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“OLIY MATEMATIKA” KAFEDRASI

Oliy matematika fanidan TAJRIBA ISHLARI TO'PLAMI

Navoiy - 2007 y.

Sizga taqdim etilayotgan ushbu tajriba ishlar to'plami "Oliy matematika" fanining texnika oliy o'quv yurtlari bakalavr tayyorlash namunaviy dasturi asosida tuzilgan (O'z.R.O va O'MTV tomonidan 27 may 2002 yilda tasdiqlangan № Q 1126). Ushbu tajriba ishlar to'plami oliy matematika fanidan tajriba ishlarini bajarishda uslubiy yordam ko'rsatadi degan umiddamiz.

Tajriba ishlar to'plami barcha texnik yo'nalishlarda ta'lim olayotgan talabalar uchun mo'ljallangan.

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Ushbu tajriba ishlar to'plami NDKI «Oliy matematika» kafedrasi yig'ilishida 17.01.2007 yil (№ 15 sonli bayonnoma) hamda KMF kengashida 29.01.2007 yil (№ 3 sonli bayonnoma), NDKI uslubiy kengashida 30.03.2007 yil (№ 6 sonli bayonnoma) muhokama etilib chop etishga va o'quv jarayonida foydalanish uchun tavsiya qilingan.

Ushbu tajriba ishlari 1991 yilda chop etilgan «Oliy matematika kursi buyicha laboratoriya ishlari» uslubiy qo'llanmasi asosida yangi o'quv dasturiga asoslanib to'ldirilgan holda qayta tayyorlandi.

SO'Z BOSHI

Oliy matematikaning turli bo'limlari bo'yicha yozilgan tajriba ishlari ushbu to'plamda jamlangan. Bu to'plamda tajriba ishlarning umumiy soni 15 ta. Chiziqli algebraik tenglamalar sistemasini yechish bo'yisha 3 ta tajriba ishi yozilgan, bunday tenglamalar sistemalarini yechishning Kramer, Gauss va teskari matrisa usullaridir.

Transendent tenglamaning haqiqiy ildizlarini talab etilgan aniqlikda taqribiy hisoblash ishida ildizni vatar va urinma (ya'ni Nyuton) usullari bilan taqribiy topish alohida- alohida ravishda ko'rsatilgan. Ish oxirida bu usullar birlashtirilgan holda berilgan.

Interpolyasiyalashga oid Lagranj va Nyutonning interpolyasion formulalari, aniq integrallarni Simpson usuli bilan taqribiy hisoblash, eng kichik kvadratlar usuli, bir argumentli funksiyaning differensiali yordamida funksiyalar qiymatlarini taqribiy hisoblash, birinchi tartibli differensial tenglamalarni Eyler usuli bilan taqribiy yechish, ikkinchi tartibli differensial tenglamalarni taqribiy yechish, aniq integralni qatorlar yordamida taqribiy hisoblash usullari, Laplas almashtirish yordamida differensial tenglamalarni yechish, Laplas almashtirish yordamida differentsial tenglamalar sistemalarini yechish va ehtimollar nazariyasi fanidan ikkita tajriba ishlari ham bu to'plamdan joy olgan.

Hisoblashlarni yengillatish maqsadida har bir tajriba ishida jadval tuzilgan. Har bir tajriba ishida namunaviy misol yoki masalalar yechish hamda talabalar mustaqil ishlari uchun misollar keltirilgan.

1-Tajriba ishi

Chiziqli algebraik tenglamalar sistemasini Kramer usuli bilan yechish

Quyida uch noma'lumli uchta chiziqli tenglamalar sistemasini Kramer usuli deb ataluvchi usul bilan yechishni ko'rib chiqamiz.

Faraz qilaylik,

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{33}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases} \qquad (1)$$

chiziqli algebraik tenglamalar sistemasi berilgan bo'lsin. (1) sistemaning asosiy aniqlovchisi (determinanti) deb, Δ bilan belgilanadigan quyidagi aniqlovchiga aytiladi:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad (2)$$

Bu aniqlovchi (1) sistemaning koeffisientlaridan tuzilgan bo'lib, biz uni noldan farqli bo'lsin deb faraz qilamiz. Endi $\Delta x_к$ (k = 1,2,3) aniqlovchilarni Δ aniqlovchining k-ustunini ozod hadlarning ustuniga quyidagicha almashtirish orqali hosil qilamiz.

$$\Delta x_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}; \quad \Delta x_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}; \quad \Delta x_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

Ma'lumki $\Delta \neq 0$ bo'lganda (1) sistema birgalikdagi sistema bo'ladi va u yagona yechimga ega bo'ladi. Bu yechim

$$x_1 = \frac{\Delta x_1}{\Delta}, \quad x_2 = \frac{\Delta x_2}{\Delta}, \quad x_3 = \frac{\Delta x_3}{\Delta} \qquad (3)$$

formulalar orqali topiladi va bu formulalar Kramer formulalari deyiladi.

Izoh: Umuman esa Kramer usuli bilan n noma'lumli n ta chiziqli тenglamalar sistemasini yechish mumkin (n-ixtiyoriy butun musbat son). $\Delta=0$ bo'lganda esa Kramer usulini qo'llash mumkin emas, chunki bu holda (3) formulalar ma'noga ega bo'lmaydi.

Misol:

$$\begin{cases} 0{,}314x_1 - 1{,}256x_2 + 0{,}125x_3 = 0{,}514 \\ 2{,}183x_1 + 0{,}958x_2 - 1{,}228x_3 = 0{,}985 \\ 1{,}327x_1 - 1{,}415x_2 + 1{,}238x_3 = 1{,}823 \end{cases}$$

uch noma'lumli uchta tenglamalar sistemasi yechilsin.

Berilgan tenglamalar sistemasining aniqlovchisini tuzamiz va hisoblaymiz:

$$\Delta=\begin{vmatrix}0,314 & -1,256 & 0,125\\ 2,183 & 0,958 & -1,228\\ 1,327 & -1,415 & 1,238\end{vmatrix}=$$

=0,314*0,958*1,238+(-1,256)*(-1,228)*1,327+0,125*(-1,415)*2,183-0,125*0,958*1,327- (-1,256)* 2,183*1,238 - 0,314*(-1,228)*(-1,415)≈4,7229

$\Delta\neq 0$, demak sistema birgalikda va yagona yechimiga ega. Δx_1, Δx_2, Δx_3 aniqlovchilarni tuzamiz va hisoblaymiz.

$$\Delta x_1=\begin{vmatrix}0,514 & -1,256 & 0,125\\ 0,985 & 0,958 & -1,228\\ 1,823 & -1,415 & 1,238\end{vmatrix}=0,514\cdot 0,958\cdot 1,238+(-1,256)\cdot(-1,228)\cdot 1,823+0,125\cdot 0,985\cdot(-1,415)-(-1,256)\cdot 0,985\cdot 1,238-0,514\cdot(-1,228)\cdot(-1,415)-0,125\cdot 0,958\cdot 1,823\approx 3,429$$

$$\Delta x_2=\begin{vmatrix}0,314 & 0,514 & 0,125\\ 2,183 & 0,985 & -1,228\\ 1,327 & 1,823 & 1,238\end{vmatrix}=0,314\cdot 0,985\cdot 1,238+0,514\cdot(-1,228)\cdot 1,327+0,125\cdot 2,183\cdot 1,823-0,125\cdot 0,985\cdot 1,327-0,314\cdot(-1,228)\cdot 1,823-0,514\cdot 2,183\cdot 1,238\approx -0,8068$$

$$\Delta x_3=\begin{vmatrix}0,314 & -1,256 & 0,514\\ 2,183 & 0,958 & 0,985\\ 1,327 & -1,415 & 1,823\end{vmatrix}=0,314\cdot 0,958\cdot 1,823+(-1,256)\cdot 0,985\cdot 1,327+0,514\cdot 2,183\cdot(-1,415)-(-1,256)\cdot 2,183\cdot 1,823-0,514\cdot 0,958\cdot 1,327-0,314\cdot 0,985\cdot(-1,415)\approx 2,1016$$

Kramer formulalari bo'yicha sistema yechimini topamiz:

$$x_1=\frac{\Delta x_1}{\Delta}\approx\frac{3,429}{4,7229}\approx 0,726;\quad x_2=\frac{\Delta x_2}{\Delta}\approx\frac{-0,8068}{4,7229}\approx -0,1708;\quad x_3=\frac{\Delta x_3}{\Delta}\approx\frac{2,1016}{4,7229}\approx 0,445$$

Tekshirish:

$$\begin{cases}0,314\cdot 0,726-1,256\cdot(-0,1708)+0,125\cdot 0,445\approx 0,514\\ 2,183\cdot 0,726+0,958\cdot(-0,1708)-1,228\cdot 0,445\approx 0,985\\ 1,327\cdot 0,726-1,415\cdot(-0,1708)+1,238\cdot 0,445\approx 1,823\end{cases}$$

Javob: $x_1\approx 0,726$ $\quad x_2\approx -0,1708$ $\quad x_3\approx 0,445$

Tajriba ishi variantlari

$$\begin{cases}4,508x_1-(2,221-n)x_2+0,532x_3=0,405\\ -2,227x_1-1,892x_2+(n+0,756)x_3=-1,813\\ 0,025nx_1+0,21x_2+2,031x_3=4,356\end{cases}$$

Izoh: Bu erda n talabaning guruh jurnalidagi tartib raqami.

Kramer usulining dasturi

```
uses crt;
LABEL 1,2;
var a:array[1..4,1..4] of real;
e:array[1..4,1..4] of real;
c:array[1..4,1..4] of real; b:array[1..4] of real;
det:array[1..4] of real; d:array[1..4] of real;
x:array[1..4] of real;
i,j,k,t1,t2,m,h:integer;
f1,f2,dd,t3:real;
```

```
begin
  ClrScr;
 Writeln('Kramer usuli');
 begin
for i:=1 to 4 do begin
for j:=1 to 4 do begin
write('A[',i,',',j,']= ');
 readln(a[i,j]);
  end;
  end;
for i:=1 to 4 do begin
write('B[',i,']= '); readln(b[i]); end;
for k:=1 to 4 do begin
t1:=0; t2:=0;
for i:=2 to 4 do begin t1:=t1+1; t2 :=0;
for j:=1  to 4  do begin
if j<>k then
begin
     t2:=t2+1;
t3:=a[i,j];
c[t1,t2]:=t3; end; end; end;
f1:=c[1,1]*c[2,2]*c[3,3]+c[1,2]*c[2,3]*c[3,1]+c[1,3]*c[2,1]*c[3,2];
f2:=-c[3,1]*c[2,2]*c[1,3]-c[2,1]*c[1,2]*c[3,3]-c[3,2]*c[2,3]*c[1,1];
det[k]:=f1+f2;
end;
dd:=a[1,1]*det[1]-a[1,2]*det[2]+a[1,3]*det[3]-a[1,4]*det[4];
for m:=1 to 4 do
begin
for i:=1 to 4 do
begin
for j:=1 to 4 do
begin
if i=m  then e[i,j]:=b[i] else e[i,j]:=a[i,j];
end;
end;
for k:=1 to 4 do
begin t1:=0;
t2:=0;
for i:=2 to 4 do
begin
t1:=t1+1;
t2:=0;
for j:=1 to 4 do begin
if j<>k then
begin   t2:=t2+1;
```

```
t3:=e[i,j];
c[t1,t2]:=t3;
end;
end;
end;
f1:=c[1,1]*c[2,2]*c[3,3]+c[1,2]*c[2,3]*c[3,1]+c[1,3]*c[2,1]*c[3,2];
f2:=-c[3,1]*c[2,2]*c[1,3]-c[2,1]*c[1,2]*c[3,3]-c[3,2]*c[2,3]*c[1,1];
det[k]:=f1+f2;
end;

if m=1 then
d[1]:=(b[1]*det[1])-(a[1,2]*det[2])+(a[1,3]*det[3])-(a[1,4]*det[4]);
if m=2 then
d[2]:=(a[1,1]*det[1])-(b[1]*det[2])+(a[1,3]*det[3])-(a[1,4]*det[4]);
if m=3 then
d[3]:=(a[1,1]*det[1])-(a[1,2]*det[2])+(b[1]*det[3])+(a[1,4]*det[4]);
if  m=4 then
d[4]:=(a[1,1]*det[1])+(a[1,2]*det[2])+(a[1,3]*det[3])-(b[1]*det[4]);
end;
for i:=1 to 4 do begin
x[i]:=d[i]/dd;
writeln('x',i,'=','',x[i]:4:2);
end;
 readln;
 readln;
  end;
  end.
```

2-Tajriba ishi

Chiziqli algebraik tenglamalar sistemasini Gauss usuli bilan yechish

Ushbu chiziqli to`rt no'malumli to`rtta tenglamalar sistemasi berilgan bo`lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = a_{15} \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = a_{25} \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = a_{35} \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = a_{45} \end{cases} \qquad (1)$$

Faraz qilaylik $a_{11} \neq 0$ (yetakchi element) bo`lsin (agar $a_{11} = 0$ bo`lsa x_1 no'malum oldidagi koeffisienti nol bo`lmagan tenglama bilan birinchi tenglamani almashtiramiz).

Birinchi tenglamaning barcha koeffisientlarini a_{11} ga bo`lsak:

$x_1+\frac{a_{12}}{a_{11}}x_2+\frac{a_{13}}{a_{11}}x_3+\frac{a_{14}}{a_{11}}x_4=\frac{a_{15}}{a_{11}}$ yoki $x_1+b_{12}^{(1)}x_2+b_{13}^{(1)}x_3+b_{14}^{(1)}x_4=b_{15}^{(1)}$ (2)

hosil bo`ladi, bu yerda $b_{1j}^{(1)}=\frac{a_{1j}}{a_{11}}$ $(j\geq 2)$

Endi (2) ni a_{21} ga ko`paytirib (1) sistemaning ikkinchi tenglamasidan, (2) ni $a_{31}$ ga ko`paytirib, o`sha sistemaning uchunchi tenglamasidan va nihoyat (2) ni $a_{41}$ ga ko`raytirib (1) sistemaning to`rtinchi tenglamasidan ayirib, quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} a_{22}^{(1)}x_2+a_{23}^{(1)}x_3+a_{24}^{(1)}x_4=a_{25}^{(1)} \\ a_{32}^{(1)}x_2+a_{33}^{(1)}x_3+a_{34}^{(1)}x_4=a_{35}^{(1)} \\ a_{42}^{(1)}x_2+a_{43}^{(1)}x_3+a_{44}^{(1)}x_4=a_{45}^{(1)} \end{cases} \quad (3)$$

bu yerda $a_{ij}^{(1)}=a_{ij}-a_{i1}\cdot b_{1j}^{(1)}$ $,(i,j\geq 2)$. (3) sistemaning yetakchi elementi $a_{22}^{(1)}\neq 0$ deb faraz qilib, birinchi tenglamaning barcha koeffisientlarini $a_{22}^{(1)}$ ga bo`lib, quyidagini hosil qilamiz:

$x_2+b_{a3}^{(2)}x_3+b_{24}^{(2)}=b_{25}^{(2)};$ (4) bu yerda $b_{2j}^{(2)}=\frac{a_{2j}^{(1)}}{a_{22}^{(1)}},(j\geq 3)$

Endi (4) ni ketma-ket $a_{32}^{(1)},a_{42}^{(1)}$ larga ko`paytirib, (3) sistemaning ikkinchi va uchunchi tenglamalaridan ayiramiz.

$$\begin{cases} a_{33}^{(2)}x_3+a_{34}^{(2)}x_4=a_{35}^{(2)} \\ a_{43}^{(2)}x_3+a_{44}^{(2)}x_4=a_{45}^{(2)} \end{cases}$$

(5) bu yerda $a_{ij}^{(2)}=a_{ij}^{(1)}-a_{i2}^{(1)}\cdot b_{2j}^{(2)},(ij\geq 3)$ (5) ning birinchi tenglamasida $a_{33}^{(2)}\neq 0$ deb (yetakchi element) faraz qilib birinchi tenglamaning barcha elementlarini $a_{33}^{(2)}$ ga bo`lsak: $x_3+b_{34}^{(3)}x_4=b_{35}^{(3)}$ (6)

tenglama hosil bo`ladi, bu yerda $b_{3j}^{(2)}=\frac{a_{3j}^{(2)}}{a_{33}^{(2)}},(j\geq 4)$ (6) ni $a_{43}^{(2)}$ ga ko`paytirib (5) ning ikkinchi tenglamasidan ayirsak $a_{44}^{(3)}x_4=a_{45}^{(3)}$ (7) hosil bo`ladi, bu yerga $a_{ij}^{(3)}=a_{ij}^{(2)}-a_{i3}^{(2)}\cdot b_{3j}^{(3)},(i,j\geq 4)$ x_4 ni (7) tenglikdan topamiz: $x_4=\frac{a_{45}^{(3)}}{a_{44}^{(3)}}=b_{45}^{(4)}$ (6), (4), (2) lardan foydalanib x_3, x_2, x_1 – larni topamiz:

$$\left.\begin{aligned} x_3&=b_{35}^{(3)}-b_{34}^{(3)}\cdot x_4 \\ x_2&=b_{25}^{(2)}-b_{24}^{(2)}x_4-b_{23}^{(2)}x_3 \\ x_1&=b_{15}^{(1)}-b_{14}^{(1)}x_4-b_{13}^{(1)}x_3-b_{12}^{(1)}x_2 \end{aligned}\right\} \quad (8)$$

Tenglamalar sistemasining yuqorida ko`rsatilgan kabi usul bilan yechilishi, ya'ni navbat bilan $x_1, x_2, x_3$ noma'lumlarni yo`qotib boorish usuli bilan yechilishi, tenglamalar sistemasini yechishning Gauss usuli deb ataladi.

Quyida tenglamalar sistemasini Gauss usuli bilan yechish sxemasini ko`rsatamiz. Hisoblashda xatoga yo`l qo`ymaslik uchun hisoblash jarayonini nazorat qilish ma'quldir. Buning uchun har bir satrning elementlari yig`indisi topiladi.

x_1	x_2	x_3	x_4	Ozod had	$\sum$	Formulalar	Sistema bo`limi
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}		1 A
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}		
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}		
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}		
1	$b_{12}^{(1)}$	$b_{13}^{(1)}$	$b_{14}^{(1)}$	$b_{15}^{(1)}$	$b_{16}^{(1)}$	$b_{ij}^{(1)}=\frac{a_{ij}}{a_{11}},(j\geq 2)$	2
	$a_{22}^{(1)}$	$a_{23}^{(1)}$	$a_{24}^{(1)}$	$a_{25}^{(1)}$	$a_{26}^{(1)}$	$a_{ij}^{(1)}=a_{ij}-a_{i1}\cdot b_{1j}^{(1)}$	
	$a_{32}^{(1)}$	$a_{33}^{(1)}$	$a_{34}^{(1)}$	$a_{35}^{(1)}$	$a_{36}^{(1)}$	$a_{24}^{(1)}=a_{24}-a_{21}\cdot b_{14}^{(1)}$	1
	$a_{42}^{(1)}$	$a_{43}^{(1)}$	$a_{44}^{(1)}$	$a_{45}^{(1)}$	$a_{46}^{(1)}$	$(i,j\geq 2)$	A_1
	1	$b_{23}^{(2)}$	$b_{24}^{(2)}$	$b_{25}^{(2)}$	$b_{26}^{(2)}$	$b_{2j}^{(2)}=\frac{a_{2j}^{(1)}}{a_{22}^{(1)}}\quad(j\geq 3)$	2
		$a_{33}^{(2)}$	$a_{34}^{(2)}$	$a_{35}^{(2)}$	$a_{36}^{(2)}$	$a_{ij}^{(2)}=a_{ij}^{(1)}-a_{i2}^{(1)}\cdot b_{2j}^{(2)}$	1
		$a_{43}^{(2)}$	$a_{44}^{(2)}$	$a_{45}^{(2)}$	$a_{46}^{(2)}$	$a_{43}^{(2)}=a_{43}^{(1)}-a_{42}^{(1)}\cdot b_{23}^{(2)}$	A_2
		1	$b_{34}^{(3)}$	$b_{35}^{(3)}$	$b_{36}^{(3)}$	$b_{3j}=\frac{a_{3j}^{(2)}}{a_{33}^{(2)}}(j\geq 4)$	2
			$a_{44}^{(3)}$	$a_{45}^{(3)}$	$a_{46}^{(3)}$	$a_{ij}^{(3)}=a_{ij}^{(2)}-a_{i3}^{2}\cdot b_{3j}^{(3)}$	1
			1	$b_{45}^{(4)}$	$b_{46}^{(4)}$	$b_{4j}^{(4)}=\frac{a_{4j}^{(3)}}{a_{44}^{(3)}}(j\geq 5)$	A_3 2
			1	x_4	$\bar{x}_4$	$x_4=b_{45}^{(4)}$	
		1		x_3	$\bar{x}_3$	$x_3=b_{35}^{(3)}-b_{34}^{(3)}\cdot x_4$	B
	1			x_2	$\bar{x}_2$	$x_2=b_{25}^{(2)}-b_{24}^{(2)}\cdot x_4-b_{23}^{(2)}\cdot x_3$	
1				x_1	$\bar{x}_1$	$x_1=b_{15}^{(1)}-b_{14}^{(1)}\cdot x_4-b_{13}^{(1)}\cdot x_3-b_{12}^{(1)}\cdot x_2$	

Namunaviy misol

Misol. Quyidagi algebraik tenglamalar sistemasi yechilsin:

$$\begin{cases}3{,}4x_1+2x_2-5{,}6x_3+4{,}8x_4=6{,}21\\ 2{,}4x_1-3{,}7x_2+8{,}5x_3-5{,}9x_4=5{,}73\\ 9{,}3x_1+8{,}9x_2-3{,}3x_3-0{,}6x_4=2{,}5\\ 0{,}7x_1-7{,}4x_2+4{,}1x_3+5{,}8x_4=3{,}4\end{cases}$$

Yechish: Sxemaning A bo'limining 1-qismiga noma'lumlar oldidagi koeffitsientlarni, ozod hadlarni va kontrol yig'indini yozamiz. 1-satrning barcha elementlarini 3,4 ga bo'lib, A bo'limning ikkinchi qismini to'ldiramiz.

A – bo'limning 1-qismini to'ldirish uchun uning elementlarini quyidagicha topamiz: A bo'limning 1-qismidagi 1-satrda turgan ixtiyoriy elementidan, shu element turgan satrning birinchi elementi bilan, shu element turgan ustunning oxirgi elementi ko'paytmasini ayirib, A bo'limning 1-qismidagi mos o'rniga yozamiz.

$$a_{22}^{(1)} = a_{22} - a_{21} \cdot b_{13}^{(1)} = -3{,}7 - 2{,}4 \cdot 0{,}588 = -5{,}1112$$

$$a_{23}^{(1)} = a_{23} - a_{21} \cdot b_{13}^{(1)} = 8{,}5 - 2{,}4(-1{,}647) = 12{,}4528$$

$$a_{34}^{(1)} = a_{34} - a_{31} \cdot b_{14}^{(1)} = -0{,}6 - 9{,}3 \cdot 1{,}412 = -13{,}7316$$

A_1 bo'limning 2-qismini topish uchun 1-qismning 1-satrining barcha elementlarining $-5{,}1112$ ga bo'lamiz.

A_2, A_3 – bo'limlar ham A_1-bo'lim to'ldirilganidek to'ldiriladi. Noma'lumlarni topish uchun, birinchi koeffisienti 1 ga teng bo'lgan satrlardan foydalanamiz. x_4 – A_3 bo'limning oxirgi elementidir:

$a_4 = b_{45}^{(4)} = -1{,}6564$ x_1, x_2, x_3 larni quyidagicha hisoblaymiz:

$$x_3 = b_{35}^{(3)} - b_{34}^{(3)} x_4 = -0{,}666 + 0{,}9799(-1{,}6564) = -2{,}2894,$$

$$x_2 = b_{25}^{(2)} - b_{24}^{(2)} x_4 - b_{23}^{(2)} x_3 = -0{,}2637 - 1{,}8173 \cdot (-1{,}6564) + 2{,}4364 \cdot (-2{,}2894) = -2{,}8314$$

$$x_1 = b_{15}^{(1)} - b_{14}^{(1)} x_4 - b_{13}^{(1)} x_3 - b_{12}^{(1)} x_2 = 1{,}826 - 1{,}412 \cdot (-1{,}6564) - 1{,}647 \cdot (-2{,}2894) - 0{,}588 \cdot (-2{,}8314) = 2{,}0591$$

Endi $x_1 = 2{,}0591$, $x_2 = -2{,}8314$, $x_3 = -2{,}2894$, $x_4 = -1{,}6564$ quyidagilarni sxemaga yozamiz. Noma'lumlarning topilgan bu qiymatlarini berilgan sistemaga quyib, tekshiramiz.

Javob: $x_1 = 2{,}0591$, $x_2 = -2{,}8314$, $x_3 = -2{,}2894$, $x_4 = -1{,}6564$

x_1	x_2	x_3	x_4	*Ozod xadlar*	*Yig'indi* $\sum_{k=1}^{5} x_k$	*Bo'limlar*
3,4	2	-5,6	4,8	6,21	10,81	A
2,4	-3,7	8,5	-5,9	5,73	7,63	
9,3	8,9	-3,3	-0,6	2,5	16,8	
0,7	-7,4	4,1	5,8	3,4	3,4	
1	0,588	-1,647	1,412	1,826	3,179	A_1
	-5,1112	12,4528	-9,2888	1,3476	-0,5996	
	3,4316	12,0171	-13,731	-14,4818	-12,7647	
	-7,8116	5,2529	4,8116	2,1218	4,3747	
	1	-2,4364	1,8173	-0,2637	0,1173	A_2
		20,3779	-19,9678	-13,5769	-13,1672	
		-13,7793	19,0076	0,0619	5,2910	
		1	-0,9799	-0,6663	-0,6462	A_3
			5,5053	-9,1192	-3,6139	
			1	-1,6564	-0,6564	
			1	-1,6564	-1,6564	
		1		-2,2894	-1,1894	
	1			-2,8314	-1,8314	
1				2,0591	3,0591	

Tajriba ishi variantlari

$$\begin{cases} 3{,}4x_1 + (2+n)x_2 - 5{,}6x_3 + 4{,}8x_4 = 6{,}21 \\ 2{,}1x_1 - 3{,}7x_2 + (8{,}5-n)x_3 - 5{,}9x_4 = 5{,}73 \\ (9{,}3-0{,}5n)x_1 + 8{,}9x_2 - 3{,}3x_3 - 0{,}6x_4 = 2{,}5 \\ 0{,}7x_1 - 4{,}3x_2 + 5{,}1x_3 + (2n-5{,}8)x_4 = 3{,}4 \end{cases}$$

Izoh: bu erda n – talabaning guruh jurnalidagi tartib raqami

Gaus usuliga tuzilgan dastur

```
program gaus;
uses crt;
var i,j,k:integer;
a,a1,a2,a3,b1,b2,b3,b4:array[1..10,1..10] of real;
x,y:array [1..10]of real;
h,h1,h2,h3,u,o:real;
begin
    clrscr;
    writeln('Matritsaning elementlarini kiriting');
    for i:=1 to 4 do begin
    for j:=1 to 5 do begin
    write ('a[',i,',',j,']=');
    read(a[i,j]);
    end;
    end;
    begin
    h:=a[1,1];
    begin
    for i:=1 to 1 do begin
    for j:=2 to 6 do begin
    b1[i,j]:=a[i,j]/h;
    end;
    end;
    end;
      begin
      for i:=2 to 4 do begin
      for j:=2 to 6 do begin
      for k:=1 to 1 do begin
      a1[i,j]:=a[i,j]-(a[i,k]*b1[k,j]);
      end;
      end;
      end;
      end;
   begin
   h1:=a1[2,2];
    begin
```

```
for i:=2 to 2 do begin
for j:=3 to 6 do begin
b2[i,j]:=a1[i,j]/h1;
end;
end;
end;
begin
 for i:=3 to 4 do begin
 for j:=3 to 6 do begin
 for k:=2 to 2 do begin
 a2[i,j]:=a1[i,j]-(a1[i,k]*b2[k,j]);
 end;
 end;
 end;
 end;
 begin
   h2:=a1[3,3];
   begin
   for i:=3 to 3 do begin
   for j:=4 to 6 do begin
   b3[i,j]:=a2[i,j]/h2;
   end;
   end;
   end;
   begin
 for i:=4 to 4 do begin
 for j:=4 to 6 do begin
 for k:=3 to 3 do begin
 a3[i,j]:=a2[i,j]-(a2[i,k]*b3[k,j]);
 end;
 end;
 end;
 end;
 begin
 h3:=a3[4,4];
 begin
 for i:=4 to 4 do begin
 for j:=5 to 6 do begin
 b4[i,j]:=a3[i,j]/h3;
 end;
 end;
 begin
 x[4]:=b4[4,5];
 x[3]:=b3[3,5]-(b3[3,4]*x[4]);
 x[2]:=b2[2,5]-(b2[2,4]*x[4])-(b2[2,3]*x[3]);
```

```
x[1]:=b1[1,5]-(b1[1,4]*x[4])-(b1[1,3]*x[3])-(b1[1,2]*x[2]);
end;
writeln;
begin
{Dasturni tekshirish protsessi}

y[1]:=a[1,1]*x[1]+a[1,2]*x[2]+a[1,3]*x[3]+a[1,4]*x[4];
y[2]:=a[2,1]*x[1]+a[2,2]*x[2]+a[2,3]*x[3]+a[2,4]*x[4];
y[3]:=a[3,1]*x[1]+a[3,2]*x[2]+a[3,3]*x[3]+a[3,4]*x[4];
y[4]:=a[4,1]*x[1]+a[4,2]*x[2]+a[4,3]*x[3]+a[4,4]*x[4];
   for i:=1 to 4 do begin
   for j:=5 to 5 do begin
   if abs(y[i]-a[i,j])<0.001 then
   begin
   for i:=1 to 4 do begin
      writeln('x[',i,']=',x[i]:1:2);
end;
readln;
readln;
end;
end;
end;
end;
end;
end;
end;
end;
end;
end.
```

3 - Tajriba ishi

Chiziqli tenglamalar sistemasini teskari matritsalar usuli bilan yechish

Ushbu uch noma'lumli chiziqli tenglamalar sistemasi berilgan bo'lsa:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2, \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

uni teskari matritsalar usuli yordamida yeching

Tenglamalar sistemasida qatnashayotgan $a_{ij}, b_i \left(i, j = 1,2,3\right)$ o'zgarmas koeffitsientlar quyidagi jadvalda berilgan.

1-jadval

Variant	a_{11}	a_{12}	a_{13}	a_{21}	a_{22}	a_{23}	a_{31}	a_{32}	a_{33}	b_1	b_2	b_3
1	4	-3	1	5	-2	7	1	2	1	-8	9	3
2	1	0	3	2	1	4	-1	1	-2	7	9	-3
3	-2	2	1	4	-5	0	3	7	1	9	-13	4
4	-1	2	4	5	0	8	-7	1	3	16	14	24
5	3	2	1	-4	1	3	-2	0	2	-7	18	10
6	0	-1	3	5	0	4	-1	-3	7	8	22	20
7	1	-1	1	-2	0	-5	2	1	3	6	13	4
8	-1	1	3	-4	0	3	-2	1	4	12	17	17
9	1	1	-2	1	3	0	5	-1	2	-7	20	-5
10	2	-1	0	3	1	4	-5	4	-3	-5	7	5
11	-3	0	1	-1	4	2	3	2	-1	-3	12	-7
12	4	2	1	-3	0	5	6	-2	1	-4	9	-11
13	55	4	1	2	-1	0	-7	-3	1	-3	-5	14
14	-1	-5	2	4	-2	3	1	0	0	-1	-22	-5
15	-2	4	3	-12	5	-7	-4	3	3	17	19	25
16	3	-3	2	1	4	0	10	5	-6	-3	2	-33
17	4	1	-2	1	5	8	0	-7	1	-13	27	-4
18	5	0	1	1	-2	3	-2	3	-4	-7	5	-5
19	-1	3	-4	4	3	1	2	3	4	-7	-2	11
20	-5	1	3	3	2	8	7	5	3	20	20	0
21	-2	4	2	1	0	7	10	-9	-1	6	19	32
22	-4	2	1	5	-2	3	4	-3	-2	13	0	-17
23	-5	3	2	0	1	7	8	-3	-2	19	22	-25
24	1	-2	3	4	-1	3	7	5	-1	5	0	-12
25	3	2	1	-2	0	10	8	-7	3	1	34	14

Namunaviy misol va ko'rsatmalar

$$\begin{cases} 3x_1 - 2x_2 + 4x_3 = 4, \\ 2x_1 + 4x_2 - 5x_3 = -15, \\ -7x_1 + x_2 - 8x_3 = -9, \end{cases}$$

algebraic tenglamalar sistemasi teskari matritsalar usullari yordamida yechilsin. Berilgan tenglamalar sistemasini $A\cdot X = B$ ko'rinishida yozib olamiz.Bu erda

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Agar A matritsaning determinanti Δ≠0 bo'lsa, u holda $A\cdot A^{-1} = E$ tenglikni qanoatlantiruvchi A^{-1} matritsa A matritsaga teskari matritsa deyiladi. Bu erda E-birlik matritsa, ya'ni

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Agar A matritsaga teskari bo'lgan A^{-1} matritsa mavjud bo'lsa, u holda berilgan sistemaning yechimi

$$X = A^{-1}\cdot B$$

formula yordamida aniqlanadi. A^{-1} matritsani aniqlash jarayoni quyidagi algoritmga ega:

1) Berilgan A matritsaning determinanti hisoblanadi

$$\Delta = \det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

2) A matritsaning a_{ij} elementlarining $A_{ij} = (-1)^{i+j}$, (i, j = 1,2,3) algebraik to'ldiruvchilari hisoblanib,

$$\tilde{A} = \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

matritsa hosil qilinadi. Bu erda M_{ij} – a_{ij} elementning minori.

3) Transponirlangan matritsani topish.

A matritsaning a_{ij} elementlarining $A_{ij} = (-1)^{i+j}$, (i, j = 1,2,3) algebraik to'ldiruv-chilari hisoblanib,

$$\tilde{A} = \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

matritsa hosil qilinadi. Bu erda M_{ij} – a_{ij} elementning minori.

4) Teskari matritsani topish

$$A^{-1} = \frac{1}{\Delta} \cdot \tilde{A} = \frac{1}{\Delta} \cdot \begin{pmatrix} \frac{A_{11}}{\Delta} & \frac{A_{21}}{\Delta} & \frac{A_{31}}{\Delta} \\ \frac{A_{12}}{\Delta} & \frac{A_{22}}{\Delta} & \frac{A_{32}}{\Delta} \\ \frac{A_{13}}{\Delta} & \frac{A_{23}}{\Delta} & \frac{A_{33}}{\Delta} \end{pmatrix}$$

formula orqali ifodalanadi.

Demak, yuqorida qayd etilganidek, dastlab,

$A = \begin{pmatrix} 3 & -2 & 4 \\ 2 & 4 & -5 \\ -7 & 1 & -8 \end{pmatrix}$ matritsaga teskari bo'lgan A^{-1} matritsani aniqlaymiz. Bizga ma'lumki, $\Delta = \begin{vmatrix} 3 & -2 & 4 \\ 2 & 4 & -5 \\ -7 & 1 & -8 \end{vmatrix} = -63 \neq 0$

$\tilde{A}$ matritsani tuzish uchun $A_{ij}(i, j = 1,2,3)$ algebraik to'ldiruvchilarni aniqlaymiz.

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 4 & -5 \\ 1 & -8 \end{vmatrix} = -27, \quad A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ -7 & 1 \end{vmatrix} = 11,$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & -5 \\ -7 & -8 \end{vmatrix} = 51, \quad A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 4 \\ 4 & -5 \end{vmatrix} = -6,$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 4 \\ -7 & 1 \end{vmatrix} = 30, \quad A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 4 \\ 2 & -5 \end{vmatrix} = 23,$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 4 \\ 1 & -8 \end{vmatrix} = -12, \quad A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 2 & 4 \end{vmatrix} = 16$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 4 \\ -7 & -8 \end{vmatrix} = 4$$

Izlangan teskari matritsa $A^{-1}=\frac{1}{\Delta}\cdot\tilde{A}=\frac{1}{-63}\begin{pmatrix}-27 & -12 & -6\\ 51 & 4 & 23\\ 30 & 11 & 16\end{pmatrix}$ ko'rinishga ega bo'ladi.

Shunday qilib, $X=A^{-1}\cdot B$ formulaga asosan

$$X=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\frac{1}{-63}\begin{pmatrix}-27 & -12 & -6\\ 51 & 4 & 23\\ 30 & 11 & 16\end{pmatrix}\cdot\begin{pmatrix}4\\ -15\\ -9\end{pmatrix}=\frac{1}{-63}\begin{pmatrix}-27\cdot 4+(-12)\cdot(-15)+(-6)\cdot(-9)\\ 51\cdot 4+4\cdot(-15)+23\cdot(-9)\\ 30\cdot 4+11\cdot(-15)+16\cdot(-9)\end{pmatrix}=\frac{1}{-63}\begin{pmatrix}126\\ -63\\ -189\end{pmatrix}=\begin{pmatrix}-2\\ 1\\ 3\end{pmatrix}$$

ni topamiz. Demak, $x_1=-2,\ x_2=-1,\ x_3=-3$ ikkala usulda ham bir xil yechimni olamiz.

Javob: $\{-2,1,3\}$

Teskari matritsaga tuzilgan dasturlar

```
Program Teskari;
uses crt;
var
  a,b,c,g,x:array[1..10,1..10] of real;
   i,j,k:integer;
 f,e,d,h:real;
  begin
    clrscr;
    Writeln('A massiv elementlarini kiriting:');
    for i:=1 to 3 do
    for j:=1 to 3  do
    begin
      textcolor(12);
      Write('a[',i,',',j,']=');
      readln(k);
      a[i,j]:=k;
    end;
      begin
         writeln('G matritsaning elementlarinikiriting');
        for i:=1 to 3 do
        for j:=1 to 1 do
        begin
          textcolor(11);
          Write('G[',i,',',j,']=');
          readln(h);
          g[i,j]:=h;
        end;
      begin
      b[1,1]:=a[2,2]*a[3,3]-a[3,2]*a[2,3];
      b[2,1]:=a[2,1]*a[3,3]-a[3,1]*a[2,3];
      b[3,1]:=a[2,1]*a[3,2]-a[3,1]*a[2,2];
```

```
b[1,2]:=a[1,2]*a[3,3]-a[3,2]*a[1,3];
b[2,2]:=a[1,1]*a[3,3]-a[3,1]*a[1,3];
b[3,2]:=a[1,1]*a[3,2]-a[3,1]*a[1,2];
b[1,3]:=a[1,2]*a[2,1]-a[2,2]*a[1,3];
b[2,3]:=a[1,1]*a[2,1]-a[2,1]*a[1,3];
b[3,3]:=a[1,1]*a[2,2]-a[2,1]*a[1,2];
d:=a[1,1]*a[2,2]*a[3,3]+a[2,1]*a[3,2]*a[1,3]+a[1,2]*a[2,1]*a[3,1];
e:=-a[1,3]*a[2,2]*a[3,1]-a[2,1]*a[3,2]*a[1,1]-a[1,2]*a[2,1]*a[3,3];
f:=d+e;
for i:=1 to 3 do
begin
for k:=1 to 3 do
begin
c[i,j]:=0;
for j:=1 to 3 do begin
c[i,j]:=c[i,j]+b[i,k]*g[k,j];
        x[i,j]:=c[i,j]/f;
          Writeln('x[',i,',',j,']=',x[i,j]:1:1);
        end;
        end;
        end;
        end;
        end;
        end.
```

4-Tajriba ishi

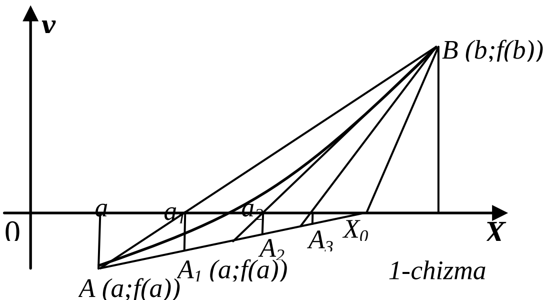
Tenglamaning haqiqiy ildizlarini taqribiy hisoblash

Ma'lumki tenglamalarning ildizlarini hamma vaqt ham aniq topib bo'lmaydi.

Agar tenglama koeffisientlari berilgan bo'lsa, uning ildizlarini istagan darajada aniqlik bilan taqribiy hisoblash mumkin. Quyida tenglama ildizlarini taqribiy hisoblashning ba'zi usullarini bayon qilamiz.

Vatarlar usuli

Ushbu $f(x)=0$ (1) tenglama berilgan bo'lib $f(x)$ funksiya $[a,b]$ kesmada uzluksiz va ikki marta differensiallanuvchi bo'lsin. Bu funksiyaning $[a,b]$ kesma chetlaridagi qiymatlari har xil ishorali bo'lsin, ya'ni $f(a)\cdot f(b)<0$. Aniqlik uchun $f(a)<0$ va $f(b)>0$ deb



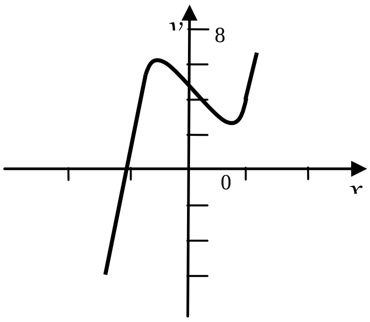
1-chizma

faraz qilamiz. $y = f(x)$ funksiya $[a,b]$ kesmada uzluksiz bo'lgani uchun uning grafigi a va b nuqtalar orasidagi birorta nuqtada ox o'qni kesadi. Bu nuqta absissasi $f(x)=0$ tenglamaning ildizlari-dan biridir ana shu ildizni topamiz. Buning uchun esa AB vatarni o'tkazamiz. (1-chizma). Bu vatarning OX o'qi bilan kesishgan nuqtasining absissasi ildizning taqribiy qiymati bo'ladi. AB vatar tenglamasini yozamiz:

$$\frac{y-f(a)}{f(b)-f(a)} = \frac{x-a}{b-a}$$

$x = a_1$ bo'lganda, $y=0$ bo'lgani uchun $\dfrac{-f(a)}{f(b)-f(a)} = \dfrac{a_1 - a}{b-a}$

bundan $a_1 = a - f(a)\dfrac{b-a}{f(b)-f(a)}$ yoki $a_1 = \dfrac{af(b)-bf(a)}{f(b)-f(a)}$ (2) ekanligini topamiz.



2- chizma

Ildizning aniqroq qiymatini topish uchun $f(a_1)$ qiymatini aniqlaymiz. Agar $f(a_1)<0$ bo'lsa, (2) formulani $[a,b]$ kesmaga qo'llanib, shu usulni takrorlaymiz. Agar $f(a_1)>0$ bo'lsa, shu formulani $[a,a_1]$kesmaga qo'llaymiz. Bu usulni bir necha marta takrorlab, ildizning yanada aniqroq a_2, a_3 va x.k. qiymatlarini topamiz.

Misol: $f(x)=x^3-3x+5=0$ tenglamaning yechimi vatar usuli bilan topilsin.

Yechish: $f(x)=x^3-3x+5$ funksiyaning grafigi *Ox* o'qini $[-2{,}5;-2]$ kesmada kesib o'tadi. (2-chizma). Bu erda

$$a=-2{,}5;\ b=-2;\ f(a)=f(-2{,}5)=-3{,}125<0;\ f(b)=f(-2)=3>0$$

$$f(a)\cdot(b)=f(-2{,}5)\cdot f(-2)<0$$

2-formulaga asosan:

$$a_1=-2{,}5-\frac{[-2-(-2{,}5)]\cdot(-3{,}125)}{3-(-3{,}125)}=-2{,}5+\frac{0{,}5\cdot 3{,}125}{6{,}125}\approx -2{,}2449$$

Endi $f(a_1)$ ni hisoblaymiz.

$$f(a_1)=f(-2{,}2449)=(-2{,}2449)^3-3(-2{,}2449)+5=0{,}4214>0$$

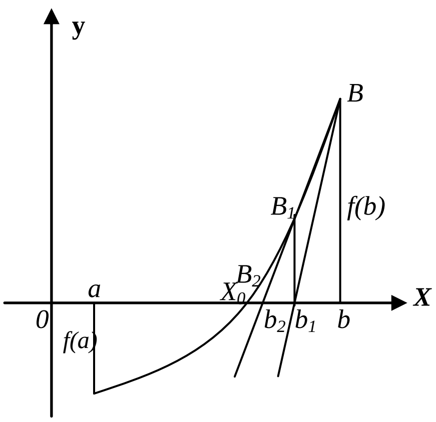
Demak, tenglamaning ildizi $[-2{,}5;-2{,}24]$ da yotadi, chunki

$$f(a)=f(-2{,}5)=-3{,}125<0$$ va
$$f(a_1)=f(-2{,}2449)=0{,}4214>0$$

(2) formulaga asosan:

$$a_2=-2{,}5-\frac{[-2{,}2449-(-2{,}5)]\cdot(-3{,}125)}{0{,}4214-(-3{,}125)}=-2{,}5+\frac{0{,}2551\cdot 3{,}125}{3{,}5464}\approx -2{,}2752$$

Demak, tenglamaning ildizi deb $x_0\approx -2{,}2752$ sonni olsa bo'ladi.



Bu erda ham vatar usuldagi kabi $f(a)<0$, $f(b)>0$ va $[a,b]$ kesmada birinchi hosilaning ishorasi o'zgarmaydi deb faraz qilamiz. Bu holda $[a,b]$ kesmada $f(x)=0$ tenglama bitta ildizga ega bo'ladi. $[a,b]$ kesmani $f(x)$ funksiyaning ikkinchi hosilasi o'z ishorasini o'zgartirmaydigan darajada kichik qilib olsin, u vaqtda funksiya grafigi bu kesmada yo faqat

3-chizma qavariq yo faqat botiq bo'ladi.

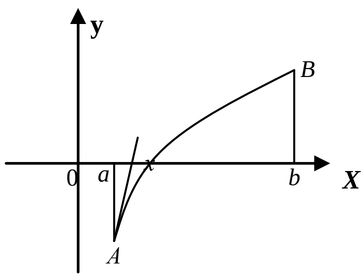
Egri chiziqqa B nuqtada urinma o'tkazamiz (3-rasm). Urinmaning *Ox* o'q bilan kesishgan nuqtasining absissasini b_1 bilan belgilasak, u ildizning taqribiy qiymati bo'ladi. Shu absissasini topish uchun B nuqtadagi urinma tenglamasini yozamiz.

$$y - f(b) = f^1(b)(x-b)$$

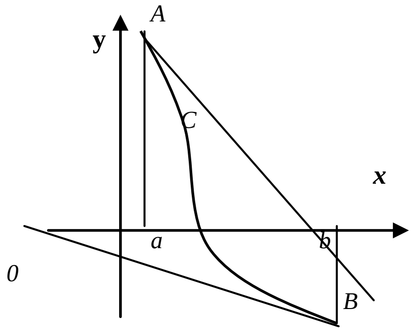
Agar $x = b_1$, desak $y = 0$ bo'ladi va

$$-f(b) = f^1(b)(b_1 - b), \quad b_1 = b - \frac{f(b)}{f^1(b)} \qquad (3)$$

bo'ladi. So'ngra b_1 nuqtada *Ox* o'qiga perpendikulyar to'g'ri chiziq o'tkazib $y = f(x)$ funksiya grafigi bilan uning kesishish nuqtasi B_1 ni topamiz. Endi $_1$ nuqtada $y = f(x)$ funksiya grafigiga urinma o'tkazib berilgan tenglama ildizining aniqroq qiymati b_2 ni topamiz. Bu usulni bir necha marta takrorlab, ildizning istalgan aniqlikdagi tarqibiy qiymatini topa olamiz. Rasmdan ko'rinib turibdiki, yoyning qaysi chegara nuqtasida $f(x)$, $f'(x)$ *va* $f''(x)$ larning ishoralari bir xil bo'lsa, o'sha nuqtada $y = f(x)$ egri chiziqqa urinma o'tkazish kerak. Bunday nuqta hamma vaqt mavjuddir, chunki $f(x)$ funksiya $[a,b]$ kesma



4- chizma



5- chizma

chekkalarida har xil ishorali qiymatlar qabul qiladi. $f''(x)$ esa o'z ishorasini o'zgartirmaydi. Bu qoida $f'(x) < 0$ bo'lganda ham o'z kuchini saqlaydi. Agar urinma intervalning chap chegarasidan o'tkazilsa, (3) formulada *b* o'rniga *a* ni qo'yish kerak: $a_1 = a - \dfrac{f(a)}{f(a)}$ (3)

Agar (*a*, *b*) intervalda C burilish nuqtasi bo'lsa, urinmalar usuli ildizning (*a*, *b*) intervaldan tashqarida yotgan taqribiy qiymatini berishi mumkin (5-chizma).

Misol: $f(x) = x^3 - 3x + 5 = 0$ tenglamaning yechimi urinma usuli bilan topilsin.

Yechish: 1-yoki 2-rasmdan foydalanib berilgan tenglamaning ildizi $[-2{,}5;\ -2]$ kesmada yotishini aniqladik. Endi urinmani qaysi nuqtada o'tkazishni aniqlaylik:

$$f`(x) = 3x^2 - 3, \qquad f`(x) = 6x$$

$$\left.\begin{array}{l} f(2,5) = -3{,}125 < 0 \\ f``(-2,5) = 6(-2,5) = -15 < 0 \end{array}\right\} \Rightarrow \begin{array}{l} f`(x) < 0 \\ f``(x) < 0 \end{array}$$

$$\left.\begin{array}{l} f(-2) = 3 > 0 \\ f``(-2) = 6(-2) = -12 < 0 \end{array}\right\} \Rightarrow \begin{array}{l} f(x) > 0 \\ f``(x) < 0 \end{array}$$

Urinmani A (-2,5; -3,125) nuqtada o’tkazamiz, (3) formulaga asosan

$$b_1 = -2{,}5 - \frac{-3{,}125}{19{,}75} = -2{,}3016 \qquad b_2 = b_1 - \frac{f(b_1)}{f``(b_1)}$$

larni hisoblaymiz. $f(b_1) = f(-2{,}3016) = (-2{,}3016)^3 - 3(-2{,}3016) + 5 = -0{,}267$

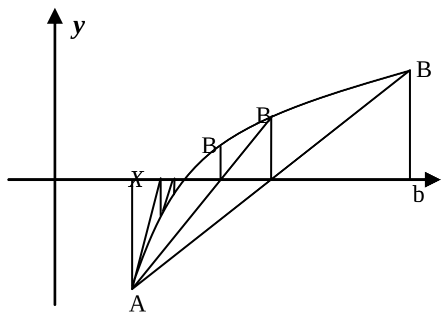
$f`(b_1) = f`(-2.3016) = 3(-2{,}3016)^2 - 3 = 12{,}87$; $\quad b_2 = -2{,}3016 - \frac{-0{,}267}{12{,}87} = -2{,}3016 + 0{,}02074 = -2{,}2808$

bo’lar ekan.

Vatar va urinmalar yoki birlashtirilgan usul

Vatarlar usuli va urinmalar usulini $[x_1, x_2]$ kesmada bir vaqtda qo’llanib, izlangan x ildizning ikki tomonida yotgan a_1 va b_1 nuqtalarni topamiz (chunki $f(a_1)$ *va* $f(b_1)$ ordinatalarning ishoralari har xil) (6-rasm). So’ngra $[a_1, b_1]$ kesmada yana vatarlar usulini va urinmalar usulini qo’llaymiz. Natijada ildizning qiymatiga yanada yaqinroq ikkita a_2 va b_2 sonlarni topamiz. Topilgan taqribiy qiymatlar orasidagi ayirma talab etilgan aniqlik darajasidan kichik bo’lguncha ishni shunday davom ettiramiz.

Birlashtirilgan usul bilan biz izlanayotgan ildizga bir vaqtda ikki tomondan yaqinlashamiz (ya'ni bir vaqtda ildizning haqiqiy qiymatidan ortiqroq va kamroq taqribiy qiymatlarni topamiz).



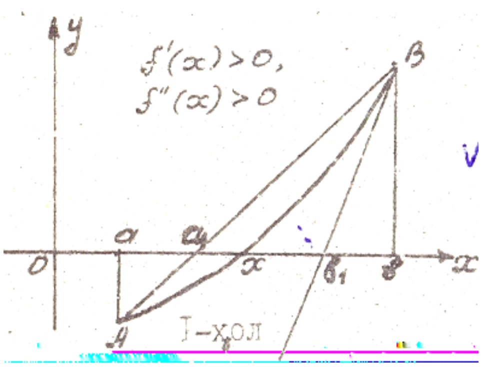
6-chizma

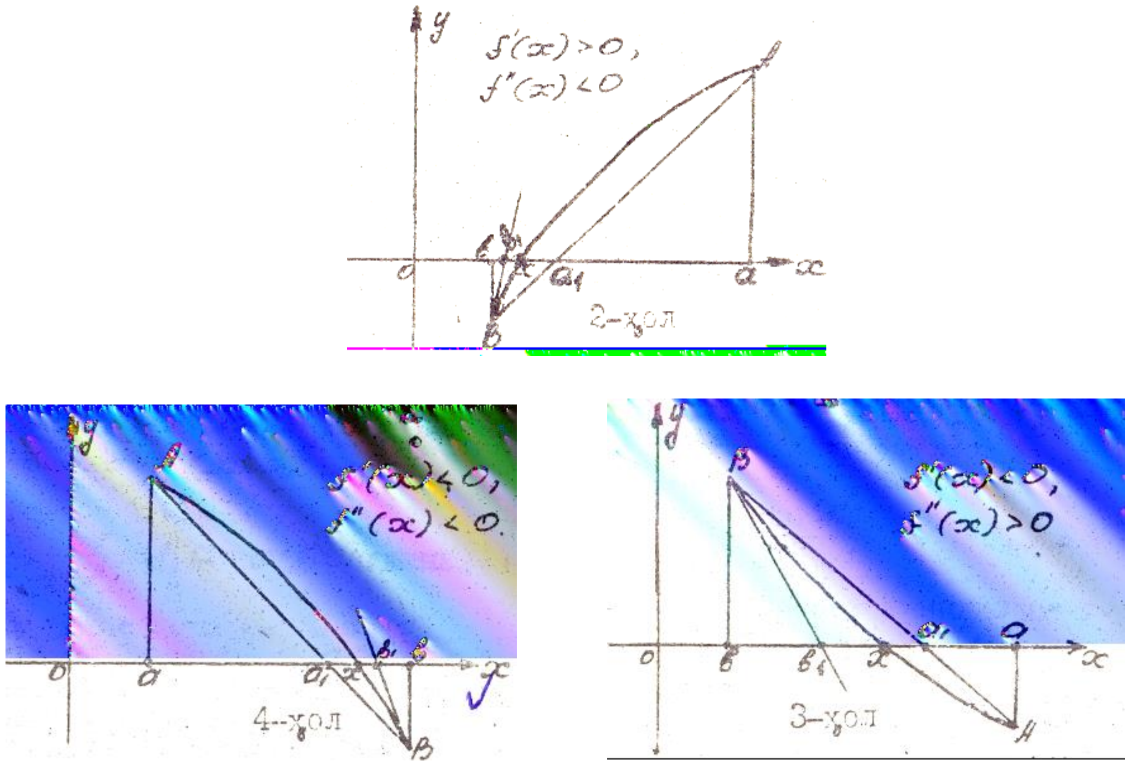
Vatarlar usuli va urinmalar usullarini berilgan aniqlikka ega bo’lmaguncha biror n marta takrorlasak, (2) va (3) formulalarning ko’rinishi quyidagicha bo’ladi.

$$a_n = a_{n-1} - f(a_{n-1})\frac{b_{n-1} - a_{n-1}}{f(b_{n-1}) - f(a_{n-1})} \qquad (4)$$

$$b_n = b_{n-1} - \frac{f(b_{n-1})}{f`(b_{n-1})} \qquad (5)$$

Agar $[a,b]$ kesmada $f(a) \cdot f(b) < 0$ bo’lib, $f'(x)$ $f''(x)$ hosilalar o’z ishorasini saqlasa, quyidagi to’rt hol bo’lishi mumkin (1-hol, 2-hol, 3-hol, 4-hol deb nomerlangan shakllarga qarang).1-hol, 2-hol, 3-hol, 4-hol





7 - chizma

1- va 4-hollarda (4) formulani chap chegaraviy nuqta uchun (5) formulani esa uning chegaraviy nuqta uchun qo'llaymiz.

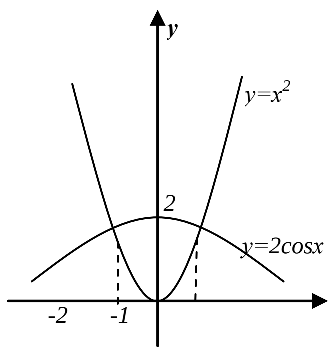
2- va 3-hollarda esa (4) formulani o'ng chegaraviy nuqta uchun, (5) formulani esa chap chegaraviy nuqta uchun qo'llaymiz.

Misol: $f(x) = x^2 - 2\cos x = 0$ tenglama berilgan bo'lsin. Uning musbat ildizi 0,0001 aniqlikda topilsin.

Yechish: Berilgan $x^2 - 2\cos x = 0$ tenglamani ikki funksiyaning tengligi ko'rinishida yozamiz:

$$x^2 = 2\cos x \quad va \quad \varphi(x) = x^2, \psi(x) = 2\cos x$$

deb belgilab olamiz. $\varphi(x) = x^2$ va $\psi(x) = 2\cos x$ funksiyalarning grafiklarini chizib, ularning kesishish nuqtalari absissalarini belgilaymiz. Ikkita ildizga ega bo'lamiz: biri – musbat, ikkinchisi – manfiydir. (8-rasm).



8- chizma

Musbat ildiz [1;1,3] kesmada yotadi. Bu kesma uchun quyidagi hollarni qaraymiz.

1) [1; 1,3] kesma uchlaridagi nuqtalarda $f(x)$ funksiyaning qiymatlari ishoralarini aniqlaymiz:

$$f(1) = 1^2 - 2\cos 1 = 1 - 2\cdot 0{,}5403 = -0{,}0806 < 0,$$
$$f(1{,}3) = 1{,}3^2 - 2\cos 1{,}3 = 1{,}69 - 2\cdot 0{,}2675 = 1{,}155 > 0$$

$f(1)\cdot f(1{,}3) < 0$ bo'lgani uchun $[1; 1{,}3]$ da $f(x) = 0$ tenglamaning ildizi bor.

2) $f(x)=x^2-2\cos x$ funksiya $(-\infty;+\infty)$ da uzluksiz, hamda $[1;1{,}3]$ da birinchi va ikkinchi hosilalarga ega. Ular $f'(x)=2x+2\sin x,\ f''(x)=2+2\cos x$, ko'rinishda bo'ladi.

3) $x=1$ va $x=1{,}3$ da birinchi va ikkinchi hosilalarning ishoralarini aniqlaymiz.

$$f`(1)=2\cdot 1+2\sin b_1=2+2\cdot 0{,}8415=3{,}6830>0,$$
$$f`(1{,}3)=2\cdot 1{,}3+2\sin 1{,}3=2{,}6+2\cdot 0{,}9636=4{,}5272>0,$$
$$f``(1)=2+2\cos 1=2+2\cdot 0{,}5403=3{,}0806>0,$$
$$f``(1{,}3)=2+2\cos 1{,}3=2+2\cdot 0{,}2675=2+0{,}5350=2{,}5350>0$$

Demak, $f'(x)>0,\ f''(x)>0$, bu esa 7-rasmdagi 1-hol bo'lib, $a=1$ va $b=1{,}3$ bo'lar ekan.

a_1 va b_1 larni (4) va (5) formulalar yordamida topimiz.

$$a_1=a-f(a)\frac{b-a}{f(b)-f(a)}=1-f(1)\cdot\frac{1{,}3-1}{f(1{,}3)-f(1)}=1-(-0{,}0806)\frac{0{,}3}{1{,}155-(-0806)}\approx 1{,}01957$$

$$b_1=b-\frac{f(b)}{f`(b)}=1{,}3-\frac{f(1{,}3)}{f`(1{,}3)}=1{,}3-\frac{1{,}155}{4{,}5272}\approx 1{,}04488$$

Ammo

$$\Delta_1=|a_1-b_1|=|1{,}01957-1{,}04488=0{,}02531>0{,}0001|$$

bo'lgani uchun bu topilgan qiymatlar izlanayotgan ildizning hali talab qilingan darajadagi aniqlikda topilmaganini ko'rsatadi.

Shuning uchun $f(a_1),\ f(b_1)$ *va* $f'(b_1)$ larni hisoblaymiz.

$$f(a_1)=f(1{,}01957)=(1{,}01957)^2-2\cdot\cos 1{,}01957=1{,}03952-2\cdot 1{,}0474\approx 0{,}00798,$$
$$f(b_1)=f(1{,}04488)=(1{,}04488)^2-2\cdot\cos 1{,}04488\approx 0{,}087758,$$
$$f`(b_1)=f`(1{,}04488)=2\cdot 1{,}04488+2\cdot\sin 1{,}04488\approx 3{,}819488$$

topilgan $f(a_1),\ f(b_1),\ f'(b_1)$ lardan foydalanib, a_2 va b_2 larni topamiz.

$$a_2=a_1-f(a_1)\cdot\frac{b_1-a_1}{f(b_1)-f(a_1)}=1{,}01957-(-0{,}007944)\cdot\frac{1{,}04488-1{,}01957}{0{,}087758-(-0{,}007944)}=1{,}01957+\frac{0{,}007544\cdot 0{,}02531}{0{,}095702}\approx 1{,}02167$$

$$b_2=b_1-\frac{f(b_1)}{f`(b_1)}=1{,}04488-\frac{0{,}087758}{3{,}819488}=1{,}02190 \qquad \Delta_2=|a_2-b_2|=|1{,}02167-1{,}02190|=0{,}00023>0{,}0001$$

Talab qilingan aniqlik hosil bo'lmadi. Shuning uchun hisoblashlarni davom ettiramiz.

$$f(a_2)=f(1{,}02157)\approx -0{,}0020765,\quad f(b_2)=f(1{,}02190)\approx 0{,}000796,\quad f`(b_2)=f`(1{,}02190)\approx 3{,}750002$$

$$a_3=a_2-f(a_2)\frac{b_2-a_2}{f(b_2)-f(a_2)}=1{,}02167-(-0{,}0020765)\cdot\frac{1{,}02190-1{,}02167}{0{,}0007976-(-0{,}0020765)}\approx 1{,}0218097$$

$$b_3=b_2-\frac{f(b_2)}{f`(b_2)}=1{,}02190-\frac{0{,}0007976}{3{,}750002}\approx 1{,}0218074;$$

$$\Delta_3=|a_3-b_3|=|1{,}0218097-1{,}0218074|=0{,}0000023<0{,}0001$$

Shunday qilib, tenglama ildizining 0,0001 aniqlikdagi taqribiy qiymati $x\approx\frac{1}{2}(1{,}0218074+1{,}0218097)=1{,}02180085$ ga teng bo'lar ekan.

Tajriba ishi variantlari

Quyidagi tenglamalarning musbat ildizlari 10^{-3} aniqlikda topilsin.

1. $f(x)=2x-\cos x$	16. $f(x)=x^3-\cos \pi x$
2. $f(x)=3(2-x)-tgx$	17. $f(x)=x^5-x^2-0{,}5$
3. $f(x)=\frac{x}{2}-\lambda gx-3$	18. $f(x)=(x-1)^2-\cos x$
4. $f(x)=\sin \pi x-\frac{x}{3}$	19. $f(x)=x^4-3x^3-5x+2$
5. $f(x)=\frac{x}{2}-\sin^2 x$	20. $f(x)=(x-1)-e^{-x}$
6. $f(x)=e^{-x}-\cos\frac{\pi}{2}x$	21. $f(x)=\lambda gx-\frac{1}{x^2}$
7. $f(x)=\sqrt{x}-e^{-x}$	22. $f(x)=2-x-ctgx$
8. $f(x)=\lambda nx+\sqrt{x}$	23. $f(x)=2x-tgx$
9. $f(x)=x-3\cos^2 1{,}04x$	24. $f(x)=2-x-2\lambda gx$
10. $f(x)=\frac{1}{x}-\cos\pi x$	25. $f(x)=3x-tg\frac{\pi}{4}x$
11. $f(x)=x^2-ctg\frac{x}{6}$	26. $f(x)=1-x-x\sqrt{x}$
12. $f(x)=2\lambda gx+x$	27. $f(x)=\sqrt{x+1}-\frac{1}{x}$
13. $f(x)=\frac{x}{3}-2-\lambda gx$	28. $f(x)=1-x-\sin x$
14. $f(x)=\lambda gx-\cos x$	29. $f(x)=3-x^2-e^{\frac{x}{2}}$
15. $f(x)=ctg\frac{\pi}{3}x-x^2$	30. $f(x)=\sin x+\cos x$

5-Tajriba ishi

Bir argumentli funksiya differentsiali yordamida funksiya qiymatini taqribiy hisoblash

$y=f(x)$ funktsiya $[a,b]$ kesmada differensiallanuvchi bo'lsin: $Lim\frac{\Delta y}{\Delta x}=f'(x)$ bundan $\frac{\Delta y}{\Delta x}=f`(x)+\varepsilon$ bu erda $\Delta x\to 0$ da $\varepsilon\to 0$ oxirgi tenglikdan esa $\Delta y=f`(x)\cdot\Delta x+\varepsilon\cdot\Delta x$ (1) hosil bo'ladi.

Shunday qilib, funksiya orttirmasi Δy va differensiali $f'(x)\Delta x$ orasidagi ayirma $\varepsilon\Delta x$ ga teng va bu miqdor Δx ga nisbatan yuqori tartibli cheksiz kichik miqdordir, shuning uchun $f`(x)\neq 0$ da $Lim\frac{\Delta y}{f`(x)\Delta x}=1$ (2)

ya'ni funksiya orttirmasi va uning differensiali ekvivalent cheksiz kichik miqdorlardir. Shuning uchun taqribiy hisoblashlarda ba'zan

$$\Delta y\approx f`(x)dx \quad (3)$$

taqribiy tenglikdan yoki yoyiqroq ko'rinishdagi ($y=x$, bo'lganda $dx=\Delta x$ ekanini e'tiborga olib)

$$f(x+\Delta x)-f(x)\approx f`(x)\Delta x \text{ yoki } f(x+\Delta x)\approx f(x)+f`(x)\Delta x \quad (4)$$

tenglikdan foydalaniladi, bu esa hisoblashni osonlashtiradi.

Namunaviy misollar yechimi

1-misol: $\cos 32^0 13'$ hisoblansin.

Yechish:

$\cos 32^0 13'$ ni $\cos(30^0 + 2^0 13')$ deb yozib, (4) ning chap qismidagi $f(x+\Delta x)$ ga asosan $x = 30^0$, $\Delta x = 2^0 13'$ ligini ko'ramiz. U holda $f(x) = \cos x$ va $f'(x) = -\sin x$ bo'ladi.

$x = 30^0$ da $f(30^0) = \cos 30^0 = \frac{\sqrt{3}}{2} \approx 0{,}866$ $f`(30^0) = -\sin 30^0 = -0{,}5$ dir

$1^0 \approx 0{,}01745$ va $1' \approx 0{,}00029$ lardan foydalanib, $\Delta x = 2^0 13'$ ning radian qiymatini topamiz. $\Delta x = 2^0 13` \approx 2 \cdot 0{,}01745 + 13 \cdot 0{,}00029 \approx 0{,}0387$ topilgan qiymatlarni $f(x+\Delta x) \approx f(x) + f`(x)\Delta x$ ga qo'yamiz. $\cos 32^0 13` = \cos(30^0 + 2^0 13`) \approx 0{,}866 + (-0{,}5) \cdot 0{,}0387 = 0{,}8463$ hosil bo'ladi. Ma'lumki masalan, **Bradis** jadvalida $\cos 3^2 13` \approx 0{,}8460$

2-misol: $\sqrt[3]{26{,}19}$ ning taqribiy qiymati topilsin.

Yechish:

Bu erda $f(x) = \sqrt[3]{x}$, $f`(x) = \frac{1}{3\sqrt[3]{x^2}}$ $x = 27, \Delta x = -0{,}81$ deb olib, $f(x+\Delta x) \approx f(x) + f`(x)\Delta x$ formulaga asosan quyidagini topamiz:

$\sqrt[3]{26{,}19} = \sqrt[3]{27 - 0{,}81} \approx \sqrt[3]{27} - \frac{0{,}81}{3\sqrt[3]{27^2}} 3 - \frac{0{,}81}{3 \cdot 9} \approx 2{,}97$ Demak, $\sqrt[3]{26{,}19} \approx 2{,}97$

Tajriba ishi variantlari

1. $\sin 25^0 13`$	*ва* $\sqrt[3]{129}$	14. $\cos 41^0 05`$	*ва* $\sqrt[4]{189}$
2. $\cos 40^0 54`$	*ва* $\sqrt[5]{183}$	15. $tg 61^0 34`$	*ва* $\sqrt[4]{213}$
3. $tg 63^0 12`$	*ва* $\sqrt[3]{189}$	16. $\sin 48^0 23`$	*ва* $\sqrt[5]{623}$
4. $\sin 47^0 36`$	*ва* $\sqrt[5]{213}$	17. $\cos 56^0 54`$	*ва* $\sqrt[7]{834}$
5. $\cos 53^0 17`$	*ва* $\sqrt[4]{196}$	18. $tg 33^0 16`$	*ва* $\sqrt[6]{628}$
6. $tg 32^0 43`$	*ва* $\sqrt[4]{213}$	19. $\sin 64^0 15`$	*ва* $\sqrt[5]{453}$
7. $\sin 61^0 02`$	*ва* $\sqrt[5]{623}$	20. $\cos 26^0 14`$	*ва* $\sqrt[4]{196}$
8. $\cos 25^0 34`$	*ва* $\sqrt[6]{329}$	21. $tg 44^0 44`$	*ва* $\sqrt[5]{123}$
9. $tg 40^0 54`$	*ва* $\sqrt[7]{834}$	22. $\sin 33^0 15`$	*ва* $\sqrt[4]{71{,}23}$
10. $\sin 32^0 43`$	*ва* $\sqrt[5]{453}$	23. $\cos 52^0 18`$	*ва* $\sqrt[5]{133}$
11. $\cos 53^0 17`$	*ва* $\sqrt[5]{213}$	24. $tg 22^0 53`$	*ва* $\sqrt[4]{281}$
12. $tg 25^0 13`$	*ва* $\sqrt[4]{196}$	25. $\sin 27^0 14`$	*ва* $\sqrt[4]{362}$
13. $\sin 27^0 23`$	*ва* $\sqrt[5]{183}$		

Birlashgan usulga tuzilgan dastur

```
uses crt;
const e=0.001;
```

```
var
   x,a,b:real;
   i:integer;
Function f(x:real):real;
begin
f:=sqrt(1.19)*0.6*exp(0.6*x)+(0.36-2*x*x*x/sqr(0.36+x*x*x));
end;
Function f1(x:real):real;
begin
 f1:=sqrt(1.19)*0.6*exp(0.6*x)+(x/0.36+x*x*x);
end;
begin clrscr;
 a:=-1;
 b:=1;
 i:=0;
 Repeat
   a:=a-f(a)*(b-a)/(f(b)-f(a));
   b:=b-f(b)/f1(b);
   i:=i+1;
   writeln('a=',a:6:6,'   b=',b:6:6, ' i =',i);
 Until abs(a-b)<e;
 writeln;
Writeln('Natija = ',(a+b)/2:6:6,'   i=',i);
readln
end.
```

6-Tajriba ishi

Interpolyatsiyalash. Lagranjning interpolyatsion formulasi

Biror hodisani o'rganishda y va x miqdorlar orasida shu hodisaning miqdor tomonini aniqlovchi funksional bog'lanish borligi aniqlangan bo'lsin; bunda $y = f(x)$ funksiya noma'lum bo'lib, lekin tajriba asosida argumentning $[a,b]$ kesmadagi x_0, x_1, x_2, …, x_n qiymatlarida funktsiyaning y_0, y_1, y_2,…,y_n qiymatlari mos kelgan bo'lsin. Bu erda asosiy masala $y = f(x)$ noma'lum funktsiyani $[a,b]$ kesmada aniq yoki taqribiy tasvirlaydigan, hisoblash uchun mumkin qadar qulay (masalan $P_n(x)$ ko'phad) shaklidagi funktsiyani topishdan iborat.

Bunday quyilgan masalani interpolyatsiyalash masalasi, $P_n(x)$-ko'phadni esa interpolyatsion ko'phad deyiladi. x_0, x_1, x_2, …, x_n lar interpolyatsiya tugunlari, tugunlar orasidagi masofa $h = x_n - x_{n-1}$ interpolyatsiya qadami deyiladi.

$P_n(x)$ interpolyatsion ko'phadni noma'lum koeffitsientlarni oson topish imkonini beradigan shaklida izlaymiz:

$$\begin{aligned} P_n(x) &= C_0(x-x_1)(x-x_2)...(x-x_n)+C_1(x-x_0)(x-x_2)...(x-x_n)+ \\ &+C_2(x-x_0)(x-x_1)(x-x_3)...(x-x_n)+...+C_n(x-x_0)(x-x_1)...(x-x_{m-1}) \end{aligned} \quad (1)$$

$C_0, C_1, \ldots, C_n$ koeffitsientlarni

$$P_n(x_0)=y_0,\ \ P_n(x_1)=y_1,...,\ P_n(x_n)=y_n \quad (2)$$

shartni qanoatlantiradigan qilib izlaymiz. (1) formulaga $x=x_0$ ni qo'ysak va (2) ni hisobga olsak, $y_0 = C_0(x_0-x_1)(x_0-x_2)...(x_0-x_n)$ ni hosil qilamiz va bundan $C_0 = \dfrac{y_0}{(x_0-x_1)(x_0-x_2)...(x_0-x_n)}$ ni topamiz.

So'ngra (1) ga $x=x_1$ ni quyib va (2) ni hisobga olib C_1 ni topamiz: $y_1 = C_1(x_1-x_0)(x_1-x_2)...(x_1-x_n)$, $C_1 = \dfrac{y_1}{(x_1-x_0)(x_0-x_2)...(x_0-x_4)}$ ni topamiz. Shu usul bilan C_2, $C_3, \ldots, C_n$ larni topamiz:

$$C_2 = \frac{y_2}{(x_2-x_0)(x_2-x_1)(x_2-x_3)...(x_2-x_n)},$$

$$C_3 = \frac{y_3}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)...(x_3-x_n)},$$

....,

$$C_n = \frac{y_n}{(x_n-x_0)(x_n-x_1)(x_n-x_2)...(x_n-x_{n-1})}$$

Topilgan $C_0, C_1, C_2, \ldots, C_n$ larni (1) ga qo'yib Lagranjning interpolyatsion formulasini topamiz:

$$P_n(x) = \frac{(x-x_1)(x-x_2)...(x-x_n)}{(x_0-x_1)(x_0-x_2)...(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)...(x-x_n)}{(x_1-x_0)(x_1-x_2)...(x_1-x_n)}y_1 + ... + \frac{(x-x_0)(x-x_1)...(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)...(x_n-x_{n-1})}y_n$$

Agar $\varphi(x)$ funktsiya $[a,b]$ kesmada $(n+1)$ - tartibli hosilaga ega bo'lsa, $\varphi(x)$ funktsiyani $P_n(x)$ ko'phad bilan almashtirishdagi Xatolik $R(x)=\varphi(x)-P_n(x)$ quyidagi tengsizlikni qanoatlantiradi:

$$|R_n(x)| < |\varphi(x)-P_n(x)| = |(x-x_0)(x-x_1)...(x-x_n)|\frac{1}{(n+1)}\max|\varphi^{(n+1)}(x)|$$

bu formula Lagranj interpolyatsion formulasi uchun baholash formulasi deyiladi.

Misol: Tajriba davomida $x_0=1$ da $y_0=2$, *da* $x_1=3$ da $y_1=1$, $x_2=4$ da $y_2=3$ va $x_3=5$ da $y_3=5$ qiymatlar qabul qilingan. $y=\varphi(x)$ funktsiyani taqribiy $P_3(x)$ ko'phad ko'rinishida topish talab qilinadi.

Yechilishi: x va y o'zgaruvchilarning qiymatlariga ko'ra jadval tuzatish. Bu erda n=3 ga teng

x	$x_0=1$	$x_1=3$	$x_2=4$	$x_3=5$
y	$y_0=2$	$y_1=1$	$y_2=3$	$y_3=6$

U holda Lagranjning interpolyatsion formulasi quyidagi ko'rinishda bo'ladi:

$$\begin{aligned} P_3(x) &= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 + \\ &+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3 \end{aligned}$$

x_0, x_1, x_2, x_3 va y_0, y_1, y_2, y_3 larning son qiymatlarini formulaga quyamiz:

$$P_3(x) = \frac{(x-3)(x-4)(x-5)}{(1-3)(1-4)(1-5)}\cdot 2 + \frac{(x-1)(x-4)(x-5)}{(3-1)(3-4)(3-5)}\cdot 1 + \frac{(x-1)(x-3)(x-5)}{(4-1)(4-3)(4-5)}\cdot 3 + \frac{(x-1)(x-3)(x-4)}{(5-1)(5-3)(5-4)}\cdot 6 =$$

$$= -\frac{x^3-9x^2+20x-3x^2+27x-60}{12} + \frac{x^3-9x^2+20x-x^2+9x-20}{4} + \frac{x^3-4x^2+3x-5x^2+20x-15}{1} + \frac{x^3-4x^2+3x-4x^2+16x-12}{4}$$

$$= \frac{1}{12}(-x^3+12x^2-47x+60+3x^3-30x^2+87x-60+12x^3+108x^2-276x+180+9x^3-72x^2+171x-108) =$$

$$= \frac{1}{12}\left(-x^3+18x^2-65x+72\right) = -\frac{1}{12}x^3 + \frac{18}{12}x^2 - \frac{65}{12}x + \frac{72}{12}$$

Shunday qilib izlangan ko'phad quyidagi ko'rinishda bo'lar ekan

$$P_3(x) = -\frac{1}{12}x^3 + \frac{3}{2}x^2 - \frac{65}{12}x + 6$$

Tekshirish

$x_0=1$ da $\quad y_0 = P_3(1) = -\frac{1}{12} + \frac{3}{2} - \frac{65}{12} + 6 = \frac{-1+18-65+72}{12} = \frac{24}{12} = 2$

$x_1=3$ da $\quad y_1 = P_3(3) = -\frac{1}{12}\cdot 27 + \frac{3}{2}\cdot 9 - \frac{65}{12}\cdot 6 = -\frac{9}{4} + \frac{27}{2} - \frac{65}{4} + 6 = \frac{-9+54-65+24}{4} = \frac{4}{4} = 1$

$x_2=4$ da $\quad y_2 = P_3(4) = -\frac{1}{12}\cdot 64 + \frac{3}{2}\cdot 16 - \frac{65}{12}\cdot 4 + 6 = -\frac{16}{3} + 24 - \frac{65}{3} + 6 = -27+30 = 3$

$x_3=5$ da $\quad y_3 = P_3(5) = -\frac{1}{12}\cdot 125 + \frac{3}{2}\cdot 25 - \frac{65}{12}\cdot 5 + 6 = \frac{-125+450-325+72}{12} = \frac{72}{12} = 6$

Nyutonning interpolyatsion formulasi $y=\varphi(x)$ funktsiyaning $x_0, x_1, x_2, ..., x_n$ $(n+1)$ ta argumenti qiymatlarida qabul qiladigan $y_0, y_1, ..., y_n$ qiymatlari bizga ma'lum bo'lsin. Bu erda argumentning qo'shni qiymatlari orasidagi ayirma o'zgarmas bo'lib, uni h orqali ifodalaylik. Shunday qilib, noma'lum funktsiya $y=\varphi(x)$ funktsiyaning argumentlari qiymatlariga mos, qiymatlari jadvaliga ega bo'lamiz.

x	x_0	$x_1=x_0+h$	$x_2=x_0+2h$	……	$x_n=x_0+nh$
y	y_0	y_1	y_2	……	y_n

Argumentning tegishli qiymatlarida darajasi n dan oshmaydigan tegishli qiymatlar qabul qiladigan $P_n(x)$ ko'phad tuzish talab etilgan bo'lsin. Bu ko'phad $\varphi(x)$ funksiyani taqriban ifodalaydi.

Birinchi, ikkinchi, …, n – tartibli chekli ayirmalarni tuzamiz:

$$\Delta y_0 = y_1 - y_0,\quad \Delta y_1 = y_2 - y_1,\quad \Delta y_2 = y_3 - y_2, ...,$$
$$\Delta^2 y_0 = \Delta y_1 = \Delta y_0,\quad \Delta^2 y_1 = \Delta y_2 - \Delta y_1, ...,$$
$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0, ...,$$
$$...$$
$$\Delta^n y_0 = \Delta^{n-1} y_1 - \Delta^{n-1} y_0.$$

x_0 va x_1 ga mos y_0 va y_1 qiymatni qabul qiladigan 1-darajali ko'phadni yozamiz: $\quad P(x) = y_0 + \Delta y_0 \frac{x-x_0}{h} \qquad (1)$

Haqiqatan ham $\quad P_1(x)\Big|_{x=x_0} = y_0,\quad P_1(x)\Big|_{x=x_1} = y_0 + \Delta y_0 \frac{h}{h} = y_0 + (y_1 - y_0) = y_1$

x_0, x_1, x_2 ga mos y_0, y_1, y_2 qiymatlarni qabul qiluvchi 2-darajali $P_2(x)$ ko'phadni yozamiz: $P_2(x)=y_0+\Delta y_0\frac{x-x_0}{h}+\frac{\Delta^2 y_0}{21}\frac{x-x_0}{h}\left(\frac{x-x_0}{h}-1\right)$ (2)

Haqiqatan,

$$P_2(x)\Big|_{x=x_0}=y_0,\quad P_2(x)\Big|_{x=x_1}=y_1,\quad P_2(x)\Big|_{x=x_2}=y_0+\Delta y_0\cdot 2+\frac{\Delta^2 y_0}{2!}\frac{2h}{h}\left(\frac{2h}{h}-1\right)=y_2$$

3-darajali ko'phad quyidagi ko'rinishda bo'ladi:

$$P_3(x)=y_0+\Delta y_0\frac{x-x_0}{h}+\frac{\Delta^2 y_0}{2!}\frac{x-x_0}{h}\left(\frac{x-x_0}{h}-1\right)+\frac{\Delta^3 y_0}{3!}\frac{x-x_0}{h}\left(\frac{x-x_0}{h}-1\right)\left(\frac{x-x_0}{h}-2\right)\qquad(3)$$

Umuman olganda $x_0, x_1, \dots, x_n$ larga mos $y_0, y_1, \dots, y_n$ qiymatlarni qabul qiluvchi n-darajali $P_n(x)$ ko'phad quyidagi ko'rinishda bo'ladi:

$$P_n(x)=y_0+\Delta y_0\frac{x-x_0}{h}+\frac{\Delta^2 y}{2!}\frac{x-x_0}{h}\left(\frac{x-x_0}{h}-1\right)+\dots+\frac{\Delta^n y_0}{n!}\frac{x-x_0}{h}\left(\frac{x-x_0}{h}-1\right)\dots\left(\frac{x-x_0}{h}-(n-1)\right)\qquad(4)$$

Bu formula Nyutonning interpolyatsion formulasi deyiladi. Agar $q=\frac{x-x_0}{h}$ belgilash kiritilsa (4) formulaning ko'rinishi quyidagicha bo'ladi:

$$P_n(x)=y_0+q\Delta y_0+\frac{q(q-1)}{2!}\Delta^2 y_0+\dots+\frac{q(q-1)\dots(q-n-1)}{n!}\Delta^n y_0$$

Nyutonning interpolyatsion formulasini qoldiq hadlarini baholash formulasi quyidagidan iborat.

$$R_n(x)=x^{n+1}\frac{q(q-1)\dots(q-n)}{(n+1)!}\cdot\varphi^{(n+1)}(\xi)$$, bu erda $q=\frac{x-x_0}{h}$, $\xi\in[x_0;x_n]$

Amalda $y=\varphi(x)$ funksiyaning analitik ko'rinishi har doim ma'lum bo'lavermaydi, shuning uchun Nyutonning interpolyatsion formulasining xatolik formulasi

$$R_n(x)\approx\frac{q(q-1)\dots(q-n)}{(n+1)1}\Delta^{n+1}y_0$$ ko'risnhda bo'ladi.

Misol: Tajriba natijasida quyidagi jadvalda keltirilgan qiymatlar aniqlangan bo'lsin. $y=\varphi(x)$ funksiyani taqribiy $P_3(x)$ ko'phad ko'rinishida topish talab qilinadi.

x	*1*	*3*	*5*	*7*
y	*-2*	*1*	*7*	*10*

Yechilishi: Avval chekli ayirmalarni tuzamiz, bu erda $x_0=1$, $y_0=-2$, $x_1=3$, $y_1=1$, $x_2=5$, $y_2=7$, $x_3=7$, $y_3=10$ ва n=3

$$\Delta y_0=y_1-y_0=1-(-2)=3$$
$$\Delta y_1=y_2-y_1=7-1=6$$
$$\Delta y_2=y_3-y_2=10-7=3$$
$$\Delta^2 y_0=\Delta y_1-\Delta y_0=6-3=3$$
$$\Delta^2 y_1=\Delta y_2-\Delta y_1=3-6=-3$$
$$\Delta^3 y_0=\Delta^2 y_1-\Delta^2 y_0=-3-3=-6$$

n=3 hol uchun Nyuton ko'phadi $P_3(x)$ quyidagi ko'rinishda bo'ladi.

$$P_3(x)=y_0+\Delta y_0\frac{x-x_0}{h}+\frac{\Delta^2 y_0}{2!}\frac{x-x_0}{h}\left(\frac{x-x_0}{h}-1\right)+\frac{\Delta^3 y_0}{3!}\frac{x-x_0}{h}\left(\frac{x-x_0}{h}-1\right)\left(\frac{x-x_0}{h}-2\right)$$

Bu erda n=2 ga teng. U holda

$$P_3(x)=-2+3\frac{x-1}{2}+\frac{3}{2!}\frac{x-1}{2}\left(\frac{x-1}{2}\right)+\frac{-6}{3!}\frac{x-1}{2}\left(\frac{x-1}{2}-1\right)\left(\frac{x-1}{2}-2\right)=-2+\frac{3}{2}x-\frac{3}{2}+\frac{3}{2}\cdot\frac{x-1}{2}\frac{x-1-2}{2}+\frac{-6}{6}\cdot\frac{x-1}{2}\frac{x-1-2}{2}\frac{x-1-4}{2}=$$

$$=-2+\frac{3}{2}x-\frac{3}{2}+\frac{3}{8}(x-1)(x-3)-\frac{1}{8}(x-1)(x-3)(x-5)=$$

$$=-\frac{7}{2}+\frac{3}{2}x+\frac{3}{8}\left(x^2-4x+3\right)-\frac{1}{8}\left(x^3-4x^2+3x-5x^2+20x-15\right)=\frac{1}{8}\left(-28+12x+3x^2-12x+9-x^3+9x^2-23x+15\right)=$$

$$=\frac{1}{8}\left(-x^3+12x^2-23x-13\right)=-\frac{1}{8}x^3+\frac{3}{2}x^2-\frac{23}{8}x-\frac{1}{2}$$

Tekshirish

$x_0=1$ da $y_0=-\frac{1}{8}+\frac{3}{2}-\frac{23}{8}-\frac{1}{2}=-\frac{24}{8}+1=-3+1=-2$

$x_1=3$ da $y_1=-\frac{27}{8}+\frac{3}{2}\cdot 9-\frac{23}{8}\cdot 3-\frac{1}{2}=-\frac{27}{8}-\frac{69}{8}+\frac{27}{2}-\frac{1}{2}=-\frac{96}{8}+\frac{26}{2}=-12+13=1$

$x_2=5$ da $y_2=-\frac{125}{8}+\frac{3}{2}\cdot 25-\frac{23}{8}\cdot 5-\frac{1}{2}=-\frac{125}{8}-\frac{115}{8}+\frac{75}{2}-\frac{1}{2}=-\frac{240}{8}+\frac{74}{2}=-30+37=7$

$x_3=7$ da $y_3=-\frac{343}{8}+\frac{3}{2}\cdot 49-\frac{23}{8}\cdot 7-\frac{1}{2}=-\frac{343}{8}-\frac{161}{}+\frac{147}{2}-\frac{1}{2}=-\frac{504}{8}+\frac{146}{2}=-63+73=10$

Tajriba ishi variantlari

Tajriba davomida x_0, x_1, x_2, x_3 argumentning mos qiymatlariga, y_0, y_1, y_2, y_3 qiymatlarini qabul qiladigan. $y=\varphi(x)$ funksiyaning taqribiy $P_4(x)$ ko'phad ko'rinishidagi **Lagranj** va **Nyuton**ning interpolyatsion formulalari tuzulsin.

№	x_0	x_1	x_2	x_3	y_0	y_1	y_2	y_3
1	1	3	5	7	1	-2	3	4
2	1	3	5	7	-3	-1	5	6
3	-3	-2	-1	0	4	2	5	7
4	-2	0	2	4	4	8	10	12
5	-2	-1	0	1	3	7	2	5
6	2	3	4	5	-1	1	5	8
7	0	2	4	6	5	7	8	10
8	1	2	3	4	-2	-1	0	4
9	0	1	2	3	3	5	6	8
10	-3	-2	-1	0	5	7	8	10
11	2	3	4	5	1	-2	3	4
12	0	2	4	6	-2	-1	0	4
13	3	5	7	9	3	0	1	5
14	1	4	7	10	-2	0	1	4
15	2	3	4	5	4	8	9	10
16	1	2	3	4	3	2	7	5
17	5	4	3	2	3	2	7	5
18	4	3	2	1	5	7	8	10
19	3	2	1	0	1	-2	3	4
20	2	1	0	-1	-1	-2	6	5
21	1	0	-1	-2	3	5	6	7
22	4	2	0	-2	3	-1	2	5

23	7	5	3	1	0	2	3	4
24	1	2	3	4	3	5	2	4
25	0	-2	1	3	4	5	7	8
26	1	3	5	7	-2	-1	0	4
27	3	5	7	9	-2	0	1	4
28	1	3	5	7	3	7	2	5
29	0	2	4	6	-3	0	1	4
30	2	3	4	5	1	-2	3	4

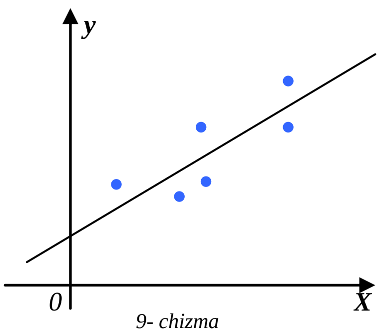
7-Tajriba ishi

Eng kichik kvadratlar usuli

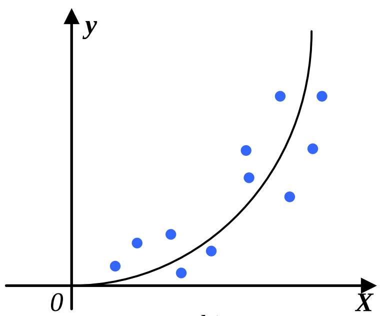
y miqdorning x miqdorga funksional bog`liqligini tajribaga asosan aniqlash talab etilsin. Tajriba natijasida argumentning $n$ taqiymatlari uchun funksiyaning ham $n$ taqiymatlari quyidagi jadvalda ko`rsatilganidekaniqlangan bo`lsin:

x	x_1	x_2	x_3	……	x_n
y	y_1	y_2	y_3	……	y_n

Funksional bog'liqlik $y=\varphi(x)$ ni tajribaga asosan aniqlash talab etilsin. Bu funksiyaning ko'rinishi nazariy mulohazalarga yoki tajribada olingan qiymatlarga mos keladigan nuqtalarning koordinatalar tekisligida qanday joylashganiga qarab aniqlanadi. Masalan, nuqtalar 9-chizma yoki 10-chizma-dagidek joylashgan bo'lsa, u vaqtda tajriba natijasida yo'l qo'yilishi mumkin bo'lgan



9- chizma



10 chizma

xatolikni hisobga olib izlanayotgan funksiyalarni mos ravishda $y=ax+b$ va $y=ax^b$ ko'rinishlarida topish tabiiy.

Shunday qilib, funksiyani $y=\varphi(x, a, b, c, \ldots)$ ko'rinishda tanlab olingach, a, b, c,…, parametrlarni shunday tanlash kerak bo'ladiki, u funksiya o'rganilayotgan hodisani biror ma'noda juda yaxshi tarzda aks ettirsin.

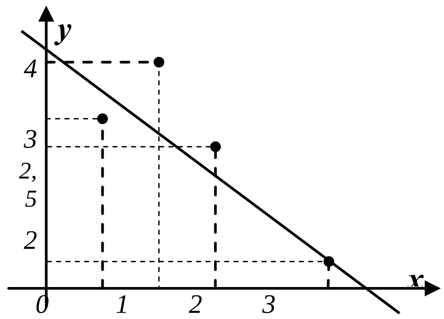
Eng kichik kvadratlar usuli quyidagichadir, tajribada olingan y_k ($k=1,\ldots n$) qiymatlar bilan mos nuqtalardagi $\varphi(x, a, b, c, \ldots)$ funksiya qiymatlari orasidagi ayirmalar kvadratlarning yig'indisini qaraymiz:

$$S(a,b,c,...)=\sum_{k=1}^{n}\left[y_k-\varphi\left(x_k a,b,c,...\right)\right]^2 \qquad (1)$$

Endi a, b, c, … parametrlarni shunday tanlab olamizki (1) yig\`indi eng kichik qiymat qabul qilsin. Demak, $\frac{\partial S}{\partial a}=0,\frac{\partial S}{\partial b}=0,\frac{\partial S}{\partial c}=0,......$tenglik bajarilishi lozim. Bu tenglamalarni yoyilgan ko'rinishida

$$\left.\begin{array}{l}\sum_{k=1}^{n}[y_k-\varphi(x_k,a,b,c,...)]\cdot\frac{\partial\varphi(x_k,a,b,c,...)}{\partial a}=0\\ \sum_{k=1}^{n}[y_k-\varphi(x_k,a,b,c,...)]\cdot\frac{\partial\varphi(x_k,a,b,c,...)}{\partial b}=0\\ ....\end{array}\right\} \qquad (2)$$

yozish mumkin. Bu erda qancha noma'lum bo'lsa, shuncha tenglama bo'ladi. $y=\varphi(x,a,b,c,...)$ funksiyani qanday aniqlashni quyidagi misolda ko'rib chiqamiz. $y=ax+b$ bo'lsin. Bu erda $S(a,b)$ funksiya quyidagi ko'rinishda bo'ladi:



11- chizma

$$S(a,b)=\sum_{k=1}^{n}\left[y_k-\left(ax_k+b\right)\right]^2 \qquad (3)$$

Bu funksiya *a* va *b* o'zgaruvchilarning funksiyasidir. Shuning uchun

$$\left.\begin{array}{l}\frac{\partial S}{\partial a}=-2\sum_{k=1}^{n}\left[y_k-\left(ax_k+b\right)\right]\cdot x_k=0\\ \frac{\partial S}{\partial b}=-2\sum_{k=1}^{n}\left[y_k-\left(ax_k+b\right)\right]=0\end{array}\right\}$$

ya'ni (2) tenglamalar sistrmasi bu holda quyidagi ko'rinishni olamiz:

$$\left.\begin{array}{l}\sum_{k=1}^{n}y_k\cdot x_k-a\cdot\sum_{k=1}^{n}x_k^2-b\sum_{k=1}^{n}x_k=0\\ \sum_{k=1}^{n}y_k-a\cdot\sum_{k=1}^{n}x_k-b\cdot n=0\end{array}\right\} \qquad (4)$$

Bu sistemadan $S(a,b)$ funksiyaga minimum qiymat beradigan *a* va *b* larning qiymatlari topiladi.

Faraz qilaylik argumentning to'rtta qiymatida (ya'ni *n=4* bo'lganda) izlangan funksiyaning тajribaga asosan тo'rtta qiymati olingan bo'lib; bu qiymatlar

x	*1*	*2*	*3*	*5*
y	*3*	*4*	*2,5*	*0,5*

jadval shaklida berilgan bo'lsin. $\varphi(x)$ funksiyani $y=ax+b$ chiziqli funksiya ko'rinishda topamiz, $S(a, b)$ uchun quyidagi ifodani tuzamiz:

$$S(a,b)=\sum_{k=1}^{n}\left[y_k-\left(ax_k+b\right)\right]^2$$

a va *b* o'zgaruvchilarning qiymatlarini (4) sistema yordamida topamiz. (4) sistemani tuzish uchun oldin quyidagilarni hisoblaymiz:

$$\sum_{k=1}^{4} y_k x_k = 3\cdot 1 + 4\cdot 2 + 2{,}5\cdot 3 + 0{,}5\cdot 5 = 21,$$

$$\sum_{k=1}^{4} x_k^2 = 1^2 + 2^2 + 3^2 + 5^2 = 39,$$

$$\sum_{k=1}^{4} x_k = 1 + 2 + 3 + 5 = 11,$$

$$\sum_{k=1}^{4} y_k = 3 + 4 + 2{,}5 + 0{,}5 = 10$$

(4) sistema ushbu ko'rinishni oladi: $\begin{cases} 21 - 39a - 11b = 0, \\ 10 - 11a - 4b = 0 \end{cases}$ bu sistemani yechib $a = -\dfrac{26}{35}$ va $b = \dfrac{159}{35}$ ekanini topamiz. Demak, izlangan funksiya $y = -\dfrac{26}{35}x + \dfrac{159}{35}$ ko'rinishda bo'lar ekan. U esa to'g'ri chiziq tenglamasidir. (11-chizmaga qarang)

Tajriba ishi variantlari

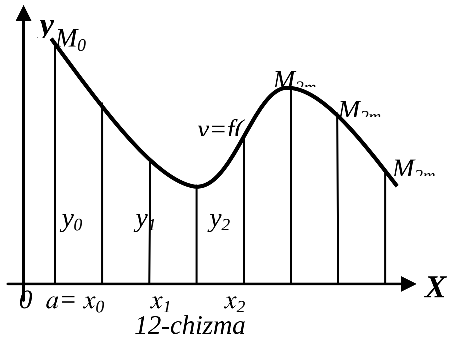
1	x	1,1	2	3,6	5,4	4
	y	10,5	8,3	5	4	3,1
2	x	0,5	2,1	5,2	6	7,3
	y	1	3	2,2	4	5,2
3	x	2,3	5	4,5	3	6
	y	8	7	5,4	3	4
4	x	1	2	3	5	7
	y	1,7	2	3	6	7,3
5	x	1	2,5	4	3,7	5
	y	2,2	3	5	4	6,3
6	x	1	2	3	4	5
	y	9,3	7	8,4	6	5,4
7	x	1,3	2	3,4	7	4
	y	1	3	6	4	9
8	x	1	2	3	4	5
	y	4,6	5,6	4	2,1	2,6
9	x	1	2	3	4	5
	y	4,8	6,8	4,3	2,3	2,8
10	x	1	2	3	4,2	6
	y	3,2	4,3	2,6	6,7	1,3
11	x	1	2,1	4,2	3,5	6
	y	3,5	4,4	2,9	0,5	1,4
12	x	1,1	2	3,5	5,7	6,9
	y	3,8	4,6	3,2	0,9	1,6
13	x	1	2	3	4,5	7
	y	2,7	3,8	2,3	0,8	0,3
14	x	1	2	2	4	5
	y	4	5	3,5	1,5	2
15	x	1	3	6	4	5
	y	4,1	5,1	3,6	1,4	2,1

16	x	2,8	3	4	5	3,4
	y	6	5,4	3,2	2	1,3
17	x	1	2	3	4	5
	y	4,4	5,4	3,9	1,9	2,4
18	x	1	2	3,2	4,5	5
	y	4,2	3,2	5,6	7,2	4,9
19	x	1,5	2	3,7	4,5	5,2
	y	4,5	2,5	6,2	7,5	3,7
20	x	1	2	3	4	5
	y	4,1	5,6	7,1	2,5	3,4
21	x	1	2,5	3,6	4,5	5
	y	3,5	4,5	6,2	7,5	8,6
22	x	1,6	2	3,4	4,5	5
	y	4,1	2,3	4,6	7,8	5,8
23	x	1,6	2	3,5	4,6	5
	y	5,6	7,2	9,5	9,3	6,5
24	x	1	2,5	3,4	4,2	5
	y	4,5	6,5	7,3	4,9	4,8
25	x	1	2,7	3,6	4,7	5
	y	4,8	5,6	7,2	8,7	9,1
26	x	1	2,7	3,8	4,7	5
	y	1,6	4,7	9,5	9,7	9,8
27	x	1,5	2,7	3,6	4	5
	y	7,9	6,5	3,7	2,8	4,7
28	x	1,7	3,2	4,1	6,3	7,8
	y	8,1	9,3	6,2	4,2	3
29	x	5,2	34	4	2	1
	y	8,3	7,1	6	5	4
30	x	3,4	5	2,1	4	1
	y	6,1	5	2,3	3	2,1

8-Tajriba ishi

Aniq integrallarni Simpson usuli yordamida taqribiy hisoblash

$\int_a^b f(x)dx$ (1) integral berilgan bo'lsin. $[a,b]$ kesmani $a=x_0$, x_1, x_2, $...x_{2m}=b$ nuqtalar bilan n=2m ta juft sondagi teng $[x_0, x_1]$, $[x_1, x_2]$,… $[x_{2m-1}, x_{2m}]$ kesmalarga ajratamiz x_0, x_1, $...x_{2m}$ nuqtalarda $y=f(x)$ funksiyaning qiymatlari y_0, y_1, y_2, … y_{2m} larni hisoblaymiz:

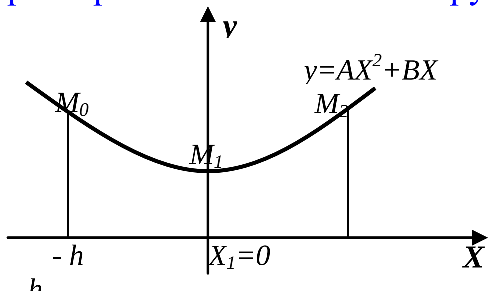


12-chizma

$y_0 = f(x_0)$, $y_1 = f(x_1)$, $y_2 = f(x_2)$,…, $y_{2m-1} = f(x_{2m-1})$,
$y_{2m} = f(x_{2m})$ (2)

$[x_0,x]$ va $[x_1,x_2]$ kesmalarga mos va berilgan y=f(x) chiziq bilan chegaralangan egri chiziqli trapetsiyaning yuzini $M_0(x_0,y_0)$, $M_1(x_1,y_1)$, $M_2(x_2,y_2)$ nuqtalardan o'tuvchi va o'qi OY o'qiga parallel bo'lgan ikkinchi darajali parabola bilan chegaralangan egri chiziqli trapetsiyaning yuzi bilan almashtiramiz (12-chizma).

O'qi Oy o'qiga parallel bo'lgan parabolaning tenglamasi $y=Ax^2+Bx+C$ ko'rinishda bo'ladi, bu erda A, B, C koeffitsientlar parabolaning berilgan uchta nuqta orqali o'tish shartidan bir qiymatli ravishda aniqlanadi (13-rasm).



13-chizma

$y=Ax^2+Bx+C$ parabola, Ox o'q va y_0, y_2, ordinatalar bilan chegaralangan egri chiziqli trapetsiyaning S yuzini aniqlaymiz. Yordamchi koordinatalar sistemasini 13- chizmada ko'rsatilganidek joylashtiramiz.

$$S = \int_{-h}^{h} (Ax^2 + Bx + C)dx = \left(A\frac{x^3}{3} + B\frac{x^2}{2} + Cx\right)\Bigg/_{-h}^{h} = \frac{h}{3}(2Ah^2 + 6C) \qquad (3)$$

bu erda $h = x_1 - x_0 = x_2 - x_1$

Demak, parabolik trapetsiyaning S yuzi A va C koeffisientlarga bog'liq ekan. Ularni esa quyidagi shartlardan foydalanib topamiz:

$$\left.\begin{aligned} &x_0 = -h \text{ bo'lsa, u holda } y_0 = Ah^2 - Bh + C \\ &x_1 = 0 \text{ bo'lsa, u holda } y_1 = C \\ &x_2 = h \text{ bo'lsa, u holda } y_2 = Ah^2 + Bh + C \end{aligned}\right\} \qquad (4)$$

Bu tenglamalardan esa (ikkinchisini 4 ga ko'paytirib)

$$y_0 + 4y_1 + y_2 = 2Ah^2 + 6C \qquad (5)$$

formulani topamiz. (3) va (5) formulalardan esa $S = \frac{h}{3}(y_0 + 4y_1 + y_2)$ (6)

formula hosil bo'ladi. Ammo yuqorida aytganlarimizga asosan

$$\int_{x_0}^{x_2} f(x)dx \approx \frac{h}{3}(y_0 + 4y_1 + y_2) \qquad (7)$$

taqribiy hisoblash formulasini yoza olamiz. (7) formulaga o'xshash formulalarni $[x_2, x_4]$, $[x_4, x_6]$,…, $[x_{2m-1}, x_{2m}]$, kesmalar uchun ham yuqoridagi usul bilan isbotlash mumkin. Agar

$$x_1 - x_0 = x_2 - x_1 = x_3 - x_2 = ... = x_{2m} - x_{2m-1} = h$$

ekanini e'tiborga olsak: $\int_{x_2}^{x_4} f(x)dx \approx \frac{h}{3}(y_2+4y_3+y_4)$

$$\int_{x_4}^{x_6} f(x)dx \approx \frac{h}{3}(y_4+4y_5+y_6)$$

....

....

$$\int_{x_{2m-2}}^{x_{2m}} f(x)dx \approx \frac{h}{3}(y_{2m-2}+4y_{2m-1}+y_{2m})$$

formulalar hosil bo'ladi.

(7) va oxirgi formulalardan esa ularning chap va o'ng tomonlarini qo'shib, chapda izlanayotgan integralni, o'ngda esa uning taqribiy qiymatini hosil qilamiz:

$$\int_a^b f(x)dx \approx \frac{h}{3}(y_0+4y_1+2y_2+4y_3+2y_4+...+2y_{2m-2}+4y_{2m-1}+y_{2m})$$

Agar $h=\frac{b-a}{2m}$ ekanini inobatga olsak,

$$\int_a^b f(x)dx \approx \frac{b-a}{2m}\left[y_0+y_{2m}+2(y_2+y_4+...y_{2m-2})+4(y_1+y_3+...+y_{2m-1})\right] \qquad (8)$$

formula hosil bo'ladi. Bu **Simpson** formulasidir, uni ba'zan parabolalar formulasi ham deyiladi. Bu erda bo'linish nuqtalarining soni 2m ixtiyoriy, lekin bu son qancha katta bo'lsa, (8) tenglikning o'ng tomonidagi yigindi-integral qiymatini shuncha aniq ifodalaydi.

Agar $f^{(4)}(x)=y^{(4)}$ mavjud va $[a,b]$ kesmada chegaralangan bo'lsa, (8) formulaning xatosini

$$|\delta_n| \le \frac{M_4(b-a)^5}{180(2m)^4} \qquad (9)$$

tengsizlik yordamida baholash mumkin, bu erda $M_4=Max\left|f^{(4)}(x)\right|$ ya'ni $f^{(4)}(x)$ funktsiyaning $[a,b]$ kesmadagi maksimumining moduli, 2(m – kesmalar soni.

Izoh: Agar u (4) ni topish mushkulroq bo'lsa, $\int_a^b f(x)dx$ integral m ning biror qiymatida hisoblanadi, so'ngra m ning ikkilangan qiymatida integral yana bir bor hisoblanadi. Agar aytaylik, verguldan so'ng uchta raqami o'zaro teng bo'lsa, hisoblangan integralning qiymatida xatolik 0,001 dan oshmaydi.

Misol: $\int_0^{\pi/2} \frac{\sin x}{x}dx$ integral Simpson formulasi yordamida 0,001 aniqlikda hisoblansin.

Yechish: Masalaning shartiga asosan yo'l qo'yiladigan xato 0,001 dan oshmasligi kerak, shuning uchun $\left[0,\frac{\pi}{2}\right]$ kesmani bo'laklarga bo'lishlar soni 2m ni (9) tengsizlikka asosan topamiz: $\frac{M_4(b-a)^5}{180(2m)^4}<0{,}001$ (10) $f(x)=\frac{\sin x}{x}$ funktsiyaning to'rtinchi tartibli hosilasini topamiz:

$f^{(4)}(x)=\frac{x^4\sin x+4x^3+\cos x-12x^2\cdot\sin x-24x\cos x+24\sin x}{x^5}$ $f^{(4)}(x)$ funktsiya $\left[0,\frac{\pi}{2}\right]$ kesmada aniqlangan. $\left|f^{(4)}(a)\right|=24$ $\left|f^{(4)}\left(\frac{\pi}{2}\right)\right|\approx 0{,}05<24$ Demak, $M_4=\max\limits_{0\le x\le\frac{\pi}{2}}\left|f^{(4)}(x)\right|=\left|f^{(4)}(0)\right|=24$

$b-a=\frac{\pi}{2}-0=\frac{\pi}{2}$ va $M_4=24$ larni (10) tengsizlikka qo'llaymiz: $\frac{24\cdot\left(\frac{\pi}{2}\right)^5}{180\cdot(2m)^4}<0{,}001$ yoki $m^4>\frac{24\left(\frac{\pi}{2}\right)^5}{180\cdot 2^4\cdot 0{,}001}\approx 250$ bundan $m\geq 4$ ni topamiz.

Hisoblash uchun m = 5 deb, $\left[0,\frac{\pi}{2}\right]$ kesmani teng 10 qismga bo'lamiz. Bizning misolimiz uchun $n=2m=10, a=0, b=\frac{\pi}{2}$, yuqoridagi (8) formulaga asosan

$$\int_0^{\pi/2}\frac{\sin x}{x}dx\approx\frac{h}{3}\left[y_0+y_{10}+2(y_2+y_4+y_6+y_8)+4(y_1+y_3+y_5+y_7+y_9)\right]$$

bo`ladi,bunda $\frac{h}{3}=\frac{b-a}{3n}=\frac{\frac{\pi}{2}-0}{3\cdot 10}=\frac{90^0}{30}=3^0\approx 0{,}0524$ $y_0, y_1, \ldots, y_{10}$ larni hisoblaymiz, buning uchun quyidagi jadvaldan foydalanamiz.

I	x_i	$\sin x_i$	$y_i=\frac{\sin x_i}{x_i}$
0	$x_0=0$	0	$y_0=1$
1	$x_1=9^0\approx 0{,}1571$	0,156454	$y_1=0{,}995879$
2	$x_2=18^0\approx 0{,}3142$	0,309055	$y_2=0{,}983625$
3	$x3=27^0\approx 0{,}4712$	0,453955	$y_3=0{,}963404$
4	$x_4=36^0\approx 0{,}6283$	0,58777	$y_4=0{,}9354925$
5	$x_5=45^0\approx 0{,}7854$	0,707102	$y_5=0{,}9003157$
6	$x_6=54^0\approx 0{,}9425$	0,80903	$y_6=0{,}8583872$
7	$x_7=63^0\approx 1{,}0821$	0,882946	$y_7=0{,}815956$
8	$x_8=72^0\approx 1{,}2566$	0,951045	$y_8=0{,}7558398$
9	$x_9=81^0\approx 1{,}4137$	0,987886	$y_9=0{,}6986531$
10	$x_{10}=90^0\approx 1{,}5708$	1,00000	$y_{10}=0{,}6366182$

Topilgan qiymatlarni Simpson formulasiga qo'yamiz:

$$\int_0^{\frac{\pi}{2}}\frac{\sin x}{x}dx\approx 0{,}0524\cdot[1+0{,}6366182+2\cdot(0{,}983625+0{,}9354925+0{,}8583872+0{,}7568398+$$
$$+4\cdot(0{,}995879+0{,}953404+0{,}9003157+0{,}815956+0{,}6986531))]=1{,}372992\approx 1{,}373$$

Demak,

$$\int_0^{\pi/2}\frac{\sin x}{x}dx\approx 1{,}373$$

Tajriba ishi variantlari

1. $\int_0^1\frac{arctgx^2}{2x}dx$	9. $\int_{-2}^{2}(2x+3)e^{-x}dx$	17. $\int_{0,3}^{2,3}\sqrt{x+1}\cdot\cos x^2dx$	25. $\int_{-1,5}^{-0,5}x^2\arccos x^2dx$

2. $\int_{0}^{0,5} \frac{x^2 dx}{1+x^4}$	10. $\int_{1}^{2} \frac{dx}{x(1+x^5)}$	18. $\int_{0,1}^{2,1} (x^2-1)e^{-x} dx$	26. $\int_{0,8}^{1,8} \frac{eg(x^2+1)}{x+1} dx$
3. $\int_{1}^{2} \frac{3x dx}{1+x^2}$	11. $\int_{1}^{2} \frac{\sqrt{4-x^3}}{x^4} dx$	19. $\int_{0}^{2} \sqrt{(5-x^2)^4} dx$	27. $\int_{-1}^{1} (1-x)\sin \pi x dx$
4. $\int_{-1}^{0} \sqrt{\arcsin x^3} dx$	12. $\int_{0,5}^{1} x^2 arctg \sqrt{x} dx$	20. $\int_{0,5}^{1,5} \frac{\sin(5x+3)}{x^2+\sqrt{x}} dx$	28. $\int_{-1}^{3} \sqrt[3]{3x^2+2} \cdot dx$
5. $\int_{0}^{1} x\sqrt{1+x^3} dx$	13. $\int_{0}^{2} \ln\left(\sqrt{1+x^2} - \cos x\right) dx$	21. $\int_{0}^{\pi/2} x^2 \cos \sqrt{x} dx$	29. $\int_{1,2}^{2,2} \left(\frac{x^2}{2}+1\right) \sin \frac{x}{2} dx$
6. $\int_{0}^{5} \frac{dx}{x+\sqrt{2x-1}}$	14. $\int_{0}^{5} \sqrt{\frac{x}{3+x^3}} dx$	22. $\int_{0,1}^{0,5} x e^{-\sqrt{x}} dx$	30. $\int_{-1}^{0} \arccos\left(x^2+3x\right) dx$
7. $\int_{0}^{1} \frac{\ln(1+x)}{1+x^2} dx$	15. $\int_{0,4}^{1,4} \sqrt{1+x^2} \sin 5x dx$	23. $\int_{1,3}^{2,3} \left(\frac{\sin x^2}{x^2} - x\right) dx$	
8. $\int_{0}^{1} \frac{e^{-x}}{1+x} dx$	16. $\int_{1,2}^{2,2} \left(\frac{x}{2}+1\right) \sin \frac{x}{2} dx$	24. $\int_{1,5}^{2,5} \left(x^5 - \sqrt{x}\right) \arcsin x dx$	

Simpson formulasi yordamida yuqoridagi berilgan integrallar 0,001 aniqlikda hisoblansin.

Izoh: Har bir integral ikki marta hisoblanadi, ya'ni *n=10* va *n=20* bo`lganda.

Simson usuliga tuzilgan dastur

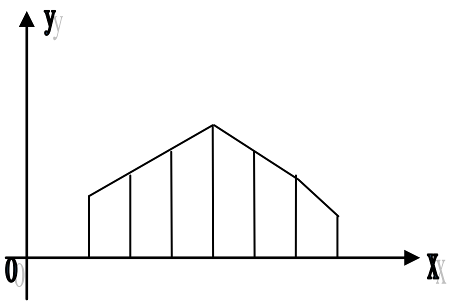
```
Uses Crt;
const
  m=5;
  n=10;
  a=1;
  b=0.5;
var
    h,xk,s1,s2,bb:real;
    i:integer;
    y,x:array [0..n] of real;
Function f(var x:real):real;
begin
   f:=1/x*sqrt(3.61-sqr(x));
end;
begin
    clrscr;
 h:=(b-a)/n;
 bb:=b;
 y[2*m]:=f(bb);
 s1:=0;
 s2:=0;
for i:=0 to m-1 do begin
  x[2*i]:=a+2*i*h;
```

```
  x[2*i+1]:=a+(2*i+1)*h ;
  y[2*i]:=f(x[2*i]);y[2*i+1]:=f(x[2*i+1]);
  s1:=s1+y[2*i];s2:=s2+y[2*i+1];
  writeln('y[',2*i,']=  ',y[2*i]:6:4,'      y[',2*i+1,']=  ',y[2*i+1]:6:4)
                    end;
   writeln('y[',2*m,']=  ',y[2*m]:6:4);

   xk:=abs(b-a)/(3*n)*(y[0]+y[2*m]+2*(s1-y[0])+4*s2);
   write(' Natija= h/3*(y0+4*y1+2*y2+4*y3+2*y4+4*y5+2*y6+4*y7+y8) =
',xk:6:4);

readln;

end.
```

9-Tajriba ish

Birinchi tartibli differentsial tenglamalarni Eyler usuli bilan taqribiy yechish

Faraz qilaylik $\frac{dy}{dx}=f(x,y)$ (1) tenglama berilgan bo'lib, uning $[x_0,b]$ kesmada $y|_{x=x_0}=y_0$ boshlang'ich shartni qanoatlantiruvchi taqribiy echimini topish talab qilinsin. Avvalo $[x_0,b]$ kesmani $x_0,\ x_1,\ x_2,\ \dots\ x_n=b$ nuqtalar bilan n ta teng bo'lakka bo'lamiz (bu erda $x_0<x_1<x_2<\dots<x_n$) va $x_1-x_0=x_2-x_1=\dots=b-x_{n-1}=\Delta x=h$ deb belgilaymiz. Demak $h=\frac{b-x_0}{n}$ bo'ladi $y=\varphi(x)$ funktsiya (1) tenglamaning biror taqribiy echimi va $y_0=\varphi(x_0),\ \ y_1=\varphi(x_1),\ \dots,\ y_n=\varphi(x_n)$ bo'lsin.



14- *chizma*

Endi $\Delta y_0=y_1-y_0,\ \ \Delta y_1=y_2-y_1,\ \dots\ \Delta y_1=y_n-y_{n-1}$ deb belgilaymiz. (1) tenglamadagi $\frac{dy}{dx}$ hosilani $x_0\ \ x_1,\ x_2\dots\ x_n$ nuqtalarda $\frac{\Delta y}{\Delta x}$ chekli ayirmalar nisbati bilan almashtiramiz: $\frac{\Delta y}{\Delta x}=f(x,y)$ (2) bundan esa $\Delta y=f(x,y)\Delta x$ (2'). Agar $x=x_0$ bo'lsa, (2) dan $\Delta y=f(x_0;y_0)\Delta x$ yoki $y_1-y_0=f(x_0,y_0)\Delta x$ hosil bo'ladi. Bu erda $x_0,\ y_0,\ \Delta x=h$ lar ma'lum, demak $y_1=y_0+f(x_0;y_0)\cdot h$ ni topamiz. $x=x_1$ da (2') tenglama

$$\Delta x_1 = f(x_1;y_1)\cdot h$$
$$y_2 - y_1 = f(x_1;y_1)\cdot h$$
$$y_2 = y_1 + f(x_1;y_1)\cdot h$$

ko'rinishni oladi. So'ngra formuladan y_2 qiymatni ma'lum x_1 y_1 h_1 sonlar orqali topiladi. Xuddi shuning kabi quyidagilarni topamiz:

$$y_3 = y_2 + f(x_2,y_2)\cdot h, \dots, y_k + f(x_k;y_k)\cdot h, \dots, y_n = y_{n+1} + f(x_{n-1};y_{n-1})\cdot h$$

Shunday qilib, x_0, x_1, x_2, ... , x_n nuqtalarda yechimning taqribiy qiymatlari y_0, y_1, x_2, y_3, ... , y_n lar topiladi. Koordinatalar tekisligida (x_0,y_0), (x_1,y_1), ... (x_n,y_n) nuqtalarni to'g'ri chiziq kesmalari bilan tutashtirib, siniq chiziqni egri chiziqning taqribiy tasvirini hosil qilamiz. Bu siniq chiziq **Eyler** siniq chizig'i deyiladi. Bu erda Eyler usuli bo'yicha qiymatlarni hisoblash formulasi

$$y_{k+1} = y_k + \Delta y_k;\ \ \Delta y_k = f(x_k;y_k)\cdot h;\ \ y_k = y(x_k);\ \ x_k = x_0 + k_h$$

bo'ladi.

Yuqoridagilarga asosan quyidagi jadvalni tuzamiz, u hisoblashlarni yengillashtiradi.

№	x_k	y_k	$f(x_k;y_k)$	$f(x_k;y_{k1})h$
0	x_0	y_0	$f(x_0;y_0)$	$\Delta y_0 = f(x_0;y_0)h$
1	x_1	$y_1=y_0+\Delta y_0$	$f(x_1;y_1)$	$\Delta y_1 = f(x_1;y_1)h$
2	x_2	$y_2=y_1+\Delta y_1$	$f(x_2;y_2)$	$\Delta y_2 = f(x_2;y_{21})h$
…	….	…………….	………	…………………….
n-1	x_{n-1}	$y_{n-1}=y_{n-2}+\Delta y_{n-2}$	$f(x_{n-1};y_{n-1})$	$\Delta y_{n-1} = f(x_{n-1};y_{n-1})h$
n	x_n	$y_n=y_{n-1}+\Delta y_{n-1}$	$f(x_n;y_n)$	———

Misol: $y' = \dfrac{x^3+y^2}{y^2}$ tenglama berilgan. [0;1] kesmada $x_o=0$, $y_o=1$ boshlang'ich shartni qanoatlantiruvchi yechim **Eyler** usuli yordamida topilsin. Qadamlar $h_1=0,1$ va $h_2=0,2$ deb olinsin. Hisob verguldan so'ng uchta raqam bilan olib borilsin.

Yechish: Hisoblash qulay bo'lsin maqsadida jadval tuzib olamiz ($h_1 = 0,1$)

1. Boshlang'ich shartlar bo'yicha jadvalning birinchi satrini to'ldirib olamiz. $x_0=0$ $y_0=1$ da

$$f(x_0;y_0) = f(0;1) = \frac{x_0^{\,3}+y_0^{\,2}}{y_0^{\,2}}\Bigg|_{y_0=1}^{x^0=0} = \frac{0^3+1^2}{1^2} = 1; \Delta y_0 = f(x_0;y_0)\cdot h = 1\cdot 0{,}1 = 0{,}1$$ bo'ladi

2. Jadvalning ikkinchi satrini to'ldiramiz.

$$x_1 = 0{,}1,\quad y_1 = y_0 + \Delta y_0 = 1 + 1{,}0 = 1{,}1$$ da

$$f(x_1;y_1) = f(0{,}1;1{,}1) = \frac{x_1^{\,3}+y_1^{\,2}}{y_1^2}\Bigg|_{y=1{,}1}^{x_1=0{,}1} = \frac{(0{,}1)^3+(1{,}1)^2}{(1{,}1)^2} \approx 1{,}1$$

$$\Delta y_1 = f(x_1,y_1)h = 1{,}1\cdot 0{,}1 = 0{,}11$$ va hokazo.

Hisoblashlarni toki [0,I] oraliqni o'tib bo'lmaguncha davom ettiramiz.

№	$x_i=x_0+ih$	$y_{i+1}=y_i+\Delta y_i$	y_i^2	x_i^3	$f(x_i;y_i)=\dfrac{x_i^3+y_0^2}{y_i^2}$	$\Delta y_2=f(x_i;y_i)h$
0	0	1,000	1,000	0	1,000	0,100
1	0,1	1,100	1,210	0,001	1,000	0,100
2	0,2	1,210	1,440	0,008	1,006	0,101
3	0,3	1,301	1,692	0,027	1,016	0,102
4	0,4	1,402	1,966	0,064	1,033	0,103
5	0,5	1,506	2,256	2,256	1,055	0,105
6	0,6	1,611	2,595	2,595	1,083	0,108
7	0,7	1,719	2,956	2,956	1,116	0,112
8	0,8	1,830	3,350	3,350	1,153	0,115
9	0,9	1,946	3,768	3,768	1,26	0,126
10	1,0	2,060	-	-	-	-

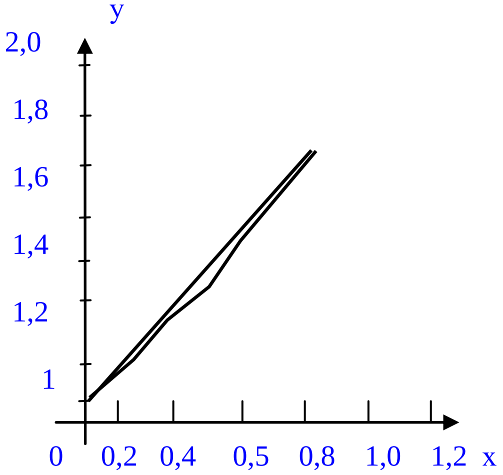
Endi h = 0,2 qadam uchun yuqoridagi kabi hisoblashlarni bajarib, jadvalni to'ldiramiz.

№	x_i	y_i	y_i^2	x_i^3	$f(x_i;y_i)$	Δy_i
0	0	1,000	1,000	0	1,000	0,200
1	0,2	1,200	1,440	0,098	1,006	0,200
2	0,4	1,400	1,960	0,054	1,030	0,210
3	0,6	1,610	2,592	0,216	1,080	0,220
4	0,8	1,830	3,350	0,512	1,100	0,210
5	1,0	2,050	-	-	-	-

Demak $\Delta = 2{,}06-2{,}05=0{,}01;$ $\delta=\dfrac{0{,}91}{2{,}05}\cdot 100\%=0{,}488\%$

Shunday qilib absolyut xato $\Delta=0{,}01$ nisbiy xato $\delta\approx 0{,}488\%\approx 0{,}5\%$

Koordinatalar tekisligida $(x_0;y_0)$, (x_1,y_1) ,…, (x_n,y_n) nuqtalarni to'g'ri chiziq kesmalari bilan tutashtirib **Eyler** siniq egri chizig'ini hosil qilamiz



15- *chizma*

Tajriba ishi variantlari

№	Tenglama	Boshlang'ich qiymat	Kesma	qadamlar
1	$y'=x+\sqrt{1+y^2}$	$y/_{x=0,2}=0{,}3$	$0{,}2\le x\le 1{,}2$	$h_1=0{,}1;\ h_2=0{,}2$
2	$y'=x-(1+\sqrt[3]{y})$	$y/_{x=1}=2$	$1\le x\le 2$	$h_1=0{,}1;\ h_1=0{,}2$
3	$y'=\dfrac{x}{2}+\dfrac{e^2}{x+y}$	$y/_{x=0,5}=2$	$0{,}5\le x\le 1{,}5$	$h_1=0{,}1;\ h_2=0{,}2$

4	$y'=\sqrt{xy^3-1}$	$y/_{x=0}=0$	$0\le x\le 1$	$h_1=0{,}2;\ h_2=0{,}2$
5	$y'=\sqrt{1+x^2+y^3}$	$y/_{x=0,3}=1$	$0{,}3\le x\le 1{,}3$	$h_1=0{,}1;\ h_2=0{,}2$
6	$y'=y^2-xy+y^2$	$y/_{x=0}=1$	$0\le x\le 0{,}5$	$h_1=0{,}05;\ h_2=0{,}1$
7	$y'=\sqrt{xy^2+1}$	$y/_{x=0}=0$	$0\le x\le 1$	$h_1=0{,}1;\ h_2=0{,}1$
8	$y'=x^2y^3-x^2$	$y/_{x=0}=1$	$0\le x\le 0{,}5$	$h_1=0{,}05;\ h_2=0{,}1$
9	$y'=\frac{1}{x^2+y^2}$	$y/_{x=0,5}=0{,}5$	$0{,}5\le x\le 1$	$h_1=0{,}05;\ h_2=0{,}1$
10	$y'=\frac{x}{x^2+y^2}$	$y/_{x=0,5}=0{,}5$	$0\le x\le 1$	$h_1=0{,}1;\ h_2=0{,}2$
11	$y'=2y^2-x^2+1$	$y/_{x=1}=0$	$1\le x\le 1{,}5$	$h_1=0{,}05;\ h_2=0{,}1$
12	$y'=e^{x^2}+xy$	$y/_{x=0}=1$	$0\le x\le 1$	$h_1=0{,}1;\ h_2=0{,}2$
13	$y'=\sin y-\sin x$	$y/_{x=0}=1$	$0\le x\le 1$	$h_1=0{,}1;\ h_2=0{,}2$
14	$y'=\frac{yx}{1+\cos x}$	$y/_{x=0}=1$	$0\le x\le 1$	$h_1=0{,}1;\ h_2=0{,}2$
15	$y'=x^2-y^2$	$y/_{x=0}=0$	$0\le x\le 0{,}5$	$h_1=0{,}05;\ h_2=0{,}1$
16	$y'=x^2y^2-1$	$y/_{x=1}=1$	$1\le x\le 1{,}5$	$h_1=0{,}05;\ h_2=0{,}1$
17	$y'=y^3-x^2$	$y/_{x=0}=0{,}5$	$1\le x\le 1{,}5$	$h_1=0{,}05;\ h_2=0{,}1$
18	$y'=\frac{1+x^2}{y}+1$	$y/_{x=0}=0$	$0\le x\le 1$	$h_1=0{,}1;\ h_2=0{,}2$
19	$y'=x^2+2y$	$y/_{x=0}=0{,}3$	$0\le x\le 1$	$h_1=0{,}1;\ h_2=0{,}2$
20	$y'=\sqrt{x}+\frac{y^2}{2}$	$y/_{x=1}=2$	$1\le x\le 2$	$h_1=0{,}1;\ h_2=0{,}2$
21	$y'=x+y^2$	$y/_{x=1}=0$	$1\le x\le 1{,}5$	$h_1=0{,}05;\ h_2=0{,}2$
22	$y'=x-y^2$	$y/_{x=0,3}=1$	$0{,}3\le x\le 0{,}8$	$h_1=0{,}05;\ h_2=0{,}2$
23	$y'=\frac{y^2-x^2}{x^2}$	$y/_{x=0}=1$	$0\le x\le 1$	$h_1=0{,}1;\ h_2=0{,}2$
24	$y'=x+\sin\frac{y}{\pi}$	$y/_{x=0}=1$	$1\le x\le 2$	$h_1=0{,}1;\ h_2=0{,}2$
25	$y'=x^3-yx+y^2$	$y/_{x=0}=0$	$0\le x\le 1$	$h_1=0{,}1;\ h_2=0{,}2$
26	$y'=1{,}3y\sqrt{yx}$	$y/_{x=1}=2{,}7$	$1\le x\le 2$	$h_1=0{,}1;\ h_2=0{,}2$
27	$y'=e^x-\frac{y}{x}$	$y/_{x=1}=1$	$1\le x\le 2$	$h_1=0{,}1;\ h_2=0{,}2$
28	$y'=2x-0{,}1y^2$	$y/_{x=0,5}=1$	$0{,}5\le x\le 1$	$h_1=0{,}05;\ h_2=0{,}2$
29	$y'=\frac{1-x}{1-x^2}$	$y/_{x=0}=1$	$0\le x\le 1$	$h_1=0{,}1;\ h_2=0{,}2$
30	$y'=3x^2-\frac{y}{x}$	$y/_{x=1}=0$	$1\le x\le 2$	$h_1=0{,}1;\ h_2=0{,}2$

Eyler usuliga tuzilgan dastur

```
uses crt;
 const
  x0=0;   y0=0.3;   a=4;   b=5.5;   h=0.1;
var
  r,x,y,x1,y1:real;   n,i:integer;
Function Eiler:real;
Begin   R:=sqrt(1+(cos(2*x1))/2)/(a*a-y*y);   Eiler:=r; end;
Begin Clrscr;
```

```
  x:=x0;   y:=y0;    n:=round((b-a)/h);
  for i:=1 to n do
   begin
     x:=x+h;  y:=y+Eiler*h;
     writeln('x[',i,']= ',x:3:1,'     Y[',i,']= ',y:6:4);
   end;
  writeln('Result      Y= ',y:6:4);
end.
```

10-Tajriba ishi

Ikkinchi tartibli differensial tenglamalarni taqribiy yechish

Ushbu $y''=F(x,y,y')$ (1) ikkinchi tartibli differensial tenglamaning $y/_{x=xo}=y_0;$ $y'/_{x=xo}=y'_0$ (2) boshlang'ich shartlarni qanoatlantiradigan yechimini topish talab qilinadi.

Yechish: ***Ketma-ket differensiallash usuli***

Izlanayotgan *y=y (x)* yechim *x-x₀* ayirmaning darajalari bo'yicha darajali qatorga yoyiladi. Bu qator har qanday darajali qator kabi o'z yig'indisining Teylor qatoridir va shu sababdan yoyilma quyidagi ko'rinishga ega:

$$y=y(x_0)+\frac{y'(x_0)}{1!}(x-x_0)+\frac{y''(x_0)}{2!}(x-x_0)^2+...\frac{y^{(n)}(x_0)}{n!}(x-x_0)^n+... \qquad (3)$$

(2) shartdan $y_0=y(x_0),y'_0=y'(x_0)$ (1) tenglamadan esa $y''(x_0)=(y'')_{x=x_0}=F(x_0;y_0;y'_0)$ hosil bo'ladi.

Endi esa (1) tenglamani ketma-ket differensiallab va har bir differensiallashdan keyin x = x $_0$ deb olib ushbularni topamiz:

$$y'''(x_0),\ y^{(IV)}(x_0),\ y^{(v)}(x_0),...y^{(n)}(x_0)$$

Bu erda jarayon yo berilgan koeffisientda uziladi yoki koeffiientlarni aniqlashning umumiy sonunini topish bilan yakunlanadi. Hosilalarning topilgan qiymatlarini (3) tenglikka qo'yamiz. x ning (3) qator yaqinlashadigan barcha qiymatlarida bu qator berilgan differensial tenglamaning yechimidir.

II. Koeffitsientlarni taqqoslash usuli

Izlanayotgan y = y(x) yechim x-x$_0$ ayirmaning darajasi bo'yicha qatorga yoyiladi. Boshlang'ich shartlardan.

$$a_0=y_0,\quad a_1=y'_0,\quad a_2=\frac{y''_0}{2!},...,a_{n-1}=\frac{y^{(n-1)}}{(n-1)!},...$$

koefisientlar topiladi. Differensial tenglikdagi y, ning hosilalari va hokazo funksiya o'rniga ularning x =x$_0$ ayirmaning darajalari bo'yicha darajali qatorlarga yoyilmalari qo'yiladi, hamda x =x$_0$ ning bir xil darajalari oldidagi koeffisientlari tenglashtiriladi va bu tenglamalardan qatorning koeffisientlari topiladi.

1-misol $y''=xy'+y$ (I) differensial tenglamaning $y/_{x=0}=0,\ y'/_{x=0}=1$ (2) boshlang'ch shartlarga ko'ra $y''(0)=0\cdot 1+0=0$ ekanini topamiz.

Ketma-ket differensiallash bilan quyidagilarni hosil qilamiz.

$$y'''=2y'+xy'' \qquad y'''(0)=2$$
$$y^{IV}=3y''+xy''' \qquad y^{IV}(0)=0$$
$$y^{V}=4y'''+xy^{IV} \qquad y^{V}(0)=8=2\cdot 4$$
$$y^{VI}=5y^{IV}+xy^{V} \qquad y^{VI}(0)=0$$
$$y^{VII}=6y^{V}+xy^{VI} \qquad y^{VII}(0)=48=2\cdot 4\cdot 6;$$

$$y^{(2n)}=(2n-1)y^{(2n-2)}+xy^{(2n-1)}, \qquad y^{(2n)}(0)=0$$
$$y^{(2n+1)}=2ny^{(2n-1)}+xy^{(2n)}, \qquad y^{(2n+1)}(0)=2\cdot 4\cdot 6\cdot 2n=(2n)!!$$

Shunday qilib

$$y=0+\frac{1}{1!}(x-0)+\frac{0}{2!}(x-0)^2+\frac{2}{3!}(x-0)^3+\frac{0}{4!}(x-0)^4+\ldots$$

yoki buni quyidagicha yozish mumkin:

$$y=\sum_{n=0}^{\infty}\frac{(2n)!!}{(2n+1)!}x^{2n+1}=\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!!}=x+\frac{x^3}{1\cdot 3}+\frac{x^5}{1\cdot 3\cdot 5}+\frac{x^7}{1\cdot 3\cdot 5\cdot 7}+\ldots$$

Bu qatorning yaqinlashish radiusini topamiz.

$$u_n=\frac{x^{2n+1}}{1\cdot 3\cdot 5\cdot 7(2n-1)(2n+1)}, \qquad u_{n+1}=\frac{x^{2n+3}}{1\cdot 3\cdot 5\cdot 7\ldots(2n+1)(2n+3)}$$

bo'lgani uchun qator yasinlashishning. Dalamber alomatiga asosan

$$\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=\lim_{n\to\infty}\left[\frac{x^{2n+3}}{1\cdot 3\cdot 5\cdot 7\ldots(2n+1)(2n+3)}\frac{1\cdot 3\cdot 5\cdot 7\ldots(2n-1)(2n+1`)}{x^{2n+1}}\right]=$$

$$\lim_{n\to\infty}\frac{x^2}{2n+3}=x^2\lim_{n\to\infty}\frac{1}{2n+3}=x^2\cdot 0=0<1$$

Demak, qator x ning har qanday qiymatida ham yaqinlanuvchi ekan, ya'ni qaralayotgan qatorning yaqinlashish radiusi $R=+\infty$ ekan.

Demak, (1) tenglamaning yechimi (3) qator shaklidagi funktsiyadir.

2-misol $y''=(y')^2+xy$ differensial tenglamaning *y(1)=1,* $y'(1)=0$ boshlang'ich shartlarni qanoatlantiruvchi yechimning darajali qatorga yoyilmasining dastlabki beshta hadi topilsin.

Yechish: x=1 qiymatni berilgan tenglamaga quyib boshlang'ich shartlarga asosan y"(1)=1 ekanini topamiz. Tenglamani ikki marta differentsiallab

$$y''=2y'y''+y+xy', \qquad y'''(1)=1,$$
$$y^{IV}=2(y'')^2+2y'y'''+2y'+xy'', \qquad y^{IV}(1)=3$$

qiymatlarini hosil qilamiz.

$$y=y(1)+\frac{y'(1)}{1!}(x-1)+\frac{y''(1)}{2!}(x-1)^2+\frac{y'''(1)}{3!}(x-1)^3+\ldots$$

bo'lgani uchun, yuqorida berilgan *y(1)=1,* *y'(1)=0* va hisoblab topilgan $y''(1)=1,\ y'''(1)$ va $y^{IV}(1)=3$ qiymatlarni oxirgi qatorga qo'yib quyidagini

$$y=1+\frac{0}{1!}(x-1)+\frac{1}{2!}(x-1)^2+\frac{1}{3!}(x-1)^3+\frac{3}{4}(x-1)^4+...=$$
$$=1+\frac{1}{2}(x-1)^2+\frac{1}{6}(x-1)^3+\frac{1}{8}(x-1)^4+...$$

hosil qilamiz.

Bu qatorning dastlabki beshta hadi yig'indisi quyidagi funksiyadir.

$$y=1+\frac{1}{2}(x-1)^2+\frac{1}{6}(x-1)^3+\frac{1}{8}(x-1)^4$$

3-misol $y''=2xy'+4y$ tenglamaning $y/_{x=0}=0,\ y'/_{x=0}=1$ boshlang'ich shartlarni qanoatlantiruvchiyechimi topilsin.

Yechish. $y=a_0+a_1x+a_2x^2+...+a_nx^n+...$ bo'lsin. Boshlang'ich shartlarga asosan $a_o=0,\ a_1=1$.

Demak, $y=x+a_2x^2+a_3x^3+...+a_nx^n+...$ ikki bor differensiallab

$$y'1+2a_2x+3a_3x^2+...+na_nx^{n-1}+...$$
$$y''=2a_2+2\cdot3a_3x+...+(n-1)na_nx^{n-2}+...$$

larni hosil qilamiz. Bu ifodalarni berilgan tenglamaga qo'yamiz:

$$2a_2+2\cdot3a_3x+...+(n-1)na_nx^{n-2}+...=$$
$$2x(1+2a_2x+3a_3x^2+...+na_nx^{n-1}+...)+4(x+a_2x^2+a_3x^3+...a_nx^n+...)$$

Endi x ning bir xil darajalari oldidagi koeffisientlarni tenglashtirib quyidagilarni hosil qilamiz:

$2a_2=0$ ya'ni $a_2=0$

$2\cdot3a_3=2+4$ ya'ni $a_3=1$

$3\cdot4a_4=4a_2+4a_2$ ya'ni $a_4=0$

$n(n-1)a_n=(n-2)\cdot2a_{n-2}+4a_{n-2}=n2a_{n-2}$, yoki $n(n-1)a_n=n2a_{n-2}$ $\quad a_n=\frac{2a_{n-2}}{n-1}$

Bundan esa quyidagilar hosil bo'ladi.

$$a_5=\frac{2\cdot1}{4}=\frac{1}{2};\quad a_6=\frac{2a_4}{5}=\frac{2\cdot0}{5}=0;\quad a_7=\frac{2a_5}{6}=\frac{1}{3!};$$
$$a_8=\frac{2a_6}{7}=\frac{0}{7}=0;\qquad a_9=\frac{2\cdot\frac{1}{3!}}{8}=\frac{1}{4!},\ ....,\qquad a_{2k}=0,$$
$$a_{2k+1}=\frac{2\cdot\frac{1}{(k-1)!}}{2k}=\frac{1}{k!}.$$

Qator koeffisientlarining topilgan bu qiymatlari o'rniga qo'yib, quyidagini hosil qilamiz:

$$y=x+x^3+\frac{x^5}{2!}+\frac{x^7}{3!}+...+\frac{x^{2k+1}}{k!}+...$$

hosil qilingan qator x ning har qanday qiymatida yaqinlashuvchi qatordir.

Bu topilgan xususiy yechimni elementar funksiyalar orqali ifodalash mumkin, haqiqatdan ham:

$$y=x(1+\frac{x^2}{1!}+\frac{x^4}{2!}+\frac{x^6}{3!}+\frac{x^8}{4!}+...)=xe^{x^2}.$$

Demak, $y=xe^{x^2}$ funksiya berilgan tenglamaning xususiy yechimi ekan.

Topshiriq variantlari

Quyida keltirilgan differensial tenglamalarni yuqorida ko’rsatilgan usullar yordamida yeching.

1	$y'' = xy$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 0$	16	$y'' = yy' - x^2$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 1$
2	$y'' = xy' - y$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 1$	17	$y'' = x\sin xy'$	$y_{\|x=1} = 0$	$y'_{\|x=0} = \frac{\pi}{2}$
3	$y'' + y' + xy = o$	$y_{\|x=0} = 0$	$y'_{\|x=0} = 0$	18	$y'' = xy' - y + 1$	$y_{\|x=0} = 0$	$y'_{\|x=0} = 0$
4	$(1 - x^2)y'' = xy'$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 1$	19	$y'' = xyy'$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 1$
5	$y'' + 2xy' = 0$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 1$	20	$xy'' = y + y'$	$y_{\|x=0} = 0$	$y'_{\|x=0} = 1$
6	$y'' = xyy'$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 1$	21	$y''(1 + x^2) = 2xy'$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 3$
7	$xy'' + y' = 0$	$y_{\|x=0} = 1$	$y'_{\|x=1} = 1$	22	$xy'' = x(y')^2 = y'$	$y_{\|x=0} = 2$	$y'_{\|x=2} = 1$
8	$y'' - 2xy' = 2e^{x2}$	$y_{\|x=0} = 1$	$y'\|_{x=0} = 0$	23	$y'' - 9y = 9$	$y\|_{x=o} = 0$	$y'\|_{x=0} = 3$
9	$y'' - y + 2\cos x = 0$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 0$	24	$2xy'' = 3y^2 + 1$	$y_{\|x=2} = 1$	$y'_{\|x=2} = -1$
10	$y'' + xy = 0$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 0$	25	$yy'' = x^2 - (y')^2$	$y_{\|x=1} = 2$	$y'_{\|x=1} = -1$
11	$y'' = xy' - y + e^x$	$y_{\|x=0} = 0$	$y'_{\|x=0} = 1$	26	$y^3 xy'' = -y'$	$y_{\|x=1} = 1$	$y'_{\|x=1} = 0$
12	$y'' = xy^2 - y'$	$y_{\|x=0} = 2$	$y'_{\|x=0} = 1$	27	$y'' = xy' + y + 1$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 0$
13	$y'' = xy^2 - y'$	$y_{\|x=0} = 1$	$y'_{\|x=0} = \frac{\pi}{3}$	28	$y'' + y' + y\sin 2x = 0$	$y_{\|x=\pi} = 0$	$y'_{\|x=\pi} = 1$
14	$y'' = xy - y$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 0$	29	$y'' - 2xy' = e^x$	$y_{\|x=0} = 2$	$y'_{\|x=0} = 1$
15	$(1 + x^2)y + xy' - y' = 0$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 1$	30	$y'' - x^2 y' = 2y(x-1)$	$y_{\|x=0} = 1$	$y'_{\|x=0} = 1$

11-Tajriba ishi

Aniq integralni qatorlar yordamida taqribiy hisoblash usullari

Ma'lumki, juda ko’p elementar funksiyalarning boshlang’ich funksiyalari mavjud bo’lsada, ularni elementar funksiya orqali ifodalab bo'lmaydi. Ammo, integral sifatidagi funksiyani darajali qatorga yoyish mumkin va integrallash chegaralarini shu qatorning yaqinlashish sohasiga tegishli bo'lsa, bu integralni ko'rsatilgan aniqlikda taqribiy hisoblash mumkin. Bir nechta misolga qaraymiz.

1-misol. $\int_0^{\frac{1}{4}} e^{-x^2}dx$ integralni 10^{-4} aniqlida hisoblansin. Bunda e^{-x^2} funksiyaning boshlangich funkiyasi elementar funksiyasi bo'lmaydi.

Yechish. Bu integralni hisoblash uchun e^x funksiya yoyilmasidagi x ni $-x^2$ almashtirib, integral ostidagi funksiyani qatorga yoyamiz:

$$e^x = 1 + x + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \cdots + \frac{x^n}{n!} + \cdots (-\infty < x < \infty) \text{ ga ko'ra}$$

$$e^{-x^2} = 1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots + (-1)\frac{x^{2n}}{n!} + \cdots + (-1)^n \frac{x^{2n}}{n!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}$$

Bu integralning ikala tomonini 0 dan $\frac{1}{4}$ gacha chegaralarda integrallab, quyidagini topamiz:

$$\int_0^{x^2} e^{-x^2}dx = \int_0^{\frac{1}{4}} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^{\frac{1}{4}} x^{2n}dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{1}{(2n+1)\cdot 4^{2n+1}} = \frac{1}{4} - \frac{1}{3\cdot 4^3} + \frac{1}{2!\cdot 5\cdot 4^5} -$$

Ishoaralari almashinuvchi sonli qator hosil bo'ldi.

Uchinchi haddan boshlab tashlashda hosil bo'ladigan xatolik absalyut qiymati bo'yicha uchunchi haddan kichik bo'ladi.

$$|\Delta| = \frac{1}{2!5\cdot 4^5} < 10^{-4} \quad \text{Demak,} \quad \int_0^{\frac{1}{4}} e^{-x^2}dx \approx \frac{1}{4} - \frac{1}{3\cdot 4^3} = 0{,}25 - 0{,}0052 = 0{,}2448$$

Quyidagi integrallarni integral ostidagi funktsiyani qatorga yoyish yordamida $0{,}001$ aniqlikda hisoblang.

1. $\int_{0,1}^{0,9} e^{\cos x}dx$	14. $\int_0^1 e^{-x^2}dx \quad (0{,}7468) \quad \varepsilon = 0{,}00001$
2. $\int_{-0,5}^{0,5} e^{arctgx}dx$	15. $\int_0^{\frac{\pi}{4}} \sin(x^2)dx \quad (0{,}1571) \ \varepsilon = 0{,}0001$
3. $\int_{-0,4}^{0,7} \ln(1+e^x)dx$	16. $\int_0^{0,5} e^{\sqrt{x}}dx \quad (0{,}81) \quad \varepsilon = 0{,}01$
4. $\int_{-0,3}^{0,7} e^{\sin x}dx$	17. $\int_0^{0,5} \frac{arctgx}{x}dx \quad (0{,}487) \quad \varepsilon = 0{,}001$
5. $\int_{-0,1}^{0,9} (1+x)^x dx$	18. $\int_0^1 \cos\sqrt{x}dx \quad (0{,}764) \quad \varepsilon = 0{,}001$
6. $\int_{-0,2}^{0,8} arctgxdx$	19. $\int_0^{0,25} \ln(1+\sqrt{x})dx \quad (0{,}071) \quad \varepsilon = 0{,}001$
7. $\int_{0,1}^{0,8} \frac{e^x - 1}{x}dx$	20. $\int_0^1 e^{-\frac{x^2}{4}}dx \quad (0{,}071) \quad \varepsilon = 0{,}0001$
8. $\int_{0,2}^{1} (1+x)e^{-x}dx$	21. $\int_0^{0,2} \frac{\sin x}{\sqrt{1-x}}dx \quad (0{,}9226) \ \varepsilon = 0{,}0001$

9. $\int_{0,10}^{0,9} \ln(x+\sqrt{1+x^2})dx$	22. $\int_0^{0,5} \frac{dx}{1+x^4}$ (0,494) $\varepsilon = 0{,}01$
10. $\int_{0,05}^{0,95} \frac{\ln(x+1)}{x}dx$	23. $\int_0^1 \frac{\ln(1+x)}{x}dx$
11. $\int_{0,01}^{0,91} \frac{arctgx}{x}dx$	24. $\int_0^1 \frac{\ln(1-x)}{x}dx$ $(\frac{\pi^2}{6})$
12. $\int_0^1 \frac{\cos x}{x}dx$	25. $\int_0^1 \ln\frac{1+x}{1-x}dx$
26. $\int_0^{\frac{\pi}{6}} \sqrt{\cos x}dx$	13. $\int_0^1 \frac{\sin x}{x}dx$ (0,94608) $\varepsilon = 0{,}0001$

12-Tajriba ishi

Laplas almashtirish yordamida differensial tenglamalarni yechish

Ma'lumki, bu tajriba ishimizda o'zgarmas koeffisientli, chiziqli differensial tenglamalarni **Laplas** almashtirishi yordamida, ya'ni operatsion hisob usulini qo'llab yechish bilan tanishamiz. Shuni ta'kidlab o'tish kerakki, operatsion hisob usuli yordamida differensial tenglamaning faqatgina berilgan boshlang'ich shartlarni qanoatlantiruvchi xususiy yechimi topiladi.

Operatsion hisob usuli yordamida differensial tenglamani yechish quyidagicha amalga oshiriladi: differensial tenglamaning har ikkala tomonidan Laplas almashtirish olinadi va natijada yechimning tasviriga nisbatan chiziqli tenglama hosil qilinadi, hosil qilingan chiziqli tenglama tasvirga nisbatan yechiladi;topilgan tasvir bo'yicha jadvaldagi formulalar yordamida uning boshlang'ich funksiyasi ya'ni differensial tenglamaning xususiy yechimi topiladi.

Misol 1. Umumiy holda $y' + a_1 y = f(t)$ ning $y(0) = y_0$ ni qanoatlantiruvchi xususiy yechimi topilsin.

Yechish. Buning uchun tasvirni differensiallash formulasidan foydalanaylik. Agar $F(p) = \int_0^{\infty} f(t)e^{-pt}dt$ бўлса, $f'(t) = pF(p) - f(0)$

$f''(t) = p^2F(p) - pf(0) - f'(0)$

$L\{y(t)\} = y(p)$ $L\{f(t)\} = F(p)$ shulardan foydalanib, $y(p) = \frac{P(p) + y_0}{a+p}$ ni hosil qilamiz. Ajratish teoremasi va tasvirlar jadvalidan *y(t)* ni aniqlaymiz.

Misol 2. $y' + 2y = t$ differensial tenglamaning $y(0)=0$ shartni qanoatlantiruvchi xususiy yechimi topilsin.

Yechish. qidirilayotgan *y=y(t)* yechimning tasvirini $\bar{y}(p)$ orqali belgilaymiz, ya'ni $\bar{y}(p) = L[y]$. Berilgan differensial tenglamaning har ikkala tomonidan Laplas almashtirish olaylik: $L[y'+2y] = L[t]$ yoki $L[y'] + 2L[y] = L[t]$ hosila tasvirining formulasiga ko'ra va $y(0)=0$ ekanligini nazarda tutsak, u holda

$L[y'] = p\bar{y}(p) - y(0) = p\bar{y}(p)$ ga erishamiz. Bundan $L[t] = \frac{1}{p^2}$ ekanligini hisobga olib, $p\bar{y}(p) + 2\bar{y}(p) = \frac{1}{p^2}$ yordamchi tenglamaga kelamiz. Bu tenglamada $y=y(t)$ yechimning $\bar{y}(p) = \frac{1}{p^2(p+2)}$ tasvirini topamiz. Endi topilgan $\bar{y}(p)$ tasvir bo'yicha boshlang'ich funksiyani aniqlaymiz. Tenglikning o'ng tomonini sodda kasrlar yig'indisi shaklida yozamiz: $\frac{1}{p^2(p+2)} = \frac{A}{p} + \frac{B}{p^2} + \frac{C}{p+2}$ bundan $1 = Ap(p+2) + B(p+2) + Cp^2$ ayniyatni hosil qilib undan noma'lum A,B,C koeffisientlarga nisbatan quyidagi chiziqli tenglamalar sistemasiga kelamiz: $\begin{cases} A+C=0 \\ 2A+B=0 \\ 2B=1 \end{cases}$ sistemani yechib, $B=\frac{1}{2}; A=-\frac{1}{4}; C=\frac{1}{4}$ ekanligini aniqlaymiz. Demak, $\bar{y}(p) = -\frac{1}{4}\frac{1}{p} + \frac{1}{2}\frac{1}{p^2} + \frac{1}{4}\frac{1}{p+2}$ yoyilmaga kelamiz va $\bar{y}(p)$ tasvirga mos keluvchi $y(p) = -\frac{1}{4} + \frac{1}{2}t + \frac{1}{4}e^{-2t}$ boshlang'ich funktsiyani tiklaymiz. Bu esa berilgan differensial tenglamaning izlangan xususiy yechimi bo'ladi. Topilgan xususiy yechimni differensial tenglamani qanoatlantirishini ko'rsatamiz:

$y' = \left(-\frac{1}{4} + \frac{1}{2}t + \frac{1}{4}e^{-2t}\right)' = \frac{1}{2}\left(1-e^{-2t}\right)$ shuning uchun

$y' + 2y = \frac{1}{2}\left(1-e^{-2t}\right) + 2\left(-\frac{1}{4} + \frac{1}{2}t + \frac{1}{4}e^{-2t}\right) = \frac{1}{2} - \frac{1}{2}e^{-2t} - \frac{1}{2} + t + \frac{1}{2}e^{-2t} = t$ Bundan tashqari $y(0) = -\frac{1}{4} + \frac{1}{4} = 0$ bo'lar ekan.

Misol. Umumiy holda ikkinchi tartibli chiziqli o'zgarmas koeffisientli bir jinsli bo'lmagan differensial tenglamani $y'' + ay' + by = \varphi(t)$ $y(0) = y_0$ va $y'(0) = y_0'$ boshlang'ich shartlarni qanoatlantiruvchi yechimining tasvirini topaylik. Buning uchun tenglamaning har ikkala tomonidan Laplas almashtirishni olamiz.

$L[y''] + aL[y'] + bL[y] = L[\varphi]$ yoki $p^2\bar{y}(p) - py(0) - y'(0) + a(p\bar{y}(p) - y(0)) + b\bar{y}(p) = L[\varphi]$ yordamchi chiziqli tenglamaga kelamiz. Bundan $y(p)\left[p^2 + ap + b\right] = L[\varphi] + py(0) + y'(0) + ay(0)$ tenglamani hosil qilamiz va uni yechib, $\bar{y}(p) = \frac{L[\varphi] + (p+a)y(0) + y'(0)}{p^2 + ap + b}$ (1) tasvirni topamiz. Agar differensial tenglamaning o'ng tomoni $\varphi(t)$ va yechimning boshlang'ich y_0, y_0' qiymatlari berilgan bo'lsa, bu tenglamadan y(t) yechimni yuqoridagi jadval yordamida tiklash mumkin.

Misol. $y'' - 2y' + y = \sin t$ differensial tenglamaning $y(0) = 1,\ y'(0) = 0$ shartlarni qanoatlantiruchi yechimi topilsin.

Yuqoridagi differensial tenglamada *a*=-2, *в*=1 va $\varphi(t) = \sin t$ deb olamiz. Bizga ma'lum tasvirga asosan $L[\varphi] = L[\sin t] = \frac{1}{p^2+1}$ Endi yuqorida ko'rsatilgan (1) formulaga asosan berilgan differensial tenglamaning tasvirini yozamiz.

$$\bar{y}(p) = \frac{p^3 - 2p^2 + p - 1}{(p^2+1)(p-1)^2}$$

Tenglikning o'ng tomonini sodda ratsional kasrlar yig'indisi ko'rinishida yozishga harakat qilamiz: $\frac{p^3-2p^2+p-1}{(p-1)^2(p^2+1)}=\frac{A}{p-1}+\frac{B}{(p-1)^2}+\frac{Cp+Д}{p^2+1}$ bu erdan A,B,C,D koeffisientlar uchun oldin ko'rilgan usulda $p^3-2p^2+p-1=A(p-1)(p^2+1)+B(p^2+1)+(Cp+Д)(p-1)^2$ ayniyatni hosil qilamiz. Natijada A,B,C,D larni topish uchun chiziqli $\begin{cases} A+C=1 \\ -A+B-2C+Д=-2 \\ A+C-2Д=1 \\ -A+B+Д=-1 \end{cases}$ tenglamalar sistemasini hosil qilamiz va uni yechib, $A=C=\frac{1}{2}; B=-\frac{1}{2}; Д=0$ ekanligiga ishonch hosil qilish mumkin. Natijada $\bar{y}(p)=\frac{1}{2}\frac{1}{p-1}-\frac{1}{2}\frac{1}{(p-1)^2}+\frac{1}{2}\frac{p}{p^2+1}$ tasvirlar yoyilmasiga erishamiz va jadvaldagi mos formulalarga asosan $y(t)=\frac{1}{2}e^t-\frac{1}{2}te^t+\frac{1}{2}\cos t$ xususiy yechimni tiklaymiz.

Tajriba ishi variantlari

1) $y'+cy=\varphi(t)$ differensial tenglamaning $y(0)=y_0$ shartni qanoatlantiruvchi yechimi topilsin:

2) $y''+ay'+by=\varphi(t)$ differensial tenglamaning $y(0)=y_0, y'(0)=y_0'$ shartlarni qanoatlantiruvchi yechimi topilsin, agar

t/r	c	y_0	$\varphi(t)$	t/r	a	b	y_0	y'_0	$\varphi(t)$
1	c=-4	$y_0=1$	$\varphi(t)=e^{3t}$	14	a=3	в=3	$y_0=1$	$y'_0=2$	$\varphi(t)=e^3$
2	c=0	$y_0=2$	$\varphi(t)=\sin t$	15	a=-7	в=10	$y_0=1$	$y_0'=1$	$\varphi(t)=5$
3	c=0	$y_0=2$	$\varphi(t)=te^{-t}$	16	a=0	в=-3	$y_0=0$	$y_0'=0$	$\varphi(t)=e^{2t}-2$
4	c=1	$y_0=1$	$\varphi(t)=5t$	17	a=-2	b=1	$y_0=1$	$y_0'=0$	$\varphi(t)=\sin t$
5	c=2	$y_0=0$	$\varphi(t)=3$	18	a=4	b=0	$y_0=0$	$y_0'=0$	$\varphi(t)=\sin 3t$
6	c=1	$y_0=1$	$\varphi(t)=t$	19	a=-3	в=2	$y_0=1$	$y_0'=0$	$\varphi(t)=\sin 2t$
7	c=2	$y_0=1$	$\varphi(t)=2t$	20	a=2	b=1	$y_0=1$	$y_0'=2$	$\varphi(t)=1$
8	c=1	$y_0=0$	$\varphi(t)=3t$	21	a=1	b=2	$y_0=1$	$y_0'=3$	$\varphi(t)=4$
9	c=3	$y_0=1$	$\varphi(t)=6t$	22	a=3	b=2	$y_0=0$	$y_0'=0$	$\varphi(t)=e^{3t}$
10	c=3	$y_0=1$	$\varphi(t)=e^t$	23	a=0	b=3	$y_0=1$	$y_0'=3$	$\varphi(t)=6$
11	c=2	$y_0=2$	$\varphi(t)=e^{2t}$	24	a=4	b=0	$y_0=0$	$y_0'=0$	$\varphi(t)=\sin 4t$
12	c=1	$y_0=0$	$\varphi(t)=4$	25	a=-3	b=1	$y_0=1$	$y_0'=0$	$\varphi(t)=\sin t$
13	c=2	$y_0=0$	$\varphi(t)=7$	26	a=2	c=2	$y_0=0$	$y_0'=0$	$\varphi(t)=7$

13-Tajriba ishi

Laplas almashtirish yordamida differentsial tenglamalar sistemalarini yechish

O'zgarmas koeffitsientli chiziqli differensial tenglamalar sistemasini operatsion hisob yordamida yechish. $\begin{cases} \frac{dx}{dt}+a_{11}x(t)+a_{12}y(t)=f_1(t) \\ \frac{dy}{dt}+a_{21}x(t)+a_{22}y(t)=f_2(t) \end{cases}$ (1) ning $y(0)=y_0,\quad x(0)=x_0$ shartni qanoatlantiruvchi yechimini operatsion usul yordamida yechaylik.

$$\begin{cases} L\{x(t)\}=x(p), L\{y(t)\}=y(p) \\ x'=px(p)-x_0,\ y'(t)=py(p)-y_0 \\ L\{f_1(t)\}=F_1(p),\ L\{f_2(t)\}=F_2(p) \end{cases} \tag{2}$$

(2) ni (1) sistemaga qo'ysak:

$$\begin{cases} (p+a_{11})x(p)+a_{12}y(p)=F_1(p)+x_0 \\ a_{21}x(p)+(p+a_{22})y(p)=F_2(p)+y_0 \end{cases} \tag{3}$$

$$\Delta=\begin{vmatrix} p+a_{11} & a_{12} \\ a_{21} & p+a_{22} \end{vmatrix} \tag{4}$$

$\Delta_1=\begin{vmatrix} F_1(p)+x_0 & a_{12} \\ F_2(p)+y_0 & p+a_{22} \end{vmatrix}$ (5) $\Delta_2=\begin{vmatrix} p+a_{11} & F_1(p)+x_0 \\ a_{21} & F_2(p)+y_0 \end{vmatrix}$ (6) $x(p)=\frac{\Delta_1}{\Delta}$, $y(p)=\frac{\Delta_2}{\Delta}$ (7)

Endi ikkinchi tartibli differensial tenglamalar sistemasini

$$\begin{cases} \frac{d^2x}{dt^2}+a\frac{dx}{dt}+a_{11}x+a_{12}y=f_1(t) \\ \frac{d^2y}{dt^2}+b\frac{dy}{dt}+a_{21}x+a_{22}y=f_2(t) \end{cases} \tag{8}$$

$$y(0)=y_0,\quad x(0)=x_0,\quad y'(0)=y'_0,\quad x'(0)=x'_0 \tag{9}$$

shartni qanoatlantiruvchi yechimini

topaylik: $\begin{cases} x''(t)=p^2x(p)-px_0-x'_0 \\ y''(t)=p^2y(p)-py_0-y'_0 \end{cases}$ (10)

(2), (9) , (10) larni hisobga olib, (8) sistemani quyidagi chiziqli algebraik sistemaga keltiramiz.

$$\begin{cases} (p^2+ap+a_{11})x(p)+a_{12}y(p)=F_1(p)+px_0+x_0'+ax_0 \\ a_{21}x(p)+(p^2+bp+a_{22})y(p)=F_2(p)+py_0+y_0'+by_0 \end{cases}$$

$$\begin{cases} x''(t)=p^2x(p)-px_0-x'_0 \\ y''(t)=p^2y(p)-py_0-y'_0 \end{cases}$$

$\Delta=\begin{vmatrix} p^2+ap+a_{11} & a_{12} \\ a_{21} & p^2+bp+a_{22} \end{vmatrix}$ $\Delta_1=\begin{vmatrix} F_1(p)+px_0+x_0'+ax_0 & a_{12} \\ F_2(p)+py_0+y_0'+by_0 & p^2+bp+a_{22} \end{vmatrix}$

$\Delta_2=\begin{vmatrix} p^2+ap+a_{11} & F_1(p)+px_0+x_0'+ax_0 \\ a_{21} & F_2(p)+py_0+y_0'+by_0 \end{vmatrix}$ $x(p)=\frac{\Delta_1}{\Delta}$ $y(p)=\frac{\Delta_2}{\Delta}$

x (t) va y(t) ni tasvirlar jadvalidan foydalanib yechimlarni topamiz.

Misol. Ushbu $\begin{cases} x' = z + y - x \\ y' = z + x - y \\ z' = x + y + z \end{cases}$ bir jinsli differensial tenglamalar sistemasining

x(0)=1, z(0)=y(0)=0 boshlang'ich shartlarini qanoatlantiruvchi yechim topilsin.
Yechish: Topilishi kerak bo'lgan *x,y,z* yechimlarning tasvirlarini mos ravishda $\bar{x}, \bar{y}, \bar{z}$ lar orqali belgilaymiz.Sistema tenglamalarining chap va o'ng tomonlaridan Laplas almashtirishi olib, quyidagi yordamchi algebraik tenglamalar sistemasiga kelamiz.

$\begin{cases} p\bar{x} - x(0) = \bar{z} + \bar{y} - \bar{x} \\ p\bar{y} - y(0) = \bar{z} + \bar{x} - \bar{y} \\ p\bar{z} - z(0) = \bar{x} + \bar{y} + \bar{z} \end{cases}$ $x(0)=1$, $y(0)= z(0)=0$ boshlang'ich shartlarni hisobga olib

mos hadlarni guruhlab, $\bar{x}, \bar{y}, \bar{z}$ tasvirlar uchun $\begin{cases} (p+1)\bar{x} - \bar{y} - \bar{z} = 1 \\ -\bar{x} + (p+1)\bar{y} - \bar{z} = 0 \\ -\bar{x} - \bar{y} + (p+1)\bar{z} = 0 \end{cases}$ chiziqli

sistemaga kelamiz. Bu sistemani Kramer usuli bilan yechamiz:

$$\bar{x} = \frac{\Delta(\bar{x})}{\Delta}; \bar{y} = \frac{\Delta(\bar{y})}{\Delta}; \bar{z} = \frac{\Delta(\bar{z})}{\Delta}$$

Bunda uchinchi tartibli aniqlovchilar quyidagicha hisoblanadi.

$$\Delta = \begin{vmatrix} p+1 & -1 & -1 \\ 1 & -(p+1) & 1 \\ 1 & 1 & 1-p \end{vmatrix} = (p+1)(p+2)(p-2) \qquad (\Delta\bar{x}) = \begin{vmatrix} 1 & -1 & -1 \\ 0 & -(p+1) & 1 \\ 0 & 1 & 1-p \end{vmatrix} = p^2 - 2$$

$$\Delta(\bar{y}) = \begin{vmatrix} p+1 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 0 & 1-p \end{vmatrix} = p \qquad \Delta(\bar{z}) = \begin{vmatrix} p+1 & -1 & 1 \\ 1 & -(p+1) & 0 \\ 1 & 1 & 0 \end{vmatrix} = (p+2)$$

Natijada noma'lum tasvirlar uchun $\bar{x}(p) = \frac{p^2 - 2}{(p+1)(p+2)(p-2)}$, $\bar{y}(p) = \frac{p}{(p+1)(p+2)(p-2)}$, $\bar{z}(p) = \frac{1}{(p+1)(p-2)}$ tengliklarni olamiz.

Ularning boshlang'ich funksiyalari quyidagicha aniqlanadi: $\bar{x}(p)$ ni sodda tasvirlar yig'indisiga ajratsak $\frac{p^2 - 2}{(p+1)(p+2)(p-2)} = \frac{A}{p+1} + \frac{B}{p+2} + \frac{C}{p-2}$ bu erdan $p^2 - 2 = (A + B + C)p^2 + (3C - B)p + 2C - 4A - 2B$ ayniyatga asosan noma'lum A,B,C koeffisientlarga nisbatan quyidagi $\begin{cases} A + B + C = 1 \\ 3C - B = 0 \\ 4A + 2B + 2C = -2 \end{cases}$ chiziqli sistemaga kelamiz.

Bu sistemani yechib, $A = \frac{1}{3}; B = \frac{1}{2}; C = \frac{1}{6}$ ekanligini aniqlaymiz.Shundayqilib jadvaldagi mos formulalarga asosan $x(t) = \frac{1}{3}e^{-t} + \frac{1}{2}e^{-2t} + \frac{1}{6}e^{2t}$ yechimni topamiz. Xuddi shuningdek $\bar{y}(p)$ ni sodda tasvirlar yig'indisi shaklida ifodalaymiz:

$\bar{y}(p) = \frac{A}{p+1} + \frac{B}{p+2} + \frac{C}{p-2}$ yuqoridek fikr yuritib, A,B,C larga nisbatan

$\begin{cases} A+B+C=0 \\ -B+3C=1 \\ -4A-2B+2C=0 \end{cases}$ chiziqli sistemani olamiz va undan $A=\frac{1}{3};\ B=-\frac{1}{2};\ C=\frac{1}{6}$ ekanligiga ishonch hosil qilamiz. Endi jadvaldagi formulalar asosida $\bar{y}(p)$ tasvirga mos $y(t)=\frac{1}{3}e^{-t}-\frac{1}{2}e^{-2t}+\frac{1}{6}e^{2t}$ boshlang'ich funksiyani topamiz. Shuningdek $\bar{z}$ tasvir uchun $\bar{z}(p)=\frac{1}{(p+1)(p-2)}=-\frac{1}{3}\frac{1}{p+1}+\frac{1}{3}\frac{1}{p-2}$ yoyilmani topib, undan $\bar{z}$ tasvirning $z(t)=-\frac{1}{3}e^{-t}+\frac{1}{3}e^{2t}$ boshlang'ich funksiyasini tiklaymiz.

Topshiriq variantlari:

Quyidagi birinchi tartibli chiziqli, o'zgarmas koeffisientli differensial tenglamalar sistemasining berilgan boshlang'ich shartlarini qanoatlantiruvchi yechimi topilsin.

$$\begin{cases} \dfrac{dx}{dt}+a_{11}x(t)+a_{12}y(t)=f_1(t) \\ \dfrac{dy}{dt}+a_{21}x(t)+a_{22}y(t)=f_2(t) \end{cases}$$

x(0)=x₀, y(0)=y₀ boshlang'ich shartlar

1. $a_{11}=4$ $a_{12}=-1$ $x(0)=1$
   $a_{21}=-2$ $a_{22}=-1$ $y(0)=2$
   $f_1(t)=t,\ f_2(t)=3$
2. $a_{11}=1$ $a_{12}=-2$ $x(0)=2$
   $a_{21}=2$ $a_{22}=-1$ $y(0)=1$
   $f_1(t)=t,\ f_2(t)=2$
3. $a_{11}=3$ $a_{12}=-1$ $x(0)=1$
   $a_{21}=-1$ $a_{22}=1$ $y(0)=0$
   $f_1(t)=3t,\ f_2(t)=1$
4. $a_{11}=1$ $a_{12}=-1$ $x(0)=1$
   $a_{21}=1$ $a_{22}=2$ $y(0)=2$
   $f_1(t)=t^2,\ f_2(t)=1$
5. $a_{11}=3$ $a_{12}=-2$ $x(0)=1$
   $a_{21}=-1$ $a_{22}=1$ $y(0)=1$
   $f_1(t)=3,\ f_2(t)=t$
6. $a_{11}=2$ $a_{12}=1$ $x(0)=0$
   $a_{21}=-1$ $a_{22}=3$ $y(0)=2$
   $f_1(t)=4,\ f_2(t)=t^2$
7. $a_{11}=1$ $a_{12}=1$ $x(0)=0$
   $a_{21}=1$ $a_{22}=1$ $y(0)=0$
   $f_1(t)=5,\ f_2(t)=2t$
8. $a_{11}=3$ $a_{12}=1$ $x(0)=1$
   $a_{21}=1$ $a_{22}=-1$ $y(0)=1$
   $f_1(t)=4t,\ f_2(t)=6$
9. $a_{11}=-1$ $a_{12}=4$ $x(0)=2$
   $a_{21}=-2$ $a_{22}=1$ $y(0)=0$
   $f_1(t)=t^2,\ f_2(t)=2t$
10. $a_{11}=2$ $a_{12}=-1$ $x(0)=1$
    $a_{21}=-1$ $a_{22}=1$ $y(0)=0$
    $f_1(t)=t^2,\ f_2(t)=1$
11. $a_{11}=3$ $a_{12}=1$ $x(0)=2$
    $a_{21}=-1$ $a_{22}=2$ $y(0)=1$
    $f_1(t)=2t,\ f_2(t)=3$
12. $a_{11}=2$ $a_{12}=-1$ $x(0)=0$
    $a_{21}=-1$ $a_{22}=3$ $y(0)=1$
    $f_1(t)=4t,\ f_2(t)=5$
13. $a_{11}=-4$ $a_{12}=-1$ $x(0)=1$
    $a_{21}=-2$ $a_{22}=1$ $y(0)=2$
    $f_1(t)=6t,\ f_2(t)=5$
14. $a_{11}=3$ $a_{12}=-1$ $x(0)=2$
    $a_{21}=-1$ $a_{22}=1$ $y(0)=1$
    $f_1(t)=2,\ f_2(t)=t$
15. $a_{11}=1$ $a_{12}=1$ $x(0)=0$
    $a_{21}=1$ $a_{22}=1$ $y(0)=0$
    $f_1(t)=2,\ f_2(t)=3t$
16. $a_{11}=2$ $a_{12}=-1$ $x(0)=1$
    $a_{21}=-1$ $a_{22}=-1$ $y(0)=2$
    $f_1(t)=6,\ f_2(t)=e^t$
17. $a_{11}=3$ $a_{12}=-2$ $x(0)=1$
    $a_{21}=1$ $a_{22}=1$ $y(0)=-1$
    $f_1(t)=2t^2,\ f_2(t)=3t$
18. $a_{11}=1$ $a_{12}=-1$ $x(0)=-1$
    $a_{21}=2$ $a_{22}=1$ $y(0)=1$
    $f_1(t)=6t,\ f_2(t)=t^2$
19. $a_{11}=1$ $a_{12}=-1$ $x(0)=2$
    $a_{21}=2$ $a_{22}=1$ $y(0)=1$
20. $a_{11}=4$ $a_{12}=-1$ $x(0)=1$
    $a_{21}=-2$ $a_{22}=-1$ $y(0)=-2$

$f_1(t)=5t,\ \ f_2(t)=e^t$ $\qquad$ $f_1(t)=2t,\ \ f_2(t)=e^{3t}$

21. Quyidagi birinchi tartibli, chiziqli, o'zgarmas koeffisientli differensial tenglamalar sistemasining berilgan boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin:

$$\begin{cases} x'+4x-y=0, & x(0)=1 \\ y'-2x-y=0 & y(0)=2 \end{cases}$$

22. $\begin{cases} x'+x+y=t, & x(0)=0 \\ y'+x+y=0 & y(0)=0 \end{cases}$

23. $\begin{cases} x'=-2x-2y-4z, & x(0)=1 \\ y'=-2x+y-2z, & y(0)=1 \\ z'=5x+2y+7z, & z(0)=1 \end{cases}$

24. Ikkinchi tartibli differensial tenglamalar sistemasining xususiy yechimi topilsin: $\begin{cases} x''+y=1, & x(0)=0, & x'(0)=0 \\ y''+x=0 & y(0)=0, & y'(0)=0 \end{cases}$

25. Ushbu $\begin{cases} x'=x+y+t \\ y'=-4x-3y+2t \end{cases}$ bir jinsli bo'lgan differensial tenglamalar sistemasining $x(0)=0,\ y(0)=0$ boshlang'ich shartlarni qanoatlantiruchi yechimi topilsin.

26. Ushbu ikkinchi tartibli $\begin{cases} x''+16y=0, & x(0)=0, & x'(0)=1 \\ x+y''=0 & y(0)=0, & y'(0)=0 \end{cases}$ differensial tenglamalar sistemasi yechilsin.

14-Tajriba ishi

Diskret tasodifiy miqdorlar va ularning taqsimot qonunlari
Diskret tasodifiy miqdorlarning sonli xarakteristikalari

Qabul qilishi mumkin bo'lgan qiymatlari ayrim ajralgan sonlar bo'lib, ularni tayin ehtimollar bilan qabul qiladigan miqdorga ***diskret*** tasodifiy miqdor deyiladi. Boshqacha qilib aytganda, diskret tasodifiy miqdorning qiymatlarini nomerlab chiqish mumkin. Diskret tasodifiy miqdorning qabul qilishi mumkin bo'lgan qiymatlarining soni chekli yoki cheksiz bo'lishi mumkin.

Diskret tasodifiy miqdorning taqsimot qonuni deb, uning qabul qilishi mumkin bo'lgan qiymatlari bilan ularga mos ehtimollar ro'yxatiga aytiladi.

x diskret tasodifiy miqdorning taqsimot qonuni quyidagicha: birinchi satri mumkin bo'lgan x qiymatlaridan, ikkinchi satri esa p ehtimollardan tuzilgan

$$\begin{matrix} x & x_1, & x_2, & x_3, \ldots, & x_n \\ p & p_1, & p_2, & p_3, \ldots, & p_n \end{matrix}$$

jadval ko'rinishida berilishi mumkin, bu erda $\sum_{i=1}^{n} p_i = 1$.

X diskret tasodifiy miqdorning taqsimot qonuni $P(x=x_i)=\varphi(x_i)$ analitik usulda yoki integral funksiya yordamida berilishi ham mumkin.

Diskret tasodifiy miqdorning taqsimot qonunini grafik usulda tasvirlash mumkin, buning uchun to'g'ri burchakli koordinatalar sistemasida $M_1(x_1;p_1)$, $M_2(x_2;p_2)$,..., $M_n(x_n;p_n)$ nuqtalar (x_i - x ning mumkin bo'lgan qiymatlari, p_i-mos ehtimollari) yasaladi va ular to'g'ri chiziq kesmalari orqali tutashtiriladi. Hosil qilingan figura taqsimot ko'pburchagi deyiladi.

Binomial taqsimot qonuni deb, har birida hodisaning ro'y berish ehtimoli *p* ga teng bo'lgan n ta erkli sinovda bu hodisaning ro'y berishlari sonidan iborat *x* diskret tasodifiy miqdorning taqsimot qonuniga aytiladi; mumkin bo'lgan x=k (hodisaning ro'y berishlari soni k) qiymatning ehtimoli $P_n(k)=C_n^k p^k q^{n-k}$ Bernulli formulasi bo'yicha hisoblanadi. Agar sinovlar soni katta bo'lib, har bir sinovda hodisaning ro'y berish ehtimoli r juda kichik bo'lsa, u holda $P_n(k)=\frac{\lambda^k e^{-\lambda}}{k!}$ taqribiy formuladan fodalaniladi, bu erda k-hodisaning n –ta erkli sinovda ro'y berish soni, $\lambda = np$ (hodisaning p ta erkli sinovda ro'y berishlari o'rtacha soni). Bu holda tasodifiy miqdor Puasson qonuni bo'yicha taqsimlangan deyiladi.

Namunaviy masala

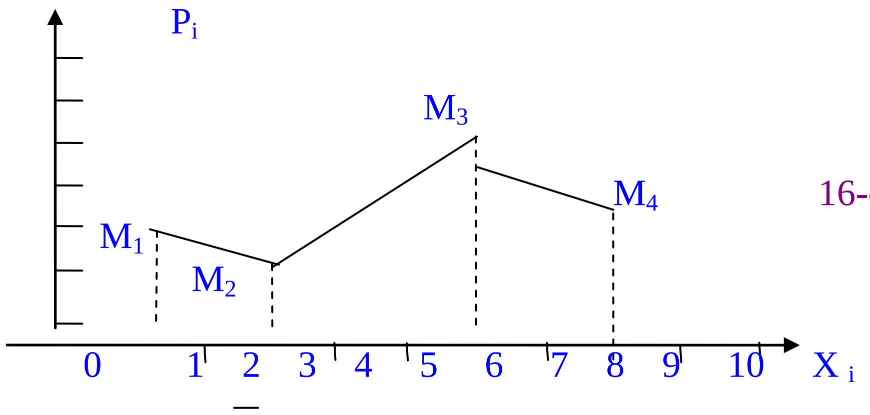
Masala: diskret tasodifiy miqdor ushbu taqsimot qonuni (qatori) bilan berilgan:

X	1	3	6	8
p	0,2	0,1	0,4	0,3

taqsimot ko'pburchagini yasang.

Yechilishi:

To'g'ri burchakli koordinatalar sistemasini yasaymiz,bunda absissalar o'qi bo'ylab mumkin bo'lgan x_i qiymatlarni, ordinatalar o'qi bo'ylab esa tegishli p_i ehtimollarni qo'yamiz. $M_1(1;0,2)$, $M_2(3;0,1)$, $M_3(6;0,4)$ va $M_4(8;0,3)$ nuqtalarni yasaymiz. Bu nuqtalarni to'g'ri chiziq kesmalari bilan tutashtirib, izlanayotgan taqsimot ko'pburchagini hosil qilamiz.



16-chizma

Topshiriq variantlari

1-variant

x diskret tasodifiy miqdor ushbu taqsimot qonuni (qatori) bilan berilgan:

x	1	3	6	8

p	0,2	0,1	0,4	0,3

taqsimot ko'pburchagini yasang.

2-variant

x diskret tasodifiy miqdor ushbu taqsimot qonuni (qatori) bilan berilgan:

x	2	4	6	8	10
p	0,1	0,4	0,2	0,1	0,2

taqsimot ko'pburchagini yasang.

3-variant

x diskret tasodifiy miqdor ushbu taqsimot qonuni (qatori) bilan berilgan:

x	3	4	6	7	8
p	0,2	0,3	0,1	0,1	0,3

taqsimot ko'pburchagini yasang.

4-variant

x diskret tasodifiy miqdor ushbu taqsimot qonuni (qatori) bilan berilgan:

x	3	4	5	6	8
p	0,3	0,1	0,2	0,1	0,3

taqsimot ko'pburchagini yasang.

5-variant

Qurilma bir-biridan erkli ishlaydigan uchta elementdan iborat. Har bir elementning bitta tajribada ishdan chiqish ehtimoli 0,2 ga teng. Bitta tajribada ishdan chiqqan elementlar sonining taqsimot qonunini tuzing.

6-variant

8 ta detal solingan qutida 6 ta standart detal bor. Tavakkaliga 2 ta detal olingan. Olingan detallar orasidagi standart detallar sonining taqsimot qonunini tuzing.

7-variant

12 ta detal solingan qutida 8 ta standart detal bor. Tavakkaliga 4 ta detal olingan. Olingan detallar orasidagi standart detallar sonining taqsimot qonunini tuzing.

8-variant

Qutidagi 6 ta detal solingan qutida 4 ta standart detal bor. Tavakkaliga 3 ta detal olingan. $\boldsymbol{x}$ diskret tasodifiy miqdor olingan detallar orasidagi standart detallar sonining taqsimot qonunini tuzing.

9-variant

Qutidagi 10 ta detal solingan qutida 3 ta standart detal bor. Tavakkaliga 4 ta detal olingan. $\boldsymbol{x}$ diskret tasodifiy miqdor olingan detallar orasidagi standart detallar sonining taqsimot qonunini tuzing.

10-variant

Quyidagi taqsimot qonuni bilan berigan $\boldsymbol{x}$ diskret tasodifiy miqdorning matematik kutilishini toping:

x	-3	5	8	5
P	0,4	0,3	0,1	0,2

11-variant

Quyidagi taqsimot qonuni bilan berigan $\boldsymbol{x}$ diskret tasodifiy miqdorning matematik kutilishini toping:

X	4	-3	7	5
P	0,2	0,3	0,1	0,4

12-variant

Agar X va Y ning matematik kutilishi ma'lum bo'lsa, Z tasodifiy miqdorning matematik kutilishini toping: Z=X +3Y, M(X)=4, M(Y)=3

13-variant

Agar X va Y ning matematik kutilishi ma'lum bo'lsa, Z tasodifiy miqdorning matematik kutilishini toping: Z=3X +2Y, M(X)=3, M(Y)=5

14-variant

X diskret tasodifiy miqdorning mumkin bo'lgan qiymatlarining ro'yxati berilgan: x_1=0 x_2=1 x_3=2 shuningdek, bu miqdorning va uning kvadratining matematik kutilishlari ma'lum: M(X)=0,2; M(X^2)=0,8. Mumkin bo'lgan $x_1,\ x_2$ va x_3 qiymatlarga mos $p_1,\ p_2$ va p_3 ehtimollarni toping.

15-variant

$\boldsymbol{x}$ diskret tasodifiy miqdorning mumkin bo'lgan qiymatlarining ro'yx ati berilgan: $x_1=-1,\ x_2=0,\ \ x_3=2$ shuningdek, bu miqdorning va uning kvadratining matematik kutilishlari ma'lum: M($\boldsymbol{x}$)=0,1; M($\boldsymbol{x}^2$)=0,9 Mumkin bo'lgan $x_1,\ x_2$ va x_3 qiymatlarga mos $p_1,\ p_2$ va p_3 ehtimollarni toping.

16-variant

Ushbu

x	-4	1	3	5
P	0,3	0,1	0,2	0,4

taqsimot qonuni bilan berilgan. $\boldsymbol{x}$ diskret tasodifiy miqdorning dispersiyasini va o'rtacha kvadratik chetlanishini toping.

17-variant

Ushbu

x	1	2	3	4
P	0,1	0,2	0,3	0,4

taqsimot qonuni bilan berilgan ***x*** diskret tasodifiy miqdorning dispersiyasini va o'rtacha kvadratik chetlanishini toping.

18-variant

Ushbu

x	-3	1	7	5
P	0,4	0,2	0,3	0,1

taqsimot qonuni bilan berilgan ***x*** diskret tasodifiy miqdorning dispersiyasini va o'rtacha kvadratik chetlanishini toping.

19-variant

Ushbu

x	-4	3	4	5
P	0,3	0,1	0,4	0,2

taqsimot qonuni bilan berilgan ***x*** diskret tasodifiy miqdorning dispersiyasini va o'rtacha kvadratik chetlanishini toping.

20-variant

Ushbu

x	2	-5	3	5
P	0,1	0,1	0,5	0,3

taqsimot qonuni bilan berilgan ***x*** diskret tasodifiy miqdorning dispersiyasini va o'rtacha kvadratik chetlanishini toping.

21-variant

Qurilma bir-biridan erkli ishlaydigan uchta elementdan iborat. Har bir elementning bitta tajribada ishdan chiqish ehtimoli 0,2 ga teng. Bitta tajribada ishdan chiqqan elementlar sonining taqsimot qonunini tuzing.

22-variant

8 ta detal solingan qutida 6 ta standart detal bor. Tavakkaliga 2 ta detal olingan. Olingan detallar orasidagi standart detallar sonining taqsimot qonunini tuzing.

23-variant

12 ta detal solingan qutida 8 ta standart detal bor. Tavakkaliga 4 ta detal olingan. Olingan detallar orasidagi standart detallar sonining taqsimot qonunini tuzing.

24-variant

Qutidagi 6 ta detal solingan qutida 4 ta standart detal bor. Tavakkaliga 3 ta detal olingan. $\boldsymbol{x}$ diskret tasodifiy miqdor- olingan detallar orasidagi standart detallar sonining taqsimot qonunini tuzing.

25-variant

Qutidagi 10 ta detal solingan qutida 3 ta standart detal bor. Tavakkaliga 4 ta detal olingan. $\boldsymbol{x}$ diskret tasodifiy miqdor-olingan detallar orasidagi standart detallar sonining taqsimot qonunini tuzing.

26-variant

$\boldsymbol{x}$ miqdorning quyidagi taqsimot qonuni bilan berilgan $\boldsymbol{x}$ diskret tasodifiy matematik kutilishini toping:

x	-3	5	8
P	0,3	0,2	0,5

27-variant

Quyidagi taqsimot qonuni bilan berigan $\boldsymbol{x}$ diskret tasodifiy miqdorning matematik kutilishini toping:

x	4	-3	5	7
P	0,2	0,3	0,1	0,4

28-variant

Agar $\boldsymbol{x}$ va $\mathbf{y}$ ning matematik kutilishi ma'lum bo'lsa, $\mathbf{z}$ tasodifiy miqdorning matematik kutilishini toping: $z=\boldsymbol{x}+3\boldsymbol{y}$, $M(\boldsymbol{x})=4$, $M(y)=3$

29-variant

Agar $\boldsymbol{x}$ va $\boldsymbol{y}$ ning matematik kutilishi ma'lum bo'lsa, z tasodifiy miqdorning matematik kutilishini toping: $z=3\boldsymbol{x}+2\boldsymbol{y}$, $M(\boldsymbol{x})=3$, $M(\boldsymbol{y})=5$

30-variant

$\boldsymbol{x}$ diskret tasodifiy miqdorning mumkin bo'lgan qiymatlarining ro'yxati berilgan: $\boldsymbol{x}_1=0$ $x_2=1$ $\boldsymbol{x}_3=2$ shuningdek, bu miqdorning va uning kvadratining matematik kutilishlari ma'lum: $M(\boldsymbol{x})=0,2$ $M(\boldsymbol{x}^2)=0,8$. Mumkin bo'lgan $\boldsymbol{x}_1$, $\boldsymbol{x}_2$ va $\boldsymbol{x}_3$ qiymatlarga mos $\boldsymbol{p}_1$, $\boldsymbol{p}_2$ va p_3 ehtimollarni toping.

31-variant

$\boldsymbol{x}$ diskret tasodifiy miqdorning mumkin bo'lgan qiymatlarining ro'yxati berilgan: $\boldsymbol{x}_1=-1$ $\boldsymbol{x}_2=0$ $\boldsymbol{x}_3=2$ shuningdek, bu miqdorning va uning kvadratining matematik kutilishlari ma'lum: $M(\boldsymbol{x})=0,1$ $M(\boldsymbol{x}^2)=0,9$ Mumkin bo'lgan $\boldsymbol{x}_1$, x_2 va x_3 qiymatlarga mos p_1, p_2 va p_3 ehtimollarni toping.

32-variant

Ushbu

x	-4	1	3	5
p	0,3	0,1	0,2	0,4

taqsimot qonuni bilan berilgan. x diskret tasodifiy miqdorning dispersiyasini va o'rtacha kvadratik chetlanishini toping.

33-variant

Ushbu

x	1	2	3	4
p	0,1	0,2	0,3	0,4

taqsimot qonuni bilan berilgan. ***x*** diskret tasodifiy miqdorning dispersiyasini va o'rtacha kvadratik chetlanishini toping.

15- tajriba ishi

Tasodifiy miqdorlar. Ehtimollar taqsimotining qonunlari

Taqsimotning integral funksiyasi deb, har bir x qiymat uchun x ta tasodifiy miqdorning x dan kichik qiymat qabul qilish ehtimolini aniqlaydigan F(x) funksiyaga aytiladi, ya'ni F(x)=P(x <x).

Ko'pincha "integral funksiya" termini o'rnida "taqsimot funksiyasi" terminidan foydalaniladi.

Integral funksiya quyidagi xossalarga ega:

1-Xossa. Integral funksiyaning qiymatlari [0;1] kesmaga tegishli: $0 \le F(x) \le 1$

2-Xossa. Integral funksiya kamaymaydigan funksiya, ya'ni $x_2 > x_1$ bo'lsa, u holda $F(x_2) \ge F(x_1)$.

1-natija. ***x*** tasodifiy miqdorning (a,b) intervalda yotgan qiymatni qabul qilish ehtimoli integral funksiyaning shu intervaldagi orttirmasiga teng:

P(a< x <b)=F(b)-F(a)

2-natija. Uzluksiz tasodifiy miqdorning bitta tayin qiymatni, masalan, x_1 qiymatni qabul qilish ehtimoli nolga teng: P(x =x_1)=0

3-Xossa. Agar x tasodifiy miqdorning barcha mumkin bo'lgan qiymatlari intervalga tegishli bo'lsa,u holda $x \le a$ bo'lganda F(x)=0; $x \ge в$ bo'lganda F(x)=1

3-natija. Quyidagi limit munosabatlar o'rinli: $\lim_{x\to-\infty} F(x) = 0$ $\lim_{x\to\infty} F(x) = 1$

Namunaviy masala yechimi

x tasodifiy miqdor quyidagi integral funksiya bilan berilgan:

$$F(x) = \begin{cases} 0, & x \le -1 \\ \frac{3}{4}x + \frac{3}{4} & -1 < x \le \frac{1}{3} \\ 1 & x > \frac{1}{3} \end{cases}$$

Sinov natijasida *x* miqdorning (0;1/3) intervalda yotgan qiymatni qabul qilish ehtimolini toping.

Yechilishi: *x* ning (a;b) intervalda yotgan qiymatni qabul qilish ehtimoli integral funksiyaning bu intervaldagi orttirmasiga teng:

P(a<*x*<b)=F(b)-F(a) Bu formulaga a=0, в=1/3 ni qo'yib, quyidagini hosil qilamiz:

$$\mathrm{P}(0<x<1/3)=F(1/3)-F(0)=\left[\frac{3}{4}x+\frac{3}{4}\right]_{x=\frac{1}{3}}-\left[\frac{3}{4}x+\frac{3}{4}\right]_{x=0}=\frac{1}{4}$$

Topshiriq variantlari

1-variant

x tasodifiy miqdor quyidagi integral funksiya bilan berilgan:

$$F(x)=\begin{cases}0, & x\le -1\\ \frac{1}{2}x+\frac{1}{2} & -1<x\le\frac{1}{4}\\ 1 & x>\frac{1}{4}\end{cases}$$

Sinov natijasida *x* miqdorning (0;1/4) intervalda yotgan qiymatni qabul qilish ehtimolini toping.

2-variant

x tasodifiy miqdor butun O*x* o'qda F(*x*)=1/2+(1/π) arctg x integral funksiya bilan berilgan. Sinov natijasida *x* miqdorning (0,1) intervalda yotadigan qiymat qabul qilish ehtimolini toping.

3-variant

x tasodifiy miqdor quyidagi integral funksiya bilan berilgan:

$$F(x)=\begin{cases}0, & x\le -2\\ \frac{1}{2}+\frac{1}{\pi}\arcsin\frac{x}{2} & -2<x\le 2\\ 1 & x>2\end{cases}$$

Sinov natijasida *x* miqdorning (-1,1) intervalda yotgan qiymatni qabul qilish ehtimolini toping.

4-variant

x tasodifiy miqdor quyidagi integral funksiya bilan berilgan:

$$F(x)=\begin{cases}0, & x\le 2\\ 0{,}5x-1, & 2<x\le 4\\ 1 & x>4\end{cases}$$

Sinov natijasida *x* miqdorning 0,2 dan kichik qiymat qabul qilish ehtimolini toping.

5-variant

x tasodifiy miqdor quyidagi integral funksiya bilan berilgan:

$$F(x)=\begin{cases}0, & x\le 2\\ 0{,}5x-1, & 2<x\le 4\\ 1 & x>4\end{cases}$$

Sinov natijasida x miqdorning 3 dan kichik qiymat qabul qilish ehtimolini toping.

6-variant

x tasodifiy miqdor quyidagi integral funksiya bilan berilgan:

$$F(x)=\begin{cases}0, & x\le 2\\ 0{,}5x-1, & 2<x\le 4\\ 1 & x>4\end{cases}$$

Sinov natijasida x miqdorning 3 dan kichik bo'lmagan qiymat qabul qilish ehtimolini toping.

7-variant

x tasodifiy miqdor quyidagi integral funksiya bilan berilgan:

$$F(x)=\begin{cases}0, & x\le 2\\ 0{,}5x-1, & 2<x\le 4\\ 1 & x>4\end{cases}$$

Sinov natijasida x miqdorning 5 dan kichik bo'lmagan qiymat qabul qilish ehtimolini toping.

8-variant

x uzluksiz tasodifiy miqdorning differensial funksiyasi berilgan:

$$F(x)=\begin{cases}0, & x\le 0\\ \cos x, & 0<x\le \pi/2\\ 0, & x>\pi/2\end{cases}$$ bo'lganda $f(x)$ integral funksiyani toping.

9-variant

x uzluksiz tasodifiy miqdorning $F(x)=\begin{cases}0, & x\le 0\\ \sin x, & 0<x\le \pi/2\\ 0 & x>\pi/2\end{cases}$ bo'lganda differensial funksiyasi berilgan. $f(x)$ integral funksiyani toping.

10-variant

x tasodifiy miqdor (0;1) intervalda F(x) = 2x differensial bilan berilgan. Bu intervaldan tashqarida $f(x)=0$. $\boldsymbol{x}$ miqdorning matematik kutilishini toping.

11-variant

x tasodifiy miqdor (0;2) intervalda F(x)=1/2x differensial bilan berilgan. Bu intervaldan tashqarida $f(x)=0$. x miqdorning matematik kutilishini toping.

12-variant

x tasodifiy miqdor (-c;c) intervalda $f(x)=\frac{1}{\pi\sqrt{c^2-x^2}}$ differensial funksiya bilan berilgan, bu intervaldan tashqarida $f(x)=0$. x ning dispersiyasini toping.

13-variant

x tasodifiy miqdor(-3;3) intervalda $f(x)=\frac{1}{\pi\sqrt{9-x^2}}$ differensial funksiya bilan berilgan, bu intervaldan tashqarida $f(x)=0$.

a) x ning dispersiyasini toping;

б) Qaysi biri ehtimolliroq sinash natijasida x<1 bo'lishimi, yoki x>1 bo'lishimi?

14-variant

x tasodifiy miqdor $(0;\pi)$ intervalda $F(x)=\frac{1}{2}\sin x$ differensial funksiya bilan berilgan, bu intervaldan tashqarida $f(x)=0$. x ning dispersiyasini toping;

PP15-variant

x tasodifiy miqdor (0;5) intervalda $F(x)=\frac{2}{25}x$ differensial funksiya bilan berilgan; bu intervaldan tashqarida $f(x)=0$. x ning dispersiyasini toping;

16-variant

x tasodifiy miqdor integral funksiya bilan berilgan;

$$F(x)=\begin{cases} 0, & x\le -2 \\ \frac{x}{4}+\frac{1}{2}, & -2<x\le 2 \\ 1 & x>2 \end{cases}$$

x miqdorning dispersiyasini toping;

17-variant

x tasodifiy miqdor $(0;\pi)$ intervalda $F(x)=\frac{1}{2}\sin x$ differensial funksiya bilan berilgan, bu intervaldan tashqarida $f(x)=0$. $y=\varphi(x)=x^2$ funksiyaning dispersiyasini dastlab y ning differentsial funksiyasini topmasdan hisoblang.

18-variant

x tasodifiy miqdor $(0;\pi/2)$ intervalda $F(x)=\cos x$ differensial funksiya bilan berilgan, bu intervaldan tashqarida $f(x)=0$. $Y=\varphi(x)=x^2$ funksiyaning dispersiyasini dastlab ***y*** ning differensial funksiyasini topmasdan hisoblang.

19-variant

x tasodifiy miqdor $x \geq 0$ bo'lganda $F(x) = \frac{x^n e^{-x}}{n!}$ differensial funksiya bilan berilgan, x <0 bo'lganda $f(x) = 0$. x ning matematik kutilishini toping.

20-variant

x tasodifiy miqdor $x \geq 0$ bo'lganda $F(x) = \frac{x^n e^{-x}}{n!}$ differensial funksiya bilan berilgan, X<0 bo'lganda $f(x) = 0$. x ning dispersiyasini toping.

21-variant

x tasodifiy miqdor quyidagi integral funksiya bilan berilgan:

$$F(x) = \begin{cases} 0, & x \leq 2 \\ 0{,}5x - 1, & 2 < x \leq 4 \\ 1 & x > 4 \end{cases}$$

Sinov natijasida x miqdorning 3 dan kichik qiymat qabul qilish ehtimolini toping.

22-variant

x tasodifiy miqdor quyidagi integral funksiya bilan berilgan:

$$F(x) = \begin{cases} 0, & x \leq 2 \\ 0{,}5x - 1, & 2 < x \leq 4 \\ 1 & x > 4 \end{cases}$$

Sinov natijasida x miqdorning 3 dan kichik bo'lmagan qiymat qabul qilish ehtimolini toping.

23-variant

x tasodifiy miqdor quyidagi integral funksiya bilan berilgan:

$$F(x) = \begin{cases} 0, & x \leq 2 \\ 0{,}5x - 1, & 2 < x \leq 4 \\ 1 & x > 4 \end{cases}$$

Sinov natijasida x miqdorning 5 dan kichik bo'lmagan qiymat qabul qilish ehtimolini toping.

24-variant

x uzluksiz tasodifiy miqdorning differensial funksiyasi berilgan:

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \cos x, & 0 < x \leq \pi/2 \\ 0, & x > \pi/2 \end{cases}$$ bo'lganda f (x) integral funksiyani toping.

25-variant

x uzluksiz tasodifiy miqdorning

$F(x)=\begin{cases}0, & x\le 0\\ \sin x, & 0<x\le \pi/2\\ 0 & x>\pi/\end{cases}$ differensial funksiyasi berilgan. f ($\boldsymbol{x}$) integral funksiyani toping.

26-variant

x tasodifiy miqdor (0;1) intervalda F(x) = 2x differensial funksiya bilan berilgan. Bu intervaldan tashqarida $f(x)=0$. x miqdorning matematik kutilishini toping.

27-variant

x tasodifiy miqdor (0;2) intervalda F(x)=1/2x differsial funksiya bilan berilgan. Bu intervaldan tashqarida $f(x)=0$. x miqdorning matematik kutilishini toping.

28-variant

x tasodifiy miqdor (-c;c) intervalda $f(x)=\dfrac{1}{\pi\sqrt{c^2-x^2}}$ differensial funksiya bilan berilgan, bu intervaldan tashqarida $f(x)=0$. x ning dispersiyasini toping.

29-variant

x tasodifiy miqdor (-3;3) intervalda $f(x)=\dfrac{1}{\pi\sqrt{9-x^2}}$ differensial funksiya bilan berilgan, bu intervaldan tashqarida $f(x)=0$.

a) x ning dispersiyasini toping;

б) Qaysi biri ehtimolliroq: sinash natijasida x <1 bo'lishimi, yoki x >1 bo'lishimi?

30-variant

x tasodifiy miqdor $(0;\pi)$ intervalda $F(x)=\dfrac{1}{2}\sin x$ differensial funksiya bilan berilgan, bu intervaldan tashqarida $f(x)=0$. x ning dispersiyasini toping;

31-variant

x tasodifiy miqdor (0;5) intervalda $F(x)=\dfrac{2}{25}x$ differensial funksiya bilan berilgan; bu intervaldan tashqarida $f(x)=0$. x ning dispersiyasini toping.

Loran qatori

Loran darajali qatorlarining umumiy ko'rinishi

$$...+\frac{c_{-n}}{(z-a)^n}+\frac{c_{-(n-1)}}{(z-a)^{n-1}}+...+\frac{c_{-2}}{(z-a)^2}+\frac{c_{-1}}{z-a}+c_o+c_1(z-a)+c_2(z-a)^2+...+c_n(z-a)^n+...$$

$$...=\sum_{n=-\infty}^{\infty}c_n(z-a)^n.$$

(1)

bo'lib, u ikki qismdan iborat:

a) to'g'ri qismi:

$$P=c_o+c_1(z-a)+c_2(z-a)^2+...+c_n(z-a)^n+... \qquad (2)$$

Ma'lumki, bu qatorning yaqinlashish sohasi $|z-a|<R$ doiradan iborat;

b) bosh qismi:

$$Q=...+c_{-n}(z-a)^{-n}+c_{-(n-1)}(z-a)^{-(n-1)}+...+c_{-2}(z-a)^{-2}+c_{-1}(z-a)^{-1}=\sum_{n=-\infty}^{-1}c_n(z-a)^n$$

(3)

Buning yaqinlashish sohasi $|z-a|>r$, ya'ni doira tashqarisidan iborat.

Shunday qilib, Loran umumiy qatorining yaqinlashish sohasi markazi a nuqtada bo'lgan r va R radiusli aylanalar orasidagi xalqadan iborat bo'ladi: $r<|z-a|<R$ Bu yerda uch hol bo'lishi mumkin:

1. Agar r < R bo'lsa, xalqa mavjud bo'lib, Loran qatori o'sha xalqa ichida absolyut yaqinlashadi.
2. Agar r = R bo'lsa, xalqa faqat aylanadan iborat bo'lib, Loran qatori uning ba'zi nuqtalaridagina yaqinlashuvchi bo'lishi mumkin.
3. Agar r > R bo'lsa, hech qanday xalqa yo'q, shu sababli Loran qatori uzoqlashuvchi bo'ladi.

Agar Loran qatori berilgan bo'lsa, uning ikki;a qismini alohida tekshirib xalqani topish lozim bo'ladi. Aksincha, agar funksiya berilgan bo'lsa, uni Loran qatoriga yoyish uchun oldindan berilgan xalqaga qarab ish ko'rish talab qilinadi.

Kompleks argumentli funksiyalar nazariyasidan ma'lumki, Loran qatorining c_n koeffitsiyenti ushbu formula orqali ifodalanadi:

$$c_n=\frac{1}{2\pi i}\oint_G\frac{f(z)dz}{(z-a)^{n+1}},\quad n=0,\pm1,\pm2,\pm3,..., \qquad (4)$$

bunda *a* son – ikkila aylana markazi, *f(z)* – xalqa ichida analitik funksiya ; G chiziq – xalqaz ichida a markazli ixtiyoriy aylanadan iborat. Odatda c_n koeffisiyenlarni (4) formula orqali topish qiyin bo'lganda, ba'zan sun'iy yo'l bilan ish ko'rishga to'g'ri keladi.

M i s o l l a r

Quyidagi Loran qatorlarining yaqinlashish sohalarini aniqlang:

1. $\sum_{n=1}^{\infty}\left(\frac{2}{z}\right)^n+\sum_{n=0}^{\infty}\left(\frac{z}{4}\right)^n$.

Yechilishi. Dastlab qatorning to'g'ri qismini tekshirib ko'raylik:

$P=\sum_{n=0}^{\infty}\frac{z^n}{4^n}$; $c_n=\frac{1}{4^n}$, $c_{n-1}=\frac{1}{4^{n-1}}$, $L=\lim_{n\to\infty}\left|\frac{c_n}{c_{n-1}}\right|=\lim_{n\to\infty}\frac{4^{n-1}}{4^n}=\frac{1}{4}$, $L=\frac{1}{4}$, $R=\frac{1}{L}=4$, $|z|<4$.

Endi qatorning bosh qismini tekshiramiz:

$$Q=\sum_{n=1}^{\infty}\frac{2^n}{z^n}=\sum_{n=-\infty}^{-1}2^{-n}z^n;\quad c_{-n}=2^n,\ c_{-(n-1)}=2^{n-1}.$$

$$r=\lim_{n\to\infty}\left|\frac{c_{-n}}{c_{-(n-1)}}\right|=\lim_{n\to\infty}\frac{2^n}{2n-1}=2,\quad |z|>2$$

Shunday qilib, biz izlayotgan soha $2<|z|<4$ xalqadan iborat ekan.

2. $\sum_{n=1}^{\infty}\frac{\sin in}{(z-i)^n}+\sum_{n=0}^{\infty}\frac{(z-i)^n}{nl}$.

Yechilishi. Qatorning to'g'ri qismi:

$P=\sum_{n=0}^{\infty}\frac{(z-i)^n}{nl}$, $a=i$, $c_n=\frac{1}{nl}$, $c_{n-1}=\frac{1}{(n-1)l}$;

$$L=\lim_{n\to\infty}\frac{(n-1)l}{nl}=\lim_{n\to\infty}\frac{1}{n}=0,\qquad R=\frac{1}{L}=+\infty,$$

$$|z-i|<+\infty;$$

Qatorning bosh qismi:

$$Q=\sum_{n=1}^{\infty}\frac{\sin in}{(z-i)^n};\quad c_{-n}=\sin in=\frac{1}{2i}\left(e^{-n}-e^{n}\right);$$

$$c_{-(n-1)}=\frac{e^{-(n-1)}-e^{n-1}}{2i};$$

$$r=\lim_{n\to\infty}\left|\frac{c_{-n}}{c_{-(n-1)}}\right|=\lim_{n\to\infty}\frac{e^{-2n}-1}{e\cdot e^{-2n}-e^{-1}}=\frac{1}{e^{-1}}=e,$$

$$r=e,\ \ |z-i|>e.$$

Demak, soha : $e<|z-i|<\infty$.

3. $f(x)=z^4\cos\frac{1}{z}$ ni z=0 nuqta atrofida Loran qatoriga yoying.

Yechilishi. Ma'lumki,

$$\cos\frac{1}{z}=1-\frac{1}{2l}\left(\frac{1}{z}\right)^2+\frac{1}{4l}\left(\frac{1}{z}\right)^4-\frac{1}{6l}\left(\frac{1}{z}\right)^6+\ldots$$

Shu sababli

$$z^4\cos\frac{1}{z}=z^4-\frac{z^2}{2l}+\frac{1}{4l}-\frac{1}{6lz^2}+\frac{1}{8lz^4}-\frac{1}{10lz^6}+\dots,R=+\infty.$$

4. $f(z)=\dfrac{1+\cos z}{z^4}$ ni z=0 nuqta atrofida Loran qatoriga yoying.

Yechilishi. Ma'lumki,

$$\cos z=1-\frac{z^2}{2l}+\frac{z^4}{4l}+\frac{z^6}{6l}+\dots.$$

Buning ikkala tomoniga 1 ni qo'shib, so'ngra z^4 ga bo'lamiz.

$$\frac{1+\cos z}{z^4}=\frac{2}{z^4}-\frac{1}{2lz^2}+\frac{1}{4l}-\frac{z^2}{6l}+\dots,R=+\infty.$$

5. $f(z)=\dfrac{\sin^2 z}{z}$ ni z=0 nuqta atrofida Loran qatoriga yoying.

Yechilishi. Ma'lumki,

$$\sin^2 z=\frac{1-\cos 2x}{2}=\frac{1}{2}-\frac{1}{2}\left[1-\frac{(2z)^2}{4l}+\frac{(2z)^4}{4l}-\frac{(2z)^6}{6l}+\dots\right]=\left[\frac{(2z)^2}{2l}-\frac{(2z)^4}{4l}+\frac{(2z)^6}{6l}-\frac{(2z)^8}{8l}+\dots\right]\cdot\frac{1}{2}.$$

Shunga asosan

$$\frac{\sin^2 z}{z}=\left[\frac{2^2\cdot z}{2l}-\frac{2^4z^3}{4l}+\frac{2^6z^5}{6l}-\frac{2^8z^7}{8l}+\dots\right]\cdot\frac{1}{2},R=+\infty$$

6. $f(z)=\dfrac{1}{(z-2)(z-3)}$ funksiyani $2<|z|<3$ xalqada Loran qatoriga yoying.

Yechilishi. Buning uchun berilgan funksiyani eng soda kasrlarga ajratib olamiz:

$$\frac{1}{(z-2)(z-3)}=\frac{A}{z-2}+\frac{B}{z-3},\quad 1=A(z-3)+B(z-2),$$

z=2 bo'lganda A=-1; z=3 bo'lganda B=1.

Berilgan xalqaga binoan ish ko'ramiz : a) $|z|>2,$ ya'ni $\dfrac{2}{|z|}<1$ bo'lishi uchun quyidagi ko'rinishda yozamiz :

$$-\frac{1}{z-2}=-\frac{1}{z}\cdot\frac{1}{1-\frac{2}{z}}=-\frac{1}{z}\left[1+\frac{2}{z}+\left(\frac{2}{z}\right)^2+\left(\frac{2}{z}\right)^3+\dots\right]=-\sum_{n=1}^{\infty}\frac{2^{n-1}}{z^n}\ ;$$

b) $|z|<3$ ya'ni $\left|\dfrac{z}{3}\right|<1$ bo'lishi uchun quyidagi ko'rinishda yozamiz :

$$\frac{1}{z-3}=-\frac{1}{3-z}=-\frac{1}{3}\cdot\frac{1}{1-\frac{z}{3}}=-\frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{z}{3}\right)^n.$$

Demak,

$$\frac{1}{(z-2)(z-3)}=-\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{2}{z}\right)^n-\frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{z}{3}\right)^n.$$

Bulardan birinchisi qatorning bosh qismi bo'lib, ikkinchisi esa to'g'ri qismidir.

7. $\frac{1}{z+z^2}$ funksiyani $0<|z|<1$ xalqada Loran qatoriga yoying.

Yechilishi.Ma'lumki,

$$\frac{1}{z+z^2}=\frac{1}{z(1+z)}=\frac{1}{z}-\frac{1}{1+z}=\frac{1}{z}-\sum_{n=0}^{\infty}(-1)^n z^n .$$

Bu qatorning bosh qismi birgina haddan iborat.

M a sh q l a r.

Quyidagi Loran qatorlarining yaqinlashish sohalarini aniqlang.

1. $\sum_{n=1}^{\infty}\frac{(3+4i)^n}{(z+2i)^n}+\sum_{n=0}^{\infty}\frac{(z+2i)^n}{6^n};$ 2. $\sum_{n=1}^{\infty}\frac{1}{n^n(z-2+i)^n}+\sum_{n=0}^{\infty}(1+in)(z-2+i)^n;$

3. $\sum_{n=1}^{\infty}\frac{1}{z^n}+\sum_{n=0}^{\infty}\frac{z^n}{2^{n+1}};$ 4. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^4z^4}+\sum_{n=1}^{\infty}\frac{z^n}{n2^n};$

5. $-\frac{1}{z-1}+\sum_{n=0}^{\infty}(-1)^n(z-1)^n$; 6. $\sum_{n=1}^{\infty}\frac{1}{z^n}+\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n;$

7. $\sum_{n=-\infty}^{-1}(n+2)i^{n+1}(z-i)^n$

Quyidagi funksiyalarni Loran qatoriga yoying:

8. $\frac{e^z}{z}$ funksiyani z=0 nuqta atrofida .

9. $\frac{e^z}{z^3}$ funksiyani z=0 nuqta atrofida .

10. $\frac{1-\cos z}{z^2}$ funksiyani z=0 nuqta atrofida .

11. $\frac{1}{z}\sin^2\frac{2}{z}$ funksiyani z=0 nuqta atrofida .

12. $\frac{1-e^{-z}}{z^3}$ funksiyani z=0 nuqta atrofida .

13. $\frac{2z+3}{z^2+3z+2}$ funksiyani $1<|z|<2$ xalqada.

14. $\frac{z^2-z+3}{z^3-3z+2}$ funksiyani $1<|z|<2$ xalqada.

15. $\frac{1}{z^2+2z-8}$ funksiyani $1<|z+2|<4$ xalqada.

16. $\frac{1}{\left(z^2-4\right)^2}$ funksiyani $1<|z+2|<\infty$ xalqada.

17. $\frac{2z-3}{z^2-3z+2}$ funksiyani $0<|z-2|<1$ xalqada.

18. $\frac{2z+1}{z^2+z-2}$ funksiyani $2<|z|<\infty$ xalqada.

19. $\frac{1}{(z-1)^2(z+2)}$ funksiyani $1<|z|<2$ xalqada.

20. $\frac{1}{\left(z^2+1\right)\left(z^2-4\right)}$ funksiyani $1<|z|<2$ xalqada.

21. $z^3 e^{\frac{1}{z}}$ funksiyani $0<|z|<\infty$ xalqada.

22. $z^2 \sin\frac{(z+1)\pi}{z}$ funksiyani $0<|z|<\infty$ xalqada.

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M U N D A R I J A