O‘ZBEKISTON RESPUBLIKACI
OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

FARG‘ONA POLITEXNIKA INSTITUTI

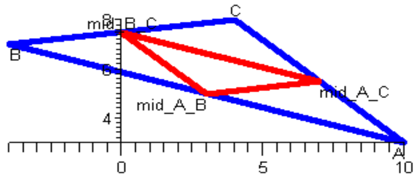
OLIY MATEMATIKA
kafedrasi

E.M.MIRZAKARIMOV

M A P L E 7
d a s t u r i yo r d a m i d a
OLIY MATEMATIKA
masalalarini echish

USLUBIY QO‘LLANMA
(1- qism)

```
> with(geometry):
> draw({T(color=blue),mT(color=red)},
style=line, axes=NONE, printtext=true);
```



Farg‘ona-2010

O'ZBEKISTON RESPUBLIKACI
OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

FARG'ONA POLITEXNIKA INSTITUTI

OLIY MATEMATIKA
kafedrasi

E.M.MIRZAKARIMOV

M A P L E 7
d a s t u r i yo r d a m i d a
OLIY MATEMATIKA
masalalarini echish

USLUBIY QO'LLANMA
(1- qism)

Institut uslubiy kengashi
tomonidan tasdiqlangan bayon
№_3_"_26_"_02_2010 yil

Farg'ona-2010

Ushbu uslubiy qo‘llanma, barcha oliy o‘quv yurtidagi texnika yo‘nalishlari bo‘yicha ta’lim olayotgan talabalar uchun, 1-semestrdagi Oliy matematika fanidan, mustaqil ishlarni MALE7 dasturidan foydalanib bajarishlari uchun mo‘ljallangan.

Uslubiy qo‘llanma Oliy matematika fanidan analitik geometriya bo‘limining chiziqli tenglamalar sistemasini echish usullari, vektorlar, tekislikda to‘g‘ri chiziq, egri chiziq, fazoda tekislik, to‘g‘ri chiziq va ikkinchn tartibli sirtlar masalalariga bag‘ishlangan. Har bir bo‘limda nazariya bo‘yicha mahlumotlar keltirilgan, ularga mos namunaviy masalalarni MALE7 dasturi yordamida echish yo‘llari ko‘rsatilgan, shuningdek mustaqil ishlash uchun variantlar tuzilgan.

Ushbu uslubiy qo‘llanma “Oliy matematika” kafedrasining uslubiy semenarida muxokama qilingan (№ 6, 18. 01. 2010)

Iqtisod va menejment fakulg‘tetining uslubiy komissiyasi tomonidan maqullangan. (№ 4, 24 . 01. 2010)

Tuzuvchi katta o‘qituvchi **M.E. MIRZAKARIMOV**

Taqrizchilar:
Tashkent axborot texnologiya universiteti Farg‘ona filiyali, “Axborot texnologiyalari” kafedrasi ditsenti, texnika fanlari nomzodi: **F. POLVONOV**
Farg‘ona davlat universiteti “Amaliy matematika va informatika” kafedrasi ditsenti, fizika-matematika fanlari nomzodi: **Sh. T. KARIMOV**
Farg‘ona politexnika instituti “Oliy matematika” kafedra mudiri fizika-matematika fanlari nomzodi,dotsent. **A. ABDURAZOQOV**

1. CHIZIQLI TENGLAMALAR SISITEMACINI ECHISH

Chiziqli tenglamalar sistemasini Gauss, Kramer va teskari matritsa usullari bilan echimini topish masalarini ko‘ramiz.

1.1-masala. Quyidagi uch nomahlumli chiziqli tenglamalar sistemasining echimini:
1) Gauss usuli, 2) Kramer, 3) Matritsa usulida toping.

$$\begin{cases} 2x_1 + 7x_2 + 13x_3 = 0 \\ 3x_1 + 14x_2 + 12x_3 = 18 \\ 5x_1 + 25x_2 + 16x_3 = 39 \end{cases} \qquad (1)$$

Echish:

1.1. Gauss usulida echish.

Etakchi tenglama uchun birinchi tenglamani olamiz. Bu tenglamadan etakchi nomahlum uchun x_1 va $a_{11}\neq 0$ ni etakchi element uchun tanlaymiz. Birinchi tenglamadagi x_1 ning koeffitsenti a_{11} ni 1 ga aylantirish uchun birinchi tenglamaning barcha qo‘shiluvchilarini $a_{11}\neq 0$ ga bo‘lamiz. Xosil bo‘lgan tenglamadan foydalanib ikkinchi va uchinchi tenglmalardan x_1 nomahlumni yo‘qotish yoki uning koeffitsentini nolg‘ga aylantirish uchun etakchi tenglamani –3 ga ko‘paytirib 2- tenglamaga qo‘shamiz, so‘ngra etakchi tenglamani –5 ko‘paytirib 3- teglamaga qo‘shamiz. Natijada quyidagicha sistemaga kelamiz:

$$\begin{cases} 2x_1 + 7x_2 + 13x_3 = 0 \\ \frac{7}{2}x_2 + \frac{15}{2}x_3 = 18 \\ \frac{15}{2}x_2 - \frac{33}{2}x_3 = 39 \end{cases}$$

Bu tenglamlar sistemasida etakchi tenglama uchu 2- tenglamani olamiz. Unda 7/2 koeffitsientli x_2 nomahlumni etakchi element uchun olib, 2- tenglamani 7/2 ga bo‘lib hosil bo‘lgan etakchi tenglamani –15 ga ko‘paytirib 3- tenglamaga qo‘shamiz:

$$\begin{cases} 2x_1 + 7x_2 + 13x_3 = 0 \\ \frac{7}{2}x_2 - \frac{15}{2}x_3 = 18 \\ -\frac{3}{7}x_3 = \frac{3}{7} \end{cases}$$

bu sistemaning 3- tenglamasidan x_3 nomahlumni topamiz. x_3 asosida 2- tenglamadan x_2 ni topamiz. x_3, x_2 lar asosida 1- tenglamadan x_1 ni topamiz.

$$x_3 = -1$$

$$x_2 = \frac{2}{7}(18 + \frac{15}{2}x_3) = \frac{2}{7}(18 - \frac{15}{2}) = 3$$

$$x_1 = -\frac{1}{2}(7x_2 + 13x_3) = -\frac{1}{2}(7*3 + 13*(-1)) = -4$$

Demak sistema echimi: $x_1=-4$, $x_2=3$, $x_3= -1$

Maple7 dasturida masalani echish.

Uch noma'lumli **chiziqli tenglamalar sistemasini oddiy va Gauss usulida echish(1.1-masala).**

$$\begin{cases} 2ő_1 + 7ő_2 + 13ő_3 = 0, \\ 3ő_1 + 14ő_2 + 12ő_3 = 18, \\ 5ő_1 + 25ő_2 + 16ő_3 = 39, \end{cases}$$

Maple7 dasturida masalalarni echishdagi amallarni bajarish uchun ishchi oynada > belgidan so'ng kerakli buyruqni yozib Enter tugmasini bosish kerak.

1. Oddiy usulida echish **(Gauss.mw).**

```
> solve( {2*x + 7*y + 13*z = 0, 3*x + 14*y + 12*z =18, 5*x + 25*y +16*z =39}, [x, y, z]);
```

[x=-4, y=3, z=-1]

2. Gauss usulida uch noma'lumli chiziqli tenglamalar sistemasini echish.

```
> with(LinearAlgebra):
A := <<2,3,5>|<7,14,25>|<13,12,16>>;
```

$$A := \begin{bmatrix} 2 & 7 & 13 \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{bmatrix}$$

```
> B := <0,18,39>;
```

$$b := \begin{bmatrix} 0 \\ 18 \\ 39 \end{bmatrix}$$

```
> GaussianElimination(A);
```

$$\begin{bmatrix} 2 & 7 & 13 \\ 0 & \frac{7}{3} & \frac{-15}{2} \\ 0 & 0 & \frac{-3}{7} \end{bmatrix}$$

```
> GaussianElimination(A,'method'='FractionFree');
```

$$\begin{bmatrix} 2 & 7 & 13 \\ 0 & 7 & -15 \\ 0 & 0 & -3 \end{bmatrix}$$

```
>ReducedRowEchelonForm(<A|b>);
```

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

3. To‘rt noma’lumli chiziqli tenglamalar sistemasini Maple7 dasturida echish

1) Oddiy usulida echish

> sys:=({1*x1-5*x2-1*x3+3*x4=-5,2*x1+3*x2+1*x3-1*x4=4, 3*x1-2*x2+3*x3+4*x4=-1,5*x1+3*x2+2*x3+2*x4=0}):
> solve(sys,{x1,x2,x3,x4});

$$\{x4 = -3,\ x2 = -1,\ x1 = 1,\ x3 = 2\}$$

2) Gauss usulida echish

$$\begin{cases} x_1 - 5x_2 - x_3 + 3x_4 = -5, \\ 2x_1 + 3x_2 + x3 - x_4 = 4, \\ 3x_1 - 2x_2 + 3x_3 + 4x_4 = -1, \\ 5x_1 + 3x_2 + 2x_3 + 2x_4 = 0. \end{cases}$$

> with(LinearAlgebra):
A := <<1,2,3,5>|<-5,3,-2,3>|<-1,1,3,2>|<3,-1,4,2>>;

$$A := \begin{bmatrix} 1 & -5 & -1 & 3 \\ 2 & 3 & 1 & -1 \\ 3 & -2 & 3 & 4 \\ 5 & 3 & 2 & 2 \end{bmatrix}$$

> b := <-5,4,-1,0>;

$$B := \begin{bmatrix} -5 \\ 4 \\ -1 \\ 0 \end{bmatrix}$$

> GaussianElimination(A,'method'='FractionFree');

$$\begin{bmatrix} 1 & -5 & -1 & 3 \\ 0 & 13 & 3 & -7 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 67/39 \end{bmatrix}$$

> ReducedRowEchelonForm(<A|b>);

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

1.2. Kramer qoidasi yordamida echish.

Berilgan tenglamalar sistema nomahlumlarning koeffitsientlari va ozod hadalari yordamida determinantlarni tuzamiz va ularni hisoblashning uchburchak yoki Sarrus usullaridan foydalanamiz. Biz (1) tenglamlar sistemasining determinantlarini tuzib, uchburchak usulida hisolab uni son qiymatlarni topamiz.

$$\Delta = \begin{vmatrix} 5 & 7 & 13 \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{vmatrix} \begin{aligned} &= 2\cdot 14\cdot 26 + 3\cdot 25\cdot 13 + 7\cdot 12\cdot 5 - 5\cdot 14\cdot 13 - 12\cdot 2\cdot 25 - 3\cdot 7\cdot 16 = \\ &= 448 + 975 + 420 - 910 - 600 - 336 = -3 \end{aligned}$$

$$\Delta x_1 = \begin{vmatrix} 0 & 7 & 13 \\ 18 & 14 & 12 \\ 39 & 25 & 16 \end{vmatrix} = 0 + 3276 + 5850 - 7098 - 0 - 2016 = 12$$

$$\Delta x_2 = \begin{vmatrix} 2 & 0 & 13 \\ 3 & 18 & 12 \\ 16 & 39 & 16 \end{vmatrix} = 576 + 0 + 1521 - 1170 - 0 - 936 = -9$$

$$\Delta x_3 = \begin{vmatrix} 2 & 7 & 0 \\ 3 & 14 & 18 \\ 5 & 25 & 39 \end{vmatrix} = 1092 + 0 + 630 - 0 - 900 - 819 = 3$$

Maple7 dasturida masalani echish.

```
> with(Student[LinearAlgebra]):
> d := <<2,3,5>|<7,14,25>|<13,12,16>>;
> d:=Determinant(d);
```

$$d = \begin{vmatrix} 5 & 7 & 13 \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{vmatrix}$$

$$d := K\ 3$$

```
> dx1:=<<0,18,39>|<7,14,25>|<13,12,16>>;
> d1:=Determinant(dx1);
```

$$dx1 = \begin{vmatrix} 0 & 7 & 13 \\ 18 & 14 & 12 \\ 39 & 25 & 16 \end{vmatrix}$$

$$d1 := 12$$

```
> dx2 := <<2,3,5>|<0,18,39>|<13,12,16>>;
> d2:=Determinant(dx2);
```

$$dx_2 = \begin{vmatrix} 2 & 0 & 13 \\ 3 & 18 & 12 \\ 16 & 39 & 16 \end{vmatrix}$$

$$d2 := \text{K } 9$$

```
> dx3 := <<2,3,5>|<7,14,25>|<0,18,39>>;
> d3:=Determinant(dx3);
```

$$dx3 = \begin{vmatrix} 2 & 7 & 0 \\ 3 & 14 & 18 \\ 5 & 25 & 39 \end{vmatrix}$$

$$d3 := 3$$

Kramer koidasiga asosnan sistema echimini topmamiz:

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{12}{-3} = -4, \qquad x_2 = \frac{\Delta x_2}{\Delta} = \frac{-9}{-3} = 3, \qquad x_3 = \frac{\Delta x_3}{\Delta} = \frac{3}{-3} = -1$$

```
> x:=d1/d;y:=d2/d;z:=d3/d;
```

$x := \text{K } 4 \quad y := 3 \quad z := \text{K } 1$

1.3. Teskari matritsa yordamida echish

Berilgan

$$\begin{cases} 2x_1 + 7x_2 + 18x_3 = 0 \\ 3x_1 + 14x_2 + 12x_3 = 18 \\ 5x_1 + 25x_2 + 16x_3 = 39 \end{cases}$$

Sistemani matritsa yordamida quyidagnicha yozamiz. Berilgan sistemaning matritsa ko‘rinishida yozilishi:

$$A \cdot X = B \qquad (*)$$

Bu erda:

$$A = \begin{pmatrix} 2 & 7 & 18 \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 18 \\ 39 \end{pmatrix}$$

(*) tenglmani quyidagicha yozamiz:

$$X = A^{-1} \cdot B$$

Bu erda:

$$A^{-1} = \frac{1}{\Delta} \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} \qquad (**)$$

Bu teskari matritsa topish formulasi bo‘lib, Δ A matritsa determnanti, A_{ij}(i,j=1,2,3) - Δ determinantning a_{ij} elementiga to‘g‘ri keluvchi algebraik to‘ldiruvchisi. Teskari matritsani topish uchun A matritsa detreminanti Δ ni tuzamiz va uning algebraik to‘ldiruvchlarini topamiz.

$$\Delta = \begin{vmatrix} 2 & 7 & 13 \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{vmatrix} = -3$$

Teskari matritsaning 1- ustun elementlari:

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 14 & 12 \\ 25 & 16 \end{vmatrix} = 224 - 300 = -76$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 12 \\ 5 & 16 \end{vmatrix} = -(48 - 60) = 12$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 3 & 14 \\ 5 & 25 \end{vmatrix} = 75 - 70 = 5$$

2-ustun elementlari:

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 7 & 13 \\ 25 & 16 \end{vmatrix} = -(112 - 325) = 213,$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 2 & 13 \\ 5 & 16 \end{vmatrix} = 32 - 65 = -33$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 2 & 7 \\ 5 & 25 \end{vmatrix} = -(50 - 35) = -15$$

3-ustun elementlari:

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 7 & 13 \\ 14 & 12 \end{vmatrix} = 84 - 182 = -98,$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 13 \\ 3 & 12 \end{vmatrix} = -(24 - 39) = 15$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 2 & 7 \\ 3 & 14 \end{vmatrix} = 28 - 21 = 7$$

A matrtitsaga teskari A^{-1} matritsani yozamiz:

$$A^{-1} = \frac{1}{-3}\begin{pmatrix} -76 & 213 & -98 \\ 12 & -33 & 15 \\ 5 & -15 & 7 \end{pmatrix}$$

$X = A^{-1} \cdot B$ tenglama echimini topamiz:

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 & -71 & 98/3 \\ -4 & 11 & -5 \\ -5/3 & 5 & -7/3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 18 \\ 39 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \\ -1 \end{pmatrix}$$

$x_1 = -4$, $x_2 = 3$, $x_3 = -1$

Maple7 dasturida masalani echish.

```
> with(Student[LinearAlgebra]):
> A := <<2,3,5>|<7,14,25>|<13,12,16>>;
```

$$A=\begin{pmatrix}2 & 7 & 18\\ 3 & 14 & 12\\ 5 & 25 & 16\end{pmatrix}$$

```
> A^(-1);
```

$$\begin{pmatrix}\frac{76}{3} & \frac{-213}{3} & \frac{-98}{3}\\ \frac{-12}{3} & \frac{-33}{3} & \frac{-15}{3}\\ \frac{-5}{3} & \frac{15}{3} & \frac{-7}{3}\end{pmatrix}$$

```
> B := <<0,18,39>>;
```

$$B:=\begin{pmatrix}0\\ 18\\ 39\end{pmatrix}$$

```
> X:=A^(-1).B;
```

$$X:=\begin{pmatrix}-4\\ 3\\ -1\end{pmatrix}$$

1.4. Matritsaviy tenglamani echish

Quyidagi matritsaviy tenglamani undaagi koeffitsent matritsaga teskari matritsa topish bilan echamiz.

$$\begin{pmatrix}1 & 3 & -1\\ 0 & 1 & -2\\ 1 & 1 & 0\end{pmatrix}X=\begin{pmatrix}13 & -4 & 6\\ 2 & 4 & 2\\ -2 & 5 & 5\end{pmatrix}$$

Yuqorida keltirilgan formula (*) va (**) formulalar

$$A\cdot X=B\ ,\quad X=A^{-1}\cdot B$$

asosida echish uchun oldin A^{-1} ni topamiz va unga B matritsani ko‘paytiramiz. Bu amalni Maple 7 dasturida quyidagicha bajaramiz:

```
> with(Student[LinearAlgebra]):
> A := <<1,0,1>|<3,1,-1>|<-1,-2,0>>;
```

$$A:=\begin{pmatrix}1 & 3 & -1\\ 0 & 1 & -2\\ 1 & 1 & 0\end{pmatrix}$$

```
> A^-1:=A^(-1);
```

$$A^{-1}:=\begin{pmatrix}\frac{2}{7} & \frac{-1}{7} & \frac{5}{7}\\ \frac{2}{7} & \frac{-1}{7} & \frac{-2}{7}\\ \frac{1}{7} & \frac{-4}{7} & \frac{-1}{7}\end{pmatrix}$$

```
> B := <<13,2,-2>|<-4,-4,5>|<6,2,5>>;
```

$$B := \begin{pmatrix} 13 & -4 & 6 \\ 2 & 4 & 2 \\ -2 & 5 & 5 \end{pmatrix}$$

> X:=A^(-1).B;

$$X := \begin{pmatrix} 2 & 3 & 5 \\ 4 & -2 & 0 \\ 1 & 1 & -1 \end{pmatrix}$$

1-topshiriq.

Quyidagi chiziqli tenglamlar sistemasini:

1) Gauss usulida eching.; 2) Kramer usulida eching.; 3) Matritsa tenglamasini teskari matritsa topish usulida eching.

1. 1) $\begin{cases} x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 - x_2 - x_3 - 2x_4 = -4 \\ x_1 + 2x_2 + 3x_3 - x_4 = -4 \\ x_1 + x_2 + x_3 - x_4 = 6 \end{cases}$ 2) $\begin{cases} 5x + 8y - z = -7 \\ x + 2y + 3z = 1 \\ 2x - 3y + 2z = 9 \end{cases}$ 3) $\begin{pmatrix} 2 & 3 & 1 \\ -1 & 2 & 4 \\ 5 & 3 & 0 \end{pmatrix} X = \begin{pmatrix} 2 & 7 & 13 \\ -1 & 0 & 5 \\ 5 & 13 & 21 \end{pmatrix}$

2. 1) $\begin{cases} x_1 + 2x_2 + 3x_3 - 2x_4 = 6 \\ x_1 - x_2 - 2x_3 - 3x_4 = 8 \\ 2x_1 + 2x_2 - x_3 + 2x_4 = 4 \\ 2x_1 - 3x_2 + 2x_3 + x_4 = -8 \end{cases}$ 2) $\begin{cases} x + 2y + z = 4 \\ 3x - 5y + 3z = 1 \\ 2x + 7y - z = 8 \end{cases}$ 3) $\begin{pmatrix} -1 & -2 & 3 \\ 2 & 3 & 5 \\ 1 & 4 & -1 \end{pmatrix} X = \begin{pmatrix} 4 & 11 & 3 \\ 1 & 6 & 1 \\ 2 & 2 & 16 \end{pmatrix}$

3. 1) $\begin{cases} x_1 + 2x_2 + 3x_3 - 4x_4 = 5 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 + 2x_2 + x_3 + 2x_4 = 1 \\ 4x_1 + 3x_2 + 2x_3 + x_4 = -5 \end{cases}$ 2) $\begin{cases} 3x + 2y + z = 5 \\ 2x + 3y + z = 1 \\ 2x + y + 3z = 11 \end{cases}$ 3) $\begin{pmatrix} 4 & -2 & 0 \\ 1 & 1 & 2 \\ 3 & -2 & 0 \end{pmatrix} X = \begin{pmatrix} 0 & -2 & 6 \\ 2 & 4 & 3 \\ 0 & -3 & 4 \end{pmatrix}$

4. 1. $\begin{cases} x_1 - 3x_3 + 4x_4 = -5 \\ x_1 - 2x_3 + 3x_4 = -4 \\ 3x_1 + 2x_2 - 5x_4 = 12 \\ 4x_1 + 3x_2 - 5x_3 = 5 \end{cases}$ 2. $\begin{cases} x_1 + 2x_2 + 4x_3 = 31 \\ 5x_1 + x_2 + 2x_3 = 29 \\ 3x_1 - x_2 + x_3 = 10 \end{cases}$ 3) $\begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 0 \\ 4 & -3 & 0 \end{pmatrix} X = \begin{pmatrix} 22 & -14 & 3 \\ 6 & -7 & 0 \\ 11 & 3 & 15 \end{pmatrix}$

5. 1. $\begin{cases} x_1 + 3x_2 + 5x_3 - 7x_4 = 12 \\ 3x_1 + 5x_2 + 7x_3 + x_4 = 0 \\ 5x_1 + 7x_2 + x_3 + 3x_4 = 4 \\ 7x_1 + x_2 + 3x_3 + 5x_4 = 16 \end{cases}$ 2. $\begin{cases} 4x - 3y + 2z = 9 \\ 2x + 5y - 3z = 4 \\ 5x + 6y - 2z = 18 \end{cases}$ 3) $\begin{pmatrix} 2 & 3 & 1 \\ 4 & -1 & 0 \\ 0 & 1 & 2 \end{pmatrix} X = \begin{pmatrix} 9 & 8 & 7 \\ 2 & 7 & 3 \\ 4 & 3 & 5 \end{pmatrix}$

6. 1. $\begin{cases} x_1+5x_2+3x_3-4x_4=20 \\ 3x_1+x_2-2x_3=9 \\ 5x_1-7x_2+10x_4=-9 \\ 3x_2-5x_3=1 \end{cases}$ 2. $\begin{cases} 2x_1-x_2-x_3=4 \\ 3x_1+4x_2-2x_3=11 \\ 3x_1-2x_2+4x_3=11 \end{cases}$ 3) $\begin{pmatrix} 5 & 1 & 2 \\ -1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} X = \begin{pmatrix} 8 & 1 & 5 \\ -2 & 2 & -1 \\ 17 & 1 & 7 \end{pmatrix}$

7. 1. $\begin{cases} 2x_1+x_2-5x_3+x_4=8 \\ x_1-3x_2-6x_4=9 \\ 2x_2-x_3+2x_4=-5 \\ x_1+4x_2-7x_3+6x_4=0 \end{cases}$ 2. $\begin{cases} x_1+x_2+2x_3=-1 \\ 2x_1-x_2+2x_3=-4 \\ 4x_1+x_2+4x_3=-2 \end{cases}$ 3) $\begin{pmatrix} 4 & 2 & 1 \\ 3 & -2 & 0 \\ 0 & -1 & 2 \end{pmatrix} X = \begin{pmatrix} 2 & 0 & 2 \\ 5 & -7 & -2 \\ 1 & 0 & -1 \end{pmatrix}$

8. 1. $\begin{cases} 2x_1-x_2+3x_3+2x_4=4 \\ 3x_1+3x_2+3x_3+2x_4=6 \\ 3x_1-x_2-x_3+2x_4=6 \\ 3x_1-x_2+3x_3-x_4=6 \end{cases}$ 2. $\begin{cases} 3x_1-x_2=5 \\ -2x_1+x_2+x_3=0 \\ 2x_1-x_2+4x_3=15 \end{cases}$ 3) $\begin{pmatrix} 1 & 4 & 2 \\ 2 & 1 & -2 \\ 0 & 1 & -1 \end{pmatrix} X = \begin{pmatrix} 4 & 6 & -2 \\ 4 & 10 & 1 \\ 2 & 4 & -5 \end{pmatrix}$

9. 1. $\begin{cases} x_1+2x_2-x_3+x_4=8 \\ 2x_1+x_2+x_3+x_4=5 \\ x_1-x_2+2x_3+x_4=-1 \\ x_1+x_2-x_3+3x_4=10 \end{cases}$ 2. $\begin{cases} 3x_1-x_2+x_3=4 \\ 2x_1-5x_2-3x_3=-17 \\ x_1+x_2-x_3=0 \end{cases}$ 3) $\begin{pmatrix} 3 & 2 & -5 \\ 4 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix} X = \begin{pmatrix} -1 & 2 & 4 \\ 0 & 3 & 2 \\ -1 & -3 & 4 \end{pmatrix}$

10. 1. $\begin{cases} 4x_1+x_2-x_4=-9 \\ x_1-3x_2+4x_3=-7 \\ 3x_2-2x_3+4x_4=12 \\ x_1+2x_2-x_3-3x_4=0 \end{cases}$ 2. $\begin{cases} x_1+x_2+x_3=2 \\ 2x_1-x_2-6x_3=-1 \\ 3x_1-2x_2=8 \end{cases}$ 3) $\begin{pmatrix} 5 & 3 & -1 \\ -2 & 0 & 4 \\ 3 & 5 & -1 \end{pmatrix} X = \begin{pmatrix} 1 & 4 & 16 \\ -3 & -2 & 0 \\ 5 & 7 & 2 \end{pmatrix}$

11. 1. $\begin{cases} 2x_1-x_2+x_3-x_4=1 \\ 2x_1-x_2-3x_4=2 \\ 3x_1-x_3+x_4=-3 \\ 2x_1+2x_2-2x_3+5x_4=-6 \end{cases}$ 2. $\begin{cases} 2x_1+x_2-x_3=1 \\ x_1+x_2+x_3=6 \\ 3x_1-x_2+x_3=4 \end{cases}$ 3) $\begin{pmatrix} 5 & -1 & 3 \\ 0 & 2 & -1 \\ -2 & -1 & 0 \end{pmatrix} X = \begin{pmatrix} 3 & 7 & -2 \\ 1 & 1 & -2 \\ 0 & 1 & 3 \end{pmatrix}$

12. 1. $\begin{cases} x_1+x_2-x_3-x_4=0 \\ x_2+2x_3-x_4=2 \\ x_1-x_2-x_4=-1 \\ -x_1+3x_2-2x_3=0 \end{cases}$ 2. $\begin{cases} 2x_1-x_2-3x_3=3 \\ 3x_1+4x_2-5x_3=8 \\ 2x_2+7x_3=17 \end{cases}$ 3) $\begin{pmatrix} 4 & 5 & -2 \\ 3 & -1 & 0 \\ 4 & 2 & 7 \end{pmatrix} X = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 3 \\ 5 & 7 & 3 \end{pmatrix}$

13. 1. $\begin{cases} 5x_1+x_2-x_4=-9 \\ 3x_1-3x_2+x_3+4x_4=-7 \\ 3x_1-2x_3+x_4=-16 \\ x_1-4x_2+x_4=0 \end{cases}$ 2. $\begin{cases} x_1+5x_2+x_3=-7 \\ 2x_1-x_2-x_3=0 \\ x_1-2x_2-x_3=2 \end{cases}$ 3) $\begin{pmatrix} 2 & -8 & 5 \\ -1 & 1 & 1 \\ -2 & -2 & -3 \end{pmatrix} X = \begin{pmatrix} 10 & -2 & 6 \\ 0 & 4 & -2 \\ -4 & -2 & 0 \end{pmatrix}$

14. 1. $\begin{cases} 2x_1+x_3+4x_4=9 \\ x_1+2x_2-x_3+x_4=8 \\ 2x_1+x_2+x_3+x_4=5 \\ x_1-x_2+2x_3+x_4=-1 \end{cases}$ 2. $\begin{cases} x-2y+3z=6 \\ 2x+3y-4z=16 \\ 3x-2y-5z=12 \end{cases}$ 3) $\begin{pmatrix} 5 & 3 & -1 \\ 2 & 0 & 4 \\ 3 & 5 & -1 \end{pmatrix} X = \begin{pmatrix} 1 & 4 & 16 \\ -3 & -2 & 0 \\ 5 & 7 & 2 \end{pmatrix}$

15. 1. $\begin{cases} 2x_1 - 6x_2 + 2x_3 + 2x_4 = 12 \\ x_1 + 3x_2 + 5x_3 + 7x_4 = 12 \\ 3x_1 + 5x_2 + 7x_3 + x_4 = 0 \\ 5x_1 + 7x_2 + x_3 + 3x_4 = 4 \end{cases}$ 2. $\begin{cases} 3x + 4y + 2z = 8 \\ 2x - y - 3z = -1 \\ x + 5y + z = 0 \end{cases}$ 3) $\begin{pmatrix} 2 & 3 & -1 \\ 4 & 5 & 2 \\ -1 & 0 & 7 \end{pmatrix} X = \begin{pmatrix} -1 & 0 & 5 \\ 2 & 1 & 3 \\ 0 & -2 & 4 \end{pmatrix}$

16. 1. $\begin{cases} x_1 + 5x_2 = 2 \\ 2x_1 - x_2 + 3x_3 + 2x_4 = 4 \\ 3x_1 - x_2 - x_3 + 2x_4 = 6 \\ 3x_1 - x_2 + 3x_3 - x_4 = 6 \end{cases}$ 2. $\begin{cases} 2x_1 - x_2 + 3x_3 = 7 \\ x_1 + 3x_2 - 2x_3 = 0 \\ 2x_2 - x_3 = 2 \end{cases}$ 3) $\begin{pmatrix} 12 & 15 & -6 \\ 0 & -3 & 0 \\ 12 & 0 & 21 \end{pmatrix} X = \begin{pmatrix} 8 & 7 & -4 \\ 3 & 1 & 6 \\ 16 & 16 & 13 \end{pmatrix}$

17. 1. $\begin{cases} x_1 - 4x_2 - x_4 = 2 \\ x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 2x_1 + 3x_2 - x_3 - x_4 = -6 \\ x_1 + 2x_2 + 3x_3 - x_4 = -4 \end{cases}$ 2. $\begin{cases} 2x_1 + x_2 + 4x_3 = 20 \\ 2x_1 - x_2 - 3x_3 = 3 \\ 3x_1 + 4x_2 - 5x_3 = -8 \end{cases}$ 3) $\begin{pmatrix} 1 & 3 & 4 \\ 6 & 6 & 5 \\ -1 & -2 & 11 \end{pmatrix} X = \begin{pmatrix} 4 & -3 & 11 \\ 0 & -3 & 4 \\ 1 & -4 & 1 \end{pmatrix}$

18. 1. $\begin{cases} 5x_1 - x_2 + x_3 + 3x_4 = -4 \\ x_1 + 2x_2 + 3x_3 - 2x_4 = 6 \\ 2x_1 - x_2 - 2x_3 - 3x_4 = 8 \\ 3x_1 + 2x_2 - x_3 + 2x_4 = 4 \end{cases}$ 2. $\begin{cases} x_1 - x_2 = 4 \\ 2x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 11 \end{cases}$ 3) $\begin{pmatrix} 8 & -5 & -1 \\ -4 & 7 & -1 \\ -4 & 1 & 5 \end{pmatrix} X = \begin{pmatrix} 5 & 4 & -1 \\ 10 & 12 & -3 \\ 0 & 1 & 1 \end{pmatrix}$

19. 1. $\begin{cases} 4x_1 - 2x_2 + x_3 - 4x_4 = 3 \\ 2x_1 - x_2 + x_3 - x_4 = 1 \\ 3x_1 - x_3 + x_4 = -3 \\ 2x_1 + 2x_2 - 2x_3 + 5x_4 = -6 \end{cases}$ 2. $\begin{cases} x_1 + 5x_2 - x_3 = 7 \\ 2x_1 - x_2 - x_3 = 4 \\ 3x_1 - 2x_2 + 4x_3 = 11 \end{cases}$ 3) $\begin{pmatrix} 3 & 2 & -5 \\ 4 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix} X = \begin{pmatrix} -1 & 2 & 4 \\ 0 & 3 & 2 \\ -1 & -3 & 4 \end{pmatrix}$

20. 1. $\begin{cases} 2x_1 - x_3 - 2x_4 = -1 \\ x_2 + 2x_3 - x_4 = 2 \\ x_1 - x_2 - x_4 = -1 \\ -x_1 + 3x_2 - 2x_3 = 0 \end{cases}$ 2. $\begin{cases} 11x + 3y - z = 2 \\ 2x + 5y - 5z = 0 \\ x + y + z = 2 \end{cases}$ 3) $\begin{pmatrix} 1 & 2 & 1 \\ 3 & -5 & 3 \\ 2 & 7 & -1 \end{pmatrix} X = \begin{pmatrix} 4 & 2 & 1 \\ 1 & -5 & 3 \\ 8 & 7 & -1 \end{pmatrix}$

21. 1. $\begin{cases} -x_1 + x_2 + x_3 + x_4 = 4 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 + 2x_2 + x_3 + 2x_4 = 1 \\ 4x_1 + 3x_2 + 2x_3 + x_4 = -5 \end{cases}$ 2. $\begin{cases} 7x + 5y + 2z = 18 \\ x - y - z = 3 \\ x + y + 2z = -2 \end{cases}$ 3) $\begin{pmatrix} -1 & 2 & 0 \\ -3 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} X = \begin{pmatrix} 5 & -1 & 3 \\ 4 & 2 & 1 \\ -1 & 0 & 2 \end{pmatrix}$

22. 1. $\begin{cases} 5x_1 + 3x_2 - 7x_3 + 3x_4 = 1 \\ x_2 - 3x_3 + 4x_4 = -5 \\ x_1 - 2x_3 - 3x_4 = -4 \\ 4x_1 + 3x_2 - 5x_3 = 5 \end{cases}$ 2. $\begin{cases} 2x + 3y + z = 1 \\ x + z = 0 \\ x - y - z = 2 \end{cases}$ 3) $\begin{pmatrix} 1 & 1 & -1 \\ 4 & -3 & 1 \\ 0 & 2 & 1 \end{pmatrix} X = \begin{pmatrix} 7 & 0 & -5 \\ 4 & 11 & 2 \\ 1 & 3 & 1 \end{pmatrix}$

23. 1. $\begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ x_1 + 2x_2 - 2x_4 = 1 \\ x_1 - x_2 - x_4 = -1 \\ -x_1 + 3x_2 - 2x_3 = 0 \end{cases}$ 2. $\begin{cases} x - 2y - 2z = 3 \\ x + y - 2z = 0 \\ x - y - z = 1 \end{cases}$ 3) $\begin{pmatrix} 2 & 3 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix} X = \begin{pmatrix} 7 & 5 & 2 \\ 1 & -1 & -1 \\ 1 & 1 & 2 \end{pmatrix}$

24. 1. $\begin{cases} 2x_1 + x_2 - x_3 + 3x_4 = -6 \\ 3x_1 - x_2 + x_3 + 5x_4 = 3 \\ x_1 + 2x_2 - x_3 + 2x_4 = 28 \\ 2x_1 + 3x_2 + x_3 - x_4 = 0 \end{cases}$ 2. $\begin{cases} 3x_1 + x_2 - 5x_3 = -7 \\ 2x_1 - 3x_2 + 4x_3 = -1 \\ 5x_1 - x_2 + 3x_3 = 0 \end{cases}$ 3) $\begin{pmatrix} 2 & 3 & 1 \\ 3 & -1 & 0 \\ 1 & 2 & -1 \end{pmatrix} X = \begin{pmatrix} 1 & 5 & 0 \\ 3 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$

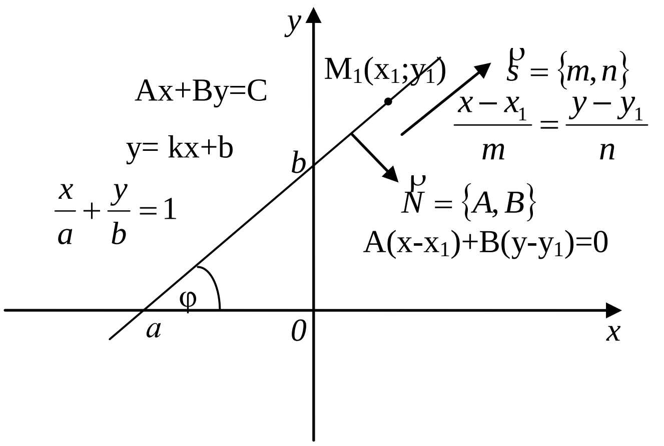
25. 1. $\begin{cases} 2x_1 - x_2 + 2x_3 + 2x_4 = -3 \\ 3x_1 + 2x_2 + x_3 - x_4 = 3 \\ x_1 - 3x_2 - x_3 - 3x_4 = 0 \\ 4x_1 + 2x_2 + 2x_3 + 5x_4 = -15 \end{cases}$ 2. $\begin{cases} x_1 - x_2 - 5x_3 = -7 \\ 2x_1 - 3x_2 + 4x_3 = -1 \\ 5x_1 - x_2 + 3x_3 = 0 \end{cases}$ 3) $\begin{pmatrix} -2 & 1 & 2 \\ 3 & 0 & 4 \\ 2 & 1 & -1 \end{pmatrix} X = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$

2. TEKISLIKDA TO‘G‘RI CHIZIQ

2.1^0. **To‘g‘ri chiziqning burchak koeffitsientli tenglamasi.**

l: y = kx + b , k=tgα, α=(l,x), b- OY dan jaratilgan kesma.

2.2^0. **To‘g‘ri chiziqning umimiy tenglmasi**. Ax + By + C=0, $k=-\frac{A}{B}$



1) C=0, Ax + By=0, $y=-\frac{A}{B}x$, koordinata boshidan o‘tadi.
2) B=0, Ax + C=0, $x=-\frac{C}{A}=a$, Ou ga parallel.
3) A=0, By + C=0, $y=-\frac{C}{B}=b$, Ox ga parallel..
4) B= C=0, Ax=0, x =0, OY – o‘qi.
5) A=C=0, By=0, y =0, OX – o‘qi.

2.3^0. To‘g‘ri chiziqning kesamalar bo‘yicha tenglamsi:

$\frac{x}{a}+\frac{y}{b}=1$, *a* –OX o‘qidan, b-OY o‘qidan kesgan kesmalari.

2.4^0. Berilgan M_1 (x_1; y_1) nuqtadan o‘tib, berilgan $\overline{N} = \{A,B\}$ vektorga perpendikulyar bo‘lgan to‘g‘ri chiziq tenglmasi:

$$A(x-x_1) + B(y-y_1) = 0$$
$$Ax + By + C = 0.$$

2.5^0. Berilgan M_1 (x_1; y_1) nuqtadan o‘tib, berilgan $\overline{S} = \{m,n\}$ vektorga parallel bo‘lgan to‘g‘ri chiziq

$$\frac{x-x_1}{m} = \frac{y-y_1}{n}$$

Agar: 1) $\overline{S} = \{0,n\}$ bo‘lsa, $\frac{x-x_1}{0} = \frac{y-y_1}{n}$, x=$x_1$ OY ga parallel.

2) $\overline{S} = \{m,0\}$ bo‘lsa, $\frac{x-x_1}{m} = \frac{y-y_1}{0}$, y=$y_1$ OX ga parallel.

2.6^0. Berilgan nuqtadan berilgan yo‘nalishda o‘tivchi to‘g‘ri chiziq tenglamasi:

1) $\frac{x-x_1}{\cos\alpha}=\frac{y-y_1}{\sin\alpha}$, birlik yo‘naltiruvchi vektor $\overrightarrow{S^0}=\left\{\frac{m}{|\vec{S}|},\frac{n}{|\vec{S}|}\right\}=\{\cos\alpha,\sin\alpha\}$.

2) $y-y_1=k(x-x_1)$, k=tgα, M_1 nuqtadagi to‘g‘ri chiziqlar dastasi.

2.7^0. Berilgan $M_1(x_1; y_1)$ va $M_2(x_2; y_2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi:

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}\ , \qquad y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

Bunda to‘g‘ri chiziq burchak koeffitsienti:

$$k=tg\alpha=\frac{y_2-y_1}{x_2-x_1}$$

2.8^0. Ikki to‘g‘ri chiziq orasidagi burchak.

1) Agar to‘g‘ri chiziqlar $y=k_1x+b_1$ va $y=k_2x+b_2$ ko‘rinishda bo‘lsa, ular orasidagi burchakni quyidagcha hisoblaymiz.

$$tg\varphi=\frac{k_2-k_1}{1+k_1k_1}$$

Agar to‘g‘ri chiziqlar $A_1x + B_1y + C_1=0$ va $A_2x + B_2y + C_2=0$ ko‘rinishda bo‘lsa,

$$tg\varphi=\frac{A_1B_1-A_2B_2}{A_1A_2+B_1B_2}$$

Agar to‘g‘ri chiziqlar parallel bo‘lsa, $k_1 = k_2$, $\frac{A_1}{A_2}=\frac{B_1}{B_2}$

Agar to‘g‘ri chiziqlar perpendikulyar bo‘lsa, $k_2=-\frac{1}{k_1}$, $A_1A_2 + B_1B_2 = 0$

2.9^0. Prallel bo‘lmagan to‘g‘ri chiziqlarning kesishish nuqtasi.

Agar to‘g‘ri chiziqlarning tenglamlari $\mathbf{A_1x + B_1y + C_1 = 0}$ va $\mathbf{A_2x + B_2y + C_2 = 0}$ ko‘rinishda berilgan bo‘lsa, ularni kesishish nuqtasini topish uchun tenglamlarni birgalikda echamiz.

$$x=\frac{\begin{vmatrix}-C_1 & B_1\\ -C_2 & B_2\end{vmatrix}}{\begin{vmatrix}A_1 & B_1\\ A_2 & B_2\end{vmatrix}}\ , \qquad y=\frac{\begin{vmatrix}A_1 & -C_1\\ A_2 & -C_2\end{vmatrix}}{\begin{vmatrix}A_1 & B_1\\ A_2 & B_2\end{vmatrix}}\ , \qquad \begin{vmatrix}A_1 & B_1\\ A_2 & B_2\end{vmatrix}\neq 0$$

2.10^0. To‘g‘ri chiziqning normal tenglamasi.

Ax + By + C = 0 bo‘lsa, $\mu=\pm\frac{1}{\sqrt{A^2+B^2}}$ normallovchi ko‘paytuvchi, $\pm$ ishora C ning ishorasiga qarama – qarshi olinadi. Bu holda to‘g‘ri chiziqning normal tenglamasi:

$$\frac{A}{\pm\sqrt{A^2+B^2}}x+\frac{B}{\pm\sqrt{A^2+B^2}}y+\frac{C}{\pm\sqrt{A^2+B^2}}=0$$

$$x\cos\beta + y\sin\beta - p = 0$$

$$\overrightarrow{N}=\{A,B\}\ \ ,\beta=(\overrightarrow{N}\wedge OX)$$

2.11^0. Berilgan $M_0(x_0 ;y_0)$ nuqtadan berilgan Ax + By + C = 0 to‘g‘ri chiziqqacha bo‘lgan masofa.

$$d = |x_0 \cos\beta + y_0 \sin\beta - p|, \qquad d = \frac{|Ax_0 + By_0 + c|}{\sqrt{A^2 + B^2}}$$

2.12^0. Ax + By + C = 0, va $A_1x + B_1y + C_1 = 0$ to‘g‘ri chiziqlar orasidagi burchaklar bissektirisalarining tenglamalari.

$$\frac{Ax + By + C}{\sqrt{A^2 + B^2}} = \pm \frac{A_1x + B_1y + C}{\sqrt{A_1^2 + B_1^2}}$$

2.13^0. Berilgan Ax + By + C=0 **va** $A_1x + B_1y + C_1 = 0$ **to‘g‘ri chiziqlarning kesishish nuqtasidan o‘tuvchi to‘g‘ri chiziqlar dastasi:**

$$\alpha(Ax + By + C) + \beta(A_1x + B_1y + C_1) = 0$$

Tekislikda to‘g‘ri chiziq tenglamlariga bog‘liq masalalar. Tekislikda uchburchak masalasi.

Quyidagi masalani echishdagi har bir bajarilgan amalni **Maple7** dasturida bajarilishini ko‘rsatib boramiz.

2.1-masala. **Tekislikda** A(10;3), B(-4;7), C(4;8) **nuqtalar koordinatalari bilan berilgan. Quyidagilarni toping(Uchburchak-1.mw).**

1) **Δ ABC tomonlarining tenglamalarini;**
2) **AB va AC tomonlar orasidagi burchakni;**
3) **AD balandlik va uning uzunligini;**
4) **AE mediana va AN bissektirsani;**
5) **ΔABC ni yuzasini hisoblang.**
6) **Tekislikda ΔABC ni quring**.

Echish:

1) Δ ABC tomonlari tenglmalarini berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglmasi:

$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$ formulasiga asosan topamiz.

AB: $\frac{y-3}{7-3} = \frac{x-10}{-4-10}$, $\frac{y-3}{4} = \frac{x-10}{-14}$ bundan 2x + 7y - 41, $k_1 = -\frac{2}{7}$

```
> with(geometry):
point(A,10,3), point(B,-4,7):
l1:=line(AB,[A,B]);
```
l1:= AB

```
> Equation(AB,[x,y]);
```
82 K 4 *x* K 14 *y* = 0

AC: $\frac{x-10}{4-10} = \frac{y-3}{8-3}$, $\frac{y-3}{5} = \frac{x-10}{-6}$ bundan 5x + 6y – 68 = 0 $y = -\frac{5}{6}x + \frac{34}{3}$, $k_2 = -\frac{5}{6}$

```
> with(geometry):
point(A,10,3),point(S,4,8):
l2:=line(AC,[A,S]);
```
l2:= AC

```
> Equation(AC,[x,y]);
```
68 K 5 *x* K 6 *y* = 0

BC: $\frac{x+4}{4+4} = \frac{y-7}{8-7}$, $\frac{x+4}{8} = \frac{y-7}{1}$ bundan x – 8y + 60 = 0 , $y = \frac{1}{8}x + \frac{15}{2}$, $k_3 = \frac{1}{8}$

```
> with(geometry):
point(B,-4,7),point(S,4,8):
l3:=line(BC,[B,S]);
```

$l3 := BC$

```
> Equation(BC,[x,y]);
```

$-60 - x + 8y = 0$

Δ ABC tomonlarining uzunliklari:

```
> with(geometry):
> triangle(ABC, [point(A,10,3), point(B,-4,7), point(C,4,8)]):
distance(A,B);distance(A,C);distance(B,C);
```

$\sqrt{212}\quad \sqrt{61}\quad \sqrt{65}$

```
> sides(ABC);
```

$[\sqrt{212}, \sqrt{65}, \sqrt{61}]$

2)AB va AC tomonlar orasidagi burchak

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1 k_1}$$

formulaga asosan topiladi:

$$k_1 = -\frac{2}{7},\ k_2 = -\frac{5}{6}: \qquad tg = \frac{-\frac{2}{7} + \frac{5}{6}}{1 - \frac{2}{7}\cdot(-\frac{5}{6})} = \frac{23}{52},\quad \varphi = arctg(\frac{23}{52}) \approx 42^0 15'$$

```
> with(geometry):
_EnvHorizontalName := 'x': _EnvVerticalName := 'y':
line(AB, 82-4*x-14*y=0), line(AC, 68-5*x-6*y = 0);
```

AB, AC

```
> phi:=FindAngle(AB, AC);
```

$\phi := \arctan\left(\frac{23}{52}\right)$

```
> phi:=evalf(phi);
```

$\phi := 0.4164386172$

3) AD balandlikni topish uchun A(10;3) nuqtadan o‘tuvchi to‘g‘ir chiziqlar dasatasi tenglaasini yozamiz, undan BC tomonga perpendikulyar bo‘lgan to‘g‘ri chiziqni ajratamiz va tenglamasini yozamiz.

$$y - y_1 = k\ (x - x_1)$$

da $x_1 = 10$, $y_1 = 3$ bo‘lsa,

$$y - 3 = k\ (x - 10)$$

bu to‘g‘ri chiziqlar dastasidan BC($k_3 = \frac{1}{8}$) ga perpendikulyarini ajratish uchun k ni quyidagi perpendikulyarlik shartidan foydalanib topamiz:

$$k = -\frac{1}{k_3} = \frac{1}{-\frac{1}{8}}, \text{ dan } \quad k = -8$$

Bu holda AD balandilik tenglamasi:

y – 3 = 8 (x – 10), 8x + y – 83 = 0 (AD)

```
> with(geometry):
triangle(ABC,[point(A,10,3), point(B,-4,7), point(C,4,8)]):
altitude(hA1,A,ABC);                hA1
> form(hA1);                        line2d
> detail(hA1);
        name of the object:  hA1
        form of the object:  line2d
        equation of the line:  -83+8*x+y = 0
```

Bu AD balandlik uzunligini topishda BC tomondan A(10;3) nuqtagacha bo‘lgan masofani, yahni AD balandilik uzunligini hisoblaymiz.

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|1 \cdot 10 + 8 \cdot 3 + 60|}{\sqrt{1^2 + (-8)^2}} = \frac{46}{\sqrt{65}} = 5.7$$

```
> with(geometry):
> assume(m<>0):
line(BC, x-8*y=-60,[x,y]): distance(A, BC);
```

$$\frac{46}{65}\sqrt{65}$$

4) AE mediana tenglamasini yozish uchun BC tomon o‘rtasi E nuqtaning koordinaialarini

$$x_0 = \frac{x_1 + x_2}{2}, \; y_0 = \frac{y_1 + y_2}{2}$$

formulaga asosan topamiz:

$$x_E = \frac{x_B + x_C}{2} = \frac{-4+4}{2} = 0, \quad y_E = \frac{y_B + y_C}{2} = \frac{7+8}{2} = \frac{15}{2}, \qquad E(0; \frac{15}{2})$$

Bu holda AE ning tenglamasi:

$$\frac{x-10}{0-10} = \frac{y-3}{\frac{15}{2}-3}, \quad \frac{x-10}{-10} = \frac{y-3}{\frac{9}{2}}, \quad \frac{x-10}{-20} = \frac{y-3}{9}, \qquad 9x + 20u - 150 = 0$$

```
> with(geometry):
triangle(ABC, [point(A,10,3), point(B,-4,7), point(C,4,8)]):
median(mA, A, ABC);                  mA
> form(mA);                          line2d

> detail(mA);
```

name of the object: mA
form of the object: line2d

*equation of the line: 75-9/2*x-10*y = 0*

> median(mA, A, ABC, E);

> form(mA); *segmebt2d*

> coordinates(E); $\left[0, \frac{15}{2}\right]$

> detail(mA);

name of the object: mA

form of the object: segment2d

the two ends of the segment: [[10, 3], [0, 15/2]]

ABC uchburchk og'irlik markazi:

> with(geometry):

s:=[point(A,10,3), point(B,-4,7), point(C,4,8)]; *ps* := [*A*, *B*, *C*]

> centroid(G,s); *G*

> form(G); *point2d*

> coordinates(G); $\left[\frac{10}{3}, 6\right]$

> detail(G); *name of the object: G*

form of the object: point2d

coordinates of the point: [10/3, 6]

Uchlari uchburchak tomonlarining ortalrida bo'lgan uchburchak.

> with(geometry):

triangle(T, [point(A,10,3), point(B,-4,7), point(C,4,8)]):

medial(mT,T); *mT*

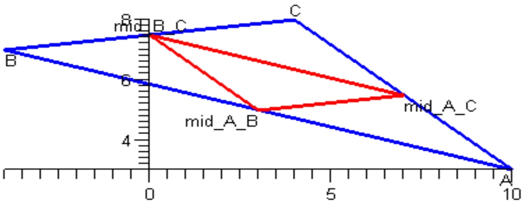
> detail(mT); *name of the object: mT*

form of the object: triangle2d

method to define the triangle: points

the three vertices: [[3, 5], [7, 11/2], [0, 15/2]]

> draw({T(color=blue),mT(color=red)},style=line,axes=NONE, rinttext=true);



Endi AN bissektirsa tenglamasin ikki xil usulda topish mumkin:

a) $$\frac{A_1x + B_1y + C}{\sqrt{A_1^2 + B_2^1}} = \pm \frac{A_2x + B_2y + C}{\sqrt{A_2^2 + B_2^2}}$$

tenglamga asosan aniqlaymiz.

Bu bissektirsa AB va AC tomonlar orasida bo‘lagligi uchun bu tomonlar tenglamalari

AB: 2x + 7y – 41 = 0 va AC: 5x + 6y – 68 = 0.

ga asosan:

$$\frac{2x + 7y - 41}{\sqrt{2^2 + 7^2}} = \pm \frac{5x + 6y - 68}{\sqrt{5^2 + 6^2}}$$

bundan:

$$\frac{2x + 7y - 41}{\sqrt{53}} = + \frac{5x + 6y - 68}{\sqrt{61}} \quad (A), \qquad \frac{2x + 7y - 41}{\sqrt{53}} = - \frac{5x + 6y - 68}{\sqrt{61}} \quad (B)$$

tenglamlardan qaysi biri A burchakning ichki burchagining bissektirsasi AD ekanligini aniqlaymiz. (A) tenglamaga B va C nuqtalarning koordinatalarini qo‘yganda chap va o‘ng kasirlar ishoralari har xil bo‘ladi. Bundan (A) tenglama ΔABC ning A ichki burchagining bissektrisasi bo‘ladi. (B) tenglamaga B va C nuqtalarning koordinatalari qo‘yilganda bir xil ishorali bo‘lgani uchun (B) tenglama qo‘shni burchak bissektrisasi bo‘ladi.

Ichki burchak bissektrisasi:

```
> with(geometry):
triangle(ABC, [point(A,10,3), point(B,-4,7), point(C,4,8)]):
```

define the ``line'' bisector bA

```
> bisector(bA, A, ABC);
```

bA

```
> Equation(bA,[x,y]);
```

$1630.535418 - 104.0420976\ x - 196.7048141\ y = 0$

```
> detail(bA);
```

name of the object: bA
form of the object: line2d
*equation of the line: 1630.535418-104.0420976*x-196.7048141*y = 0*

Qo‘shni burchakning (qo‘shma) bissektrisasi;

```
> with(geometry):
```

triangle(ABC,[point(A,10.,3),point(B,-4,7),point(C,4,8)]):

define the external bisector bA

> ExternalBisector(bA, A, ABC); *bA*

> Equation(bA,[x,y]); 1654.921848 K 196.7048141 *x* C 104.0420976 *y* = 0

> detail(bA); *name of the object: bA*

form of the object: line2d

*equation of the line: 1654.921848-196.7048141*x+104.0420976*y = 0*

> bisector(ibA,A,ABC):

ArePerendicular(bA,ibA); *true*

b) A burchak bissektrisasini BC tomon bilan kesishish nuqtasi N ning koordinatalarini topamiz. Geometriya kursidan mahlumki, uchburchak burchagining bissektrisasi burchak qarshisidagi tomonni burchakka yopishpgan tomonlarga proportsional bo‘laklarga bo‘ladi.

Demak: $\lambda = \frac{|CN|}{|NB|} = \frac{|AC|}{|AB|}$ dan λ dani son qiymatini topib($\lambda=\frac{|AC|}{|AB|}=\frac{\sqrt{212}}{\sqrt{61}}$),

$$x_N = \frac{x_C + \lambda x_B}{1+\lambda}, \quad y_N = \frac{y_C + \lambda y_B}{1+\lambda}$$

formulalarga asosan N(x, y) nuqtani topamiz va ikki nuqtadan o‘tuvchi to‘g‘ri chiziq formulasiga asosan: AN bissektrisa tenglamisini tuzamiz:

$$\frac{x - x_A}{x_N - x_A} = \frac{y - y_A}{y_N - y_A}.$$

bA bissektrisani BC tomon bilan kesishish nuqtasi N koordinatalari:

> restart;

> with(geometry):

triangle(ABC, [point(A,10.,3.), point(B,-4,7), point(C,4,8)]):

define the ``segment" bisector bA

> bisector(bA, A, ABC,N); *bA*

> form(bA); *segmebt2d*

> OnSegment(N, B, C, sqrt(212/61.)); *N*

> coordinates(N); [1.206942951 , 7.650867870]

> detail(bA); *name of the object: bA*

form of the object: segment2d

the two ends of the segment: [[10., 3.], [1.206942951, 7.650867870]]

5) Δ ABC ning yuzini quyidagicha hisoblaymiz.

a) $S=\frac{1}{2}ah$, (bunda *a*- asos, *h*-asosga tushirilgan balandlik) formulasiga asosan hisoblaymiz. Asos uchun BC tomon uzunligi:

$$|BC|=\sqrt{(4+4)^2+(8-7)^2}=\sqrt{65}=8.062 \text{ ni}$$

va balandlik uchun 3) punktdagi AD uzunligi $|AD|=\frac{46}{\sqrt{65}}=5.7$ ekanidan,

Δ ABC ning yuzasi: $S=\frac{1}{2}|BC||AD|=\frac{1}{2}\sqrt{65}\frac{46}{\sqrt{65}}=23$,

b) $$S=\pm\frac{1}{2}\begin{vmatrix} x_2-x_1 & y_2-y_1 \\ x_3-x_1 & y_3-y_1 \end{vmatrix}$$

formulaga uchburchak ABC ning uchlarining koordinatalarini qo‘yib uni yuzasini topamiz.

$$S=\pm\frac{1}{2}\begin{vmatrix} -4-10 & 7-3 \\ 4-10 & 8-3 \end{vmatrix}=\pm\frac{1}{2}\begin{vmatrix} -14 & 4 \\ -6 & 5 \end{vmatrix}=23$$

> with(geometry):

triangle(ABC, [point(A,10,3), point(B,-4,7), point(C,4,8)]):

area(ABC); ***23***

***A** nuqtadan BC tamonga parallel:*

> with(geometry):

triangle(ABC, [point(A,10,3), point(B,-4,7), point(C,4,8)]): line(BC, [B,C]);

BC

> arallelLine(lA,A,BC); *lA*

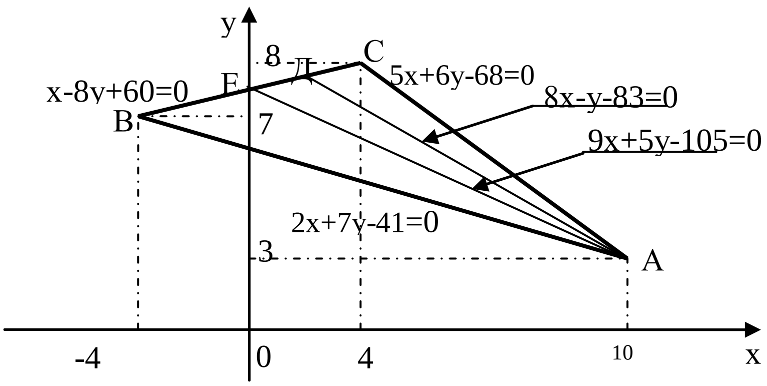
> Equation(lA,[x,y]); K 14 K *x* C 8 *y* = 0

> detail(lA); *name of the object: lA*

form of the object: line2d

*equation of the line: -14-x+8*y = 0*

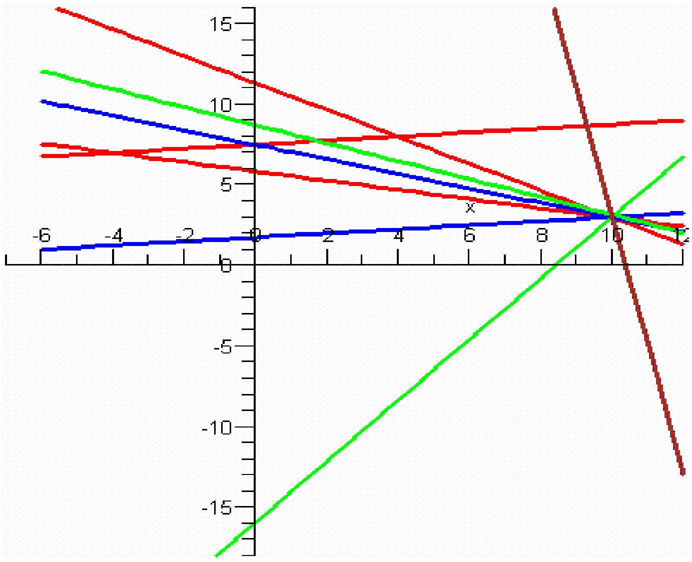
ABC uchburchakni qurish:



ABC uchburchakni tamonlari tenglamalariga asosan qurish;

> with(plots):

> plot([(41-2*x)/7,(68-5*x)/6,(60+x)/8,(75-9/2*x)/10, (14+x)/8,(197*x-1655.)/104.1, (-104.1*x+1631.)/187, (83-8*x)], x=-6..12, color=[red,red,red,blue,blue,green,green, brown], style=[line], thickness=2, view=[-7..12,-18..16]);

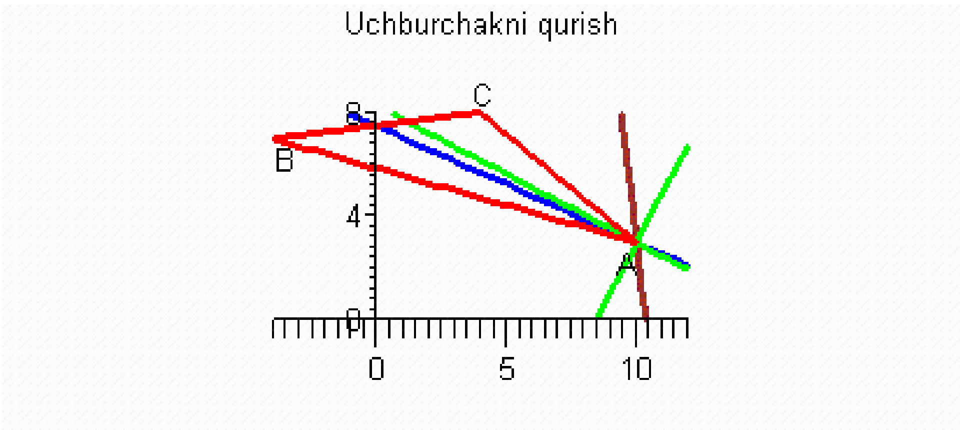


ABC uchburchakni uchining koordinatalariga asosan kesmalari bo'yicha qurish:

> triangle(ABC,[point(A,10,3),point(B,-4,7),point(C,4,8)]): altitude(hA1,A,ABC), median(mA, A, ABC), bisector(bA, A, ABC),ExternalBisector(bqA, A, ABC);

$$hA1,\ mA,\ bA,\ bqA$$

> draw([ABC(color=red),hA1(color=brown),mA(color=blue), bA(color=green), bqA(color=green)],title=`Uchburchakni qurish`,style=patch,thickness=2, rinttext=true, view=[-4..12,0..8]);



Mustaqil ishlash uchun tekislikda to‘g‘ri chiziq tenglamalariga doir masalalar

Nazorat ishi uchun variant namunalari

1 – variant.

1) C(3;5) nuqta AB kesmani AC:CB=3:4 kabi nisbatda bo‘lib, oxirgi uchi B(-1;1) bo‘lsa A uchini koordinatalarini toping.
2) Koordinatalar boshidan va A nuqtadan o‘tuvchi to‘g‘ri chiziq burchak koeffitsienti ¾ bo‘lib, A nuqta koordinatalar boshidan 10 birlik masofada joylashgan bo‘lsa, A nuqtaning koordinatalarini toping.
3) (3:4) nuqtadan o‘tuvchi va OY o‘qidan b=2 kesma ajratuvchi to‘g‘ri chiziq tenglamasini tuzing.
4) A (4;-3) va B(2;-2) nuqtalardan o‘tuvchi to‘g‘ri chiziq bilan 3x+2y+4 to‘g‘ri chiziq orasidagi burchakni toping.

2 – variant

1) AB kesma A (-3;-3) va B(1;2) nuqtalar bilan berilgan. AB:BC=5:3 bo‘lishligi uchun AB kesmani qanday S nuqtagacha davom ettirish kerak?
2) (-5;1) nuqtadan o‘tuvchi to‘g‘ri chiziq Ou o‘qida 6 ga teng kesma ajratadi. Bu to‘g‘ri chiziqning tenglamasini tuzing.
3) A(4;3) nuqtadan o‘tuvchi va 3x+2y+4=0 to‘g‘ri chiziqka perpendikulyar bo‘lgan to‘g‘richiziq tenglamasini tuzing.
4) Uchburchakning uchlari berilgan : A(2;-1), B(-7;3) va C(-1;-5). S burchak bissektrisasining tenglamasini yozing.

3 – variant

1) Agar uchburchakning uchlari A(7;-4), B(-1;8) va C(-12;-1) bo‘lsa, uchburchakning medianalarining kesishish nuqtasini toping.
2) Uchburchak tomonlarining tenglmalar 3x-2y-1=0, 5x+4y-31=0 va x-8y-15=0 bo‘lsa, ichki burchaklarni toping.
3) (-2;6) va (3;-3) nuqtaldardan o‘tuvchi to‘g‘ri chiziqka perpendikulyar bo‘lgan va (2;4) nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

4 – variant

1) Uchlari A(4;2) B(6;5) va C(-5;4) nuqtalarda bo‘lgan, uchuburchak balandliklar tenglamasini tuzing.
2) Uchlari A(-2;2) B(6;4) va C(-5;4) nuqtalarda bo‘lgan uchburchakning AC tomooni bilan A uchidan o‘tkazilgan medianasi orasidagi burchak topilsin.
3) A(-8;-2), B(2;10) va C(4;4) nuqtalar berilgan bo‘lsa, ∆ABC ogilik markazalari topilisin.

2-topshiriq.

Uchburchakning uchlari A,B,C nuqtalar koordinatalari bilan berilgan. Topish kerak:

1) Δ ABC tomonlarining tenglamalarini va uzunliklarini;
2) AB va AC tomonlar orasidagi burchak bissektirsa tenglamasini
3) AD balandlik tenglamasi va uzunligini
4) AE mediana tenglamsini
5) Δ ABC ni ichki burchaklarini
6) Δ ABC ning yuzini

№	1	2	3	4	5	6
Koordinata	A (2;2), B (3;3), C (-4;4)	A (1;3), B (2;1), C (2;-1)	A (1; -1), B (6;1), C (4;3)	A (-1;1), B (6;-1), C (4;-3)	A (2;-1), B (6;-1), C (-4;3)	A (1;-6), B (2;1), C (0;1)
№	7	8	9	10	11	12
Koordinata	A (2;1), B (-2;1), C (2;0)	A (1;2), B (-1;-2), C (0;-3),	A (1;-2), B (-1;-1), C (15;-1)	A (2;2), B (-2;-2), C (0;2),	A (3;-1), B (1;-2), C (3;-3)	A (1;5), B (2;-2), C (-4;2)
№	13	14	15	16	17	18
Koordinata	A (1;4), B (2;3), C (-4;2)	A (1;1), B (2;2), C (3;-3)	A (2;2), B (1;1), C (-1;5)	A (-4;4), B (3;3), C (0;1)	A (1;2), B (2;1), C (2;0)	A (3;3), B (-3;3), C (0;6)
№	19	20	21	22	23	24
Koordinata	A (2;2), B (4;1), C (2;0)	A (2;1), B (3;-3), C (1;4)	A (1;3), B (4;1), C (0;5)	A (1;4), B (1;1), C (2;0)	A (-4;3), B (3;2), C (3;0)	A (1;1), B (-1;2), C (4;3)
№	25	26	27	28	29	30
Koordinata	A (-6;7), B (2;1), C (0;-1)	A (4;3), B (0;4), C (1;1)	A (4;1), B (1;-5), C (2;2)	A (6;-2), B (1;0), C (3;0)	A (2;-5), B (0;3), C (1;1)	A (3;-3), B (-1;0), C (2;1)

3. FAZODA ANALITIK GEOMETRIYA

3.1. TEKISLIK TENGLAMASI.

(Analitik geom-FAZO.mw).

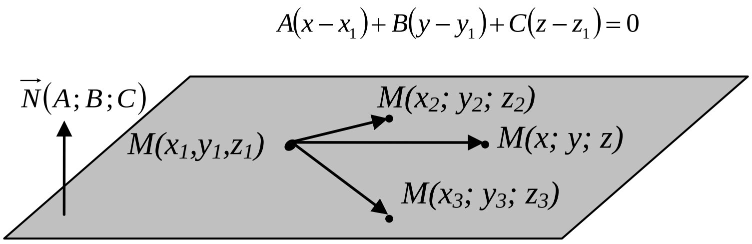
1. M_1 (x_1; y_1;z_1) nuqtadan o'tuvchi va $\vec{N}=\{A,B,C\}$ vektorga perpendikulyar tekislik tenglamasi:

$$A(x-x_1)+B(y-y_1)+C(z-z_1)=0 \qquad (1)$$

2. Tekislikning umumiy tenglamasi.

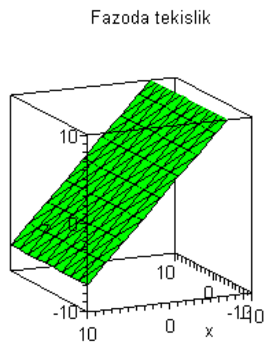
$Ax+By+Cz+D=0$ (2)

$\vec{N}=\{A,B,C\}$ vektor (1) va (2) tekisliklarga normal vektor deyiladi.



(2) tenglamaning maxsus xollari:

1) A=0 bo‘lsa, By + Cz + D = 0 tekislik 0x o‘qqa parallel.
2) B=0 bo‘lsa, Ax + Cz + D = 0 tekislik 0y o‘qqa parallel.
3) C=0 bo‘lsa, Ax + By + D = 0 tekislik 0z o‘qqa parallel.

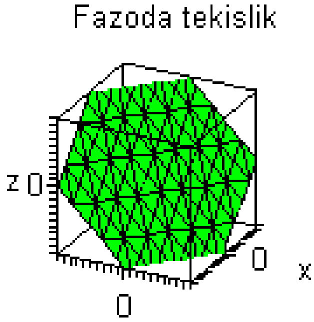


0x o‘qqa parallel tekislik y+z=3 tenglamacb , bo‘yicha qurish.

```
> with(geom3d):
> plane(p,y+z=3,[x,y,z]):
> draw([p(color=green)],title=`Fazoda tekslik`,style=patch);
```

4) D=0 bo‘lsa, Ax+ By + Cz = 0 koordinatalar boshidan o‘tuvchi tekislik:

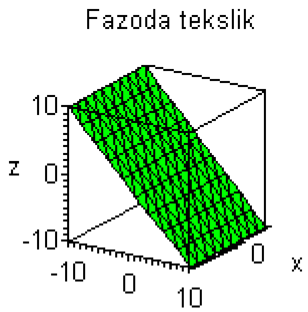
a) A=0 da By + Cz = 0 0x o‘qidan o‘tadi;
b) B=0 da Ax + Cz = 0 0y o‘qidan o‘tadi;
v) A=0 da By + Cz = 0 0x o‘qidan o‘tadi;



Koordinatala boshisan otuvchi $x \mathrm{C} y \mathrm{C} z = 0$ *tekislik tenglamasi boyicha qurish*:

```
> with(geom3d):
> plane(p,x+y+z=0,[x,y,z]):
> draw([p(color= green)], title=` Fazoda tekslik `,style=patch);
```

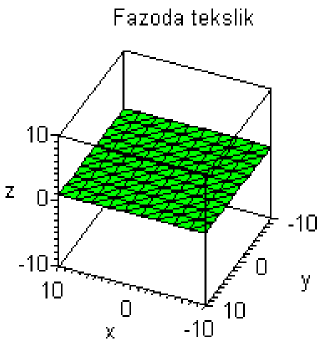
5) A=0, B=0 da Cz+D=0, 0xy tekisligiga parallel.
A=0, C=0 da By+D=0, 0xz tekisligiga parallel.
C=0, B=0 da Ax+D=0, 0yz tekisligiga parallel.
Ox o‘qdqn o‘tuvchi tekislik y+z=0 bo‘yicha qurish.




```
> with(geom3d):
> plane(p,y+z=0,[x,y,z]):
> draw([p(color= green)],title=` Fazoda tekslik`,
style=patch);
```

6) x=0, y=0,z=0 tenglamalar mos ravishda Y0Z, 0ZX, 0XY koordinatalar tekisliklarini ifodalaydi.

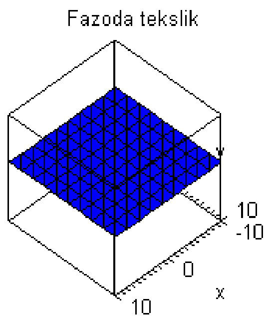
Oxy koordinata tekisligiga parallel bolgan $z = 1$ *tekni tenglamasi boyicha qurish*:




```
> with(geom3d):
> plane(1,z=1,[x,y,z]):
 > draw([1(color= green)],title=` Fazoda tekslik `,
style=patch);
```

Oxy koordinata tekisligi $z = 0$ *tenglamasi boyicha qurish*:

```
> with(geom3d):
> plane(p,z=0,[x,y,z]):
> draw([p(color= green)],title=` Fazoda tekslik `,
style=patch);
```



3. Tekislikning koordinatalar o‘qlaridan ajratgan kesimlari bo‘yicha tenglamasi:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad (3)$$

4. Ikki tekislik orasidagi burchak.

$$\cos\varphi=\pm\frac{\vec{N}_1\vec{N}_2}{|\vec{N}_1|*|\vec{N}_2|}=\pm\frac{A_1A_2+B_1B_2+C_1C_2}{|\vec{N}_1|*|\vec{N}_2|} \qquad (4)$$

(4) dan: $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$ parallelik sharti

$A_1A_2+B_1B_2+C_1C_2=0$ perpendikulyarlik sharti

5. $M_0(x_0, y_0, z_0)$ nuqtadan $Ax+By+Cz+D=0$ **tekislikgacha bo‘lgan masofa**:

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{|\vec{N}|} \qquad (5)$$

> with(geom3d):
Distance from a point to a plane
> assume(A<>0);
plane(p,A*x+B*y+C*z+D=0, [x,y,z]):
point(D,[x1,y1,z1]):
d:=distance(D,p);

$$d := \frac{|x1\ A\sim + y1\ B + z1\ C + D|}{\sqrt{A\sim^2 + B^2 + C^2}}$$

6. Ikkita kesishuvchi tekisliklar kesishish chizigidan o‘tuvchi tekisliklar dastasi tenglamasi:

$$\alpha(A_1x+B_1y+C_1z+D_1)+\beta(A_2x+B_2y+C_2z+D_2)=0 \qquad (6)$$

3.2. TO‘G‘RI CHIZIQ TENGLAMALARI.

(Analitik geom-FAZO.mw).

1. A (*a*, *b*, *c*) nuqtadan o‘tib, $\vec{S}=\{m,n,p\}$ **yunaltiruvchi vektorga parallel bo‘lgan to‘g‘ri chiziq tenglamasi**:

$$\frac{x-a}{m}=\frac{y-b}{n}=\frac{z-c}{p} \qquad (7)$$

(7) tenglama kononik tenglama deyiladi.

> with(geom3d):
Let the straight line ass through the point A=[x1,y1,z1] and has direction-cosines (or ratios) [l,m,n]
> assume(l<>0): # the direction-ratios is a non-zero vector
point(A,[a,b,c]): s := [l,m,n]:
define the line l1 that asses through A and has [l,m,n] as its direction-ratios
> line(l1,[A,s]):
detail(l1);

name of the object: l1

form of the object: line3d

*equation of the line: [_x = a+_t*l, _y = b+_t*m, _z = c+_t*n]*

2. To‘g‘ri chiziqning pparametrik tenglamasi .

$$\begin{cases} x = mt + a \\ y = nt + b \\ z = pt = c \end{cases} \quad (8)$$

3. Ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi.

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \quad (9)$$

4. To'g'ri chiziqning umumiy tenglamasi.

$$\begin{cases} A_1 x + B_1 y + C_1 z + D_1 = 0 \\ A_2 x + B_2 y + C_2 z + D_2 = 0 \end{cases} \quad (10)$$

Bu to'g'ri chiziqni kononik ko'rinishga keltirish formulasi:

$$\frac{x-x_1}{\begin{vmatrix} B_1 C_1 \\ B_2 C_2 \end{vmatrix}} = \frac{y-y_1}{-\begin{vmatrix} A_1 C_1 \\ A_2 C_2 \end{vmatrix}} = \frac{z-0}{\begin{vmatrix} A_1 B_1 \\ A_2 B_2 \end{vmatrix}} \quad (11)$$

bu erda x_1 va y_1 lar (9) tenglamada z=0 bo'lganda sistemani echib topiladi.

Tekisliklarning parallelligi

```
> with(geom3d):
```
Define two planes 1, 2
```
> assume(A1<>0, A2<>0, A1<>A2, t<>0);
plane(p1,A1*x+B1*y+C1*z+D1,[x,y,z]);
```
p1
```
> plane(p2,A2*x+B2*y+C2*z+D2,[x,y,z]);
```
p2

Find the condition that makes 1 and 2 arallel to each other
```
> Arearallel(p1,p2,'cond');
```
FAIL
```
> cond;
```
$$(B1\,C2\,K\;\; C1\,B2 = 0)\ \&and\,(C1\,A2{\sim}K\;\; A1{\sim}\,C2 = 0)$$
$$\&and\,(A1{\sim}\,B2\,K\;\; B1\,A2{\sim} = 0)$$

```
> additionally(op(cond));  Arearallel(p1,p2);
```
true

5. (9) tenglamadan bir marta *y* ni, ikkinchi marta *x* ni yo'qotib , to'g'ri chiziqni proektsiyalari tenglamalarini yozamiz.

$$\begin{cases} x = mz + a \\ y = nz + b \end{cases} \quad (12)$$

(12) dan $\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-0}{1}$ ni yozish mumkin.

3.3. TO'G'RI ChIZIQ VA TEKISLIK
(Analitik geom-FAZO.mw).

1. $\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-c}{p}$ **to'g'ri chiziq va** $Ax + By + Cz + D = 0$ **tekislik orasidagi burchak.**

$$\sin\alpha = \frac{\vec{N}\vec{S}}{|\vec{N}| \cdot |\vec{S}|} = \frac{|Am + Bn + Cp|}{|\vec{N}| \cdot |\vec{S}|} \quad (13)$$

Am+Bn+C=0 parallelik sharti; $\frac{A}{m} = \frac{B}{n} = \frac{C}{p}$ perpendikulyarlik sharti.

2. Tekislik bilan to'g'ri chiziqni kesishish nuqtasi. To'g'ri chiziqni

$$x = m\,t + a\,, \quad y = n\,t + b\,, \quad z = \; t + c$$

pparametrik ko‘rinishga keltirib $Ax + By + Cz + D = 0$ tekislik tenglamasidagi x, y, z larni o‘rniga qo’yib t_0 pparametrni topib, to‘g‘ri chiziq tenglamasidan x_0, y_0, z_0 kesishish nuqtalarini koordinatalarini topamiz.

$$\begin{vmatrix} a-a_1 & b-b_1 & c-c_1 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix} = 0 \qquad (14)$$

*(x-1)/2=(y+1)/(-1)=(z-3)/4 to‘g‘ri chiziq bilan x+2*y+z=6 tekislikni kesishish nuqtas**i**.*

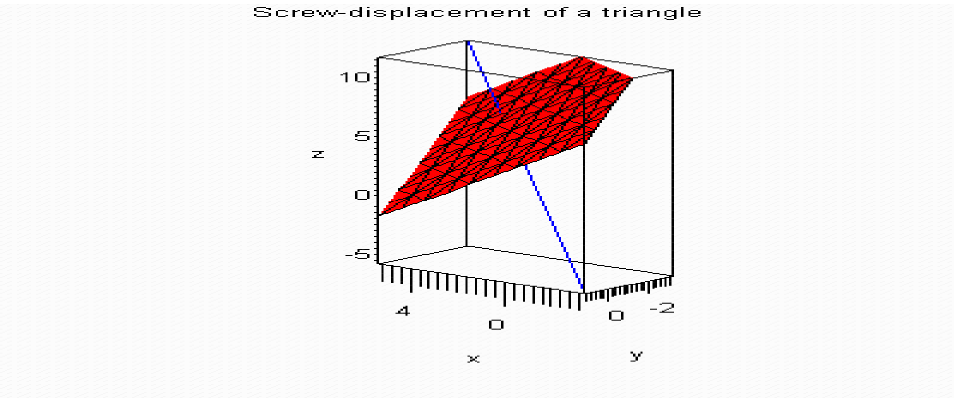
```
> with(geom3d):
> line(l,[point(o,1,-1,3),[2,-1,4]]): plane(p,x+2*y+z=6,[x,y,z]):
projection(l1,l,p);
```
l1

```
> Equation(l1,'t');
```
$$\left[\frac{5}{3}+\frac{4}{3}t,\ \frac{1}{3}-\frac{7}{3}t,\ \frac{11}{3}+\frac{10}{3}t,\right]$$

```
> intersection(M,l,p);
```
M

```
> coordinates(M);
```
[3, K 2, 7]

```
> draw([l(color=blue),p(color=red)],title=`Screw-displacement of a triangle`,
style=patch);
```



3.4. FAZODA UCHBURCHAK MASALASI

3.1-masala . Fazoda A, B, S nuqtalar koordinatalari bilan berilgan.

A(7, 2, 2), B(5, 7, 7), C(4, 6, 10)

Quyidagilarni toping(Analitik geom-FAZO.mw).

1. **)ΔABC tomonlarining tenglamalarini;**
2. **)AB va AC tomonlar orasidagi burchakni;**
3. **)AD balandlik va uning uzunligini;**
4. **)AM mediana va AN bissektirsani;**
5. **)Δ ABC ni yuzasini hisoblang.**
6. **)Fazoda ΔABC ni quring**.

Echish: Bu masala 2.1-masalaga o‘xshash(faqat koordinatalar soni bilan farq qiladi) bo‘lganligi uchun uni masalalarni to’g’ridan-to’g’ri Maple 7 dasturida echishni ko‘rsatamiz.

1.*ΔABC tomonlarining tenglamalarini topish*;

```
> restart;
> with(geom3d):
> point(A,7,2,2), point(B,5,7,7), point(C,4,6,10):
```
Define the line that asses through two points A and B

```
> line(AB,[A,B]);line(AC,[A,C]);line(BC,[B,C]);
```
AB AC BC

```
> Equation(AB,t);Equation(AC,t);Equation(BC,t);
```
[7 K 2 *t*, 2 C 5 *t*, 2 C 5 *t*] [7 K 3 *t*, 2 C 4 *t*, 2 C 8 *t*] [5 K *t*, 7 K *t*, 7 C 3 *t*]

AB va AC tomonlar orasidagi burchakni topish;

> line(AB,[A,B]); line(AC,[A,C]); AB AC

> FindAngle(AB, AC); $\arccos\left(\frac{11}{267}\sqrt{6}\sqrt{89}\right)$

AD balandlik, kesishish nuqtasi va uning uzunligini topish;

> with(geom3d):

Define triangle ABC with vertices A, B and C.

> triangle(ABC, [point(A,7,2,2),point(B,5,7,7), point(C,4,6,10)]):

Find the altitude of ABC at A

> altitude(hA1,A,ABC); *hA1*

> form(hA1); *lint3d*

> detail(hA1);

name of the object: hA1

form of the object: line3d

*equation of the line: [_x = 7-10/11*_t, _y = 2+67/11*_t, _z = 2+19/11*_t]*

> altitude(hA1,A,ABC,H); *hA1*

> coordinates(H); $\left[\frac{67}{11}, \frac{89}{11}, \frac{41}{11}\right]$

> form(hA1); *segment3d*

> detail(hA1);

name of the object: hA1

form of the object: segment3d

the 2 ends of the segment: [[7, 2, 2], [67/11, 89/11, 41/11]]

> DefinedAs(hA1); $[A, H]$

> distance(A,H); $\frac{15}{11}\sqrt{22}$

AN bissektirsani va AM medianani topish:

> altitude(bA, A, ABC); *bA*

> detail(bA);

name of the object: bA

form of the object: line3d

*equation of the line: [_x = 7-10/11*_t, _y = 2+67/11*_t, _z = 2+19/11*_t]*

define the ``segment" bisector bA

> altitude(bA, A, ABC, N);

> coordinates(N); $\left[\frac{67}{11}, \frac{89}{11}, \frac{41}{11}\right]$

> detail(bA);

name of the object: bA

form of the object: segment3d

the 2 ends of the segment: [[7, 2, 2], [67/11, 89/11, 41/11]]

> distance(A,N); $\frac{15}{11}\sqrt{22}$

> altitude(mA, A, ABC); *mA*

> form(mA); *lint3d*

> detail(mA);

name of the object: mA

form of the object: line3d

*equation of the line: [_x = 7-10/11*_t, _y = 2+67/11*_t, _z = 2+19/11*_t]*

> altitude(mA, A, ABC, M); *mA*

> coordinates(M); $\left[\frac{67}{11}, \frac{89}{11}, \frac{41}{11}\right]$

> form(mA); *segment3d*

> detail(mA);

name of the object: mA

form of the object: segment3d

the 2 ends of the segment: [[7, 2, 2], [67/11, 89/11, 41/11]]

> distance(A,M); $\frac{15}{11}\sqrt{22}$

Δ ABC ni yuzasini hisoblash. Ogirlik markazini topish.

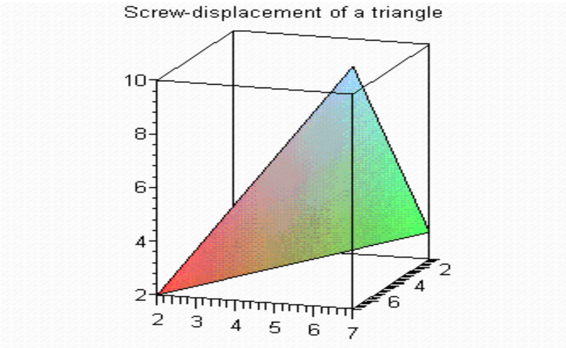
> with(geom3d):
Define triangle ABC with vertices A, B and C
> triangle(ABC, [point(A,7,2,2),point(B,5,7,7),point(C,4,6,10)]):
Find the area of ABC
> area(ABC); $\frac{15}{2}\sqrt{2}$

> centroid(G,ABC); *G*

> coordinates(G); $\left[\frac{16}{3}, 5, \frac{19}{3}\right]$

Fazoda ΔABC uchburchakni qurinsh.

> with(geom3d):
> triangle(T1,[point(A,7,2,2),point(B,1,7,3), point(C,4,6,10)]):
> draw([T1(color=blue)], title=`Screw-dislacement of a triangle`, style=patch);



3.5. FAZODA PIRAMIDA MASALASI.

Quyidagi masalani echishdagi har bir bajarilgan amalni **Maple7** dasturida bajarilishini ko‘rsatib boramiz.

3.2-masala. Fazoda A, B, C, D nuqtalar koordinatalari bilan berilgan.

A(7, 2, 2), B(5, 7, 7), C(4, 6, 10), D(2, 3, 7)

Quyidagilarni toping(Analitik geom-FAZO.mw).

1) $\overrightarrow{AB}$ **vektor proektsiyalari va yunalishini;**

2) $\overrightarrow{AB}\cdot\overrightarrow{AC}, \overrightarrow{AB}\times\overrightarrow{AC}$ **ko‘paytmalarni;**

3) ΔABC yuzasini;

4) ABCD piramida xajmini;

5) ABC yokining tenglamasi;

6) AD to‘g‘ri chiziq bilan ABC tekislik orasidagi burchak;

7) D uchidan ABC yokiga o‘tkazilgan perpendikulyar tenglamasi va uzunligi , kesishuvchi nuqtasi.

8) ABCD piramida chizmasini

Echish.

1) $\overrightarrow{AB}$ vektor kordinatalarini topish uchun:

$$X = a_x = x_2 - x_1, \qquad Y = a_y = y_2 - y_1, \qquad Z = a_z = z_2 - z_1$$

formulalarga asosan: $a_x = 5 - 7 = -2, \quad a_y = 7 - 2 = 5, \quad a_z = 7 - 2 = 5$

Demak, $\overrightarrow{AB}$ ={-2; 5; 5} , $|\overrightarrow{AB}| = \sqrt{(-2)^2 + 5^2 + 5^2} = 3\sqrt{6}$

> with(geom3d):

> distance(A,B); distance(A,C); distance(A,D);

$$3\sqrt{6} \quad \sqrt{89} \quad \sqrt{51}$$

$\overrightarrow{AB}$ vektor yunalishini , yahni koordinata o‘qlari bilan xosil kilgan burchaklarini

$$\cos\alpha = \frac{a_x}{|\overrightarrow{AB}|}, \quad \cos\beta = \frac{a_y}{|\overrightarrow{AB}|}, \quad \cos\gamma = \frac{a_z}{|\overrightarrow{AB}|}$$

yunaltiruvchi kosinuslardan foydalanamiz.

$$\cos\alpha = -\frac{2}{3\sqrt{6}}, \qquad \cos\beta = \frac{5}{3\sqrt{6}}, \qquad \cos\gamma = \frac{5}{3\sqrt{6}},$$

> with(geom3d):

> point(A,7,2,2),point(B,5,7,7), point(C,4,6,10), point(D,2,3,7):

> with(LinearAlgebra):

```
> v := <a,b,c>;
```

$$v := (a)e_x + (b)e_y + (c)e_z$$

```
> VectorNorm(v,2,conjugate=false);
```

$$\sqrt{a^2 + b^2 + c^2}$$

```
> v1 := <5-7,7-2,7-2>;
```

$$v1 := -2e_x + 5e_y + 5e_z$$

```
> VectorNorm(v1,2,conjugate=false);
```

$$3\sqrt{6}$$

```
> Normalize(<a,b,c>,Euclidean,conjugate=false);
```

$$\begin{bmatrix} \dfrac{a}{\sqrt{a^2+b^2+c^2}} \\ \dfrac{b}{\sqrt{a^2+b^2+c^2}} \\ \dfrac{c}{\sqrt{a^2+b^2+c^2}} \end{bmatrix}$$

```
> Normalize(v1,Euclidean,conjugate=false);
```

$$\begin{bmatrix} -\frac{1}{9}\sqrt{6} \\ \frac{5}{18}\sqrt{6} \\ \frac{5}{18}\sqrt{6} \end{bmatrix}$$

2) $\overrightarrow{AB}$ ={-2; 5; 5} , $\overrightarrow{AC} = \{-3,4,8\}$ dan

$$\overrightarrow{AB}\cdot\overrightarrow{AC} = (-2)\cdot(-3) + 5\cdot 4 + 5\cdot 8 = 66$$

```
> vAC := <4-7,6-2,10-2>;
```

$$vAC := -3e_x + 4e_y + 8e_z$$

```
> AB.AC:=Dotroduct( <5-7,7-2,7-2>, <4-7,6-2,10-2> );
```

$$VectorCalculus:-.(AB,\ AC) := 66$$

$\overrightarrow{AB}\times\overrightarrow{AC}$ vektor ko‘paytmani topamiz:

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 5 & 5 \\ -3 & 4 & 8 \end{vmatrix} = \begin{vmatrix} 5 & 5 \\ 4 & 8 \end{vmatrix}\vec{i} - \begin{vmatrix} -2 & 5 \\ -3 & 8 \end{vmatrix}\vec{j} - \begin{vmatrix} -2 & 5 \\ -3 & 4 \end{vmatrix}\vec{k} = 20\vec{i} + \vec{j} + 7\vec{k}$$

```
> A := <<i,l,o>|<j,m,p>|<k,n,q>>;
```

$$A := \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ l & m & n \\ o & p & q \end{vmatrix}$$

```
> Determinant(A);
```

$$i\,m\,q - i\,n\,p + l\,k\,p - l\,j\,q + o\,j\,n - o\,k\,m$$

> axb := <<i,-2,-3>|<j,5,4>|<k,5,8>>;
$$axb = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 5 & 5 \\ -3 & 4 & 8 \end{vmatrix}$$

> Determinant(axb);
$$20\, i \mathrm{C}\ 7\, k \mathrm{C}\ j$$

> Crossroduct(<a, b, c>, <d, e, f>);
$$(b\, f \mathrm{K}\ c\, e) e_x \mathrm{C}\ (c\, d \mathrm{K}\ a\, f) e_y \mathrm{C}\ (a\, e \mathrm{K}\ b\, d) e_z$$

> N:=Crossroduct(<5-7,7-2,7-2>, <4-7,6-2,10-2>); $N := \begin{bmatrix} 20 \\ 1 \\ 7 \end{bmatrix}$

3) ΔABC yuzasini : $\overrightarrow{AB} \times \overrightarrow{AC}$ ko‘paytmadan topamiz:
$$S = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\sqrt{20^2 + 1^2 + 7^2} = \frac{15\sqrt{2}}{2}$$

> modN:=VectorNorm(N,2,conjugate=false); $modN := 15\sqrt{2}$

> s:=modN/2; $s := \frac{15}{2}\sqrt{2}$

> with(geom3d):
Define triangle ABC with vertices A, B and C
> triangle(ABC, [point(A,7,2,2),point(B,5,7,7), point(C,4,6,10)]):
Find the area of ABC
> area(ABC); $\frac{15}{2}\sqrt{2}$

4) ABCD piramida xajmini $V = \frac{1}{6}\left|\vec{a} \cdot \vec{b} \cdot \vec{c}\right| = \pm \frac{1}{6}\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$ formulaga asosan
$$V = \frac{1}{6}\overrightarrow{AB} \cdot \overrightarrow{AC} \cdot \overrightarrow{AD} = \pm \frac{1}{6}\begin{vmatrix} -2 & 5 & 5 \\ -3 & 4 & 8 \\ -5 & 1 & 5 \end{vmatrix} = \pm \frac{1}{6}(64) = \frac{64}{6} = \frac{32}{3}$$

<u>*ABCD iramida xajmini toish*</u>:

> abc := <<-5,-2,-3>|<1,5,4>|<5,5,8>>;
$$abc := \begin{vmatrix} -2 & 5 & 5 \\ -3 & 4 & 8 \\ -5 & 1 & 5 \end{vmatrix}$$

> Determinant(abc); **-64**

> VABCD:={ABC(Determinant(abc))/6}; *VABCD* := $4\frac{32}{3}$5

Tetroeder

> with(geom3d):
> point(A,7,2,2),point(B,5,7,7), point(C,4,6,10), point(D,2,3,7):

> gtetrahedron(TABCD, [A, B, C, D]); *TABCD*

detail(TABCD);

name of the object: TABCD

form of the object: gtetrahedron3d

the 4 vertices: [[7, 2, 2], [5, 7, 7], [4, 6, 10], [2, 3, 7]]

the 4 faces: [[[7, 2, 2], [5, 7, 7], [4, 6, 10]], [[7, 2, 2], [5, 7, 7], [2, 3, 7]], [[7, 2, 2], [4, 6, 10], [2, 3, 7]], [[5, 7, 7], [4, 6, 10], [2, 3, 7]]]

> volume(TABCD); $\frac{32}{3}$

4)ABC yokning tenglamasini uch nuqtadan o'tgan tekislik tenglamasiga asosan topamiz:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = \begin{vmatrix} x-7 & y-2 & z-2 \\ -2 & 5 & 5 \\ -3 & 4 & 8 \end{vmatrix} = 0$$

$$\begin{vmatrix} 5 & 5 \\ 4 & 8 \end{vmatrix}(x-7) - \begin{vmatrix} -2 & 5 \\ -3 & 8 \end{vmatrix}(y-2) + \begin{vmatrix} -2 & 5 \\ -3 & 4 \end{vmatrix}(z-2) = 0$$

$$20(x-7) + (y-2) + 7(z-2) = 0$$

$$20x + y + 7z - 156 = 0$$

Piramida yoqlarining tenglamalari

> with(geom3d):
> point(A,7,2,2),point(B,5,7,7), point(C,4,6,10), point(D,2,3,7):
Define the plane that asses through three given points A, B, C
> plane(ABC,[A,B,C],[x,y,z]):ABC:=Equation(ABC);

ABC := K 156 C 20 *x* C *y* C 7 *z* = 0

>plane(ABD,[A,B,D],[x,y,z]):ABD:=Equation(ABD);

ABD := K 156 C 20 *x* K 15 *y* C 23 *z* = 0

>plane(BCD,[B,C,D],[x,y,z]):BCD:=Equation(BCD);

BCD := K 4 C 12 *x* K 9 *y* C *z* = 0

>plane(ACD,[A,C,D],[x,y,z]):ACD:=Equation(ACD);

ACD := K 68 C 12 *x* K 25 *y* C 17 *z* = 0

AD qirrani ABC tekislikdagi royesiyasi

> with(geom3d):
> point(A,7,2,2),point(B,5,7,7), point(C,4,6,10),point(D,2,3,7):
> line(AD,[A,D]); plane(,[A,B,C]); *AD*

> projection(l1,AB,); *l1*

> detail(l1);

Warning, assume that the arameter in the parametric equations is _t
Warning, assuming that the names of the axes are _x, _y, and _z

name of the object: l1

form of the object: line3d

*equation of the line: [_x = 7-2*_t, _y = 2+5*_t, _z = 2+5*_t]*

5) AD kirra bilan ABC asos tekisligi orasidagi burchakni

$$\sin\theta = \frac{Am + Bn + Cp}{\sqrt{A^2 + B^2 + C^2}\cdot\sqrt{m^2 + n^2 + p^2}}$$

formulaga asosan : AD to‘g‘ri chiziq tenglamasini yozamiz

$$AD: \frac{x-7}{-5} = \frac{y-2}{1} = \frac{z-2}{5}$$

Bunda m = -5 ; n = 1 ; p= 5.
ABC yoq tenglamasi: $20x + y + 7z - 156 = 0$ bunda A=20, B= 1, C=7

Bu holda $\sin\theta = \frac{20(-5)+1\cdot 1+7\cdot 5}{\sqrt{20^2+1^2+7^2}\cdot\sqrt{(-5)^2+1^2+5^2}} = -\frac{64}{15\sqrt{5}\cdot\sqrt{51}} = -\frac{64}{15\sqrt{225}};$

$$\sin\theta = -\frac{64}{15\sqrt{225}} = -0.4356, \quad \theta \approx$$

> with(geom3d): point(A,7,2,2),point(B,5,7,7), point(C,4,6,10), point(D,2,3,7): plane(ABC,[A,B,C],[x,y,z]); line(AD,[A,D]); FindAngle(ABC,AD); evalf(FindAngle(ABC,AD));

ABC *AD* K arcsin $\left(\frac{32}{765}\sqrt{102}\right)$ - 0.4361609542

7) $D(2;3;7)$ nuqtadan o‘tgan to‘g‘ri chiziq $\frac{x-2}{m} = \frac{y-3}{n} = \frac{z-7}{p}$

ABC asos tekisligigi perpendikulyar bo‘lsa; $\frac{A}{m} = \frac{B}{n} = \frac{C}{p}$ shartga asosan.

$$\frac{20}{m} = \frac{1}{m} = \frac{7}{m},$$

dan $m = 20, \quad n = 1, \quad p = 7$ bo‘ladi. Bu holda balandlik-perpendikulyar tenglamasi:

$$\frac{x-2}{20} = \frac{y-3}{1} = \frac{z-7}{7}$$

Endi balandlik bilan asosning kesisish nuqtasini topamiz. Balandlik tenglamasini pparametrik ko‘rinishda ezamiz.

$$x = 20t + 2, \quad y = t + 3, \quad z = 7t + 7$$

> with(geom3d):
> point(D,2,3,7), plane(ABC,-156+20*x+y+7*z = 0,[x,y,z]):
> line(HD,[D,ABC]); *HD*
> Equation(HD,'t'); [2 C 20 *t*, 3 C *t*, 7 C 7 *t*]

Bu x, y, z larni ABC tekislik tenglamasi $20x + y + 7z - 156 = 0$ ga qo'yib t ni hisoblaymiz.
$20(20t+2)+(t+3)+7(7t+7)-156=0$ tenglamadan $t=\frac{64}{450}=\frac{32}{225}=0.1422$ buni balandlikning pparametrik tenglamasidagi t ni o‘rniga qo‘yib, kesishish nuqtasining koordinatalarini topamiz:

$$x = 20t + 2 = 20\cdot\frac{32}{225}+2=\frac{218}{45}\approx 4.84$$

$$y = t + 3 = \frac{32}{225}+3=\frac{707}{225}\approx 3.14$$

$$z = 7t + 7 = 7\cdot\frac{32}{225}+7=\frac{899}{225}\approx 7.99$$

Balandlik uzunligini, nuqtadan tekislikkacha masofa formulasiga asosan topamiz:

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}=\frac{|20x_D+1y_D+7z_D-156|}{\sqrt{20^2+1^2+7^2}}=\frac{|20\cdot 2+1\cdot 3+7\cdot 7-156|}{\sqrt{450}}=\frac{64}{\sqrt{450}}=\frac{64}{15\sqrt{2}}=3.0139$$

DH balandlik bilan ABC tekislikni kesishish nuqtasi

```
> rojection(H,D,ABC);
```
H
```
> coordinates(H);
```
$$\left[\frac{218}{45},\frac{707}{225},\frac{1799}{225}\right]$$

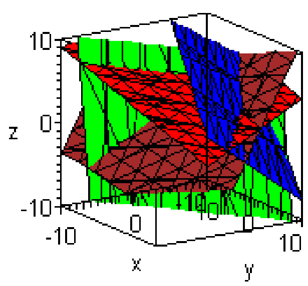
the length of the erendicular is

```
> distance(D,H); evalf(distance(D,H));
```
$\frac{32}{15}\sqrt{2}$ 3.016988932

Piraidani yoq tekisliklari bo‘yicha qurish.

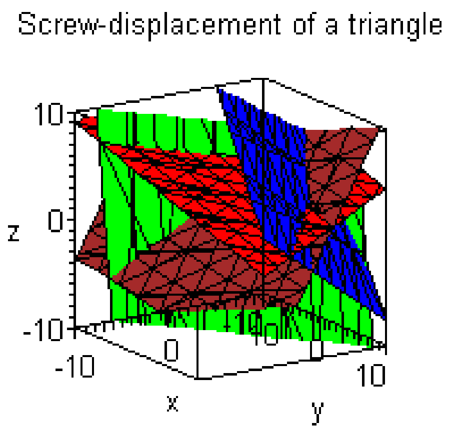
```
> with(geom3d):
> point(A,7,2,2),point(B,5,7,7), point(C,4,6,10),point(D,2,3,7):
> plane(ABC,[A,B,C]): plane(ABD,[A,B,D]):  plane(BCD,[B,C,D]):plane(ACD,[A,C,D]):
> draw([ABC(color=blue),ABD(color=red),BCD(color=green), ACD(color=brown)],title=
`Screw-dislacement of a triangle`,style=patch);
```

Screw-displacement of a triangle



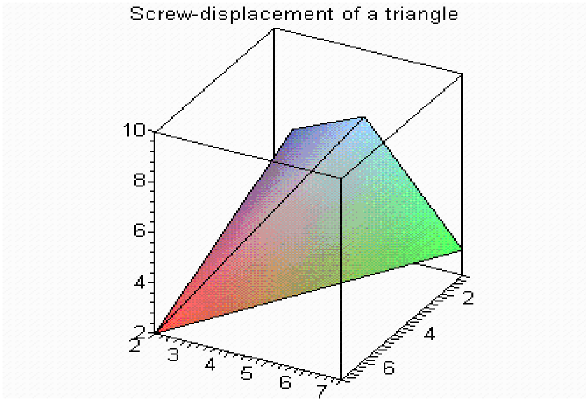
Piraidani yoq tetenglamalari bo‘yicha qurish.

```
> with(geom3d):
> point(A,7,2,2),point(B,5,7,7), point(C,4,6,10),point(D,2,3,7):
> plane(ABC,-156+20*x+y+7*z = 0,[x,y,z]):
        plane(ABD,-156+20*x-15*y+23*z = 0,[x,y,z]):
        plane(BCD,-4+12*x-9*y+z = 0,[x,y,z]):
        plane(ACD,-68+12*x-25*y+17*z = 0,[x,y,z]):
> draw([ABC(color=blue),ABD(color=red),BCD(color=green),
ACD(color=brown)],title=`Screw-dislacement of a triangle`,style=patch);
```

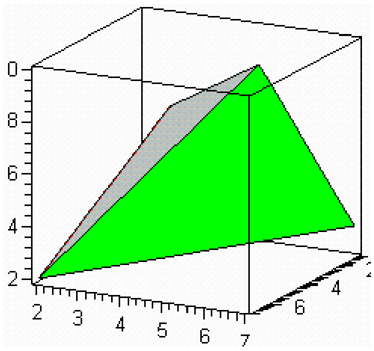


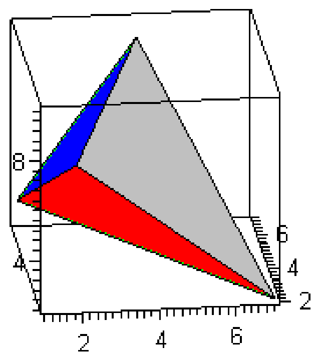
Piraidani yoq- uchburchaklari bo'yicha qurish.

```
> with(geom3d):
> triangle(T1,[point(A,7,2,2),point(B,1,7,3), point(C,4,6,10)]):
    triangle(T2,[point(A,7,2,2),point(B,1,7,3),point(D,2,3,7)]):
    triangle(T3,[point(A,7,2,2),point(D,2,3,7),point(C,4,6,10)]):
    triangle(T4,[point(D,2,3,7),point(B,1,7,3), point(C,4,6,10)]):
> draw([T1(color=blue),T2(color=red),T3(color=green),
T4(color=blue)],title=`Screw-dislacement of a triangle`,style=patch);
```



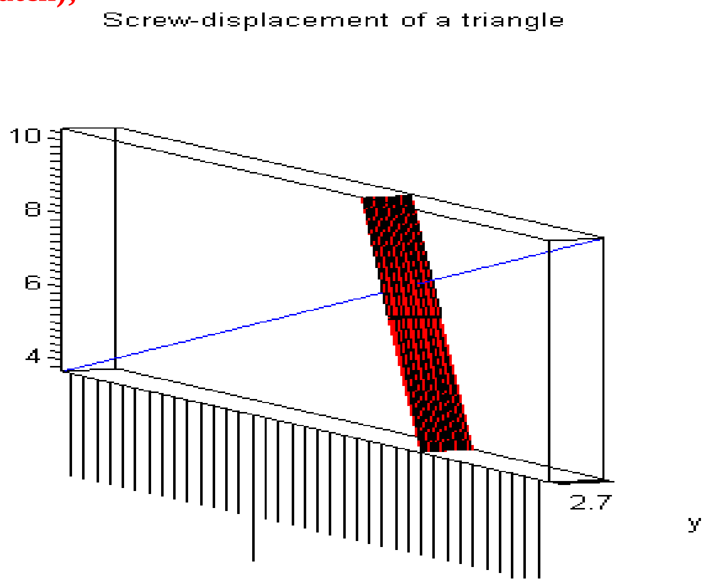

```
> with(plottools):with(plots):
> display(polygon([[7,2,2],[1,7,3],[4,6,10]],color=green),
polygon([[7,2,2],[1,7,3],[2,3,7]],color=red),
polygon([[7,2,2],[2,3,7],[4,6,10]],color=gray),
polygon([[2,3,7],[1,7,3],[4,6,10]],color=blue),
axes=frame, color=blue, orientation=[-45, 60]);
```





Berilgan nuqtadan o'tuvchi to'g'ri chiziqni berilgan tekislik bilan kesishish nuqtasini topish:
D nuqtadan o'tiuvchi to'g'ri chiziq bilan ABC tekislik kesishuvi:

```
> restart;
> with(geom3d):
> point(D,2,3,7), plane(ABC,-156+20*x+y+7*z = 0,[x,y,z]):
> line(HD,[D,ABC]);
```
HD
```
> rojection(H,HD,ABC);
```
H
```
> Equation(HD,'t');
```
$[2+20t, 3+t, 7+7t]$
```
> intersection(H,HD,ABC);
```
H
```
> coordinates(H);
```
$\left[\frac{218}{45}, \frac{707}{225}, \frac{1799}{225}\right]$
```
> draw([HD(color=blue),ABC(color=red)],title=`Screw-dislacement of a triangle`,style=patch);
```



ABC va ABD yoqlar kesishishidan hosil bolgan AB togri chiziq:

```
> with(geom3d):
> point(A,7,2,2),point(B,5,7,7), point(C,4,6,10), point(D,2,3,7):
> plane(ABC,[A,B,C],[x,y,z]);plane(ABD,[A,B,D],[x,y,z]);
```
ABC ABD
```
> line(AB,[ABC,ABD]);
```
AB
```
> AB:=Equation(AB,'t');
```
$AB := [\frac{39}{5} + 128t, -320t, -320t]$

AB qirra tenglamasi:

```
> restart;
> with(geom3d):
> plane(ABC, -156+20*x+y+7*z=0,[x,y,z]):
plane(ABD, -156+20*x-15*y+23*z=0,[x,y,z]):
line(AB,[ABC,ABD]);
```
AB
```
> AB:=Equation(AB,'t');
```
$AB := \left[\frac{39}{5} + 128t, -320t, -320t\right]$

Berilgan nuqtadan berilgan vektor yo'nalishda o'tuvchi to'g'ri chiziq

```
> with(geom3d):
```

Let the straight line ass through the point A=[x1,y1,z1] and has direction-cosines (or ratios) [l,m,n]

```
>assume(l<>0): # the direction-ratios is a non-zero vector
point(A,[x1,y1,z1]): v := [l,m,n]:
```

define the line l1 that asses through A and has [l,m,n] as its direction-ratios

```
> line(l1,[A,v]):
detail(l1);
```

name of the object: l1
form of the object: line3d
*equation of the line: [_x = x1C _t*l, _y = y1C _t*m, _z = z1C _t*n]*

Tekisliklarning paralleligi:

```
> with(geom3d):
```

Define two planes 1, 2

```
> assume(A1<>0, A2<>0, A1<>A2, t<>0);
plane(p1,A1*x+B1*y+C1*z+D1,[x,y,z]);
```

p1

```
> plane(p2,A2*x+B2*y+C2*z+D2,[x,y,z]);
```

p2

Find the condition that makes 1 and 2 arallel to each other

```
> AreParallel(p1,p2,'cond');
```

FAIL

```
> cond;
```

$(B1\,C2 K\ C1\,B2 = 0)$ &and $(C1\,A2\sim K\ A1\sim C2 = 0)$ &and $(A1\sim B2 K\ B1\,A2\sim = 0)$

```
> additionally(o(cond));
AreParallel(p1,p2);
```

true

Nazorat ishi uchun variant namunalari

1-variant

1. 0x o'qidan va $M_1(2;-4;-3)$ nuqtadan o'tuvchi tekislik tenglamasi ezilsin.
2. (-4; 3;0) nuqtadan o'tuvchi va $\begin{cases} x-2y+z=4 \\ 2x+y-z=0 \end{cases}$ to'g'ri chiziqka parallel bo'lgan to'g'ri chiziq tenglamalari ezilsin .
3. (3;1;-1) nuqtaning x+2y+3z-30=0 tekislikdagi proektsiyasi topilsin.

2-variant

1. $M_1(3;-2;-7)$ nuqtadan o'tib 2x-3z+5=0 tekislikka parallel bo'lgan tekislik tenglamasi yozilsin.
2. $\begin{cases} x-y+z-40=0 \\ 2x+y-2z+5=0 \end{cases}$ to'g'ri chiziqni kanonik ko'rinishga keltiring.
3. $x=2t-1,\quad y=t+2,\quad z=1-t$ to'g'ri chiziq bilan $3x-2y+z=3$ tekislik orasidagi burchak topilsin.

3-variant

1. 0z o‘qqa parallel hamda, $M_1(2;2;0)$ va $M_2(4;0;0)$ nuqtalardan o‘tuvchi tekislik tenglamasi yozilsin.
2. $x-2y+2z-5=0$ tekislikka parallel va undan 2 birlik uzoklikda bo‘lgan tekisliklar tenglamalari yozilsin.
3. $\begin{cases}3x-y+2z+9=0\\ x+z-3=0\end{cases}$ to‘g‘ri chiziqni proektsiyalar buyicha tenglamasi ezilsin.

4-variant

1. $M_1(5;3;7)$ nuqtadan 3x-2y-5=0 tekislikkacha bo‘lgan masofa topilsin.
2. $\begin{cases}3x-y+2z-9=0\\ x+z-3=0\end{cases}$ to‘g‘ri chiziqdan va $M_1(4;2;3)$ nuqtadan o‘tuvchi tekislik tenglamasi ezilsin.
3. (2;3;0) nuqtadan o‘tib, $3x-4y-z-1=0$ tekislikka parallel tekislik tenglamasi topilsin.

Fazoda $A_1,\ A_2,\ A_3,\ A_4$ nuqtalar koordinatalari bilan berilgan.
Quyidagilarni topish talab qilinadi

3-topshiriq

1. $\overrightarrow{A_1A_2},\ \overrightarrow{A_1A_3},\overrightarrow{A_1A_4}$ **vektorlar proektsiyalari va yunalishini.**
2. $\overrightarrow{A_1A_2},\ \overrightarrow{A_1A_3}$ **vektorlar orasidagi burchak**
3. $\overrightarrow{A_1A_2}\cdot\overrightarrow{A_1A_3},\ \overrightarrow{A_1A_2}\times\overrightarrow{A_1A_3}$ **ko‘paytmalarni.**
4. $\Delta A_1A_2A_3$ **yuzasini.**
5. $A_1A_2A_3A_4$ **piramida xajmini.**
6. A_4 **uchidan** $A_1A_2A_3$ **yoqiga tushirilgan balandlikni**

4-topshiriq

1. A_1A_2 **to‘g‘ri chiziqning kononik, pparametrik va yo‘nalish bo‘yicha tenglamasi**
2. A_1A_2 *ва* A_1A_3 **to‘g‘ri chiziqlar orasidagi burchak**
3. $A_1A_2A_3$ **yokning tenglamasi.**
4. A_1A_4 **qirra bilan** $A_1A_2A_3$ **tekislik orasidagi burchak.**
5. A_4 **uchidan** $A_1A_2A_3$ **yoqqa tushurilgan balandlik tenglamasi, kesishish nuqtasi va uzunligi.**
6. $A_1A_2A_3A_4$ **piramida shakli chizilsin.**

№	**1**	**2**	**3**	**4**	**5**
Koordinata	A_1 (3, 0, 0) A_2 (-2, 4, 1) A_3(2, 3, 2) A_4(4, 5, 6)	A_1(1, 3, -1) A_2(3,-2,0) A_3(3,3,3) A_4(-4, 4, 4)	A_1 (1, -1, 1) A_2(2,2,2) A_3(3, 3, -3) A_4(4, -4, 4)	A_1 (2, 2, 1) A_2 (3, 3, 2) A_3(4, 4, -3) A_4(5, 5, 4)	A_1 (3, 3, 2) A_2 (2, -3, -3) A_3(4, -4, 3) A_4(4, -5, 2)
№	6	7	8	9	10
Koordina	A_1 (1, 3, 3) A_2(2,2,-3) A_3(3, 3, -4) A_4(4, 4, 7)	A_1(2,2,3) A_2(-1,0,1) A_3(0,1,1) A_4(3, 0,7)	A_1 (2, 2, 3) A_2 (-1, 0, 1) A_3(0, 1, 1) A_4(3, 0, 7)	A_1(2, 2, 3) A_2(-1,0,1) A_3(0, 1, 1) A_4(3, 0, 7)	A_1(2,2,3) A_2(-1,0,1) A_3(0,1,1) A_4(3, 0, 7)

ta					
№	11	12	13	14	15
Ko or di na ta	A_1 (1, 3, 3) A_2(1, 2, 0) A_3(4, 1, 0) A_4(5, 0, 7)	A_1 (2, 2, 1) A_2 (3, 2, 1) A_3(5, 1, 0) A_4(4, 1, 4)	A_1(2,2,1) A_2(3,2,1) A_3(5, 1, 0) A_4(4, 1, 4)	A_1 (2, 2, 1) A_2 (3, 2, 1) A_3(5, 1, 0) A_4(4, 1, 4)	A_1 (3, 3, 1) A_2 (3, 1, 4) A_3(4, -1, -4) A_4(2, -1, -4)
№	16	17	18	19	20
Ko or di na ta	A_1 (4, 4, 0) A_2 (5, 0, 5) A_3(3, 0, 3) A_4(2, 2, 6)	A_1 (2, 1, 1) A_2 (3, -3, 0) A_3(4, 4, 0) A_4(2, 2, 6)	A_1 (2, 0, 1) A_2 (-2, 1,-2) A_3(4,-4, 0) A_4(3,-3, 6)	A_1 (-2, 0,-2) A_2 (3, 0,-3) A_3(4, 0, 1) A_4(2, 6, -2)	A_1 (-3, 0, 1) A_2 (3, 2, 1) A_3(2, 1, 0) A_4(-2,-1, 6)
№	21	22	23	24	25
Ko or di na ta	A_1 (2, -1, 0) A_2 (4, 0,-1) A_3(1, 0, 1) A_4(-1, 1,-1)	A_1 (3, 1, 0) A_2 (-3,-1, 1) A_3(1, 4, -1) A_4(2,-2,-3)	A_1 (1, 4, 0) A_2 (-2,-1, 4) A_3(3, 0,-3) A_4(5, 0,-2)	A_1 (0, 4, 1) A_2 (-4, 0,-1) A_3(3,-2, 1) A_4(-5, 2, 6)	A_1 (1, 2, 0) A_2 (2,-1,-3) A_3(1, 0, 3) A_4(1, 2, 7)
№	26	27	28	29	30
Ko or di na ta	A_1 (-1,-3, 1) A_2 (-2, 1, 3) A_3(-1, 0,-3) A_4(1,-2, 7)	A_1 (5,-6,-1) A_2 (2,-2,-4) A_3(-2, 0, 1) A_4(2, 1, 7)	A_1 (-5, 6,-4) A_2 (-4, -5, 1) A_3(2, 4, 6) A_4(1, 1, 5)	A_1 (-4,-5, 2) A_2 (3,-, 0) A_3(2, 1, 4) A_4(1, 5, -3)	A_1 (2,-5,-3) A_2 (3, 2, 1) A_3(-2, 0, 1) A_4(4, 5,-3)

4. IKKINCHI TARTIBLI EGRI CHIZIQLAR

4.1. Aylana

Ta'rif. ***Tekislikning markaz deb atalgan nuqtasidan barobar uzunlikda yotuvchi nuqtalarning geometrik o'rniga aylana deb ataladi.***

Ta'rifga asosan aylananing kononik tenglamasini uning ixtiyoriy nuqtasidan markazigacha masofa formulasiga asosan quyidagicha yozamz:

$$(x - a)^2 + (y - b)^2 = R^2 \qquad (1)$$

C($a;b$) – aylana markazi, R- aylana radiusi. (1) tenglama yoyilmasini quyidagicha yozamiz:

$$x^2 + y^2 + mx + ny + \ = 0 \qquad (2)$$

Bu (2) tenglamadan (1) tenglamaga o'tish uchun chap tomonidagi ifodani to'la kvadratini ajratamiz.

$$(x + \frac{m}{2})^2 + (y + \frac{n}{2})^2 = \frac{m^2}{4} + \frac{n^2}{4} - p \qquad (3)$$

Bu erda: $a = -\frac{m}{2}, \ b = -\frac{n}{2} \quad R = \frac{m^2}{4} + \frac{n^2}{4} - p$

Quyidagi masalani echishdagi har bir bajarilgan amalni **Maple7** dasturida bajarilishini ko'rsatib boramiz.

4.1-masala. **A(-1;3), B(0;2) va C(1;-1) nuqtalar berilgan.**

Topish kerak(Aylana-1.mw):

1) **A, B, C nuqtalardan o'tuvchi aylana tenglamasini**
2) **Topilagn aylananing koordinatalar o'qlari bilan kesiishish nuqtalarini**
3) **Aylananing y = x + 1 to'g'ri chizigi bilan kesishish nuqtasini**

4) **A nuqtaga o‘tkazilgan urinma tenglamasini**.

Echish : 1) Aylananing $x^2 + y^2 + mx + ny + \ = 0$ umuimy tenglamasiga A, B va C nuqtalarning koordinatalarini qo‘yamiz.

A: $(-1)^2 + 3^2 + m(-1) + n3 + \ = 0$
B: $0^2 + 2^2 + m0 + 2n + \ = 0$
C: $1^2 + (-1)^2 + m1 + n(-1) + \ = 0$

bundan

$$\begin{cases} -m + 3n + p + 10 = 0 \\ 0m + 2n + p + 4 = 0 \\ m - n + p + 2 = 0 \end{cases}$$

sistemani echib m, n, larni aniqlaymiz: m = 8, n = 2, p= -8
Demak, izlanayotgan aylana tenglamasi:

$x^2 + y^2 + 8x + 2y - 8 = 0$ yoki $(x+4)^2 + (y+1)^2=25$, C(-4;-1), R=5

Uch nuqtadan o‘tuvchi aylana tenglamasi;

```
> restart;
> with(geometry):
_EnvHorizontalName := x: _EnvVerticalName := y:
```

define circle c1 from three distinct points:

```
> circle(c1,[point(A,-1,3), point(B,0,2), point(C,1,-1)], 'centername'=O1):
center(c1), coordinates(center(c1));
```

O1, [K 4, K 1]

```
> radius(c1);
```

$\sqrt{25}$

```
>area(c1);
```

25 p

```
> Equation(c1);
```

K 8 C x^2 C y^2 C 8 *x* C 2 *y* = 0

```
> detail(c1);
```

name of the object: c1
form of the object: circle2d
name of the center: O1
coordinates of the center: [-4, -1]
radius of the circle: 25^(1/2)
*equation of the circle: -8+x^2+y^2+8*x+2*y = 0*

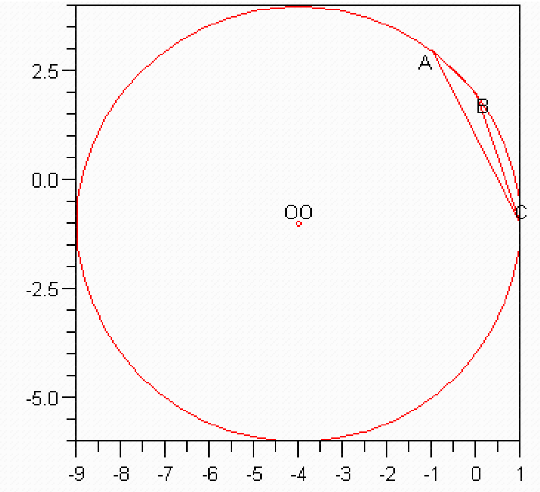
Uch nuqtadan o‘tuvchi aylanani qurish:

```
> with(geometry):
triangle(T, [point(A,-1,3), point(B,0,2), point(C,1,-1)]):
circumcircle(Elc, T, 'centername' = OO);
detail(Elc);
```

name of the object: Elc
form of the object: circle2d
name of the center: OO
coordinates of the center: [-4, -1]
radius of the circle: 25^(1/2)
*equation of the circle: -8+x^2+y^2+8*x+2*y = 0*

```
draw({Elc,T},rinttext=true);
```

Elc



2) $x^2 + y + 8x + 2y - 8 = 0$ aylanani Ox o'qi bilan kesishish nuqtalarini topish uchun tenglamada y = 0 deb olmiz va

$$x^2 + 8x - 8 = 0$$

tenglamani yozib,

$$x_1 = -4 - 2\sqrt{6}, \quad x_2 = -4 + 2\sqrt{6}$$

ABCitsalarini topamiz:

$$A_1(-4-2\sqrt{6};0),\ A_2(-4+2\sqrt{6};0)\ .$$

Topilgan aylananing koordinata o'qlari bilan kesishish nuqtasi:

OX o'qi bilan kesishish nuqtasi:

> evalf(solvefor({-8+x^2+y^2+8*x+2*y = 0, y=0}));

$$[\ \{y = 0.,\ x = 0.898979486\ \},\ \{y = 0.,\ x = K\ 8.898979486\ \}\ ]$$

3)Oy o'qi bilan kesishish nuqtalarini topish uchun tenglamada x = 0 deb olmiz va

$$y^2 + 2y - 8 = 0$$

tenglamani yozib,

$$y_1 = -4,\ y_2 = 2$$

ordinatalarini topamiz:

$$B_1(0;-4),\ B_2(0;2),.$$

OY o'qi bilan kesishish nuqtasi:

> evalf(solvefor({-8+x^2+y^2+8*x+2*y = 0, x=0}));

$$[\ \{y = 2.,\ x = 0.\ \},\ \{x = 0.,\ y = K\ 4.\ \}\ ]$$

4) $x^2 + y^2 + 8x + 2y - 8 = 0$ tenglamani y = x + 1 to'g'ri chiziq bilan kesishish nuqtasini topish uchun aylana tenglamasida y o'rniga x+1 ni qo'yib, $2x^2 + 12x - 5 = 0$ tenglamani echib,

$$x_1 = \frac{-6+\sqrt{46}}{2},\quad x_1 = \frac{-6-\sqrt{46}}{2}$$

topamiz. Bulardan :

$$y_1 = x_1 + 1 = -2 + \sqrt{11.5} \quad y_2 = x_2 + 1 = -2 - \sqrt{11.5}$$

Demak, aylana bilan to'g'ri chiziqning kesishish nuqtalari:

$$M_1(-3+\sqrt{11.5};\ -2+\sqrt{11.5}),\ M_2(-3-\sqrt{11.5};\ -2-\sqrt{11.5})$$

Aylana bilan to'ri chiziq kesishish nuqtasi

```
> restart;
> with(geometry):
> line(l2, -x+y=1.,[x,y]),circle(c, -8.+x^2+y^2+8*x+2.*y=0,[x,y]):
intersection(H, l2, c, [M1,M2]);
```

$[M1, M2]$

```
> H;
```

$[M1, M2]$

```
> detail(H);
```

name of the object: M1
form of the object: point2d
coordinates of the point: [.391164992, 1.391164992]
name of the object: M2
form of the object: point2d
coordinates of the point: [-6.391164990, -5.391164990]

Aylana va to'g'ri chiziq tenglamalarini birgalirda yechish yordamida toish:

```
> solvefor( {-8+x^2+y^2+8*x+2*y = 0, -x+y=1} );
```

4 y = K 2 C $\frac{1}{2}\sqrt{46}$, x = K 3 C $\frac{1}{2}\sqrt{46}$ 5 4 y = K 2 K $\frac{1}{2}\sqrt{46}$, x = K 3 K $\frac{1}{2}\sqrt{46}$ 5

```
> evalf(solvefor( {-8+x^2+y^2+8*x+2*y = 0, -x+y=1} ));
```

[{y = 1.391164992, x = 0.391164992}
, {y = K 5.391164992, x = K 6.391164992}]

4)Aylananing biror nuqtasiga o'tkazilgan urinma aylananing shu nuqtasidagi radiusiga perpendikulyar bo'lgani uchun, A(-1;3) nuqtasidagi urinmani shu nuqtadagi to'g'ri chiziqlar dastasi

$$y - 3 = K(x + 1)$$

tenglamasidan R=CA radiusga perpendikulyar bo'lganini ajratamiz. Radius tenlamasining burchak koeffitsienti

$$K_R = \frac{Y_A - Y_C}{X_A - X_C} = \frac{3+1}{-1+4} = \frac{4}{3}$$

dan urinma burchak koeffitsienti K topamiz: $K = -\frac{1}{K_R} = -\frac{3}{4}$ bo'ladi.

Bu holda A(-1;3) nuqtadagi urinma tenglamasi: $y - 3 = -\frac{3}{4}(x+1)$ yoki 3x + 4u – 9 = 0.

```
> with(geometry):
point(A,-1,3), circle(c,-8+x^2+y^2+8*x+2*y=0,[x,y]);
```

A,s

```
> tangentc(l, A, c);
```

l

```
> form(l);
```

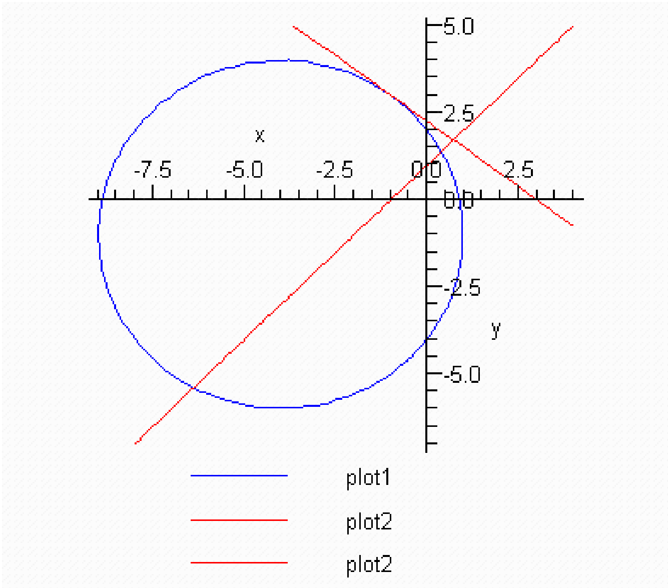
line2d

```
> Equation(l);
```

K 9 C 3 x C 4 y = 0

Aylana, urinma va kesishuvchi chiziqlarni qurish:

```
> restart;
> with(plots):
> imlicitplot([-8+x^2+y^2+8*x+2*y = 0, -x+y=1,-9+3*x+4*y = 0], x=-10..4, y=-7..5,
color=[blue, red,red], legend=[plot1,plot2,plot2]);
```



Masala yechimini grafikda ifodalash.

```
> restart;
> with(geometry):
> point(A,-1,3),point(B,0,2),point(C,1,-1),point(OO,-4,-1);
```

A, B, C, OO

```
> point(A1,-4-sqrt(24),0), point(A2,-4+sqrt(24),0);         point(B1,0,-4), point(B2,0,2);
```

$A1, A2 \quad B1, B2$

```
> point(M1,-3-sqrt(46)/2,-2-sqrt(46)/2),  point(M2,-3+sqrt(46)/2,-2+sqrt(46)/2);
```

$M1, M2$

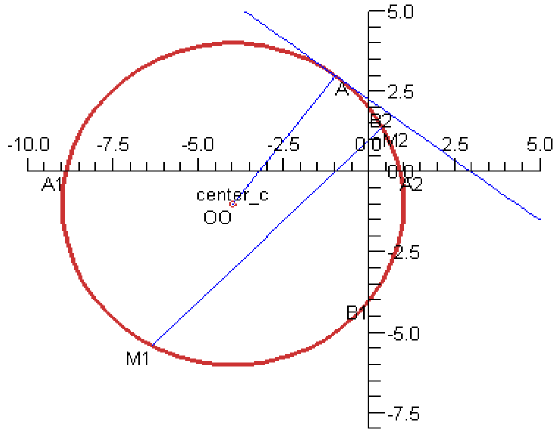
```
> line(ur,3*x+4*y=9,[x,y]), line(l2,y-x=1,[x,y]);
```

$ur, l2$

```
> circle(c, -8.+x^2+y^2+8*x+2.*y=0,[x,y]):
segment(rad1,[OO,A]),segment(l4,[B1,B2]);
segment(l5,[A1,A2]),segment(l6,[M1,M2]);
```

$rad1, l4 \quad l5, l6$

```
> draw({c(color=orange, thickness=2), rad1,ur,l4,l5,l6}, color=blue,axes=BOX,
style=LINE, symbol=DIAMOND, rinttext=true, view=[-10..5,-8..5]);
```



QO'SHIMCHA MACALALAR.

Quyidagi masalalarni echishmni to'g'ridan-to'g'ri **Maple7** dasturida bajarilishini ko'rsatamiz.
*Uchburchak tomonlarining o'rtalaridan o'tkazilgan aylana***:**

> with(geometry):
_EnvHorizontalName := x: _EnvVerticalName := y:
> triangle(T, [point(A,-1,3), point(B,0,2), point(C,1,-1)]):
EulerCircle(Elc,T,'centername'=o); *Elc*
> detail(Elc);

name of the object: Elc

form of the object: circle2d

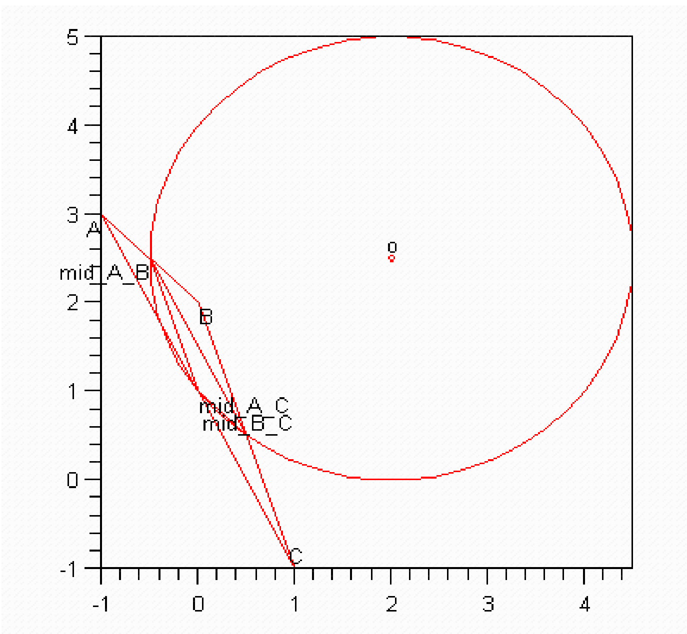
name of the center: o

coordinates of the center: [2, 5/2]

*radius of the circle: 1/4*25^(1/2)*4^(1/2)*

*equation of the circle: 4+x^2+y^2-4*x-5*y = 0*

> medial(T1,T):
> draw({T,T1,Elc}, rinttext=true);



Aylanalar kesishgan nuqtalaridan o'tkazilgan aylana:

```
> with(geometry):
_EnvHorizontalName := 'x': _EnvVerticalName := 'y':
circle(c1, x^2 + y^2 = 1, 'centername' = o1):
circle(c2, [point(A,3,3), 4], 'centername' =o2):
CircleOfSimilitude(c,c1,c2,'centername'=o); s
> detail(c);
```

name of the object: c

form of the object: circle2d

name of the center: o

coordinates of the center: [-1/5, -1/5]

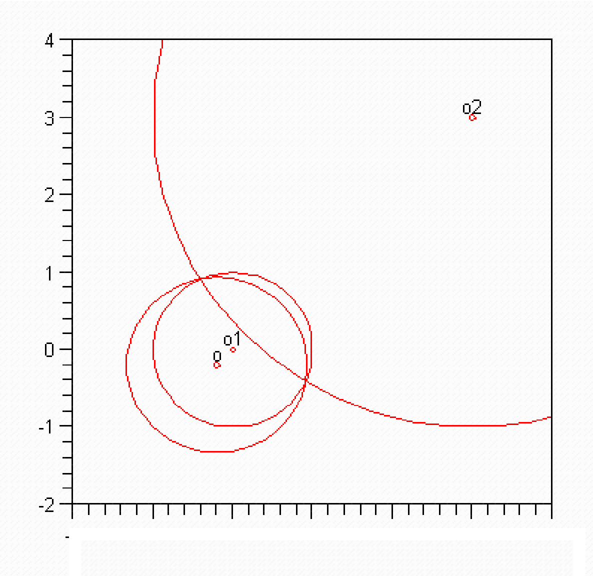
radius of the circle:

*1/50*128^(1/2)*25^(1/2)*

equation of the circle: -

*6/5+x^2+y^2+2/5*x+2/5*y = 0*

```
> draw({c,c1,c2},rinttext= true,
view=[-2..4,-2..4]);
```

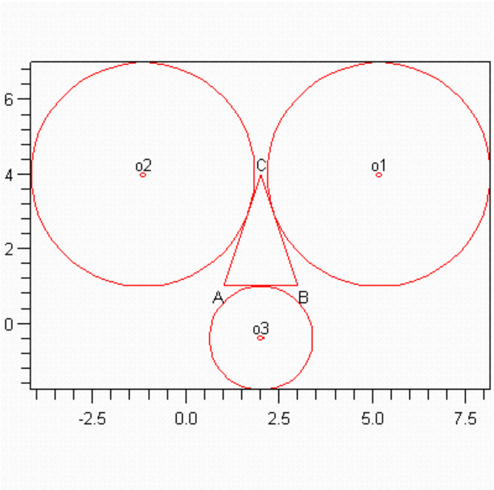


Uchburchak tashqi tomonlariga urinuvchi aylanalar:

```
> restart;
> with(geometry):
s := point(A,1,1),point(B,3,1),
point(C,2,4): triangle(T,[s]);   T
>excircle(obj,T, [c1(o1),c2(o2),c3(o3)]);
```

$$[c1, c2, c3]$$

```
> draw({o(obj),T},rinttext=true);
```



Uchburchakka ichki chizilgan aylana:

```
> with(geometry):
triangle(T, [point(A,0,0), point(B,2,0),
point(C,1,3)]):
Gergonnepoint(G, T);   G
> detail(G);
```

name of the object: inc

form of the object: circle2d

name of the center: o

coordinates of the center: [2.748677468, 2.839772364]

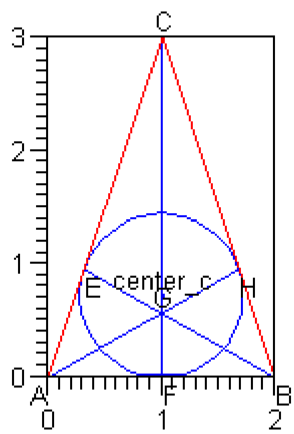
radius of the circle: 1.749794557

equation of the circle: 12.55775391+_x^2+_y^2-

*5.497354936*_x-5.679544728*_y = 0*

draw the icture of the above definition for the triangle T

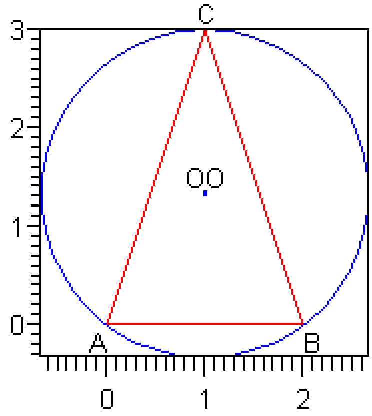
```
> incircle(c,T):
segment(sg1,A,rojection(H,center(c),line(tm,[B,C]))):
segment(sg2,B,rojection(E,center(c),line(tm,[C,A]))):
```



```
segment(sg3,C,rojection(F,center(c),line(tm,[A,B]))):
draw({sg1,sg2,sg3,c(color= blue,style=OINT),
G(symbol=DIAMOND),T(color=red)},color=blue,rinttext=true);
```

Uchburchakka tashqi chizilgan aylana:

```
> with(geometry):
triangle(T, [point(A,0,0), point(B,2,0), point(C,1,3)]):
circumcircle(Elc, T, 'centername' = OO);
detail(Elc);
draw({Elc(color=blue),T},printtext=true);
```



name of the object: Elc

form of the object: circle2d

name of the center: OO

coordinates of the center: [1, 4/3]

*radius of the circle: 1/9*25^(1/2)*9^(1/2)*

*equation of the circle: _x^2+_y^2-2*_x-8/3*_y = 0*

Uchburchakka ichki va tashqi chizilgan aylana:

```
> with(geometry):
triangle(T, [point(A,-1,3.), point(B,5,2), point(C,1,-1)]):     > circumcircle(Elc, T,
'centername' = OO);
```
Elc

```
Gergonnepoint(G, T);
```
G

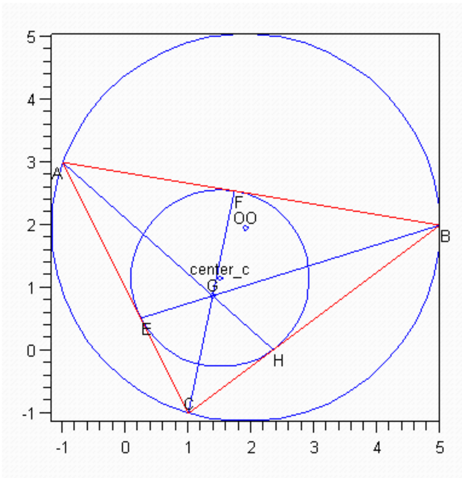
```
> coordinates(OO);
```
[1.909090909 , 1.954545454]

```
> coordinates(G);
```
[1.391235831 , 0.8742435189]

```
> incircle(c,T);
```
c

```
> radius(Elc); radius(c);
```
3.091243298   1.414345456

```
> detail(G);
```



name of the object: G

form of the object: point2d

coordinates of the point:

[1.391235831, .8742435189]

draw the icture of the above definition for the triangle T

```
> incircle(c,T):
segment(sg1,A,rojection(H,center(c),line(tm,[B,C]))):
segment(sg2,B,rojection(E,center(c),line(tm,[C,A]))):
segment(sg3,C,rojection(F,center(c),line(tm,[A,B]))):
draw({Elc,sg1,sg2,sg3,c(color=blue,style=line),G(symbol
=DIAMOND),T(color=red)},color=blue,rinttext=true);
```

Aylanadan tashqaridagi nuqtadan o'tkazilgan urinma tenglamalari:

```
> restart;
> with(geometry):
point(A,3,1),circle(c,x^2+y^2+8*x+2*y-8=0,[x,y]);
```
A, c

> TangentLine(obj, A, c, [l1, l2]); *[l1, l2]*

> form(l1), evalf(Equation(l1));

$$line2d,\ 4.550992298\ x - 2.699716487\ y - 10.95326040 = 0.$$

> form(l2), evalf(Equation(l2));

$$line2d,\ -2.437784750\ x - 4.696509929\ y + 12.00986418 = 0.$$

4.2. Ellips

Ta'rif: ***Ellips deb, har bir nktasidan berilgan F va F$_1$ nuqtasigacha (fokuslarigacha) masofalarning yigindisi o'zgarmas 2a miqdorga teng bo'lgan nuqtalarning geometrik o'rniga aytiladi*.**

Ellipsning kanonik tenglamasi:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (4)$$

Bu markazi koordinata boshida bo'lib, koordinata o'qlariga nisbatan simmetrik bo'ladi. *a*- katta va b-kichik yarim o'qlar bo'lib, *a*>b bo'lsa F va F$_1$ fokuslar Ox o'qida joylashgan bo'ladi va markazdan $c=\sqrt{a^2-b^2}$ masofada bo'ladi. $\frac{c}{a}=\varepsilon<1$ nisbat ellipsning ekstsentrisiteti deyiladi. Ellipsning M(x;u) nuqtasidan fokuslarigacha bo'lgan masofalar faqal radiuslari deyiladi va

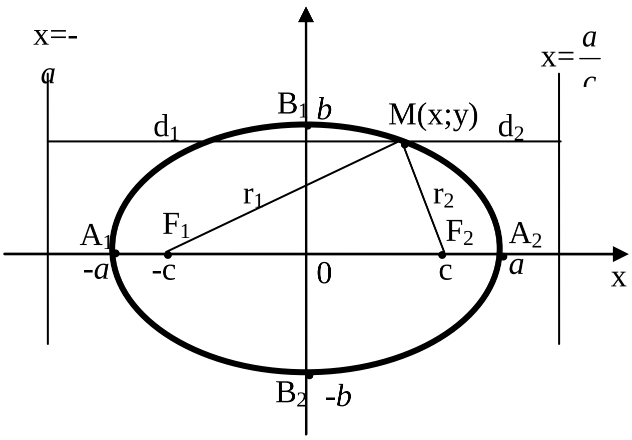
$$r = a-\varepsilon x, \qquad r_1 = a+\varepsilon x \qquad (5)$$

formulalar bilan aniqlanadi.

Agar *a*<b bo'lsa, fokuslar OU o'qida bo'lib,

$c=\sqrt{a^2-b^2}$, $\varepsilon=\frac{c}{b}$, r = b ± εy bo'ladi.

Markaz (x$_0$, u$_0$) nuqtada bo'lgan ellips tenglamasi:



$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1 \qquad (6)$$

uning simmetrik o'qlari_(x$_0$;y$_0$) nuqtadan o'tuvchi to'g'ri chiziqlari koordinatalariga parallel bo'ladi. *a*,*b* o'qlari_(x$_0$;y$_0$) nuqtaga nisbatan simmetrik o'qlar bo'yicha joylashtiriladi.

Quyidagi masalani echishdagi har bir bajarilgan amalni **Maple7** dasturida bajarilishini ko'rsatib boramiz.

4.2-masala. $A(\sqrt{3},\sqrt{6})$, $B(3,\sqrt{2})$ **nuqtalar berilgan. Topish kerak (Ellis-1.mw):**

1) A va B nuqtalardan o'tuvchi, markazi koordinatalar boshidan bo'lgan ellips tanglamasini;

2) Ellips fokuslarini, ekstsentrisitetini;

3) $B(3,\sqrt{2})$ **nuqtaning fokus radius vektorlarini ;**

4) Ellipsning y=x+1 to'g'ri chiziq bilan kesishishdan hosil bo'lgan kesma uzunligini ;

5) Ellipsning direktirsasini tenglamasini.

Echish: 1) Markaza koordinatalar boshida bo'lagn koordinata o'qlariga simmetrik ellips tenglamsini yozamiz:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Agar bu ellips A va B nuqtalarda utsa, bu nuqtalar koordinatalarini tenglamaga *a* va *b* larni

$$\begin{cases} 3b^2 + 6a^2 = a^2b^2 \\ 9b^2 + 2a^2 = a^2b^2 \end{cases}$$

tenglamlar sistemasini echib a^2=12, b^2=8 larni topamiz. u=a^2, v=b^2 deb:

```
> evalf(solvefor( {3/u+6/v=1, 9/u+2/v=1} ));
```

{u=12, v=8}

Bu holda izlanayotgan ellips tenglamasi:

$$\frac{x^2}{12} + \frac{y^2}{8} = 1$$

bo‘ladi. Ularning koordinatalari: $A_1(-2\sqrt{3},0)$, $A_2(+2\sqrt{3},0)$, $B_1(0,-2\sqrt{2})$, $A_1(0,+2\sqrt{2})$

2)Ellips fokuslarini $ń = \pm\sqrt{a^2 - b^2}$ formulaga asosan: s = -2, c = +2

Bundan F_1(-2;0), F_2(+2;0) bo‘ladi. Ektsentrisentini topish uchun $\varepsilon = \frac{ń}{a}$ formuladan:

$$\varepsilon = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} < 1 \text{ bo‘ladi.}$$

3) Ellipsning direktrissalar tenglamalari $x = -\frac{a}{\varepsilon}$, $x = +\frac{a}{\varepsilon}$ ekanidan:

$a = 2\sqrt{3}$, $\varepsilon = \frac{1}{\sqrt{3}}$ larga asosan: x = - 6, x = + 6.

```
> with(geometry):
> _EnvHorizontalName := 'x': _EnvVerticalName := 'y':
ellipse(e10,x^2/12+y^2/8=1):
center(e10), coordinates(center(e10));
```

center_e10, [0, 0]

Fokuslarining koordinatalari:

```
> foci(e10), ma(coordinates,foci(e10));
```

[*foci_1_e10*, *foci_2_e10*], [[K 2, 0], [2, 0]]

Katta va kichik yarim o‘qlari:

```
> a:=MajorAxis(e10)/2; b:=MinorAxis(e10)/2;
```

$a := 2\sqrt{3}$ $b := 2\sqrt{2}$

Directrissa va ektsentrisenti

```
> Directrix(e10);dir:=a^2/sqrt(a^2-b^2);
```

Directrix(*e10*) $dir := 6$

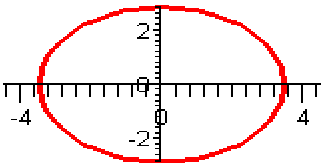
```
Eccentricity(e10); ecc:=sqrt(a^2-b^2)/a;
```

Eccentricity(*e10*) $ecc := \frac{1}{3}\sqrt{3}$

```
>detail(e10);
```

name of the object: e10
form of the object: ellipse2d
center: [0, 0]
foci: [[-2, 0], [2, 0]]
*length of the major axis: 4*3^(1/2)*
*length of the minor axis: 4*2^(1/2)*
*equation of the ellipse: 1/12*x^2+1/8*y^2-1 = 0*

```
> draw({e10},rinttext=true,view=[-5..5,-3..3]);
```



4) $B(3,\sqrt{2})$ nuqtadagi radius vektorlar $r_1 = a - \varepsilon x$, $r_2 = a + \varepsilon x$ formulalar asosida topamiz. x =3, $a = 2\sqrt{3}$, $\varepsilon = \dfrac{1}{\sqrt{3}}$ ekanidan:

$$r_1 = 2\sqrt{3} - \frac{1}{\sqrt{3}}3 = 2\sqrt{3} - \sqrt{3} = \sqrt{3} \qquad r_1 = 2\sqrt{3} + \frac{1}{\sqrt{3}}3 = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$$

> r1:=a-ecc*x2;r2:=a+ecc*x2;

$$r1 := 2\sqrt{3}\ \mathrm{K}\ \frac{1}{3}\sqrt{3}\ x2 \qquad r2 := 2\sqrt{3}\ \mathrm{C}\ \frac{1}{3}\sqrt{3}\ x2$$

> x2:=3; r1:=a-ecc*x2;r2:=a+ecc*x2; $x2 := 3 \quad r1 := \sqrt{3} \quad r2 := 3\sqrt{3}$

5) y=x+1 to‘g‘ri chiziq bilan topilgan $\dfrac{x^2}{12} + \dfrac{y^2}{8} = 1$ ellipsning kesishish nuqtalarini topish uchun

$$\begin{cases} \dfrac{x^2}{12} + \dfrac{y^2}{8} = 1 \\ y = x + 1 \end{cases}$$

tenglamlar sistemasini echamiz:

$$\frac{x^2}{12} + \frac{(x+1)^2}{8} = 1 \text{ yoki } 2x^2 + 3(x+1)^2 = 24$$

bundan

$$5x^2 + 6x - 21 = 0$$

$$x_1 = \frac{-3-\sqrt{114}}{5} = -\frac{3}{5} - \frac{\sqrt{114}}{5}, \qquad x_2 = \frac{-3+\sqrt{114}}{5} = -\frac{3}{5} + \frac{\sqrt{114}}{5}$$

$$y_1 = x_1 + 1 = \frac{2}{5} - \frac{\sqrt{114}}{5}, \qquad y_2 = x_2 + 1 = \frac{2}{5} + \frac{\sqrt{114}}{5}$$

demak, ellips bilan to‘g‘ri chiziqni kesishish nuqtalari:

$$M_1\left(-\frac{3}{5} - \frac{\sqrt{114}}{5}, \frac{2}{5} - \frac{\sqrt{114}}{5},\right), \quad M_2\left(-\frac{3}{5} + \frac{\sqrt{114}}{5}, \frac{2}{5} + \frac{\sqrt{114}}{5},\right).$$

> solvefor({x^2/12+y^2/8=1,y-x=1});

$$[\ 4y = \frac{2}{5}\ \mathrm{C}\ \frac{1}{5}\sqrt{114},\ x = \mathrm{K}\ \frac{3}{5}$$
$$\mathrm{C}\ \frac{1}{5}\sqrt{114}\ 5\ 4y = \frac{2}{5}\ \mathrm{K}\ \frac{1}{5}\sqrt{114},\ x = \mathrm{K}\ \frac{3}{5}\ \mathrm{K}\ \frac{1}{5}\sqrt{114}\ 5$$

> evalf(solvefor({x^2/12+y^2/8=1,y-x=1}));

[{y = K 1.735415650, x = K 2.735415650}
, {y = 2.535415650, x = 1.535415650}]

Bu M_1, M_2 nuqtalar oarsidagi ellips vatarining uzunligini topamiz:

$$d = (M_1 M_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

formuladan

$$d=\sqrt{(-\frac{3}{5}+\frac{1}{5}\sqrt{114}+\frac{3}{4}+\frac{1}{5}\sqrt{114})^2+(\frac{2}{5}+\frac{1}{5}\sqrt{114}-\frac{2}{5}+\frac{1}{5}\sqrt{114})^2}=$$

$$=\frac{1}{5}\sqrt{4\cdot(114)+4\cdot(114)}=\frac{2}{5}\sqrt{228}\approx 6.04$$

```
> point(M1,-3/5-sqrt(114)/5,2/5-sqrt(114)/5),
point(M2,-3/5+sqrt(114)/5,2/5+sqrt(114)/5);
d:=distance(M1, M2);evalf(d);
```

$M1,\ M2\quad d:=\frac{1}{25}\sqrt{912}\sqrt{25}\qquad 6.039867548$

6)Ellipsning $B(3,\sqrt{2})$ nuqtadagi urinma tenglamasi

$$\frac{x\,x_0}{a^2}+\frac{y\,y_0}{b^2}=1$$

formulaga asosan, $x_0 = 3$, $y_0=\sqrt{2}$ bo‘lganda quyidagini yozamiz.

$$\frac{3x}{12}+\frac{\sqrt{2}y}{8}=1$$

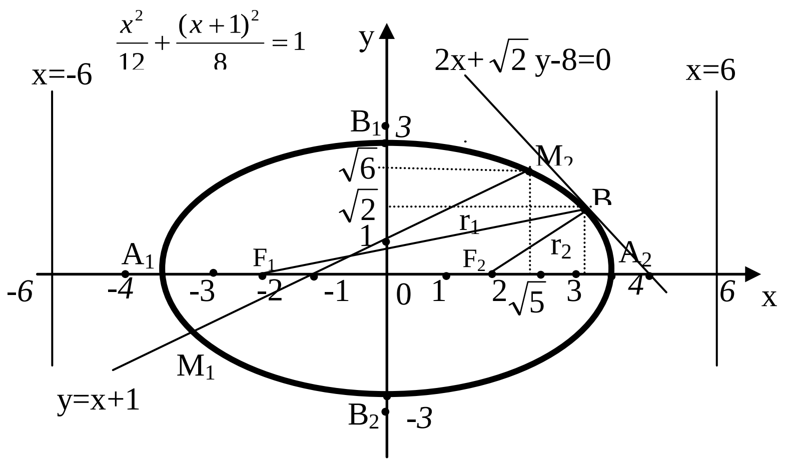
hosil bo‘lagn to‘g‘ri chiziq , ellipsga urinma bo‘lishlik shartini tekshiramiz:

$A^2a^2+B^2b^2=C^2$ ga asosan: A=2, V=$\sqrt{2}$, s=-8; a=2$\sqrt{3}$, b=2$\sqrt{2}$

$$2^2\cdot 12+(\sqrt{2})^2\cdot 8=(-8)^2$$

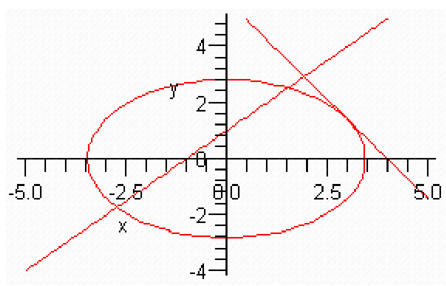
48+16=64, 64=64.

Masalada topilgan chiziqlarni quramiz.



1) *Ellips, urinma va kesishuvchi chiziqlarni qurish:*

```
> with(plots):
  imlicitplot([x^2/12+y^2/8=1,y-x=1,3*x/12+y*sqrt(2)/8=1], x=-5..5, y=-5..5);
```



2) *Masala shartlari bo'yicha to'liq grafikni qurish:*

> restart;
> with(geometry):
> point(F1,-2,0),point(F2,2,0), point(B,3,sqrt(2)); $F1, F2, B$

> point(A1,-sqrt(12),0), point(A2,sqrt(12),0); $A1, A2$

> point(B1,0,-sqrt(8)), point(B2,0,sqrt(8)); $B1, B2$

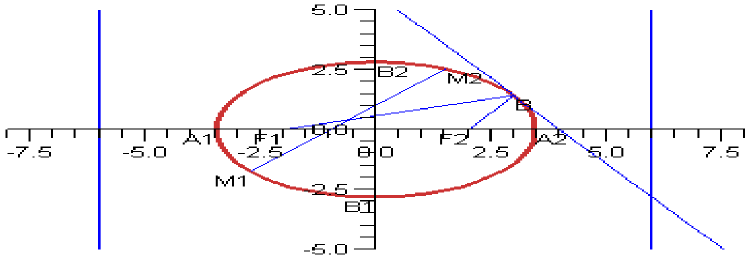
> point(M1,-3/5-sqrt(114)/5,2/5-sqrt(114)/5),
point(M2,-3/5+sqrt(114)/5,2/5+sqrt(114)/5); $M1, M2$

> line(l1,3*x/12+y*sqrt(2)/8=1,[x,y]), line(l2,y-x=1,[x,y]), line(l3,x=-6,[x,y]), line(l4,x=6,[x,y]); $l1, l2, l3, l4$

>ellipse(e10,x^2/12+y^2/8=1,[x,y]);
segment(fr1,[F1,B]),segment(fr2,[F2,B]);
segment(l5,[A1,A2]),segment(l6,[M1,M2]), segment(l7,[B1,B2]);

$e10 \quad fr1, fr2 \quad l5, l6 \quad l7$

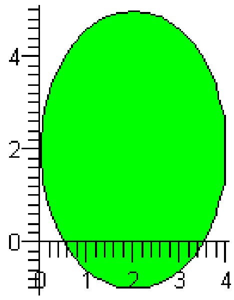
>draw({e10(color=orange ,thickness=2), fr1,fr2,l1,l3,l4,l5,l6,l7}, color=blue, axes=BOX, style=LINE,symbol=DIAMOND, rinttext=true,view=[-8..8,-5..5]);



Markazi (x0,y0) nuqtada bo'lgan ellis masalalari:

> restart;
> with(plottools):
with(plots):
> el:= (x-x0)^2/a^2 + (y-y0)^2/b^2 = 1;

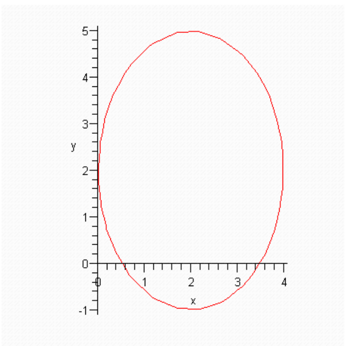
$$el := \frac{(x - x0)^2}{a^2} + \frac{(y - y0)^2}{b^2} = 1$$



> a := 2: b := 3: x0 := 2: y0:=2:
> elli:= ellipse([x0,y0], a, b, filled=true, color=blue):
> dislay(elli, scaling=constrained);
> eq := (x-x0)^2/a^2 + (y-y0)^2/b^2 = 1;

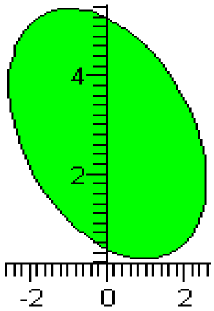
$$eq := \frac{1}{4}(x - 2)^2 + \frac{1}{9}(y - 2)^2 = 1$$

> imlicitplot(eq, x=-6..6,y=-6..6, scaling=constrained);



Ellisin i/4 burchakka burish.

> dislay(rotate(elli, i/4));

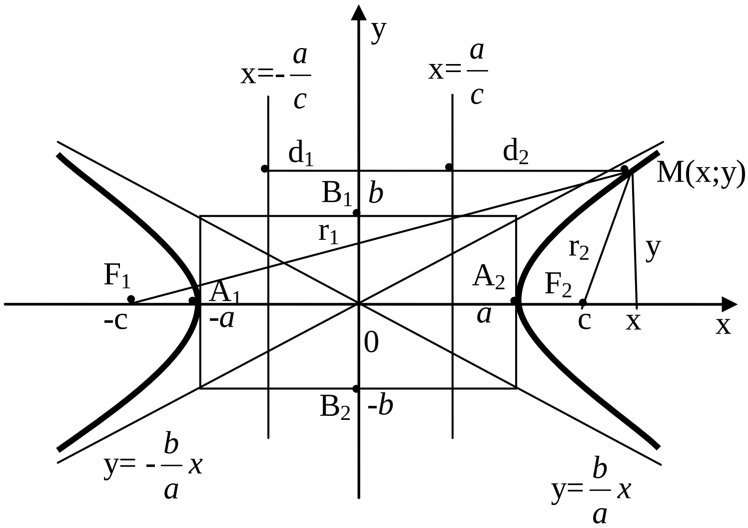


4.3. Giperbola

Ta'rif: ***Giperbola deb shunday nuqtalarning geometrik o'rniga aytiladiki, ularning har biridan berilgan ikki F va F_1 nuqtalargacha (fokuslargacha) bo'lgan masofalar ayirmasining ABColyut qiymati o'zgarmas 2a ($0<2a<FF_1$) mikdordan iborat.***

Giperbolaning kononik tenglamasi:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad (7)$$



(7) tenglama bilan berilgan giperbola koordinata o'qlariga nisbatan simmetrik. Giperbola Ox o'qini A(a;0), A_1(-a;0) nuqialarda kesadi bu nuqialar giperbolaning uchlar deb ataladi. Ou o'q bilan kesishmaydi. *a* – yarim xakiky o'q, b esa mavhum yarim o'q deyiladi. $c=\sqrt{a^2+b^2}$ pparametr markazidan fokusgacha bo'lgan masofani bildiradi. Giperbola shaklini aniqlovchi $\frac{c}{a}=\varepsilon>1$ uning ekstsentrisiteti deyiladi. Giperbolaning asimtotalari $y=\pm\frac{b}{a}x$ to'g'ri chiziqlar aniqlanadi. M(x,u) nuqtadan fokuslarigacha bo'lgan masofalar (faqal – radiuslar): r $=|\varepsilon x - a|$, $r_1=|\varepsilon x + a|$ (8)

formula bilan topiladi. Agar a = b bo'lsa giperbola teng tomonli giperbola deyiladi.

Uning tenglamasi $x^2 - u^2 = a^2$ va asimtotasi u=± x .

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \text{ va } \frac{y^2}{b^2}-\frac{x^2}{a^2}=1$$

giperbolalar qo'shma giperbolalar deyiladi.

$$\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=1$$

markazi (x_0 ; u_0) nuqtada bo'lagan giperbola tenglamasi. Uning simmetrik o'qlari (x_0;u_0) nuqtadan o'tuvchi to'g'ri chiziqlari koordinatalariga parallel bo'ladi. *a*,*b* o'qlari (x_0;u_0) nuqtaga nisbatan simmetrik o'qlar bo'yicha joylashtiriladi.

Quyidagi masalani echishdagi har bir bajarilgan amalni **Maple7** dasturida bajarilishini ko'rsatib boramiz.

4.3-masala. Berilgan A(6;3), B($5\sqrt{2}$,-4) nuqtalar berilgan.

Topish kerak(Gierbola-1.mw):

1) **Fokuslari Ox o'qida yotuvchi A va V nuqtadan o'tuvchi giperbola tenglamasini;**
2) **Giperbola ekstsentrisitentini, fokuslarini, direktrissasini;**
3) **2y – x = 0 to'g'ri chiziq bilan kesishish nuqtalari orasidagi masfalarni;**
4) **A(b, 3) nuqtasidagi urinma tenglamasini**.

Echish: 1)Berilgan A(6;3), B($5\sqrt{2}$,-4) nuqtalardan o'tuvchi giperbola tenglamasi: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ deb olsak, tenglamadagi *a*, b larni topish uchun A va V nuqtalar koordinatalarini tenglamalar sistemasiga qo'yamiz:

$$\begin{cases}\frac{6^2}{a^2}-\frac{3^2}{b^2}=1\\ \frac{(5\sqrt{2})^2}{a^2}-\frac{(-4)^2}{b^2}=1\end{cases}\quad \text{yoki}\quad \begin{cases}36b^2-9a^2=a^2b^2\\ 50b^2-16a^2=a^2b^2\end{cases}$$

bu sistemani echib, $a^2 = 18$, $b^2 = 9$ ni topamiz. Giperbola uchlarinig kordinatalari:

$$a=-3\sqrt{2},\ a=+3\sqrt{2}.$$

Bu erda $2a=6\sqrt{2}$ - xakikiy o'qi, $2b=6$ esa giperbolaning mavxum o'qi, chunki giperbola Ou o'qi bilan kesishmaydi. $u=a^2, v=b^2$ deb:

> **solvefor({36/u+9/v=1, 50/u+16/v=1});** {u=18, v=-9}

> **valf(solvefor({36/u+9/v=1, 50/u+16/v=1}));** {u=18, v=-9}

Giperbolaning uchlari A_1 va A_2 larning koordinatalari.

> **vertices(g3), ma(coordinates,vertices(g3));**

$$[vertex\_1\_g3,\ vertex\_2\_g3],\ [[-3\sqrt{2},0],\ [3\sqrt{2},0]]$$

2)Giperbolaning fokuslarining koordinatasini $c=\pm\sqrt{a^2+b^2}$ formulaga asosan: $a^2 = 18$, $b^2 = 9$ bo'lganda

$$c=-\sqrt{18+9}=-\sqrt{27},\ c=+\sqrt{27}=3\sqrt{3}$$

bundan $F_1(-3\sqrt{3};0)$ va $F_2(3\sqrt{3};0)$ bo'ladi.

> **foci(g3), ma(coordinates,foci(g3));**

$$[foci\_1\_g3,\ foci\_2\_g3],\ [[-3\sqrt{3},0],\ [3\sqrt{3},0]]$$

Giperbola ekstsentrisitetini $\varepsilon=\frac{c}{a}>1$ fromulaga asosan: $\varepsilon=\frac{3\sqrt{3}}{\sqrt{18}}=\frac{3\sqrt{3}}{3\sqrt{2}}=\sqrt{\frac{3}{2}}>1$.

Giperbola direktrisasini tenglamasini $x=\pm\frac{a}{\varepsilon}$ formulaga asosan : $x=\pm\frac{3\sqrt{2}}{\frac{\sqrt{3}}{\sqrt{2}}}=\pm2\sqrt{3}$.

> **a:=3*sqrt(2); b:=3;** $a:=3\sqrt{2}\quad b:=3$

> **Directrix(g3);dir:=a^2/sqrt(a^2+b^2); Eccentricity(e10); ecc:=sqrt(a^2+b^2)/a;**

$$Directrix(g3)\quad dir:=2\sqrt{3}\quad Eccentricity(e10)\quad ecc:=\frac{1}{2}\sqrt{3}\sqrt{2}$$

> **x2:=6; r1:=ABC(ecc*x2-a);r2:=ABC(ecc*x2+a);**

$$x2 := 6 \quad r1 := 3\sqrt{3}\sqrt{2} - 3\sqrt{2} \quad r2 := 3\sqrt{3}\sqrt{2} + 3\sqrt{2}$$

Asimtotalari $y=\pm\frac{b}{a}x$ tenglamaga asosan: $y=\pm\frac{3}{3\sqrt{2}}x=\pm\frac{\sqrt{2}}{2}x$.

> asymtotes(g3), ma(Equation,asymtotes(g3));

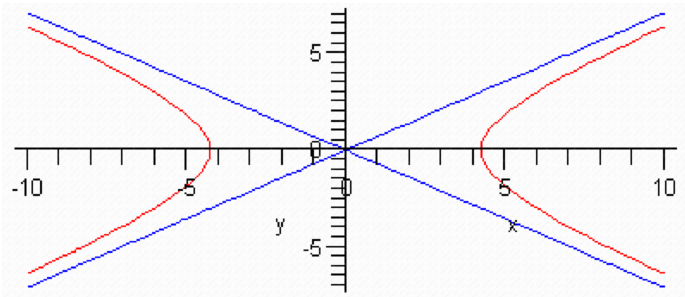
$$[asymtote\_1\_g3\ asymtote\_2\_g3],\ \left[y+\frac{1}{2}\sqrt{2}x=0,\ y-\frac{1}{2}\sqrt{2}x=0\right]$$

> with(geometry):
> _EnvHorizontalName := 'x': _EnvVerticalName := 'y':
hyerbola(g3,x^2/18+y^2/(-9)=1):
center(g3), coordinates(center(g3)); *center_g3*, [0, 0]

> detail(g3);

name of the object: g3
form of the object: hyerbola2d
center: [0, 0]
*foci: [[-3*3^(1/2), 0], [3*3^(1/2), 0]]*
*vertices: [[-3*2^(1/2), 0], [3*2^(1/2), 0]]*
*the asymtotes: [y+1/2*2^(1/2)*x = 0, y-1/2*2^(1/2)*x = 0]*
*equation of the hyerbola: 1/18*x^2-1/9*y^2-1 = 0*

> with(plots):
imlicitplot([x^2/18+y^2/(-9)=1,y+1/2*sqrt(2)*x=0, y-1/2*sqrt(2)*x=0], x=-10..10, y=-10..10,color=[red,blue,blue]);



3)Topilgan $\frac{x^2}{18}-\frac{y^2}{9}=1$ giperbola bilan 2y-x =0 to‘g‘ri chiziq bilan kesishish nuqtalarni topish uchun tenglamalarni birgalikda echamiz:

$$\begin{cases} x^2-2y^2=18 \\ 2y-x=0 \end{cases}$$

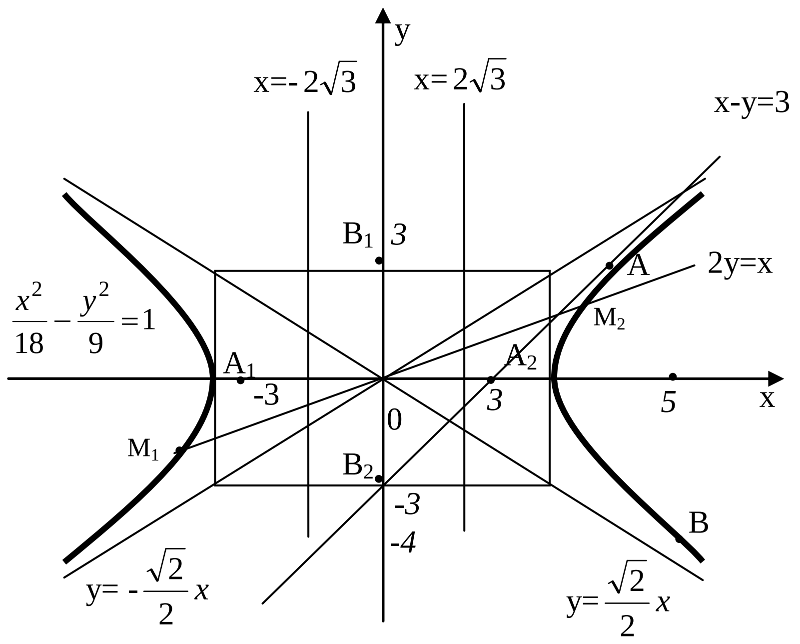
Bundan. x = 2y; $(2y)^2 - 2y^2 = 18$, $4y^2 - 2y^2 = 18$, $2y^2 = 18$, y=$\pm$3 bundan x=$\pm$6 kesishish nuqtalarining koordinatalri $M_1(-6,-3)$, $M_2(6,3)$.

> solvefor({x^2/18+y^2/(-9)=1,2*y-x=0});

$$[\{y=3,\ x=6\},\ \{y=-3,\ x=-6\}]$$

Ular orasidagi masofa: $d=|M_1M_2|=\sqrt{(6+6)^2+(3+3)^2}=6\sqrt{5}$.

4)A(6,3) nuqtadagi urinmani $\frac{xy_0}{a^2}-\frac{yy_0}{b^2}=1$ tenglamaga asosan: $x_0=6$, $y_0=3$, $a^2=18$, $b^2=9$ bo‘lganda, $\frac{6x}{18}-\frac{3y}{9}=1$ yoki $\frac{x}{3}-\frac{y}{3}=1$ bundan: x – y = 3. To‘g‘ri chiziq x – y = 3 urinma bo‘lish sharti $A^2a^2-B^2b^2=C^2$ ni tekshiramiz: $(1)^2\cdot 18-(-1)^2\cdot 9=9$, 18 – 9 = 9, 9 = 9.
Masalada topilgan chiziqlarni quramiz.



Masala shartlari bo‘yicha to‘liq qurish:

```
>restart;
> with(geometry):
> point(F1,-sqrt(27),0),point(F2,sqrt(27),0),point(A,6,3);
```
F1, F2, A

```
> point(A1,-sqrt(18),0), point(A2,sqrt(18),0);            point(B1,0,-3),
point(B2,0,3);
```
A1, A2   B1, B2

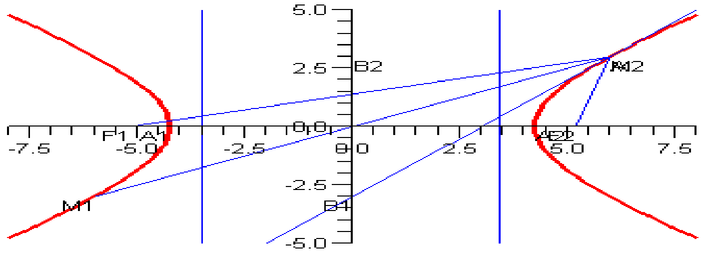
```
> point(M1,-6,-3),point(M2,6,3);
```
M1, M2

```
:=distance(M1, M2);evalf(d);
```
$d:=\sqrt{180}$ 13.41640786

```
> line(l1,x-y=3,[x,y]), line(l2,2*y-x=0,[x,y]),
  line(l3,x=-sqrt(12),[x,y]), line(l4,x=sqrt(12),[x,y]);
```
l1, l2, l3, l4

```
> hyerbola(g3,x^2/18-y^2/9=1,[x,y]);
segment(fr1,[F1,A]),segment(fr2,[F2,A]); segment(l5,[A1,A2]), segment(l6,[M1,M2]),
segment(l7,[B1,B2]);
```
g3   fr1 , fr2   l5, l6, l7

> draw({g3(color=orange, thickness=2),fr1,fr2,l1,l3,l4,l5,l6,l7}, color=blue, axes=BOX,style=LINE,symbol=DIAMOND, rinttext=true,view= [-8..8,-5..5]);

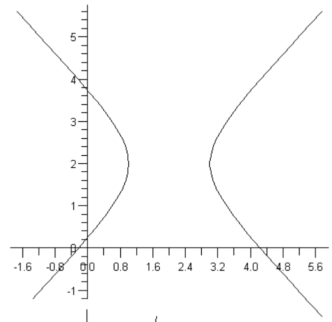


<u>*Markazi C(x_0,y_0) da, o'qlari a,b bo'lgan gierbolani qurish.*</u>

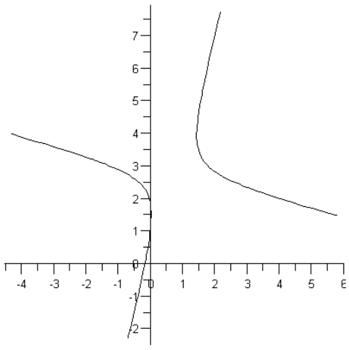
> restart;
> with(plottools):
with(plots):
> eq := (x-x0)^2/a^2 - (y-y0)^2/b^2 = 1;

$$eq := \frac{(x - x0)^2}{a^2} - \frac{(y - y0)^2}{b^2} = 1$$

> a := 1: b := 1: x0 := 2: y0 := 2:
Generate the hyerbola described by the equation above,
> h := hyperbola([x0,y0], a, b, -2..2):
dislay(h);



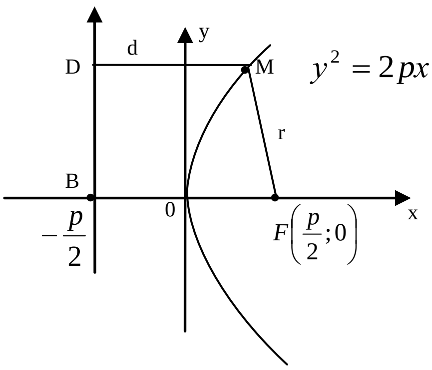
<u>*Giperbolanini i/6 burchakka burish*</u>.
> dislay(rotate(h, i/6));

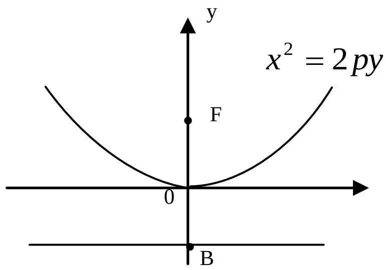


4.4. Parabola.

Ta'rif. ***Berilgan nuqta (fokus)dan va berilgan chiziq (direktrisasi)dan bir xil uzoqlikda bo'lgan nuqtalarning geometrik o'rni parabola deyiladi***.
Parabolaning kanonik tenglamasi quyidagi ko'rinishga ega:

1. $y^2=2x$ – Ox o'qiga nisbatan simmetrik
2. $x^2=2y$ – Oy o'qiga nisbatan simmetrik..





. . Har ikkala holda ham

parabolaning uchi, yahni simmetrik o‘qida yotuvchi nuqtasi, koordinatalar boshida bo‘ladi.

$y^2 = 2px$ parabola $F(\frac{p}{2};0)$ fokus va $x = -\frac{p}{2}$ direktrisaga ega; uning M(x;u) nuqtasining fokal radiusi $r = x + \frac{p}{2}$

1) $(y - y_0)^2 = 2p(x - x_0)$, uchi ($x_0$;$u_0$) nuqtada bo‘lgan parabola simmetrik o‘qi $y = y_0$ Ox o‘qiga parallel.

2) $(x - x_0)^2 = 2p(y - y_0)$, uchi ($x_0$;$u_0$) nuqtada bo‘lgan parabola simmetrik o‘qi $x = x_0$ Oy o‘qiga parallel.

Quyidagi masalani echishdagi har bir bajarilgan amalni **Maple7** dasturida bajarilishini ko‘rsatib boramiz.

4.4-masala. A(1;2) va M(4;8) nuqtalar berilgan. Topish kerak(Parabola-1.mw)::

1) **Uchi A(1;2) nuqtada simmetrik o‘qi Ox ga parallel va M(4;8) nuqtadan o‘tuvchi parabola tenglamasini;**
2) **topilgan parabola fokusini va direktrisasini, M(4;8) nuqtadagi radius vektorini;**
3) **koordinata o‘qlari bilan kesishish nuqtasini;**
4) **parabolaning M(4;8) nuqtasiga o‘tkazilgan urinma tenglamasini;**
5) **y = 2x to‘g‘ri chiziq bilan kesishish nuqtalarini**.

Echish: 1) Uchi (a,b) nuqtada, simmetriya o‘qi Ox o‘qqa parallel bo‘lgan tarmoklari unga yo‘nalgan parabola tenglamasi:

$$(y-b)^2 = 2(x-a).$$

Chunki, berilgan masaladagi parabolada yotuvchi M(4;8) nuqta parabola uchi A(1;2) dan o‘ng tomonda joylashgan.
Masala shartiga binoan parabola A(1;2) nuqtadan o‘tganligi uchun *a*=1, b=2 bo‘ladi va. parabola tenglamasi:

$$(y-2)^2 = 2(x-1).$$

Bu tenglamadagi pparametrni parabolaning M(4;8) nuqtadan o‘tishidan foydalanib topamiz.

$$M: x = 4,\ y = 8;\quad (8 - 2)^2 = 2\ (4 - 1),\quad = 6.$$

Bu holda masala shartiga asosan izlanayotgan parabola tenglamasi

$$(y - 2)^2 = 12\ (x - 1) \quad \text{yoki} \quad y^2 - 4y - 12x + 16 = 0$$

```
> restart;
> with(geometry):
> ar1:=(y-b)^2=2**(x-a);
```
$par1 := (y - b)^2 = 2\,p\,(x - a)$

```
> a:=1;b:=2;
```
$a := 1 \quad b := 2$

```
> ar1:=(y-b)^2=2**(x-a);
```
$par1 := (y - 2)^2 = 2\,p\,(x - 1)$

```
> x:=4; y:=8;
```
$x := 4 \quad y := 8$

```
> evalf(solvefor({ar1},x=4,y=8 ));
```
$p = 6.$

2)Topilgan $(y - 2)^2 = 12\ (x - 1)$ ko‘rinishdagi parabola fokusini quyidagtcha topamiz.

$$F(a+\frac{p}{2};\ b) = F(1+\frac{b}{2};\ 2) = F(4;2).$$

Direktrisa tenglamasi: $x = a - \frac{p}{2} = 1 - \frac{6}{2} = -2$, x = -2.

Radius –vektorini A(4;8) nuqta uchun topamiz: $r = x + \frac{p}{2} - a = 4 + \frac{6}{2} - 1 = 6.$

> p:=6; x:=4; a:=1; r:=x+p/2-a; $p := 6 \quad x := 4 \quad a := 1 \quad r := 6$

3) Parabolaning koordinata o‘qlari bilan kesishish nuqtasini topish uchun:
a)x=0, da $(y - 2)^2 = 12\,(0 - 1) = -12$ hosil bo‘lgan tenglik o‘rinli emas, shuning uchun, parabola Ou o‘qi bilan kesishmaydi.

> evalf(solve({x=0,(y-2)^2=12*(x-1)}));

$$\{x = 0.,\ y = 2.000000000 \ \ C\ \ 3.464101615\ \ I\}$$

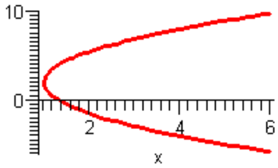
b)u=0 da $(0 - 2)^2 = 12\,(x - 1)$ dan $x - 1 = \frac{4}{12}$, $x = \frac{4}{3}$.

Demak, parabola Ox o‘qi bilan $(\frac{4}{3}, 0)$ nuqtada kesishadi.

> evalf(solve({y=0,(y-2)^2=12*(x-1)})); $\{x = 1.333333333\ ,\ y = 0.\}$

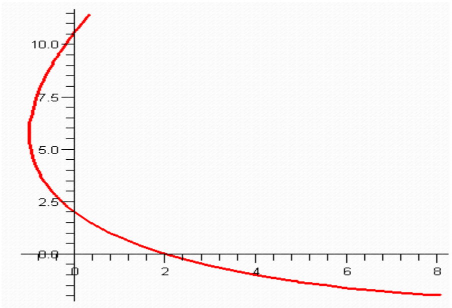
> with(plots):

imlicitplot((y-2)^2=12*(x-1), x=-6..6, y=-10..10,thickness=2);



Topilgan parabolani i/6 ga burish.

> with(plottools):with(plots):
> display(rotate(imlicitplot((y-2)^2=12*(x-1), x=-6..6, y=-10..10,thickness=2), i/6));



4) M(4;8) nuqtaga o‘tkazilgan urinma tenglamasini $(y - y_0)(y_0 - b) = (x - x_0)$ tenglamag asosan: $x_0 = 4$, $y_0 = 8$, b = 2, = 6 bo‘lganda

$$(y - 8)(8 - 2) = 6\,(x - 4), \quad y - 8 = x - 4,$$
$$y = x + 4.$$

5)u = 2x to‘g‘ri chiziq bilan kesishish nuqtasini

$$\begin{cases}(y-2)^2 = 12(x-1)\\ y = 2x\end{cases}$$

sistemani echib topamiz.

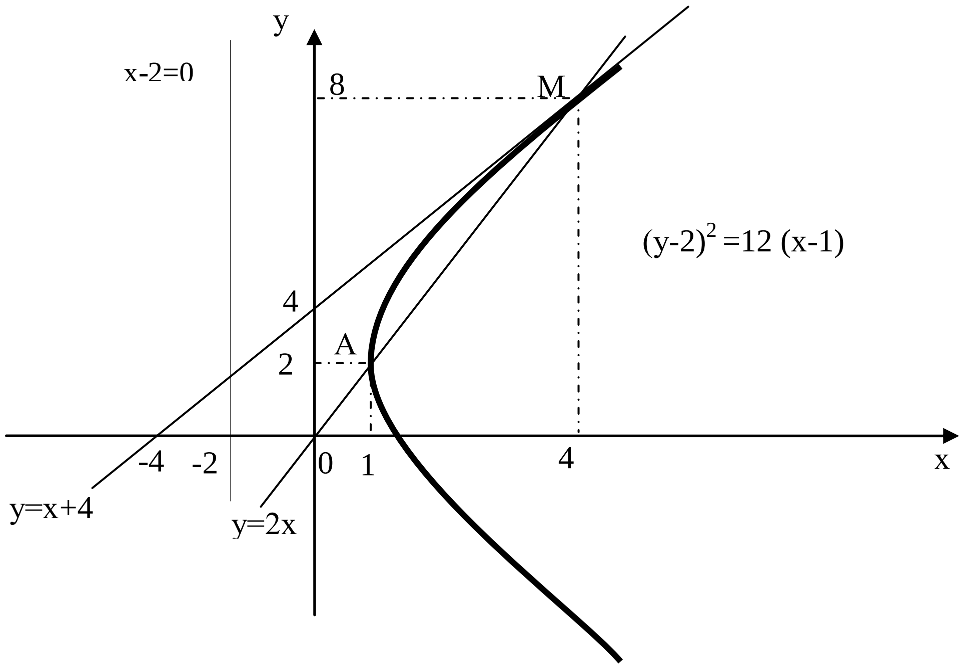
$$y = 2x,\ (2x - 2)^2 = 12(x - 1),\quad 4(x - 1)^2 = 12(x - 1),\ (x - 1)^2 = 3(x - 1),$$
$$(x - 1)(x - 4) = 0,\ x_1 = 1,\ x_2 = 4;$$
$$y_1 = 2x_1 = 2,\quad y_2 = 2*4 = 8.$$

Kesishish nuqtalari: (1;2), (4;8).

```
> evalf(solve({-2*x+y=0,(y-2)^2=12*(x-1)}));
```

$$\{y = 2.,\ x = 1.\},\ \{y = 8.,\ x = 4.\}$$

Masalada topilgan chiziqlarni quramiz:



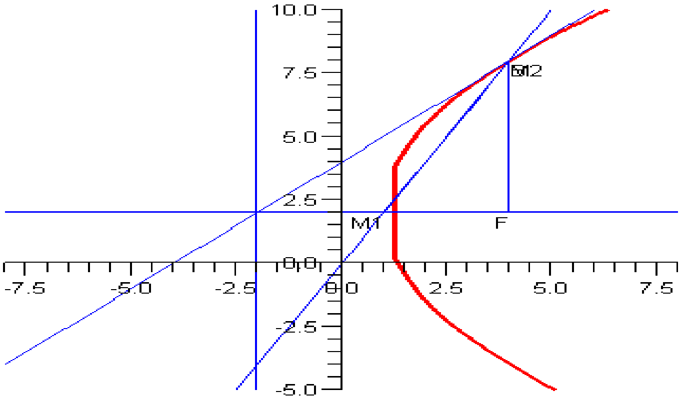

```
> restart;
> with(geometry):
> point(A,1,2),point(B,4,8);
```
A, B

```
> point(M1,1,2),point(M2,4,8); point(F,4,2);
 d:=distance(M1, M2); evalf(d);
```
M1, M2 $d := \sqrt{45}$ 6.708203932

```
> line(l2,y=2*x,[x,y]),line(l3,x=-2,[x,y]),
line(l4,y=2,[x,y]),line(l1,y=x+4,[x,y]);
```
l2, l3, l4, l1

```
> parabola(p5,y^2-12*x-4*y+16=0,[x,y]),
segment(r,[F,B]),segment(l6,[M1,M2]);
```
p5, r, l6

```
> draw({5(color=red ,thickness=2),r,l1,l2,l3,l4,l6}, color=blue, axes=BOX,
style=LINE,symbol=DIAMOND, rinttext=true,view=[-8..8,-5..10]);
```



4.5. Ikkinchi tartibli egri chiziq turini umumiy tenglama bo‘yicha aniqlash.

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \qquad (1)$$

Bu (1) tenglama ikkinchi tartibli egri chiziqning umumiy tenlamasi deb ataladi.

I. Bu tenglama koeffitsientlariga qarab egri chiziq turini aniqlaymiz. Biz (1) tenglamada V=0 bo‘lgan holdagi quyidagi tenglamani ko‘ramiz.

$$Ax^2 + Cy^2 + Dx + Ey + F = 0 \qquad (2)$$

Agar (2) tenglamada A=6 yoki A=C=1 yoki A=C$\neq$ 0 bo‘lsa,

$$Ax^2 + Ay^2 + Dx + Ey + F = 0 \qquad (3)$$

Bu (3) tenglama aylananing umumiy tenglamasi deb ataladi. (3) tenglamani kanonik ko‘rinishga keltirish uchun *x* va *y* li kushiluvchilar asosiga tula kvadratlarga keltiramiz:

$$(x+\frac{D}{2A})^2 + (y+\frac{E}{2A})^2 = \frac{D^2+E^2}{4A^2} = F \qquad (4)$$

Bu erda $a= -\frac{D}{2A}$; b=$-\frac{E}{2A}$, $\frac{D^2+E^2}{4A^2} - F = R^2$ deb olsak, aylananing

$$(x - a)^2 + (y - b)^2 = R^2$$

kanonik tenglamasiga kelamiz.

II. (2) tenglamada $A \neq C \neq 0$ bo‘lsa, x va u lar ishtirok etgan kushiluvchilarni tula kvadratlarini ajratamiz:

$$A(x+\frac{D}{2A})^2 + C(y+\frac{E}{2C})^2 = \frac{D^2}{4A} + \frac{E^2}{4C} - F \qquad (5)$$

bu erda: $x_0 = -\frac{D}{2A}$, $y_0 = -\frac{E}{2C}$ va $\Delta = \frac{D^2}{4A} + \frac{E^2}{4C} - F$ (6)

deb olsak

$$A(x-x_0)^2 + C(y-y_0)^2 = \Delta \qquad (7)$$

tenglamaga kelamiz. Bu markaziy egri chiziq deb ataladi va $O'(x_0; y_0)$ markazi bo‘lib, egri chiziq markaziga nisbatan simmetrik bo‘ladi. x = x_0 va y = y_0 to‘g‘ri chiziqlar egri chiziqning simmetriya o‘qlari bo‘ladi.

(7) tenglama koeffitsientlari buyicha egri chiziq turini quyidagi jadvaldan aniqlaymiz.

	Δ>0	Δ=0	Δ<0
AC>0	Xakikiky ellips	Nuqta	Mavxum ellips
AC<0	Xakikiy giperbola	Kesuvchi to‘g‘ri chiziqlar	Kushma giperbola

1) Xaqiqiy ellips:

$$\frac{(x-x_0)^2}{\frac{\Delta}{A}}+\frac{(y-y_0)^2}{\frac{\Delta}{C}}=1$$

2) Nuqta: $A(x-x_0)^2+C(y-y_0)^2=0$ yoki x=x_0 , u=u_0, (x_0; u_0)

3) Mavxum ellips: $\frac{A(x-x_0)^2}{\Delta}+\frac{C(y-y_0)^2}{\Delta}\neq -1$

4) Xaqiqiy giperbola: $\frac{(x-x_0)^2}{\frac{\Delta}{A}}+\frac{(y-y_0)^2}{\frac{\Delta}{(-C)}}=1$

5) Kesishuvchi to‘g‘ri chiziqlar jufti: $(\sqrt{A}x-\sqrt{-C}y)(\sqrt{A}x+\sqrt{-C}y)=0$

6) Qo‘shma giperbola: $\frac{(x-x_0)^2}{\Delta/(-A)}-\frac{(y-y_0)^2}{\Delta/(-C)}=-1$

III. Agar (2) tenglamada AC=0 va $A^2+C^2\neq 0$ bo‘lsin. Aniqlik uchun A≠0, S≠0, D≠0 bo‘lsa

$$C(y+\frac{E}{2C})^2=-Dx-F+\frac{E^2}{4C} \tag{8}$$

parabola tenglamasiga ega bo‘lamiz. Bunda

$$x_0=-\frac{F}{D}+\frac{E^2}{4DC},\qquad y_0=-\frac{E}{2C},\qquad 2p=-\frac{D}{C} \tag{9}$$

deb olsak,

$$(y-y_0)^2=2p(x-x_0) \tag{10}$$

bu uchi $(x_0;y_0)$ nuqtada simmetrik o‘qi $y=y_0$ bo‘lgan parabola tenglamasi. Koordinata o‘qlari bilan kesishish nuqtalari:

$$y=0,\ x=x_0+y^2/(2p);\qquad x=0,\ ó=ó_0\pm\sqrt{2đ(-ő_0)}$$

Quyidagi masalani echishdagi har bir bajarilgan amalni **Maple7** dasturida bajarilishini ko‘rsatib boramiz.

4.5-masala. $9x^2+16y^2-90x+32y+97=0$ **ikkinchi tartibli egri chiziq turini, fokuslarini aniqlang. Shaklini chizing(Ikkinchi tart egr chiz.mw):**.

Echish . Masala shartiga asosan, A=9, C=16, D= -90, E=32, F=97. Bunda A≠S≠0

$$x_0=-\frac{D}{2A}=-\frac{-90}{2\cdot 9}=5\qquad y_0=-\frac{E}{2C}=-\frac{32}{2\cdot 16}=-1$$

$$\Delta=\frac{D^2}{4A}+-\frac{E^2}{4C}-F=\frac{(-90)^2}{4\cdot 9}+\frac{32^2}{4\cdot 16}-97=144$$

Demak, A*C=144>0, Δ=144>0 bo‘lganligi uchun berilgan chiziq tenglamasi

$$\frac{(x-5)^2}{16}+\frac{(y+1)^2}{9}=1$$

xakikiy ellips tenglamasini beradi.

Agar $\begin{cases} x' = x-5 \\ y' = y+1 \end{cases}$ *еки* $\begin{cases} x' = x+5 \\ y' = y-1 \end{cases}$ formulalar koordinata o‘qlarini $0'(5;-1)$ nuqtaga parallel kuchirishni ifodalaydi. Bu holda yangi koordinatalar sistemasida ellips tenglamasi quyidagicha bo‘ladi.

$$\frac{x'^2}{16} + \frac{y'^2}{9} = 1$$

Bu ellipsning yarim o‘qlari $a = 4,\ b = 3$ *маркази* $0'(5;-1)$, *фокал уки* $0x'$ *марказдан фокуслар гача масофа* $C = \sqrt{16-9} = \sqrt{7}$

Fokuslari :

1. Yangi sistema buyicha: $F_1: \ x' = -\sqrt{7}, \ y = 0; \ F_2: \ x' = \sqrt{7}, \ y' = 0;$
2. Eski sistema buyicha : $F_1: \begin{cases} x = x' + 5 = -\sqrt{7} + 5 \\ y = y' - 1 = -1 \end{cases}$ $F_2: \begin{cases} x = x' + 5 = \sqrt{7} + 5 \\ y = y' - 1 = -1 \end{cases}$

Endi ellipsning koordinata o‘qlari bilan kesishish nuqtasini topishuchun berilgan tenglamadan x=0 va y=0 bo‘lganda quyidagi tenglamalarni topamiz:

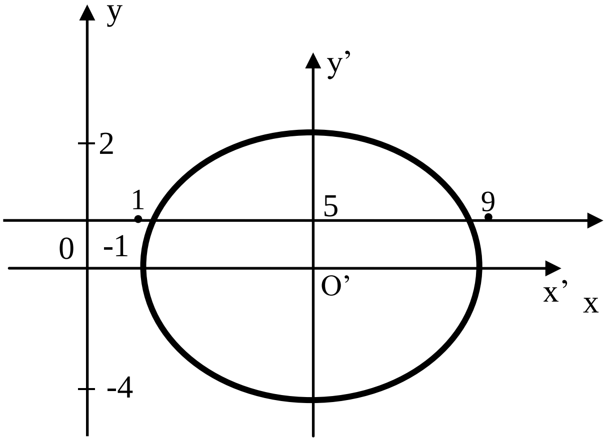
$9x^2 - 90x + 97 = 0, \quad .16y^2 + 32y + 97 = 0$

ikkinchi tenglama kompleks ildizlarga ega bo‘lgani uchun ellips 0y o‘qi bilan kesishmaydi.

Birinchi tenglamani echib,

$x_1 = 5 - \frac{8}{3}\sqrt{2} \approx 1.3, \quad x_2 = 5 + \frac{8}{3}\sqrt{2} \approx 8.7$

ildizlarni topamiz. Bu $(x_1;0)$ va $(x_2;0)$ nuqtalar ellipsning 0x o‘qi bilan kesishish nuqtalaridir.




```
> with(geometry):
> EnvHorizontalName := 'x':
_EnvVerticalName := 'y':
conic(e1,9.*x^2+16*y^2-90*x+32*y+97=0,[x,y]):
> form(e1);
```
ellipse2d

```
> detail(e1);
```
name of the object: e1

form of the object: ellipse2d

center: [5.000000000, -1.]

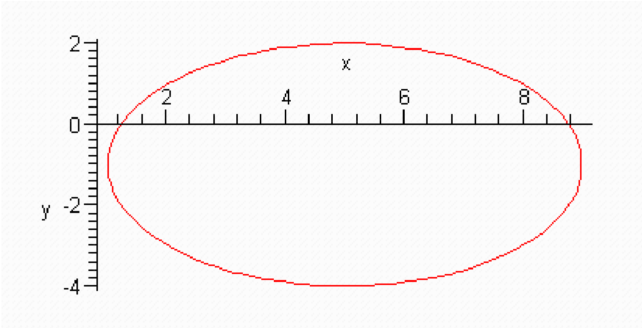
foci: [[2.354248689, -1.], [7.645751311, -1.]]

length of the major axis: 8.000000000

length of the minor axis: 6.000000000

*equation of the ellipse: 97.+9.*x^2+16*y^2-90*x+32*y = 0*

```
> with(plots):
imlicitplot(9*x^2+16*y^2-90*x+32*y+97=0, x=0..10, y=-6..6);
```



Quyidagi masalalarda berilgan **ikkinchi tartibli egri chiziq turini** aniqlashni **Maple7** dasturida bajarilishini ko‘rsatamiz.

Aylana:

```
> with(geometry):
> conic(c4,x^2-6*x+13+y^2-4*y-9,[x,y]):
> form(c4);                     circle2d
> detail(c4);
```

name of the object: c4

form of the object: circle2d

name of the center: center_c4

coordinates of the center: [3, 2]

radius of the circle: 9^(1/2)

*equation of the circle: x^2-6*x+4+y^2-4*y = 0*

Gierbola:

```
> with(geometry):
> conic(c9,3.*x^2-4*x*y-2*x+4*y-5=0,[x,y]);   c9
> detail(c9);
```

name of the object: c9

form of the object: hyerbola2d

center: [.9999999998, 1.000000000]

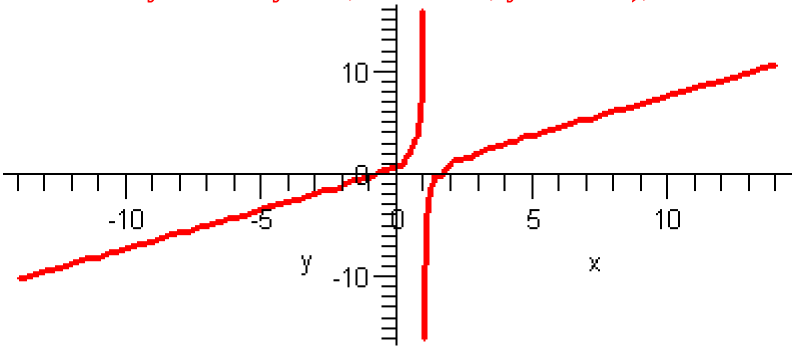
foci: [[2.999999999, 0.], [-1.000000000, 2.000000000]]

vertices: [[1.894427191, .5527864043], [.1055728088, 1.447213596]]

*the asymtotes: [-.6708203932*x+.8944271910*y-.2236067975 = 0, -1.118033989*x+1.118033988 = 0]* *equation of the hyerbola: -5.+3.*x^2-4.*x*y-2.*x+4.*y = 0*

> with(plots):

imlicitplot(3*x^2-4*x*y-2*x+4*y-5=0, x=-14..14, y=-16..16);



Nuqta:

> with(geometry):

> conic(c5,x^2+y^2-4*x-10*y+29 = 0,[x,y]):

> form(c5); *point2d*

> detail(c5);

name of the object: c5

form of the object: point2d

coordinates of the point: [2, 5]

Ikkilangan to'g'ri chiziq:

> with(geometry):

> conic(c6,x^2-2*x*y+2*x+y^2-2*y+1,[x,y]):
conic/classify: "degenerate case: a double line"

> form(c6); *line2d*

> detail(c6);

name of the object: c6

form of the object: line2d

*equation of the line: -1/2*x*2^(1/2)+1/2*2^(1/2)*y = 0*

parallel chiziqlar:

> with(geometry):

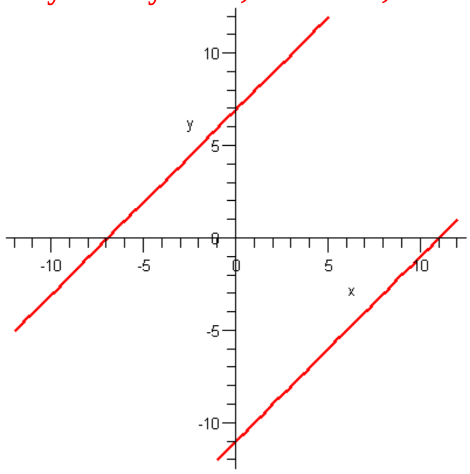
> conic(c7,x^2-2*x*y-4*x+y^2+4*y-77,[x,y]);

> form(c7); *FAIL*
> detail(c7);

[name of the object: Line_1_c7
form of the object: line2d
equation of the line:
*-1/2*x*2^(1/2)C 1/2*2^(1/2)*yC 11/2*2^(1/2) = 0, name of the object: Line_2_c7*
form of the object: line2d
equation of the line:
*-1/2*x*2^(1/2)C 1/2*2^(1/2)*y-7/2*2^(1/2) = 0]*

> with(plots):

imlicitplot(x^2-2*x*y-4*x+y^2+4*y-77=0, x=-12..12, y=-12..12);



Kesishuvchi to'g'ri chiziqlar:

> conic(c8,11*x^2+24*x*y+4*y^2+26*x+32*y+15=0,[x,y]);

conic/classify: "degenerate case: two intersecting lines"

[Line_1_c8, Line_2_c8]

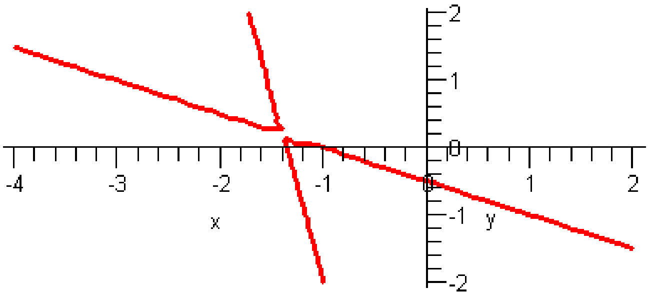
> form(c8); *FAIL*

> detail(c8);

[
name of the object: Line_1_c8
form of the object: line2d
*equation of the line: xC 2*yC 1 = 0, name of the object: Line_2_c8*
form of the object: line2d
*equation of the line: -11/5*x-2/5*y-3 = 0]*

> with(plots):

imlicitplot(11*x^2+24*x*y+4*y^2+26*x+32*y+15=0, x=-4..2, y=-2..2);



parabola:

> with(geometry):

> conic(c17,x^2-2*x*y+4*x+y^2+4*y-7,[x,y]); *c17*

> form(c17); *parabola2d*

> detail(c17);

name of the object: c17

form of the object: arabola2d

vertex: [7/8, 7/8]

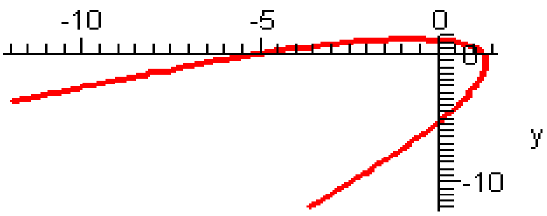
focus: [3/8, 3/8]

*directrix: 1/2*2^(1/2)*x+1/2*y*2^(1/2)-11/8*2^(1/2) = 0*

*equation of the arabola: x^2-2*x*y+4*x+y^2+4*y-7 = 0*

> with(plots):

imlicitplot(x^2-2*x*y+4*x+y^2+4*y-7, x=-12..12, y=-12..12);
Warning, the name changecoords has been redefined



4.6. Ikkinchi tartibli chiziqni, uning qutb koordinat sistemasidagi tenglamacidan aniqlash

Ikkinchi tartibli chiziklardan ellips, giperbola na parabolalarning oldingi bo‘limlarda bayon etilgan xossalaridan foydalanib, ularning kutb koordinatalardagi tenglamasini keltirib chiqarish mumkin.

$$r = \frac{p}{1 - \varepsilon \cos \varphi}$$

Buerda ε- elstsuntrisitet, – faqal pparametr. Ellips va giperbola uchun $=b^2/a$.

a) ε=0 bo‘lsa, u aylanani aniqlaydi va φ bu holda 0<φ<π oraliqdagi barcha qiymatlarni qabul qiladi;

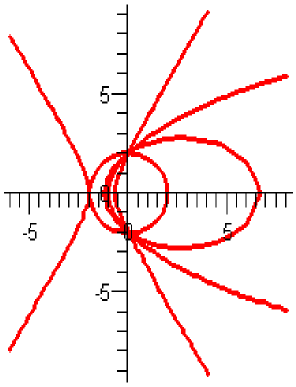
b) 0<ε<1 bo‘lsa, u ellipsni aniqlaydi va φ bu holda 0<φ<π oraliqdagi barcha qiymatlarni qabul qiladi;

v) ε = 1 bo‘lsa, u parabolani aniklaydi va φ bu holda 0< φ <2 π oralikdagi barcha kiymatlarni qabul qiladi;

s) ε >1 bo‘lsa, u giperbolani aniqlaydi. Bu holda φ qaysi oraliqda o‘zgarishi quyidagicha aniqlanadi. $2\varphi_0$ — asimptotalar orasidagi tarmoq joylashgan burchak bo‘lsin, u holda qaralayotgan tarmoq uchun: $\varphi_0 < \varphi < 2\pi - \varphi_0$

<u>Qutub koordinatalar sistemasida 2-tartibli egri chiziqni qurish</u>: *Ko‘rsatilgan holatlar bo‘yicha qiymatlar berib tuzilgan funktsiyalar bo‘yicha grafiklarini quramiz.*

```
> restart;
> with(plots):
> imlicitplot({r=1/(1-0*cos(theta)),r=1/(1-0.7*cos(theta)), r=1/(1-1*cos(theta)),r=1/(1-2*cos(theta))}, r=-10..10, theta=0..2*i, coords=olar,thickness=2);
```



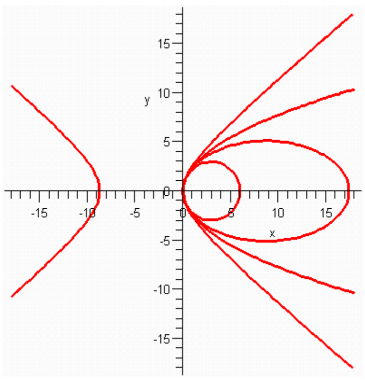
$y^2 = 2px - (1-\varepsilon^2)x^2$ rekurent formulaga asosan egri chiziq turini quyidagicha aniqlayiz:

a) ε=0 bo‘lsa, u aylanani aniqlaydi;
b) 0<ε<1 bo‘lsa, u ellipsni aniqlaydi;
v) ε = 1 bo‘lsa, u parabolani aniklaydi;
s) ε >1 bo‘lsa, u giperbolani aniqlaydi.

Ko‘rsatilgan holatlar bo‘yicha qiymatlar berib tuzilgan funktsiyalar bo‘yicha gafiklarini quramiz.

<u>Ekstsentrisiset asosida qurish:</u>

```
> with(plots):
imlicitplot([-y^2+6*x-(1-0^2)*x^2,-y^2+6*x-(1-0.81^2)*x^2, -y^2+6*x-(1-1^2)*x^2,-y^2+6*x-(1-1.3^2)*x^2], x=-18..18, y=-18..18,numpoints=2000,thickness=2);
```



4.7.Ajoyib egri chiziqlarni qurish

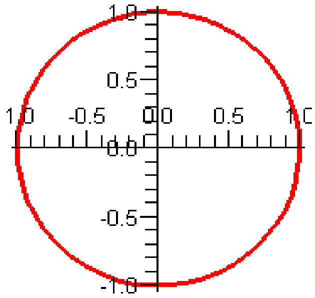
Aylana: *Dekart kordinatalar sistemasidagi tenglamasi:* $x^2+y^2=a^2$

Qutub kordinatalar sistemasidagi tenglamasi: ***r=a***, ***r=acosφ***, ***r=asinφ***

parametrik tenglamasi: ***x=a cost, y=asint, a=1***

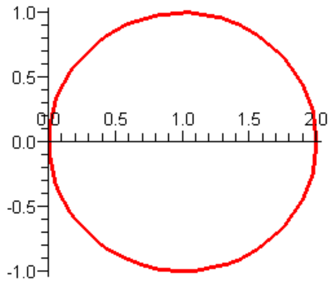
> with(plots):

> imlicitplot({r=1}, r=-14..14, phi=0..2*i, coords=olar, thickness=2);



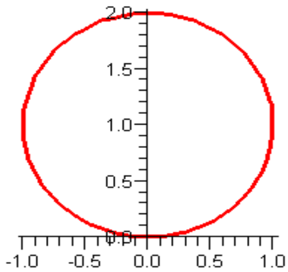
> with(plots):

>imlicitplot({r=2*cos(phi)}, r=-14..14, phi=0..2*i, coords=olar, thickness=2);



> with(plots):

> imlicitplot({r=2*sin(phi)}, r=-14..14, phi=0..2*i, coords=olar, thickness=2);



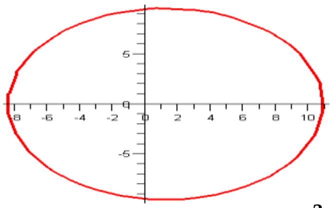
Ellis: *Dekart kordinatalar sistemasidagi tenglamasi:* $x^2/a^2+y^2/b^2=1$

Qutub kordinatalar sistemasidagi tenglamasi: $r=\dfrac{b^2/a}{1-\varepsilon\cos\varphi}$,(0<ε<1)

parametrik tenglamasi: ***x=a cost, y=bsint, a=1***

> with(plots):

> imlicitplot({r=(19/2)/(1-0.13*cos(phi))}, r=-14..14, phi=0..2*i, coords=olar,thickness=2);

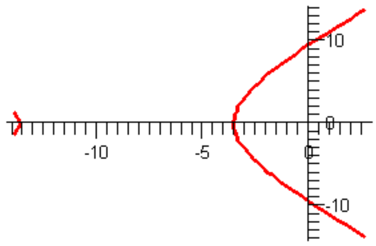


Giperbola: *Dekart kordinatalar sistemasidagi tenglamasi:* $x^2/a^2 - y^2/b^2 = 1$

Qutub kordinatalar sistemasidagi tenglamasi: $r = \dfrac{b^2/a}{1-\varepsilon\cos\varphi}$, $(\varepsilon>1)$

parametrik tenglamasi: ***x=a sect, y=btgt, a=1***

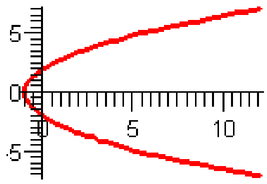
> with(plots):
> imlicitplot({r=(19/2)/(1-1.7*cos(phi))}, r=-14..14, phi=0..2*i, coords=olar,thickness=2);



arabola: *Dekart kordinatalar sistemasidagi tenglamasi:* $2x - y^2 = 0$

Qutub kordinatalar sistemasidagi tenglamasi: $r = \dfrac{b^2/a}{1-\varepsilon\cos\varphi}$, $(\varepsilon=1)$

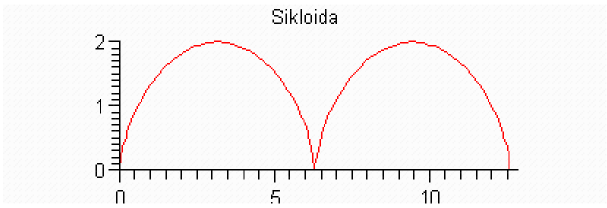
> with(plots):
> imlicitplot({r=2/(1-1*cos(phi))}, r=-14..14, phi=0..2*i, coords=olar,thickness=2);



Sikloida *Dekart kordinatalar sistemasidagi tenglamasi:* $x^3+y^3-3axy=0$

parametrik tenglamasi: ***x=a*(t-sint), y=a*(1-cost), a=1***

> with(plots):
>plot([1*(t-sin(t)),1*(1-cos(t)),t=0..4*i],title=`Sikloida`);



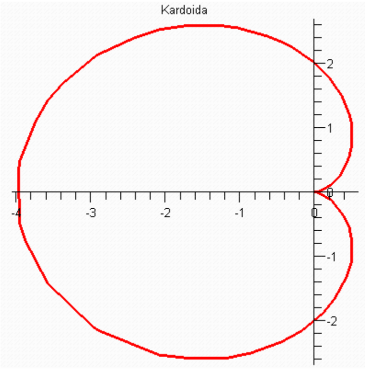
Kardoida

Dekart kordinatalar sistemasidagi tenglamasi: $(x^2+y^2+2ax)^2=4a^2(x^2+y^2)$

Qutub kordinatalar sistemasidagi tenglamasi: ***r=2a(1-cosφ),a=1***

parametrik tenglamasi: ***x=2acost-acos2t, y=2asint-asin2t***

> restart;
> with(plots):
> imlicitplot({r = 2*1*(1-cos(phi))}, r=-6..6, phi=0..2*i, coords=olar,thickness=2,title=`Kardoida`);



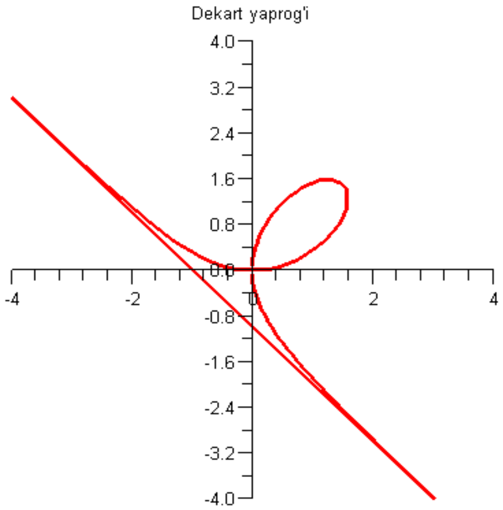
Dekart yaprog'I

Dekart kordinatalar sistemasidagi tenglamasi: $x^3+y^3-3axy=0$.

Qutub kordinatalar sistemasidagi tenglamasi: $r = \dfrac{3a\cos\varphi\sin\varphi}{\cos^3\varphi + \sin^3\varphi}$.

parametrik tenglamasi: $x=3at/(t^3+1)$, $y=3at^2/(t^3+1)$, $a=1$.

```
> with(plots):
plot([3*1*t/(t^3+1), 3*1*t^2/(t^3+1), t=-14*i..14*i],-4..4, -4..4, title=`Dekart yarog'i`);
```

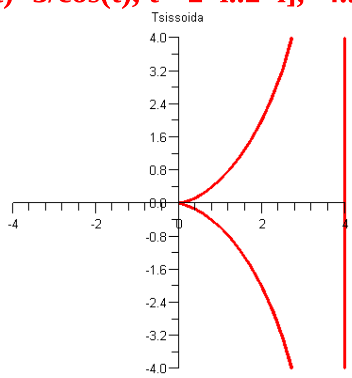


Tsissoida *Dekart kordinatalar sistemasidagi tenglamasi:* $y^2 = \dfrac{x^3}{a-x}$

Qutub kordinatalar sistemasidagi tenglamasi: $r = atg\varphi\sin\varphi$

parametrik tenglamasi: $x=2a\sin^2 t$, $y=2a\sin^3 t/\cos(t)$, $a=2$

```
> with(plots):
plot([2*2*sin(t)^2, 2*2*sin(t)^3/cos(t), t=-2*i..2*i], -4..4, -4..4,title=`Tsissoida`);
```



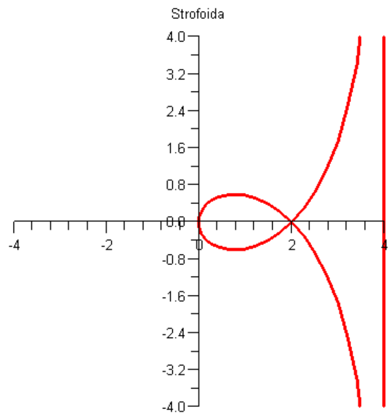
Strofoid

Dekart kordinatalar sistemasidagi tenglamasi: $y^2 = \frac{x(a-x)^2}{2a-x}$

Qutub kordinatalar sistemasidagi tenglamasi: $r = \frac{a(1 \pm \sin \varphi)}{\cos \varphi}$

parametrik tenglamasi: ***x=2(1+sint), y=2(1+sint)sint/cost***

```
> with(plots):
plot([2*(1+sin(t)), 2*(1+sin(t))*sin(t)/cos(t), t=0..2*i],
 -4..4, -4..4,title=`Strofoida`);
```

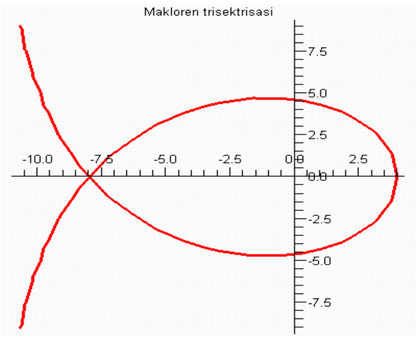


<u>*Makloren trisektrisasi*</u>

Dekart kordinatalar sistemasidagi tenglamasi: $x(x^2+y^2)= a\,(y^2-3\,x^2)$

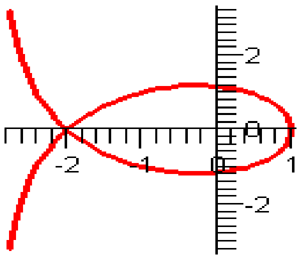
Qutub kordinatalar sistemasidagi tenglamasi: $r=4/\cos(\varphi/3)$

```
> restart;
> with(plots):
> imlicitplot({r=4/cos(phi/3)}, r=-14..14, phi=0..3*i,
coords=olar,thickness=2,title=`Makloren trisektrisasi`);
```



Animatsiya yordamida gafikni qurish:

```
> with(plots):
>animatecurve([1/cos(t/3),t,t=-4..4],coords=olar,frames=60,
numpoints=100,thickness=2);
```



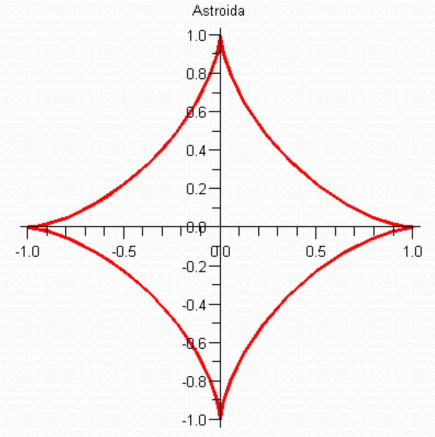
Astroida

Dekart kordinatalar sistemasidagi tenglamasi: $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

parametrik tenglamasi: ***x=acos³t, y=asin³t,(a=1)***

> with(plots):

> plot([1*cos(t)^3, 1*sin(t)^3, t=0..4*i], thickness=2, title=`Astroida`);



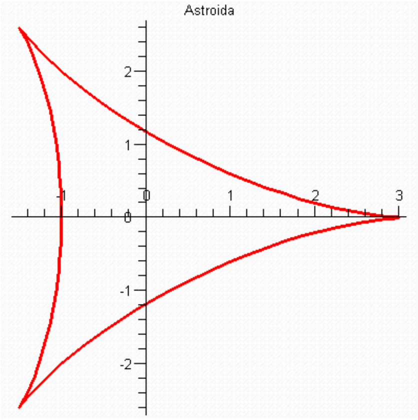
Shteyner egri chizig/i

Dekart kordinatalar sistemasidagi tenglamasi: $(x^2+y^2)^2+8ax(3y^2-x^2)+18a^2(x^2+y^2)=27a^4$

parametrik tenglamasi: ***x=2acos(t/3)+acos(2t/3), y=2asin(t/3)-asin(2t/3) (a=1)***

> with(plots):

> plot([2*1*cos(t/3)+1*cos(2*t/3), 2*1*sin(t/3)-1*sin(2*t/3), t=0..6*i],thickness=2,title=`Astroida`);



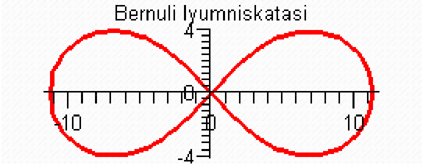
Bernuli lyumniskatasi

Dekart kordinatalar sistemasidagi tenglamasi: $(x^2+y^2)^2=2a^2(x^2-y^2)$

Qutub kordinatalar sistemasidagi tenglamasi: $r^2=2a^2\cos(2\varphi)$, ***(a=8)***

> with(plots):

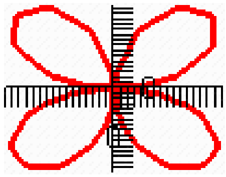
> imlicitplot({r^2=2*8^2*cos(2*phi)}, r=-16..16, phi=0..1*i, coords=olar,thickness=2,title=`Bernuli lyumniskatasi`);



Gul yarog'I *Qutub kordinatalar sistemasidagi tenglamasi:* ***r= sin(kφ)*** *egri chiziqni* **k=2,3,5/2,1/2,1/3** *bo'lgan hollar uchun gaafiklarini quramiz:*

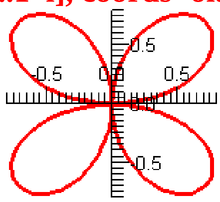
1) ***r=sin(2φ)***

```
> with(plots):
> imlicitplot({r=sin(2*phi)}, r=-14..14, phi=0..2*i, coords=olar,thickness=2,title=`Gul yarog‘i`);
```



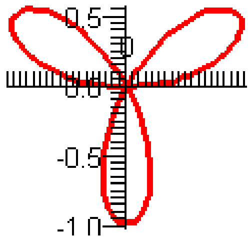
Animatsiya yordamida gafikni qurish:

```
> with(plots):
> animatecurve([sin(2*t),t,t=-1*i..1*i], coords=olar, frames=60,numpoints=100);
```



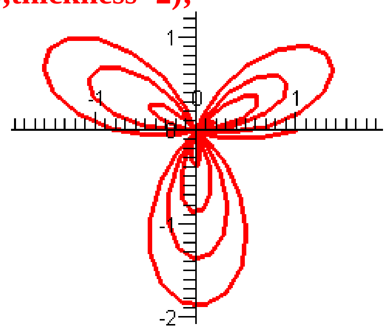
2) ***r=sin(3φ)***

```
> restart;
> with(plots):
> imlicitplot({r=sin(3*phi)}, r=-4..4, phi=0..1*i, coords=olar,thickness=2,title=`Gul yarog‘i`);
```



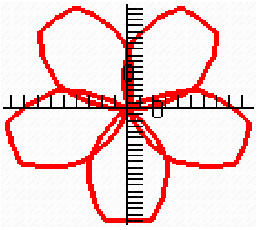
Radiusini o‘zgartirish bilan animatsiya yordamida gafikni qurish:

```
> with(plots):
>animatecurve([u+sin(3*t),t,t=0..4*i, u=1..3],coords=olar, frames=60,numpoints=100,thickness=2);
```



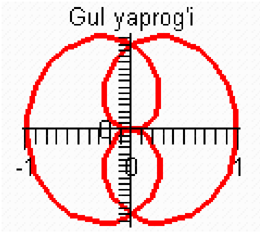
3) ***r=sin(5φ/3)***

```
> with(plots):
> imlicitplot({r=sin(5*phi/3)}, r=-4..4, phi=0..3*i, coords=olar,thickness=2,title=`Gul yarog‘i`);
```



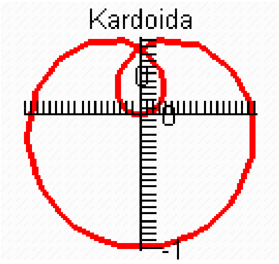
4) *r=sin((1/2)φ)*

```
> with(plots):
> imlicitplot({r=sin((1/2)*theta)}, r=-4..4, theta=0..4*i,
coords=olar,thickness=2,title=`Gul yarog‘i`);
```



5) *r=sin((1/3)φ)*

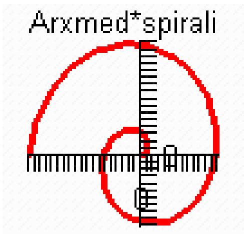
```
> with(plots):
> imlicitplot({r=sin((1/3)*theta)}, r=-14..14, theta=0..3*i,
coords=olar,thickness=2,title=`Kardoida`);
```



Arxmed spirali:

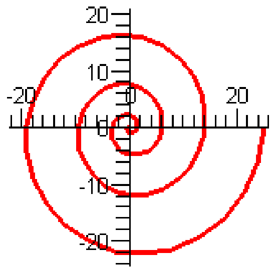
Qutub kordinatalar sistemasidagi tenglamasi: ***r= aφ, (a=1)***

```
> with(plots):
> imlicitplot({r=1*hi}, r=-14..14, hi=0..3*i,
coords=olar,thickness=2,title=`Arxmed*sirali`);
```



Animatsiya yordamida gafikni qurish:

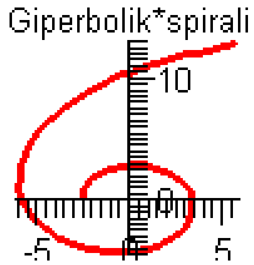
```
> with(plots):
>animatecurve([u*t,t,t=1..8*i],coords=olar,frames=60, numpoints=100,thickness=2);
```



Giperbolik spirali:

Qutub kordinatalar sistemasidagi tenglamasi: ***r= a/φ, (a=15)***

```
> with(plots):
> imlicitplot({r=15/phi}, r=-14..14, phi=0..3*i,
coords=olar,thickness=2,title=`Gierbolik*sirali`);
```



Nazorat ishi uchun variant namunalari

1-variant.

1. Markazi (5;-7) nuqtada bo'lgan va (2;-3) nuqtadan o'tuvchi aylana tenglamasini tuzing.
2. Agar ellipsning ikkita uchi A_1(-6; 0) va A_2(6; 0) nuqtalarda, fokuslari esa, (-4; 0), (4; 0) koordinatalarda bo'lgan tenglamasini tuzing.
3. Uchlari A_1 (-3; 0) va A_2 (3; 0) nuqtalarda, fokuslari (-5; 0) va (5; 0) nuqtalarda bo'lgan giperbola tenglamasini tuzing.
4. Uchi koordinatalar boshida , 0x o'qqa simmetrik bo'lgan va A (1; -2) nuqtadan o'tuvchi parabola tenglamasini tuzing.

2-variant.

1. Markazi (-2; 3) nuqtada bo'lgan, koordinatalar boshidan o'tuvchi aylana tenglamasini tuzing.
2. Fokuslari (-4; 0), (4; 0) nuqtalarda etgan, ekstsentrisiteti ε=0.8 bo'lgan ellipsning tenglamasini tuzing.
3. Fokuslari va ekstsentrisiteti F (± 20; 0), ε=5/3 bo'lgan giperbola tenglamasini yozing.
4. $y^2 = 12x$ parabola funktsiyasidan simmetrik o'qiga perpendikulyar bo'lib o'tuvchi vatar uzunligini toping.

3-variant

1. $3x^2 + 3y^2 - 18x - 10y - 48 = 0$ aylananing koordinata o'qlari bilan kesishish nuqtalarini toping.
2. Fokuslari 0x o'qida, katta yarim o'qi 14 ga ekstsentrisiteti ε=2 / 3 bo'lgan ellips tenglamasini tuzing.
3. Fokuslari 0x o'qida bo'lgan giperbolaning xakikiy o'qi 8 ga teng bo'lsa va (8; 6) nuqtadan utsa, uning tenglamasini tuzing.
4. $y^2 = 16x$ parabolaning $4x - 3y + 8 = 0$ to'g'ri chiziq bilan kesishish nuqtasini toping.

4-variant

1. $x^2 + y^2 - 4x + 2y - 29 = 0$ aylana bilan $x - y - 1 = 0$ to'g'ri chiziqni kesishish nuqtalarini koordinatalarini toping.
2. Agar fokuslari 0x o'qida bo'lgan ellipsning yarim o'qlarining yigindisi 8 ga, fokuslar orasidagi masofa ham 8 ga teng bo'lsa, uning tenglamasini tuzing.

3. Agar giperbola fokuslarining koordinatalari (-8; 0) va (8; 0) bo‘lib, asimtotalari $y = \pm\sqrt{3}x$ bo‘lsa, uning tenglamasini tuzing.
4. $y^2 - 6y - 8x - 7 = 0$ parabola berilgan. Parabola o‘qining tenglamasini tuzing.

5-topshiriq

Quyidagicha tenglama bilan berilgan ikkinchi tartibli egri chiziqlar uchun quyidagilar topilsin:

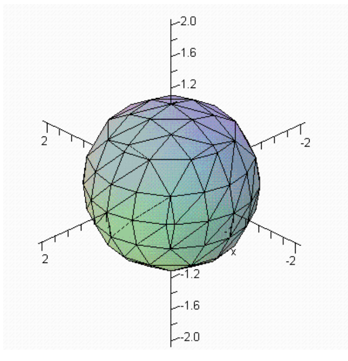
1) **Egri chiziqning kanonik ko‘rinishdagi tenglamasi;**
2) **Fokuslarining koordinatalari;**
3) **Direktrisa va asimptota (giperbola uchun) tenglamalari;**
4) **Koordinata o‘qlari bilan kesishish nuqtalari;**
5) **Topilgan egri chiziqni qurish**.

1. $16x^2 + 25y^2 + 32x - 100y - 284 = 0$
2. $x^2 + 2y^2 + 4x - 6y - 1 = 0$
3. $2x^2 + y^2 + 4x = 0$
4. $2x^2 + y^2 - 8y + 2x + 58 = 0$
5. $x^2 + 2y^2 + 6x - 20y + 84 = 0$
6. $3x^2 + 6x + 2y + 1 = 0$
7. $x^2 + y^2 + 4x = 0$
8 $7x^2 - 6x - y + 2 = 0$
9. $2x^2 + 2y^2 - 4x - 3 = 0$
10. $x^2 - y^2 - 6x + 10 = 0$
11. $x^2 + 4y^2 + 8y - 5 = 0$
12. $-2x^2 + 4y^2 + 8y - 3x + 8 = 0$
13. $3x^2 - 5y^2 + 6x + 25y = 0$
14. $16x^2 + 25y^2 - 32x + 50y - 359 = 0$
15. $36x^2 + 36y^2 - 36x - 24y - 23 = 0$
16. $x^2 - 8y^2 + 2x + 36y = 0$
17. $4x^2 + 3y^2 - 8x + 12y - 32 = 0$
18. $16x^2 - 9y^2 - 64x - 54y - 161 = 0$
19. $9x^2 + 16y^2 + 90x + 32y - 367 = 0$
20. $x^2 + y^2 - 26x + 30y + 313 = 0$
21. $y^2 = 2x^2 + 3y + 13$
22. $x^2 + 9y^2 - 6x - 27 = 0$
23. $4x^2 - 9y^2 + 3x - 7y + 12 = 0$
24. $x^2 - 2x + y + 1 = 0$
25. $y^2 + 4y - 12 = 4x^2 - 5x$
26. $x^2 - 4x - 5 = y^2 + 2y$
27. $x^2 + y^2 - 2xy + 3y + 13 = 0$
28. $2x^2 + y^2 - 6xy + 3y + 1 = 0$
29. $x^2 + 2y^2 - 2xy + 3x + 1 = 0$
30. $x^2 - 2y^2 - 6xy + y + 3 = 0$

5. IKKINCHI TARTIBLI SIRTLAR

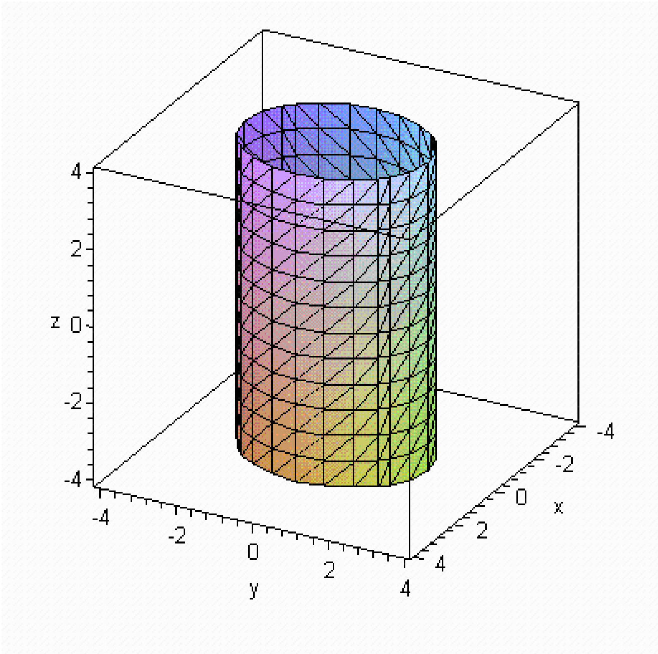
Shar : x^2 C y^2 C $z^2 = 1$

```
> with(plots):
imlicitplot3d([x^2+y^2+z^2=1], x=-2..2, y=-2..2, z=-2..2, grid=[13,13,13]);
```



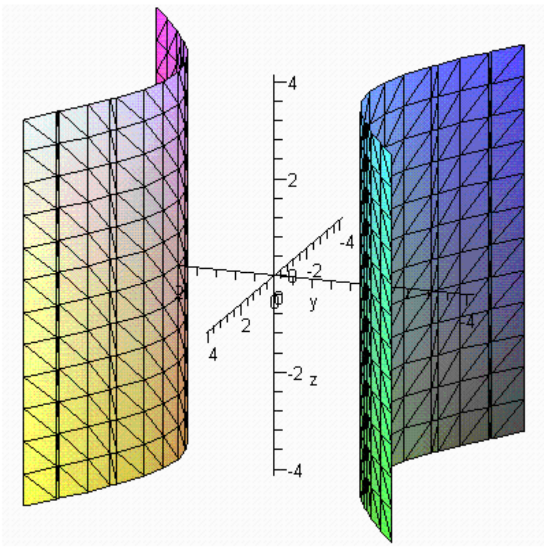
Elliptik tsilindr: $\frac{1}{4}x^2 \mathrm{C} \frac{1}{6}y^2 = 1$

> with(plots):
imlicitplot3d([x^2/4+y^2/6=1], x=-4..4, y=-4..4, z=-4..4, grid=[13,13,13]);



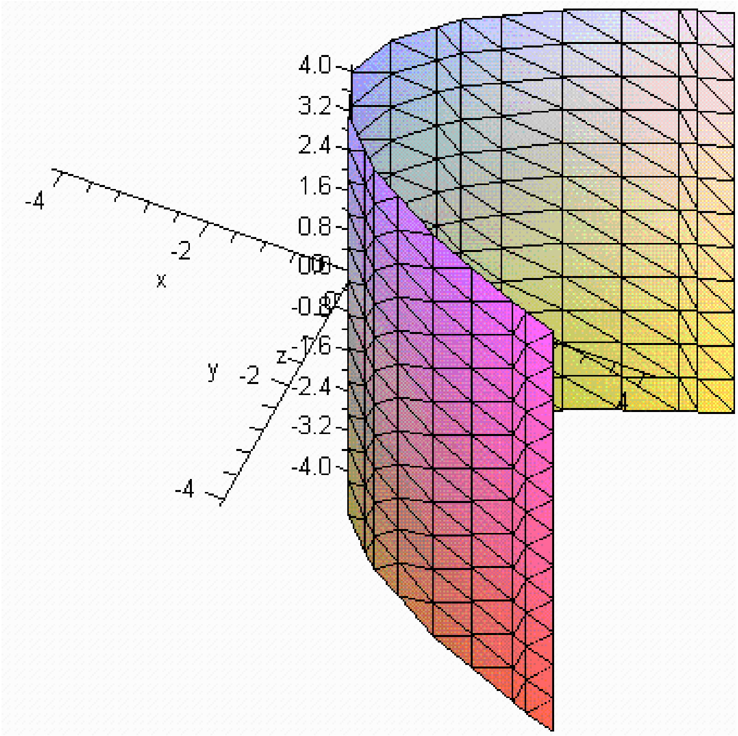
Giperbolik tsilindr: $\frac{1}{4}x^2 \mathrm{K} \frac{1}{6}y^2 = 1$

> with(plots):
imlicitplot3d([x^2/4-y^2/6=1], x=-4..4, y=-4..4, z=-4..4, grid=[13,13,13]);



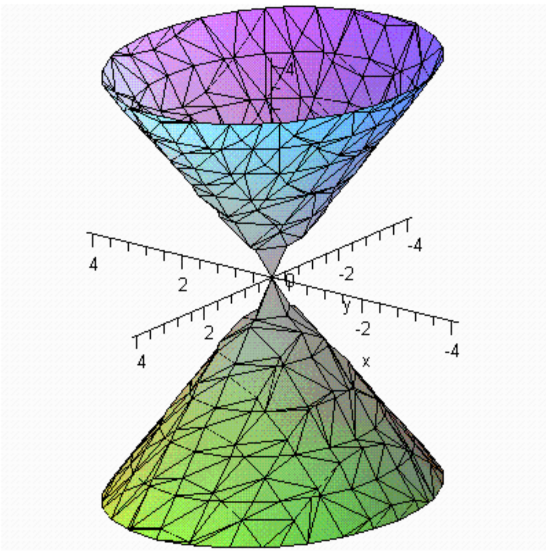
Parabolik tsilindr: $y^2 = 2\,x$

```
> with(plots):
imlicitplot3d([y^2=2*x], x=-4..4, y=-4..4, z=-4..4, grid=[13,13,13]);
```



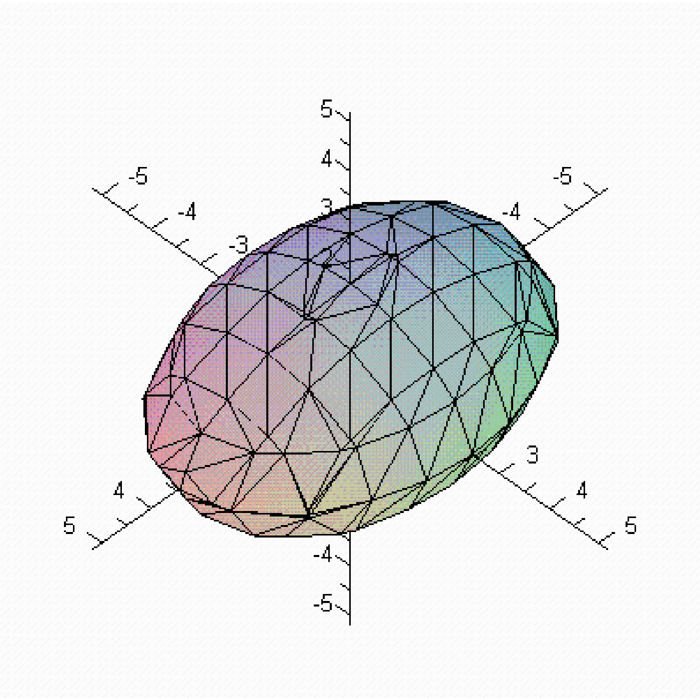
Elliptik konus: $\frac{1}{4}\,x^2 + \frac{1}{6}\,y^2 = \frac{1}{8}\,z^2$

```
> with(plots):
imlicitplot3d([x^2/4+y^2/6=z^2/8], x=-4..4, y=-4..4,     z=-4..4, grid=[13,13,13]);
```



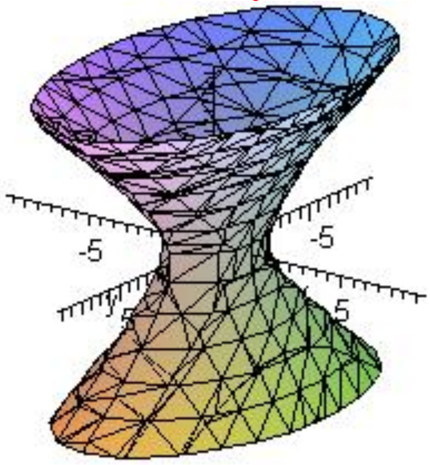
Ellipsoid:: $\frac{1}{4}x^2 \text{ C } \frac{1}{6}y^2+\frac{1}{3}z^2=1$

>with(plots):
imlicitplot3d([x^2/12+y^2/6+z^2/3=1], x=-5..5, y=-5..5, z=-5..5, grid=[13,13,13]);



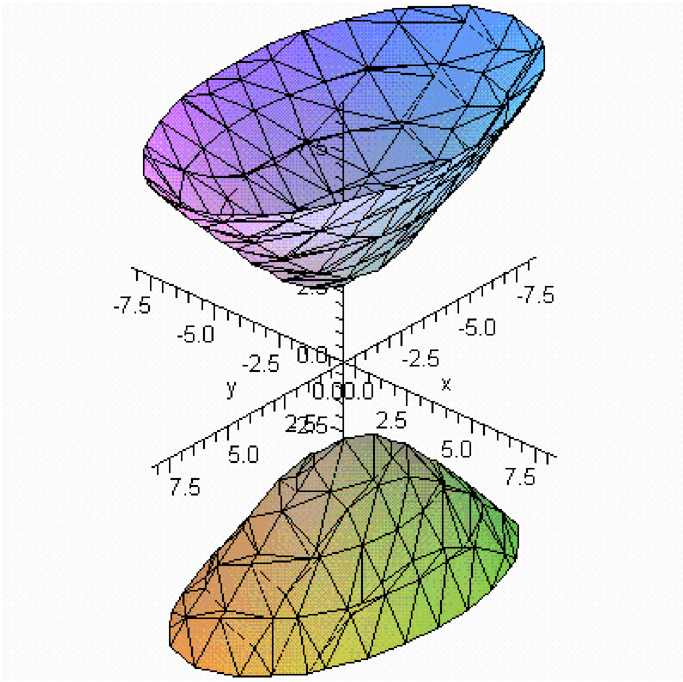
Bir pallalielliptikgiperboloid:: $\frac{1}{6}x^2 \text{ C } \frac{1}{2}y^2 \text{ K } \frac{1}{9}z^2=1$

>with(plots):
imlicitplot3d([x^2/6+y^2/2-z^2/9=1], x=-8..8, y=-8..8, z=-8..9, grid=[13,13,13]);



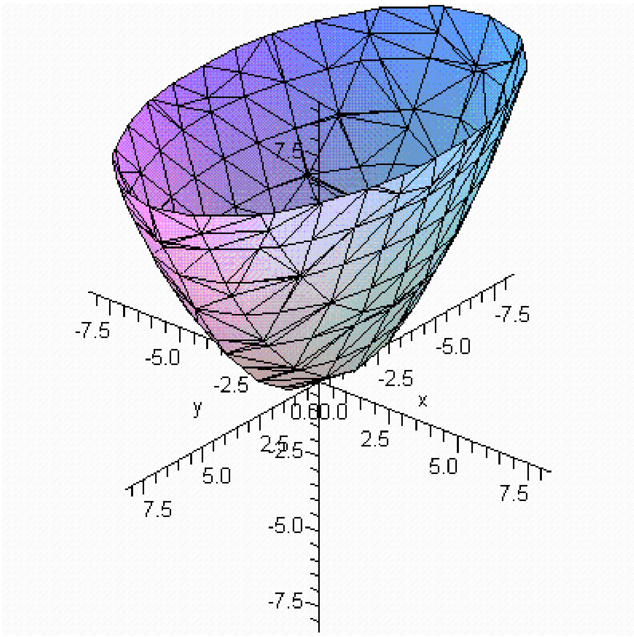
Ikki pallalielliptik giperboloid: $\frac{1}{6}x^2 + \frac{1}{2}y^2 - \frac{1}{8}z^2 = -1$

>with(plots):
imlicitplot3d([x^2/6+y^2/2-z^2/8=-1], x=-8..8, y=-8..8, z=-8..9, grid=[13,13,13]);



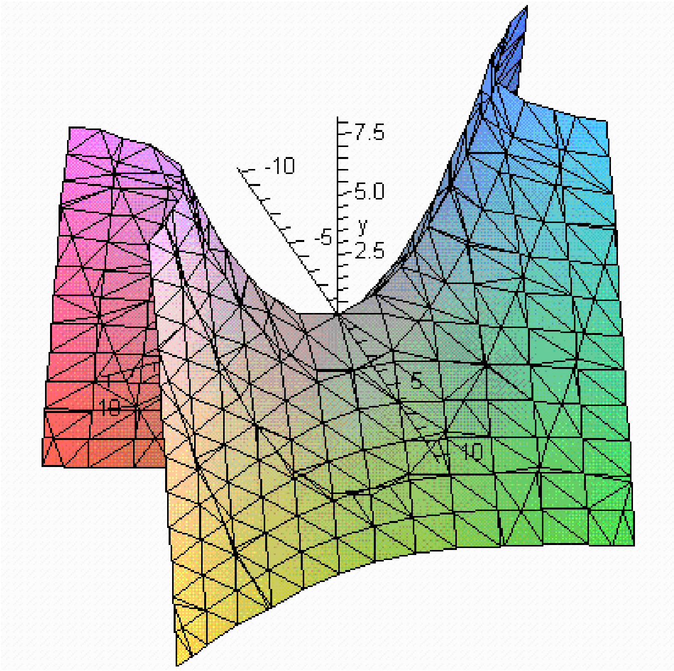
Elliptik paraboloid: $\frac{1}{6}x^2 + \frac{1}{2}y^2 = z$

> with(plots):
imlicitplot3d([x^2/6+y^2/2=z], x=-8..8, y=-8..8, z=-8..9, grid=[13,13,13]);



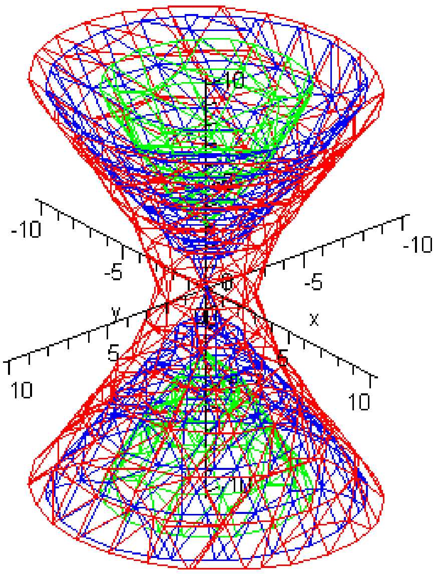
Giperbolik paraboloid:: $\frac{1}{6}x^2 - \frac{1}{2}y^2 = z$

> with(plots):
imlicitplot3d([x^2/6-y^2/2=z], x=-10..10, y=-10..10, z=-8..8, grid=[13,13,13]);



Bir-birini ichiga joylashgan bir allali gierboloid, konus va ikki allali gierboloid.

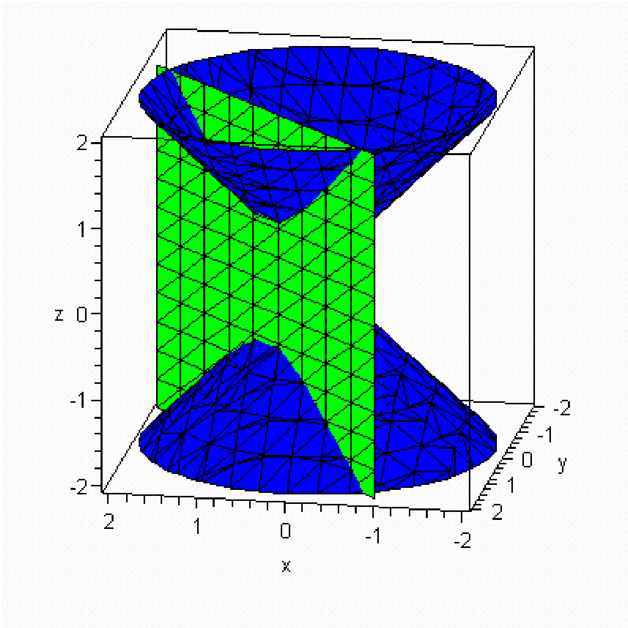
> with(plots):
imlicitplot3d([x^2/4+y^2/4=z^2/10,x^2/2+y^2/2-z^2/10=-1,x^2/4+y^2/5-z^2/10=1], x=-10..10, y=-10..10, z=-10..10, grid=[13,13,13],color=[blue,green,red]);



Sirtlarni kesishishiga misol

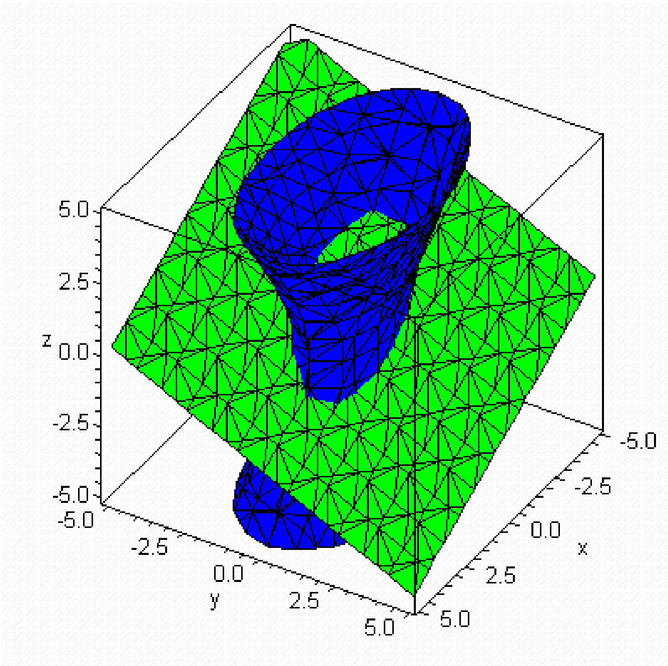
Konus bilan tekislik:

```
> with(plots):
imlicitplot3d([x^2+y^2=z^2,x+y=1], x=-2..2, y=-2..2, z=-2..2,
grid=[13,13,13],color=[blue,green], scaling=constrained, axes=boxed);
```



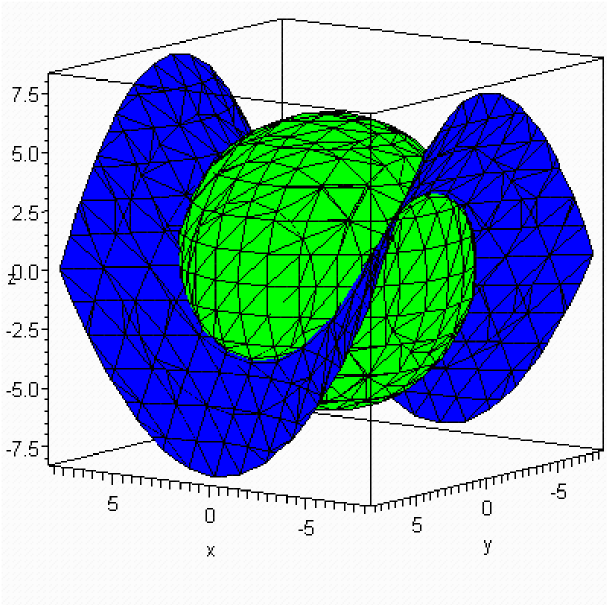
Ellitik gierboloid va tekislik:

```
> with(plots):
imlicitplot3d([x^2/6+y^2/2-z^2/9=1,x+y+2*z=0.5], x=-5..5, y=-5..5, z=-5..5,
grid=[13,13,13],color=[blue,green], scaling=constrained, axes=boxed);
```



Gierboloik araboloid va shar:

```
> with(plots):
imlicitplot3d([x^2/8-y^2/8=z,x^2+y^2+z^2=40], x=-8..8, y=-8..8, z=-8..8, grid=[13,13,13],
color=[blue,green],scaling=constrained, axes=boxed);
```



Adabiyot

1. Jo‘raev T.J, Sahdullaev A., Xudoyberganov G., Mansurov X., Vorisov A. “Oliy matematika asoslari” I,II T. «O‘zbekiston» 1985
2. Tojiev Sh.I. “Oliy matematikadan masalalar yechish” T. «O‘zbekiston” 2002
3. Latipov X. va boshqalar «Analitik geometriya va chizikli algebra» T. «O‘zbekiston”1995
4. SahdullaevA., Xudoyberganov G. Mansurov X., G’ulomov R. «Matematik analizdan misol va masalalar to‘plami» T. «O‘zbekiston» 1992y.
5. Danko R.E. A.G.oov., G.U.Kojevnikova Oliy matematika. Misol va masalalar. I qism T.2007
6. Davlatov A. Jo`raqulov R. Oliy matematikadan misol va masalalar. T.2007
7. Atahonov K.U., Erzin V.A. Hodjayev B. Matematik analizdan misol va masalalar to`lami. T.20
8. Minorskiy V.P.. «Oliy matematikadan masalalar to‘plami» T.1977
9. Soatov Yo. “Oliy matematika”, I,II, T. «O‘zbekiston»1983
10. Rajaov F., S. Mashariova, R.Madrahimov. Oliy matematika. T.2007

Mundarija