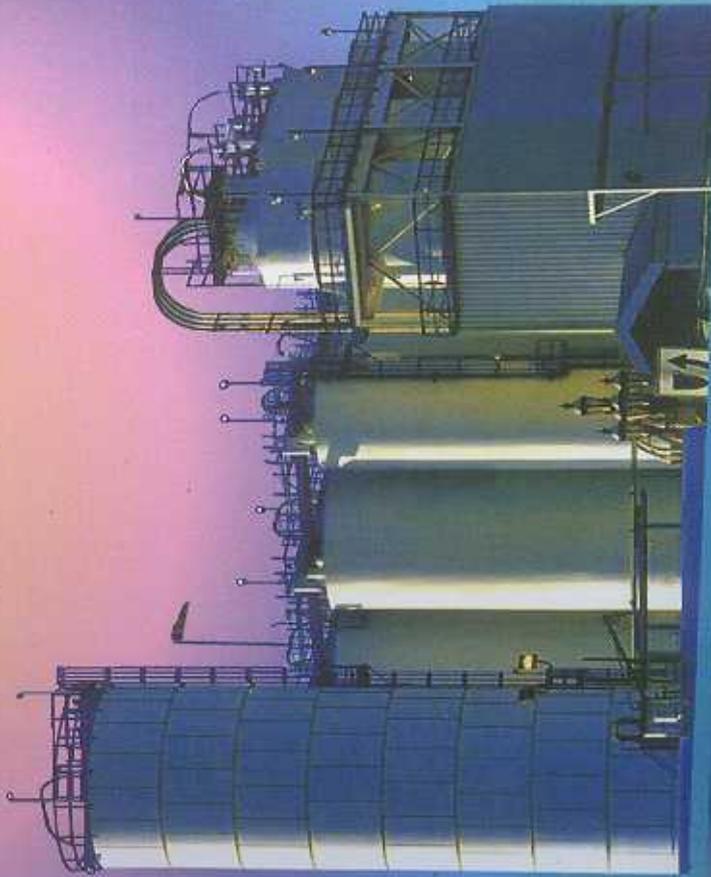


OLIV O'QUV YURTLARI UCHUN

K. ISMAYILOV

ELASTIK-PLASTIK QOBIQLAR



**Cho'lon nomidagi
nashriyot-matbaa ijodiy uyi**

ISBN 978-9943-05-223-9



9 789943 052239

1-81

K. ISMAYILOV

ELASTIK-PLASTIK QOBIQLAR

Oliy o'quv yurtlari uchun o'quv qo'llanma

Qo'llanma

FARG'ONA DAVLAT
UNIVERSITETI
AXBOROT RESURS MARKAZI
QABUL QILISH YIG'ISH VA
KATALOGLASH BO'LIMI

Cho'pon nomidagi nashriyot-matbaa ijodiy uyi
Toshkent — 2008

Mas'ul muharrir:

T.Sh. Shirinqulov — akademik.

Taqrizchilar:

X.X. Xudaynazarov — texnika fanlari doktori, professor,

Z.S. Sirojiddinov — texnika fanlari doktori, professor.

SO'ZBOSHI

Mustaqil Respublikamizda aviasozlik, mashinasozlik, gidrotexnik va fuqaro qurilishida ko'pgina hollarda yupqa devorli tizimlarni ishlatish katta samara bermoqda. Inshootlarni loyihalashda optimal, eng samarali va yuqori mustahkamlikka ega bo'lgan turli xil shakldagi qobiqlarga e'tibor qaratilmoqda. Qobiqlar nazariyasi, konstruksiya ishlarini chuqur taxlil qilish asosida, amaliy hisoblash usullarini va soddalashgan nazariyasini ishlab chiqish bo'yicha ishlar jadal rivojlanmoqda.

Shuning uchun ham ushbu o'quv qo'llanmada elastiklik chegarasidan keyin qobiqlarning kuchlanganlik deformatsiya holati va ustuvorlik nazariyasi muammolari qarang.

Elastiklik chegarasidan keyin qobiq oddiy yuklangan va uning materiali siqilmaydi, deb faraz qilinsa, bu kichik elastik-plastik deformatsiya nazariyasidan foydalanishga imkon beradi.

Tekis taralgan yuk bilan yuklangan qobiqning elastik-plastik muvozanat holatini ifodalovchi chiziqli bo'lmagan asosiy differensial tenglama A.A. Ilyushinning «elastik yechim» usulidan farg qiluvchi ketma-ket yaqinlashish usulining yangi varianti asosida yechilgan. Yangi usulning mohiyati shundan iboratki, birinchi yaqinlashuvda eguvchi moment va zo'riqish kuchlarining chiziqli bo'lmagan komponentlarini elastik-plastik uchastka uzunligi bo'yicha uch-burchak qonuni asosida o'zgarsin deb qaraladi. Bu usulning afzalligi shundaki, u birinchi yaqinlashuvning o'zidayoq fizik nochizizlikni e'tiborga olib, elastik-chiziqli yechimga bog'liq bo'lmagan yechimni olishga imkon beradi.

Elastiklik chegarasidan keyin siqilgan sterjen ustuvorligini yo'qotishining pastki chegarasi, sterjenning cheksiz kichik egilgan muvozanat holatda tutib turuvchi kritik kuchlanish bilan aniqlanishi, uning muvozanat shakli ikkilangan (bifurkatsiya) paytida bo'lishi ko'rsatilgan. Elastiklik chegarasidan keyin siqilgan sterjenning cheksiz kichik egilishida kesuvchi modul uning bo'yilama tolalarining

O'quv qo'llanmada turli yuklar ta'siridagi va konturi bo'yicha turli chegara shartli elastik va elastik-plastik qobiqlar hisobi asoslari qisqacha keltirilgan.

Ushbu o'quv qo'llanma O'zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligi tomonidan tasdiqlangan namunaviy dastur asosida universitet va texnika oliy o'quv yurtlarining talabalari uchun mo'ljallangan.

I 4306020900 - 54 - 2008
360(04) - 2008

ISBN 978-9943-05-223-9

© Cho'pon nomidagi nashriyot-matbaa ijodiy uyi, 2008- y.

yuklanish, shuningdek, yuksizlanish qismlarining cheksiz kichik uchastkasida, material siqilish diagrammasining kritik nuqtasiga urinma bo'yicha sijishi hamda kesuvchi modulning yuklanish qismida kamayishi, yuksizlanish qismida esa ortishi ko'rsatilgan. Shunday qilib, bifurkatsiya vaqtidagi kritik nuqta muhim nuqta emasligi tasdiqlanadi.

Bu natijalar elastiklik chegarasidan keyin siqilgan sterjen ustuvorlik nazariyasida fundamental ahamiyatga ega. Bu muhim xulosa asosida muallif plastik deformatsiya nazariyasi doirasida siqilgan sterjen uchun ustuvorlik tenglamasini yechishni va bu yondashuvni siqilgan doiraviy, to'g'ri burchakli plastinkalar, silindrik va sferik qobiqlar uchun umumlashtirdi. Qurilish po'lati uchun elastiklik chegarasidan keyin Berlin Dalemsk laboratoriyasida olingan klassik natijalarni tasdiqlovchi, siqilish diagrammasining analitik ifodasi tenglamasi taklif etilgan.

O'quv qo'llanmada konturi turli xil mahkamlangan bo'ylama yo'nalish bo'yicha siqilgan yopiq silindrik qobiqlar, siqilgan silindrik panel, tashqi bosim ta'sirida bo'lgan yopiq silindrik qobiq va sferik qobiqlar ustuvorlik masalalarining fundamental yechimlari olingan. Muallif qo'lyozma bilan tanishib, qimmatli maslahatlarini bildirgan akademik T.Sh.Shirinqulovga va texnika fanlari doktori, professor Z.S.Sirojiddinov, texnika fanlari doktori, professor X.X.Xudaynazarovlarga o'z minnatdorchiligini bildiradi.

O'quv qo'llanma to'g'risidagi barcha fikr va mulohazalarni muallif mamnuniyat bilan qabul qiladi.

QOBIQLAR NAZARIYASINING ASOSIY TENGLAMALARI

1.1. Asosiy tushunchalar. Qobiqlarning geometriyasi va ularning tasnifi

Qobiq deb, qalinligi h qolgan ikki o'lchamlariga nisbatan kichik bo'lgan egri sirt bilan chegaralangan jisimga aytiladi. Qobiqning qalinligini teng ikkiga bo'luvchi tekislik, uning o'rta sirti deb ataladi.

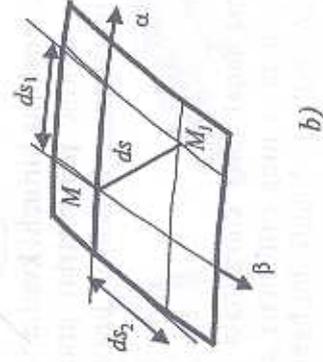
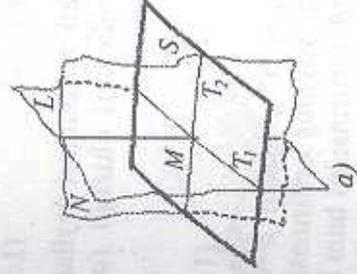
Agar qobiq h qalinligi eng kichik radius R_{min} ga nisbati $1/1500 \leq (h/R_{min}) \leq 1/20$ bo'lsa yupqa, agar bu nisbat $(h/R_{min}) > 1/2$ bo'lsa, qalin qobiqlar deb ataladi.

Yupqa qobiqlar nazariyasi Kirxgof-Lyav gipotezasiga binoan quyidagicha asoslandi:

— deformatsiyagacha qobiq o'rta sirtiga perpendikulyar bo'lgan element deformatsiyadan keyin ham o'rta sirtiga perpendikulyarligicha qoladi, qobiq qalinligi o'zgarmaydi;

— qobiqning o'rta sirtiga parallel bo'lgan qatlamlar orasida bosim hosil bo'lmaydi deb qaraladi.

Berilgan S egri sirtga normal qilib, uning ixtiyoriy biror M nuqtasidan L hamda N tekisliklarini o'tkazish mumkin (1.1- a chizma). Bu tekisliklarning S sirt bilan kesishgan egri chiziqlari T_1 va T_2 ekstremal qiymatlarga ega bo'ladi va ular bosh yo'nalishlar deb ataladi. Bu egri chiziqlarning radiuslari mos ravishda R_1 va R_2 bo'lsin.



1.1- chizma.

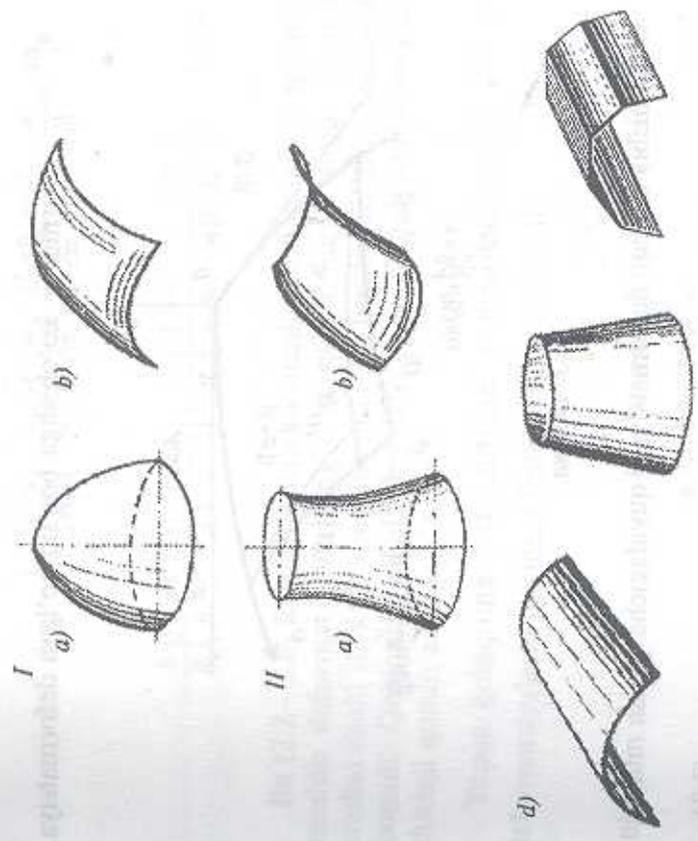
parametrlarni sirt bo'yicha yoy uzunligi deb qarash mumkin. Bosh R_1 va R_2 radiuslarga teskari bo'lgan kattaliklar tegishli bosh $K_1 = 1/R_1$, $K_2 = 1/R_2$ egriliklar deb ataladi. Sirt nazariyasida, tegishli bosh radiuslarga teskari bo'lgan K_1 va K_2 egriliklardan tashqari Gauss egriligi katta ahamiyatga ega.

Qobiq o'rta sirtning biror «M» nuqtasi bosh egriliklarning ko'paytmasiga qobiqning Gauss egriligi deyiladi.

$$G = K_1 K_2 \quad (1.5)$$

Qobiqlar Gauss egriligiga qarab quyidagi turlarga bo'linadi:

1. Gauss egriligi musbat, ya'ni $G > 0$ sferik, elliptik va parabolik qobiqlar (1.2 - a chizma).
2. Gauss egriligi manfiy, ya'ni $G < 0$ giperbolik qobiqlar (1.2 - v chizma);
3. Gauss egriligi nol, ya'ni $G = 0$ silindrik va konussimon qobiqlar (1.2 - d chizma).



1.2- chizma.

Qobiqlarning radiusiga teskari bo'lgan kattalik uning egriligi deb ataladi. O'tkazilgan L va N tekisliklar ortogonal tekisliklar bo'lgani uchun M nuqtadagi bosh yo'nalishlar ham ortogonaldir (1 - b chizma). Berilgan sirtga bir qancha egri chiziqlar o'tkazish mumkin, bu egri chiziqlarning har bir nuqtasiga o'tkazilgan urinma bosh yo'nalishlar bo'yicha yo'naladi.

Dekart koordinata x, y, z sistemasida S egri sirtini quyidagi tenglama orqali ifodalash mumkin:

$$\begin{aligned} X &= X(\alpha, \beta); \\ Y &= Y(\alpha, \beta); \\ Z &= Z(\alpha, \beta). \end{aligned} \quad (1.1)$$

Bu funksiyalar ma'lum bo'lsa, A va B Lyame parametrlari deb ataluvchi parametrlarni quyidagi formuladan topish mumkin:

$$\begin{aligned} A^2 &= \left(\frac{\partial X}{\partial \alpha}\right)^2 + \left(\frac{\partial Y}{\partial \alpha}\right)^2 + \left(\frac{\partial Z}{\partial \alpha}\right)^2; \\ B^2 &= \left(\frac{\partial X}{\partial \beta}\right)^2 + \left(\frac{\partial Y}{\partial \beta}\right)^2 + \left(\frac{\partial Z}{\partial \beta}\right)^2. \end{aligned} \quad (1.2)$$

Berilgan M nuqtadan, M , nuqta egri sirt bo'yicha cheksiz kichik masofada tursin. Bu cheksiz kichik kesma, sirt nazariyasida chiziqli element deb yuritiladi. Juda kichik chiziqli element kvadratini ortogonal koordinata sistemasida quyidagicha aniqlaymiz:

$$\begin{aligned} dS^2 &= dS_1^2 + dS_2^2; \\ dS^2 &= A^2 d\alpha^2 + B^2 d\beta^2; \\ dS_1 &= A d\alpha; \quad dS_2 = B d\beta. \end{aligned} \quad (1.3)$$

Bu tenglama birinchi kvadratlilik sirt shakli tenglamasi deyiladi. A va B Lyame parametrlar umumiy holda egri chiziqli (α, β) koordinatalarning funksiyasi hisoblanadi, ya'ni:

$$\begin{aligned} A &= A(\alpha, \beta); \\ B &= B(\alpha, \beta). \end{aligned} \quad (1.4)$$

Agar faqat bitta koordinata o'zgarsa, α o'zgaruvchi $\beta = const$ bo'lsa, unda α ning ortirmasi $d\alpha$ bo'lib, yoyning ortirmasi esa $dS_1 = A d\alpha$ bo'ladi. Xuddi shu kabi $dS_2 = B d\beta$ ortirmasini ham hosil qilish mumkin. Shunday qilib, koordinata chiziqlari $\alpha = const, \beta = const$ bo'lganda, ortirmasi tegishli $d\alpha = 1, d\beta = 1$ bo'lganda A va B

1.2. Qobiqdagi deformatsiya va ko'chishlar

Qobiq o'rta sirtining α koordinata yo'nalishidagi deformatsiyani aniqlashni ko'rib chiqamiz (1.3- chizma). Nisbiy chiziqli ε_1 deformatsiyasi uch qismdan iborat bo'ladi, ya'ni:

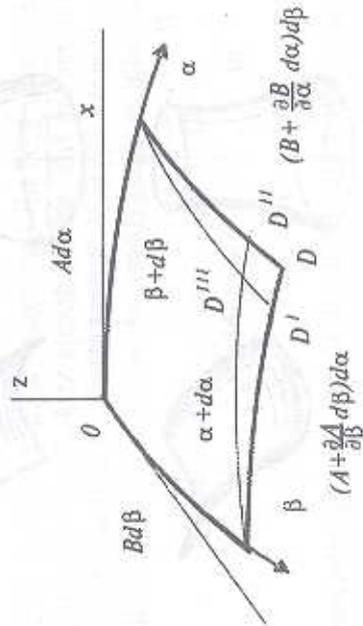
$$\varepsilon_1 = \varepsilon_1^I + \varepsilon_1^{II} + \varepsilon_1^{III} \quad (2.1)$$

Bu yerda:

$\varepsilon_1^I = \frac{1}{A} \frac{\partial u}{\partial \alpha}$ — ko'chishning $\beta = const$ koordinata chizig'i bo'yicha o'zgarishidan aniqlanadi.

$\varepsilon_1^{II} = \frac{1}{AB} \frac{\partial A}{\partial \beta} v$ — ko'chishning $\alpha = const$ koordinata chizig'i bo'yicha o'zgarishi va birinchi kvadrat shakli ko'effitsiyenti o'zgarishidan aniqlanadi.

$\varepsilon_1^{III} = \frac{w}{R_1}$ normal w ko'chishga bog'liq bo'lgan deformatsiya.



1.3- chizma.

Unda nisbiy chiziqli deformatsiyani quyidagicha yozish mumkin:

$$\varepsilon_1 = \frac{\partial u}{A \partial \alpha} + \frac{1}{AB} \frac{\partial A}{\partial \beta} v + \frac{w}{R_1} \quad (2.2)$$

Xuddi shuningdek, β koordinata bo'yicha nisbiy chiziqli deformatsiyani quyidagicha ifodalash mumkin:

$$\varepsilon_2 = \frac{\partial v}{B \partial \beta} + \frac{1}{AB} \frac{\partial B}{\partial \alpha} u + \frac{w}{R_2} \quad (2.3)$$

Qobiq o'rta sirtining siljish deformatsiyasi egriligining o'zgarishi va buralishi deformatsiyalari quyidagicha ifodalanadi:

$$\gamma = \frac{B}{A} \frac{\partial}{\partial \alpha} \left(\frac{v}{B} \right) + \frac{A}{B} \frac{\partial}{\partial \beta} \left(\frac{u}{A} \right) \quad (2.4)$$

$$\chi_1 = -\frac{1}{A} \frac{\partial}{\partial \alpha} \left(\frac{1}{A} \frac{\partial w}{\partial \alpha} - \frac{u}{R_1} \right) - \frac{1}{AB} \frac{\partial A}{\partial \beta} \left(\frac{1}{B} \frac{\partial w}{\partial \beta} - \frac{v}{R_2} \right);$$

$$\chi_2 = -\frac{1}{B} \frac{\partial}{\partial \beta} \left(\frac{1}{B} \frac{\partial w}{\partial \beta} - \frac{v}{R_2} \right) - \frac{1}{AB} \frac{\partial B}{\partial \alpha} \left(\frac{1}{A} \frac{\partial w}{\partial \alpha} - \frac{u}{R_1} \right) \quad (2.5)$$

$$\lambda = -\frac{1}{AB} \left(\frac{\partial^2 w}{\partial \alpha \partial \beta} - \frac{1}{A} \frac{\partial A}{\partial \beta} \frac{\partial w}{\partial \alpha} - \frac{1}{B} \frac{\partial B}{\partial \alpha} \frac{\partial w}{\partial \beta} \right) +$$

$$+ \frac{1}{R_1} \left(\frac{1}{B} \frac{\partial u}{\partial \beta} - \frac{1}{AB} \frac{\partial A}{\partial \beta} u \right) + \frac{1}{R_2} \left(\frac{1}{A} \frac{\partial v}{\partial \alpha} - \frac{1}{AB} \frac{\partial B}{\partial \alpha} v \right) \quad (2.6)$$

Bu (2.2—2.6) formulalardan ko'rinadiki, deformatsiya egrilik va buralish deformatsiyalarining o'zgarishi umumiy holda birinchi kvadrat shakl A va B ko'effitsiyentlariga bog'liq bo'lgan hadlardan iboratdir. Qobiqning qalinligi bo'ylab ε_1 , ε_2 va γ siljish deformatsiyalari chiziqli qonun asosida o'zgaradi.

Yupqa qobiqning o'rta sirtidan z masofadagi $\varepsilon_1^{(z)}$, $\varepsilon_2^{(z)}$, $\gamma^{(z)}$ deformatsiyalarni quyidagi formuladan aniqlanadi:

$$\begin{aligned} \varepsilon_1^{(z)} &= \varepsilon_1 + z \chi_1, \\ \varepsilon_2^{(z)} &= \varepsilon_2 + z \chi_2, \\ \gamma^{(z)} &= \gamma + 2z \lambda. \end{aligned} \quad (2.7)$$

1.3. Qobiq kesimdagi ichki kuchlar va momentlar

Berilgan qobiqdan, uning o'rtasiga normal bo'lgan α , $\alpha + d\alpha$; va β , $\beta + d\beta$ tekisliklar yordamida kichik element ajratib olamiz (1.4-shakl). Normal kuchlanishlarni σ_1 , σ_2 , o'rtasiga tekislikka parallel ravishda ta'sir etuvchi urinma kuchlanishlarni τ_{12} , τ_{21} , o'rtasiga tekislikka normal yo'nalishda ta'sir etuvchi urinma kuchlanishlarni τ_{13} , τ_{31} bilan belgilaymiz.

Uzunlik o'lchov birligiga to'g'ri keluvchi zo'riqish kuchlarini hamda momentlarini aniqlaymiz. Uzunlik o'lchov birligiga to'g'ri keluvchi zo'riqish kuchlarini N_1 , N_2 , urinma kuchlarni S_{12} , S_{21} , ko'ndalang kuchlarni Q_1 , Q_2 , eguvchi momentlarni M_1 , M_2 hamda τ_{12} , τ_{21} urinma kuchlanishlarning qobiq kesimi qalinligi bo'yicha notekis taqsimlanishi natijasida hosil bo'lgan buruvchi momentlarni M_{12} , M_{21} bilan belgilaymiz.

Zo'riqish N_1 kuchini aniqlash uchun ajratilgan elementar

$dF = Bd\beta \left(1 + \frac{z}{R_2}\right) dz$ yuzadagi dN_1 miqdorini quyidagicha ifodalaymiz:

$$dN_1 = Bd\beta \left(1 + \frac{z}{R_2}\right) \sigma_1 dz. \quad (3.1)$$

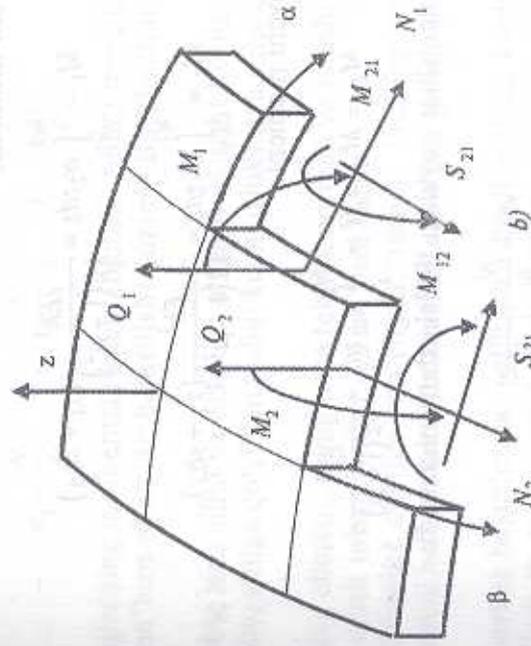
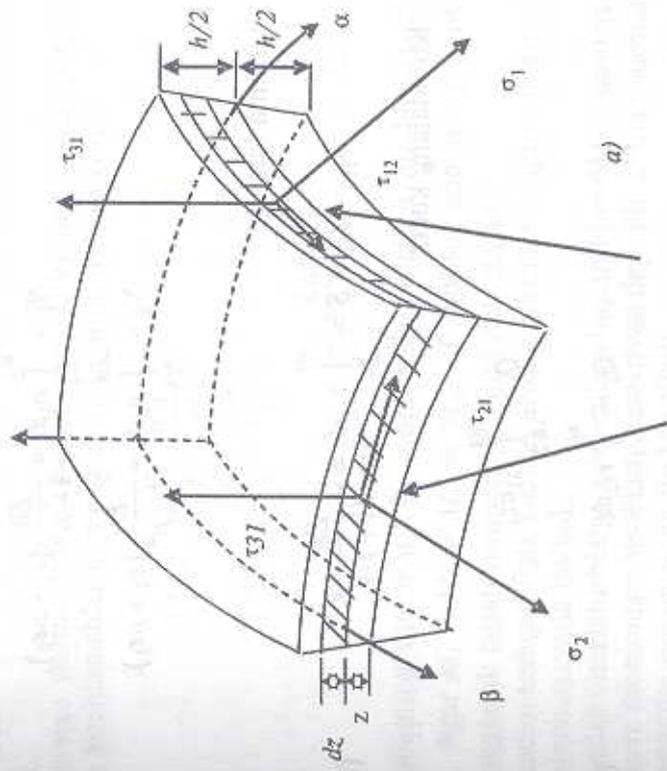
Yupqa qobiqlar uchun z/R_1 , z/R_2 kattaliklarni birga nisbatan e'tiborga olmasa ham bo'ladi. Haqiqatan ham qobiq qalinligi uning egrilik radiusiga nisbatan juda kichik bo'ladi. Unda:

$$dN_1 = Bd\beta \sigma_1 dz \quad (3.2)$$

(2.7) formulani e'tiborga olib, kuchlanish bilan deformatsiya orasidagi bog'lanishni ifodalovchi Guk qonuni quyidagicha yoziladi:

$$\begin{aligned} \sigma_1 &= \frac{E}{1-\mu^2} (\epsilon_1 + \mu\epsilon_2); \\ \sigma_2 &= \frac{E}{1-\mu^2} (\epsilon_2 + \mu\epsilon_1); \\ \tau_{12} = \tau_{21} &= \frac{E}{2(1+\mu)} \gamma. \end{aligned} \quad (3.3)$$

1.4-b chizmada ichki kuchlarning musbat yo'nalishi ko'rsatilgan. Uzunlik o'lchov birligiga to'g'ri keluvchi bu ichki kuch komponentlari quyidagi formulalardan aniqlanadi.



1.4-chizma.

Normal kuch:

$$N_1 = \int_{-h/2}^{h/2} \sigma_1 dz = \frac{Eh}{1-\mu^2} (\varepsilon_1 + \mu\varepsilon_2), \quad (3.4)$$

$$N_2 = \int_{-h/2}^{h/2} \sigma_2 dz = \frac{Eh}{1-\mu^2} (\varepsilon_2 + \mu\varepsilon_1).$$

Urinnma kuch:

$$S_{12} = S_{21} = S = \int_{-h/2}^{h/2} \tau_{12} dz = \frac{Eh}{2(1+\mu)} \gamma. \quad (3.5)$$

Ko'ndalang kuch:

$$Q_1 = \int_{-h/2}^{h/2} \tau_{31} dz; \quad (3.6)$$

$$Q_2 = \int_{-h/2}^{h/2} \tau_{32} dz.$$

Eguvchi moment:

$$M_1 = \int_{-h/2}^{h/2} \sigma_2 z dz = \frac{Eh^3}{12(1-\mu^2)} (\chi_1 + \mu\chi_2). \quad (3.7)$$

$$M_2 = \int_{-h/2}^{h/2} \sigma_1 z dz = \frac{Eh^3}{12(1-\mu^2)} (\chi_2 + \mu\chi_1).$$

Burovchi moment:

$$M_{21} = M_{12} = H = \int_{-h/2}^{h/2} \tau_{21} z dz = \frac{Eh^3}{12(1+\mu)} \lambda. \quad (3.8)$$

Kuchlanishlar quyidagi ifodalardan aniqlanadi:

$$\sigma_1 = \frac{N_1}{h} + \frac{M_1}{I} z;$$

$$\sigma_2 = \frac{N_2}{h} + \frac{M_2}{I} z;$$

$$\tau_{12} = \frac{S_{12}}{h} + \frac{M_{12}}{I} z. \quad (3.9)$$

Bu yerda $I = h^3/12$ eni bir birlikka to'g'ri keluvchi kesimning o'rta tekislikka urinma bo'lgan o'qqa nisbatan inersiya momenti.

Inersiya momentini e'tiborga olib, (3.9) formulaga $z = h/2$ ni qo'yib kuchlanishlarning eng katta qiymatlarini topamiz:

$$(\sigma_1)_{\max} = \frac{N_1}{h} + \frac{6M_1}{h^2};$$

$$(\sigma_2)_{\max} = \frac{N_2}{h} + \frac{6M_2}{h^2};$$

$$(\sigma_3)_{\max} = (\tau_{12})_{\max} = \frac{S_{12}}{h} + \frac{6M_{12}}{h^2}. \quad (3.10)$$

Qobiqning momentli va momentsiz holatlari bir-biridan farq qiladi. Agar $M_1 = M_2 = M_{12} = M_{21} = 0$ bo'lsa, qobiqning bunday kuchlanganlik holati momentsiz holat deb ataladi.

Bundan tashqari fizik kuchlar asosan normal va siljitivchi kuchlar bilan muvozanatda bo'ladi.

Bunday cheklanishlarga asosan qobiqlarni hisoblash nazariyasi, qobiqlarni momentsiz hisoblash nazariyasi deyiladi. Unda yuqoridagi kuchlanishlar quyidagicha ifodalaniladi:

$$\sigma_1 = \frac{N_1}{h}; \quad \sigma_2 = \frac{N_2}{h}; \quad \tau_{12} = \frac{S_{12}}{h}. \quad (3.11)$$

Qobiqlarning momentsiz kuchlanganlik holati mavjud bo'lishi uchun ma'lum shartlar bajarilishi lozim. Bu shartlar quyidagilardir:

1. Qobiq sirti silliq va uzluksiz o'zgarib borishi lozim.
2. Qobiq sirtiga qo'yilgan tashqi kuch uzluksiz va silliq o'zgarishi lozim.
3. Qobiq chetlari, uning o'rta tekislik normalini yo'nalishi bo'yicha erkin ko'chish imkoniyatiga ega bo'lishi lozim.
4. Qobiqning chetiga qo'yilgan tashqi kuchlar uning o'rta sirtiga normal bo'lgan tekislikda tashkil etuvchilarga ega bo'lmaydigan bo'lishi lozim.

Bu shartlar bajarilmasa, egilishdan hosil bo'lgan kuchlanishlar katta bo'ladi va qobiqlarni hisoblashda uni e'tiborga olish kerak. Bunday hollarda qobiqlar momentli qobiqlar nazariyasi bilan hisoblanadi.

1.4. Qoʻbiq muvozanat differensial tenglamasi

Qoʻbiqdan $\alpha = \text{const}$, $\alpha + d\alpha = \text{const}$ va $\beta + d\beta = \text{const}$ normal kesimlar yordamida ajratib olingan biror kichik elementning muvozanatini tekshirib koʻramiz (1.5- chizma). Qoʻbiq sirti boʻyicha tashqi kuchlar silliq va tekis taqsimlangan boʻlsin.

Elementga taʼsir etuvchi barcha kuchlarning koordinata α , β , z oʻqlaridagi proyeksiyalarini q_1, q_2, q_3 lar bilan belgilaymiz. Elementning $\alpha + d\alpha = \text{const}$, $\beta + d\beta = \text{const}$ tomonlari quyidagicha orttir-maga ega boʻladi:

$$\begin{aligned} DD^I &= \frac{\partial B}{\partial \alpha} d\alpha \, d\beta; \\ DD^{II} &= \frac{\partial A}{\partial \beta} d\alpha \, d\beta. \end{aligned} \quad (4.1)$$

Xuddi shuningdek, zoʻriqish kuchlari ham α , β koordinatalarining uzluksiz funksiyalari boʻlganligi uchun orttirmaga ega boʻladi.

Chetki 0C — tomonida normal kuch $N_1 B d\beta$, chetki FD — tomonida normal kuch quyidagicha ifodalanadi:

$$\begin{aligned} \left(N_1 + \frac{\partial N_1}{\partial \alpha} d\alpha \right) \left(B d\beta + \frac{\partial B}{\partial \alpha} d\alpha d\beta \right) &= N_1 B d\beta + \\ + \frac{\partial N_1}{\partial \alpha} B d\alpha d\beta + N_1 \frac{\partial B}{\partial \alpha} d\alpha d\beta &+ \frac{\partial N_1}{\partial \alpha} \frac{\partial B}{\partial \alpha} d\alpha d\alpha d\beta. \end{aligned} \quad (4.2)$$

Bu ifodadagi oxirgi had juda ham kichik boʻlganligi uchun uni eʼtiborga olmaymiz va bunda:

$$\frac{\partial N_1}{\partial \alpha} B d\alpha d\beta + N_1 \frac{\partial B}{\partial \alpha} d\alpha d\beta = \frac{\partial}{\partial \alpha} (N_1 B) d\alpha d\beta \quad (4.3)$$

ekanligini eʼtiborga olib, chetki FD — tomoniga taʼsir etuvchi normal kuchni quyidagicha ifodalaymiz:

$$N_1 B d\beta + \frac{\partial}{\partial \alpha} (N_1 B) d\alpha d\beta. \quad (4.4)$$

Xuddi shuningdek, chetki FD va CD tomonlaridagi zoʻriqish kuchlarni aniqlaymiz:

1. Barcha kuchlarni 0x oʻqiga proyeksiyalab muvozanat tenglamasini tuzamiz.

Normal kuchlarning 0x oʻqidagi proyeksiyasi:

$$N_1 B d\beta + \frac{\partial}{\partial \alpha} (N_1 B) d\alpha d\beta - N_1 B d\beta = \frac{\partial}{\partial \alpha} (N_1 B) d\alpha d\beta. \quad (4.5)$$

Sijituvchi S_{12} kuchning 0x oʻqidagi proyeksiyasi:

$$\begin{aligned} S_{12} A d\alpha + \frac{\partial}{\partial \beta} (S_{12} A) d\alpha d\beta - S_{12} A d\alpha &= \\ = \frac{\partial}{\partial \beta} (S_{12} A) d\alpha d\beta. \end{aligned} \quad (4.6)$$

Chetki CD tomonga taʼsir etuvchi normal kuch:

$$N_2 A d\alpha + \frac{\partial}{\partial \beta} (N_2 A) d\alpha d\beta, \quad (4.7)$$

Chetki 0F tomonga nisbatan juda kichik ψ burchakka burilgan bu normal kuch 0x oʻqiga proyeksiya beradi.

Quyidagi nisbatni eʼtiborga olib, bu juda kichik ψ burchakni 1.5- chizmadan aniqlaymiz:

$$\begin{aligned} \text{tg } \psi &= \frac{DD^{III} DD^{II}}{FD^{III}} \approx \frac{DD^I}{FD^{III}}; \\ \psi &\approx \text{tg } \psi = \frac{\frac{\partial B}{\partial \alpha} d\alpha d\beta}{A d\alpha} \end{aligned} \quad (4.8)$$

ekanligini eʼtiborga olib, (4.7) ifodani quyidagi koʻrinishga keltiramiz:

$$\begin{aligned} - \left[N_2 A d\alpha + \frac{\partial}{\partial \beta} (N_2 A) d\alpha d\beta \right] \psi &= \\ = -N_2 \frac{\partial B}{\partial \alpha} d\alpha d\beta - \frac{\partial}{\partial \beta} (N_2 A) \frac{\partial B}{\partial \alpha} \frac{1}{A} d\alpha^2 d\beta^2. \end{aligned} \quad (4.9)$$

Bu ifodaning oʻng tomonidagi oxirgi had juda ham kichik boʻlganligi uchun eʼtiborga olmaymiz. Unda:

$$-N_2 \frac{\partial B}{\partial \alpha} d\alpha d\beta \quad (4.10)$$

Xuddi shuningdek, chetki FD tomonga ta'sir etuvchi siljituvcchi kuchning Ox o'qidagi proyeksiyasi:

$$\left[S_{21} B d\beta + \frac{\partial}{\partial \alpha} (S_{21} B) d\alpha d\beta \right] \psi^*, \quad (4.11)$$

Bu yerda:

$$\psi^* = \frac{\frac{\partial A}{\partial \beta} d\alpha d\beta}{B d\beta} \quad (4.12)$$

Unda bu kuchning proyeksiyasi:

$$S_{21} \frac{\partial A}{\partial \beta} d\alpha d\beta \quad (4.13)$$

Kesuvchi Q_1 kuchning Ox o'qidagi proyeksiyasi, yuqori tartibli cheksiz kichik hadlarni e'tiborga olmasak, quyidagicha ifodalanadi:

$$\frac{1}{R_1} Q_1 A B d\alpha d\beta \quad (4.14)$$

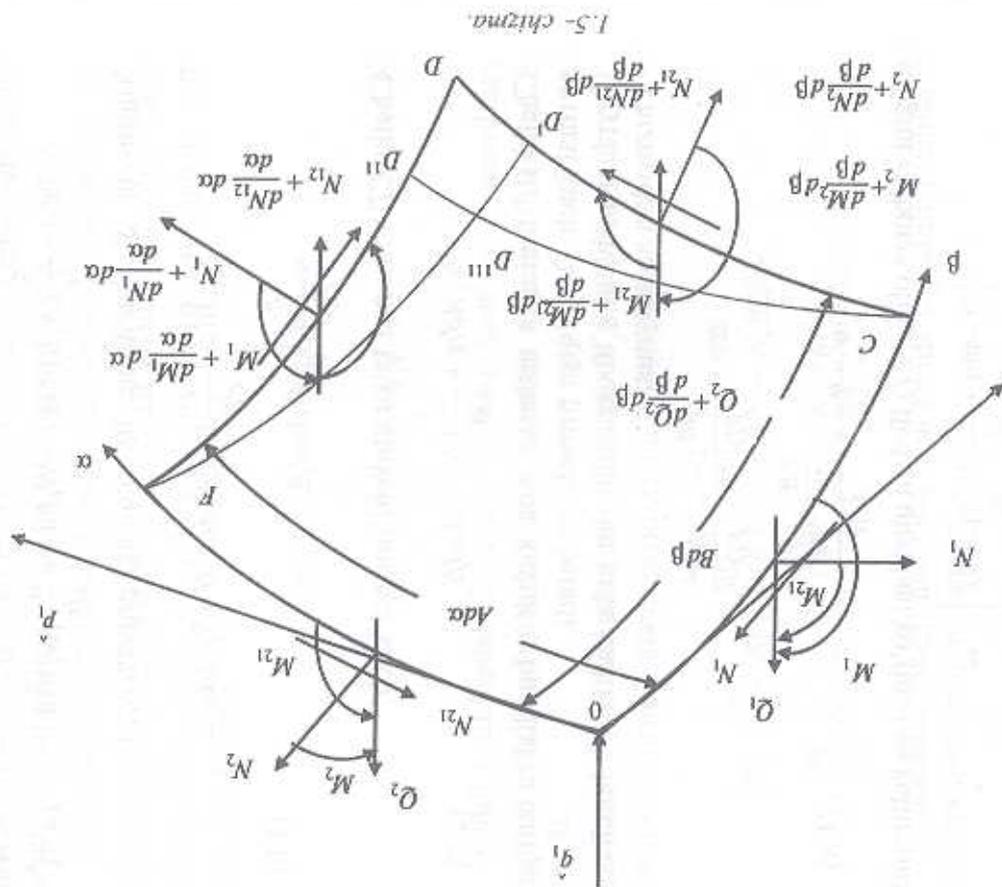
Sirtqi kuchning Ox o'qidagi proyeksiyasi:

$$q_1 A B d\alpha d\beta \quad (4.15)$$

Yuqorida aniqlangan barcha kuchlarni qo'shib va hosil qilingan natijani $d\alpha$, $d\beta$ bo'lib, Ox o'qi bo'yicha yo'nalgan kuchlar muvozanat tenglamasini hosil qilamiz:

$$\begin{aligned} \frac{\partial}{\partial \alpha} (N_1 B) + \frac{\partial}{\partial \beta} (S_{12} A) - N_2 \frac{\partial B}{\partial \alpha} + \\ + S_{21} \frac{\partial A}{\partial \beta} + Q_1 \frac{A B}{R_1} + A B q_1 = 0. \end{aligned} \quad (4.16)$$

2. Xuddi shu usul bilan Oy koordinata o'qi bo'yicha muvozanat tenglamasini ham tuzamiz:



$$\frac{\partial}{\partial \beta} (N_2 A) + \frac{\partial}{\partial \alpha} (S_{21} B) - N_1 \frac{\partial A}{\partial \beta} + S_{12} \frac{\partial B}{\partial \alpha} + Q_2 \frac{AB}{R_2} + AB q_2 = 0. \quad (4.17)$$

3. Koordinata Oz o'qiga nisbatan muvozanat tenglamasini tuzamiz.

Ko'ndalang Q_1 kuchining proyeksiyasi:

$$-Q_1 B d\beta + \left[Q_1 B d\beta + \frac{\partial}{\partial \alpha} (Q_1 B) d\alpha d\beta \right] \cos \varphi_1. \quad (4.18)$$

φ_1 - burchak juda ham kichik bo'lgani uchun $\cos \varphi_1 = 1$ bo'ladi. Unda (4.18) ifoda quyidagicha bo'ladi:

$$\frac{\partial}{\partial \alpha} (Q_1 B) d\alpha d\beta.$$

Chetki CD tomondagi normal kuchning Oz o'qidagi proyeksiyasi:

$$- \left[N_1 B d\beta + \frac{\partial}{\partial \alpha} (N_1 B) \right] \frac{Ad\alpha}{R_1}. \quad (4.19)$$

Bu ifodani soddalashtirib, quyidagi ko'rinishga keltiramiz:

$$-N_1 \left(\frac{AB}{R_1} \right) d\alpha d\beta.$$

Chetki OF va CD tomonlariga ta'sir etuvchi kuchlar ham xuddi quyidagicha tashkil etuvchilarni beradi:

$$\frac{\partial}{\partial \beta} (Q_2 A) d\alpha d\beta; \text{ va } -N_2 \frac{AB}{R_2} d\alpha d\beta. \quad (4.20)$$

Sirtqi kuchning Oz o'qidagi proyeksiyasi:

$$q_3 AB d\alpha d\beta. \quad (4.21)$$

Yuqorida aniqlangan barcha kuchlarni qo'shib va natijani $d\alpha$, $d\beta$ bo'lib, Oz o'qi bo'yicha yo'nalgan kuchlar muvozanat tenglamasini hosil qilamiz:

$$\frac{\partial}{\partial \alpha} (Q_1 B) + \frac{\partial}{\partial \beta} (Q_2 A) - N_1 \frac{AB}{R_1} - N_2 \frac{AB}{R_2} + q_3 AB. \quad (4.22)$$

4. Koordinata Ox o'qiga nisbatan barcha kuchlardan olingan momentlarning algebraik yig'indisini nolga tenglaymiz:

$$\frac{\partial (M_{21} B)}{\partial \alpha} + \frac{\partial (M_2 A)}{\partial \beta} - M_1 \frac{\partial A}{\partial \beta} + M_{12} \frac{\partial B}{\partial \alpha} - Q_2 AB = 0. \quad (4.23)$$

5. Xuddi shuningdek, Oy o'qiga nisbatan barcha kuchlardan olingan momentlarning algebraik yig'indisini nolga tenglaymiz:

$$\left(\frac{\partial M_{12} A}{\partial \beta} \right) + \frac{\partial (M_1 B)}{\partial \alpha} - M_2 \frac{\partial B}{\partial \alpha} + M_{21} \frac{\partial A}{\partial \beta} - Q_1 AB = 0. \quad (4.24)$$

6. Urinma kuchlanishlarning juftlik qonuni asosida Oz o'qiga nisbatan barcha zo'riqlardan olingan momentlar statikaning muvozanat tenglamasini aynan qanoatlantiradi.

Shunday qilib, qobiq uchun 5 ta muvozanat tenglamalar sistemasini hosil qildik:

$$\begin{aligned} \frac{\partial}{\partial \alpha} (N_1 B) + \frac{\partial}{\partial \beta} (S_{12} A) - N_2 \frac{\partial B}{\partial \alpha} + S_{21} \frac{\partial A}{\partial \beta} + Q_1 K_1 AB + q_1 AB = 0. \end{aligned} \quad (4.25)$$

$$\begin{aligned} \frac{\partial}{\partial \beta} (N_2 A) + \frac{\partial}{\partial \alpha} (S_{21} B) - N_1 \frac{\partial A}{\partial \beta} + S_{12} \frac{\partial B}{\partial \alpha} + Q_2 K_2 AB + q_2 AB = 0. \end{aligned} \quad (4.26)$$

$$\frac{1}{AB} \left[\frac{\partial}{\partial \alpha} (Q_1 B) + \frac{\partial}{\partial \beta} (Q_2 A) \right] - N_1 K_1 - N_2 K_2 + q_3 = 0. \quad (4.27)$$

$$\left(\frac{\partial M_{21} B}{\partial \alpha} \right) + \frac{\partial (M_2 A)}{\partial \beta} - M_1 \frac{\partial A}{\partial \beta} + M_{12} \frac{\partial B}{\partial \alpha} - Q_2 AB = 0. \quad (4.28)$$

$$\left(\frac{\partial M_{12} A}{\partial \beta} \right) + \frac{\partial (M_1 B)}{\partial \alpha} - M_2 \frac{\partial B}{\partial \alpha} + M_{21} \frac{\partial A}{\partial \beta} - Q_1 AB = 0. \quad (4.29)$$

Bu tenglamalarga 10 ta $N_1, N_2, S_{12}, S_{21}, M_1, M_2, M_{12}, M_{21}, Q_1, Q_2$ noma'lumlar kiradi.

Agar momentlar $M_1 = M_2 = M_{21} = 0$ va ko'ndalang kuch $Q_1 = Q_2 = 0$ ga teng bo'lsa, (4.25-4.29) tenglamalardan momentsiz qobiqlar nazariyasi tenglamalari kelib chiqadi:

$$\frac{\partial}{\partial \alpha} (N_1 B) + \frac{\partial}{\partial \beta} (S_{21} A) - N_2 \frac{\partial B}{\partial \alpha} + S_{21} \frac{\partial A}{\partial \beta} + q_1 AB = 0. \quad (4.30)$$

$$\frac{\partial}{\partial \beta} (N_2 A) + \frac{\partial}{\partial \alpha} (S_{21} B) - N_1 \frac{\partial A}{\partial \beta} + S_{12} \frac{\partial B}{\partial \alpha} + q_2 AB = 0. \quad (4.31)$$

$$\frac{N_1}{R_1} + \frac{N_2}{R_2} + q_3 = 0. \quad (4.32)$$

1.5. Momentsiz nazariya asosida aylanish sirtli qobiqlar hisobi

Sferik koordinatalar sistemasida ixtiyoriy aylanish sirtli qobiq 1.6- chizmada keltirilgan. Chizmadan quyidagilarni aniqlash mumkin:

$$r = R_2 \sin \varphi; \quad ab = ds_2 = R_2 \sin \varphi \, d\varphi; \quad (5.1)$$

$$ad = ds_1 = R_1 d\varphi; \quad A = R_1; \quad B = r. \quad (5.1)$$

Simmetrik qobiq uchun A va B Lyame koeffitsiyentlari faqat o'zgaruvchining funksiyasi bo'lib, θ koordinataga bog'liq emas. Bu holda differensial muvozanat (4.30-4.32) tenglamalar quyidagi ko'rinishda ifodalanadi:

$$\frac{\partial}{\partial \varphi} (rN_1) + R_1 \frac{\partial S}{\partial \theta} - N_2 R_1 \cos \varphi + q_1 r R_1 = 0. \quad (5.2)$$

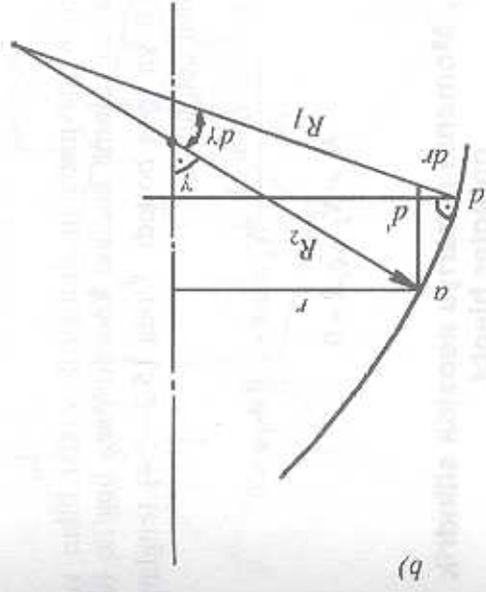
$$R_1 \frac{\partial N_2}{\partial \theta} + \frac{\partial}{\partial \varphi} (rS) + SR_1 \cos \varphi + q_2 r R_1 = 0. \quad (5.3)$$

$$\frac{N_1}{R_1} + \frac{N_2}{R_2} + q_3 = 0. \quad (5.4)$$

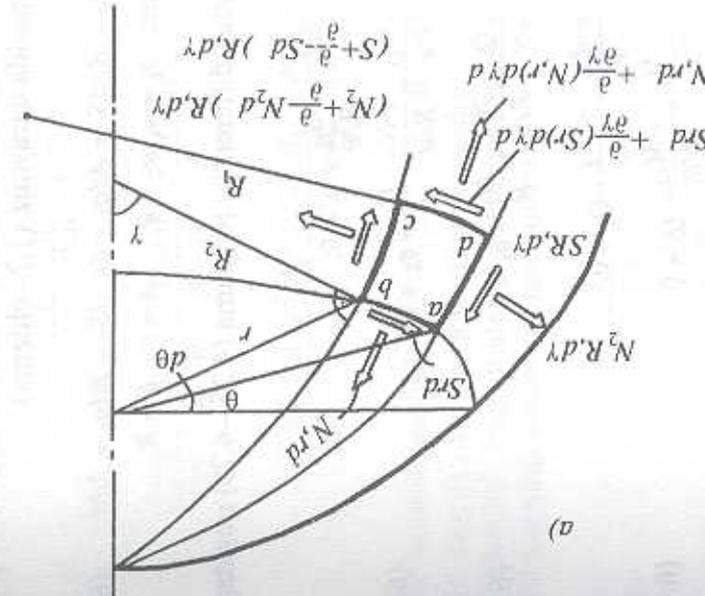
Xususiy holda sferik qobiq uchun $R_1 = R_2 = R$ va $Ad\alpha = R d\varphi$; $Bd\beta = (R \sin \varphi) d\theta$ ekanligini aniqlash qiyin emas. Unda $\alpha = \varphi$; $\beta = \theta$; $A = R$; $B = R \sin \varphi$ bo'ladi. Qobiqning momentsiz nazariyasi tenglamalarini quyidagicha yozamiz:

$$\frac{\partial}{\partial \varphi} (N_1 \sin \varphi) + \frac{\partial S_{12}}{\partial \theta} - N_2 \cos \varphi + q_1 R \sin \varphi = 0. \quad (5.5)$$

$$\frac{\partial}{\partial \theta} (S_{21} \sin \varphi) + \frac{\partial N_2}{\partial \theta} + S_{12} \cos \varphi + q_2 R \sin \varphi = 0. \quad (5.6)$$



1.6- chizma.



$$N_1 + N_2 + q_3 R = 0. \quad (5.7)$$

Agar qobiq o'qqa nisbatan simmetrik yuklar bilan yuklangan bo'lsa, normal va urinma kuchlar koordinataga bog'liq bo'lmaydi va $S_{12} = S_{21} = 0$ ga teng bo'ladi. Unda (5.2—5.4) tenglamalardan quyidagi ikkita qoladi:

$$\frac{\partial(N_1 \sin \varphi)}{\partial \varphi} - N_2 \cos \varphi + q_1 R \sin \varphi = 0. \quad (5.8)$$

$$N_1 + N_2 + q_3 R = 0. \quad (5.9)$$

1.6. Momentli nazariya asosida silindrik qobiqlar hisobi

Uzunligi ℓ va radiusi R bo'lgan silindir shaklidagi qobiqni x, θ koordinata sistemasida qaraymiz (1.7- chizma).

Unda:

$$R_1 = \infty; R_2 = R, dS_1 = Ad\alpha_1 = dx; dS_2 = Bd\alpha_2 = R d\theta \quad (6.1)$$

bo'ladi, demak: $\alpha_1 = x; \alpha_2 = \theta; A = 1; B = R$.

Unda muvozanat differensial tenglama (4.25—4.29) quyidagicha ifodalanadi:

$$\frac{\partial N_1}{\partial x} + \frac{\partial S_{12}}{R \partial \theta} + q_1 = 0;$$

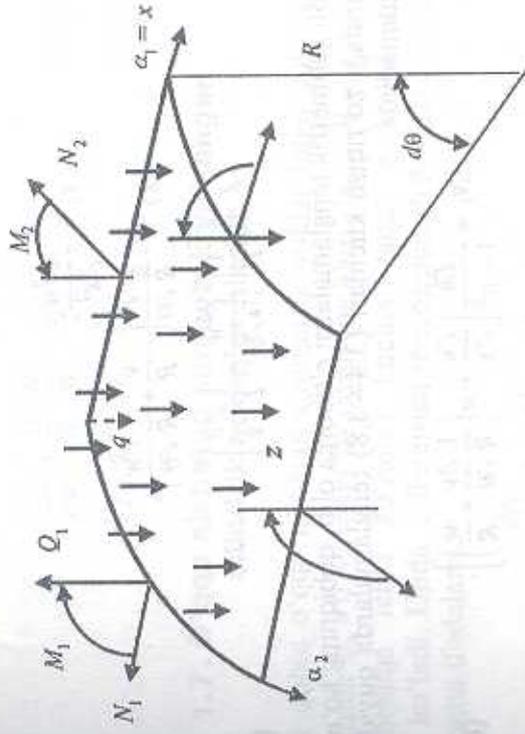
$$\frac{\partial S_{21}}{\partial x} + \frac{\partial N_2}{R \partial \theta} - \frac{Q_2}{R} + q_2 = 0. \quad (6.2)$$

$$\frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{R \partial \theta} + \frac{N_2}{R} + q_3 = 0. \quad (6.3)$$

$$\frac{\partial M_{21}}{\partial x} + \frac{\partial M_2}{R \partial \theta} - Q_2 = 0.$$

$$\frac{\partial M_1}{\partial x} + \frac{\partial M_{12}}{R \partial \theta} - Q_1 = 0. \quad (6.4)$$

Bu 5 ta tenglamalar sistemasidan kesuvchi kuchni yo'qotamiz, unda:



1.7- chizma.

$$\frac{\partial N_1}{\partial x} + \frac{\partial S_{12}}{R \partial \theta} + q_1 = 0. \quad (6.5)$$

$$\frac{\partial S_{12}}{\partial x} + \frac{1}{R} \frac{\partial N_2}{\partial \theta} + \frac{1}{R^2} \frac{\partial M_2}{\partial \theta} + \frac{1}{R} \frac{\partial M_{12}}{\partial x} + q_2 = 0. \quad (6.6)$$

$$\frac{\partial^2 M_1}{\partial x^2} + \frac{1}{R^2} \frac{\partial^2 M_2}{\partial \theta^2} + \frac{2}{R} \frac{\partial^2 M_{12}}{\partial x \partial \theta} + \frac{N_2}{R} + q_3 = 0. \quad (6.7)$$

Bu uchta tenglamada 6 ta $N_1, N_2, M_1, M_2, M_{12}, M_{21}$ noma'lum bo'lib, masala uch marta statik aniqmasdir.

Geometrik tenglamalarini yuqoridagi (2.2—2.7) tenglamalarga asosan quyidagi ifodalarni hosil qilamiz:

$$\epsilon_1 = \frac{\partial u}{\partial x};$$

$$\epsilon_2 = \frac{1}{R} \frac{\partial v}{\partial \theta} + \frac{w}{R};$$

$$\gamma = \frac{\partial v}{\partial x} + \frac{1}{R} \frac{\partial u}{\partial \theta}. \quad (6.8)$$

$$\chi_1 = -\frac{\partial^2 w}{\partial x^2};$$

$$\chi_2 = -\frac{1}{R} \frac{\partial}{\partial \theta} \left(-\frac{v}{R} + \frac{1}{R} \frac{\partial w}{\partial \theta} \right);$$

$$\lambda = -\frac{1}{R} \left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{1}{2} \frac{\partial v}{\partial x} \right).$$

(6.9)

Bu geometrik tenglamalarni e'tiborga olib, qobiqning ko'ndalang kesimdagi zo'riqish kuchlari (3.4—3.8) tenglamalarda quyidagicha ifodalanadi:

$$N_1 = \frac{Eh}{1-\mu^2} \left[\frac{\partial u}{\partial x} + \mu \left(\frac{1}{R} \frac{\partial v}{\partial \theta} + \frac{w}{R} \right) \right]. \quad (6.10)$$

$$N_2 = \frac{Eh}{1-\mu^2} \left[\frac{1}{R} \frac{\partial v}{\partial \theta} + \mu \frac{\partial u}{\partial x} \right]. \quad (6.11)$$

$$S = S_{21} = S_{12} = \frac{Eh}{2(1+\mu)} \left[\frac{1}{R} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} \right]. \quad (6.12)$$

$$M_1 = -D \left[\frac{\partial^2 w}{\partial x^2} + \frac{\mu}{R^2} \left(-\frac{\partial v}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2} \right) \right]. \quad (6.13)$$

$$M_2 = -D \left[\frac{1}{R^2} \left(-\frac{\partial v}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2} \right) + \mu \frac{\partial^2 w}{\partial x^2} \right]. \quad (6.14)$$

$$M_{12} = M_{21} = H = -D(1-\mu) \frac{1}{R} \left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{1}{2} \frac{\partial v}{\partial x} \right);$$

$$D = \frac{Eh^3}{12(1-\mu^2)}. \quad (6.15)$$

Agar $M_1 = M_2 = M_{12} = M_{21} = 0$ bo'lsa, (6.5—6.7) tenglamalar soddalashadi va momentsiz qobiq nazariyasi tenglamalarini hosil qilamiz:

$$\frac{\partial N_1}{\partial x} + \frac{1}{R} \frac{\partial S_{12}}{\partial \theta} + q_1 = 0. \quad (6.16)$$

$$\frac{\partial S_{21}}{\partial x} + \frac{1}{R} \frac{\partial N_2}{\partial \theta} + q_2 = 0, \quad (6.17)$$

$$N_2 + q_3 R = 0. \quad (6.18)$$

1.7. O'qqa nisbatan simmetrik yuklangan silindrik qobiqlar hisobi

Agar qobiq o'qqa nisbatan simmetrik yuklar bilan yuklangan bo'lsa, unda $q_2 = 0$ va ichki zo'riqish kuchlari $M_{12} = H = 0$; $S_{12} = 0$ bo'lib, boshqa ichki zo'riqish kuchlari faqat x koordinataga bog'liq bo'ladi. Unda differensial muvozanat (6.5—6.7) tenglama quyidagicha ifodalanadi:

$$\begin{aligned} \frac{dN_1}{dx} + q_1 &= 0; \\ \frac{d^2 M_1}{dx^2} + \frac{N_2}{R} + q_3 &= 0. \end{aligned} \quad (7.1)$$

Bu ikki tenglamada uchta N_1, N_2, M_1 noma'lum bo'lib, masala bir marta statik aniqlanmasdir.

Birinchi muvozanat tenglamadan aniqlaymiz:

$$N_1 = -q_1 x + C. \quad (7.2)$$

Bu masalani ko'chishga nisbatan yechamiz: qaralayotgan masala o'qqa simmetrik bo'lgani uchun doiraviy ko'chish $v=0$ bo'lib, u, w - ko'chishlar x koordinataning funksiyasidir. Unda geometrik tenglama (6.8—6.9) quyidagicha ifodalanadi:

$$\epsilon_1 = \frac{du}{dx};$$

$$\epsilon_2 = -\frac{w}{R};$$

$$\gamma = 0;$$

$$\chi_1 = \frac{d^2 w}{dx^2};$$

$$\chi_2 = 0;$$

$$\lambda = 0. \quad (7.3)$$

Unda zo'riqish (6.10—6.15) kuchlari quyidagicha ifodalanadi:

$$N_1 = \frac{Eh}{1-\mu^2} \left(\frac{du}{dx} - \mu \frac{w}{R} \right);$$

$$N_2 = \frac{Eh}{1-\mu^2} \left(-\frac{w}{R} + \mu \frac{du}{dx} \right). \quad (7.4)$$

Eguvchi momentlar:

$$M_1 = -D \frac{d^2 w}{dx^2};$$

$$M_2 = -\mu D \frac{d^2 w}{dx^2}. \quad (7.5)$$

Bu (7.5) ifodani e'tiborga olib, (6.4) formuladan quyidagini aniqlaymiz:

$$Q_1 = -D \frac{d^3 w}{dx^3};$$

$$Q_2 = 0. \quad (7.6)$$

Bundan keyin $N_1 = 0$ deb qaraymiz va (7.4) formuladan quyidagilarni aniqlaymiz:

$$\frac{du}{dx} = \mu \frac{w}{R};$$

$$N_2 = -\frac{Ehw}{R}. \quad (7.7)$$

Unda (7.7) tenglamani e'tiborga olib, (7.1) tenglamaning ikkinchi tenglamasi quyidagicha ifodalanadi:

$$D \frac{d^4 w}{dx^4} + \frac{Ehw}{R} = q_3. \quad (7.8)$$

Bu ifodaga:

$$q_3 = q; \quad \beta = \frac{Eh}{4DR^2} = \frac{3(1-\mu^2)}{R^2 h^2} \quad (7.9)$$

belgilashlar kiritamiz va uni quyidagicha ifodalaymiz:

$$\frac{d^4 w}{dx^4} + 4\beta^4 w = \frac{q}{D}. \quad (7.10)$$

Bu tenglamaning yechimi quyidagicha ifodalanadi:

$$w = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + e^{\beta x} (C_3 \cos \beta x + C_4 \sin \beta x) + w^*, \quad (7.11)$$

bu yerda w^* — bir jinsli bo'lmagan tenglamaning xususiy yechimi. Agar qobiq juda ham uzun $x \rightarrow \infty$ bo'lsa, $C_3 = C_4 = 0$ bo'ladi, unda:

$$w = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + w^*. \quad (7.12)$$

Umumiy holda o'zgarmas C_1, C_2, C_3, C_4 koeffitsiyentlar qobiqning chegara shartlaridan aniqlanadi.

1.8. Chetki qirg'og'i bo'ylab yuklangan yarim cheksiz silindr shaklidagi qobiq hisobi

Uzun silindr shaklidagi qobiq chetki qirg'og'i bo'ylab tekis yoyilgan eguvchi moment M_0 va ko'ndalang kuch Q_0 bilan yuklangan (1.8- chizma). Bu masalada $N_1=0$; $q_1=q_2=q_3=0$ bo'lgani uchun differensial tenglamaning (7.10) yechimi quyidagicha aniqlanadi:

$$w = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x). \quad (8.1)$$

Bu funksiyani differensiallab aylanish burchak deformatsiyasi, eguvchi moment va ko'ndalang kuch ifodalarini quyidagicha ifodalaymiz:

$$\frac{\partial w}{\partial x} = \beta e^{-\beta x} [-C_1 (\cos \beta x + \sin \beta x) + C_2 (\cos \beta x - \sin \beta x)]. \quad (8.2)$$

$$M_1 = -D \frac{\partial^2 w}{\partial x^2} = -2D\beta^2 e^{-\beta x} [C_1 \sin \beta x - C_2 \cos \beta x]. \quad (8.3)$$

$$Q_1 = \frac{dM_1}{dx} = -2D\beta^3 e^{-\beta x} [C_1 (\cos \beta x - \sin \beta x) + C_2 (\cos \beta x + \sin \beta x)]. \quad (8.4)$$

$$N_2 = \frac{Ehw}{R} = -\frac{Eh}{R} 2De^{-\beta x} [C_1 \cos \beta x + C_2 \sin \beta x]. \quad (8.5)$$

Bu ifodalardagi o'zgarmas C_1 va C_2 koeffitsiyentlarini $x=0$ kesimning yuklanish shartidan aniqlaymiz:

Qobiqning $x=0$ bo'lgan qirg'og'ida salqilik va momentlar o'zining eng katta qiymatiga erishadi, ya'ni:

$$w_{\max} = w(0) = -\frac{Q_0 + \beta M_0}{2D\beta^3} < 0;$$

$$(M_1)_{\max} = M_0; \quad M_2 = \mu M_1. \quad (8.12)$$

Koordinata x ning ortishi bilan M_1 , M_2 eguvchi momentlar, w salqilik hamda σ_1 , σ_2 kuchlanishlar tez so'nib boradi (1.8- b chizma).

M_1 momentni nolga tenglab, moment ishorasining o'zgarish nuqtasini belgilovchi x_0 uzunlikni aniqlovchi shartni hosil qilamiz:

$$\operatorname{tg} \beta x_0 = -\frac{\beta M_0}{Q_0 + \beta M_0}. \quad (8.13)$$

Xuddi shuning kabi $w=0$ shartdan:

$$\operatorname{tg} \beta x_0^* = -\frac{Q_0 + \beta M_0}{2D\beta^3} = \frac{1}{\operatorname{tg} \beta x_0}. \quad (8.14)$$

Agar qobiq uchun uning momentsiz holati shartlarining birinchi ikkitasi bajarilib qolgan ikkitasi bajarilmasa, unda qobiq kuchlanganlik holati va qirg'og' effekti yig'indisidan iborat, deb qarash mumkin. Bu holatda qobiq hisobi oldin tashqi kuchdan momentsiz nazariya asosida hisoblashga keltiriladi. So'ngra esa qirg'og' effekti masalasi yechiladi, shundan keyin zo'riqish va momentlarni qo'shib umumiy yechim aniqlanadi.

1.9. Yassisimon qobiqlar nazariyasi asoslari

Yassi qobiq deb, ko'tarish balandligi f qolgan chiziqli o'lchamlariga nisbatan kichik bo'lgan qobiqqa aytiladi (1.9- chizma) $f/a \leq 1.5$.

Yassi qobiqlar, asosan, bozorlar, ishlab chiqarish binolari va stadionlarni yopishda keng qo'llaniladi.

V. Z. Vlasovning yassi qobiq nazariyasi quyidagi gipotezalarga asoslanadi:

1. Qobiq shunchalik yassi deb olinadiki, uning geometrik sirti taxminan uning proyeksiya tekisligidagi geometriyasiga to'g'ri keladi.

$$M_1 = M_0; \quad (8.6)$$

$$Q_1 = Q_0.$$

$$2\beta^2 C_2 = \frac{M_0}{D}; \quad 2\beta^3 (C_1 + C_2) = -\frac{Q_0}{D}. \quad (8.7)$$

Bundan:

$$C_1 = -\frac{Q_0 + \beta M_0}{2\beta^3 D}; \quad C_2 = \frac{M_0}{2\beta^2 D}. \quad (8.8)$$

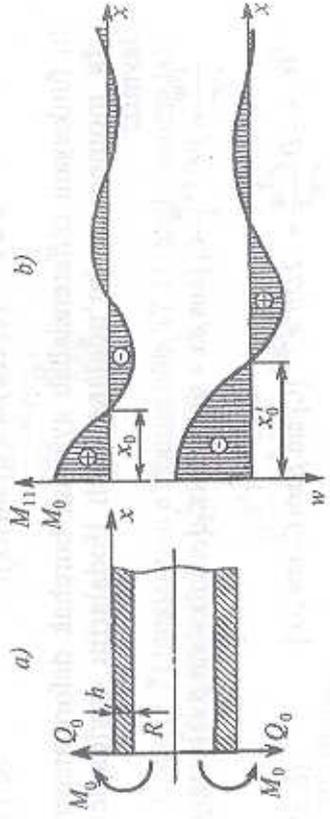
Unda salqilik funksiyasi ifodasi:

$$w = \frac{e^{-\beta x}}{2D\beta^3} [\beta M_0 \sin \beta x - (Q_0 + \beta M_0) \cos \beta x]. \quad (8.9)$$

Eguvchi moment ifodasi:

$$M_1 = \frac{e^{-\beta x}}{\beta} [\beta M_0 \cos \beta x + (Q_0 + \beta M_0) \cos \beta x]; \quad (8.10)$$

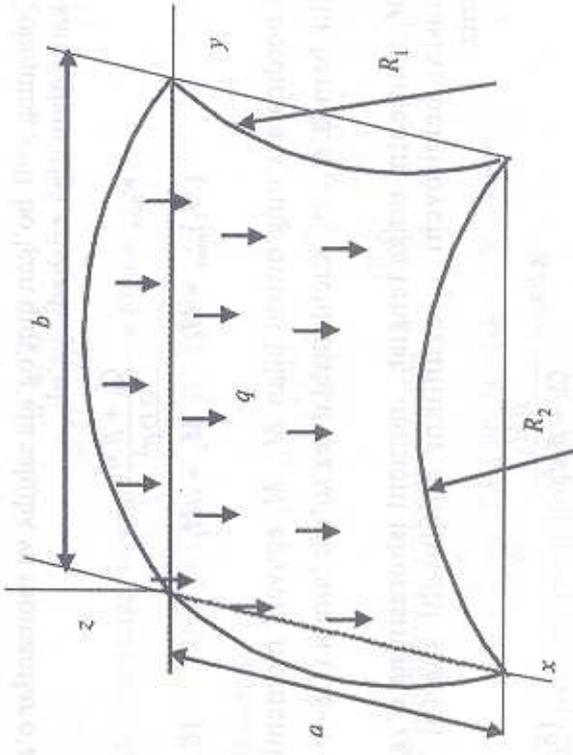
$$M_1 = \mu \frac{e^{-\beta x}}{\beta} [\beta M_0 \cos \beta x + (Q_0 + \beta M_0) \cos \beta x].$$



1.8- chizma.

Kuchlanishlarni (3.9) formulaga asosan quyidagicha aniqlanadi:

$$\sigma_1 = \frac{M_1}{I} z; \quad \sigma_2 = \frac{M_2}{I} z; \quad \tau_{12} = 0. \quad (8.11)$$



1.9- chizma.

Demak:

$$(ds)^2 = A^2 (d\alpha)^2 + B^2 (d\beta)^2 \quad (9.1)$$

o'rniga: $(ds)^2 = (dx)^2 + (dy)^2$

$$(ds)^2 = (dx)^2 + (dy)^2 \quad (9.2)$$

deb hisoblash mumkin, unda: $A = B = 1; \alpha = x; \beta = y$.

2. Muvozanat differensial tenglamada $1/R_1$ va $1/R_2$ koeffitsiyentlarga ko'paytirilgan kesuvchi kuchlari bor hadlarni tashlab yuborish lozim.
3. Egilish deformatsiyasi ifodalarida qobiq o'rta sirti normal bo'yicha hosil bo'lgan deformatsiyani qoldirish kifoya.

Bu gipotezalarni e'tiborga olib, yassi qobiqlar muvozanat differensial tenglamalari quyidagicha ifodalanadi:

$$\frac{\partial N_1}{\partial x} + \frac{\partial S_{12}}{\partial y} + q_1 = 0;$$

$$\frac{\partial S_{21}}{\partial x} + \frac{\partial N_2}{\partial y} + q_2 = 0. \quad (9.3)$$

$$\frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} + \frac{N_1}{R_1} + \frac{N_2}{R_2} + q_3 = 0; \quad (9.4)$$

$$Q_1 = \frac{\partial M_1}{\partial x} + \frac{\partial M_{12}}{\partial y};$$

$$Q_2 = \frac{\partial M_{21}}{\partial x} + \frac{\partial M_2}{\partial y}. \quad (9.5)$$

Yuqoridagi gipotezalarga asoslanib deformatsiyalarni quyidagicha aniqlaymiz:

$$\xi_1 = \frac{\partial u}{\partial x} - \frac{w}{R_1};$$

$$\xi_2 = \frac{\partial v}{\partial y} - \frac{w}{R_2};$$

$$2\gamma = \frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}. \quad (9.6)$$

$$\chi_1 = \frac{\partial^2 w}{\partial x^2};$$

$$\chi_2 = \frac{\partial^2 w}{\partial y^2};$$

$$\lambda = \frac{\partial^2 w}{\partial x \partial y}. \quad (9.7)$$

Deformatsiyaning uzluksizlik tenglamasi:

$$\frac{\partial^2 \epsilon_1}{\partial x^2} + \frac{\partial^2 \epsilon_2}{\partial y^2} - 2 \frac{\partial^2 \gamma}{\partial x \partial y} = \frac{1}{R_1} \frac{\partial^2 w}{\partial y^2} + \frac{1}{R_2} \frac{\partial^2 w}{\partial x^2}. \quad (9.8)$$

Fizik tenglamalarni quyidagicha ifodalash mumkin:

$$N_1 = \frac{Eh}{1-\mu^2} (\epsilon_1 + \mu \epsilon_2);$$

$$N_2 = \frac{Eh}{1-\mu^2} (\epsilon_2 + \mu \epsilon_1);$$

$$S = \frac{Eh}{2(1+\mu)} \gamma. \quad (9.10)$$

Yuqoridagi (9.10) tenglamadan deformatsiya komponentlarini aniqlaymiz:

$$\begin{aligned}\varepsilon_1 &= \frac{1}{Eh}(N_1 - \mu N_2); \\ \varepsilon_2 &= \frac{1}{Eh}(N_2 - \mu N_1); \\ \gamma &= \frac{1 + \mu}{Eh} S_{12}.\end{aligned}\quad (9.20)$$

Bu (9.20) ifodaga zo'riqish kuchi o'rimga (9.17) tenglamadan kuchlanishlar funksiyasini qo'yib, quyidagilarni hosil qilamiz:

$$\begin{aligned}\varepsilon_1 &= \frac{1}{Eh} \left(\frac{\partial^2 \varphi}{\partial y^2} - \mu \frac{\partial^2 \varphi}{\partial x^2} \right); \\ \varepsilon_2 &= \frac{1}{Eh} \left(\frac{\partial^2 \varphi}{\partial x^2} - \mu \frac{\partial^2 \varphi}{\partial y^2} \right); \\ \gamma &= -\frac{1 + \mu}{Eh} \frac{\partial^2 \varphi}{\partial x \partial y}.\end{aligned}\quad (9.21)$$

Deformatsiyaning uzluksizlik (9.5) tenglamasidagi deformatsiyani zo'riqish kuchlari (9.17) orqali, keyin zo'riqish kuchlarini kuchlanish funksiyasi orqali ifodalab, quyidagi tenglamani hosil qilamiz:

$$\nabla^4 \varphi = -Eh \left(K_1 \frac{\partial^2 w}{\partial y^2} + K_2 \frac{\partial^2 w}{\partial x^2} \right).\quad (9.22)$$

Shunday qilib yassi qobiqning asosiy tenglamalar sistemasini hosil qilamiz:

$$D\nabla^4 w = q + K_1 \frac{\partial^2 w}{\partial y^2} + K_2 \frac{\partial^2 w}{\partial x^2}.\quad (9.23)$$

$$\nabla^4 \varphi = -Eh \left(K_1 \frac{\partial^2 w}{\partial y^2} + K_2 \frac{\partial^2 w}{\partial x^2} \right).\quad (9.24)$$

Bu tenglamalardan ikkita φ va w funksiyalarni aniqlash mumkin. Bu differensial tenglamalar sistemasini ikki masalasi: elastiklik nazariyasi tekis masalasini va plastinkaning egilish haqidagi masalalarni o'z ichiga oladi.

$$M_1 = -D(\chi_1 + \mu\chi_2) = -D \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right).\quad (9.11)$$

$$M_2 = -D(\chi_2 + \mu\chi_1) = -D \left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right).\quad (9.12)$$

$$M_{12} = M_{21} = H = -D(1 - \mu)\lambda = -D(1 - \mu) \frac{\partial^2 w}{\partial x \partial y}.\quad (9.13)$$

$$\begin{aligned}Q_1 &= -D \frac{\partial}{\partial x} (\nabla^2 w); \\ Q_2 &= -D \frac{\partial}{\partial y} (\nabla^2 w).\end{aligned}\quad (9.14)$$

Tenglamalar sistemasini (9.3—9.14)ni ikki noma'lumli ikkita tenglamaga keltirish mumkin. Buning uchun $q_1 = q_2 = 0$; $q_3 = q$ deb qaraymiz. Unda (9.3) tenglamani quyidagicha ifodalaymiz:

$$\begin{aligned}\frac{\partial N_1}{\partial x} + \frac{\partial S_{12}}{\partial y} &= 0; \\ \frac{\partial S_{21}}{\partial x} + \frac{\partial N_2}{\partial y} &= 0.\end{aligned}\quad (9.15)$$

(9.4) tenglamaga (9.5) ifodalarni qo'yib, quyidagini hosil qilamiz:

$$\frac{\partial^2 M_1}{\partial x^2} + \frac{\partial^2 M_{12}}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} + \frac{N_1}{R_1} + \frac{N_2}{R_2} + q = 0.\quad (9.16)$$

Tekis masaladagi kabi kuchlanish φ funksiyasini kiritamiz:

$$N_1 = \frac{\partial^2 \varphi}{\partial y^2}; \quad N_2 = \frac{\partial^2 \varphi}{\partial x^2}; \quad S_{12} = -\frac{\partial^2 \varphi}{\partial x \partial y}.\quad (9.17)$$

Unda (9.15) tenglamalar qanoatlanadi. Eguvchi moment bilan burovchi moment ifodalarni (9.11—9.13) formuladan (9.10) ifodaga qo'yib siqilishda egilgan o'rta sirt tenglamasini hosil qilamiz:

$$D\nabla^4 w = q + N_1 K_1 + N_2 K_2.\quad (9.18)$$

Bu yerda:

$$K_1 = 1/R_1; \quad K_2 = 1/R_2.\quad (9.19)$$

Haqiqatan ham tekis plastinka $K_1 = K_2 = 0$ uchun bo'lsa, bizga ma'lum bo'lgan tenglamalarga ega bo'lamiz:

$$D\nabla^4 w = q; \quad \nabla^4 \varphi = 0. \quad (9.25)$$

Bu tenglamaning birinchisi plastinka uchun Sofi-Jermen tenglamasi bo'lib, ikkinchisi esa elastiklik nazariyasi tekis masalasi bigormonik tenglamasidir.

Tenglamalar (9.23—9.24) sistemasini sonli yoki taqribiy usullardan birini qo'llab yechish mumkin.

Yassi qobiq uchun masalani yechishni Bubnov-Galerkin usuli bilan qarab chiqamiz. Kuchlanish φ funksiyasi bilan salqilik w funksiyasini cheksiz qator ko'rinishida qabul qilamiz:

$$\varphi = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{mn} \varphi_{mn};$$

$$w = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} B_{mn} w_{mn}. \quad (9.26)$$

Bu yerda φ_{mn} va w_{mn} funksiyalar barcha chegara shartlarini qanoatlantirishi lozim. O'zgarmas A_{mn} va B_{mn} koeffitsiyentlar Bubnov-Galerkinning taqribiy usuli tenglamalaridan:

$$\int_0^a \int_0^b (D\nabla^4 w - q\nabla_k^2 \varphi) w_{mn} dx dy = 0;$$

$$\int_0^a \int_0^b (\nabla^4 \varphi + E h \nabla_k^2 w) \varphi_{mn} dx dy = 0 \quad (9.27)$$

aniqlanadi.

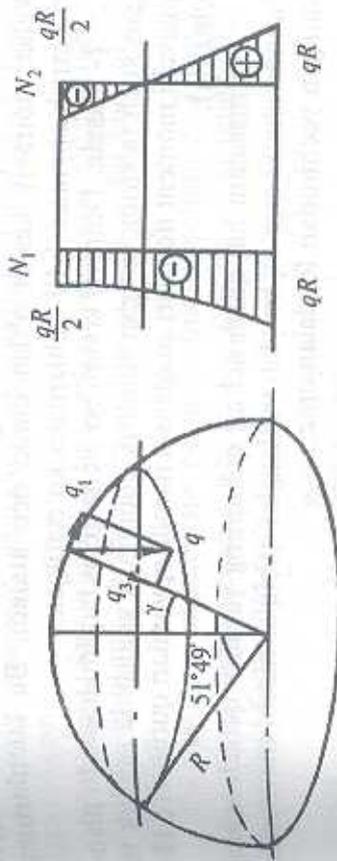
Bu yerda: $\nabla_k^2 = K_1 \frac{\partial^2}{\partial y^2} + K_2 \frac{\partial^2}{\partial x^2}$ Vlasov operatori.

Bu tenglamalardan A_{mn} va B_{mn} parametrlar aniqlanadi. Shunday qilib, yassi qobiqlar hisobi ikkita noma'lum w va φ funksiyalarni aniqlashga keltiriladi.

MASALALAR

1- masala. Xususiy og'irligi q ta'sirida bo'lgan aylanish sirtli qobiq (kupola) hisobini qaraymiz (1.10- chizma).

Bir birlik yuzaga to'g'ri kelgan og'irlik q kuchining tashkil etuvchilarini quyidagicha yozish mumkin:



1.10- chizma.

$$q_1 = q \sin \varphi; \quad q_2 = q \cos \varphi.$$

Bu kuch ifodalarini yuqoridagi (5.8-5.9) tenglamalarga qo'yib

$$\frac{\partial}{\partial \varphi} (N_1 \sin \varphi) - N_2 \cos \varphi + qR \sin^2 \varphi = 0.$$

$$N_1 + N_2 + qR \cos \varphi = 0$$

tenglamani hosil qilamiz.

Bu tenglamaning yechimi quyidagi ko'rinishga ega:

$$N_1 = -\frac{qR}{1 + \cos \varphi};$$

$$N_2 = -qR \left(\cos \varphi - \frac{1}{1 + \cos \varphi} \right).$$

Bu formuladan ko'rinadiki, meridional zo'riqish kuchi N_1 barcha kesimida manfiy bo'ladi.

Aylana N_2 zo'riqish kuchi manfiy va musbat bo'lishi mumkin.

Aylana sirtli qobiq (kupol)ning cho'qqisida $\varphi=0$ bo'lsa, $N_1 = N_2 = -qR/2$ bo'ladi, agar $\varphi=\pi/2$ bo'lsa, $N_1 = -qR$, $N_2 = qR$. Aylanish burchak

$\varphi \approx 51^{\circ}40'$ ga teng bo'lsa, aylana zo'riqish kuchi o'z ishorasini o'zgartiradi.

Aylana N_2 zo'riqish kuchining ishorasi o'zgarish joyiga to'g'ri kelsa, doiraviy kesim o'tish choki deb ataladi. Bu kuchlarning o'zgarish epyurasi 1.10- chizmada ko'rsatilgan.

2- masala. Doiraviy kesimi bo'yicha tekis taqsimlangan to'plan-gan kuch ta'sirida bo'lgan silindrik qobiqning egilishida salqilik va eguvchi moment ifodalari aniqlansin hamda epyuralari qurilsin (1.11- chizma).

Bu masalani hal qilish uchun qobiqning yarmini qarash kifoya hamda chetki qirg'oq bo'ylab yuklangan qobiqlar hisobi mavzusida olingan yechimdan foydalanamiz.

Qobiqning chetki qirg'oqdagi kesuvchi kuch (1.11- a chizma) $Q_0 = -0,5p$, qirg'oqdagi M_0 momentni chegara shartidan aniqlaymiz. Ya'ni, $x=0$ bo'lganda

$$\text{Ya'ni, } x=0 \text{ bo'lganda: } \frac{dw}{dx} = 0 \text{ bo'ladi;}$$

$$\begin{aligned} \frac{dw}{dx} = & -\beta \frac{e^{-\beta x}}{2D\beta^3} [\beta M_0 \sin \beta x - (Q_0 + \beta M_0) \cos \beta x] + \\ & + \frac{e^{-\beta x}}{2D\beta^3} [\beta^2 M_0 \cos \beta x + \beta(Q_0 + \beta M_0) \sin \beta x]. \end{aligned}$$

$$\left(\frac{dw}{dx} \right)_{x=0} = \frac{M_0}{2D\beta} \left(2 + \frac{Q_0}{\beta M_0} \right) = 0; \quad M_0 = -\frac{Q_0}{2\beta} = \frac{p}{4\beta}.$$

Unda salqilik tenglamasi ifodasini quyidagicha ifodalaymiz:

$$w = \frac{p}{8D\beta^2} e^{-\beta x} (\sin \beta x + \cos \beta x).$$

Eng katta salqilik:

$$w_{\max} = \frac{p}{8D\beta^2} = \frac{pR^2}{2Eh} \beta.$$

Salqilik funksiyasining x bo'yicha ikkinchi tartibli hosilasi quyidagicha yoziladi:

$$\frac{d^2 w}{dx^2} = \frac{q}{4D\beta} e^{-\beta x} (\sin \beta x - \cos \beta x).$$

Eguvchi moment ifodasi:

$$M_1 = -D \frac{d^2 w}{dx^2} = \frac{p}{4\beta} e^{-\beta x} (\cos \beta x - \sin \beta x).$$

Eguvchi moment va salqilik epyuralari 1.11- d chizmada ko'rsatilgan.

3- masala. Suyuqlik bilan to'ldirilgan vertikal vaziyatda turgan silindrik qobiq $q_3 = -P = -\gamma(h-x)$ bosim ostida bo'ladi (1.12- chizma).

Bu masalani momentsiz nazariya bilan yechamiz. Zo'riqish kuchlari formulaga asosan $N_1 = 0$; $N_2 = -q_3 R = \gamma R(1-x)$ ga teng bo'ladi. Yuqoridagi formulaga asosan:

$$w = -\frac{RN_2}{Eh} = -\frac{\gamma R^2}{Eh} (1-x).$$

Masalani momentli nazariya asosida yechamiz. Silindrik qobiqni cheksiz uzun deb qabul qilamiz, unda (7.12) yechimdan foydalanish mumkin:

$$w = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + w^*.$$

Differensial (7.10) tenglamaning xususiy yechimi:

$$w^* = -\frac{\gamma}{4D\beta^4} (\ell - x) = -\frac{\gamma R^2}{2Eh} (\ell - x)$$

bo'lganligi uchun salqilik quyidagicha ifodalanadi:

$$w = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) - \frac{\gamma R^2}{2Eh} (\ell - x).$$

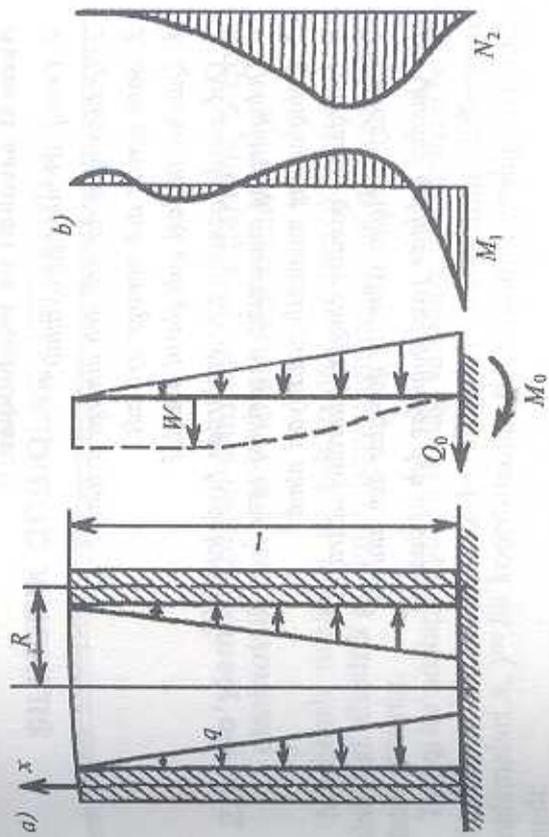
C_1 va C_2 o'zgarmas koeffitsiyentlarni $x=0$ bo'lganda $w=0$; $dw/dx=0$ bo'lish chegaralaridan aniqlaymiz, ya'ni:

$$C_1 = \frac{\gamma R\ell}{Eh};$$

$$C_2 = \frac{\gamma R^2}{Eh\beta} (\ell\beta - 1).$$

Unda salqilik ifodasi:

$$w = \frac{\gamma R^2}{Eh\beta} \left\{ e^{-\beta x} [(\ell\beta - 1) \sin \beta x + \beta \cos \beta x] - \beta(\ell - x) \right\}.$$



1.12- chizma.

Ichki zo'riqish kuchlarini (7.4—7.5) formulalarga asosan aniqlaymiz:

$$N_1 = 0;$$

$$N_2 = \frac{\gamma R}{\beta} \left\{ e^{-\beta x} [(\ell\beta - 1) \sin \beta x + \ell\beta \cos \beta x] - \beta(\ell - x) \right\};$$

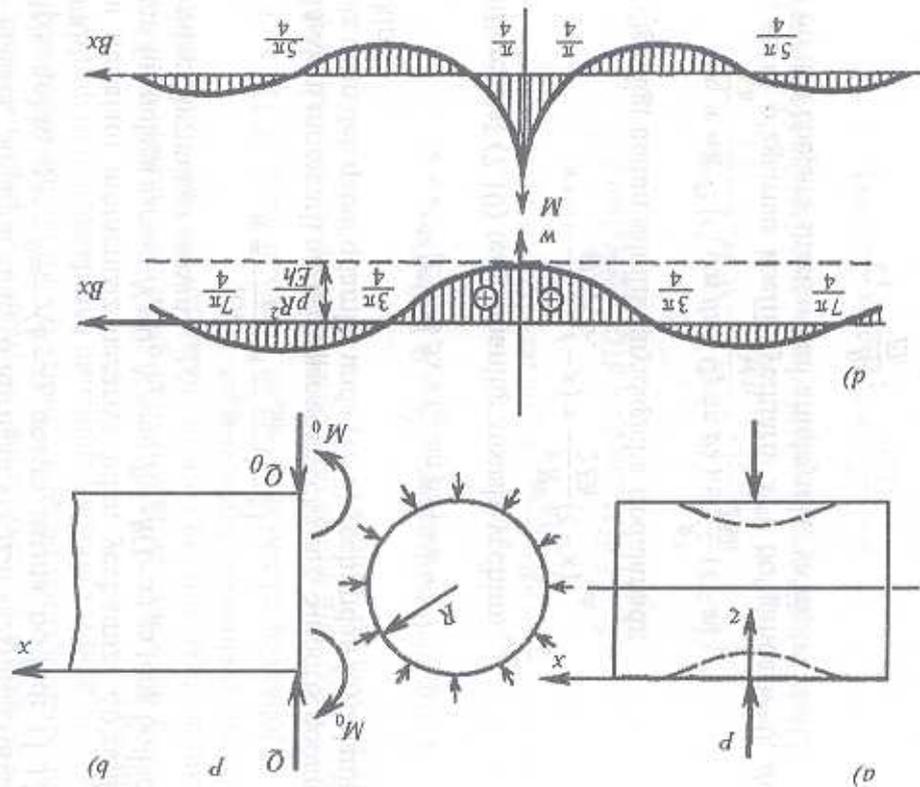
$$M_1 = \frac{\gamma R E h}{\sqrt{12(1-\nu^2)}} \left\{ e^{-\beta x} \left[\sin \beta x - \frac{1}{\ell\beta} (\ell\beta - 1) \cos \beta x \right] \right\};$$

$$M_2 = \nu M_1.$$

Eguvchi moment M_1 qo'biqning mahkamlangan $x=0$ qismida eng katta qiymatga erishadi, ya'ni

$$(M_1)_{\max} = \frac{\gamma R E h}{\sqrt{12(1-\nu^2)}} \left(1 - \frac{1}{\ell\beta} \right).$$

Zo'riqish kuchi N_2 eng katta qiymati $dN_2/dx=0$ bo'lgan kesimida bo'ladi. Bu miqdorni aniqlovchi shartni beradi. Shundan keyin yuqoridagi formuladan $(N_2)_{\max}$ aniqlanadi.



1.11- chizma.

Nazorat savollari va topshiriqlari:

1. Qobiq deb nimaga aytiladi?
2. Qobiqning o'rti sirti deb nimaga aytiladi?
3. Bosh egrilik deb nimaga aytiladi?
4. Qanday turdagi qobiqlarni bilasiz?
5. Qobiqlarda qanday gipotezalardan foydalaniladi?
6. Qobiqlarning momentli nazariyasi nima?
7. Qobiqlarning momentli nazariyasi nima?
8. Qobiqlarda qanday chegara shartlari uchraydi?
9. Qanday qobiqlar silindrik qobiqlar deb ataladi?
10. Qanday qobiqlar yassi qobiqlar deb ataladi?

II BOB. SILINDRIK QOBIQLAR USTUVORLIGI

2.1. Bo'ylama siqilgan silindrik qobiqlarning o'qqa simmetrik qavarish holatidagi ustuvorligi

Momentli nazariya asosida o'qqa simmetrik deformatsiya holatidagi silindrik qobiqlar ustuvorligining asosiy munosabatlarini eslatib o'tamiz.

Tashqi ko'ndalang q yuk ta'siridagi va radiusi R , qalinligi h bo'lgan silindrik qobiq uchun (2.1- chizma) muvozanat differensial tenglamalari $x, y=R\theta$ koordinatalar sistemasida quyidagicha ifodalanadi:

$$\begin{aligned} \frac{dQ_x}{dx} + \frac{N_y}{R} + q &= 0; \\ \frac{dM_x}{dx} - Q_x &= 0. \end{aligned} \quad (1.1)$$

Zo'riqish kuchlari va ularning musbat yo'nalishlari 2.1- chizmada ko'rsatilgan.

(1.1) differensial tenglamalardan ko'ndalang Q_x kuchni yo'qotib, natijada quyidagi munosabatni hosil qilamiz:

$$\frac{d^2 M_x}{dx^2} + \frac{N_y}{R} + q = 0. \quad (1.2)$$

Aylana yo'nalishi bo'yicha nisbiy ε_y deformatsiya uchun Guk qonunining quyidagi bog'lanishi mavjud:

$$\varepsilon_y = \frac{1}{Eh} (N_y - \mu N_x). \quad (1.3)$$

Nisbiy deformatsiya ε_y bilan qobiq ichki normali bo'yicha yo'nalgan musbat normal ko'chish w orasidagi bog'lanish

$$\varepsilon_y = -\frac{w}{R} \quad (1.4)$$

ni e'tiborga olib, (1.3) formuladan zo'riqish kuchi ifodasini aniqlaymiz:

$$N_y = \mu N_x - \frac{Eh}{R} w. \quad (1.5)$$

Salqilik funksiyasi w bilan eguvchi moment M_x quyidagi bog'lanish bilan ifodalanadi:

$$M_x = -D \frac{d^2 w}{dx^2}. \quad (1.6)$$

Bu yerda: D — qobiqning silindrik bikirligi, $D = \frac{Eh^3}{12(1-\mu^2)}$. Hosil qilingan (1.5) — (1.6) ifodalarni e'tiborga olib (1.2) formula asosida izlanayotgan salqilik funksiyasiga nisbatan to'rtinchi tartibli differensial tenglamani hosil qilamiz:

$$D \frac{d^4 w}{dx^4} + \frac{Eh}{R^2} w - q = \frac{\mu}{R} N_x. \quad (1.7)$$

Bu (1.7) tenglama (1.1) formulaning ikkinchisiga muvofiq, qobiqdan ajratib olingan kichik elementning sirt normaliga nisbatan muvozanat shartini ifodalashini eslatib o'tamiz.

Bo'yлама yo'nalish bo'yicha doimiy N_x kuch bilan siqilgan silindrik qobiq ustuvorlik differensial tenglamasini, o'qqa simmetrik ravishda oddiy bo'rtib (2.2- chizma) chiqqanda (1.7) munosabatlarga asoslanib hosil qilish qiyinchilik tug'dirmaydi. Buning uchun $N_x = -N$ deb qabul qilib, ustuvorlik masalasida siquvchi kuch musbat deb qabul qilinganligini e'tiborga olamiz. Bundan tashqari ko'ndalang q kuchni keltirilgan ko'ndalang kuch

$$q = -N \frac{d^2 w}{dx^2}$$

bilan almashiramiz, chunki bo'yлама siquvchi kuch qobiq ustuvorligini yo'qotishi natijasida bo'rtib chiqqan sirti normaliga proyeksiya beradi:

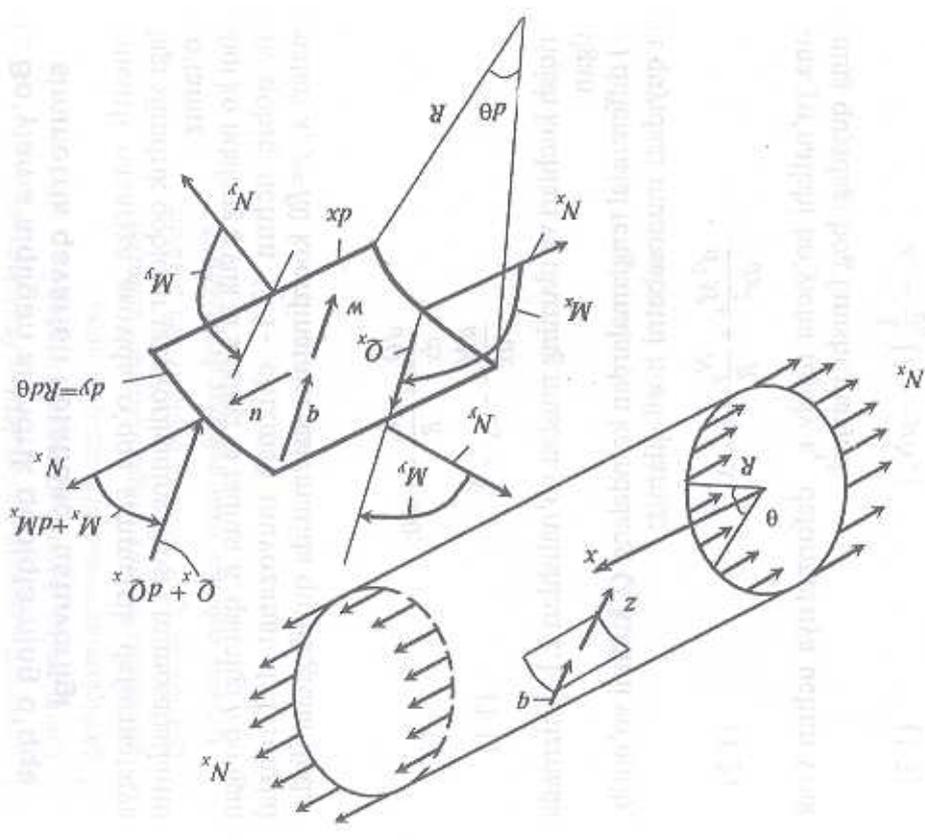
$$D \frac{d^4 w}{dx^4} + N \frac{d^2 w}{dx^2} + \frac{Eh}{R^2} w = -\frac{\mu}{R} N. \quad (1.8)$$

Bu (1.8) tenglama bir jinsli bo'lmagan tenglama bo'lib, uning umumiy yechimi tegishli bir jinsli tenglamaning umumiy yechimi va bir jinsli bo'lmagan tenglamaning xususiy yechimlari yig'indisidan iborat.

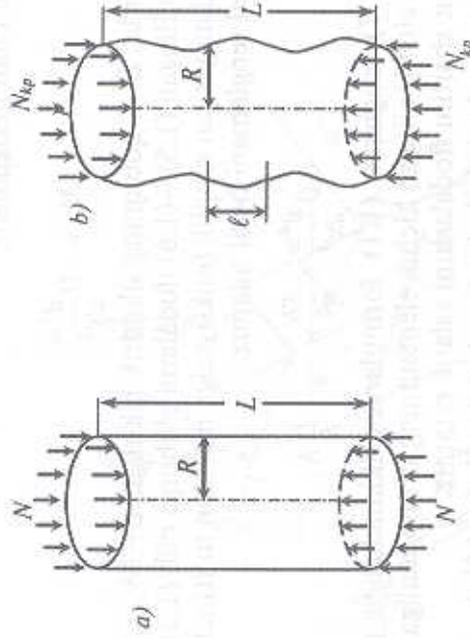
Qobiqning qaralayotgan holatida N yuk doimiy bo'lgani uchun tenglamaning xususiy yechimi quyidagi formuladan aniqlanadi:

$$w^* = -\frac{\mu R}{Eh} N. \quad (1.9)$$

2.1- chizma.



Bu esa bo'ylama siqilgan qobiqning dastlabki moment(siz muvozanat holatiga to'g'ri keladi, chunki qobiqning salqiligi (1.9) yechim uchun doimiydir.



2.2- chizma.

Shunday qilib, qobiqning o'qqa simmetrik bo'rtib chiqib egilgan yangi muvozanat shakliga mos keluvchi ustuvorlik tenglamasi quyidagicha ifodalanaadi:

$$D \frac{d^4 w}{dx^4} + N \frac{d^2 w}{dx^2} + \frac{Eh}{R^2} w = 0. \quad (1.10)$$

Bo'ylama siqilgan silindrik qobiqning o'qqa simmetrik deformatsiya holatida kritik kuchi aniqlashni ko'rib chiqamiz. Silindrik qobiqni yetarlicha uzun, ya'ni $L > R$ deb qabul qilib, (1.10) tenglama yechimini quyidagi ko'rinishda qabul qilamiz:

$$w = f \sin \lambda x. \quad (1.11)$$

Bu yerda f — ixtiyoriy o'zgarmas koeffitsiyent; λ — qobiq ustuvorligini yo'qotishdagi yarim to'lqinlar ℓ uzunligiga bog'liq bo'lgan parametrlar bo'lib, quyidagi bog'lanish orqali ifodalanaadi:

$$\lambda = \frac{\pi}{\ell}. \quad (1.12)$$

Salqilik (1.11) ifodasini ustuvorlik tenglamasi (1.10) ga qo'yamiz va uni qanoatlantirishini talab qilib, quyidagini hosil qilamiz:

$$D \lambda^4 - N \lambda^2 + \frac{Eh}{R^2} = 0.$$

Bundan qobiqning ustuvorligini yo'qotish (1.11) shakliga to'g'ri keluvchi bo'ylama kuch qiymatini topamiz:

$$N = D \lambda^2 + \frac{Eh}{R^2} \frac{1}{\lambda^2}. \quad (1.13)$$

Izlanayotgan kritik kuchning miqdorini aniqlash uchun λ^2 bo'yicha bo'ylama N kuchning minimum shartini tuzamiz:

$$\frac{dN}{d\lambda^2} = D - \frac{Eh}{R^2} \frac{1}{\lambda^4} = 0.$$

Bundan N — funksiyaning minimumiga to'g'ri keluvchi λ^2 qiymatini aniqlaymiz:

$$\lambda_{kp}^2 = \sqrt{\frac{Eh}{DR^2}}. \quad (1.14)$$

Bu (1.14) ifodani e'tiborga olib, (1.13) formula asosida izlanayotgan siquvchi kritik kuchi aniqlaymiz:

$$N_{kp} = 2 \sqrt{\frac{EhD}{R^2}}. \quad (1.15)$$

Puasson koeffitsiyenti $\mu=0,3$ bo'lganda silindrik bikirligini e'tiborga olsak, kritik kuch ifodasi:

$$N_{kp} = 0,605 \frac{Eh^2}{R}. \quad (1.16)$$

Unda kritik kuchlanish ifodasini quyidagicha ifodalash mumkin:

$$\sigma_{kp} = k \frac{Eh}{R}. \quad (1.17)$$

Bunda: $k = 0,605$ — ustuvorlik koeffitsiyenti.

Qobiqning bo'rtib chiqishidagi normal salqilik deformatsiyasi uzunligini (1.14) va (1.12) ifodalash asosida quyidagicha aniqlaymiz:

$$\ell = \pi \sqrt{\frac{DR^2}{Eh}}. \quad (1.18)$$

Agar (1.18) formuladan aniqlanadigan bo'rtib chiqish salqilik deformatsiyasi uzunligi ℓ qobiq umumiy uzunligi L bo'ylab butun

son tarzida joylashsa, kritik kuch qiymati kalta qobiq uchun ham (1.16) formuladan aniqlanadi.

Aks holda kritik kuch miqdori katta bo'ladi.

2.2. Yassi qobiqlar momentli nazariyasi asosiy munosabatlari. Ustuvorlik tenglamasi

Simmetrik bo'lmagan deformatsiya holatida bo'lgan silindrik qobiqlarning momentli nazariya doirasida asosiy tenglamasini keltirib chiqaramiz.

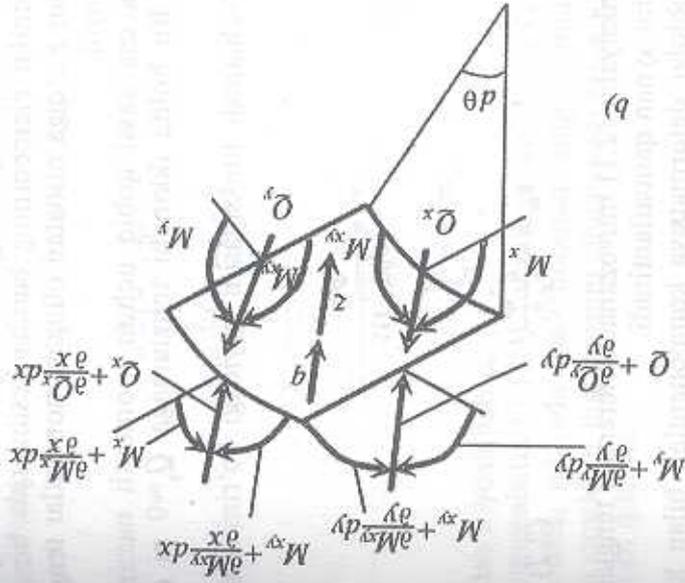
Qobiq intensivligi $q=const$ bo'lgan teng taralgan ko'ndalang yuk ta'sirida bo'lsin.

Qobiqdan o'lchamlari dx va $dy=Rd\theta$ bo'lgan cheksiz kichik elementni ajratib olamiz. Ajratib olingan element tomonlariga normal N_x, N_y , urinma N_{xy} , ko'ndalang Q_x, Q_y kuchlar, eguvchi momentlar M_x, M_y hamda buruvchi momentlar M_{xy}, M_{yx} va tashqi ko'ndalang q kuch ta'sir etsin.

Zo'riqish kuchlarining musbat yo'nalishlari 2.3- chizmada ko'rsatilgan.

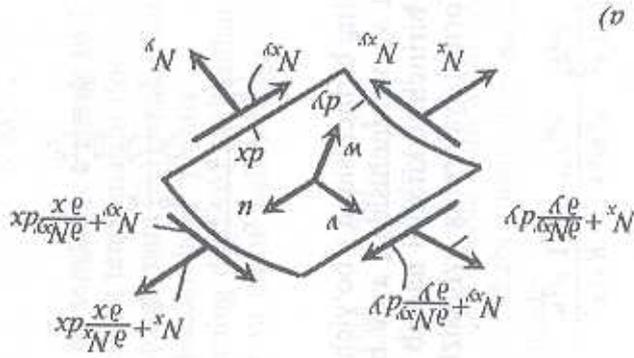
Qaralayotgan silindrik qobiq statik muvozanat holatda bo'lgani uchun undan ajratib olingan elementga ta'sir etuvchi zo'riqish va tashqi kuchlar statik muvozanat tenglamalarini qanoatlantirishi shart, ya'ni quyidagi tengliklar bajariladi:

$$\begin{aligned} \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= 0; \\ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} - \frac{Q_y}{R} &= 0; \\ \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x &= 0; \\ \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y &= 0; \\ \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{N_y}{R} + q &= 0. \end{aligned} \quad (2.1)$$



(b)

2.3- chizma.



(a)

Taqriban urinma kuchlar N_{xy} va N_{yx} hamda M_{xy} va M_{yx} burovchi momentlar elementning barcha kesimlarida bir xil deb hisoblab, ulardan z o'qiga nisbatan olingan momentlar tenglamasi e'tiborga olinmaydi.

Keyinchalik esa yassi qobiq uchun momentli nazariyadan foydalanamiz. Bu holda ikkinchi tenglamaga $Q_y=0$ ni qo'yish lozim.

$\Phi(x, y)$ kuchlanish funksiyasini quyidagi ko'rinishda qabul qilamiz:

$$\begin{aligned}\sigma_x &= \frac{\partial^2 \Phi}{\partial y^2}, \\ \sigma_y &= \frac{\partial^2 \Phi}{\partial x^2}, \\ \tau_{xy} &= -\frac{\partial^2 \Phi}{\partial x \partial y}.\end{aligned}\quad (2.2)$$

Bu (2.2) funksiyalar (2.1) muvozanat differensial tenglamaning birinchi ikkitasini aynan qanoatlantiradi.

Silindrik qobiqlar deformatsiya komponentlari bilan ko'chish komponentlari u, v, w orasidagi bog'lanishlar quyidagi formulalardan aniqlanadi:

$$\begin{aligned}\xi_x &= \frac{\partial u}{\partial x}, \\ \xi_y &= \frac{\partial v}{\partial y} - \frac{w}{R}, \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}.\end{aligned}\quad (2.3)$$

Bu (2.3) tenglamaning birinchisidan y bo'yicha, ikkinchisidan x bo'yicha ikki marta va uchinchisidan x, y bo'yicha aralash hosilalarini olamiz va birinchi ikkitasini hadlab qo'shib, uchinchisini esa ayirib, deformatsiyalarning uzluksizlik tenglamasini hosil qilamiz:

$$\frac{\partial^2 \xi_y}{\partial x^2} + \frac{\partial^2 \xi_x}{\partial y^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = -\frac{1}{R} \frac{\partial^2 w}{\partial x^2}.\quad (2.4)$$

Deformatsiya va kuchlanish orasidagi bog'lanishni ifodalovchi Guk qonunining

$$\begin{aligned}\varepsilon_x &= \frac{1}{E}(\sigma_x - \mu\sigma_y); \\ \varepsilon_y &= \frac{1}{E}(\sigma_y - \mu\sigma_x); \\ \gamma_{xy} &= \frac{1}{G}\tau_{xy}; \\ G &= \frac{E}{2(1-\mu)}\end{aligned}\quad (2.5)$$

ifodalardan foydalanamiz.

Bu munosabatlarni (2.4) formulaga qo'yib va (2.2) formulani e'tiborga olib, natijada yassi qobiqlar salqiligi bilan kuchlanish funksiyasi orasidagi bog'lanishni ifodalovchi birinchi asosiy tenglamani hosil qilamiz:

$$\nabla^2 \nabla^2 \phi + \frac{E}{R} \frac{\partial^2 w}{\partial x^2} = 0.\quad (2.6)$$

Hosil qilingan bu tenglama o'zining fizik ma'nosi bilan deformatsiyaning uzluksizlik tenglamasini ifodalaydi.

Ikkinchi asosiy tenglamani (2.1) muvozanat differensial tenglamalarning oxirgi uchtasidan hosil qilamiz.

Bu tenglamalarning dastlabki ikkitasidan Q_x va Q_y ko'ndalang kuchlarni topamiz va ularni uchinchisiga qo'yib, quyidagini hosil qilamiz:

$$\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \frac{N_y}{R} + q = 0.\quad (2.7)$$

Yassi qobiqlar nazariyasida ham xuddi yuqqa plastinkalarning egilish nazariyasidagi kabi momentlar salqilik funksiyasi orqali quyidagicha ifodalanadi:

$$M_x = -D \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right);$$

$$M_y = -D \left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right);$$

$$M_{xy} = -D(1 - \mu) \frac{\partial^2 w}{\partial x \partial y}. \quad (2.8)$$

Kuchlanish funksiyasi (2.2) va momentlar (2.8) ifodalari ni e'tiborga olib, natijada quyidagini hosil qilamiz:

$$D \nabla^2 \nabla^2 w - \frac{h}{R} \frac{\partial^2 \phi}{\partial x^2} - q = 0. \quad (2.9)$$

Bu tenglama fizik ma'nosi bo'yicha qobiqning muvozanat tenglamasini ifodalaydi.

Shunday qilib, silindrik qobiqlar hisobi ikkita (2.6), (2.9) differensial tenglamalarni birgalikda integrallab, noma'lum bo'lgan normal w salqilik va kuchlanish $\Phi(x, y)$ funksiyasini aniqlashga keltiriladi.

Agar (2.6) tenglamani x bo'yicha ikki marta differensiallab, (2.9) tenglamani $\nabla^2 \Delta^2$ bigarmonik operator bo'yicha differensiallab, bu tenglamalardan $\Phi(x, y)$ kuchlanish funksiyasini yo'qotish yo'li bilan qobiqlar momentli texnik nazariyasining asosiy tenglamalarini salqilik funksiyasiga nisbatan bitta umumiy tenglamaga keltiramiz:

$$D \nabla^2 \nabla^2 \nabla^2 w + \frac{hE}{R^2} \frac{\partial^4 w}{\partial x^4} - \nabla^2 \nabla^2 q = 0. \quad (2.10)$$

Bu yerda:

$$\nabla^2 \nabla^2 \nabla^2 \nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^4 =$$

$$= \frac{\partial^8}{\partial x^8} + 4 \frac{\partial^8}{\partial x^6 \partial y^2} + 6 \frac{\partial^8}{\partial x^4 \partial y^4} + 4 \frac{\partial^8}{\partial x^2 \partial y^6} + \frac{\partial^8}{\partial y^8}$$

bo'ladi.

Yassi qobiq nazariyasi umumiy tenglamasidan, tegishli uch ustuvorlik masalalari tenglamalariga o'tish qiyin emas. Bu holatda salqilik w funksiyasi qobiq ustuvorligini yo'qotishda yangi muvozanat holatiga to'g'ri keladi deb hisoblab, q yukning qobiq ustuvorligini yo'qotishida qiyshayish natijasida hosil bo'lgan, keltirilgan q^* yukning qobiq sirti normal bo'yicha o'qi, halqa va urinma kuchlar proyeksiyalari bilan almashiramiz.

Agar qobiq faqat siquvchi N_1 bo'yilama yuk bilan yuklangan bo'lsa, unda keltirilgan yukni plastinka uchun keltirib chiqarganimizdek, quyidagicha ifodalaymiz:

$$q_1^* = -N_1 \frac{\partial^2 w}{\partial x^2}. \quad (2.11)$$

Xuddi shuningdek, faqat doimiy urinma yuk S bilan yuklangan qobiqda keltirilgan yuk uchun

$$q_2^* = -2S \frac{\partial^2 w}{\partial x \partial y} \quad (2.12)$$

ifodani yozish mumkin.

Agar qobiq doimiy tashqi p bosim bilan yuklangan bo'lsa, unda qobiqning halqa yo'nalishi v bo'yicha hosil bo'ladigan dastlabki momentsiz holatiga to'g'ri keluvchi siquvchi N_2 kuchni aniqlaymiz:

$$N_2 = pR. \quad (2.13)$$

Tashqi p bosimning ortishi bilan N_2 kuch ham ortib, natijada qobiq ustuvorligini yo'qotadi.

Bu holatga to'g'ri keluvchi kritik kuch quyidagi miqdorni qabul qiladi:

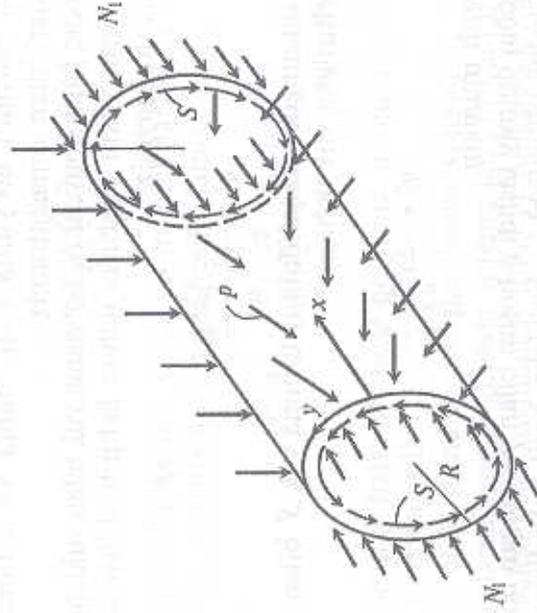
$$q_3 = -N_2 \frac{\partial^2 w}{\partial y^2}.$$

Bu yerda N_2 — siquvchi kuch musbat deb qabul qilingan. (2.10) tenglamadagi q kuchni keltirilgan kuchga

$$q^* = q_1^* + q_2^* + q_3^* = - \left(N_1 \frac{\partial^2 w}{\partial x^2} + 2S \frac{\partial^2 w}{\partial x \partial y} + N_2 \frac{\partial^2 w}{\partial y^2} \right). \quad (2.14)$$

almashirib, natijada silindrik qobiqning izlanayotgan ustuvorlik differensial tenglamasini hosil qilamiz:

$$D\nabla^2\nabla^2\nabla^2\nabla^2 w + \frac{Eh}{R^2}\frac{\partial^4 w}{\partial x^4} + \nabla^2\nabla^2 \left(N_1 \frac{\partial^2 w}{\partial x^2} + N_2 \frac{\partial^2 w}{\partial y^2} + 2S \frac{\partial^2 w}{\partial x\partial y} \right) = 0. \quad (2.15)$$



2.4- chizma.

Bu tenglama bo'yilama kuch N_1 , urinma kuch S va tashqi p bosim kombinatsiyasi ta'sirida bo'lgan qobiq ustuvorlik differensial tenglamasidir (2.4- chizma).

2.3. Bo'yilama siqilgan qobiqlarning o'qqa nosimmetrik deformatsiya holidagi ustuvorligi

Doimiy bo'yilama N kuchlar bilan siqilgan, radiusi R va uzunligi L bo'lgan silindrik qobiqning ustuvorlik masalasini qarab chiqamiz.

Qaralayotgan holat uchun (2.15) ustuvorlik tenglamasi quyidagi ko'rinishda ifodalanadi:

$$D\nabla^2\nabla^2\nabla^2\nabla^2 w + \frac{Eh}{R^2}\frac{\partial^4 w}{\partial x^4} + N\nabla^2\nabla^2\frac{\partial^2 w}{\partial x^2} = 0. \quad (3.1)$$

Qobiqlar ustuvorligini yo'qotishda hosil bo'lgan normal w ko'chishni quyidagi ko'rinishdan qidiramiz:

$$w = f \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{R}. \quad (3.1a)$$

Bunda: f — doimiy koeffitsiyent m, n — butun sonlar. Bularning birinchisi qobiq uzunligi L bo'yicha hosil bo'lgan salqilik funksiyasi yarim to'liqlar sonini, ikkinchisi esa ko'ndalang kesim aylana $2\pi R$ uzunligi bo'yicha hosil bo'lgan to'liqlar sonlarini ifodalaydi.

Salqilik ifodasini (3.1) differensial tenglamaga qo'yamiz va uning bajarilishini talab qilamiz:

$$D \left[\left(\frac{m\pi}{L} \right)^2 + \left(\frac{n}{R} \right)^2 \right]^4 + \frac{Eh}{R^2} \left(\frac{m\pi}{L} \right)^4 - N_1 \left(\frac{m\pi}{L} \right)^2 \left[\left(\frac{m\pi}{L} \right)^2 + \left(\frac{n}{R} \right)^2 \right]^2 = 0. \quad (3.2)$$

(3.2) tenglamadan qobiqning ustuvorligini yo'qotish shakliga to'g'ri keluvchi siquvchi kuch miqdorini aniqlaymiz:

$$N = D\lambda^2 + \frac{Eh}{R^2\lambda^2}. \quad (3.2)$$

Bu yerda:

$$\lambda = \frac{L}{m\pi} \left[\left(\frac{m\pi}{L} \right)^2 + \left(\frac{n}{R} \right)^2 \right]. \quad (3.3)$$

Qobiqni cheksiz uzun deb qabul qilamiz. Bunda N siquvchi kuchni λ^2 o'zgaruvchining uzluksiz funksiyasi deb qarash mumkin. Bu funksiyaning minimumiga kritik N_{kp} kuch qiymati to'g'ri keladi.

Kritik kuch (3.2) tenglamasi (1.13) tenglamaga aniq to'g'ri kelganligi uchun ham kritik kuch (1.16) formula bilan, λ^2 — parametrlar miqdori (1.14) formula bilan aniqlanadi.

Demak, bo'yilama siqilgan uzun silindrik qobiq o'qiga nisbatan simmetrik va nosimmetrik shaklda ustuvorligini yo'qotishda kritik kuch bir xil bo'lar ekan.

Qobiqning bo'yilama va yo'nalishi bo'yicha (3.1a) salqilik funksiyasi yarim to'liqlari uzunliklari:

$$\ell_x = \frac{L}{m}; \quad \ell_y = \frac{\pi R}{n}. \quad (3.4)$$

Unda (3.3) formula quyidagicha ifodalanganadi:

$$\lambda = \lambda_{kp} \sqrt{\frac{1}{\ell_x^2} + \frac{1}{\ell_y^2}} \quad (3.5)$$

(1.14) ifodada $\lambda = \lambda_{kp}$ bo'lgan qiymatida ℓ_x va ℓ_y kattalik o'zaro faqat bitta (3.5) munosabat bilan bog'langan va ularni bir-biriga bog'liq bo'lmagan holda aniqlab bo'lmaydi. Agar ℓ_x/ℓ_y nisbat oldindan ma'lum bo'lsa, kritik (1.16) kuchga to'g'ri keluvchi yarim to'lqinlar uzunliklari ℓ_x va ℓ_y ni aniqlash mumkin. Uzun silindrik qobiq ustuvorligini yo'qotishda uning sirtida kvadrat shaklida botiq va qavariq to'lqinlar hosil bo'lishiga $\ell_x = \ell_y = \ell$ to'g'ri keladi. Unda (1.14) formulani e'tiborga olib, (3.5) ifodani quyidagicha yozish mumkin:

$$\frac{2\pi}{\ell} = 4 \sqrt{\frac{Eh}{DR^2}} \quad (3.6)$$

Saltqlik funksiyasi uchun yarim to'lqinlar uzunligini aniqlaymiz:

$$\ell = \frac{\pi R}{n}$$

Qobiqning ko'ndalang kesimi aylanasini uzunligi bo'yicha hosil bo'lgan to'lqinlar sonini (3.6) formuladan aniqlaymiz:

$$n = \frac{1}{2} \sqrt{\frac{EhR^2}{D}}$$

O'zgarmas koeffitsiyent $\mu = 0,3$ bo'lsa, $D = Eh^3/12(1 - \mu^2)$ bikirlikni e'tiborga olib:

$$n = 0,91 \sqrt{\frac{R}{h}} \quad (3.7)$$

Yupqa qobiq uchun $R/h = 1000$ miqdor, qalin qobiq uchun esa, $R/h = 100$ miqdor to'g'ri keladi. Bu formulalardan ko'rinadiki, $\ell_x = \ell$ shart bajarilganda, uzun qobiqning ustuvorligini yo'qotishda hosil bo'lgan yarim to'lqinlar soni real qobiqlar uchun juda katta bo'ladi.

Qobiq uzunligi L qisqara borganda, (nolga intilganda) (3.5) ifodaning o'ng tomoni $\lambda = \lambda_{kp}$ miqdordan katta bo'ladi, ya'ni juda katta qobiqlar uchun (3.7) ifoda o'z kuchini yo'qotadi. Bunday qobiqlar uchun birinchi (3.2) boshlang'ich munosabatni e'tiborga olish lozim. Qobiq uzunligi L nolga intilganda, (3.3) ifodaga asosan λ miqdor

cheksiz ortadi. Bu holda (3.2) formulaning ikkinchi hadi nolga intiladi, shuning uchun ham juda qisqa qobiqlar uchun siquvchi kuch quyidagi bog'lanishdan aniqlanadi:

$$N = D\lambda^2 \quad (3.8)$$

Kuchning kritik miqdori λ^2 parametrlarining kichik qiymatiga to'g'ri keladi. Bu kichik qiymat $n=0$ bo'lganda hosil bo'lib, u (1.11) o'qqa simmetrik shaklida ustuvorlikni yo'qotishga mos keladi va $m=1$ bo'lgan, ya'ni qobiqning uzunligi bo'yicha saltqlik deformatsiyasi bitta yarim sinusoida joylashgan holga mosdir.

Bu holda $\lambda = \frac{\pi}{L}$ bo'ladi va shunday qilib juda kalta qobiq uchun izlanayotgan kritik kuch ifodasini hosil qilamiz:

$$N_{kp} = \frac{\pi^2 D}{L^2} \quad (3.9)$$

Bu formuladan ko'rinadiki, uchlari bilan sharnirli mahkamlangan siqilgan sterjen ustuvorligini yo'qotishda kritik kuchni (3.9) formuladan silindrik D bikirlikni egilishdagi EI bikirlikka almashirib aniqlash mumkin.

Demak, qisqa qobiqni halqa yo'nalishi bo'yicha eni bir birlikka teng bo'lgan o'zaro uzluksiz tutashgan vertikal sterjenlar to'plamidan iborat deb qarash mumkin.

Kritik kuchlanish (1.17) formulaga asosan qobiqning L uzunligiga bog'liq bo'lmagan, o'zgarmas bo'lib qoladi.

Lekin uzun qobiq o'qining egilishi natijasida xuddi uchlari bilan sharnirli mahkamlangan L uzunlikdagi sterjen siqilishdagi kabi ustuvorligini yo'qotishi mumkin. Bunda tegishli kritik kuchlanish Eyer formulasidan aniqlanadi:

$$\sigma_{kp} = \frac{\pi^2 E}{c} \quad (3.10)$$

Bunda: c - sterjen egiluvchanligi.

Sterjen uzunligining ortishi bilan uning egiluvchanligi ham ortadi. Bunda kritik kuchlanish kamayadi. Juda ham uzun qobiqlar uchun kuchlanish miqdori (1.17) qiymatidan ham kam bo'ladi.

Bunday holda (1.17) formula o'z kuchini yo'qotadi va kritik kuchlanish (3.10) formuladan aniqlanadi.

Yuqorida aytilganlardan shuni xulosa qilish mumkin, bo'y lama siqilgan silindrik qobiqlar uchun (1.17) formula asosida kritik

kuchlanishni aniqlash faqat o'rtacha uzunlikdagi qobiqlar uchun to'g'ri natija beradi.

Kritik kuchlanishni juda qisqa qobiq uchun (3.9) munosabatdan, juda uzun qobiq uchun esa (3.10) munosabatdan foydalanib aniqlash lozim. Bu (3.9) va (3.10) ifodalari (1.17) formulaning ishlatilish chegarasini aniqlash imkoniyatini beradi.

A.S. Volmir monografiyasida tegishli tahlil qilib, bu formuladan L/R parametrining quyidagi chegarada foydalanish mumkinligini ko'rsatgan:

$$1,38\sqrt{\frac{h}{R}} < \frac{L}{R} < 0,52\sqrt{\frac{R}{h}}$$

Agar $\frac{R}{h} = 100$ bo'lsa, $0,14 < \frac{L}{R} < 5,7$ hosil bo'ladi. Bundan ko'rinadiki, umuman aytganda, o'rtacha uzunlikdagi qobiq tushunchasi qobiq geometrik parametrlarining yetarlicha katta o'zgarish chegarasini qamrab oladi.

Yuqorida olingan ustuvorlik koeffitsiyentining nazariy miqdori $k=0,605$, tajriba yo'li bilan olingan ustuvorlik koeffitsiyenti miqdoriga nisbatan juda katta orttirilgan.

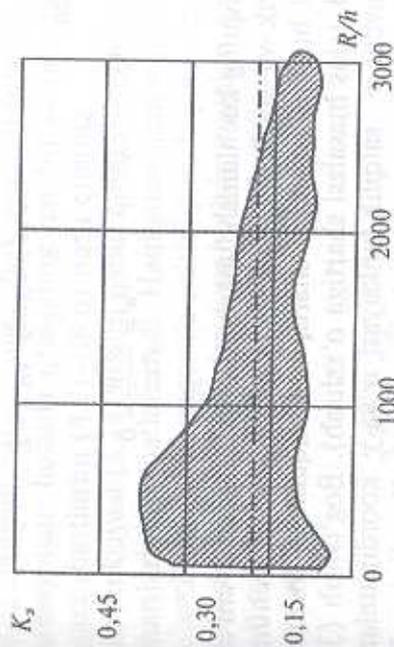
Bo'yлама siqilgan silindrik qobiq uchun R/h nisbatlarining har xil qiymatlarida tajriba yo'li bilan olingan ustuvorlik koeffitsiyenti 2.1-jadvalda keltirilgan.

2.1-jadval

R/h	250	500	750	1000	1500
k_s	0,18	0,14	0,12	0,10	0,09

2.1-jadvaldan ko'rinadiki, R/h nisbatning ortishi bilan ustuvorlik koeffitsiyenti miqdorining kamayishini, nisbatan qalin qobiq ($R/h=250$) uchun nazariy yo'li bilan olingan ustuvorlik koeffitsiyenti, tajriba yo'li bilan olingan koeffitsiyentdan uch martadan ham ortiqroqdir.

Shuni aytib o'tish lozimki, bir xil namunadan yasalgan juda ko'p qobiqlarni siqib sinash tajribalarini o'tkazish natijasida olingan ustuvorlik koeffitsiyentining o'rtacha miqdori 2.1-jadvalda keltirilgan. Haqiqatda esa, bunday ko'p sonli namunalarni ustuvorlikka sinashda ustuvorlik koeffitsiyenti miqdori yetarlicha katta chegarada yoyilganli-



2.5-chizma.

gini ko'ramiz. Tajribalar yo'li bilan olingan ustuvorlik koeffitsiyenti miqdorining o'zgarish chegarasi kengligini 2.5-chizma ham tasdiqlaydi.

2.4. Bo'yлама siqilgan qobiq chegaralarining kritik kuchga ta'siri

Yuqorida ko'rsatilganidek, silindrik qobiq ustuvorlik masalasi, (2.6) va (2.9) tenglamalar sistemasini birgalikda integrallashga keltirilib, undagi ko'ndalang q yuk keltirilgan ko'ndalang yuk bilan almashiriladi, unda bu tenglamalar sistemasini bo'yлама yo'nalish bo'yicha siqilgan hol uchun quyidagicha ifodalaymiz:

$$\nabla^2 \nabla^2 \phi + \frac{E}{R} \frac{\partial^2 w}{\partial x^2} = 0;$$

$$D \nabla^2 \nabla^2 w - \frac{h}{R} \frac{\partial^2 \phi}{\partial x^2} + N \frac{\partial^2 w}{\partial x^2} = 0. \quad (4.1)$$

Bu yerda: N – siquvchi kuch. Bu (4.1) tenglamalarining har biri x va y o'zgaruvchilar bo'yicha to'rtinchi darajalidir. Shuning uchun ham sistemasining umumiy darajasi sakkizga tengdir. Yuqoridagilarni e'tiborga olib, ustuvorlik masalasi yechishda silindrik qobiqning har bir uchida to'rtta bir jinsli chegaralar shartlari qo'yilgan bo'lishi lozim.

Uchlari sharnirli tayangan qobiq chegaralar shartlarini qanoatlantiruvchi yechim sifatida w salqilik (3.1a) funksiyasini qabul qilamiz. Unda qobiq chegaralar shartlari

bo'lsa,

$$x=0 \text{ va } x=L$$

$$w = \frac{\partial^2 w}{\partial x^2} = 0 \quad (4.2)$$

bo'ladi.

Qobiqning kuchlanish funksiyasi orqali aniqlanuvchi tangensial, geometrik va statik miqdorlarga nisbatan (4.2) yechimga to'g'ri keluvchi bir jinsli chegara shartlarini aniqlash qiyin emas (shart, elastiklik tekis masalasi shartiga o'xshash). Bog'lanish (3.1a) bilan aniqlanuvchi w salqilik funksiyasi x va y koordinatalar bo'yicha sinusoida qonuni asosida o'zgaruvchi bo'lganligi uchun (4.1) tenglamalar sistemasi yechimiga to'g'ri keluvchi Φ kuchlanish funksiyasi ham xuddi shu qonun asosida o'zgaradi:

$$\Phi \sim \sin \frac{m\pi x}{L} \sin \frac{ny}{R} \quad (4.3)$$

Demak, (2.2) munosabatni e'tiborga olib, qobiq ustuvorligini yo'qotishida qobiq kuchlanganlik holati uchun kuchlanishlarning o'zgarish qonunini quyidagicha ifodalaymiz:

$$\begin{aligned} \sigma_x &\sim \sin \frac{m\pi x}{L} \sin \frac{ny}{R}; \\ \sigma_y &\sim \sin \frac{m\pi x}{L} \sin \frac{ny}{R}; \\ \tau_{xy} &\sim \cos \frac{m\pi x}{L} \cos \frac{ny}{R}. \end{aligned} \quad (4.4)$$

Bu ifodalarni e'tiborga olib, (2.3) va (2.5) formulalar asosida qabul qilingan (3.1a) normal ko'chish w ifodasi shakliga to'g'ri keluvchi x va y koordinatalar bo'yicha u va v ko'chishlarning o'zgarish qonunini aniqlaymiz:

$$\begin{aligned} u &\sim \cos \frac{m\pi x}{L} \sin \frac{ny}{R}; \\ v &\sim \sin \frac{m\pi x}{L} \cos \frac{ny}{R}. \end{aligned} \quad (4.5)$$

(4.4) va (4.5) bog'lanishlar, tangensial hisoblashlar miqdoriga nisbatan qobiqning uchlari bo'yicha quyidagi bir jinsli chegara shartlariga keltiriladi:

$$\text{agar } x=0 \text{ va } x=L \text{ bo'lsa, } \sigma_x = \nu = 0 \text{ bo'ladi.} \quad (4.6)$$

Yuqorida aytilganlarni e'tiborga olsak, qobiq ustuvorlik masalasi uchun qaralayotgan holatda qobiqning har bir uchida to'rtta bir jinsli chegara shartlarini (4.2)—(4.6) hosil qilamiz.

Salqilik funksiyasi (3.1a) uchun qabul qilingan yechim bir jinsli chegara shartlarini qanoatlantiradi. Haqiqatan ham siqilgan qobiqning $x=0$, $x=L$ uchlari yuqorida keltirilgan chegara shartlaridan boshqa chegara shartlari variantlari ham bo'lishi mumkin. Qobiqning egilishida hisoblash miqdorlari uchun uning $x=0$, $x=L$ uchlari quyidagi chegara shartlari ham bajarilishi mumkin:

$$\begin{aligned} w = \frac{\partial w}{\partial x} &= 0; \\ Q_x^* - N \frac{\partial w}{\partial x} &= M_x = 0, \\ Q_x^* &= \frac{\partial w}{\partial x} = 0. \end{aligned} \quad (4.7)$$

Ulardan birinchisi qobiq uchlari qistirib mahkamlangan, ikkinchisi qobiq uchlari erkin, uchinchisi qobiq uchlari bir vaqtning o'zida aylanish burchagi va keltirilgani kesuvchi kuch nolga tenglik shartlarini ifodalaydi.

Qobiq uchlari tangensial hisoblash miqdorlariga nisbatan, (4.6) ifodadan tashqari, xuddi shuningdek, quyidagi uch xil chegara shartlarini ham yozish mumkin:

$$\begin{aligned} u = \nu &= 0; \\ \sigma_x = \tau_{xy} &= 0; \\ \tau_{xy} = u &= 0. \end{aligned} \quad (4.8)$$

Bular qobiq uchlari tangensial ko'chishlarga nisbatan qistirib mahkamlangan, qobiq uchlari bu ko'chishlarga nisbatan erkin, qobiq uchlari halqa ν ko'chishlarga nisbatan erkin va bo'ylama u ko'chishga nisbatan mahkamlanishiga to'g'ri keladi.

Kritik kuch miqdoriga silindrik qobiq chegara shartlari qanday ta'sir qiladi, degan savolga javob berish uchun bo'ylama siqilgan silindrik qobiq masalasini yechishda mumkin bo'lgan barcha bir jinsli chegara shartlari variantlarini qarash lozim.

Yopiq silindrik qobiq yechimining halqa koordinatasi bo'yicha davriylik shartini e'tiborga olib, izlanayotgan salqilik w va kuchlanish Φ funksiyalari quyidagi ko'rinishda izlanadi:

2.5. Kritik kuch miqdoriga boshlang'ich xatolik va kritik holatgacha bo'lgan qobiq momentli holatining ta'siri

Silindrik qobiq

$$w_0 = \frac{4f_0}{L^2} x(L-x) \quad (5.1)$$

formula bilan aniqlanuvchi o'qqa simmetrik boshlang'ich xatolikka ega bo'lsin, deb faraz qilamiz.

Agar $x=L/2$ bo'lsa, $w_0=f_0$ ni hosil qilamiz. Shuning uchun ham f_0 — koeffitsiyent qobiq o'rta kesimidagi boshlang'ich salqilning (salqilik cho'qqisi) maksimal miqdorini ifodalaydi.

Taxminan qobiq meridiani egrilik radiusi ρ_1 quyidagi formuladan topiladi:

$$\frac{1}{\rho_1} = \frac{\partial^2 w_0}{\partial x^2} = -\frac{8f_0}{L^2}. \quad (5.2)$$

Bu ρ_1 miqdor qobiqning birinchi bosh egrilik radiusini ifodalaydi. Ikkinchi egrilik radiusi ρ_2 qobiq sirtidan normal bo'yicha uning o'qigacha bo'lgan masofa sifatida aniqlanadi. 2.6- chizmada u ρ_1 orqali belgilangan. Qobiqning boshlang'ich w_0 salqilini qobiq radiusiga nisbatan kichik deb hisoblaymiz ($f_0 \ll R$). Unda qobiq sirti normal va uning o'qiga normal orasidagi burchak θ ham kichik miqdor bo'ladi va unda $r_1=r$ deb qabul qilamiz. Qobiq ixtiyoriy kesimi radiusi r oxirgi kesim radiusidan juda kam farq qilishini e'tiborga olib, taxminan $\rho_2=R$ deb olamiz.

Qaralayotgan bu holda radiuslar markazi qobiqning turli tomonlarida bo'ladi, bu esa qobiqning manfiy egriligiga to'g'ri keladi ($\rho_1, \rho_2 < 0$).

Qobiqning momentsiz birinchi boshlang'ich kritik holati uchun muvozanat sharti uning sirti normaliga kuchlarning proyeksiyasi:

$$-\frac{N_1}{\rho} + \frac{N_2}{R} = 0. \quad (5.3)$$

Bu yerda: $\rho = -\rho_1 = \frac{8f_0}{L^2}$ musbat miqdor hisoblanadi.

Bo'yilama yo'nalish bo'yicha N_1 kuch bilan siqilgan qobiqning manfiy gauss egriligi tufayli (5.3) formulaga asosan halqa yo'nalish bo'yicha N_2 siquvchi kuch hosil bo'ladi:

$$w = \chi_1(x) \sin \frac{\pi y}{R};$$

$$\Phi = \chi_2(x) \sin \frac{\pi y}{R}.$$

Bu funksiyalarni (4.1) tenglamaga qo'yib, y o'zgaruvchisini aniqlaymiz va natijada ustuvorlik tenglamasini izlanayotgan χ_1, χ_2 funksiyalarga nisbatan ikkita sakkizinchi tartibli oddiy differensial tenglamalar sistemasiga keltiramiz.

Tenglamaning umumiy yechimini yozamiz va unda integrallash natijasida hosil bo'lgan o'zgarimaslarni qobiq uchlaridagi bir jinsli chegara shartlaridan aniqlaymiz.

Shunday qilib, ustuvorlik masalasini yechish uchun izlanayotgan algebraik bog'lanishni hosil qilamiz.

Bu bog'lanishdan aniqlanuvchi kritik kuch miqdori qobiq uchlari bo'yicha mumkin bo'lgan har bir chegara shartlari variantlari uchun har xil bo'ladi.

Oldingi mavzularda kritik kuchni qobiq uchlaridagi chegara shartlaridan foydalanib aniqlash yaxshi o'rganildi.

Ko'pgina chegara shartlarining barcha variantlarida ustuvorlik koeffitsiyenti qiymati (1.17) klassik formulaga yaqin bo'lib, $k=0,605$ qobiq uchlari sharnirli mahkamlangan chegara shartiga to'g'ri keladi.

Faqat bitta chegara sharti uchun:

$$x = 0; \quad x = L;$$

$$w = \frac{\partial^2 w}{\partial x^2} = 0; \quad \sigma_x = \tau_{xy} = 0.$$

Siqilgan silindrik qobiqda ustuvorlik koeffitsiyentining nazariy qiymati klassik qiymatining taxminan yarmiga teng bo'ladi. Bu holat uchun normal w ko'chishga nisbatan uchlari bo'yicha sharnirli tayangan shart bajariladi, tangensial u va v ko'chishlar uchun esa uchlari erkin bo'lgan shart bajariladi. Oxirgi shart amaliyotda real konstruksiyalarda qo'llanilmaydi.

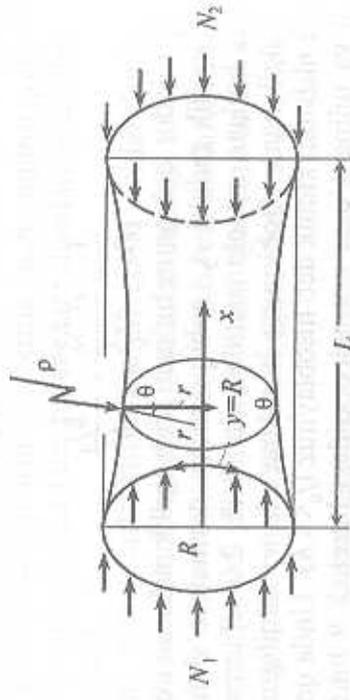
Shunday qilib, siqilgan silindrik qobiq uchlaridagi har xil chegara shartlari kritik kuch miqdoriga deyarli ta'sir ko'rsatmaydi.

Bizga ma'lumki, sterjen va plastinkaning ustuvorlik masalarida esa buning teskarisi edi, ya'ni sterjen uchlaridagi (plastinka konturidagi) chegara shartlari kritik kuch miqdoriga ta'sir ko'rsatishini bilamiz.

$$N_2 = \frac{R}{\rho} N_1, \quad (5.4)$$

Ideal qobiq uchun deformatsiya (2.3) munosabati kabi boshlang'ich salqilikka ega bo'lgan qobiq uchun quyidagi formuladan aniqlanadi:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} + \frac{w}{\rho}; \\ \varepsilon_y &= \frac{\partial v}{\partial y} - \frac{w}{R}; \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}. \end{aligned} \quad (5.5)$$



2.6- chizma.

Bu munosabatlardan u va v ko'chishlarni yo'qotib, deformatsiyaning uzluksizlik tenglamasini hosil qilamiz:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = -\frac{1}{R} \frac{\partial^2 w}{\partial x^2} + \frac{1}{\rho} \frac{\partial^2 w}{\partial y^2}. \quad (5.6)$$

(2.5) va (2.2) bog'lanishlarni e'tiborga olib, uzluksizlik tenglamasini salqilik w funksiyasi va kuchlanish Φ funksiyasi orqali quyidagicha ifodalaymiz:

$$\frac{1}{E} \nabla^2 \nabla^2 \Phi = -\frac{1}{R} \frac{\partial^2 w}{\partial x^2} + \frac{1}{\rho} \frac{\partial^2 w}{\partial y^2}. \quad (5.7)$$

Bu tenglama yangi muvozanat holatiga tegishli bo'lib, qobiqning ustuvorligini yo'qotishda hosil bo'lgan salqilik w funksiyasi va kuchlanish Φ funksiyasi bilan aniqlanadi. Boshlang'ich salqilikka

ega bo'lgan qobiq ustuvorlik tenglamasining ikkinchisi (2.9) tenglama asosida aniqlanadi. Unda kesuvchi q yukni keltirilgan kesuvchi q^* yuk bilan almashtiriladi. Qaralayotgan holatda N_1 kuchdan tashqari N_2 kuchni ham e'tiborga olish lozim va (2.11), (2.14) formulalarga muvofiq quyidagini hosil qilamiz:

$$q^* = -N_1 \left(\frac{\partial^2 w}{\partial x^2} + \frac{R}{\rho} \frac{\partial^2 w}{\partial y^2} \right).$$

Natijada quyidagi tenglamaga ega bo'lamiz:

$$D \nabla^2 \nabla^2 w - \frac{h}{R} \frac{\partial^2 \Phi}{\partial x^2} + N_1 \left(\frac{\partial^2 w}{\partial x^2} + \frac{R}{\rho} \frac{\partial^2 w}{\partial y^2} \right) = 0. \quad (5.8)$$

(5.7) va (5.8) tenglamalardan kuchlanish funksiyasini yo'qotib, boshlang'ich w_0 salqilikni e'tiborga oluvchi normal ko'chish w funksiyasiga nisbatan izlanayotgan ustuvorlik tenglamasini hosil qilamiz:

$$D \nabla^2 \nabla^2 \nabla^2 w + \frac{Eh}{R^2} \left(\frac{\partial^4 w}{\partial x^4} - \frac{R}{\rho} \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) + N_1 \nabla^2 \nabla^2 \left(\frac{\partial^2 w}{\partial x^2} + \frac{R}{\rho} \frac{\partial^2 w}{\partial y^2} \right) = 0. \quad (5.9)$$

Qobiq ustuvorligini yo'qotishda salqilik funksiyasini (3.1a) ko'rinishda izlaymiz va ustuvorlik tenglamasini qanoatlantirishini talab qilamiz. Natijada tegishli siquvchi kuch miqdorini aniqlaymiz:

$$N_1 = \frac{D(\alpha^2 + \beta^2)^2}{\alpha^2 + \frac{R}{\rho} \beta^2} + \frac{Eh}{R^2} \frac{\left(\alpha^4 - \frac{R}{\rho} \alpha^2 \beta^2 \right)}{(\alpha^2 + \beta^2)^2 \left(\alpha^2 + \frac{R}{\rho} \beta^2 \right)}. \quad (5.10)$$

Bu yerda quyidagi belgilashlar kiritilgan:

$$\begin{aligned} \alpha &= \frac{m\pi}{L}; \\ \beta &= \frac{n}{R}. \end{aligned} \quad (5.11)$$

Agar (5.10) munosabatga $\rho = \infty$ ni qo'ysak, bu boshlang'ich salqilik yo'q holatiga to'g'ri kelib, (5.10) formula (3.2) formulaga o'tadi, siquvchi N_1 kuch esa ideal silindrik qobiq zo'riqishiga to'g'ri keladi.

Yuqoridagi formulalardan ko'rinadiki, (5.10) formulada ρ mavjud bo'lgan had har ikki qo'shiluvchi hadlarning qiymatini kamaytiradi va natijada kritik yuk ham kamayadi.

Bu qaralayotgan qobiq botiq (gauss egriligi manfiy) bo'lganligi, binobarin, boshlang'ich salqilik qobiq markaziga yo'nalganligi bilan bog'liqdir.

Agar (5.1) boshlang'ich w_0 salqilikni teskari ishora bilan olinsa, unda qobiq qavariq, ya'ni gauss egriligi musbat bo'ladi ($\rho, \rho_2 > 0$). Egriлик ρ radiusi ishorasi teskari o'zgariganda ham siqilgan qobiqqa to'g'ri keluvchi kritik kuch miqdori (5.10) formuladan aniqlanadi. Bu holatda N_1 kritik kuch miqdori ideal qobiqdagi kuch miqдорidan ortiq bo'ladi.

Qobiq boshlang'ich salqiligining kritik kuch miqдорiga ta'sirini (5.10) bog'lanishga asosan tadqiq qilamiz. Aniqlik kiritish uchun, faraz qilaylikki, qobiq ustuvorligini yo'qotishida uning sirtida kvadrat qavariq va botiq ezilish hosil bo'lsin. Bu holda (3.4) formulaga asosan quyidagini hosil qilamiz:

$$\ell_x = \ell_y = \frac{\pi R}{n}. \quad (5.12)$$

Bu yerda n — qobiqning doiraviy ko'ndalang kesimi bo'yicha hosil bo'lgan butun to'lqinlarning soni. Qobiq L uzunligini ustuvorlikni yo'qotishda hosil bo'lgan yarim to'lqinlar (5.12) uzunligiga teng deb qabul qilib, (5.10) va (5.11) formulalarga asosan siquvchi kuch uchun quyidagi ifodaga ega bo'lamiz:

$$N = \frac{4D \left(\frac{n}{R}\right)^2}{1 + \frac{R}{\rho}} + \frac{Eh}{4R^2} \left(1 + \frac{R}{\rho}\right) \left(\frac{n}{R}\right)^2. \quad (5.13)$$

(5.2) va (5.12), bog'lanishlarga asosan quyidagini hosil qilamiz:

$$\frac{1}{\rho} = 8 \frac{n^2 \alpha h}{\pi^2 R^2}. \quad (5.14)$$

Bu yerda quyidagi belgilash kiritilgan:

$$f_0 = \alpha h. \quad (5.15)$$

Bu yerda: h — qobiq qalinligi, α — doimiy koeffitsiyent bo'lib, maksimal boshlang'ich f_0 salqilikning qobiq qalinligiga bo'lgan nisbatini ifodalaydi.

Yuqorida ko'rsatilganidek, (5.12) shartning bajarilishida to'lqinlar soni n miqdorga to'g'ri keluvchi siquvchi kritik kuchning miqdori (5.9) ifodadan aniqlanadi. Bu aytilganlarni e'tiborga olib, (5.13) bog'lanishni quyidagi ko'rinishga keltiramiz:

$$N = \frac{4 \cdot 0,91^2}{1 + \frac{8 \cdot 0,91^2}{\pi^2} \alpha} D + \frac{1}{4 \cdot 0,91^2} \frac{1 - \frac{8 \cdot 0,91^2}{\pi^2} \alpha}{1 + \frac{8 \cdot 0,91^2}{\pi^2} \alpha} \frac{Eh^2}{R}.$$

Puasson koeffitsiyenti $\mu=0,3$ bo'lganda esa:

$$N = \frac{1}{1 + 0,671\alpha} [0,303 + 0,302(1 - 0,671\alpha)] \frac{Eh^2}{R}. \quad (5.16)$$

Agar $\alpha=0$ bo'lsa, boshlang'ich salqilikning bo'lmagan holatiga to'g'ri kelib, bu formula ustuvorlik koeffitsiyentini klassik $k=0,605$ miqdorga keltiradi. Agar boshlang'ich salqilik mavjud bo'lsa, ustuvorlik koeffitsiyenti salmoqli kamayadi.

Masalan $\alpha=1$ bo'lsa, ya'ni boshlang'ich f_0 salqilikning maksimal qiymati qobiq h qalinligiga teng bo'ladi, (5.16) formulaga asosan $k=0,24$ bo'lishini aniqlaymiz. Agar $\alpha=0,5$, $\alpha=0,25$ bo'lsa, tegishlicha $k=0,377$, $k=0,475$ bo'ladi.

Keltirilgan natijalardan ko'rinadiki, boshlang'ich xatoliklarning bo'lishi, hattoki, juda kichik miqdorda bo'lsa ham, ustuvorlik koeffitsiyentining ideal holatidagi qobiq ustuvorlik koeffitsiyentiga nisbatan juda ko'p kamayishga olib keladi.

Yuqorida keltirilgan faktlarni 2.6- chizmada keltirilgan qobiq uchun kritik holatigacha fazoviy konstruksiya sifatida ishlatilishidan tushuntirish mumkin.

Bo'ylama siqilgan qobiqning muvozanat sharti asosida unda kritik kuchni salmoqli darajada kamaytiruvchi siquvchi kuchlanish hosil bo'ladi.

Bu masalada qobiq sifat jihatidan to'g'ri o'qli sterjendan farq qiladi. Biz 1 bobda to'g'ri o'qli sterjening markaziy siqilishida ustuvorligini yo'qotishda boshlang'ich salqilikning kritik kuch miqdoriga ta'sir qilmasligi aytib o'tilgan edi.

Shular bilan birga e'tiborni shunga qaratish kerakki, kritik holatgacha bo'ylama siqilgan qobiq ustuvorligini yo'qotishi vaqtida qo'shimcha salqilik hosil bo'ladi, qobiq materiali Puasson μ koeffitsiyenti ta'siri natijasida yuz beradi. Guk qonuni bo'yicha bo'ylama deformatsiyani quyidagicha yozamiz:

$$\varepsilon = \frac{\sigma}{E}$$

Shuningdek, halqa kuchini yo'q deb qarab, Guk qonuniga asosan quyidagi tenglikni yozish mumkin:

$$\varepsilon_y = -\mu \varepsilon_x$$

Qobiqning normal w salqiligi quyidagi munosabatdan aniqlanadi:

$$w = \xi_y R = \mu \frac{\sigma}{E} R$$

Qobiq ustuvorligini yo'qotish vaqtda bo'yлама normal σ kuchlanish (1.17) formula bilan aniqlangan miqdorini qabul qiladi. Shunday qilib, kritik holatda eng katta salqilik $w=0,605\mu R$ bo'ladi.

Agar $\mu=0,3$ bo'lsa, $w=0$ bo'ladi. Bunday salqilik deformatsiyasi miqdori kritik holatgacha siqilgan qobiqning $x=0$, $x=L$ uchlariida salqilik deformatsiyasi nolga tenglik shartida, uning markaziy qismida hosil bo'ladi. Bu kritik holatgacha egilish hatto qobiqning berilgan birinchi boshlang'ich shakli ideal silindrik bo'lsa ham ustuvorlik k koeffitsiyenti miqdoriga, albatta, ta'sir etadi.

Shuni aytib o'tish muhim ahamiyatga egaki, to'g'ri o'qli siqilgan sterjenlar yoki siqilgan to'g'ri burchakli plastinkalar ustuvorligini yo'qotish vaqtda, berilgan birinchi dastlabki to'g'ri chiziqli shaklining ustuvorligini yo'qotish vaqtigacha to'la saqlaydi. Siqilgan plastinka yangi momentli egilish muvozanat shakli faqat ustuvorligini yo'qotgan vaqtda hosil bo'ladi.

Qobiq uchun bu jihat butunlay boshqacha kechadi. Bunda qobiq geometriyasining murakkabligidan qobiq egilishi bilan xarakterlanuvchi berilgan birinchi boshlang'ich momentsiz va momentli muvozanat shakllariga ajralmasligini ko'rish qiyin emas. Real silindrik qobiq kritik holatigacha siqilganda, qobiq momentli kuchlanganlik holat bilan xarakterlanuvchi egilish hosil bo'ladi. Siquvchi kuchning ortishi bilan salqilik va momentlar ham uzluksiz ortib boradi.

Bu mavzuda kritik kuchni aniqlashda kritik holatgacha siqilgan momentli holat e'tiborga olinmaydi. Agar qobiq boshlang'ich xato-ligini va momentli holatini e'tiborga olsak, kritik kuchni kamaytirish bundan ham salmoqliroq bo'ladi.

Masalan, monografiyada qobiqning kritik holatigacha egilishda momentli holatiga va $\alpha=h/R$ nisbatning o'zgarishiga bog'liq bo'lgan ustuvorlik k koeffitsiyenti miqdori 2.2- jadvalda keltirilgan.

α	0	0,10	0,2	0,3	0,4	0,5
k	0,605	0,35	0,290	0,230	0,196	0,180

Yuqorida aytilganlarni mulohaza qilib, bo'yлама siqilgan qobiq boshlang'ich xatoligi, kritik holatgacha uning momentligi kritik kuch klassik miqdorining keskin kamayishiga olib kelishini ta'kidlash mumkin.

Bu xatolik miqdori uncha katta bo'lmasa-da, haqiqatan u tasodifiy xarakterga ega va qiyin boshqarishga ega bo'lgan kattalik, bu esa bo'yлама siqilgan qobiq kritik kuchi miqdorining yetarlicha tarqoqligiga (sochilishiga) olib keladi. Buni bo'yлама siqilgan real qobiq ustida olib borilgan tajribalar to'liq tasdiqlaydi (2.5- chizma).

2.6. Bo'yлама siqilgan silindrik qobiqning katta salqiligi ni e'tiborga olganda ustuvorlik

Bo'yлама siqilgan qobiqning o'qqa nosimmetrik bo'rtib chiqishi holatiga to'g'ri keluvchi (3.1a) salqilik funksiyasi qobiqning bir-biridan bir xil uzoqlikda joylashgan parallel va meridianni ifodalovchi chiziqlarda nolga aylanadi. Haqiqatda esa real silindrik qobiqning sirtida romb shaklidagi chuqur ezilishlar hosil bo'ladi. Yuklanish natijasida qobiq birinchi holatdan aynan o'xshash ikkinchi holatga o'ny o'tishi natijasida ezilish sakrash tarzida ro'y beradi. Hosil bo'lish vaqtida ularning chuqurliklarini qobiq qalinligiga taqqoslasa bo'ladi. Bu ma'lumotlar qobiq ustuvorlik masalalarida, qobiq katta salqilik deformatsiyasini e'tiborga oluvchi nochiziqli nazariyadan foydalanish lozim ekanligini tasdiqlaydi.

Bo'yлама siqilgan silindrik qobiqlarning nochiziq masalalarida izlanayotgan salqilik w funksiyasi va kuchlanish Φ funksiyasiga nisbatan ikkita tenglamalar sistemasi quyidagicha ifodalanadi:

$$\frac{1}{E} \nabla^2 \nabla^2 \Phi + \frac{1}{R} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 = 0;$$

$$\frac{D}{h} \nabla^2 \nabla^2 w - \frac{1}{R} \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 \Phi}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - 2 \frac{\partial^2 \Phi}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} = 0. \quad (6.1)$$

Bu formulalarning uchinchi va undan keyingi hadlari, ya'ni salqilik va kuchlanish funksiyalarining ikkinchi hosilalari ko'paytmasi mavjud bo'lgan hadlari tenglamaning chiziqli bo'lmagan qismini ifodalaydi.

Shuni aytib o'tish lozimki, nochiziqli tenglamalarni keltirib chiqarish ham chiziqli (4.1) tenglamalarni keltirib chiqarish kabi bajariladi. Lekin (6.1) tenglamalarning birinчисini hosil qilishda deformatsiya bilan ko'chishlar orasidagi (2.3) bog'lanishlarni, albatta, quyidagi chiziqli bo'lmagan munosabat bilan almashtirish lozim:

$$\begin{aligned}\varepsilon_x &= \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2; \\ \varepsilon_y &= \frac{\partial v}{\partial y} - \frac{w}{R} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2; \\ \gamma_{xy} &= \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}.\end{aligned}\quad (6.2)$$

Bu ifodalardagi oxirgi qo'shiluvchilar nochiziqli hadlar bo'lib, ular plastinkadagi (2.3) va (2.4) bog'lanishlar kabi hosil qilinadi. Tenglamalar (6.1) sistemasining ikkinchi tenglamasini hosil qilishda ko'ndalang q yuk, keltirilgan q^* yuk (2.14) bilan almashtiriladi. Bunda N_1 , N_2 va S kuchlar ma'lum deb qaraladi va kuchlanish funksiyasi orqali ifodalanadi:

$$\begin{aligned}N_1 &= h \frac{\partial^2 \Phi}{\partial y^2}; \\ N_2 &= h \frac{\partial^2 \Phi}{\partial x^2}; \\ S &= -h \frac{\partial^2 \Phi}{\partial x \partial y}.\end{aligned}$$

(6.1) tenglamalar sistemasining aniq yechimini topish juda qiyinligi sababli, uning taqribiy yechimi variatsion metodlarga asoslangan quyidagi mulohazalar asosida aniqlanishi mumkin. Izlanayotgan salqilik funksiyasini quyidagi ko'rinishda qidiramiz:

$$w = f_1 \sin \frac{m\pi x}{L} \sin \frac{ny}{R} + f_2 \sin^2 \frac{m\pi x}{L} \sin^2 \frac{ny}{R} + f_3. \quad (6.3)$$

Bu yerda: L , R — tegishli qobiq uzunligi va radiusi; m va n butun miqdorlarni qabul qiladi; f_1 , f_2 , f_3 — aniqlanishi lozim bo'lgan o'zgarmas koeffitsiyentlar.

(6.1) tenglamalar sistemasining birinchi tenglamasiga (6.3) funksiyani qo'yamiz. Hosil bo'lgan tenglamani integrallab, undan kuchlanish funksiyasining analitik ifodasini topamiz:

$$\varphi = \Phi_0(x, y, m, n, f_0, f_1, f_2) - \frac{\sigma y^2}{2}, \quad (6.4)$$

Bu yerda: Φ_0 — birorta ma'lum funksiya bo'lib, u (6.3) funksiya bilan birgalikda; σ — qobiq siqilishidagi o'rtacha siquvchi kuchlanish. (6.1) tenglamalar sistemasining birinчисini qanoatlantiradi.

Kuchlanishlar σ_x va σ_y (2.2) formuladan aniqlangandan keyin nisbiy deformatsiyani quyidagi formuladan aniqlaymiz:

$$\varepsilon_y = \frac{1}{E} (\sigma_y - \mu \sigma_x) = \frac{\partial v}{\partial y}.$$

Ko'chish v uzluksizlik shartini qanoatlantirishi lozim, ya'ni:

$$\int_0^{2\pi R} \frac{\partial v}{\partial y} dy = 0.$$

Bundan (6.4) formuladagi doimiy f_0 koeffitsiyentni aniqlaymiz. (6.1) sistemasining ikkinchi tenglamasini qisqacha quyidagi ko'rinishda yozamiz:

$$A(w, \varphi) = 0.$$

Bu yerda: A — tegishli differensial operator. O'zgarmas f_1 , f_2 koeffitsiyentlarni aniqlash uchun Bubnov-Galerkin usuliga asosan quyidagi bog'lanishlarni yozish mumkin:

$$\left. \begin{aligned} \int_0^{2\pi R} \int_0^L A(w, \varphi) \sin \frac{m\pi x}{L} \sin \frac{ny}{R} dx dy &= 0, \\ \int_0^{2\pi R} \int_0^L A(w, \varphi) \sin^2 \frac{m\pi x}{L} \sin^2 \frac{ny}{R} dx dy &= 0. \end{aligned} \right\} \quad (6.5)$$

Bu ifodalarni integrallash natijasida quyidagi munosabatlarni hosil qilamiz.

$$\left. \begin{aligned} \varphi_1(f_1, f_2, m, n, \sigma) &= 0, \\ \varphi_2(f_1, f_2, m, n, \sigma) &= 0 \end{aligned} \right\} \quad (6.6)$$

Bu ifodalardan f_2 parametrni yo'qotib, quyidagi nochiziq munosabatni hosil qilamiz:

$$\sigma = \varphi(m, n, f_1).$$

Grafik usulida m va n uchun shunday butun qiymatlarni aniqlaymizki, unda σ kuchlanish minimal qiymatini qabul qiladi. Bu qiymatlarni m^* va n^* bilan belgilab olamiz. Unda:

$$\sigma = \varphi(m^*, n^*, f_1).$$

Bundan teskari munosabatni topamiz:

$$f_1 = f_1(m^*, n^*, \sigma).$$

Undan keyin (6.6) ifoda asosida

$$f_1 = f_2(m^*, n^*, \sigma) \text{ va } f_0 = f_0(m^*, n^*, \sigma)$$

bog'lanishlarni aniqlaymiz.

Unda izlanayotgan funksiyani quyidagi ko'rinishda aniqlaymiz:

$$\Phi = \Phi_0(x, y, m^*, n^*, \sigma) - \frac{\sigma y^2}{2},$$

va (2.2) formula bo'yicha kuchlanishni (2.7) formuladan foydalanib, deformatsiyani aniqlaymiz.

Deformatsiya ε_x orqali bo'yлама yuk bilan siqilgan qobiq uchlarining yaqinlashishini aniqlaymiz:

$$\Delta = \int_0^L \frac{\partial u}{\partial x} dx = u|_L - u|_0.$$

Bu oxirgi ifoda bilan izlanayotgan nochiziqli munosabat:

$$\bar{\Delta} = f(\bar{\sigma}), \quad (6.7)$$

qobiq uzayishi:

$$\bar{\Delta} = \frac{\Delta}{L}$$

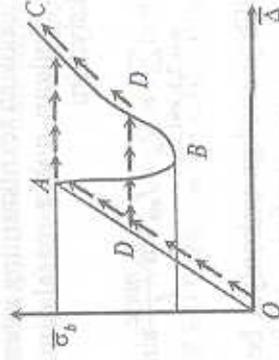
va o'lchovsiz siquvchi kuchlanish:

$$\bar{\sigma} = \frac{\sigma R}{Eh}.$$

orasidagi bog'lanish ko'rsatilgan.

Bu bog'lanishning taxminiy grafigi 2.7- chizmada keltirilgan.

Bu grafigdagi boshlang'ich OA to'g'ri chiziqli uchastkasi qobiqning dastlabki muvozanat holatiga to'g'ri keladi. Uchastkaning yuqorigi A nuqtasi ustuvorlik koeffitsiyentining $k=0,605$ miqdoriga to'g'ri keladi. Bu esa chiziqli nazariya asosidagi kritik kuchning bo'yлама siqilishidagi «klassik» miqdori hisoblanadi. Nochiziqli masalada bu ko'rsatilgan miqdor bilan yuqori kritik kuch aniqlanadi. Pastga qarab tushuvchi AB uchastka qobiqning noustuvorlik holatiga to'g'ri keladi va u amalga oshirilmaydi. Kritik kuch miqdori yuqori kritik kuch miqdoriga yetganda A nuqtadan C nuqtaga o'ny sakrab o'tadi, ya'ni BC egri chiziq ustuvorlik holatiga to'g'ri keluvchi yangi BC egri chiziq bo'yicha ifodalanadi. Bu real qobiq uchun o'ny o'tishi kuzatiladi. Egri chiziqning B nuqtasi kritik kuchning pastki chegarasiga to'g'ri keladi. Har qanday qobiq boshlang'ich xatolikka va boshqa kamchiliklarga ega bo'ladi, unda haqiqatda qobiqning yuklanishi A nuqtaga yetmaydi va OA chiziqdan BC chiziqqa sakrash pastki va yuqorigi kritik kuch oraliq'idagi $\sigma_H \sigma$ miqdorida DD' to'g'ri chiziq bo'yicha to'y beradi.



2.7- chizma.

Qobiqning bo'yлама siqilishida katta salqilik hosil qilish holati ma'lum ma'noda bir to'g'ri chiziqda yotmagan ikki sterjendan iborat bo'lgan birlashgan nuqtasiga to'plangan kuch qo'yilgan fermanın holatiga o'xshab ketadi (2.8- chizma).

Yukning o'sishi bilan A tugun pastga siljiydi va A tugunning ko'chishi ortadi. Yukning ma'lum bir qiymatida tugun holati B nuqtada bo'lganda, muvozanat holati noustuvor bo'ladi va tugun B holatdan K holatga o'ntadi. Bunda tugunning ko'chishi birdaniga katta miqdorga ortadi. Yukning keyinchalik ortishi bilan salqilik ham uzluksiz ortib boradi. Yuk bilan tugun ko'chishi orasidagi grafik 2.9- chizmada keltirilgan. Grafikning pastga tushgan BC egri chiziq noustuvor muvozanat sohasiga to'g'ri keladi. Tugunning C holatiga

F yukning nolga teng miqdoriga to'g'ri keladi. CDE uchastkada yuk manfiy. Fermaning C va E nuqtalar ko'chirlari oralig'ida ushlab turish uchun unga teskari ishorali yuk qo'yilishi lozim. Qaralayotgan bu holda yuklanish fermaning holatini tasvirovchi egri chiziq yuk manfiy bo'lgan qismida ham mavjud bo'ladi (2.9- chizma).

Ferma misolida egri chiziqning F kuch musbat qiymatiga ega bo'lgan sohada egri chiziq to'liq olish mumkin. Agar 2.10- chizmada ko'rsatilganidek ferma tuguni elastik prujina bilan mahkamlangan bo'lsa.

Shunday qilib, bo'yama siqilgan qobiqlar hisob masalasi nochiziqli nazariya asosida pastki kritik kuch miqdorini aniqlashga keltiriladi. Yuqorida ko'rilgan masalada bu miqdor $\sigma=0,284$ ga teng edi. Agar salqilik (2.72) funksiyasi o'rimga quyidagi funksiya:

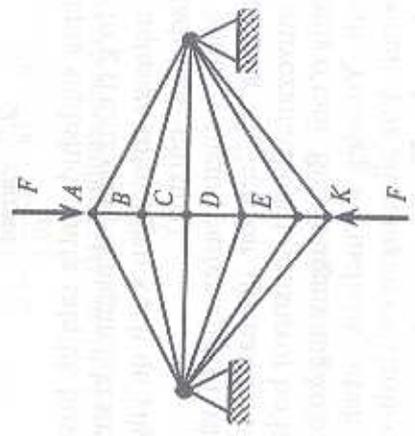
$$w = f_1 \sin \frac{m\pi x}{L} \sin \frac{ny}{R} + f_2 \sin^2 \frac{m\pi x}{L} + f_3$$

olinsa, unda kritik kuchning pastki chegarasi miqdori juda kichik $\sigma=0,124$ bo'ladi. Bu natijalar bundan ikki yuz yil oldin olingan.

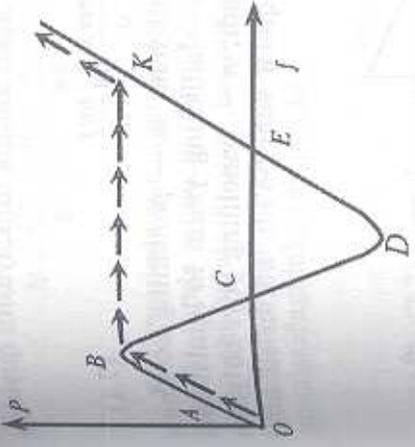
Hozirgi davrda hisoblash texnikasining intensiv rivojlanishi oxirgi vaqtlarda bo'yama siqilgan qobiq masalasini nochiziqli nazariya asosida salqilik funksiyasini

$$w = \sum_{m=1,2,3, \dots} \sum_{n=1,2,3, \dots} f_{mn} \sin \frac{m\pi x}{L} \sin \frac{ny}{R} \quad (6.8)$$

ko'rinishda qabul qilib yechishga imkoniyat berdi.



2.8- chizma.



2.9- chizma.

2.10- chizma.

Kritik kuchning pastki chegarasini hisoblash natijalari shuni ko'rsatadiki, kritik kuch pastki chegarasi ma'lum bir aniq qiymatga ega emas va u qator hadning ortishi bilan kamayib boradi. Kritik kuchning pastki miqdori (6.8) ifoda qator hadlar soniga bog'liq holda quyidagilarga:

$$m = n = 10 \text{ bo'lsa, } \bar{\sigma} = 0,085; \quad m = n = 12 \text{ bo'lsa, } \bar{\sigma} = 0,07;$$

$$m = n = 14 \text{ bo'lsa, } \bar{\sigma} = 0,0526; \quad m = n = 16 \text{ bo'lsa, } \bar{\sigma} = 0,0429,$$

teng bo'ladi.

Shunday qilib, bo'yama siqilgan qobiq ustuvorlik masalasida nochiziqli nazariyadan foydalanish nazariy va tajriba natijalarining to'g'ri kelmasligini yetarlicha tushuntirib berishga imkoniyat yaratib bera olmadi. Bu natijalarning to'g'ri kelmasligini chiziqli nazariya asosida ham yetarlicha ko'rsatgan edik. Lekin shuni aytib o'tish lozimki, chiziqli bo'lmagan nazariya qobiqlarning kritik holatida katta deformatsiyasini hisoblashda, albatta, muhim ahamiyatga ega.

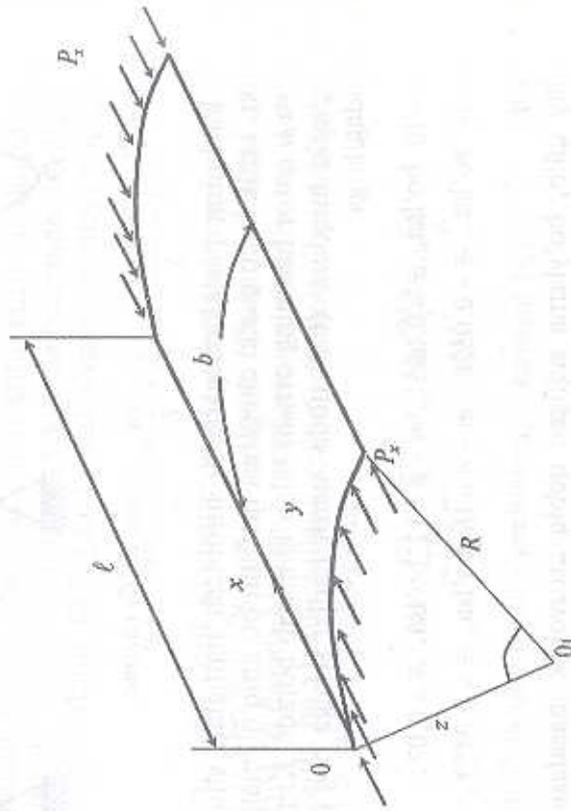
2.7. Silindrik panellar usuvorligi

Egri chiziqli konturi bo'yicha tekis taralgan F o'zgarmas kuch orqali bo'yama siqilgan doiraviy silindrik panel ustuvorlik masalasini qaraymiz. Silindrik panelning a uzunligi va eni b — doira bo'ylab ikki to'g'ri chiziqli konturi orasidagi masofa. Plastinka butun konturi bo'yicha sharnirli mahkamlangan bo'lsin (2.11- chizma). Panelning

ko'rinishda qidiramiz:

$$w = f \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{R} \quad (7.1)$$

Bunda: f — o'zgarmas koeffitsiyent; m — panelning bo'ylama yo'nalish bo'yicha hosil bo'lgan salqilikning yarim to'lqinlar soni bo'lib, butun sonlarni qabul qiladi; n — panelning yoyi bo'yicha hosil bo'lgan salqilik yarim to'lqinlar soni bo'lib, butun sonlarni qabul qiladi.



2.11- chizma.

Yuqoridagi (7.1) munosabatni (3.1) ustuvorlik tenglamasiga qo'yamiz va uni qanoatlantirishdan (7.1) salqilik funksiyasiga tegishli siquvchi yukning ifodasini aniqlaymiz:

$$N = \frac{Da^2}{(m\pi)^2} \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right]^2 + \frac{Eh \left(\frac{m\pi}{a} \right)^2}{R^2 \left(\frac{m\pi}{a} \right)^2} \left[\frac{1}{\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2} \right] \quad (7.2)$$

Agar $b \ll R$ bo'lsa, panel juda yassi bo'ladi. Bu holatda n bo'yicha siquvchi kuchning eng kichik qiymati $n=1$ bo'lganda hosil bo'ladi, ya'ni:

$$N = \frac{D\pi^2}{b^2} \left(\frac{mb}{a} + \frac{a}{mb} \right)^2 + \frac{Eh}{R^2} \left(\frac{mb}{a} + \frac{a}{mb} \right)^2 \quad (7.3)$$

Bu formulada butun parametr m qiymati shunday tanlab olinishi lozimki, unda siquvchi N yuk eng kichik qiymatni qabul qilsin. Uchinchi bobda konturi bo'yicha sharmirli mahkamlangan va bir yo'nalish bo'yicha siqilgan to'g'ri burchakli plastinka ustuvorligi masalasi degidek, aniqlaymiz:

$$m = \frac{a}{b}$$

Unda (7.3) formulaga asosan izlanayotgan kritik kuch quyidagicha ifodalanadi:

$$N_{kp} = 4 \frac{D\pi^2}{b^2} + \frac{Ehb^2}{4R^2} \quad (7.4)$$

Agar panel radius R cheksizlikka intilsa, (7.4) formula $k=4$ bo'lganda, konturi bilan sharmirli mahkamlangan bir yo'nalish bo'yicha siqilgan plastinka kritik kuch formulasiga o'tadi. Bundan shuni xulosa qilib aytish mumkinki, (7.4) formuladagi ikkinchi qo'shiluvchi panelning siqilishida kritik kuch miqdoriga panel egriligining ta'sirini ifodalaydi.

2.8. Tashqi kuch ta'siridagi silindrik qobiqlar ustuvorligi

Tashqi $p = \text{const}$ bosim ta'siridagi (2.4- chizma) qobiqqa tegishli ustuvorlik tenglamasini $N_1 = 5=0$ bo'lganligi sababli (2.15) asosida, (2.13) formulani e'tiborga olib, hosil qilamiz:

$$D\nabla^2 \nabla^2 \nabla^2 w + \frac{Eh}{R^2} \frac{\partial^4 w}{\partial x^4} + pR\nabla^2 \nabla^2 \frac{\partial^2 w}{\partial y^2} = 0. \quad (8.1)$$

Qobiqni kritik holatgacha momentsiz det hisoblab, qobiq ustuvorligini yo'qotishdagi salqilik funksiyasini (3.1a) formula ko'rinishida izlaymiz.

Salqilik (3.1a) funksiyasi ifodasini (8.1) formulaga qo'yib va uni qanoatlantirishini talab qilib, p yukni ifodalovchi bog'lanishni aniqlaymiz.

$$pR = \frac{DR^2}{n^2} \left[\left(\frac{m\pi}{L} \right)^2 + \left(\frac{n}{R} \right)^2 \right]^2 + \frac{Eh}{R^2} \left[\left(\frac{m\pi}{L} \right)^2 + \left(\frac{n}{R} \right)^2 \right] \frac{n^2}{R^2} \quad (8.2)$$

Bu yukning eng kichik qiymati $m=1$ bo'lganda, ya'ni salqilik funksiyasining bitta yarim to'liqini qobiq uzunligi bo'yicha joylashsa hosil bo'ladi. Unda quyidagi tenglik:

$$pR = \frac{Dn^2}{R^2} \left[1 + \left(\frac{\pi R}{nL} \right)^2 \right]^2 + \frac{\pi^4 EhR^4}{n^6 L^4} \left[1 + \left(\frac{\pi R}{nL} \right)^2 \right] \quad (8.3)$$

uchun shartni qabul qilamiz:

$$\left[\left(\frac{\pi R}{nL} \right)^2 \right] \ll 1. \quad (8.4)$$

Bu shart shuni ko'rsatadiki, ustuvorlikni yo'qotishda n yetarlicha katta, ya'ni qobiqning aylanasiz uzunligi bo'yicha ko'p to'liqin hosil bo'ladi. Bunday tipdagi to'liqlarning hosil bo'lishi o'rtacha uzunlikdagi silindrik qobiqlar uchun xarakterlidir. Buni tajriba ham tasdiqlaydi.

Bu aytilganlarni e'tiborga olib, (8.2) bog'lanishni quyidagi oddiy ko'rinishga keltiramiz:

$$pR = \frac{Dn^2}{R^2} + \frac{\pi^4 EhR^4}{L^4} \frac{1}{n^6}. \quad (8.5)$$

Bunda pR ni n ning katta qiymatlarida n^2 o'zgaruvchining uzluksiz funksiyasi deb qarash mumkin:

$$\frac{\partial(pR)}{\partial n^2} = \frac{D}{R^2} - \frac{3\pi^4 EhR^4}{L^4} \frac{1}{n^8} = 0.$$

Bundan pR funksiyaning minimumiga to'g'ri keluvchi izlanayotgan miqdorni aniqlaymiz:

$$n^8 = \frac{3\pi^4 EhR^6}{DL^4}. \quad (8.6)$$

Bu (8.6) formuladan aniqlanadigan n — miqdorning katta

ekanligiga ishonch hosil qilish uchun $\mu=0,3$ bo'lganda, $D=Rh^3/12(1-\mu^2)$ bikirlikni e'tiborga olib quyidagi formulani hosil qilamiz:

$$n = 2,9 \sqrt[4]{\alpha_1 \alpha_2}.$$

Bu yerda:

$$\alpha_1 = \frac{R}{L}, \quad \alpha_2 = \frac{R}{h}.$$

Jumladan, $\alpha_1=1$, $\alpha_2=500$ qiymatlarda real qobiqlar uchun $n=14$ ekanligini aniqlaymiz. Bu natija qabul qilingan (8.4) shartni to'la qanoatlantiradi.

Qobiq ustuvorligini yo'qotishdagi kritik bosimni (8.6) ifodani e'tiborga olib, (8.5) asosida aniqlaymiz:

$$p_{kp}R = \frac{Dn^2}{R^2} \left(1 + \frac{\pi^4 EhR}{L^4} \frac{1}{n^8} \right) = \frac{4}{3} \frac{n^2 D^2}{R^2}. \quad (8.7)$$

$\mu=0,3$ bo'lganda, $D=Rh^3/12(1-\mu^2)$ ni e'tiborga olib, quyidagini hosil qilamiz:

$$p_{kp}R = 0,92E \frac{R}{L} \left(\frac{h}{R} \right)^{3/2}. \quad (8.8)$$

Siquvchi halqaning kritik kuchlanishi uchun quyidagi miqdorni hosil qilamiz:

$$\sigma_{kp} = \frac{N_2}{h} = \frac{p_{kp}R}{h} = 0,92 \frac{ER}{L} \left(\frac{h}{R} \right)^{3/2}. \quad (8.9)$$

Bu bog'lanish Popkovich formulasi deb ataladi.

Juda uzun $L \gg R$ qobiqlar uchun (8.6) formulaga tegishli n son kichik bo'ladi. Bu holda (8.5) ifodadagi pR kattalikni n^2 o'zgaruvchining uzluksiz funksiyasi deb qarab bo'lmaydi. Lekin L/R nisbatning katta qiymatlarida bu formulaning ikkinchi qo'shiluvchisini birinchi hadiga nisbatan e'tiborga olmasa ham bo'ladi. Natijada:

$$pR = \frac{n^2 D}{R^2}$$

hosil bo'ladi.

Izlanayotgan kritik kuch $n=2$ da aniqlanadi:

$$p_{kp} = \frac{4D}{R^3}. \quad (8.10)$$

Bu kritik bosimga to'g'ri keluvchi salqilik ifodasi:

$$w = f \sin \frac{2y}{R}.$$

Bunda f — doimiy koeffitsiyent.

Ustuvorlikni yo'qotishning qaralayotgan holatida normal w ko'chish bo'ylama koordinata x ga bog'liq bo'lmaydi (tekis deformatsiya holati).

Agar (8.10) formuladagi silindrik qobiqning bikirigini egilishidagi EI halqaning bikiriligiga almashtirsak, unda bir-biridan ajratilgan halqalarning siqilishidagi kritik kuch miqdorini olamiz. Lekin olingan bu bog'lanish son koeffitsiyenti bilan siqilgan halqaning kritik kuch ifodasidagi koeffitsiyentidan farq qiladi [2]:

$$p_{kp} = \frac{3EI}{R^3}.$$

Shunday qilib, (8.10) formula taqribiy xarakterga egadir. Demak, bundan ko'rinadiki, (8.1) ustuvorlik tenglamasi o'rtacha uzunlikdagi qobiqlar uchun haqiqatan ham to'g'ri ekan.

Uzun qobiqlarning ustuvorlik masalasini yechishda qobiqlarning momentli nazariyasidan foydalanish lozim.

Juda ham qisqa qobiqlar uchun n katta qiymatlarni qabul qiladi. Bunday holda (8.3) formuladagi ikkinchi hadni birinчисiga nisbatan e'tiborga olmasak ham bo'ladi.

Unda quyidagini hosil qilamiz:

$$pR = k \frac{D}{R^2}. \quad (8.11)$$

Bu yerda:

$$k = \left[\left(\frac{\pi R}{L} \right)^2 \frac{1}{n} + n \right]^2.$$

Bu ifodani n o'zgaruvchining funksiyasi sifatida qaraymiz va k kattalikning minimal qiymatini aniqlash shartini tuzamiz:

$$\frac{\partial k}{\partial n} = - \left(\frac{\pi R}{L} \right)^2 \frac{1}{n^2} + 1 = 0.$$

Bundan :

$$n = \frac{\pi R}{L}; \quad k = 4 \left(\frac{\pi R}{L} \right)^2. \quad (8.12)$$

Shunday qilib, kritik kuch miqdori tashqi bosim ostidagi juda qisqa qobiq uchun (8.11) ifodani va (8.12) bog'lanishni e'tiborga olib, quyidagi formuladan aniqlanadi:

$$N_2 = Rp_{kp} = 4 \frac{\pi^2 D}{L^2}.$$

Bundan ko'rinadiki, bu bog'lanish $k=4$ bo'lganda, plastinka formula bilan, ya'ni konturi bo'yicha sharmirli mahkamlangan, bir yo'nalish bo'yicha siqilgan to'g'ri burchakli plastinka siqilishidagi kritik kuch miqdori bilan bir xil bo'ladi.

Xulosa qilib shuni aytish mumkinki, tashqi bosim ostidagi qobiq, bo'ylama siqilgan qobiqqa qaraganda nazariy yo'l bilan olingan kritik kuch tajriba natijalariga yaqin ekan.

2.9. Silindrik qobiqning buralishdagi ustuvorligi

Qobiq chetidagi uchlari ko'ndalang kesim aylanasini bo'yicha burovchi momentga

$$H=2pR^2S$$

keltiruvchi tekis taralgan urinma S kuch bilan yuklangan bo'lsin. Bunda $N_1=N_2=0$ bo'lgan holga tegishli ustuvorlik tenglamasi (1.15) formula asosida quyidagicha ifodalanadi:

$$D\nabla^2\nabla^2 w + \frac{Eh}{R^2} \frac{\partial^4 w}{\partial x^4} + 2hr\nabla^2\nabla^2 \frac{\partial^2 w}{\partial x\partial y} = 0. \quad (9.1)$$

Bu yerda: τ — urinma kuchlanish bo'lib, $S=\tau h$ tenglik o'rinlidir. Qobiqqa qo'yilgan burovchi kuchning bunday holatida aniq yechim olish mumkin emas, chunki (9.1) tenglamada o'zgaruvchilar bo'linmasdir.

Masalaning taqribiy yechimini olish uchun salqilik funksiyasini quyidagicha qabul qilamiz:

$$w = f \sin \frac{n}{R} (x - cy) \sin \frac{\pi x}{L}. \quad (9.2)$$

Bu bog'lanish qobiq ustuvorligini yo'qotishi holatida uning sirtida qiya to'lqinlarning hosil bo'lishiga to'g'ri keladi. Bunga (9.2) ifoda bilan uzun plastinkalar ustuvorlik masalasini taqribiy yechishda qabul qilingan (3.1a) funksiyalarining o'xshashligidan ham ishonch hosil qilsa bo'ladi.

III bobdagi kabi o'zgarmas urinma kuch bilan yuklangan qobiq to'la energiyasini (9.2) formula asosida aniqlaymiz va uni nolga tenglab, kritik urinma kuchlanishni aniqlash uchun quyidagi formulani hosil qilamiz:

$$\tau_{kp} = 0,78 \frac{Eh}{R} \sqrt{\frac{hR}{L^2}} \quad (9.3)$$

Shuni aytib o'tish lozimki, real qobiqlarning burilishdagi ustuvorligi bo'yicha olingan nazariy natijalar eksperiment natijalari bilan ancha yaxshi to'g'ri keladi.

Tajriba natijasida aniqlangan kritik kuchlanish miqdori nazariy (9.3) natijalarning 0,8—0,9 miqdoriga to'g'ri keladi.

Nazorat savollari va topshiriqlar

1. Bo'ylama o'q bo'yicha siqilgan silindrik qobiqlarning simmetrik qavariq holatida ustuvorlik tenglamasi qanday ifodalanadi?
2. Bo'ylama o'q bo'yicha siqilgan silindrik qobiqlarda kritik kuch ifodasi nimaga teng?
3. Bo'ylama o'q bo'yicha siqilgan silindrik qobiqlar egilishida salqilik deformatsiyasi nimaga teng?
4. Normal va urinma kuchlanishlarning kuchlanish funksiyasi orqali ifodasini yozing.
5. Deformatsiya komponentlari qanday ifodalanadi?
6. Deformatsiyaning uzluksizlik tenglamasi ifodasi qanday yoziladi?
7. Bo'ylama va urinma kuchlar ta'sirida bo'lgan silindrik qobiq ustuvorlik differensial tenglamasi ifodasini yozing.
8. Kritik kuch miqdoriga qobiq chegara shartlari ta'siri qanday bo'ladi?
9. Kritik kuch miqdoriga boshlang'ich xatolik qanday ta'sir ko'rsatadi?
10. Bo'ylama siqilgan silindrik qobiqda katta salqilikni e'tiborga olganda masalani yechish tartibi qanday bo'ladi?
11. Tektis taralgan yukdan siqilgan silindrik panel uchun kritik kuch ifodasi qanday bo'ladi?
12. Tashqi kuch ta'sirida bo'lgan silindrik qobiq uchun kritik kuch ifodasini yozing.

13. Silindrik qobiqning boshlang'ich xatoliklarini aniqlash uchun qanday tenglamalar yoziladi? 14. Silindrik qobiqning boshlang'ich xatoliklarini aniqlash uchun qanday tenglamalar yoziladi? 15. Silindrik qobiqning boshlang'ich xatoliklarini aniqlash uchun qanday tenglamalar yoziladi?

III BOB.

PLASTIKLIK NAZARIYASI ASOSIY QONUNLARI

3.1. Plastiklik sharti

Bizga ma'lumki, umumlashgan Guk qonuni bir o'qli kuchlan-ganlik holatidagi Guk qonuni asosida keltirib chiqariladi. Shuning uchun ham umumlashgan Guk qonuni yuklanishning faqat boshlang'ich davrida plastik deformatsiya hosil bo'lganda haqqoniy bo'ladi.

Yuklanishning boshlang'ich davrida umumlashgan Guk qonuni haqqoniy bo'lgani uchun plastik deformatsiyaning paydo bo'lishi faqat kuchlanish bilan aniqlanadi. Shularga asosan plastiklik shartini kuchlanish tenzor komponentlarining biror funksiyasi ko'rinishida yozish mumkin. Demak, o'z-o'zidan ko'rinib turibdiki, izotrop materiallar uchun plastik deformatsiyaning hosil bo'lishi koordinatalar sistemasining tanlanishiga bog'liq emas. Shuning uchun ham plastiklik shartini tenzor kuchlanishlar funksiyasi ko'rinishida yozish mumkin:

$$f[I_1(D_\sigma); I_2(D_\sigma); I_3(D_\sigma); I_3(D_\sigma)] = 0. \quad (1.1)$$

Ko'pgina tajribalarning ko'rsatishicha, hamma tomonlari bilan siqilgan yoki cho'zilgan material elastik deformatsiyalanib, plastik deformatsiya hosil bo'lmaydi. Shuning uchun ham plastiklik sharti deviator kuchlanishning ikkinchi va uchinchi invarianti funksiyasi ko'rinishida ifodalanadi:

$$f[I_2(D_\sigma); I_3(D_\sigma)] = 0. \quad (1.2)$$

Koordinata $\sigma_1, \sigma_2, \sigma_3$ sistemasida jismning birorta nuqtasining kuchlanganlik holatini vektor komponentlari xarakterlaydi. Tenglamasi:

$$\sigma_1 + \sigma_2 + \sigma_3 = 0 \quad (1.3)$$

bo'lgan tekislik koordinata boshidan va bosh o'qlarga bir xil burchak ostida o'tadi.

Shubhasiz, bu tekislikda yotuvchi har qanday vektor biron-bir kuchlanganlik holatining deviator kuchlanishini xarakterlaydi.

3.2. Maksimal urinma kuchlanishning doimiylik sharti. Tresk - Sen - Venan sharti

Birinchi bo'lib 1868-yili fransuz muhandisi Tresk bosim ostida metallarning teshikdan oqib o'tishi tajribasini o'tkazdi. Bu tajriba natijasida oquvchanlik holatida muhitning barcha nuqtalarida maksimal urinma kuchlanish bir xil bo'lib, qaralayotgan material uchun o'zgarmas va sof siljishda materialning oquvchanlik chegarasiga teng ekanligini aniqladi.

Sen-Venan tomonidan bu shartning matematik ifodasi tekis masala uchun taqdim etildi. M. Levi esa bu shartni plastiklik nazariyasining fazoviy masalalari uchun umumlashtirdi.

Fazoviy kuchlanganlik holati uchun bu shart quyidagicha ifodalanadi:

$$\begin{aligned} 2|\tau_1| &= |\sigma_1 - \sigma_2| = \sigma_T, \\ 2|\tau_2| &= |\sigma_2 - \sigma_3| = \sigma_T, \\ 2|\tau_3| &= |\sigma_3 - \sigma_1| = \sigma_T. \end{aligned} \quad (2.1)$$

Agar $\sigma_1 \geq \sigma_2 \geq \sigma_3$ bo'lsa, yuqoridagi bog'lanishlardan faqat bittasi qoladi:

$$2|\tau_{\max}| = |\sigma_1 - \sigma_3| = \sigma_T. \quad (2.2)$$

Sen-Venan-Levining bu sharti materiallar qarshiligi kursida eng katta urinma kuchlanish mustahkamlik nazariyasi nomi bilan ataladi. Umuman olganda, bu nom unchalik to'g'ri emas, chunki mustahkamlik va plastiklik tushunchalari mutlaq boshqa-boshqa tushunchalardir. Plastik holatning hosil bo'lishi material mustahkamligining batamom tugadi degani emas.

Tresk-Sen-Venan shartidan cho'zilishdagi oquvchanlik chegarasi σ_T va sof siljishdagi oquvchanlik chegarasi orasida quyidagi bog'lanish mavjudligi kelib chiqadi

$$\begin{aligned} (\sigma_1 = \tau, \sigma_2 = 0; \sigma_3 = -\tau, \tau_{\max} = \tau): \\ \sigma \\ \sigma_T = 2\tau \end{aligned} \quad (2.3)$$

Bu (2.2) shartni koordinatalar $\sigma_1, \sigma_2, \sigma_3$ sistemasida o'qi deviator tekisligiga perpendikulyar bo'lgan olti qirrali prizmani ifodalaydi (3.2- chizma).

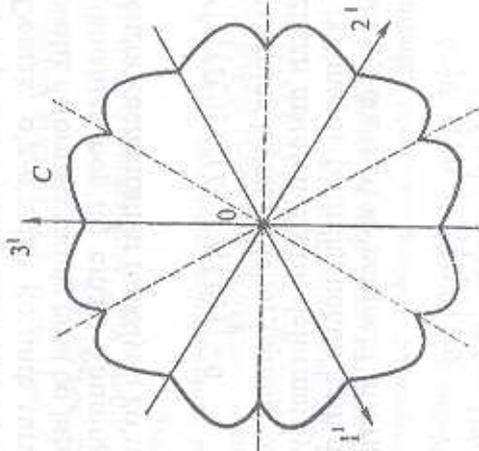
Shuning uchun ham (1.3) tekislikni deviator tekisligi deb ataymiz. Tegishli $\sigma_1, \sigma_2, \sigma_3$ o'qlarning deviator tekislikdagi proyeksiyalarni $1', 2'$ va $3'$ bilan belgilaymiz.

(1.2) tenglamaga o'rtacha normal kuchlanish σ_0 kirmaganligi uchun koordinatalar $\sigma_1, \sigma_2, \sigma_3$ sistemasida (1.2) tenglama o'qi deviator tekisligiga perpendikulyar bo'lgan silindrni tasvirlaydi. Demak, bu silindrning deviator tekisligidagi izini qarash kifoyadir. Bu egri chiziq «C» (3.1- chizma) quyidagi xususiyatlarga ega bo'lishi lozim:

1) yetarlicha katta kuchlanishda plastik deformatsiya hosil bo'lgani uchun egri chiziq koordinata boshidan o'tmaydi;

2) koordinata boshidan chiquvchi nur egri chiziq bilan faqat bir marta kesishishi lozim (aks holda, plastiklik shartini qanoatlantiruvchi ikkita o'xshash kuchlanganlik holati mavjud bo'ladi, bu mumkin emas);

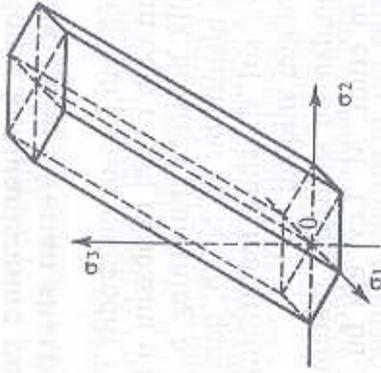
3) egri chiziq $1', 2', 3'$ koordinata o'qlariga simmetrik bo'lishi shart, chunki plastiklik shartiga bosh kuchlanishlar simmetrik kirishi shart;



3.1- chizma. Plastiklik egri chizig'i.

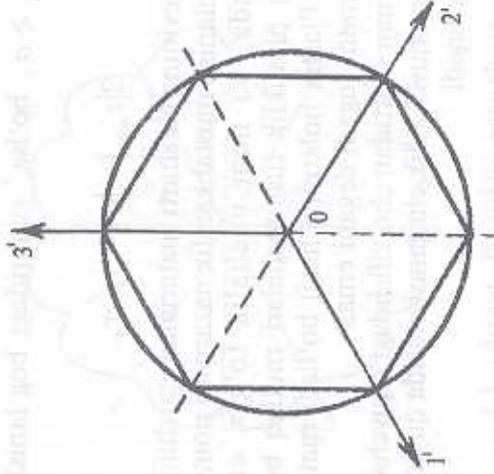
4) egri chiziq $1', 2', 3'$ o'qlariga perpendikulyar bo'lgan to'g'ri chiziq'larga nisbatan simmetrik bo'lishi shart, chunki cho'zilish va siqilishda materialning mexanik xossalari bir xil deb faraz qilinib, Bausshenger effekti e'tiborga olinmaydi.

Yuqorida aytilganlardan ko'rinadiki, egri chiziq (3.1- chizmada) ko'rsatilganidek, 12 ta bir xil yoylardan iborat bo'ladi.



3.2- chizma. Koordinata sistemasida o'qi deviator tekisligiga perpendikulyar bo'lgan olti burchakli prizma.

Prizmaning deviator tekisligi bilan kesishishidan, $\sigma_T/\sqrt{2}$ radiusi aylanaga ichki chizilgan olti burchak hosil bo'ladi (3.3- chizma).



3.3- chizma. Koordinata $\sigma_1, \sigma_2, \sigma_3$ sistemasida deviator tekisligida olti burchak va aylana.

Tresk-Sen-Venan plastiklik shartining kamchiligi shundan iboratki, plastik deformatsiyaning hosil bo'lishida oraliq bosh kuchlanish σ_2 ning ta'siri e'tiborga olinmaydi.

3.3. Urinma kuchlanish intensivligining doimiylik sharti. Mizes-Guber sharti

Tresk-Sen-Venanning plastiklik nazariyasidan uch o'lchamli masalalarni hal qilishda ba'zi bir matematik qiyinchiliklar tug'ildi. Bu qiyinchiliklarni bartaraf qilish uchun Mizes $\sigma_1, \sigma_2, \sigma_3$, koordinatalar sistemasidagi olti burchakli prizmani doiraviy silindr bilan almashtirishni taklif qildi (3.4- chizma). Bu silindr tenglamasi:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_T^2 \quad (3.1)$$

Bu materiallar qarshiligi kursida energetik mustahkamlik nazariyasi nomi bilan yuritiladi. Silindrning deviator tekisligi bilan kesishgan kesimi olti burchakka tashqi chizilgan aylananani beradi (3.3- chizma).

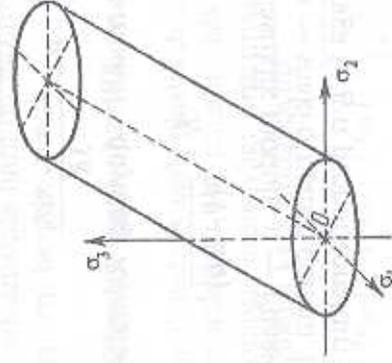
Bu plastiklik sharti Mizesgacha deformatsiyaning potensial energiyasini e'tiborga olib, Guber tomonidan taklif qilingan edi. Shuning uchun ham u Guber-Mizes plastiklik sharti deb yuritiladi.

Mizes-Guber shartining chap tomoni kuchlanishlar intensivligini ifodalashini e'tiborga olib, quyidagini hosil qilamiz:

$$\sigma_1 = \sigma_T \quad (3.2)$$

Ya'ni kuchlanish intensivligi materialning cho'zilishdagi oquvchanlik chegarasiga yetganda plastik deformatsiya hosil bo'ladi.

Mizes-Guber sharti Tresk-Sen-Venan shartiga qaraganda umumiy bo'lib, uni fazoviy kuchlanganlik holati uchun ham qo'llash mumkin.



3.4- chizma. Koordinata $\sigma_1, \sigma_2, \sigma_3$ sistemasida o'qi deviator tekisligiga perpendikulyar bo'lgan silindr.

Mizes urinma kuchlanishning doimiylik shartini aniq, (2.1) shartni esa taqribiy deb hisobladi. Lekin ko'p sonli tajribalar Mizes (2.1) sharti urinma kuchlanishning doimiylik shartiga qaraganda polikristall materiallar uchun to'g'ri kelishimi tasdiqlaydi.

Sof siljish holida Mizes-Guber sharti quyidagicha bo'ladi:

$$\tau_T = \frac{\sigma_T}{\sqrt{2}} \approx 0,577\sigma_T. \quad (3.3)$$

Tajribalarning ko'rsatishicha, plastik deformatsiya sof siljishda ($\tau_{max}/\sigma = (0,56...0,60)\sigma$) bo'lganda hosil bo'ladi. Demak, bundan ham ko'rinadiki, urinma kuchlanish doimiylik shartiga ko'ra, urinma kuchlanishlar intensivligi doimiylik $0,5\sigma_T$ sharti tajriba natijalariga yaqindir.

3.4. Kichik elastik-plastik deformatsiya nazariyasi

Kichik elastik-plastik deformatsiya nazariyasi izotrop materiallar uchun quyidagi uchta qonun asosida qurilgan:

1. Birinchi qonun — hajm o'zgarish qonuni. Jismning hajmiy deformatsiyasi elastik bo'lib, o'rtacha normal kuchlanishga to'g'ri proporsionaldir:

$$\sigma_{yp} = K\theta = 3K\varepsilon_{yp}. \quad (4.1)$$

Hajmiy deformatsiya bilan normal kuchlanishlar quyidagi bog'lanishlar orqali ifodalanadi:

$$\theta = \frac{\sigma_x + \sigma_y + \sigma_z}{3K}. \quad (4.2)$$

Hajmiy elastiklik moduli quyidagi formuladan aniqlanadi:

$$K = \frac{E}{3(1-2\mu)}. \quad (4.3)$$

Bu yerda: E — elastiklik moduli; μ — Puasson koeffitsiyenti.
2. Ikkinchi qonun — shakl o'zgarish qonuni. Deviator kuchlanish deviator deformatsiyaga to'g'ri proporsionaldir:

$$D_\sigma = \psi D_\varepsilon. \quad (4.4)$$

Bu tenglik skalyar ko'rinishda quyidagicha yoziladi:

$$\begin{aligned} \sigma_x - \sigma_{yp} &= \psi(\varepsilon_x - \varepsilon_{yp}); & \tau_{xy} &= \psi\gamma_{xy}/2; \\ \sigma_y - \sigma_{yp} &= \psi(\varepsilon_y - \varepsilon_{yp}); & \tau_{yz} &= \psi\gamma_{yz}/2; \\ \sigma_z - \sigma_{yp} &= \psi(\varepsilon_z - \varepsilon_{yp}); & \tau_{zx} &= \psi\gamma_{zx}/2. \end{aligned} \quad (4.5)$$

Bu ψ — parametрни kuchlanish va deformatsiya intesivligi orqali ifodalaymiz:

$$\begin{aligned} \sigma_i &= \frac{1}{\sqrt{2}} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)} = \\ &= \frac{1}{\sqrt{2}} \sqrt{\psi^2 \left[(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + 3(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2) \right]} = \\ &= \frac{1}{\sqrt{2}} \psi \sqrt{2} \varepsilon_i. \end{aligned} \quad (4.6)$$

Bundan:

$$\psi = \frac{2\sigma_i}{3\varepsilon_i}. \quad (4.7)$$

3. Uchinchi qonun — kuchlanish intensivligi deformatsiya intesivligi funksiyasi bo'lib, kuchlanganlik holati turlariga bog'liq bo'lmaydi:

$$\sigma_i = \Phi(\varepsilon_i). \quad (4.8)$$

Kichik elastik-plastik deformatsiya nazariyasi munosabatlari faqat oddiy yuklanishda to'g'ri bo'ladi. Lekin oddiy kuchlanishga yaqin bo'lgan murakkab yuklanishda ham bu nazariya tajriba natijalariga yaqin bo'lgan natijalarni beradi.

3.5. Plastik oquvchanlik nazariyasi

Plastik oquvchanlik nazariyasida plastik deformatsiyalar ortirmasi bilan kuchlanishlar orasidagi bog'lanishni isbotsiz qabul qilamiz.

Plastik deformatsiya intesivligi ortirmasi, ifodasi ham xuddi deformatsiya intesivligi kabi ifodalanadi:

$$\bar{d}\varepsilon_{yp} = \frac{\sqrt{2}}{3} \sqrt{\left[\frac{d\varepsilon_{yp} - d\varepsilon_{yp}}{3} \right]^2 + \left[\frac{d\varepsilon_{yp} - d\varepsilon_{yp}}{3} \right]^2 + \left[\frac{d\varepsilon_{yp} - d\varepsilon_{yp}}{3} \right]^2 + \frac{3}{2} \left[\frac{d\gamma_{xyp}}{2} \right]^2 + \left[\frac{d\gamma_{yxp}}{2} \right]^2 + \left[\frac{d\gamma_{zxp}}{2} \right]^2} \quad (5.1)$$

Shuni aytib o'tish lozimki, plastik deformatsiya orttirmasi intensivligi $d\epsilon_{sp}$ plastik deformatsiya intensivligi orttirmasi $d\epsilon_{ip}$ ga teng emas.

Izotrop materiallar uchun plastik oqish nazariyasi quyidagi farazlarga asoslanadi:

1. Hajmiy deformatsiya o'rtacha normal kuchlanishga to'g'ri proporsional. Bu cheklanish elastik deformatsiyasida ham ishlatilgan edi:

$$\epsilon_{sp} = \frac{1}{3K} \sigma_{yp} \text{ yoki } d\epsilon_{sp} = \frac{1}{3K} d\sigma_{yp}. \quad (5.2)$$

3. Deformatsiya orttirmasi, elastik:

$$d\epsilon_x; d\epsilon_y; d\epsilon_z; d\gamma_{xy}; d\gamma_{yz}; d\gamma_{zx}; d\gamma_{yx}; d\gamma_{zy}; d\gamma_{zx}$$

va plastik deformatsiyalar

$$d\epsilon_{sp}; d\epsilon_{ip}; d\epsilon_{ep}; d\gamma_{xyp}; d\gamma_{yxp}; d\gamma_{xyp}$$

orttirmalari yig'indisiga teng:

$$d\epsilon_x = d\epsilon_{x3} + d\epsilon_{sp}; d\gamma_{xy} = d\gamma_{xy3} + d\gamma_{xyp};$$

$$d\epsilon_y = d\epsilon_{y3} + d\epsilon_{sp}; d\gamma_{yz} = d\gamma_{yz3} + d\gamma_{yxp};$$

$$d\epsilon_z = d\epsilon_{z3} + d\epsilon_{sp}; d\gamma_{zx} = d\gamma_{zx3} + d\gamma_{xyp}. \quad (5.3)$$

Bundan birinchi cheklanish asosida quyidagi tenglik kelib chiqadi:

$$d\epsilon_{yxp} = 0. \quad (5.4)$$

3. Plastik deformatsiya orttirmasi deviator komponentlari kuchlanish deviator komponentlariga to'g'ri proporsional:

$$D_{d\epsilon_p} = d\lambda D_\sigma. \quad (5.5)$$

Bu ifoda skalyar ko'rinishda quyidagicha yoziladi:

$$d\epsilon_{sp} = d\lambda(\sigma_x - \sigma_{yp}); \frac{1}{2} d\gamma_{xyp} = d\lambda \tau_{xy};$$

$$d\epsilon_{yp} = d\lambda(\sigma_y - \sigma_{yp}); \frac{1}{2} d\gamma_{yxp} = d\lambda \tau_{yx};$$

$$d\epsilon_{zp} = d\lambda(\sigma_z - \sigma_{yp}); \frac{1}{2} d\gamma_{zyp} = d\lambda \tau_{xz}. \quad (5.6)$$

$d\lambda$ parametr ifodasini kichik elastik-plastik deformatsiya nazariyasi mavzusida Ψ parametrni aniqlashda qilingan mulohazalardan foydalanib, quyidagini hosil qilamiz:

$$d\lambda = \frac{3 d\epsilon_{ip}}{2 \sigma_i}. \quad (5.7)$$

4. Kuchlanish intensivligi plastik deformatsiya orttirmasi intensivligi integrali funksiyasi bo'lib, kuchlanganlik holati turiga bog'liq bo'lmaydi:

$$\sigma_i = \Phi \left(\int d\epsilon_{ip} \right). \quad (5.8)$$

Materialning cho'zilish diagrammasi bo'yicha Φ funksiyani aniqlashni ko'rib chiqamiz. Bir o'q bo'yicha cho'zilishda:

$$\sigma_x = \sigma_y = 0; \sigma_z = \sigma; \tau_{xy} = \tau_{yz} = \tau_{xz} = 0;$$

$$d\epsilon_{xp} = d\epsilon_{yp} = -d\epsilon_{zp}/2 = -d\epsilon_p/2$$

bo'ladi.

$$\text{chunki } d\epsilon_{xp} = d\epsilon_{yp} \text{ va } d\epsilon_{xp} + d\epsilon_{yp} + d\epsilon_{zp} = 0.$$

Demak, kuchlanish intensivligi va plastik deformatsiya orttirmasi intensivligi quyidagicha bo'ladi:

$$\sigma_i = \sigma; d\epsilon_{ip} = d\epsilon_p. \quad (5.9)$$

Bundan kelib chiqadiki:

$$\int d\epsilon_{ip} = \int d\epsilon_p = \epsilon_p.$$

Shunday qilib, bir o'q bo'yicha cho'zilgan holda to'rtinchi cheklanish quyidagi ko'rinishda bo'lar ekan:

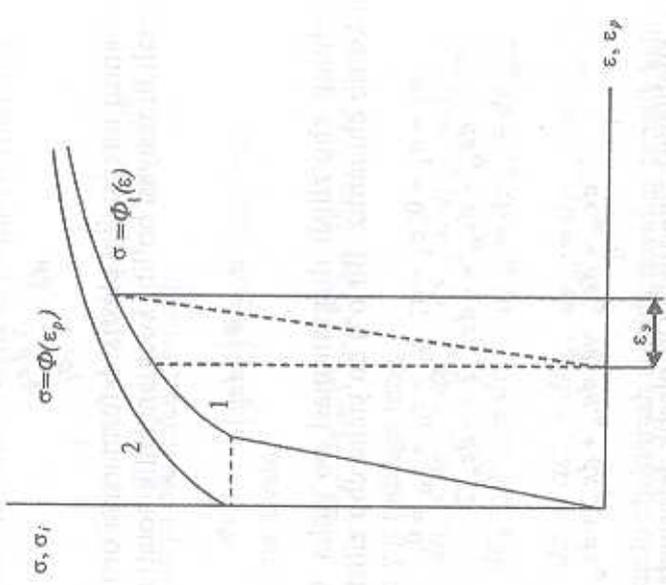
$$\sigma = \Phi \left(\int d\epsilon_{ip} \right) = \Phi(\epsilon_p). \quad (5.10)$$

Bu bog'lanishning grafisini cho'zilish diagrammasi (3.5- chizma, 1 egri chiziq) asosida qurish qiyinchilik tug'dirmaydi. Buning uchun

Nazorat savollari va topshiriqlar

1. Plastiklik kriteriyasi qanday ifodalangani?
2. Tresk-Sen-Yenan plastiklik sharti mohiyati nimadan iborat?
3. Mizes-Guber plastiklik sharti mohiyati nimadan iborat?
4. Kichik elastik-plastik deformatsiya nazariyasi birinchi qonunini aytib bering.
5. Kichik elastik-plastik deformatsiya nazariyasi ikkinchi qonunini aytib bering.
6. Kichik elastik-plastik deformatsiya nazariyasi uchinchi qonunini aytib bering.
7. Oquvchanlik nazariyasi birinchi qonunini aytib bering.
8. Oquvchanlik nazariyasi ikkinchi qonunini aytib bering.
9. Oquvchanlik nazariyasi uchinchi qonunini aytib bering.
10. Oquvchanlik nazariyasi to'rtinchi qonunini aytib bering.
11. Oddiy yuklanishda qaysi nazariyadan foydalanish mumkin?
12. Oquvchanlik nazariyasidan qanday holda foydalanish mumkin?

diagrammaning barcha nuqtalaridagi elastik deformatsiya miqdorini topamiz va bu nuqtalarni chap tomonga ϵ_s masofaga surish lozim. Shu usul bilan qurilgan 2 egri chiziq (3.5- chizma) $\Phi(\int d\epsilon_p)$ funksiyasi grafigi bo'ladi.



3.5- chizma. Yuklanish va yuklanishda deformatsiyalanish diagrammasi.

Plastik oquvchanlik nazariyasi tenglamalari differensial ko'rinishda bo'lib, kichik elastik-plastik deformatsiya nazariyasi tenglamalariga nisbatan ancha murakkabdir.

Oddiy yuklanishda bu ikki nazariya bir xil natija berishini nazariya va tajribalarda isbotlangan.

Murakkab yuklanish holida, plastik oquvchanlik nazariyasida olingan natijalar bilan tajriba yo'li orqali olingan ma'lumotlar juda yaxshi mos keladi.

Shuning uchun ham murakkab yuklanish holidagi masalalarni yechishda bu nazariya keng qo'llaniladi.

ELASTIKLIK CHEGARASIDAN KEYIN QOBIQLARNING KUCHLANGANLIK VA DEFORMATSIYA HOLATI

4.1. Yupqa qobiqlar nazariyasining asosiy munosabatlari

Qobiqning yuklanishi elastiklik chegarasidan keyin oddiy yuklangan va uning materiali siqilmaydi, deb faraz qilamiz, bu kichik elastik-plastik deformatsiya nazariyasidan foydalanishga imkoniyat beradi.

Yassi qobiq o'rtta sirt tekisligining shunday deformatsiyasini qaraymizki, u faqat normal bo'yicha hosil bo'lgan salqilikdan paydo bo'ladi, bunda salqilik qobiq qalinligidan kichik bo'ladi.

Geometrik munosabatlari:

$$\begin{aligned} \varepsilon_1 &= \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 - K_1 w; & \varepsilon_2 &= \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 - K_2 w; \\ \gamma &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}; & \lambda &= -\frac{\partial^2 w}{\partial x \partial y}. \end{aligned} \quad (1.1)$$

Muvozanat tenglamalar:

$$\begin{aligned} \frac{\partial N_1}{\partial x} + \frac{\partial S}{\partial y} = 0; \quad \frac{\partial N_2}{\partial y} + \frac{\partial S}{\partial x} = 0; \\ \frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} - N_1 \left(K_x + \frac{\partial^2 w}{\partial x^2} \right) - N_2 \left(K_y + \frac{\partial^2 w}{\partial y^2} \right) - 2S \frac{\partial^2 w}{\partial x \partial y} = 0; \quad (1.2) \\ \frac{\partial M_1}{\partial x} + \frac{\partial H_2}{\partial y} = Q_1; \quad \frac{\partial M_2}{\partial y} + \frac{\partial H_1}{\partial x} = Q_2; \end{aligned}$$

Zo'riqish kuchlari va eguvchi momentlar ifodalari:

$$\begin{aligned} N_1 &= \frac{4}{3} Eh \left(\varepsilon_1 + \frac{1}{2} \varepsilon_2 \right) + \Delta N_1; \\ N_2 &= \frac{4}{3} Eh \left(\varepsilon_2 + \frac{1}{2} \varepsilon_1 \right) + \Delta N_2; \\ S &= \frac{1}{3} Ehy + \Delta S. \end{aligned} \quad (1.3)$$

$$\begin{aligned} M_1 &= -\frac{Eh^3}{9} \left(\chi_1 + \frac{1}{2} \chi_2 \right) + \Delta M_1; \\ M_2 &= -\frac{Eh^3}{9} \left(\chi_2 + \frac{1}{2} \chi_1 \right) + \Delta M_2. \end{aligned} \quad (1.4)$$

Bu yerda:

$$\begin{aligned} N_1 &= -\frac{4}{3} \left[(Eh - I_1) \left(\varepsilon_1 + \frac{1}{2} \varepsilon_2 \right) - \left(\chi_1 + \frac{1}{2} \chi_2 \right) I_2 \right]; \\ N_2 &= -\frac{4}{3} \left[(Eh - I_1) \left(\varepsilon_2 + \frac{1}{2} \varepsilon_1 \right) - \left(\chi_2 + \frac{1}{2} \chi_1 \right) I_2 \right]; \\ \Delta S &= -\frac{1}{3} \left[(Eh - I_1) \lambda - 2\gamma I_2 \right]. \end{aligned} \quad (1.5)$$

$$\begin{aligned} M_1 &= -\frac{4}{3} \left[\chi_1 + \frac{1}{2} \chi_2 \right] \left(\frac{Eh^3}{9} - I_3 \right) - \left(\varepsilon_1 + \frac{1}{2} \varepsilon_2 \right) I_2; \\ M_2 &= -\frac{4}{3} \left[\chi_2 + \frac{1}{2} \chi_1 \right] \left(\frac{Eh^3}{9} - I_3 \right) - \left(\varepsilon_2 + \frac{1}{2} \varepsilon_1 \right) I_2; \\ \Delta H &= -\frac{2}{3} \left[\left(\frac{Eh^3}{9} - I_3 \right) \lambda - 2\gamma I_2 \right]. \end{aligned} \quad (1.6)$$

$$I_1 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\sigma_i}{\varepsilon_i} dz; \quad I_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\sigma_i}{\varepsilon_i} z dz; \quad I_3 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\sigma_i z^2}{\varepsilon_i} dz. \quad (1.7)$$

Integral ostiga kiruvchi deformatsiya intensivligi ifodasi quyidagi formuladan aniqlanadi:

$$\begin{aligned} \varepsilon_i &= \frac{2}{\sqrt{3}} \sqrt{P_\varepsilon - 2\varepsilon_1 P_\varepsilon + \varepsilon_2^2 P_\varepsilon}; & P_\varepsilon &= \varepsilon_1^2 + \varepsilon_1 \varepsilon_2 + \varepsilon_2^2 + \frac{1}{4} \gamma^2; \\ P_\varepsilon &= \varepsilon_1 \chi_1 + \varepsilon_2 \chi_2 + \frac{1}{2} (\varepsilon_2 \chi_1 + \varepsilon_1 \chi_2) + \frac{1}{2} \lambda \gamma; \\ P_\varepsilon &= \chi_1^2 + \chi_1 \chi_2 + \chi_2^2 + \frac{1}{4} \lambda^2. \end{aligned} \quad (1.8)$$

Bunda kuchlanish intensivligi bilan deformatsiya intensivligi orasidagi bog'lanish quyidagicha ifodalansin:

$$\sigma_i = E \varepsilon_i \left(1 - \lambda + \lambda \frac{\varepsilon_i}{\varepsilon_1} \right); \quad \lambda = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{\operatorname{tg} \alpha} = 1 - \frac{E_1}{E}. \quad (1.9)$$

Kuchlanish funksiyasini kiritamiz:

$$N_1 = -\frac{\partial^2 \varphi}{\partial x^2}; \quad N_2 = -\frac{\partial^2 \varphi}{\partial y^2}; \quad S = -\frac{\partial^2 \varphi}{\partial x \partial y}. \quad (1.10)$$

Unda yuqoridagi formulalardan foydalanib, birinchi paragrafdagi tenglamalarni φ , w noma'lumlarga nisbatan ikkita chiziqli bo'lmagan tenglamalar sistemasiga keltiramiz:

$$\begin{aligned} & \frac{1}{Eh} \nabla^2 \nabla^2 \varphi - K_2 \frac{\partial^2 w}{\partial x^2} - K_1 \frac{\partial^2 w}{\partial y^2} = \\ & = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + \Phi_1(x, y); \\ & - \left(K_2 \frac{\partial^2 w}{\partial x^2} + K_1 \frac{\partial^2 w}{\partial y^2} \right) + D \nabla^2 \nabla^2 w = \\ & = \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 \varphi}{\partial y^2} - 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 \varphi}{\partial y \partial x} + \\ & + q(x, y) + \Phi_2(x, y). \end{aligned} \quad (1.11)$$

Bu formulada quyidagi belgilashlar kiritilgan:

$$\begin{aligned} \Phi_1(x, y) &= \frac{1}{Eh} \left[\frac{\partial^2}{\partial y^2} \left(\frac{\Delta N_1 - \frac{1}{2} \Delta N_2}{\partial x^2} + \frac{\partial^2}{\partial x^2} \left(\frac{\Delta N_2 - \frac{1}{2} \Delta N_1}{\partial y^2} - 3 \frac{\partial^2 \Delta H}{\partial x \partial y} \right) \right) \right]; \\ \Phi_2(x, y) &= \frac{\partial^2 \Delta M_1}{\partial x^2} + \frac{\partial^2 \Delta M_2}{\partial y^2} + 2 \frac{\partial^2 \Delta H}{\partial x \partial y}; \\ D &= \frac{Eh^3}{9}. \end{aligned} \quad (1.12)$$

Bu tenglamalar sistemasini ketma-ket yaqinlashish metodining yangi varianti asosida yechamiz.

4.2. O'qqa simmetrik deformatsiyalanuvchi silindrik qobiqlarning elastik-plastik holati

Plastik deformatsiya nazariyasi doirasida silindrik qobiqning elastik-plastik muvozanat holatini ifodalovchi asosiy differensial tenglama quyidagi ko'rinishda bo'ladi:

$$\frac{d^4 w}{dx^4} + 4w = \frac{4R}{Eh} q + \frac{2R}{Eh} N_1 - \frac{4R}{E} \frac{\Delta N_2}{h} - \frac{6R}{E} \frac{d^2 \Delta M_1}{h^2 dx^2}. \quad (2.1)$$

Bu yerda:

$x = \beta s = \sqrt{3/2} Rh$ s — o'lchamsiz izlanayotgan koordinata bo'lib, qobiqning bo'ylama yo'nalishi s koordinatasiga proporsionaldir;

R , h — tegishli qobiq radiusi va qalinligi;

E — proporsionallik moduli;

q_z — qobiq sirtiga normal yo'nalgan yuk;

N_1 — bo'ylama kuch.

Funksiyalar ΔN_2 , ΔM_1 — tegishli halqa kuch va eguvchi momentning chiziqli bo'lmagan komponentlari bo'lib, ular quyidagi ifodalardan aniqlanadi:

$$\begin{aligned} \Delta N_2 &= (Eh - I_1) \varepsilon_2; \\ \Delta M_1 &= \left(\frac{Eh^3}{9} - \frac{4I_1 I_3 - I_2^2}{3I_1} \right) \chi - \frac{I_2}{I_1} N_1. \end{aligned} \quad (2.2)$$

$$\epsilon_2 = -\frac{w}{R}; \quad \chi = -\frac{3}{2Rh} \frac{d^2 w}{dx^2}. \quad (2.3)$$

$$I_1 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\sigma_I}{\epsilon_I} dz;$$

$$I_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\sigma_I}{\epsilon_I} z dz;$$

$$I_3 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\sigma_I}{\epsilon_I} z^2 dz;$$

$$\epsilon_I = \frac{2}{\sqrt{3}} \sqrt{\epsilon_1 + \epsilon_2 + 2\chi} (\epsilon_1 + 0.5\epsilon_2) + z^2 \chi^2. \quad (2.4)$$

Bu ifodalarda kuchlanish intensivligi σ_p deformatsiya ϵ_I intensivligining biron-bir funksiya ko'rinishida beriladi.

Halqa kuchni va eguvchi momentni elastik-plastik uchastkaga tegishli kesimdagi deformatsiyalar orqali quyidagicha ifodalaymiz:

$$N_1 = \frac{4}{3} [(\epsilon_1 + 0.5\epsilon_2) I_1 + \chi I_2];$$

$$N_2 = E h \epsilon_2 - \Delta N_2; \quad M_1 = \frac{E h^3}{9} \chi - \Delta M_1. \quad (2.5)$$

Chiziqli bo'lmagan asosiy muvozanat differensial tenglamaning yechimini quyidagi ko'rinishda ifodalaymiz:

$$w(x) = w(0) F_1(x) + \frac{1}{\beta} v(0) F_2(x) - \frac{6R}{E h^2} M_1(0) F_3(x) - \frac{6R}{E h^2 \beta} Q_1(0) F_4(x) - \frac{6R}{E h^2} Y(x) - \frac{4R}{E h} Z(x). \quad (2.6)$$

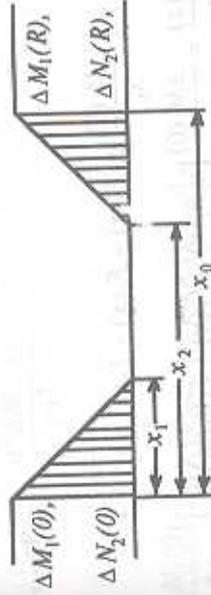
Bu yerda: F_1, F_2, F_3, F_4 — A.N. Krilov funksiyalari; $w(0), v(0), M_1(0), Q_1(0)$ — boshlang'ich parametrlar;

$$Y(x) = \int_0^x F_2(x, \alpha) \Delta M_1(\alpha) d\alpha;$$

$$Z(x) = \int_0^x F_4(x, \alpha) \Delta N_2(\alpha) d\alpha. \quad (2.7)$$

Masalani yechishda ketma-ket yaqinlashish usulining yangi usulidan foydalanamiz, ya'ni birinchi yaqinlashuvda eguvchi moment va zo'riqish kuchlarining chiziqli bo'lmagan komponentlarini elastik-plastik uchastkasi bo'yicha uchburchak qonuni asosida o'zgarish, deb faraz qilamiz. Chunki plastik deformatsiya elastik-plastik uchastka uzunligi bo'yicha uchburchak qonuni asosida tarqalishini tajribalar ham tasdiqlaydi (4.1 - chizma).

Bundan salqilik ifodasidagi $Y(x)$ va $Z(x)$ funksiyalar uchastkalar



4.1 - chizma. Eguvchi moment va halqa kuchlarining nochiq komponentlari o'zgarishi.

bo'yicha quyidagilarga teng bo'ladi:

a) birinchi uchastkada $0 \leq x \leq x_1$

$$Y(x) = \frac{\Delta M_1(0)}{x_1} [x_1 F_3(x) - F_4(x)];$$

$$\frac{dY(x)}{dx} = \frac{\Delta M_1(0)}{x_1} [x_1 F_2(x) - F_3(x)];$$

$$\frac{d^2 Y(x)}{dx^2} = \frac{\Delta M_1(0)}{x_1} [x_1 F_1(x) - F_2(x)];$$

$$\frac{d^3 Y(x)}{dx^3} = -\frac{\Delta M_1(0)}{x_1} [4x_1 F_4(x) + F_1(x) - 1] + \frac{d \Delta M_1(x)}{dx};$$

$$\frac{d^4 Y(x)}{dx^4} = -4Y(x) + \frac{d^2 \Delta M_1(x)}{dx^2};$$

$$\begin{aligned}
 Z(x) &= \frac{\Delta N_2(0)}{4x_1} [x_1 - x - x_1 F_1(x) + F_2(x)]; \\
 \frac{dZ(x)}{dx} &= \frac{\Delta N_2(0)}{4x_1} [4x_1 F_4(x) + F_1(x) - 1]; \\
 \frac{d^2 Z(x)}{dx^2} &= \frac{\Delta N_2(0)}{x_1} [x_1 F_3(x) - F_4(x)]; \\
 \frac{d^3 Z(x)}{dx^3} &= \frac{\Delta N_2(0)}{x_1} [x_1 F_2(x) - F_3(x)]; \\
 \frac{d^4 Z(x)}{dx^4} &= -4Z(x) + 4\Delta N_2(x);
 \end{aligned}
 \tag{2.8}$$

b) ikkinchi uchastkada $x_1 \leq x \leq x_0$

$$\begin{aligned}
 Y(x) &= \frac{\Delta M_1(0)}{x_1} [x_1 F_3(x) - F_4(x) + F_4(x, x_1)]; \\
 \frac{dY(x)}{dx} &= \frac{\Delta M_1(0)}{x_1} [x_1 F_2(x) - F_3(x) + F_3(x, x_1)]; \\
 \frac{d^2 Y(x)}{dx^2} &= \frac{\Delta M_1(0)}{x_1} [x_1 F_1(x) - F_2(x) + F_2(x, x_1)]; \\
 \frac{d^3 Y(x)}{dx^3} &= -\frac{\Delta M_1(0)}{x_1} [4x_1 F_4(x) + F_1(x) + F_1(x, x_1)]; \\
 \frac{d^4 Y(x)}{dx^4} &= -4Y(x); \\
 Z(x) &= \frac{\Delta N_2(0)}{4x_1} [-x_1 F_1(x) + F_2(x) - F_2(x, x_1)]; \\
 \frac{dZ(x)}{dx} &= \frac{\Delta N_2(0)}{4x_1} [4x_1 F_4(x) + F_1(x) - F_1(x, x_1)]; \\
 \frac{d^2 Z(x)}{dx^2} &= \frac{\Delta N_2(0)}{x_1} [x_1 F_3(x) - F_4(x) + F_4(x, x_1)]; \\
 \frac{d^3 Z(x)}{dx^3} &= \frac{\Delta N_2(0)}{x_1} [x_1 F_2(x) - F_3(x) + F_3(x, x_1)]; \\
 \frac{d^4 Z(x)}{dx^4} &= -4Z(x);
 \end{aligned}
 \tag{2.9}$$

d) uchinchi uchastkada $x_2 \leq x \leq x_0$

$$\begin{aligned}
 Y(x) &= \frac{\Delta M_1(0)}{x_1} [x_1 F_3(x) - F_4(x) + F_4(x, x_1)] + \frac{\Delta M_1(x_0)}{x_0 - x_2} F_4(x, x_2); \\
 \frac{dY(x)}{dx} &= \frac{\Delta M_1(0)}{x_1} [x_1 F_2(x) - F_3(x) + F_3(x, x_1)] + \frac{\Delta M_1(x_0)}{x_0 - x_2} F_5(x, x_2); \\
 \frac{d^2 Y(x)}{dx^2} &= \frac{\Delta M_1(0)}{x_1} [x_1 F_1(x) - F_2(x) + F_2(x, x_1)] + \frac{\Delta M_1(x_0)}{x_0 - x_2} F_2(x, x_2); \\
 \frac{d^3 Y(x)}{dx^3} &= -\frac{\Delta M_1(0)}{x_1} [4x_1 F_4(x) + F_1(x) + F_1(x, x_1)] + \\
 &+ \frac{\Delta M_1(x_0)}{x_0 - x_2} [F_4(x, x_2) - 1] + \frac{d\Delta M_1(x)}{dx}; \\
 \frac{d^4 Y(x)}{dx^4} &= -4Y(x) + \frac{d^2 \Delta M_1(x)}{dx^2}.
 \end{aligned}
 \tag{2.10}$$

$$\begin{aligned}
 Z(x) &= \frac{\Delta N_2(0)}{4x_1} [-x_1 F_1(x) + F_2(x) - F_2(x, x_1)] + \\
 &+ \frac{\Delta N_2(x_0)}{4(x_0 - x_2)} [x - x_2 - F_2(x, x_2)]; \\
 \frac{dZ(x)}{dx} &= \frac{\Delta N_2(0)}{4x_1} [4x_1 F_4(x) + F_1(x) - F_1(x, x_1)] + \frac{\Delta N_2(x_0)}{4(x_0 - x_2)} [1 - F_1(x, x_2)]; \\
 \frac{d^2 Z(x)}{dx^2} &= \frac{\Delta N_2(0)}{x_1} [x_1 F_3(x) - F_4(x) + F_4(x, x_1)] + \frac{\Delta N_2(x_0)}{(x_0 - x_2)} [F_4(x, x_2)]; \\
 \frac{d^3 Z(x)}{dx^3} &= \frac{\Delta N_2(0)}{x_1} [x_1 F_2(x) - F_3(x) + F_3(x, x_1)] + \frac{\Delta N_2(x_0)}{(x_0 - x_2)} [F_3(x, x_2)]; \\
 \frac{d^4 Z(x)}{dx^4} &= -4Z(x) + 4\Delta N_2(x_0).
 \end{aligned}$$

Agar silindrik qobiqning chap va o'ng qirralarida $M_1(0)$, $M_2(0)$ eguvchi moment va ko'ndalang kuchlar $Q_1(0) = Q_2(0)$ bo'lsa, unda bu chegara shartlariga ko'ra (2.6—2.10) formulalardan boshlang'ich parametrlarni aniqlaymiz (4.1 - chizma):

$$w(0) = \frac{3R}{2Eh^2} \frac{AF_5(x_0) - BF_4(x_0)}{F_3^2(x_0) - F_2(x_0)F_4(x_0)},$$

$$v(0) = \frac{3R\beta}{2Eh} \frac{BF_5(x_0) - AF_2(x_0)}{F_3^2(x_0) - F_2(x_0)F_4(x_0)}, \quad (2.11)$$

$$A = M_1(x_0) - \Delta M_1(x_0) - \Delta M_1(0)F_1(x_0) - \frac{1}{\beta}Q_1(0)F_2(x_0) - \frac{\Delta M_1(0)}{x_1} [x_1F_1(x_0) - F_2(x_0) + F_2(x_0, x_1)] - \frac{\Delta M_1(x_0)}{x_0 - x_2} F_2(x_0, x_1) - \frac{2}{3} \frac{\Delta N_2(0)}{h} \frac{1}{x_1} [x_1F_3(x_0) - F_4(x_0) + F_4(x_0, x_1)] - \frac{2}{3} h \frac{\Delta N_2(x_0)}{x_0 - x_2} F_4(x_0, x_2); \quad (2.12)$$

$$B = \frac{1}{\beta}Q_1(0) + 4\Delta M_1(0)F_4(x_0) - \frac{1}{\beta}Q_1(0)F_1(x_0) - \frac{\Delta M_1(0)}{x_1} [F_1(x_0, x_1) - F_1(x_0) - 4x_1F_4(x_0)] - \frac{\Delta M_1(x_0)}{x_0 - x_2} [F_1(x_0, x_2) - 1] - \frac{2}{3} h \frac{\Delta N_2(0)}{x_1} [x_1F_2(x_0) - F_3(x_0) + F_3(x_0, x_1)] - \frac{2}{3} h \frac{\Delta N_2(x_0)}{x_0 - x_2} F_3(x_0, x_2). \quad (2.13)$$

Agar silindr yetarlicha uzun bo'lsa, boshlang'ich parametrlar (2.11) soddalashtirib, quyidagicha ifodalalanadi:

$$w(0) = \frac{3R}{Eh^2} \left\{ \frac{DM_1(0) + \frac{1}{\beta}Q_1(0) + DM_1(0) \left[1 - \frac{q(x_1)}{x_1} \right] + \frac{h}{3} DN_2(0) \left[1 - \frac{x(x_1)}{x_1} \right]}{DM_1(0) + \frac{1}{\beta}Q_1(0) + DM_1(0) \left[1 - \frac{q(x_1)}{x_1} \right] + \frac{h}{3} DN_2(0) \left[1 - \frac{x(x_1)}{x_1} \right]} \right\};$$

$$e^{-x_1} \cos x_1 = \theta(x_1);$$

$$e^{-x_1} \sin x_1 = \zeta(x_1).$$

$$\varphi(x_1) = \theta(x_1) + \varepsilon(x_1);$$

$$\psi(x_1) = \theta(x_1) - \zeta(x_1).$$

$$v(0) = \frac{6R\beta}{Eh^2} \left[M_1(0) + \frac{1}{2\beta}Q_1(0) - \frac{1}{2}\Delta M_1(0) \frac{1 - \varphi(x_1)}{x_1} + \frac{h}{6}\Delta N_2(0) \frac{1 - \varphi(x_1)}{x_1} \right]. \quad (2.14)$$

Unda salqilik funksiyasi (2.6) ifodasini 2.8–2.12 formulalarni e'tiborga olib, ba'zi bir almashtirishlardan keyin uzun silindrik qobiq uchun quyidagi ko'rinishga keltiramiz:

$$w(x) = -\frac{3R}{Eh^2} \left\{ \frac{1}{\beta}Q_1(0)\theta(x) + \Delta M_1(0)h_0(x_1, x) \right\} + h\Delta N_2(0)c_0(x_1, x) + \frac{R}{2Eh}N_1. \quad (2.15)$$

Bu yerda $0 \leq x \leq x_1$ bo'lganda:

$$h_0(x_1, x) = \frac{1}{4x_1} \{ 2[2 - \theta(x_1)]\theta(x) - \psi(x_1)\psi(x) - \varphi(x_1) - \varphi(x) \};$$

$$c_0(x_1, x) = \frac{1}{12x_1} \{ 4(x_1 - x) - \varphi(x_1)\psi(x) - 2\xi(x_1)\theta(x) + \psi(x_1)\theta(x) - \varphi(x_1) - \varphi(x) \}. \quad (2.16)$$

$x > x_1$ bo'lganda:

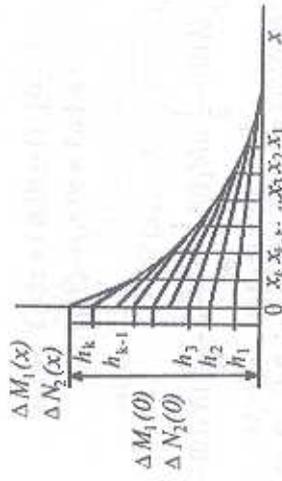
$$h_0(x_1, x) = \frac{1}{4x_1} \{ 2[2 - \theta(x_1)]\theta(x) - \psi(x_1)\psi(x) - \varphi(x - x_1) \};$$

$$c_0(x_1, x) = \frac{1}{12x_1} \{ -\varphi(x_1)\psi(x) - 2\xi(x_1)\theta(x) + \psi(x - x_1) \}. \quad (2.17)$$

Salqilik funksiyasi ifodasi $w(x)$ ni aniqlash mumkin bo'ladi, qachonki agar $\Delta M_1(x)$, $\Delta N_2(x)$ chiziqli bo'lmagan komponentlar elastik-plastik soha x_1 uzunligi bo'yicha uchburchak qonuni asosida o'zgarsa.

Masalani yechish jarayonida aniqlanishi lozim bo'lgan va chiziqli bo'lmagan komponentlar keyingi yaqinlashuvlarda siniq chiziqli funksiya ko'rinishida approksimatsiyalanib har biri n uchburchakdan iborat bo'ladi va quyidagi ko'rinishda bo'ladi (4.2- chizma).

Masalani bunday holda qaraganda $b_0(x_1, x)$, $c_0(x_1, x)$ funksiyalarni tegishlicha $f_0(x_1, x)$, $g_0(x_1, x)$ funksiyalar bilan almashtirish lozim:



4.2- chizma. Eguvchi moment va bo'ylama kuchlarning ikkinchi yaqinlashuvda o'zgarish qonuni.

$$f_0(x_1, x) = \frac{1}{\Delta M_1(0)} \sum_{k=1}^n h_k b_0(x_k, x);$$

$$g_0(x_1, x) = \frac{1}{\Delta N_2(0)} \sum_{k=1}^n h_k c_0(x_k, x). \quad (2.18)$$

Uzun yupqa silindrik qobiqning elastik-plastik kuchlanganlik deformatsiyalanganlik holatini hisoblash uchun boshlang'ich kesim-dagi xarakterli deformatsiya intensivligi $\varepsilon_0, \varepsilon_2, \varepsilon_0$ miqdorlarini hamda elastik-plastik uchastka uzunligi x_1 ni aniqlash zarur.

Demak, bu $\varepsilon_1, \varepsilon_2, \varepsilon_0$ va x_1 parametrlarni aniqlash uchun quyidagi to'rtta chiziqli bo'lmagan algebraik tenglamalar sistemasi mavjud:

$$N_1(0) = f_1[\varepsilon_1(0), \varepsilon_2(0), \varepsilon_0(0)];$$

$$M_1(0) = f_2[\varepsilon_1(0), \varepsilon_2(0), \varepsilon_0(0)];$$

$$\varepsilon_{10}(0) = \left| \frac{w(0)}{R} \right| = \frac{3R}{Eh^2} \left\{ [M_1(0) + \Delta M_1(0)] + \frac{1}{\beta} Q_1(0) - \Delta M_1(0) b_0(x_1, 0) + h \Delta N_2(0) c_0(x_1, 0) \right\} + \frac{R}{2Eh} N_1(0). \quad (2.19)$$

$$\frac{2}{\sqrt{3}} \sqrt{\varepsilon_1^2(x_1) + \varepsilon_2(x_1) \varepsilon_0(x_1) + \lambda \chi(0) \left[\varepsilon_1(x_1) + \frac{1}{2} \varepsilon_2(x_1) \right] + \frac{\lambda^2}{4} \chi^2(x_1)} = \varepsilon_T.$$

Bu yerda ε_T — oquvchanlik chegarasiga teng bo'lgan deformatsiya. Bu tenglamalar sistemasining birinchi ikkita σ kuchlanish intensivligi bilan ε deformatsiya intensivligi orasidagi bog'lanish berilgandan keyin tuziladi. Bu bog'lanishdan A.A. Ilyushin tomonidan

kashf etilgan bikiirikliklar aniqlanadi. Bo'ylama kuch $N_1=0$ bo'lsa, chiziqli bo'lmagan algebraik tenglamalar sistemasi (2.19) uchtagacha kamayadi va deformatsiya intensivligi qobiq ko'ndalang kesimi balandligi bo'yicha simmetrik taqsimlanadi. Shu sababli birinchi holatda qobiq balandligi bo'yicha qirrasiga yaqin bo'lgan tolalarida ikkita bir xil plastik soha hosil bo'ladi va markaziy tolalari atrofida esa elastik soha bo'ladi. Ikkinchi holatda esa qobiq qalinligi bo'yicha plastik soha bo'ladi.

Halqa kuch va eguvchi momentni elastik-plastik uchastkaga tegishli kesim dagi deformatsiya intensivligining maksimal va minimal qiymatlari orqali quyidagicha ifodalaymiz:

$$m_1(x) = \frac{\sqrt{3}}{6} \left[\frac{2(1-\lambda)\sqrt{1-\gamma^2}}{3} + \frac{\lambda}{\sqrt{1-\gamma^2}} \left(1 - \frac{\gamma^2}{\sqrt{1-\gamma^2}} \ln \Omega \right) \frac{\gamma \alpha^2 \sqrt{1-\beta^2}}{1-\gamma^2} \frac{1+2\beta^2}{3} \right];$$

$$n_1(x) = \beta(1-\lambda) + \lambda \frac{\gamma}{\sqrt{1-\gamma^2}} \left(\sqrt{1-\beta^2} + \ln \Omega \right);$$

$$\Delta m_1(x) = \frac{\sqrt{3}}{9} \frac{\sqrt{1-\gamma^2}}{\alpha} - m_1(x); \quad \Delta n_1(x) = \beta - n_1(x);$$

$$\Omega = \frac{1 + \sqrt{1-\gamma^2}}{1 + \sqrt{1-\beta^2}} \frac{\beta}{\gamma}; \quad \gamma = \alpha\beta = \frac{\varepsilon_{10}}{\varepsilon_{11}}. \quad (2.20)$$

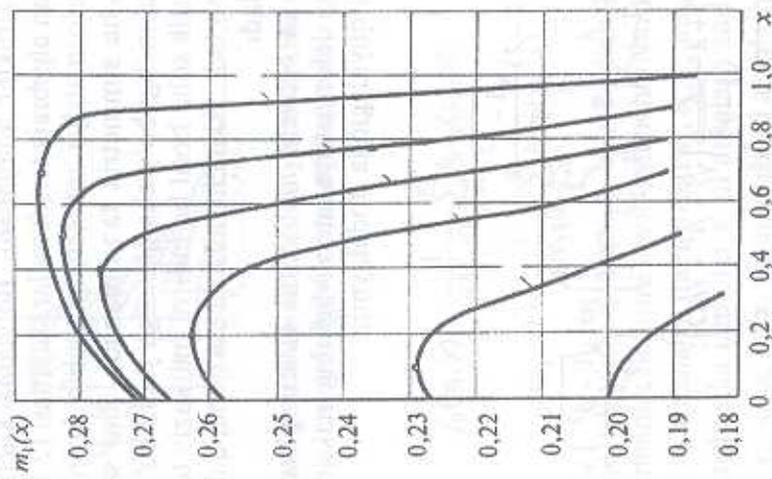
$\beta > 1$

$$m_1(x) = \frac{\sqrt{3}}{6} \left[\frac{2}{3} (1-\lambda) \frac{\sqrt{1-\gamma^2}}{\alpha} + \frac{\lambda}{\sqrt{1-\gamma^2}} \left(1 - \frac{\gamma^2}{\sqrt{1-\gamma^2}} \ln \Omega \right) \right];$$

$$n_1(x) = \beta(1-\lambda) + \lambda \frac{\gamma}{\sqrt{1-\gamma^2}} (\ln \Omega);$$

$$\Delta m_1(x) = \frac{\sqrt{3}}{9} \frac{\sqrt{1-\gamma^2}}{\alpha} - m_1(x); \quad \Delta n_1(x) = \beta - n_1(x);$$

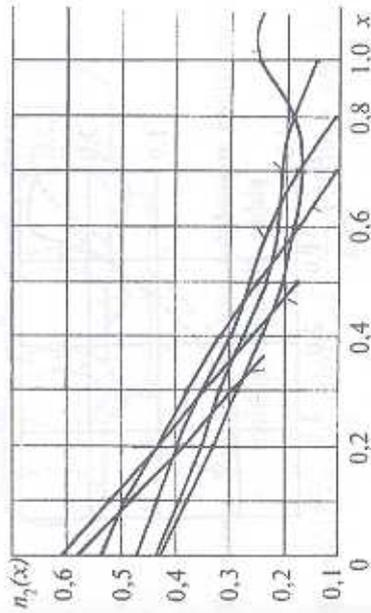
$$\Omega = \frac{1 + \sqrt{1-\gamma^2}}{\gamma}; \quad \gamma = \alpha\beta = \frac{\varepsilon_{10}}{\varepsilon_{11}}. \quad (2.21)$$



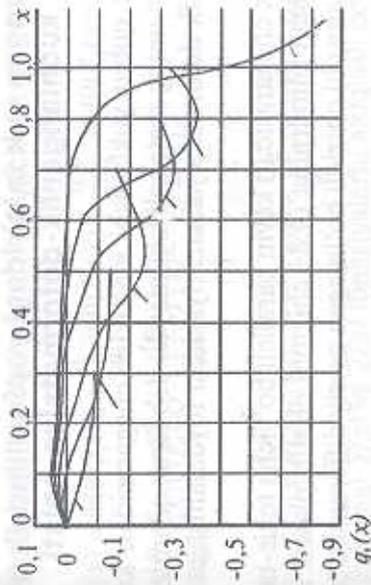
4.3- chizma. Bo'ylama eguvchi momentning qirg'och effekti sohasida elastik-plastik uchastka uzunligidan o'zgarish grafigi.

Bu 2.20—2.21 bog'lanishlar miqdori $0 \leq \lambda \leq 1$ o'zgaruvchi parametr λ materialning chiziqi puxtalanishini hisobga oladi. Agar λ parametri bir birlikka yaqinlashsa ham unda u yoki bu masalalarni yechish mumkin bo'ladi. Shuning uchun ham yechayotgan masalamizda ideal-plastik model qaralgan, ya'ni $\lambda=1$. Ideal-plastik jism darajali puxtalanish $\sigma_1 - \epsilon_1$ diagrammasining xususiy holidir va shuning uchun ham A.A.Ilyushinning jism oddiy yuklanish teoremasiga zid emas.

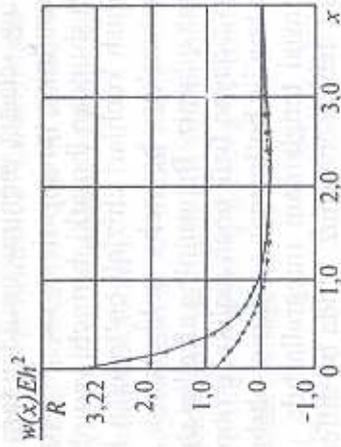
Hisoblash natijalari 4.3—4.5 chizmalarda qirg'och effekt zonasida bo'ylama eguvchi moment, halqa kuch va ko'ndalang kuchlarning elastik-plastik zonasining turli uzunliklari uchun keltirilgan 4.5—5.7 chizmalarda bo'ylama eguvchi moment va nisbiy salqilikning silindr bo'ylama yo'nalishi bo'yicha elastik-plastik zonada $\lambda=0,9$ bo'lgandagi o'zgarish grafigi keltirilgan.



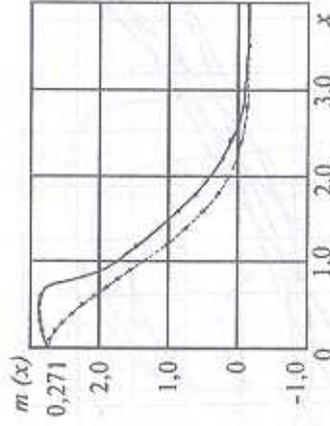
4.4- chizma. Halqa kuchning qirg'och effekti sohasida elastik-plastik uchastka uzunligidan o'zgarish grafigi.



4.5- chizma. Ko'ndalang kuchning qirg'och effekti sohasida elastik-plastik uchastka uzunligidan o'zgarish grafigi.



4.6- chizma. Nisbiy salqilikning qobiq yasovchisi bo'ylab elastik-plastik uchastka uzunligi 0,9 bo'lganda o'zgarish grafigi.



4.7- *chizma*. Bo'ylama eguvchi momentning qobiq yasovchisi bo'ylab elastik-plastik uchastka uzunligi 0,9 bo'lganda o'zgarish grafiqi.

4.3. Elastiklik chegarasidan keyin paralleli bo'yicha to'plangan yuk ta'siridagi yopiq silindirik qobiq kuchlanganlik-deformatsiya holati

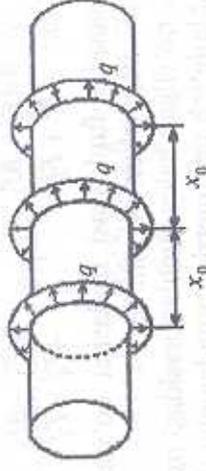
Silindirik qobiq yuklanishi elastiklik chegarasidan keyin oddiy yuklangan va uning materiali siqilmaydi, deb faraz qilamiz, bu kichik elastik-plastik deformatsiya nazariyasidan foydalanishga imkoniyat beradi.

Elastiklik chegarasidan keyin paralleli bo'yicha tekis-taralgan yuk bilan yuklangan silindrimning (4.8- *chizma*) elastik-plastik muvozanat holatini ifodalovchi chiziqli bo'lmagan asosiy differensial tenglamani:

$$\frac{d^4 w}{dx^4} + 4w = -\frac{6R}{Eh^2} \frac{d^4 \Delta M_1}{dx^4} - \frac{4R}{Eh} \frac{\Delta N_2}{\Delta x} \quad (3.1)$$

ni A.A. Ilyushinning «elastik yechim» usulidan farq qiluvchi ketma-ket yaqinlashish usulining yangi varianti asosida yechamiz. Bu yangi usulning mohiyati shundan iboratki, birinchi yaqinlashuvda eguvchi moment va zo'riqish kuchlari chiziqli bo'lmagan komponentlarini elastik-plastik uchastkasi bo'yicha uchburchak qonuni asosida o'zgarsin, deb faraz qilamiz. Bu usulning afzalligi shundaki, birinchi yaqinlashuvning o'zidayoq fizik nochiziqlikni e'tiborga olib, elastik-chiziqli yechimga bog'liq bo'lmagan yechimni olishga imkon beradi.

Asosiy differensial tenglamani integrallashda o'ng tomondagi funksiyani ma'lum deb hisoblaymiz. Unda bu differensial tenglama elastik tayanchda yotuvchi balka tenglamasiga to'g'ri keladi, uning umumiy yechimi quyidagicha:



4.8- *chizma*. Paralleli bo'yicha to'plangan radial bosim ostidagi silindirik qobiq.

$$w(x) = w(0)F_1(x) + \frac{1}{\beta} v(0)F_2(x) - \frac{6R}{Eh^2} (M_1(0) + \Delta M_1(0)) \cdot$$

$$F_3(x) - \frac{6R}{Eh^2 \beta} \left(Q_1(0) + \beta \frac{d\Delta M_1(0)}{dx} \right) F_4(x) + \bar{w}_1(x) + \bar{w}_2(x). \quad (3.2)$$

Bu salqilik ifodasidagi xususiy yechimni Koshi integrali shaklida izlaymiz. Unda salqilik ifodasini quyidagicha yozish mumkin:

$$w(x) = w(0)F_1(x) + \frac{1}{\beta_0} v(0)F_2(x) - \frac{6R}{Eh^2} M_1(0)F_3(x) - \frac{6R}{Eh^2 \beta_0} Q_1(0)F_4(x) - \frac{6R}{Eh^2} Y(x) - \frac{4\beta}{Eh} Z(x); \quad (3.3)$$

$$Y(x) = \int_0^x F_2(x, \alpha) \Delta M_1(\alpha) d\alpha; \quad Z(x) = \int_0^x F_4(x, \alpha) \Delta N_2(\alpha) d\alpha.$$

Bu ifodadagi $Y(x)$ va $Z(x)$ funksiyalarni aniqlash uchun integraldagi $\Delta N_2(x)$ funksiya ifodalarini bilish lozim. Integral ostidagi ifodalar ketma-ket yaqinlashish usuli bilan aniqlanadi, chunki ular izlanayotgan salqilik funksiyasi bilan murakkab bog'lanishda.

Birinchi taqribiy yaqinlashishda qaralayotgan masalada $\Delta N_2(x)$ $\Delta M_1(x)$ funksiyalarni uchburchak qonuni asosida o'zgarsin deb qabul qilamiz. Unda $Y(x)$ va $Z(x)$ funksiyalar va uning differensiallari (2.9—2.10) formulalardan aniqlanadi.

Elastiklik chegarasidan keyin radial halqa kuch ta'siridagi yopiq silindirik qobiq salqiligini, zo'riqish kuchini va eguvchi momentlarini aniqlash uchun uning uzunligi x_0 ga teng bo'lgan biror uchastkasini qaraymiz. Qobiqning chegara shartlari:

$$Q(0) = Q(x_0) = q/2 \quad \text{va} \quad v(0) = v(x_0) = 0. \quad (3.4)$$

Chegara shartlaridan hosil bo'lgan tenglamalardan boshlang'ich parametrlarni quyidagicha ifodalaymiz:

$$W(0) = \frac{3R}{2EH^2} \frac{4AF_4 + BF_2}{4F_4^2 + F_2^2}; \quad M_1(0) = \frac{AF_2 - BF_4}{4F_4^2 + F_2^2} - \Delta M_1. \quad (3.5)$$

Bu ifodani salqilik funksiyasi ifodasiga qo'ysak, masalaning yechimini umumiy hol uchun olamiz.

Agar silindrga qo'yilgan halqa kuchlar orasidagi masofa yetarlicha uzoq bo'lsa, boshlang'ich parametrlar quyidagicha ifodalanadi:

$$w(0) = \frac{3R}{2EH^2} \left[\frac{\Delta M_1(0)}{x_1} \frac{1 - \varphi(x_1)}{x_1} + \frac{2h}{3} \Delta N_2(0) \frac{1 - \psi(x_1)}{x_1} + \frac{q}{2\beta} - \frac{2h}{3} \Delta N_2(0); \right]$$

$$M_1(0) = \frac{1}{2} \left[\Delta M_1(0) \frac{1 - \varphi(x_1)}{x_1} - \frac{1}{3} \Delta N_2(0) \frac{1 - \varphi(x_1)}{x_1} - \frac{q}{2\beta} \right] - \Delta M_1(0). \quad (3.6)$$

Bularni e'tiborga olib, salqilik tenglamasini va uning hosilalarini quyidagicha ifodalaymiz:

1) birinchi uchastkada $0 \leq x \leq x_1$:

$$w(x) = -\frac{3R}{4EH^2} \left[\frac{q}{\beta} \varphi(x) - \Delta M_1(0) \frac{\Phi_0(x_1, x)}{x_1} - \frac{h}{3} \Delta N_2(0) \frac{\Psi_0(x_1, x)}{x_1} + \frac{4}{3} h \Delta N_2(0) \left[1 - \frac{x}{x_1} \right] \right]. \quad (3.7)$$

$$\frac{dw(x)}{dx} = \frac{3R}{4EH^2} \left[\frac{2q}{\beta} \zeta(x) + \Delta M_1(0) \frac{\Phi_1(x_1, x)}{x_1} - \frac{h}{3} \Delta N_2(0) \frac{\Psi_1(x_1, x)}{x_1} + \frac{4}{3} h \Delta N_2(0) \right]. \quad (3.8)$$

$$w(0) \frac{d^2 w(x)}{dx^2} = \frac{3R}{2EH^2} \left[\frac{q}{\beta} \psi(x) - \Delta M_1(0) \frac{\Psi_0(x_1, x)}{x_1} + \frac{h}{3} \Delta N_2(0) \frac{\Phi_0(x_1, x)}{x_1} + \frac{\lambda}{3} \Delta N_2(0) \frac{\Phi_0(x_1, x)}{x} \right]. \quad (3.9)$$

$$\frac{d^3 w(x)}{dx^3} = \frac{3R}{2EH^2} \left[-\frac{2q}{\beta} \theta(x) + \Delta M_1(0) \frac{\Psi_1(x_1, x)}{x_1} + \frac{h}{3} \Delta N_2(0) \frac{\Phi_1(x_1, x)}{x_1} \right]. \quad (3.10)$$

Bu yerda:

$$\Phi_0(x_1, x) = 2\varphi(x) - \varphi(x_1, x) - \varphi(x_1)\theta(x) - \psi(x_1)\zeta(x). \quad (3.11)$$

$$\Psi_0(x_1, x) = 2\psi(x) - \psi(x_1, x) - \psi(x_1)\theta(x) - \varphi(x_1)\zeta(x). \quad (3.12)$$

$$\Phi_1(x_1, x) = 4\zeta(x) - \zeta(x_1, x) - \varphi(x_1)\varphi(x) - \psi(x_1)\psi(x). \quad (3.13)$$

$$\Psi_1(x_1, x) = 4\theta(x) + 2\theta(x_1, x) - \psi(x_1)\varphi(x) - \varphi(x_1)\psi(x). \quad (3.14)$$

2) ikkinchi uchastka $x \geq x_1$.

Ikkinchi uchastkada (3.7)–(3.10) formulalarga o'xshash ifodani olish uchun quyidagi o'zgartirishlar kiritilishi lozim. (3.7) formuladagi oxirgi hadni tashlab yuborib, (3.11)–(3.14) formuladagi funksiyalarni $\theta(x, x)$, $\zeta(x, x)$ tegishlicha funksiyalar $\theta(x, x_1)$, $\zeta(x, x_1)$ bilan almashtirish kerak.

Salqilik funksiyasi uchun olingan $w(x)$ ifoda egrilik $\chi(x)$, bo'ylama eguvchi moment $M_1(x)$ va ko'ndalang kuch $Q_1(x)$ ifodalarini quyidagicha yozishga imkon beradi:

1) birinchi uchastka $0 \leq x \leq x_1$:

$$\chi_1(x) = -\frac{d^2 w}{ds^2}(s) = -\beta^2 \frac{d^2 w}{dx^2}(x) = \frac{3}{2Rh} \frac{d^2 w}{dx^2}(x) = -\frac{9R}{4EH^3} \left[\frac{q}{\beta} \psi(x) - \Delta M_1(0) \frac{\Psi_0(x_1, x)}{x_1} + \frac{h}{3} \Delta N_2(0) \frac{\Phi_0(x_1, x)}{x_1} \right]. \quad (3.15)$$

$$M_1(x) = \frac{EH^3}{9} \chi_1(x) - \Delta M_1(x) = -\frac{1}{4} \left[\frac{q}{\beta} \psi(x) - \Delta M_1(0) \frac{\Psi_0(x_1, x)}{x_1} + \frac{h}{3} \Delta N_2(0) \frac{\Phi_0(x_1, x)}{x_1} \right] - \Delta M_1(x) \quad (3.16)$$

$$Q_1(x) = \frac{dM_1(s)}{ds} = \beta \frac{dM_1(x)}{dx} = -\frac{\beta}{4} \left[\frac{2q}{\beta} \theta(x) - \Delta M_1(0) \left(\frac{\Psi_1(x_1, x)}{x_1} - \frac{1}{x_1} \right) - \frac{h}{3} \Delta N_2(0) \frac{\Phi_1(x_1, x)}{x_1} \right] \quad (3.17)$$

II) ikkinchi uchastkada $x \gg x_1$:

$$x_1(x) = -\frac{9R}{4EH^3} \left[\frac{q}{\beta} \psi(x) - \Delta M_1(0) \frac{\Psi_0(x, x_1)}{x_1} + \frac{h}{3} \Delta N_2(0) \frac{\Phi_0(x, x_1)}{x_1} \right] \quad (3.18)$$

$$M_1(x) = -\frac{1}{4} \left[\frac{q}{\beta} \psi(x) - \Delta M_1(0) \frac{\Psi_0(x, x_1)}{x_1} + \frac{h}{3} \Delta N_2(0) \frac{\Phi_0(x, x_1)}{x_1} \right] - \Delta M_1(x) \quad (3.19)$$

$$Q_1(x) = \frac{\beta}{4} \left[\frac{2q}{\beta} \theta(x) - \Delta M_1(0) \frac{\Psi_1(x, x_1)}{x_1} - \frac{h}{3} \Delta N_2(0) \frac{\Phi_1(x, x_1)}{x_1} \right] \quad (3.20)$$

Halqa kuchni va eguvchi momentni elastik-plastik uchastkaga tegishli kesimdagi deformatsiya intensivligining maksimal va minimal qiymatlari orqali quyidagicha ifodalaymiz:

$$N_2 = Eh \left[1 - \lambda \left(1 - \frac{\mu_0}{\mu_1} \right) - \epsilon n \Omega \right] \quad \Delta N_2 = Eh \lambda \left[\left(1 - \frac{\mu_0}{\mu_1} \right) - \frac{\epsilon n \Omega}{\mu_1} \right]$$

$$M_1 = -\frac{\sqrt{3}}{9} Eh^2 \left\{ 1 - \lambda \left[\left(1 - \left(\frac{\mu_0}{\mu_1} \right)^3 \right) - \frac{3}{2} \frac{\epsilon_T}{\mu_1} \left(\frac{\epsilon_{11}}{\mu_1} - \frac{\mu_0 \epsilon_T}{\mu_1^2} - \frac{\epsilon_{10}^2}{\mu_1^2} \right) \right] \right\} \mu_1$$

$$M_1 = -\frac{\sqrt{3}}{9} Eh^2 \lambda \left\{ \left[1 - \left(\frac{\mu_0}{\mu_1} \right) \right] - \frac{3}{2} \frac{\epsilon_T}{\mu_1} \left[\frac{\epsilon_{11}}{\mu_1} - \frac{\mu_0 \epsilon_T}{\mu_1^2} - \frac{\epsilon_{10}^2}{\mu_1^2} \right] \right\} \mu_1 \quad (3.21)$$

Shunday qilib, qaralayotgan qobiqning kuchlanganlik-deformatsiya holatini aniqlovchi chiziqli bo'lgan bog'lanishlar $w, v,$

Q, M (3.7—3.21) sistemasidagi uchta noma'lum parametrlarni, ya'ni elastik-plastik zona uzunligi x_1 va o'z navbatida qaralayotgan kesimdagi deformatsiya intensivligining maksimal va minimal qiymatlari orqali ifodalanuvchi $\Delta N_2(x), \Delta M_1(x)$ halqa kuchi va eguvchi momentlarning chiziqli bo'lgan komponentlarini aniqlashga keltiriladi. Demak, kuchning berilgan qiymatida x_1 , parametrlarni aniqlash uchun uchta tenglama tuzish lozim. Ular quyidagilardir:

1- tenglama boshlang'ich $x=0$ kesimdagi salqilik funksiyasini silindr radius R ga bo'lganda deformatsiya intensivligining minimal qiymatiga tengligidan tuziladi:

$$\epsilon_{10}(0) = \epsilon_2(0) = -\frac{w(0)}{R} = \frac{3}{4EH^2} \left[\frac{q - \Delta M_1(0)}{\beta} \frac{\Phi_0(x_1, 0)}{x_1} - \frac{h}{3} \Delta N_2(0) \frac{\Psi_0(x_1, 0)}{x_1} - \frac{4h}{3} \Delta N_2(0) \right] \quad (3.22)$$

2- tenglama boshlang'ich $x=0$ kesimda deformatsiya intensivligining maksimal qiymati bilan shu kesimdagi deformatsiya intensivligining minimal qiymati va egrilik deformatsiyasi qiymatlari orasidagi bog'lanishdan tuziladi:

$$\epsilon_{11}(0) = \sqrt{\epsilon_{10}^2(0) + [h x_1(0)]^2} / 3. \quad (3.23)$$

Bu yerda:

$$x_1(0) = -\frac{3}{4EH^3} \left[\frac{q}{\beta} - \Delta M_1(0) \frac{\Phi_0(x_1, 0)}{x_1} + \frac{h}{3} \Delta N_2(0) \frac{\Psi_0(x_1, 0)}{x_1} \right]$$

3- tenglama elastik-plastik uchastkaning $x=x_1$ kesimidagi deformatsiya intensivligining maksimal qiymati oquvchanlik deformatsiyasiga tengligidan tuziladi:

$$\epsilon_{11}(x_1) = \sqrt{\epsilon_{10}^2(x_1) + [h x_1(x_1)]^2} / 3 = \epsilon_T. \quad (3.24)$$

Bu yerda:

$$\epsilon_{10}(x_1) = -\frac{3}{4EH^2} \left[\frac{q}{\beta} \varphi(x_1) - \Delta M_1(0) \frac{\Phi_0(x_1, x_1)}{x_1} - \frac{h}{3} \Delta N_2(0) \frac{\Psi_0(x_1, x_1)}{x_1} \right]$$

$$x_1(x_1) = -\frac{9}{4EH^3} \left[\frac{q}{\beta} \psi(x) - \Delta M_1(0) \frac{\Psi_0(x_1, x_1)}{x_1} + \frac{h}{3} \Delta N_2(0) \frac{\Phi_0(x_1, x_1)}{x_1} \right]$$

Bu tenglamalar sistemasini yechishda kuch miqdori P dan ko'ra elastik-plastik uchastkada uzunligi x_1 qiymatini berish maqsadga muvofiqdir, unda bu sistemadagi ancha murakkab giperbola-trigo-nometrik funksiya orqali ifodalanuvchi hadlar birdaniga aniq bo'lib qoladi, ya'ni kuch miqdorini va boshlang'ich kesimdagi deformatsiya intensivligining maksimal va minimal qiymatlarini aniqlashga to'g'ri keladi.

Qo'biq materialini Prandtl diagrammasi bilan aniqlanuvchi ideal plastik modelini qabul qilamiz.

Bunda (3.23)–(3.24) tenglamalar sistemasidan kuchni yo'qotish yo'li bilan γ va β parametrlarga nisbatan quyidagi ikkita chiziqli bo'lmagan algebraik tenglamaga kelamiz:

$$\beta = \frac{\lambda}{4r_1} \left\{ \frac{4}{3} r_3 + \frac{1}{3} (\beta - r_3) [f_0(x_1, 0) + g_2(x_1, 0)] + (r_2 - \beta r_1) [f_0(x_1, 0) - g_2(x_1, 0)] \right\} + \frac{1 - \lambda}{3r_1} \beta. \quad (3.25)$$

$$\sqrt{\beta^2(x_1, x_1) + [hr_{\lambda 1}(x_1, x_1)]^2} / \beta = 1. \quad (3.26)$$

Bu yerda:

$$\beta(x_1, x_1) = \frac{3}{4} \lambda \left\{ \frac{4}{3} r_3 \varphi(x_1) + (\beta r_1 - r_2) [f_0(x_1, x_1) - f_0(x_1, 0)] \varphi(x_1) \right\} - \frac{1}{3} (\beta - r_3) [g_2(x_1, x_1) - g_2(x_1, 0)] \varphi(x_1) - (1 - \lambda) \beta \varphi(x_1). \quad (3.27)$$

$$hr_{\lambda 1}(x_1, x_1) = -\frac{9\lambda}{4} \left\{ \frac{4r_3 \psi(x_1)}{3} + (\beta r_1 - r_2) [g_0(x_1, x_1) - f_0(x_1, 0)] \psi(x_1) \right\} + \frac{1}{3} (\beta - r_3) [f_2(x_1, x_1) + g_2(x_1, 0)] \psi(x_1) - \frac{1}{3} (1 - \lambda) \beta \psi(x_1). \quad (3.28)$$

O'ltchovsiz birlikka o'tamiz:

$$\beta = \frac{\varepsilon_{10}}{\varepsilon_T}; \quad \alpha = \frac{\varepsilon_T}{\varepsilon_{11}}; \quad \gamma = \alpha \beta = \frac{\varepsilon_{10}}{\varepsilon_{11}}.$$

$$P = \frac{q}{Eh^2 \varepsilon_T \beta}; \quad n_2 = \frac{N_2}{Eh \varepsilon_T};$$

$$m_1 = \frac{M_1}{Eh^2 \varepsilon_T}; \quad \Delta n_2 = \frac{\Delta N_2}{Eh \varepsilon_T};$$

$$\Delta m_1 = \frac{\Delta M_1}{Eh^2 \varepsilon_T}. \quad (3.29)$$

$$m_1 = -r_1 \beta (1 - \lambda) - \lambda r_2; \quad n_2 = \beta (1 - \lambda) + \lambda r_3. \quad (3.30)$$

$$\Delta m_1 = \lambda (1 - r_1 \beta + r_2). \quad (3.31)$$

$$\Delta n_2 = \lambda (\beta - r_3). \quad (3.32)$$

$$\frac{l_1}{Eh} = 1 - \lambda + \lambda \frac{r_3}{\beta}; \quad \frac{l}{Eh^3} = \frac{1 - \lambda}{12} + \frac{\lambda}{12} \frac{r_2}{\beta r_1}. \quad (3.33)$$

Bu ifodalardagi r_1, r_2, r_3 funksiyalar quyidagi formulalardan aniqlanadi:

$$a) \quad \beta = \frac{\varepsilon_{10}}{\varepsilon_T} \leq 1; \quad r_1 = \frac{\sqrt{3} \sqrt{1 - \gamma^2}}{\gamma}. \quad (3.34)$$

$$r_2 = \frac{\sqrt{3}}{6} \frac{\gamma^2}{1 - \gamma^2} \left[\frac{\sqrt{1 - \gamma^2}}{\gamma^2} - \ell n \Omega - \frac{(1 - 2\beta^2) \sqrt{1 - \beta^2}}{3\beta^2} \right]. \quad (3.35)$$

$$r_3 = \frac{\gamma}{1 - \gamma^2} \left[\sqrt{1 - \beta^2} + \ell n \Omega \right]. \quad (3.36)$$

Bu yerda:

$$\Omega = \frac{\beta (1 + \sqrt{1 - \gamma^2})}{\gamma (1 + \sqrt{1 - \beta^2})};$$

$$b) \quad \beta = \frac{\varepsilon_{10}}{\varepsilon_T} \geq 1; \quad r_1 = \frac{\sqrt{3} \sqrt{1-\gamma^2}}{9 \gamma} \quad (3.37)$$

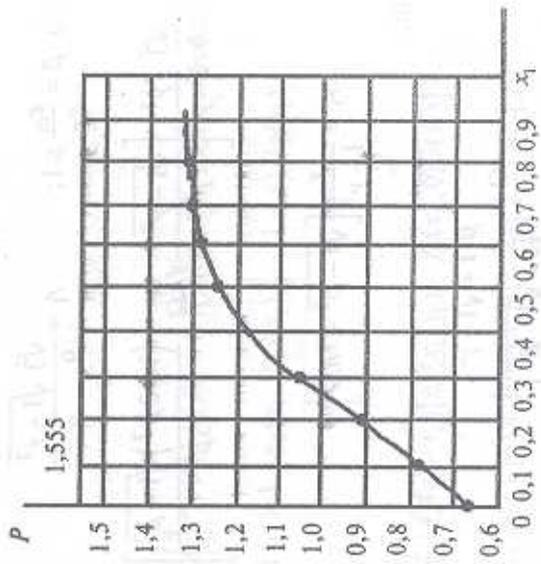
$$r_2 = \frac{\sqrt{3}}{6} \frac{\gamma^2}{1-\gamma^2} \left[\frac{\sqrt{1-\gamma^2}}{\gamma^2} - \ln \Omega \right] \quad (3.38)$$

$$r_3 = \frac{\gamma}{1-\gamma^2} \ln \Omega, \quad \Omega = \frac{1 + \sqrt{1-\gamma^2}}{\gamma} \quad (3.39)$$

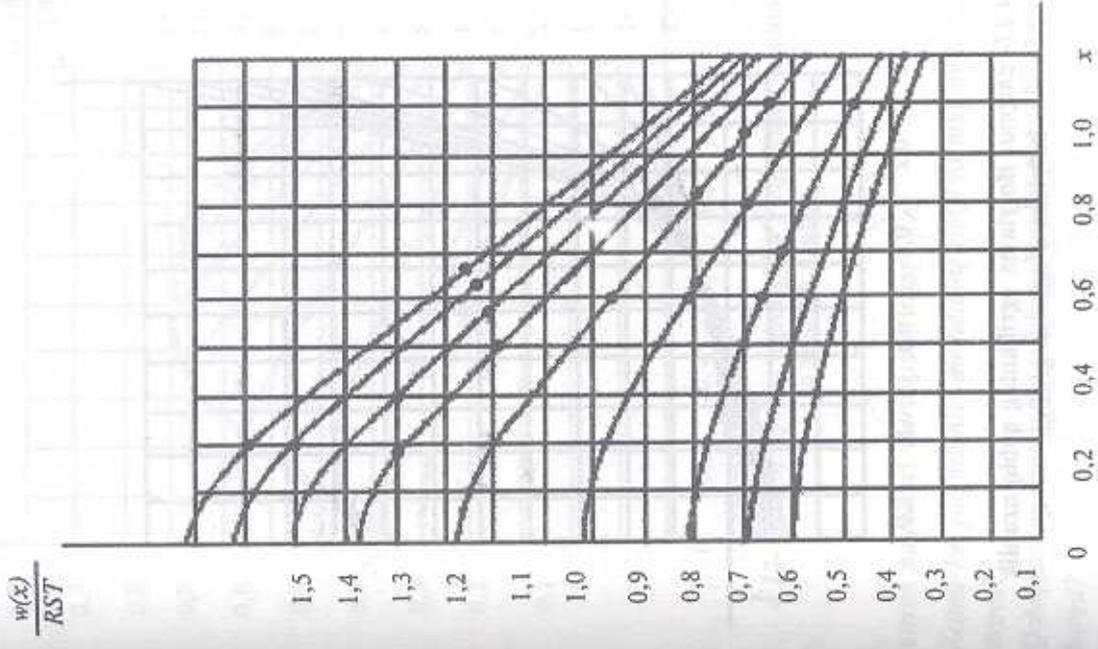
Quyidagi (3.23)–(3.24) tenglamalar sistemasi yechilgandan keyin har bir yaqinlashuvda yuk parametri miqdorini quyidagi ifodadan aniqlaymiz:

$$P = \lambda \left\{ r_2 f_0(x_1, 0) + \frac{1}{3} r_3 [4 - g_2(x_1, 0)] \right\} - \beta \left[r_1 f_0(x_1, 0) - \frac{1}{3} g_2(x_1, 0) \right] + \frac{4}{3} (1-\lambda) \beta \quad (3.40)$$

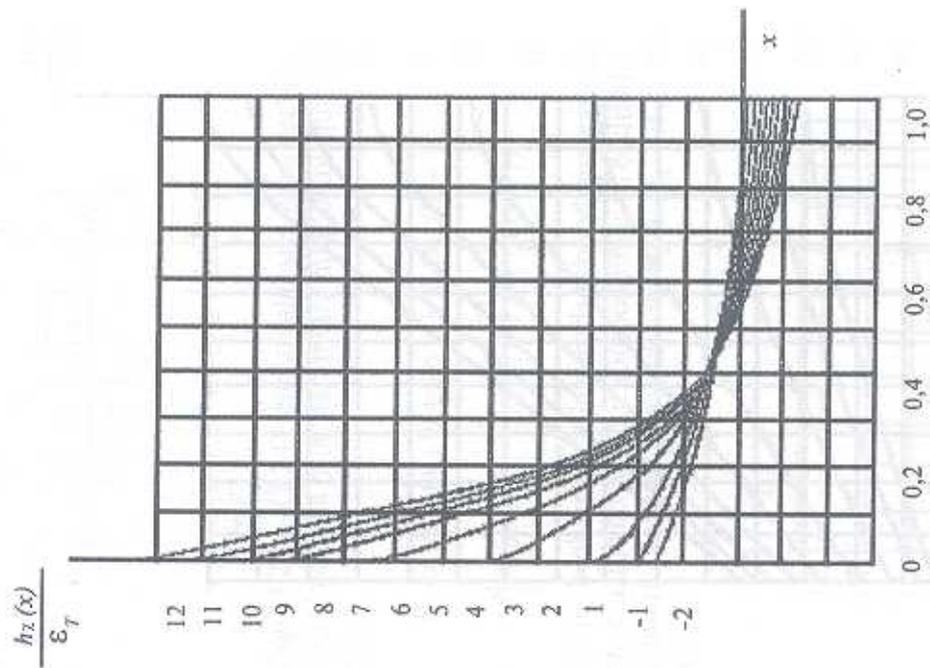
Qobiqning boshlang'ich kesimidagi bo'yлама eguvchi moment, halqa kuch, chiziqli bo'lmagan komponentlar va bikiiriklari aniqlanib, hisob natijalari quyidagi 4.8, 4.9, 4.10, 4.11, 4.12- chizmalarda keltirilgan.



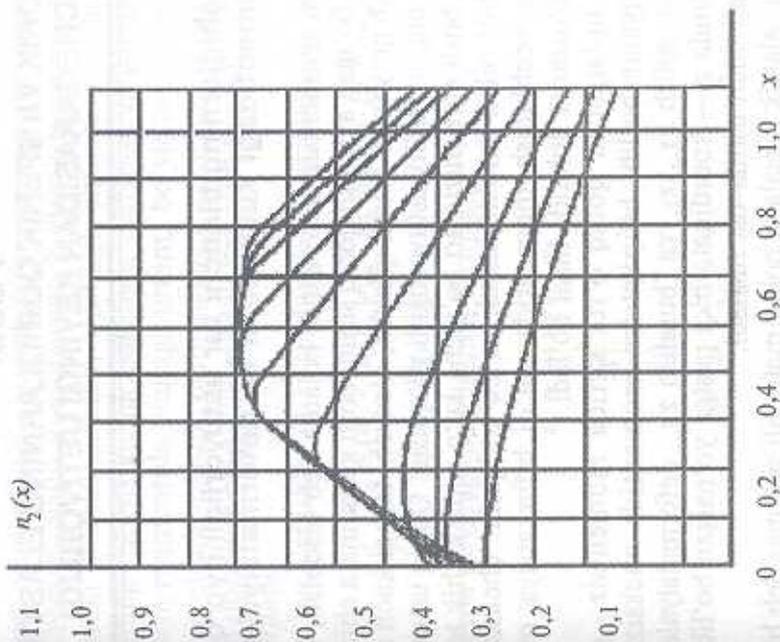
4.9- chizma. Elastik-plastik uchastka uzunligining yuk parametri dan o'zgarish grafi.



4.10- chizma. Qobiq uzunligi bo'yicha salqilikning o'zgarish grafi.



4.11- chizma. Bo'yama egrilikning qobiq uzunlik bo'yicha o'zgarish grafiqi.



4.12- chizma. Qobiq uzunligi bo'yicha halqa kuchining o'zgarish grafiqi.

Nazorat savollari va topshiriqlar

1. Ketma-ket yaqinlashish metodining yangi varianti mohiyati nimalardan iborat?
2. Qobiqlarning muvozanat tenglamalarining elastiklik chegarasidan farqi bormi?
3. Ko'rilgan masalalarda qobiqlarning elastik chegarasidan keyin qanday effekti aniqlangan?

SILINDRIK VA SFERIK QOBIQLARNING ELASTIKLIK CHEGARASIDAN KEYINGI USTIVORLIGI

5.1. Qobiqlarning birinchi tur ustuvorligini yo'qotish momentidagi kuchlanish va deformatsiyalar

Qobiqni momentsiz muvozanat holatda deb hisoblab, qalinligi o'zgarmas bo'lgan aylinish qobiq sirtini bosh koordinata chiziqlariga x, y keltirib qo'yamiz. Faraz qilaylik, o'suvchi tashqi kuch ta'sirida bo'lgan qobiq, mahalliy ustuvorligini yo'qotsin. Qobiq ustuvorligini yo'qotishi bosh egri chiziqlari bo'yicha ko'p sonli kichik to'lqinlar hosil bo'lishi bilan xarakterlanadi va bu to'lqinlar cheksiz kichik egilishning boshlanish momentida, ya'ni bifurkatsiya momenti vaqtida muvozanat holatida hosil bo'ladi.

Kritik holatgacha qobiq o'rta sirtida momentsiz $\varepsilon_x, \varepsilon_y, \gamma$ deformatsiyalar bo'lib, bifurkatsiya momentida cheksiz kichik qo'shimcha egilish $z\chi_x, z\chi_y$ va buralish $z\chi_{xy}$ deformatsiyalari hosil bo'ladi. Bunda z — koordinata o'qi pastga yo'nalgan bo'lib, qobiq sirti normalii bilan ustma-ust tushadi.

Chiziqli-elastik masalada qobiq materiali siqilmas deb hisoblab, Puasson koeffitsiyentini 0,5 ga teng deb qabul qilamiz.

Qobiqning cheksiz kichik egilishida ko'ndalang kesimning koordinata boshidan z masofadagi nuqta atrofidagi kuchlanishlar:

$$\begin{aligned}\sigma_x &= \frac{4}{3} E \left[\varepsilon_x + \frac{1}{2} \varepsilon_y - z\chi_x - \frac{1}{2} z\chi_y \right]; \\ \sigma_y &= \frac{4}{3} E \left[\varepsilon_y + \frac{1}{2} \varepsilon_x - z\chi_y - \frac{1}{2} z\chi_x \right]; \\ \tau &= \frac{1}{3} E [\gamma - 2z\chi_{xy}]\end{aligned}\quad (1.1)$$

formulardan aniqlanadi.

Bu formulalarni tuzishda siquvchi kuchlanishlar musbat deb qabul qilingan, qobiq elementi pastga qarab qavarib egilsin, unda $\chi_x, \chi_y, \chi_{xy}$ funksiyalar musbatdir.

(1.1) formuladan ko'rinadiki, bifurkatsiya momenti vaqtida ko'ndalang kesimning yuqori qismida yuklanish ($z < 0$), pastki qismida

($z > 0$) yuksizlanish ro'y beradi, bunda yuklanish hamda yuksizlanish qismlarida kuchlanish deformatsiyaga proporsional bo'lib qoladi. Proporsionallik koeffitsiyenti E elastik modul bo'ladi.

Yuqorida ko'rsatilganidek, elastiklik chegarasidan keyin qobiq elementining cheksiz kichik egilishida yuklanish va yuksizlanish M_0 bifurkatsiya momentida nuqtaga urinma bo'yicha ro'y beradi.

Kichik elastik-plastik deformatsiya nazariyasiga asosan (1.1) ifodadagi doimiy E elastiklik moduli kesuvchi modul ifodalari bilan almashtiriladi.

Unda kuchlanishlar (1.1) formulalarni quyidagicha yozamiz:

$$\begin{aligned}\sigma_x &= (a_x - zb_x) \psi_x; \quad \sigma_y = (a_y - zb_y) \psi_y; \\ \tau &= (a_{xy} - zb_{xy}) \psi_{xy}.\end{aligned}\quad (1.2)$$

Bu yerda quyidagi belgilashlar kiritilgan:

$$\begin{aligned}a_x &= \frac{4}{3} \left(\varepsilon_x + \frac{1}{2} \varepsilon_y \right); \\ a_y &= \frac{4}{3} \left(\varepsilon_y + \frac{1}{2} \varepsilon_x \right); \\ a_{xy} &= \frac{1}{3} \gamma; \\ b_x &= \frac{4}{3} \left(\chi_x + \frac{1}{2} \chi_y \right); \\ b_y &= \frac{4}{3} \left(\chi_y + \frac{1}{2} \chi_x \right); \\ b_{xy} &= \frac{2}{3} \chi_{xy}.\end{aligned}\quad (1.3)$$

$$\begin{aligned}\psi_x &= \psi_0 \left[1 + \frac{zb_x}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right]; \\ \psi_y &= \psi_0 \left[1 + \frac{zb_y}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right]; \\ \psi_{xy} &= \psi_0 \left[1 + \frac{zb_{xy}}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right].\end{aligned}\quad (1.5)$$

(1.5) funksiya (1.1) formuladagi E elastik modulini o'zi bilan almashiradi.

Bu yerda: E_k — elastiklik chegarasidan keyin $\sigma_i - \varepsilon_i$ siqilish diagrammasining M_0 nuqtasidagi urinma modul; ε_0 — shu nuqtadagi deformatsiya intensivligi; ψ_0 — bifurkatsiya vaqtidagi M_0 nuqtadagi tegishli kesuvchi modul.

5.2. Bifurkatsiya momentida qobiq kesimlaridagi bo'ylama kuchlar, eguvchi va burovchi momentlar

Qobiqning qalinligini h bilan belgilab, (1.2) formulani e'tiborga olib, bo'ylama N_x , N_y kuchlar, urinma S kuch, eguvchi momentlar M_x va M_y burovchi H moment uchun quyidagi ifodalarni yozamiz:

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dz = a_x \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x dz - b_x \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_y dz, \quad (2.1)$$

$$N_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y dz = a_y \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_y dz - b_y \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x dz.$$

$$S = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau dz = a_{xy} \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_{xy} dz - b_{xy} \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_{xy} dz. \quad (2.2)$$

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z dz = a_x \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x z dz - b_x \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_y z dz;$$

$$M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y z dz = a_y \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_y z dz - b_y \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x z dz. \quad (2.3)$$

$$H = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau z dz = a_{xy} \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_{xy} z dz - b_{xy} \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_{xy} z^2 dz. \quad (2.4)$$

Qobiq ko'ndalang kesim bikiirligi tushunchalarini kiritamiz. Cho'zilish yoki siqilishdagi bikiirliklar:

$$I_{1x} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x dz. = \psi_0 \int_{-\frac{h}{2}}^{\frac{h}{2}} dz + \frac{b_x}{\varepsilon_0} (\psi_0 - E_k) \int_{-\frac{h}{2}}^{\frac{h}{2}} z dz = \psi_0 h;$$

$$I_{1y} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_y dz = \psi_0 h; \quad I_{1xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_{xy} dz = \psi_0 h. \quad (2.5)$$

Aralash bikiirliklar:

$$I_{2x} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x z^2 dz. = \psi_0 \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 dz + \frac{b_x}{\varepsilon_0} (\psi_0 - E_k) \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 dz = \frac{b_x h^3}{12 \varepsilon_0} (\psi_0 - E_k);$$

$$I_{2y} = \frac{b_y h^3}{12 \varepsilon_0} (\psi_0 - E_k); \quad I_{2xy} = \frac{b_{xy} h^3}{12 \varepsilon_0} (\psi_0 - E_k).$$

Eglish va buuralishdagi bikiirliklar:

$$I_{3x} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x z^2 dz. = \psi_0 \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 dz + \frac{b_x}{\varepsilon_0} (\psi_0 - E_k) \int_{-\frac{h}{2}}^{\frac{h}{2}} z^3 dz = \frac{\psi_0 h^3}{12};$$

$$I_{3y} = \frac{\psi_0 h^3}{12}; \quad I_{3xy} = \frac{\psi_0 h^3}{12}. \quad (2.7)$$

Shuni aytib o'tish lozimki, chiziqli elastik masalalarda aralash bikiirik (2.6) bo'lmaydi, chunki bunga tegishli bo'lgan integral nolga teng bo'ladi.

Qobiq ko'ndalang kesimi bikiirligi uchun olingan (2.5) — (2.7) ifodalar bo'ylama kuchlar, urinma kuch, eguvchi momentlar va burovchi moment uchun natijaviy formulalarini olishga imkon beradi:

$$N_x = a_x I_{1x} - b_x I_{2x} = \psi_0 h a_x - \frac{b_x^2 h^3}{12 \varepsilon_0} (\psi_0 - E_k). \quad (2.8)$$

$b_x = \frac{1}{3} \left(z_x + \frac{1}{2} z_y \right)$ miqdor va uning kvadratini cheksiz kichik bo'lgani uchun (2.8) formulaning ikkinchi hadini tashlab yuborish mumkin.

Natijada bo'ylama va urinma kuchlar uchun quyidagi ifodalarga ega bo'lamiz:

$$\begin{aligned} N_x &= \psi_0 h a_x = \frac{4}{3} \psi_0 h \left(\varepsilon_x + \frac{1}{2} \varepsilon_y \right); \\ N_y &= \psi_0 h a_y = \frac{4}{3} \psi_0 h \left(\varepsilon_y + \frac{1}{2} \varepsilon_x \right); \\ S &= \psi_0 h a_{xy} = \frac{1}{3} \psi_0 b y. \end{aligned} \quad (2.9)$$

Bu formulalarda $\varepsilon_x, \varepsilon_y$ qobiq o'rta sirtining bo'ylama deformatsiyalari, γ esa shu sirtning siljish deformatsiyasidan. Bu deformatsiyalar kritik holatgacha qobiqning momentsiz holatiga to'g'ri keladi. Endi eguvchi momentlar va burovchi moment ifodalarni aniqlaymiz. Biz qobiqning egilgan elementi qavariqligi pastga qarab va salqilik ham z o'qi bo'yicha pastga yo'nalgan bo'lsin, deb faraz qilamiz. (2.3) va (2.4) formulalarning chap va o'ng qismlari bir xil ishorali bo'lishi uchun bu formulalar ishorasini teskari ishoraga almashtirish lozim:

$$\begin{aligned} M_x &= -(a_x I_{2x} - b_x I_{3x}) = \frac{\psi_0 h^3}{12} b_x \left[1 - \frac{a_x}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right]; \\ M_y &= \frac{\psi_0 h^3}{12} b_y \left[1 - \frac{a_y}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right]. \end{aligned} \quad (2.10)$$

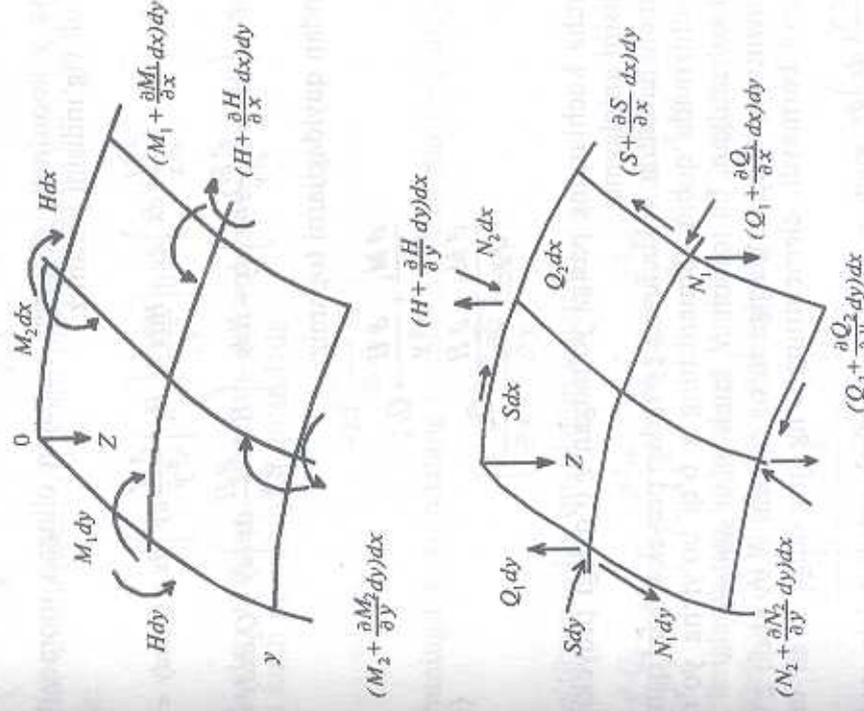
$$H = \frac{\psi_0 h^3}{12} b_{xy} \left[1 - \frac{a_{xy}}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right]. \quad (2.11)$$

(2.10) hamda (2.11) formulalar bilan hisoblanuvchi eguvchi va burovchi momentlar cheksiz kichik qiymat bo'lib, ularga cheksiz kichik egrilik deformatsiyalar $\chi_x, \chi_y, \chi_{xy}$ bilan (1.4) formulalardan aniqlanuvchi b_x, b_y, b_{xy} ko'paytuvchilar kiradi.

5.3. Qobiq elementining muvozanat tenglamasi

Qobiq elementini x, y, z koordinatalar sistemasiga joylashtiramiz (5.1- chizma).

Qobiqni dastlabki momentsiz holatida deb qaraymiz. Qobiq o'rta sirti qirrasiga ta'sir etuvchi siquvchi kuchlar N_1, N_2 va urinma kuch S qo'yilgan bo'lsin. Bu zo'riqish kuchlardan tashqari cheksiz kichik eguvchi momentlar M_1, M_2 va cheksiz kichik burovchi moment H ta'sir etsin. Bu kuchlar va momentlar qobiqning bifurkatsiya momenti vaqtida cheksiz kichik egilishidan hosil bo'ladi.



5.1- chizma. Qobiqning o'rta sirtiga ta'sir etuvchi zo'riqishlar.

Kuchlarning x va y o'qlardagi proyeksiyalari yig'indisini yozamiz:

$$N_1 dy - \left(N_1 + \frac{\partial N_1}{\partial x} dx \right) dy + S dx - \left(S + \frac{\partial S}{\partial y} dy \right) dx = 0;$$

$$N_2 dx - \left(N_2 + \frac{\partial N_2}{\partial y} dy \right) dx + S dy - \left(S + \frac{\partial S}{\partial x} dx \right) dy = 0.$$

bulardan quyidagilarni hosil qilamiz:

$$\frac{\partial N_1}{\partial x} + \frac{\partial S}{\partial y} = 0_1;$$

$$\frac{\partial N_2}{\partial y} + \frac{\partial S}{\partial x} = 0_2. \quad (3.1)$$

x va y koordinata o'qlariga nisbatan olingan momentlarning algebraik yig'indisini yozamiz:

$$M_1 dy - \left(N_1 + \frac{\partial M_1}{\partial x} dx \right) dy + H dx - \left(H + \frac{\partial H}{\partial y} dy \right) dx + Q_1 dx dy = 0;$$

$$M_2 dx - \left(M_2 + \frac{\partial M_2}{\partial y} dy \right) dx + H dy - \left(H + \frac{\partial H}{\partial x} dx \right) dy + Q_2 dx dy = 0.$$

bundan quyidagilarni topamiz:

$$\frac{\partial M_1}{\partial x} + \frac{\partial H}{\partial y} = Q_1;$$

$$\frac{\partial M_2}{\partial y} + \frac{\partial H}{\partial x} = Q_2. \quad (3.2)$$

Barcha kuchlarning pastga yo'nalgan z o'qidagi proyeksiyalari yig'indisini aniqlaymiz.

Birinchi navbatda N_1 kuchning z o'qidagi proyeksiyasini qaraymiz. 5.2-chizmada qobiq elementning x o'qi bo'yicha yo'nalgan tomoni ko'rsatilgan, bu tomon N_1 kuch bilan siqilgan.

Elementning chap qirg'oqiga ta'sir etuvchi $N_1 dy$ kuch z o'qiga proyeksiya bermaydi, elementning o'ng qirg'oqiga ta'sir etuvchi

$$\left(N_1 + \frac{\partial N_1}{\partial x} dx \right) dy$$

kuch z o'qiga α burchak ostida ta'sir etadi. Bu

burchak ikki qismdan iborat: birinchi qismi $\frac{1}{R_x} = K_x$ deformatsiya-

gacha x o'qi bo'yicha element egriligidan hosil bo'ladi; ikkinchi qismi $\frac{\partial^2 w}{\partial x^2}$ ustuvorlikni yo'qotishda cheksiz kichik egilish natijasida paydo bo'lgan qo'shimcha egrilikdan hosil bo'ladi.

Shuning uchun ham 5.2-chizmada ko'rsatilgan α burchak

$\alpha = K_x + \frac{\partial^2 w}{\partial x^2}$ ga teng. Unda N_1 kuchning z o'qidagi proyeksiyasi quyidagicha bo'ladi:

$$-\left(N_1 + \frac{\partial N_1}{\partial x} dx \right) dy \left(K_x + \frac{\partial^2 w}{\partial x^2} \right) dx = -N_1 \left(K_x + \frac{\partial^2 w}{\partial x^2} \right) dx dy.$$

Xuddi shuningdek, N_2 kuch proyeksiyasini ham quyidagicha topamiz:

$$-N_2 \left(K_y + \frac{\partial^2 w}{\partial y^2} \right) dx dy.$$

Bulardan tashqari, buralish deformatsiyasidan hosil bo'lgan urinma kuch proyeksiyasi qo'shiladi:

$$-2S \frac{\partial^2 w}{\partial x \partial y} dx dy.$$

Shuningdek, ko'ndalang kuchlar proyeksiyalari yig'indisi

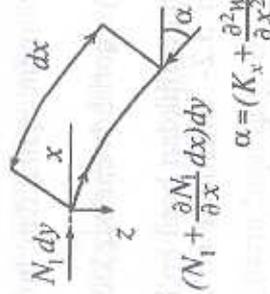
$$\left(\frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} \right) dx dy$$

ni e'tiborga olib, quyidagini topamiz:

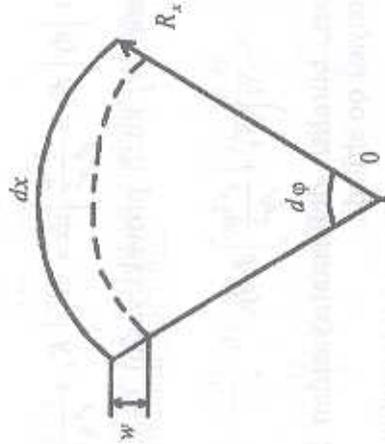
$$\frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} - N_1 \left(K_x + \frac{\partial^2 w}{\partial x^2} \right) - N_2 \left(K_y + \frac{\partial^2 w}{\partial y^2} \right) - 2S \frac{\partial^2 w}{\partial x \partial y} = 0. \quad (3.3)$$

Bu tenglamadagi ko'ndalang kuchlarni (3.2) ifoda yordamida almashtirib, (3.3) tenglamani quyidagi ko'rinishda yozamiz:

$$\frac{\partial^2 M_1}{\partial x^2} + 2 \frac{\partial^2 H}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} - N_1 \left(K_x + \frac{\partial^2 w}{\partial x^2} \right) - N_2 \left(K_y + \frac{\partial^2 w}{\partial y^2} \right) - 2S \frac{\partial^2 w}{\partial x \partial y} = 0. \quad (3.4)$$



5.2- chizma. Bo'ylama kuchlarni aniqlash uchun.



5.3- chizma. Qo'biq elementning deformatsiyalanmagan va deformatsiyalangan holati.

Eguvchi va burovchi momentlarni $w(x, y)$ salqilik funksiyasi orqali ifodalaymiz.

(1.4) formulaga kiruvchi egilishdagi egrilik χ_x, χ_y va buralishdagi χ_{xy} egrilik deformatsiyalarini (5.1 - chizma) quyidagi munosabatlardan aniqlaymiz:

$$\chi_x = -\frac{\partial^2 w}{\partial x^2}, \quad \chi_y = -\frac{\partial^2 w}{\partial y^2}, \quad \chi_{xy} = -\frac{\partial^2 w}{\partial x \partial y} \quad (3.5)$$

(2.1) va (2.11) formulalarni quyidagi shaklda ifodalash mumkin:

$$\begin{aligned} M_1 &= -D_0 \left[1 - \frac{a_x}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right] \left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \frac{\partial^2 w}{\partial y^2} \right]; \\ M_2 &= -D_0 \left[1 - \frac{a_y}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right] \left[\frac{\partial^2 w}{\partial y^2} + \frac{1}{2} \frac{\partial^2 w}{\partial x^2} \right]. \end{aligned} \quad (3.6)$$

$$H = -\frac{1}{2} D_0 \left[1 - \frac{a_{xy}}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right] \frac{\partial^2 w}{\partial x \partial y} \quad (3.7)$$

Bu bog'lanishlar asosida (3.4) tenglamadagi birinchi uch had ifodasini quyidagicha yozamiz:

$$\begin{aligned} \frac{\partial^2 M_1}{\partial x^2} + 2 \frac{\partial^2 H}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} &= -D_0 \nabla^2 \nabla^2 w + D_0 \left(1 - \frac{E_k}{\psi_0} \right) \cdot \\ &\cdot \left(\frac{a_x}{\varepsilon_0} \frac{\partial^4 w}{\partial x^4} + \frac{a_y}{\varepsilon_0} \frac{\partial^4 w}{\partial y^4} + \frac{a_x + 2a_{xy} + a_y}{2\varepsilon_0} \frac{\partial^4 w}{\partial x^2 \partial y^2} \right). \end{aligned} \quad (3.8)$$

(3.8) ifodani e'tiborga olib, (3.4) tenglamani quyidagi ko'rinishda ifodalaymiz:

$$\begin{aligned} D_0 \nabla^2 \nabla^2 w - \\ - D_0 \left(1 - \frac{E_k}{\psi_0} \right) \left(\frac{a_x}{\varepsilon_0} \frac{\partial^4 w}{\partial x^4} + \frac{a_y}{\varepsilon_0} \frac{\partial^4 w}{\partial y^4} + \frac{a_x + 2a_{xy} + a_y}{2\varepsilon_0} \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) + \\ + N_1 \left(K_x + \frac{\partial^2 w}{\partial x^2} \right) + N_2 \left(K_y + \frac{\partial^2 w}{\partial y^2} \right) + 2S \frac{\partial^2 w}{\partial x \partial y} = 0. \end{aligned} \quad (3.9)$$

5.4. Deformatsiyaning uzluksizlik tenglamasi

Kritik holatgacha qo'biqning o'rta sirti deformatsiyalari $\varepsilon_x, \varepsilon_y, \gamma$ cheksiz kichik egilishning hosil bo'lishida (bifurkatsiya momentida) salqilik funksiyasi $w(x, y)$ bilan bog'langan bo'lishi shart.

Bizga ma'lumki, plastinkaning tekis egilishida uning o'rta sirti deformatsiyasi quyidagi formula bilan ifodalanadi:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2; \\ \varepsilon_y &= \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2; \\ \gamma &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}. \end{aligned} \quad (4.1)$$

Qobiqlarning egilishida esa (4.1) tenglamaning birinchi ikkitasiga egrilik deformatsiyasini saqlilik funksiyasi $w(x, y)$ orqali ifodalovchi qo'shimcha hadlar qo'shiladi.

O'lchami dx bo'lgan element egilishini qaraymiz (5.3- chizma). Element markazga qarab $w(x, y)$ kattalikka ko'chganda x o'qi bo'yicha deformatsiyalanadi:

$$\epsilon_x = \frac{(R_x - w)d\varphi - R_x d\varphi}{R_x d\varphi} = -\frac{w}{R_x} = -K_x w.$$

$$\epsilon_y = -K_y w.$$

Xuddi shuningdek, y o'qi bo'yicha deformatsiya:

(4.1) formulalarni qobiq elementi uchun umumlashtirib, quyidagicha yozamiz:

$$\epsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 - K_x w;$$

$$\epsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 - K_y w;$$

$$\gamma = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}. \quad (4.2)$$

Bu ifodalardan u va v ko'chishlarni yo'qotib, deformatsiyaning uzluksizlik tenglamasini hosil qilamiz:

$$\begin{aligned} \frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} - \frac{\partial^2 \gamma}{\partial x \partial y} &= \\ = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - K_x \frac{\partial^2 w}{\partial y^2} - K_y \frac{\partial^2 w}{\partial x^2}. \end{aligned} \quad (4.3)$$

$\epsilon_x, \epsilon_y, \gamma$ deformatsiyalar σ_x, σ_y, τ kuchlanishlar orqali quyidagicha ifodalanadi:

$$\epsilon_x = \frac{1}{\psi_0} \left(\sigma_x - \frac{1}{2} \sigma_y \right); \quad \epsilon_y = \frac{1}{\psi_0} \left(\sigma_y - \frac{1}{2} \sigma_x \right); \quad \gamma = \frac{3}{\psi_0} \tau. \quad (4.4)$$

Agar bizga ma'lum bo'lgan bog'lanishlar yordamida $\Phi(x, y)$ kuchlanish funksiyasini kiritsak:

$$\sigma_x = \frac{\partial^2 \Phi}{\partial y^2}; \quad \sigma_y = \frac{\partial^2 \Phi}{\partial x^2}; \quad \tau = -\frac{\partial^2 \Phi}{\partial x \partial y}. \quad (4.5)$$

(4.3) tenglamaning chap tomonidagi uch hadni quyidagi ko'rinishga keltiramiz:

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} - \frac{\partial^2 \gamma}{\partial x \partial y} = \frac{1}{\psi_0} \nabla^2 \nabla^2 \Phi. \quad (4.6)$$

Bu formulani e'tiborga olib, (4.3) deformatsiyaning uzluksizlik tenglamasi quyidagicha ifodalanadi:

$$\frac{1}{\psi_0} \nabla^2 \nabla^2 \Phi = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - K_x \frac{\partial^2 w}{\partial y^2} - K_y \frac{\partial^2 w}{\partial x^2}. \quad (4.7)$$

5.5. Qobiq ustuvorlik tenglamasi

Kuchlanish funksiyasi $\Phi(x, y)$ orqali aniqlanuvchi bo'yama kuchlar N_1, N_2 , siquvchi bo'lgani uchun manfiy ishora bilan olinadi:

$$N_1 = h\sigma_x = -h \frac{\partial^2 \Phi}{\partial y^2};$$

$$N_2 = h\sigma_y = -h \frac{\partial^2 \Phi}{\partial x^2};$$

$$S = \tau h = -h \frac{\partial^2 \Phi}{\partial x \partial y}. \quad (5.1)$$

(5.1) ifodani (3.9) tenglamaga qo'yamiz:

$$\frac{D_0}{h} \nabla^2 \nabla^2 w -$$

$$- \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \left(\frac{a_x \partial^4 w}{\epsilon_0 \partial x^4} + \frac{a_y \partial^4 w}{\epsilon_0 \partial y^4} + \frac{a_x + 2a_y + a_y}{2\epsilon_0} \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) +$$

$$+ \sigma_x \frac{\partial^2 w}{\partial x^2} + \sigma_y \frac{\partial^2 w}{\partial y^2} + 2\tau \frac{\partial^2 w}{\partial x \partial y} - K_x \frac{\partial^2 \Phi}{\partial y^2} - K_y \frac{\partial^2 \Phi}{\partial x^2} = 0. \quad (5.2)$$

(5.2) muvozanat tenglamasi bilan (4.7) deformatsiyaning uzluksizlik tenglamasini bitta umumiy ustuvorlik tenglamasiga keltirish maqsadida, (5.2) ifodaga $\Delta_1 \Delta_2$ operator bilan ta'sir etamiz:

$$\begin{aligned}
& \frac{D_0}{h} \nabla^2 \nabla^2 \nabla^2 \nabla^2 w - \\
& - \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \nabla^2 \\
& \nabla^2 \left(\frac{a_x}{\xi_0} \frac{\partial^4 w}{\partial x^4} + \frac{a_y}{\xi_0} \frac{\partial^4 w}{\partial y^4} + \frac{a_x + 2a_{xy} + a_y}{2\xi_0} \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) + \\
& + \sigma_x \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial x^2} + \\
& + \sigma_y \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial y^2} + 2\tau \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial x \partial y} \\
& - K_x \nabla^2 \nabla^2 \frac{\partial^2 \Phi}{\partial y^2} - K_y \nabla^2 \nabla^2 \frac{\partial^2 \Phi}{\partial x^2} = 0.
\end{aligned} \quad (5.3)$$

(5.3) ifodaning oxirgi ikki hadini quyidagicha yozish mumkin:

$$\begin{aligned}
& -K_x \nabla^2 \nabla^2 \frac{\partial^2 \Phi}{\partial y^2} - K_y \nabla^2 \nabla^2 \frac{\partial^2 \Phi}{\partial x^2} = \\
& = -K_x \frac{\partial^2}{\partial y^2} \nabla^2 \nabla^2 \Phi - K_y \frac{\partial^2}{\partial x^2} \nabla^2 \nabla^2 \Phi = \\
& = - \left(K_x \frac{\partial^2}{\partial y^2} + K_y \frac{\partial^2}{\partial x^2} \right) \nabla^2 \nabla^2 \Phi.
\end{aligned} \quad (5.4)$$

(4.7) formulaning o'ng tomonidagi cheksiz kichik hadlarni e'tiborga olmasdan $\Delta^2 \Delta^2$ ifodani (5.4) formulaga qo'yib quyidagini hosil qilamiz:

$$\begin{aligned}
& - \left(K_x \frac{\partial^2}{\partial y^2} + K_y \frac{\partial^2}{\partial x^2} \right) \nabla^2 \nabla^2 \Phi = \\
& = \psi_0 \left(K_x^2 \frac{\partial^4 w}{\partial y^4} + 2K_x K_y \frac{\partial^4 w}{\partial x^2 \partial y^2} + K_y^2 \frac{\partial^4 w}{\partial x^4} \right).
\end{aligned} \quad (5.5)$$

Bunday almashtirishlardan keyin (5.3) tenglama elastiklik chegarasidan keyin yassi qobiqning umumiy ustuvorlik tenglamasini ifodalaydi:

$$\begin{aligned}
& \frac{D_0}{h} \nabla^2 \nabla^2 \nabla^2 \nabla^2 w - \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \nabla^2 \\
& \nabla^2 \left(\frac{a_x}{\xi_0} \frac{\partial^4 w}{\partial x^4} + \frac{a_y}{\xi_0} \frac{\partial^4 w}{\partial y^4} + \frac{a_x + 2a_{xy} + a_y}{2\xi_0} \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) + \\
& + \sigma_x \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial x^2} + \sigma_y \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial y^2} + 2\tau \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial x \partial y} \\
& + \psi_0 \left(K_x^2 \frac{\partial^4 w}{\partial y^4} + 2K_x K_y \frac{\partial^4 w}{\partial x^2 \partial y^2} + K_y^2 \frac{\partial^4 w}{\partial x^4} \right) = 0.
\end{aligned} \quad (5.6)$$

Agar bu tenglamada $E = \psi_0 = E_k$ deb olsak, unda bu tenglama chiziqli elastik qobiq ustuvorlik tenglamasiga o'tadi (Puaason koeffitsiyenti 0,5 bo'lganda):

$$\begin{aligned}
& \frac{D_0}{h} \nabla^2 \nabla^2 \nabla^2 \nabla^2 w + \sigma_x \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial x^2} + \sigma_y \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial y^2} + 2\tau \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial x \partial y} + \\
& + E \left(K_x^2 \frac{\partial^4 w}{\partial y^4} + 2K_x K_y \frac{\partial^4 w}{\partial x^2 \partial y^2} + K_y^2 \frac{\partial^4 w}{\partial x^4} \right) = 0.
\end{aligned} \quad (5.7)$$

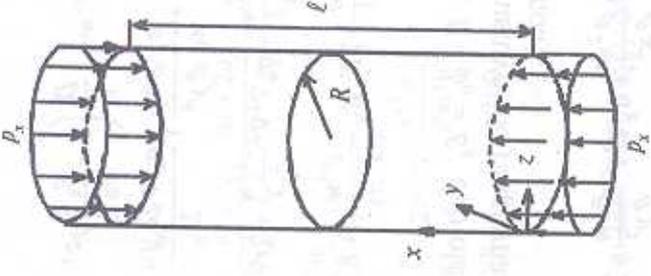
5.6. Bo'ylama siqilgan yopiq silindrik qobiqda o'qqa simmetrik to'lqin hosil bo'lishidagi ustuvorlik

O'q bo'yicha siquvchi tekis taralgan P_x kuch ta'sirida bo'lgan silindrik qobiqni qaraymiz (5.4- chizma).

Ustivorlikni yo'qotishda to'lqinlarning hosil bo'lishi o'qqa simmetrik bo'lsin deb hisoblaymiz, unda $K_x = 0$; $K_y = \frac{1}{\lambda}$ ekanligini e'tiborga olib umumiy tenglama (5.6) ni quyidagi ko'rinishga keltiramiz:

$$\frac{D_0 \partial^8 w}{h \partial x^8} - \left(1 - \frac{E_k}{\psi_0} \right) \frac{D_0 \partial_x \partial^8 w}{h \epsilon_0 \partial x^8} + \frac{\psi_0 \partial^4 w}{R^2 \partial x^4} + P_x \frac{\partial^6 w}{\partial x^6} = 0. \quad (6.1)$$

Kritik holatgacha unga to'g'ri keluvchi M_0 nuqtada siquvchi kuchdan quyidagi deformatsiyalar hosil bo'ladi:



5.4- chizma. Bo'ylama siqilgan qobiq.

$$\varepsilon_x; \varepsilon_y = -\frac{1}{2}\varepsilon_x; \varepsilon_z = -(\varepsilon_x + \varepsilon_y) = -\frac{1}{2}\varepsilon_x.$$

Unda M_0 nuqtada deformatsiya intensivligi quyidagicha bo'ladi:

$$\varepsilon_r = \varepsilon_0 = \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2} = \frac{\sqrt{2}}{3} \sqrt{\frac{9}{2}\varepsilon_x^2} = \varepsilon_x.$$

$a_x; a_y; a_{xy}$ kattaliklar (1.3) formula asosida quyidagi qiymatlarni qabul qiladi:

$$a_x = \frac{4}{3} \left(\varepsilon_x + \frac{\varepsilon_y}{2} \right) = \frac{4}{3} \left(\varepsilon_x - \frac{\varepsilon_x}{4} \right) = \varepsilon_x; \quad a_y = a_{xy} = 0.$$

Unda (6.1) ustuvorlik tenglamasi quyidagi ko'rinishda bo'ladi:

$$\frac{D_0 E_k}{h \psi_0} \frac{\partial^8 w}{\partial x^8} + \frac{\psi_0}{R^2} \frac{\partial^4 w}{\partial x^4} + P_x \frac{\partial^6 w}{\partial x^6} = 0. \quad (6.2)$$

Salqilik funksiyasi $w(x)$ ni quyidagi qator ko'rinishda izlaymiz:

$$w(x) = \sum A_m \sin \frac{m\pi x}{\ell}. \quad (6.3)$$

Bu qator silindr qirg'oqlaridagi chegara shartlarini qanoatlantiradi.

(6.3) ifodani (6.2) tenglamaga qo'yib, kritik kuchlanishni aniqlovchi ifodani hosil qilamiz:

$$P_{kcp} = \frac{D_0 E_k}{h \psi_0} \left(\frac{m\pi}{\ell} \right)^2 + \frac{\psi_0}{R^2} \left(\frac{m\pi}{\ell} \right)^2 \quad (6.4)$$

Kritik kuchning minimal miqdorini aniqlash uchun (6.4) funksiyani $\lambda = \left(\frac{m\pi}{\ell} \right)^2$ parametr bo'yicha minimallashtiramiz.

(6.4) funksiyani quyidagicha ifodalab olamiz:

$$P_{kcp} = \frac{D_0 E_k}{h} \lambda + \frac{\psi_0}{R^2} \lambda. \quad (6.5)$$

$\partial P_{kcp} / \partial \lambda$ hosilani nolga tenglaymiz:

$$\frac{\partial P_{kcp}}{\partial \lambda} = \frac{D_0 E_k}{h} - \frac{\psi_0}{R^2} \lambda^2 = 0.$$

Bundan:

$$\lambda = \frac{3}{Rh} \sqrt{\frac{\psi_0}{E_k}} = \left(\frac{m\pi}{\ell} \right)^2. \quad (6.6)$$

(6.6) ifodani (6.5) formulaga qo'yib, kritik kuchlanishning minimal miqdorini topamiz:

$$P_{kcp} = \frac{2}{3} \sqrt{E_k \psi_0} \frac{h}{R}. \quad (6.7)$$

Agar $E = \psi_0 = E_k$ teng deb olsak, unda (6.7) ifoda chiziqli elastik masala uchun ma'lum bo'lgan formulaga o'tadi:

$$P_{kcp} = \frac{2}{3} E \frac{h}{R}.$$

(6.7) kritik kuchlanishni kritik deformatsiya orqali ifodalab:

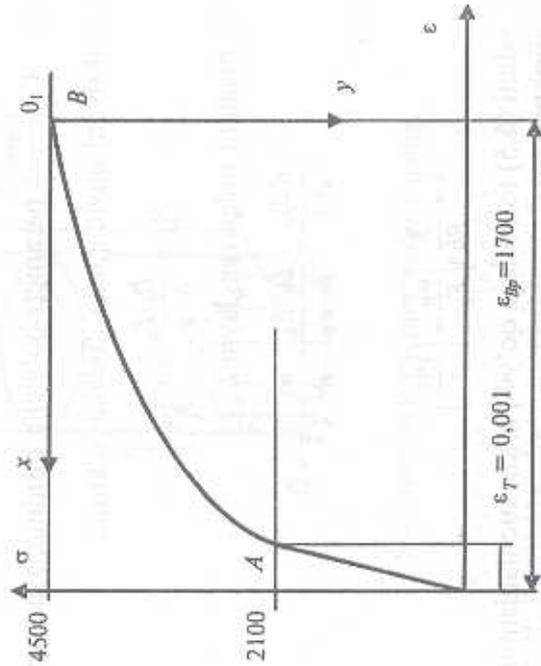
$$P_{kcp} = \psi_0 \varepsilon_{kcp}. \quad (6.8)$$

(6.7) formulani grafiklarni qurish uchun qulay bo'lgan shaklga keltiramiz:

$$\frac{R}{h} = \frac{2}{3} \frac{1}{\varepsilon_{kcp}} \sqrt{\frac{E_k}{\psi_0}}. \quad (6.9)$$

Olingan natijalarni qurilish po'latidan yasalgan yopiq silindrik qobiq uchun qo'llaymiz. Siqilish diagrammasi $\sigma - \epsilon$ 5.5- chizmada keltirilgan.

5.6- chizmada elastiklik chegarasidan keyin qobiqning nisbiy qalinligi R/h bilan kritik deformatsiya orasidagi bog'lanish grafigi, uzluqli chiziq bilan shu bog'lanishlar uchun chiziqli-elastik masala uchun keltirilgan.



5.5- chizma. Qurilish po'latining cho'zilish-siqilish diagrammasi.

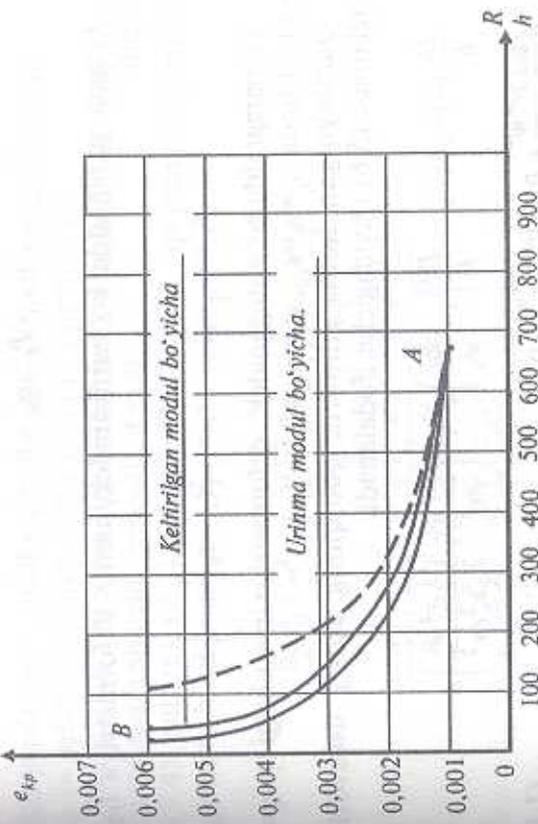
Proporsionallik chegarasiga to'g'ri keluvchi ϵ_{kp} kritik deformatsiyani 0,100 ga teng deb qabul qilamiz. 5.6- chizmada (6.9) formula asosida hisoblashlar natijasida qurilgan grafik keltirilgan.

Ustivorligini yo'qotishiga to'g'ri keluvchi bo'yлама yarim to'liqlar soni (6.6) formuladan aniqlanadi:

$$m = \frac{\sqrt{3}}{\pi} \frac{\ell}{R} \sqrt{\frac{R}{h}} \sqrt{\frac{\psi_0}{E_k}} \quad (6.10)$$

Faraz qilaylik, silindr $\epsilon_{kp} = 0,002$ kritik deformatsiyada ustivorligini yo'qotsin. Hisoblar bo'yicha bu kritik deformatsiyaga

$$\sqrt{E_k/\psi_0} = 0,501, \quad \frac{R}{h} = 167 \text{ to'g'ri keladi.}$$



5.6- chizma. O'qqa simmetrik to'liqin hosil bo'lganda kritik deformatsiyaning qobiq nisbiy qalinligiga bog'liqlik grafigi.

Bundan tashqari $\frac{\ell}{R} = 10$ bo'lsin deb hisoblab, (6.10) formuladan yarimta to'liqlar sonini aniqlaymiz:

$$m = \frac{1,732}{\pi} \cdot 10 \cdot 12,92 \cdot 1,41 = 100.$$

Agar $\epsilon_{kp} = 0,002$ kritik deformatsiyada masala chiziqli elastik bo'lsa, unda (6.10) formulaga $E = \psi_0 = E_k$ va $\sqrt{\psi_0/E_k} = 1$ qo'yamiz va yarim to'liqlar soni kamayib:

$$m = \frac{1,732}{\pi} \cdot 10 \cdot 12,92 \cdot 1 = 71$$

ga teng bo'ladi.

5.7. Bo'yлама siqilgan yopiq silindrik qobiqning ikki yo'nalish bo'yicha yarim to'liqlarini hisobga olganda ustuvorlik

Bo'yлама yo'nalish bo'yicha siqilgan silindrik qobiq ustuvorligini yo'qotishda bo'yлама va aylana to'liqlar hosil bo'lsin. Kritik holatgacha qobiqda faqat bitta siquvchi kuchlanish bo'ladi:

$$\sigma_x = P_x; \sigma_y = 0; \tau_{xy} = 0.$$

O'tgan paragrafda ko'rsatilganidek, uchta deformatsiya hosil bo'ladi:

$$\varepsilon_x, \quad \varepsilon_y = -\frac{1}{2}\varepsilon_x; \quad \varepsilon_z = -(\varepsilon_x + \varepsilon_y) = -\frac{1}{2}\varepsilon_x.$$

5.6 paragrafda ko'rsatilganidek, deformatsiya intensivligi $\varepsilon_i = \varepsilon_0 = \varepsilon_x$ va (1.3) ifoda $\alpha_x = \varepsilon_x = \varepsilon_0$, $\alpha_y = \alpha_{xy} = 0$ bo'ladi.

Qaralayotgan holda silindrik qobiqning asosiy ustuvorlik tenglamasi (5.6) quyidagicha ifodalanadi:

$$\begin{aligned} & \frac{D_0}{h} \nabla^2 \nabla^2 \nabla^2 w - \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \nabla^2 \nabla^2 \left(\frac{\partial^4 w}{\partial x^4} + \frac{1}{2} \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) + \\ & + \frac{\psi_0}{R^2} \frac{\partial^4 w}{\partial x^4} + P_x \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial x^2} = 0. \end{aligned} \quad (7.1)$$

(7.1) tenglamaning ikkinchi hadidagi:

$$\frac{\partial^2}{\partial x^2} \left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \frac{\partial^2 w}{\partial y^2} \right)$$

kattalikni quyidagicha ifodalab olamiz:

$$\frac{\partial^2}{\partial x^2} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{2 \partial y^2} \right) = \nabla^2 \nabla^2 w - \frac{3 \partial^4 w}{2 \partial x^2 \partial y^2} - \frac{\partial^4 w}{\partial y^4}. \quad (7.2)$$

(7.2) ifodani (7.1) tenglamaga qo'yib, ustuvorlik tenglamasini sodda ko'rinishda ifodalab olamiz:

$$\begin{aligned} & \frac{D_0}{h} \frac{E_k}{\psi_0} \nabla^2 \nabla^2 \nabla^2 w + \\ & + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \nabla^2 \nabla^2 \left[\frac{3 \partial^4 w}{2 \partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right] + \\ & + \frac{\psi_0}{R^2} \frac{\partial^4 w}{\partial x^4} + P_x \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial x^2} = 0. \end{aligned} \quad (7.3)$$

Salqilik funksiyasini $w(x, y)$ quyidagi trigonometrik qator ko'rinishida izlaymiz:

$$w(x, y) = \sum_m \sum_n C_{mn} \sin \frac{m\pi x}{l} \sin \frac{n y}{R}. \quad (7.4)$$

Bu yerda: m — silindr o'qi bo'ylab yarimta to'liqlar soni; n — aylanish yo'nalishi bo'yicha to'liqlar soni.

(7.4) salqilik funksiyasi qirg'oqi bo'yicha sharnirli mahkamlangan silindr chegara shartini qanoatlantiradi.

(7.4) qatorni (7.3) tenglamaga qo'yib, quyidagi algebraik tenglamani hosil qilamiz:

$$\begin{aligned} & \frac{D_0 E_k}{h \psi_0} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2}{R^2} \right)^4 + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \left(\frac{m^2 \pi^2}{\ell^2} + \right. \\ & \left. + \frac{n^2}{R^2} \right)^2 \left(\frac{3 m^2 \pi^2}{2 \ell^2} + \frac{n^2}{R^2} \right) + \frac{\psi_0 m^4 \pi^4}{R^2 \ell^4} - \\ & - P_x \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2}{R^2} \right)^2 \frac{m^2 \pi^2}{\ell^2} = 0. \end{aligned} \quad (7.5)$$

Bundan hosil qilamiz:

$$\begin{aligned} P_x = & \frac{D_0 E_k}{h \psi_0} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2}{R^2} \right)^2 \frac{\ell^2}{m^2 \pi^2} + \frac{\psi_0}{R^2} \frac{1}{\left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2}{R^2} \right)^2} \frac{\ell^2}{m^2 \pi^2} + \\ & + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \frac{n^2}{R^2} \frac{\ell^2}{m^2 \pi^2} \left(\frac{3 m^2 \pi^2}{2 \ell^2} + \frac{n^2}{R^2} \right). \end{aligned} \quad (7.6)$$

Bundan kritik kuch ifodasini aniqlaymiz:

$$P_x = P_1 + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \frac{n^2 \ell^2}{R^2 m^2 \pi^2} \left(\frac{3 m^2 \pi^2}{2 \ell^2} + \frac{n^2}{R^2} \right). \quad (7.7)$$

Bu yerda:

$$P_1 = \frac{D_0 E_k}{h \psi_0} \lambda + \frac{\psi_0}{R^2} \frac{1}{\lambda}; \quad \lambda = \frac{\ell^2}{m^2 \pi^2} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2}{R^2} \right)^2. \quad (7.8)$$

P_1 funksiya minimal bo'lgandagi λ parametrlar miqdorini izlaymiz:

$$\frac{\partial P_1}{\partial \lambda} = \frac{D_0 E_k}{h \psi_0} - \frac{\psi_0}{R^2} \frac{1}{\lambda^2} = 0.$$

Bundan:

$$\lambda = \frac{3}{Rh} \sqrt{\frac{\psi_0}{E_k}} \quad (7.9)$$

P_1 funksiyaning minimal miqdori:

$$(P_1)_{\min} = \frac{D_0}{h} \frac{E_k}{\psi_0} \frac{3}{Rh} \sqrt{\frac{\psi_0}{E_k}} + \frac{\psi_0}{R^2} \frac{Rh}{3} \sqrt{\frac{E_k}{\psi_0}} + \frac{h}{3R} \sqrt{E_k \psi_0} + \frac{h}{3R} \sqrt{E_k \psi_0} = \frac{2h}{3R} \sqrt{E_k \psi_0} \quad (7.10)$$

(7.6) asosiy formulani quyidagicha ifodalab olamiz:

$$P_x = \frac{2h}{3R} \sqrt{E_k \psi_0} + \frac{3D_0}{2h} \left(1 - \frac{E_k}{\psi_0} \right) \frac{n^2}{R^2} \left(1 + \frac{2}{3} \frac{n^2}{m^2 \pi^2} \frac{\ell^2}{R^2} \right) \quad (7.11)$$

(7.8) va (7.9) ifodalar asosida bo'ylama yarim to'lqinlar soni m bilan aylana yo'nalishi bo'yicha to'lqinlar soni n ni bog'lovchi tenglamani hosil qilamiz:

$$\frac{\ell^2}{m^2 \pi^2} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2}{R^2} \right) = \frac{3}{Rh} \sqrt{\frac{\psi_0}{E_k}} = \lambda \quad (7.12)$$

Bundan quyidagini topamiz:

$$\frac{n^2}{R^2} = \frac{m\pi}{\ell} \left[\sqrt{\lambda} - \frac{m\pi}{\ell} \right] \quad (7.13)$$

$\frac{m\pi}{\ell} = \alpha \sqrt{\lambda}$ ekanligini e'tiborga olib:

$$\frac{n^2}{R^2} = \lambda \alpha (1 - \alpha) \quad (7.14)$$

$$\frac{n^2}{m^2 \pi^2} \frac{\ell^2}{R^2} = \frac{\lambda \alpha (1 - \alpha)}{\lambda \alpha^2} = \frac{1 - \alpha}{\alpha} \quad (7.15)$$

(7.14) va (7.15) bog'lanishlarni (7.12) ustuvorlik tenglamasiga qo'yib, quyidagiga ega bo'lamiz:

$$P_x = \frac{2h}{3R} \sqrt{E_k \psi_0} \left[1 + \frac{1}{4} \left(\frac{\psi_0}{E_k} - 1 \right) (2 + \alpha) (1 - \alpha) \right] \quad (7.16)$$

(7.16) tenglamaga P_x bilan $P_x = \psi \epsilon_x$ bog'lanishda bo'lgan bo'ylama deformatsiya ϵ_x ni kiritib, (7.16) tenglamani quyidagicha yozamiz:

$$\frac{3R}{2h} \epsilon_x = \left[1 - \frac{1}{4} (2 + \alpha) (1 - \alpha) \right] \sqrt{\frac{E_k}{\psi_0}} \frac{1}{4} (2 + \alpha) (1 - \alpha) \sqrt{\frac{E_k}{\psi_0}} \quad (7.17)$$

Yangi belgilash kiritamiz:

$$\beta = \frac{1}{4} (2 + \alpha) (1 - \alpha) \quad (7.18)$$

unda (7.1) tenglamani quyidagi shaklda yozamiz:

$$\frac{3R}{2h} \epsilon_x = (1 - \beta) \delta + \frac{\beta}{\delta} \quad (7.19)$$

Bu yerda:

$$\delta = \sqrt{E_k / \psi_0} \quad (7.20)$$

$$f = \frac{3R}{2h} \epsilon_x$$

funksiyani δ bo'yicha minimallashtiramiz:

$$\frac{\partial f}{\partial \delta} = 1 - \beta - \frac{\beta}{\delta^2} = 0$$

bundan:

$$\delta^2 = \frac{E_k}{\psi_0} \frac{\beta}{1 - \beta}; \quad \delta = \sqrt{\frac{\beta}{1 - \beta}} \quad (7.21)$$

(7.19) ifodani (7.21) formulaga qo'yib, $f = \frac{3R}{2h} \epsilon_{sp}$ minimal miqdorini topamiz (bo'ylama deformatsiyaning minimal qiymatini ϵ_{sp} bilan belgilaymiz):

$$\frac{3R}{2h} \epsilon_{sp} = 2 \sqrt{\beta (1 - \beta)} \quad (7.22)$$

(7.21) ifodadan quyidagilarni topamiz:

$$\beta = \frac{\delta^2}{1 + \delta^2} \quad (7.23)$$

$$1 - \beta = \frac{1}{1 + \delta^2} \quad (7.24)$$

(7.19) formula quyidagi ko'rinishda bo'ladi:

$$\frac{3R}{2h} \varepsilon_{kp} = 2 \frac{\delta}{1 + \delta^2} = 2 \frac{\sqrt{E_k/\psi_0}}{1 + \frac{E_k}{\psi_0}} = 2 \frac{\sqrt{\psi_0 E_k}}{\psi_0 + E_k} \quad (7.25)$$

Natijada elastiklik chegarasidan keyin yopiq silindrik qobiqlar ustuvorligini tadqiqot qilish uchun ikkita fundamental tenglamalarga ega bo'lamiz:

$$\frac{R}{h} = \frac{4}{3} \frac{1}{\varepsilon_{kp}} \frac{\sqrt{\psi_0 E_k}}{\psi_0 + E_k} \quad (7.26)$$

$$P_{kp} = \frac{4}{3} \psi_0 \frac{\sqrt{\psi_0 E_k}}{\psi_0 + E_k} \frac{h}{R} \quad (7.27)$$

Agar (7.27) formulaga $\psi_0 = E_k = E$ qo'yusak, chiziqli elastik masala uchun ma'lum bo'lgan kritik kuch formulasini hosil qilamiz:

$$P_{kp} = \frac{2}{3} E \frac{h}{R} \quad (7.28)$$

5.7- chizmada uchta egri chiziq tasvirlangan. Bu egri chiziqlar elastiklik chegarasidan keyin kritik deformatsiya ε_{kp} bilan nisbiy qalinligi R/h orasidagi bog'lanishni ifodalaydi. Uzlukli egri chiziq qobiq materiali chiziqli elastik $\varepsilon_{kp} > 0,0010$ bo'lganda ifodalaydi.

Bo'ylama yarim to'lqinlar soni m ni topish uchun (7.15) formula orqali m soni bilan bog'liq bo'lgan α parametrimni aniqlaymiz. (7.19) kvadrat tenglamani α nisbatan yechib, quyidagi ifodani hosil qilamiz:

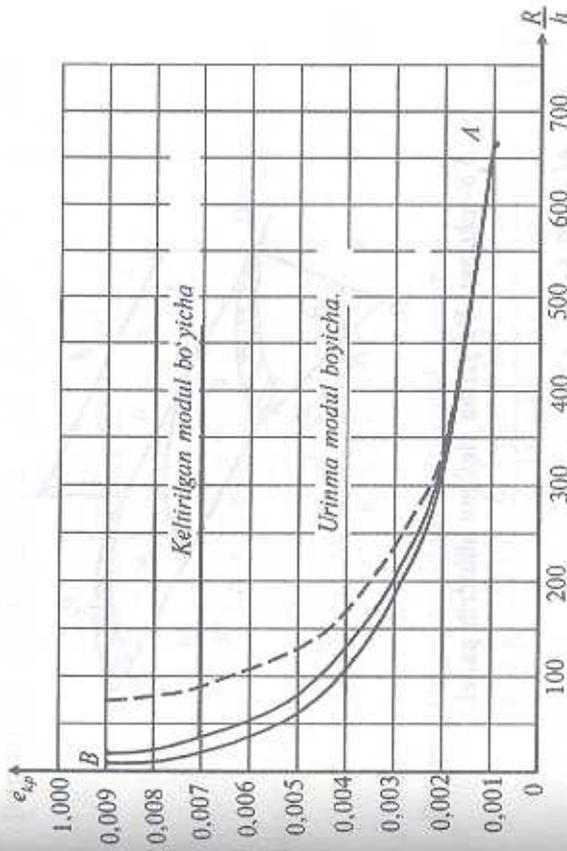
$$\alpha = \frac{1}{2} \left[\sqrt{\frac{9-7\delta^2}{1+\delta^2}} - 1 \right] \quad (7.29)$$

(7.13) va (7.14) formulalardan foydalanib, bo'ylama yarim to'lqinlar sonini aniqlaymiz:

$$m = \frac{\sqrt{3}}{2\pi} \frac{l}{R} \sqrt{\frac{R}{h}} \frac{1}{\sqrt{\delta}} \left[\sqrt{\frac{9-7\delta^2}{1+\delta^2}} - 1 \right] \quad (7.30)$$

(7.14) bog'lanishdan (7.30) formulani e'tiborga olib, qobiq aylana bo'yi bo'ylama yarim to'lqinlar sonini topamiz:

$$n = \frac{\sqrt{3}}{2} \sqrt{\frac{R}{h\delta}} \left[\sqrt{\frac{9-7\delta^2}{1+\delta^2}} - 1 \right] \left(3 - \sqrt{\frac{9-7\delta^2}{1+\delta^2}} \right) \quad (7.31)$$



5.7- chizma. Ikki yo'nalish bo'yicha to'lqinlarning hosil bo'lishida kritik deformatsiyaning qobiq nisbiy qalinligiga bog'liqlik grafigi.

Chiziqli-elastik masalada $\delta=1$ bo'ladi va m yoki n sonlarning har biri nolga teng bo'ladi, bu holda to'lqinlarning hosil bo'lishi masalasi yechilmasdan qoladi.

5.8. Siqilgan silindrik panel ustuvorligi

Oldingi paragrafda olingan natijalarni siqilgan silindrik panellar ustuvorligi masalasi uchun umumlashtirish mumkin.

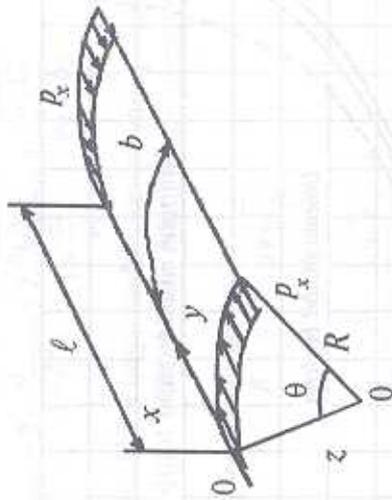
Siqilgan panel bo'ylama yo'nalish bo'yicha l uzunlikka ega va aylana yo'nalishi bo'yicha eni b bo'lsin deb qaraymiz (5.8- chizma). Panelning kuchlanganlik holati:

$$\sigma_x = P_x; \quad \sigma_y = 0; \quad \tau_{xy} = 0.$$

Deformatsiyalar:

$$\varepsilon_x; \quad \varepsilon_y = -\frac{1}{2} \varepsilon_x; \quad \varepsilon_z = -(\varepsilon_x + \varepsilon_y) = -\frac{1}{2} \varepsilon_x; \quad \gamma_{xy} = 0.$$

Deformatsiya intensivligi o'tgan paragrafdagidek $\varepsilon_r = \varepsilon_x = \varepsilon_0$ bo'lib, (1.3) funksiyalar quyidagicha bo'ladi:



5.8- chizma. Bo'ylama siqilgan silindirik panel.

$$a_x = \frac{4(\epsilon_x + 0,5\epsilon_y)}{3} = \frac{4}{3} \left(\epsilon_x - \frac{\epsilon_x}{4} \right) = \epsilon_x; \quad a_y = a_{xy} = 0.$$

Ustivorlik tenglamasi esa siqilgan yopiq qobiq tenglamasi kabi bo'lib, (7.3) munosabat bilan aniqlanadi. Qobiq bo'ylama qirg'og'i bilan sharmirli tayangan deb qabul qilamiz. Ustivorlikni yo'qotishdagi salqilik funksiyasi $w(x, y)$ ni quyidagi qator ko'rinishida olamiz:

$$w(x, y) = \sum_m \sum_n C_{mn} \sin \frac{m\pi x}{\ell} \sin \frac{n\pi y}{b}. \quad (8.1)$$

Bu qatorning m va n indekslariga to'g'ri keluvchi bitta hadini (7.3) tenglamaga qo'yib, siquvchi kritik kuchni aniqlash uchun quyidagi algebraik tenglamani hosil qilamiz:

$$\begin{aligned} & \frac{D_0 E_k}{h \psi_0} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^4 + \\ & + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^2 \frac{n^2 \pi^2}{b^2} \left(\frac{3m^2 \pi^2}{2\ell^2} + \frac{n^2 \pi^2}{b^2} \right) + \\ & + \frac{\psi_0 m^4 \pi^4}{R^2 \ell^4} - P_x \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^2 \frac{m^2 \pi^2}{\ell^2} = 0. \end{aligned} \quad (8.2)$$

Bundan P_x ni aniqlaymiz:

$$\begin{aligned} P_x = & \frac{D_0 E_k}{h \psi_0} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^2 \frac{\ell^2}{m^2 \pi^2} + \\ & + \frac{\psi_0}{R^2} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^2 \frac{\ell^2}{m^2 \pi^2} + \\ & + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \frac{n^2 \ell^2}{b^2 m^2} \left(\frac{3m^2 \pi^2}{2\ell^2} + \frac{n^2 \pi^2}{b^2} \right). \end{aligned} \quad (8.3)$$

Bu tenglamani quyidagi ko'rinishda yozamiz:

$$P_x = P_1 + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \frac{n^2 \ell^2}{b^2 m^2} \left(\frac{3m^2 \pi^2}{2\ell^2} + \frac{n^2 \pi^2}{b^2} \right). \quad (8.4)$$

$$P_1 = \frac{D_0 E_k}{h \psi_0} \lambda + \frac{\psi_0}{\lambda R^2}; \quad \lambda = \frac{\ell^2}{m^2 \pi^2} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right). \quad (8.5)$$

(8.5) funksiyaning λ parametrlar bo'yicha minimal qiymatini aniqlab, (7.9) ko'rinishdagi munosabatni topamiz. (8.5) formulaga (7.9) ifodani qo'yamiz:

$$(P_1)_{\min} = \frac{2h}{3R} \sqrt{E_k \psi_0}. \quad (8.6)$$

Unda (8.4) tenglama quyidagi ko'rinishga ega bo'ladi:

$$P_x = \frac{2h}{3R} \sqrt{E_k \psi_0} + \frac{3D_0}{2h} \left(1 - \frac{E_k}{\psi_0} \right) \frac{n^2 \pi^2}{b^2} \left(1 + \frac{2n^2 \ell^2}{3m^2 b^2} \right). \quad (8.7)$$

(8.5) va (7.10) asosida m va n sonlari orasidagi bog'lanishni aniqlaymiz:

$$\frac{\ell^2}{m^2 \pi^2} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^2 = \frac{3}{Rh} \sqrt{\frac{\psi_0}{E_k}} = \lambda. \quad (8.8)$$

Bu tenglamadan quyidagini topamiz:

$$\frac{n^2 \pi^2}{b^2} = \frac{m\pi}{\ell} \left[\sqrt{\lambda} - \frac{m\pi}{\ell} \right]. \quad (8.9)$$

α parametrni quyidagicha kiritamiz:

$$\frac{m\pi}{\ell} = \alpha\sqrt{\lambda}. \quad (8.10)$$

Unda (8.9) tenglamadan quyidagi ifodani topamiz:

$$\frac{n^2\pi^2}{b^2} = \lambda\alpha(1-\alpha) = \frac{3}{Rh}\sqrt{\frac{\psi_0}{E_k}}\alpha(1-\alpha). \quad (8.11)$$

Bundan tashqari (8.10) va (8.11) formulalardan foydalanib quyidagini topamiz:

$$\frac{n^2}{m^2} \frac{\ell^2}{b^2} = \frac{\lambda\alpha(1-\alpha)}{\lambda\alpha^2} = \frac{1-\alpha}{\alpha}. \quad (8.12)$$

(8.11) va (8.12) ifodalarni (8.7) ustuvorlik tenglamasiga qo'yib, quyidagi formulani aniqlaymiz:

$$P_x = \frac{2h}{3R}\sqrt{E_k\psi_0} + \frac{3}{2}\left(1 - \frac{E_k}{\psi_0}\right)\frac{1}{9}\psi_0 h^2 \frac{3}{Rh}\sqrt{\frac{\psi_0}{E_k}}(1-\alpha)\alpha\left[1 + \frac{2(1-\alpha)}{3}\frac{1-\alpha}{\alpha}\right] = \frac{2h}{3R}\sqrt{E_k\psi_0} + \frac{1}{6}\sqrt{E_k\psi_0}\frac{\psi_0}{E_k}\left(1 - \frac{E_k}{\psi_0}\right)(2+\alpha)(1-\alpha); \quad (8.13)$$

$$P_x = \frac{2h}{3R}\sqrt{E_k\psi_0}\left[1 + \frac{1}{4}\left(\frac{\psi_0}{E_k} - 1\right)(2+\alpha)(1-\alpha)\right].$$

Bu formula siqilgan yopiq silindrik qobiqqa tegishli bo'lgan (7.16) ustuvorlik tenglamasi bilan to'liq mos keladi. Shuning uchun ham siqilgan silindrik qobiq uchun olingan (7.26) va (7.27) fundamental tenglamalar silindrik panel uchun ham o'rinni bo'ladi.

Shuningdek, 5.7- chizmada ko'rsatilgan grafik ham o'rinni bo'ladi, (8.13) ga kiruvchi α parametr (7.29) bog'lanishdan aniqlanadi. (8.10) asosida bo'ylama to'liqlar sonini aniqlaymiz:

$$m = \frac{\sqrt{3}}{2\pi} \frac{\ell}{R} \sqrt{\frac{R}{h}} \frac{1}{\sqrt{\delta}} \left[\sqrt{\frac{9-7\delta^2}{1+\delta^2}} - 1 \right]. \quad (8.14)$$

Panel eni b bo'yicha yarim to'liqlar sonini (8.11) formula asosida aniqlaymiz:

$$n = \frac{\sqrt{3}R}{2h} \sqrt{h\delta} \sqrt{\sqrt{\frac{9-7\delta^2}{1+\delta^2}} - 1} \left[3 - \sqrt{\frac{9-7\delta^2}{1+\delta^2}} \right]. \quad (8.15)$$

Silindrik panel ustuvorligi bo'yicha olingan natijalar panel b enini aniqlovchi θ burchakning katta qiymatlarida o'rinni bo'ladi. Ustuvorlik nazariyasida chiziqli-elastic silindrik panel burchagi θ , panel eni b va panel qalinligi h orasida $\frac{\theta b}{h} \geq 12$ tengsizlik o'rnatilgan.

Elastiklik chegarasidan keyin silindrik panel ustuvorligida bu tengsizlik saqlanadi, deb hisoblaymiz.

5.9. Sirtqi bosim ta'siridagi yopiq silindrik qobiq ustuvorligi

Uzunligi ℓ va aylana radiusi R bo'lgan yopiq silindrik qobiq sirti aylananing markaziga radial yo'nalgan tekis taralgan q bosim ta'sirida bo'lsin (5.9- chizma).

Kritik holatgacha qobiq momentsiz holatda bo'lsin, shuning uchun q bosim radial kuchlanishni hosil qiladi [6]:

$$\sigma_y = P_y = \frac{qR}{h}. \quad (9.1)$$

Qolgan boshqa kuchlanishlar nolga teng.

Bu σ_y kuchlanishdan bo'ylama deformatsiyalar hosil bo'ladi:

$$\varepsilon_x = -\frac{1}{2}\varepsilon_y; \quad \varepsilon_z = -(\varepsilon_x + \varepsilon_y) = -\frac{1}{2}\varepsilon_y.$$

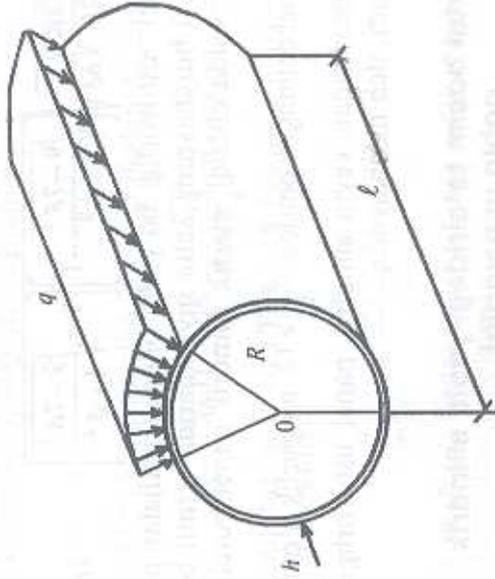
Deformatsiya intensivligi oldingi paragrafdagi kabi $\varepsilon_y = \varepsilon_x = \varepsilon_z = \varepsilon_0$ bo'ladi.

(1.3) funksiyalar esa quyidagiga teng bo'ladi:

$$a_y = \frac{4}{3}\left(\varepsilon_y + \frac{\varepsilon_x}{2}\right) = \frac{4}{3}\left(\varepsilon_y - \frac{\varepsilon_y}{4}\right) = \varepsilon_y; \quad a_x = a_{xy} = 0.$$

Qobiq bifurkatsiya momentidagi muvozanat holatida (9.6) ustuvorlik tenglamasi quyidagi ko'rinishga ega bo'ladi:

$$\frac{D_0}{h} \nabla^2 \nabla^2 \nabla^2 \nabla^2 w - \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0}\right) \nabla^2 \nabla^2 \left(\frac{\partial^4 w}{\partial y^4} + \frac{1}{2} \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) + \frac{\psi_0}{R^2} \frac{\partial^4 w}{\partial x^4} + P_y \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial y^2} = 0. \quad (9.2)$$



5.9- chizma. Tashqi bosim ta'sirida bo'lgan silindirik qobiq.

Tenglamaning ikkinchi hadidagi qavs ichidagi ifodani quyidagicha yozamiz:

$$\left(\frac{\partial^4 w}{\partial y^4} + \frac{1}{2} \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) = \nabla^2 \nabla^2 w - \frac{\partial^4 w}{\partial x^4} - \frac{3}{2} \frac{\partial^4 w}{\partial x^2 \partial y^2}.$$

Unda (9.2) ustuvorlik tenglamasini quyidagi ko'rinishda yozish mumkin:

$$\frac{D_0 E_k}{h \psi_0} \nabla^2 \nabla^2 \nabla^2 \nabla^2 w + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \nabla^2 \nabla^2 \left(\frac{3 \partial^4 w}{2 \partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial x^4} \right) + \frac{\psi_0}{R^2} \frac{\partial^4 w}{\partial x^4} + P_y \nabla^2 \nabla^2 \frac{\partial^2 w}{\partial y^2} = 0. \quad (9.3)$$

Saltqlik funksiyasini sinus bo'yicha ikki qator ko'rinishida qabul qilamiz:

$$w(x, y) = \sum_{m, n} C_{mn} \sin \frac{m \pi x}{l} \sin \frac{n y}{R}. \quad (9.4)$$

(9.4) ifoda silindirik qobiq qirg'og'i bilan sharnirli tayangan chegara shartlarini qanoatlantiradi.

(9.3) ustuvorlik tenglamasiga (9.4) qatorning m va n indeksli hadlarini qo'yib, quyidagi algebrik tenglamaga keltiramiz:

$$\begin{aligned} & \frac{D_0 E_k}{h \psi_0} \left(\frac{m^2 \pi^2}{l^2} + \frac{n^2}{R^2} \right)^4 + \\ & + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \left(\frac{m^2 \pi^2}{l^2} + \frac{n^2}{R^2} \right)^2 \frac{m^2 \pi^2}{l^2} \left(\frac{3 n^2}{2 R^2} + \frac{m^2 \pi^2}{l^2} \right) + \\ & + \frac{\psi_0}{R^2} \frac{m^4 \pi^4}{l^4} - P_y \left(\frac{m^2 \pi^2}{l^2} + \frac{n^2}{R^2} \right)^2 \frac{n^2}{R^2} = 0. \end{aligned} \quad (9.5)$$

Adabiyotlarda [9] ko'rsatilishicha, sirtqi kuch bilan siqilgan silindr bo'ylama yo'nalish bo'yicha bitta-yarimta to'lqin hosil qilib, ustuvorligini yo'qotishi, tajribalar natijalari bilan aniqlangan, (9.5) tenglamaga $m=1$ ni qo'yib, uni quyidagicha yozamiz:

$$\begin{aligned} & \frac{D_0 E_k}{h \psi_0} \left(\frac{\pi^2}{l^2} + \frac{n^2}{R^2} \right)^4 + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \left(\frac{\pi^2}{l^2} + \frac{n^2}{R^2} \right)^2 \frac{\pi^2}{l^2} - \\ & \left(\frac{3 n^2}{2 R^2} + \frac{\pi^2}{l^2} \right) + \frac{\psi_0}{R^2} \frac{\pi^4}{l^4} - P_y \left(\frac{\pi^2}{l^2} + \frac{n^2}{R^2} \right)^2 \frac{n^2}{R^2} = 0. \end{aligned} \quad (9.6)$$

(9.6) tenglamadan P_y aniqlab, izlanayotgan kattalikni quyidagi ifoda orqali yozamiz:

$$\begin{aligned} P_y = & \frac{D_0 E_k}{h \psi_0} \left(\frac{\pi^2}{l^2} + \frac{n^2}{R^2} \right)^2 \frac{R^2}{n^2} + \\ & + \frac{\psi_0}{R^2} \frac{1}{\left(\frac{\pi^2}{l^2} + \frac{n^2}{R^2} \right)^2} \frac{R^2}{n^2} + \frac{\psi_0}{l^4} \frac{\pi^4}{R^4} - \frac{\pi^4}{l^4} \frac{n^4}{R^4} + \\ & + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \frac{\pi^2 R^2}{l^2 n^2} \left(\frac{\pi^2}{l^2} + \frac{3 n^2}{2 R^2} \right). \end{aligned} \quad (9.7)$$

(9.7) tenglamadan keyinchalik foydalanish qulay bo'lishi uchun uni quyidagicha ko'rinishga keltiramiz:

$$P_y = P_1 + \frac{\psi_0}{R^2} \frac{\pi^4 - n^4}{\left(\frac{\pi^2}{\ell^2} + \frac{n^2}{R^2}\right)^2} + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0}\right) \frac{\pi^2 R^2}{\ell^2 n^2} \left(\frac{3n^2}{2R^2} + \frac{\pi^2}{\ell^2}\right). \quad (9.8)$$

Bu yerda:

$$P_1 = \frac{D_0}{h} \frac{E_k}{\psi_0} \lambda + \frac{\psi_0}{R^2} \frac{1}{\lambda}. \quad (9.9)$$

$$\lambda = \frac{R^2}{n^2} \left(\frac{\pi^2}{\ell^2} + \frac{n^2}{R^2}\right). \quad (9.10)$$

P_1 qiymatning minimal bo'lish shartidan parametr λ miqdorini aniqlaymiz:

$$\lambda = \frac{3}{Rh} \sqrt{\frac{\psi_0}{E_k}}. \quad (9.11)$$

(9.11) ifodani (9.9) formulaga qo'yib:

$$(P_1)_{\min} = \frac{2h}{3R} \sqrt{\psi_0 E_k} \quad (9.12)$$

ni hosil qilamiz.

(9.7) tenglamaga kiruvchi ikkinchi funksiyani quyidagicha ifodalaymiz:

$$P_2 = \frac{\psi_0}{R^2} \frac{\pi^4 - n^4}{\left(\frac{\pi^2}{\ell^2} + \frac{n^2}{R^2}\right)^2} \frac{n^2}{R^2} = \frac{\psi_0}{R^2} \frac{\pi^4 R^4}{n^4 \ell^4} - 1 \quad (9.13)$$

(9.10) ifodadan aniqlaymiz:

$$\frac{\pi^2}{\ell^2} = \frac{n}{R} \sqrt{\lambda} - \frac{n^2}{R^2}. \quad (9.14)$$

(9.14) ifodadan $\frac{\pi^4 R^4}{n^4 \ell^4}$ miqdorni aniqlaymiz:

$$\frac{R^4 \pi^4}{\ell^4 n^4} = \left(\frac{n\sqrt{\lambda}}{R} - \frac{n^2}{R^2}\right)^2 \frac{R^4}{n^4} = \frac{R^2}{n^2} \lambda - 2 \frac{R}{n} \sqrt{\lambda} + 1. \quad (9.15)$$

(9.15) ni (9.13) ifodaga qo'yib, quyidagini hosil qilamiz:

$$P_2 = \frac{\psi_0}{R^2} \frac{\lambda - 2 \frac{R}{n} \sqrt{\lambda}}{\lambda} = \frac{\psi_0}{R^2} \left(\frac{R^2}{n^2} - \frac{2R}{n\sqrt{\lambda}}\right). \quad (9.16)$$

(9.16) funksiyani n bo'yicha minimallashtiramiz:

$$\frac{\partial P_2}{\partial n} = \frac{\psi_0}{R^2} \left(-2 \frac{R^3}{n^3} + 2 \frac{R}{n^2} \frac{1}{\sqrt{\lambda}}\right) = 0; \quad -\frac{R}{n} + \frac{1}{\sqrt{\lambda}} = 0;$$

Bundan

$$n = R\sqrt{\lambda}. \quad (9.17)$$

P_2 funksiyaning minimal qiymati quyidagiga teng bo'ladi:

$$(P_2)_{\min} = \frac{\psi_0}{R^2} \left(\frac{1}{\lambda} - \frac{2}{\lambda}\right) = -\frac{\psi_0}{R^2 \lambda} = -\frac{h}{3R} \sqrt{\psi_0 E_k}. \quad (9.18)$$

(9.7) asosiy tenglamaning oxirgi, ya'ni uchinchi hadini quyidagicha ifodalaymiz:

$$P_3 = \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0}\right) \frac{\pi^2 R^2}{\ell^2 n^2} \left(\frac{3n^2}{2R^2} + \frac{\pi^2}{\ell^2}\right) = \frac{\pi^2 h^2}{6\ell^2} (\psi_0 - E_k) \left(1 + \frac{2R^2}{3n^2} \frac{\pi^2}{\ell^2}\right). \quad (9.19)$$

(9.19) formuladan n^2 ni yo'qotib, (9.17) formulaga asosan

$$\frac{R^2}{n^2} = \frac{R^2}{R^2 \lambda} = \frac{1}{\lambda} = \frac{Rh}{3} \sqrt{\frac{E_k}{\psi_0}} \quad (9.20)$$

hosil qilamiz.

(9.20) ifodani e'tiborga olib, P_3 ni quyidagi ko'rinishga keltiramiz:

$$P_3 = \frac{\pi^2 R^2 h^2}{6\ell^2} (\psi_0 - E_k) \left(1 + \frac{2\pi^2 h R^2}{9R} \frac{\pi^2}{\ell^2} \sqrt{\frac{E_k}{\psi_0}}\right). \quad (9.21)$$

(9.8) kritik kuchlanish formulasi quyidagi ko'rinishga ega bo'ladi:

$$P_{\psi_0} = P_y = P_1 + P_2 + P_3 = \frac{1}{3} \frac{h}{R} \sqrt{E_k \psi_0} + \frac{\pi^2 R^2 h^2}{6\ell^2 R^2} (\psi_0 - E_k) \left(1 + \frac{2\pi^2 R^2 h}{9} \frac{\pi^2}{\ell^2} R \sqrt{\frac{E_k}{\psi_0}}\right). \quad (9.22)$$

(9.1) formuladan foydalanib, sirtqi q_{kp} bosimning kritik miqdori ifodasini aniqlaymiz:

$$q_{kp} = \frac{1}{3} \sqrt{\frac{E_k \psi_0}{R^2} + \frac{\pi^2 R^2 h^3}{6 \ell^2 R^3}} (\psi_0 - E_k) \left(1 + \frac{2\pi^2 R^2 h}{9 \ell^2 R} \sqrt{\frac{E_k}{\psi_0}} \right). \quad (9.23)$$

Agar $E = E_k = \psi_0$ bo'lsa, bu murakkab formula soddalashib chiziqli-elastik masala uchun quyidagi ko'rinishga ega bo'ladi:

$$q_{kp} = \frac{E h^3}{3 R^2}. \quad (9.24)$$

Silindrik qobiqlarning ustuvorlik masalalari bo'yicha bu formula adabiyotlarda keltirilmagan, lekin [9] monografiyada q_{kp} sirtqi bosim

$$q_{kp} = \frac{E h^3}{12(1-\mu^2) R^3} \quad (9.25)$$

ko'rinishda berilgan.

Bu yerda: μ — Puasson koeffitsiyenti.

Agar (9.25) formulaga $\mu=0,5$ ni qo'ysak va (9.17) formuladan foydalanib chiziqli-elastik masala uchun $n^2 = R^2 \frac{3}{Rh}$ ekanligini e'tiborga olsak, unda (9.25) ifoda (9.24) tenglama bilan to'liq mos keladi.

5.10. Tekis siqilgan sferik qobiqlar ustuvorligi

Radiusi R , qalinligi h bo'lgan yupqa sferik qobiq, sirti bo'yicha tekis taralgan q kuch ta'sirida bo'lsin. Unda sferaning devorlarida siquvchi kuchlanish hosil bo'ladi:

$$\sigma = \frac{q R}{2 h}. \quad (10.1)$$

Sferik qobiqdan uzunligi ℓ va eni b bo'lgan biror sferik panelni ajratib olamiz.

Panelning kritik holatigacha kuchlanganlik holati:

$$\sigma_x = \sigma_y = \sigma; \quad \sigma_z = 0.$$

Panelning bo'ylama deformatsiyalari:

$$\varepsilon_y = \varepsilon_x = \varepsilon; \quad \varepsilon_z = -(\varepsilon_x + \varepsilon_y) = -2\varepsilon_x.$$

Kuchlanish intensivligi:

$$\sigma_r = \frac{\sqrt{2}}{2} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2} = \frac{\sqrt{2}}{2} \sqrt{2\sigma^2} = \sigma.$$

Deformatsiya intensivligi:

$$\varepsilon_r = \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2} = \frac{\sqrt{2}}{3} \sqrt{18\varepsilon^2} = 2\varepsilon.$$

Kesuvchi model kritik holatda, bifurkatsiya oldida:

$$\frac{\sigma_r}{\varepsilon_r} = \frac{\sigma}{2\varepsilon} = \frac{2\psi_0 \varepsilon}{2\varepsilon} = \psi_0.$$

(1.3) funksiya quyidagini qabul qiladi:

$$a_x = \frac{4}{3}(\varepsilon_x + \varepsilon_y) = 2\varepsilon = \varepsilon_l; \quad a_y = a_x; \quad a_{xy} = 0.$$

Sferik panel egriliklari $K_x = K_y = 1/R$ bo'ladi.

Sferik panel (5.10- chizma) siquvchi kuchlanish ta'sirida mahalliy ustuvorligini yo'qotsin va uning sirtida qarama-qarshi ikki yo'nalishlar bo'yicha kichik to'liqlar hosil bo'lishi mumkin.

Ustivorlikning asosiy tenglamasi sferik qobiq uchun quyidagi ko'rinishga ega bo'ladi:

$$\frac{D_0}{h} \nabla^2 \nabla^2 \nabla^2 w - \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \nabla^2 \nabla^2 \left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + \frac{\psi_0}{R^2} \nabla^2 \nabla^2 w + \sigma \nabla^2 \nabla^2 w = 0.$$

Bu tenglamada

$$\left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = \nabla^2 \nabla^2 w - \frac{\partial^4 w}{\partial x^2 \partial y^2}$$

ekanligini e'tiborga olib, ustuvorlik tenglamasini

$$\frac{D_0}{h} \frac{E_k}{\psi_0} \nabla^2 \nabla^2 \nabla^2 w + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \nabla^2 \nabla^2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\psi_0}{R^2} \nabla^2 \nabla^2 w + \sigma \nabla^2 \nabla^2 w = 0. \quad (10.2)$$

ko'rinishda yozamiz.

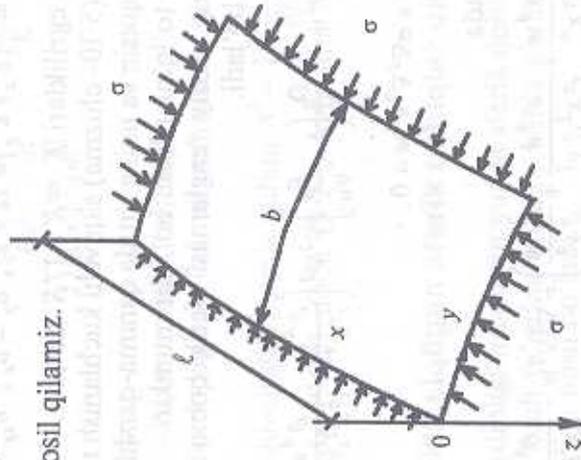
Sferik panel bifurkatsiya momentida mahalliy ustuvorligini cheksiz kichik to'liqlar hosil qilib yo'qotsin deb hisoblaymiz. Bu to'liqlar sinuslar qonuniga asosan hosil bo'lsin:

$$w(x, y) = \sum_m \sum_n C_{mn} \sin \frac{m\pi x}{\ell} \sin \frac{n\pi y}{b}. \quad (10.3)$$

(10.3) qatarning m va n indeksli hadlarini (10.2) ifodaga qo'yib:

$$\begin{aligned} & \frac{D_0}{h} \frac{E_k}{\psi_0} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^4 + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^2 \frac{\pi^4 m^2 n^2}{\ell^2 b^2} + \\ & + \frac{\psi_0}{R^2} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^2 - \\ & - \sigma \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^3 = 0 \end{aligned} \quad (10.4)$$

tenglamani hosil qilamiz.



5.10- chizma. Siquvchi kuchlanishlar ta'siridagi sterik panel.

Bundan:

$$\begin{aligned} \sigma = & \frac{D_0}{h} \frac{E_k}{\psi_0} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^2 + \\ & + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \frac{\pi^4 m^2 n^2}{\left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^2 \ell^2 b^2} + \frac{\psi_0}{R^2} \left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^2 \end{aligned} \quad (10.5)$$

kelib chiqadi.

Bu ifodani quyidagi ko'rinishda yozamiz:

$$\sigma = P_1 + \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \frac{\pi^4 m^2 n^2}{\left(\frac{m^2 \pi^2}{\ell^2} + \frac{n^2 \pi^2}{b^2} \right)^2 \ell^2 b^2}. \quad (10.6)$$

Bu yerda:

$$(P_1)_{\min} = \frac{2h}{3R} \sqrt{\psi_0 E_k}. \quad (10.7)$$

(10.6) ustuvorlik tenglamasini quyidagi ko'rinishda ifodalaymiz:

$$\sigma = \frac{2h}{3R} \sqrt{\psi_0 E_k} + P_2; \quad (10.8)$$

Bu yerda:

$$P_2 = \frac{D_0}{h} \left(1 - \frac{E_k}{\psi_0} \right) \frac{\pi^4 m^2 n^2}{\ell^2 b^2 \lambda} = \frac{1}{27} \sqrt{\psi_0 E_k} \left(1 - \frac{E_k}{\psi_0} \right) \frac{\pi^4 m^2 n^2}{\ell^2 b^2} R \lambda^3. \quad (10.9)$$

α parametрни quyidagi formula asosida kiritamiz:

$$\left(\frac{m\pi}{\ell} \right)^2 = \alpha \lambda. \quad (10.10)$$

Unda:

$$\frac{n^2 \pi^2}{b^2} = \left[\lambda - \left(\frac{m\pi}{\ell} \right)^2 \right] = \lambda (1 - \alpha) \quad (10.11)$$

bo'ladi.

(10.10) ifodani (10.9) formulaga qo'yamiz.

$$P_2 = \frac{1}{3} \sqrt{E_k \psi_0} \left[\left(\frac{\psi_0}{E_k} - 1 \right) \frac{h}{R} \alpha (1 - \alpha) \right]. \quad (10.12)$$

P_2 funksiyaning α bo'yicha minimumini topib:

$$\frac{\partial P_2}{\partial \alpha} = \frac{1}{3} \sqrt{E_k \psi_0} \left[\left(\frac{\psi_0}{E_k} - 1 \right) \frac{h}{R} (1 - 2\alpha) \right] = 0$$

tenglamani hosil qilamiz va undan $\alpha = 0,5$ ni aniqlaymiz.

Unda P_2 funksiyaning minimal qiymati:

$$P_2 = \frac{1}{12} \sqrt{E_k \psi_0} \left(\frac{\psi_0}{E_k} - 1 \right) \frac{h}{R} \quad (10.13)$$

bo'lad.

(10.9) formulaga (10.13) ni qo'yisak, kritik kuchlanish $\alpha\lambda p$ ifodasini quyidagi formula orqali yozish mumkin:

$$\sigma_{kp} = \frac{2}{3} \sqrt{E_k \psi_0} \frac{h}{R} \left[1 + \frac{1}{8} \left(\frac{\psi_0}{E_k} - 1 \right) \right]. \quad (10.14)$$

(10.1) formuladan foydalanib, sferik qobiqning kritik bosimini aniqlaymiz:

$$q_{kp} = \frac{4}{3} \sqrt{E_k \psi_0} \left(\frac{h}{R} \right)^2 \left[1 + \frac{1}{8} \left(\frac{\psi_0}{E_k} - 1 \right) \right]. \quad (10.15)$$

Agar bu formulaga $\psi_0 = E_k = E$ va Poisson koeffitsiyentining 0,5 qiymatini qo'yisak, u bizga ma'lum bo'lgan chiziqli-elastik masala formulasini beradi:

$$q_{kp} = \frac{4}{3} E \left(\frac{h}{R} \right)^2.$$

Nazorat savollari va topshiriqlar

1. Qobiqlar uchun kesuvchi modul ifodalarini yozing.
2. Qobiqlar ustuvorlik tenglamasini yozing.
3. Qobiqlar uzluksizlik tenglamasi ifodasini yozing.
4. Qobiqlar umumiy ustuvorlik tenglamasini yozing.
5. Bo'yлама siqilgan silindrik qobiq ustuvorlik tenglamasini yozing.
6. Bo'yлама siqilgan silindrik qobiq ustuvorlik kritik kuch ifodasini yozing.
7. Bo'yлама siqilgan silindrik qobiq yarim to'liqlarini sonini aniqlovchi formulani yozing.
8. Bo'yлама siqilgan yopiq silindrik qobiq ikki yo'nalish bo'yicha yarim to'liqlar hosil qilib ustuvorlik yo'qotishda kritik kuch ifodasini yozing.
9. Bo'yлама siqilgan yopiq silindrik qobiq ikki yo'nalish bo'yicha yarim to'liqlar hosil qilib, ustuvorlik yo'qotishda yarim to'liqlar sonini aniqlovchi formulani yozing.
10. Silindrik panel uchun kritik kuch ifodasini yozing.
11. Sirtqi bosim ta'siridagi silindrik qobiq uchun kritik kuch ifodasini yozing.
12. Tekis siqilgan sferik qobiq uchun kritik kuch ifodasini yozing.

FOYDALANILGAN ADABIYOTLAR

1. Александров А. В., Потапов В. Д. Основы теории упругости и пластичности, М. Высшая школа. 1990.
2. Алфутов Н.А. Основы расчёта на устойчивость упругих систем. М.: Машиностроение. 1978.
3. Безухов Н. И. Основы теории упругости, пластичности и ползучести. -М.: Высшая школа, 1968.
4. Безухов Н.И. Примеры и задачи по теории упругости пластичности и ползучести. -М.: Высшая школа, 1965.
5. Власов В.З. Общая теория оболочек и её приложения в технике.-М.: Гостехиздат, 1949.
6. Вольмир А.С. Гибкие пластинки и оболочки. -М.: Гостехиздат, 1956.
7. Вольмир А.С. Устойчивость деформируемых систем. М.: Физматгиз. 1961.
8. Грач С.А. Расчёт круглых пластин. -Фрунзе.: Мектеп, 1981.
9. Динник А.Н. Устойчивость упругих систем. ОНТИ, 1935.
10. Зубчанинов В.Г. Основы теории упругости и пластичности.- М.: Высшая школа, 1990.
11. Исмаилов К. Устойчивость сжатых стержней, пластин и оболочек за пределом упругости. ФАН. Самарканд. 2003.
12. Ismayilov K. Elastik siqilgan steyjenlar, plastinkalar va qobiqlarning ustuvorligi. — Samarqand. 2005.
13. Ismayilov K. Siqilgan steyjenlar, plastinkalar va qobiqlarning elastiklik chegarasidan keyingi ustuvorligi. O'zbekiston Respublikasi faylasuflar milliy jamiyati nashriyoti, — Toshkent, 2006.
14. Качанов Л.М. Основы теории пластичности. М.:Наука, 1969.
15. Колкунов Н.В. Основы расчёта упругих оболочек. -М.: Высшая школа, 1987.
16. Лурье А.И. Статика тонкостенных упругих оболочек. Гостехиздат, 1947.
17. Малинин Н.Н. Прикладная теория пластичности и ползучести.- М.: Машиностроение, 1975.
18. Пановка Я.Г., Губанов И.И. Устойчивость и колебания упругих систем. М.:Наука. 1987.

MUNDARIJA

So'zboshi	3
1 bob. QOBIQLAR NAZARIYASINING ASOSIY TENGLAMALARI	
1.1. Asosiy tushunchalar. Qobiqlarning geometriyasi va ularning tasnifi.....	5
1.2. Qobiqdagi deformatsiya va ko'chishlar.....	8
1.3. Qobiq kesimdagi ichki kuchlar va momentlar.....	10
1.4. Qobiq muvozanat differensial tenglamasi.....	14
1.5. Momenttsiz nazariya asosida aylanish sirtli qobiqlar hisobi.....	20
1.6. Momentli nazariya asosida silindrik qobiqlar hisobi.....	22
1.7. O'qqa nisbatan simmetrik yuklangan silindrik qobiqlar hisobi.....	25
1.8. Chetki qirg'oqi bo'ylab yuklangan yarim cheksiz silindrik shaklidagi qobiq hisobi.....	27
1.9. Yassisimon qobiqlar nazariyasi asoslari.....	29
Masalalar.....	34
2 bob. SILINDRIK QOBIQLAR USTUVORLIGI	
2.1. Bo'yлама siqilgan silindrik qobiqlarning o'qqa simmetrik qavarish holatidagi ustuvorligi	41
2.2. Yassi qobiqlar momentli nazariyasi asosiy munosabatlari. Ustuvorlik tenglamasi.....	46
2.3. Bo'yлама siqilgan qobiqlarning o'qqa nosimmetrik deformatsiya holidagi ustuvorligi.....	52
2.4. Bo'yлама siqilgan qobiq chegara shartlarining kritik kuchga ta'siri.....	57
2.5. Kritik kuch miqdoriga boshlang'ich xatolik va kritik holatgacha bo'lgan qobiq momentli holatining ta'siri.....	61
2.6. Bo'yлама siqilgan silindrik qobiqning katta salqilgini e'tiborga olganda ustuvorlik	67
2.7. Silindrik panellar ustuvorligi.....	73
2.8. Tashqi kuch ta'siridagi silindrik qobiqlar ustuvorligi.....	75
2.9. Silindrik qobiqning buralishdagi ustuvorligi.....	79

**3 bob. PLASTIKLIK NAZARIYASI
ASOSIY QONUNLARI**

3.1. Plastiklik sharti.....	81
3.2. Maksimal urinma kuchlanishning doimiylik sharti. Tresk-Sen-Venan sharti.....	83
3.3. Urinma kuchlanish intensivligining doimiylik sharti. Mizes-Guber sharti.....	85
3.4. Kichik elastik-plastik deformatsiya nazariyasi.....	86
3.5. Plastik oquvchanlik nazariyasi.....	87

**4 bob. ELASTIKLIK CHEGARASIDAN KEYIN QOBIQLARNING
KUCHLANGANLIK VA DEFORMATSIYA HOLATI**

4.1. Yupqa qobiqlar nazariyasining asosiy munosabatlari.....	92
4.2. O'qqa simmetrik deformatsiyalanuvchi silindrik qobiqlarning elastik-plastik holati.....	95
4.3. Elastiklik chegarasidan keyin paralleli bo'yicha to'plangan yuk ta'siridagi yopiq silindrik qobiq kuchlanganlik-deformatsiya holati.....	106

**5 bob. SILINDRIK VA SFERIK QOBIQLARNING ELASTIKLIK
CHEGARASIDAN KEYINGI USTUVORLIGI**

5.1. Qobiqlarning birinchi tur ustuvorligini yo'qotish momentidagi kuchlanish va deformatsiyalar.....	118
5.2. Bifurkatsiya momentida qobiq kesimlaridagi bo'ylama kuchlar, eguvchi va buruvchi momentlar.....	120
5.3. Qobiq elementining muvozanat tenglamasi.....	123
5.4. Deformatsiyalarning uzluksizlik tenglamasi.....	127
5.5. Qobiq ustuvorlik tenglamasi.....	129
5.6. Bo'ylama siqilgan yopiq silindrik qobiqda o'qqa simmetrik to'liq hosil bo'lishidagi ustuvorlik.....	131
5.7. Bo'ylama siqilgan yopiq silindrik qobiqning ikki yo'nalish bo'yicha yarim to'liqlarini hisobga olganda ustuvorlik.....	135
5.8. Siqilgan silindrik panel ustuvorligi.....	141
5.9. Sirtqi bosim ta'siridagi yopiq silindrik qobiq ustuvorligi.....	145
5.10. Tekis siqilgan sferik qobiqlar ustuvorligi.....	150
6. Foydalanilgan adabiyotlar.....	155

K. ISMAYILOV

**ELASTIK-PLASTIK
QOBIQLAR**

Oliti o'quv yurtlari uchun o'quv qo'llanma

*Muharrir To'liqin Alimov
Rassom Aiyona Delyagina
Badiiy muharrir Shuhrat Odilov
Texnik muharrir Yelena Tolochko
Musahhih Mahmuda Usmonova*

Kompyuterda sahifalovchi Dilnoza Mansurova

Bosishga ruxsat etildi 11.08.2008. Bichimi 60×90^{1/16}. Tayms TAD gamitulasi. Sharti
b.t. 10.0. Nashr b.t. 8.0. Shartnoma № 54—2008. 500 nusxada. Buyurtma № 549.

O'zbekiston Matbuot va axborot agentligining Cho'lpon nomidagi nashriyot-matbaa
ijodiy uyi. 100129, Toshkent, Navoiy ko'chasi, 30-uy.

«Shoakbar» xususiy ilmiy ishlab chiqarish korxonasi. 100031. Toshkent, To'g'on
Rejimetov ko'chasi, 1a.

