O‘ZBEKISTON RESPUBLIKASI
OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

FARG‘ONA DAVLAT UNIVERSITETI

MATEMATIKADAN AMALIY MAShG‘ULOTLAR

uslubiy ko‘llanma

Farg‘ona -2009

H.Mahmudov, Sh.Sobirov, T.Bakirov. Matematikadan amaliy mashg‘ulotlar uchun uslubiy ko‘llanma (kunduzgi bo‘lim bakalavr yo‘nalishlari uchun), Farg‘ona-2009, 48 b.

Ushbu o‘quv - uslubiy qo‘llanma bakalavriyatning kunduzgi bo‘lim bakalavr yo‘nalishi uchun mo‘ljallangan. Amaliy mashg‘ulotlarni ўtkazish uchun har bir mavzu uchun bitta varianti namunasi echimi bilan berilgan va mustaқil echish uchun topshiriqlar keltirilgan.

Taqrizchilar: A.Abdurazzaqov - fizika-matematika fanlari nomzodi, dotsent
A.A.Shermatov - fizika-matematika fanlari nomzodi, dotsent

Ushbu uslubiy qo‘llanma universitet o‘quv-uslubiy Kengashining 2009 yil __________ ____ - yig‘ilishi qarori bilan nashrga tavsiya etilgan.

KIRISh

O‘zbekiston Respublikasining “Ta’lim to‘g‘risida”gi qonuni, “Kadrlar tayyorlash milliy dasturi” va O‘zbekiston Respublikasi Vazirlar Mahkammasining “Uzluksiz ta’lim tizimi uchun davlat ta’lim standartlarini ishlab chiqish va joriy etish to‘g‘risida”gi qarorlari qabul qilingan. Bu hujjatlar oliy va o‘rta maxsus o‘quv yurtlari talabalarining bilim darajasini sifat jihatdan yanada yaxshilashga, amaliy malakalarini yuksaltirishga qaratilgandir.

Ushbu uslubiy qullanmalar nomatematik yo‘nalish talabalari uchun amaliy mashg‘ulotlarni o‘tkazish bo‘yicha tavsiyalar va nazorat ishlarining variantlari berilgan. Ko’llanmada 15 ta mashgulot ko’rib chikilgan bo’lib, xar bir mashgulot uchun namunaviy misol va masala echib kursatilgan. Zarur xollarda formula va qoidalar uchun tushuntirish berilgan. $z=x^3+8u^3-6xu+5$ funktsiyaning ekstrimumlari hisoblansin.

Bo’lim oxirida nazorat mashgulotlari uchun 10 ta variantdan iborat o‘xshash misol va masalalar ko‘rsatilgan.Variantlardagi misol va masalalarni echish uchun kerak bo‘ladigan nazariy materiallarni o‘qituvchilar tomonidan yozilgan ma’ruzalar matni hamda talabalar tomonidan olingan konspektlar va quyidagi adabiyotlardan topish mumkin:

1. YO.U.Soatov «Oliy matematika» I q. T. «O‘qituvchi», 1992-496 b.
2. YO.U.Soatov «Oliy matematika» II q. T. «O‘qituvchi», 1994-416 b.
3. YO.U.Soatov «Oliy matematika» III q. T. «O‘qituvchi», 1996-640 b.
4. T.Jo‘raev, A.Sa’dullaev va boshqalar. «Oliy matematika asoslari» T. «O‘zbekiston» 1994 -280 b.
5. Danko P.Ye., Popov A.G., Kojevnikova T.Ya. «Vqsshaya matematika v uprajneniyax i zadachax» 1-2 ch. M.: Vqssh.shkola 1986, - 304 s.

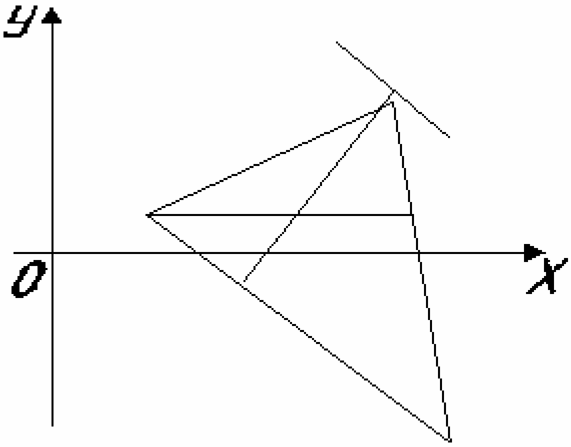
Yuqoridagi keltirilgan adabiyotlardan olingan bilimlar natijasida amaliy mashg‘ulotlar muvaffaqiyatli bajariladi va shu bilan birga bo‘lim bo‘yicha oraliq va yakuniy nazoratlarga ham puxta tayyorgarlik ko‘riladi. Talabalar tomonidan bajarilishi lozim bo‘lgan misol va masalalarning raqamlari o‘qituvchi tomonidan belgilanadi.

ANALITIK GEOMETRIYA VA CHIZIQLI ALGEBRA

1-AMALIY MAShG'ULOT
TEKISLIKDA DEKART KOORDINATALARI SISTEMASI VA TO'G'RI CHIZIQ TENGLAMALARI.

Masala. Uchlari A (4; 1), B (16; -8) va C (14; 6) nuqtalarda bo'lgan ABC uchburchak berilgan. ABC uchburchakda

1) BC tomonning uzunligini toping;
2) AB va BC tomonning tenglamasini tuzing;
3) B burchakni radianlarda hisoblang;
4) Uchburchakning C uchidan o'tuvchi va AB tomonga parallel hamda perpendikulyar bo'lgan to'g'ri chiziqlar tenglamasini tuzing;
5) ABC uchburchak yuzini hisoblang.



Yechish: Avval ABC uchburchakni chizib olamiz.

1. Ikkita M_1 $(x_1;u_1)$ va M_2 $(x_2;u_2)$ nuqtalar orasidagi masofa ushbu

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (1)$$

formula yordamida topiladi. Masala shartiga ko'ra B (16, -8), C (14;6):

x_1=16; x_2=14; y_1=-8; y_2=6.

Bu qiymatlarni (1) formulaga qo'yib, topamiz:

$$|BC| = \sqrt{(14-16)^2 + (6+8)^2} = \sqrt{(-2)^2 + (14)^2} = \sqrt{200} = 10\sqrt{2}$$

Demak, $|BC|$ $=10\sqrt{2}$ (uzunlik birlik).

2. Ikki M_1 va M_2 nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \qquad (2)$$

ko'rinishga ega. Bu formuladan foydalanib AB tomonning tenglamasini tuzamiz.

Shartga ko'ra A (4;1), V (16; -8); x_1=4, y_2=16, y_1=1, y_2=-8.

U holda $\dfrac{x-4}{16-4}=\dfrac{y-1}{-8-1}$; $\dfrac{x-4}{12}=\dfrac{y-1}{-9}$; $3x-12=-4y+4$;

$$y=-\frac{3}{4}x+4$$

Shunga o‘xshash BC tomonning tenglamasi ham tuziladi.

$$\frac{x-4}{14-16}=\frac{y-(-8)}{6-(-8)};\qquad \frac{x-16}{-2}=\frac{y+8}{14};\qquad y+8=-7x+112;$$

$$y=-7x+104$$

3. To‘g‘ri chiziqni burchak koeffitsientli tenglamasi

$y=k_1x+b_1$ va $y=k_2x+b_2$

ko‘rinishga ega bo‘lgan ikkita to‘g‘ri chiziq orasidagi α burchak ushbu

$$tga=\frac{k_2-k_1}{1+k_1\cdot k_2} \qquad (3)$$

formulaga asosan topiladi. Masalaning 2-qismida ko‘rdikki, BC tomon tenglamasi y=-7x+104, AB tomon tenglamasi y=-¾x+4, bulardan

$k_1=-7$, $k_2=-¾$

Bu qiymatlarni (3) ga qo‘yib

$$tga=\frac{-\frac{3}{4}+7}{1+\left(-\frac{3}{4}\right)\cdot(-7)}=1;\qquad tga=1;\qquad a=arctg1=\frac{p}{4};$$

$$a=\angle B=\frac{p}{4}$$

ekanini topamiz.

2. $M_0(x_0,y_0)$ nuqtadan berilgan yo‘nalish bo‘yicha o‘tuvchi to‘g‘ri chiziq tenglamasi:

$$y-y_0=k(x-x_0) \qquad (4)$$

ko‘rinishda edi. Masala shartiga ko‘ra C (14; 6) $x_0=14$ va $y_0=6$. Bu koordinatalarni (4) formulaga qo‘yib va AB tomon tenglamasida burchak koeffitsienti k=¾ ekanini hisobga olib, ikki to‘g‘ri chiziqning parallellik ($k_1=k_2$) va perpendikulyarlik shartidan $\left(k_1=-\dfrac{1}{k_2}\right)$ foydalanib, uchburchakning C uchidan AB tomonga parallel to‘g‘ri chiziq tenglamasi

$$y-6=-\frac{3}{4}\cdot(x-14);\qquad \text{yoki}\qquad y=-\frac{3}{4}x+\frac{33}{2},$$

shuningdek AB ga perpendikulyar bo‘lgan $\left(k=\dfrac{4}{3}\right)$ to‘g‘ri chiziq tenglamasi

$$y-6=\frac{4}{3}\cdot(x-14)\quad \text{yoki}\quad y=\frac{4}{3}x-\frac{38}{3}$$

ko‘rinishga ega bo‘ladi.

3. ABC uchburchakni yuzini topish uchun AK balandlik uzunligini topishimiz kerak. Buning uchun $M_0(x_0, y_0)$ nuqtadan Ax+By+C=0 to‘g‘ri chiziqqacha bo‘lgan masofani

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} \qquad (5)$$

formuladan foydalanamiz. Bunda $x_0=4$, $y_0=1$. BC tenglamani esa 7x+y-104=0 ekanini e'tiborga olib, (5) formulaga asosan masofani topamiz.

$$d = \frac{|7 \cdot 4 + 1 - 104|}{\sqrt{49+1}} = \frac{|-75|}{5\sqrt{2}} = \frac{15}{\sqrt{2}} = \frac{15\sqrt{2}}{2} \text{ (uz.b.)}$$

Demak, ABC uchburchak yuzi

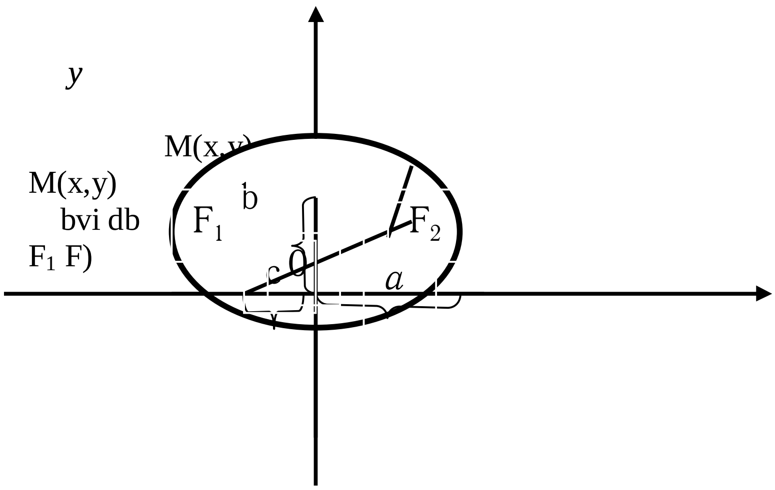
$$S = \frac{1}{2}|BC| \cdot d = \frac{1}{2} 10\sqrt{2} \cdot \frac{15\sqrt{2}}{2} = 75 \text{ (kv. birligi)}$$

2- mashg'ulot.
Tekisliklikda egri chiziq tenglamalar
E L L I P S

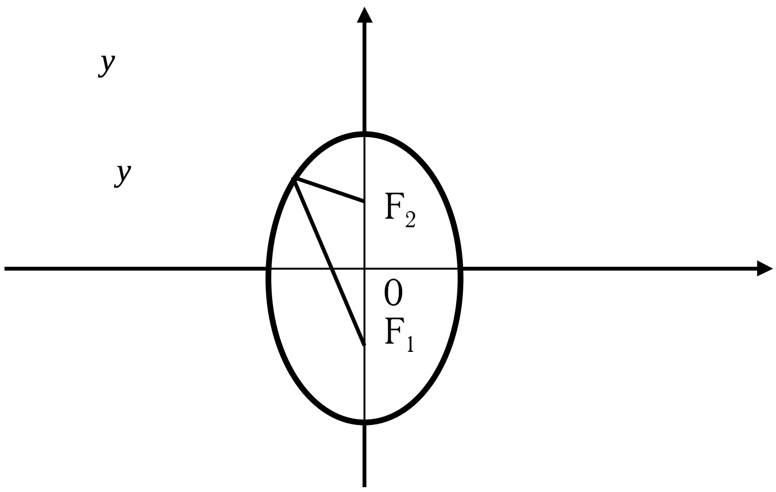
Ixtiyoriy nuqtasidan fokuslar deb ataluvchi berilgan ikki nuqtagacha masofalari yig‘indisi o‘zgarmas (bu yig‘indi fokuslar orasidagi masofadan katta) bo‘lgan nuqtalarning geometrik o‘rni tekislikda **ellips** deb ataladi.

Ellips tenglamasi fokuslar abstsissa o‘qida yotganda rasmdagi

shaklda bo‘lib, uning tenglamasi $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a>b)$



bunda a - katta yarim o‘q uzunligi, b- kichik yarim o‘q uzunligi a,b,c parametrlar $a^2 - b^2 = c^2$ (bunda s- fokuslar orasidagi masofa yarmi) munosabat bilan bog‘langan. $e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a} < 1$ qiymat ellipsning ekstsentrisiteti deb ataladi. Agar fokuslar ordinata o‘qida yotsa ellipsning shakli:



tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (b>a) kabi ko‘rinishga ega. Ellipsni $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ tenglamasiga asosan yasash uchun: 1) Ellipsning a va b yarim o‘qlari bo‘yicha uchlari $A_1(-a;0)$, $A_2(a;0)$, $B_1(0;-b)$, $B_2(0;b)$.

2) Bu nuqtalardan koordinata o‘qlariga parallel to‘g‘ri chiziqlar o‘tkazib, hamda markazi koordinatalar boshida bo‘lgan to‘g‘ri to‘rtburchak yasaladi.

3) A_1,A_2,B_1,B_2 nuqtalar orqali o‘tuvchi egri chiziq chiziladi:

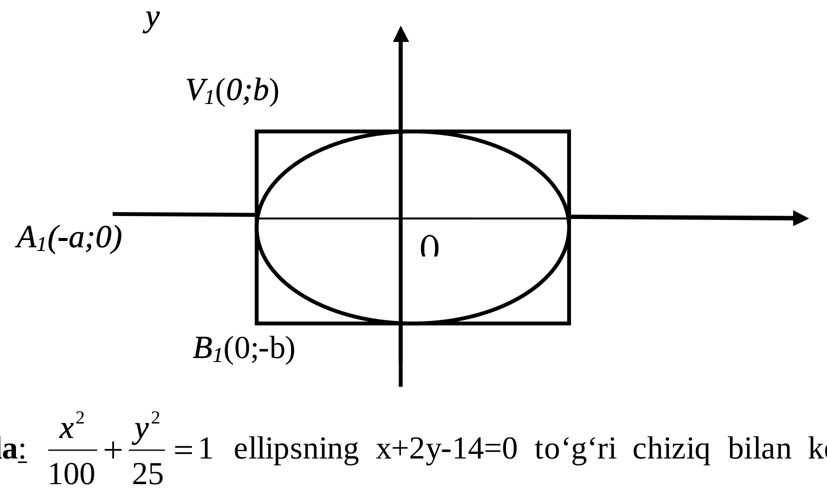
4) Biror $N(x_1;y_1)$ nuqtaning ellipsda yotish yotmasligi

$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}=1$ munosabatini tekshirish orqali aniqlanadi:

a) Agar $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}>1$ munosabat o‘rinli bo‘lsa, H nuqta ellips tashqarida.

b) Agar $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}<1$ munosabat o‘rinli bo‘lsa H nuqta ellips ichida yotadi.

$|MF_2|=r_2$ - o‘ng fokal radius-vektori $r_2=a-ex$ formula bilan va $|MF_1|=r_1$ - chap fokal radius-vektor $r_1=a+ex$ formula bilan topiladi.



Masala: $\frac{x^2}{100}+\frac{y^2}{25}=1$ ellipsning x+2y-14=0 to‘g‘ri chiziq bilan kesishgan nuqtalari orasidagi masofani, ellipsning ekstsentritetini kesishgan nuqtalarning fokal radius-vektorlarini toping, chizmada tasvirlang.

Yechish: Ma’lumki ikkita chiziqning kesishgan nuqtasini topish uchun berilgan ikki tenglamani sistema qilib echamiz:

$$\begin{cases}\dfrac{x^2}{100}+\dfrac{y^2}{25}=1\\ x+2y-14=0\end{cases} \Leftrightarrow \begin{cases}x^2+4y^2=100\\ x+2y=14\end{cases} \Leftrightarrow \begin{cases}x^2+4y^2=100\\ x=14-4y\end{cases} \Leftrightarrow (14-2y)^2+4y^2=100$$

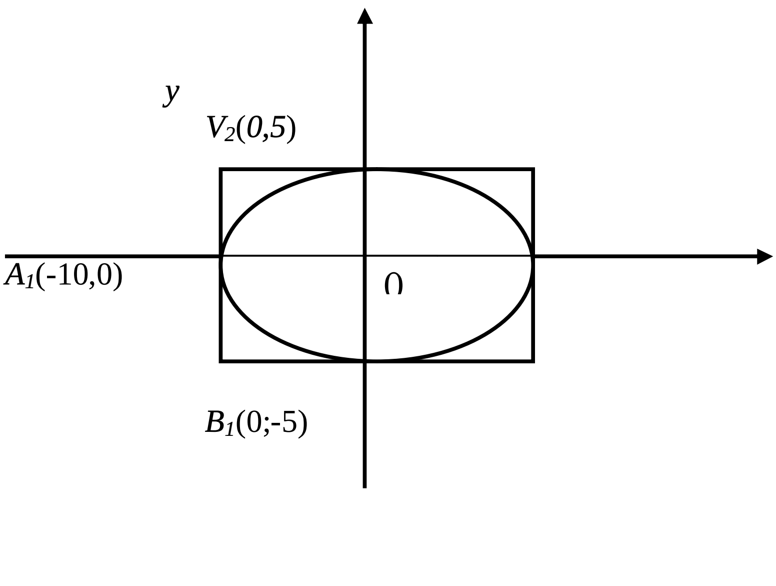
$4(7-y)^2+4y^2=100 \Leftrightarrow (7-y)^2+y^2=25 \Leftrightarrow 49-14y+y^2+y^2=25 \Leftrightarrow 2y^2-14y+24=0 \Leftrightarrow$
$y^2-7y+12=0, y_{1,2}=\frac{7}{2}\pm\frac{1}{2}, y_1=4,\ y_2=3,\ x_1=14-2\quad y_1=6,$
$x_2=8$,
demak, kesishgan nuqtalar $M_1(6;4)$ va $M_2(8;3)$. M_1va M_2 nuqtalar orasidagi masofa $|M_1M_2|=\sqrt{(8-6)^2+(3-4)^2}=\sqrt{4+1}=\sqrt{5}=2.2$ (kami bilan) ga teng. Endi ellipsning ekstsentritetini

$e=\frac{c}{a}=\frac{\sqrt{a^2-b^2}}{a}$ formula bilan topamiz

$$e=\frac{\sqrt{10^2-5^2}}{10}=\frac{\sqrt{100-25}}{10}=\frac{\sqrt{75}}{10}=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}$$

Fokal radius vektorlarni $r_2=a$-ex va $r_1=a$+ex formulalar bilan topamiz.

M_1 ning o‘ng fokal radius $r_2=10-\frac{\sqrt{3}}{2}6=10-3\sqrt{3}$ chap fokal radius $r_1=10+\frac{\sqrt{3}}{2}6=10+3\sqrt{3}$. Xuddi shunday M_2 uchun $r_2=10-\frac{\sqrt{3}}{2}8=$
$=10-4\sqrt{3}$ va $r_1=10+\frac{\sqrt{3}}{2}8=10+4\sqrt{3}$, natijalarni hosil qilamiz. Endi masalaning chizmasini yasaymiz.

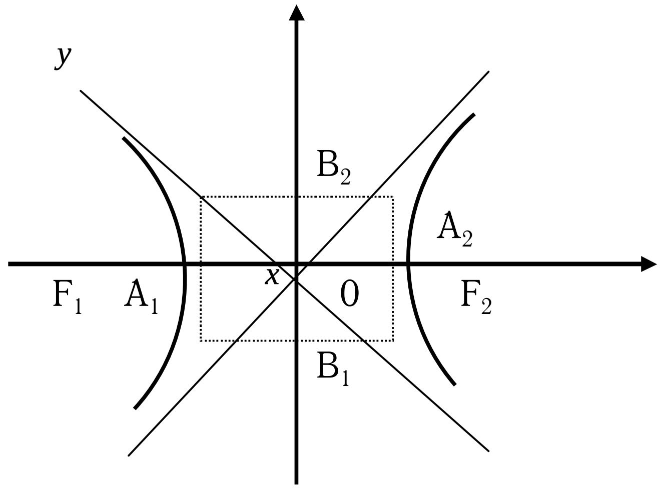


GIPERBOLA

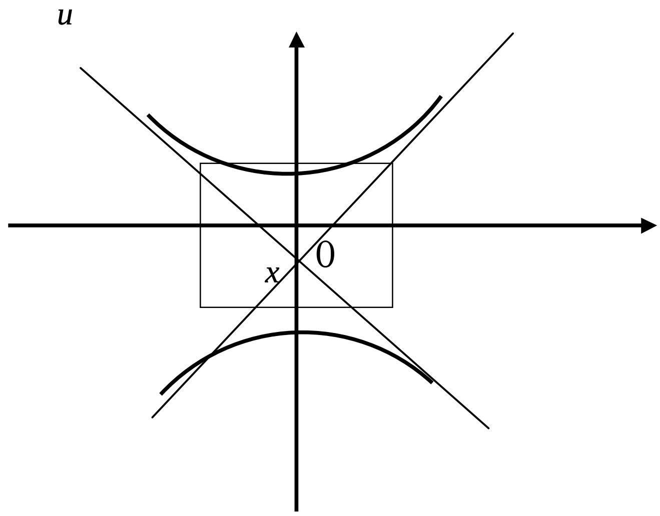
Ixtiyoriy nuqtasidan fokuslar deb ataluvchi berilgan ikki nuqtagacha masofalari ayirmasining absolyut qiymati o‘zgarmas bo‘lgan (fokuslar orasidagi masofadan kichik) nuqtalarning geometrik o‘rni tekislikda **giperbola** deb ataladi.

Fokuslar obtsissa o‘qida yotganda quyidagi shaklda bo‘lib, tenglamasi

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$$



Bunda *a*- haqiqiy yarim o‘q uzunligi, b- mavhum yarim o‘q uzunligi. *a*,b,c parametrlar orsidagi bog‘lanish $b^2 = c^2 - a^2$, bunda s fokuslar orasidagi masofani yarmi. Giperbola ekstsentriteti $e=\frac{c}{a}=\frac{\sqrt{a^2+b^2}}{a}>1$ ga teng. Assimptotalari u=$\pm\frac{b}{a}x$ tenglamalar bilan ifodalanadi. Teng tomonli giperbola assimptotalari tenglamalari u=$\pm x$. Fokuslar ordinata o‘qida yotganda grafigi



shaklda bo‘lib, uning tenglamasi $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ yoki $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$,

a-haqiqiy yarimo‘q uzunligi, b- mavhum yarimo‘q uzuligi. Assimptotalari u=$\pm\frac{b}{a}x$ bo‘lgan.

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbola tenglamasi bo‘yicha uni quyidagicha chiziladi:

1. *a* va b yarim o‘qlar topiladi va ular bo‘yicha $A_1(-a;0)$, $A_2(a;0)$, $B_1(0;-b)$, $B_2(0;b)$ nuqtalar belgilanadi.

2. Tomonlari 2a va 2b bo‘lib, markazi koordinatalar boshida bo‘lgan to‘g‘ri to‘rtburchak yasaladi.
3. Bu to‘rtburchakning diogannallari, ya’ni giperbola assimptotalari yasaladi.
4. $A_1(-a;0)$ va $A_2(a;0)$ uchlardan o‘tuvchi va assimptotalarga yaqinlashib boruvchi egri chiziqlar chiziladi.
5. $|MF_2| = r_2$ - o‘ng fokal radius-vektori $r_2=a$ - ex, $|MF_1| = r_1$ - chap fokal radius-vektor r_1=ex+a formula bilan topiladi.

Masala: Giperbolaning ekstsentrisiteti $\sqrt{2}$ ga teng va M($\sqrt{3};\sqrt{2}$) nuqtadan o‘tuvchi giperbola tenglamasi tuzilsin va chizma yasalsin.

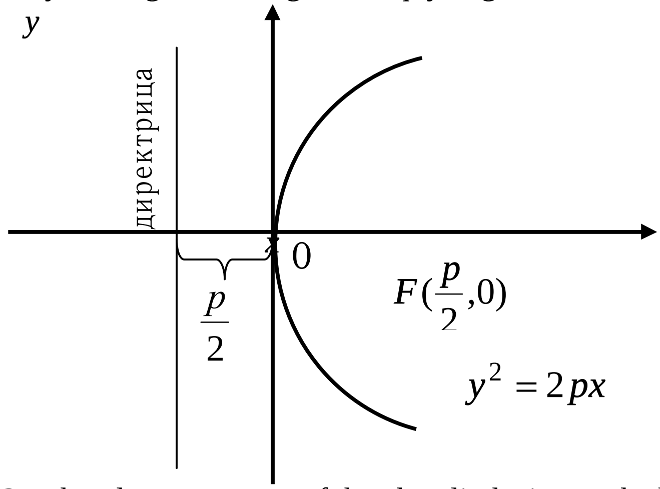
YeCHISh: Ekstsentrisitet ta’rifiga ko‘ra $\frac{c}{a}=\sqrt{2}$, yoki $s^2=2a^2$. Lekin $s^2=a^2+b^2$, demak $a^2+b^2=2a^2$ yoki $a^2=b^2$, ya’ni giperbola teng tomonli M nuqtaning giperbolada yotganligidan

$$\frac{(\sqrt{3})^2}{a^2}-\frac{(\sqrt{2})^2}{b^2}=1 \Leftrightarrow \frac{3}{a^2}-\frac{2}{b^2}=1 \Leftrightarrow \frac{3}{a^2}-\frac{2}{a^2}=1 \Leftrightarrow \frac{1}{a^2}=1 \Leftrightarrow a^2=1$$

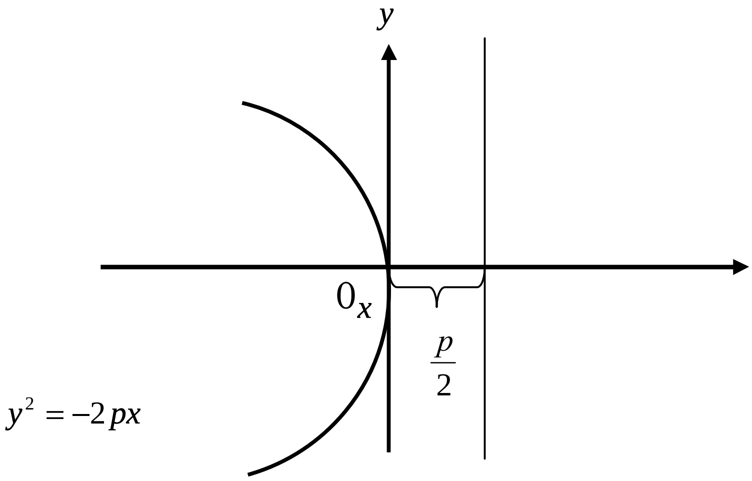
demak $\frac{x^2}{1}-\frac{y^2}{1}=1$ yoki $x^2 - y^2=1$, a=1, b=1 larga asosan tomonlari 2 va 2 bo‘lgan simmetriya markazi koordinatalar boshida bo‘lgan giperbolaning asosiy to‘rtburchagi kvadratni yasab, uning diogonallari- giperbolaning assimptotalarini o‘tkazamiz va giperbolani yasaymiz.

PARABOLA

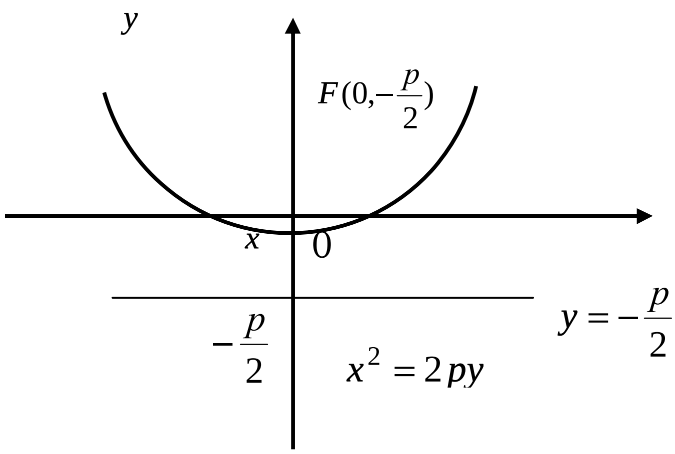
Ixtiyoriy nuqtasidan fokuslar deb ataluvchi berilgan nuqtagacha va direktrissa deb ataluvchi berilgan to‘g‘ri chiziqqacha masofalari bir xil bo‘lgan nuqtalarning geometrik o‘rni tekislikda **parabola** deb ataladi (fokus direktrissada yotmaydi). OX o‘qi fokusdan o‘tib, direktrissaga perpendikulyar bo‘lganda uning shakli quyidagicha bo‘ladi:

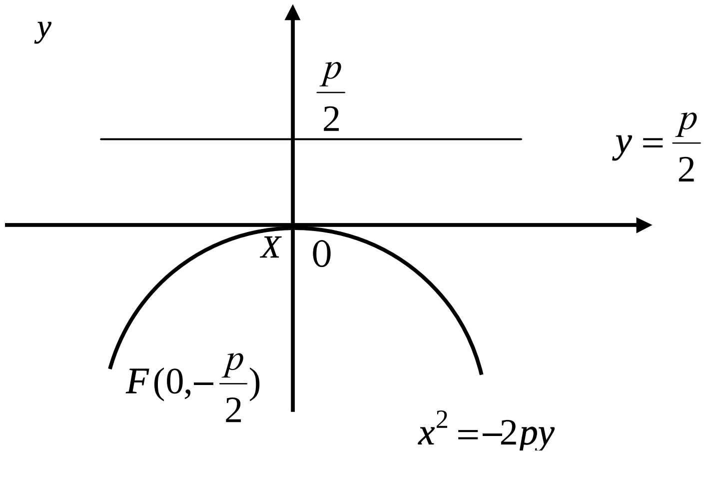


Tenglamasi y^2=2rx, bunda r- parametr - fokusdan direktrissagacha bo‘lgan masofa. r>0 da parabolaning uchi koordinatalar boshida, simmetriya o‘qi OX va tarmoqlari o‘ngga qaragan. Tenglamasi y^2 = - 2rx (r>0) bo‘lgan parabolaning uchi koordinatalar boshida simmetriya o‘qi OX va tarmoqlari chapga qaragan.



OY o‘qi fokusdan o‘tib, direktrissa perpendikulyar bo‘lgan holda x^2=2ru parabolaning tarmoqlari yuqoriga qaragan, x^2 = - 2ru parabola- niki esa pastga qaragan, har ikki holda ham simmetriya o‘qi OY va uchi koordinatalar boshida yotadi.

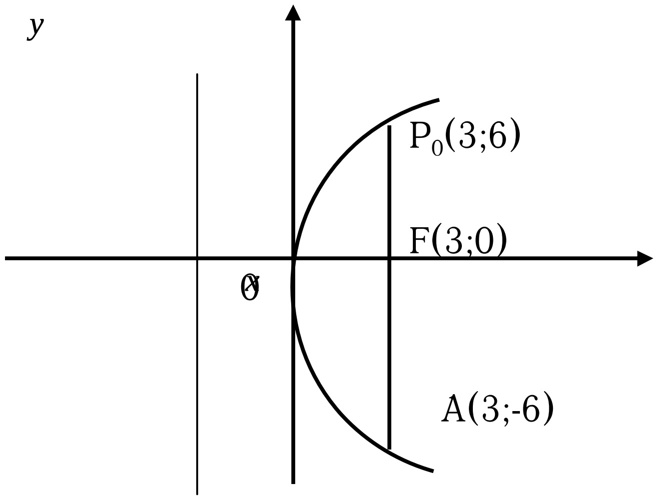




Masala. u^2=12x parabola berilgan. Fokusdan o‘tib, OY o‘qqa perpendikulyar bo‘lgan vektor uzunligi topilsin, parabola yasalsin.

YeCHISh: Shartga ko‘ra vatar fokusdan o‘tadi va parabola o‘qiga perpendikulyar, shuning uchun vatarning parabola bilan kesishgan nuqtalari abtsissasi fokus abtsissasi bilan bir xil bo‘ladi. Parabola tenglamasidan fokus koordinatalarini topamiz.

u^2=12x, 2r=12 $\Rightarrow \frac{p}{2}=3 \Rightarrow F(3,0)$. Vatarning parabola bilan kesishgan nuqtalarini topish uchun parabola tenglamasiga x=3 ni qo‘yamiz:



Demak kesishgan nuqtalar R$_0$(3;6) va A(3;-6). Vatar uzunligi R$_0$ A=2FR$_0$= $2\cdot 6$ =12

3-mashg‘ulot.

CHiziqli tenglamalar sistemasini echish uchun Gauss va Kramer usuli.

Misol. Berilgan tenglamalar sistemasini kami bilan ikki xil usulda eching.

$$\begin{cases} 2x_1+x_2-x_3=1 \\ 3x_1+2x_2-2x_3=1 \\ x_1-x_2+2x_3=5 \end{cases}$$

YeCHISh. I. **Gauss usuli** (noma’lumlarni ketma-ket yo‘qotish usuli). Birinchi tenglamani 2 ga bo‘lamiz.

$$\begin{cases} x_1+0{,}5x_2-0{,}5x_3=0{,}5 \\ 3x_1+2x_2-2x_3=1 \\ x_1-x_2+2x_3=5 \end{cases}$$

Endi hosil bo‘lgan sistemada 1 chi tenglamani 3 ga ko‘paytirib 2-chi tenglamadan va I chi tenglamani o‘zini 3 tenglamadan ayiramiz.

$$\begin{cases} x_1+0{,}5x_2-0{,}5x_3=0{,}5 \\ 0{,}5x_2-0{,}5x_3=-0{,}5 \\ -1{,}5x_2+2{,}5x_3=4{,}5 \end{cases}$$

Ikkinchi tenglamaning barcha hadlarini 0,5 ga bo‘lib

$$\begin{cases} x_1 + 0{,}5x_2 - 0{,}5x_3 = 0{,}5 \\ x_2 - x_3 = -1 \\ -1{,}5x_2 + 2{,}5x_3 = 4{,}5 \end{cases}$$

sistemani hosil qilamiz va shu ikkinchi tenglamani -1,5 ga ko‘paytiramiz va uchinchi tenglamadan ayiramiz va natija ushbu

$$\begin{cases} x_1 + 0{,}5x_2 - 0{,}5x_3 = 0{,}5 \\ \qquad x_2 - x_3 = -1 \\ \qquad\quad x_3 = 3 \end{cases}$$

sistemani hosil qilamiz. Bundan esa ketma-ket quyidagilarni topamiz.

$$x_3 = 3 \quad ; \quad x_2 = -1 + x_3 = -1 + 3 = 2;$$
$$x_1 = 0{,}5 = 0{,}5x_2 + 0{,}5x_3 = 0{,}5 - 1 + 1{,}5 = 1, \quad x_1 = 1$$

Kramer usuli. Noma’lumlar oldida turgan koeffitsentlardan tuzilgan asosiy determinantni tuzamiz va uni hisoblaymiz.

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 3 & 2 & -2 \\ 1 & -1 & 2 \end{vmatrix} = 8 - 2 + 3 + 2 - 4 - 6 = 1$$

Endi yordamchi $\Delta_{x_1}, \Delta_{x_2}, \Delta_{x_3}$ determinantlarni tuzib hisoblaymiz

$$\Delta_{x_1} = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & -2 \\ 5 & -1 & 2 \end{vmatrix} = 4 - 10 + 1 + 10 - 2 - 2 = 1,$$

$$\Delta_{x_2} = \begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 5 & 2 \end{vmatrix} = 4 - 2 - 15 + 1 + 20 - 6 = 2,$$

$$\Delta_{x_3} = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & -1 & 5 \end{vmatrix} = 20 + 1 - 3 + 2 - 15 + 2 = 3.$$

Kramer formulasiga asosan

$$x_1 = \frac{\Delta_{x_1}}{\Delta} = \frac{1}{1} = 1, x_2 = \frac{\Delta_{x_2}}{\Delta} = \frac{2}{1} = 2, x_3 = \frac{\Delta_{x_3}}{\Delta} = \frac{3}{1} = 3$$

qiymatlarni hisoblab topamiz.

4-mashg'ulot .
Funktsiya aniqlanish sohasini topish.

Misol. $y=\sqrt{1-x^2}+\log_3 x$
funktsiyaning aniqlanish sohasini toping.

YECHISH. Mazkur funktsiya ikkita funktsiya yig'indisidan iborat. Ulardan birini aniqlanish sohasi $1-x^2\geq 0$ munosabatni qanoatlantiradigan barcha haqiqiy sonlar to'plamidan iborat. Ya'ni, M_1 to'plam $-1\leq x\leq 1$ tengsizlikni qanoatlantiradigan barcha x nuqtalar to'plamidir. Shunday qilib, $M_1=[-1,1]$

Ikkinchi qo'shiluvchi $\log_3 x$ funktsiyasining aniqlanish sohasi $M_2=(0,+\infty)$ intervaldan iborat. Demak, berilgan funktsiyaning aniqlanish sohasi barcha $x\in M_1$, $x\in M_2$ nuqtalardan iborat M to'plamdir, yoki $Dy=M_1\cap M_2=(0,1]$ bo'ladi.

AMALIY MAShG'ULOTLAR UCHUN MISOL VA MASALALAR

I-topshiriq.

Masalalardan ABC uchburchak uchlarining koordinatalari berilgan.
ABC uchburchakda қuydagilar aniқlansin:
1) AB tomonining uzunligini toping.
2) AB va BC tomonlarining tenglamasini tuzing.
3) B burchakni radianlarda hisoblang.
4) Uchburchakning C uchidan o'tib AB tomonga parallel, perpendikulyar bo'lgan to'g'ri chiziq tenglamalarini tuzing.
5) Shaklni chizing va ABC uchburchak yuzasini hisoblang.

1.1. A (-5;-3)	B (4; -12),	C (8;0)
1.2. A (-5; 7)	B (7; -2),	C (11;20)
1.3. A (-12;-1),	B (0; -10),	C (4; 12)
1.4. A (-7; 4),	B (5; -5),	C (3; 9)
1.5. A (0; 3),	B (12;-6),	C (10; 8)
1.6. A (-5; 9),	B (7; 0),	C (5; 14)
1.7. A (-10; 5),	B (2 ; -4),	C (0; 10)
1.8. A (6; 8),	B (6; -1),	C (4; 13)
1.9. A (-2; 7),	B (10; -2),	C (8; 12)
1.10. A (-3; 10),	B (9; 1),	C (7;15)

2-topshiriq.

Ellipsga doir masalalarni eching.
2.1. O'qlari $2a=10$,$2b=4$ bo'lgan, fokuslari OY o'qida yotuvchi ellips tenglamasi tuzilsin.
2.2. Fokuslari orasidagi masofasi 10 ga teng bo'lgan (ular OX o'qida yotadi) va katta o'qi 12 ga teng bo'lgan ellips tenglamasini tuzing.
2.3 Yarimo'qlari yig'indisi 25, fokuslarini koordinatalari ($\pm 5;0$) bo'lgan ellips tenglamasi tuzilsin.
2.4 Fokuslari OX o'qida yotib ($2;2\sqrt{2}$) nuqtadan o'tuvchi va kichik o'qi 6 ga teng bo'lgan ellips tenglamasi tuzilsin.

2.5.($\sqrt{2}$;2) va (2;$\sqrt{3}$) nuqtalardan o‘tuvchi fokuslari OX o‘qida yotuvchi ellips tenglamasi tuzilsin.

2.6. $\frac{x^2}{25}+\frac{y^2}{9}=1$ va $\frac{x^2}{16}+\frac{y^2}{81}=1$ ellipsning uchlari koordinatalarini va o‘qlari uzunliklarini toping.

2.6. $\frac{x^2}{12}+\frac{y^2}{3}=1$ va $\frac{x^2}{10}+\frac{y^2}{26}=1$ ellipsning fokuslari koordinatalarini va fokuslari orasidagi masofani toping.

2.7. $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellipsda shunday nuqta topingki, uning fokal radius vektorlari ayirmasi 6,4 ga teng bo‘lsin.

2.8. $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellipsning chap fokusi va quyi uchidan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

2.9. $x+5=0$ to‘g‘ri chiziqda shunday nuqta topingki, bu nuqta $\frac{x^2}{20}+\frac{y^2}{4}=1$ ellipsning chap fokusidan va yuqori uchidan baravar uzoqlikda bo‘lsin.

3-topshiriq.

Giperbolaga doir misollarni eching:

3.1. Fokuslari OX o‘qida yotgan, haqiqiy o‘qi 12 va fokuslari orasidagi masofa 20 bo‘lgan giperbola tenglamasi tuzilsin va chizma yasalsin.

3.2.Fokuslari OX da yotgan, mavhum o‘qi 8ga va ekstsentrisiteti $\frac{3\sqrt{5}}{5}$ ga teng bo‘lgan giperbola tenglamasi tuzilsin va chizma yasalsin.

3.3.Agar: 1) yarimo‘qlar yig‘indisi 7 va fokuslar orasidagi masofa 10 ga teng bo‘lsa, 2) yarimo‘qlar ayrimasi 4 va fokuslar orasidagi masofa 40 bo‘lsa giperbola tenglamasi tuzilsin va chizma yasalsin.

3.4.Fokuslari ($\pm$5;0), assimptotalari $y=\pm\frac{4}{3}x$ bo‘lgan giperbola tenglamasi tuzilsin va chizma yasalsin.

3.5.Fokuslar OY o‘qida bo‘lib, (-$\sqrt{3}$;-$\sqrt{5}$) nuqtadan o‘tuvchi teng tomonli bo‘lgan giperbola tenglamasi tuzilsin va yasalsin.

3.6. $\frac{x^2}{9}-\frac{y^2}{16}=-1$ giperbolaning uchlari, fokuslari va asimptotalari topilsin.

3.7.Giperbolaning assimptotalari orasidgi burchak 60^0. Uning ekstsentrisiteti topilsin.

3.8. $\frac{x^2}{64}-\frac{y^2}{36}=1$ giperbolaning chap tarmog‘ida shunday nuqta topingki, o‘ng fokal radius vektori 18 ga teng bshlsin.

3.9.Agar giperbolaning ekstsentrisiteti 2ga teng bo‘lsa va fokuslari $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellipsning fokuslari bilan ustma-ust tushsa giperbolaning tenglamasi qanday ko‘rinishda bo‘ladi.

3.10.M(0;-1) nuqta va $3x^2$-$4y^2$=12 giperbolaning uchidan to‘g‘ri chiziq o‘tkazilgan. Bu to‘g‘ri chiziqning giperbola bilan kesishgan ikkinchi nuqtasi topilsin.

4-topshiriq.

Misollarda parabolaga doir misollarni eching:

4.1.y^2=16x parabola bilan 2x-y+2=0 to‘g‘ri chiziq kesishgan nuqtalari topilsin.

4.2.y^2=12x va x^2=12y parabolalarning kesishgan nuqtalari topilsin.

4.3.Uchi koordinatalar boshida, simmetriya o‘qi OY da bo‘lgan 1 va 3 chorak bissektrisasidan $8\sqrt{2}$ uzunlikdagi vatar kesuvchi parabola tenglamasi yozilsin.

4.4.Fokusi 4x-3y-4=0 to‘g‘ri chiziqning OX o‘qi bilan kesishgan nuqtasida bo‘lgan parabola tenglamasini yozing.

4.5. y^2=8x parabolada shunday nuqtani topingki, uning direktrissadan parabolagacha bo‘lgan masofasi 4ga teng bo‘lsin.

4.7.Uchi koordinatalar boshida, simmetriya o‘qi OX bo‘lgan va y=x to‘g‘ri chiziqdan $4\sqrt{2}$ uzunlikdagi vatar ajratuvchi parabola tenglamasi tuzilsin.

4.8.y^2=2x parabola koordinatalar boshidan o‘tuvchi to‘g‘ri chiziqdan 3/4 uzunlikdagi vatar ajratadi. Shu to‘g‘ri chiziq tenglamasi tuzilsin.

4.9.Simmetriya o‘qiga perpendikulyar bo‘lib, uchi va fokusi orasidagi masofani teng ikkiga bo‘luvchi vatarning uzunligi 1 ga teng bo‘lgan parabola tenglamasi tuzilsin.

4.10y^2=32x parabolada shunday nuqtani topingki, undan 4x+3y+10=0 to‘g‘ri chiziqqacha masofa 2ga teng bo‘lsin.

4.11.y^2=3x parabola bilan y=x to‘g‘ri chiziqlarni kesishish nuqtalarining koordinatalarini toping.

5-topshiriq.

Misollarda chiziqli tenglamalar sistemasini har birini kamida 2 xil usul bilan eching.

5.1. $\begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 11 \end{cases}$ 5.2. $\begin{cases} 2x_1 - x_2 - x_3 = 4 \\ 3x_1 + 4x_2 - 2x_3 = 11 \\ 3x_1 - 2x_2 + 4x_3 = 11 \end{cases}$

5.3. $\begin{cases} 7x_1 - 5x_2 = 31 \\ 4x_1 + 11x_2 = -43 \\ 2x_1 + 3x_2 + 4x_3 = -20 \end{cases}$ 5.4. $\begin{cases} x_1 - x_2 + 3x_3 = -1 \\ 2x_1 - x_2 + 2x_3 = -4 \\ 4x_1 + x_2 + 4x_3 = -2 \end{cases}$

5.5. $\begin{cases} x_1 - 4x_2 - 2x_3 = -3 \\ 3x_1 + x_2 + x_3 = 5 \\ -3x_1 + 5x_2 + 6x_3 = 7 \end{cases}$ 5.6. $\begin{cases} 4x_1 - 3x_2 + 2x_3 = 9 \\ 2x_1 + 5x_2 - 3x_3 = 4 \\ 5x_1 + 6x_2 - 2x_3 = 18 \end{cases}$

5.7. $\begin{cases} x_1 + x_2 - x_3 = 1 \\ 8x_1 + 3x_2 - 6x_3 = 2 \\ -4x_1 - x_2 + 3x_3 = -3 \end{cases}$ 5.8. $\begin{cases} x_1 - 2x_2 + 3x_3 = 6 \\ 2x_1 + 3x_2 - 4x_3 = 20 \\ 3x_1 - 2x_2 - 5x_3 = 6 \end{cases}$

5.9. $\begin{cases} 3x_1 + 4x_2 + 3x_3 = -3 \\ 2x_1 - x_2 + 3x_3 = -1 \\ x_1 + 5x_2 + x_3 = 0 \end{cases}$ 5.10 $\begin{cases} x_1 + 2x_2 + 4x_3 = 31 \\ 5x_1 + x_2 + 2x_3 = 20 \\ 3x_1 - x_2 + x_3 = 10 \end{cases}$

MATEMATIK ANALIZ VA DIFFERENTSIAL XISOB

5-mashg‘ulot.
Limitlar nazariyasi

Misol. Lopital qoidasidan foydalanmay ushbu limitlarni hisoblang.

$$\lim_{x\to\infty}\frac{5x^2+6x+1}{6x^2+4x+2} \quad \text{va} \quad \lim_{x\to 0}\frac{1-\cos x}{x^2}$$

YECHISH. Birinchi misolda $\frac{\infty}{\infty}$, ikkinchisida esa $\frac{0}{0}$ ko‘rinishdagi aniqmaslikka olib keladigan limitlar ekani ko‘rinib turibdi. Bularni to‘g‘ridan-to‘g‘ri hisoblab bo‘lmaydi. Shuning uchun bu aniqlanmasliklarni «ochishga», ya’ni funktsiya sur’ati va maxrajida ba’zi bir shakl almashtirishlar bajarishga to‘g‘ri keladi. Birinchi misolda kasrning sur’at va maxrajini x^2 ga bo‘lamiz. Natijada:

$$\lim_{x\to\infty}\frac{5x^2+6x+1}{6x^2+4x+2}=\lim_{x\to\infty}\frac{5+\frac{6}{x}+\frac{1}{x^2}}{6+\frac{4}{x}+\frac{2}{x^2}}=\frac{\lim\limits_{x\to\infty}\left(5+\frac{6}{x}+\frac{1}{x^2}\right)}{\lim\limits_{x\to\infty}\left(6+\frac{4}{x}+\frac{2}{x^2}\right)}=\frac{5}{6}$$

ekanini hisoblab topamiz. Ikkinchi misolda esa suratida $1-\cos x=2\sin^2\frac{x}{2}$ shakl almashtirish ishlatamiz va $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$ ajoyib limitdan foydalanamiz, ya’ni:

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}=\lim_{x\to 0}\left[\frac{1}{2}\cdot\frac{\sin\frac{x}{2}}{\frac{x}{2}}\cdot\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right]=\frac{1}{2}\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\cdot\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}=\frac{1}{2}\cdot 1\cdot 1=\frac{1}{2}$$

6-mashg’ulot.
Funktsiyani uzluksizligi va limiti.

Misol. $y=\frac{x}{x-4}$ funktsiya - ax ning $x_1=4$ va $x_2=5$ qiymatlari berilgan. Argumentning berilgan qiymatlarida funktsiyaning:

1. Uzluksiz yoki uzilishga ega ekanini aniqlang.
2. Uzilish nuqtalarida bir tomonli limitlarni hisoblang.
3. Sxematik chizmasini chizing.

YECHISH. $y=\frac{x}{x-4}$ funktsiya x=4 nuqtada uzilishga ega. CHunki funktsiya bu nuqtada aniqlanmagan.

Uni o‘ng va chap limitlarini hisoblaymiz.

$$\lim_{x\to 4-0}\frac{x}{x-4}=-\infty, \qquad \lim_{x\to 4+0}\frac{x}{x-4}=+\infty,$$

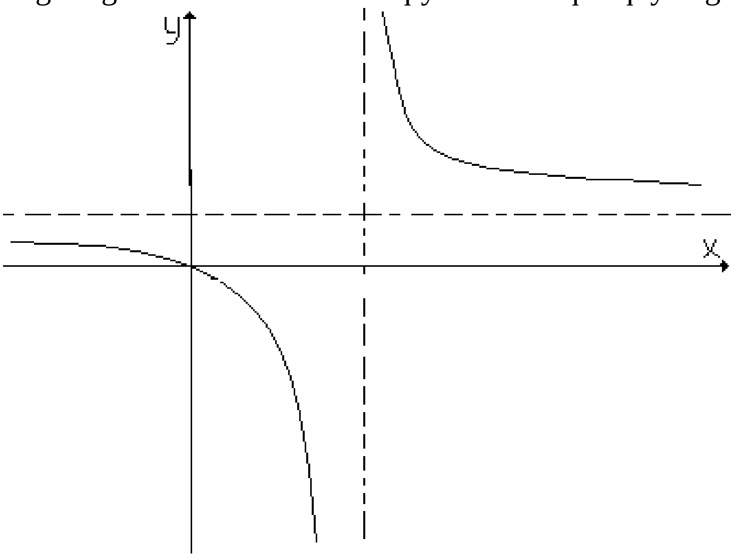
Demak, chapdan ham o‘ngdan ham funktsiya chekli limitga ega emas ekan. Bundan esa bu uzilishni II-tur uzilish ekanligi kelib chiqadi. Endi funktsiyaning $x_2=5$ nuqtadagi xususiy va limitik qiymatlarini topamiz.

$$y(5)=\frac{5}{5-4}=5$$

$$\lim_{x\to 5}\frac{x}{x-4}=\frac{\lim\limits_{x\to 5}x}{\lim\limits_{x\to 5}(x-4)}=\frac{5}{1}=5$$

Demak, funktsiya x=5 nuqtada uzluksiz.

Funktsiya grafigining sxematik chizmasini qiymatlari orqali quyidagicha chizamiz.



7 mashg‘ulot.
Funktsiya xosilasi.

Misol. $y=\ln tg\frac{x}{2}$ funktsiyaning hosilasini va $f'\left(\frac{p}{3}\right)$, $f'\left(\frac{p}{2}\right)$, $f'\left(\frac{p}{4}\right)$ qiymatlarini aniqlang.

YECHISH. Murakkab funktsiyadan hosila olish qoidasiga binoan:

$$y'=\frac{1}{tg\frac{x}{2}}\cdot\left(tg\frac{x}{2}\right)'=\frac{1}{tg\frac{x}{2}}\cdot\frac{1}{\cos^2\frac{x}{2}}\cdot\left(\frac{x}{2}\right)'=\frac{1}{2tg\frac{x}{2}\cdot\cos^2\frac{x}{2}}=\frac{1}{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\frac{1}{\sin x}$$

a) $f'\left(\frac{p}{3}\right)=\frac{1}{\sin\frac{p}{3}}=\frac{1}{\frac{\sqrt{3}}{2}}=\frac{2}{\sqrt{3}}=\frac{2}{3}\sqrt{3}$ b) $f'\left(\frac{p}{2}\right)=\frac{1}{\sin\frac{p}{2}}=\frac{1}{1}=1$

v) $f'\left(\frac{p}{4}\right)=\frac{1}{\sin\frac{p}{4}}=\frac{1}{\frac{\sqrt{2}}{2}}=\frac{2}{\sqrt{2}}=\sqrt{2}$

8-mashg'ulot.
Funktsiyani hosila yordamida to'la tekshirish.

Misol. $y=\frac{x^3+4}{x^2}$ funktsiyaning differentsial hisob usullari bilan tekshiring va bu tekshirish natijalaridan foydalanib uning grafigini yasang.

YECHISH. Tekshirishni quyidagi tartibda olib boramiz.

1. Berilgan funktsiya OX o'qining x=0 nuqtasidan tashqari barcha nuqtalarida aniqlangan, ya'ni

$$Dy=(-\infty;\ 0)\cup(0;\ +\infty)$$

2. Funktsiya juft ham emas, toq emas, davriy ham emas, chunki

$$f(-x)\neq f(x),\ f(-x)\neq -f(x) \text{ va } f(x+T)\neq f(x),\ T\neq 0$$

3. Egri chiziq abstsissa o'qi bilan $x=-\sqrt[3]{4}$ nuqtada kesishadi.
4. Aniqki, $\lim\limits_{x\to 0} y=\infty$, bundan esa x=0 chiziq, ya'ni OY o'q funktsiya grafigning vertikal asimptotasi ekani kelib chiqadi. Funktsiyaning og'ma asimptotasini aniqlaymiz.

$$\kappa=\lim_{x\to 0}\frac{f(x)}{x}=\lim_{x\to 0}\frac{x^3+4}{x^3}=\lim_{x\to 0}\left(1+\frac{4}{x^3}\right)=1$$

$$b=\lim_{x\to 0}[f(x)-kx]=\lim_{x\to 0}\left[\frac{x^3+4}{x^2}-x\right]=\lim_{x\to 0}\frac{4}{x^2}=0$$

Demak, og'ma asimptota tenglamasi y=kx+b formulaga asosan y=x. ekan.

5. Funktsiyaning ekstremumlari, o'sish va kamayish intervallari.

$$y=x+\frac{4}{x^2},\quad y'=1-\frac{8}{x^3}=\frac{x^3-8}{x^3}$$

Funktsiyaning kritik nuqtalarini topamiz. Buning uchun y`=0 tenglamani echamiz.

$$x^3-8=0,\qquad (x-2)(x^2+2x+4)=0,$$

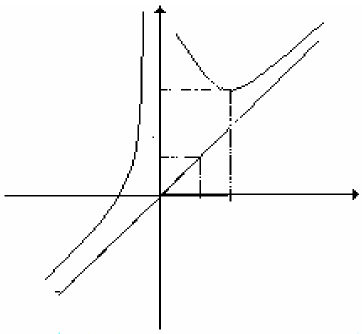
x-2=0 $\Rightarrow$ x=2; $x^2+2x+4=0$ kvadrat tenglama haqiqiy ildizga ega emas, chunki $D=b^2-4ac=2^2-4\cdot1\cdot4=-12<0$. Demak, x=2 kritik nuqta. x=0 esa funktsiyaning uzilish nuqtasi. x=0 va x=2 nuqtalar sonlar o'qini $(-\infty;\ 0)$, (0; 2), $(2;\ +\infty)$ intervallarga ajratadi. $(-\infty;0)$ va $(2;\ +\infty)$ intervallarda y`>0 (masalan, y`(-1)=9>0, y`(3)=$\frac{19}{27}$>0), (0; 2) intervalda y`<0 (masalan, y`(1)=-7<0) bo'ladi. Bundan birinchi va uchinchi oraliqda funktsiya o'suvchi, ikkinchi oraliqda esa kamayuvchi bo'lishi kelib chiqadi. Funktsiyaning ikkinchi tartibli hosilasini kritik nuqtda ishorasini aniqlaymiz.

$$y''=\frac{24}{x^4},\qquad y''(2)=\frac{3}{2}>0$$

Demak, funktsiya x=2 nuqtada minimum qiymatga ega.

6. $x \in Dy$ uchun y``>0 bo‘lganligidan egri chiziq botiq bo‘ladi. Egilish nuqta mavjud emas.

7. Tekshirish natijalariga asoslanib funktsiya grafigini yasaymiz.



AMALIY MAShG‘ULOTLAR UCHUN MISOL VA MASALALAR

1-topshiriq.

Misollarda berilgan funktsiyaning aniqlanish sohasini toping va juft yoki toqligini tekshiring.

1.1 $f(x)=\sqrt{9-x^2}+\dfrac{1}{x-3}$; 1.2. $f(x)=\sqrt{x^2-16}+\lg(x-4)$;

1.3. $f(x)=\dfrac{x}{x^2-9}+\sqrt{x}$; 1.4. $f(x)=\sqrt{x+2}+\sqrt[4]{2-x}$;

1.5. $f(x)=\sqrt{x-1}+\sqrt{4-x}$; 1.6. $f(x)=1+\ln x+\sqrt{x^2-25}$;

1.7. $f(x)=\dfrac{2x}{x^2+2x-15}+\sqrt[3]{x}$; 1.8. $f(x)=\dfrac{1}{x^2-4}+\sqrt{x^2-4}$;

1.9. $f(x)=\lg(x-4)+\dfrac{1}{x-4}$; 1.10. $f(x)=\dfrac{x}{x+4}+\lg(x+4)$.

2-topshiriq.

Misollarda ko‘rsatilgan limitlarni hisoblang.

2.1. *а*) $\lim\limits_{x\to 1}\dfrac{x^2-3x+3}{x^2-4x+3}$, *б*) $\lim\limits_{x\to\infty}\dfrac{n^2-5n+3}{2n^2+6n-9}$,

в) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{tg 5x}$, *г*) $\lim\limits_{x\to\infty}\left(\dfrac{x-1}{x}\right)^{x-1}$.

2.2. *а*) $\lim\limits_{x\to 1}\dfrac{x^6-1}{x^3-1}$, *б*) $\lim\limits_{x\to\infty}\dfrac{5-2x-3x^2}{x^2+x+3}$,

в) $\lim\limits_{x\to 0}\dfrac{tg\, ax}{\sin bx}$, *г*) $\lim\limits_{x\to\infty}\left(\dfrac{x+2}{x}\right)^{x+4}$.

2.3. *а*) $\lim\limits_{x\to 3}\dfrac{x^2-2x-3}{x^2-5x+6}$, *б*) $\lim\limits_{x\to\infty}\dfrac{2x^3+2x+1}{3x^2-4x+2}$, в) $\lim\limits_{x\to\infty}\left(\dfrac{4n+1}{4n}\right)^{n-5}$, г) $\lim\limits_{x\to 0}\dfrac{\sin px}{\sin 12x}$.

2.4. а) $\lim\limits_{n\to\infty}\dfrac{2n^3+n-4}{3+n-4n^3}$, б) $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{x-1}$

в) $\lim\limits_{x\to 0}\dfrac{tg^2 x}{5x^2}$, г) $\lim\limits_{x\to\infty}\left(\dfrac{2x-2}{2x}\right)^{-x}$,

2.5. а) $\lim\limits_{x\to 2}\dfrac{x^2-4x+4}{x^2-4}$, б) $\lim\limits_{x\to\infty}\dfrac{4x^2-10x+8}{10-11x-6x^2}$,

в) $\lim\limits_{x\to\infty}\left(\dfrac{2x+2}{2x}\right)^{\frac{x}{2}}$, г) $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{8x^2}$.

2.6. а) $\lim\limits_{x\to -2}\dfrac{x^3+8}{x^2+x-2}$, б) $\lim\limits_{x\to\infty}\left(\dfrac{x+2}{x-2}\right)^2$,

в) $\lim\limits_{x\to 0}\dfrac{\sin 10x}{tg 30x}$, г) $\lim\limits_{x\to\infty}\dfrac{2x^2-3x+1}{3x^2+4x+4}$.

2.7. а) $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{x-2}$, б) $\lim\limits_{n\to\infty}\dfrac{10n+15n^2+100n^3}{6n^3+9n-100}$,

в) $\lim\limits_{x\to\infty}\left(\dfrac{8x-5}{8x}\right)^{-x}$, г) $\lim\limits_{x\to 0}\dfrac{\ln(1+90x)}{x}$.

2.8. а) $\lim\limits_{x\to\infty}\dfrac{3x^2-5x-2}{2x^2-x-6}$ б) $\lim\limits_{x\to\infty}\dfrac{2x^2-3x+1}{3x^2+4x+4}$

в) $\lim\limits_{n\to\infty}\left(\dfrac{9n+10}{9n}\right)^{-n}$, г) $\lim\limits_{x\to 0}\left[\dfrac{1}{x}\ln(1+80x)\right]$.

2.9. а) $\lim\limits_{x\to -1}\dfrac{x^2-10x-11}{x^2-1}$, б) $\lim\limits_{x\to\infty}\dfrac{(x-2)^2}{x^2+8x-9}$,

в) $\lim\limits_{n\to\infty}\left(\dfrac{n-4}{n}\right)^{n+8}$, г) $\lim\limits_{x\to 0}\dfrac{1+\cos x}{x^2}$.

2.10. а) $\lim\limits_{x\to 1}\dfrac{x^2-3x+2}{x-1}$, б) $\lim\limits_{x\to\infty}\dfrac{x^2-10x+11}{(x+2)^2}$,

в) $\lim\limits_{x\to 0}\dfrac{tg^2 x}{x\sin 2x}$, г) $\lim\limits_{n\to\infty}\left(\dfrac{8n-4}{3n}\right)^{8n}$.

3-topshiriq.

Misollarda $y=f(x)$ funktsiya va x argumentning ikkita x_1, x_2 qiymati berilgan. Argumentning berilgan qiymatlarida funktsiyaning
1) uzluksiz yoki uzulishga ega ekanini aniqlang:
2) uzulish nuqtalarida bir tomonli limitlarni hisoblang;
3) Sxematik chizmasini chizing.

3.1. $y=\dfrac{4x}{x-1}, \qquad x_1=1, \qquad x_2=3$

3.2. $y=\dfrac{4x}{x-2}, \qquad x_1=2, \qquad x_2=5$

3.3. $y=\dfrac{4x}{x-4}, \qquad x_1=-2, \qquad x_2=4$

3.4. $y=\dfrac{x}{x-3}, \qquad x_1=-2, \qquad x_2=3$

3.5. $y=\dfrac{5x}{x-5}, \qquad x_1=-1, \qquad x_2=5$

3.6. $y=\dfrac{5x}{x+1}, \qquad x_1=-1, \qquad x_2=3$

3.7. $y=\dfrac{4x}{3+x}, \qquad x_1=-3, \qquad x_2=1$

3.8. $y=\dfrac{4x}{x+2}, \qquad x_1=-2, \qquad x_1=2$

3.9 $y=\dfrac{5x}{x+4}, \qquad x_1=-4, \qquad x_2=4$

3.10 $y=\dfrac{4x}{x+5}, \qquad x_1=-5, \qquad x_2=5$

4-topshiriq.

Misollarda differentsiallashning asosiy qoidalaridan foydalanib, $y=f(x)$ funktsiyaning hosilasini toping, xususiy qiymatlarini hisoblang.

4.1.a) $f(x)=(x-2)^4 \quad ; \qquad f'(0), \quad f'(2), \quad f'(-2), \quad f'(p)$

b) $y=e^{\sqrt{x}}(x^2+3x)$ v) $y=\sqrt{1-x^2}+x\cdot\sin x$

g) $y=\ln\dfrac{x-1}{2x+1}$

4.2. a) $f(x)=(x+1)^3$; $f'(0)$, $f'(1)$, $f'(-1)$, $f'(a+b)$

b) $y=\sqrt{\sin 2x}$, v) $y=e^{3x}\sin x$ g) $y=tg\left(\sqrt{x^2-2}\right)$

4.3. a) $f(t)=t^3-2t-e^{2t}$; $f'(0)$, $f'(1)$, $f'(-1)$, $f'(\ln 2)$

b) *y=lnsinx+e^{2x}* v) $y = arctg\sqrt{x-1}$ g) *y=sin²(2x+3)*

4.4. a) $s(t) = qt^2/2;$ $s'(0),$ $s'(1),$ $s'(\sqrt{2}),$ $s'(5)$

b) $y = (1-\cos x)\sin 2x$, v) $y = \dfrac{1+xarctgx}{\sqrt{1+x}}$, g) $y = x^3 e^{\frac{1}{x}}$

4.5. a) $s(t)=t^2+5t-8;$ $s'(0),$ $s'(-1),$ $s'(a),$ $s'(a+b)$

b) $y=arcsin2x,$ v) $y=x^2tgx,$ g) $y = \sqrt{1+\dfrac{1}{x}}$

4.6. a) $j(t) = 3t^2+4t+1,$ $j'(1),$ $j'\left(\dfrac{1}{2}\right),$ $j'(-1),$ $j'(p)$

b) $y = \sqrt{(x-1)^3}$, v) $y = e^x - 3x^2 \ln x$, g) $y = \dfrac{e^x \sin x}{x}$

4.7. a) $f(j) = \sin j + \cos^2 j;$ $f'\left(\dfrac{p}{4}\right),$ $f'\left(-\dfrac{p}{4}\right),$ $f'\left(\dfrac{p}{2}\right)$

b) $y = \dfrac{e^x+1}{e^x-1}$ v) $y = \arcsin\sqrt{3x}$ g) $y = \ln\dfrac{\sqrt{a^2+x^2}}{\sqrt{a^2-x^2}}$

4.8. a) $j(s) = 2s^3+3s^2-4;$ $j'(0),$ $j'(-1),$ $j(\sqrt{2})$

b) $y = \dfrac{x-\sin x}{\sqrt{x}}$ v) $y = arctgx - \dfrac{1}{x}$ g) $y = \ln\left(e^{2x}+1\right)$

4.9. a) $y = \dfrac{x^3}{e^x}$ b) $y = \cos^2 x^3$

v) $f(x) = 3x^3-5x^2+3;$ $f(0),$ $f'\left(\dfrac{p}{2}\right),$ $f'\left(-\dfrac{p}{2}\right)$

g) $y = \ln(\ln x)$

4.10. a) $j(x) = ctg^2 x + \sin^2 x;$ $j'(0),$ $j'\left(\dfrac{p}{4}\right),$ $j'\left(-\dfrac{p}{4}\right)$

b) $y = \sqrt{1+\sin^2 4x}$, v) $y = arctg^2 x$ g) $y = e^x\sqrt{1-e^{2x}}$

5-topshiriq.

Masalalarda funktsiyalarni differentsial hisob metodlari bilan tekshiring va bu tekshirish natijalaridan foydalanib, ularning grafiklarini yasang. Bu masalalarda funktsiyani tekshiring deganda quyidagi ishlarni bajarish ko‘zda tutiladi.

1. Funktsiyaning aniqlanish sohasini topish.
2. Funktsiyaning juft toqligini, davriyligini aniqlash.
3. Funktsiya grafigining koordinata o‘qlari bilan kesishish nuqtalarini topish.
4. Funktsiya uzluksizligini tekshirish (uzilishga ega bo‘lsa, uzilish nuqtalari va uzilish turini aniqlash) egri chiziq asimptotalarini topish.
5. Funktsiya ekstremumlarini topish, uni o‘sish, kamayish intervallarini aniqlash.
6. Qavariqlik, botiqlik intervallarini aniqlash, egilish nuqtalarini topish.
7. Yuqoridagi chiqarilgan xulosalardan foydalanib, funktsiya grafigini chizing.

5.1. $y=\frac{x^2+1}{x}$ 5.2. $y=\frac{1}{1+x^2}$ 5.3. $y=\frac{(x-1)^2}{1+x^2}$

5.4. $y=\frac{x^3+1}{x^2}$ 5.5. $y=e^{\frac{1}{x}}$ 5.6. $y=\frac{x}{3-x^2}$

5.7 $y=\frac{1}{1-x^2}$ 5.8. $y=\left(\frac{x+1}{x-1}\right)^2$ 5.9. $y=xe^{-x}$

5.10. $y=x\ln x$

INTEGRAL XISOB

9-mashg’ulot.
Boshlang’ich funktsiya va aniqmas integralni hisoblash.

Misol. Quyidagi aniqmas integralni hisoblang.

a) $\int\frac{\sin 2x}{\sqrt{3-\cos^4 x}}dx$ b) $\int\left(2\sin\frac{x}{2}+3\right)^2\cos\frac{x}{2}dx$

v) $\int x\sin xdx$

YeCHISh.

a) $c’s^2x=t$ desak, - 2 c’sx sinx dx = dt; sin2x dx = - dt bo‘ladi, u holda

$$\int\frac{\sin 2x}{\sqrt{3-\cos^4 x}}dx=-\int\frac{dt}{\sqrt{3-t^2}}=-\arcsin\frac{t}{\sqrt{3}}+C=-\arcsin\frac{\cos^2 x}{\sqrt{3}}+C$$

bo‘ladi, bunda S=c’nts

b) $2\sin\frac{x}{2}+3=t$ desak, $\cos\frac{x}{2}dx=dt$ bo‘ladi, u holda

$$\int\left(2\sin\frac{x}{2}+3\right)^2\cos\frac{x}{2}dx=\int t^2dx=\frac{1}{3}t^3+C=\frac{1}{3}\left(2\sin\frac{x}{2}+3\right)^3+C$$ bo‘ladi.

v) $u=x$, $dv=\sin xdx$ desak, $du=dx$, $v=-\cos x$ bo‘ladi.

U holda bo‘laklab integrallash qoidasiga binoan

$$\int x\cdot\sin xdx=x\cdot\cos x+\int\cos xdx=-x\cdot\cos x+\sin x+C$$ bo‘ladi.

Misol. Eng sodda bo‘lgan ratsional kasr ko‘rinishidagi funktsiyalar integrallari uchta tipi quyidagi formulalar yordamida hisoblanadi:

$$\int \frac{A}{x-a}dx = A\ln|x-a| + C$$

$$\int \frac{A}{(x-a)^m}dx = \frac{A}{1-m}\ \frac{1}{(x-a)^{m-1}} + C$$, m - natural son, $m¹1$

$$\int \frac{dx}{x^2+p10+q} = \frac{2}{\sqrt{4q-p^2}} arctg\ \frac{2x+p}{\sqrt{4q-p^2}} + C$$, bunda r^2*-4q < 0*

Masalan, $\int \frac{dx}{2x^2-2x+3}$

Ko‘rinishdagi integralni hisoblash uchun uni uchinchi tipdagi integral ko‘rinishga keltiramiz.

$$\int \frac{dx}{2x^2-2x+3} = \frac{1}{2}\int \frac{dx}{x^2-x+\frac{3}{2}} = \frac{1}{2}\int \frac{dx}{\left(x-\frac{1}{2}\right)^2+\left(\frac{3}{2}-\frac{1}{4}\right)} = \frac{1}{2}\int \frac{d\left(x-\frac{1}{2}\right)}{\left(x-\frac{2}{2}\right)^2+\left(\frac{\sqrt{5}}{2}\right)^2} =$$

$$= \frac{1}{2}\cdot\frac{2}{\sqrt{5}} arctg \frac{x-\frac{1}{2}}{\frac{\sqrt{5}}{2}} + C = \frac{1}{\sqrt{5}} arctg \frac{2x-1}{\sqrt{5}} + C$$

Undan tashqari ratsional kasrlar maxrajini ko‘paytuvchilarga ajratib integrallash usuli ham bor. Bu usulni

$$\int \frac{x^2+2x+6}{(x-1)(x-2)(x-4)}dx$$

ko‘rinishdagi integralni hisoblashda qo‘llaymiz.

Kasr maxrajidagi *(x-1), (x-2), (x-4)* ikki hadlar birinchi darajali bo‘lgani uchun integralni 1-tipdagi integral ko‘rinishiga keltirish mumkin. Ya’ni:

$$\frac{x^2+2x+6}{(x-1)(x-2)(x-4)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-4}$$

A,B,C noma’lum koeffitsientlar deb, ularni aniqlaymiz.

$x^2+2x+6=A(x-2)(x-4)+B(x-1)(x-4)+C(x-1)(x-2)$ bundan esa.

$x^2+2x+6=A(x^2-6x+8)+B(x^2-5x+4)+C(x^2-3x+2)$ o‘ng tomondagi ifodani x ning darajalari bo‘yicha gruppalab chiqsak,

$x^2+2x+6=(A+B+C)x^2+(-6A-5B-3C)x+(8A+4B+2C)$ x - ning bir xil darajalari oldida turgan koeffitsientlarni bir-biriga tenglab,

$$\begin{cases} A+B+C=1 \\ -6A-5B-3C=2 \\ 8A+4B+2C=6 \end{cases}$$

sistemaga kelamiz.

Bu sistemani echib A=3, B=-7, S=5 ekanini topamiz.

Shunday qilib

$$\frac{x^2+2x+6}{(x-1)(x-2)(x-4)}=\frac{3}{x-1}-\frac{7}{x-2}+\frac{5}{x-4}$$

Tenglikni ikkala tomonini integrallab,

$$\int\frac{x^2+2x+6}{(x-1)(x-2)(x-4)}dx=3\int\frac{dx}{x-1}-7\int\frac{dx}{x-2}+5\int\frac{dx}{x-4}=$$

$$=3\ln|x-1|-7\ln|x-2|+5\ln|x-4|+C=\ln\left|\frac{(x-1)^3(x-4)^5}{(x-2)^7}\right|+C$$

ekanini topamiz.

10-mashg‘ulot.

Aniq integralni hisoblash.

<u>Misol</u>. Berilgan aniq integrallarni hisoblang.

a) $\int\limits_0^{x/2}\cos^5 x\cdot\sin 2x dx$ b) $\int\limits_0^1 x\cdot e^{-x}dx$

Yechish. a) $\int\limits_0^{p/2} cos^5 x\cdot\sin 2x dx=\int\limits_0^{p/2}\cos^5 x 2\sin x\cdot\cos x dx=$

$$=2\int\limits_0^{p/2}\cos^6 x\sin x dx=-2\int\limits_0^{p/2}\cos^6 x d(\cos x)=-\frac{2}{7}\cos^7 x\Big|_r^{p/2}=-\frac{2}{7}\left(\cos^7\frac{p}{2}-1\right)=-\frac{2}{7}$$

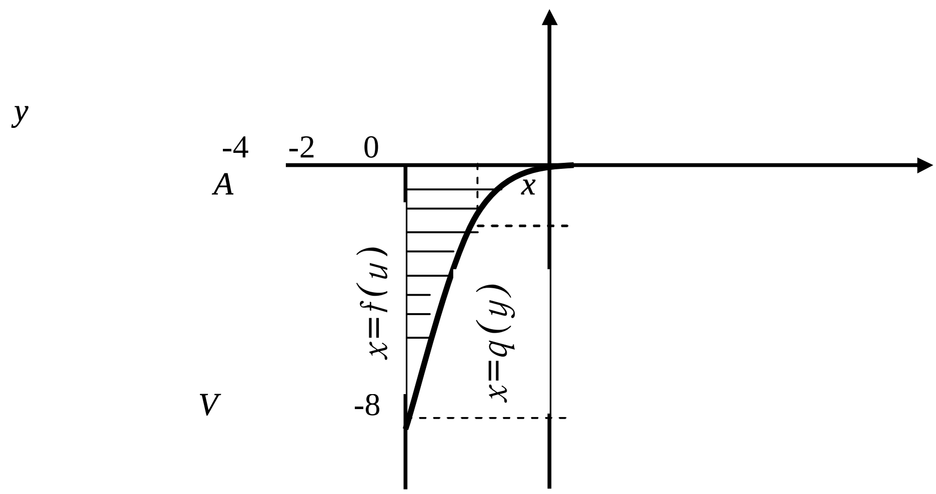
b) $\int\limits_0^1 x\cdot e^{-x}dx$ integralni bo‘laklab integrallaymiz.

u=x, dv=e^{-x}*dx* desak, *du=dx, v=-e*$^{-x}$ ekanini topamiz.

U holda $\int\limits_0^1 x\cdot e^{-x}dx=-xe^{-x}\,/_0^1+\int\limits_0^1 e^{-x}dx=-e^{-1}-e^{-x}\,/_0^1=-2e^{-1}+1=\frac{e-2}{e}$

<u>Misol</u>. $y=x\sqrt{-x},\quad x=-4, y=0$ chiziqlar bilan chegaralangan figuraning 0y o‘qi atrofida aylanishdan hosil bo‘lgan jism hajmini hisoblang.

Yechish. $y=x\sqrt{-x}$



Funktsiyaning grafigi yarim kubik parabolaning pastki tarmog‘idir. OAB figurani ordinata o‘qi atrofida aylanishidan hosil bo‘lgan jismning hajmi

$V=p\int_a^b[f^2(y)-q^2(y)]dy$ formula bilan hisoblanadi. *f(y)=-4* va $\boldsymbol{y=x\sqrt{-x}}$ dan *x=q(y)=-y*$^{2/3}$ ekanligini e’tiborga olsak,

$$\boldsymbol{V}=p\int_{-8}^{0}\left[\boldsymbol{(-4)}^2-\left(-\boldsymbol{y}^{\frac{2}{3}}\right)^2\right]\boldsymbol{dy}=16p\int_{-8}^{0}\boldsymbol{dy}-p\int_{-8}^{0}\boldsymbol{y}^{\frac{4}{3}}\boldsymbol{dy}=16p\cdot 8-\frac{3}{7}p\boldsymbol{y}^{\frac{7}{3}}\Big|_{-8}^{0}=$$

$$=128p+\frac{3}{7}\boldsymbol{(-8)}^{\frac{7}{3}}=128p-\frac{38{,}4}{7}p=73\frac{1}{7}p \text{ (kub.b.)}$$

Demak, $\boldsymbol{V}=73\frac{1}{7}p$ (kub.b.)

AMALIY MAShG‘ULOTLARNI UTKAZISh UCHUN MISOL VA MASALALAR.

I-topshiriq.

Misollarda berilgan aniqmas integrallarni hisoblang

1.1.a) $\int\frac{2x^3}{\sqrt[3]{x^4+1}}dx$; b) $\int x\cdot\cos 2x dx$; v) $\int\frac{dx}{1+\sqrt{x+1}}$.

1.2.a) $\int\frac{\sin x}{3\cos^5 x}dx$; b) $\int x\cdot\sin 2x dx$; v) $\int\frac{x^2dx}{\sqrt{x-1}}$.

1.3.a) $\int\frac{x^4}{4\sqrt{4+x^5}}dx$; b) $\int x\cdot 5^x dx$; v) $\int\frac{dx}{x\sqrt{x+1}}$.

1.4.a) $\int\frac{5\cos x}{\sqrt[3]{\sin^5 x}}dx$; b) $\int x\cdot arctgx dx$; v) $\int\frac{2xdx}{x^4+3}$

1.5.a) $\int x^2\sqrt[5]{x^2+3}dx$; b) $\int x\cdot e^{2x}dx$; v) $\int\frac{x^2+1}{x^2-1}dx$

1.6.a) $\int\frac{\sqrt{1+\ln x}}{x}dx$; b) $\int 3x\cos x dx$; v) $\int\frac{dx}{1+\sqrt[3]{x+1}}$.

1.7.a) $\int\sin 2x\cos 3x dx$; b) $\int x^2\cdot e^x dx$; v) $\int\frac{dx}{1+\sqrt{x}}$

1.8.a) $\int\cos 3x\sin 5x dx$; b) $\int x\cdot a^{-x}dx$; v) $\int\frac{\sqrt{x+1}}{x}dx$

1.9.a) $\int\sin 3x\sin 4x dx$; b) $\int x\cdot\arccos x dx$; v) $\int\frac{x+1}{\sqrt{x-2}}dx$.

1.10.a) $\int \sin x \cdot \sin 5x dx$; b) $\int x \cdot e^{3x} dx$; v) $\int \frac{xdx}{\sqrt{2x-3}}$

Ko‘rsatma:

$\int \sin ax \cos bxd$; $\int \sin ax \cdot \sin bxd$; $\int \cos ax \cdot \cos bxdx$

ko‘rinishdagi integrallarni topishda quyidagi

$$\sin ax \cdot \cos bx = \frac{1}{2}[\sin(a+b)x + \sin(a-b)x]$$

$$\sin ax \cdot \sin bx = \frac{1}{2}[\cos(a-b)x - \cos(a+b)x]$$

$$\sin ax \cdot \cos bx = \frac{1}{2}[\cos(a+b)x + \cos(a-b)x]$$

trigonometrik formulalardan foydalanish qulayroq.

2-topshiriq.

Misollarda berilgan integrallarni ratsional kasr funktsiyalarni integrallash metodida foydalanib hisoblang.

2.1 a) $\int \frac{dx}{x^2+2x-15}$; b) $\int \frac{dx}{x(x^2+4)}$

2.2 a) $\int \frac{dx}{1-x^2}$; b) $\int \frac{dx}{x(x^2+25)}$

2.3 a) $\int \frac{dx}{x^2-4}$; b) $\int \frac{(3x^2+8)dx}{3x^3+4x^2+4x}$

2.4 a) $\int \frac{dx}{a^2-x^2}$; b) $\int \frac{2x^5+6x^3+1}{x^4+3x^2}dx$

2.5. a) $\int \frac{2x+1}{x^2+x-6}dx$; b) $\int \frac{dx}{x^2(x^2+1)}$

2.6.a) $\int \frac{3x-4}{x^2+x-6}dx$; b) $\int \frac{dx}{x^2(x^2+16)}$

2.7.a) $\int \frac{3dx}{x^2+3x-6}$; b) $\int \frac{dx}{x^2(x^2+49)}$

2.8.a) $\int \frac{6x-4}{x^3-4x}dx$; b) $\int \frac{dx}{x^2(x^2+4)}$

2.9.a) $\int \frac{6x^2-13x+4}{x^3-3x^2+x}dx$; b) $\int \frac{dx}{x^2(x^2+9)}$

2.10a) $\int \frac{(7x-3)}{x^3+2x^2-3x}dx$; b) $\int \frac{dx}{x^3-8}$

3-topshiriq.

Misollarda berilgan aniq integrallarni hisoblang.

3.1. a) $\int_0^e \frac{\ln^2 x}{x}dx$;   b) $\int_4^5 (4-x)^3 dx$

3.2. a) $\int_0^{p/2} 4\cdot e^{\cos x}\cdot \sin x dx$;   b) $\int_0^1 \frac{dx}{(3x+1)^4}$

3.3. a) $\int_0^p 3\cdot \cos^2 x \sin x dx$;   b) $\int_0^3 \sqrt[3]{3x-1}dx$

3.4. a) $\int_{-3}^{-1} \frac{5dx}{(2-x)^3}$;   b) $\int_0^5 \sqrt{(2x-1)^3}dx$

3.5. a) $\int_{3/2}^4 \frac{3dx}{\sqrt{1+2x}}$;   b) $\int_0^{-1/2} e^{-2x}dx$

3.6.a) $\int_0^2 (2x+1)^4 dx$;   b) $\int_0^1 x\cdot e^{x^2}dx$

3.7.a) $\int_0^{1/2} \frac{(x+1)}{1+x^2}dx$;   b) $\int_0^{p/4} \sin 2xdx$

3.8.a) $\int_{\frac{3}{2}}^{\sqrt{3}} \frac{dx}{\sqrt{9-x^2}}$;   b) $\int_0^p \sin\frac{x}{3}dx$

3.9.a) $\int_0^{3/2} \frac{2dx}{4+9x^2}$;   b) $\int_{p/12}^{p/8} \frac{\sin^2 4x+\cos^2 2x}{\sin^2 4\cdot \cos^2 2x}$ dx

3.10.a) $\int_{p/2}^{p} \frac{5\sin xdx}{(1-\cos x)}$;   b) $\int_{p/6}^{p/4} \cos\left(2x-\frac{p}{6}\right)dx$

4 – topshiriq.

Misollarda berilgan chiziqlar bilan chegaralangan yuzalar, hajmlar va sirtlar hisoblansin.

4.1.$y=2-x^4$ va $y=x^2$ chiziqlar bilan chegaralangan figuraning ox o‘qi atrofida aylanishidan hosil bo‘lgan jismning hajmi hisoblansin.

4.2.$y=x^2$ va $y=\sqrt{x}$ parabolalar bilan chegaralangan figuraning ox o‘qi atrofida aylanishidan hosil bo‘lgan jismning hajmi hisoblansin.

4.3.$y=-x^2+4$ parabola va $2x-y+1=0$ to‘g‘ri chiziq bilan chegaralangan figuraning yuzi topilsin.

4.4. $2y=3\sqrt{4-x^2}$ yarim ellips va $2x-y^2+1=0$ parabola bilan chegaralangan shaklning oy o‘qi atrofida aylanishidan hosil bo‘lgan jismning hajmi hisoblansin.

4.5. $y=\frac{a}{2}\left(e^{\frac{x}{2}}-e^{-\frac{x}{2}}\right)$ zanjir chiziqning x=a va x = -a nuqtalar orasida joylashgan qismining ox o‘q atrofida aylanishidan hosil bo‘lgan sirt yuzini toping.

4.6.y=lnsinx egri chiziqni x=π/2 dan x=π/3 gacha bo‘lgan chegaradagi yoy uzunligi topilsin.

4.7.y=sinx sinusoidaning bir tarmog‘i (0 dan 2π gacha) ox o‘qi atrofida aylanadi. Buning aylanishidan hosil bo‘lgan yon sirt yuzi topilsin.

4.8.y=x^3 kubik parabolaning OX o‘qi atrofida aylanishidan hosil bo‘lgan sirtning x=0 dan x=1 gacha bo‘lgan chegaradagi yuzi topilsin.

4.9.Doira aylanasi o‘z diametrida aylanadi, hosil bo‘lgan sferaning sirti topilsin.

4.10.y=x^2 parabola va y=2x to‘g‘ri chiziq orasidagi yuzni toping.

KO’P O’ZGARUVCHILI FUNKTSIYa VA XUSUSIY XOSILALAR

11-mashg‘ulot.

Ko’p o’zgaruvchili funktsiya.

Misol. Ikki o‘zgaruvchili funktsiyalarning aniqlanish sohasi topilsin:

a) $\boldsymbol{z}=\frac{5}{\boldsymbol{x}^2+\boldsymbol{y}^2}$ b) $\boldsymbol{z}=\sqrt{1-\boldsymbol{x}^2-\boldsymbol{y}^2}$

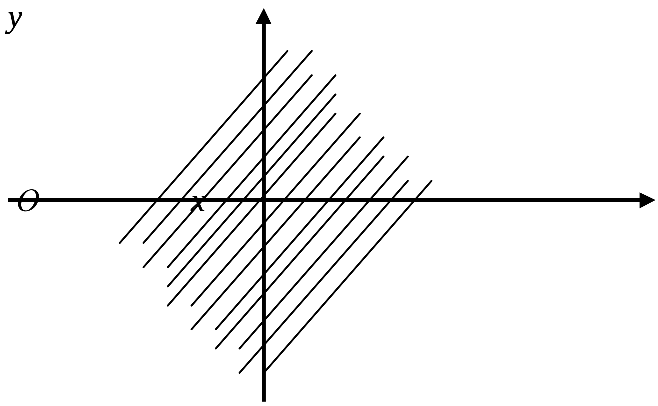
YeCHISh. a) Berilgan funktsiya kasr ko‘rinishda. Kasr funktsiya ma’noga ega bo‘lishi uchun, uning maxraji noldan farqli bo‘lishi zarur.

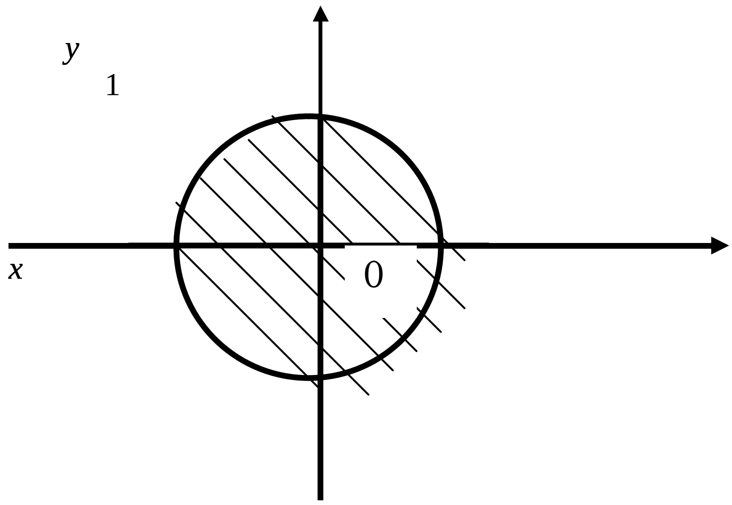
$x^2+y^2=0$ tenglik faqat $x=0$ va $y=0$ dagina bajariladi. Demak, berilgan funktsiyaning aniqlanish sohasi (0;0) nuqtadan tashqari barcha sonlar tekisligidan iborat.

b) $\boldsymbol{z}=\sqrt{1-\boldsymbol{x}^2-\boldsymbol{y}^2}$ ko‘rinishdagi funktsiya ma’noga ega bo‘lishi uchun kvadrat ildiz ostidagi ifoda manfiy bo‘lmasligi kerak, ya’ni

$1-x^2-y^2$ ³*0*, bundan esa x^2+y^2£*1* tengsizlikka ega bo‘lamiz. Bu markazi koordinatalar boshida va radiusi $r=1$ bo‘lgan, hamda $x^2+y^2=1$ aylanani ham o‘z ichiga oluvchi yopiq soha (doira)dan iborat.

Tekislikda bu ikki funktsiyaning aniqlanish sohasini quyidagicha tasvirlash mumkin.





12- mashg'ulot
Xususiy xosila va tula differentsial

Misol. Funnktsiyalarning birinchi tartibli xususiy hosilalari va to'la differentsiallari hisoblansin:

a) $z = \ln\left(x + \sqrt{x^2 + y^2}\right)$ b) $z = \arcsin\sqrt{x - y}$

Yechish. A) $z = \ln\left(x + \sqrt{x^2 + y^2}\right)$

Birinchi tartibli xususiy hosilalarni topish uchun,

$$(\ln u)' = \frac{1}{u}u' \qquad \textit{ва} \qquad \left(\sqrt{u}\right)' = \frac{1}{2\sqrt{u}}u'$$

formulalardan foydalanamiz.

$$\frac{\partial z}{\partial x} = \left[\ln\left(x + \sqrt{x^2 + y^2}\right)\right]_x' = \frac{1}{x + \sqrt{x^2 + y^2}}\left(x + \sqrt{x^2 + y^2}\right)_x' =$$

$$= \frac{1}{x + \sqrt{x^2 + y^2}}\left(1 + \frac{2x}{2\sqrt{x^2 + y^2}}\right) = \frac{1}{x + \sqrt{x^2 + y^2}} \cdot \frac{x + \sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2}} = \frac{1}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial z}{\partial y} = \left[\ln\left(x + \sqrt{x^2 + y^2}\right)\right]_y' =$$

$$= \frac{1}{x + \sqrt{x^2 + y^2}}\left(0 + \frac{2y}{2\sqrt{x^2 + y^2}}\right) = \frac{y}{\sqrt{x^2 + y^2}\left(x + \sqrt{x^2 + y^2}\right)}$$

Ma'lumki, *z (x,y)* funktsiya berilgan bo'lsa, uning to'la differentsiali

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dx$$

formula orqali ifodalanadi.

Shunga ko‘ra

$$dz = \frac{1}{\sqrt{x^2+y^2}}dx + \frac{y}{\sqrt{x^2+y^2}\left(x+\sqrt{x^2+y^2}\right)}dy$$

b) Xususiy hosilani aniqlash uchun

$$(\arcsin u)' = \frac{1}{\sqrt{1-u^2}}u'$$

formuladan foydalanamiz.

$$\frac{\partial z}{\partial x} = \frac{1}{\sqrt{1-(x-y)}}\cdot\frac{1}{2\sqrt{x-y}}, \qquad \frac{\partial z}{\partial y} = \frac{1}{\sqrt{1-(x-y)}}\cdot\frac{1}{2\sqrt{x-y}}$$

To‘la differentsialni hisoblash formulasiga asosan

$$dz = \frac{1}{2\sqrt{(x-y)(1-x+y)}}(dx-dy)$$

13 – mashg’ulot
Ikki o‘zgaruvchili funktsiya ekstremumi.

Aytaylik, z=f(x,y) funktsiya M(x,y) nuqta atrofida differentsiallanuvchi va bu nuqtada ikkinchi tartibli xususiy hosilalarga ega bo‘lsin.

I. funktsiyaning birinchi tartibli $z^1{}_x(x,y)$ va $z^1{}_u(x,y)$ xususiy hosilalarini topamiz.

2.Ushbu

$$\begin{cases} z_x\,(x,y)=0 \\ z_y\,(x,y)=0 \end{cases}$$

tenglamalar sistemasini echib. M_0 (x_0,y_0) kritik nuqta (yoki nuqtalarni) topamiz.

3. $M_0(x_0,y_0)$ nuqtada $A=z''_{xx}(M_0)$, $B= z''_{xu}(M_0)$, $C= z''_{uu}(M_0)$ qiymatlarini hisoblaymiz va $\Delta=AC - B^2$ ifodani ishorasini aniqlaymiz. Agar:

a) $\Delta>0$ bo‘lsa u holda $M_0(x_0,y_0)$ nuqtada funktsiya ekstrimumga ega bo‘ladi.(A>0 holda minimum A<0 holda maksimum)

b) $\Delta<0$ bo‘lsa, u holda $M_0(x_0,y_0)$ nuqtada funktsiya ekstrimumga ega bo‘lmaydi.

v) $\Delta=0$ bo‘lsa funktsiya $M_0(x_0,y_0)$ nuqtada funktsiya ekstrimumga ega bo‘lishi ham mumkin va ega bo‘lmasligi ham mumkin, ya’ni masala ochiq qoladi.

Masala. $z=x^3+8u^3-6xu+5$ funktsiyaning ekstrimumlari hisoblansin.

***Yechish*: *I.* $z'_x = 3x^2 - 6u$, $z'_y = 24u^2 - 6x$**

$\begin{cases} 3x^2 - 6y = 0 \\ 24y^2 - 6x = 0 \end{cases}$ sistemani echib $M_1(0,0)$, $M_2\ (1, \frac{1}{2})$ kritik nuqtalarini topamiz. Har ikkala M_1 va M_2 nuqtalar haqiqatdan ham kritik nuqtalardir, chunki berilgan funktsiya XOY tekislikning barcha nuqtalarida aniqlangan.

3. A.B va C - larni topamiz:

$z''_{xx} = 6x$, $z''_{uu} = 48u$, $z''_{xu} = -6$. $M_1(0,0)$ nuqta uchun $A = z''_{xx}(0,0) = 0$, $B = z''_{xu}(0,0) = 6$, $S = z''_{uu}(0,0) = 0$, ya'ni $\Delta = AC - B^2 = -36 < 0$, demak, (b) holga ko'ra, funktsiya $M_1(0,0)$ nuqtalar ekstrimumga ega emas.

$M_2\ (1, \frac{1}{2})$ nuqta uchun $A = z''_{xx}\ (1, \frac{1}{2}) = 6$, $A = z''_{xu}\ (1, \frac{1}{2}) = -6$, $C = z''_{uu}(1, \frac{1}{2}) = 24$, ya'ni $\Delta = AC - B^2 = 6 \cdot 24 - (-6)^2 = 144 - 36 = 108 > 0$ demak, (*a)* holga ko'ra funktsiya $M_2(1, \frac{1}{2})$ nuqtada ekstrimumga ega, chunki $A = 6 > 0$. U holda

$z_{min} = z(M_2) = z\ (1, \frac{1}{2}) = 4$ bo'ladi.

AMALIY MAShG'ULOTLARNI UTKAZISh UCHUN MISOL VA MASALALAR.

1-topshiriq.

Misollarda berilgan funktsiyalarning aniqlanish sohasini toping va aniqlanish sohasini geometrik tasvirlang.

1.1.a) $z = \ln(x^2 + y + 1)$; b) $z = \dfrac{1}{\sqrt{x^2 + y^2 - 4}}$

1.2.a) $z = \dfrac{1}{\sqrt{2x - 2}} + \dfrac{1}{y + 3}$; b) $z = \ln(y^2 - x - 1)$

1.3.a) $z = \sqrt{x - 1} + \sqrt{2y - 2}$; b) $z = \dfrac{x}{y - x^2}$

1.4.a) $z = \ln(6 - 6x^2 - 3y^2)$; b) $z = \sqrt{1 - x^2 - y^2}$

1.5.a) $z = \sqrt{\dfrac{x^2}{9} - \dfrac{y^2}{16} - 4}$; b) $z = \dfrac{x}{x + y} - \dfrac{y}{x - y}$;

1.6.a) $z = \dfrac{1}{\sqrt{9 - x^2 - y^2}}$; b) $z = \ln(x^2 + y^2 - 36)$

1.7.a) $z = \sqrt{4x - 10y}$; b) $z = \dfrac{x}{x^2 + y^2 - 25}$

1.8.a) $z = \sqrt{\dfrac{x^2}{36} - \dfrac{y^2}{25} - 1}$; b) $z = \ln(3y - 30x)$

1.9.a) $z=\dfrac{x}{\sqrt{y-x^2}}-\dfrac{y}{y-x}$; b) $z=\ln(2xy)$

1.10.a) $z=\dfrac{x}{25-x^2-y^2}$; b) $z=\ln(8-2x^2-4y^2)$

2-topshiriq.

Misollarda berilgan: z=f(x,y) funktsiyaning birinchi tartibli xususiy hosilalarini va to‘la differentsialini toping.

2.1.a) $z=\ln(x^2+y^2)$; b) $z=tg\dfrac{x}{y}$

2.2.a) $z=\sin(x^2+y^2)$; b) $z=\ln\dfrac{x}{y}$

2.3.a) $z=cos(x^2+y^2)$; b) $z=ln(2x+2y)$

2.4.a) $z=tg(2x+3y)$; b) $z=e^{-x^2y^2}$

2.5.a) $z=ctg(4y-3x)$; b) $z=a^{-6y^2x^3}$

2.6.a) $z=\dfrac{x^2}{x^2-y^2}$; b) $z=arcc's(yx)$

2.7.a) $z=\dfrac{x^2+y^2}{y^2}$; b) $z=arcsin(xy)$;

2.8.a) $z=\sqrt{x^2+y^2}$; b) $z=\dfrac{x^2+y}{x-y}$

2.9.a) $z=\sqrt{2x^2-y^2}$; b) $z=arctg(xy)$;

2.10.a) $z=\ln(4x+\sqrt{y})$; b) $z=e^{x^3+y^3}$

3-topshiriq.

Misollarda berilgan z=f(x,y) funktsiyaning ekstremumi mavjud yoki mavjud emasligini tekshiring. Agar funktsiyaning ekstremumi mavjud bo‘lsa, ularni hisoblang.

3.1. $z=x^2-xy+y^2+9x-6y+20$

3.2. $z=y\sqrt{x}-y^2-x+6y$

3.3. $z=x^3-8y^3-6xy+1$

3.4. $z=2xy-4x-2y$

3.5. $z=3x+6y-x^2-xy-y^2$

3.6. $z=2x^3-xy^2+5x^2+y^2$

3.7. $z=x^3+8y^3-6xy+5$

3.8. $z=(x-1)^2+2y^2$

3.9. $z=2x^3-x^2+xy^2-4x+3$

3.10. $z=3x+6y-x^2-xy-y^2$

DIFFERYENTSIAL TENGLAMALAR, KARRALI INTEGRAL VA QATORLAR NAZARIYaSI

14 – mashg'ulot
Sodda differentsial tenglamalarni echish

Aytaylik quyidagi

$$u^{(n)}+r_1u^{(n-1)}+r_2u^{(n-2)}+...+r_{n-1}u'+r_nu=0 \qquad (1)$$

o'zgarmas koeffitsientli, chiziqli bir jinsli differentsial tenglama berilgan bo'lsin. Uning echimlari

$$r^n+p_1r^{n-1}+p_2r^{n-2}+...+p_{n-1}r+P_n=0 \qquad (2)$$

harakteristik tenglama yordamida topiladi. (2) tenglamani tuzish esa, (1) da u - ni bir bilan almashtirib, hosila tartiblarini ularga mos r larning darajalari bilan almashtirib, r_i - koeffitsientlarini esa o'zgarishsiz qoldirish orqali bajariladi. Harakteristik tenglama ildizlariga qarab, quyidagi echimlar hosil qilinadi:

I. Agar (2) harakteristik tenglama haqiqiy va har xil r_1, r_2,..., r_n - ildizlariga ega bo'lsa, (I) tenglamalar umumiy echimi:

$$y=C_1\cdot e^{r_1x}+C\cdot_2 e^{r_1x}+...+C\cdot_n e^{r_nx} \qquad (3)$$

kabi bo'ladi. Bunda $S_1,S_2,..., S_n$ lar ixtiyoriy o'zgarmas sonlar.

II. agar (2) xarakteristik tenglamaning ildizlari karrali bo'lmagan $r=a\pm ib$ kompleks ko'rinishda bo'lsa, (I) tenglamaning (3) umumiy echimidagi har bir juft mos hadlar quyidagi qo'shiluvchilar bilan almashadi:

$$e^{ax}(C_1\cos bx+C_2\sin bx) \qquad (4)$$

III. Agar (2) harakteristik tenglama K-karrali r_1-haqiqiy ildizga ega bo'lsa, (I) tenglamaninig umumiy echimining shu ildizga mos keluvchi qismi (qo'shiluvchisi)

$$y=e^{r_1x}(C_1+C_2x+C_3x^2+...+C_Kx^{K-1}) \quad (5)$$

kabi bo'ladi.

IV. Agar (2) harakteristik tenglama K-karrali $r=a\pm ib$ ko'rinishdagi qo'shma kompleks ildizlarga ega bo'lsa, (I) tenglama bu umumiy echimining bu ildizlarga mos keluvchi qismi.

$$y=e^{ax}\big[(C_1+C_2x+...+C_Kx^{K-1})\cos bx+$$
$$(C_{K+1}+C_{K+2}+...+C_{2K}x^{K-1})\sin bx\big]$$

kabi bo'ladi.

Misol . u'''-6y''+13u'=0 tenglama ildizlarini toping. Uning harakteristik tenglamasi quyidagi ko'rinishga ega.

$r(r^2-6r+13)=0$; $\qquad r_1=0$

$r^2-6r+13=0$; $\qquad r_{2,3}=3\pm 2i$

U holda (3) va (4) formulalarga ko'ra umumiy echim

$y=C_1+e^{3x}(C_2\cos 2x+C_3\sin 2x)$ kabi ko'rinishga ega bo'ladi.

Misol: $u''+6y'+5u=25x^2-2$ (1)

tenglamaning umumiy echimi topilsin.

YeCHISh: I) Berilgan (I) tenglamaga mos bir jinsli chiziqli bo'lgan

$$u''+6y'+5u=0 \qquad (2)$$
tenglamaning umumiy echimini topamiz: $u''+6y'+5u=0$ tenglamaning xarakteristik tenglamasi $r^2+6r+5=0$, $r_1=-5$ va $r_2=-1$ haqiqiy va karrali bo‘lmagan ildizlariga ega, shuning uchun (2) ning umumiy echimi

$$y_0 = C_1 \cdot e^{-5x} + C_2 \cdot e^{-x} \qquad (3)$$

bo‘ladi.

2) (I) - bir jinsli bo‘lmagan differentsial tenglamaning xususiy echimini topamiz. Bu erda (I) ning o‘ng tomoni $g(x)=25x^2-2$ ikkinchi darajali ko‘phad bo‘lgani uchun xususiy echimini $u_1=Ax^2+Bx+s$ ko‘rinishida izlaymiz. Bundan $u_1'=2Ax+B;$ $u_1''=2A$. Bu qiymatlarni (I) ga qo‘yamiz:
$2A+6(2Ax+B)+5(Ax^2+Bx+S)=25x^2-2$ yoki $2A+6(2Ax+B)+5(Ax^2+Bx+S)=25x^2-2$
Tenglikning har ikkala tomonidagi x-ning bir xil darajali koeffitsentlarini tenglashtirib.

$$\begin{cases} 5A=25 \\ 12A+5B=0 \\ 2A+6B+5C=-2 \end{cases}$$ sisitemani hosil kilamiz.

Bundan $A=5$, $B=-12$, $C=12$ bo‘ladi. (I) tenglamaning xususiy echimi
$u_1=5x^2-12x+12$ va umumiy echimi esa
$u=u_0+u_1=S_1e^{-5x}+S_2e^{-x}+5x^2-12x+12$ ko‘rinishga ega bo‘ladi.

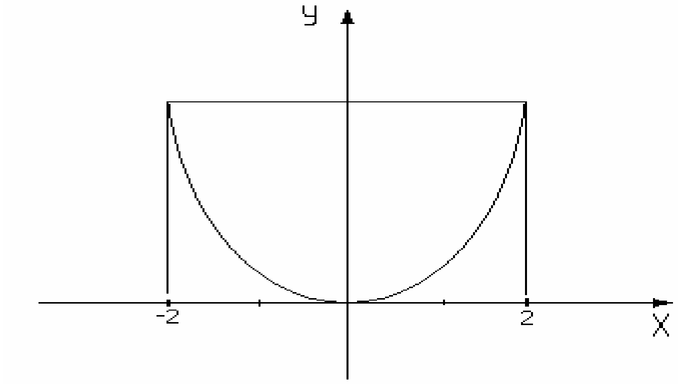
15 – mashg’ulot
Karrali integlar

Misol: $\int_{-2}^{2}dx\int_{x^2}^{4}f(x,y)dy$ integralda integrallash tartibi o‘zgartirilsin, integrallash sohasi chizmada tasvirlansin.

YeCHISh: Umuman, integrallash tartibini o‘zgartirish, bevosita hisoblash ishlariga ketadigan vaqtni tejashga imkon beradi.

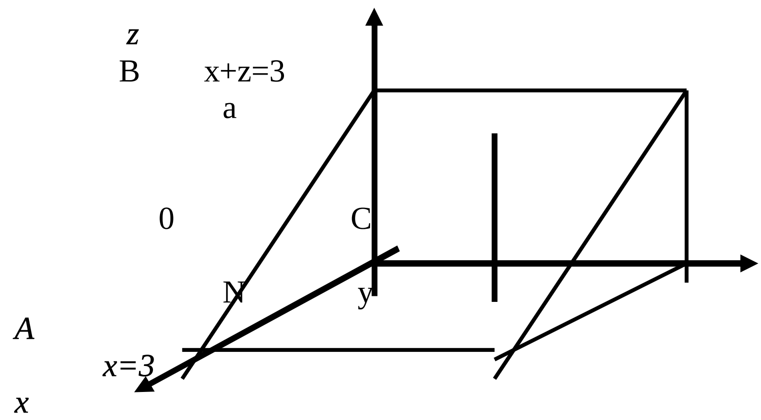
I) Berilgan integrallarni chegaralari yordamida integrallash sohasini aniqlaymiz. Bu soha $x=-2$, $x=2$, $y=x^2$, $y=4$ chiziqlar bilan chegaralangan. Bu sohani tekislikdagi koordinatalar sistemasida yasab, OAB parabolik sigmentni hosil qilamiz, bu sigment OY-o‘qiga simetrikdir.

Endi esa integrallash tartibini o‘zgartiramiz, ya’ni oldin x bo‘yicha keyin u bo‘yicha integrallaymiz. Ichki integral chegaralarini parabola tenglamasini x-ga nsibatan echib topamiz:



a) $\iiint\limits_D \frac{dxdydz}{(x+y+z+1)^3}$ hisoblansin, bunda D - soha $x+z=3$, *y =2, x=0, y=0, z=0* teksliklar bilan chegaralangan.

YeCHISh: a) koordinatalar sistemasida ko‘rsatilgan tekisliklarni yasaymiz, natijada uchburchakli prizmani hosil qilamiz



$$\iiint_D f(x,y,z)dxdydz = \iint_{D_{xy}} dxdy \int_{z=j_1(x,y)}^{z=j_2(x,y)} f(x,y,z)dz$$ *формулага кура*

$$\iiint_D \frac{dxdydz}{(x+y+z+1)^3} = \iint_{D_{xy}} dxdy \int_{z_m=0}^{z_0=3-x} f(x,y,z+1)^{-3}dz = \iint_{D_{xy}} dxdy \int_0^{3-x} f(x,y,z+1)^{-3}\, d(x,y,z+1) =$$

$$= \iint_{D_{xy}} \left[\frac{(x+y+z+1)}{-2}\right]_{z_n=0}^{z_0=3-x} (x,y,z+1)^{-3}dz = \frac{1}{2}\int_0^3 dx\int_0^2 [(x+y+1)^{-2} - (y+4)^{-2}\ ]dy =$$

$$= \frac{1}{2}\int_0^3\left[\left(\frac{1}{y+4}-\frac{1}{x+y+1}\right)\Big/_{y=0}^{y=2}\right]dx = \frac{1}{2}\int_0^3\left(\frac{1}{y+4}-\frac{1}{x+3}-\frac{1}{12}\right)dx = \frac{1}{2}\left[\ln\left|\frac{x+1}{x+3}\right|-\frac{x}{12}\right]\Big/_0^3 =$$

$$\frac{1}{2}\left[\left(\ln\frac{4}{6}-\ln\frac{1}{3}\right)-\left(\frac{1}{4}-0\right)\right] = \frac{1}{2}\left(\ln 2-\frac{1}{4}=\frac{1}{2}\ \frac{4\ln 2-1}{4}\right) = \frac{4\ln 2-1}{8}$$

b) $\int_{(0,0)}^{(1,1)} 3x^2ydx+(x^3+1)dy$ integralni *(0; 0), (I; I)* nuqtalarini tutashtiruvchi, *y=x* va $y=x^2$ chiziqlar bo‘yicha hisoblansin.

YeCHISh: I) bu erda *y=x* bo‘lgani uchun *dy=dx* va *x* ning o‘zgarish chegarasi [0, 1] bo‘lganida

$$\int_{(0,0)}^{(1,1)} 3x^2ydx+(x^3+1)dy = \int_0^1\left(3x^2xdx+(x^3+1)dx\right) = \int_0^1 (4x^3+1)dx = (x^4+x)/_0^1 = 2$$

2) Bu erda $y=x^2$ bo‘lgani uchun *dy=2xdx*, $0\le x\le 1$ bo‘ladi, u holda

$$\int_{(0,0)}^{(1,1)} 3x^2ydx+(x^3+1)dy = \int_0^1\left(3x^2x^2dx+(x^3+1)2xdx\right) = \int_0^1 (4x^3+1)dx = (x^4+x)/_0^1 = 2$$

Izoh. Bu erda har ikkala yo‘l bo‘yicha olingan integrallarning bir xil bo‘lishiga sabab $\frac{\partial(3x^2y)}{\partial y} = \frac{\partial(x^2+1)}{\partial x} = 3x^2$ ekanligidadir, ya’ni integrallash yo‘liga bog‘liq bo‘lmasligining zaruriy va etarli sharti bajariladi.

16-mashg’ulot

Qatorlar nazariyasi

$\sum_{n-1}^{\infty} \frac{n^5}{2^n}$ sonli qatorni yaqinlashishini Dalamber belgisi bo‘yicha tekshiring. Ma’lumki, $a_1+a_2+...+ a_n+ a_{n+1}+...$ sonli qator berilgan bo‘lsa, $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = l$ ni tekshiramiz, bunda uch hol bo‘ladi, agar $l<1$ bo‘lsa, qator yaqinlashadi, $l>1$ da uzoqlashadi, $l=1$ da esa, yaqinlashish yoki uzoqlashish masalasi ochiq qoladi.

$a_n = \frac{n^5}{2^n}$, $\quad a_{n+1} = \frac{(n+1)^5}{2^{n+1}}$ bo‘lgani uchun

$l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^5}{2^n 2}}{\frac{n^5}{2^n}} = \frac{1}{2}\lim_{n\to\infty}\left(\frac{(n+1)}{n}\right)^5 = \frac{1}{2}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^5 = \frac{1}{2}\cdot 1 = \frac{1}{2} < 1$ demak, qator yaqinlashuvchi.

b) $\sum_{n=2}^{+\infty} \frac{1}{n\cdot \ln^3 n}$ sonli qatorni yaqinlashishi tekshirilsin.

YeCHISh. Bu qatorni tekshirish masalasi xosmas integrallar tushunchasi bilan bog‘liqdir. Berilgan qatorga mos birinchi jins xosmas integralni tuzamiz, agar xosmas integral yaqinlashuvchi bo‘lsa, berilgan qator ham yaqinlashadi.

$$\int_2^{+\infty} \frac{dx}{x\ln^3 x} = \lim_{b\to+\infty}\int_2^b (\ln x)^{-3} d(\ln x) = \lim_{b\to+\infty}\left(-\frac{1}{2\ln^2 x}\right)\Big/_2^b = \lim_{b\to\infty}\left(\frac{1}{2\ln^2 2} - \frac{1}{2\ln^2 b}\right) = \frac{1}{2\ln^2 2} - 0 = \frac{1}{2\ln^2 2}$$

oxirigi ifoda chekli son, demak xosmas integral yaqinlashuvchi, shuning uchun qator ham yaqinlashuvchi bo‘ladi.

AMALIY MAShG‘ULOTLAR UCHUN MISOL VA MASALALAR

1-topshiriq.

Misollarda berilgan bir jinsli o‘zgarmas koeffitsientlari differentsial tenglamalarning umumiy echish topilsin.

1.1. $y^{II}- y'-2y=0$	*1.6.* $y^{IV}+a^4y=0$
1.2. $y^{II} - y'=0$	*1.7.* $y^{IV}+5y^{II}+4y=0$
1.3. $y^{II} +25y=0$	*1.8.* $y^{III}-2y^{II}+y^{I}=0$
1.4. $y^{II} - 4y^2+4y=0$	*1.9.* $y^{II}-4y^{I}+13y=0$
1.5. $y^{IV} - 2y^{III}+y^{II}=0$	*1.10.* $y^{II}-7y^{I}+16=0$

2-topshiriq.

Misollarda berilgan birjinsli bo‘lmagan differentsial tenglamalarning umumiy echimlari topilsin:

2.1. $y^{II}-6y^{I}+8y=x\cdot e^{x}$
2.2. $y^{II}-4y^{I}+4y=x^{2}$
2.3. $y^{II}-y^{I}-2y=3\cdot e^{2x}$
2.4. $y^{II}-2y^{I}+y=9\cdot e^{-2x}$
2.5. $y^{II}-3y^{I}=3\cdot e^{3x}$
2.6. $y^{II}-4y^{I}+5y=5x-4$
2.7. $y^{II}+y=6\cdot \sin 2x$
2.8. $y^{II}+5y^{I}+6y=12c\text{'}s2x$
2.9. $y^{II}-2y^{I}+y=1+x$
2.10. $y^{II}-4y^{I}+13y=26x+5$

3-topshiriq.

Misollarda integrallardagi integrallash tartibi o‘zgartirilsin. Integrallash sohasini chizmada tasvirlang.

3.1. $\int_{0}^{1}dx\int_{2x^2}^{3-x}f(x,y)dy$ 3.2. $\int_{0}^{3/2}dy\int_{2y^2}^{y+3}f(x,y)dx$ 3.3. $\int_{-1}^{0}dx\int_{2x^2}^{x+3}f(x,y)dy$

3.4. $\int_{0}^{1}dy\int_{2y^2}^{3-y}f(x,y)dx$ 3.5. $\int_{-3/2}^{0}dx\int_{2x^2}^{3-x}f(x,y)dy$ 3.6. $\int_{0}^{4}dy\int_{5y/4}^{\sqrt{9+y^2}}f(x,y)dx$

3.7. $\int_{0}^{4}dx\int_{3x/4}^{\sqrt{25-x^2}}f(x,y)dy$ 3.8. $\int_{-4}^{0}dy\int_{-\sqrt{9+y^2}}^{-5y/4}f(x,y)dx$ 3.9. $\int_{0}^{1}dx\int_{-2}^{x^2+1}f(x,y)dy$

3.10. $\int_{0}^{4}dy\int_{3y/4}^{\sqrt{25-x^2}}f(x,y)dx$

4-topshiriq.

Masalalarda berilgan ikki va uch o‘lchovli integrallarni hisoblang.

4.1. $\iint\limits_{Д} x^2y^2\sqrt{1-x^3y^3}\,dxdy$ hisoblansin.

Bu erda D soha $x^3+y^3=1$ chiziq va koordinata o‘qlari bilan chegaralangan.

4.2. $\int_{0}^{-1}dx\int_{0}^{e-x-1}dy\int_{0}^{x+y+e}\frac{\ln(z-x-y)}{(x-e)(x+y-e)}dz$ integralni hisoblang.

4.3. $\iiint\limits_{Д} \frac{dxdydz}{(x+y+z+1)^3}$ integral hisoblansin.

Bu erda D soha *x=0,y=0,z=0, x+y+z=1* tekisliklar bilan chegaralangan.

4.4. $\int\limits_{l} (x-y)dx+(x+y)dy$ egri chiziqli integralni hisoblang, bu erda *l* uchlari A(0,0), V(1,0) va S(0,1) nuqtalarda bo‘lib, soat strelkasining yo‘nalishiga qarshi harakat qiluvchi uchburchak konturidir.

4.5. $\int\limits_{AB} \frac{y^2+1}{y}dx - \frac{x}{y^2}dy$ egri chiziqli integralni hisoblang, bu erda AV kesma A(1,2) va V (2,4) nuqtalarni tutashtiruvchi to‘g‘ri chiziq kesmasi.

4.6 (1,2) va (2,4) nuqtalarni birlashtiruvchi va ixtiyoriy yo‘l bo‘yicha olingan $\int\limits_{(1,2)}^{(2,4)} 2xy^3dx+3x^2y^2dy$ egri chiziqli integral hisoblansin.

4.7. (0,0) va (3,4) nuqtalarni birlashtiruvchi va ixtiyoriy yo‘l bilan olingan $\int\limits_{(0,0)}^{(3,4)} (x^2-y)dx+(y^2-x)dy$ egri chiziqli integral hisoblansin.

4.8.(1,2) va (3,6) nuqtalarni birlashtiruvchi va ixtiyoriy yo‘l bo‘yicha olingan $\int\limits_{(1,2)}^{(3,6)} (3x^2y+1)dx+(x^3-2)dy$ egri chiziqli integral hisoblansin.

4.8. $z=0,\ z=2x;\ x+y=3,\ x=\frac{\sqrt{y}}{2}$ sirtlar bilan chegaralangan jismning hajmi hisoblansin.

4.10. $x=0,\ y=0;\quad x+y=2,\ y=\sqrt{1-z}$ sirtlar bilan chegaralangan jismning hajmi hisoblansin.

5-topshiriq.

Misollarda berilgan sonli qatorlar yaqinlashish tekshirilsin.

5.1. $\sum\limits_{n=1}^{\infty}(-1)^n\frac{n}{6n-5}$, 5.2. $\sum\limits_{n=1}^{\infty}\frac{n^2}{2^n}$

5.3. $\sum\limits_{n=1}^{\infty}\frac{1}{\ln^n(n+1)}$, 5.4. $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\sin\frac{a}{n}$

5.5. $\sum\limits_{n=1}^{\infty}\frac{(-1)^{n-1}\ln(n-1)}{\sqrt{n}}$. 5.6. $\sum\limits_{n=1}^{\infty}\frac{\sin na}{n\sqrt{n}}$

5.7. $\sum\limits_{n=1}^{\infty}\frac{2\cdot5\cdot8...(3n-1)}{1\cdot5\cdot9...(4n-3)}$, 5.8. $\sum\limits_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$

5.9. $\sum_{n=1}^{\infty}\frac{1}{n^2+n}$, 5.10. $\sum_{n=1}^{\infty}\frac{1}{n\cdot\ln^2 n}$

16-mashg'ulot.
Tasodifiy hodisa ehtimolini xisoblash.

Misol. Guruhda 15 ta talaba bo'lib, ulardan 10 tasi yaxshi baholarga, 5 tasi a'lo bahoga o'qiydi. Tasodifan 5 ta talaba ajratildi. Ular orasida 3 ta a'lo va 2 ta yaxshi bahoga o'qiydigan talabalar bo'lishi ehtimolini toping.

YeCHISh. Elementar hodisalar fazosi 15 ta talabani 5 tadan gruppalashlar soniga, ya'ni S_{15}^5 ga teng. So'ralgan hodisa uchun imkoniyat yaratuvchi elementar hodisalar esa 5 ta ajratilganlar ichida 3 tasi a'lo va 2 tasi yaxshi bahoga o'qishi zarur. Uchta a'lochi talabani guruhdagi barcha a'lochilar, ya'ni 5 ta a'lochilar ichidan ajratib olish mumkin. Ikkita yaxshi o'qiydigan talabani esa 10 ta talaba ichidan olish mumkin. Demak, axtarilayotgan hodisa soni $S_5^3 S_{10}^2$ ga teng. U holda klassik ehtimollik ta'rifiga asosan

$$P(A)=\frac{C_5^3 C_{10}^2}{C_{15}^5}=\frac{5!}{3!2!}\ \frac{50!}{2!\ 8!}\ \frac{5!\,10!}{15!}=\frac{150}{1001}=0{,}15$$

Misol. Birinchi yashikda 6 ta oq va 4 ta qora, ikkinchi yashikda 8 ta oq va 3 ta qora shar bor. Birinchi yashikdan tasodifan bitta shar olinib ikkinchisiga tashlandi. Ikkinchi yashikdan tasodifan olingan sharni oq chiqish ehtimolini toping.

YeCHISh. Masala to'la ehtimollik formulasiga asosan echiladi. Buning uchun quyidagi gipotezalarni tuzamiz:

N_1 - birinchisidan ikkinchisiga oq shar tashlandi;

N_2 - birinchisidan ikkinchisiga qora shar tashlandi.

Bu gipotezalarni yuz berish ehtimoli klassik ta'rifga asosan $P(H_1)=\frac{6}{10}$, $P(H_2)=\frac{4}{10}$ ga teng.

A orqali ikkinchi yashikdan olingan sharni oq chiqish hodisasini belgilaymiz. Shartli ehtimollik formulasiga asosan

$$P(A/H_1)=\frac{9}{12},\quad P(A/H_2)=\frac{8}{12}$$

To'la ehtimollik formulasiga asosan

$$P(A)=\sum_{i=1}^{2}p(H_i)\ p(A/H_i)=p(H_1)p(A/H_1)+p(H_2)p(A/H_2)=$$

$$=\frac{6}{10}\cdot\frac{9}{12}+\frac{4}{10}\cdot\frac{8}{12}=\frac{86}{120}=\frac{43}{60}$$

17- mashg‘ulot.
Bernulli formulasi uchun limit teoremalar.

Misol. O‘zaro bog‘liqsiz tajribalarning har birida hodisaning yuz berish ehtimoli 0,7 ga teng. Agar 2100 ta tajriba o‘tkazilgan bo‘lsa, shundan 1500 tasida kuzatilayotgan hodisani yuz berish ehtimolini toping.

Yechish. Muavr-Laplas lokal teoremasiga asosan Bernulli formulasini taqribiy hisoblash formulasi quyidagicha

$$P_n(m) \approx \frac{1}{\sqrt{npq}} \cdot j(x)$$

Bu erda $j(x) = \frac{1}{\sqrt[1]{2p}} e^{-x^{2/2}}$, $x = \frac{k - np}{\sqrt{npq}}$ - Laplas funktsiyasidir. (juft funktsiya φ(-x)=φ(x)). Laplas funktsiyasi qiymatlarini jadval asosida topish mumkin.

Masala shartiga asosan n=2100, k=1500, r=0,7,
q=1 - r = 1 - 0,7 = 0,3

$$x = \frac{k - np}{\sqrt{npq}} = \frac{1500 - 2100 \cdot 0{,}7}{\sqrt{2100 \cdot 0{,}7 \cdot 0{,}3}} = \frac{30}{21} = 1{,}43$$

φ(x) funktsiyani x=1,43 dagi qiymatini jadval asosida topsak

φ(1,43)=0,1435

U holda 2100 ta tajribada hodisaning 1500 marta ro‘y berish ehtimoli

$$P_{2100}(1500) = \frac{1}{\sqrt{2100 \cdot 0{,}7 \cdot 0{,}3}} \cdot j(1{,}43) = \frac{0{,}1435}{21} = 0{,}007$$

18-mashg‘ulot

Tasodifiy miqdor sonli xarakteristikalari.

Misol.

X-tasodifiy miqdorning taqsimoti quyidagicha berilgan

x_l	-1	0	1	2
r_i	0,3	0,4	0,2	0,1

Matematik kutilma, dispersiya va o‘rta kvadratik og‘ish qiymatlari topilsin.

Yechish. Ma’lumki agar tasodifiy miqdor taqsimot qonuni.

x_l	x_1	x_2	....	x_n
r_i	r_1	r_2		r_n

kabi berilsa matematik kutilmasi

$$Mx = x_1 p_1 + x_2 p_2 + ... + x_n p_n = \sum_{i=1}^{n} x_i p_i$$, kabi, dispersiya $Dx=Mx^2-(Mx)^2$,

o‘rta kvadratik og‘ish qiymati $s = \sqrt{Dx}$ ko‘rinishda hisoblanadi.

Masala shartiga, asosan

$$Mx = \sum_{i=1}^{4} x_i p_i = -1 \cdot 0{,}3 + 0 \cdot 0{,}4 + 1 \cdot 0{,}2 + 2 \cdot 0{,}1 = -0{,}3 + 0{,}2 + 0{,}2 = 0{,}1$$

$$Mx^2=\sum_{i=1}^{4}x_1^2\cdot p_i=(-1)^2\cdot 0{,}3+0^2\cdot 0{,}4+1^2\cdot 0{,}2+2^2\cdot 0{,}1=0{,}3+0{,}2+0{,}4=0{,}9$$

$Dx=Mx^2-(Mx)^2=(0{,}9)^2-(0{,}1)^2=0{,}81-0{,}01=0{,}8$, $\quad s=\sqrt{0{,}8}\approx 0{,}9$.

<u>Misol</u> X- tasodifiy miqdor taqsimot funktsiyasi

$$F(x)=\begin{cases}0 & \text{агар} \quad x\le 0\\ \dfrac{x}{2} & \text{агар} \quad 0<x\le 2\\ 1 & \text{агар} \quad x>2\end{cases}$$

tasodifiy miqdor zichlik funktsiyasini, matematik kutilma va dispertsiyasini toping.

YeCHISh. Tasodifiy miqdor taqsimoti funktsiyasi va zichlik funktsiyasi orasida *F'(x)=r(x)* munosabat o‘rinli. Shunga ko‘ra

$$F'(x)=p(x)=\begin{cases}0 & \text{агар} \quad x\notin[0{,}2]\\ \dfrac{1}{2} & \text{агар} \quad x\in[0{,}2]\end{cases}$$

Uzluksiz tasodifiy miqdor matematik kutilmasi

$$Mx=\int_{-\infty}^{\infty}x\cdot p(x)dx$$

munosabat orqali hisoblanadi.

$$Mx=\int_{-\infty}^{\infty}x\cdot p(x)dx=\int_{0}^{2}x\cdot\frac{1}{2}dx=\frac{1}{2}\int_{0}^{2}xdx=\frac{1}{2}\frac{x^2}{2}\Big|_0^2=1$$

Dispersiyani hisoblash uchun

$$Mx^2=\int_{-\infty}^{\infty}x^2\cdot p(x)dx$$

qiymatni hisoblaymiz

$$Mx^2=\int_{0}^{2}x^2\cdot\frac{1}{2}dx=\frac{1}{2}\cdot\frac{x^3}{3}\Big|_0^2=\frac{8}{6}=\frac{4}{3}$$

U holda tasodifiy miqdor dispersiyasi va o‘rta kvadratik og‘ishi

$$Dx=Mx^2-(Mx)^2=\frac{4}{3}-1^2=\frac{1}{3},\ s=\sqrt{Dx}=1/\sqrt{3}$$

19-mashg'ulot.
Ishonchililik intervalini xisoblash.

<u>Misol</u>. Hajmi n=100, o'rta qiymati x=10,43 va o'rta kvadratik og'ishi σ=5 ga teng bo'lgan normal taqsimot matematik kutilmasi a uchun 0,95 ishonchlilik ehtimoli bilan ishonchlilik intervalini toping.

Yechish: Ishonchlilik intervali formulasiga asosan

$$\bar{x} - t_a \frac{s}{\sqrt{n}} < a < \bar{x} + t_a \cdot \frac{s}{\sqrt{n}}$$

Bu erda $\bar{x}$ - tanlama o'rta qiymati
α - ishonchlilik ehtimoli bo'lib,
t_a qiymat jadval asosida topiladi.
σ - o'rta kvadratik og'ish
n - tanlanma hajmi

Jadvalga asosan α =0,95 ehtimollik uchun t_a =1,96.
U holda

$$10{,}43 - 1{,}96 \cdot \frac{5}{\sqrt{100}} < a < 10{,}43 + 1{,}96 \cdot \frac{5}{\sqrt{100}}$$

10,43 - 1,96 - 0,2 < a < 10,43+1,96 ·0,2

9,45 < a < 11,41

AMALIY MAShG'ULOTLAR UCHUN MISOL VA MASALALAR
1-topshiriq.

Yashikda N ta detal bo'lib ulardan n tasi oliy navli. Tasodifan m ta detal ajratib olindi. Ajratib olingan detallar ichida k tasi oliy navli chiqish ehtimolini toping.

1.1.	N=16	n=11	m=6	k=3
1.2.	N=16	n=12	m=5	k=2
1.3.	N=16	n=10	m=5	k=3
1.4.	N=20	n=15	m=6	k=3
1.5.	N=20	n=14	m=6	k=2
1.6.	N=18	n=13	m=7	k=4
1.7.	N=18	n=13	m=6	k=4
1.8.	N=18	n=13	m=5	k=3
1.9.	N=21	n=16	m=7	k=3
1.10.	N=21	n=18	m=5	k=2

2-topshiriq.

Uchta yashikni har birida n ta oq va m ta qora sharlar bor. Tasodifan birinchisidan ikkinchisiga, ikkinchisidan uchinchisiga bittadan shar tashlandi. Uchinchi yashikda olingan sharni oq chiqish ehtimolini toping.

2.1.	n=6	m=4
2.2.	n=7	m=5
2.3.	n=6	m=5
2.4.	n=8	m=3

2.5. n=8 m=4
2.6. n=6 m=3
2.7. n=7 m=4
2.8. n=7 m=6
2.9. n=8 m=5
2.10 n=8 m=6

3-topshiriq.

O‘zaro bog‘liqsiz-tajribalar ketma-ketligining har birida, A hodisaning ro‘y berish ehtimoli r ga teng. O‘tkazilgan n ta tajribadi A hodisaning k marta ro‘y berish ehtimolini toping.

3.1. n=2000 k=1400 r=0,6
3.2. n=2000 k=1300 r=0,6
3.3. n=2000 k=1200 r=0,6
3.4. n=2100 k=1300 r=0,7
3.5. n=2100 k=1400 r=0,7
3.6. n=2100 k=1500 r=0,6
3.7. n=2200 k=1400 r=0,6
3.8. n=2200 k=1500 r=0,7
3.9. n=2200 k=1600 r=0,6
3.10. n=2300 k=1500 r=0,7

4-topshiriq.

X tasodifiy miqdor taqsimoti berilgan. Matematik kutilmasi, dispersiyasi va o‘rta kvadratik og‘ish qiymati topilsin

4.1.

x_i	-2	2	3	4
r_i	0,3	0,4	0,2	0,1

4.2.

x_i	-1	0	2	4
r_i	0,2	0,3	0,3	0,2

4.3.

x_i	-2	-1	0	1
r_i	0,1	0,2	0,3	0,4

4.4.

x_i	-1	0	3	4
r_i	0,1	0,2	0,4	0,3

4.5.

x_i	0	1	2	3
r_i	0,1	0,4	0,4	0,1

4.6.

x_i	-2	-1	0	3
r_i	0,3	0,5	0,1	0,1

4.7.

x_i	1	2	4	6
r_i	0,3	0,2	0,4	0,1

4.8.

x_i	0	1	4	7
r_i	0,5	0,2	0,2	0,1

4.9.

x_i	-1	0	1	2
r_i	0,6	0,2	0,1	0,1

4.10.

x_i	1	2	4	5
r_i	0,1	0,3	0,5	0,1

5-topshiriq.

X - uzluksiz tasodifiy miqdor taqsimot funktsiyasi F(x) teng. Zichlik funktsiyasi, matematik kutilma va dispersiya qiymatlari aniqlansin.

5.1. $\boldsymbol{F(x)} = \begin{cases} 0, & \boldsymbol{x} \le 0 \\ \boldsymbol{x}^2 & \boldsymbol{x} \in [0,1] \\ 1, & \boldsymbol{x} > 1 \end{cases}$

5.2. $\boldsymbol{F(x)} = \begin{cases} 0, & \boldsymbol{x} \le 1 \\ (\boldsymbol{x}^2 - \boldsymbol{x})/2 & \boldsymbol{x} \in (1,2] \\ 1, & \boldsymbol{x} > 2 \end{cases}$

5.3. $F(x) = \begin{cases} 0, & x \le 0 \\ x^3 & 0 < x \le 1 \\ 1, & x > 1 \end{cases}$

5.4. $F(x) = \begin{cases} 0, & x \le 0 \\ 3x^2 \_ 2x & 0 < x \le \frac{1}{3} \\ 1, & x > \frac{1}{3} \end{cases}$

5.5. $F(x) = \begin{cases} 0, & x \le 2 \\ \frac{x}{2} - 1 & 2 < x < 4 \\ 1, & x > 4 \end{cases}$

5.6. $F(x) = \begin{cases} 0, & x \le 0 \\ \frac{x^2}{9} & 0 < x < 3 \\ 1, & 3 > 3 \end{cases}$

5.7. $F(x) = \begin{cases} 0, & x \le 0 \\ \frac{x^2}{4} & 0 < x \le 2 \\ 1, & x > 2 \end{cases}$

5.8. $F(x) = \begin{cases} 0, & x \le -\frac{p}{2} \\ \cos x & -\frac{p}{2} < x \le 0 \\ 1, & x > 0 \end{cases}$

5.9. $F(x) = \begin{cases} 0, & x \le 0 \\ 2 \cdot \sin x & 0 < x \le \frac{p}{6} \\ 1, & x > \frac{p}{6} \end{cases}$

5.10. $F(x) = \begin{cases} 0, & x \le \frac{3p}{4} \\ \cos 2x & \frac{3p}{4} < x \le p \\ 1, & x > p \end{cases}$

6-topshiriq.

Tanlanma hajmi n ga teng bo‘lganda, normal taqsimot uchun 0,95 ishonchlilik koeffitsenti bilan matematik kutilma ***a*** uchun ishonchlilik intervalini toping. Bunda tanlanma o‘rta qiymati $\bar{x}$ va o‘rta kvadratik og‘ish σ ga teng deb olinsin.

6.1. $\bar{x}$ =75,17	a =36	σ = 6
6.2. $\bar{x}$ =75,16	a =49	σ = 7
6.3. $\bar{x}$ =75,15	a =64	σ = 8
6.4. $\bar{x}$ =75,13	a =100	σ = 10
6.5. $\bar{x}$ =75,11	a =144	σ = 12
6.6. $\bar{x}$ =75,09	a =196	σ = 14
6.7. $\bar{x}$ =75,14	a =81	σ = 9
6.8. $\bar{x}$ =75,12	a =121	σ = 11
6.9. $\bar{x}$ =75,10	a =169	σ = 13
6.10. $\bar{x}$ =75,08	a =225	σ = 15