O`zbekiston Respublikasi Vazirlar Mahkamasi huzuridagi Toshkent Islom universiteti "Informatika va axborot texnalagiyalari" yo`nalishi I-kurs talabasi

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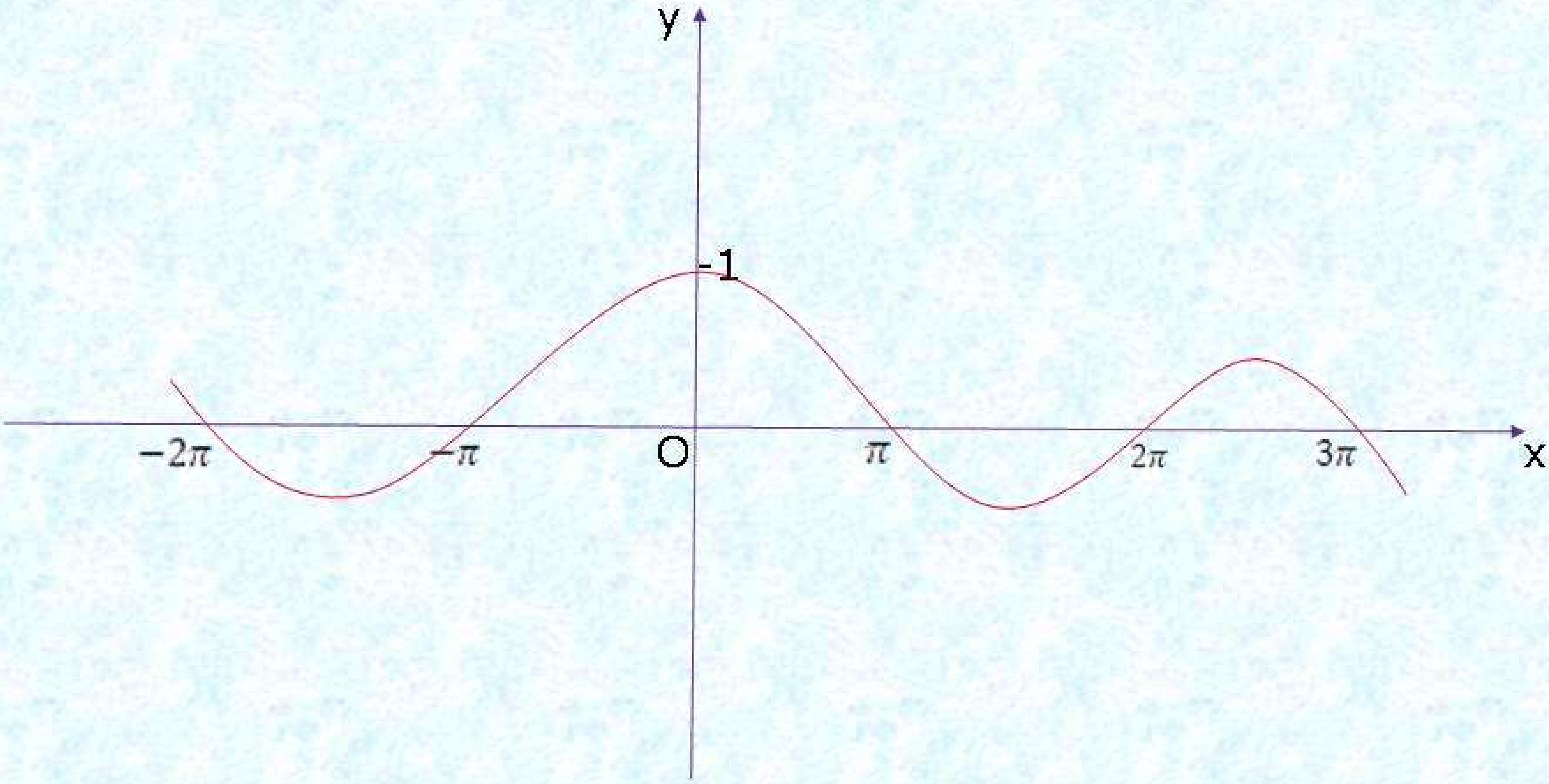
Mavzu: Birinchi ajoyib limit.

$y = \frac{sinx}{x}$ *funksiyaning grafigi.*



Misollar.

$$\lim_{x\to 0}\frac{tgx}{x} = \lim_{x\to 0}\frac{sinx}{x}\cdot\frac{1}{cosx} =$$

$$= \lim_{x\to 0}\frac{sinx}{x}\cdot\lim_{x\to 0}\frac{1}{cosx} = 1\cdot\frac{1}{1} = 1$$

$$\lim_{x\to 0}\frac{sinkx}{kx} = \lim_{x\to 0} k\frac{sinkx}{kx} = k\lim_{x\to 0}\frac{\sin(kx)}{(kx)} = k\cdot 1 = k$$

$$(k = conct)$$

$$\lim_{x\to 0}\frac{1-cosx}{x}=\lim_{x\to 0}\frac{2sin^2\frac{x}{2}}{x}=$$

$$=\lim_{x\to 0}\frac{sin\frac{x}{2}}{\frac{x}{2}}sin\frac{x}{2}=1\cdot 0=0$$

$$\lim_{x\to 0}\frac{sin\alpha x}{sin\beta x}=\lim_{x\to 0}\frac{\alpha}{\beta}\cdot\frac{\frac{sin\alpha x}{\alpha x}}{\frac{sin\beta x}{\beta x}}=\frac{\alpha}{\beta}\frac{\lim_{x\to 0}\frac{sin\alpha x}{\alpha x}}{\lim_{x\to 0}\frac{sin\beta x}{\beta x}}=\frac{\alpha}{\beta}\cdot\frac{1}{1}=\frac{\alpha}{\beta}$$

$$(\alpha = conct, \beta = const).$$

E’tiboringiz uchun katta rahmat.

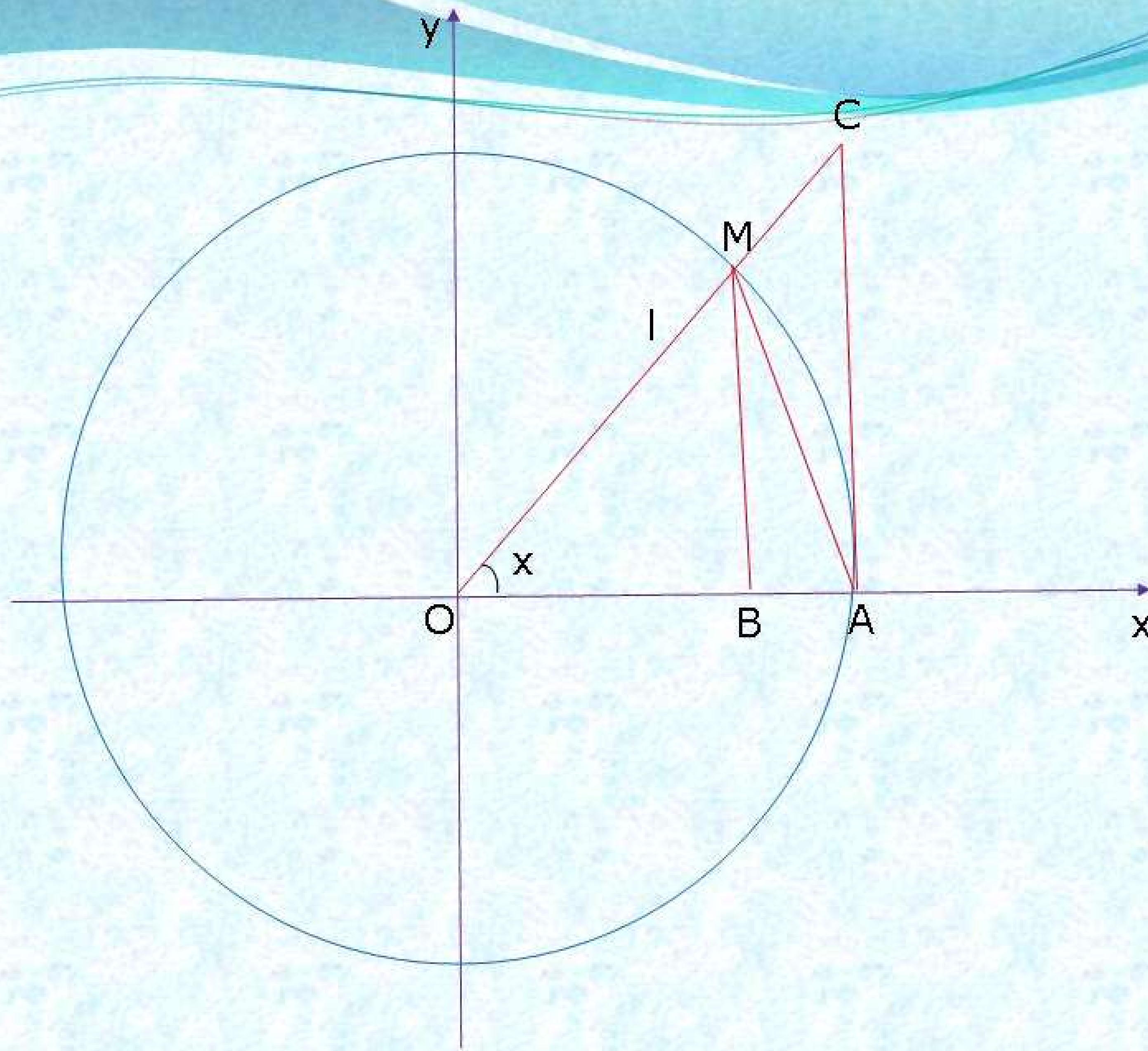
Reja.

1. Birinchi ajoyib limit teoremasi.
2. Birinchi ajoyib limit teoremasining isboti.
3. Birinchi ajoyib limit teoremasiga oid misollar.

Birinchi ajoyib limit teoremasi.

Teorema. $\frac{\sin x}{x}$ funksiya $x \to o$ da 1 ga teng limitiga ega.

Teorema. Agar uchta *u=u(x)* ,*z=z(x)*, *v=v(x)* funksiyaning tegishli qiymatlari orasida $u \le z \le v$ tengsizliklar bajarilsa , bunda $x \to a$ *u(x)* va *v(x)* birgina b limitga intilsa ,u holda da $x \to a$ *z=z(x)* ham shu limitga intiladi.



1-rasm

Isboti. $\frac{sinx}{x}$ funksiya *x=0 da aniqlanmagan, chunki kasrning surat va maxraji nolga aylanadi.Bu funkisiyaning* $x \to 0$ *dagi limitini topamiz.Radiusi 1ga teng bo`lgan aylanani qaraymiz (1-rasm);*

- MOB markaziy burchakni x bilan belgilaymiz, bunda $0 < x < \frac{\pi}{2}$. 1-rasmdan bevosita quydagilar chiqadi:
Uchburchak MOA yuzi$<$ MOA sektor yuzi$<$ Uchburchak COA yuzi .
Uchburchak MOA yuzi=$\frac{1}{2} \cdot OA \cdot MB = \frac{1}{2} \cdot 1 \cdot sinx = \frac{1}{2} sinx$
Uchburchak MOA sektor yuzi=$\frac{1}{2} OA \cdot \overset{\frown}{MA} = \frac{1}{2} \cdot 1 \cdot x = \frac{1}{2} x.$
Uchburchak COA yuzi=$\frac{1}{2} OA \cdot AC = \frac{1}{2} \cdot 1 \cdot tgx = \frac{1}{2} \cdot tgx.$

- (1)tengsizlik $\frac{1}{2}$ ga qisqartirilgandan so`ng bunday yoziladi:

$$sinx < x < tgx.$$

- Hamma hadlarni *sinx* ga bo`lamiz:

$$1 < \frac{x}{sinx} < \frac{1}{cosx}.$$

- Yoki

$$1 > \frac{sinx}{x} > cosx.$$

- Biz bu tengsizlikni $x > 0$ faraz qilib chiqardik:

$$\frac{\sin(-x)}{(-x)} = \frac{sinx}{x}$$ va $$\cos(-x) = cosx$$

- Ekanligini e'tiborga olib, bu tengsizlikni $x > 0$ da ham to`gri degan xulosaga kelamiz. Ammo

$$\lim_{x\to 0} cosx = 1 \qquad \lim_{x\to 0} 1 = 1$$

- Demak, $\frac{\sin x}{x}$ o`zgaruvchi shunday ikki miqdor oralig`idaki ularning ikkalssi ham birgina limitga intiladi va u limit 1 ga teng;Shunday qilib 2-teoremasiga asosan

$$\lim_{x \to 0} \frac{sinx}{x} = 1.$$