

Orginal hisob va uning bazi tadbiqlari

Laplas almashtirishlari va hossalari:

Agar haqiqiy o'zgaruvchi f(t) funksita uchun quyidagi shartlar bajarilsa:

- 1. $t < 0$ da $f(t) = 0$
- 2. shunday $M > 0$ va $S > 0$ o'zgarmas sonlar mavjud bo'lsaki va $|f(t)| < M \cdot e^{S \cdot t}$ bo'lsa
- 3. f(t) bo'lakli-uzluksiz yani chekli intervalda chekli sondagi birinchi tur uzilish nuqtalariga ega bo'lsa, u vaqtda quyidagi hosmas integralga Laplas almashtirishlari deyiladi va bunday yoziladi:

$$F(P) = \int_0^{\infty} e^{-P \cdot t} f(t) dt$$

F(P) funksiya f(t) funksiyaning laplas tasviri deb ataladi. f(t) funksiya esa orginal deb aytiladi

Asosiy hossalari:

- 1. Tasvirning chiziqlilik hossasi:
Ixtiyoriy c_k $k=1,2,3..n$ o'zgarmas sonlar uchun quyidagi tenglik o'rinni:

$$\sum_{k=1}^n C_k f(t)_k = \sum_{k=1}^n C_k F_k(P)$$

- 2. Orginal argumentni musbat songa ko'paytirish xossasi. O'xshashlik teoremasi:
Har qanday o'zgarmas musbat $a > 0$ son uchun quyidagi tenglik o'rinni

$$f(at) = \left(\frac{1}{a} F\left(\frac{P}{a}\right) \right)$$

- 3. Orginal argumentni musbat $\tau > 0$ vaqtga kechikish xossasi. Kechikish teoremasi:
Agar orginal $f(t - \tau)$ ga kechiksa tasvir $e^{-P \cdot \tau}$ ga ko'paytiriladi:

$$f(t - \tau) = \left(e^{-P \cdot \tau} F(P) \right)$$

- 4. Tasvirning kechikish hossasi. Siljish teoremasi: Agar orginal $e^{\alpha t}$ ga ko'paytirilsa α ga kechikadi:
 $\left(e^{\alpha t} f(t) \right) = F(P - \alpha)$

- 5. Orginalni difrensiallash hossasi. Agar f(t) va uning hosilalari $f^{(k)}(t)$ orginal $k=1,2,3..n$ bo'lsa $f^{(k)}(t) = P^k F(P) - P^{k-1} f(0) - P^{k-2} f'(0) - - f^{(k-1)}(0)$ bo'ladi. Hususan:

6. Orginalni integrallash hossasi

$$f(t) = P \cdot F(P) - f(0)$$
$$\int_0^t f(\tau) d\tau + \frac{1}{P} F(P)$$

- 7. Tasvirni diffrensiallash hossasi. Tasvirni argument P bo'yicha diffrensiallasak orginal -t ga ko'paytiriladi.

$$F^{(n)}(P) = (-1)^n t^n f(t)$$
$$\int_P^{\infty} F(p) dp + \frac{1}{t} f(t)$$

- 8. Tasvirni integrallash hossasi.

9. Tasvirlarni ko'paytirish hossasi. Borel teoremasi.

Agar $f(t) = F(P)$ va $g(t) = G(P)$ bo'lsa $F(P) \cdot G(P) = f \times g + \int_0^t f(\tau) g(t - \tau) d\tau$

$f \times g$ simvol f(t) va g(t) funksiyalarni kompozitsiyasi (o'ramasi) deb ataladi.

- 10. Dramel integrali. Agar $f(t) = F(P)$ va $g(t) = G(P)$ bo'lsa $P \cdot F(P) \cdot G(P) = f(0) g(t) + (f \times g)(t)$ bo'ladi. Bu yerda $(f \times g)(t) = \int_0^t \frac{d}{dt} f(t - \tau) g(\tau) d\tau$

F(P)	f(t)	F(P)	f(t)
$\frac{p}{p^2 + \omega^2}$	$\cos \omega t$	$\frac{1}{p}$	1
$\frac{p}{p^2 - \omega^2}$	$\ch \omega t$	$\frac{1}{e + \alpha}$	$e^{-\alpha t}$
$\frac{1}{p(p + \alpha)^2}$	$\frac{1 - (1 + \alpha t)e^{-\alpha t}}{\alpha^2}$	$\frac{1}{p - \ln a}$	a^t
$\frac{1}{p^2(p + \alpha)}$	$\frac{e^{-\alpha t} + \alpha t - 1}{\alpha^2}$	$\frac{1}{p^2}$	t
$\frac{1}{p(p^2 + \omega^2)}$	$\frac{1}{\omega^2}(1 - \cos \omega t)$	$\frac{n!}{p^{n+1}}$	t^n
$\frac{2}{p(p^2 + 4)}$	$\sin(t)^2$	$\frac{1}{(p + \alpha)^{n+1}}$	$\frac{t^n}{n!} e^{-\alpha t}$
$\frac{p^2 + 2}{p(p^2 + 4)}$	$\cos(t)^2$	$\frac{\omega}{p^2 + \omega^2}$	$\sin \omega t$
$\frac{1}{p^2(p^2 + \omega^2)}$	$\frac{1}{\omega^3}(\omega t - \sin \omega t)$	$\frac{\omega}{p^2 - \omega^2}$	$\sh \omega t$
$\frac{1}{p^n}$	$\frac{t^{n-1}}{(n - 1)!}$	$\frac{1}{(p + \alpha)(p + \beta)}$	$\frac{(e^{-\alpha t} - e^{-\beta t})}{\beta - \alpha}$
$\arctg \cdot \frac{\omega}{p}$	$\frac{1}{t} \sin \omega t$	$\frac{p + k}{(p + \alpha)^2}$	$[(k - \alpha)t + 1]e^{-\alpha t}$
$\ln\left(\frac{p - \alpha}{p - \beta}\right)$	$\frac{1}{t}(e^{\beta t} - e^{\alpha t})$	$\frac{\omega}{(p + \alpha)^2 + \omega^2}$	$e^{-\alpha t} \sin \omega t$
$\ln\left(\frac{p^2 + \alpha^2}{p^2}\right)$	$\frac{2}{t}(1 - \cos \alpha t)$	$\frac{p + \alpha}{(p + \alpha)^2 + \omega^2}$	$e^{-\alpha t} \cos \omega t$
$\frac{p}{(p^2 + \omega^2)^2}$	$\frac{t}{2\omega} \sin \omega t$	$\frac{1}{p(p + \alpha)(p + \beta)}$	$\frac{1}{\alpha\beta} - \frac{\beta e^{-\alpha t} - \alpha e^{-\beta t}}{\alpha\beta(\beta - \alpha)}$
		$\frac{p + k}{(p + \alpha)(p + \beta)}$	$\frac{(k - \alpha)e^{-\alpha t} - (k - \beta)e^{-\beta t}}{\beta - \alpha}$
		$\frac{p + k}{p^2 + \omega^2}$	$\frac{1}{\omega} \cdot \sqrt{k^2 + \omega^2} \cdot \sin(\omega t + \phi) - t$

