx$ bo'ladi.

Misol-2. $y=4\sqrt[3]{x^2}+\frac{2}{x^5}-\frac{3}{\sqrt{x}}$ $y^1=?$; $dy=?$

Funksiyani berilishini $\sqrt[n]{x^k}=x^{\frac{k}{n}}$ ва $\frac{1}{x^n}=x^{-n}$ tengliklar yordamida quyidagicha yozish mumkin: $y=4x^{\frac{2}{3}}+2x^{-5}-3x^{-\frac{1}{2}}$ 1 misoldagidek mulohaza bilan hosila jadval formulalaridan foydalanib

$$y=4*\frac{2}{3}x^{\frac{2}{3}-1}+2*(-5)x^{-5-1}-3(-\frac{1}{2})x^{-\frac{1}{2}-1}=\frac{8}{3}x^{-\frac{1}{3}}-10x^{-6}+\frac{3}{2}x^{-\frac{3}{2}}=\frac{8}{3\sqrt[3]{x}}-\frac{10}{x^6}+\frac{3}{2\sqrt{x^3}}$$ hamda

$$dy=\left(\frac{8}{3\sqrt[3]{x}}-\frac{10}{x^6}+\frac{3}{2\sqrt{x^3}}\right)dx$$

Misol-3. $y=x^3\ln x$, $y^1=?$, $dy=?$

(19) formulaga asosan $y^1=(x^3)^1\ln x+(\ln x)^1x^3= x^3\ln x+x^3/x=3x^2\ln x+x^2$

$dy=x^2(3\ln x+1)dx$

Misol-4. $y=\frac{\cos x}{1+2\sin x}, y^1=?, dy=?$

(21) formuladan, ya'ni kasrni (bo'linmani) hosilasini topish formulasiga asosan

$$y^1=\left(\frac{(\cos x)^1(1+2\sin x)-\cos x(1+2\sin x)^1}{(1+2\sin x)^2}\right)=\left(\frac{-\sin x(1+2\sin x)-\cos x*2\cos x}{(1+2\sin x)^2}\right)=\left(\frac{-\sin x-2(\sin^2 x+\cos^2 x)}{(1+2\sin x)^2}\right)=-\frac{(\sin x+2)}{(1+2\sin x)^2}$$

$$dy=-\frac{(\sin x+2)}{(1+2\sin)^2}dx$$

Misol-5. $y=\ln^3x$, $y^1=?$, $dy=?$, $t=\ln x$ deb belgilasak, bu erda (22)ga asoan $y^1=3t^2$, $t^1_x=1/x$ bo'lganligidan $y^1=3t^2\frac{1}{x}=\frac{3\ln^2 x}{x}, dy=\frac{3\ln^2 x}{x}dx$

bo'ladi.

53 . Mustaqil yechish uchun misollar.

1-20 misollarda quyidagi funksiyalarni hosilalarini va differensiallarini toping.

$1.a)\ y=5x^3+8\sqrt[3]{x}-\frac{4}{3x^3}-2$ $\qquad b)\ y=\ln x\cdot\arcsin x$

$c)\ y=\frac{2-x^2}{x^3-6x}$ $\qquad d)\ y=(2x^2+3)$

$2.a)\ y=2x^5+\sqrt[5]{x^3}-\frac{1}{4x^3}-3$ $\qquad b)\ y=\ln x\cdot tgx$

$c)\ y=\dfrac{x+3}{x^2-6x-9}$ $\quad$ $d)\ y=\sqrt[3]{1+\cos x}$

$3.a)\ y=5x^7+10\sqrt[5]{x^2}-\dfrac{5}{3x^6}+6$ $\quad$ $b)\ y=x^3\ln(x+1)\cdot tgx$

$c)\ y=\dfrac{x^3+3}{3x+1}$ $\quad$ $d)\ y=3^{\arcsin x}$

$4.a)\ y=8x^5+3\sqrt[3]{x^2}-\dfrac{2}{5x^4}-8$ $\quad$ $b)\ y=4tgx\cdot arctgx$

$c)\ y=\dfrac{\sin x}{x^3+5}$ $\quad$ $d)\ y=(ctgx-3)^4$

$5.a)\ y=2x+\sqrt[3]{x}+\dfrac{8}{x^3}-1$ $\quad$ $b)\ y=(x^2+1)arctgx$

$c)\ y=\dfrac{1-x^2}{x^3-3x}$ $\quad$ $d)\ y=(x^2-\ln x)^4$

$6.a)\ y=2x^3-2\sqrt[5]{x^2}-\dfrac{1}{x}+8$ $\quad$ $b)\ y=x^3\arcsin x$

$c)\ y=\dfrac{3x+2}{x^2-6x+3}$ $\quad$ $d)\ y=\sqrt[5]{(x^3+4x-5)^2}$

$7.a)\ y=5x^4-\dfrac{8}{x^3}-2\sqrt[5]{x}-6$ $\quad$ $b)\ y=2x^3\cdot tgx$

$c)\ y=\dfrac{\sin x}{x-\cos x}$ $\quad$ $d)\ y=(\ln\sin x-x)^2$

$8.a)\ y=2x^2-4\sqrt{x}-\dfrac{1}{3x^2}+5$ $\quad$ $b)\ y=x^3arctgx$

$c)\ y=\dfrac{\cos x}{2+\sin x}$ $\quad$ $d)\ y=(3x^2+e^{2x})^2$

$9.a)\ y=3x+3\sqrt[4]{x}-\dfrac{2}{3x^2}-3$ $\quad$ $b)\ y=x^2\arcsin x$

$c)\ y=\dfrac{\ln x}{x+3}$ $\quad$ $d)\ y=\ln\sqrt{2x-3}$

$10.a)\ y=-3x^2+4\sqrt[9]{x^2}+\dfrac{8}{8x^3}-6$ $\quad$ $b)\ y=(x^2+2)\ln x$

$c)\ y=\dfrac{x-6}{\ln x}$ $\quad$ $d)\ y=arctg\sqrt{x+1}$

$11.a)\ y=3x^2-\dfrac{3}{x^2}-\dfrac{1}{3\sqrt[3]{x^2}}+7$ $\quad$ $b)\ y=x^3(\ln x+3)$

$c)\ y=\dfrac{x+2\cos x}{x^3}$ $\quad$ $d)\ y=arctg\left(\dfrac{x+1}{x-1}\right)$

$12.a)\ y=3x^4-\dfrac{3}{x^4}+\dfrac{7}{\sqrt{x}}-5$ $\qquad b)\ y=2x^3\arccos x$

$c)\ y=\dfrac{x^2-1}{x^3+2}$ $\qquad d)\ y=\arccos\sqrt{1-x}$

$13.a)\ y=3x^3+\dfrac{3}{x^3}-\dfrac{2}{\sqrt[3]{x^2}}+3$ $\qquad b)\ y=\ln x\cdot arcctgx$

$c)\ y=\dfrac{x^3}{x^3+3}$ $\qquad d)\ y=\ln(x^2+8)$

$14.a)\ y=2x^5+\sqrt[5]{x^3}-\dfrac{4}{x^2}+3$ $\qquad b)\ y=x^4\sin x$

$c)\ y=\dfrac{\sin x}{1+tgx}$ $\qquad d)\ y=e^{\arccos x}$

$15.a)\ y=5x^3-\dfrac{5}{3x^2}+10\sqrt[5]{x^2}-6$ $\qquad b)\ y=x^2\sin x$

$c)\ y=\dfrac{2\sin x}{1+\cos x}$ $\qquad d)\ y=\arcsin\dfrac{1}{x}$

$16.a)\ y=8x^5-\dfrac{2}{5x^3}+3\sqrt[4]{x^3}+7$ $\qquad b)\ y=x^6\cos x$

$c)\ y=\dfrac{1+\sin x}{1-\cos x}$ $\qquad d)\ y=\arcsin\dfrac{x}{2}$

$17.a)\ y=2x^5-\dfrac{1}{3x^3}+\sqrt[5]{x^3}-7$ $\qquad b)\ y=e^x\ln x$

$c)\ y=\dfrac{\arcsin x}{1-x^2}$ $\qquad d)\ y=\cos x-2\sin^2 x$

$18.a)\ y=2x^3-\dfrac{1}{x}+4\sqrt[5]{x^2}-8$ $\qquad b)\ y=x^3 arctgxe^x\ln x$

$c)\ y=\dfrac{1+x}{1-\sqrt{x}}$ $\qquad d)\ y=arctg\sqrt{x}$

$19.a)\ y=5x^3-\dfrac{2}{3x^2}+3\sqrt[8]{x}+5$ $\qquad b)\ y=x^5\ln x$

$c)\ y=\dfrac{\cos x}{2+\sin x}$ $\qquad d)\ y=\arccos(3x^3)$

$20.a)\ y=7x^5-\dfrac{4}{x^3}+\sqrt[3]{x}-6$ $\qquad b)\ y=\sin x\cdot\cos x$

$c)\ y=\dfrac{\ln x}{x^3-1}$ $\qquad d)\ y=e^{2x^3}$

21. a) $y=\left(3x-4\sqrt[3]{x}+2\right)^4$, $\qquad$ b) $y=\dfrac{4x+7tgx}{\sqrt{1+9x^2}}$,

v) $y = \cos 3x \cdot e^{\sin x}$, g) $y = \ln arctg\, 2x$.

22. a) $y = \left(3x^3 - 2\sqrt[3]{x^2} - 1\right)^2$, b) $y = \dfrac{\arcsin 3x}{1 - 8x^2}$,

v) $y = 2^{3x} tg\, 2x$, g) $y = \cos \ln 5x$.

23. a) $y = \left(x^2 - \dfrac{1}{x^3} + 5\sqrt{x}\right)^4$, b) $y = \dfrac{\arcsin 7x}{x^4 + e^x}$,

v) $y = e^{tgx} \ln 2x$, g) $y = \cos \sqrt{x^2 + 3}$.

24. a) $y = \left(4x^2 - \dfrac{3}{\sqrt{x}} + 4\right)^3$, b) $y = \dfrac{\sin 2x}{\cos 5x}$,

v) $y = 2^{8x} tg\, 3x$, g) $y = \arcsin \ln 4x$.

25. a) $y = \left(x^5 - \sqrt[3]{x} + 1\right)^5$, b) $y = \dfrac{\sqrt{1 - 4x^2}}{2^x + tgx}$,

v) $y = e^{ctgx} \cdot \sin 4x$, g) $y = \sin \ln 5x$

26. a) $e = \left(6x^2 - \dfrac{2}{x^4} + 5\right)^2$, b) $y = \dfrac{\cos 3x}{\sqrt{3x^2 + 4}}$,

v) $y = 3^{tgx} \arcsin(x^2)$, g) $y = \ln \sin 6x$.

27. a) $y = \left(x^3 - 4\sqrt[4]{x^3} + 2\right)^3$, b) $y = \dfrac{arctg\, 7x}{2 - 9x^2}$,

v) $y = e^{ctg} \cos 6x$, g) $y = \sin \ln 2x$.

28. a) $y = \left(x^2 - 2\sqrt[5]{x} + 4\right)^4$, b) $y = \dfrac{x^3 + e^x}{\sqrt{4 - 9x^5}}$,

v) $y = 4^{\cos x} arctg\, 2x$, g) $y = \ln \cos 5x$.

29. a) $y = \left(3x^5 - \dfrac{5}{x^3} - 2\right)^5$, b) $y = \dfrac{\cos 6x}{\sin 3x}$,

v) $y = e^{x^3} tg\, 7x$, g) $y = \arcsin \ln 2x$.

30. a) $y = \left(x^4 + 2\sqrt[3]{x} + 1\right)^2$, b) $y = \dfrac{\sqrt{3 - 5x^3}}{e^x - ctgx}$,

v) $y = 2^{\sin x} \arcsin 2x$, g) $y = \ln \cos 7x$.

54.HOSILANI IQTISODIY MASALALARGA TADBIQLARI.

Faraz qilaylik ishlab chiqarish xarajatlari maxsulot miqdori y ning funksiyasidan iborat bo'lsin $y = k(x)$. Xarajat miqdori $x + \Delta x$ ortsa unga mos keluvchi maxsulot miqdori $k(x + \Delta x)$ bo'ladi, ya'ni $\Delta k = k(x + \Delta x) - k(x)$. Bundan o'rtacha maxsulot ishlab chiqarishini o'zgarish tezligi $\dfrac{\Delta k}{\Delta x} = \dfrac{k(x + \Delta x) - k(x)}{\Delta x}$ Yoki, birlik xarajatiga mos keluvchi ishlab chiqarilgan maxsulot miqdorini me'yoriy chegarasi deyiladi.

Misol 1. boqiladigan mollarni o'sish vazni y (kg), ularga beriladigan oziqa miqdori X ga bogliq bo'ladi. Berilayotgan makkajo'xori miqdori X (kg) (12 foiz oqsil saqlaganda) mollarni o'sish vazni u bilan quyidagicha funksional boglanishga ega bo'lsa,

$$y = -0.0001\ x^2 + 0.433\ x - 3.062$$

$x = 100$ kg makkajo'xori berilganda qoramol vazni o'sishini aniqlang.

Echish. $y + \Delta y = -0{,}0001\ (x + \Delta x)^2 + 0{,}433\ (x + \Delta x) - 3{,}062$ bundan Δy ni topsak

$\Delta y = 0{,}433\ \Delta x - 0{,}0002\ x \cdot \Delta x - 0{,}0001\ \cdot \Delta x^2$

bundan $\frac{\Delta y}{\Delta x} = 0{,}433 - 0{,}0002\ x - 0{,}0001\ \cdot \Delta x$, va $y' = \lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 0{,}433 - 0{,}0002\ x$

Natijada boqilayotgan molning o'sish vazni $x = 100$ kg berilganda

$y'(100) = 0{,}433 - 0{,}0002 \cdot 100 = 0{,}413$ kg. ga oshar ekan.

Misol – 2. faraz qilaylik biror maxsulotning talab baxosi $y = 20 - x$ bo'lsin, bu erda x- talab, y- bahosi. Natajada mahsulotni sotishdan keladigan daromad ham taqriban 4 birlikka ortar ekan:

$U = x \cdot y = x(20 - x)$, $U' = 20 - 2x$, $U'(8) = 4$.

$y = f(x)$ funksiya berilgan bo'lsin, Δx- erkli o'zgaruvchi x ni orttirmasi bo'lsa, u holda erkli o'zgaruvchi x ni nisbiy orttirmasi $\frac{\Delta y}{x}$ bo'ladi. Erksiz o'zgaruvchini nisbiy orttirmasi esa $\frac{\Delta y}{y}$. Natajada bu ikki miqdorlarning nisbatlari

$$\frac{\Delta y}{y} : \frac{\Delta x}{x} = \frac{\Delta y}{\Delta x} \cdot \frac{x}{y}$$

Funksiya egiluvchanligi $E_x(y)$ (x–erkli o'zgaruvchiga nisbatan) quyidagicha aniqlanadi: $\lim\limits_{\Delta x \to 0} \left(\frac{\Delta y}{y} : \frac{\Delta x}{x} \right) = \frac{x}{y} \cdot \frac{dy}{dx} = \frac{x}{y} \cdot y'$

Ya'ni $E_x(y) = \frac{x}{y} \cdot \frac{dy}{dx}$ ekan.

Demak, $y = f(x)$ funksiyani x –ga nisbatan egiluvchanligi shu funksiyani erkli x o'zgaruvchini bir foiziga nisbatan o'sishini bildiradi.

Misol – 3. $y = x - 5$ funksiyani egiluvchanligini toping,

$$E_x(y) = \frac{x}{y} \cdot y' = \frac{x}{x-5} \cdot 1 = \frac{x}{x-5}.$$

Masalan, $x = 10$ bo'lganda, funksiya egiluvchanligi $E_{10}(5) = \frac{10}{10-5} = \frac{10}{5} = 2$.

Demak, agar x erkli o'zgaruvchi bir foizga ortsa, u holda funksiya y ikki foizga ortadi.

Biror mahsulotga talab U ni shu mahsulot bahosi V ning funksiyasidan iborat deb qarash mumkin: $U = f(v)$

Agar Δv - baho orttirmasi, ΔU - talab orttirmasi bo'lsa, bahoning nisbiy o'zgaruvchanligi $\frac{\Delta v}{v}$, talabni nisbiy o'zgaruvchanligi $\frac{\Delta u}{u}$ bo'ladi. Natijada quyidagi nisbatan $\frac{\Delta u}{u} : \frac{\Delta v}{v}$ maxsulot narxi bir foizga oshganda, maxsulotga talabni nisbiy o'zgarishi deb qarash mumkin.

Bundan $\lim\limits_{\Delta v \to 0}\left(\frac{\Delta u}{u}:\frac{\Delta v}{v}\right)=\frac{v}{u}\cdot\frac{du}{dv}=\frac{v}{u}\cdot u'$ ya'ni $E_T = E_v(u)=\frac{v}{u}\cdot\frac{du}{dv}$ miqdorga mahsulotga talabni uning bahosi bir foizga oshgandagi egiluvchanligi deyiladi.

Ko'pgina hollarda mahsulot narxi oshganida ungsha talab kamayishi sababli $\frac{du}{dv}<0$ bo'ladi. Bunday hollarda manfiylikdan qutilish uchun, talabning egiluvchanligi $E_T = E_v(u)=-\frac{v}{u}\cdot\frac{du}{dv}$ deb olish mumkin.

1. Agar $E_v(u)=1$ bo'lsa, u holda maxsulot narxi 1 foizga oshsa, mos ravishda mahsulotga talabni 1 foizdan ko'p kamayishi kuzatiladi ya'ni talab egiluvchan bo'ladi;

2. Agar $E_v(u)=1$ bo'lsa, mahsulot narxi 1 foizga oshishi talabni ham 1 foizga kamayishiga olib keladi ya'ni talab bartaraf bo'ladi;

3. Agar $0<E_v(u)<1$ bo'lsa, u holda 1 foiz mahsulot narxining oshishi, mahsulotga talabni 1 foizdan kam kamayishiga olib keladi, ya'ni talab egiluvchan bo'lmaydi.

Misol – 4. mahsulotga talab $y=4-x$ shaklida bo'lsin. $\frac{dy}{dx}=-1<0$ bo'lganligi uchun (funksiya) talabni egiluvchanligi

$E_x(y)=-\frac{x}{y}\cdot\frac{dy}{dx}=-\frac{x}{4-x}(-1)=\frac{x}{4-x}$ bo'ladi.

Agar $x=2$ bo'lsa $E_T=\frac{2}{4-2}=1$ bo'ladi.

Buni ma'nosi, agar smahsulot narxi 2 so'm bo'laganda uning bahosini 1 foizga oshirilishi, unga talabni 1 foizga kamayishiga olib keladi.

TALABNING DAROMADGA NISBATAN EGILUVCHANLIGINI ANIQLASh.

r - daromad, q - talab miqdori bo'lsin. Unda $q=f(r)$

Δr - daromadni oshishi, Δq - mahsulotga talabni oshishi bo'lib, mahsulotga nisbatan talabning egiluvchanligi quyidagicha bo'ladi:

$E_r(q)=\frac{r}{q}\cdot\frac{dq}{qr}$ bu erda $\lim\limits_{\Delta r\to 0}\left(\frac{\Delta q}{q}:\frac{\Delta r}{r}\right)=\frac{r}{q}\cdot\frac{dq}{qr}$.

Misol – 5. faraz qilaylik davlat biror maxsulot narxini 10 foizga oshirsin. Agar talabni egiluvchanligi 0,2 bo'lsa, u holda shu maxsulotga talabni 2 foizga kamayishiga olib keladi. Bu tadbir natajasida yuqorida keltirilgan formulalar yordamida davlat daromadini 8 foizga oshishi kelib chiqadi.

55.Parametrik tenglamalar bilan berilgan funksiyaning hosilasi

Agar y funksiyaning erkli o'zgaruvchi x ga bogliqligi yordamchi o'zgaruvchi (parametr) t yordamida berilgan bo'lsa,

$$\begin{cases} x = x(t), \\ y = y(t), \end{cases} \text{ u holda } \quad y'_x = \frac{y'_t}{x'_t}; \quad y''_{xx} = \frac{(y'_x)'_t}{x'_t}.$$

56. Mavzuga doir namunaviy misollarni yechimi.

Misol-1. Funksiya parametrik berilgan:

$$\begin{cases} x = t^2 - 1 \\ y = t^3 + 5 \end{cases}$$

y ning x bo'yicha birinchi va ikkinchi hosilalarini aniqlang.

Yechilishi. Quyidagiga egamiz:

$$y'_x = \frac{y'_t}{x'_t} = \frac{3t^2}{2t} = \frac{3}{2}t, \quad y''_{xx} = \frac{d}{dx}(y'^{\,t}_x) = \frac{d}{dt}(y'_x)\cdot\frac{dt}{dx} = \frac{\frac{d}{dt}(y'_x)}{x'_t} = \frac{\frac{d}{dt}\left(\frac{3}{2}t\right)}{2t} = \frac{3}{4t}.$$

57. Mustaqil yechish uchun misollar.

Masalalarda parametrik berilgan funksiyalar uchun ko'rsatilagan hosilalarni topíng.

1. $\begin{cases} x = 3t - 2, \\ y = t^3 + t. \end{cases}$ $\frac{dy}{dx}$ ni toping. 2. $\begin{cases} x = \sqrt[3]{t}, \\ y = \sqrt[4]{t}. \end{cases}$ $\frac{dy}{dx}$ ni toping.

3. $\begin{cases} x = e^{-2t}, \\ y = e^{t}. \end{cases}$ $\left(\frac{dv}{dx}\right)_{t=1}$ ni toping. 4. $\begin{cases} x = 5\sin t, \\ y = 5\cos t. \end{cases}$ $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{3}}$ ni toping.

5. $\begin{cases} x = \ln(1 + t^2), \\ y = arc\ ctg\ t. \end{cases}$ $\left(\frac{dy}{dx}\right)_{t=-\frac{1}{6}}$ ni toping. 6. $\begin{cases} x = a(\sin t - t\cos t), \\ y = a(\cos t + t\sin t). \end{cases}$ $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}}$ ni toping.

7. $\begin{cases} x = \dfrac{3t}{1 + t^3} \\ y = \dfrac{3t^2}{1 + t^3} \end{cases}$ $\left(\frac{dy}{dx}\right)_{t=0}$ va $\left(\frac{dy}{dx}\right)_{t=1}$ larni toping.

8. $\begin{cases} x = t^2 + 3 \\ y = t^5 - 7 \end{cases}$ $\left(\frac{dy}{dx}\right)$ va $\frac{a^2 y}{dx^2}$ larni toping.

9. $\begin{cases} x = 3\cos t, \\ y = 3\sin t. \end{cases}$ $\frac{dy}{dx}$ va $\frac{d^2 y}{dx^2}$ larni toping.

10. $\begin{cases} x = e^{-\varphi}, \\ y = e^{3\varphi}. \end{cases}$ $\left(\frac{d^2 y}{d\varphi^2}\right)_{\varphi=0}$ va $\left(\frac{d^2 y}{d\varphi^2}\right)_{\varphi=\frac{1}{5}}$ larni toping.

11. $\begin{cases} x = \ln t, \\ y = t^2 \end{cases}$ $\left(\frac{dy}{dx}\right)_{t=1}$ va $\left(\frac{d^2 y}{dx^2}\right)_{t=1}$ larni toping.

58.Hosilaning mexanika masalalariga tatbiqlari

$y = y(x)$ funksiyaning argumentning $x = x_0$ nuqtasida hisoblangan $y'(x_0)$ hosilasi bu funksiyaning erkli o'zgaruvchi x ga nisbatan $x = x_0$ nuqtadagi o'zgarish tengligidan iboratdir.

Xususiy holda agar to'gri chiziqli xarakatda o'tilgan yo'l s va t vaqt orasidagi munosabat $s = s(t)$ formula bilan ifodalanadigan bo'lsa, u holda t

vaqtning istalgan momentdagi harakat tezligi $\frac{ds}{dt}$ dan, tezlanish $\frac{d^2s}{dt^2}$ dan iborat bo'ladi.

59.Mavzuga doir namunaviy misollarni yechimi.

Misol-1. Nuqta $s=\frac{1}{3}t^3+2t^2-t$ (s - metrlarda, t - sekundlarda ifodalanadi) qonun bo'yicha to'gri chiziqli harakat qilyapti. Harakat bolangandan 1 sek. o'tgandan keyingi harakat tezligi va tezlanishini toping.

Echilishi. To'gri chiziqli harakat tezligiyo'ldan vaqt bo'yicha olingan hosilaga teng: $v(t)=\frac{ds}{dt}=t^2+4t-1$. Bu erdan $v(1)=4$ (m/sek).

To'gri chiziqli harakat tezlanish yo'ldan vaqt bo'yicha olingan ikkinchi tartibli hosilaga teng:
To'gri chiziqli harakat tezlanish yo'ldan vaqt bo'yicha olingan ikkinchi tartibli hosilaga teng:

$a(t)=\frac{d^2s}{dt^2}=2t+4$. Demak, $a(1)=6(м/сек^2)$

Misol-2 .Silindr asosining radiusi 3/m sek tezlik bilan ortadi, balandligi esa 2m/sek tezlik bilan kamayadi. Silindr hajmning o'zgarish tezligi qanday?
Yechilishi: Silindr hajmi $V=\pi r^2 h$, bu erda r - asos radiusi, h - silindr balandligi, v, r va h lar t ga bogliqligini etiborga olib, tenglikning ikala tomonini t vaqt bo'yicha differensiallaymiz:

$$\frac{dV}{dt}=\pi\left(2r\frac{dr}{dt}h+r^2\frac{dh}{dt}\right).$$

Shartga ko'ra $\frac{dr}{dt}=3$ (m/sek), $\frac{dh}{dt}=-2$ (m/sek) bo'lganligi uchun

$$\frac{dV}{dt}=\pi(6rh-2r^2).$$

Hosil qilingan formula silindr hajmining o'zgarish tezligini bildiradi.

60. **Mustaqil yechish uchun misollar.**

1. Nuqtaning to'gri chiziqli harakat qonuni $s=1+t^2-\frac{1}{4}t^4$ (s - metrlarda, t - sekuntlarda) formula orqali berilgan. Vaqtning $t=0$, $t=2$, $t=3$ momentlaridagi harakat tezligi va tezlanishini toping.
2. Nuqta abssissalar o'qi bo'yicha $x=\frac{t^4-4t^3+2t^2-12t}{4}$ (x - metrlarda, t - sekuntlarda) qonun bilan harakat qilyapti. Vaqtning qaysi momentida u to'xtaydi?
3. Massasi 25 k bo'lgan jism $s=\ln(1+t^2)$ qonuni bilan to'gri chiziqli harakat qilmoqda. Jismning harakat boshlangandan 2 sek.o'tgandan keyingi kinenik energiyasi $\left(\frac{mv^2}{2}\right)$ ni toping.

4. Doira radiusi 5m/sek tezlik bilan o’zgaradi. Uning aylanasining uzunligi qanday tezlik bilan o’zgaradi?
5. Kvadrat tomoni 3m/sek tezlik bilan ortadi. Uning tomoni4 m ga teng bo’lganda kvadrat yuzining o’zgarish tezligi qanday?
6. Nuqta $y = \sqrt{x}$ parabola bo’ylab harakat qilmoqda. Uning koordinatalaridan qaysi biri tezroq o’zgaradi?

Nuqta s (s - metrlarda, t- sekundlarda ifodalanadi) qonun bo’yicha to’gri chiziqli harakat qilyapti. Harakat bolangandan t q3 sek. o’tgandan keyingi harakat tezligi va tezlanishini toping.

1. $s(t) = 2x^4 - 3x^2 + x - 2$, 2. $s(t) = 3x^4 - 2x^2 - x + 1$, 3. $s(t) = 4x^4 - 3x^2 - 2x - 1$,
4. $s(t) = x^4 + x^2 - 3x + 1$, 5. $s(t) = 2x^4 - 2x^2 + x - 2$, 6. $s(t) = 3x^4 - x^2 + 2x + 1$,
7. $s(t) = 4x^4 - 3x^2 - x + 2$, 8. $s(t) = 2x^4 + 4x^2 - 5x - 1$, 9. $s(t) = 3x^4 + x^2 - 2x + 1$, .
10. $s(t) = 4x^4 + 2x^2 - 7x - 3$, 11. $s(t) = 2x^4 - 3x^3 + x^2 - 2$, 12. $s(t) = 3x^4 - 2x^3 - x^2 + 1$, .
13. $s(t) = 4x^4 - 3x^3 - 2x^2 - 1$, 14. $s(t) = x^4 - 3x^3 - 3x^2 + 1$, 15. $s(t) = 2x^4 - 2x^3 + x^2 - 2$,

61.Funksiya differensiali va uni tatbiqlari.

$y = y(x)$ funksiyaning dy differensiali deb funksiya orttirmasining erkli o’zgaruvchi x ning orttirmasi Δx ga proporsional bo’lgan bosh qismiga aytiladi.Erkli o’zgaruvchi x ning dx diffrensiali uning orttirmasi Δx ga teng: $dx = \Delta x$ Diffrensiallanuvchi istalgan $y = y(x)$ funksiyaning diffrensiali uning hosilasini erkli o’zgaruvchi diffrensialiga ko’paytmasiga teng: $dy = y'(x)dx$ yoki $y(x + \Delta x) \approx y(x) + y'(x) \cdot \Delta x$.

62. Mavzuga doir namunaviy misollarni yechimi.

Misol-1. $x = 3$ va $\Delta x = 0{,}01$ da $y = x^2 - x + 1$ funksiyaning orttirmasi Δy ni va diffrensiali dy ni toping. Funksiya orttirmasini uning diffrensiali bilan almashtirishda yo’l qo’yiladigan absalyut va nisbiy xatolar qanday?

Yechilishi. Quyidagi egamiz: $\Delta y = y(x + \Delta x) - y(x) = y(3 + 0{,}01) - y(3) = \left[(3 + 0{,}01)^2 - (3 + 0{,}01) + 1\right] - (3^2 - 3 + 1) = 0{,}0501$

funksiya diffrensialini (1) formula bo’yicha topamiz:

$$dy = y'(x)\Delta x = y'(3) \cdot 0{,}01 = (2x - 1)_{x=3} \cdot 0{,}01 = (2 \cdot 3 - 1) \cdot 0{,}01 = 0{,}05 .$$

Absalyut xato:

$|dy - \Delta y| = |0{,}05 - 0{,}0501| = 0{,}0001$. Nisbiy xato: $\left|\frac{dy - \Delta y}{\Delta y}\right| = \frac{0{,}0001}{0{,}0501} \approx 0{,}002 = 0{,}2$ %

Misol-2. Agar kubning hajmi 27m^3 dan 2,72m^3 ga o’zgargan bo’lsa, kub qarrasi qanchaga (taqriban) ortadi?

Yechilishi. Agar y - kub qirrasi bo'lsa, uning hajmi $V = y^3$. Shundy qilib, masala $V = 27$ va $\Delta V = 27{,}2 - 27 = 0{,}2$ bo'lganda $y = \sqrt[3]{V}$ funksiya orttirmasi Δy ni topishga keltiriladi. Δy orttirmani Ushbu taqribiy tenglikdan foydalanib, topamiz:

$$\Delta y \approx dy = y' dV = \frac{1}{\sqrt[3]{V^2}} \Delta V = \frac{1}{3 \cdot 9} \cdot 0{,}2 = 0{,}007 \text{ (m)}$$

Misol-3. Kvadrat tomonini o'lchashda 1% xatolikka yo'l qo'yilgan. Topilgan taqribiy qiymat bo'yicha kvadrat yuzi xisoblangan. Bunda qanday xatolikka yo'l qo'yilgan?

Yechilishi. Agar kvadrat tomonining aniq qiymatini x, uning o'lchash natijasida topilgan qiymatini $x + dx$ deydigan bo'lsak, u holda o'lchash xatoligi $dx = \Delta x = \pm 0{,}01 x$ bo'ladi. Kvadrat yuzi S ni o'lchashda yo'l qo'yilgan ΔS xato quyidagicha bo'ladi:

$$\Delta S \approx dS = d(x^2) = 2x \cdot dx = 2x(\pm 0{,}01 x) = \pm 0{,}02 x^2 = \pm 0{,}02 S,$$

bu yuzining 2% ini tshkil etadi.

Misol-4. $\sqrt[3]{8{,}01}$ ni taqribiy hisoblang.

Yechilishi. $y = \sqrt[3]{x}$ funksiyani qarab chiqib, $x = 8$, $\Delta x = 0{,}01$ deymiz. U holda (2) formuladan foydalanib, topamiz: $y(8{,}01) = \sqrt[3]{8{,}01}$;

$$y(8 + 0{,}01) \approx y(8) + y'(8) \cdot 0{,}01,$$

ya'ni

$$y(8{,}01) \approx \sqrt[3]{8} + \frac{1}{3\sqrt[3]{64}} \cdot 0{,}01 = 2 + \frac{1}{3 \cdot 4} \cdot 0{,}01 = 2 + \frac{0{,}01}{12} = 2{,}0008 .$$

Shunday qilib, $\sqrt[3]{8{,}01} \approx 2{,}0008$.

63. Mustaqil yechish uchun misollar.

Masalalarda funksiyalarning argument va uning orttirmasining ixtiyoriy qiymatlari uchun diffrensiallarini toping.

2. $y = \frac{1}{x^2}$. 3. $y = \frac{x + 2}{x - 1}$. 4. $y = arc\ tg\ 2x$.

5. $y = \ln(1 + x^2)$. 6. $y = \sin^2 x$. 7. $y = 5x^2 \arccos \frac{1}{x}$. 8. $y = \frac{tg \sqrt{x}}{\sqrt{x}}$.

9. $y = ax + b$ chiziqli funksiyaning dy differensiali va Δy orttirmasi argumentning istalgan qiymatida bir xilda ekanligini ko'rsating.

10. $x = 9$ va $\Delta x = 0{,}2$ da $y = \sqrt{x}$ funksiyaning orttirmasi va diffrensialini toping. Funksiya orttirmasi uning diffrensiali bilan almashtirishda hosil bo'ladigan absolyut va nisbiy xatolarni hisoblang.

11. . Agar kvadrat yuzini 16 m^2 dan 15b82 m^2 ga kamaytirsak, kvadrat tomoni (taqriban) qanchaga o'zgaradi?

14. Doira radiusini hisoblashda 1% xatolikka yo'l qo'yilgan. Doira yuzini $S = \pi r^2$ formula bo'yicha hisoblashda (bu erda r - radiusi o'lchash natijasida hosil qilangan) xatolik doira yuzining 2% ni tashkil etishini ko'rsating.

15. Funksiyaning differensialidan foydalanib quyidagi taqribiy tengliklarning to'griligini isbot qiling:

a) $(1+a)^R \approx 1+Ra$; b) $\sqrt[R]{1+a} \approx 1+\frac{R}{a}$; v) $\ln(1+a) \approx a$.

Masalalarda funksiyaning differensialidan foydalanib berilgan ifodalarning taqribiy qiymatlarini toping.

16. $\sqrt[4]{17}$. 17. $\lg 10{,}08$. 18. $\cos 32^\circ$. 19. $tg\, 44^\circ 52'$.

20. $arc\ \sin\ 0{,}48$. 21. $\sqrt[3]{26{,}97}$. 22. $0{,}96^3$.

24. $y=\ln x$ funksiyaning absalyut xatosini uning argument absolyut xatomi orqali ifodalang.

64. Funksiyani tula tekshirishga hosilani qo'llanilishi.

Bizga chekli hosilalarga ega bo'lgan uqf(x) funksiya berilgan bo'lsin. Bu funksiyani hosila yordamida tekshirishga qisqacha to'xtalib o'tamiz. Bu uqf(x) funksiyani chekli hosilalarga ega bo'lganda monotonlik oraliqlarini, ekstremum qiytmatlarini, egilish va bukilish nuqtalarini, botiqlik va qavariqlik oraliqlarini topib yanada chuqurroq xususiyatlarini o'rganish va aniqroq shaklini chizish mumkin.

1. Funksiyani monotonlik oraliqlarini topish. Berilgan u=f(x) funksiyadan birinchi tartibli hosila olamiz. $y^1=f^1(x)$. y=f(x) funksiya monotonligini yetarli shartlari quyidagicha:

a) Agar biror oraliqni barcha nuqtalarida $f^1(x)\geq 0$ tengsizlik bajarilsa, funksiya shu oraliqda o'suvchi bo'ladi.

b) Agar biror oraliqni barcha nuqtalarida $f^1(x)\leq 0$ tengsizlik bajarilsa, funksiya shu oraliqda kamayuvchi bo'ladi.

2. Funksiyani ekstremum qiytmatlarini topish.

Ferma teoremasiga asosan y=f(x) funksiya x_0 nuqtada differensiallanuvchi bo'lib, shu nuqtada ekstremum qiytmatlarga ega bo'lsa, u holda $f^1(x_0)=0$ bo'ladi. Demak, y=f(x) funksiya ekstremum qiytmatga ega bo'lishi uchun $f^1(x)=0$ bajarilishi zaruriy shartdir. Bu $f^1(x)=0$ tenglamani yechib $x_0,x_1,...,x_n$ kritik nuqtalarini topamiz.

Shuni unutmaslik kerakli zaruriy shart $x=x_0$ nuqtada bajarilishidan hamma vaqt ham funksiyani shu nuqtada ekstremumga egaligi kelib chiqmaydi.

Buni uchun y=f(x) funksiya ekstremumi mavjudligining etarli shartini tekshirish lozim. Etarli shartda 1 chi va 2 chi qoidalar mavjud.

a) **Funksiya ekstremumi mavjudligining etarli shart 1-qoidasi.** $X=x_0$ kritik nuqtani etarli kichik $(x_0-\varepsilon;x_0+\varepsilon)$ $(\varepsilon>0)$ atrofida y=f(x) funksiya hosilasi $f^1(x)$ ishorasini
hisoblaymiz.

Agar

$$\left\{\begin{array}{l} \mathbf{f\,(x_0-\varepsilon)>0\ \ (+)} \\ \mathbf{f\,(x_0+\varepsilon)<0\ \ (-)} \end{array}\right. \Rightarrow \textbf{max ega.}$$

ya'ni $x=x_0$ kritik nuqta atrofida funksiya hosilasi o'z ishorasini (+) dan (-) ga o'zgartirsa u=f(x) funksiya shu $x=x_0$ nuqtada max ega bo'ladi.

Agar

$$\begin{cases} \mathbf{f\,(x_0-\varepsilon)<0\ \ (-)} & \\ & \Rightarrow \textbf{min ega.} \\ \mathbf{f\,(x_0+\varepsilon)>0\ \ (+)} & \textbf{bo'ladi.} \end{cases}$$

Bordiyu $f^1(x)$ xqx_0 nuqta atrofida $(x_0-\varepsilon;x_0+\varepsilon)$o'z ishorasini o'zgartirmasa y=f(x) funksiya ekstremumga ega bo'lmaydi.

b) **Funksiya ekstremumi mavjudligining yetarli shart 2-qoidasi**. Qoida bilan u=f(x) funksiyani ekstremumga egaligini tekshirish uchun bu funksiya yuqori tartibli chekli hosilalarga ega bo'lishi kerak. Agar $x=x_0$ kritik nuqtada barcha $f^{(2k-1)}(x_0)=0$ toq tartibli hosilalarga nolga teng bo'lib $f^{(2k)}(x_0)\neq 0$ bo'lmasa u holda $f^{(2k)}(x_0>0$ bo'lsa min ega, $f^{(2k)}(x_0)<0 \Rightarrow$ max ega bo'ladi.

Agar $f^{(2k-1)}(x_0)\neq 0$ bo'lsa (barcha undan oldingi hosilalar nolga teng shartda) funksiya ekstremumga ega bo'lmaydi.

Yuqorida tavsiya etilgan adabiyotlardan foydalanib y=f(x) funksiyani egilish, bukilish nuqtalarini, botiqlik va qavariqlik oraliqlarini, asimptotalarini topishni o'rganib olishingizni tavsiya etamiz.

65. Mavzuga doir namunaviy masalani yechimi.

Misol. Ushbu $y=\frac{1}{3}\cdot x^3 - x^2 - 3x + 2$ funksiyani hosila yordamida o'sish va kamayish oraliqlarini, ekstremum qiymatlarini, egilish, bukilish nuqtalarini hamda qavariqlik va botiqlik oraliqlarini topib shaklini chizing.

Yechilishi: a) Bu funksiya R xaqiqiy sonlar o'qida aniqlangan va uzluksiz bo'lib uni hosilasi

$$y^1 = \left(\frac{1}{3}\cdot x^3 - x^2 - 3x + 2\right)^1 = \frac{1}{3}\cdot x^2 - 2x - 3 = x^2 - 2x - 3$$

b) ekstremum mavjudligini zaruriy shartiga asosan

$u^1 = x^2 - 2x - 3 = 0;\ x_1 = -1;\ x_2 = 3$

Bular funksiyani kritik (stasionar) nuqtalaridir.

y^1=f(x) $x\in(-\infty:+\infty)$ da mavjud bo'lgani uchun y= f(x) ; $x_1=-1$ va $x_2=3$ dan boshqa kritik qiymatlari yo'q.

Funksiya ekstremumi mavjudligini etarli shart I-qoidasiga asosan har bir kritik nuqtani etarli kichik atrofida funksiya hosilasini ishorasini o'zgarishini tekshiramiz, $\varepsilon=0{,}1$ bo'lsa.

v) $x_1=-1$ da (-1,1*-0,9)u^1=f(-1,1)=x^2-2x-3=(x+1)(x-3)=(-1,1+1)(-1,1-3)>0 (+)

u^1=f(-0,9)=(0,9+1)(-0,9-3)<0 (-)

Demak, $x_1=-1$ nuqtada funksiya max ega.

Ikkinchi $x_2=3$ da (2,9; 3,1) y^1=f(2,9)=(2,9+1)(2,9-3)<0

y^1=f (3,1)=(3,1+1)(3,1-1)>0 (+)

Demak x=3 nuqta berilgan funksiya min ga ega.

Natijada

$$y_{\max} = f(-1) = -\frac{1}{3} - 1 + 3 + 2 = 3\frac{2}{3}$$

$y_{\min} = f(3) = -9 + 2 = -7$

b) Berilgan funksiyani monotonlik oraliqlarini topamiz $y^1=(x+1)(x-3)$, $y^1 \geq 0$ va $y^1 \leq 0$ shartlardan x(-∞;-1]da $y = \frac{1}{3} \cdot x^3 - x^2 - 3x + 2$ funksiya o'sadi, x∈ [-1;3] da kamayadi. x(3;+ ∞;) da o'sadi.

$A(-1;3\frac{2}{3})\max$ nuqtasi B(3;7)min nuqtasi.

d) Egilish va bukilish nuqtalarini topamiz:$y^{11}=(x^2-2x-3)^1=2x-2=2(x-1)$

$y^{11}=2(x-1)=0 \Rightarrow x=1$

$y^{11}(0,9)=2(0,9-1)<0$

$y^{11}(1,1)=2(1,1)-1>0$

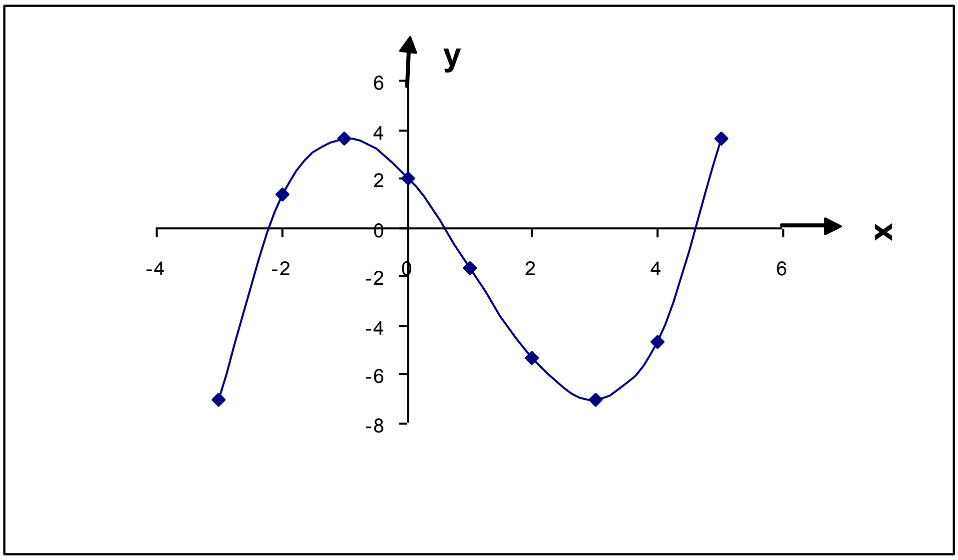
Demak, xq1 bukilish nuqtasi. Shunday qilib x(-∞;1) da funksiya qavariq x(1;+∞) da botiq ekan.

$$y = f(1) = \frac{1}{3} - 1 - 3 + 2 = -\frac{5}{3}$$

$P\left(1;-\frac{5}{3}\right)$ bukilish nuqtasi

$e)\ y = \frac{1}{3} \cdot x^3 - x^2 - 3x + 2$ grafigini chizamiz. Aniqroq chizish uchun funksiyani bir necha nuqtalardagi qiymatlarini hisoblaymiz:

x	-3	-2	-1	0	1	2	3	4
u	-7	4/3	11/3	2	-5/3	-16/3	-7	-14/3



66. Mustaqil yechish uchun misollar.

Quyida 1-20 misollarda berilgan funksiyalarni ekstremum qiymatlarini o'sish va kamayish oraliqlarini egilish, bukilish nuqtalarini, qabariqlik va botiqlik oraliqlarini topib shaklini chizing.

$1)\ y = \frac{1}{2}\cdot x^3 + 3x^2 - 7$

$2)\ y = \frac{1}{3}\cdot x^3 - \frac{3}{2}x^2 - 4x + 10$

$3)\ y = \frac{1}{4}\cdot x^3 - \frac{3}{2}x^2 + 2$

$4)\ y = \frac{1}{5}\cdot x^3 - \frac{9}{5}x^2 + 3x + 3$

$5)\ y = \frac{1}{6}\cdot x^3 - \frac{3}{2}x^2 + 8$

$6)\ y = -\frac{1}{2}\cdot x^3 + 6x - 1$

$7)\ y = -\frac{1}{3}\cdot x^3 + x^2 + 3x - 2$

$8)\ y = -\frac{1}{4}\cdot x^3 + \frac{9}{8}x^2 + 3x - 6$

$9)\ y = \frac{1}{3}\cdot x^3 - x^2 - 3x - 4$

$10)\ y = -\frac{1}{6}\cdot x^3 + \frac{9}{2}x + 2$

$11)\ y = \frac{1}{3}\cdot x^3 - x^2 - 3x + 4$

$12)\ y = \frac{1}{8}\cdot x^3 - \frac{3}{8}x^2 - 3x + 6$

$13)\ y = \frac{1}{3}\cdot x^3 - 2x^2 + 6$

$14)\ y = \frac{2}{3}\cdot x^3 - 3x^2 + 5$

$15)\ y = \frac{1}{8}\cdot x^3 + \frac{3}{8}x^2 - 3x - 4$

$16)\ y = -0.2\cdot x^3 - 1.2x^2 + 3x + 8$

$17)\ y = \frac{1}{3}\cdot x^3 - \frac{3}{2}x^2 + 4x + 5$

$18)\ y = -0.2\cdot x^3 + 1.2x^2 + 3x - 7$

$19)\ y = -\frac{1}{9}\cdot x^3 + 3x - 2$

$20)\ y = -\frac{1}{6}\cdot x^3 + \frac{3}{2}x^2 - 10$

67. Funksiyaning eng katta va eng kichik qiymatlari.

Biror $[a,\ b]$ kesmada uzluksiz bo'lgan funksiyaning katta va eng kichik qiymatlarini topish uchun bu funksiyaning kesma oxiridagi va bu kesmaga tegishli (bu holda funksiyaning birinchi tartibli hosilasi nolga aylanadigan yoki mavjud bo'lmaydigan nuqtalar shular jumlasidandir) barcha kritik nuqtalardagi qiymatlarini hisoblash kerak. Topilgan qiymatlar ichidan eng katta va eng kichiklari funksiyaning kesmadagi eng katta va eng kichik qiymatlari bo'ladi.

Tekshirilayotgan funksiya $[a,\ b]$ kesmaning ba'zi nuqtalarida uzulishga ega bo'lsa yoki cheksiz intervalda berilgan bo'lsa, u holda qo'shimcha ravishda uning xarakterini uzulish nuqtalari atrofida va $x \to \pm\infty$ da qarab chiqish kerak.

68.Mavzuga doir namunaviy misolni yechimi.

1. Quyidagi funksiyalarning eng katta va eng kichik qiymatlarini toping:

1) $f(x) = x^3 - 3x^2 + 4$, $[1,\ 3]$ kesmada;

2) $\varphi(x) = x + \frac{1}{x}$, $[-2,\ 2]$ kesmada;

Echilish. 1) $f(x)$ funksiya $[1, \quad 3]$ kesmada uzluksiz. Differensiallab, topamiz:

$$f'(x) = 3x^2 - 6x \ .$$

Bu holda $f'(x)$ hosila nolga teng bo'lgan nuqtalargina, ya'ni $x = 0$ va $x = 2$ nuqtalar kritik nuqtalar bo'ladi. $[1, \quad 3]$ kesmaga bu nuqtalarning biri, ya'ni $x = 2$ nuqta tegishlidir. $f(x)$ funksiyaning $x = 2$ nuqtadagi va kesma uchlari $x = 1$ va $x = 3$ dagi qiymatlarini hisoblaymiz:

$$f(2) = 0\,, \qquad f(1) = 2\,, \qquad f(3) = 4\ .$$

Shunday qilib funksiyaning eng katta qiymati 4 ga teng va funksiya unga kesmaning o'ng uchida $x = 3$ nuqtada erishadi; funksiyaning eng kichik qiymati nolga teng bo'lib, unga kesmaning ichki nuqtasi $x = 2$ da erishadi.

2) $\varphi(x)$ funksiya $[-2, \quad 2]$ kesmaga tegishli $x = 0$ nuqtada uzilishga ega. Funksiya xarakterini uzilish nuqtasi atrofida tekshiramiz:

$$\lim_{x \to +0} \varphi(x) = +\infty \qquad \qquad \lim_{x \to -0} \varphi(x) = -\infty$$

Demak, $x = 0$ nuqta atrofida $\varphi(x)$ funksiya absolyut qiymati jihatidan musbat hamda manfiy istalgancha katta qiymatlarga erishadi, va binobarin, na eng katta va na eng kichik qiymatga ega bo'ladi.

69. Mustaqil yechish uchun misollar.

Masalalarda berilgan funksiyalarning eng katta va eng kichik qiymatlarini toping.

1. $y = 2x^2 - 8x + 1$, $\quad [0,3]$ kesmada.
2. $y = x^5 - 5x^3 - 8$, $\quad [0,2]$ kesmada.
3. $y = \sqrt{9 - x^2}$, $\quad [-3, \quad 3]$ kesmada.
4. $y = \arcsin x^2$, $\quad \left[-\frac{\sqrt{2}}{2} \ \frac{\sqrt{2}}{2}\right]$ kesmada.
5. $y = x \ln x$, $\quad [1, \quad e]$ kesmada.
6. $y = \dfrac{1}{1 + x^2}$, $\quad (-\infty, +\infty)$ intervalda.
7. $y = \dfrac{1}{\sin x}$, $\quad \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ kesmada.
8. $y = \operatorname{arctg} x - \ln \sqrt{x}$, $\quad (1, +\infty)$ intrvalda.
9. $y = 2x + \cos 2x$, $\quad \left[-\frac{3\pi}{4}, \pi\right]$ kesmada.

70.Funksiyaning eng katta va eng kichik qiymatlarini topishga doir amaliy masalalar.

Miqdorlarning eng katta va eng kichik qiymatlarini xisoblashga doir masalalarni echishda eng avval masalada qanday miqdorning eng katta (yoki eng kichik) qiymati topilayotganligini aniq bilib olish kerak. Ana shu miqdor tekshirilayotgan funksiya bo'ladi. So'ngra, o'zgarishlaridan funksiyaning ham o'zgarishi kelib chiqadigan miqdorlardan birini erkli o'zgaruvchi deb qabul qilib, funksiyani u orqali ifodalash kerak.

71. Mavzuga doir namunaviy masalani yechimi.

Misol-1. Radiusi R bo'lgan sharga eng katta hajmga ega bo'lgan silindrni ichki chizing.

Yechilishi. Sharning balandligi, asos radiusi va xajmini mos ravishda h, r va V bilan belgilaymiz. U holda shar hajmi:

$V = \pi r^2 h$. $r^2 = R^2 - \frac{h^2}{4}$ ekanligini hisobga olsak, silindr xajmi uchun ifoda hosil qilamiz:

$$V = \pi\left(R^2 - \frac{h^2}{4}\right)h = \pi\left(R^2 h \frac{h^3}{4}\right).$$

Shunday qilib, masala ushbu $V(h) = \pi\left(R^2 h - \frac{h^3}{4}\right)$

funksiyaning $(0,2R)$ dagi eng katta qiymatini topishga keltiriladi. Bu funksiyaning hosilasinim topamiz: $V'(h) = \pi\left(R^2 - \frac{3}{4}h^2\right)$. $V'(h)$ ni nolga tenglab, $(0,2R)$ intervalga tegishli bo'lgan yagona $h_0 = \frac{2}{\sqrt{3}}R$ kritik nuqtani hosil qilamiz, bu nuqtada $V(h)$ funksiyaning o'zining eng katta qiymatiga ega bo'ladi. $V(h_0) = \frac{4\pi R^3}{3\sqrt{3}}$

Shunday qilib, balandligi $h = \frac{2}{\sqrt{3}}R$ bo'lgan silindr eng katta hajmga ega bo'lar ekan.

Misol-2. Elektr lampochkasini OB vertikal to'gri chiziq bo'ylab siljitish mumkin. Gorizontal tekislikning A nuqtasida eng ko'p yoritilganlikka ega bo'lish uchun lampochkani tekislikdan qanday masofada joylashtirish kerak?

Yechilishi. Yoritilganlik $J = c\frac{\sin\varphi}{r^2}$

formula yordamida hisoblanadi, bu erda $r = AB$, $\varphi = \angle OAB$; $c = const$ (V manbaning yoruglik kuchi). Erkli o'zgaruvchi uchun (uning o'zgarishi bilan lampochkadan tekislikkacha bo'lgan masofa, demak, J yoritilganlik ham o'zgaradi) h kattalikning o'zini, φ yoki r ni tanlash mumkin. Erkli o'zgaruvchi uchun φ burchakni qabul qilib va

$r = \frac{a}{\cos\varphi}$ ekanligidan foydalanib, J ning φ orqali ancha sodda ifodasini hosil qilamiz: $J = \frac{c}{a^2}\sin\varphi\cos^2\varphi$.

Hosil qilingan $J(\varphi)$ funksiyaning erkli o'zgarvchi φ ning o'zgarish oraligi $\left(0, \frac{\pi}{2}\right)$ dagi eng katta qiymatini topamiz. $J(\varphi)$ ni diffrensiallab, topamiz:

$$J'(\varphi) = \frac{c}{a^2}(\cos^2\varphi - 2\sin^2\varphi\cos\varphi) = 2\frac{c}{a^2}\cos^3\varphi\left(\frac{1}{2} - tg^2\varphi\right).$$

$J'(\varphi) = 0$ tenglamani yechib, $J(\varphi)$ funksiya $\left(0, \frac{\pi}{2}\right)$ intervalda yagona kritik nuqta $\varphi_0 = arctg \frac{1}{\sqrt{2}}$ ga ega ekanligini ko'ramiz. $\left(0, \frac{\pi}{2}\right)$ oraliqning uchlarida $J(\varphi)$ funksiya noga teng va $J(\varphi_0) > 0$ bo'lganligi sababli $\varphi = \varphi_0$ da $J(\varphi)$ yoritilganlik eng katta bo'ladi.

Shunday qilib, $h = atg\,\varphi_0 = \frac{a}{\sqrt{2}}$ bu izlanayotgan kattalikdir.

72. Mustaqil yechish uchun misollar.

1) Berilgan musbat m sonni shunday ikkita qo'shiluvchiga ajratingki, ularning ko'paytmasi eng katta bo'lsin.
2) Ikkita sonning ayirmasi 7 ga teng. Bu sonlar qanday bo'langanda ularning ko'paytmasi eng kichik bo'ladi?
3) uzunligi 20 sm bo'lgan simni bukib, eng katta yuzga ega bo'lgan to'gri to'rtburchak yasash talab qilinadi. Bu to'gri to'rtburchakning o'lchamlari qanday bo'ladi?
4) Perimetrlari 20 sm bo'lgan barcha to'gri to'rtburchaklar ichidan diagonali eng kichik bo'lganini ajrating.
5) To'gri burchakli uchburchakning gipotenuzasi c ga teng. Bu burchakning yuzi eng katta bo'lishi uchun uning katetlari qanday bo'lishi kerak?
6) Radiusi R bo'lgan yarim doiraga eng katta perimetrli to'gri to'rtburchakni ichki chizing.
7) Radiusi r bo'lgan berilgan doiraga ichki chizilgan barcha teng yonli uchburchaklar ichida eng katta perimetrga teng tomonli uchburcha ega ekanligini ko'rsating.
8) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipsga ichki chizilgan eng katta yuzli to'gri to'rtburchakninng tomonlarini toping.
9) Kitob saxifasidagi bosilgan tekst (satrlar orasidagi oraliqlarni ham qo'shib hisoblaganda) 192 sm^2 yuzni egallashi kerak. Yuqori va pastki xoshiyalar 4 sm dan, o'ng va chap tomondagilari esa 3 sm dan bo'lishi kerak. Agar faqat qogozni tejash e'tiborga olinadigan bo'lsa, kitob sahifasining eng maqsadga muvofiq o'lchamlri qanday bo'lishi kerak?
10) Eng kichik to'la sirtga ega bo'lib, hajmi 10m^3 bo'lgan silindrning o'lchamlarini aniqlang.
11) Tomoni 42 sm bo'lgan kvadrat temir listning uchlaridan teng kvadratlar qirqib olib tashlandi va qolgan qismidan ustki ochiq quticha yasaldi. qutichaning sigimi eng katta bo'lishi uchun qirqib olib tashlangan kvadratlarning tomoni qanday bo'lishi kerak?
12) Radiusi R ga teng bo'lgan berilgan sharga eng katta yon irtga ega bo'lgan silindrni ichki chizing.
13) Suvning qalin devordagi teshik orqali o'tishida har sekundda sarf bo'lishi $Q = cy\sqrt{h - y}$ formula bo'yicha aniqlanadi, bu erda y - teshik diametri, h -

uning eng quyi nuqtasining chuqurligi, c - birorta o'zgarmas son, y ning qanday qiymatida Q eng katta bo'ladi?

14) Tashqi zanjirdagi galvanik elementdan olinadigan tok quvvati N eng katta bo'lishi uchun tashqi zanjirning qarshiligi element qarshiligi r ga teng bo'lishi kerakligini ko'rsating.

73. Lopital qoidasi.
Lopital qoidasiga namunaviy misolni yechimi

Lopital qoidasi $\left(\frac{0}{0}\right), \left(\frac{\infty}{\infty}\right), 0^0, 1^\infty$ shaklidagi aniqmasliklarni ochishda qo'llaniladi. Xususan $\left(\frac{0}{0}\right)$ holda agar $f(x), g(x)$ funksiyalar uchun $x=a$ nuqtada aniqlanmagan $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0$, ammo shu nuqtani biror atrofida aniqlangan va chekli hosilalarga ega bo'lsa $g^1(x) \neq 0$ u xolda $\lim\limits_{x\to a}\frac{t(x)}{g(x)} = \lim\limits_{x\to a}\frac{f^1(x)}{g^1(x)}$. o'rinli bo'ladi.

Misol-1 Ushbu $\lim\limits_{x\to \frac{\Pi}{4}} (tgx)^{tg2x}$ limitini xisoblang.

Echish: 1^∞ shaklidagi aniqmaslikni logarifmlash yordamida $\left(\frac{0}{0}\right)$ shaklidagi aniqmaslikka keltiramiz:

$$У = (tgx)^{tg2x}, \qquad \ln y = tg2x \ln tgx = \frac{\ln tgx}{(tg2x)^{-1}}$$

$$\lim_{x\to\frac{\Pi}{4}} \ln y = \lim_{x\to\frac{\Pi}{4}} \frac{\ln tgx}{(tg2x)^{-1}} = \lim_{x\to\frac{\Pi}{4}} \frac{(lutgx)}{((tg2x)^{-1})^1} = \lim_{x\to\frac{\Pi}{4}} \frac{\frac{1}{tgx}\cdot\frac{1}{Cos^2 x}}{\frac{2}{Cos^2 2x}\cdot(tg2x)^{-2}} = -\lim_{x\to\frac{\Pi}{4}} Sin\,2x = -1$$

Natijada $\lim\limits_{x\to\frac{\Pi}{4}} (tgx)^{tg2x} = \frac{1}{e}$

74. Mustaqil yechish uchun misollar

Lopital qoidasidan foydalanib quyidagi limitlarni xisoblang.

1). $\lim\limits_{x\to\infty}\frac{\ln x}{x^\varepsilon}$ $(\varepsilon > 0)$, 2). $\lim\limits_{x\to+\infty} x^n e^{-x^2}$

3). $\lim\limits_{x\to 1} x^{\frac{1}{x-1}}$ 4). $\lim\limits_{x\to 0}\frac{1-\cos(x^2)\ln x}{x^2 Sin(x^2)}$

5). $\lim\limits_{x\to 0}\frac{xctgx - 1}{x^2}$ 6). $\lim\limits_{x\to 0}\frac{a^x - a^{\sin x}\ln x}{x^3}$ $(a > 0)$

7). $\lim\limits_{x\to +0} |\ln x|^{2x}$ 8). $\lim\limits_{x\to +0}\left(\frac{1}{x}\right)^{\sin x}$

9). $\lim\limits_{x \to 0} \frac{e^{\alpha x} - Cos\ \alpha x}{e^{\beta x} - Cos\ \beta x}$ 10). $\lim\limits_{x \to +0} (1 + x) \ln x$

75. Aniqmas integral, xossalari. Integrallash usullari.

Biz o'tgan mavzularda funksiya va uni hosilasini topish masalasini ya'ni differensiallash masalalarini o'rgandik. Unga teskari amal ya'ni funksiya hosilasi berilsa uni boshlang'ich funksiyasini topish masalasini o'rganamiz.

Maktab dasturidan ma'lumki F(x) funksiya f(x) ni boshlang'ich funksiyasi deyiladi agar $F^1(x)= f(x)$ tenglik qaralayotgan soha uchun o'rinli bo'lsa. Boshlang'ich funksiya integrallash amali yordamida topiladi.

$\int f(x)dx=F(x)+C$ bu erda $(F(x)+C)^1=F^1(x)=f(x)$

Aniqmas integrallarni echishda quyidagi xossalaridan foydalaniladi:

1) $\int kf(x)dx=k\int f(x)dx$ (k - o'zgarmas son)

2) $\int[af(x)\pm bg(x)]dx$qa$\int f(x)dx\pm b\int g(x)dx$

Agar f(x) funksiya qaralayotgan sohada uzluksiz bo'lsa, bu funksiya shu sohada integrallanuvchi bo'ladi. Aslida integrallanuvchi funksiyalar sinfi yanada kengroq. Chunki, u=f(x) funksiyani biror nuqtada hosilaga ega bo'lishi uchun uni uzluksiz bo'lishidan tashqari $\Delta u/\Delta x$ nisbatning limiti shu nuqtada mavjud bo'lishi kerak edi. Yuqoridagi mulohazalardan ko'rinib turibdiki integrallanuvchi bo'lishi uchun funksiyani uzluksiz bo'lishi etarli.

Integrallarni echishda quyidagi integrallar jadvalidan foydalaniladi.

Integrallash jadvali.

$1. \int dx = x + C$

$2. \int x^n dx = \frac{x^{n+1}}{n+1} + C$

$3. \int \frac{1}{x} dx = \ln|x| + C$

$4. \int a^x dx = \frac{a^x}{\ln a} + C$

$5. \int e^x dx = e^x + C$

$6. \int \cos xdx = \sin x + C$

$7. \int \sin xdx = -\cos x + C$

$8. \int \frac{1}{\cos^2 x} dx = tgx + C$

$9. \int \frac{1}{\sin^2 x} dx = -ctgx + C$

$10. \int \frac{dx}{\sqrt{1-x^2}} = \begin{matrix} \arcsin x + C \\ -\arccos x + C \end{matrix}$

$11. \int \frac{dx}{1+x^2} = \begin{matrix} arctgx + C \\ -arcctgx + C \end{matrix}$

$12. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$

$13. \int \frac{dx}{\sqrt{x^2 \pm a}} = \ln \left| x + \sqrt{x^2 \pm a} \right| + C$

$14. \int \frac{xdx}{\sqrt{a \pm x^2}} = \pm\sqrt{a \pm x^2} + C$

$15. \int \frac{xdx}{x^2 \pm a} = \frac{1}{2} \ln \left| x^2 \pm a \right| + C$

$16. \int [af(x) \pm bg(x)]dx = a\int f(x)dx \pm b\int g(x)dx$

$17.\int udv = uv - \int vdu$ bu bo'laklab integrallash formulasi.

$18.\int \frac{dx}{x^2+a^2} = \frac{1}{a} arctg \frac{x}{a} + C$

Aniqmas integrallarni echishda 1) bevosita integrallash;
2) o'zgaruvchini almashtirish usuli bilan integrallash;
3) bo'laklab integrallash; 4) rasional kasrlarni intagrallash; 5) trigonometrik funksiyalarni integrallash va boshqa ko'plab usullar mavjud.

76. Mavzuga doir namunaviy misollarni yechimi.

Quyidagi integrallashlarni echib, to'g'riligini differensiallash yordamida tekshiring.
1) Bevosita integrallash usuli.
Misol- 1. I=$\int [3x^4+2x^3-4]dx$

Yechish. Bu integral bevosita integrallash usuli bilan yechiladi, ya'ni berilgan integralni uni xossalaridan foydalanib yozganimizda yuqorida keltirilgan jadval interallarga keladi. Bizni misolda integralni xossasiga asosan, ya'ni yig'indining integrali integrallar yig'indisiga tengligidan (16) formulaga qarang: I=$\int 3x^4dx+\int 2x^3dx-\int 4dx$= o'zgarmasini integral belgisidan chiqarib yozish mumkinligidan =$3\int x^4dx+2\int x^3dx-4\int dx$. Endi 1 ,2 formulalarga asosan talab qilingan integral $3x^{4+1}/(4+1)+2x^{3+1}/(3+1)-4x+c=(3x^5/5)+(x^4/2)-4x+c$ bo'ladi.
Boshlang'ich funksiyani to'g'ri topilganligini tekshiramiz:

$$\left(\frac{3x^5}{5}+\frac{x^4}{2}-4x+c\right)^1 = \frac{3}{5}(x^5)^1+\frac{1}{2}(x^4)^1-4(x)^1+c^1 = \frac{3}{5}5x^4+\frac{1}{2}*4x^3-4+0 = 3x^4+2x^3-4$$

misol.2.

$\int (5x^3 - \frac{3}{x^4} + 2\sqrt[3]{x})dx$ bu yerda $\frac{1}{x^4} = x^{-4}$ $\sqrt[3]{x} = x^{\frac{1}{3}}$ bo'lgandigidan berilgan integral Endi yuqorida echilgan 1 misoldagidek mulohaza bilan yechiladi:

$$= 5\int x^3dx - 3\int x^{-4}dx + 2\int x^{\frac{1}{3}}dx = \frac{5x^4}{4} - 3\frac{x^{-3}}{-3} + 2\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + c = \frac{5x^4}{4} + x^{-3} + \frac{3}{2}x^{\frac{4}{3}} + c$$

Tekshirish:

$$\left(\frac{5x^4}{4}\right)^1 + (x^{-3})^1 + (\frac{3}{2}x^{\frac{4}{3}})^1 + c^1 = 5x^3 - 3x^{-4} + 2x^{\frac{1}{3}}$$

2) O'zgaruvchini almashtirish usuli bilan intagrallash.

Ba'zi integrallarni to'g'ridan-to'g'ri jadval integrallarga keltirib, ya'ni bevosita integrallash usuli bilan (misol1 va 2 ga qarang) yechib bo'lmaydi. Bunday hollarda ma'lum bir belgilash kiritib, bevosita integrallashgan berilgan integralni keltirish mumkin.

misol. 3. $\int (2x+3)^5dx$

2x+3=U   dU=2dx

$$= \int u^{5} * \frac{1}{2} du = \frac{1}{2} \int u^{5} du = \frac{1}{2} * \frac{u^{6}}{6} + c = \frac{1}{12} (2x+3)^{6} + c$$

Tekshirish: $\left[\frac{1}{12} (2x+3)^{6} + c \right]^{1} = \frac{12(2x+3)^{5}}{12} = (2x+3)^{5}$

Misol.4.

$$\int \frac{dx}{\cos^{2}(4x+3)}$$

bundan ham U=4x+3 deb belgilaymiz, bundan dU=4dx

$$dx = \frac{1}{4} du$$

$$= \int \frac{1}{\cos^{2} u} * \frac{1}{4} du = \frac{1}{4} \int \frac{1}{\cos^{2} u} du = \frac{1}{4} tgu + c = \frac{1}{4 tg(4x+3)} + c$$

Tekshirish:

$$\left(\frac{1}{4} tg(4x+3) \right)^{1} \frac{1}{4} * \frac{1}{\cos^{2}(4x+3)} * (4x+3)^{1} = \frac{4}{4\cos^{2}(4x+3)} = \frac{1}{\cos^{2}(4x+3)}$$

3) Bo’laklab integrallash.

Agarda integral belgisi ostida teskari trigonometrik, logarifmik yoki algebraik funksiya bilan transsendent funksiyalar ko’paytmasidan iborat ifodalar qatnashgan bo’lsa, bunday integrallarni bo’laklab integrallash formulasidan foydalanib echish mumkin:

$\int$ udv=uv-$\int$ vdu   (18)

Misol -5. $\int \frac{\ln x}{x^{5}} dx$

Bo’laklab integrallash (18)formulasidan foydalanamiz, buning uchun y=lnx va dv=dx/x^5 bedgilasak dy=dx/x,

$$V = \int \frac{dx}{x^{5}} = \frac{x^{-5+1}}{-5+1} = \frac{x^{-4}}{-4} = -\frac{1}{4x^{4}}$$ u holda

$$\int \frac{\ln x}{x^{5}} dx = -\frac{1}{4x^{4}} \ln x + \frac{1}{4} \int \frac{1}{x^{4}} * \frac{1}{x} dx = -\frac{\ln x}{4x^{4}} + \frac{1}{4} \int x^{-5} dx = -\frac{\ln x}{4x^{4}} + \frac{1}{4} * \frac{x^{-5+1}}{-5+1} + c = -\frac{\ln x}{4x^{4}} - \frac{x^{-4}}{16} + c = -\frac{\ln x}{4x^{4}} - \frac{1}{16x^{4}} + c$$

Tekshirish

$$\left(-\frac{\ln x}{4x^{4}} - \frac{1}{16x^{4}} + c \right) = -\frac{(\ln x)^{1} \cdot x^{4} - (x^{4})^{1} \ln x}{4x^{8}} - \frac{1}{16}(x^{-4})^{1} + 0 = -\frac{\frac{1}{x} x^{4} - 4x^{3} \ln x}{4x^{8}} + \frac{1}{4} x^{-5} = -\frac{1}{4} x^{-5} + \frac{\ln x}{x^{5}} + \frac{1}{4} x^{-5} = -\frac{\ln x}{x^{5}}$$

Binamial differensialni integrallash.

$\int x^{m}(a+Bx^{n})^{P} dx$ integrallarni yechishda quyidagi hollar bo’lishi mumkin: 1). Agar r butun bo’lsa, $\sqrt[\lambda]{x} = t$ almashtirish yordamida yechiladi. 2). Agar $\frac{m+1}{n} = q$ butun bo’lsa $\sqrt[\upsilon]{a+Bx^{n}} = t$ almashtirish bilan yechiladi. 3). Agar r+q=p$\frac{m+1}{n}$ butun bo’lsa, $\sqrt[\upsilon]{\frac{a+Bx^{n}}{x^{n}}} = t$ almashtirish yordamida yechiladi.

Misol-6.

$$\int\sqrt[3]{\frac{1+\sqrt[4]{x}}{\sqrt{x}}}dx=\int x^{-\frac{1}{2}}(1+x^{\frac{1}{4}})^{\frac{1}{3}}dx \qquad \text{mq-}\frac{1}{2},\ \text{nq}\frac{1}{4},\ \text{pq}\frac{1}{3}$$

1). R-butun emas 2). $\frac{m+1}{n}=\frac{1-\frac{1}{2}}{\frac{1}{4}}=2$

Demak $\sqrt[3]{1+\sqrt[4]{x}}=t$ $\qquad \sqrt[4]{x}=t^3-1$ $\quad$ xq(t^3-1)^4 $\quad$ dxq4$\cdot 3t^2(t^3-1)^3dt$

$$\int\frac{t}{(t^3-1)^2}12t^2(t^3-1)^3dt=12\int\left(t^6-t^3\right)dt=\frac{12t^7}{7}-3t^4+c\ \frac{12}{7}\sqrt[3]{\left(1+\sqrt[4]{x}\right)^7}-3\cdot\sqrt[3]{\left(1+\sqrt[4]{x}\right)^4}+c$$

Misol- 7. Quyidagi rasional kasrni integrallang

$$\int\frac{x\ dx}{(x-1)(x^2+1)}$$

Yechish. Berilgan integralni rasional kasrlarni integrallaga doir nazariy ma'lumotlar asosida sodda kasrlarga yoyamiz: $\int\frac{x\ dx}{(x-1)(x^2+1)}=A\int\frac{dx}{x-1}+\int\frac{BX+c}{x^2+1}dx$

Bundan $\frac{x}{(x-1)(x^2+1)}=\frac{A}{x-1}+\frac{BX+C}{x^2+1}$. Aniqmass koeffisientlar usuli Bilan ayniyatligiga asoslanib noma'lum A, B, S larni tenglamalar sistemasini echib topamiz:

$x=A(x^2+1)+B(x^2-x)+C(x-1)$, $\quad (A+B)x\ +(-B+C)x+A-C=x$

$$\begin{array}{l} x^2: \\ x: \\ x^0: \end{array}\begin{cases} A+B=0 \\ -B+C=1 \\ A-C=0 \end{cases}\Rightarrow\begin{cases} B=-A \\ A=C \\ -B+C=1, \end{cases} \qquad \begin{array}{ll} A=-B=\frac{1}{2}, & C=\frac{1}{2} \\ -2B=1, & B=-\frac{1}{2} \end{array}$$

Topilganlarga asosan berilgan integralni yechimi

$$\int\frac{x\ dx}{(x-1)(x^2+1)}=\frac{1}{2}\int\frac{dx}{x-1}-\frac{1}{2}\int\frac{x-1}{x^2+1}dx=\frac{1}{2}\ln(x-1)-\frac{1}{2}\int\frac{x\ dx}{x^2+1}+\frac{1}{2}\int\frac{dx}{x^2+1}=$$

$$=\frac{1}{2}\ln(x-1)-\frac{1}{4}\int\frac{d(x^2+1)x}{x^2+1}+\frac{1}{2}arctgx\ =\frac{1}{2}\ln(x-1)-\frac{1}{4}\ln(x^2+1)+\frac{1}{2}arctgx\ +c.$$

77. Mustaqil yechish uchun misollar.

1-20 Ushbu aniqmas integrallarni yechib, to'g'riligini differensiallash yordamida tekshiring.

1. a) $\int\left(3x^4+\frac{2}{x^3}-\sqrt[4]{x^3}\right)dx$ $\qquad$ b) $\int\frac{x^2dx}{4-2x^3}$ $\qquad$ c) $\int$x cosxdx

2. a) $\int\left(5x^5+2\sqrt[3]{x^2}-\frac{8}{x^2}\right)$ b) $\int\frac{xdx}{\sqrt[3]{3-x^2}}$ c) $\int x\,e^x dx$

3. a) $\int\left(4x^2+3\sqrt{x}-\frac{3}{\sqrt{x}}\right)dx$ b) $\int e^{3x-4}dx$ c) $\int x\sin x\,dx$

4. a) $\int\left(3x^2+4\sqrt{x}-\frac{4}{\sqrt[3]{x}}\right)dx$ b) $\int\frac{\cos x}{\sin^2 x}dx$ c) $\int x^2\ln x\,dx$

5. a) $\int\left(7x^6-3\sqrt[3]{x}+\frac{5}{\sqrt[3]{x^2}}\right)dx$ b) $\int\sin x\cos^5 x\,dx$ c) $\int x^3\ln x\,dx$

6. a) $\int\left(1-5\sqrt[4]{x^3}+\frac{3}{\sqrt[6]{x}}\right)dx$ b) $\int\cos(4x+3)dx$ c) $\int\frac{\ln x}{x^4}dx$

7. a) $\int\left(\sqrt[5]{x^2}+\frac{2x^3}{3}+\frac{1}{x^3}\right)dx$ b) $\int e^x x^3\,dx$ c) $\int x^2\sin x\,dx$

8. a) $\int\left(1+2\sqrt[4]{x}-\frac{3x^4}{5}\right)dx$ b) $\int\frac{3xdx}{(2x^2-5)^5}$ c) $\int x^2\cos x\,dx$

9. a) $\int\left(\sqrt[5]{x}-\frac{2}{7}x^6+\frac{8}{x^4}\right)dx$ b) $\int\frac{dx}{\sqrt[3]{3-x}}$ c) $\int 3x\,e^x\,dx$

10. a) $\int\left(2\sqrt[6]{x}-\frac{5x^6}{7}+\frac{3}{\sqrt[6]{x}}\right)dx$ b) $\int \mathrm{tg}3x\,dx$ c) $\int 3x\sin x\,dx$

Aniqmas integrallarni yeching.

1) $\sqrt{\cos x}\sin xdx$

11) $\int\frac{\sin x}{\cos^2 x}dx$

2) $\int(\ln x)^3\frac{dx}{x}$

12) $\int\frac{x^2}{2x^3+3}dx$

3) $\int\frac{arctgx}{1+x^2}dx$

13) $\int\sqrt{5x^4+3x^3}dx$

4) $\int\frac{\cos x}{\sqrt[3]{\sin x}}dx$

14) $\int x^2e^{x^3+1}dx$

5) $\int e^{-x^2}xdx$

15) $\int\frac{x^3}{\sqrt{8x^4-1}}dx$

6) $\int\frac{x}{2+x^4}dx$

16) $\int\frac{x}{2x^2+3}dx$

7) $\int\sqrt{\ln x}\cdot\frac{dx}{x}$

17) $\int\arcsin^2 x\frac{dx}{\sqrt{1-x^2}}$

8) $\int\frac{x}{\sqrt{1-2x^2}}dx$

18) $\int\frac{\sqrt{arcrgx}}{1+x^2}dx$

9) $\int\frac{x}{2x^4+5}dx$

19) $\int\frac{\ln x+3}{x}dx$

10) $\int\frac{dx}{x\ln x}$

20) $\int\sqrt{1+2x^2}xdx$

21) $\int\frac{4x-1}{x^2-4x+8}dx$

31) $\int\frac{8x-7}{x^2+10x+29}dx$

22) $\int \frac{5x+8}{x^2-2x+5}dx$

23) $\int \frac{3x-2}{x^2+4x+8}dx$

24) $\int \frac{8x-3}{x^2-6x+10}dx$

25) $\int \frac{7x+3}{x^2-4x+5}dx$

26) $\int \frac{9x+10}{x^2-6x+10}dx$

27) $\int \frac{3x+10}{x^2-8x+10}dx$

28) $\int \frac{3x+7}{x^2+8x+17}dx$

29) $\int \frac{5x-2}{x^2-2x+5}dx$

30) $\int \frac{7x-3}{x^2-6x+13}dx$

32) $\int \frac{11x-3}{x^2+6x+13}dx$

33) $\int \frac{10x-7}{x^2-8x+20}dx$

34) $\int \frac{3x+11}{x^2-16x+68}dx$

35) $\int \frac{5x+16}{x^2+2x+17}dx$

36) $\int \frac{3x-11}{x^2-8x+20}dx$

37) $\int \frac{17x+5}{x^2-12x+40}dx$

38) $\int \frac{12x-7}{x^2+16x+65}dx$

39) $\int \frac{8x-7}{x^2-2x+17}dx$

40) $\int \frac{17x-3}{x^2+8x+32}dx$

41) $\int \ln xdx$

42) $\int (2x+1)\sin 3xdx$

43) $\int (x-1)e^{2x}dx$

44) $\int x\cos 2xdx$

45) $\int arctg\ 2xdx$

46) $\int (5x+1)\ln xdx$

47) $\int (8x-2)\sin 5xdx$

48) $\int (x-3)e^{-2x}dx$

49) $\int \sqrt{x}\ln 3xdx$

50) $\int (2x+8)e^{-7x}dx$

51) $\int x^3\ln xdx$

52) $\int (3x+7)\cos 5xdx$

53) $\int (12x+2)\sin 3xdx$

54) $\int \sqrt[3]{x}\ln 2xdx$

55) $\int x\sin 8xdx$

56) $\int \arccos xdx$

57) $\int \arcsin 2xdx$

58) $\int (2x-1)\cos 3xdx$

59) $\int (8x-10)\sin 7xdx$

60) $\int \ln 8xdx$

61) $\int \frac{x}{x^3+1}dx$

62) $\int \frac{x+20}{x^3-8}dx$

63) $\int \frac{3x+1}{x(x^2+1)}dx$

64) $\int \frac{2x+5}{x^3+2x}dx$

65) $\int \frac{3x-1}{x^3+3x}dx$

66) $\int \frac{8x+5}{(x+1)(x^2+2)}dx$

67) $\int \frac{7x-2}{(x-3)(x^2+1)}dx$

68) $\int \frac{5x-11}{x(x^2+4)}dx$

69) $\int \frac{3x}{(x+1)(x^2+3)}dx$

70) $\int \frac{2x}{x^3-1}dx$

75) $\int \frac{x}{(x+5)(x^2+3)}dx$

76) $\int \frac{x+1}{(x-1)(x^2+4)}dx$

71) $\int \frac{3x-1}{x(x^2+3)}dx$

72) $\int \frac{5x-1}{x^3+1}dx$

73) $\int \frac{2x-1}{x^3-x}dx$

74) $\int \frac{2x+5}{x^3-4x}dx$

77) $\int \frac{x}{(x-3)(x^2+10)}dx$

78) $\int \frac{2x+5}{x(x^2+6)}dx$

79) $\int \frac{x-3}{(x+2)(x^2+5)}dx$

80) $\int \frac{x-2}{(x+2)(x^2+3)}dx$

78. Aniq integral va uni tatbiqlari

Aniq integral bevosita amaliy masalalarni yechishdan xususan murakkab figuralarni yuzalarini, jismlarni xajmini hisoblashlardan kelib chiqqan.

Aniq integralni hisoblash uchun avvalo aniqmas integral yechilib, uning boshlang'ich funksiyasi F(x) topiladi va unga Nyuton-Leybnis formulasi qo'llaniladi, ya'ni

$$\int_a^b f(x)dx=F(x)\Big|_a^b=F(b)-F(a)$$

Bu formulani inegral hisobining asosiy formulasi ham deyiladi.

Masalan, aniq integral yordamida y=f(x) (f(x)≥0) funksiya va y=0, x=a, x =b to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapesiya yuzasini, aylanma jismni hajmini hisoblash mumkin. Aniq integrallarni yechishda uni quyidagi xossalaridan foydalaniladi:

1. $\int_a^b f(x)dx=\int_a^b f(t)dt=\int_a^b f(y)dy$ 2. $\int_a^b [C_1f(x)\pm C_2g(x)dx=C_1\int_a^b f(x)dx\pm C_2\int_a^b g(x)dx$

3. $\int_a^b f(x)dx=-\int_b^a f(x)dx$ 4. $\int_a^a f(x)dx=0$

5. Agar a<c< b bo'lsa $\int_a^b f(x)dx=\int_a^c f(x)dx+\int_a^b f(x)dx$

Aniq integral yordamida yuza, yoy uzunligini va hajimlarni hisoblash.

Ma'lumki $y=f(x)$, $f(x)\geq 0$ egri chiziq ox o'qi va $x=a$, $x=в$ to'gri chiziqlar bilan chegaralangan egri chiziqli trapesiya yuzasi quyidagi formula $S=\int_a^в f(x)dx$ bilan hisoblanadi. Umumiy holda yuzani hisoblash formulasi $S=\int_a^в If(x)Idx$.

Agar egri chiziq $x=\varphi(t)$, $y=\varphi(t)$ parametrik tenglamalar bilan berilgan bo'lsa $\alpha\leq t\leq\beta$ figura yuzasi

$$dx = \phi^1(t)dt\ ,\ S = \int_a^в ydx = \int_\alpha^\beta \psi(t)\varphi^1(t)dt$$

Qutub koordinatalari sistemasida $g = g(\varphi)\ ,\ \alpha < \varphi < \beta$ chiziqlar bilan chegaralangan yuza

$$S = \frac{1}{2}\int_\alpha^\beta g^2(\varphi)d\varphi$$

AB egri chiziq y=f(x)tenglama bilan berilgan bo'lib, f(x) $[a,s]$kesmada uzluksiz va uni barcha ichki nuqtalarida birinchi tartibli hosilalarga ega bo'lsin,u holda AB yoyni uzunligi

$$l = \int_a^в \sqrt{1+\left[f^1(x)\right]^2}dx$$

Agar egri chiziq x=x(t)va y=(t), $\alpha \le t \le \beta$. A=x(α),b=x(β). parametrik tenglamalar bilan berilgan bo'lib, x(t) va u(t) birinchi tartibli hosilalarga ega bo'lsa, bu holda egri chiziq yoyining uzunligi

$$l = \int_\alpha^\beta \sqrt{(x^1(t))^2 + (y^1(t))^2}dt$$

Agar egri chiziq $\rho = \rho(\varphi)\quad \alpha \le \varphi \le \beta$ tenglama bilan qutb koordinatalar sistemasida berilgan bo'lsa, u holda bu egri chiziq yoyining uzunligi

$l = \int_\alpha^\beta \sqrt{\rho^2 + (\rho^1)^2 d\varphi}$ formula bilan hisoblanadi.

Shu figurani ox o'qi atrofida aylanishidan hosil bo'lgan aylanma jismni hajmini hisoblash formulasi B_{ox}

$$B_{ox}=\pi \int_a^b [(f(x) - \varphi(x)]^2dx.$$

79. Mavzuga doir namunaviy misollarni yechimi.

Quyidagi aniq integrallarni hisoblang.

Misol-1.

$$\int_1^5 x^2dx = \frac{x^3}{3} = \frac{5^3}{3} - \frac{1^3}{3} = \frac{125-1}{3} = 41\frac{1}{3}$$

Misol-2. $\int_1^3 [5x^3-3x^2+6]dx=$

Bir necha funksiya yig'indisi va ayirmasining integrali integrallar yig'indisi va ayirmasiga tengligidan

$$= 5\int_1^3 x^3dx - 3\int_1^3 x^2dx + 6\int_1^3 dx = \frac{5}{4}x^4 - 3\cdot\frac{1}{3}x^3 + 6x = \frac{5}{4}(3^4 - 1^3) + 6(3-1) = 100 - 26 + 12 = 86$$

Misol-3.

$$\int_0^1 dx/(1+x^2)=\operatorname{arctg}x\Big|_0^1=\operatorname{arctg}1-\operatorname{arctg}0=\pi/4$$

Misol-4. Quyidagi parabola $y=-x^2+5x-4$ hamda $y-x+1=0$ tugri chiziqlar bilan chegaralangan yuza hisoblansin.

Yechish. Talab qilingan yuzani topish uchun parabola bilan to'g'ri chiziqni kesishish nuqtalarini topamiz:

$$\begin{cases} y=-x^2+5x-4 \\ y=x-1 \end{cases}$$

$x-1=-x^2+5x-4$, $x^2-4x+3=0$

kvadrat tenglamani yechib,

$x_{1/2}=2\pm\sqrt{4-3}=2\pm1$ $x_1=1$, $x_2=3$ ekanligidan

kesishish nuqtalari A(0;1), B(3;2) bo'ladi

Topilayotgan yuzani grafik shaklida izohlash uchun parabolani to'la tekshirish yordamida chizamiz: $y^1=(-x^2+5x-4)^1=-2x+5=0$, $x=2{,}5$ kritik nuqtasi va bu nuqta atrofida

$$\begin{cases} y^1(2)=-2*2+5=1>0 \\ y^1(3)=-2*3+5=-1<0 \end{cases}$$

bo'lganligidan funksiya max ega $y_{max}=y(2{,}5)=-(2{,}5)^2+5*2{,}5-4=2{,}25$, D(2,5;2,25)

OX uqini kesishish nuqtalari (y=0) da

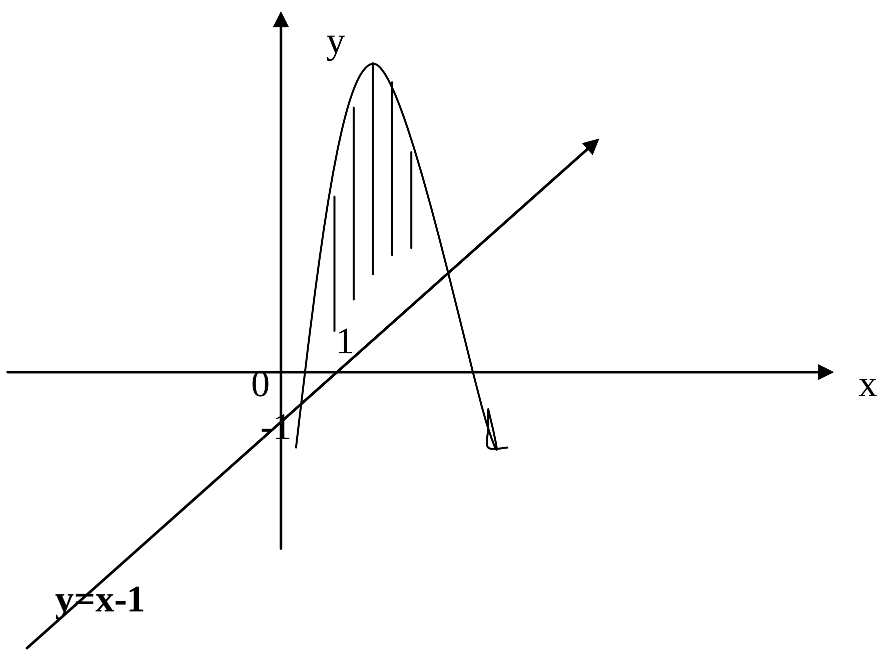
$-x^2+2x-4=0$, $x^2-5x+4=0$, $x_{1,2}=(5+3)/2$, $x_1=1$, $x_2=4$ bo'ladi, hamda x=2 bo'lganda u=3, x=3, u=2 bo'lganligidan C(2;2), B(3;2) nuqtalardan parabola o'tadi. Y=x-1 to'g'ri chiziq

X	0	1
Y	-1	0

M(0;-1), A(1;0) nuqtalardan o'tadi.

U holda (20) formulaga asosan talab qilingan yuza:

$$S=\int_1^3[-x^2+5x-4-x+1]dx=-\int_1^3 x^2dx+4\int_1^3 xdx-3\int_1^3 dx=-x^3/3\Big|_1^3+2x^2\Big|_1^3-3x\Big|_1^3=-1/3(3^3-1^3)+2(3^2-1^2)-3(3-1)=4/3 \text{ kv.birlik.}$$

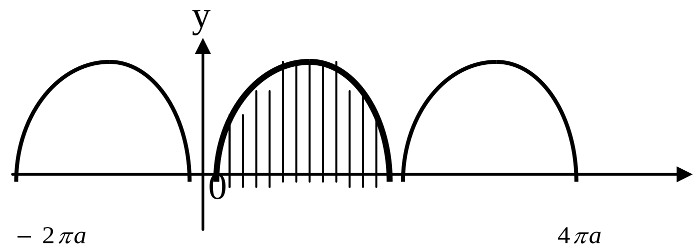


Misol-5. $x = a(t - \sin t)$, $y = a(1 - \cos t)$ sikloidaning bir arki va ox o'qi bilan chegaralangan yuzasini hisoblang.

Yechish: $0 \le t \le 2\pi$ oraliqda o'zgarganda, $0 \le x \le 2\pi a$ o'zgaradi, $dx = x^1(t)dt = a(1 - Cost)dt$ bo'lganligidan yuqorida parametrik holda yuzani hisoblang formulasidan

$$S = \int_0^{2\pi a} ydx = \int_0^{2\pi} a(1 - Cost)a(1 - Cost)dt = a^2 \int_0^{2\pi} (1 - Cost)^2 dt = a^2 \int_0^{2\pi} (1 - 2Cost + Cos^2 t)dt =$$

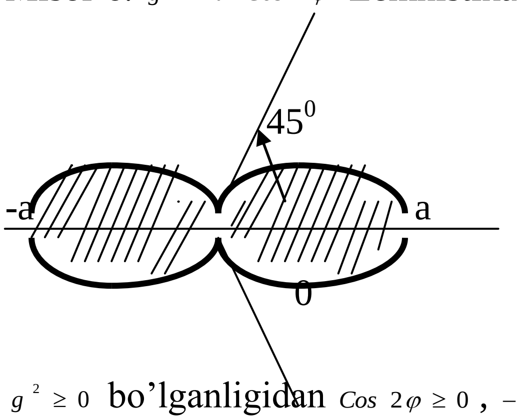
$$a^2((t - 2S\text{int})\Big|_0^{2\pi} + \int_0^{2\pi} \frac{1 + Cos\,2t}{2} dt) = 2\pi a^2 + \frac{1}{2}a^2 t\Big|_0^{2\pi} + \frac{1}{4}a^2 Sin\,2t\Big|_0^{2\pi} = 2\pi a^2$$



Qutub koordinatalari sistemasida $g = g(\varphi)$, $\alpha < \varphi < \beta$ chiziqlar bilan chegaralangan yuza

$$S = \frac{1}{2}\int_\alpha^\beta g^2(\varphi)d\varphi$$

Misol-6. $g^2 = a^2 Cos\,2\varphi$ Lemnistika bilan chegaralangan yuzani hisoblang



$g^2 \ge 0$ bo'lganligidan $Cos\,2\varphi \ge 0$, $-\frac{\pi}{2} \le 2\varphi \le \frac{\pi}{2}$ $-\frac{\pi}{4} \le \varphi \le \frac{\pi}{4}$.

$$\frac{S}{2} = \frac{1}{2}\cdot\frac{1}{2}\int_{-\pi/4}^{\pi/4} a^2 Cos\,2\varphi d\varphi = \frac{a^2}{4}\int_{-\pi/4}^{\pi/4} Cos\,2\varphi d\varphi = \frac{a^2}{2}, \quad \underline{S = a^2}$$

Misol -7. Bizga ma'lumki aylana yoyining uzunligi eq2PR ga teng.

$$\begin{cases} x^1 = RCost \\ y^1 = RS\text{int} \end{cases} \qquad 0 \le t \le 2\pi$$ aylana yoyini uzunligi

(1) formula orqali topilsin.

$x^1 = RCost \qquad (x^1)^2 = R^2 Cos^2 t$

$y^1 = RS\text{int} \qquad (y^1)^2 = R^2 Sin^2 t$

y xolda $e = \int_0^{2\pi} \sqrt{R^2(\cos^2 t + \sin^2 t)}dt = R\int_0^{2\pi} dt = 2PR$

Misol-8. $\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}$ tenglamalarni sistemasini bilan berilgan egri chiziqning uzunligi aniqling.

Yechish: x^1=a(1-cos) y^2=sint

$x^{12}+y^{12}=a^{2(}1\text{-cost+})^2+a^2\sin^2t=a^2-2a^2\cos t+a^2\cos^2t+a^2\sin^2t=a^2-2a^2\cos t+a^2=2a^2(1-\cos t)$

$\sqrt{x^{12}+y^{12}} = a\sqrt{2(1-\cos t)}$

$$e = \int_0^{2\pi} a\sqrt{2(1-\cos t)}dt = a\int_0^{2\pi}\sqrt{2\cdot 2\sin^2\frac{t}{2}}dt = 2a\int_0^{2\pi}\sin\frac{t}{2}dt = 4a\int_0^{2\pi}\sin\frac{t}{2}d\left(\frac{t}{2}\right) = 14a\cos\frac{t}{2}\Big|_0^{2\pi} = -4a(-1-1) = 8a$$

80. Mustaqil yechish uchun misollar
Quyidagi aniq integrallarni hisoblang

81. $\int_0^{\frac{1}{4}} \frac{\sin\frac{x}{2}}{\sqrt{x}}dx$; 82. $\int_0^1 x^4 e^{-\frac{1}{2}}dx$;

83. $\int_0^{\frac{1}{9}} \sqrt{x}e^{-\sqrt{x}}dx$; 84. $\int_0^{\frac{1}{2}} \frac{\sin(x^2)}{x}dx$;

85. $\int_0^{\frac{1}{3}} x\ln(1+\sqrt{x})dx$; 86. $\int_0^{\frac{1}{4}} x^2 e^{-\sqrt{x}}dx$;

87. $\int_0^{\frac{1}{4}} \frac{\sin 4x}{\sqrt{x}}dx$; 88. $\int_0^{\frac{1}{4}} \sqrt{x}\cos 2x dx$;

89. $\int_0^{\frac{1}{5}} x^3 e^{-x^3}dx$; 90 . $\int_0^{\frac{1}{4}} \frac{\sin 2x}{\sqrt{x}}dx$;

91. $\int_0^{\frac{1}{8}} \frac{\sin(x^2)}{x}dx$; 92. $\int_0^{\frac{1}{9}} \sqrt{x}\sin\frac{x}{3}dx$;

93. $\int_0^1 \sqrt{x}e^{-x^2}dx$; 94. $\int_0^{\frac{1}{4}} \frac{\sin\sqrt{x}}{\sqrt{x}}dx$;

95. $\int_0^{\frac{1}{4}} x^2\ln(1+x)dx$; 96. $\int_0^{\frac{1}{3}} \sqrt{x}\sin\sqrt{x}dx$;

97. $\int_0^{\frac{1}{2}} x\cos\sqrt{x}dx$; 98. $\int_0^{\frac{1}{4}} xe^{-\frac{1}{2}x^3}dx$;

99. $\int_0^1 \sqrt{x}e^{-x^3}dx$; 100. $\int_0^{\frac{1}{2}} x\ln(1+x^2)dx$.

1-20. Quyidagi chiziqlar bilan chegaralangan figura yuzasini toping va shaklini chizing.

1. $y=2x^2-3x+3$ va $y=2x+1$
2. $y=3x^2-7x+5$ va $y=3x+2$
3. $y=4x^2-3x+3$ va $y=x+2$
4. $y=3x^2+3x+2$ va $y=x+2$
5. $y=x^2-5x+2$ va $y=x-3$
6. $y=x^2-x-30$ va $y=5-3x$
7. $y=2x^2-5x-1$ va $y=2x+3$
8. $y=5x^2-6x+2$ va $y=2x-1$
9. $y=x^2-5x+2$ va $y=x+2$
10. $y=5x-7-x^2$ va $y=3-2x$
11. $y=3x-x^2+3$ va $y=x+3$
12. $y=x^2-4x+2$ va $y=2x+2$
13. $y=2x^2-2x+1$ va $y=5x-2$
14. $y=x^2-4x+10$ va $y=3x-2$
15. $y=3x^2-x+2$ va $y=3x+1$
16. $y=x^2-5x+10$ va $y=3x+10$
17. $y=x^2+x+5$ va $y=3-2x$
18. $y=3x^2-x-3$ va $y=3x-4$
19. $y=2x^2-x+3$ va $y=2x+2$
20. $y=2x^2+x+2$ va $y=2x+3$

Quyidagi chiziqlar bilan chegaralangan figura yuzalarini hisoblang

21) $y=\frac{1}{2}x^2-x+1$; $y=-\frac{1}{2}x^2+3x+6$.

22) $y=2x^2-6x+1$; $y=-x^2+x-1$.

23) $y=\frac{1}{2}x^2+x+2$; $y=-\frac{1}{2}x^2-5x+7$.

24) $y=\frac{1}{3}x^2-2+4$; $y=-\frac{2}{3}x^2-x+2$.

25) $y=\frac{1}{3}x^2-3x+2$; $y=-\frac{2}{3}x^2-2x+4$.

26) $y=x^2-5x-3$; $y=-3x^2+2x-1$.

27) $y=2x^2+6x-3$; $y=-x^2+x+5$.

28) $y=x^2-2x-5$; $y=-x^2-x+1$.

29) $y=3x^2+5x-1$; $y=-x^2+2x+1$.

30) $y=\frac{1}{4}x^2-2x-5$; $y=-\frac{3}{4}x^2-x+1$.

31) $y=x^2+3x-1$; $y=-x^2-2x+5$.

32) $y=\frac{1}{2}x^2+3x-2$; $y=-\frac{1}{2}x^2-x+3$

33) $y=2x^2-6x+3$;

34) $y=2x^2+3x+1$;

$y=-2x^2+x+5.$ $y=-x^2-2x+9.$

35) $y=x^2-3x-4$; 36) $y=2x^2-6x-2$;

$y=-x^2-x+8$ $y=-x^2+x-4.$

37) $y=\frac{1}{2}x^2-3x-1$; 38) $y=x^2-2x-4$;

$y=\frac{1}{2}x^2-x+2.$ $y=-x^2-x+2.$

39) $y=2x^2+4x-7$; 40) $y=\frac{1}{2}x^2-3x-2$;

$y=-x^2-x+1.$ $y=-\frac{1}{2}x^2-7x+3.$

Qutub koordinatalar sistemasida va parametrik tenglamalar bilan berilgan figura yuzalarini, yoy uzunliklarini hisoblang.

1. Astroida bilan chegaralangan figura yuzasini hisoblang
$x=aCos^3t,\ y=Sin^3t.$
2. $x=a(t^2-1),\ y=в(4t-t^3)\quad(a>0, в>0)$ chiziqlar bilan chegaralangan ilmoq figura yuzasini xisoblang
3. $x=\frac{1}{3}t(3-t^2),\quad y=t^2$ chiziqlar bilan chegaralangan ilmoq figura yuzasini hisoblang.
4. Кardoida bilan chegaralangan yuzani hisoblang
$$x=a(2Cost-Cos\,2t),\ y=a(2S\,\text{int}-Sin\,2t)$$
5. Ellips yuzani toping
$$x=aCost\quad,\ y=вS\,\text{int}$$
6. $g=aSin\,2\varphi$ egri chiziq bilan chegaralangan yuzani hisoblang.
7. $g=2aCos\,3\varphi$ egri chiziq va $g=a$ doira orasida joylashgan yuzani hisoblang.
8. $g^2=2aCos\,4\varphi$ egri chiziq bilan chegaralangan yuzani hisoblang.
9. $g=2\sqrt{3}aCos\,\varphi\quad,\ g=2aSin\,\varphi$ chiziqlar bilan chegaralangan yuzani hisoblang.
10. $g=2-Cos\,\varphi,\ g=Cos\,\varphi$ chiziqlar orasidagi yuzani hisoblang.

11) $r=2\cos\varphi$, 12) $r=2\sin\varphi$

$\varphi=0,\ \varphi=\frac{\pi}{4}.$ $\varphi=0,\ \varphi=\frac{\pi}{3}.$

13) $r=8(1+\cos\varphi)$, 14) $r=10\cos(\varphi-\frac{\pi}{4})$,

$\varphi=0,\ \varphi=\frac{\pi}{4},$ $\varphi=\frac{\pi}{4},\ \varphi=\frac{\pi}{2}.$

15) $r=2\cos\varphi$; 16) $r=1-\cos 2\varphi$,

$r=4\cos\varphi.$ $\varphi=0,\ \varphi=\frac{\pi}{6}.$

17) $r=\sqrt{\cos 2\varphi}$, 18) $r=2\varphi$,

$\varphi = 0,\ \varphi = \frac{\pi}{6}.$ $\qquad$ $\varphi = 0,\ \varphi = \frac{\pi}{2}.$

19) $r = 2 + \cos\varphi,$ $\qquad$ 20) $r = e^{\varphi},$

$\varphi = 0,\ \varphi = \frac{\pi}{6}.$ $\qquad$ $\varphi = 0,\ \varphi = \frac{\pi}{4}.$

Yoy uzunligini hisoblash

1) $y = \frac{1}{3}x\sqrt{x},$ $\qquad$ 2) $r = 1 - \cos\varphi,$

$0 \le x \le 12.$ $\qquad$ $0 \le \varphi \le \frac{\pi}{2}.$

3) $y = \ln\sin x,$ $\qquad$ 4) $r = 2(1 + \cos\varphi),$

$\frac{\pi}{3} \le x \le \frac{\pi}{2}.$ $\qquad$ $0 \le \varphi \le \frac{\pi}{2}.$

5) $y = 1 - \ln\cos x,$ $\qquad$ 6) $y = \ln x,$

$0 \le x \le \frac{\pi}{6}.$ $\qquad$ $\frac{3}{4} \le x \le 1.$

7) $r = a\sin\varphi$ $\qquad$ 8) $y = \ln\cos x,$

$0 \le \varphi \le \frac{\pi}{2}.$ $\qquad$ $0 \le \varphi \le \frac{\pi}{3}.$

9) $r = a\cos\varphi,$ $\qquad$ 10) $y = \frac{1}{3}(x+1)\sqrt{x+1},$

$0 \le \varphi \le \frac{\pi}{2}.$ $\qquad$ $0 \le x \le 1.$

81. Aniq integrallarni taqribiy hisoblash formulalari.

$f(x)$ funksiya murakkab bo’lsa uning boshlangich funksiyasini topish qiyin bo’ladi, unda berilgan funksiyaning integralini taqribiy hisoblashga to’gri keladi.

1.To’gri to’rtburchaklar formulasi. $f(x)$ funksiya [a,b] segmentda berilgan va uzluksiz bo’lsin. [a,b] segmentni $x_0=a<x_1<x_2<\ldots<x_{n-1}<x_n=b$ nuqtalar yordamida n ta teng bo’lakka bo’lamiz.

$\Delta X_R = X_{R+1} - X_R = \frac{в - a}{n},$ $\qquad$ $X_R = a + R\frac{в - a}{n}(R = 0,1,2,\ldots,n$ bo’ladi.

$$\int_a^в f(x)dx = \frac{a - в}{n}\left[f(x_0) + f(x_1) + f(x_2) + \ldots + f(x_{n-1})\right] = \frac{в - a}{n}\sum_{R=0}^{n-1} f(x_R)$$

bu formula to’gri to’rtburchakli formula deyiladi.

2.Trapesiyalar formulasi. [a,b]segmentni yuqoridagi n ta teng bo’lakka bo’lib f(x) funksiyaning $[x_R; x_{R+1}]$ segment bo’yicha olingan aniq integralini quyidagicha

$$\int_{x_R}^{x_{R=1}} f(x)dx \approx \frac{f(x_R) + f(x_{R+1})}{2}\Delta x_R = \frac{f(x_R) + f(x_{R+1})}{2}\frac{в - a}{n}$$

Bunday taqribiy formulani har bir $[x_R; x_{R+1}]$ (R=0,1,2,…,n-1) segmentga nisbatan yozib so’ng ularni xadlab qo’shib topamiz.

$$\int_a^в f(x)dx \approx \frac{в-a}{n}[f(x_0)+2f(x_1)+2f(x_2)+...+2f(x_{n-1})+f(x_n)]$$

bu formula trapesiyalar formulasi deb ataladi.

3.Parabolalar (Simpson) formulasi. $f(x)$ funksiya [a;b] segmentda berilgan va uzluksiz bo'lsin. [a;b] segmentni

$$x_0 = a \prec x_1 \prec x_2 \prec x_{2n-2} \prec x_{2n-1} \prec x_{2n} = в$$

nuqtalar yordamida 2n ta teng bo'lakka bo'lamiz. f(x) funksiyaning [x_{2R}; x_{2R+2}] segment bo'yicha aniq integralini quyidagicha

$$\int_{x_{2R}}^{x_{2R+2}} f(x)dx \approx \frac{в-a}{6}[f(x_{2R})+4f(x_{2R+1})+f(x_{2R+2})]$$

$$\int_a^в f(x)dx \approx \frac{в-a}{6n}[(f(x_0)+f(x_{2n}))+4(f(x_1)+f(x_3)+...+f(x_{2n-1}))+2(f(x_2)+f(x_4)+...+f(x_{2n-2}))]$$

bu formula parabolalar (Simpson) formulasi deyiladi.

82. Mavzuga doir namunaviy misollarni yechimi.

Misol. $\int_0^1 e^{-x^2}dx$ aniq integral taqribiy hisoblansin.

[0; 1] segmentni ushbu x_0=0, x_1=0,2, x_2=0,4, x_3=0,6, x_4=0,8, x_5=1,0 nuqtalar yordamida 5 ta teng bo'lakka bo'lamiz. So'ng f(x)=l^{-x2} funksiyaning shu nuqtalardagi qiymatlarini hisoblaymiz:

$f(x_0) = f(0) = e^0 = 1{,}00000$, $f(x_1) = f(0{,}2) \approx 0{,}96079$, $f(x_2) = f(0{,}4) \approx 0{,}85214$,

$f(x_3) = f(0{,}6) \approx 0{,}69768$, $f(x_5) = f(1) \approx 0{,}36788$, $f(x_4) = f(0{,}8) \approx 0{,}52729$.

Berilgan $\int_0^1 e^{-x^2}dx$ integralni to'gri to'rtburchak, trapesiya hamda Simpson formulalari bo'yicha taqribiy hisoblaymiz.

A) to'gri to'rtburchaklar formulasi bo'yicha

$$\int_0^1 e^{-x^2}dx \approx \frac{1-0}{5}[f(0)+f(0{,}2)+f(0{,}4)+f(0{,}6)+f(0{,}8)] =$$

$$= \frac{1}{5}(1{,}00000 + 0{,}96079 + 0{,}85214 + 0{,}69768 + 0{,}52729) = \frac{1}{5}(4{,}03790) = 0{,}80758$$

bo'ladi. Demak, $\int_0^1 e^{-x^2}dx \approx 0{,}80758$;

b) trapesiyalar formulasi bo'yicha

$$\int_0^1 e^{-x^2}dx \approx \frac{1-0}{2\cdot 5}[f(0)+2f(0{,}2)+2f(0{,}4)+2f(0{,}6)+2f(0{,}8)+f(1)] =$$

$$= \frac{1}{10}(1{,}00000 + 2\cdot 0{,}96079 + 2\cdot 0{,}85214 + 2\cdot 0{,}69768 + 0{,}52729) = 0{,}74805$$

bo'ladi. Demak, $\int_0^1 e^{-x^2}dx \approx 0{,}74805$;

v) Simpson formulasi bo'yicha

$$\int_0^1 e^{-x^2}dx \approx 0{,}74682 \;;$$

83. Mustaqil yechish uchun misollar

Quyidagi aniq integrallarni 1) trapesiya, 2) Simpson formulalari bilan 10^{-3} aniqlik bilan hisoblang.

1. $\int_0^1 \frac{x^3dx}{1+x^8}$, 2. $\int_0^2 \sqrt{1+x^3}\,dx$

3. $\int_0^1 \frac{1}{\sqrt{1+x^3}}dx$, 4. $\int_0^1 e^{-x^2}dx$

5. $\int_0^{0,5} \frac{dx}{\sqrt{1+x^4}}$, 6. $\int_2^3 \frac{dx}{\ln x}$

7. $\int_0^1 \frac{arctgx}{x}dx$, 8. $\int_0^{0,8} \frac{Sinx}{x}dx$

84. Xosmas integrallar.

1. Birinchi tur xosmas integral.

y=f(x) funksiya x∈[a;+∞) oraliqda aniqlangan va har qanday chekli [a;b] oraliqda integrallanuvchi bo'lsin.

$\int_a^{+\infty} f(x)dx = \lim_{B\to+\infty}\int_a^B f(x)dx$ Xosmas integral bo'lib, B→+∞ da chekli limitga ega bo'lsa, xosmas integral yaqinlashuvchi, aks xolda uzoqlashuvchi bo'ladi.

Xosmas integrallarni yaqinlashuvchanligini tekshirishda Koshi kriteriyasi, taqqoslash teoremalari, Dseixle va boshqa belgilar mavjud.

1). Taqqoslash belgisi. Agar (a;+∞) oraliqda f(x)≥0, g(x))≥0 nomanfiy integrallanuvchi funksiyalar uchun f(x_0)≤c=(x) (c>0) tengsizlik o'rinli bo'lsa, $\int_a^{+\infty} q(x)dx$ integralni yaqinlashuvchanligidan $\int_a^{\infty} f(x)dx$ integralni yaqinlashuvchanligi kelib chiqadi va $\int_a^{+\infty} f(x)dx$ integralni uzoqlashuvchanligidan $\int_a^{\infty} d(x)dx$ ni uzoqlashuvchanligi kelib chiqadi.

masalan $\int_1^{+\infty} f(x)dx$ xosmas integral f(x)=$\frac{1}{x^{\lambda}}$ ko'rinishida bo'lsa, λ>1 bo'lganda yaqinlashuvchanligi λ≤1 bo'lganda uzoqlashuvchi bo'ladi.

2). Agar $\int_a^{+\aleph} f(x)q(x)dx$ xosmas integralda =(x)↓x→+∞ da uzluksiz differensiallanuvchi funksiya kamayib =(x)→0 intilsa va f(x) chegaralangan boshlangich $|F(x)| \le c\angle\infty$ funksiyaga ega bo'lsa, berilgan xosmas integral yaqinlashadi.

3). Agar $\int_a^{+\infty}|f(x)|dx$ (1) yaqinlashuvchi bo'lsa, $\int_a^{+\infty} f(x)dx$ integral absalyut yaqinlashuvchi.
(2) integral yaqinlashuvchi bo'lib, (1) integral uzoqlashuvchi bo'lsa, (2) integral shartli yaqinlashuvchi integral bo'ladi.
Birinchi tur xosmas integralni absolyut yaqinlashuvchanligidan har qachon uni yaqinlashuvchanligi kelib chiqadi.

2. Ikkinchi tur xosmas integral.

y=f(x) funksiya [a;b-E] (E>0) kesmada chegaralangan va integrallanuvchi bo'lib, [a;v] kesmada chegaralangan bo'lsa, $\int_a^B f(x)dx$ 2-tur xosmas integral deyiladi.

Agar $\int f(x)dx = \lim_{E\to 0}\int_a^{B-E} f(x)dx$ bu limit chekli mavjud bo'lsa, 2-tur xosmas integral yaqinlashuvchi, aks xolda uzoqlashuvchi bo'ladi.
2-tur xosmas integrallarni yaqinlashuvchanligini yuqorida keltirilgan kriteriyalar yordamida tekshirish mumkin, chunki t=$\frac{1}{x-B}$ almashtirish yordamida 2-tur xosmas integral 1-tur xosmas integralga keltiriladi.
2-tur xosmas integrallarni yaqinlashuvchiligini tekshirishda taqqoslash teoremasidan foydalanish qulay. Masalan $f(x)\le q(x)=\frac{1}{(x-B)^\lambda}$ ko'rinishdagi chegaralanmagan funksiya bo'lsa, u xolda 2-tur xosmas integral λ<1 bo'lganda yaqinlashuvchi, λ≥1 bo'lganda uzoqlashuvchi bo'ladi.

85. Mavzuga doir namunaviy misollarni yechimi.

Quyidagi xosmas integrallarni yaqinlashuvchiligini tekshiring

Misol-1. Birinchi tur xosmas integralni yaqinlashuvchiligini tekshiring

$$\int_1^{+\infty}\frac{1}{x^2+1}dx \le \int_1^{+\infty}\frac{1}{x^2 dx} = \lim_{B\to+\infty}\int_i^B x^{-2}dx = \lim_{B\to+\infty}\frac{x^{-2+1}}{-2+1}\Bigg| = \lim_{B\to+\infty}\left[-\frac{1}{x}\Bigg|\right] = \lim_{B\to+\infty}\left[-\frac{1}{B}+1\right] = 1\angle\infty$$

Demak, berilgan xosmas integral taqqoslash teoremasiga asosan yaqinlashuvchi.

Misol-2. $\int_1^2\frac{1}{\sqrt{x-1}}dx = \lim_{E\to 0}\int_{1+E}^2\frac{1}{(x-1)^{1_2}}dx$ chunki f(x)q$\frac{1}{(x-1)^{1_2}}$ [1;2] oraliqda chegaralanmagan funksiya bo'lganligidan berilgan xosmas integral 2-tur xosmas integral bo'ladi, hamda taqqoslash teoremasiga asosan $\lambda=\frac{1}{2}\angle 1$ bo'lganligidan yaqinlashuvchanligi kelib chiqadi. haqiqatdan ham

$$= \lim_{E\to 0}\int_{1+E}^2 (x-1)^{-\frac{1}{2}}d(x-1) = \lim_{E\to 0}\frac{(x-1)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Big|_{1+E}^2 = \lim_{E\to 0}2\sqrt{x-1}\Big|_{1+E}^2 = 2\lim_{E\to 0}\left[1-\sqrt{1+E-1} = 2\angle\infty\right]$$

86. Mustaqil uchun yechish misollar

Quyidagi I – tur xosmas integrallarni yaqinlashuvchi yoki uzoqlashuvchiligini tekshiring.

1. $\int_1^{\infty}\frac{dx}{x+x^3}$, 2. $\int_1^{\infty}\frac{1}{x^2}e^{-\frac{1}{x}}dx$, 3. $\int_1^{\infty}\frac{dx}{x\sqrt{1+x^2}}$,

4. $\int_e^{\infty}\frac{dx}{x\ln^2 x}$, 5. $\int_{-\infty}^{\infty}\frac{dx}{x^2+4x+5}$, 6. $\int_1^{\infty}\frac{arctgx}{1+x^2}dx$,

7. $\int_1^{\infty}xCosdx$, 8. $\int_1^{\infty}\frac{dx}{\sqrt{4+x^2}}$, 9. $\int_2^{\infty}\frac{xdx}{x^4+1}$

10. K – ning qanday qiymatlarida xosmas integral yaqinlashuvchi bo'ladi?

$$\int_2^{\infty}\frac{dx}{x\ln^k x}$$

Quyidagi II – tur chegaralanmagan funksiyalarni xosmas integrallarini yaqinlashuvchi yoki uzoqlashuvchiligini tekshiring.

1. $\int_0^{2}\frac{dx}{\sqrt[3]{(x-1)^2}}$, 2. $\int_0^{2}\frac{dx}{x\sqrt{\ln x}}$, 3. $\int_0^{\frac{1}{2}}\frac{dx}{x\ln^2 x}$,

4. $\int_0^{1}\frac{dx}{\sqrt[5]{x^2}}$, 5. $\int_{-1}^{1}\frac{3x^2+2}{\sqrt[3]{x^2}}dx$

6. K – ning qanday qiymatlarida quyidagi integral yaqinlashadi?

$$\int_1^{2}\frac{dx}{x\ln^k x}$$

87. Ikki o'zgaruvchili funksiya. Aniqlanish, o'zgarish sohalari, limiti va xususiy hosilalari. Yo'nalish bo'yicha hosila. Gradient. Ikki argumentli funksiya ekstremumi

Amaliyotda bir necha erksiz o'zgaruvchilarga bog'liq bo'lgan miqdorlar bilan ish ko'rishga to'g'ri keladi. Masalan, paxtadan olinadigan hosildorlik faqat tuproq unumdorligiga bog'liq bo'lmay balki, suvga, o'g'itga, obi-havoga va boshqa bir qancha miqdorlarga bog'liq bo'ladi. Misol 1. To'g'ri to'rtburchakni yuzi ikki argumentning, ya'ni eni x va bo'yi u ni funksiyasidir z=xy.

D={(x;y):x∈R,Y∈R} juft-juft (x;y) haqiqiy sonlar to'plamidan iborat bo'lsin.

Ta'rif. Agar D to'plamdan olingan har bir (x; u) juftlikka biror qonun qoida yordamida z soni mos qo'yilsa D to'plamda z=f(x, y) ikki o'zgaruvchili funksiya aniqlangan deyiladi.

D to'plamni ikki o'zgaruvchili funksiyaning aniqlanish sohasi bo'ladi. Ikki o'zgaruvchili funksiyaning grafigi fazodagi sirtni ifodalaydi.

Ta'rif. Z=f(x; y) funksiya D sohada aniqlangan bo'lib, $M_0(x_0, u_0)\in D$ bo'lsin. D sohaning $M_0(x_0; u_0)$ nuqtaga yaqin barcha M(x; u) nuqtalari uchun, yetarlicha kichik $\varepsilon>0$ da $|f(x, y)-a|<\varepsilon$ tengsizlik bajarilsa, u holda a sonini f(x; y) funksiyaning $M_0(x_0; y_0)$ dagi limiti deyiladi va uni limf(x; y)qa kabi yoziladi.

$x\to x_0$ $u\to u_0$

Agar limf(x; y)=f(x_0; y_0) bo'lsa, u holda f(x; y) funksiya

$x\to x_0$ $y\to y_0$

(x_0; y_0) nuqtada uzluksiz deb ataladi.

Ikki o'zgaruvchili funksiyaning xususiy hosilasi va differensiali.

Ikki o'zgaruvchili funksiyaning hususiy orttirmalari mos ravishda **$\Delta_x z=f(x+\Delta x, y)-f(x, y)$(1), $\Delta_u z=f(x, y+\Delta y)-f(x, y)$ (2)** ôîðìóëàëàð îðqàëè òîïèëàäè, òûëà îðòòèðìàñè ýñà $\Delta f=f(x+\Delta x, y+\Delta y)-f(x, y)$ (3) orqali ifodalanadi.

Ta'rif. Agar $\Delta x\to 0$ da $\Delta_x f(x_0; y_0)/\Delta x$ nisbatning limiti mavjud bo'lsa, bu limit f(x, y) funksiyaning (x_0; y_0) nuqtadagi x argumenti bo'yicha xususiy hosilasi deyiladi va $f^1_x(x_0; y_0)$

Xuddi shunga o'xshash agar $\Delta y\to 0$ da $\Delta_y f(x_0; y_0)/\Delta y$ nisbatning limiti mavjud bo'lsa, bu limit f(x, y) funksiyaning (x_0; y_0) nuqtadagi u argumenti bo'yicha xususiy hosilasi deyiladi $f^1_y(x_0; y_0)$ belgilanadi:

Z=f(x, y) funksiyaning birinchi tartibli xususiy hosilasidan yana bir marta olingan xususiy hosila ikkinchi tartibli xususiy hosila deyiladi va Z^{11}_{xx}; Z^{11}_{yy}; Z^{11}_{xy}; Z^{11}_{yx} ko'rinishda belgilanadi.

Z=f(x, y) funksiyaning to'liq orttirmasi deb uning hamma argumentlari o'zgarishi natijasida olingan orttirmaga aytiladi, ya'ni $z=f(x+\Delta x, y+\Delta y)-f(x, y)$.

Z=f(x,y) funksiya biror M(x_0, u_0) nuqtada differensiallanuvchi bo'lsin. Z=f(x;y) funksiya to'liq orttirmasining Δx va Δy ga nisbatan chiziqli bosh qismi, ya'ni

$f^1_x\Delta x+f^1_u\Delta y$ ni f(x; y) funksiyaning M(x_0, u_0) nuqtadagi to'la differensiali deyiladi $dz=\partial z/\partial x\, dx+\partial z/\partial y\, dy$.

Yo'nalish bo'yicha hosila

Skalyar maydonda differensiallanuvchi U=f(x; y; z) funksiya berilgan bo'lsin. Shu maydonga tegishli M(x; y; z) x, u nuqtani va shu nuqtadan chiquvchi e nurni qaraymiz. L nur bilan $\bar{n}$ birlik $|\bar{n}|=1$ vektor bir xil yo'nalishda ega bo'lsin:

$\bar{n}=\bar{i}\cos\alpha+\bar{j}\cos\beta+\bar{k}\cos\gamma$

α, β, γ mos ravishda n birlik vektorni ox, ou, oz o'qlari bilan tashkil etgan burchaklari.

M_1 ($x+\Delta x$; $y+\Delta y$; $z+\Delta z$) l nurda yotuvchi boshqa nuqtasi bo'lsin.

U funksiyani skalyar maydondagi orttirmasi $\Delta_e y=f(x+\Delta x; y+\Delta y; z+\Delta z)-f(x;y;z)$

U=f(x;y;z) funksiyasi M(x;y;z) nuqtadan l yo'nalish bo'yicha hosilasi

$\Delta l=MM_1\to$ o da $\frac{\Delta_e u}{\Delta e}$ nisbatni limitiga aytiladi

$$\frac{\partial u}{\partial e}=\lim_{\Delta l\to 0}\frac{\Delta_e u}{\Delta e} \qquad (1)$$

Agar $\Delta y=f^1_x(x;y;z)\Delta x+f^1_u(x;y;z)\Delta y+f^1_z(x;y;z)\Delta z+\omega(\Delta x;\Delta y;\Delta z)$ ekanini va $\frac{\omega}{\rho}\to 0$,

Agar $\rho=\sqrt{\Delta x^2+\Delta y^2+\Delta z^2}\to 0$, e'tiborga olsak $\Delta x=\Delta l\cos\alpha$, $\Delta y=\Delta l\cos\beta$, $\Delta z=\Delta l\cos\gamma$ ekanligidan (1) dan l yo'nalish bo'yicha olingan hosila quyidagicha bo'ladi: $\frac{\partial u}{\partial e}=\frac{\partial\alpha}{\partial x}\cos\alpha+\frac{\partial\alpha}{\partial y}\cos\beta+\frac{\partial\alpha}{\partial z}\cos\gamma$

Agar sklyar maydon silliq bo'lsa, maydonning skalyar funksiyasi zqf (x;y) bo'lib, $\bar{n}$ birlik viktor o x y tekislikda yotadi ($\cos\alpha=0$) bo'ladi bu holda natijada $\bar{n}=\bar{i}\cos\alpha+\bar{j}\cos\beta$,bo'lib, bu holda yo'nalish bo'yicha hosila $\frac{\partial z}{\partial e}=\frac{\partial f}{\partial x}\cos\alpha+\frac{\partial f}{\partial y}\sin\alpha$

Gradient. Skalyar maydonda y=f(x;y;z) funksiya berilgan bo'lsin. Shu skalyar maydonning M(x;y;z) nuqtasining gradienti vektordan iborat bo'lib, u quyidagicha aniqlanadi:

$$\operatorname{Grad}f(x;y;z)=\frac{\partial f}{\partial x}i+\frac{\partial f}{\partial y}\bar{j}+\frac{\partial f}{\partial z}\bar{k}$$

Funksiya gradienti va l yo'nalish bo'yicha olingan hosila orasida quyidagicha bog'lanish mavjud: $np_e\ \operatorname{grad}u=\frac{\partial u}{\partial e}$; U=f(x;y;z) funksiyasini gradienti vektordan iborat bo'lib, shu M(x;y;z) nuqtadagi skalyar maydonning eng katta o'sish tezligini bildiradi va uni moduli o'zish tezligiga teng bo'ladi.

88. Mavzuga doir namunaviy misollarni yechimi.

Misol-1. Quyidagi funksiyani Aniqlanish sohasini toping $Z=(4-x^2-y^2)^{0.5}$. Bu funksiya aniqlangan bo'lishi uchun $4-x^2-y^2\geq 0\Rightarrow x^2+y^2\leq 4$ bo'lishi kerak. Bu esa markazi O(0; 0) va radiusi 2 ga teng bo'lgan doira ichidagi nuqtalar to'plamidan iborat bo'ladi.

Misol-2. $U=x^2-2xz+y^2$ funksiyani M(1;2;-1) nuqtadan $M_1(2;4;-3)$ nuqtaga qarab yo'nalish bo'yicha hosilasini toping.

Yechish $\overline{MM_1}=i+2j-2k$ vektirga mos yo'nalishdosh $\bar{n}$ birlik vektorni koordinatalarini topamiz.

$$\bar{n}=\frac{i+2j-2\bar{k}}{\sqrt{(1)^2+(2)^2+(-2)^2}}=\frac{1}{3}\bar{i}+\frac{2}{3}\bar{j}-\frac{2}{3}\bar{k}$$

Bunda yo'naltiruvchi kosinuslari $\cos\alpha=\frac{1}{2},\cos\beta=\frac{2}{3},\cos\gamma=-\frac{2}{3}$.

Xususiy hosilalarini hisoblaymiz:

$U^1_x=2x-2z$, $y^1_y=2y$, $y^1_z=-2x$.

$U^1_x(1;2;-1)=4$, $u^1_y(1;2;-1)=4$ $y^1_z(1;2;-1)=-2$

Natijada l yo'nalish bo'yicha olingan hosila.

$$\frac{\partial u}{\partial e}=u^1_x\cos\alpha+u^1_x\cos\beta+u^1_z\cos\gamma=4\cdot\frac{1}{3}+4\cdot\frac{2}{3}-2\cdot\left(-\frac{2}{3}\right)=\frac{16}{3}$$

Misol-3. $u=x^2+2y^2-z^2-5$ funksiyani gradientini M(2;-1;1) nuqtada hisoblang

$$\frac{\partial u}{\partial x}=2x\ ,\quad \frac{\partial u}{\partial y}=4y\ ,\quad \frac{\partial u}{\partial z}=-2z\ ,\quad \left.\frac{\partial u}{\partial x}\right|_m=4,\left.\frac{\partial u}{\partial y}\right|_m=-4,\left.\frac{\partial u}{\partial z}\right|_m=-2\ .$$

Natijada M(2;-1;-1) nuqtadagi gradienti grad U=$4\bar{i}-4\bar{j}-2\bar{k}$ bo'ladi.

89. **Mustaqil yechish uchun misollar**

Quyidagi ikki o’zgruvchili funksiyalarni $D(z)$ aniqlanish sohalarini toping:

1. $z=\dfrac{4}{x^2+y^2}$, 2. $z=\sqrt{xy}$, 3. $z=\dfrac{xy}{y-x}$

4. $z=\sqrt{16-x^2-y^2}$, 5. $z=\sqrt{x^2+y^2-36}$,

6. $z=\sqrt{x}+\sqrt{y}$, 7. $z=\ln(xy)$.

Quyidagi funksiyalarni birinchi va ikkinchi tartibli xususiy hosilalarini toping va aralash hosilalar tengligi haqidagi teoremani bajarilishini tekshiring:

1. $z=3\sin(x^2+y^2)-5x^2y-7$; 2. $z=8\ln(xy^2)-10x^2y-8x$;

3. $z=2e^{3x+y^2}-2x^2+y^29y$; 4. $z=8\cos(xy^2)-3x-12x^4y$;

5. $z=3\sqrt{x^2+y^2}-5xy^3+8y$; 6. $z=x\sin(xy)+8x^2-7x$;

7. $z=0{,}5\ln(x^2+y^2)-9x^3y+2x$; 8. $z=3\sqrt{x+2y}-3x^4y-8x-2$;

9. $z=8e^{x+y^3}-3xy^3+7x-3$; 10. $z=8\ln(x^2+y^2)-6x^2y^3+8x-1$.

Quyidagi funksiyalarni M(x;y;z) nuqtada ox o’qini musbat yo’nalishi bilan α bo’rchak tashkil qiluvchi y yo’nalish bo’yicha hosilalarini va gradientini toping:

11. $z=\frac{1}{2}x^2+\frac{1}{4}xy^3,\quad M(1,-1),\quad \alpha=\frac{\pi}{4}$;

12. $z=tgx+x-2\sin y,\quad M\left(\dfrac{\pi}{4},\dfrac{\pi}{3}\right),\quad \alpha=\dfrac{\pi}{4}$;

13. $z=3x^2y+\sqrt{xy},\quad M(2,2),\quad \alpha=\dfrac{\pi}{6}$;

14. $z=2\cos(x+y)+2x,\quad M\left(\dfrac{\pi}{6},\dfrac{\pi}{6}\right),\quad \alpha\ \dfrac{\pi}{3}$;

15. $z=x\sin(x+y)-1,\quad M\left(\dfrac{\pi}{6},\dfrac{\pi}{6}\right),\quad \alpha\ \dfrac{\pi}{4}$;

16. $z=\ln(x^2+y^2),\quad M(3,4),\quad \alpha=\dfrac{\pi}{6}$;

17. $z=\dfrac{x^3}{3}+\dfrac{y^4}{4},\quad M(1,-2),\quad \alpha\ \dfrac{\pi}{4}$;

18. $z=xtgy+\cos x,\quad M\left(\dfrac{\pi}{6},\dfrac{\pi}{4}\right),\quad \alpha=\dfrac{\pi}{4}$;

19. $z=\ln(x+2y)-xy,\quad M(1,1),\quad \alpha=\dfrac{\pi}{3}$;

20. $z=e^{x^2-y^2},\quad M(2,2),\quad \alpha=\dfrac{\pi}{6}$.

90. Ikki argumentli funksiya ekstremumi mavjudligining zaruriy va yetarli shartlari.

Ekstremum mavjudligining zaruriy sharti. Ikki o'zgaruvchili z=f(x,y) funksiya $M_o(x_o,u_o)$ nuqtada ekstremumga ega bo'lish uchun uning bu nuqtadagi hususiy hosilalari mavjud bo'lib $\partial z/\partial x=0$,
$\partial z/\partial y=0$ bo'lishi zaruriydir.
Quyidagi belgilashlarni kiritamiz:
$A = f_{xx}(x_0,y_0)$, $B = f_{xu}(x_0,y_0)$, $S= f_{uu}(x_0,y_0)$ $D=AC-B^2$
shu nuqtadagi ikkinchi tartibli xususiy hosilalari uzluksiz bo'lsin. 1) Agar D>0 bo'lib A>0 bo'lsa Z=f(x,y) funksiya $M_o(x_o,y_o)$ nuqtada minimumga; 2) D>0 bo'lib A<0 bo'lsa, shu nuqtada f(x,y) funksiya maksimumga ega bo'ladi. 3) D<0 bo'lsa,z=f(x,y) funksiya ekstremumga erishmaydi. 4) D=0 bo'lsa z=f(x,y) funksiya ekstremumga egaligi ochik qoladi va qo'shimcha tekshirish talab qilinadi.

91. Mavzuga doir namunaviy misollarni yechimi.

Masala-1. Xajmi B=32 m^3 ga teng bo'lgan parallelepiped asosining va yon yoklarining yuzlari yigindisi eng kichik bo'lishi uchun parallelepipedning ulchamlari qanday bo'lishi kerak
Yechish. Parallelepipedning o'lchamlarini mos ravishda x, y va h orqali belgilaymiz. Bizga ma'lumki B_n=xyh, ó õîëäà 32 $ì^2$=xyh. Bundan h=32/xy kelib chiqadi. Parallelepiped asosining va yon yonlarini yuzini quyidagi ikki uzgaruvchili funksiya orqali yozish mumkin, ya'ni
S=S(x,y)=xy+2xh+2yh=xy+2x(32/xy)+2y(32/xy)=xy+64(x/xy+y/xy)=
=xy+64(1/x+1/y), x≠0, y≠0.
Xususiy hosilalarni olamiz

$S_x=y+64(-1/x^2)=y-64/x^2$,

$S_y=x+64(1/y^2)=x-64/y^2$,

Hususiy hosilalarni nolga tenglaymiz (zaruriy shart) va sistema tuzamiz:

$$\begin{cases} y-64/x^2=0 \\ x-64/y^2=0 \end{cases} \Rightarrow \begin{cases} y=64/x^2 \\ x-x^4/64=0 \end{cases} \Rightarrow \begin{cases} y=64/x^2 \\ 64x-x^4=0 \end{cases} \Rightarrow \begin{cases} y=64/x^2 \\ x(64-x^3)=0 \end{cases} \quad \begin{matrix} y=64/x^2 \\ 4^3-x^3=0 \end{matrix}$$

x≠0 , x=4 y=4 Yechimga ega bo'lamiz. U holda h=32/xy=32/16=2

Ikkinchi tartibli hususiy hosilalarni topamiz:
$S_{xx}=128/x^3$, $A=S_{xx\ Xoq4}=128/4^3=2$ A=2 $S_{yy}=128/y^3$, $C=S_{yy\ (Yo=4)}=128/4^3=2$ C=2
$S_{xy}=1$ $B=S_{xe\ Xoq4}=1$, B=1 $D=AC-B^2=2*2-1=3>0$
U holda 2 teoremaga ko'ra $M_0(4,4)$ nuqtada S(x,y) funksiya minimumga ega bo'ladi.
$S_{min}=S(4,4)=4*4+64(1/4+1/4)=16+32=48$ m^2. Demak yuza eng kichik bo'lishi uchun parallelepipedni o'lchamlari 4, 4 va 2 bo'lishi kerak ekan.

92. Mustaqil yechish uchun misollar

Quyidagi ikki o’zgaruvchili funksiyalarni ekstremum qiymatlarini toping:

21. $z = x^2 + xy + y^2 - 3x - 6y - 2$; 22. $z = 2x^2 - xy + y^2 - 3x - y + 1$;
23. $z = 3x^2 + 2xy + y^2 - 2x - 2y + 3$; 24. $z = 2x^2 + xy - y^2 - 7x + 5y + 2$;
25. $z = x^2 - 3xy + y^2 - 2x + 6y + 1$; 26. $z = 3x^2 + xy + 6y^2 - 6x - y + 9$;
27. $z = x^2 + 3xy + 2y^2 - 4x + 6y - 2$; 28. $z = 4x^2 + 2xy + y^2 - 2x - 4y + 1$;
29. $z = 0{,}5x^2 + xy + y^2 - x - 2y + 8$; 30. $z = 8x^2 - xy + 2y^2 - 16x + y - 1$;
31. $z = 2x^2 + 3y^2 - 2xy + 2x - 16y + 3$; 32. $z = 6xy - 2x^2 - y^2 - 14x + 5$;
33. $z = 2x^2 - y^2 + 3xy - 2x + 7y + 6$; 34. $z = 10xy + 3x^2 - 2y^2 - 26x + 18y - 1$;
35. $z = 3x^2 + 2y^2 - 2xy + 18x + 8y - 1$; 36. $z = 3 - 3x^2 + 5y^2 - 8xy + 4x + 26y$;
37. $z = 2x^2 - 3xy^2 - 2xy + 8x + 10y - 6$; 38. $z = 5x^2 + 3y^2 + 2xy - 18x - 10y + 4$;
39. $z = 5 - 7x^2 - 5y^2 + 2xy - 34x + 34y$; 40. $z = 2x^2 - 3y^2 + 2xy - 10x + 16y - 7$;

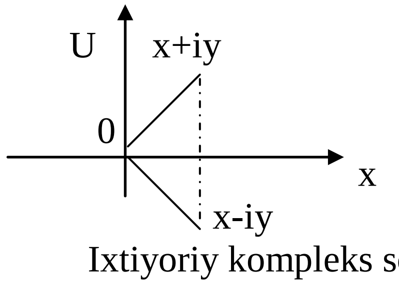
Quyidagi ikki o’zgaruvchili funksiyalarni shartli ekstremumlarini toping:

1. $z = x^2 + y^2 - xy + x + y - 4$, quyidagi shartda $x + y + 3 = 0$
2. $z = \frac{1}{x} + \frac{1}{y}$, quyidagi shartda $x + y = 2$
3. $z = xy^2$, quyidagi shartda $x + 2y = 1$
4. $z = 2x + y$ quyidagi shartda $x^2 + y^2 = 1$

93. Kompleks sonlar.

Kompleks son umumiy holda z=x+iy ko’rinishida yoziladi. Bu erda x-kompleks soning xaqiqiy qismi, y- esa mavhum qismi bo’lib x,y haqiqiy sonlar, i-mavxum birlik bo’lib i=$\sqrt{-1}$. Tekislikda ox-xaqiqiy sonlar o’qi; oy-mavxum sonlar o’qi deyiladi. Kompleks sonlarni ifodalovchi tekestlik kompleks tekislik deyiladi. Kompleks sonning moduli $|z|$ deyiladi $|z| = \sqrt{x^2 + y^2}$.

Z=x+iy va z=x-iy sonlar o’zoro qo’shma sonlar deyiladi va z va $\bar{z}$ shaklida ham yoziladi.
Demak $\bar{z}$ va z sonlar xaqiqiy OX o’qqa simmetrik nuqtalar bo’ladi



Ixtiyoriy kompleks soni trigonometrik shaklda yozish mumkin:

$z = |z|(\cos\varphi + i\sin\varphi)$ bu erda $|z| = \sqrt{x^2 + y^2}$ $\frac{x}{|z|} = \cos\varphi$, $\frac{y}{|z|} = \sin\varphi$ $x = |z|\cos\varphi;\ y = |z|\sin\varphi$.

φ - kompleks sonning argumenti deyiladi va argz shaklida yoziladi. Argument $-\pi \prec \varphi \le \pi$ oraliqdagi qiymati uning bosh qismi deyiladi $\arg z = \arg z + 2\pi n, n \in Z$

Kompleks sonlarning ko'paytmasi

$Z_1 * Z_2 = r_1 * r_2(\cos\varphi_1 + i\sin\varphi_1)(\cos\varphi_2 + i\sin\varphi_2) = r_1 * r_2[(\mathrm{cis}\varphi_1 * \cos\varphi_2 - \sin\varphi_1 \sin\varphi_2) + i(\sin\varphi_1 \cos\varphi_2 + \cos\varphi_1 \sin\varphi_1)] = r_1 r_2(\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2))$

Muavr formulasi. Yuqoridagi natijasiga ko'ra:

$$[r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi),\ n \in Z$$

Ikki kompleks sonning bo'linmasining moduli ularning bo'linmasiga, argumenti esa suratini argumentidan maxrajini argumentini ayirmasiga teng $Z_1=r_1(\cos\varphi_1 + i\sin\varphi_1)$, $z_2=r_2(\cos\varphi_2 + i\sin\varphi_2)$ bo'lsa,

$$\frac{z_1}{z_2} = \frac{r_1(\cos(\varphi_1-\varphi_2) + i\sin(\varphi_1-\varphi_2))}{r_2(\cos^2\varphi_2 + \sin^2\varphi_2)} = \frac{r_1}{r_2}\left(\cos(\varphi_1-\varphi_2) + i\sin(\varphi_1-\varphi_2)\right)$$

Kompleks sondan ildiz chiqarish. $\sqrt[n]{z} = \rho(\cos\psi + i\sin\psi)$ bo'lsin. Bu yerda $z=r(\cos\varphi + i\sin\varphi)$ yuqoridagiga asosan:

$$\sqrt[n]{z} = \sqrt[n]{|z|}\left(\cos\frac{\varphi+2\pi k}{n} + i\sin\frac{\varphi+2\pi k}{n}\right),\ k=0,1,2,3,\dots n-1$$

Eyler formulasi $e^{ix}=\exp(ix)=\cos x + i\sin x$

94. Mavzuga doir namunaviy misollarni yechimi.

Kompleks sonlar ustida arifmetik amallarni bajarish

1) kompleks sonlarni qo'shish $z_1 \pm z_2=(x_1 \pm iy_1) \pm (x_2 \mp iy_2) =(x_1 \pm xi)+i(y_1 \pm y_2)$

2) komplekssonlarni ko'paytirish

$z_1*z_2=(x_1-iy_1)(x_2+iy_2)=x_1x_2+ix_1y_2+iy_1x_2+(i_2)y_1y_2=(x_1x_2-y_1y_2)+i(x_1y_2+y_1x_2)$

$z=x+iy$ $\bar{z}=x-iy$ bo'lsa u holda $z\bar{z}=(x+iy)(x-iy)=(x)^2-(iy)^2=x^2+y^2$.

kompleks sonlarni bo'lish

3) $\dfrac{Z_1}{Z_2} = \dfrac{(x_1x_2+y_1y_2)+i(x_2y_1-y_2x_1)}{x_2^2+y^2}$

Misollar: $z_1=2+3i$ $z_2=1-2i$

1) $z_1+z_2=(2+i3)+(1-i2)=3+i(3-2)=3+i$

2) $z_1z_2=(2+i3)*(1-i2)=2-4i+3i-6i^2=2-i+6=8-i$

3) $\dfrac{z_1}{z_2} = \dfrac{2+i3}{1-i2} = \dfrac{(2+i3)(1+i2)}{(1-i2)(1+i2)} = \dfrac{2+4i+3i-6}{1+4} = \dfrac{-4+7i}{5}$

<u>Misollar-4.</u> 3+3i trigonometrik shaklida yozing.

X=3 y=3 $r=\sqrt{9+9}=3\sqrt{2}$ y

$\cos\varphi = \dfrac{3}{3\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ $\varphi = \dfrac{\pi}{4}$, $3+3i=3\sqrt{2}(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4})$

Quyidagi darajalarni xisoblang. Muavr formulasiga ko'ra

$\left(\dfrac{3}{2} - \dfrac{\sqrt{3}}{2}i\right)^{10}$, avval $\dfrac{3}{2} - \dfrac{\sqrt{3}}{2}i$ ni kompleks shaklida yozamiz.

$x = \frac{3}{2}$ $y = -\frac{\sqrt{3}}{2}$ $r = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}$

x

$\left(\frac{3}{2} - \frac{\sqrt{3}}{2} i\right)^{10} = \left[\sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\left(-\frac{4}{6}\right)\right)\right]^{10} = 3^5\left(\cos\left(10\frac{\pi}{6}\right) - i\sin\left(10\frac{\pi}{6}\right)\right) =$

q $3^5\left(\cos 5\frac{\pi}{3} - i\sin 5\frac{\pi}{3}\right) = 3^5\left(\cos\left(270 + \frac{\pi}{6}\right) - i\sin\left(270 + \frac{\pi}{6}\right)\right) =$

q$3^5\left(\sin\frac{\pi}{6} + \cos\frac{\pi}{6}\right) = 3^5\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$, Demak: $\left(\frac{3}{2} - i\frac{\sqrt{3}}{2}\right)^{10} = 3^5\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$\sqrt{i}$ sondan ildiz chiqaring. Trigonometrik shaklida yozamiz.

I=0$\cdot x + 1 \cdot i$ r=1 $tg\,\varphi = \frac{y}{x}$ $\varphi = \frac{\pi}{2}$, iq$\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$,

z_kq$\sqrt{i} = \sqrt{\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}} = \cos\frac{\frac{\pi}{2} + 2\pi k}{2} + i\sin\frac{\frac{\pi}{2} + 2\pi k}{2}$, kq0,1

$z_1 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$, $z_2 = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = \cos\left(\frac{\pi}{4} + \pi\right) + i\sin\left(\frac{\pi}{4} + \pi\right)$

$= -\cos\frac{\pi}{4} - i\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

Misol-5. Quyidagi akslashlarda z_0 nuqtani aksini (tasvirini) toping. A) $\omega = z$, $z_0 = i$; б) $\omega = \bar{z} / z$, $z_0 = 2 + 3i$

Namunaviy misolni echimi.

$|z| = \sqrt{2}$ $\omega = z^2$?

Yechish. Berilganga asosan $\mathrm{Re}\,\omega = x^2 - y^2$, $\mathrm{Im}\,\omega = 2xy$.

Tenglamalar sistemasida x, y yo'qotsak

$$\left.\begin{array}{l} u = x^2 - y^2, \\ v = 2xy, \\ x^2 + y^2 = 2, \end{array}\right\}$$

$u^2 + v^2 = (x^2 + y^2)^2 = 4$. Demak, bu nov tekisligida Markazi koordinatalar boshida, radiusi 2 ga teng aylanani ifodalaydi.

95.Mustaqil yechish uchun misollar.

Amallarni bajaring:

1) a) (3+i)+(-3-8i); b) (1-i)-(7-3i)-(2+9i)+(6-2i). c) –i$\sqrt{5} \cdot i\sqrt{5}$;

d) (3+4i)$\cdot(3 - 4i)$; e) $\left(\frac{3}{2} - \frac{1}{3}i\right) \cdot \left(\frac{1}{3} - \frac{4}{3}i\right)$, f) $\frac{2}{2i}, \frac{1+i}{1-i}, \frac{2-3i}{4+5i}, \frac{\sqrt{5}+i}{\sqrt{5}-2i}$

2)Quyidagi kompleks sonlarni modullarini toping. A) $z = 4 + 3i$;

b) $z = \cos\alpha - i\sin\alpha$;   c) $z = -2 + 2\sqrt{3}i$.

3)Quyidagi sonlarni trigonometrik shaklda yozing.

a) $\sqrt{3} - i, \frac{\sqrt{3}}{2} - \frac{1}{2}i$, b) $z = -1 - i\sqrt{3}$;   s) $z = -\sqrt{2} + i\sqrt{2}$.

4) Quyidagi sonlarni algebra shakda yozing.

$5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right); 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

5)bo'lishni bajaring.

$\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) : 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

6) Darajaga ko'taring.

a) $\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^6$, в) $\left[2\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)\right]^8$, c) $(-1 + i\sqrt{3})^{60}$.

d) $(\sqrt{3} - 3i)^{60}$;   e) $\left(\frac{1-i}{1+i}\right)^4$

7) Sonlardan ildiz chiqaring.

a) $\sqrt[3]{i}$, б) $\sqrt[4]{4}$, c) $\sqrt[4]{-2 + 2i\sqrt{3}}$. d) $\sqrt[4]{1 - i}$;   e) $\sqrt[4]{1}$.

8) Tengliklarini isbotlang. A) $z + \bar{z} = 2\,\mathrm{Re}\,z$;   b) $z - \bar{z} = 2i\,\mathrm{lm}z$;

v) $|\bar{z}| = |z|$.

9)Quyidagi tenglamalar bilan qanday egri chiziqlar berilgan. A) $|z - z_0| = r$, $r > 0$,

b) $|z + c| + |z - c| = 2a, a > c$ haqiqiy sonlar.

V) $\mathrm{Re}(1/\omega) = 1/2, \mathrm{Im} = 1/4, \omega = x + 2yi$

g) $\mathrm{Im}\,z^2 = 2$;   d) $\mathrm{Im}\left(\frac{z}{x^2}\right) = 1$?

96. Differensial tenglama. Asosiy tushunchalar. Sodda differensial tenglamalarni yechish.

Differensial tenglamalar turli sohalarga jumladan qishloq xo'jalik masalalarini echishda ham qo'llaniladi. Masalan o'simliklarni o'sish jarayonlarini Gompersni differensial tenglamalari yordamida yozish mumkin.

Ta'rif. Differensial tenglama deb erkli o'zgaruvchi x, noma'lum funksiya u va uning turli tartibli hosilalari qatnashgan tenglamaga aytiladi.

Differensial tenglamani umumiy shaklda quyidagicha yozish mumkin:

$$F(x; y; y^1; y^{11}; ...; y^{(n)}) = 0 \qquad (1).$$

Agar noma'lum funksiya u bir argumentli bo'lsa oddiy differensial tenglama, deyiladi. Differensial tenglamani tartibi unga kiruvchi eng yuqori hosilaning tartibi orqali aniqlanadi.

Differensial tenglamani yechimi deb, differensial tenglamaga qo'yganda uni ayniyatga aylantiruvchi y=f(x) funksiyaga aytiladi.

Misol. $y^1=2x$ differensial tenglamani yechimi $y=x^2+s$ bo'lib, integral egri chiziqlari parabolalardan iborat ekan. Topilgan $y=f(x, c)$ umumiy echimidan, $x=x_0$ bo'lganda $y(x_0)=y_0$ shartni qanoatlantiruvchi yechimiga differensial tenglamani xususiy yechimi deyiladi.

Birinchi tartibli o'zgaruvchilari ajralgan va ajraladigan differensial tenglamalar.

a) Birinchi tartibli eng sodda differensial tenglamalarga o'zgaruvchilarga ajralgan $f_1(x)dx+f_2(y)dy=0$ hol kiradi. Bu tenglama bevosita integrallash orqali echimi topiladi $\int f_1(x)dx+\int f_2(y)dy=c$.

Misol. $xdx+ydy=0$, integrallaymiz: $\int xdx+\int ydy=c$, $x^2/2+y^2/2=s$, $x^2+y^2=R^2$ bo'lib, integral egri chiziqlari konsentrik aylanalarni beradi.

b) O'zgaruvchilarga ajraladigan differensial tenglamani umumiy ko'rinishi quyidagicha bo'ladi: $f_1(x)f_2(y)dx+f_3(x)f_4(y)dy=0$. Bu tenglama $f^1{}_2(y)*f_3(x)\neq0$ shartda $f^1(y)f_3(x)$ bo'lish natijasida o'zgaruvchilarga ajralgan differensial tenglamaga keltirilib, integrallash yordamida umumiy yechimi topiladi:

$(f_1(x)/f_3(x))dx+(f_4(y)/f_2(y))dy=0$; $\int(f_1(x)/f_3(x))dx+\int(f_4(y)/f_2(y))dy=c$.

Birinchi tartibli chiziqli differensial tenglama.

Noma'lum funksiya va uning hosilalariga nisbatan chiziqli bo'lgan ushbu $y^1+p(x)y=(x)$ (1) ko'rinishdagi tenglamaga birinchi tartibli chiziqli differensial tenglama deyiladi, bunda $p(x)$ va $q(x)$ uzluksiz funksiyalar. (1) tenglamaning echimini $y=u(x)v(x)$ $=uv$ ko'rinishda izlanadi. Natijada bu tenglamani umumiy echimi quyidagi formula bilan topiladi **$y=uv=e^{-\int pdx}$ $(C+\int=e^{\int pdx}$ dx).**

Ikkinchi tartibli bir jinsli o'zgarmas koeffisientli chiziqli differensial tenglama.

Ikkinchi tartibli bir jinsli o'zgarmas koeffisientli chiziqli differensial tenglamani umumiy ko'rinishi quyidagicha

$$y^{11}+py^1+qy=0 \quad (1),$$

bunda p va q - o'zgarmas haqiqiy sonlar. Tenglamaning umumiy yechimini topish uchun uning ikkita chiziqli erkli xususiy yechimini topish yetarlidir. Tenglamani yechimini $y=e^{kx}$ shaklda izlaymiz, bu erda k - nolga teng bo'lmagan o'zgarmas son $y=e^{kx}$ (1) differensial tenglama umumiy yechimi $K^2+pK+q=0$ xarakteristik tenglama yechimiga bilan bogliq bo'lib, agar

1. D>0 bo'lsa, K_1 va K_2 xaqiqiy echimlar u holda (1) tenglama umumiy yechimi $y=C_1e^{\kappa_1x}+C_2e^{\kappa_2x}$ ko'rinishida bo'ladi.

2. D=0 bo'lsa $K_1=K_2=K$ bo'lib, $y=C_1e^{\kappa_1x}+C_2e^{\kappa x}$ umumiy yechimga ega bo'lamiz.

3.D<0 bo'lsa, $K_{1/2}=\alpha\pm\beta i$ bo'lib, (1) ning umumiy yechimi $y=e^{\alpha x}(C_1Cos\,\beta x+C_2Sin\,\beta x)$ ko'rnishida bo'ladi.

Bir jinsli bo'lmagan differensial tenglama

Bir jinsli bo'lmagan differensial tenglamani umumiy ko'rinishi quyidagicha

$$y^{11} + py^{1} + qy = f(x) \qquad (2).$$

TEOREMA (2) differensial tenglamaning umumiy yechimi unga mos bo'lgan (1) differensial tenglamaning umumiy yechimi bilan (2) differensial tenglamaning xususiy yechimi yigindisiga teng.

Bir jinsli bo'lmagan (2) differensial tenglama xususiy yechimlarini topish masalasini ko'raylik u $f(x)$ funksiyani ko'rinishiga qarab tanlanadi:

1-xususiy hol. $f(x) = ae^{mx}$ bo'lsin. Xususiy yechimni ham $z = A\cdot e^{mx}$ ko'rinishda qidiramiz. $z^{1} = Am\,e^{mx}$, $z^{II} = Am^{2}e^{mx}$. O'rniga qo'yamiz. ((2)ga) $Am^{2}e^{mx} + pAme^{mx} + q\cdot Ae^{mx} = a\cdot e^{mx}$,

$$A(m^{2} + pm + q) = a$$

Bunda 1) m xarakteristik tenglama ildizi bo'lmasa, ya'ni $m^{2} + pm + q \neq 0$ bo'lsa

$A\dfrac{a}{m^{2} + pm + q}$, demak $z = \dfrac{ae^{mx}}{m^{2} + pm + q}$

m xarakteristik tenglamani ildizi, ya'ni $m^{2} + pm + q = 0$. U holda (2) tenglamani xususiy echimi $z = Axe^{mx}$ $(m_1 \neq m_2)$ yoki $z = Ax^{2}e^{mx}$ $(m_1 = m_2)$ ko'rinishida izlanadi.

2 – xususiy xol: $f(x) = PCos\ \varphi x + Q\sin\ \varphi x$ bo'lsin. Xususiy yechimni $z = ACos\ \varphi(x) + BSin\ \varphi(x)$ ko'rnishda qidiramiz.

$z^{1} = A\varphi Sin\ \varphi x + B\varphi Cos\ \varphi x$, $\qquad z^{II} = -A\varphi^{2}Cos\ \varphi x - B\varphi^{2}Sin\ \varphi x$ bularni tenglamaga qo'ysak,

$$-A\varphi^{2}Cos\ \varphi x - B\varphi^{2}Sin\ \varphi x + p(-A\varphi Sin\ \varphi x + B\varphi Cos\ \varphi x) +$$
$$+ q(ACos\ \varphi x + BSin\ \varphi x) = PCos\ \varphi + QSin\ \varphi x,$$

bundan

$(-A\varphi^{2} + Bp\varphi + Aq)Cos\ \varphi x + (-B\varphi^{2} - Ap\varphi + Bq)Sin\ \varphi x = PCos\ \varphi x + QSin\ \varphi x$ xosil bo'ladi. Bu oxirgi tenglik mos koeffisentlarini tenglashtirsak:

$P = A(q - \varphi^{2}) + BP\varphi$, $\qquad Q = -AP\varphi + B(q - \varphi^{2})$ larga ega yuo'lamiz, bulardan umuman aytganda, A va B larni topamiz. R=0, $q = \varphi^{2}$ bo'lgan xolda esa, xususiy yechim $z = x(ACos\ \varphi x + BSin\ \varphi x)$ ko'rinishda qidiriladi.

3- xususiy xol: $f(x) = ax^{2} + вx + c$ bo'lsin. Xususiy echimni ham $z = Ax^{2} + Bx + C$ ko'rinishda qidiramiz. $z^{1} = 2Ax + B$ va $z^{II} = 2A$ larni tenglamaga qo'yamiz.

$$2A + 2pAx + pB + Aqx^{2} + Bqx + Cq = ax^{2} + вx + C$$

$$Aqx^{2} + (2pA + Bq)x + (2A + Bq + Cq) = ax^{2} + вx + C$$

Bundan Aq=a, 2Ar+Bq=b, 2A+B=+S=qc (3). Bunda, agar $q \neq 0$ bo'lsa A,B,S lar tayin qiymatlar qabul qiladi. Agar q=0 bo'lsa xarakteristik tenglama O yechimga ega u holda (3) sistema birgalikda emas. Bu holda r$\neq 0$ deb xususiy yechimni $z = x(Ax^{2} + Bx + c)$ ko'rinishida qidiriladi.

97. Differerensial tenglamalarga doir namunaviy misollarni yechimi.

Misol-1. Quyidagi differensial tenglamalarni umumiy echimini toping

a) 3x2ydx+2√4-x^3dx=0 b) y^1-(y/x)=x^2 v) y^{11}+3y^1-4y=0

Yechish. a) 3x^2ydx+2√4-x^3dx=0

Ma'lumki, φ(x)dx+φ(y)dx=0 o'zgaruvchilarga ajraladigan differensial tenglama bo'lib, integrallash yordamida yechimlari topiladi. Bizning misolda y√4-x^3≠0 deb tenglikni ikkala tomonini shu ifodaga bo'lamiz:

(3x^2ydx)/(y√4-x^3)+(2√4-x^3)/ (y√4-x^3)=0 bundan, (3x^2dx)/(√4-x^3)+(2/y)dy=0

-d(4-x^3)/(4-x^3)$^{(1/2)}$+(2/y)dx=0, integrallasak

-∫(4-x^3)$^{(1/2)}$∗d(4-x^3)+2∫(dx/y)=S

∫y^ndy=($y^{(n+1)}$)/(n+1)+C (n≠-1) va ∫(dy/y)=ln│y│+C

Jadval integrallarga asosan

-(4-x^3)$^{(-1/2)+1}$/(-(1/2)+1)+2ln│y│=C,

-(4-x^3)$^{(1/2)}$/(1/2)+2ln│y│=C yoki

2ln y =C+2(√4-x^3)

lny=S_1+(√4-x^3)=lnS_2+(√4-x^3) potensirlasak, natijada umumiy yechimi

u=S_2 $e^{(\sqrt{4-x3})}$ bo'ladi.

b) y^1 - y/x=x^2 chiziqli differensial tenglamani yechimini topamiz.

Ma'lumki, chiziqli differensial tenglama quyidagicha edi

u^1+r(x)u=q(x) Bizning misolda r(x)=-(1/x); q(x)=x^2

Nazariyadan ma'lumki, ([1]) chiziqli differensial tenglamani umumiy yechimi quyidagi formula bilan topiladi

$$y=e^{-\int p(x)dx}[\int q(x)\, e^{\int p(x)dx}\, dx+C]$$

Bizning misol uchun yechimi quyidagicha bo'ladi:

$y=e^{\int(1/x)dx}[\int x^2\, e^{-\int 1/xdx}\, dx+C]=e^{lnx}[\int x^2\, e^{-lnx}\, dx+C]=$

$y=e^{-\int p(x)dx}[\int x^2\, e^{(lnx)-1}\, dx+C]=\ \ x[\int x^2\, e^{(lnx)-1}\, dx+C]$

$=x[\int x^2\ 1/xdx+C]= =x[\int xdx+C]= =x[(x^2/2)+C]=(x^3/2)+C*x$

Bu erda $a^{\log_a x}$=x formuladan foydalandik.

v) u^{11}+3u^1-4u=0 ikkinchi tartibli o'zgarmas koeffisientli bir jinsli chiziqli differensial tenglamani yechimini u=e^{kx} shaklida izlaymiz. U xolda y^1=$(e^{kx})^1$= ke^{kx}; y^{11}=k^2 e^{kx} berilgan differensial tenglamaga qo'ysak k^2e^{kx}+3ke^{kx}–4e^{kx}=0 ; e^{kx}(k^2+3k–4)=0 ;

e^{kx}≠0 bo'lganligdan k^2+3^k–4=0

xarakteristik tenglama xosil bo'ladi. Bu xarakteristik tenglamani yechib $К_1$= (-3+ 5)/ 2 =1, $К_2$ =(-3- 5)/ 2q -4

natijada berilgan differensial tenglamaning xususiy yechimlari

y_1= e^x y = e $^{-4x}$ buladi.

Malumki, umumiy yechimi chiziqli bog'lanmagan xususiy yechimlarining yig'indisidan iborat buladi.

y = s_1y_1+ s_2 y_2 = s $_1e^x$ + s $_2e^{-4x}$

Demak talab qilingan yechim y= s $_1e^x$ + s $_2e^{-4x}$ ekan.

Bir jinsli bo'lmagan differensial tenglamalarni eching.

Misol-2: $y^{II}-3y^{1}+2y=e^{3x}$ umumiy yechimini topamiz.

$y^{II}-3y^{1}+2y=0$ bir jinsli tenglamani yechamiz.

$\kappa^{2}-3\kappa+2=0$ xarakteristik tenglama ildizlari $k_1=1$, $k_2=2$

Umumiy yechimi $y=C_1e^{x}+C_2e^{x}$. M=3, xarakteristik tenglama yechimi bo'lmagani uchun xususiy yechimi $z=Ae^{3x}$ ko'rinishda qidiramiz. $z^{1}=3Ae^{3x}$, $z^{II}=9Ae^{3x}$. Bularni birjinslilimas tenglamaga qo'yib topamiz.

$9Ae^{3x}-3\cdot 3Ae^{3x}+2Ae^{3x}=e^{3x}$ 2A=1 A=$\frac{1}{2}$

Demak , xususiy yechim: $y=C_1e^{x}+C_2e^{2x}+\frac{1}{2}e^{3x}$ bo'ladi.

Misol-3. $y^{II}-6y^{1}+9y=Cosx$ birjinslimas tenglama umumiy yechimi topilsin.

$y^{II}-6y^{1}+9y=0$ ni ko'ramiz. Uning xarakteristik tenglamasi:

$k^2-6k+9=0$, bunda $\kappa_{1/2}=3\pm\sqrt{9-9}=3,$ demak umumiy yechimi $y=e^{3x}(C_1+C_2x)$ bo'ladi. Berilgan birjinslimas differensial tenglama xususiy yechimini

$z=ACosx+BSinx$ ko'rinishda qidiramiz.

$z^{1}=-ASinx+BCosx$, $z^{II}=ACosx-BSinx$ larni tenglamaga qo'ysak,

$-ACosx-BSinx+6ASinx-6BCosx+9ACosx+9BSinx=Cosx$

Chapda va o'ngda Cosx va Sinx lar oldidagi koeffisentlarini tenglashtirib, xosil qilamiz:

$\begin{cases}8A-6B=1\\6A+8B=0\end{cases}$, bundan $A=\frac{2}{25},\quad B=-\frac{3}{50}.$

Demak $z=\frac{2}{25}Cosx-\frac{3}{50}Sinx$, va berilgan birjinslimas tenglama umumiy echimi

$y=e^{3x}(C_1+C_2x)+\frac{2}{25}Cosx-\frac{3}{50}Sinx$ bo'ladi.

Misol-4. $y^{II}-2y^{1}+10y=3x+4$ tenglama umumiy yechimini topamiz.

$y^{II}-2y^{1}+10y=0$ ni ko'ramiz $\kappa^{2}-2\kappa+10=0\quad\Rightarrow$

$\kappa_{1/2}=\frac{2\pm\sqrt{4-4\cdot 10}}{2}=\frac{2\pm 6i}{2}=1\pm 3i$. Umumiy yechimi $y=e^{x}(C_1Cos\,3x+C_2Sin\,3x)$

Birjinslimas tenglama xususiy yechimini z=Ax+B ko'rnishda qidiramiz. $Z^1=A$, $z^{II}=0$ larni tenglamaga qo'yib, $-2A+10(Ax+B)=3x+4$ va bundan $10Ax-2A+10B=3x+4$ ni xosil qilamiz. X ning bir xil darajalari koeffisentlarini tenglashtirib: 10A=3, -2A+10B=4 larni topamiz. Bulardan $A=\frac{3}{10},\quad B=\frac{23}{50}$ larni topamiz.

O'rinlariga qo'ysak: $z=\frac{3}{10}x+\frac{23}{50}$ va berilgan birjinslimas tenglama umumiy yechimi $y=e^{x}(C_1Cos\,3x+C_2Sin\,3x)+\frac{3}{10}x+\frac{23}{50}$ ga ega bo'lamiz.

98. Mavzuga doir mustaqil yechish uchun misollar

1-20.Quyidagi differensial tenglamalarni umumiy yechimlarini toping.

1.a) $(3x-1)dx + y^2 dx = o$ b) $y' - 4/x = x+1/x$ v) $y''+5y'+6y=0$

2.a) $y' = 1+y^2/1+x^2$ b) $y'+ y/x = x\ e^{x/2}$ v) $y'' - 7y'+ 6y = 0$

3. a) $\sqrt[3]{1-2x^3+x^6}\,dy = x^2 y^2 dx$ b) $y' - 2y = y^{2x}$ v) $y'' + 2y' - 15y = 0$

4. a) $y' = (x-1)e^x$ b) $y' - y = x^2$ v) $y'' + 2y' - y = 0$

5. a) Cos x dy/dx = y / In y b) $y' - 2xy = x^3$ v) $y'' + 36y = 0$

6. a) $y' = tgx * tgy$ b) $xy' - y = x^2$ Cos x v) $y'' - 2y' + 17y = 0$

7. a) $(1+x^2)dy + y\,dx = 0$ b) $y' + 2xy = xe^{-x2}$ v) $y'' + 6y' + 10y = 0$

8. a) $y\ y' + xe^{-xe} = 0$ b) y'Cosx+y=1- Sin x v) $y'' - 6y' + 9y = 0$

9. a) $(1+e^{2x})y^2\,d\,y = e^x dx$ b) $y' + 2xy = 1+2x^2$ v) $y'' - 2y' = 0$

10.a) $y' = 2+y$ b) $y' - y = 2x - x^2$ v) $4y'' + 3y' - 7y = 0$

11. a) $x\,y' - u = 0$ b) $y' - 4\,y/x = x+1/x$ v) $y'' + 5y' - 6y = 0$

12. a) $x+xy+y'(y+xy) = 0$ b) y'+ucosx=sinx*cosx v) $y'' - 8y' + 7y = 0$

13. a) $x^2 y^1 + y = 0$ b) $y^1 + tgx\ y = (1/\cos x)$ v) $y'' - 25y = 0$

14. a) $2yx^2 d\,y = (1+x^2)\,dx$ b) $y' + 1/x\ y\ qx$ v) $y'' + 2y' - 15y = 0$

15. a) $2y'\sqrt{x} = y$ b) $y'' + 2y' + y = 0$ v) $y' - 3y/x = -x$

16. a) $y' = (2y+1)\,ctg\,x$ b) $y' - y\,tg\,x = 1/\cos x$ v) $y'' - 6y' + 9y = 0$

17. a) $x^2 y' + y^2 = 0$ b) $y' + 2xy = 2xe^{-x2}$ v) $y'' + 236y = 0$

18. a) $(x^2 - 4)\,y' = 2xy$ b) $y' + 2/x\ y = x^2$ v) $y'' + 2y' - 35y = 0$

19. a) $y' + yctg\,x = 0$ b) $y' + 2/x\ y = x^2$ v) $y'' + 6y' + 10 = 0$

20. a) $y'(x^2 + x) = 2u + 1$ b) $y''(2y' + 2y) = 0$ v) $y' + (y/x) = (1+x)$

Berilgan differensial tenglamalarni quyidagi boshlangich shartlarni qanoatlantiruvchi xususiy yechimlarini toping

21. $y' + 2xy = 3x^2 e^{-2}$, $y(0) = 0$; 22. $xy' - y = y^2 \cdot \sin x$, $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$;

23. $y' + 2xy = x\ln e^{-x^2}$, $y(1) = 0$; 24. $y' - yctgx = \frac{1}{\sin x}$, $y\left(\frac{\pi}{2}\right) = 0$;

25. $xy' - y = x^2 \sin x, y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$; 26. $y'\cos^2 x + y = tgx$, $y(0) = 0 - 1$;

27. $y' + \frac{1}{x}y = xy^2$; $y(1) = 1$; 28. $(1 + x^2)y' + y = y^2 arctx$, $y(0) = 1$;

29. $y' + 3x^2 y = x^3 e^{x^3}$, $y(0) = 0$; 30. $xy' - y = x^2 \cos x$, $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$;

31. $y'\sin^2 x + y = ctgx$, $y\left(\frac{\pi}{2}\right) = 1$; 32. $\sqrt{1 - x^2} \cdot y' + y = \arcsin x \cdot y^2$, $y(0) = -1$;

33. $y' - \frac{2x}{1 + x^2}y = arctg^2 x$, $y(0) = 0$; 34. $y' + 3ytg\,3x = \sin 6x$, $y(0) = \frac{1}{3}$;

35. $(1 + x^2)y' + y = arctgx$, $y(0) = 1$; 36. $y' + \frac{2xy}{1 + x^2} = 4\frac{arctgx}{\sqrt{1 + x^2}}\sqrt{y}$, $y(0) = 1$;

37. $y' + \frac{1}{\sin^2 x} \cdot y = y^2 \frac{ctgx}{\sin^2 x}$, $y\left(\frac{\pi}{2}\right) = 1$; 38. $y' + \frac{2}{x}y = x^2 y^2$, $y(1) = 1$;

39. $y'+\frac{1}{\sqrt{1+x^2}}y=4\frac{arcsibx}{\sqrt{1+x^2}}$, $y(0)=-1$; 40. $y'\cos^2 x+y=y^2 tgx$, $y(0)=-1$.

Bir jinsli differensial tenglamalarni yeching

1. $y'=\frac{x+8y}{8x+y}$; 2. $xy\quad y'=x^2+y^2$; 3. $y'=\frac{x+y}{x-y}$; 4. $xy'+xtg\frac{y}{x}=0$;

5. $xy'+y\ln\frac{y}{x}=0$; 6. $y'=\frac{y}{x}+\sin\frac{y}{x}$; 7. $xyy'=x^2-y^2$; 8. $(x-y)y'=2x+y$;

9. $xy'y\ln^2\frac{y}{x}=0$; 10. $xy'\ln\frac{y}{x}=x+y\ln\frac{y}{x}$; 11. $xy'=y+2xctg\frac{y}{x}$; 12. $y'=\frac{x+2y}{2x-y}$;

13. $xyy'=2x^2+y^2$; 14. $x^2y'=y^2+xy+x^2$; 15. $(2x+y)y'=x+2y$;

16. $y'=\frac{y}{x}+\sqrt{1-\frac{y^2}{x^2}}$; 17. $(x-2y)y'=(x+y)$; 18. $xy'=y+3x\sin\frac{y}{x}$;

19. $xy'=y+y\ln\frac{y}{x}$; 20. $(3x+y)y'=x+3y$. .

To'liq diffrensial tenglamalarni yeching.

1. $(e^x+y+Siny)dx+(e^y+x+xCosy)dy=0$. 2. $(x+y-1)dx+(e^y+x)dy=0$

3. $(xCosy-ySiny)dy+(xSiny+yCosy)dx=0$ 4. $(x+Siny)dx+(xCosy+Siny)dy=0$

5. $(y+e^xSiny)dx+\left(\frac{1}{2}x^2+xCosy\right)dy=0$ 6. $(xy+Siny)dx+\left(\frac{1}{2}x^2+xCos\right)dy=0$

7. $(x^2+Siny)dx+(1+xCos)dy=0$ 8. $ye^xdx+(y+e^x)dy=0$

9. $(e^xSiny+x)dx+(e^xCosy+y)dy=0$ 10. $(7x+3y)dx(3x-5y)dy=0$

Berilgan differensial tenglamalarni quyidagi boshlangich shartlarni qanoatlantiruvchi xususiy yechimlarini toping

41. a) $y''-7y'+10y=0$; $y(0)=2$; $y'(0)=-1$; b) $y''-2y'=3x^2+1$;

42. a) $y''+2y'+10y=0$; $y\left(\frac{\pi}{2}\right)=0$; $y'\left(\frac{\pi}{2}\right)=1$; b) $y''-5y'+6y=2xe^{-x}$

43. a) $y''-6y'+9y=0$; $y(0)=1$; $y'(0)=0$; b) $y''-8y'+=(x-1)e^{2x}$

44. a) $y''-8y'+7y=0$; $y(0)=2$; $y'(0)=0$; b) $y''-6y'+8y=3e^{4x}$

45. a) $y''-9y=0$; $y(\pi)=0$; $y'(\pi)=1$; b) $y''-2y'-3y=xe^{4x}$

46. a) $y''-7y'+12y=0$; $y(0)=2$; $y'(0)=-2$; b) $y''+y'-2y=(x+2)e^{-2x}$;

47. a) $y''-9y'=0$; $y(0)=1$; $y'(0)=-3$; b) $y''+2y'-8y=(3x+1)e^{2x}$;

48. a) $y''-3y'+2y=0$; $y(0)=0$; $y'(0)=1$; b) $y''+7y'=2x^2+x$;

49. a) $y''-5y'+6y=0$; $y(0)=5$; $y'(0)=0$;

50. a) $y''-2y'+5y=0$; $y(0)=-1$; $y'(0)=0$; b) $y''+3y'-10y=2x^2e^x$;

51. a) $y''+16y=0$; $y(\pi)=-1$; $y'(\pi)=0$; b) $y''+2y'=x^2-3x+1$;

52. a) $y''+10y'+25y=0$; $y(0)=1$; $y'(0)=1$; b) $y''-5y'-24y=(2x+3)e^x$;

53. a) $y''-6y'=0$; $y(0)=2$; $y'(0)=-2$; b) $y''-2y'-3y=8e^{3x}$

54. a) $y''-4y'+4y=0$; $y(0)=1$; $y'(0)=3$; b) $y''+2y'-3y=-2e^{3x}$;

55. a) $y''-8y'+15y=0$; $y(0)=1$; $y'(0)=-2$; b) $y''+8y'=(x^2+1)e^{-x}$;

56. a) $y''-4y'+17y=0$; $y\left(\frac{\pi}{2}\right)=0$; $y\left(\frac{\pi}{2}\right)=1$; b) $y''+4y'+3y=-xe^{-x}$;

57. a) $y''-2y'+y=0$; $y(1)=0$; $y'(1)=-2$; b) $y''-2y'-3y=(x+2)e^{-x}$;

58. a) $y''+y=0$; $y(\pi)=-1$; $y'(\pi)=-4$; b) $y''+y'-6y=2(x-1)e^{2x}$;

59. a) $y''-7y'+6y=0$; $y(0)=2$; $y'(0)=0$; b) $y''-4y'=2x^2-3x+1$;

60. a) $y''+8y'+16y=0$; $y(0)=1$; $y'(0)=0$; b) $y''-5y'+6y=2xe^{3x}$.

99. O'zgarmas koeffisentli chizili differensial tenglamalar sistemasini yechish

$$\begin{cases} \dfrac{dx_1}{dt}=a_{11}x_1+a_{12}x_2+...+a_{1n}x_n \\ \dfrac{dx_2}{dt}=a_2x_1+a_{22}x_2+...+a_{2n}x_n \\ \dfrac{dx_n}{dt}=a_{n1}x_1+a_{n2}x_2+...+a_{nn}x_n \end{cases} \qquad (1)$$

$x_1, x_2, \ldots x_n$ noma'lum funksiyalar.

Chap tomoni birinchi tartibli no'malum funksiyalarni xosilalarini o'ng tomonida uni xosilalari qatnashmagan tenglamalar sistemasi chiziq differensial tenglamalar sistemasi deyiladi.

Bu sestemadan ketma-ket xosilalar olib, qolganlarini boshqa tenglamalardan o'rniga qo'yish natijasida bita n- tartib yechiladi.

Biz uni yechishni boshqacha usul bilan ko'rsatamiz.

$$x_1=\alpha_1e^{k.t}, \qquad x_2=\alpha_2e^{k.t}, \quad ,\ldots, \quad x_n=\alpha_ne^{k.t},$$

shaklda yechimlarni izlaymiz va (1)ga qo'ysak

$$\{k\alpha_1e^{kx}=a_{11}\alpha_1e^{\kappa t}+a_{12}\alpha_2e^{kt}+...+a_{1n}\alpha_ge^{kx}$$

$$\{k\alpha_2e^{kx}=a_{21}\alpha_1e^{\kappa t}+a_{22}\alpha_2e^{kt}+...+a_{1n}\alpha_ge^{kx}$$

$$\{k\alpha_ne^{kx}=a_{n1}\alpha_1e^{\kappa t}+a_{n2}\alpha_2e^{kt}+...+a_{nn}\alpha_ne^{kx}$$

e^{kt} ga qisqartirib, quyidagi algebrik tenglamalar sistemasiga ega bo'lamiz.

$$\begin{cases} (a_{11}-\kappa)\alpha_1+a_{12}\alpha_2+..+a_{1n}\alpha_n=0 \\ a_{21}+(a_{22}-k)\alpha_2+..+a_{2n}\alpha_n=0 \\ a_{n1}x_1+a_{n2}x_2+...+(a_{nn}-k)\alpha_n=0 \end{cases} \qquad (2)$$

$$\Delta(k)=\begin{vmatrix} \dfrac{a_{11}-k \quad a_{12}\ldots a_{1n}}{a_{n1} \quad a_{n2}\ldots(a_{nn}-k)} \end{vmatrix}$$

Agar $\Delta(k)\neq 0 =)$ $x_1(t)=x_2(t)=...=x_n(t)=0$ tirival yechimga ega bo'ladi. Shu sababli $\Delta(k)=0$

Bu xarakteristik tenglamani yechib ildizlarini topamiz.

1).Xarakter tenglama ildizlari xaqiqiy $k_1, k_2, .., k_n$ xar xil bo'lsa

$$\begin{cases} x_1(+)=c_1\alpha_2^{(1)}e^{k_1t}+c_2\alpha_1^{(2)}e^{\kappa_2t}+..+c_n\alpha_1^{(r)}e^{k_n}t \\ x_n(t)=c_1\alpha_n^1e^{k_1t}+....+c_n\alpha_n^{(n)}e^{k_nt} \end{cases}$$

100. Mavzuga doir namunaviy misolni yechimi.

Differensial tenglamalar sistemasini yeching

Misol-1 : $\begin{cases} \dfrac{dx_1}{dt} = 2x_1 + 2x_2 \\ \dfrac{dx_2}{dt} = x_1 + 3x_2 \end{cases}$ Sistemani umumiy yechimini toping

Yechish: Xarakteristik tenglamasini tuzamiz.

$$\begin{vmatrix} 2-\kappa & 2 \\ 1 & +3-\kappa \end{vmatrix} = 0 + (2-\kappa)(+3-\kappa) - 2 = 0$$

$$\kappa^2 - 5\kappa + 4 = 0 \qquad \kappa_1 = 1, \kappa_2 = 4$$

$\kappa_1 = 1$ uchun $\begin{cases} (2-1)\alpha_1^{(1)} + 2\alpha_2^{(1)} = 0 \\ \alpha_1^{(1)} + (3-1)\alpha_2^{(1)} = 0 \end{cases} =) \begin{cases} \alpha_1^{(1)} + 2\alpha_2^{(1)} = 0 \\ \alpha_1^{(1)} + 2\alpha_2^{(1)} = 0 \end{cases}$

Sistemani umumiy yechimi quyidagicha bo'ladi.

$$\begin{cases} x_1(t) = c_1 e^{t} + c_2 e^{4t} \\ x_2(t) = c_1 e^{t} = c_2^1 e^{4t} \end{cases}$$

Misol -2. Quyidagi differensial tenglamalar sistemasini $x(0) = 2$, $y(0) = 0$ boshlangich shartni qanoatlantiruvchi yechimini toping:

$$\begin{cases} \dfrac{dx}{dt} = x + y \\ \dfrac{dy}{dt} = x - y \end{cases}$$

Yechish. Birinchi tenglamani t bo'yicha differensialab, hosil bo'lgan tenglamaga ikkinchi tenglamani qo'ysak, quyidagicha sodda tenglama hosil bo'ladi:

$$\frac{d^2x}{dt^2} = \frac{dx}{dt} + \frac{dy}{dt}, \qquad \frac{d^2x}{dt^2} = x + y + x - y$$

$\dfrac{d^2x}{dt^2} - 2x = 0$. O'zgarmas koeffisentli 2-chi tartibli, bir jinsli differensial tenglamani mos xarakteristik tenglama $k^2 - 2 = 0$ $\quad k_{1,2} = \pm\sqrt{2}$ bo'lganlidan, umumiy yechimi

$$x = c_1 e^{\sqrt{2}t} + c_2 e^{-\sqrt{2}t}$$

birnchi tenglamadan

$$y = \frac{dx}{dt} - x = \sqrt{2}C_1 e^{\sqrt{2}t} - \sqrt{2}C_2 e^{-\sqrt{2}t} - C_1 e^{\sqrt{2}t} - C_2 e^{-\sqrt{2}t} = C_1(\sqrt{2}-1)e^{\sqrt{2}t} - C_2(\sqrt{2}+1)e^{-\sqrt{2}t}.$$

Boshlangich shartga asosan:

$$\begin{cases} C_1 + C_2 = 2 \\ \sqrt{2}(C_1 - C_2) - (C_1 + C_2) = 0 \end{cases} \Rightarrow \quad C_1 = \frac{(\sqrt{2}+2)}{2}, \quad C_2 = \frac{(2-\sqrt{2})}{2}$$

Natijada sistemaning xususiy yechimi:

$$\begin{cases} x(t) = \left(\dfrac{\sqrt{2}}{2}+1\right)e^{\sqrt{2}t} + \left(1-\dfrac{\sqrt{2}}{2}\right)e^{-\sqrt{2}t} \\ y(t) = \dfrac{\sqrt{2}}{2}e^{\sqrt{2}t} - \dfrac{\sqrt{2}}{2}e^{-\sqrt{2}t} \end{cases}.$$

101. Mavzuga doir mustaqil yechish uchun misollar

Quyidagi differensial tenglamalar sistemasini yeching:

1) $\begin{cases} \dfrac{dx}{dt} = 2x + y, \\ \dfrac{dy}{dt} x + 2y \end{cases}$, $x(0) = 1, \quad y(0) = 3$. 2) $\begin{cases} \dfrac{dx}{dt} = \dfrac{x}{2x+3y} \\ \dfrac{dy}{dt} = \dfrac{y}{2x+3y} \end{cases}$, $x(1) = 1, \quad y(0) = 2$.

3) $\begin{cases} \dfrac{dx}{dt} = 4x + 6y \\ \dfrac{dy}{dt} = 2x - 3y \end{cases}$ 4) $\begin{cases} \dfrac{dx}{dt} = e^{3t} - y \\ \dfrac{dy}{dt} = 2e^{3t} - x \end{cases}$

5) $\begin{cases} \dfrac{dx}{dt} = 3x - 2y \\ \dfrac{dy}{dt} = 2x - y \end{cases}$ 6) $\begin{cases} \dfrac{dx}{dt} = 7x + 6y \\ \dfrac{dy}{dt} = -x + 2y \end{cases}$

7) $\begin{cases} \dfrac{dx_1}{dt} = 7x_1 + 3x_2 \\ \dfrac{dx_2}{dt} = 6x_1 + 4x_2 \end{cases}$ 8) $\begin{cases} \dfrac{dx_1}{dt} = x_1 - 4x_2 \\ \dfrac{dx_2}{dt} = x_1 + x_2 \end{cases}$

9) $\begin{cases} \dfrac{dx_1}{dt} = 3x_1 + x_2 \\ \dfrac{dx_2}{dt} = -4x_1 - x_2 \end{cases}$ 10) $\begin{cases} \dfrac{dx_1}{dt} = x_1 - 2x_2 \\ \dfrac{dx_2}{dt} = x_1 - x_2 \end{cases}$.

102. Qatorlar nazariyasi

Sonli qatorlar. Asosiy tushunchalar. Qator yaqinlashishining zarur va yetarli shartlari.

Bizga $U_1, U_2, ..., U_n ...$, cheksiz sonlar ketma-ketligi berilgan bo'lsin. Ushbu yig'indii $U_1+U_2+...+U_n+...= \sum U_n$ (1)sonli qator deyiladi. $U_1, U_2, ..., U_n, ...$ sonlar qatorning hadlari, U_n esa qatorning umumiy hadi deyiladi. Agar qatorning n-chi hadi berilgan bo'lsa, u holda qator berilgan deyiladi. (1) ko'rinishdagi cheksiz qatorlar bilan o'rta maktab algebrasidan tanishmiz. Masalan cheksiz kamayuvchi geometrik professiya hadlaridan tuzilgan ushbu qator $a+aq^1+aq^2+...+aq^{n-1}+...$(2) qatorga misol bo'ladi. (2) qatorning umumiy hadi $U_n=a=^{n-1}$ formula bilan ifodalanadi. Bu yerda a qatorning birinchi hadi, q- maxraji deyiladi. Ushbu 1+1/2+1/3+...+1/n+... (3) garmonik qator deyiladi. (3) qatorning umumiy hadi $U_n=1/n$ (n=1, 2, ...).

(1) qatorning dastlabki n ta hadlari yig'indisidan tuzilgan ushbu $S_n=U_1+U_2+...+U_n$ yig'indini qaraymiz. Agar n cheksiz oshganda S_n yig'indi biror S

chekli limitga intilsa, u holda (1) qatorni yaqinlashuvchi va S sonni qatorning yig'indisi deyiladi. $\lim S_n=S$ kabi belgilanadi. Agar n cheksiz ortganda S_n yig'indi cheksiz ortsa yoki limit mavjud bo'lmasa, u holda (1) qatorni uzoqlashuvchi bo'ladi.

Bu usul bilan ixtiyoriy berilgan qatorni yaqinlashuvchi yoki uzoqlashuvchi ekanligini hamma vaqt ham tekshirib bo'lmaydi. Shu sababli qator yaqinlashishining zarur va etarli shartlari mavjud. (1) sonli qator yaqinlashuvchi bo'lishi uchun uning umumiy hadini limiti nolga teng bo'lishi zaruriydir: $\lim_{n\to\infty} a_n = 0$. (4)

Agar (1) sonli qator uchun (4) qator yaqinlashuvining zaruriy sharti bajarilmasa u holda qator uzoqlashuvchi bo'ladi, lekin zaruriy shart bajarilishidan hamma vaqt ham berilgan sonli qatorni yaqinlashuvchiligi kelib chiqmaydi. Masalan ushbu garmonik qator uchun $1+1/2+1/3+...+1/n+...$ zaruriy shart bajariladi

$$\lim_{n\to\infty} 1/n = 0$$

aslida esa bu qator uzoqlashuvchidir.

Qator yaqinlashishining yetarli alomatlariga taqqoslash teoremalari, Dalamber, Koshi, integral va boshqa alomatlar kiradi. Biz ularga qisqacha to'xtalib o'tamiz.

1)**Taqqoslash teoremasi**. Bizga ikkita $\sum U_n$, $\sum V_n$ musbat hadli ($U_n>0$, $V_n>0$) sonli qatorlar berilgan bo'lsin.

Teorema. Agar $U_1+U_2+...+U_n+...$(1) qatorning hadlari musbat bo'lib, $V_1+V_2+...+V_n+...$(2) yaqinlashuvchi qatorning mos hadlaridan kichik bo'lsa, ya'ni $U_n<V_n$, u holda (1) qàtor yaqinlashuvchi bo'ladi.

Natija. Agar $U_1+U_2+...+U_n+...$ (1) va $V_1+V_2+...+V_n+...$, ($V_n\geq 0$) (2), qatorlar uchun $U_n>V_n$ (n=1, 2,...) bo'lib, (2) qator uzoqlashuvchi bo'lsa, u holda (1) qator ham uzoqlashuvchi bo'ladi.

2. Dalamber alomati. Bizga musbat hadli $\sum U_n=U_1 + U_2 +...+U_n +...$qator berilgan bo'lsin.

Agar $\lim U_{n+1}/U_n=D$ bo'lib D<1 bo'lsa, berilgan qator yaqinlashuvchi, D>1 bo'lsa berilgan qator uzoqlashuvchi, D=1 bo'lsa, masala ochiq qoladi, qo'shimcha tekshirish kerak.

3. Koshi alomati. Musbat xadli $\sum U_n=U_1+U_2+....+U_n+.....$ (1) qator berilgan bo'lsin. Natija $\lim (U^{1/n})=k$, limit mavjud bo'lib, k<1, bo'lsa(1) qator yaqinlashadi,aksincha ê>1 bo'lsa qator uzoqlashadi.

4. **Integral alomat.** Hadlari musbat va kamayuvchi $U_1+U_2+....+U_n+...$ (1) qator berilgan bo'lsin. Agar (1) qator uchun shunday f(x) funksiya topish mumkin bo'lsaki,$U_n=f(n)$ bo'lib, $\int_1^{\infty} f(x)dx=A$, , bu erda A chekli son bo'lsa u holda, (1) qàtor yaqinlashadi, aks holda uzoqlashuvchi bo'ladi.

Ishora almashinuvchi qatorlar. Leybnis teoremasi. Absolyut va shartli yaqinlashuvchi qatorlar.

Ushbu sonli qatorga $a_1-a_2+a_3-a_4+....+(-1)^n*a_n$ +.... (bunda $a_n>0$ (n=1,2,3,...)) ishora almashinuvchi qator deyiladi. Masalan $1-1/2+1/3-1/4+...(-1)^{n-1}*1/n+...$ qator ishora almashinuvchi qatordir. Ishora almashinuvchi qatorlarni yaqinlashuvchi yoki uzoqlashuvchiligini tekshirishda quyidagi Leybnis teoremasidan foydalaniladi.

Teorema. Ushbu $a_1-a_2+a_3-a_4+....+(-1)^n*a_n + ...(a_n>0)$ ishora almashinuvchi qator berilgan bo'lsin. Agar bu qatorning hadlari uchun quyidagi shartlar bajarilsa:

1) $a_1>a_2>a_3>.....a_n>...$

2) lim $a_n=0$,bo'lsa, u xolda (1) ishora almashinuvchi qator yaqinlashuvchi bo'ladi .

Darajali qatorlar. Darajali qatorni yaqinlashish radiusini topish.

Ushbu har bir hadi X sohada aniqlangan $f_n(x)$ funksiyalardan tuzilgan qator $\sum f_n(x)=f_1(x)+f_2(x)+f_3(x)+...+f_n(x)+...$ (1)

funksional qator deyiladi. $x_0\in$ X nuqtani olib, (1) funksional qator hadlarining shu nuqtadagi $f_n(x_o)$ qiytmatlarini hisoblaymiz. Natijada (1) funksional qator ushbu

$\sum f_n(x_0)=f_1(x_0)+f_2(x_0)+...+f_n(x_0)+...$ sonli qatorga aylanadi. Agar $\sum f_n(x_0)$ sonli qator yaqinlashuvchi bo'lsa, u holda $\sum f_n(x)$ funksional qator x_0 nuqtada yaqinlashuvchi deyiladi. Agar $\sum f_n(x_0)$ sonli qator uzoqlashuvchi bo'lsa, u holda $\sum f_n(x)$ funksional qator x_0 nuqtada uzoqlashuvchi deyiladi. Demak, funksional qatorlardan xususiy holda biz o'rgangan sonli qatorlar hosil bo'ladi.

Darajali qatorlar. Abel teoremasi.

Ushbu $\sum a_nx^n=a_0+a_1x+a_1x^2+a_2x^3+...+a_nx^n+...$ (3) ko'rinishdagi qator darajali qator deb ataladi, bunda a_0, a_1, a_2, ..., a_n, ... lar o'zgarmas sonlar bo'lib, ular darajali qatorning koeffisientlari deyiladi. Demak, darajali qatorlar funksional qatorlarning $f_n(x)=a_nx^n$ bo'lgandagi xususiy holidan iborat.

Teorema. (Abel teoremasi). Agar $\sum a_nx^n=a_0+a_1x+a_1x^2+a_2x^3+...+a_nx^n$Q... (3) darajali qator x ning $x=x_0$ $(x_0\neq 0)$ qiytmatida yaqinlashuvchi bo'lsa, |x| ning $|x| < x_0$ tengsizlikni qanoatlantiruvchi barcha qiytmatlarida darajali qator absolyut yaqinlashuvchi bo'ladi.

(3) darajali qator {x} to'plamning har bir nuqtasida yaqinlashuvchi bo'lsin. Bu {x} to'plamni yuqoridan chegarasi mavjud. Uni R bilan belgilaymiz R=sup{x}. Ko'rsatish mumkinki, x ning ushbu |x| <R tengsizlikni qanoatlantiruvchi qiytmatlarida (3) qator yaqinlashuvchi, x ning |x| >R tengsizlikni qanoatlantiruvchi qiytmatlarida (4) qator uzoqlashuvchi bo'ladi. Ushbu (-R, R) interval (3) darajali qatorning yaqinlashish intervali, R esa yaqinlashish radiusi deb ataladi. Agar darajali qator xq0 nuqtada yaqinlashuvchi bo'lib, boshqa barcha nuqtalarda uzoqlashuvchi bo'lsa, u holda yaqinlashish radiusi R=0 deb olinadi. Agar darajali qator barcha nuqtalarda yaqinlashuvchi bo'lsa, Rq+∞ deb olinadi.

Darajali qator x= -R, x=R nuqtalarda yaqinlashuvchi yoki uzoqlashuvchi bo'lishi mumkin. (3) darajali qatorning yaqinlashish radiusi Dalamber belgisiga asosan quyidagi formuladan topiladi:

$R=\lim |a_n/a_{n+1}|$ (4).

Absalyut va shartli yaqinlashuvchi qatorlar.

Bizga sonli qator $\sum a_n= a_1+a_2+a_3+....a_n+....$ (1) berilgan bo'lsin.

Teorema Agar (1) qatorning absolyut qiytmatlaridan tuzilgan $\sum|a_n| =|a_1|+ |a_2|+|a_3|+....+|a_n|+....$(2) qator yaqinlashuvchi bo'lsa, u holda (1) qator absalyut yaqinlashuvchi bo'ladi.

Demak, agar $\sum| a_n|=|a_1| +|a_2|+|a_3|+....+|a_n|+....$ qator yaqinlashuvchi bo'lsa u xolda $\sum a_n =a_1+a_2+.... a_n+...$ qator absolyut yaqinlashuvchi qator bo'ladi. Agar $\sum a_n$ qator yaqinlashuvchi bo'lib $\sum|a_n|$ qator uzoqlashuvchi bo'lsa, u holda $\sum a_n$ qator shartli yaqinlashuvchi qator deyiladi.

Agar $\sum a_nx^n$ darajali katorning yakinlashish radiusi R(R>0) bo'lsa,darajali qator $[- \alpha, \alpha]$ segmentda $(0<\alpha<R)$ tekis yaqinlashuvchi bo'ladi.

Tekis yaqinlashuvchi darajali qatorlarni $[a,b] \subset (-R, R)$ da hadlab integrallash, hadlab differensiallash mumkin.

Teylor va Makloren qatorlari.Funksiyalarni Makloren qatoriga yoyish. Qatorlarni taqribiy hisoblashlarga tadbiqlari

Bu mavzuda bosh maqsad murakkab ko'rinishga ega bo'lgan y=f(x) funksiyani uni differensiallanuvchiligidan foydalanib sodda butun ko'rsatkichli ko'phad bilan almashtirib yozish masalalari qisqacha yoritiladi.

Teorema. Agar f(x) funksiya biror]-R,R [oraliqda istalgan tartibli chekli hosilalarga ega bo'lib,IxI<R soxada biror o'zgarmas S son uchun $|f^{(n)}(x)| <C$ tengsizlik bajarilsa, u xolda f(x) funksiyani (2) Makloren qatoriga yoyish mumkin

$$f(x)=f(0)+f^1(0)/1!x+f^{11}(0)/2!x^2+...+f^{(p)}(n)(0)/n!x^n+... \quad (2)$$

Masalan, agar f(x) funksiya yoki uning hosilalari x=0 nuqtada mavjud bo'lmasa, bunday funksiyani qatorga yoyib bo'lmaydi. Masalan lnx, ctgx,1/x funksiyalarni makloren qatorga xq0 nuqtada yoyib bo'lmaydi. Bunga f(x)= lnx, f(x)=ctgx kabi funksiyalar misol bo'ladi. Bunday hollarda berilgan funksiyani Teylor qatoriga yoyish mumkin.

f(x) funksiya x_0 nuqta atrofida aniqlangan bo'lib.shu nuqtada uning barcha hosilalari mavjud bo'lsin.U holda bu funksiyani x_0 nuqtada Teylor qatoriga yoyilmasi quyidagicha bo'ladi:

$$f(x)=\sum f^{(n)}(a)/n!(x-a)^n=f(a)+f^1(a)/1!(x-a)+f^{11}(a)/2!(x-a)^2+....+f^n(a)/n!(x-a)^n+.... \quad (3)$$

Bundan ya'ni Teylor qatoridan xususan a=0 bo'lganda Makloren qatori hosil bo'ladi.

Funksiyalarning darajali qatorga yoyilmasidan foydalanib bu funksiyalarning ayrim nuqtalardagi xususiy qiytmatlarni va boshlang'ich funksiyasini elementar funksiyalar orqali ifodalanmaydingan aniq integrallarni istalgan aniqlikda taqribiy hisoblash mumkin.

103. Mavzuga doir namunaviy misollarni yechimi.

Misol-1. Ushbu $1/(1*2)+1/(2*3)+...+1/(n(n+1))+...+$ qatorni yaqinlashuvchiligi aniqlansin. Ko'rsatish mumkinki qatorni umumiy hadi uchun

$1/(n(n+1))=1/n-1/(n+1)$. Qatorni xar-bir qo'shiluvchisiga ushbu yoyilmadan foydalanib o'rniga qo'ysak, birinchi va oxirgi hadidan tashqari barchasi qisqarib $S_n=1-1/(n+1)$.Bundan $\lim S_n=1$. Demak berilgan qator yaqinlashuvchi.

Misol-2. Quyidagi $\sum 1/n^k$ umumlashgan garmonik qatorni qaraymiz. Bu qator $\sum 1/n^k$ taqqoslash teoremasiga asosan $k>1$ bo'lsa yaqinlashuvchi, $k\leq 1$ bo'lganda uzoqlashuvchi bo'ladi.

Misol-3. $1/\sqrt{1}+1/\sqrt{2}+1/\sqrt{3}+.......$ qatorning yaqinlashishi yoki uzoqlashuvchiligi tekshirilsin.

Yechish. Berilgan qatorni $1+1/2+1/3+...$ garmonik qator bilan taqqoslaymiz. Natijaga ko'ra $1/\sqrt{n}>1/n$ $(n=2,3,...)$ garmonik qatorning uzoqlashuvchi ekanligidan, berilgan qatorni uzoqlashuvchiligi kelib chiqadi

Misol.4. $a/1+a^2/2+a^3/3+...+a^n/n+.....$ $a>0$ qatorning yaqinlashishi tekshirilsin

Yechish: $U_n=a^{n+1}/(n+1)$ $U_n=a^n/n$, $U_{n+1}/U_n=n/(n+1)*a$; $\lim U_{n+1}/U_n=a*\lim n/(n+1)=a$

Dalamber alomatiga ko'ra $0<a<1$ bo'lsa, berilgan qator yaqinlashuvchi, aksincha $a>1$ bo'lsa, uzoqlashuvchi bo'ladi.

Ìèñîë-5. Quyidagi sonli qatorni yaqinlashishini tekshiring:

$$\sum_{n=1}^{\infty} p / 3^p = 1/3+2/3^2 +3/3^2 + ...+ p/3^p +........$$

Dalamber alomatiga asosan:

$$d= \lim_{n\to\infty} (a_{n+1})/ (a_n)= \lim_{n\to\infty}((p+1) / (3^{p+1}))/(n/3^n)= \lim_{n\to\infty}((p+1)3^n/(n3^{p+1}))=$$

$$= \lim_{n\to\infty} (1+1/n)*(1/3)=(1/3)<1$$ demak berilgan qator yaqinlashuvchi.

Misol-6. Ushbu $\sum 1/n^2=1/1^2+1/2^2+1/3^2+....1/n^2+...$ qator yaqinlashuvchiligi tekshirilsin

Yechish. Dalamber alomatiga asosan

$\lim U_{n+1}/U_n=\lim n^2/(n+1)^2 q\lim 1/(1+1/n^2)=1$.berilgan qatorni yaqinlashuvchi yoki uzoqlashuvchi ekanligini àniqlamaydi.

Integral alomat bilan tekshiramiz: 1) qatorni xadlari musbat va kamayuvchi

2） $U_n=1/n^2=f(n),f(x)=1/x^2$

$$\int_1^{\propto} f(x)dx=\int_1^{\propto} 1/x^2dx=\lim_{V\to\propto} \int_1^V f(x)dx=\lim(-1/x)\Big|_1^V =1$$

Demak berilgan qator yaqinlashuvchidir.

Misol-7. Ushbu $1-1/2+1/3-1/4+...$ (-1)n-1*1/n+.... ishora almashinuvchi qatorni yaqinlashuvchiligini tekshiring.

Bu qator uchun Leybnis toremasini shartlarini bajarilishini tekshiramiz

1) $1>1/2>1/3>.... >1/n>....$

2） $\lim 1/n=0$. Demak, bu qator Leybnis teoremasiga asosan yaqinlashuvchi ekan.

Misol- 8. Ushbu $1-1/2+1/3-1/4+...(-1)^{n-1}*1/n+....$ ishora almashinuvchi qatorni absalyut yaqinlashuvchiligini tekshiring.

Bu qatorni absalyut qiytmatlaridan tuzilgan qator, biz o'rgangan garmonik qator bo'lib 1+1/2+1/3+...+1/n+... u uzoqlashuvchi qator edi. Ammo berilgan ishora almashinuvchi qator Leybnis teoremasiga asosan yaqinlashuvchi. Shu sababli bu qator shartli yaqinlashuvchi qator bo'ladi.

Misol-9. Ushbu $\sum x^n=1+x+x^2+...+x^n+...$ darajali qatorning yaqinlashish radiusi va yaqinlashish intervali topilsin.

Yuqoridagi (4) formulaga ko'ra R=lim 1/l =1. Demak, berilgan darajali qatorning yaqinlashish radiusi R=1, yaqinlashish intervali (-1, 1) bo'ladi.

Misol. 2. Darajali qatorni yaqinlashish radiusini toping:

$\sum_{n=1}^{\infty} x^n/(n* 2^n)$ Ma'lumki, $\sum_{n=1}^{\infty} (a_n x^n)$ darajali qatorni

yaqinlashish radiusi quyidagi formula bilan topiladi

$$R=\lim_{n\to\infty} |a_n|/|a_{n+1}|$$

Bizning misolda $a_n=(1/n)*(1/2^n)$; $a_{n+1}=(1/n+1)*(1/2^{n+1})$ bo'lganligidan yaqinlashish radiusi: $R=\lim_{n\to\infty}|(a_n/a_{a+1})|=\lim_{n\to\infty}|((1/n2^n)/(1/(n+1)*(1/2^{n+1}))|=$

$=\lim_{n\to\infty}(n+1)* 2^{n+1}/(n* 2^n) = 2\lim_{n\to\infty}(1+1/n)=2$

Demak, yaqinlashish oraligi $|x|<R=2$, $-2<x<2$ oraliqdagi barcha x lar uchun darajali qator yaqinlashuvchi bo'ladi.

Misollar-10. $f(x)=e^x$, $f(x)=\sin x$, $f(x)=\cos x$, va $f(x)=(1+x)^m$ funksiyalarni Makloren qatoriga yoyilmalari quyidagicha bo'ladi
(barcha $x\in R$ lar uchun):

$\mathbf{e^x=1+x/1!+x^2/2!+x^3/3!+...+x^n/n!+...}$

$\mathbf{\sin x=x/1!-x^3/3!+x^5/5!-...+(-1)^{n-1}\ x^{2n-1}/(2n-1)! + ...}$

$\mathbf{\cos x=1- x^2/2!+x^4/4! - ... +(-1)^n\ x^{2n}/(2n)! +...}$ yoyilmalar o'rinlidir.

$f(x)=(1+x)^m$ (m-butun, kasr, musbat yoki manfiy son bo'lishi mumkin) funksiyani
1 < x < 1 oraliqda Makloren qatoriga yoyilmalari quyidagicha bo'ladi

$\mathbf{(1+x)^m= 1+m/1!x + m(m-1)/2!x^2 + ... +m(m-1)... (m-n+1)\ x^n/n!+...}$

yoyilma o'rinlidir. Bu binominal qator deyiladi.

Bundan xususan a) m=1/2 bo'lganda $\mathbf{(1+x)^{1/2} \approx 1+x/2 - x^2/8}$

b) m=1/3 bo'lganda $\mathbf{(1+x)^{1/3} \approx 1+x/3 - x^2/9}$

taqribiy ildiz chiqarish formulalari kelib chiqadi.

Misol-11. e^x, sinx, cosx funksiyalarning qiytmatlarini taqribiy hisoblash.

1) $f(x) =e^x$ funksiyaninng x=0,2 dagi takribiy qiytmati 0,0001 aniqlida hisoblang.

Yechish: $f(x)=e^x$ funksiyani makloren qatoriga yoyilmasiga
$e^x=1+x/1!+x^2/2!+x^3/3!+...+x^n/n!+...$ x=0,02 qo'ysak $e^{0,2}=1+0,2+(0,2)^2/2!+$ $+(0,2)^3/3!+(0,2)^4/4!+...=1+0,2+0,002+0,008/6+0,0016/24+...=1+0,2+0,002+0,00$ 13+0,000066+...+=1,2213 demak $e^{0,2}=1,2213$. Uni aniq qiytmati $e^{0,2}=1,2214027$

2) f(x)=sinx funksiyani x=1 nuqtadagi qiytmati 0,0001 aniqlikda hisoblansin

Yechish: f(x)=sinx funksiyani Makloren qatoriga yoyilmasidan: $Sinx=x-x^3/3!+x^5/5!+..+(-1)^{n-1}*x^{2n-1}/(2n-1)!+.=1-0,1666+0,008333-0,00019+..=0,8417$ demak

$Sin1=Sin180^0/\pi=Sin57^0 18^1 =0,8417$

Binomial qator yordamida ildizlarni taqribiy hisoblash.

Misol-12. √516 ni 0,0001 Aniqlik bilan hisoblang

Yechish:Binomial qatordan foydalanish maqsadida √516 ni ushbu ko'rinishda yozib olamiz:

$^3\sqrt{}516=^3\sqrt{}512+4=^3\sqrt{}8^3+4=^3\sqrt{}8^3(1+4/83)=8*^3\sqrt{}1+0,0078=8(1+0,0078)^{1/3}$

$^3\sqrt{}516=8*(1+0,0078)^{1/3}=8+0,0204-0,000054008+..0,0001$ aniqlikda hisoblash uchun qatorning ikkita xadi bilan chegaralanish yetarlidir. $^3\sqrt{}516 =8,0204$ Aniq qiymati $^3\sqrt{}516 =8,0207793$

104. Mavzuga doir mustaqil yechish uchun misollar.

1-20.Quyidagi misollarda a) sonli qatorni yaqinlashishini Dalamber yoki Koshi, integral alomatlari bilan tekshiring: b) darajali qatorni yaqinlashish radiusini toping: v) berilgan funksiyani makloren qatoriga yoying.

1. $a) 1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}+...$ $\quad б)\sum_{n=1}^{\infty}\frac{x^n}{n}$ $\quad в) f(x)=\cos x$

2. $a) \frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+...+\frac{n}{2^n}+...$ $\quad б)\sum_{n=0}^{\infty}\frac{x^n}{n}$

$в) f(x)=\ln(1-x+x^2)$

3. $a) 1+\frac{4}{2}+\frac{9}{3}+\frac{1}{4}...$ $\quad б)\sum_{n=1}^{\infty}\frac{x^n}{3^n(n+1)}$ $\quad в) f(x)=\cos^2 x$

4. $a) \frac{1}{100}+\frac{2}{100^2}+\frac{3}{100^3}+...$ $\quad б)\sum_{n=1}^{\infty}\frac{x^n}{5^n}$ $\quad в) f(x)=\sin^2 x$

5. $a) \frac{2}{3}+\frac{4}{9}+\frac{6}{27}+...+\frac{2n}{3^n}$ $\quad б)\sum_{n=1}^{\infty}\frac{x^n}{n4^n}$ $\quad в) f(x)=\ln(1+x)$

6. $a) 1+\frac{2}{2}+\frac{4}{3}+...+\frac{2^{n-1}}{n}+...$ $\quad б)\sum_{n=1}^{\infty}\frac{x^n}{n2^n}$ $\quad в) f(x)=\sin x$

7. $a) 1+\frac{3}{2\cdot 3}+\frac{3^2}{2^2\cdot 5}+...+\frac{3^n}{2^n(2n+1)}+...$ $\quad б)\sum_{n=1}^{\infty}\frac{x^n}{6^n}$ $\quad в) f(x)=\cos x$

8. $a) 1+\frac{3}{4^2}+\frac{1}{7^2}+...+\frac{3^n}{(3n+1)^2}+...$ $\quad б)\sum_{n=1}^{\infty}\frac{n}{n+1}\cdot\frac{x^n}{2^n}$

$в) f(x)=arctgx$

9. $a) \frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+...+\frac{2n-1}{2^n}+...$ $\quad б)\sum_{n=1}^{\infty}\frac{n+1}{n+2}\cdot\frac{x^n}{3^n}$ $\quad в) f(x)=\arcsin x$

10. а) $1+\frac{2}{2^2}+\frac{1}{2^4}+\ldots+\frac{1}{2^n}+\ldots$ б) $\sum_{n=1}^{\infty}\frac{1}{n}\cdot\frac{x^n}{4^n}$ в) $f(x)=\arccos x$

11. а) $1+\frac{1}{8}+\frac{1}{27}+\ldots+\frac{1}{n^3}+\ldots$ б) $\sum_{n=1}^{\infty}\frac{1}{n^2}\cdot\frac{x^n}{2^n}$ в) $f(x)=e^x$

12. а) $1+\frac{1}{5}+\frac{1}{13}+\ldots+\frac{1}{2^n-3}+\ldots$ б) $\sum_{n=1}^{\infty}\frac{1}{n^3}\cdot\frac{x^n}{3^n}$ в) $f(x)=e^{3x}$

13. а) $1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots+\frac{1}{(2n-1)^2}+\ldots$ б) $\sum_{n=1}^{\infty}\frac{x^n}{n^2}$ в) $f(x)=\sin\frac{x}{2}$

14. а) $\frac{1}{2^2}+\frac{1}{4^2}+\ldots+\frac{1}{(2n)^2}+\ldots$ б) $\sum_{n=1}^{\infty}\frac{x^n}{n^3}$ в) $f(x)=\cos\frac{x}{2}$

15. а) $1+\frac{3}{8}+\frac{5}{27}+\ldots+\frac{2n-1}{n^3}+\ldots$ б) $\sum_{n=1}^{\infty}\frac{x^n}{(2n)^2\cdot 2^{2n}}$ в) $f(x)=\sin(x^2)$

16. а) $\frac{2}{1}+\left(\frac{3}{3}\right)^2+\left(\frac{4}{5}\right)^3+\ldots+\left(\frac{n+1}{2n-1}\right)^n+\ldots$ б) $\sum_{n=1}^{\infty}\frac{x^n}{n^n}$ в) $f(x)=\cos(x^2)$

17. а) $\frac{1}{2}+\left(\frac{2}{5}\right)^3+\left(\frac{3}{8}\right)^5+\ldots+\left(\frac{n}{3n-1}\right)^{2n-1}+\ldots$ б) $\sum_{n=1}^{\infty}\frac{x^n}{(n+1)^n}$ в) $f(x)=e^{-4x}$

18. а) $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}+\ldots$ б) $\sum_{n=1}^{\infty}\frac{x^n}{(2n+1)^n}$ в) $f(x)=\ln(1+x)$

19. а) $1+\frac{2}{3}+\frac{3}{5}+\ldots+\frac{n}{2n-1}+\ldots$ б) $\sum_{n=1}^{\infty}\frac{x^n}{n}\cdot\left(\frac{2}{3}\right)^n$ в) $f(x)=\frac{1}{1-x}$

20. а) $\frac{(1)^2}{2}+\frac{(2)^2}{2^4}+\frac{(3)^2}{2^9}+\ldots+\frac{(n)^2}{2^{n^2}}+\ldots$

21. a) $\sum_{n=1}^{\infty}\frac{2^n}{n^2}$; b) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n+5}$; v) $\sum_{n=1}^{\infty}\frac{n}{5^n}x^n$;

22. a) $\sum_{n=1}^{\infty}\frac{1}{n5^{n+1}}$; b) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2+3}$; v) $\sum_{n=1}^{\infty}\frac{3^n}{n}x^n$;

23. a) $\sum_{n=1}^{\infty}\frac{1}{(n+2)5^n}$; b) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2}$; v) $\sum_{n=1}^{\infty}\frac{2^n}{n+1}x^n$;

24. a) $\sum_{n=1}^{\infty}\frac{n}{3^{n+2}}$; b) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$; v) $\sum_{n=1}^{\infty}\frac{2^n}{n}x^n$;

25. a) $\sum_{n=1}^{\infty}\frac{3^n}{n^2}$; b) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n+1}$; v) $\sum_{n=1}^{\infty}\frac{n}{2^n}x^n$;

26. a) $\sum_{n=1}^{\infty}\frac{1}{(n+1)7^n}$; b) $\sum_{n=1}^{\infty}(-1)^n\frac{2}{n^2+1}$; v) $\sum_{n=1}^{\infty}\frac{nx^n}{3^n}$;

27. a) $\sum_{n=1}^{\infty}\frac{7^n}{n+3}$; b) $\sum_{n=1}^{\infty}(-1)^n\frac{2}{\sqrt{n}}$; v) $\sum_{n=1}^{\infty}\frac{1}{n5^n}x^n$;

28. a) $\sum_{n=1}^{\infty}\frac{2^n}{(1+n)^2}$; b) $\sum_{n=1}^{\infty}(-1)^n\frac{3}{n+2}$; v) $\sum_{n=1}^{\infty}\frac{x^n}{7^n}$;

29. a) $\sum_{n=1}^{\infty} \frac{n+2}{2^n}$; b) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$; v) $\sum_{n=1}^{\infty} \frac{n}{5^n}$;

30. a) $\sum_{n=1}^{\infty} \frac{5^{n+1}}{n^2}$; b) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n+3}$; v) $\sum_{n=1}^{\infty} \frac{x^n}{n7^{n+1}}$;

31. a) $\sum_{n=1}^{\infty} \frac{n^2}{5^{n+1}}$; b) $\sum_{n=1}^{\infty} (-1)^n \frac{2}{n+2}$; v) $\sum_{n=1}^{\infty} \frac{x^n}{n+2}$;

32. a) $\sum_{n=1}^{\infty} \frac{n}{3^{n+2}}$; b) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$; v) $\sum_{n=1}^{\infty} \frac{x^n}{n7^n}$;

33. a) $\sum_{n=1}^{\infty} \frac{2^n}{(n+2)^2}$; b) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{(n+2)^2}$; v) $\sum_{n=1}^{\infty} \frac{x^n}{n+5}$;

34. a) $\sum_{n=1}^{\infty} \frac{1}{2^n \cdot n}$; b) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$; v) $\sum_{n=1}^{\infty} nx^n$;

35. a) $\sum_{n=1}^{\infty} \frac{n+1}{2^n}$; b) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt[3]{n}}$; v) $\sum_{n=1}^{\infty} \frac{n}{2^n} x^n$;

36. a) $\sum_{n=1}^{\infty} \frac{1}{n5^n}$; b) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+2}$; v) $\sum_{n=1}^{\infty} \frac{n}{5^n} x^n$;

37. a) $\sum_{n=1}^{\infty} \frac{3^n}{(n+2)^2}$; b) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n+1}}$; v) $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$;

38. a) $\sum_{n=1}^{\infty} \frac{n}{5^{n+1}}$; b) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt[3]{n+2}}$; v) $\sum_{n=1}^{\infty} \frac{n}{7^n} x^n$;

39. a) $\sum_{n=1}^{\infty} \frac{1}{(n+1)3^n}$; b) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n+5}}$; v) $\sum_{n=1}^{\infty} \frac{n+1}{8^n} x^n$;

40. a) $\sum_{n=1}^{\infty} \frac{8^n}{n^2+1}$; b) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+3}$; v) $\sum_{n=1}^{\infty} \frac{x^n}{n7^{n+1}}$;

Quyidagi ishora almashinuvchi qatorlarni absalyut yoki shartli yaqinlashuvchiligini tekshiring:

1. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + (-1)^{n+1} \frac{1}{n} + \ldots$

2. $1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \ldots + (-1)^{n+1} \frac{1}{(2n-1)^2} + \ldots$

3. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$, 4. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\ln(n+1)}$

Hadlab differensiallab yoki integrallab quyidagi darajali qatorlarni yigindisini toping:

1. $x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots. + \frac{x^n}{n} + \ldots$

2. $1 + 2x + 3x^2 + \ldots + (n+1)x^n + \ldots$

3. $1 - 3x^2 + 5x^4 - \ldots + (-1)^{n-1}(2n-1)x^{2n-2} + \ldots$

105. Fure qatorlari

$\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n Cosnx+b_n \sin nx)$ (1) trigonometrik qator deyiladi. a_0, a_n va b_n trigonometrik qatorni koeffisientlari.

Agar (1) qator f(x) funksiyaga yaqinlashsa, bu funksiya T=2π davrga ega bo'lgan funksiyadan iborat bo'ladi, $f(x+2\pi)=f(x)$

Teskari masalani qaraymiz, 2π davriga ega bo'lgan f(x)funksiyani shu funksiyaga yaqinlashuchi (1) trigonometrik qator shaklida yozish mumkinmi?

Faraz qilaylik shunday qator mavjud bo'lsin:

$$f(x)=\frac{a_o}{2}+\sum_{n=1}^{\infty}(a_n Cosnx+b_n Sinnx)$$ (-π;π) oraliqda integrallasak.

$$\int_{-\Pi}^{\Pi} f(x)dx=\frac{a_o}{2}\int_{-\Pi}^{\Pi} dx+\sum_{1}^{\infty}\left(\int_{-\Pi}^{\Pi} a_n Cosnxdx+\int_{-\Pi}^{\Pi} b_n \sin nxdx\right) a_o=\frac{1}{\Pi}\int_{-\Pi}^{\Pi} f(x)dx$$

$a_к$, $в_к$ larni topish uchun tenglikni ikkala tomonini Coskx ga ko'paytirib topamiz:

$$a_к=\frac{1}{\Pi}\int_{-\Pi}^{\Pi} f(x)Coskxdx \quad , \quad b_к=\frac{1}{\Pi}\int_{-\Pi}^{\Pi} f(x)Coskxdx \qquad (2)$$

(2) formula bilan aniqlangan $a_к$, $в_к$ Fure koeffisientlari, (1) qator Fure qatori deyiladi.

106. Mavzuga doir namunaviy misolni yechimi.

Misol-1: $f(x)=x,$ -π < x<π

Bu funksiya bo'lakli monoton va chegaralangan funksiya bo'lganligi uchun uni Fure qatoriga yoyib yozish mumkin; (2) formulaga asosan Fure koeffisentlarini hisoblaymiz.

$$a_o=\frac{1}{\Pi}\int_{-\Pi}^{\Pi} xвx=\frac{1}{\Pi}\frac{x^2}{2}\Big/_{-\Pi}^{\Pi}=0$$

$$a_к=\frac{1}{\Pi}\int_{-\Pi}^{\Pi} xCosk\ xos=\frac{\Pi}{\Pi}\left[x\frac{\sin kx}{k}\Bigg|_{-\Pi}^{\Pi}-\frac{1}{к}\int_{-\Pi}^{\Pi}\sin kxdx-\right]=0$$

$$b_к=\frac{1}{\Pi}\int_{-\Pi}^{\Pi} xSin\ kxxd=\frac{1}{\Pi}\left[-x\frac{Coskx}{k}\Bigg|_{-\Pi}^{\Pi}+\frac{1}{к}\int_{-\Pi}^{\Pi} Coskxdx-\right]=(-1)^{k+1}\frac{2}{k}$$

$$f(x)=2\sum_{k=1}^{\infty}\frac{Sinkx}{k}=2\left[\frac{Sin}{1}-\frac{Sin\ 2x}{2}+....+(-1)^{k+1}\frac{\sin kx}{k}+...\right]$$

Bu tenglik funksiyani uzilish nuqtalaridan tashqarii barcha nuqtalarida o'rinlidir.

Misol-2. $f(x)=\frac{\pi}{4}-\frac{x}{2}$ funksiyani $[0;\ \pi]$ kesmada kosinus bo'yicha Fure qatoriga yoying.

Yechish. Fure koeffisentlarini hisoblaymiz.

$$a_0=\frac{2}{\pi}\int_0^{\pi} f(x)dx=\frac{2}{\pi}\int_0^{\pi}\left(\frac{\pi}{4}-\frac{x}{2}\right)dx=\frac{2}{\pi}\left(\frac{\pi}{4}x-\frac{x^2}{2}\right)/_0^{\pi}=0,$$

$$a_n=\frac{2}{\pi}\int_0^{\pi} f(x)Cosnxdx=\frac{2}{\pi}\int_0^{\pi}\left(\frac{\pi}{4}-\frac{x}{2}\right)Cosxdx$$

Bo'laklab integralash formulasiga asosan $u=\pi-2x,\quad dv=Cosnxdx\quad,\quad du=-2dx$ $v=\frac{1}{n}Sinnx$

$$a_n=\frac{1}{2\pi}\left[\left((\pi-2x)\frac{Sinnx}{n}\right)\Big|_0^{\pi}+\frac{2}{\pi}\int_0^{\pi}Sinnxdx\right]=\frac{1}{2\pi}\left(-\frac{2}{n^2}Cosx\right)\Big|_0^{\pi}=\frac{1}{n^2\pi}(1-Cosn\pi)=\begin{cases}0\\ \dfrac{2}{n^2\pi}\end{cases}$$

agar n – juft bo'lsa,
agar n – toq bo'lsa.

Natijada berilgan funksiyani kosinus bo'yicha $[0;\ \pi]$ oraliqda Fure qatoriga yoyilmasi quyidagicha bo'ladi:

$$\frac{\pi}{4}-\frac{x}{2}=\frac{2}{\pi}\left[Cosx+\frac{Cos\,3x}{3^2}+\frac{Cos\,5x}{5^2}+....\right]$$

107. Mustaqil yechish uchun misollar.

Mustaqil echish uchun misollar

Quyidagi $f(x)$ funksiyalarni $[0;\ \pi]$ kesmada kosinus bo'yicha Fure qatoriga yoying.

1. $f(x)=x-2$; 2. $f(x)=1-2x$;
3. $f(x)=3x$; 4. $f(x)=2x+1$;
5. $f(x)=1-x$; 6. $f(x)=-x-1$;
7. $f(x)=2x-1$; 8. $f(x)=\pi-x$;
9. $f(x)=\pi-2x$; 10. $f(x)=\frac{\pi}{2}+x$;
11. $f(x)=3x+1$; 12. $f(x)=-\pi+\frac{1}{4}x$;
13. $f(x)=-2x+3$; 14. $f(x)=7x-1$;
15. $f(x)=\pi x+2$; 16. $f(x)=\pi x+1$;
17. $f(x)=x-\frac{\pi}{2}$; 18. $f(x)=\frac{\pi}{2}2x$;
19. $f(x)=8x+\frac{\pi}{2}$; 20. $f(x)=3x-8$;

Quyidagi funksiyalarni 2π davrli Fure qatoriga yoying.

21) a) f(x)=x $(-\pi,\pi)$; b) $f(x)=\frac{\pi-x}{2}$ $(0,2\pi)$;

v) f(x) $=|x|$ $(-\pi,\pi)$; g) $f(x)=\begin{cases}0,-\pi<x\le 0,\\ x,0<x\le\pi;\end{cases}$

d) $f(x)=\begin{cases}x,-\pi<x\le 0,\\ 0,0<x<\pi;\end{cases}$ e) $f(x)=\sin ax,\ -\pi<x<\pi;$

j) $f(x)=chax,\quad -\pi<x<\pi.$

22) Quyidagi funksiyalarni Sinus va kosinus bo’yicha Fure qatorlariga yoying: $f(x) = x, 0 < x < \pi;$ $f(x) = \dfrac{\pi - 2x}{4}, 0 < x < \pi.$

23) f(x) Funksiyani 2L davrli (-L,L) oraliqqa $f(x) = |x|$ Fure qatoriga yoying.

108. Karrali integrallar.

Ikki o’zgaruvchili $z = f(x;y)$ funksiya yopiq chegaralangan D sohada aniqlangan va uzuluksiz bo’lsa, bu funksiyadan olingan ikki karali integral mavjud va quyidagicha yoziladi:

$$\iint\limits_D f(x;y)d\delta$$

Hususan D sohada $f = (x;y) > 0$ bo’lsa, ikki karrali integral yuqoridan $z = f(x;y)$ sirt bilan yon tomonidan oz o’qiga paralel va quyidan xou tekislikdagi D soha bilan chegaralangan silindirik jismning hajmiga teng bo’ladi

$$V = \iint\limits_D f(x;y)d\delta .$$

Ikki karrali integralni asosiy xossalari:

1) $\iint\limits_D [f_1(x;y) \pm g_1(x;y)]d\delta = \iint\limits_D f_1(x;y)d\delta \pm \iint\limits_D g_1(x;y)d\delta$ 2) $\iint\limits_D cf(x;y)d\delta = c\iint\limits_D f(x;y)d\delta$,

(S- o’zgarmas). 3) Agar integrallash sohasi D=D_1VD_2 bo’lsa

$$\iint\limits_D (x;y)d\delta = \iint\limits_{D_1} f(x;y)d\delta + \iint\limits_{D_2} f(x;y)d\delta$$

Dekart koordinatalar sistemasida ikki karali integralni quyidagicha yozish mumkin

$$\iint\limits_D f(x;y)d\delta = \iint\limits_D f(x;y)dxdy .$$

a) $D:\{a \le x \le b, c \le y \le d\}$, to’gri to’rtburchakli soha bo’yincha ikki karrali integral quyidagicha takroriy integralga keltirib hisoblanadi.

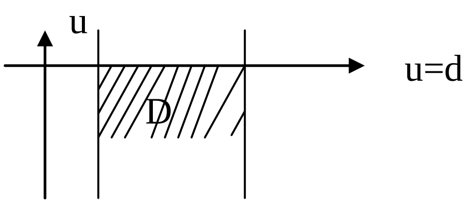
$$\iint\limits_D f(x,y)dxdy = \int_a^b dx \int_c^d f(x,y)dy = \int_c^d dy \int_a^b f(x,y)dx .$$

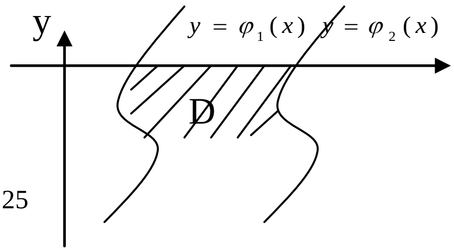
b) Agar D soha $D = \{y = \varphi_1(x),\ y = \varphi_2(x)\}$ $\varphi_1(x) \le \varphi_2(x)$ $a \le x \le b$ $\}$, egri chiziqlar bilan chegaralangan bo’lsa ikki karali integral quyidagi takroriy integralga keltirib echiladi.

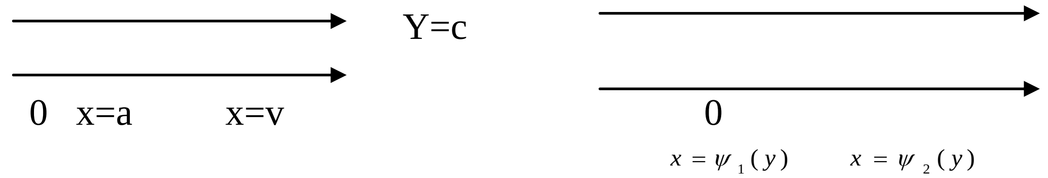
$$\iint\limits_D f(x,y)dxdy = \int_a^b dx \int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)dy .$$

Hususan D soha $D = \{x = \varphi_1(y),\ x = \varphi_2(y)\}$, $\varphi_1(y) \le \varphi_2(y)$ $c \le y \le d,$ $\}$ bo’lsa

$$\iint\limits_D f(x,y)dxdy = \int_c^d dy \int_{\varphi_1(y)}^{\varphi_2(y)} f(x,y)dx$$







Silliq figura yuzasini karali integral yordamida xisoblash. Agar D soxa $a \le x \le в,\quad \gamma_1(x) \le \gamma_2(x)$ bo'lsa

$$S=\int_a^в dx\int_{\gamma_1(x)}^{\gamma_2(x)}dy$$

Qutib koordinatalar sistemasida D soxa $\alpha \le \theta \le \beta,\quad \gamma_1(\theta) \le g \le \gamma_2(\theta)$ bo'lsa $S=\int_\alpha^\beta d\theta\int_{\gamma_1(\theta)}^{\gamma_2(\theta)} g\ dg$

c).Karrali integrallarda $x=\varphi(u,v),\quad y=\psi(u,v),$ almashtirish yordamida yangi o'zgaruvchiga $\iint_D f(x;y)dxdy=\iint_{D^1} f(x(u;v);\,y(u;v))\left|J\right|dudv$ o'tiladi. Integrallash sohasi D sohadan Δ - sohaga o'zgaradi. Bu erda J – Yakobian

$$J=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}.$$

d).Ikki karrali integralni qutub koordinatalar sistemasida $x=gCos\ \theta$, $y=gSin\ \theta$ almashtirishdan foydalanib quyidagicha hisoblash mumkin.

$\iint_D f(x;y)dxdy=\iint_D f(gCos\ \theta;\,gSin\ \theta)gdgd\ \theta$. Agar $\alpha<\theta<\beta$, $g_1(\theta)\le g\le g_2(\theta)$ bo'lsa

$\iint_D F(g,\theta)gdgd\ \theta=\int_\alpha^\beta d\theta\int_{g_1(\theta)}^{g_2(\theta)}F(g;\theta)gdg$ bu erda $F(g;\theta)=f(gCos\ \theta, vgSin\ \theta)$.

Qutub koordintalar sistemasida Yakobian

$$J=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}=\begin{vmatrix}Cos\ \theta & -gSin\ \theta\\ Sin\ \theta & gCos\ \theta\end{vmatrix}=g\quad .$$

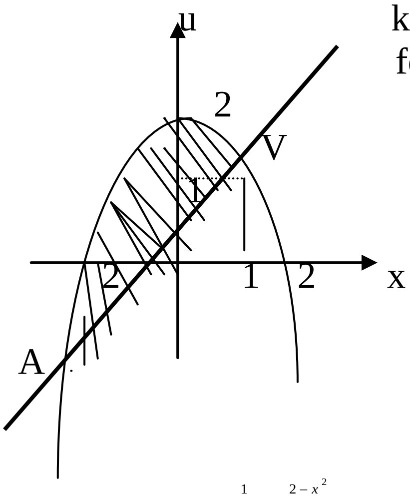
109. Mavzuga doir namunaviy misolni yechimi.

Misol- 1. $\iint_D (x-y)dxdy$ ikki karral iintegralni $y=2-x^2$, $y=2x-1$ chiziqlar bilan chegaralangan D soha bo'yicha eching.

Yechish. D integrallash sohasini topamiz, buning uchun $y=2-x^2$, $y=2x-1$ chiziqlarni kesishish nuqtalarini topamiz:

$2x-1=2-x^2$, $x^2+2x-3=0$, $D=в^2-4ac=4-4\cdot 1\cdot(-3)=16>0$,

$x_1=\frac{-2-4}{2}=-3$, $x_2=\frac{-2+4}{2}=1$ $\quad y_1=-7$, $y_2=1$ $\quad A(-3;-7)$, $B(1;1)$ kesishish nuqtalari bo'lganligidan D soha quyidagicha bo'ladi.



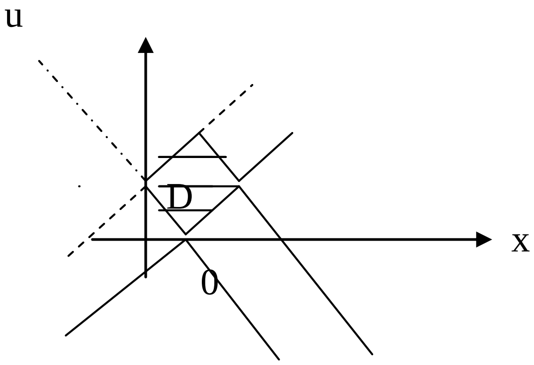
karrali integralni yuqorida keltirilgan formulalardan foydalanib hisoblaymiz:

$$\iint_D (x-y)dxdy = \int_{-3}^{1} dx \int_{2x-1}^{2-x^2}(x-y)dy = \int_{-3}^{1}\left[xy - \frac{y^2}{2}\right]\Big|_{2x-1}^{2-x^2} dx = \int_{-3}^{1}[\,(x(2-x^2) - \frac{(2-x^2)}{2} - x(2x-1) +$$

$$+\frac{(2x-1)^2}{2}]dx = \int_{-3}^{1}[-\frac{1}{2}x^4 - x^3 + 2x^2 + x - \frac{3}{2}]dx = [-\frac{x^5}{10} - \frac{1}{4}x^4 + \frac{1}{2}x^3 + \frac{1}{2}x^2 - \frac{3}{2}x]\Big|_{-3}^{1} = 4\frac{4}{15}$$

Misol -2. Quyidagi karrali integralni o'zgaruvchilarni almashtirib hisoblang. $\iint_D (x+y)^3(x-y)^2\,dxdy$ agar D soha quyidagi to'gri chiziqlar bilan chegaralangan kvadratdan iborat bo'lsa,

$$\begin{cases} x+y=1 \\ x-y=1 \end{cases} \qquad \begin{cases} x+y=3 \\ x-y=-1 \end{cases}$$



Yechish. $\begin{cases} x+y=u \\ x-y=1 \end{cases}$ belgilasak

$x=\frac{1}{2}(u+v),\quad y=\frac{1}{2}(u-v)$ bo'lganidan Yakobian

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{2},$$

Demak, $\iint_D (x+y)^3(x-y)^2\,dxdy = \frac{1}{2}\iint_{D^1} u^3v^2\,dudv$

D^1 Soha kvadratdan iborat bo'ladi. D^1 soha $\begin{matrix} 1 \le u \le 3 \\ -1 \le v \le 1 \end{matrix}$

bo'laganida $\frac{1}{2}\iint_{D^1} u^3v^2\,dudy = \frac{1}{2}\int_1^3 u^3du\int_{-1}^{1} v^2dv = \frac{1}{2}\int_1^3 u^3(\frac{1}{3}+\frac{1}{3})du = \frac{1}{3}\cdot\frac{u^4}{4}\Big|_1^3 = \frac{20}{3}$.

Misol- 3. Ikki karrali integralni qutub koordinatalar sistemasiga o'tib hisoblang, $\iint_D \sqrt{x^2+y^2}\,dxdy$, D soha $x^2+y^2 \le R^2$ doirani I-chi chorakdagi qismi.

Yechish. $x = gCos\ \theta \qquad y = gSin\ \theta$ bo'lganida

$$\iint_D \sqrt{x^2 + y^2}\,dxdy = \iint_D \sqrt{g^2 Cos^2\theta + g^2 Sin^2\theta}\, gdgd\theta = \int_0^{\pi/2} d\theta \int_0^R g^2 dg = \frac{1}{3}\int_0^{\pi/2}\left(g^3\Big|_0^R\right)d\theta = \frac{R^3}{3}\int_0^{\pi/2} d\theta =$$

$$= \frac{R^3}{3}\cdot\frac{\pi}{2} = \frac{\pi R^3}{6}.$$

110. Mustaqil yechish uchun misollar.

Quyidagi ikki karrali integrallarni takroriy integralga keltirib hisoblang.

1) $\iint_D (x+y)dxdy\ ;\quad 0 \le x \le 1,\ \ 1 \le y \le 2.$

2) $\iint_D xe^{xy}dxdy\ ;\quad 0 \le x \le 1,\ \ -1 \le y \le 0.$

3) $\iint_D (x-y)dxdy\ ;\quad x = 0,\ y = 0,\ x = y = 2$

4) $\iint_D xydxdy\ ;\quad y = 0,\ x = y,\ x = 1$

5) $\iint_D \frac{x}{y+1}dxdy\ ;\quad y = x^2 + 1,\ y = 4x,\ x = 0$

6) $\iint_D xdxdy\ ;\quad y = x^3\quad x + y = 2\ x = 0,$

7) $\iint_D \sin(x+y)dxdy\ ;\quad x = y,\ x + y = \frac{\pi}{2}$, uq0.

8) $\iint_D x^2(y-x)dxdy\ ;\quad y = x^2,\ \ x = y^2.$

9) $\iint_D x^2y^3dxdy\ ;\quad 1 - x^2 = y,\ y = 0.$

10) $\iint_D xdxdy\ ;\quad y = x^3\quad x + y = 2\ x = 0,$

11) $\iint_D ydxdy$, D- doira markazi koordinatalar boshida radiusi R bo'lgan.

Quyidagi ikki karali integrallarni D soha bo'yincha hisoblang:

12) $\iint_D \frac{dxdy}{(x+y)^2}$, $\qquad D = \{3 \le x \le 4{,}1 \le y < 2\}$

13) $\iint_D (x+2y)dxdy$, $\qquad D = \left\{\begin{matrix} y^2 - 4 \le x \le 5 \\ -3 \le y \le 3 \end{matrix}\right\}$

14) $\iint_D r^2 \sin^2\varphi drd\varphi$, $\qquad D = \left\{\begin{matrix} 0 \le r \le \cos\varphi \\ -\pi/2 \le \varphi \le \pi/2 \end{matrix}\right\}$

Integrallash sohasini chizib, integrallash tartibini o'zgartiring:

15) $I = \int_{-6}^{2} dy \int_{(y^2-4)/4}^{2-y} f(x,y)dx$

16) $I = \int_0^3 dx \int_{x/3}^{2x} f(x,y)dy$

17) $I = \int_1^2 dx \int_{x/3}^{x} f(x,y)dy$

18) $I = \int_0^1 dx \int_{x^4}^{x^2} f(x,y)dy$

19) $I = \int_0^{c\sqrt{2}} dy \int_{y^2/(2c^2)}^{\sqrt{3-\frac{y^2}{c^2}}} f(x,y)dx$

20) $I = \int_0^1 dx \int_0^x (x+y^2)dy + \int_1^2 dx \int_0^{2-x}(x+y^2)dy$

21) $I = \int_0^1 dx \int_{x^2}^{x} xy^2dy$

111. Karrali integrallarni tatbiqlari.
Mavzuga doir namunaviy misolni yechimi.

Misol-1. $x^2+y^2=9$, $x^2+z^2=9$ sirtlar bilan chegaralangan jismni birinchi oktantada joylashgan qismini hajmini hisoblang.
Yechish. Ikkita doiraviy silindr bilan chegaralangan jismni xajmini birinchi oktanta bo'yicha hisoblash talab etilgan.

$$V=\int_0^3 dx\int_0^{\sqrt{9-x^2}}\sqrt{9-x^2}\,dy=\int_0^3\sqrt{9-x^2}\,dx\int_0^{\sqrt{9-x^2}}dy=\int_0^3\sqrt{9-x^2}\left(y\Big|_0^{\sqrt{9-x^2}}\right)dx=$$

$$=\int_0^3(9-x^2)dx=\left(9x-\frac{x^3}{3}\right)\Big|_0^3=27-\frac{27}{3}=18$$

kub birlik.

112. Mustaqil yechish uchun misollar.

Quyidagi chiziqlar bilan chegaralangan yuzani ikki karrali integral yordamida hisoblang:

1) $y^2=10x+25$, $y^2=-6x+9$ parabolalar bilan chegaralangan figura yuzasini hisoblang.

2) $\int_0^1 dx\int_0^{1-x}(1-x-y)dy$, ikki karrali integral bilan berilgan jismning hajmini hisoblang.

3) Birinchi oktantada silindr va tekisliklar bilan chegaralangan jismning hajmini hisoblang $x^2+z^2=a^2$ $y=0$, $z=0, y=x$

4) V silindr $x^2+y^2=2ax$, paraboloid $x^2+z^2=2az$, va $z=0$, tekislik bilan chegaralangan jismni V hajmini hisoblang.

4) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ tekislikni birinchi oktantadagi qismini yuzasini hisobalang $(x>0, y>0, z>0)$.

5) $x^2+y^2+z^2=a^2$, sharni $\frac{x^2}{a^2}+\frac{y^2}{в^2}=1$ $(в<a)$ eliptik silindr bilan kesilgan qismini hajmini toping

6) $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\le 1$. Elipsoidning hajmini hisoblang

113. Ikki karrali integralni mexanika masalalarini yechishga qo'llanilishi.

x o y tekislikda, D sohaga ega bo'lgan plastinkani $\gamma=\gamma(x;y)$ o'zgaruvchan zichli sirt massasini hisoblash formulasi

$$M=\iint_D y\gamma(x;y)dxdy\ ,\ M_y=\iint_D x\gamma(x;y)dxdy$$

Xususan plastinkani zichligi bir jinsli bo'lsa $\gamma=Const$ bo'ladi.
Plastinkani og'irlik markazini koordinatalarini quyidagicha formulalar bilan hisoblash mumkin.

$$\bar{x}=\frac{M_x}{M},\quad \bar{y}=\frac{M_y}{M}$$

Plastinkani bir jinsli bo'lganda og'irlik markazini koordinata-lari,

$$\bar{x}=\frac{\iint_D xdxdy}{S}\ ,\ \bar{y}=\frac{\iint_D ydxdy}{S}$$

bu erda S D sohani yuzasi.
Ox, Ou o’qlariga nisbatan inersiya momentlari I_x, I_y

$$I_x = \iint_D y^2 \gamma(x;y)dxdy \quad , \quad I_y = \iint_D x^2 \gamma(x;y)dxdy$$

Koordinatalar boshiga nisbatan inersiya momenti

$$I_0 = \iint_D (x^2 + y^2)\gamma(x;y)dxdy = I_x + I_y$$

114. Mavzuga doir namunaviy misolni yechimi.

Misol-1. Quyidagi parabolalar bilan chegaralangan figurani $y^2 = x, x^2 = y$ ogirlik markazini toping.

Yechish.Parabolalar bilan chegaralangan figurani ogirlik markazini koordinatalarini topish uchun, xosil bo’lgan figurani m massasini va statik momentlarini m_x, m_y hisoblaymiz:

$$m = \iint_{Д} dxdy = \int_0^1 dx \int_{x^2}^{\sqrt{x}} dy = \int_0^1 \left[\sqrt{x} - x^2\right]dx = \left(\frac{x^{3/2}}{\frac{3}{2}} - \frac{x^3}{3} \right)\Bigg|_0^{\,\substack{0\\1}} = \frac{2^{x^2}}{3} - \frac{1}{3} = \frac{1}{3},$$

$$m_x = \iint_{Д} ydxdy = \int_0^1 dx \int_{x^2}^{\sqrt{x}} ydy = \frac{3}{20}, \qquad m_y = \iint_{Д} xdxdy = \int_0^1 xdx \int_{x^2}^{\sqrt{x}} dy = \frac{3}{20}.$$

Natijada figurani ogirlik markazini kordinatalari:

$$\bar{x} = \frac{m_y}{m} = \frac{\frac{3}{20}}{\frac{1}{3}} = \frac{9}{20}, \ \bar{y} = \frac{m_x}{m} = \frac{\frac{3}{20}}{\frac{1}{3}} = \frac{9}{20}$$

Misol-2. Ox o’qiga nisbatan qutub koordinatasida $S = a(1 + \cos\theta)$ bilan chegaralangan figurani inersiya momentini hisoblang.

Yechish. Ma’lumki, Dekart kordinatalari sistemasida inersiya momenti quyidagi formula bilan hisoblanadi $J_x = \iint_{Д} y^2 dxdy$

Bu formulada qutib koordinatalar sistemasiga o’tsak

$$J_x = \iint_{Д} g^2 \cdot Sin^2\theta \cdot gdgd\theta = \int_0^{2\pi} Sin^2\theta d\theta \int_o^{a(1+\cos\theta)} g^3 dg = \frac{1}{4}\int_o^{2\pi} \sin^2\theta \cdot g^4 \Bigg|_o^{a(1+\cos\theta)} d\theta =$$

$$= \frac{1}{4}a^4 \int_0^{2\pi} \sin^2\theta(1 + \cos\theta)^4 d\theta = \frac{a^4}{4}\int_0^{2\pi} \sin^2\theta(1 + 4Cos\theta + 6\cos^2\theta + 4\cos^3\theta + \cos^4\theta)d\theta = \frac{21}{32}\Pi a^4$$

115. Mustaqil yechish uchun misollar.

1) $D = \left\{ \frac{x^2}{a^2} + \frac{y^2}{b^2} =\leq 1, y > 0 \right\}$, Ellipsning yuqori yarim qismini, zichligi $g \equiv 1$ bo’lganda ogirlik markazini toping.

2) $\Omega = \{x^2 + y^2 + z^2 \leq a^2, z \geq 0\}$

Yarim sharni $\rho(x,y,z)=c\sqrt{x^2+y^2+z^2}$. bo'lganda, ogirlik markazini toping.
3) $x=a(t-\sin t),\quad y=a(1-\cos t)\quad (0\le t\le\pi)$. Siklonidaning yarim arkini ogirlik markazini toping.
4) Bir jinsli figuraning og'irlik markazini toping, $S=\{0\le y\le 4-x^2, -2\le x\le 2\}$
5) $x^2+y^2+z^2=a^2$ Sferani birinchi oktantadagi qismini , $(x\le 0, y\le 0, z\le 0)$, $g=\sqrt{x^2+y^2}$ bo'lganda massasini hisoblang.

Quyidagi egri chiziqlar bilan chegaralangan figurani ogirlik markazlarini va koordinatalar o'qlariga nisbatan inersiya momentlarini hisoblang:
6) $y=2x^3\qquad y^2=2x$,
7) $x^2+y^2=13$ xy=6, x>0,
8) $x^2+y^2+z^2=R^2, z\ge 0$
9) $z=1-x^2-y^2,\quad z\ge 0$ Paraboloid.
10) $x=0,\ y=0,\ z=0,\ x=1,\ y=1,\ x+y+z=3$ tekisliklar bilan chegaralangan prizma.
11) $2az=x^2+y^2\quad x^2+y^2+z^2=3a^2$ paraboloid va sharni umumiy qismi.
12) $x+2y-z=1$ tekislik va koordinatalar tekisliklari bilan chegaralangan tetraedr.

116. **Egri chiziqli integrallar.**

L soha bilan chegaralangan egri chiziqli integral $\int_L Pdx+Qdy$.

Prametrik tenglama bilan berilgan bo'lsa, egri chiziqli integral quyidagicha hisoblanadi:

$$\int_L Pdx+Qdy=\int_{t_0}^{T}[M(\varphi(t);\psi(t))\varphi'(t)+N(\varphi(t);\psi(t))\psi'(t)]dt\ ,\quad x=\varphi(t),\quad y=\psi(t),$$

Agar $P(x;y),\qquad Q(x;y)$ ikki o'zgaruvchili funksiyalar $\frac{\partial P}{\partial x},\frac{\partial Q}{\partial y}$ uzluksiz xususiy hosilalarga D yopiq sohada ega bo'lsa (L soha chegarasida ham) egri chiziqli integral uchun quyidagi Grin formu-lasi o'rinlidir

$$\int_L Pdx+Qdy=\iint_D\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dxdy$$

Bu erda egri chiziqli integral uchun $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$ tenglikni bajarilsa, berilgan egri chiziqli integralni integrallash yo'liga bogliq bo'lmaydi.

Agar $P=P(x;y;z),\quad Q=Q(x;y;z),\quad R=R(x;y;z)$ funksiyalar L yopiq kontur bilan chegaralangan S sirtda uzluksiz xususiy xosilalarga ega bo'lsa, u xolda egri chiziqli integralni hisoblashda quyidagi Ostrogradskiy – Gauss va Stoks formulalaridan foydalaniladi:

$$\iint_{bl}(PCos\ \alpha+QCos\ \beta+RCos\ \gamma)ds=\iiint_{(V)}\left(\frac{\partial P}{\partial x}+\frac{\partial O}{\partial y}+\frac{\partial R}{\partial z}\right)dxdydz$$

$$\int_L Pdx+Qdy+Rdz=\iint_s\left[\left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}\right)Cos\ \alpha+\left(\frac{\partial Q}{\partial z}-\frac{\partial R}{\partial x}\right)Cos\ \beta+\left(\frac{\partial Q}{\partial x}-\frac{\partial Q}{\partial y}\right)\cos\gamma\right]ds$$

Bu erda $\cos\alpha$, $Cos\ \gamma$, $Cos\ \lambda$ S sirt normalini yo'naltiruvchi kosinuslari.

Agar $\overline{F} = P(x;y;z)\overline{\iota} + Q(x;y;z)\overline{j} + R(x;y;z)\overline{k}$ vektor funksiya bo'lsa, Bu erda P,Q,R mos ravishda $\overline{F}$ vektorni o'qlardagi proeksiyalari. Vektor maydoning divergensiyasi quyidagi formula bilan

$$div\,\overline{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

vektor maydonining rotori ushbu formula bilan xisoblanadi.

$$rot\,\overline{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial x}\right)\overline{r} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\overline{i} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\overline{k}$$

117. Mavzuga doir namunaviy misolni yechimi.

Misol-1. $\overline{F}(x;\ y) = (2xy + y)\overline{i} + (3x^2 - 1)\overline{j}$ o'zgaruvchan kuch bilan $y = 3x^2 + x$ parabolani $M(1;\ 4)$ va $N(2;\ 14)$ nuqtalarini birlashtiruvchi egri chiziq yoyi bo'yicha bajarilgan ishni toping.

Yechish. $A = \int_L (2xy + y)dx + (3x^2 - 1)dy$ egri chiziqli integralni hisoblash talab etiladi.

$1 \le x \le 2$, $y = 3x^2 + x$, $dy = d(3x^2 + x) = (6x + 1)dx$ bo'lganligidan, berilgan integral quyidagicha bo'ladi.

$$A = \int_1^2 \left[2x(3x^2 + x) + (3x^2 + x) + (3x^2 - 1)(6x + 1)\right]dx = \int_1^2 \left[24x^3 + 8x^2 - 5x - 1\right]dx = $$
$$= (6x^4 + \frac{8}{3}x^3 - \frac{5}{2}x^2 - x)\Big/_1^2 = 90\ \frac{1}{6}$$

Misol-2. Quyidagi $M(1;\ 1)$ va $N(2;\ 3)$ nuqtalarni birlashtiruvchi kontur bo'yicha olingan egri chiziqli interalni hisoblang va uni integrallash yo'liga bogliq emasligini ko'rsating.

$$G = \int_L (x^2 + 3xy)dx + (\frac{3}{2}x^2 + y)dy$$

Yechish. $P(x;\ y) = x^2 + 3xy$, $Q(x;\ y) = \frac{3}{2}x^2 + y$ uchun $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ tenglikni bajarilishidan $\frac{\partial P}{\partial y} = 3x = \frac{\partial Q}{\partial Q} = 3x$ integrallash yo'liga bogliq emasligi kelib chiqadi.

$1 \le x \le 2$ oraliqda o'zgarganida $y = 1$, $dy = 0$, $1 \le y \le 3$ o'zgarganida $x = 2$, $dx = 0$ bo'lganlidan

$$G = \int_1^2 (x^2 + 3x)dx + \int_1^3 (6 + y)dy = (\frac{x^3}{3} + \frac{3}{2}x^2)\Big/_1^2 + (6y + \frac{y^2}{2})\Big/_1^3\ 22\ \frac{5}{6}.$$

Misol-3. $\int_\Gamma xyds$ G- A = (-1,0), B = (1,0), C = (0;1). Uchburchak bo'yicha egri chiziqli integralni hisoblang. AV: tenglamasi $y = 0$, BC : tenglamasi $x + y = 1$, AC : tenglamasi $y - x = 1$

Echish.

$$\int_{AB} xyds = 0; \quad \int_{AB} xyds = \int_{CB} xyds = \int_0^1 x(1-x)\sqrt{2}dx = \sqrt{2}/6;$$

$$\int_{CA} xyds = \int_{AC} xyds = \int_{-1}^0 x(1+x)\sqrt{2}dx = -\sqrt{2}/6; \quad \int_{\Gamma} xyds = \int_{AB} xyds + \int_{BC} xyds + \int_{CA} xyds = 0\frac{\sqrt{2}}{6} - \frac{\sqrt{2}}{6} = 0.$$

118. Mustaqil yechish uchun misollar.

$M(a;в)$ va $N(c;d)$ nuqtalarni birlashtiruvchi berilgan egri chiziqli kontur bo'yicha quyidagi egri chiziqli integrallarni integrallash yo'liga bogliq emasligini ko'rsatib hisoblang.

1. $F = (3x + y^2)\bar{i} + Q(5xy + 1)\bar{j}$, L - $y = x^2 + 2x$; $M(0;0)$, $N(1;3)$.

2. $\overline{F} = (8x^2 + y)\bar{i} - (3x^2 y + x)\bar{j}$, L - $y = 2x^2 + 1x$; $M(0;1)$, $N(2;9)$.

3. $\overline{F} = (3xy + x^2)\bar{i} + (8y - x)\bar{j}$, L - $y = x^3$; $M(0;0)$, $N(2;8)$.

4. $\overline{F} = (8x^2 y + x)\bar{i} + (-2y + 1)\bar{j}$; L - $y = 7x^3 + 2x$; $M(0;0)$, $N(2;32)$.

5. $\overline{F} = (5x^2 - y)\bar{i} - (xy + 1)\bar{j}$; L - $y = x^2 + x$ $M(1;2)$, $N(3;5)$.

6. $\overline{F} = (-xy + 3)\bar{i} + (2x^2 - y)\bar{j}$; L - $y = 3x^2 + x$; $M(1;4)$, $N(3;30)$.
7. $\overline{F} = (x^2 y + x)\bar{i} + (3x^3 - y)\bar{j}$; L - $y = x^3 + 1$; $M(0;1)$, $N(1;2)$.

8. $\overline{F} = (-3y + x^2)\bar{i} + (2xy + 1)\bar{j}$; L - $y = x^3 + 2$; $M(1;3)$, $N(2;10)$.

9. $\overline{F} = (xy + x^3)\bar{i} + (5x^2 + y)\bar{j}$; L - $y = x^2 + x$; $M(1;2)$, $N(3;12)$.

10. $\overline{F} = (2x^2 y + 1)\bar{i} + (3x + y)\bar{j}$; L - $y = 3x^2 + 2$; $M(2;14)$, $N(3;29)$.

Quyidagi misollarda $A(1;2)$ $B(3;5)$ nuqtalarni birlashtirivchi AB kesma bo'yincha egri chiziqli integrallarni hisoblang:

11. $\int (x^3 - 2y)dx - (2x - 5)dy$; 12. $\int (1 + 2xy)dx + (x^2 + y)dy$;

13. $\int (x^2 - y)dx - (x - 3y)dy$; 14. $\int (3 + xy)dx + (\frac{1}{2}x^2 + 2y)dy$;

15. $\int (5x - 2y)dx - (2x - y)dy$; 16. $\int (3x^2 - y)dx - (x + 3y)dy$;

17. $\int (4xy + 3)dx + (2x^2 - y)dy$; 18. $\int (4 + xy^2)dx + (x^2 y + 2y)dy$;

19. $\int (2xy + 8)dx + (x^2 + 2y)dy$; 20. $\int (5x^3 - 3y)dx - (y^2 - 3x)dy$

21) $\int_{\Gamma} \frac{ds}{x - y}$, G- $A = (0,-2)$ $B = (4,0)$ Nuqtalarni birlashtirivchi AB kesma bo'yincha egri chiziqli integralni hisoblang.

22) $\int_{\Gamma} xyds$, G- to'gri to'rtburchakni uchlarini koordinatalari

$A=(0,0), B=(4,0), C=(4,2), D=(0,2)$ bo'lsa, to'gri to'rtburchak konturi bo'yicha egri chiziqli integralni hisoblang.

23) $\int\limits_{\Gamma} xyds$ G- Ellipsni

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, x\ge 0, y\ge 0.$ I- chorakdagi qismi

24) $\int\limits_{\Gamma} xyds$ G- $y=2\sqrt{x}$ $0\le x\le 1.$ bo'yicha egri chiziqli integralni hisoblang.

25) $\int\limits_{\Gamma} (x-y)ds$ G- $x^2+y^2=2ax$ aylana bo'yicha egri chiziqli integralni hisoblang.

26) $x=a(t-\sin t), \quad y=a(1-\cos t) \quad (0\le t\le \pi)$. Siklondaning yarim arkini ogirlik markazini toping.

27) z o'qiga nisbatan Γ vint chiziqni birinchi sirtmogini inersiya momentini hisoblang $y=0$, $y=0$ tekisliklarga nisbatan.

$x=a\cos t, y=a\sin t, z=bt \, (0\le t\le 2\pi)$

28) $\int\limits_{\Gamma} (y^2dx+x^2dy),$ Ikkinchi tur egri chiziqli integralni Γ - ellipsni

$x=aCost, \ y=вSint$ yuqori yarmida soat strelkasi bo'yicha hisoblang.

29) $\int\limits_{\Gamma} xdy$

G- to'gri chiziq $x+y=2$ ox, oy o'qlar bilan hosil qilgan uchburchak konturi bo'yicha soat strelkasiga teskari yo'nalishda hisoblang

30) $\int\limits_{\Gamma} (y^2dx+x^2dy),$

a) G- tekislikning yuqori qismida joylashgan $y=4-x^2,$ yoyida soat strelkasi bo'yicha olingan holda b) G- (-2,0), (0,4), (2,0); siniq chiziq bo'yicha egri chiziqli integralni hisoblang.

31) $\int\limits_{\Gamma} (xdy+ydx),$

a) G- $y=\sqrt{x};$ parabola yoyi b) G- $y=x^2$ parabola yoyi va (0,0) (1,1) nuqtalarni birlashtiruvchi kesmada soat strelkasi bo'yincha olingan egri chiziqli integralni hisoblang.

Grin formulasi bilan quyidagi integrallarni hisoblang.

32) $\int\limits_{\Gamma} ((1-x^2)ydx+x(1+y^2)dy).$ Grin formulasi bilan Γ -yopiq kontr bo'yicha olingan integral shu kontur bilan chegaralangan Ω bo'yicha almashtiring.

33) $\int\limits_{\Gamma} ((e^{xy}+2x\cos y)dx+(e^{xy}-x^2\sin y)dy).$

34) $\int\limits_{\Gamma} ((xy+x+y)dx+x(xy+x-y)dy)$, Γ - ellips $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ bo'yicha hisoblang.

35) $I_1=\int\limits_{\Gamma} ((x+y)^2dx-(x-y)^2dy)$ Γ_1 $y=x^2$ parabola yoyi,

$I_2 = \int\limits_{\Gamma_2} ((x+y)^2 dx - (x-y)^2 dy)$ $\Gamma_1 - (0;0)$ va $(1;1)$ nuqtalarni birlashtiruvchi kesma bo'lsa, $I_1 - I_2$ ni hisoblang.

36) $\int\limits_{\Gamma} (-x^2 ydx + xy^2 dy)$, Γ - aylanada $x^2 + y^2 = R^2$ musbat yo'nalish bo'yicha hisoblang.

119. Sirt integrallari
Birinchi va ikkinchi tur sirt integrallari.

Agar tenglamasi parametrik shaklda $x = x(u,v)$, $y = y(u,v)$, $z = z(u,v)$ yoki vektor shiklida $r(u,v) = x(u,v)i + y(u,v)j + z(u,v)k$,
$(u,v) \in \Omega$ berilgan funksiya $x, y, z \in \Omega$ sohada uzluksiz xususiy hosilalarga ega bo'lsa, S- sirt yuzasi quyidagi ikki karali integral yordamida hisoblanadi.

$$S = \iint\limits_{\Omega} \sqrt{EG - F^2}\, dudv = \iint\limits_{\Omega} \sqrt{|r_u|^2 |r_v|^2 - (r_u, r_v)^2}\, dudv ,$$

Bu erda $E = |r_u|^2 = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2$ $G = |r_v|^2 = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2$

$F = (r_u, r_v) = \frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial z}{\partial u}\frac{\partial z}{\partial v}$, Ma'lumki

$$S = \iint\limits_{\Omega} |r_u x r_v|\, dudv , \quad r_u x r_v = \begin{vmatrix} i & j & k \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \end{vmatrix}, \quad |r_u x r_v| = \sqrt{\left[\frac{D(y,z)}{D(u,v)}\right]^2 + \left[\frac{D(z,x)}{D(u,v)}\right]^2 + \left[\frac{D(x,y)}{D(u,v)}\right]^2} .$$

Bu formulalar bilan yuzalarni hisoblash r_u va r_v vektorlar $(r_u, r_v) = 0$, ortagonal bo'lgan holda qo'lay hisoblanadi.

$$S = \iint\limits_{\Omega} |r_u| \cdot |r_u|\, dudv .$$

Stoks formulasi vektor sirkulyasiyasi va rot $\bar{a}$ vektorlar oqimini Γ - konturni S sirti orasida boglinishni ifodalovchi formula hisoblanadi.

$$\int\limits_{\Gamma} (adl) = \int\limits_{\Gamma} (Pdx + Qdy + Rdz) =$$

$$= \int\limits_{S} \begin{vmatrix} \cos\alpha & \cos\beta & \cos\gamma \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} dS = \int\limits_{S} (n, rota)\, dS$$

120. Mavzuga doir namunaviy misolni yechimi.

Misol-1. $x = u\cos v$, $y = u\sin v$, $z = 4v$
$(0 \le u \le 3, 0 \le v \le \pi)$. Sirt yuzasini hisoblang.

Yechish.

$r_u = (\cos v, \sin v\, 0,)$, $r_v = (-u\sin v,\ u\cos v, 4)$;

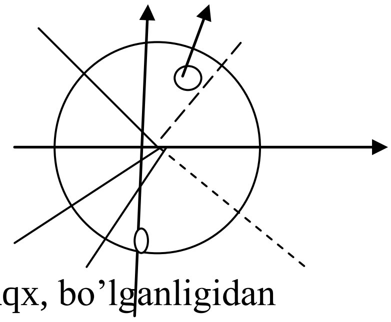
$(r_u, r_v) = 0$, $|r_u|^2 = 1$, $|r_v| = 16 + u^2$. Formulaga asosan

$$S = \iint_{\Omega} 1\cdot\sqrt{16+u^2}\,dudv = \int_0^{\pi} dv \int_0^3 \sqrt{16+u^2}\,du = \pi\int_0^3 \sqrt{16+u^2}\,du =$$

$$\pi\left[\frac{u}{2}\sqrt{16+u^2} + 81\ln(u+\sqrt{16+u^2})\right]\Bigg|_0^3 = \frac{\pi}{2}(15+16\ln 2)$$

Misol-2.Stoks formulasi bilan egri chiziqli integrallarni hisoblang.

$\int_{\Gamma}(ydx + zdy + xdz)$, Γ - aylana $x^2+y^2+z^2=b^2$, sferani $x+y+z=0$, tekislik kesganda hosil bo'lgan aylanada soat strelkasiga teskari yo'nalishda olingan kontur. Echish:



$x+y+z=0$, $x^2+y^2+z^2=b^2$,

$\cos\alpha = \cos\beta = \cos\gamma = 1/\sqrt{3}$

$F(x,y,z) \equiv x+y+z=0$,

$F'_x = F'_y = F'_z = 1$, $|gradF| = \sqrt{3}$).

$\overline{a}$ vektor uchun bu holda Rqu, Qqz, Rqx, bo'lganligidan

$$\int_{\Gamma}(ydx + zdy + xdz) = -\int_S(\cos\alpha + \cos\beta + \cos\gamma)dS = -\frac{3}{\sqrt{3}}\int_S dS$$

Bu yerda S- b-radiusli aylana, $x+y+z=0$ tekislikda yotuvchi

$$\int_{\Gamma}(ydx + zdy + xdz) = -\frac{3}{\sqrt{3}}\pi b^2 = -\sqrt{3}\pi b^2 .$$

Misol-3. Quyidagi vektor maydonning divergensiyasini toping.

$\overline{F} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$

Yechish: Divergensiyani xisoblash formulasiga asosan

$$\frac{dP}{dx} = 2x,\qquad \frac{dQ}{dy} = 2y,\qquad \frac{dR}{dz} = 2z \qquad div\,\overline{F} = 2x + 2y + 2z = 2(x+y+z).$$

Misol-4. $\overline{F} = -wy\,\overline{i} + wxj$ vektorni $x = aCost$, $y = aS\text{int}$ aylanani musbat yo'nalishi bo'yicha olingan sirkulyasiyasini xisoblang.

Yechish: Vektorni sirkulyasiyasi xisoblash formulasidan

$$\int_{Д}\overline{F}dr = \int_L -wydx + wxdy = w\int_0^{2П}(a^2\sin^2 t + a^2Cos^2 t)dt = 2Пa^2 w$$

121. Mustaqil yechish uchun misollar.

1) quyidagi II- tur sirt integrallarini Gauss Ostrogradskiy formulasi yordamida $\iiint_G divadG = \int_S(a,n)dS$, Agar S sirtni n- tashqi normalining birlik vektori bo'lsa, G – soha bo'yicha shakl almashtring.

2.) $\int_S(xydxdy + yzdydz + xzdxdz)$; 3) $\int_S(x^2dydz + y^2dxdz + z^2dxdy)$;

4) $\int\limits_{S} \frac{x\cos\alpha + y\cos\beta + z\cos\gamma}{\sqrt{x^2 + y^2 + z^2}} dS$ 5) $\int\limits_{S} \left(\frac{\partial u}{\partial x}\cos\alpha + \frac{\partial u}{\partial y}\cos\beta + \frac{\partial u}{\partial z}\cos\gamma \right) dS$

6) $u = x^2 + 2y^2 + 3z^2 + xy - 6z.$ Funksiya gradientini hisoblang.

7) $u = x^3 + y^3 + z^3 - 3xyz.$ Funksiya gradientini hisoblang.

8) $a = \{x, y, z\},$ vektor gotorini (x, y, z) nuqtada hisoblang.

9) $a = \{z + y, x, y\},$ vektor gotorini hisoblang.

10) Quyidagi vektorlarni a) $a = x^2 i + y^2 j + z^2 k;$ b) $a = yzi + xzj + xyk;$
v) $a = zi + xzj + xyk$ R_3 – fazoda potinsialga egami?

11) $a = (x^2, y^2, z^2)$ Vektorni divergensiyasini hisoblang.

12) $a = f(r)\frac{r}{r},$ Vektorni divergensiyasini hisoblang.
$r = |r|,$ $r = xi + yi + zk,$ f- differensiallanuvchi funksiya

13) $u = x^2 + y^2 + z^2$ bo’lsa , $div(grad(u)) = ?$

14) Ctoks formulasi bilan sirkulyasiyani hisoblang.
$\int\limits_{\Gamma} ((y - z)dx + (z - x)dy + (x - y)dz),$
$x^2 + y^2 = 1,$ silindirni $x + z = 1$ tekislik bilan kesishdan hosil bo’lgan, ellips bo’ylab soat strelkasiga teskari yo’nalishda olingan kontur, z o’qini musbat yo’nalishi bo’yincha qaralganda.

15) $\int\limits_{\Gamma} ((y + z)dx + (z + x)dy + (x + y)dz),$
Stoks formula bilan hisoblang. Agar
G- aylana, $x^2 + y^2 + z^2 = b^2,$ $x + y + z = 0$ bo’lsa.

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