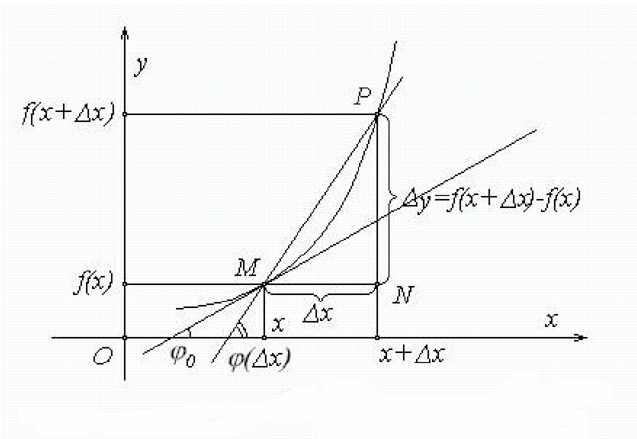
TOSHKENT VILOYAT DAVLAT PEDAGOGIKA INSTITUTI
MATEMATIKA KAFEDRASI

Ismailov Sh.N.

Matematik tahlil

II

(pedagogika institutlari talabalari uchun o'quv-uslubiy qo'llanma)



readlib(singular): singular(f(x),x);

Angren -2006

Annotatsiya

Ushbu o'quv qo'llanma «Matematika va informatika» bakalavriat yo'nalishi bo'yicha «Matematik tahlil» fanidan tuzilgan bo'lib, hosila va differensial, ularning geometrik va fizik ma'nolari; differensial hisobning asosiy teoremalari; integrallash usullari, aniq integral va uning tadbiqlari o'rganilishi mo'ljallangan.

Qo'llanma 2- semestrga doir 38-soatli ma'ruzalar kursining matni, oraliq nazorat savollarini, mustaqil ish uchun ko'p variantli individual topshiriqlarni o'z ichiga olgan. Qo'llanmadagi deyarli barcha masalalar yechish metodikasi Maple® komp'yuter dasturiga tayangan.

Tuzuvchi: fizika-matematika fanlari nomzodi **Ismailov Sh.N.**

Taqrizchi: fizika-matematika fanlari nomzodi , dots. Bekmatov Sh.

O'quv-uslubiy qo'llanma ______________ Ilmiy Kengashida ko'rib chiqilgan va nashrga tavsiya qilingan.

2006 yil «_______» ______________ -sonli majlis bayoni.

Mundarija

KIRISh

O'zbekiston Respublikasining «Ta'lim to'g'risida»gi Qonuni, «Kadrlar tayyorlash milliy dasturi» va «Davlat ta'lim standartlari» talablaridan kelib chiqqan holda matematik tahlil fanini o'qitishning umumiy maqsad va vazifalari kelib chiqadi.

Pedagogika oliy o'quv yurtlarida matematik tahlil fanini o'qitishning asosiy vazifasi-ta'limning insonparvarlashuvi va ijtimoiylashuviga erishish; hozirgi zamon sharoitlaridan kelib chiqqan holda har bir bo'lajak mutaxassisni uning mehnat faoliyati va kundalik hayoti uchun zarur bo'lgan matematik bilim, ko'nikma va malakani berishdan iborat.

2 – semestrda qo'yidagi bilimlarni egallagash ko'zda tutilgan:

- hosila va differensial, ularning geometrik va fizik ma'nolari;
- differensial hisobning asosiy teoremalari;
- aniq integral va uning tatbiqlari;

Talabalar 2 – semestr bo'yicha quyidagi ko'nikmalar hosil qilishi lozim:

- funksiyaning hosilasi va differensialini hisoblash, ularga oid tatbiqiy masalalarni yechish;
- hosila yordamida funksiyani to'la tekshirish va grafigini chizish;
- aniqmas va aniq integrallarga doir misollar yechish;
- aniq integralni geometrik va fizik kattaliklarni hisoblashga tatbiq qila olish;

«Matematik tahlil» fanining o'qitilishida simvolik hisob-kitoblarga muljallangan Maple komp'yuter dasturidan foydalaniladi. Bu asosan amaliy mashg'ulotlar paytida va individual topshiriqlar bajarilganda amalga oshiriladi.

Qo'llanmani tuzishda asosan Nizomiy nomli TVDPU professor-o'qituvchilari tomonidan tuzilgan namunaviy dastur va metodik materiallardan foydalanilgan. Ayrim metodik materiallar Internetdan olingan.

Mazkur qo'llanma g'oyasi tuzuvchi O'z R Prezidentining «Iste'dod» jamg'armasi granti sohibi sifatida Shanxay Davlat Universitetida (Ismailov Sh., 2004 y.) axborot texnologiyalarni ta'limga joriy etish bo'yicha malaka oshirish kursini o'taganlarida mazkur universitetda matematik tahlil fanini o'qitish jarayonini ko'zatish paytida vujudga keldi.

1,2-ma’ruzalar

Hosilaning ta’rifi, uning geometrik va mexanik ma’nolari.
Differensiallanuvchi funksiyaning uzluksizligi.

Reja:
1. Funksiya hosilasining ta’rifi.
2. Bir tomonli hosilalar.
3. Hosilaning geometrik va mexanik ma’nolari.
4. Differensiallanuvchi funksiyaning uzluksizligi.

Adabiyot :
1. [1] - 182-188 betlar;
2. [2] –109-113 betlar;

Funksiya hosilasining ta’rifi.

$y=f(x)$ funksiya $(a;b)$ intervalda aniqlangan bo’lsin. Bu intervalga tegishli x_0 nuqta olib, unga shunday Δx $(\Delta x\geq 0)$ orttirma beraylikki, $x_0+\Delta x\in(a;b)$ bo’lsin. Natijada $y=f(x)$ funksiya ham x_0 nuqtada

$$\Delta y=\Delta f(x_0)=f(x_0+\Delta x)-f(x_0)$$

orttirmaga ega bo’ladi.

1-ta’rif. Agar $\Delta x\to 0$ da $\frac{\Delta y}{\Delta x}$ nisbatning limiti

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x_0)-f(x_0)}{\Delta x}$$

mavjud va chekli bo’lsa, bu limit $y=f(x)$ funksiyaning x_0 nuqtadagi *hosilasi* deyiladi va $f'(x_0)$, yoki $\frac{dy}{dx}\Big|_{x=x_0}$ orqali belgilanadi.

Demak,

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} \qquad (1)$$

Bunda $x_0+\Delta x=x$ deb olaylik. Unda $\Delta x=x-x_0$ va $\Delta x\to 0$ da $x\to x_0$ bo’lib, natijada

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}=\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}$$

tenglikni hosil qilamiz. Demak, $y=f(x)$ funksiyaning x_0 nuqtadagi hosilasi $x\to x_0$ da

$$\frac{f(x)-f(x_0)}{x-x_0}$$

nisbatning limiti sifatida ham ta’riflanishi mumkin:

$$f'(x_0)=\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0} \qquad (2)$$

Agar $y=f(x)$ funksiya $(a;b)$ intervalning har bir x nuqtasida hosilaga ega bo’lsa, bu hosila x o’zgaruvchining funksiyasi bo’ladi.

Bunda $y'(x)=f'(x)=\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$

. *Maple* dasturida $f'(x)$ hosila **diff(f(x),x); D(f(x));** operatorlar yordamida topiladi.

Eslatma. Agar x_0 nuqtada

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=+\infty \text{ yoki } \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=-\infty$$

shart bajarilsa, bunday holda $y=f(x)$ funksiyaning x_0 nuqtadagi hosilasi $+\infty$ (yoki $-\infty$) ga teng deb qabul qilingan.

Misollar. 1. $f(x)=C=const$ bo'lsin. Bu funksiyaning $\forall x\in R$ nuqtadagi orttirmasi $\Delta y=f(x+\Delta x)-f(x)=C-C=0$ bo'lib, undan

$$y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=0$$

kelib chiqadi. Demak, doimiy funksiyaning hosilasi nolga teng.

2. $f(x)=x$ bo'lsin. Bu funksiya uchun $\frac{\Delta y}{\Delta x}=\frac{(x+\Delta x)-(x)}{\Delta x}=1$ bo'lib, undan $f(x)=x$ funksiyaning ixtiyoriy $x\in R$ nuqtadagi hosilasining 1 ga teng bo'lishini topamiz.

3. $f(x)=\sin x$ funksiyaning ixtiyoriy $x\in R$ nuqtadagi hosilasini hisoblaylik. Bu funksiya uchun

$$\Delta y=\sin(x+\Delta x)-\sin x=2\cos(x+\frac{\Delta x}{2})\sin\frac{\Delta x}{2}$$

bo'lib,

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\cos(x+\frac{\Delta x}{2})\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}=\cos x$$

bo'ladi. Demak, $(\sin x)'=\cos x,\ x\in R$.

4. $y=x^{\alpha}, x>0$

$$\Delta y=(x+\Delta x)^{\alpha}-x^{\alpha}=x^{\alpha}\left[(1+\frac{\Delta x}{x})^{\alpha}-1\right]$$

$$y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{(1+\frac{\Delta x}{x})^{\alpha}-1}{\Delta x}=x^{\alpha-1}\cdot\lim_{\Delta x\to 0}\frac{(1+\frac{\Delta x}{x})^{\alpha}-1}{\frac{\Delta x}{x}}=\alpha\cdot x^{\alpha-1}$$

Bu yerda $\lim_{x\to 0}\frac{(1+x)^{\alpha}-1}{x}=\alpha$ tenglikdan foydalanildi.

Bir tomonli hosilalar.

2-ta'rif: Agar $y=f(x)$ funksiya x_0 nuqtaning biror o'ng (chap) atrofida aniqlangan bo'lib, ushbu chekli yoki cheksiz $\lim_{\Delta x\to +0}\frac{\Delta y}{\Delta x}$ ($\lim_{\Delta x\to -0}\frac{\Delta y}{\Delta x}$)

limit mavjud bo'lsa, bu limit $y=f(x)$ funksiyaning x_0 nuqtadagi *o'ng* (*chap*) *hosilasi* deb ataladi va $f'(x_0+0)$ ($f'(x_0-0)$) kabi belgilanadi.

Odatda funksiyaning o'ng va chap hosilalari *bir tomonli hosilalar* deb ataladi

Funksiya hosilasi haqidagi 1- va 2- ta'riflardan hamda funksiya limiti haqidagi teoremalardan quyidagi teorema kelib chiqadi:

Teorema . Agar $y=f(x)$ funksiya x_0 nuqtaning biror atrofida aniqlangan bo'lsa, u holda uning $f'(x_0)$ hosilaga ega bo'lishi uchun $f'(x_0+0)$, $f'(x_0-0)$ lar mavjud va $f'(x_0+0)=f'(x_0-0)$ tenglik o'rinli bo'lishi zarur va yetarlidir.

Misol. $f(x)=|x|$ ni qaraylik. Bu funksiyaning $x=0$ nuqtadagi orttirmasi $\Delta y=|\Delta x|$ bo'lib, agar $\Delta x<0$ bo'lsa $\frac{\Delta y}{\Delta x}=-1$, va agar $\Delta x>0$ bo'lsa $\frac{\Delta y}{\Delta x}=1$ bo'ladi. Bu holda

$$\lim_{\Delta x\to+0}\frac{\Delta y}{\Delta x}=1\ ,\quad \lim_{\Delta x\to-0}\frac{\Delta y}{\Delta x}=-1$$

Demak, $f(x)=|x|$ funksiyaning $x=0$ nuqtadagi o'ng hosilasi 1 ga, chap hosilasi -1 ga teng, ammo bu funksiya $x=0$ nuqtada hosilaga ega emas, chunki

$$f'(-0)\neq f'(+0).$$

Hosilaning geometrik va mexanik ma'nolari

a) *Hosilaning geometrik ma'nosi.*

$y=f(x)$ funksiya $(a;b)$ intervalda aniqlangan bo'lib, $x\in(a;b)$ nuqtada $f'(x)$ hosilaga ega bo'lsin. Shu funksiya grafigining $M(x,f(x))$ nuqtasida urinma o'tkazish masalasini qaraylik (1- chizma).

M urinish nuqtasi berilgan. Izlanayotgan urinma tenglamasini tuzish uchun uning burchak koeffitsienti $k=tg\varphi$ ni topish kifoya. Buning uchun grafik ustida M nuqtadan farqli $P(x+\Delta x,f(x+\Delta x))$ nuqtani olib, bu nuqtalar orqali MP kesuvchi o'tkazamiz.

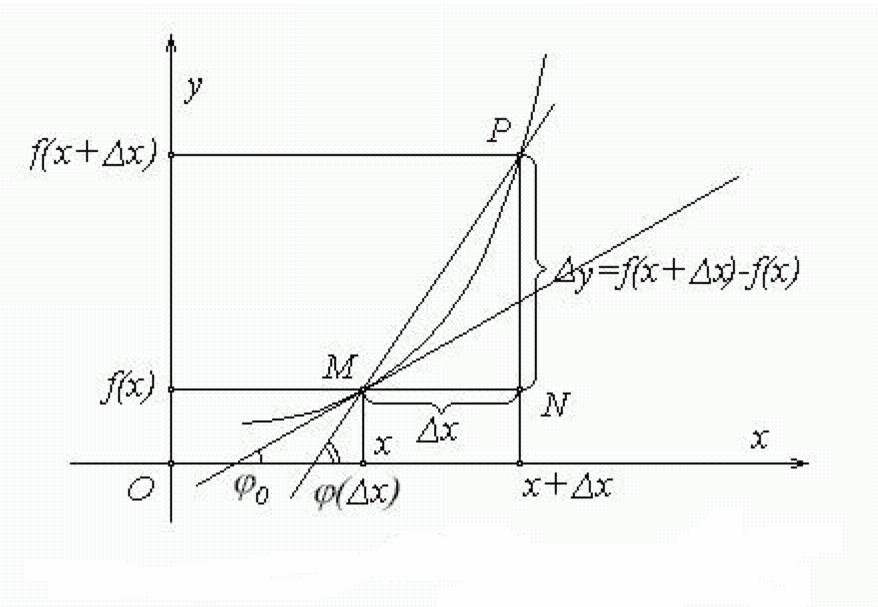
Bu kesuvchi Ox o'qi bilan tashkil etgan burchakni φ bilan belgilaylik. Ravshanki, φ burchak Δx ga bog'liq bo'ladi: $\varphi=\varphi(\Delta x)$.

Agar MP kesuvchining P nuqta $y=f(x)$ funksiya grafigi bo'ylab M ga intilgandagi (ya'ni $\Delta x\to0$ dagi) limit holati mavjud bo'lsa, kesuvchining bu limit holati shu funksiya grafigiga M nuqtada o'tkazilgan *urinma* deb ataladi.

ΔNMR dan

$$tg(\varphi(\Delta x))=\frac{PN}{MN}=\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}\qquad(3)$$

$$\lim_{\Delta x\to0}tg(\varphi(\Delta x))=tg(\varphi_0)\qquad(4)$$



1-chizma

(3) va (4) tengliklardan $f'(x)=\lim\limits_{\Delta x\to0}\dfrac{f(x_0+\Delta x)-f(x_0)}{\Delta x}=tg\varphi_0$

bo'lishini topamiz. Shunday qilib, $f(x)$ funksiya $x\in(a;b)$ nuqtada $f'(x)$ hosilaga ega bo'lsa, bu funksiya grafigiga $M(x,f(x))$ nuqtada o'tkazilgan urinma mavjud. Funksiyaning x nuqtadagi hosilasi $f'(x)$ esa bu urinmaning burchak koeffitsientini ifodalaydi. Urinmaning tenglamasi esa ushbu

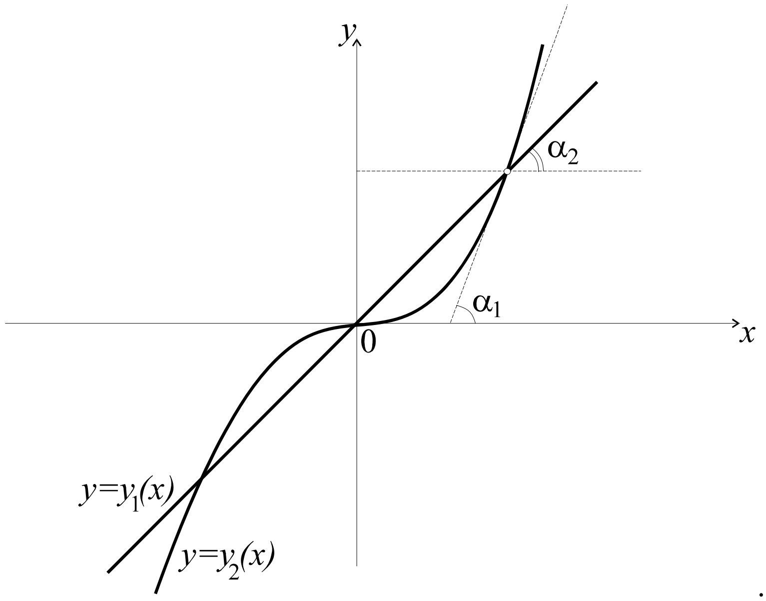
$$Y=f(x)+f'(x)(X-x)$$

ko'rinishda bo'ladi.

Misol. $y_1=x$ va $y_2=x^3$ funksiyalar grafiklari qaysi burchak ostida kesishadi?

Yechish. $x=x^3;\ x(x-1)(x+1)=0;\ x_1=0,\ x_2=-1,\ x_3=1.\ y_1'=1,\ y_2'=3x^2.$

1. $x_1 = 0, y_1' = 1, y_2' = 0.$ $\operatorname{tg}\alpha_1 = \frac{1-0}{1+0} = 1 \Rightarrow \alpha_1 = \frac{\pi}{4}$



2. $x_2 = -1, y_1' = 1, y_2' = 3.$ $\operatorname{tg}\alpha_2 = \frac{3-1}{1+3} = \frac{1}{2} \Rightarrow \alpha_2 = \operatorname{arctg}\frac{1}{2}.$

3. Simmetriya bo'lgani uchun $\alpha_3 = \alpha_2 = \operatorname{arctg}\frac{1}{2}.$

Bu yerda $\operatorname{tg}\alpha = \operatorname{tg}(\alpha_1 - \alpha_2) = \frac{\operatorname{tg}\alpha_1 - \operatorname{tg}\alpha_2}{1 + \operatorname{tg}\alpha_1 \operatorname{tg}\alpha_2}$ formula ishlatildi

. Maple dasturi yordamida bu misol qo'yidagicha yechiladi:

```
>solve(x=x^3,x);
 0   1   -1
>a:=diff(x,x); b:=diff(x^3,x);
 a:=1
 b:=3*x^2
>arctan(subs(x=0,(a-b)/(1+a*b)));
 1/4*Pi
>arctan(subs(x=1,(b-a)/(1+a*b)));
 arctan(1/2)
>arctan(subs(x=-1,(b-a)/(1+a*b)));
 arctan(1/2)
```

b) Hosilaning mexanik ma'nosi.

Moddiy nuqtaning to'g'ri chiziqli harakat qonuni $s=f(t)$ ga ko'ra uning $t=t_0$ paytdagi oniy tezligini topish masalasini qaraylik. Nuqtaning t_0 va $t_0+\Delta t$ ($\Delta t>0$) vaqtlar orasida bosib o'tgan yo'li $\Delta s=f(t_0+\Delta t)-f(t_0)$ bo'lgani uchun uning shu vaqt oraliqidagi o'rtacha tezligi

$$\frac{\Delta s}{\Delta t} = \frac{f(t_0+\Delta t)-f(t_0)}{\Delta t}$$

bo'ladi. $\Delta t \to 0$ da $\frac{\Delta s}{\Delta t}$ nisbatning limiti moddiy nuqtaning t_0 momentdagi oniy tezligi v ni ifodalaydi.

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \lim_{\Delta t \to 0} \frac{f(t_0+\Delta t)-f(t_0)}{\Delta t}$$

hosila ta'rifiga ko'ra

$$\lim_{\Delta t\to 0}\frac{f(t_0+\Delta t)-f(t_0)}{\Delta t}=f'(t_0)$$

Demak, $s=f(t)$ funksiyaning t_0 nuqtadagi hosilasi mexanik nuqtai nazardan $s=f(t)$ qonun bilan harakat qilayotgan moddiy nuqtaning t_0 momentdagi oniy tezligini bildiradi.

Differensiallanuvchi funksiyaning uzluksizligi

3-ta'rif: Agar $y=f(x)$ funksiya $x=x_0$ nuqtada chekli $f'(x_0)$ hosilaga ega bo'lsa, u shu nuqtada *differensiallanuvchi* deyiladi.

Teorema . $(a;b)$ intervalda aniqlangan $y=f(x)$ funksiya $x_0\in(a;b)$ nuqtada uzluksiz bo'lishi uchun chekli $f'(x_0)$ hosilaga ega bo'lishi yetarlidir.

Haqiqatan ham funksiya x_0 da differensialnuvchi bo'lgani sababli shu nuqtada chekli $f'(x_0)$ hosila mavjud. Hosila ta'rifiga ko'ra,

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x},$$

ya'ni $\Delta x\to 0$ da $\frac{\Delta y}{\Delta x}\to f'(x_0)$

Endi $\alpha=\frac{\Delta y}{\Delta x}-f'(x_0)$ miqdorni kiritamiz. $\Delta x\to 0$ da $\frac{\Delta y}{\Delta x}$ o'zgaruvchi bilan limitning ayirmasi cheksiz kichik bo'lgani uchun, α o'zgaruvchi miqdor bo'lib, u Δx ga bog'liq va $\Delta x\to 0$ da nolga intiladi.

Bu tenglikdan topamiz: $\Delta y=f'(x_0)\,\Delta x+\alpha\Delta x$. Bundan

$$\lim_{\Delta x\to 0}\Delta y=\lim_{\Delta x\to 0}(f'(x_0)\,\Delta x+\alpha\Delta x)=0$$

kelib chiqadi. Demak, $y=f(x)$ funksiya x_0 nuqtada uzluksizdir.

Funksiyaning uzluksiz bo'lishi uchun chekli hosilaning mavjud bo'lishi faqat yetarli shart bo'lib, zaruriy shart emas, ya'ni funksiyaning biror nuqtada uzluzsizligidan uning shu nuqtada chekli hosilaga ega bo'lishi har doim kelib chiqavermaydi. Masalan, $y=|x|$ funksiya $x=0$ nuqtada uzluksiz, ammo u shu nuqtada hosilaga ega emas.

Oraliq nazorat savollari:

1. Funksiyaning hosilasi qanday ta'riflanadi?
2. Ta'rif bo'yicha funksiya hosilasini topish uchun nima ishlar bajariladi?
3. Funksiyaning hosilasi funksiya bo'lishini qanday tushunasiz?
4. Hosilaning geometrik va mexanik ma'nolari nimalardan iborat?
5. $f(x)=x^2\sin\frac{1}{x}$ funksiya $x=0$ da hosilaga egami ?
6. Funksiyaning bir tomonli hosilalari qanday ta'riflanadi?
7. Funksiyaning nuqtada hosilaga ega bo'lishining zaruriy va yetarli shartlari nimadan iborat?
8. Funksiya grafigiga o'tkazilgan urinma qanday ta'riflanadi?
9. Hosilaning geometrik ma'nosi nimadan iborat?
10. Egri chiziqga o'tkazilgan urinma tenglamasi qanday yoziladi?
11. Egri chiziqga o'tkazilgan normal tenglamasi qanday yoziladi?
12. Hosilaning mexanik ma'nosi nimadan iborat?
13. Differensiallanuvchi funksiyaning uzluksizligi qanday isbotlanadi?
14. Uzluksiz funksiya differensiallanuvchi bo'ladi-mi? Javobingizni asoslang.

Tayanch tushunchalar: funksiya hosilasi, bir tomonli hosilalar, hosilaning geometrik va mexanik ma'nolari, uzluksizlik.

3,4- ma’ruzalar.

Yig’indi, ko’paytma va bo’linmaning hosilasi. Murakkab funksiyaning hosilasi. Teskari funksiyaning hosilasi.

Asosiy elementar funksiyalarning hosilalari.

Reja:

1. Yig’indi, ko’paytma va bo’linmaning hosilasi.
2. Murakkab funksiyaning hosilasi
3. Teskari funksiyaning hosilasi.
4. Elementar funksiyalarning hosilalari.
5. Hosilalar jadvali.

Adabiyot :

1. [1] - 188-196 betlar;
2. [2] - 115-120 betlar;

Teorema . Agar $u(x)$ va $v(x)$ funksiyalar x nuqtada differensialanuvchi bo’lsa, u holda ularning yig’indisi, ayrimasi, ko’paytmasi va bo’linmasining (agar $v(x)\neq 0$ bo’lsa) bu nuqtada ham differensiallanuvchi bo’lib, qo’yidagi formulalar o’rinli:

$$[u(x)\pm v(x)]' = u'(x)\pm v'(x),$$

$$[u(x)v(x)]' = u'(x)v(x)+u(x)v'(x),$$

$$\left[\frac{u(x)}{v(x)}\right]' = \frac{u'(x)v(x)-u(x)v'(x)}{v^2(x)}.$$

Isbot. Δu ,Δv va Δy orqali $u(x)$, $v(x)$ va $y(x)$ funksiyalarning x nuqtadagi Δx ga mos orttirmalarni belgilaymiz

$u(x)$ va $v(x)$ funksiyalar x nuqtada differensiallanuvchi bo’lsa, u holda ular bu nuqtada uzluksiz bo’lib, qo’yidagi formulalar o’rinli:

$$\lim_{\Delta x\to 0}\frac{\Delta u}{\Delta x} = u'(x),\ \lim_{\Delta x\to 0}\frac{\Delta v}{\Delta x} = v'(x),\ \lim_{\Delta x\to 0} u(x+\Delta x) = u(x)\ ,\ \lim_{\Delta x\to 0} v(x+\Delta x) = v(x) \quad (1)$$

1) $y(x) = u(x)\pm v(x)$ bo’lsin. Ravshanki,

$$\Delta y = y(x+\Delta x)-y(x) = [u(x+\Delta x)\pm v(x+\Delta x)]-[u(x)\pm v(x)] =$$
$$[u(x+\Delta x)-u(x)]\pm[v(x+\Delta x)-v(x)] = \Delta u\pm\Delta v$$

Demak, agar $\Delta x\neq 0$ bo’lsa,

$$\frac{\Delta y}{\Delta x} = \frac{\Delta u}{\Delta x}\pm\frac{\Delta v}{\Delta x}$$

formula o’rinli.

$\Delta x\to 0$ bo’lsa, (1) ga ko’ra oxirgi formulaning o’ng qismi $u'(x)\pm v'(x)$ ga intiladi. Demak, chap tarafning $\Delta x\to 0$ dagi limiti ham mavjud va hosilaning ta’rifiga qo’ra bu limit $y'(x)$ ga teng. Demak, $y'(x) = u'(x)\pm v'(x)$.

2) $y(x) = u(x)v(x)$ bo’lsin. Ravshanki,

$$\Delta y = y(x+\Delta x) - y(x) = u(x+\Delta x)v(x+\Delta x) - u(x)v(x) =$$
$$[u(x+\Delta x)v(x+\Delta x) - u(x+\Delta x)v(x)] + [u(x+\Delta x)v(x) - u(x)v(x)] =$$
$$u(x+\Delta x)[v(x+\Delta x) - v(x)] + v(x)[u(x+\Delta x) - u(x)]$$

Demak, agar $\Delta x \neq 0$ bo'lsa,

$$\frac{\Delta y}{\Delta x} = u(x+\Delta x)\frac{\Delta v}{\Delta x} \pm v(x)\frac{\Delta u}{\Delta x}$$

formula o'rinli.

$\Delta x \to 0$ bo'lsa, (1) ga ko'ra oxirgi formulaning o'ng qismi $u'(x)v(x) + u(x)v'(x)$ ga intiladi. Demak, chap tarafning $\Delta x \to 0$ dagi limiti ham mavjud va hosilaning ta'rifiga qo'ra bu limit $y'(x)$ ga teng. Demak, $y'(x) = u'(x)v(x) + u(x)v'(x)$.

3) $y(x) = \dfrac{u(x)}{v(x)}$ bo'lsin. Ravshanki,

$$\Delta y = y(x+\Delta x) - y(x) = \frac{u(x+\Delta x)}{v(x+\Delta x)} - \frac{u(x)}{v(x)} = \frac{u(x+\Delta x)v(x) - v(x+\Delta x)u(x)}{v(x+\Delta x)v(x)} =$$
$$\frac{[u(x+\Delta x)v(x) - u(x)v(x)] + [u(x)v(x) - v(x+\Delta x)u(x)]}{v(x+\Delta x)v(x)} =$$
$$\frac{v(x)[u(x+\Delta x) - u(x)] - u(x)[v(x+\Delta x) - v(x)]}{v(x+\Delta x)v(x)} = \frac{v(x)\Delta u - u(x)\Delta v}{v(x+\Delta x)v(x)}$$

Demak, agar $\Delta x \neq 0$ bo'lsa,

$$\frac{\Delta y}{\Delta x} = \frac{v(x)\dfrac{\Delta u}{\Delta x} - u(x)\dfrac{\Delta v}{\Delta x}}{v(x+\Delta x)v(x)}$$

formula o'rinli.

$\Delta x \to 0$ bo'lsa, (1) ga ko'ra oxirgi formulaning o'ng qismi $\dfrac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$ ga intiladi. Demak, chap tarafning $\Delta x \to 0$ dagi limiti ham mavjud va hosilaning ta'rifiga qo'ra bu limit $y'(x)$ ga teng. Demak, $y'(x) = \dfrac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$.

Teorema isbotlandi.

Murakkab funksiyaning hosilasi

Teorema **.** $x=\varphi(t)$ funksiya t_0 nuqtada, $y = f(x)$ funksiya esa $x_0 = \varphi(t_0)$ nuqtada differensiallanuvchi bo'lsin. U holda $y = f(\varphi(t))$ murakkab funksiya t_0 nuqtada differensiallanuvchi bo'lib, qo'yidagi formula o'rinli:

$$\{f[\varphi(t_0)]\}' = f'(x_0)\varphi'(t_0).$$

Isbot. $y = f(\varphi(t))$ bo'lsin. Δy va Δx orqali $y = f(\varphi(t))$ va $x=\varphi(t)$ funksiyalarning Δt ga mos orttirmalarni belgilaymiz

U holda $\Delta t \to 0$ da $\Delta x \to 0$ va $\dfrac{\Delta y}{\Delta x} \to f'(x_0)$, $\dfrac{\Delta x}{\Delta t} \to \varphi'(t_0)$, $\dfrac{\Delta y}{\Delta t} \to \{f[\varphi(t_0)]\}'$ bo'ladi.

Bu holda

$$\frac{\Delta y}{\Delta t}=\frac{\Delta y}{\Delta x}\frac{\Delta x}{\Delta t}$$

$\Delta t\to 0$ bo'lsa, $\Delta x\to 0$ bo'ladi (2) ga ko'ra oxirgi formulaning o'ng qismi $f'(x_0)\varphi'(t_0)$ ga intiladi. Demak, chap tarafning $\Delta t\to 0$ dagi limiti ham mavjud va hosilaning ta'rifiga qo'ra bu limit $\{f[\varphi(t_0)]\}'$ ga teng. Teorema isbotlandi.

Teskari funksiyaning hosilasi.

$y=f(x)$ funksiya $(a;b)$ intervalda aniqlangan bo'lib, bu funksiya teskari funksiyaning mavjudligi xaqidagi teoremaning barcha shartlarini qanoatlantirsin.

Teorema : $y=f(x)$ funksiyaning $(a;b)$ da aniqlangan, uzluksiz va qat'iy o'suvchi (qat'iy kamayuvchi) bo'lsin. Agar $y=f(x)$ funksiya $x_0\in(a;b)$ nuqtada $f'(x_0)\neq 0$ hosilaga ega bo'lsa, bu funksiyaga teskari $x=\varphi(y)$ funksiya x_0 nuqtaga mos bo'lgan y_0 ($y_0=f(x_0)$) nuqtada hosilaga ega va $\varphi'(y_0)=\frac{1}{f'(x_0)}$ tenglik o'rinli bo'ladi.

Isbot. Teskari funksiya argumentining $y=y_0$ qiymatiga $\Delta y\neq 0$ orttirma beramiz. U holda $x=\varphi(y)$ funksiya ham mos Δx orttirmaga ega bo'ladi: $x_0+\Delta x=\varphi(y_0+\Delta y)$, $x_0=\varphi(y_0)$, $\Delta y\neq 0$. Bunda $\Delta x\neq 0$ bo'ladi. Chunki $\Delta x=0$ bo'lsa, x ning bitta $x=x_0$ qiymatiga u ning $y_0\neq y_0+\Delta y$ bo'lgan ikkita u_0 va $y_0+\Delta y$ qiymati mos kelgan bo'lar edi. Shuning uchun $\frac{\Delta x}{\Delta y}$ nisbatni $\frac{\Delta x}{\Delta y}=\frac{1}{\frac{\Delta y}{\Delta x}}$ ko'rinishda yozish mumkin.

Endi agar har qanday qonun bo'yicha $\Delta y\to 0$ bo'lsa, u holda $x=\varphi(y)$ funksiya uzluksizligidan orttirma ham $\Delta x\to 0$. Biroq yozilgan tenglikning o'ng tomonidagi maxraj $f'(x_0)\neq 0$ limitga intiladi, demak, chap tomon uchun ham teskari $\frac{1}{f'(x_0)}$ qiymatga teng limit mavjuddir; buning o'zi $\varphi'(y_0)$ hosiladan iboratdir.

Demak, teskari funksiyani differensiallash qoidasi

$$\varphi'(y_0)=\frac{1}{f'(x_0)} \qquad \text{yoki} \qquad x'_y=\frac{1}{y'_x}$$

formula bilan ifodalanadi.

Elementar funksiyalarning hosilalari.

1^0. $y=\log_a x$ ($a>0$, $a\neq 1$, $x>0$) .

Bu funksiya $x=a^u$ funksiyaga nisbatan teskari funksiya bo'lgani uchun teskari funksiyani differensiallash qoidasiga ko'ra

$$y'_x=\frac{1}{x'_y}=\frac{1}{a^y\ln a}=\frac{1}{x\ln a}$$

Xususan, $(\ln x)'=\frac{1}{x}$

2^0. 1-ma'ruzada biz $(\sin x)'=\cos x$ formula o'rinli bo'lishini isbot qilganmiz.

Shunga o'xshash $(\cos x)'=-\sin x$ formula ham isbotlanadi .

Endi $y=tgx$ funksiyaning hosilasini bo'linmani differensiallash qoidasidan foydalanib topamiz:

$$(tgx)'=\left(\frac{\sin x}{\cos x}\right)'=\frac{(\sin x)'\cos x-\sin x(\cos x)'}{\cos^2 x}=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}$$

Xuddi shunga o'xshash quyidagi formulalar ham isbotlanadi:

$$(ctgx)'=-\frac{1}{\sin^2 x},(\sec x)'=\frac{\sin x}{\cos^2 x},(\cos ecx)'=-\frac{\cos x}{\sin^2 x}$$

Ushbu y=arc*sinx* funksiyani olaylik. Bu funksiya *x*=siny funksiyaga teskari bo'lib, uni $\left(\frac{-\pi}{2};\frac{\pi}{2}\right)$ intervalda qarasak, teskari funksiyaning hosilasini topish qoidasiga ko'ra

$$u'=(ars\sin x)'=\frac{1}{(\sin y)'}=\frac{1}{\cos y}=\frac{1}{\sqrt{1-\sin^2 y}}=\frac{1}{\sqrt{1-x^2}}$$

kelib chiqadi. Demak

$$(\arcsin x)'=\frac{1}{\sqrt{1-x^2}}\,(-1<x<1).$$

*X*uddi shu yo'l bilan quyidagi formulalar ham isbotlanadi:

$$(\arccos x)'=-\frac{1}{\sqrt{1-x^2}}\,(-1<x<1)$$

$$(arctgx)'=\frac{1}{1+x^2}$$

$$(arcctgx)'=-\frac{1}{1+x^2}$$

<u>Misol.</u> a) $y=x^3+\ln x$; b) $y=(x^3-5)\sin x$; c) $y=\frac{x^3-5}{x^4+3}$ funksiyalarning hosilasini toping.

<u>Yechish.</u> a) $y'=(x^3)'+(\ln x)'=3x^2+\frac{1}{x}$;

. Maple dasturi yordamida bu misol qo'yidagicha yechiladi:

```
>diff(x^3+ln(x))),x);
 3*x^2+1/x
```

b) $y'=(x^3-5)'\sin x+(x^3-5)\sin' x=3x^2\sin x+(x^3-5)\cos x$

c) $y'=\frac{(x^3-5)'(x^4+3)-(x^3-5)(x^4+3)'}{(x^4+3)^2}=\frac{3x^3(x^4+3)-(x^3-5)\cdot 4x^4}{(x^4+3)^2}=\frac{-x^6+20x^3+9x^2}{(x^4+3)^2}$

Hosilalar jadvali.

Biz ushbu bandda elementar funksiyalar hosilalari uchun topilgan formulalarni jamlab, ularni jadval sifatida keltiramiz:

1. $(x^{\mu})'=\mu x^{\mu-1}$ $(x>0)$;
2. $(a^x)'=a^x \ln a$ $(a>0, a\neq 1)$;
3. $(\log_a x)=(\frac{1}{x\ln a}$ $(x>0, a>0, a\neq 1)$; Xususan, $(\ln x)'=\frac{1}{x}$ $(x>0)$.
4. $(\sin x)'=\cos x$;
5. $(\cos x)'=-\sin x$;
6. $(tgx)'=\frac{1}{\cos^2 x}(x\neq\frac{\pi}{2}+k\pi, k\in Z)$
7. $(ctgx)'=-\frac{1}{\sin^2 x}(x\neq k\pi; k\in Z)$;
8. $(arc\sin x)'=\frac{1}{\sqrt{1-x^2}}(-1<x<1)$;

9. $(arc\cos x)' = -\frac{1}{\sqrt{1-x^2}}\ (-1 < x < 1)$;

10. $(arctgx)' = \frac{1}{1+x^2}$;

11. $(arcctgx)' = -\frac{1}{1+x^2}$.

Oraliq nazorat savollari

1. Teskari funksiyaning hosilasi *x*aqidagi teoremani isbotlang.
2. $y=\sqrt{1-x^2}$ funksiyaga nisbatan teskari funksiya qaysi to'plamda differensiallanuvchi bo'ladi ?
3. $y=log_x a$ (a>0) funksiyaning hosilasini toping.
4. Yig'indi, ko'paytma va bo'linmaning hosilasi *x*aqidagi teoremani isbotlang.
5. Murakkab funksiyaning hosilasi qanday xisoblanadi?
6. Teskari funksiyaning hosilasi haqida qanday teorema mavjud?
7. Asosiy elementar funksiyalarning hosilalari qanday keltirib chiqariladi?

Tayanch tushunchalar: Teskari funksiya; hosila; elementar funksiya.

5-ma'ruza.
Differensiallanuvchanlik va differensial.
Differensial formasining invariantligi. Differensial yordamida taqribiy hisoblashlar

Reja:

1. Differensiallanuvchanlik va differensial.
2. Differensial formasining invariantligi.
3. Taqribiy hisoblashlar

Adabiyot: [1] - 196-201 betlar; [2] - 111-113 betlar;

Differensiallanuvchanlik va differensial.

$y=f(x)$ funksiya $(a;b)$ intervalda aniqlangan bo'lsin.

1-ta'rif. Agar $y=f(x)$ funksiyaning $x_0 \in (a;b)$ nuqtadagi $\Delta y=f(x_0+\Delta x)-f(x_0)$ $(x_0+\Delta x \in (a;b))$ orttirmasini

$$\Delta y = A\Delta x + \alpha\Delta x \qquad (1)$$

ko'rinishda yozish mumkin bo'lsa $y=f(x)$ funksiya x_0 nuqtada differensiallanuvchi deb ataladi. Bunda A - biror son bo'lib, Δx ga bog'liq emas, α esa Δx ga bog'liq va $\Delta x \to 0$ da $\alpha=\alpha(\Delta x)\to 0$.

Bu ta'rifning 1-ma'ruzada berilgan differensiallanuvchanlikning avvalgi ta'rifi bilan ekvivalentligini quyidagi teorema ko'rsatadi.

Teorema . $y=f(x)$ funksiyaning $x_0 \in (a;b)$ nuqtada differensiallanuvchi bo'lishi uchun uning shu nuqtada chekli hosilaga ega bo'lishi zarur va yetarli.

Isbot. Zarurligi. $y=f(x)$ funksiya x_0 nuqtada differensiallanuvchi bo'lsin, ya'ni uning orttirmasini (1) ko'rinishda yozish mumkin. Shu tenglikning ikkala tomonini Δx ga bo'lib, so'ng limitga o'tsak,

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=A$$

tenglikni hosil qilamiz. Bundan $x_0\in(a;b)$ nuqtada hosilaning mavjudligi va $f'(x_0)$=A bo'lishi kelib chiqadi.

Yetarliligi. Chekli $f'(x_0)$ mavjud bo'lsin, ya'ni $\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=f'(x_0)$.

Bundan ($\Delta x\to 0$ da $\frac{\Delta y}{\Delta x}$ o'zgaruvchi bilan limitining ayirmasi cheksiz kichik bo'lgani uchun) $\frac{\Delta y}{\Delta x}=$ $f'(x_0)+\alpha$ yoki $\Delta y=f'(x_0)\Delta x+\alpha\Delta x$.

Bu tenglikdagi α miqdor Δx ga bog'liq va $\Delta x\to 0$ da $\alpha\to 0$. Demak $y=f(x)$ funksiya $x_0\in(a;b)$ nuqtada differensiallanuvchi bo'lib, $A=f'(x)$ bo'ladi. Teorema isbot bo'ldi.

$y=f(x)$ funksiya $(a;b)$ intervalda aniqlangan bo'lib, $x_0\in(a;b)$ nuqtada differensiallanuvchi bo'lsin. Demak, funksiyaning x_0 nuqtadagi orttirmasi $\Delta y=A\Delta x+\alpha\Delta x$ ko'rinishda yozilishi mumkin. Bunda $A=f'(x_0)$ bo'ladi.

2- ta'rif. $y=f(x)$ funksiyasi orttirmasi Δu ning Δx ga nisbatan chiziqli bosh qismi $A\Delta x=f'(x_0)\Delta x$ berilgan $y=f(x)$ funksiyaning x_0 nuqtadagi *differensiali* deyiladi va u dy yoki $df(x_0)$ orqali belgilanadi: $dy=df(x_0)=A\Delta x=f'(x_0)\Delta x$. Xususan, $y=x$ funksiya uchun $dy=dx=x'\Delta x=\Delta x$ bo'lganidan erkli o'zgaruvchining differensiali uning orttirmasiga teng bo'ladi: $dx=\Delta x$.

Buni nazarga olsak, y=$f(x)$ funksiya differensialining formulasi

$$dy=f'(x_0)dx \qquad (2)$$

bo'ladi.

$f(x)$ va $\varphi(x)$ funksiyalari $(a;b)$ intervalda aniqlangan bo'lib, $x\in(a;b)$ nuqtada ularning differensiallari $df(x)$, $d\varphi(x)$ mavjud bo'lsin. U holda $f(x)\pm\varphi(x)$, $f(x)\varphi(x)$ va $\frac{f(x)}{\varphi(x)}$ $(\varphi(x)\neq 0)$ funksiyalarning ham shu $x\in(a;b)$ nuqtada differensiallari mavjud va ular uchun quyidagi

$$d(f(x)\pm\varphi(x))=df(x)\pm d\varphi(x),$$
$$d(f(x)\,\varphi(x))=\varphi(x)df(x)+f(x)d\varphi(x),$$
$$d\left(\frac{f(x)}{\varphi(x)}\right)=\frac{\varphi(x)df(x)-f(x)d\varphi(x)}{\varphi^2(x)}\quad(\varphi(x)\neq 0)$$

formulalar o'rinli.

Masalan, shu formulalarning birinchisining o'rinligini ko'rsataylik. Funksiya differensialining (2) ko'rinishdagi ifodalanishidan va funksiyaning hosilalarini topish qoidalaridan foydalansak

$$d(f(x)\pm\varphi(x))=(f(x)\pm\varphi(x))'dx=f'(x)dx\pm\varphi'(x)dx=df(x)\pm d\varphi(x).$$

*X*uddi shunga o'*x*shash qolgan formulalar ham isbotlanadi.

Matematik induktsiya usulidan foydalanib, qo'shiluvchilar hamda ko'paytuvchilar soni i*x*tiyoriy chekli son bo'lgan holda ham tegishli formulalar o'rinli bo'lishini ko'rsatish mumkin.

Differensial formasining invariantligi.

Agar x erkli o'zgaruvchi hisoblansa, u holda dy differensial (2) formula bilan ifodalanadi.

Endi murakkab funksiyaning differensialini topamiz. Faraz qilaylik, $x=\varphi(t)$ funksiya $(\alpha;\beta)$ intervalda, $y=f(x)$ funksiya esa $(a;b)$ intervalda aniqlangan bo'lib, bu funksiyalar yordamida y=$f(\varphi(t))=F(t)$ murakkab funksiya tuzilgan bo'lsin (bunda, albatta, $t\in(\alpha;\beta)$ da $x=\varphi(t)\in(a;b)$ bo'lishi talab qilinadi).

Murakkab funksiyaning hosilasini topish formulasidan foydalanib, shu murakkab funksiyaning differensialini topamiz:

$$dy=d(f(\varphi(t)))=(f(\varphi(t)))'dt=f'(\varphi(t))\varphi'(t)dt.$$

Ammo $\varphi(t)$ ni x bilan almashtirsak va $\varphi'(t)dt$ ifoda x t ning funksiyasi sifatidagi differensiali ekanini eslasak, natijada $dy=f'(x)dx$ ga ega bo'lamiz, ya'ni differensialning avvalgi ko'rinishiga qaytamiz.

Shunday qilib *x*atto, avvalgi erkli o'zgaruvchi bilan almashtirilganda ham, differensialning ko'rinishi saqlanishi mumkin ekan. Biz har vaqt x erkli yoki erksiz o'zgaruvchiligiga qaramay, funksiyaning differensialini (2) ko'rinishda yozishga *x*aqlimiz; faqat farqi shundaki, agar erkli o'zgaruvchi deb t olingan bo'lsa, u holda dx i*x*tiyoriy Δx orttirmani bildirmasdan, balki t ning funksiyasi sifatida x ning differensialini bildiradi. Bu *x*ossa differensial ko'rinishning *invariantligi* deyiladi.

Taqribiy hisoblashlar

$\Delta y = dy + o(\Delta x),\ \delta(\Delta x) = \dfrac{\Delta y - dy}{\Delta x}$ - nisbiy xato.

U holda

$$f(x+\Delta x)\approx f(x)+f'(x)\Delta x$$

taqribiy formula o'rinli

Misol.

$\sqrt{1,02}\approx 1+0,01=1,01,\ \sqrt{x+\Delta x}-\sqrt{x}\approx\left(\sqrt{x}\right)'\Delta x,\ \left(\sqrt{x}\right)'=\dfrac{1}{2}\dfrac{1}{\sqrt{x}},\ x=1,\ \Delta x=0,22.$

Oraliq nazorat savollari

1. Differensiallanuvchanlikning ikki *x*il ta'riflarining ekvivalent-ligini isbotlang .
2. Funksiya differensialini ta'riflang .
3. Funksiyaning differensiali bilan uning orttirmasi orasida qanday farq bor ?
4. Differensialning geometrik ma'nosi nimadan iborat ?
5. Chekli hosilaga ega funksiyaning differensiallanuvchiligi qanday isbotlanadi?
6. Differensiallanuvchi funksiyaning chekli hosilasi mavjudligi qanday ko'rsatiladi?
7. Differensial formulasi qanday yoziladi?
8. Qanday funksiya uchun $dy=\Delta y$ tenglik o'rinli?
9. Yig'indining (ko'paytmaning, bo'linmanind) differensiali uchun qanday qoidalar mavjud?
10. Murakkab funksiyaning differensiali qanday hisoblanadi?
11. Differensial formasining invariantligi deganda nimani tushunasiz?

Tayanch tushunchalar: hosila; differensiallanuvchanlik; differensial; invariantlik.

6-ma'ruza.

Logarifmik hosila. Daraja-ko'rsatkichli funksiyaning hosilasi. Parametrik va oshkormas ko'rinishda berilgan funksiyalarning differensiallash.

Reja:

1. Logarifmik hosila.
2. Daraja-ko'rsatkichli funksiyaning hosilasi.
3. Parametrik ko'rinishda berilgan funksiyani differensiallash.
4. Oshkormas funksiyaning hosilasi.

Adabiyot :

1. [1] - 196 bet;
2. [2] - 128 bet;

Logarifmik hosila.

$y=f(x)$ funksiya $(a;b)$ intervalda chekli hosilaga ega va $\forall x\in(a;b)$ uchun noldan farqli bo'lsin.

Bu funksiyani logarifmlab yangi $z=ln|f(x)|$ funksiyasini tuzamiz. Endi murakkab funksiyaning hosilasi uchun tegishli formuladan foydalanib topamiz: $z'=\frac{y'}{y}$. Bundan $y'=yz'$. Avval berilgan funksiya logarifmining hosilasini topishdan iborat bo'lgan bu usulni qo'llanish ko'pincha hisoblashni birmuncha soddalashtiradi.

Masalan, $y=\frac{(x+1)^2\sqrt{x-1}}{(x+3)^4}$ funksiyaning hosilasini topaylik. Buning uchun tenglikning ikkala tomonini logarifmlaymiz:

$$lny=2ln(x+1)+\frac{1}{2}ln(x-1)-4ln(x+3)$$

Bu tenglikning ikkala tomonini differensiallaymiz:

$\frac{y'}{y}=\frac{2}{x+1}+\frac{1}{2(x-1)}-\frac{4}{x+3}$. Bu tenglikning ikkala tomonini u ga ko'paytirib va u o'rniga $\frac{(x+1)^2\sqrt{x-1}}{(x+3)^4}$ ifodani qo'yib, ushbu natijani hosil qilamiz:

$$u'=\frac{(x+1)^2\sqrt{x-1}}{(x+3)^4}\left(\frac{2}{x+1}+\frac{1}{2(x-1)}-\frac{4}{x+3}\right)$$

Ta'rif. Berilgan $y=f(x)$ funksiya natural logarifmining x bo'yicha hosilasidan iborat $\frac{y'}{y}=(\ln y)'$ ifoda *logarifmik hosila* deyiladi.

Misol. $y=\frac{(x-1)^{\frac{1}{2}}(x+3)^5(x+2)^7}{\sqrt[3]{(x+1)^2(x-2)}}$. y' - ?

Yechish: $\ln y=\frac{1}{2}\ln(x-1)+5\ln(x+3)+7\ln(x+2)-\frac{2}{3}\ln(x+1)-\frac{1}{3}\ln(x-2);$

$$(\ln y)'=\frac{y'}{y}=\frac{1}{2(x-1)}+\frac{5}{x+3}+\frac{7}{x+2}-\frac{2}{3(x+1)}-\frac{1}{3(x-2)};$$

$$y'=\frac{(x-1)^{\frac{1}{2}}(x+3)^5(x+2)^7}{\sqrt[3]{(x+1)^2(x-2)}}\left[\frac{1}{2(x-1)}+\frac{5}{x+3}+\frac{7}{x+2}-\frac{2}{3(x+1)}-\frac{1}{3(x-2)}\right].$$

Maple dasturi yordamida bu misol qo'yidagicha yechiladi:

```
>simplify(diff(((x-1)^(1/2)*(x+3)^5*(x+2)^7)/((x+1)^2*(x-2))^
(1/3),x));
1/2*(23*x^4+12*x^3-143*x^2-16*x+100)*(x+3)^4*(x+2)^6/((x+1)^2*
(x-2))^(1/3)/(x-2)/(x+1)/(x-1)^(1/2)
```

Daraja-ko'rsatkichli funksiyaning hosilasi.

Asosi ham, daraja ko'rsatkichi ham x ning funksiyalaridan iborat bo'lgan funksiya, ya'ni $y=(u(x))^{v(x)}$ $(u(x)>0)$ ko'rinishdagi har bir funksiya *daraja-ko'rsatkichli funksiya* deyiladi.

Endi logarifmik hosila yordamida daraja-ko'rsatkichli funksiyaning hosilasini topaylik.

$u(x)>0$ va $v(x)$ funksiyalar $u'(x)$ va $v'(x)$ hosilalarga ega bo'lsin. $y=(u(x))^{v(x)}$ tenglikdan $\ln y=v(x)\ln u(x)$ tenglik kelib chiqadi.

Buni x bo'yicha differensiallaymiz:

$\frac{1}{y}y'=v'(x)\ln u(x)+v(x)\frac{1}{u(x)}u'(x)$. Bundan

$$y'=y\left(v'(x)\ln u(x)+\frac{v(x)}{u(x)}u'(x)\right)=(u(x))^{v(x)}\left(v'(x)\ln u(x)+\frac{v(x)}{u(x)}u'(x)\right)$$

kelib chiqadi. Demak,

$$\left((u(x))^{v(x)}\right)'=v(x)(u(x))^{v(x)-1}u'(x)+(u(x))^{v(x)}v'(x)\ln u(x)$$

Shunday qilib, daraja-ko'rsatkichli funksiyaning hosilasi ikkita qo'shiluvchidan iborat; agar differensiallashda u^v darajali funksiya deb qaralsa, birinchi qo'shiluvchi chiqadi; agar u^v ko'rsatkichli funksiya deb qaralsa ikkinchi qo'shiluvchi chiqadi.

Misol. $y=x^x$ bo'lsa, u' ni toping.

Yechim: $(x^x)'=xx^{x-1}+x^x\ln x=x^x(1+\ln x)$..

. Maple dasturi yordamida bu misol qo'yidagicha yechiladi:

```
> simplify (diff(x^x),x));
 x^x*(1 +lnx)
```

Parametrik ko'rinishda berilgan funksiyaning hosilasi.

Ko'pincha x o'zgaruvchining y funksiyasi bitta $y=f(x)$ tenglama bilan berilmasdan, balki x va y larni parametr deb ataladigan uchinchi t o'zgaruvchining funksiyalari sistemasi

$$\begin{cases} x=\varphi(t) \\ y=\psi(t) \end{cases} \qquad (1)$$

orqali beriladi. Bunday sistema orqali aniqlangan funksiya *parametrik* ko'rinishda berilgan funksiya deyiladi.

$\frac{dx}{dt}=\varphi'(t)\neq 0$ va $\frac{dy}{dx}=\psi'(t)$ hosilalar mavjud bo'lsin. (1) ko'rinishda berilgan funksiyaning $\frac{dy}{dx}$ hosilasini hisoblash uchun uni dy va dx ning nisbati deb qarasak, quyidagiga ega bo'lamiz:

$$\frac{dy}{dx}=\frac{\psi'(t)dt}{\varphi'(t)dt}=\frac{\psi'(t)}{\varphi'(t)}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$$

Misol. $\begin{cases} x=a\cos t \\ y=b\sin t \end{cases}$. $y'(x)$ - ?

Yechish: $y'(x)=\frac{(b\sin t)'}{(a\cos t)'}=-\frac{b\cos t}{a\sin t}=-\frac{b}{a}\operatorname{ctg} t$. $y'(x)$ ni $y(x)$ ga o'hshatib, parametrik ko'rinishda yozsa bo'ladi: $\begin{cases} x=a\cos t \\ y'(x)=-\frac{b}{a}\operatorname{ctg} t \end{cases}$

. Maple dasturi yordamida bu misol qo'yidagicha yechiladi:

```
>S:=diff(b*sin(t),t)/diff(a*cos(t),t);
 S:=-b*cos(t)/a/sin(t)
```

Oshkormas funksiyaning hosilasi.

$y(x)$ funksiya $F(x,y)=0$ tenglama ko'rinishida berilgan bo'lsin. Funksiyaning bunday ko'rinishda berilishi *oshkormas* deyiladi.

$F(x,y(x))=0$ ayniyatni ikkala tarafini x bo'yicha differensiallab, $y'(x)$ nisbatan chiziqli tenglamani hosil qilamiz va undan $y'(x)$ ni topamiz.

Misol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. $y-$ x ning funksiyasi deb $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ayniyatni ikkala tarafini x bo'yicha differensiallab qo'yidagini hosil qilamiz: $\frac{2x}{a^2}+\frac{2yy'}{b^2}=0 \Rightarrow \frac{2y}{b^2}y'=-\frac{2x}{a^2} \Rightarrow y'=-\frac{b^2x}{a^2y}$.

. Maple dasturi yordamida bu misol qo'yidagicha yechiladi:

```
>Z:=diff(x^2/a^2+y(x)^2/b^2,x);
 Z:= 2*x/a^2+2*y(x)/b^2*diff(y(x),x)
>Q:=solve(Z=0,diff(y(x),x));
 Q:= -x*b^2/y(x)/a^2
```

Oraliq nazorat savollari:

1. Funksiyaning logarifmik hosilasi nima?
2. Daraja-ko'rsatkichli funksiyaning hosilasi qanday hisoblanadi?
3. Parametrik va oshkormas ko'rinishda berilgan funksiyalarning hosilasi qanday topiladi?

Tayanch tushunchalar: Logarifmik hosila; daraja-ko'rsatkichli funksiya; funksiyaning parametrik va oshkormas ko'rinishi.

7,8 - ma'ruzalar.

Differensial hisobning asosiy teoremalari. Lopital qoidasi.

Reja:

1. Ferma teoremasi
2. Roll teoremasi.
3. Lagranj va Koshi teoremalari.
4. Lopital qoidasi.

Adabiyot: [1] - 211-214 betlar; 2. [2] - 135-146 betlar;

Ferma teoremasi.

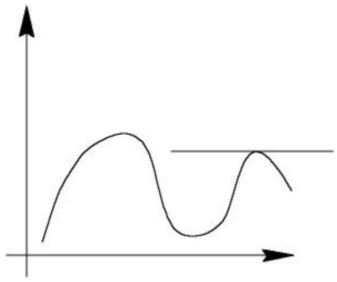
1-***Teorema*** (Ferma teoremasi). $y=f(x)$ funksiya biror $(a;b)$ intervalda aniqlangan va ichki $c\in(a;b)$ nuqtada eng katta (eng kichik) qiymatga erishsa va shu nuqtada chekli $f'(c)$ hosila mavjud bo'lsa, u holda $f'(c)=0$ bo'ladi.

Isbot. $f(c)$ qiymat $y=f(x)$ funksiyaning $(a;b)$ intervalda eng katta qiymati bo'lsa, $\forall x\in(a;b)$ $f(x)\le f(c)$ o'rinli. Shartga ko'ra bu c nuqtada chekli $f'(c)$ hosila mavjud. Ravshanki,

$$f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}=\lim_{x\to c-0}\frac{f(x)-f(c)}{x-c}=\lim_{x\to c+0}\frac{f(x)-f(c)}{x-c}.$$

Ammo $x<c$ bo'lganda, $\frac{f(x)-f(c)}{x-c}\ge 0 \Rightarrow f'(c)\ge 0$, va $x>c$ bo'lganda

(2-chizma)



$\frac{f(x)-f(c)}{x-c} \leq 0 \Rightarrow f'(c) \leq 0$ bo'lishidan $f'(c)=0$ kelib chiqadi. Shu bilan teorema isbot bo'ldi. Ferma teoremasi sodda geometrik ma'noga ega. U $y=f(x)$ funksiya grafigiga $(c,f(c))$ nuqtada o'tkazilgan urinmaning Ox o'qiga parallel bo'lishini ifodalaydi.

Roll teoremasi.

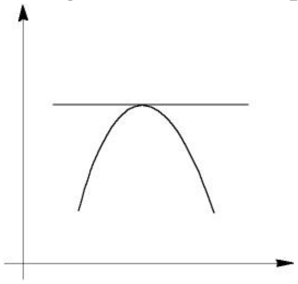
2-***Teorema*** (Roll teoremasi). $y=f(x)$ funksiya $[a;b]$ segmentda aniqlangan, uzluksiz va $f(a)=f(b)$ bo'lsin. Agar bu funksiya $(a;b)$ intervalda chekli $f'(x)$ hosilaga ega bo'lsa, u holda kamida bitta shunday c ($a<c<b$) nuqta topiladiki, $f'(c)=0$ bo'ladi.

Isbot. $y=f(x)$ funksiya $[a;b]$ segmentda uzluksiz. Demak, Veyershtrassning birinchi teoremasiga ko'ra bu oraliqda funksiya o'zining eng katta qiymati M va eng kichik qiymati m ga erishadi.

1) $m=M$ bo'lsin. Bunda $f(x)=const$ bo'ladi. Ravshanki, bu holda $\forall\ c \in(a;b)$ uchun $f'(c)=0$ bo'ladi.

2) $m \neq M$ bo'lsin. Bu holda $f(a)=f(b)$ bo'lgani uchun $y=f(x)$ funksiya o'zining eng katta qiymati M, eng kichik qiymati m larning kamida bittasiga $[a;b]$ segmentning ichki c ($a< c <b$) nuqtasida erishadi. Ferma teoremasiga asosan bu nuqtada $f'(c)=0$ bo'ladi. Teorema isbot bo'ldi.

$y=f(x)$ funksiya Roll teoremasining barcha shartlarini qanoatlantirsin. U holda bu funksiya tasvirlagan egri chiziqda shunday $(c,f(c))$ nuqta topiladiki, egri chiziqqa uning bu nuqtasida o'tkazilgan urinma 0x o'qiga parallel bo'ladi. (3- chizma).



3 - chizma

Shuni alohida ta'kidlash lozimki, Roll teoremasidagi barcha shartlar yoki bu shartlardan birortasi bajarilmasa, u holda funksiyaning hosilasi $\forall x\in(a;b)$ uchun nolga aylanmasligi mumkin. Lekin barcha shartlar yoki shartlardan birortasi bajarilmaganda ham $(a;b)$ da hosilani nolga aylantira digan nuqtalar bo'lishi mumkin, ya'ni Roll teoremasidagi shartlar yetarli bo'lib, zaruriy emasdir.

Masalan, 1) $f(x)=1-|x|$ funksiya $[-1;1]$ segmentda uzluksiz bo'lib, bu funksiya uchun $f(-1)=f(1)=0$ bo'ladi. Ammo bu funksiyaning hosilasi $(-1;1)$ intervalning birorta nuqtasida ham nolga aylanmaydi. Bunga sabab $f(x)=1-|x|$ funksiya $x=0$ nuqtada hosilaga ega emas.

2) $f(x)=x$ funksiya $[0;1)$segmentda uzluksiz bo'lib , $(0;1)$ da chekli hosilaga ega va $(0;1)$ interval ning barcha nuqtalarida $f'(x)=1$. Bu funksiya uchun Roll teoremasi xulosasining o'rinli bo'lmasligi bu funksiya uchun $f(a)=f(b)$
shartning bajarilmasligidandir.

3. $f(x)=\begin{cases} x, & агар\ x\in[0;1), \\ 0, & x=1 \end{cases}$ Bu funksiya uchun Roll teoremasining uzluksizlik sharti bajarilmay, qolgan shartlar bajariladi. Shu bilan birga berilgan funksiyaning hosilasi $(0;1)$ da nolga aylanmaydi.

Lagranj va Koshi teoremalari

3-***Teorema***. (Lagranj teoremasi). $f(x)$ funksiya $[a;b]$ da aniqlangan, uzluksiz va $(a;b)$ da chekli $f'(x)$ hosilaga ega bo'lsa, u holda kamida bitta shunday c ($a< c <b$) nuqta topiladiki, bu nuqtada

$$f'(c)=\frac{f(b)-f(a)}{b-a} \qquad (1)$$

bo'ladi.

Isbot. Shartga ko'ra $y=f(x)$ funksiya $[a;b]$ segmentda uzluksiz bo'lib, uning ichki nuqtalarida chekli $f'(x)$ hosilaga ega. Bu funksiya yordamida quyidagi $\Phi(x) = f(x) - f(a) - \frac{f(b)-f(a)}{b-a}(x-a)$ funksiyani kiritamiz. Ravshanki, bu $\Phi(x)$ funksiya $[a;b]$ segmentda aniqlangan va uzluksiz bo'lib, $(a;b)$ da esa

$$\Phi'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$$

hosilaga ega. $\Phi(x)$ funksiyaning $x=a$ va $x=b$ nuqtalardagi qiymatlarini hisoblaymiz: $\Phi(a)=\Phi(b)=0$. Demak, $\Phi(x)$ funksiya Roll teoremasining barcha shartlarini qanoatlantiradi. U holda a va b orasida c ($a< c <b$) nuqta topiladiki $\Phi'(c)=0$ bo'ladi. Shunday qilib,

$$0=\Phi'(c)=f'(c)-\frac{f(b)-f(a)}{b-a}$$

va bundan esa isbot qilinishi kerak bo'lgan (1) formula kelib chiqadi.

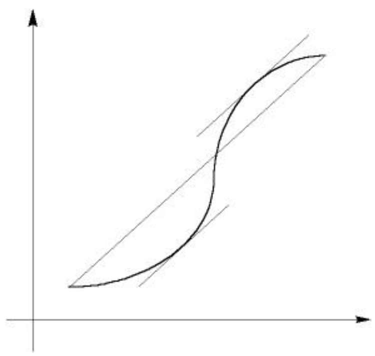
Isbot qilingan (1) formulani boshqacha ham yozish mumkin. Buning uchun $a< c <b$ tengsizliklarni e'tiborga olib, $\frac{c-a}{b-a}=\theta$ $(0<\theta<1)$ deb belgilasak, unda $c=a+(b-a)\theta$ $(0<\theta<1)$ bo'ladi. Natijada (1) formula ushbu

$$f(b)-f(a)=f'(a+\theta(b-a))(b-a)$$

ko'rinishga keladi. Bu formulada $\Delta x>0$ da $a=x$, $b=x+\Delta x$, $\Delta x<0$ da esa $a=x+\Delta x$, $b=x$ deb, topamiz:

$$f(x+\Delta x)-f(x)=f'(x+\theta\Delta x)\Delta x$$

Bu formula chekli orttirmalar formulasi deb ataladi.



4 – chizma

Lagranj teoremasining geometrik ma'nosiga kelsak, $y=f(x)$ funksiya grafigida (4-chizma) shunday M nuqta mavjudki, bu nuqtadagi urinmaning $f'(c)$ burchak koeffitsienti AB kesuvchining burchak koeffitsientiga teng :

$$\frac{f(b)-f(a)}{b-a}=f'(c)$$

Boshqacha aytganda, grafikda shunday $M(c,f(c))$ nuqta mavjudki, bu nuqtadagi urinma AB kesuvchiga parallel bo'ladi.

Agar (1) formulada $f(a)=f(b)$ deb olinsa, u holda $f'(c)=0$ ($a< c <b$) bo'lib, Lagranj teoremasidan Roll teoremasining kelib chiqishini ko'ramiz.

Natija. (Doimiylik sharti). $f'(x)=0 \Rightarrow f(x)=const$

Isbot. $f'(x)=0$ bo'lsin. Ixtiyoriy $(a;b)$ oraliqda $\frac{f(b)-f(a)}{b-a}=f'(c)$ =0 bo'ladi. Demak, barcha $a<b$ uchun $f(a)=f(b)$, ya'ni $y=f(x)$ funksiya doimiydir.

4-***Teorema*** (Koshi teoremasi). $f(x)$ va $\varphi(x)$ funksiyalar $[a;b]$ da aniqlangan, uzluksiz, $(a;b)$ da differensiallanuvchi bo'lib, $\forall x\in(a;b)$ uchun $\varphi'(x)\neq 0$ bo'lsa, $(a;b)$ da kamida bitta c nuqta mavjud bo'ladiki, ushbu tenglik o'rinli bo'ladi:

$$\frac{f(b)-f(a)}{\varphi(b)-\varphi(a)}=\frac{f'(c)}{\varphi'(c)}, \quad a< c <b$$

Bunda $\varphi(b)-\varphi(a)\neq 0$. Chunki, agar $\varphi(b)=\varphi(a)$ bo'lsa, Roll teoremasiga ko'ra $(a;b)$ dagi biror nuqtada $\varphi'(x)=0$ bo'lar edi. Bu esa berilgan ($\varphi'(x)\neq 0$, $a< c <b$) shartga ziddir. Teoremani isbot qilish uchun ushbu

$$\Phi(x)=f(x)-f(a)-\frac{f(b)-f(a)}{\varphi(b)-\varphi(a)}(\varphi(x)-\varphi(a))$$

yordamida funksiyani kiritib, Lagranj teoremasini isbot qilishdagi kabi yo'l tutiladi.

Xususan $\varphi(x)=x$ bo'lganda Koshi teoremasidan Lagranj teoremasi kelib chiqadi.

Lopital qoidasi.

Tegishli funksiyalarning hosilalari mavjud bo'lganda $\frac{0}{0},\frac{\infty}{\infty},0\cdot\infty,\infty-\infty,0^0,\infty^0,1^\infty$ ko'rinishdagi aniqmasliklarni ochish masalasi engillashadi. Odatda hosilalardan foydalanib aniqmasliklarni ochish *Lopital qoidalari* deb ataladi.

1^0. $\frac{0}{0}$ ko'rinishdagi aniqmaslik.

Ma'lumki $x\to a$ da $f(x)\to 0$, $\varphi(x)\to 0$ bo'lsa, $\frac{f(x)}{\varphi(x)}$ nisbat $\frac{0}{0}$ ko'rinishdagi aniqmaslikni ifodalaydi.

5-***Teorema***. $(a;b]$ da uzluksiz $f(x)$ va $\varphi(x)$ funksiyalar berilgan bo'lib,

1) $(a;b]$ da $f'(x)$ va $\varphi'(x)$ mavjud, hamda $\varphi'(x)\neq 0$;

2) $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a}\varphi(x)=0$;

3) $\lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)}=\mu$, ($\mu$- chekli yoki cheksiz)

shartlar bajarilsin. U holda $\lim\limits_{x\to a}\frac{f(x)}{\varphi(x)}=\mu$ bo'ladi.

Isbot. Har ikkala funksiyani x=a nuqtada f(a)=φ(a)=0 deb aniqlaymiz. Endi $[a;b]$ da $f(x)$ va $\varphi(x)$ uchun Koshi teoremasining shartlari bajariladi:

$$\frac{f(x)}{\varphi(x)}=\frac{f(x)-f(a)}{\varphi(x)-\varphi(a)}=\frac{f'(c)}{\varphi'(c)},\qquad a<c<x\leq b$$

Ravshanki, $x\to a$ da $c\to a$.

$\frac{f(x)}{\varphi(x)}=\frac{f'(c)}{\varphi'(c)}$ da $x\to a$ da limitga o'tamiz: $\lim\limits_{x\to a}\frac{f(x)}{\varphi(x)}=\lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)}=\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}=\mu$,

demak, $\lim\limits_{x\to a}\frac{f(x)}{\varphi(x)}=\mu$.

Eslatma. $\lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)}$ mavjud bo'lmasligi ham mumkin.

Misol: $f(x)=x^2\cos\frac{1}{x}$, $g(x)=\sin x$

$$\lim_{x\to 0}\frac{x^2\cos\frac{1}{x}}{\sin x}=\lim_{x\to 0}\frac{x}{\sin x}\lim_{x\to 0}x\cos\frac{1}{x}=0,$$

$$\lim_{x\to 0}\frac{f'(x)}{g'(x)}=\lim_{x\to 0}\frac{2x\cos\frac{1}{x}+\sin\frac{1}{x}}{\cos x}=\text{ mavjud emas.}$$

2^0. $\frac{\infty}{\infty}$ ko'rinishdagi aniqmaslik.

Ma'lumki $x \to a$ da $f(x) \to \infty$, $\varphi(x) \to \infty$ bo'lsa, $\dfrac{f(x)}{\varphi(x)}$ nisbat $\dfrac{\infty}{\infty}$ ko'rinishdagi aniqmaslikni ifodalaydi.

6- ***Teorema***. $(a;b)$ intervalda $f(x)$ va $\varphi(x)$ funksiyalar uchun ushbu shartlar bajarilgan bo'lsin:

1) $\lim\limits_{x \to a} f(x) = \infty$, $\lim\limits_{x \to a} \varphi(x) = \infty$;

2) $(a;b)$ da chekli $f'(x)$ va $\varphi'(x)$ hosilalar mavjud va $\varphi'(x) \neq 0$;

3) $\lim\limits_{x \to a} \dfrac{f'(x)}{\varphi'(x)} = \mu$, ($\mu$- chekli yoki cheksiz).

U holda $\lim\limits_{x \to a} \dfrac{f(x)}{\varphi(x)} = \lim\limits_{x \to a} \dfrac{f'(x)}{\varphi'(x)} = \mu$ tenglik o'rinli bo'ladi.

$0 \cdot \infty, \infty - \infty, 0^0, \infty^0, 1^\infty$ ko'rinishdagi aniqmasliklarni ham Lopital qoidasi asosida hisoblash mumkin, buning uchun ularning har birini oldin $\dfrac{0}{0}$ yoki $\dfrac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarning biriga keltirish kerak.

Misol. $\lim\limits_{x \to \infty} (2^{\frac{1}{x}} - 1)x = \infty$

Yechish. Bu $0 \cdot \infty$ ko'rinishdagi aniqmaslik, buni avval $\dfrac{0}{0}$ yoki $\dfrac{\infty}{\infty}$ ko'rinishdagi aniqmaslikka keltirib, so'ngra Lopital qoidasini qo'llaymiz:

$$\lim_{x \to \infty} (2^{\frac{1}{x}} - 1)x = \lim_{x \to \infty} = \lim_{x \to \infty} \frac{\left(2^{\frac{1}{x}} - 1\right)'}{\left(\frac{1}{x}\right)'} = \lim_{x \to \infty} \frac{2^{\frac{1}{x}} \ln 2 \frac{1}{x^2}}{-\frac{1}{x^2}} = \lim_{x \to \infty} 2^{\frac{1}{x}} \ln 2 = \ln 2$$

Misol. $\lim\limits_{x \to 0} \dfrac{\ln(\cos x)}{\ln(\cos 2x)}$.

Yechish. $$\lim_{x \to 0} \frac{\ln(\cos x)}{\ln(\cos 2x)} = \lim_{x \to 0} \frac{-\frac{\sin x}{\cos x}}{-\frac{2\sin 2x}{\cos 2x}} = \frac{1}{2} \lim_{x \to 0} \frac{\sin x \cos 2x}{\sin 2x \cos x} = \frac{1}{2} \lim_{x \to 0} \frac{\sin x}{\sin 2x} \cdot \lim_{x \to 0} \frac{\cos 2x}{\cos x} =$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{\cos x}{2\cos 2x} \cdot 1 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

. ***Maple yordamida bu misol quyidagicha yechiladi:***

```
>limit(ln(cos(x))/ln(cos(2*x)),x=0);
 1/4
```

Misol. $\lim\limits_{x \to 1} (2 - x)^{\mathrm{tg} \frac{\pi x}{2}} = A$.

Yechish. $\ln A=\lim\limits_{x\to 1}\ln(2-x)^{\operatorname{tg}\frac{\pi x}{2}}=\lim\limits_{x\to 1}\operatorname{tg}\frac{\pi x}{2}\ln(2-x)=\lim\limits_{x\to 1}\frac{\sin\frac{\pi x}{2}}{\cos\frac{\pi x}{2}}\ln(2-x)=\lim\limits_{x\to 1}\sin\frac{\pi x}{2}\cdot$

$\cdot\lim\limits_{x\to 1}\frac{\ln(2-x)}{\cos\frac{\pi x}{2}}=\lim\limits_{x\to 1}\frac{\frac{1}{2-x}}{\frac{\pi}{2}\sin\frac{\pi x}{2}}=\frac{2}{\pi}\;\Rightarrow A=e^{\frac{2}{\pi}}.$

. *Maple yordamida bu misol quyidagicha yechiladi:*

```
>limit((2-x)^tan(Pi*x/2),x=1);
 exp(2/Pi)
```

Misol. $\lim\limits_{x\to 0}x\operatorname{ctg}\pi x$.

Yechish. $\lim\limits_{x\to 0}x\operatorname{ctg}\pi x=\lim\limits_{x\to 0}\frac{\operatorname{ctg}\pi x}{\frac{1}{x}}=\lim\limits_{x\to 0}\frac{-\frac{\pi}{\sin^2\pi x}}{-\frac{1}{x^2}}=\lim\limits_{x\to 0}\frac{\pi x^2}{\sin^2\pi x}=\lim\limits_{x\to 0}\frac{\pi^2x^2}{\sin^2\pi x}\cdot\frac{1}{\pi}=\frac{1}{\pi}.$

. *Maple yordamida bu misol quyidagicha yechiladi:*

```
>limit(x*cot(Pi*x),x=0);
 1/Pi
```

Oraliq nazorat savollari.

1. Differensial hisobning qanday asosiy teoremalarini bilasiz?
2. Ferma teoremasining geometrik ma’nosi nimadan iborat?
3. Ferma teoremasining sharti va hulosasi nimadan iborat?
4. Roll teoremasining sharti va hulosasi nimadan iborat?
5. Roll teoremasining geometrik ma’nosi nimadan iborat?
6. Roll teoremasi qanday isbotlanadi?
7. Lagranj teoremasining sharti va hulosasi nimadan iborat?
8. Lagranj teoremasining geometrik ma’nosi nimadan iborat?
9. Lagranj teoremasi qanday isbotlanadi?
10. Chekli orttirmalar formulasi qanday yoziladi?
11. Koshi teoremasining sharti va hulosasi nimadan iborat?
12. Nima uchun Roll, Lagranj va Koshi teoremalarini o’rta qiymat haqidagi teoremalar deyiladi?
13. Lopital qoidasi nima uchun ishlatiladi?
14. Lopital qoidasi qanday aytiladi?
15. Lopital qoidasi yordamida qanday aniqmasliklarni ochish mumkin?
16. $f(x)=x^2$, $\varphi(x)=x^3$ funksiyalarga [-1;1] da Koshi formulasini qo’llab bo’ladimi?

Tayanch tushunchalar: Differensial hisobning qanday asosiy teoremalari, aniqmasliklar.

9-ma’ruza.

Yuqori tartibli hosila va differensial. Ikkinchi tartibli hosilaning mexanik ma’nosi. Leybnits formulasi

Reja:

1. Yuqori tartibli hosila
2. Ikkinchi tartibli hosilaning mexanik ma’nosi.
3. Leybnits formulasi
4. Yuqori tartibli differensial.

Adabiyot: [1] - 196-201 betlar; [2] - 111-113 betlar;

Yuqori tartibli hosila

$y=f(x)$ funksiya $(a;b)$ intervalda aniqlangan bo’lib, xar bir $x\in(a;b)$ nuqtada $f'(x)$ hosilaga ega bo’lsin. Shu bilan $f':(a,b)\to R$ ko’rinishdagi yangi funksiya aniqlanadi. Agar $f'(x)$ funksiya o’zi xar bir $x\in(a;b)$ nuqtada $(f'(x))'$ hosilaga ega bo’lsa, u $y=f(x)$ funksiyaga nisbatan *ikkinchi tartibli hosila* deyiladi va $f''(x)$, $\frac{d^2f}{dx^2}$ orqali belgilanadi. Induktiv tarzda mulohazani davom ettirsak, qo’yidagiga ega bo’lamiz:

Agar $f(x)$ funksiyaning (n-1) – tartibli $f^{(n-1)}(x)$ hosilasi aniqlangan va differensiallanuvchi bo’lsa, uning n – tartibli hosilasi $f^{(n)}(x)=\left(f^{(n-1)}(x)\right)'$ formula bilan aniqlanadi.

Bunda $f^{(\circ)}(x)=f(x)$ deb qabul qilingan.

. *Maple* dasturida $f^{(n)}(x)$ hosila **diff(*f*(*x*),*x*\$*n*);** operator yordamida topiladi.

Misol. $\frac{\partial^{24}}{\partial x^{24}}(e^x(x^2-1))$.

```
> diff(exp(x)*(x^2-1),x$24):
> collect(%,exp(x));
```

$$e^x(x^2+48x+551)$$

Bu yerda **collect** operatori ixchamlash maqsadida qo’llanildi.

Belgilash: $C^n(E)$ – E to’plamning har bir nuqtasida n – tartibli hosilaga ega bo’lgan funksiyalar sinfi.

<u>Misollar:</u>

1. $y=x^{\alpha}$, $y'=\alpha\cdot x^{\alpha-1}$, $y''=\alpha\cdot(\alpha-1)x^{\alpha-2}$, $y^{(3)}=\alpha(\alpha-1)(\alpha-2)x^{\alpha-3}$, …, $y^{(n)}=\alpha(\alpha-1)\ldots(\alpha-n+1)x^{\alpha-n}$.

$\alpha=m$ – natural son bo’lsa, $\left(x^m\right)^{(m)}=m!$, $\left(x^m\right)^{(n)}=0$, agar $n>m$.

2. $y=\sin x$, $y'=\cos x=\sin\left(x+\frac{\pi}{2}\right)$, $y''=\cos\left(x+\frac{\pi}{2}\right)=\sin\left(x+\frac{\pi}{2}+\frac{\pi}{2}\right)$, …, $(\sin x)^{(n)}=\sin\left(x+n\frac{\pi}{2}\right)$.

3. $\begin{cases}x=\varphi(t)\\ y=\psi(t)\end{cases}$ parametrik qurinishda berilgan funksiyaning ikkinchi tartibli hosilasi quyidagicha aniqlanadi.

$$y_x'=\frac{y_t'}{x_t},\quad y_{xx}'=\frac{\left(\frac{y_t}{x_t}\right)_t}{x_t}=\frac{y_{tt}x_t-y_tx_{tt}}{x^3{}_t}.$$

Endi biz 5 -ma’ruzada berilgan funksiyalarning ikkinchi tartibli hosilalarini topamiz

1) $y=(\ln x)^{\frac{1}{x^2}}$, $y'=\dfrac{(\ln x)^{\frac{1}{x^2}}}{x^3}\left(\dfrac{1}{\ln x}-2\ln\ln x\right).$

Differensiallaymiz y' $\Rightarrow$ $y'=\dfrac{(\ln x)^{\frac{1}{x^2}}}{x^3}\left(\dfrac{1}{\ln x}-2\ln\ln x\right)\Rightarrow$

$$y''=\frac{\left((\ln x)^{\frac{1}{x^2}}\right)' x^3-3x^2(\ln x)^{\frac{1}{x^2}}}{x^6}\cdot$$

$$\cdot\left(\frac{1}{\ln x}-2\ln\ln x\right)+\frac{(\ln x)^{1/x^2}}{x^3}\left(-\frac{1}{x\ln^2 x}-\frac{2}{x\ln x}\right)=\left[\frac{(\ln x)^{\frac{1}{x^2}}}{x^6}\left(\frac{1}{\ln x}-3\ln\ln x\right)-\right.$$

$$\left.-\frac{2}{x^4}(\ln x)^{\frac{1}{x^2}}\right]\cdot\left(\frac{1}{\ln x}-2\ln\ln x\right)-\frac{(\ln x)^{\frac{1}{x^2}}}{x^3}\cdot\left(\frac{1}{x\ln^2 x}+\frac{2}{x\ln x}\right)=\frac{(\ln x)^{\frac{1}{x^2}}}{x^4}\cdot$$

$$\cdot\left[\left(\frac{1}{x^2\ln x}-\frac{2\ln\ln x}{x^2}-3\right)\left(\frac{1}{\ln x}-2\ln\ln x\right)-\frac{1}{\ln^2 x}-\frac{2}{\ln x}\right].$$

. Maple dasturi yordamida bu misol qo’yidagicha yechiladi:

```
>factor(diff(ln(x)^(1/x^2),x$2));

 ln(x)^(1/x^2)*(4*ln(ln(x))^2*ln(x)^2-

 4*ln(ln(x))*ln(x)+1+6*x^2*ln(ln(x))*ln(x)^2-5*x^2*ln(x)-x^2)/

 x^6/ln(x)^2
```

Bu yerda **factor** operatori ko’patuvchilarga ajratish uchun ishlatiladi.

2) $\begin{cases} x=a\cos t\\ y=b\sin t\end{cases}$ $\begin{cases} x=a\cos t\\ y'(x)=-\dfrac{b}{a}\operatorname{ctg} t\end{cases}$

$$y''=\frac{\dfrac{d}{dt}y'(x)}{\dfrac{dx}{dt}}=\frac{\dfrac{b}{a}\dfrac{1}{\sin^2 t}}{-a\sin t}=-\frac{b}{a^2}\frac{1}{\sin^3 t}.\qquad \begin{cases} x=a\cos t\\ y''(x)=-\dfrac{b}{a^2}\dfrac{1}{\sin^3 t}\end{cases}$$

Maple dasturi yordamida bu misol qo’yidagicha yechiladi:

```
>S:=diff(b*sin(t),t)/diff(a*cos(t),t);

 S:=-b*cos(t)/a/sin(t)

>diff(S,t)/diff(a*cos(t),t);
 -(b/a+b*cos(t)^2/a/sin(t)^2)/a/sin(t)
```

***3*).** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ y'=-\frac{b^2x}{a^2y}$.

$$y''=\left(-\frac{b^2x}{a^2y}\right)'=-\frac{b^2}{a^2}\cdot\ \cdot\frac{y-y'x}{y^2}=-\frac{b^2}{a^2y^2}\left(y+\frac{b^2x^2}{a^2y}\right).\ \text{Demak, } y''=-\frac{b^2}{a^2y^2}\left(y+\frac{b^2x^2}{a^2y}\right).$$

Maple dasturi yordamida bu misol qo'yidagicha yechiladi:

```
>Z:=diff(x^2/a^2+y(x)^2/b^2,x);
 Z:= 2*x/a^2+2*y(x)/b^2*diff(y(x),x)
>Q:=solve(Z=0,diff(y(x),x));
 Q:= -x*b^2/y(x)/a^2
>subs(diff(y(x),x)=Q,diff(Q,x));
 -b^2/y(x)/a^2-x^2*b^4/y(x)^3/a^4
```

Bu yerda solve va subs operatorlari mos ravishda tenglamani yechish uchun va o'ringa qo'yish uchun qo'llanildi.

Ikkinchi tartibli hosilaning mexanik ma'nosi.

Moddiy nuqtaning to'g'ri chiziqli harakat qonuni $s=f(t)$ bo'lsin.

Ma'lumki, $s=f(t)$ funksiyaning t_0 nuqtadagi 2-tartibli hosilasi mexanik nuqtai nazardan $s=f(t)$ qonun bilan harakat qilayotgan moddiy nuqtaning t_0 momentdagi oniy tezligini bildiradi.

$s=f(t)$ funksiyaning t_0 nuqtadagi 2-tartibli hosilasi mexanik nuqtai nazardan $s=f(t)$ qonun bilan harakat qilayotgan moddiy nuqtaning $t=t_0$ paytdagi oniy tezlanishini bildiradi.

Leybnits formulasi.

Teorema (Leybnits**).** Agar $u(x)$ va $v(x)$ funksiyalar x nuqtada n – tartibli hosilaga ega bo'lsa, u holda

$$(u\cdot v)^{(n)}=\sum_{m=0}^{n}C_n{}^m u^{(n-m)}v^{(m)}$$

formula o'rinli.

Eslatma. $C_n^k=\frac{n!}{k!(n-k)!}$

Isbotda matematik induktsiya usuli qo'llaniladi.

Eslatma. Leybnits formulasi daraja yetarli kichik bo'lganda qulay. Katta darajalarda komp'yuter vositalariga murojaat qilish maqsadga muvofiq.

Yuqori tartibli differensial.

$y=f(x)$ funksiya $dy=f'(x)dx$ differensiali - x va dx o'zgaruvchilarga bog'liq.

Agar $f'(x)$ - differensiallanuvchi , dx orttirma o'zgarmas bo'lsa, u holda dy funksiyaning orttirmasi mavjud bo'lib, u $d(dy)=d(f'(x)dx)=[f''(x)dx]\Delta x=f''(x)dx^2$ formula yordamida aniqlanadi.

Agar $\Delta x=dx$, bo'lsa n - tartibli differensial $d^n y=d\left(dy^{n-1}\right)$ rekkurent formula yordamida aniqlanadi, bunda $d^n y=f^{(n)}(x)dx^n$, $f^{(n)}=\frac{d^n y}{dx^n}$.

Agar $x=\varphi(t)$ bo'lsa, u holda: $y=f(x),\ x=\varphi(t),\ dy=f'(x)dx,\ dx=\varphi'(t)dt$,

$$d^2y = d\left(f'(x)dx\right) = df' \cdot dx + f' \cdot d(dx) = f''dx^2 + f'd^2x = f''dxdx + f'd\left(\varphi'(t)dt\right) =$$
$$= f''dx^2 + f'\varphi''dt^2$$
va *x*.k.

Oraliq nazorat savollari.

1. Yuqori tartibli hosila deganda nimani tushunasiz?
2. Ikkinchi tartibli hosilaning mexanik ma'nosi nimadan iborat?
3. Ikkita funksiya ko'paytmasining n-tartibli hosilasi uchun qanday formula o'rinli?
4. sin*x*, cos*x*, a^x, $\frac{1}{x+1}$ funksiyalarning *n*-tartibli hosilalari uchun qanday formulalar o'rinli?
5. Yuqori tartibli differensial nima?

Tayanch tushunchalar. Yuqori tartibli hosila, ikkinchi tartibli hosila, yuqori tartibli differensial.

10-ma'ruza.

Teylor va Makloren formulalari. Ba'zi bir elementar funksiyalar uchun Makloren formulalalari.

Reja :

1. Teylor va Makloren formulalari.
2. Elementar funksiyalar uchun Makloren formulasi.

Adabiyot: [1] - 214-226 betlar; [2] - 146-150 betlar;

Teylor va Makloren formulalari.

$y=f(x)$ funksiya $(a;b)$ intervalda aniqlangan bo'lsin.

Agar $y=f(x)$ funksiya $x_0 \in (a;b)$ nuqtada differensiallanuvchi bo'lsa, u holda uning $\Delta y=f(x_0+\Delta x)-f(x_0)$ ($x_0+\Delta x\in(a;b)$) orttirmasini

$$\Delta y= A\Delta x + \alpha\Delta x \qquad (1)$$

ko'rinishda yozish mumkin, bu yerda $A = f'(x_0)$, α esa Δx ga bog'liq va $\Delta x\to 0$ da $\alpha=\alpha(\Delta x)\to 0$, ya'ni $\alpha=o(\Delta x)$.

Shunday qilib $x= x_0+\Delta x$ belgilash kiritsak, (1) formulani

$$f(x)=f(x_0)+f'(x_0)(x- x_0)+ o(x - x_0),\ x\in(a;b)$$

ko'rinishda yozish mumkin.

Boshqacha aytganda, shunday chiziqli $P_1(x)=f(x_0)+f'(x_0)(x- x_0)$ mavjudki,

$$f(x)= P_1(x) + o(x - x_0),\ x\in(a;b)$$

formula o'rinli bo'lib, $P_1(x_0)=f(x_0)$, $(P_1(x_0))'=f'(x_0)$ tengliklar bajariladi.

Endi yuqoridagini umumlashtiramiz.

Berilgan $y=f(x)$ funksiya $x_0 \in(a;b)$ nuqtada n - tartibli xosilaga ega bo'lsin. Shunday $P_n(x)$ haqiqiy koeffitsientli ko'phad topilishi kerakki, natijada

$$f(x)= P_n(x) + o((x - x_0)^n),\ x\in(a;b) \qquad (2)$$

formula o'rinli bo'lib,

$$(P_n(x_0))^{(j)}=f^{(j)}(x_0),\ k=0,1,\ldots,n, \qquad (3)$$

tengliklar bajariladi.

Ushbu $P_n(x)$ ko'phadni

$$P_n(x)=a_0+a_1(x- x_0)+a_2(x- x_0)^2+\ldots+a_n(x- x_0)^n$$

ko'rinishda izlaymiz. Bunda a_0, a_1, ... , a_n va x_0 lar o'zgarmas *haqiq*iy sonlar, n $\in$N. Bu ko'phadni n marta differensiallab topamiz:

$$P'_n(x)=a_1+2a_2(x- x_0)+\ldots+na_n(x- x_0)^{n-1},$$

$$P''_n(x)=2a_2+3\cdot 2a_3(x- x_0)+\ldots+n(n-1)a_n(x- x_0)^{n-2},$$

- -

$P^{(n)}{}_n(x)=n(n-1)(n-2)...2a_n$

Bu tengliklarda $x=x_0$ deb olinsa, unda berilgan $R_n(x)$ ko'phad va uning hosilalari $P_n^{(j)}(x)$ (k=1,...,n) ning x_0 nuqtadagi qiymatlari topiladi:

$R_n(x_0)=a_0$, $R'_n(x_0)=a_1$, $R''_n(x_0)=2!a_2$,..., $R_n^{(n)}(x_0)=n!a_n$.

Bulardan (3) tengliklardan foydalanib, ketma-ket

$a_0=f(x_0)$, $a_1=\dfrac{f'(x_0)}{1!}$, $a_2=\dfrac{f''(x_0)}{2!}$,..., $a_n=\dfrac{f^{(n)}(x_0)}{n!}$ koeffitsientlarni topamiz. Endi $P_n(x)=a_0+a_1(x-x_0)+a_2(x-x_0)^2+...+a_n(x-x_0)^n$ (2) ni qanoatlantirganligini ko'rsatish uchun

$$r_n(x)=f(x)-P_n(x) \qquad (4)$$

funksiyani kiritamiz. (3) ga ko'ra $(r_n(x_0))^{(j)}=0$, k=0,1,…,n, tengliklar bajariladi.

$\dfrac{r_n(x)}{(x-x_0)^n}$, $x\to x_0$, aniqmaslikni ochish uchun n -marta Lopital' qoidasini qo'llaymiz, ya'ni 8-ma'ruzadagi 5-teoremadan foydalanamiz:

$$\lim_{x\to x_0}\frac{r_n(x)}{(x-x_0)^n}=\lim_{x\to x_0}\frac{r_n'(x)}{n(x-x_0)^{n-1}}=...=\lim_{x\to x_0}\frac{r_n^{(n-1)}(x)}{n!(x-x_0)}=\lim_{x\to x_0}\frac{r_n^{(n)}(x)}{n!}=\frac{r_n^{(n)}(x_0)}{n!}=0$$

Bundan $r_n(x)=o((x-x_0)^n)$, $x\in(a;b)$ ekanligi kelib chiqadi.

Demak, quyidagi teorema isbotlandi:

Teorema. $y=f(x)$ funksiya $x_0\in(a;b)$ nuqtada n - tartibli xosilaga ega bo'lsin. u holda $x\to x_0$ da

$$f(x)=f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+...+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+r_n(x) \qquad (5)$$

bo'ladi, bu yerda $r_n(x)=o((x-x_0)^n)$ - qoldiq had .

Natija. $y=f(x)$ funksiya $x_0\in(a;b)$ nuqtada (n+1) - tartibli xosilaga ega bo'lsin. u holda $x\to x_0$ da

$$f(x)=f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+...+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+O((x-x_0)^{n+1}) \quad (6).$$

Haqiqatdan ham, (5) ga qura

$$f(x)=f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+...+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+\frac{f^{(n+1)}(x_0)}{(n+1)!}+o((x-x_0)^{n+1}).$$

$\dfrac{f^{(n+1)}(x_0)}{(n+1)!}+o((x-x_0)^{n+1})=O((x-x_0)^{n+1})$ bo'lgani uchun (6) formula o'rinli.

(5) formuladan $r_n(x)=f(x)-\sum_{k=0}^{n}\dfrac{(x-x_0)^k}{k!}f^k(x_0)$ kelib chiqadi. Bu qoldiq hadning ixchamroq ifodasini izlash maqsadida, uni

$r_n(x)=\dfrac{(x-x_0)^{n+1}}{(n+1)!}Q(x)$ ko'rinishda izlaymiz. Quyidagi yordamchi funksiyani kiritamiz:

$$\varphi(t)=f(t)-\sum_{k=0}^{n}\frac{(x-t)^k}{k!}f^k(t)-\frac{(x-x_0)^{n+1}}{(n+1)!}Q(x)$$

$\varphi(t)$ funksiya $[x_0,x]$ da Roll teoremasining shartlarini qanoatlantiradi: $\varphi(t)$ differensiallanuvchi va $\varphi(x_0)=\varphi(x)=0$. Shuning uchun oldin $\varphi'(t)$ ni hisoblaymiz:

$\varphi'(t)= \sum_{k=0}^{n}\frac{(x-t)^k}{k!}f^{(k+1)}(t)+\sum_{k=0}^{n}\frac{(x-t)^{k-1}}{(k-1)!}f^{k}(t)+\frac{(x-t)^n}{n!}Q(x)\;=$

$=-f^{(n+1)}(t)\;\frac{(x-t)^n}{n!}+\frac{(x-t)^n}{n!}Q(x).$

Roll teoremasiga ko'ra shunday $s\in(x_0;x)$ mavjudki, $\varphi'(c)=0$ bo'ladi, ya'ni $-f^{(n+1)}(c)$ $\frac{(x-c)^n}{n!}+\frac{(x-c)^n}{n!}Q(x)=0.$

Bundan $Q(x)=f^{(n+1)}(c)$ kelib chiqadi. Demak ,

$$r_n(x)=\frac{(x-x_0)^{n+1}}{(n+1)!}f^{(n+1)}(c).$$

Bu *qoldiq hadning Lagranj formasi* deyiladi.

Shunday qilib $y=f(x)$ funksiya uchun x_0 nuqtadagi Teylor formulasi quyidagichadir:

$$f(x)=\quad f(x_0)+\quad\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+\ldots+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n\quad+\frac{f^{(n+1)}(x_0)(c)}{(n+1)!}(x-x_0)^{n+1},$$

$c\in(x_0;x)$.

Garchi Lagranj shaklidagi qoldiq had soddalik ma'nosida yaxshilashni talab etmasa ham, ayrim hollarda qoldiq hadni baholash uchun bu shakl qo'llashga yaramasdan boshqa soddaroq bo'lmagan shakllarga murojat etishga to'g'ri kelar ekan. Masalan, ulardan bittasi, Koshi shaklidagi qoldiq had:

$$r_n(x)=\frac{f^{(n+1)}(x_0+\theta(x-x_0))}{n!}(1-\theta)^n(x-x_0)^{n+1},$$

ba'zi hollarda $(1-\theta)^n$ ko'paytuvchining borligi sababli foydaliroq bo'ladi.

$y=f(x)$ funksiyaning Teylor formulasida $x_0=0$ deb olinsa, ushbu

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\ldots+\frac{f^{(n)}(0)}{n!}x^n+r_n(x)\quad(7)$$

formula hosil bo'ladi. Bu holda qoldiq had $r_n(x)$ quyidagicha:

a) *Koshi ko'rinishida*: $r_n(x)=\frac{x^{n+1}(1-\theta)^n}{n!}f^{n+1}(\theta x)$,

b) *Lagranj ko'rinishda*: $r_n(x)=\frac{x^{n+1}}{(n+1)!}f^{n+1}(\theta x)$, $(0<\theta<1)$

yozilishi mumkin.

Yuqoridagi (7) formula $y=f(x)$ funksiyaning Makloren formulasi deb ataladi.

. *Maple* dasturida $y=f(x)$ ni x_0 nuqta atrofida Teylor formulasiga yoyish

taylor(<f(x)>,x=<x0>,<n>+1);operator yordamida amalga oshiriladi.

Elementar funksiyalar uchun Makloren formulasi.

1^0. $f(x)=e^x$ bo'lsin. Bu funksiya uchun $f^{(n)}(x)=e^x$ va $f(0)=1$, $f^{(n)}(0)=1$ (n=1, 2, ...). U holda

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\ldots+\frac{x^n}{n!}+r_n(x)$$

bo'lib, uning qoldiq hadi esa Lagranj ko'rinishida quyidagicha yoziladi

$$r_n(x)=\frac{x^{n+1}}{(n+1)!}e^{\theta x},\ (0<\theta<1)\ .$$

Har bir $x\in[-a;a]$ $(a>0)$ da $e^{\theta x}<e^a$ bo'lishini e'tiborga olsak, unda

$|r_n(x)| < \frac{a^{n+1}}{(n+1)!}e^a$ tengsizlik kelib chiqadi va n→∞ da $\frac{a^{n+1}}{(n+1)!}e^a$ ifoda, va demak, $r_n(x)$ ham nolga intiladi. Natijada $f(x)=e^x$ funksiya uchun quyidagi

$$e^x \approx 1+\frac{x}{1!}+\frac{x^2}{2!}+...+\frac{x^n}{n!}$$

taqribiy formulaga ega bo'lamiz.

2^0. $f(x)=\sin x$ bo'lsin. Bu funksiya uchun

$$f^{(n)}(x)=\sin(x+n\frac{\pi}{2}) \text{ va } f(0)=0,\ f^{(n)}(0)=\sin\frac{n\pi}{2} = \begin{cases} 0, & n=2m, \\ (-1)^{m-1}, & n=2m-1 \end{cases}$$

Shuning uchun (6) formulada $n=2m$ deb

$$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+...+(-1)^{m-1}\frac{x^{2m-1}}{(2m-1)!}+r_{2m}(x)$$

hosil qilamiz. Bu formulaning qoldiq hadi Lagranj ko'rinishida quyidagicha yoziladi:

$$r_n(x)=\frac{x^{2m+1}}{(2m+1)!}\sin(\theta x+m\pi+\frac{\pi}{2}) \quad (0<\theta<1).$$

Ravshanki, $\forall x\in[-a;a]$ $(a>0)$ *da* $|r_{2m}(x)| \le \frac{a^{2m+1}}{(2m+1)!}$ bo'lib, n→ ∞ da $\frac{a^{2m+1}}{(2m+1)!}$ ifoda, va demak, $r_{2m}(x)$ ham nolga intiladi.

Misol. $\varepsilon=10^{-5}$ aniqlikda $\sin 28°$ ni qiymatini toping..

Yechish: $y=\sin x,\ x_0=30°=\frac{\pi}{6},\ \Delta x=-2°=-\frac{\pi}{90}$. $|R_n| = \left.\frac{|(\sin y)^{n+1}|}{(n+1)!}\right|_{y=x_0+\theta(x-x_0)=x_0+\theta\Delta x}$.

$\cdot\left(\frac{\pi}{90}\right)^{n+1} \le \frac{1}{(n+1)!}\left(\frac{\pi}{90}\right)^{n+1}$. $|R_1| \le \frac{\pi^2}{90\cdot 180} \approx \frac{9}{100} > 10^{-5}$. $|R_2| \le \frac{\pi^3}{90^3\cdot 6} \approx \frac{27}{90^3\cdot 6} = \frac{1}{162000} < 10^{-5}$.

$n=2$ ta had kerakli aniqlikni ta'minlaydi.

$$\sin 28° \approx \frac{1}{2}-\frac{\sqrt{3}}{2}\frac{\pi}{90}-\frac{1}{4}\left(\frac{\pi}{90}\right)^2 = \frac{1}{2}-\frac{\sqrt{3}\pi}{180}-\frac{\pi^2}{32400} \cong \frac{1}{2}-0{,}03023-0{,}00030 \approx$$

$\approx 0{,}46947$.

. Maple dasturi yordamida bu misol qo'yidagicha yechiladi:

```
>C:=taylor(sin(x),x=Pi/6,5);
 C:=series(1/2+(1/2*3^(1/2))*(x-1/6*Pi)-1/4*(x-1/6*Pi)^2+(-
 1/12*3^(1/2))*(x-1/6*Pi)^3+1/48*(x-1/6*Pi)^4+O((x-1/6*Pi)^5),x=
 -(-1/6*Pi),5)
>V:=subs(x=7/45*Pi,C); convert(evalf(C),polynom);
V:=1/2-1/180*3^(1/2)*Pi-1/32400*Pi^2+1/8748000*3^(1/2)*
 Pi^3+1/3149280000*Pi^4+O(-1/5904900000*Pi^5)
```

.4694715632

3^0. $f(x)=\cos x$ funksiyaning Makloren formulasi quyidagicha yoziladi:

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+...+(-1)^m\frac{x^{2m}}{2m!}+r_{2m+1}(x)$$

Uning qoldiq hadi Lagranj ko'rinishda quyidagicha yoziladi:

$$r_{2m+1}(x)=\frac{x^{2m+2}}{(2m+2)!}\cos(\theta x+(m+1)\pi) \quad (0<\theta<1)$$

4^0. $f(x)=(1+x)^{\mu}$ bo'lsin, bunda $\mu\in R$. Bu funksiyaning n-tartibli hosilasi uchun $f^{(n)}(x)=\mu(\mu-1)\ldots(\mu-n+1)(1+x)^{\mu-n}$ formulaga egamiz. Ravshanki, $f(0)=1$, $f^{(n)}(0)=\alpha(\alpha-1)...(\alpha-n+1)$. $f(x)=(1+x)^{\mu}$ funksiyaning Makloren formulasi quyidagicha yoziladi:

$$(1+x)^{\mu}=1+\frac{\mu}{1!}x+\frac{\mu(\mu-1)}{2!}x^2+...+\frac{\mu(\mu-1)...(\mu-n+1)}{n!}x^n+r_n(x)$$

Koldiq hadni baholashda uni ushbu

$r_n(x)=\dfrac{\mu(\mu-1)...(\mu-n+1)}{n!}(1+\theta x)^{\mu-n-1}(1-\theta)^n x^{n+1}$ Koshi ko'rinishda olish qulay. Endi $|x|<1$ bo'lganda

$$|r_n(x)|=\left|\mu(1-\frac{\mu}{1})(1-\frac{\mu}{2})...(1-\frac{\mu}{n})\right|(1+\theta x)^{\mu-1}\left|\frac{1-\theta}{1+\theta x}\right|^n|x|^{n+1}\le$$

$$\le\left|\mu(1-\frac{\mu}{1})(1-\frac{\mu}{2})...(1-\frac{\mu}{n})\right|(1+\theta x)^{\mu-1}|x|^{n+1}$$

bo'lib, $n\to\infty$ da nolga intiladi.

Oraliq nazorat savollari

1. Peano, Lagranj va Koshi ko'rinishlardagi qoldiq hadli Teylor formulasini keltiring.
2. Teylor formulasini differensiallar formulasida ifodalang.
3. Makloren formulasini keltiring.
4. $f(x)=ln(1+x)$ funksiyasining Makloren formulasini yozing.
5. Funksiya uchun Teylor formulasi qanday yoziladi?
6. Teylor formulasining qoldiq hadi nima?
7. Qoldiq hadning Lagranj formasi qanday yoziladi?
8. Teylor va Makloren formulalari nimasi bilan farq qiladi?
9. e^x, $\sin x$, $\cos x$, $(1+x)^{\mu}$ funksiyalar uchun Makloren formulalari qanday ko'rinishga ega?

Tayanch tushunchalar: Teylor va Makloren formulalari, qoldiq had, qoldiq hadning Peano, Lagranj va Koshi ko'rinishlari.

11-ma'ruza

Funksiyaning monotonlik sharti. Maksimum va minimumlar. Ekstremumlarni topish.

Reja:

1. Funksiyaning monotonlik sharti.
2. Maksimum va minimumlar. Ekstremumlarni topish.

Funksiyaning monotonlik sharti.

Ta'rif. $y=f(x)$ funksiya $(a;b)$ intervalda aniqlangan bo'lsin. Agar ihtiyoriy $x_1\le x_2$ tengsizlikni qanoatlantiradigan $x_1, x_2\in(a,b)$ nuqtalar uchun $f(x_1)\le f(x_2)$ ($f(x_1)\ge f(x_2)$) tengsizlik bajarilsa, u holda f funksiya (a,b) oraliqda o'*suvchi* (*kamayuvchi*) funksiya deyiladi, (a,b) oraliq esa *monotonlik oralig'i* deyiladi

Ta'rif. $y=f(x)$ funksiya ($a;b$) intervalda aniqlangan bo'lsin. Agar ihtiyoriy $x_1 < x_2$ tengsizlikni qanoatlantiradigan $x_1, x_2 \in (a,b)$ nuqtalar uchun $f(x_1)< f(x_2)$ ($f(x_1)>f(x_2)$) tengsizlik bajarilsa, u holda f funksiya (a,b) oraliqda *qat'iy o'suvchi* (*kamayuvchi*) *funksiya* deyiladi.

Teorema. $y=f(x)$ funksiya ($a;b$) intervalda aniqlangan va differensiallanuvchi bo'lsin. $y=f(x)$ funksiya ($a;b$) intervalda o'suvchi (kamayuvchi) bo'lishi uchun shu intervalda $f'(x)\geq 0$ ($f'(x)\leq 0$) tengsizlik bajarilishi zarur va yetarli.

Agar ($a;b$) intervalda $f'(x)>0$ ($f'(x)< 0$) tengsizlik bajarilsa, u holda $y=f(x)$ funksiya ($a;b$) intervalda qat'iy o'suvchi (kamayuvchi) bo'ladi.

Isbot. Zarurligi. $y=f(x)$ funksiya ($a;b$) intervalda o'suvchi (kamayuvchi) bo'lsin. U holda ihtiyoriy $x_0 \in (a,b)$ uchun $\Delta x > 0$ da $\Delta y = f(x_0 + \Delta x) - f(x_0) \geq 0$ ($\Delta y \leq 0$) bo'ladi. Demak, $\frac{\Delta y}{\Delta x} \geq 0$ ($\frac{\Delta y}{\Delta x} \leq 0$) bo'ladi. $\Delta x \to 0$ da limitga o'tsak, $f'(x_0)\geq 0$ ($f'(x_0)\leq 0$) tengsizlikni hosil qilamiz.

Yetarliligi. $a < x_1 < x_2 < b$ bo'lsin. Lagranj formulasiga kura $f(x_2) - f(x_1) = f'(c)(x_2 - x_1)$ bo'ladi, bu yerda $a<c<b$. $\forall x \in (a,b)$ $f'(x)\geq 0$ shartiga ko'ra $f'(c)\geq 0$ bo'ladi. $x_1 < x_2$ bo'lgani uchun $f(x_2) \geq f(x_1)$ bo'ladi. Demak, $y=f(x)$ o'sadi.

Xuddi shunday, agar $\forall x \in (a,b)$ $f'(x)\leq 0$ bo'lsa $f'(c) \leq 0$ bo'ladi. $x_1 < x_2$ bo'lgani uchun $f(x_2) \leq f(x_1)$ bo'ladi. Demak, $y=f(x)$ kamayadi.

Agar $\forall x \in (a,b)$ $f'(x)>0$ bulsa $f'(c) >0$ bo'ladi. Bundan $f(x_2)> f(x_1)$, ya'ni funksiyaning qat'iy o'sishi kelib chiqadi.

Agar $\forall x \in (a,b)$ $f'(x)<0$ bulsa $f'(c) <0$ bo'ladi. Bundan $f(x_2) < f(x_1)$, ya'ni funksiyaning qat'iy kamayishi kelib chiqadi.

Teorema isbotlandi.

Misol. $y = x^3 - x^2 - x + 2$ funksiyaning monotonlik oraliqlarini toping.

Yechish. $y' = (x^3 - x^2 - x + 2)' = 3x^2 - 2x - 1 = 3(x-1)(x+\frac{1}{3})$. Demak, $(-\frac{1}{3}, 1)$ da funksiya kamayadi, $(-\infty, -\frac{1}{3}) \cup (1, +\infty)$ da esa o'sadi.

Maksimum va minimumlar. Ekstremumlarni topish

Ta'rif. f funksiya x_0 nuqtaning biror atrofida aniqlangan bo'lsin. Agar x_0 nuqtaning yetarlicha kichik atrofiga tegishli bo'lgan barcha x lar uchun $f(x_0)\geq f(x)$ ($f(x_0)>f(x)$) tengsizlik bajarilsa, u holda x_0 nuqta lokal maksimum (qat'iy lokal maksimum) nuqta deyiladi.

Agar x_0 nuqtaning yetarlicha kichik atrofiga tegishli bo'lgan barcha x lar uchun $f(x_0)\leq f(x)$ ($f(x_0)<f(x)$) tengsizlik bajarilsa, u holda x_0 nuqta lokal minimum (qat'iy lokal minimum) nuqta deyiladi.

Maksimum va minimum so'zlari o'rniga ekstremum termini ham yuritiladi.

Teorema (lokal ekstremumning zaruriy sharti). f funksiya x_0 lokal ekstremum nuqtasining biror atrofida aniqlangan bo'lsin. U holda yo $f'(x_0)=0$ *yo* $f'(x_0)$ mavjud emas.

Isbot. f funksiya , biror (x_0 , $x_0 + \delta$) intervalda aniqlangan bo'lib , uning ichki x_0 nuqtasida eng katta (yoki eng kichik) qiymatga erishadi.

Ferma teoremasiga kura agar nuqtada chekli $f'(x_0)$ hosila mavjud bo'lsa, u holda $f'(x_0)=0$ bo'ladi.

Teorema isbotlandi.

$f(x)= x^3$ funksiya misoli $f'(x_0)=0$ sharti x_0 nuqta ekstremum nuqtasi bo'lishi uchun yetarli emasligini ko'rsatadi.

Teorema (qat'iy lokal ekstremumning birinchi yetarli sharti). *f* funksiya x_0 lokal ekstremum nuqtasining biror atrofida aniqlangan va differensiallanuvchi bo'lsin.

Agar ($x < x_0 \Rightarrow f'(x) < 0$) va ($x > x_0 \Rightarrow f'(x) < 0$) $\Rightarrow$ ekstremum yo'q;

Agar ($x < x_0 \Rightarrow f'(x) < 0$) va ($x > x_0 \Rightarrow f'(x) > 0$) $\Rightarrow$ x_0 - lokal minimum;

Agar ($x < x_0 \Rightarrow f'(x) > 0$) va ($x > x_0 \Rightarrow f'(x) < 0$) $\Rightarrow$ x_0 - lokal maksimum;

Agar ($x < x_0 \Rightarrow f'(x) > 0$) va ($x > x_0 \Rightarrow f'(x) > 0$) $\Rightarrow$ ekstremum yo'q.

Bu teoremaning isboti Lagranj formulasidan bevosita kelib chiqadi (mustaqil tekshirind)

Teorema (qat'iy lokal ekstremumning ikkinchi yetarli sharti). *f* funksiya x_0 nuqtaning biror $U(x_0)$ atrofida aniqlangan va ushbu atrofda tartiblari $n-1$ gacha bo'lgan hosilalarga x_0 nuqtada esa $f^{(n)}(x_0)$ hosilaga ega bo'lsin.

$f^{(n)}(x_0) = ... = f^{(n-1)}(x_0) = 0$ va $f^{(n)}(x_0) \neq 0$ shartlar bajarilsin.

U holda

A) agar n -toq bo'lsa , u holda x_0 ekstremum nuqta emas,

B) agar n -juft bo'lib , $f^{(n)}(x_0) > 0$ bajarilsa, u holda x_0 lokal minimum nuqtasi.

Agar n -juft bo'lib , $f^{(n)}(x_0) < 0$ bajarilsa, u holda x_0 lokal maksimum nuqtasi.

Isbot: Teylor formulalariga ko'ra

$$f(x) - f(x_0) = f^{(n)}(x_0)(x - x_0)^n + \alpha(x)(x - x_0)^n, \ (\alpha \to 0 \quad x \to x_0 \text{ da})$$

$$f(x) - f(x_0) = \left(f^{(n)}(x_0) + \alpha(x)\right)(x - x_0)^n,$$

Agar n toq bo'lsa, $(x - x_0)^n$ ifoda x_0 nuqtadan o'tganda o'z ishorasini o'zgartiradi, demak $f(x) - f(x_0)$ ifoda ham ishorasini o'zgartiradi, ya'ni ekstremum yo'q.

Agar n -juft bo'lib , $f^{(n)}(x_0) > 0$ bajarilsa, $(x - x_0)^n$ ifoda x_0 nuqtadan o'tganda o'z ishorasini o'zgartirmaydi, demak $f(x) - f(x_0)$ ifoda har doim musbat bo'lib qoladi ya'ni x_0 lokal minimum nuqtasi.

Agar n -juft bo'lib , $f^{(n)}(x_0) < 0$ bajarilsa, $(x - x_0)^n$ ifoda x_0 nuqtadan o'tganda o'z ishorasini o'zgartirmaydi, demak $f(x) - f(x_0)$ ifoda har doim manfiy bo'lib qoladi, ya'ni x_0 lokal maksimum nuqtasi.

Teorema isbotlandi.

Natija. Agar *f'*(x_0)=0 bo'lib, *f''*(x_0) <0 (*f''*(x_0)>0) bajarilsa, u holda *f* funksiya x_0 nuqtada lokal maksimumga (lokal minimumga) erishadi.

Izox. Funksiyaning segmentdagi eng katta va eng kichik qiymatlarini topish uchun uning barcha kritik nuqtalari topiladi va ular lokal ekstremumlikka tekshiriladi. Bundan keyin hosil bo'lgan qiymatlar chegaraviy nuqtalardagi qiymatlar bilan solishtiriladi.

. Misol. $y = \frac{1}{2}(x^2 - \frac{1}{2})\arcsin x + \frac{x}{4}\sqrt{1-x^2} - \frac{\pi}{12}x^2$ funksiyaning Maple yordamida ekstremumlarini topamiz.

```
> readlib(extrema):
> y:=(x^2-1/2)*arcsin(x)/2+x*sqrt(1-x^2)/4-
  Pi*x^2/12:
> extrema(y,{},x,'s');s;
```

$$\{0, -\frac{1}{24}\pi + \frac{1}{16}\sqrt{3}\}$$

$$\{\{x = 0\}, \{x = \frac{1}{2}\}\}$$

Bunda funksiyaning ekstremum nuktalari va ekstremal qiymatlari topildi. Konkretlashtirsh uchun **maximize** va **minimize** operatorlar qo'llanladi.

> readlib(maximizf):readlib(minimizf):

> ymax:=maximize(y,{x});

$$ymax := 0$$

> ymin:=minimize(y,{x});

$$ymin := -\frac{1}{24}\pi + \frac{1}{16}\sqrt{3}$$

Javob: "Ekstremumlar: $\max y(x) = y(0) = 0$, $\min y(x) = y(1/2) = -\pi/24 + \sqrt{3}/16$."

Oraliq nazorat savollari.

1. Qanday nuqta statsionar nuqta deyiladi?
2. Qanday nuqta kritik nuqta deyiladi?
3. Statsionar va kritik nuqtalar orasida qanday boqlanish mavjud?
4. Funksiyaning nuqtadagi monotonligini qanday tushunasiz?
5. Funksiyaning nuqtadagi va to'plamdagi monotonlik shartlari nimadan iborat?
6. Funksiyaning to'plamdagi monotonlik shartlari qanday isbotlanadi?
7. Hosila yordamida monotonlikka tekshirish uchun nima ishlar bajariladi?
8. Funksiyaning maksimumi, minimumi nima?
9. Funksiyaning ekstremumi deganda nimani tushunasiz?
10. Ekstremumning zaruriy sharti nimadan iborat?
11. Ekstremumning zaruriy sharti qanday isbotlanadi?
12. Ekstremumning qanday yetarli shartlarini bilasiz?
13. Ekstremumning I yetarli sharti nimadan iborat? Nima uchun bu shart zaruriy emas? Javobingizni asoslang.
14. Ekstremumning I yetarli sharti qanday isbotlanadi?
15. Birinchi tartibli hosila yordamida ekstremumga qanday tekshiriladi?
16. Ekstremumning II yetarli sharti nimadan iborat?
17. Yuqori tartibli hosila yordamida ekstremumga qanday tekshiriladi?
18. Funksiyaning segmentdagi eng katta va eng kichik qiymatlarini topish uchun nima ishlar bajariladi?

12-ma'ruza.
Funksiyaning qavariqligi va botiqligi, burilish nuqtasi.

Reja :

1. Funksiyaning qavariqligi va botiqligi
2. Burilish nuqtasi.

Adabiyot: [1] - 238-245 betlar; [2] - 158-167 betlar;

Funksiyaning qavariqligi va botiqligi, burilish nuqtasi.

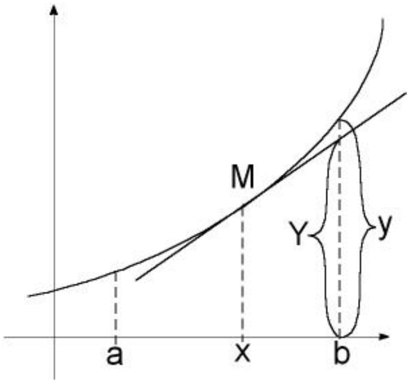
($a;b$) da differensiallanuvchi $y=f(x)$ funksiyaning tekislikdagi grafigi bo'lgan $y=f(x)$ egri chiziq berilgan bo'lsin.

1-ta'rif. Agar ($a;b$) da egri chiziqning barcha nuqtalari uning har qanday urinmasidan pastda bo'lsa, u holda shu intervalda egri chiziq qavariq deb ataladi

2- ta'rif. Agar $(a;b)$ da egri chiziqning barcha nuqtalari uning har qanday urinmasidan yuqorida bo'lsa, u holda shu intervalda egri chiziq botiq deb ataladi.

$f(x)$ funksiyaning $f'(x)$ xosilasini uzluksiz deb faraz qilib, quyidagi teoremalarni isbot qilamiz.

1-***Teorema***. Agar $y=f(x)$ egri chiziq $x_0 \in (a;b)$ da qavariq (botiq) bo'lsa, u holda shu nuqtada $f''(x_0) \leq 0$ ($f''(x_0) \geq 0$) bo'ladi.



Isbot. $y=f(x)$ egri chiziq $(x_0, f(x_0))$ nuqtasida qavariq bo'lsin. U holda shu nuqtada o'tkazilgan $Y-f(x_0)=f'(x_0)(x-x_0)$ urinma egri chiziqdan yuqorida yotadi. Shuning uchun $y=f(x)$ egri chiziq va urinmaning x_0 nuqtaning biror atrofiga mos keladigan ordinatalarining ayirmasi:

$$u-Y=f(x)-f(x_0)-f'(x_0)(x-x_0) \leq 0 \qquad (1)$$

bo'ladi. Bundan $\varphi(x)=y-Y$ deb olsak, x_0 nuqtada nolga teng bo'lib, uning biror atrofida manfiy bo'lgani uchun $\varphi(x)$ funksiyasining shu nuqtada maksimumga ega ekanligi kelib chiqadi. Demak, $\varphi''(x_0) \leq 0$ bo'lishi kerak, shuning uchun $\varphi(x)=f(x)-f(x_0)-f'(x_0)(x-x_0)$ tenglikdan $f''(x) \leq 0$ kelib chiqadi. Teorema isbot bo'ldi.

2-***Teorema***. Agar $x_0 \in (a;b)$ da $f''(x_0)<0$ ($f''(x_0)>0$) bo'lsa, u holda shu nuqtada $y=f(x)$ egri chiziq qavariq (botiq) bo'ladi.

Isbot. Bu yerda ham $\varphi(x)=y-Y$ funksiyani tekshiramiz. Shartga ko'ra $f''(x)<0$ va $f''(x)=\varphi''(x)$ bo'lgani uchun $\varphi''(x)<0$ bo'ladi. Bundan $\varphi(x)$ funksiyaning x_0 nuqtada maksimumga ega ekanligi kelib chiqadi. Demak, $\varphi(x_0)=0$ bo'lgani uchun x_0 nuqtaning biror atrofida $\varphi(x)<0$ bo'ladi. Bu holda $u-Y<0$ tengsizligi o'rinli va shu bilan egri chiziqning $x_0 \in (a;b)$ da qavariq ekanligi isbot qilinadi.

3-ta'rif. Agar $y=f(x)$ funksiya $V\delta^{-}(x_0)$ oraliqda botiq (qavariq) bo'lib, $V\delta^{+}(x_0)$ oraliqda esa qavariq (botiq) bo'lsa, u holda x_0 funksiyaning burilish nuqtasi deb ataladi.

3-***Teorema***. Agar $f''(x_0)=0$ bo'lsa, yoki $f''(x_0)$ mavjud bo'lmasa va x_0 nuqtadan o'tishda $f''(x)$ ning ishorasi o'zgarsa, u holda x_0 funksiyaning burilish nuqtasi bo'ladi.

Haqiqatan ham $x<x_0$ bo'lganda $f''(x)<0$ ($f''(x)>0$) va $x>x_0$ bo'lganda $f''(x)>0$ ($f''(x)<0$) bo'lsin. U vaqtda 2-teoremaga asosan $x<x_0$ bo'lganda $y=f(x)$ funksiya qavariq (botiq) va $x>x_0$ bo'lganda esa botiq (qavariq) bo'ladi. Demak, x_0 - funksiyaning burilish nuqtasidir.

$F''(x_0)=0$ bo'lishi yoki $f''(x_0)$ mavjud bo'lmasligi egilish nuqtasi mavjudligining faqat zaruriy sharti bo'lib, yetarli sharti bo'la olmaydi. Masalan, $y=x^4$ funksiya uchun $y'=4x^3$, $y''=12x^2$ va $y''(0)=0$ bo'ladi. Lekin $x=0$ burilish nuqta emas.

*X*ulosa. $y=f(x)$ funksiya ikkinchi tartibli hosilalarga ega bo'lsa, uning grafigidagi qavariq va botiq qismlarni aniqlash uchun $f''(x)<0$ va $f''(x)>0$ tengsizliklarni yechamiz. $f''(x)$ qaysi oraliqda manfiy (musbat) bo'lsa, o'sha oraliqda egri chiziq qavariq (botiq) bo'ladi.

Burilish nuqtasini topish uchun $f''(x)=0$ tenglamani yechamiz va $f''(x)$ mavjud bo'lmagan nuqtalarni aniqlaymiz. Bu nuqtalarning qaysi birining atrofida (chap va o'ng tomonlarida) $f''(x)$ har *x*il ishorali bo'lsa, shu nuqta funksiyaning burilish nuqtasi bo'ladi.

1. Qanday funksiya qavariq (botiq) deyiladi?
2. Funksiyaning burilish nuqtasi nima?
3. Burilishga gumon nuqtalar qanday aniqlanadi?
4. Funksiyani qavariqlik-botiqlikka qanday tekshiriladi?

13-ma'ruza.

Asimptotalar. Funksiyani tekshirish va grafigini yasash sxemasi

Reja :

1. Asimptotalar
2. Funksiyani tekshirish va grafigini yasash sxemasi

Adabiyot: [1] - 238-245 betlar; [2] - 158-167 betlar;

Asimptotalar

$y=f(x)$ tenglama orqali K egri chiziq va birorta L to'g'ri chiziq berilgan bo'lsin.

Ta'rif. Agar K egri chiziqning M nuqtasi biror tarmoq bo'ylab cheksiz uzoqlashganda bu nuqtadan L to'g'ri chiziqqacha bo'lgan masofa nolga intilsa, u holda L to'g'ri chiziq K egri chiziqning asimptotasi deyiladi.

Faraz qilaylik, $y=f(x)$ funksiya grafigining bir tarmog'i uchun L asimptota mavjud bo'lsin. U holda L to'g'ri chiziqning tenglamasi uning $0x$ o'qqa perpendikulyar yoki og'maligiga qarab quyidagicha bo'ladi: 1) $x=a$; 2) $y=kx+b$.

1-hol. L to'g'ri chiziq vertikal asimptota bo'lib, $|MA|=|x-a|\to 0$ da $f(x)\to\infty$ bo'lishi kerak. Demak, L vertikal asimptota bo'lishi uchun shunday a son mavjud bo'lishi kerakki, uning uchun $\lim\limits_{x\to a} f(x)=\infty$ o'rinli bo'lsin.

2-holda K chiziqning og'ma asimptotasi $y=kx+b$ mavjud bo'lsin. U holda $\lim\limits_{x\to\infty} MP=0$ bo'lishi kerak. $y=kx+b$ tenglamadagi k va b ni aniqlash uchun ta'rifdan foydalanamiz. MR kesma K egri chiziqning M nuqtasidan asimptotagacha bo'lgan masofani ifoda qiladi. Ravshanki, $MR=QM\cos\alpha(\alpha\neq\frac{\pi}{2})$, shuning uchun $\lim\limits_{x\to\infty} MP=0 \iff \lim\limits_{x\to\infty} QM=0$.

MQ=MB-QB=$f(x)$-($kx+b$) bo'lgani uchun $\lim\limits_{x\to\infty} QM=0$ dan

$\lim\limits_{x\to\infty}(f(x)-(kx+b))=0$ yoki $\lim\limits_{x\to\infty}(\frac{f(x)}{x}-k-\frac{b}{x})=0$ kelib chiqadi , bundan esa

$\lim\limits_{x\to\infty}\frac{f(x)}{x}-k-\lim\limits_{x\to\infty}\frac{b}{x}=0$ yoki $k=\lim\limits_{x\to\infty}\frac{f(x)}{x}$ kelib chiqadi. k topilgach,

$\lim\limits_{x\to\infty}(f(x)-kx-b)=0$ dan $b=\lim\limits_{x\to\infty}(f(x)-kx)$ ni topamiz.

Demak, $y=f(x)$ egri chiziqning $y=kx+b$ og'ma asimptotasi mavjud bo'lishi uchun

$$k=\lim_{x\to\infty}\frac{f(x)}{x} \quad \text{va} \quad b=\lim_{x\to\infty}(f(x)-kx) \qquad (2)$$

limitlar mavjud bo'lishi zarurdir.

Endi $y=f(x)$ funksiya grafigi $y=kx+b$ og'ma asimptotaga ega bo'lishi uchun (2) limitlarning o'rinli bo'lishining yetarligini ko'rsatamiz.

(2) limitlar o'rinli bo'lsin. U holda $\lim\limits_{x\to\infty}(f(x)-kx)=b$ dan

$f(x)-kx-b=MQ\to 0$ kelib chiqadi. Demak, $x\to\infty$ da MR ham nolga intiladi. Bu esa $y=kx+b$ to'g'ri chiziq $f(x)$ funksiya grafigining og'ma asimptotasi ekanini bildiradi.

Misol. Ushbu $f(x)=\frac{x^3}{(x-1)^2}$ funksiya berilgan bo'lsin. Bu funksiya grafigi uchun $x=1$ to'g'ri chiziq vertikal asimptota bo'ladi.

Berilgan funksiya uchun

$$k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{x^2}{(x-1)^2}=1,\ b=\lim_{x\to\infty}(f(x)-kx)=\lim_{x\to\infty}\left(\frac{x^3}{(x-1)^2}-x\right)=2.$$

Demak, berilgan funksiya grafigining og'ma asimptotasi $y=x+2$ to'g'ri chiziqdan iborat.

Funksiyani tekshirish va grafigini yasash sxemasi Funksiyani tekshirish va grafigini yasash quyidagi sxema bo'yicha amalga oshiriladi:

1^0. Funksiyaning aniqlanish soxasini topish.
2^0. Funksiyaning juft, toq hamda davriyligini aniqlash.
3^0. Funksiyani uzluksizlikka tekshirish. Asimptotalar.
4^0. Funksiyani monotonlikka tekshirish.
5^0. Funksiyani ekstremumga tekshirish.
6^0. Funksiya grafigining qavariq hamda botiqligini aniqlash, burilish nuqtalarini topish.
7^0. Funksiyaning xarakterli nuqtalarini topish.
8^0. Funksiyaning grafigini yasash.

Misol. y=$\frac{x^3}{3-x^2}$ funksiyani to'la tekshiring va grafigini chizing.

Yechish: 1) Funksiyaning aniqlanish sohasini topamiz. Berilgan funksiya kasr – ratsional funksiya bo'lganligi sababli, u maxraji noldan farqli bo'ladigan barcha nuqtalarda aniqlangan:
$3-x^2 \neq 0,\ x \neq \pm\sqrt{3}$.

Demak, D(y)=$(-\infty; -\sqrt{3})\cup(-\sqrt{3};\ \sqrt{3})\cup(\sqrt{3};+\infty)$

2) Funksiya toq, chunki ixtiyoriy $x \in D(y)$ uchun $-x \in D(y)$ va $f(-x)= -\frac{x^3}{3-x^2} = -f(x)$, demak funksiya grafigi koordinatalar boshiga nisbatan simmetrik. Shuning uchun uni $[0;\ \sqrt{3})\cup(\sqrt{3};+\infty)$ to'plamda tekshirish yetarli.

3) x=$\sqrt{3}$ nuqtada chap va o'ng tomonli limitlarini hisoblaymiz:
$f(\sqrt{3}-0)=+\infty$, $f(\sqrt{3}+0)=-\infty$. Demak, $\sqrt{3}$ - 2-tur uzilish nuqtasi va x=$\sqrt{3}$ to'g'ri chiziq funksiya grafigining vertikal asimptotasi ekan.

Endi og'ma asimptotasini izlaymiz:

$$k=\lim_{x\to\infty}\frac{f(x)}{x} = \lim_{x\to+\infty}\frac{x^2}{3-x^2} = -1,$$

$$b=\lim_{x\to+\infty}(f(x)-kx)= \lim_{x\to+\infty}\left(\frac{x^2}{3-x^2}+x\right)= \lim_{x\to+\infty}\frac{3x}{3-x^2}=0.$$

Demak $y = -x$ to'g'ri chiziq og'ma asimptota ekan.

4) Funksiyani xosila yordamida monotonlik oraliqlarini izlaymiz:

$$y'=\left(\frac{x^3}{3-x^2}\right)' = \frac{3x^2(3-x^2)+2x^4}{(3-x^2)^2} = \frac{9x^2-x^4}{(3-x^2)^2} = \frac{x^2(3-x)(3+x)}{(3-x^2)^2}$$

Funksiya xosilasi $x=0$ *va* $x=\pm 3$ nuqtalarda nolga aylanadi.

Intervallar usulidan fodalanib, $(0;\ \sqrt{3})\cup(\sqrt{3};\ 3)$ da funksiya o'suvchi $(3;+\infty)$ da funksiya kamayuvchi bo'lganiga amin bo'lamiz.

5) Funksiyaning $x=0$ *va* $x=\pm 3$ kritik nuqtalari statsionar nuqtalaridan iborat, chunki qaralayotgan to'plamda hosila mavjud.

$(0;\ \sqrt{3})$, $(\sqrt{3};\ 3)$ va $(3;\ \infty)$ oraliqlarda hosila ishorasini tekshiramiz.

X	$(0;\ \sqrt{3})$	$(\sqrt{3};3)$	3	$(3;+\infty)$
y'	+	+	0	-
y	↗	↗	max	↘

va x=3 da funksiya lokal maksimumga ega: y_{max}=y(3)=$-\frac{9}{2}$

6) Ikkinchi tartibli hosila yordamida funksiya grafigining botiqlik va qavariqlik oraliqlarini topamiz:

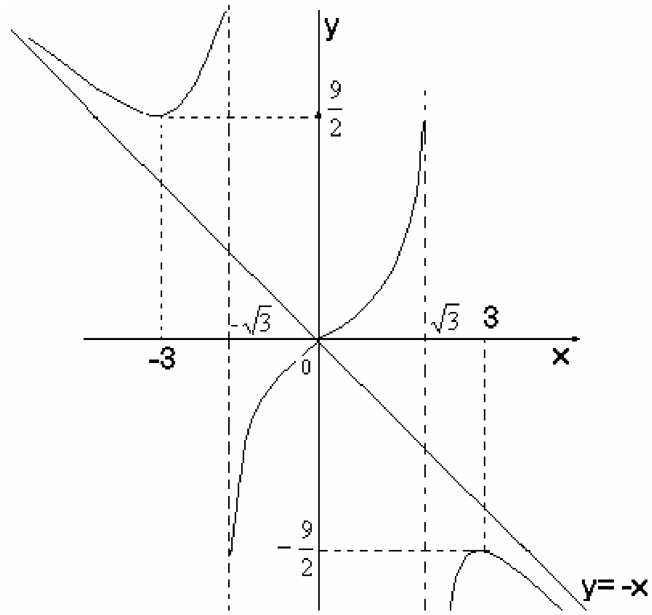
$$y''=\left(\frac{9x^2-x^4}{(3-x^2)^2}\right)'=\frac{6x(9+x^2)}{(3-x^2)^3}$$

y″=0 tenglamadan burilish nuqtasini topamiz: $6x(9+x^2)=0$, x=0.

Bu nuqtada funksiya burilishga ega, chunki x<0 da y″<0 va $\sqrt{3}$ >x>0 da y″>0. Demak, (0; $\sqrt{3}$)∪($\sqrt{3}$; +∞) da y″ ning ishorasini tekshiramiz: (0; $\sqrt{3}$) da y″>0, grafik botiq, ($\sqrt{3}$;+∞) da y″<0, bunda grafik qavariq.

7) Funksiya grafigining koordinata o'qlari bilan kesishish nuqtalarini topamiz. y=0 dan x=0, x=0 dan y=0 kelib chiqadi. Demak, grafik koordina-talar boshidan o'tadi.

7) Olingan natijalardan foydalanib, avval funksiya grafigini [0; $\sqrt{3}$)∪($\sqrt{3}$, +∞) oraliqda chizamiz, so'ngra uni koordinatalar boshiga nisbatan simmetrik almashtiramiz. Berilgan funksiyaning grafigi 8- chizmada tasvirlangan.



1. Asimptota nima? Uning qanday turlari mavjud?
2. Vertikal asimptota qanday izlanadi?
3. Og'ma asimptota mavjudligining zaruriy va yetarli sharti nimadan iborat?
4. Og'ma asimptota qanday izlanadi?
5. Funksiyani to'la tekshirish va grafigini chizish uchun nima ishlar bajariladi?

Tayanch tushunchalar: asimptota, funksiyani to'la tekshirish va grafigini yasash sxemasi.

14-ma'ruza.

Boshlang'ich funksiya va aniqmas integral. Asosiy integrallar jadvali.

Dars rejasi:

1) Integral hisobning asosiy vazifasi.
2) Boshlang'ich funksiyaning ta'rifi va misollar.
3) Aniqmas integral va uning xossalari.

4) Asosiy integrallar jadvali.

Adabiyot :

[1] - 257-262 betlar;
[2] - 174-178 betlar;
[4] - 403-412 betlar;
[5] - 114-116 betlar;
[6] - 47-48 betlar.

Integral hisobning asosiy vazifasi.

Differensial hisobning asosiy masalalaridan biri berilgan $y=f(x)$ funksiyaga ko'ra uning hosilasi $f'(x)$ ni topishdan iborat edi. Bu masalaning teskarisi, ya'ni hosilasiga ko'ra funksiyaning o'zini tiklash masalasi katta ahamiyatga ega bo'lib, integral hisobning asosiy masalalaridan hisoblanadi.

Boshlang'ich funksiya ta'rifi va misollar.

Ta'rif. Agar $F(x)$ funksiya biror yopiq oraliqda uzluksiz bo'lib, oraliqning ichki nuqtalarida

$F'(x)=f(x)$ yoki $dF(x)=f(x)dx$

bo'lsa, shu oraliqda $F(x)$ funksiya $f(x)$ ning boshlang'ich funksiyasi deyiladi. Boshqacha aytganda, biror sohada hosilasi $f(x)$ ga teng bo'ladigan $F(x)$ funksiya shu sohada $f(x)$ ning boshlang'ich funksiyasi deyiladi.

Misollar.

1) $F(x)=2\sqrt{x}$ funksiya $[0;+\infty)$ da $f(x)=\frac{1}{\sqrt{x}}$ funksiyaning boshlang'ich funksiyasidir, chunki $[0;+\infty)$ ning ichki nuqtalarida $(2\sqrt{x})'=\frac{1}{\sqrt{x}}$ bo'ladi;

2) $F(x)=x^2$ funksiya $(-\infty;+\infty)$ da $f(x)=2x$ ning boshlang'ich funksiyasidir, chunki $(-\infty;+\infty)$ ning ixtiyoriy nuqtasida $(x^2)'=2x$ bo'ladi;

3) $F(x)=\arcsin x$ $[-1;1]$ sohada $f(x)=\frac{1}{\sqrt{1-x^2}}$ uchun boshlang'ich funksiyadir, chunki $(-1;1)$ da $(\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$

Teorema . Biror D sohada $F_1(x)$ va $F_2(x)$ funksiyalar $f(x)$ ning boshlang'ich funksiyalari bo'lsa, u holda ularning ayirmasi o'zgarmas bo'ladi.

Isboti. Shartga ko'ra D ning ichki nuqtalarida $F_1(x)=f(x)$ va $F_2(x)=f(x)$. Differensiallash qoidasiga ko'ra

$(F_1(x)-F_2(x))'=F_1'(x)-F_2'(x)=f(x)-f(x)=0$. Demak, $F_1(x)-F_2(x)=C$- o'zgarmas.

Aniqmas integral va uning xossalari.

Oldingi teoremaga kura, berilgan $f(x)$ funksiyaning D sohadagi barcha boshlang'ich funksiyalarining umumiy ko'rinishi $F(x)+C$ bo'ladi.

Ta'rif. Berilgan $f(x)$ funksiyaning D sohadagi barcha boshlang'ich funksiyalarining majmuasi berilgan funksiyaning D sohadagi aniqmas integrali deyiladi va $\int f(x)dx$ orqali belgilanadi, bunda ∫- integral belgisi, $f(x)$-integral ostidagi funksiya, $f(x)dx$ -integral ostidagi ifoda deb ataladi.

Ta'rifga ko'ra $\forall x\in D$ uchun $\int f(x)dx=F(x)+C$, bunda $F(x)$ funksiya $f(x)$ ning biror boshlang'ich funksiyasi, C - o'zgarmas va ixtiyoriy.

Masalan, $(-\infty;+\infty)$ da $f(x)=\cos x$ bo'lsin. $(\sin x)'=\cos x$ bo'lgani uchun $\int \cos x dx=\sin x+C$ bo'ladi. Berilgan $f(x)$ funksiyaning aniqmas integralini topish amali integrallash deb ataladi.

. Maple dasturida integrallash **int(f, x)** va **Int(f, x)** operatorlar yordamida amalga oshiriladi, bunda **Int** operatori berilgan integralni matematik formulasini ekranga yozadi, **int(f, x)** operatori esa uning qiymatini topadi.

Misol. $\int (1+\cos(x))^2 dx$ ni hisoblang.

```
> Int((1+cos(x))^2, x=0..Pi)=
  int((1+cos(x))^2, x=0..Pi);
```

$$\int (1+\cos(x))^2 dx = \frac{3}{2}\text{x}+2\sin(\text{x})+\frac{1}{2}\cos(\text{x})\sin(\text{x})$$

Aniqmas integrallarning *x*ossalari.

1) $d\int f(x)dx=f(x)dx$ yoki $(\int f(x)dx)'=f(x)$;

2) $\int dF(x)=F(x)+C$;

3) Agar $f(x)$ ning boshlanich funksiyasi mavjud bo'lsa, u holda i*x*tiyoriy k son uchun $\int kf(x)dx=k\int f(x)dx$ bo'ladi;

4) Agar $f(x)$ va $g(x)$ larning boshlang'ich funksiyalari mavjud bo'lsa, u holda $\int (f(x)\pm g(x))dx=\int f(x)dx \pm \int g(x)dx$ bo'ladi.

1-*x*ossaning isboti. Ta'rifga ko'ra $d\int f(x)dx=d(F(x)+C)=dF(x)=F'(x)dx=f(x)dx$.

2- *x*ossaning isboti. $\int dF(x)=\int f(x)dx=F(x)+C$.

3-*x*ossaning isboti. 1-*x*ossaga ko'ra $(\int kf(x)dx)'=kf(x)$ bo'ladi. Bundan tashqari $(k\int f(x)dx)'=k(\int f(x)dx)'=kf(x)$ bo'ladi. Demak, $\int kf(x)dx$ va $k\int f(x)dx$ ifodalar birgina $kf(x)$ ning boshlang'ich funksiyalari to'plamidan iboratligi kelib chiqadi.

4-*x*ossaning isboti. $(\int (f(x)+g(x))dx)'=f(x)+g(x)$ va

$(\int f(x)dx+\int g(x)dx)'=(\int f(x)dx)'+(\int g(x)dx)'=f(x)+g(x)$

Demak, $\int (f(x)+g(x))dx$ va $\int f(x)dx+\int g(x)dx$ lar bitta to'plamni ifodalashi kelib chiqadi.

Asosiy integrallar jadvali.

Integrallash jarayonini i*x*chamlashda asosiy integrallar jadvali va ba'zi bir usullar qo'l keladi.

1. $\int x^\alpha dx=\frac{x^{\alpha+1}}{\alpha+1}+C$, $\alpha\in R$, $\alpha\neq -1$;

2. $\int \frac{dx}{x}=ln|x|+C$, $x\neq 0$;

3. $\int a^x dx=\frac{a^x}{\ln a}+C$, $a\neq 1$, $a>0$;

4. $\int \cos x dx=\sin x+C$;

5. $\int \sin x dx=-\cos x+C$;

6. $\int \frac{dx}{\cos^2 x}=tgx+C$;

7. $\int \frac{dx}{\sin^2 x}=-stgx+C$;

8. $\int \frac{dx}{\sqrt{1-x^2}}$ =arcsin$x$+C , $|x|<1$;

9. $\int \frac{dx}{1+x^2}$=arctgx+C;

10. $\int \frac{dx}{x^2-a^2}=\frac{1}{2a}\, ln|\frac{x-a}{x+a}|$+C, $x\neq\pm a$, $a\neq 0$;

*X*ulosa. Integral hisobning asosiy masalasi deb hosilaga ko'ra funksiyaning o'zini tiklash masalasiga aytiladi. Bu masalani o'rganishda boshlang'ich funksiya, aniqmas integral kabi tushunchalar va integrallash qoidalari qo'l keladi .

Oraliq nazorat savollari:

1. Integral hisobning asosiy masalasi nimadan iborat?
2. Qanday funksiyaga boshlang'ich funksiya deyiladi?
3. Berilgan funksiyaning boshlang'ich funksiyasi yagonami? Javobingizni asoslang.
4. Boshlang'ich funksiyaning umumiy ko'rinishi qanday yoziladi?
5. Aniqmas integral deb nimaga aytiladi?
6. Aniqmas integral qanday belgilanadi?
7. Integral ostidagi ifoda, funksiya deb nimaga aytiladi?
8. Aniqmas integralning qanday xossalarini bilasiz?
9. Aniqmas integralning differensiali nimaga teng?
10. Differensialning aniqmas integrali nimaga teng?
11. Yig'indining integrali nimaga teng?
12. $\int kf(x)dx = k\int f(x)dx$ tenglik qanday isbotlanadi?
13. Darajali funksiyaning aniqmas integrali nimaga teng?
14. Ko'rsatkichli funksiyaning aniqmas integrali nimaga teng?
15. cos$x$ funksiyaning aniqmas integrali nimaga teng?
16. sin$x$ funksiyaning aniqmas integrali nimaga teng?
17. Qanday funksiyaning boshlang'ich funksiyasi tgx ga teng?
18. Qanday funksiyaning boshlang'ich funksiyasi stgx ga teng?

Tayanch tushunchalar: Differensial; hosila; differensiallash qoidalari; elementar funksiyalarning differensiallash jadvali.

15-ma'ruza.

Aniqmas integralda o'zgaruvchini almashtirish usuli. Bo'laklab integrallash.

Dars rejasi:

1. Bevosita integrallash;
2. O'zgaruvchini almashtirish;
3. Bo'laklab integrallash;

Adabiyot : [1] – 262–265 betlar; [2] — 178-181 betlar;

Bevosita integrallash

Bu usul shundan iboratki, boshlangich funksiyani aniqlashda berilgan integral asosiy integrallar jadvalida bo'lmasligi mumkin, lekin ba'zi o'zgartirishlar natijasida jadvaldagi integrallardan biriga keltiriladi.

Misollar.

1) $\int$ cos2xdx=$\frac{1}{2}\int$cos2xd(2x)=$\frac{1}{2}$ sin2x +C;

2) $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\int \frac{d\left(\frac{x}{a}\right)}{1+\left(\frac{x}{a}\right)^2} = \frac{1}{a}$ arc$tg\frac{x}{a}$+C;

3) $\int$ *tgxdx*= $\int \frac{\sin x}{\cos x}dx$ =-$\int \frac{d\cos x}{\cos x}$ = - *ln* | cos*x* | +C;

4) $\int(\sqrt{x}$ -sin*x*)*dx*= $\int x^{\frac{1}{2}}dx$ -$\int$ sin*xdx*= $\frac{2}{3}x^{\frac{3}{2}}$+cos*x* +C.

O'zgaruvchini almashtirish

Ushbu $\int f(x)dx$ integralni hisoblash talab qilinsin. *x* o'zgaruvchini *t* ga biror *x*=φ(*t*) munosabatdan foydalanib almashtiramiz. Bunda φ'(*t*) uzluksiz va *x*=φ(*t*) ga nisbatan teskari funksiya *t*=ψ (*x*) mavjud deb faraz qilinadi.

Endi *x*=φ (*t*), *dx*=φ'(*t*)*dt* ifodalarni $\int f(x)dx$ ga qo'yamiz :

$\int f(x)dx=\int f(\varphi(t))\varphi'(t)dt$.

Bu yerda φ(*t*) ni shunday tanlash kerakki, o'ng tomondagi integral soddaroq bo'lsin. Agar *f*(φ(*t*))φ'(*t*) funksiyaning boshlang'ich funksiyalaridan biri *F*(*t*) bo'lsa,

$\int f(x)dx= \int f(\varphi(t))\varphi'(t)dt=F(t)+C= F(\psi(x))+C$

kelib chiqadi.

Misollar.

1. I=$\int \sin^3 x\cos x dx$ berilgan bo'lsin.

t=sin*x* deb olsak ⇒ *dt*=cos*xdx* bo'lib,

I=$\int t^3dt=\frac{t^4}{4}$ + S= $\frac{1}{4}\sin^4x$+C bo'ladi.

2. $\int \frac{xdx}{\sqrt{a^2-x^2}}$ = |*x*=*a*sin*t* , *dx*=*a*cos*tdt* | = $\int \frac{a\sin t\cdot a\cos t}{\sqrt{a^2-a^2\sin^2 t}}dt =$

$= a\int \frac{\sin t\cos t}{\cos t}dt$ =-*a*cos*t* + *C* =-$a\sqrt{1-\sin^2 t}$ +*C* =-$a\sqrt{1-\left(\frac{x}{a}\right)^2}$ +*C*=-$\sqrt{a^2-x^2}$ +C

Bo'laklab integrallash

Ma'lumki, *d*(*uc*)=*udv*+*vdu* yoki *udv*=*d*(*uc*)-*vdu*.
Bu tenglikni ikkala tomonini integrallasak

$\int udv=\int d(uc)- \int vdu$, *yoki* $\int udv=uv - \int vdu$

formulasi hosil bo'ladi. Bunday integrallash bo'laklab integrallash deyiladi.

Yuqoridagi formulani qo'llanish mo*x*iyati quyidagidan iborat:
$\int f(x)dx$ ni hisoblash uchun *f*(*x*)*dx* ni *u*(*x*) va *v'*(*x*)*dx* larning ko'paytmasi shaklida shunday yozish kerakki, natijada $\int vdu$ integral berilgan integralga qaraganda osonroq hisoblanadigan bo'lsin.

Misollar.

1. *I*=$\int x\cos x dx$ berilgan. *u*=*x*, *du*=*dx*, *u*=sin*x*, *dv*=cos*xdx* belgilashlarni kiritamiz. U holda *I* =$\int udv$=*x*sin*x*-$\int \sin x dx$=*x*sin*x*+cos*x*+*C*.

2. *I*=$\int lnxdx$ bo'lsin. *u*=*lnx*, *du*=$\frac{dx}{x}$, *v*=*x*, *dv*=*dx*,

$$I=\int udv=xlnx-\int x\frac{dx}{x}=xlnx-x+C.$$

Izoh.

$P_n - n$ - tartibli ko'phad bo'lsin.

1) Agar integral ostidagi funksiya

$$\begin{cases} P_n(x)\sin\alpha x \\ P_n(x)\cos\alpha x \\ P_n(x)e^{\alpha x} \end{cases}$$

ko'rinishga ega bo'lsa u sifatida P_n ni olish maqsadga muvofiq.

2) Agar integral ostidagi funksiya

$$\begin{cases} P_n(x)\ln x \\ P_n(x)\arcsin x \\ P_n(x)arctg\, x \end{cases}$$

ko'rinishga ega bo'lsa u sifatida $\begin{cases} P_n(x)\ln x \\ P_n(x)\arcsin x \\ P_n(x)arctg\, x \end{cases}$ larni olish maqsadga muvofiq.

3) Agar integral ostidagi funksiya $e^{\alpha x}\sin\beta x,\ e^{\alpha x}\cos\beta x$ ko'rinishga ega bo'lsa bo'laklab integrallashni ikki marotaba o'tqazish maqsadga muvofiq. Bundan keyin integralga nisbatan chiziqli tenglama hosil bo'ladi.

$$\mathrm{I}=\int e^{ax}\cos bxdx=\left|\begin{array}{l} u=e \\ v'=\cos bx \\ v=\dfrac{1}{b}\sin by \\ u'=ae^{x} \end{array}\right|=\frac{e^{ax}\sin by}{b}-\int e^{ax}\sin bx=\frac{e^{ax}\sin by}{b}-\frac{a}{b}\left[\frac{1}{b}e^{ax}+\frac{a}{b}\mathrm{I}\right]$$

. Maple dasturida **student** paketiga mansub

> **intparts(%,x^n);**

operatori qo'llaniladi. Uni moxiyatini misolda tushunsa bo'ladi:

$\int x^3\sin xdx$ integralni Maple yordamida bo'laklab integrallash yordamida topamiz.

> **restart; with(student): J=Int(x^3*sin(x),x);**

$$J=\int x^3\sin(x)dx$$

> **J=intparts(Int(x^3*sin(x),x),x^3);**

$$J=-x^3\cos(x)-\int -3x^2\cos(x)dx$$

> **intparts(%,x^2);**

$$J=-x^3\cos(x)+3x^2\sin(x)+\int -6x\sin(x)dx$$

> **intparts(%,x);**

$$J=-x^3\cos(x)+3x^2\sin(x)+6x\cos(x)-\int 6\cos(x)dx$$

> **value(%);**

$$J=-x^3\cos(x)+3x^2\sin(x)+6x\cos(x)-6\sin(x)$$

Oraliq nazorat savollari:

1. Bevosita integrallash deganda nimani tushunasiz?
2. O'zgaruvchini almashtirish usuli bilan integralni hisoblaganda nima ishlar bajariladi?
3. O'zgaruvchini almashtirish usulida yordamchi $x=\varphi(t)$ funksiyaga qanday shartlar qo'yiladi?
4. Bo'laklab integrallash deganda nimani tushunasiz?
5. $\int P_n(x)\sin xdx$ qanday integrallanadi?
6. $\int P_n(x)arctgxdx$ qanday integrallanadi?
7. $\int P_n(x)\ln xdx$ qanday integrallanadi?

Tayanch tushunchalar: Integrallash qoidalari; asosiy integrallar jadvali, boshlang'ich funksiya; aniqmas integral.

16,17-ma'ruzalar.

Ratsional funksiyalar va ularni integrallash.

Reja : Ratsional funksiyalar va ularni integrallash.

Adabiyot : [1] - 266-276 betlar; [2] - 181-186 betlar ;

Ratsional funksiyalar va ularni integrallash.

Integralni hisoblash uchun umumiy usullar bo'lmagani uchun ayrim funksiyalar sinflarini integrallash yo'llari o'rganilgan. *X*ozir biz ana shunday funksiyalar sinflaridan biri bilan tanishib chiqamiz.

Algebradan ma'lumki, $R(x)=a_0x^n+a_1x^{n-1}+...+a_{n-1}x+a_n$ ko'p*x*ad butun ratsional funksiya deb ataladi.

Butun ratsional funksiyani integrallash quyidagicha bajariladi:

$$\int R(x)dx=\int a_0x^ndx+\int a_1x^{n-1}dx+...+\int a_{n-1}xdx+\int a_ndx=$$

$$=\frac{a_0}{n+1}x^{n+1}+\frac{a_1}{n}x^n+...+a_{n-1}\cdot\frac{x^2}{2}+a_nx+C.$$

Endi

$$f(x)=\frac{P(x)}{Q(x)}=\frac{a_0x^n+a_1x^{n-1}+...+a_{n-1}x+a_n}{b_0x^m+b_1x^{m-1}+...+b_{m-1}x+b_m}\quad (a_0\neq 0, b_0\neq 0)$$

kasr ratsional funksiyalarni integrallash masalasiga o'tamiz.

Agar $n<m$ bo'lsa, u holda $f(x)$ - to'g'ri, $n\geq m$ bo'lsa $f(x)$ - noto'g'ri kasr ratsional funksiya deyiladi.

Ixtiyoriy noto'g'ri ratsional kasrni butun ratsional funksiya va to'g'ri ratsional kasr yig'indisi ko'rinishida ifodalash mumkin. Shuning uchun $\frac{P(x)}{Q(x)}$ ni noto'g'ri ratsional funksiya deb qarashimiz mumkin. Uning maxraji

$Q(x)=(x-b_1)^{\alpha_1}\cdot...\cdot(x-b_m)^{\alpha_m}(x^2+p_1x+q_1)^{\beta_1}\cdot...\cdot(x^2+p_nx+q_n)^{\beta_n}$ bo'lsin. U holda algebra kursida $\frac{P(x)}{Q(x)}$ to'g'ri ratsional kasr elementar (sodda) kasrlar yig'indisi shaklida yozilishi ko'rsatiladi :

$$\frac{P(x)}{Q(x)}=\frac{B_1^{(1)}}{(x-b_1)}+\frac{B_2^{(1)}}{(x-b_1)^2}+\ldots+\frac{B_{\alpha_1}^{(1)}}{(x-b_1)^{\alpha_1}}+\ldots+\frac{B_1^{(m)}}{(x-b_m)}+\frac{B_2^{(m)}}{(x-b_m)^2}+\ldots+\frac{B_{\alpha_m}^{(m)}}{(x-b_1)^{\alpha_m}}+$$

$$+\frac{M_1^{(1)}x+N_1^{(1)}}{(x^2+p_1x+q_1)}+\frac{M_2^{(1)}x+N_2^{(1)}}{(x^2+p_1x+q_1)^2}+\ldots+\frac{M_{\beta_1}^{(1)}x+N_{\beta_1}^{(1)}}{(x^2+p_1x+q_1)^{\beta_1}}+\ldots+\frac{M_1^{(n)}x+N_1^{(n)}}{(x^2+p_nx+q_n)}+$$

$$+\frac{M_2^{(n)}x+N_2^{(n)}}{(x^2+p_nx+q_n)^2}+\ldots+\frac{M_{\beta_n}^{(n)}x+N_{\beta_n}^{(n)}}{(x^2+p_nx+q_n)^{\beta_n}}$$

bu yerda $B_k^{(i)}, M_r^{(l)}, N_r^{(l)}$ - haqiqiy sonlar.

Bu ayniyatni umumiy maxrajga keltirib va hosil bo'lgan kasrlarning suratlarini tenglashtirib, hosil qilingan tenglikning ikkala tomonidagi x larning bir xil darajalari oldidagi koeffitsientlarni tenglashtirsak, yuqoridagi noma'lum koeffitsientlarga nisbatan m ta chiziqli tenglamalar sistemasi hosil bo'ladi. Shu sistemani yechib, noma'lum koeffitsientlarni topamiz. Noma'lum koeffitsientlarni topganimizdan keyin $\frac{P(x)}{Q(x)}$ ratsional kasrni integrallash masalasi yuqoridagi ayniyatda qatnashgan sodda kasrlarni integrallash masalasiga keltiriladi.

Shu sababli ratsional kasrni integrallash uchun quyidagi sodda ratsional kasrlarini integrallash kifoya:

1). $\frac{B}{x-b}$; 2). $\frac{B}{(x-b)^\alpha}$; 3). $\frac{Mx+N}{x^2+px+q}$; 4). $\frac{Mx+N}{(x^2+px+q)^\beta}$.

(p^2 -4q<0). Bu yerda *B, M, N, a, p* va *q* lar haqiqiy, α, β-natural sonlar. Biz shu bilan shug'ullanamiz.

$\frac{B}{x-b}$ va $\frac{B}{(x-b)^\alpha}$ larni integrallashda $t=x-b$ almashtirishni kiritish kifoya.

$\frac{Mx+N}{x^2+px+q}$ ni integrallash uchun maxrajdan to'la kvadratni ajratamiz:

$x^2+px+q=\left(x+\frac{p}{2}\right)^2+\left(q-\frac{p^2}{4}\right)$ va $a=\sqrt{q-\frac{p^2}{4}}$, $t=x+\frac{p}{2}$ belgilashlar kiritamiz:

$$\int\frac{Mx+N}{x^2+px+q}dx=\int\frac{Mt+\left(N-M\frac{p}{2}\right)}{t^2+a^2}dt=\frac{M}{2}\int\frac{d(t^2+a^2)}{t^2+a^2}+\left(N-M\frac{p}{2}\right)\int\frac{dt}{t^2+a^2}=$$

$$=\frac{M}{2}\ln(t^2+a^2)+\frac{\left(N-M\frac{p}{2}\right)}{a}arctg\frac{t}{a}+C$$

Demak, $\int\frac{Mx+N}{x^2+px+q}dx=\frac{M}{2}\ln(x^2+px+q)+\frac{2N-Mp}{2a}arctg\frac{2x+p}{\sqrt{4q-p^2}}$ +C.

$\frac{Mx+N}{(x^2+px+q)^\beta}$ ni integrallash uchun qo'yidagi ishlarni amalga oshiramiz:

$$\frac{Mx+N}{(x^2+px+q)^\beta}=\left|\begin{matrix} t=x+\frac{p}{2} \\ a=\sqrt{q-\frac{p^2}{4}}\end{matrix}\right|=\frac{Mt+\left(N-M\frac{p}{2}\right)}{(t^2+a^2)^\beta}, \text{ demak}$$

$$\int \frac{Mx+n}{\left(x^2+px+q\right)^{\beta}}dx = M\int \frac{t}{\left(t^2+a^2\right)^{\beta}}dt + \left(N - M\frac{p}{2}\right)\int \frac{dt}{\left(t^2+a^2\right)^{\beta}} = M\mathrm{I}_1 + \mathrm{I}_{\beta},$$

$$M\int \frac{d\left(t^2+a^2\right)}{\left(t^2+a^2\right)^{\beta}} = \begin{cases} \frac{1}{2}\frac{1}{1-\beta}\left(t^2+a^2\right)^{-\beta+1}, \beta \neq 1 \\ \frac{1}{2}\ln\left(t^2+a^2\right), \beta = 1 \end{cases}.$$

$$\mathrm{I}_{\beta} = \int \frac{dt}{\left(t^2+a^2\right)^{\beta}} = \left| \begin{array}{c} u = \frac{1}{\left(t^2+a^2\right)^{\beta}} \\ v' = 1 \\ v = t \end{array} \right| = \frac{t}{\left(t^2+a^2\right)^{\beta}} + 2\beta\int \frac{t^2 dt}{\left(t^2+a^2\right)^{\beta+1}} =$$

$$= \frac{t}{\left(t^2+a^2\right)^{\beta}} + 2\beta\int \frac{\left(t^2+a^2\right) - a^2}{\left(t^2+a^2\right)^{\beta+1}}dt = \frac{t}{\left(t^2+a^2\right)^{\beta}} + 2\beta\mathrm{I}_{\beta} - 2\beta a^2\mathrm{I}_{\beta+1}$$

Bundan qo'yidagi rekkurent formulaga ega bo'lamiz:

$$\mathrm{I}_{\beta+1} = \frac{2\beta-1}{2\beta a^2}\mathrm{I}_{\beta} + \frac{1}{2\beta a^2}\frac{t}{\left(t^2+a^2\right)^{\beta}},$$

$$\mathrm{I}_1 = \int \frac{dt}{t^2+a^2} = \frac{1}{a}arctg\frac{t}{a} + C$$

Misol :

$\int \frac{x^3+1}{x(x-1)^3}dx$ hisoblansin .

Yechish: $\frac{x^3+1}{x(x-1)^3} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3}$, bundan $x^3+1=$ $A(x-1)^3+Bx(x-1)^2+Cx(x-1)+Dx$ va x ning bir $x$il darajalari oldidagi koeffitsientlarni tenglashtirsak quyidagi $\begin{cases} A+B=1, \\ -3A-2B+C=0, \\ 3A+B-C+D=0, \\ -A=1 \end{cases}$ sistemaga kelamiz. Bundan $A=-1$, $B=2$, $C=1$, $D=2$ bo'ladi.

Demak , $\frac{x^3+1}{x(x-1)^3} = \frac{-1}{x} + \frac{2}{x-1} + \frac{1}{(x-1)^2} + \frac{2}{(x-1)^3}$, va

$$\int \frac{x^3+1}{x(x-1)^3}dx = -\int \frac{dx}{x} + 2\int \frac{dx}{x-1} + \int \frac{dx}{(x-1)^2} + 2\int \frac{dx}{(x-1)^3} =$$

$$= -\ln|x| + 2\ln|x-1| - \frac{1}{x-1} - \frac{1}{(x-1)^2} + C = \ln\frac{(x-1)^2}{|x|} - \frac{x}{(x-1)^2} + C.$$

Oraliq nazorat savollari :

1. Sodda kasrlarning ko'rinishi va ularni integrallash;
2. Ratsional funksiya nima?
3. To'g'ri va noto'g'ri kasr ratsional funksiyalar nima?
4. Butun ratsional funksiya va uni integrallash;
5. To'g'ri kasr sodda kasrlarga qanday yoyiladi?
6. Noto'g'ri kasrni butun qismini ajratib yozish;

Tayanch tushunchalar: Asosiy integrallar jadvali; integrallash qoidalari; Integrallash usullari. to'g'ri ratsional kasrning sodda kasrlar yig'indisi shaklida yozilishi; sodda kasrlar; ularning integrali; asosiy integrallar jadvali.

18, 19-ma'ruzalar.

Ayrim irratsional va transtsendent ifodalarni integrallash.

Reja :

1. Ayrim trigonometrik ifodalarni integrallash.
2. Kasr - chiziqli irratsionallikni integrallash.
3. Kvadratik irratsionallikni integrallash. Eyler almashtirishlari.
4. Differensial binomni integrallash.
5. Ayrim foydali almashtirishlar

Adabiyot: [1] - 276-280 betlar; [2] - 186-187 betlar;

Ayrim trigonometrik ifodalarni integrallash.

$R(x,y)$ - ratsional ifoda bo'lsin. $R(\sin x, \cos x)$ trigonometrik ifodani integrallash uchun $t = tg\frac{x}{2}$ universal almashtirishni kiritish maqsadga muvofiq, bunda $\sin x = \dfrac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = \dfrac{2t}{1+t^2}$, $\cos x = \dfrac{1-t^2}{1+t^2}$,

$x = 2\,\mathrm{arctg}\,t$, $dx = \dfrac{2dt}{1+t^2}$

$\int R(\sin x, \cos x)dx = \int R\left(\dfrac{2t}{1+t^2}, \dfrac{1-t^2}{1+t^2}\right)\dfrac{2dt}{1+t^2}$ bo'ladi.

Ya'ni $R(\sin x, \cos x)$ trigonometrik ifodani integrallash ratsional funksiyani integrallashga olib kelinar ekan.

Ayrim xususiy xollarda universal almashtirining o'rniga boshqa almashtirishlarni kiritish maqsadga muvofiq, bunda hisob-kitoblar keskin kamayadi.

$\int R(\cos^2 x, \sin^2 x)dx = \int R\left(\dfrac{1}{1+t^2}, \dfrac{t^2}{1+t^2}\right)\dfrac{dt}{1+t^2}$, bu yerda $t = tgx$, $\cos^2 x = \dfrac{1}{1+t^2}$, $\sin^2 x = \dfrac{t^2}{1+t^2}$,

$dx = \dfrac{dt}{1+t^2}$.

$\int R(\cos x, \sin^2 x)\sin xdx$ va $\int R(\cos^2 x, \sin x)\cos xdx$ xollarida $t = \cos x$ yoki mos ravishda $t = \sin x$ almashtirishni kiritsak $\int R(t, 1-t^2)dt$ va $\int R(1-t^2, t)dt$ integrallarni hosil qilamiz.

$\int \sin^2 x\cos^2 xdx$, $\int \sin^2 x\cos^4 xdx$, $\int \sin^2 x\cos^3 xdx$, $\int \sin 3x\cos 5xdx$ kabi integrallarni hisoblashda $\sin^2\Delta = \dfrac{1-\cos 2\Delta}{2}$, $\cos^2\Delta = \dfrac{1+\cos 2\Delta}{2}$, $\sin\Delta\cos\Delta = \dfrac{\sin 2\Delta}{2}$.

$\sin\Delta\cos\Pi = (\sin(\Delta+\Pi)+\sin(\Delta-\Pi))$, $\cos\Delta\cos\Pi = (\cos(\Delta+\Pi)+\cos(\Delta-\Pi))$,

$\sin\Delta\sin\Pi = (\cos(\Delta-\Pi)-\cos(\Delta+\Pi))$

trigonometrik formulalardan foydalanish maqsadga muvofiq.

Bundan avval har qanday ratsional funksiyaning boshlang'ich funksiyalari elementar funksiya bo'lishini va ularni hisoblash usullarini ko'rib chiqdik. Lekin har qanday irratsional funksiyaning boshlang'ich funksiyalari elementar funksiya bo'lavermaydi. Biz hozir boshlang'ich funksiyalari elementar bo'ladigan ba'zi bir sodda irratsional funksiyalarni integrallash bilan shug'ullanamiz. Ular asosan biror almashtirish yordamida ratsional funksiyaga keltiriladi.

Kasr - chiziqli irratsionallikni integrallash.

$R\left(x, \sqrt[n]{\dfrac{ax+b}{cx+d}}\right)$, ($a,b,c,d \in$ R , $n>0$ - butun) funksiyani integrallash masalasini qaraymiz.

$\sqrt[n]{\dfrac{ax+b}{cx+d}}$ funksiya kasr chiziqli irratsionallik deyiladi.

$ad-bc\neq 0$ bo'lsin. $t=\sqrt[n]{\dfrac{ax+b}{cx+d}}$ almashtirishdan so'ng ratsional ifodadan integralni hosil qilamiz. Haqiqatdan ham

$$t=\sqrt[n]{\frac{ax+b}{cx+d}} \Rightarrow x\left(ct^n-a\right)=b-dt^n;$$

$$x=\frac{b-dt^n}{ct^n-a};\ dx=\frac{-d\cdot nt^{n-1}\left(ct^n-a\right)-\left(b-dt^n\right)cnt^{n-1}}{\left(ct^n-a\right)^2}dt=\frac{\left(ad-bc\right)nt^{n-1}}{\left(ct^n-a\right)^2}dt.$$

Demak, $\displaystyle\int R\left(x,\sqrt[n]{\frac{ax+b}{cx+d}}\right)dx=\int R\left(\frac{b-t^n d}{ct^n-a},t\right)\frac{\left(ad-bc\right)nt^{n-1}}{\left(ct^n-a\right)^2}dt$.

Misol. $\displaystyle\int\sqrt{\frac{1+x}{1-x}}\frac{dx}{1-x}$.

Endi $I=\displaystyle\int R(x,(\frac{ax+b}{cx+d})^{\alpha_1},\ldots,(\frac{ax+b}{cx+d})^{\alpha_n})dx$, bunda R- o'z argumentlarining ratsional funksiyasi, a, *b*, s, va *d* lar haqiqiy sonlar va $\alpha_1,\alpha_2,\ldots,\alpha_n$ ratsional sonlar bo'lib, ularning umumiy maxraji *m* bo'lsin.

Bu holda $t=\sqrt[m]{\dfrac{ax+b}{cx+d}}$ yoki $t^m=\dfrac{ax+b}{cx+d}$ almashtirishni kiritsak, so'ng $x=\dfrac{t^m d-b}{a-ct^m}$ va $dx=\dfrac{m(ad-bc)t^{m-1}dt}{(a-ct^m)^2}$ kelib chiqadi. Natijada, berilgan integral *t* ga nisbatan ratsional funksiyani integrallashga keltiriladi.

$$I=\int R(\frac{dt^m-b}{a-ct^m},t^{\alpha_1\cdot m},\ldots,t^{\alpha_n m})\frac{m(ad-bc)t^{m-1}}{(a-ct^m)^2}dt$$

Bundan avval R ning argumentlari irratsional ifodalardan tashkil bo'lsa, endi argumentlar ratsional funksiyalarga keltirildi. Qisqacha qilib yozsak,

I=$\int R_1(t)dt$, bu yerda $R_1(t)$ - ratsional funksiya. Avvalgi natijalarga ko'ra bunday integral elementar funksiyalar orqali ifodalanadi,

ya'ni $R(x,(\dfrac{ax+b}{cx+d})^{\alpha_1},\ldots,(\dfrac{ax+b}{cx+d})^{\alpha_n})$ ko'rinishdagi irratsional funksiyalar integrallanuvchi bo'ladi.

Misol:

I= $\displaystyle\int\frac{dx}{\sqrt{2x+5}-\sqrt[3]{2x+5}}$ berilgan. Bunda $\alpha_1=\frac{1}{2},\alpha_2=\frac{1}{3}\Rightarrow$ $m=6$, $t^6=2x+5$, $x=\frac{1}{2}(t^6-5)$, $dx=3t^5dt$, $\sqrt{2x+5}=t^3$, $\sqrt[3]{2x+5}=t^2$. Shuning uchun

$$I=\int\frac{3t^5dt}{t^3-t^2}=3\int(t^2+t+1+\frac{1}{t-1})dt=t^3+\frac{3}{2}t^2+3t+3\ln|t-1|+C=$$

$$=\sqrt{2x+5}+\frac{3}{2}\sqrt[3]{2x+5}+3\sqrt[6]{2x+5}+3\ln|\sqrt[6]{2x+5}-1|+C$$

Kvadratik irratsionallikni integrallash. Eyler almashtirishlari.

$R\left(x,\sqrt{ax^2+bx+c}\right),\ D\neq 0$.
Eylerning 1 - almashtirishi:
$D=b^2-4ac<0$.
$a>0$ bo'lsin. $t=\sqrt{ax^2+bx+c}+x\sqrt{a}$ almashtirishdan so'ng ratsional ifodadan integralni hosil qilamiz. Haqiqatdan ham $t-x\sqrt{a}=\sqrt{ax^2+bx+c}\ \Rightarrow\ t^2-2xt\sqrt{a}+ax^2=ax^2+bx+c$

$bx+c=t^2-2\sqrt{a}\cdot tx$. Bundan $x=\dfrac{t^2-c}{b+2\sqrt{a}\cdot t}$; $\sqrt{ax^2+bx+c}=t-\sqrt{a}\,\dfrac{t^2-c}{b+2\sqrt{a}\cdot t}=\dfrac{t^2\sqrt{a}+bt+c\sqrt{a}}{b+2\sqrt{a\cdot t}}$;

$$dx=2\frac{t\left(b+2\sqrt{a}\cdot t\right)-\left(t^2-c\right)\sqrt{a}}{\left(b+2\sqrt{a}\cdot t\right)^2}dt=2\frac{t^2\sqrt{a}+bt+c\sqrt{a}}{\left(b+2\sqrt{a}\cdot t\right)^2}dt.$$

Demak. $\int R\left(x,\sqrt{ax^2+bx+c}\right)dx=\int R\left(\dfrac{t^2-c}{b+2\sqrt{a}\cdot t},\dfrac{t^2\sqrt{a}+bt+c\sqrt{a}}{b+2\sqrt{a\cdot t}}\right)\cdot 2\dfrac{t^2\sqrt{a}+bt+c\sqrt{a}}{\left(b+2\sqrt{a}\cdot t\right)^2}dt$

Eylerning 2 - almashtirishi:
$D>0$ demak, $ax^2+bx+c=a(x-x_1)(x-x_2)$,

$t=\dfrac{\sqrt{ax^2+bx+c}}{x-x_1}\ \Rightarrow\ t^2(x-x_1)=a(x-x_2),\ x=\dfrac{-ax_2+t^2x_1}{t^2-a}$,

$\sqrt{ax^2+bx+c}=t(x-x_1)=t\left(\dfrac{a(x_1-x_2)}{t^2-a}\right),\ dx=\dfrac{2a(x_1-x_2)t}{\left(t^2-a\right)^2}dt$.

Misol. $\int\dfrac{dx}{x+\sqrt{x^2+x+1}}=2\int\dfrac{t^2+t+1}{t(1+2t)^2}dt$,

$t=\sqrt{x^2+x+1}+x\ \Rightarrow\ t^2-2tx+x^2=x^2+x+1,\ t^2=x(1+2t)+1\ \Rightarrow\ x=\dfrac{t^2-1}{1+2t}$,

$$dx=2\frac{t(1+2t)-\left(t^2-1\right)}{(1+2t)^2}dt=2\frac{t^2+t+1}{(1+2t)^2}dt,$$

$$\frac{t^2+t+1}{t(1+2t)^2}=\frac{A}{t}+\frac{B}{1+2t}+\frac{C}{(1+2t)^2}=\frac{A(1+2t)^2+Bt(1+2t)+Ct}{t(1+2t)^2},$$

$4A+2B=1,\ B=-2,\ A=1,\ -\dfrac{1}{2}C=\dfrac{1}{4}-\dfrac{1}{2}+1,\ C=-\dfrac{3}{2}$.

Differensial binomni integrallash.

$x^m\left(a+bx^n\right)^p dx$ ko'rinishdagi ifoda , bu yerda a,b - haqiqiy, n,m,p - ratsional sonlar, differensial binom deyiladi.

Bunday ifoda integral ostida bo'lsa, u holda faqat qo'yidagi 3-xol uchun integral elementar funksiyalar orqali ifodalanar ekan. Bunday faktni birinchi bo'lib N'yuton aytib o'tgan, 19 asr o'rtalarida esa Chebishev buni isbotlagan edi.

1 - hol. p - butun. $t=\sqrt[N]{x}$ almashtirishdan so'ng ratsional ifodadan integralni hosil qilamiz, bu yerda N–m va n sonlar maxrajlarining eng kichik umumiy karralisi.

2 - hol. $\dfrac{m+1}{n}$ - butun. $t=\left(a+bx^n\right)^{\frac{1}{N}}$ almashtirishdan so'ng ratsional ifodadan integralni hosil qilamiz, bu yerda N– p kasrning maxraji.

3 - hol. $\dfrac{m+1}{n}+p$ - butun. $t=\sqrt[N]{\dfrac{a}{x^n}+b}$ almashtirishdan so'ng ratsional ifodadan integralni hosil qilamiz, bu yerda N– p kasrning maxraji.

Misol. $\int x^5 (1-x^2)^{-\frac{1}{2}} dx$, $m=5, n=2, p=-\dfrac{1}{2}$, $t=(1-x^2)^{\frac{1}{2}} \to x=\sqrt{1-t^2}$.

Ayrim foydali almashtirishlar

R – ratsional funksiyalar bo'lsin.

$\int R(x, \sqrt[n]{ax+b})dx$ ko'rinishdagi integralda qo'yidagi almashtirish qo'llaniladi: $ax+b=t^n$.

Misol: $\int \dfrac{x+1}{3+\sqrt[3]{3x+1}} dx$.

$\int \dfrac{Mx+N}{(x-p)\sqrt{ax^2+bx+c}} dx$ ko'rinishdagi integralda quyidagi almashtirish qo'llaniladi: $x-p=1/t$.

$\int R(x, \sqrt{a^2-x^2})dx$ ko'rinishdagi integralda qo'yidagi almashtirish qo'llaniladi: $x=a\sin t$, bunda $t=\arcsin x$, $\sqrt{a^2-x^2}=a\cos t$, $dx=a\cos t dt$.

Misol: $\int x^2\sqrt{4-x^2}dx$.

$\int R(x, \sqrt{a^2+x^2})dx$ ko'rinishdagi integralda qo'yidagi almashtirish qo'llaniladi: $x=atgt$, bunda $t=arctgx$, $\sqrt{a^2+x^2}=\dfrac{a}{\cos t}$, $dx=\dfrac{adt}{\cos^2 t}$.

Misol: $\int \dfrac{dx}{\sqrt{(5+x^2)^3}}$.

$\int R(x, \sqrt{x^2-a^2})dx$ ko'rinishdagi integralda qo'yidagi almashtirish qo'llaniladi: $x=\dfrac{a}{\sin t}$, bunda $t=\arcsin\dfrac{a}{x}$, $\sqrt{x^2-a^2}=\dfrac{a\cos}{\sin t}$, $dx=\dfrac{-a\cos t dt}{\sin^2 t}$.

Misol: $\int \sqrt{x^2-3}dx$.

Oraliq nazorat savollari:

1. Irratsional funksiyaga misollar keltiring ;
2. Qanday almashtirish yordamida ularni ratsional funksiyalar ko'rinishida yozish mumkin?
3. Qanday hollarda binomial differensialning integrali elementar funksiyalar orqali ifodalanadi?
4. Qanday integrallarni hisoblashda Eyler almashtirishlari ishlatiladi?
5. Trigonometrik funksiyalarni integrallashning umumiy usuli nimadan iborat?

Tayanch tushunchalar: trigonometrik ifodalar, kasr - chiziqli irratsionallik, kvadratik irratsionallik, Eyler almashtirishlari. differensial binom

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Individual vazifalar-1
Variant №1.

1) Hosilani toping:

a) $y=\dfrac{2\left(3x^3+4x^2-x-2\right)}{15\sqrt{1+x}}$,

b) $y=x-\ln\left(2+e^x+2\sqrt{e^{2x}+e^x+1}\right)$

c) $y=\sqrt{x}\ln\left(\sqrt{x}+\sqrt{x+a}\right)-\sqrt{x+a}$,

d) $y=\sin\sqrt{3}+\dfrac{1}{3}\dfrac{\sin^2 3x}{\cos 6x}$,

e) $y=\operatorname{arctg}\dfrac{\operatorname{tgx}-\operatorname{ctgx}}{\sqrt{2}}$

f) $y=\dfrac{1}{4\sqrt{5}}\ln\dfrac{2+\sqrt{5}\operatorname{thx}}{2-\sqrt{5}\operatorname{thx}}$,

g) $y=\left(arctgx\right)^{\left(\frac{1}{2}\right)\ln\left(arctgx\right)}$,

h) $y=\dfrac{1}{24}\left(x^2+8\right)\sqrt{x^2-4}+\dfrac{x^4}{16}\arcsin\dfrac{2}{x}$, $x>0$,

i) $y=\dfrac{x\arcsin x}{\sqrt{1-x^2}}+\ln\sqrt{1-x^2}$,

j) $y=\dfrac{1}{\sin\alpha}\ln\left(tgx+ctg\alpha\right)$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}\operatorname{tg}\left(x^3+x^2\sin\dfrac{2}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing:

$y=\dfrac{4x-x^2}{4}$, $x_0=2$.

4) dy , d^2y - ?

$y=x\arcsin\dfrac{1}{x}+\ln\left|x+\sqrt{x^2-1}\right|$, $x>0$.

5) Differensial yordamida taqribiy hisoblang:

$y=\sqrt[3]{x}$, $x=7{,}76$.

6) Hosilani toping y'_x:

$$\begin{cases}x=\dfrac{3t^2+1}{3t^3}\\ y=\sin\left(\dfrac{t^3}{3}+t\right)\end{cases}.$$

7) n- tartibli hosilani toping: $y=xe^{\alpha x}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases}x=\cos 2t\\ y=2\sec^2 t\end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\dfrac{x^3+4}{x^2}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=-\frac{x^2}{2}+2x+\frac{8}{x-2}+5,\quad [-2,\ 1].$$

Variant №2.

1) Hosilani toping:

a) $y=\frac{(2x^2-1)\sqrt{1+x^2}}{3x^3}$,

b) $y=e^{2x}\frac{2-\sin 2x-\cos 2x}{8}$,

c) $y=\ln\left(x+\sqrt{a^2+x^2}\right)$,

d) $y=\cos\ln 2+\frac{1}{3}\frac{\cos^2 3x}{\sin 6x}$,

e) $y=\arcsin\frac{\sqrt{x}-2}{\sqrt{5x}}$,

f) $y=\frac{shx}{4ch^4x}+\frac{3chx}{8ch^2x}+\frac{3}{8}arctg(shx)$,

g) $y=\left(\sin\sqrt{x}\right)^{\ln(\sin\sqrt{x})}$,

h) $y=\frac{4x+1}{16x^2+8x+3}+\frac{1}{\sqrt{2}}arctg\frac{4x+1}{\sqrt{2}}$,

i) $y=4\ln\frac{x}{1+\sqrt{1-4x^2}}-\frac{\sqrt{1-4x^2}}{x^2}$,

j) $y=x\cos\alpha+\sin\alpha\ln\sin(x-\alpha)$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}\arcsin\left(x^2\cos\frac{1}{9x}\right)+\frac{2}{3}x, & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing:

$y=2x^2+3$, $x_0=-1$.

4) dy , d^2y -?

$y=tg\left(2\arccos\sqrt{1-x^2}\right)$, $x>0$.

5) Differensial yordamida taqribiy hisoblang:

$y=\sqrt[3]{x^3+7x}$, $x=1{,}012$.

6) Hosilani toping y'_x:

$$\begin{cases}x=\sqrt{1-t^2}\\ y=tg\sqrt{1-t}\end{cases}.$$

7) n -nchi tartibli hosilani toping: $y=\sin 2x+\cos(x+1)$.

8) Ikkinchi tartibli hosilani toping

$$\begin{cases}x=\sqrt{1-t^2}\\ y=\frac{1}{t}\end{cases}.$$

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\frac{x^2-x+1}{x-1}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=8x+\frac{4}{x^2}-15,\quad \left[\frac{1}{2},\ 2\right].$$

Variant №3.

1) Hosilani toping:

a) $y=\frac{x^4-8x^2}{2(x^2-4)}$,

b) $y=\frac{-e^{3x}}{3sh^3x}$,

c) $y=2\sqrt{x-4}\ln\left(2+\sqrt{x}\right)$,

d) $y=\sin^3\cos 2-\frac{\cos^2 30x}{60\sin 60x}$,

e) $y=\mathrm{arctg}\frac{\sqrt{1+x^2}-1}{x}$,

f) $y=\frac{2}{3}\mathrm{cthx}-\frac{\mathrm{chx}}{3sh^3x}$,

g) $y=(\sin x)^{5e^x}$,

h) $y=\ln\left(2x-3+\sqrt{4x^2-12x+10}\right)-\sqrt{4x^2-12x+10}\,\mathrm{arctg}(2x-3)$,

i) $y=x(2x^2+5)\sqrt{x^2+1}+3\ln\left(x+\sqrt{x^2+1}\right)$,

j) $y=\mathrm{arctg}\left(\frac{\cos x}{\sqrt[4]{\cos 2x}}\right)$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}\mathrm{arctg}\left(x\cos\frac{1}{5x}\right), & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing:

$$y=\frac{x^{29}+6}{x^4+1},\ x_0=1.$$

4) dy, d^2y - ?

$$y=\sqrt{1+2x}-\ln\left(x+\sqrt{1+2x}\right).$$

5) Differensial yordamida taqribiy hisoblang:

$$y=\sqrt[5]{x^2},\ x=1{,}03.$$

6) Hosilani toping y'_x:

$$\begin{cases}x=\sqrt{2t-t^2}\\ y=\frac{1}{\sqrt[3]{(t-1)^2}}\end{cases}.$$

7) n -nchi tartibli hosilani toping:

$$3^{2x+5}.$$

8) Ikkinchi tartibli hosilani toping

$$\begin{cases}x=\sin t-t\cos t\\ y=\cos t+t\sin t\end{cases}.$$

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\frac{2}{x^2+2x}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=\sqrt[3]{2(x+2)^2(x-4)}+3,\quad [-4,\ 2].$$

Variant №4.

1) Hosilani toping:

a) $y=\dfrac{2x^2-x-1}{3\sqrt{2+4x}}$,

b) $y=\operatorname{arctg}\left(e^x-e^{-x}\right)$,

c) $y=\ln\dfrac{x^2}{\sqrt{1-ax^4}}$,

d) $y=\cos^2\sin 3+\dfrac{\sin^2 29x}{29\cos 58x}$,

e) $y=\arccos\dfrac{x^2-4}{\sqrt{x^4+16}}$,

f) $y=-\dfrac{\operatorname{ch}x}{2\operatorname{sh}^2x}-\dfrac{1}{2}\ln\operatorname{th}\dfrac{x}{2}$,

g) $y=(\arcsin x)^{e^x}$,

h) $y=(3x+1)^4\arcsin\dfrac{1}{3x+1}+\left(3x^2+2x+1\right)\sqrt{9x^2+6x}$, $3x+1>0$

i) $y=x^3\arcsin x+\dfrac{x^2+2}{3}\sqrt{1-x^2}$,

j) $y=\operatorname{arctg}\dfrac{\sqrt{\sqrt{x^4+1}-x^2}}{x}$, $x>0$.

2) Hosilaning ta’rifiga asoslanib, f’(0) ni toping:

$$f(x)=\begin{cases}\ln\left(1-\sin\left(x^3\sin\dfrac{1}{x}\right)\right), & x\neq 0\\ 0, & x=0\end{cases}.$$

3) dy , d^2y - ?:

$$y=\frac{x^{16}+9}{1-5x^2},\ x_0=1.$$

4) dy i d^2y:

$$y=x^2\operatorname{arctg}\sqrt{x^2-1}-\sqrt{x^2-1}.$$

5) Differensial yordamida taqribiy hisoblang: $y=x^5$, $x=2{,}997$.

6) Hosilani toping y'_x :

$$\begin{cases}x=\arcsin(\sin t)\\ y=\arccos(\cos t)\end{cases}.$$

7) n -nchi tartibli hosilani toping: $y=\log_3(x+5)$.

8) Ikkinchi tartibli hosilani toping

$$\begin{cases}x=2(t-\sin t)\\ y=4(2+\cos t)\end{cases}.$$

9) Funksiyani to’la tekshiring va grafigini yasang: $y=\dfrac{4x^2}{3+x^2}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=x^2+4x+\frac{16}{x+2}-9,\quad [-1,\ 2].$$

Variant №5.

1) Hosilani toping:

a) $y=\frac{(1+x^8)\sqrt{1+x^8}}{12x^{12}}$,

b) $y=\frac{e^x}{2}\left[(x^2+1)\cos x+(x-1)^2\sin x\right]$,

c) $y=\ln\left(\sqrt{x}+\sqrt{x+1}\right)$,

d) $y=\sin\ln\frac{1}{2}+\frac{\sin^2 25x}{25\cos 50x}$,

e) $y=\sqrt{\frac{2}{3}}\operatorname{arctg}\frac{3x-1}{\sqrt{6x}}$,

f) $y=\frac{1}{2}\left[\frac{\operatorname{shx}}{\operatorname{ch}^2x}+\operatorname{arctg}(\operatorname{shx})\right]$,

g) $y=(\ln x)^{3^x}$,

h) $y=\arcsin e^{-2x}+\ln\left(e^{2x}+\sqrt{e^{4x}-1}\right)$,

i) $y=3\arcsin\frac{3}{4x+1}+2\sqrt{4x^2+2x-2}$, $4x+1>0$

j) $y=\frac{1}{a(1+a^2)}\left[\operatorname{arctg}(a\cos x)+a\ln\operatorname{tg}\frac{x}{2}\right]$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}\sin\left(x\sin\frac{3}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing:

$$y=\sqrt{x}-3\sqrt[3]{x},\ x_0=64.$$

4) dy , d^2y - ? $\quad y=\arccos\frac{1}{\sqrt{1+2x^2}}$, $x>0$.

5) Differensial yordamida taqribiy hisoblang:

$$y=\sqrt{x^3},\ x=0{,}98.$$

6) Hosilani toping y'_x:

$$\begin{cases}x=\ln\left(t+\sqrt{t^2+1}\right)\\ y=t\sqrt{t^2+1}\end{cases}.$$

7) n -nchi tartibli hosilani toping: $y=\frac{x}{x+1}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases}x=\operatorname{arctgt}\\ y=\frac{t^2}{2}\end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\frac{12x}{9+x^2}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin : $y=x^2+\frac{16}{x}-16,\quad [1,\ 4]$.

Variant №6.

1) Hosilani toping:

a) $y=\frac{x^2}{2\sqrt{1-3x^4}}$,

b) $y=e^{\sin x}\left(x-\frac{1}{\cos x}\right)$,

c) $y=\ln\frac{a^2+x^2}{a^2-x^2}$,

d) $y=\operatorname{ctg}\sin\frac{1}{13}-\frac{1}{48}\frac{\cos^2 24x}{\sin 48x}$,

e) $y=\frac{1}{4}\ln\frac{x-1}{x+1}-\frac{1}{2}\operatorname{arctg}x$,

f) $y=\frac{\operatorname{sh}x}{2\operatorname{ch}^2 x}+\frac{1}{2}\operatorname{arctg}(\operatorname{sh}x)$,

g) $y=x^{\arcsin x}$,

h) $y=\frac{1}{x}\sqrt{1-4x^2}+\ln\frac{1+\sqrt{1-4x^2}}{2x}$,

i) $y=\sqrt{1+x^2}\operatorname{arctg}x-\ln\left(x+\sqrt{1+x^2}\right)$,

j) $y=\frac{1}{2}\ln\frac{1+\cos x}{1-\cos x}-\frac{1}{\cos x}-\frac{1}{3\cos^3 x}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}\sin\left(e^{x^2\sin\frac{5}{x}}-1\right)+x, & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing:

$$y=\frac{x^2-2x-3}{4},\ x_0=4.$$

4) dy , d^2y - ?:

$$y=\arccos\frac{x^2-1}{x^2\sqrt{2}}.$$

5) Differensial yordamida taqribiy hisoblang:

$$y=\sqrt{4x-3},\ x=1{,}78.$$

6) Hosilani toping y'_x:

$$\begin{cases}x=\sqrt{2t-t^2}\\ y=\arcsin(t-1)\end{cases}.$$

7) n -nchi tartibli hosilani toping: $y=2^{kx}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases}x=\cos t\\ y=\sin^4\frac{t}{2}\end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\frac{x^2-3x+3}{x-1}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin : $y=4-x-\frac{4}{x^2},\quad [1,\ 4].$

Variant №7.

1) Hosilani toping:

a) $y=\dfrac{(x^2-6)\sqrt{(4+x^2)^3}}{120x^5}$, f) $y=-\dfrac{shx}{2ch^2x}-\dfrac{1}{shx}-\dfrac{3}{2}arctgshx$,

b) $y=\dfrac{1}{m\sqrt{ab}}arctg\left(e^{mx}\sqrt{\dfrac{a}{b}}\right)$, g) $y=(ctg3x)^{2e^x}$,

c) $y=\ln^2(x+\cos x)$, h) $y=\sqrt{49x^2+1}arctg7x-\ln\left(7x+\sqrt{49x^2+1}\right)$,

d) $y=\ln\cos\dfrac{1}{3}+\dfrac{\sin^2 23x}{23\cos 46x}$, i) $y=2\arcsin\dfrac{2}{3x+4}+\sqrt{9x^2+24x+12}$, $3x+4>0$

e) $y=\dfrac{(1+x)arctg\sqrt{x}}{x^2}+\dfrac{1}{3x\sqrt{x}}$, j) $y=\dfrac{3^x((\ln 3)\sin 2x-2\cos 2x)}{\ln^2 3+4}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}x^2\cos\dfrac{4}{3x}+\dfrac{x^2}{2}, & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing:

$$y=x^2+8\sqrt{x}-32,\ x_0=4.$$

4) dy , d^2y - ?

$$y=\ln\left(\cos^2 x+\sqrt{1+\cos^4 x}\right).$$

5) Differensial yordamida taqribiy hisoblang:

$$y=x^7,\ x=2{,}002.$$

6) Hosilani toping y'_x:

$$\begin{cases}x=ctg(2e^t)\\ y=\ln tge^t\end{cases}.$$

7) n -nchi tartibli hosilani toping: $y=a^{2x+3}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases}x=e^t\\ y=\arcsin t\end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\dfrac{4-x^3}{x^2}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=\sqrt[3]{2(x-2)^2(8-x)}-1,\quad [0,\ 6].$$

Variant №8.

1) Hosilani toping:

a) $y=\dfrac{(x^2-8)\sqrt{x^2-8}}{6x^3}$,

b) $y=\dfrac{e^{x^3}}{1+x^3}$,

c) $y=\ln^3(1+\cos x)$,

d) $y=\cos\ln 13-\dfrac{1}{44}\dfrac{\cos^2 22x}{\sin 44x}$,

e) $y=\dfrac{x^3}{3}\arccos x-\dfrac{2+x^2}{9}\sqrt{1-x^2}$,

f) $y=\dfrac{3}{2}\ln \operatorname{th}\dfrac{x}{2}+\operatorname{ch}x-\dfrac{\operatorname{ch}x}{2\operatorname{sh}^2 x}$,

g) $y=x^{e^{\operatorname{tg}x}}$,

h) $y=\ln\left(e^{3x}+\sqrt{e^{6x}-1}\right)+\arcsin e^{-3x}$,

i) $y=x(2x^2+1)\sqrt{x^2+1}-\ln\left(x+\sqrt{x^2+1}\right)$,

j) $y=2\dfrac{\cos x}{\sin^4 x}+3\dfrac{\cos x}{\sin^2 x}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}\operatorname{arctg}\left(x^3-x^{3/2}\sin\dfrac{1}{3x}\right), & x\neq 0\\ 0, & x=0\end{cases}.$$

3) sostavit' uravnenie kasatel'noy:

$$y=\frac{3x-2x^3}{3},\ x_0=1.$$

4) dy , d^2y - ?

$$y=\ln\left(x+\sqrt{1+x^2}\right)-\sqrt{1+x^2}\operatorname{arctg}x.$$

5) Differensial yordamida taqribiy hisoblang:

$$y=\frac{1}{\sqrt{x}},\ x=4{,}16.$$

6) Hosilani toping y'_x:

$$\begin{cases}x=\ln \operatorname{ctg}x\\ y=\dfrac{1}{\cos^2 t}\end{cases}.$$

7) n -nchi tartibli hosilani toping: $y=\dfrac{5x+1}{13(2x+3)}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases}x=\cos t+t\sin t\\ y=\sin t-t\cos t\end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\dfrac{x^2-4x+1}{x-4}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=\frac{2(x^2+3)}{x^2-2x+5},\quad [-3,\ 3].$$

Variant №9.

1) Hosilani toping:

a) $y=\dfrac{4+3x^3}{x\sqrt[3]{(2+x^3)^2}}$,

b) $y=x-\ln(1+e^x)-2e^{\frac{x}{2}}\operatorname{arctg}e^{\frac{x}{2}}-\left(\operatorname{arctg}e^{\frac{x}{2}}\right)^2$,

c) $y=\ln\dfrac{x^2}{1-x^2}$,

d) $y=\sqrt{\operatorname{tg}4}+\dfrac{\sin^2 21x}{21\cos 42x}$,

e) $y=\dfrac{1}{2\sqrt{x}}+\dfrac{1+x}{2x}\operatorname{arctg}\sqrt{x}$,

f) $y=\dfrac{1}{2}\operatorname{arctg}(\operatorname{sh}x)-\dfrac{\operatorname{sh}x}{2\operatorname{ch}^2x}$,

g) $y=(\operatorname{tg}x)^{4e^x}$,

h) $y=\dfrac{1}{\sqrt{2}}\operatorname{arctg}\dfrac{2x+1}{\sqrt{2}}+\dfrac{2x+1}{4x^2+4x+3}$,

i) $y=\ln\left(x+\sqrt{x^2+1}\right)-\dfrac{\sqrt{1+x^2}}{x}$,

j) $y=\dfrac{\ln(\operatorname{ctg}x+\operatorname{ctg}\alpha)}{\sin\alpha}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}\sin x\cos\dfrac{5}{x}, & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing:

$$y=8\sqrt[4]{x}-70,\ x_0=16.$$

4) dy , d^2y - ?

$$y=\frac{\ln|x|}{1+x^2}-\frac{1}{2}\ln\frac{x^2}{1+x^2}.$$

5) Differensial yordamida taqribiy hisoblang: $y=\sqrt[3]{x}$, $x=8{,}36$.

6) Hosilani toping y'_x: $\begin{cases}x=\operatorname{arctg}e^{\frac{t}{2}}\\ y=\sqrt{e^t+1}\end{cases}$.

7) n -nchi tartibli hosilani toping: $y=\dfrac{4}{x}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases}x=\cos t\\ y=\ln\sin t\end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\dfrac{2x^3+1}{x^2}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin : $y=2\sqrt{x}-x,\quad [0,\ 4]$.

Variant №10.

1) Hosilani toping:

a) $y=\sqrt[3]{\dfrac{\left(1+x^{3/4}\right)^2}{x^{3/2}}}$,

b) $y=x-e^{-x}\arcsin e^{-x}-\ln\left(1+\sqrt{1-e^{2x}}\right)$,

c) $y=\ln \operatorname{tg}\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$,

d) $y=\operatorname{ctg}\cos 5-\dfrac{1}{40}\dfrac{\cos^2 20x}{\sin 40x}$,

e) $y=\dfrac{3+x}{2}\sqrt{x(2-x)}+3\arccos\sqrt{\dfrac{x}{2}}$,

f) $y=\dfrac{8}{3}\operatorname{cth}2x-\dfrac{1}{3\operatorname{ch}x\cdot\operatorname{ch}^3x}$,

g) $y=(\cos 5x)^{e^x}$,

h) $y=\ln\left(5x+\sqrt{25x^2+1}\right)-\sqrt{25x^2+1}\cdot\operatorname{arctg}5x$,

i) $y=\sqrt{1-3x-2x^2}+\dfrac{3}{2\sqrt{2}}\arcsin\dfrac{4x+3}{\sqrt{17}}$,

j) $y=\dfrac{2^x(\sin x+\cos x\ln 2)}{1+(\ln 2)^2}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases} x+\arcsin\left(x^2\sin\dfrac{6}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing::

$$y=\frac{x^2}{10},\ x_0=2.$$

4) dy , d^2y - ?

$$y=\ln\left(e^x+\sqrt{e^{2x}-1}\right)+\arcsin e^{-x}.$$

5) Differensial yordamida taqribiy hisoblang: $y=\dfrac{1}{\sqrt{2x^2+x+1}}$, $x=1{,}016$.

6) Hosilani toping y'_x:
$$\begin{cases} x=\ln\sqrt{\dfrac{1-t}{1+t}}\\ y=\sqrt{1-t^2}\end{cases}.$$

7) n -nchi tartibli hosilani toping: $y=\ln(1+x)$.

8) Ikkinchi tartibli hosilani toping $\begin{cases} x=t-\sin t\\ y=2-\cos t\end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\dfrac{(x-1)^2}{x^2}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=1+\sqrt[3]{2(x-1)^2(x-7)},\quad [-1,\ 5].$$

Variant №11.

1) Hosilani toping:

a) $y=\frac{x^6+x^3-2}{\sqrt{1-x^3}}$,

b) $y=\ln\left(e^x+\sqrt{e^{2x}-1}\right)+\arcsin e^{-x}$,

c) $y=\ln\sqrt[4]{\frac{1+2x}{1-2x}}$,

d) $y=\frac{\mathrm{tg}\ln 2\cdot\sin^2 19x}{19\cos 38x}$,

e) $y=\frac{4+x^4}{x^3}\mathrm{arctg}\frac{x^2}{2}+\frac{4}{x}$,

f) $y=\frac{1-8\mathrm{ch}^2x}{4\mathrm{ch}^4x}$,

g) $y=\left(x^3+4\right)^{\mathrm{tg}x}$,

h) $y=\arcsin e^{-4x}+\ln\left(e^{4x}+\sqrt{e^{8x}-1}\right)$,

i) $y=\sqrt{(4+x)(1+x)}+3\ln\left(\sqrt{4+x}+\sqrt{1+x}\right)$,

j) $y=\frac{5^x\left(\sin 3x\ln 5-3\cos 3x\right)}{9+\ln^2 5}$.

2) Hosilaning ta’rifiga asoslanib, f’(0) ni toping:

$$f(x)=\begin{cases}\mathrm{arctg}x\sin\frac{7}{x}, & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing:

$$y=\sqrt[3]{x^2}-20,\ x_0=-8.$$

4) dy , d^2y - ?

$$y=x\sqrt{4-x^2}+4\arcsin\frac{x}{2}.$$

5) Differensial yordamida taqribiy hisoblang: $y=\sqrt{4x-1}$, $x=2{,}56$.

6) Hosilani toping y'_x :
$$\begin{cases}x=\ln\frac{1}{\sqrt{1-t^4}}\\ y=\arcsin\frac{1-t^2}{1+t^2}\end{cases}.$$

7) n -nchi tartibli hosilani toping: $y=\frac{x}{9(4x+9)}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases}x=t+\sin t\\ y=2+\cos t\end{cases}$.

9) Funksiyani to’la tekshiring va grafigini yasang: $y=\frac{x^2}{(x-1)^2}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin : $y=x-4\sqrt{x}+5,\quad [1,\ 9]$.

Variant №12.

1) Hosilani toping:

a) $y = \dfrac{(x^2-2)\sqrt{4+x^2}}{24x^3}$,

b) $y = x + \dfrac{8}{1+e^{x/4}}$,

c) $y = \ln\sin\dfrac{2x+4}{x+1}$,

d) $y = \dfrac{\sin tg\frac{1}{7}\cos^2 16x}{32\sin 32x}$,

e) $y = \arcsin\sqrt{\dfrac{x}{x+1}} + arctg\sqrt{x}$,

f) $y = -\dfrac{1}{4}\arcsin\dfrac{5+3chx}{3+5chx}$,

g) $y = x^{\sin x^3}$,

h) $y = \dfrac{2x-1}{4x^2-4x+3} + \dfrac{1}{\sqrt{2}} arctg\dfrac{2x-1}{\sqrt{2}}$,

i) $y = \ln\dfrac{\sqrt{x^2-x+1}}{x} + \sqrt{3} arctg\dfrac{2x-1}{\sqrt{3}}$,

j) $y = \dfrac{\cos x}{\sin^2 x} - 2\cos x - 3\ln tg\dfrac{x}{2}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x) = \begin{cases} 2x^2 + x^2\cos\dfrac{1}{9x}, & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

3) Urinmaning tenglamasini tuzing::

$$y = \frac{1+3x^2}{3+x^2},\ x_0 = 1.$$

4) dy , d^2y - ?

$$y = \ln tg\frac{x}{2} - \frac{x}{\sin x}.$$

5) Differensial yordamida taqribiy hisoblang: $y = \sqrt[3]{x},\ x = 7{,}64$.

6) Hosilani toping y'_x : $\begin{cases} x = \sqrt{1-t^2} \\ y = \dfrac{t}{\sqrt{1-t^2}} \end{cases}.$

7) n -nchi tartibli hosilani toping: $y = 7^{5x}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases} x = \sin t \\ y = \ln\cos t \end{cases}.$

9) Funksiyani to'la tekshiring va grafigini yasang: $y = \left(1+\dfrac{1}{x}\right)^2$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y = \sqrt[3]{2x^2(x-3)},\quad [-1,\ 6].$$

Variant №13.

1) Hosilani toping:

a) $y=\frac{1+x^2}{2\sqrt{1+2x^2}}$,

b) $y=x-3\ln\left[\left(1+e^{x/6}\right)\sqrt{1+e^{x/3}}\right]-3\operatorname{arctg}e^{x/6}$,

c) $y=\log_{16}\log_5 \operatorname{tg}x$,

d) $y=\frac{\cos\operatorname{tg}\frac{1}{3}\sin^2 15x}{15\cos 30x}$,

e) $y=\frac{1}{2}\sqrt{\frac{1}{x^2}-1}-\frac{\arccos x}{2x^2}$,

f) $y=\frac{1}{\sqrt{8}}\arcsin\frac{3+\operatorname{ch}x}{1+3\operatorname{ch}x}$,

g) $y=\left(x^4+5\right)^{\operatorname{ctg}x}$,

h) $y=\ln\left(e^{5x}+\sqrt{e^{10x}-1}\right)+\arcsin\left(e^{-5x}\right)$,

i) $y=4\arcsin\frac{4}{2x+3}+\sqrt{4x^2+12x-7}$,

$2x+3>0$

j) $y=\frac{4^x\left((\ln 4)\sin 4x-4\cos 4x\right)}{16+\ln^2 4}$.

2) Hosilaning ta’rifiga asoslanib, f’(0) ni toping:

$$f(x)=\begin{cases} x^2\cos^2\left(\frac{11}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}.$$

3) Costavit’ uravnenie normali:

$$y=\frac{1+\sqrt{x}}{1-\sqrt{x}},\ x_0=4.$$

4) dy , d^2y - ?

$$y=2x+\ln\left|\sin x+2\cos x\right|.$$

5) Differensial yordamida taqribiy hisoblang: $y=x^7,\ x=1{,}996$.

6) Hosilani toping y'_x: $\begin{cases} x=\arcsin\sqrt{1-t^2} \\ y=(\arccos t)^2 \end{cases}$.

7) n -nchi tartibli hosilani toping: $y=\ln(3x+1)$.

8) Ikkinchi tartibli hosilani toping $\begin{cases} x=\sqrt{t-3} \\ y=\ln(t-2) \end{cases}$.

9) Funksiyani to’la tekshiring va grafigini yasang: $y=\left(\frac{x-1}{x+1}\right)^2$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=\frac{2\left(-x^2+7x-7\right)}{x^2-2x+2},\quad [1,\ 4].$$

Variant №14.

1) Hosilani toping:

a) $y=\frac{\sqrt{x-1}(3x+2)}{4x^2}$,

b) $y=x+\frac{1}{1+e^x}-\ln(1+e^x)$,

c) $y=\log_4\log_2 tgx$,

d) $y=\frac{\cos ctg3\cdot\cos^2 14x}{28\sin 28x}$,

e) $y=6\arcsin\frac{\sqrt{x}}{2}+\frac{6+x}{2}\sqrt{x(4-x)}$,

f) $y=-\frac{shx}{2ch^2x}+\frac{3}{2}\arcsin(thx)$,

g) $y=(\sin x)^{5^{x/2}}$,

h) $y=\frac{1}{\sqrt{2}}arctg\frac{x-1}{\sqrt{2}}+\frac{x-1}{x^2-2x+3}$,

i) $y=2\arcsin\frac{2}{3x+1}+\sqrt{9x^2+6x-3}$,

$3x+1>0$,

j) $y=\frac{3^x(4\sin 4x+kn3\cos 4x)}{16+\ln^2 3}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}2x^2+x^2\cos\frac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing::

$$y=\frac{x^2-3x+3}{3},\ x_0=3.$$

4) dy , d^2y - ?

$$y=\sqrt{ctgx}-\sqrt{tg^3\frac{x}{3}}.$$

5) Differensial yordamida taqribiy hisoblang: $y=\sqrt[3]{x}$, $x=8{,}24$.

6) Hosilani toping y'_x: $\begin{cases}x=(1+\cos^2 t)^2\\ y=\frac{\cos t}{\sin^2 t}\end{cases}$.

7) n -nchi tartibli hosilani toping: $y=\frac{4+15x}{5x+1}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases}x=\cos^2 t\\ y=tg^2 t\end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\frac{3x^4+1}{x^3}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin : $y=x-4\sqrt{x+2}+8,\quad[-1,\ 7]$.

Variant №15.

1. Hosilani toping:

a) $y=\frac{\sqrt{(1+x^2)^3}}{3x^3}$,

b) $y=e^{\alpha x}\left[\frac{1}{2a}+\frac{a\cos 2bx+2b\sin 2bx}{2(a^2+4b^2)}\right]$

c) $y=x\frac{\cos\ln x+\sin\ln x}{2}$

d) $y=8\sin ctg3+\frac{1}{5}\frac{\sin^2 5x}{\cos 10x}$,

e) $y=\frac{x}{2\sqrt{1-4x^2}}\arcsin 2x+\frac{1}{8}\ln(1-4x^2)$,

f) $y=arctg\frac{\sqrt{sh2x}}{chx-shx}$,

g) $y=x^{29^x}29^x$,

h) $y=\sqrt{9x^2-12x+5}arctg(3x-2)-\ln(3x-2-\sqrt{9x^2-12x}$

i) $y=x^3\arccos x-\frac{x^2+2}{3}\sqrt{1-x^2}$,

j) $y=x\cos\alpha+\sin\alpha\ln\sin(x-\alpha)$.

2. Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}arctg\left(x\cos\frac{1}{5x}\right), & x\neq 0\\ 0, & x=0\end{cases}.$$

3. Urinmaning tenglamasini tuzing.

$$y=\frac{x^{16}+9}{1-5x^2},\ x_0=1.$$

4. dy , d^2y - ?. $y=\arccos\frac{1}{\sqrt{1+2x^2}}$, $x>0$.

5. Differensial yordamida taqribiy hisoblang:

$$y=\sqrt{4x-3},\ x=1{,}78.$$

6. Hosilani toping y'_x :

$$\begin{cases}x=ctg(2e^t)\\ y=\ln tge^t\end{cases}.$$

7. n-nchi tartibli hosilani toping: $y=\ln(3x+1)$.

8. Ikkinchi tartibli hosilani toping $\begin{cases}x=\cos t+t\sin t\\ y=\sin t-t\cos t\end{cases}$.

9. Funksiyani to'la tekshiring va grafigini yasang: $y=\frac{2x^3+1}{x^2}$.

10. Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=\sqrt[3]{2(x-2)^2(5-x)},\quad [1,\ 5].$$

Variant №16.

1. Hosilani toping:

a) $y=\frac{2\left(3x^{3}+4x^{2}-x-2\right)}{15\sqrt{1+x}}$,

f) $y=\frac{1}{2}\left[\frac{shx}{ch^{2}x}+arctg(shx)\right]$,

b) $y=e^{2x}\,\frac{2-\sin 2x-\cos 2x}{8}$,

g) $y=x^{\arcsin x}$,

c) $y=2\sqrt{x-4}\,\ln\left(2+\sqrt{x}\right)$,

h) $y=\sqrt{49x^{2}+1}\,arctg7x-\ln\left(7x+\sqrt{49x^{2}+1}\right)$,

e) $y=\arccos\frac{x^{2}-4}{\sqrt{x^{4}+16}}$,

i) $y=\ln\left(e^{3x}+\sqrt{e^{6x}-1}\right)+\arcsin e^{-3x}$,

j) $y=\ln\left(x+\sqrt{x^{2}+1}\right)-\frac{\sqrt{1+x^{2}}}{x}$,

l) $y=\frac{2^{x}\left(\sin x+\cos x\ln 2\right)}{1+\left(\ln 2\right)^{2}}$

2. Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases} arctg\left(x\cos\frac{1}{5x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}.$$

3. Urinmaning tenglamasini tuzing::

$$y=\frac{1+3x^{2}}{3+x^{2}},\ x_{0}=1.$$

4. dy , $d^{2}y$ - ?

$$y=2x+\ln\left|\sin x+2\cos x\right|.$$

5. Differensial yordamida taqribiy hisoblang: $y=\sqrt[3]{x}$, $x=8{,}24$.

6. Hosilani toping y'_{x} :

$$\begin{cases} x=\frac{3t^{2}+1}{3t^{3}} \\ y=\sin\left(\frac{t^{3}}{3}+t\right) \end{cases}.$$

7. n -nchi tartibli hosilani toping: $y=\frac{2x+5}{13(3x+1)}$.

8. Ikkinchi tartibli hosilani toping $\begin{cases} x=tgt \\ y=\frac{1}{\sin 2t} \end{cases}$.

9. Funksiyani to'la tekshiring va grafigini yasang: $y=\frac{x^{3}-4}{x^{2}}$.

10. Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=\sqrt[3]{2x^{2}(x-6)},\quad [-2,\ 4].$$

Variant №17.

1) Hosilani toping:

a) $y=\frac{\sqrt{2x+3}(x-2)}{x^2}$,

b) $y=e^{\alpha x}\frac{(\alpha\sin\beta x-\beta\cos\beta x)}{\alpha^2+\beta^2}$,

c) $y=\lg\ln\operatorname{ctg}x$,

d) $y=\frac{1}{3}\cos\operatorname{tg}\frac{1}{2}+\frac{1}{10}\frac{\sin^2 10x}{\cos 20x}$,

e) $y=\operatorname{arctg}x+\frac{5}{6}\ln\frac{x^2+1}{x^2+4}$,

f) $y=-\frac{\operatorname{sh}3x}{\sqrt{\operatorname{ch}6x}}$,

g) $y=x^{3^x}2^x$,

h) $y=\frac{x+2}{x^2+4x+6}+\frac{1}{\sqrt{2}}\operatorname{arxtg}\frac{x+2}{\sqrt{2}}$,

i) $y=\sqrt{x^2+1}-\frac{1}{2}\ln\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+1}$,

j) $y=\ln\frac{\sqrt{2}+\operatorname{th}x}{\sqrt{2}-\operatorname{th}x}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}\frac{e^{x^2}-\cos x}{x}, & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing.

$$y=\frac{x^2-3x+6}{x^2},\ x_0=3.$$

4) dy , d^2y - ?

$$y=\operatorname{arcth}\frac{x^2-1}{x}.$$

5) Differensial yordamida taqribiy hisoblang: $y=x^{21},\ x=0{,}998$.

6) Hosilani toping y'_x :
$$\begin{cases}x=\frac{3t^2+1}{3t^3}\\ y=\sin\left(\frac{t^3}{3}+t\right)\end{cases}.$$

7) n -nchi tartibli hosilani toping: $y=2^{3x+5}$.

8) Ikkinchi tartibli hosilani toping
$$\begin{cases}x=\frac{\cos t}{1+2\cos t}\\ y=\frac{\sin t}{1+2\cos t}\end{cases}.$$

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\frac{1}{x^4-1}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=-\frac{x^2}{2}+\frac{8}{x}+8,\quad [-4,\ -1].$$

Variant №18.

1) Hosilani toping:

a) $y=\left(1-x^2\right)\sqrt[5]{x3+\frac{1}{x}}$,

b) $y=2(x-2)\sqrt{1+e^x}-2\ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right)$,

c) $y=\ln\arcsin\sqrt{1-e^{2x}}$,

d) $y=\sqrt[3]{ctg2}-\frac{1}{20}\frac{\cos^2 10x}{\sin 20x}$,

e) $y=\arcsin\frac{x-2}{(x-1)\sqrt{2}}$,

f) $y=\frac{chx}{\sqrt{sh2x}}$,

g) $y=x^{e^{ctgx}}$,

h) $y=\ln\frac{1+2\sqrt{-x-x^2}}{2x+1}+\frac{4}{2x+1}\sqrt{-x-x^2}$,

i) $y=\ln\sqrt[3]{\frac{x-1}{x+1}}-\frac{1}{2}\left(\frac{1}{2}+\frac{1}{x^2-1}\right)arctgx$,

j) $y=arctg\frac{\sqrt{2tgx}}{1-tgx}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}e^{x\sin 5x}, & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing::

$$y=\frac{1}{3x+2},\ x_0=2.$$

4) dy , d^2y - ?

$$y=\ln\left|x^2-1\right|-\frac{1}{x^2-1}.$$

5) Differensial yordamida taqribiy hisoblang: $y=\sqrt[3]{x},\ x=1{,}21$.

6) Hosilani toping y'_x : $\begin{cases}x=\arcsin\sqrt{t}\\ y=\sqrt{1+\sqrt{t}}\end{cases}$.

7) n -nchi tartibli hosilani toping: $y=\frac{2x+5}{13(3x+1)}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases}x=\sqrt{t}\\ y=\sqrt[3]{t-1}\end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y=-\left(\frac{x}{x+2}\right)^2$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=3-x-\frac{4}{(x+2)^2},\quad [-1,\ 2].$$

Variant №19.

1) Hosilani toping:

a) $y=\frac{(2x^2+3)\sqrt{x^2-3}}{9x^3}$,

b) $y=2\frac{\sqrt{2^x-1}-\mathrm{arctg}\sqrt{2^x-1}}{\ln 2}$,

c) $y=\ln\arccos\sqrt{1-e^{4x}}$,

d) $y=\mathrm{ctg}\cos 2+\frac{1}{6}\frac{\sin^2 6x}{\cos 12x}$,

e) $y=\sqrt{1-x^2}-x\arcsin\sqrt{1-x^2}$,

f) $y=-\frac{\mathrm{sh}x}{1+\mathrm{ch}x}$,

g) $y=x^{e^{\cos x}}$,

h) $y=\ln\left(4x-1+\sqrt{16x^2-8x+2}\right)-\sqrt{16x^2-8x+2}\,\mathrm{arctg}(4x-1)$,

i) $y=x\ln\left(\sqrt{1-x}+\sqrt{1+x}\right)+\frac{1}{2}(\arcsin x-x)$,

j) $y=\frac{6^x(\sin 4x\ln 6-4\cos 4x)}{16+\ln^2 6}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}3^{x^2\sin(2/x)}, & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing.

$y=2x^2-3x+1,\ x_0=1$.

4) dy, d^2y - ?

$$y=\mathrm{arctg}\left(\mathrm{tg}\frac{x}{2}+1\right).$$

5) Differensial yordamida taqribiy hisoblang: $y=x^{11},\ x=1{,}021$.

6) Hosilani toping y'_x: $\begin{cases}x=(\arcsin t)^2\\ y=\frac{t}{\sqrt{1-t^2}}\end{cases}$.

7) n -nchi tartibli hosilani toping: $y=\sqrt{x}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases}x=\sqrt{t-1}\\ y=\frac{t}{\sqrt{t-1}}\end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\frac{x^3-32}{x^2}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin $(1+x^2)\ln(1+x)$

Variant №20.

1) Hosilani toping:

a) $y=\frac{x-1}{(x^2+5)\sqrt{x^2+5}}$,

b) $y=\ln(e^x+1)+\frac{18e^{2x}+27e^x+11}{6(e^x+1)}$,

c) $y=\ln\left(bx+\sqrt{a^2+b^2x^2}\right)$,

d) $y=\cos ctg2-\frac{1}{16}\frac{\cos^2 8x}{\sin 16x}$,

e) $y=\operatorname{arctg}\frac{\sqrt{1-x}}{1-\sqrt{x}}$,

f) $y=\sqrt[4]{\frac{1+thx}{1-thx}}$,

g) $y=x^{2^x}5^x$,

h) $y=3x-\ln\left(1+\sqrt{1-e^{6x}}\right)-e^{-3x}\arcsin\left(e^{3x}\right)$,

i) $y=\operatorname{arctg}\sqrt{x^2-1-\frac{\ln x}{\sqrt{x^2-1}}}$,

j) $y=\frac{ctgx+x}{1-xctgx}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases} e^{x\sin\left(\frac{3}{5}x\right)}-1, & x\neq 0 \\ 0, & x=0 \end{cases}.$$

3) Urinmaning tenglamasini tuzing::

$$y=\frac{x}{x^2+1},\ x_0=-2.$$

4) dy , d^2y - ?

$$y=\ln\left|2x+2\sqrt{x^2+x}+1\right|.$$

5) Differensial yordamida taqribiy hisoblang: $y=\sqrt{x^2+x+3}$, $x=1{,}97$.

6) Hosilani toping y'_x : $\begin{cases} x=t\sqrt{t^2+x} \\ y=\ln\frac{1+\sqrt{1+t^2}}{t} \end{cases}$.

7) n -nchi tartibli hosilani toping: $y=\lg(x+4)$.

8) Ikkinchi tartibli hosilani toping $\begin{cases} x=tgt \\ y=\frac{1}{\sin 2t} \end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\frac{3x-2}{x^3}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=\frac{-2x(2x+3)}{x^2+4x+5},\quad [1,\ 4].$$

Variant №21.

1) Hosilani toping:

a) $y = \frac{(2x+1)\sqrt{x^2 - x}}{x^2}$,

b) $y = \frac{1}{2}\ln\left(e^{2x} + 1\right) - 2\operatorname{arctg} e^{x}$,

c) $y = \ln\left(\arccos\frac{1}{\sqrt{x}}\right)$,

d) $y = \frac{\cos\ln 7 \cdot \sin^2 7x}{7\cos 14x}$,

e) $y = \left(2x^2 + 6x + 5\right)\operatorname{arctg}\frac{x+1}{x+2} - x$,

f) $y = \frac{1}{6}\ln\frac{1 - \operatorname{sh}2x}{2 + \operatorname{sh}2x}$,

g) $y = x^{e^{\sin x}}$,

h) $y = \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{3x-1}{\sqrt{2}} + \frac{1}{3}\frac{3x-1}{x^2 - 2x + 1}$,

i) $y = 3\arcsin\frac{3}{x+2} + \sqrt{x^2 + 4x - 5}$,

j) $y = \left(1 + x^2\right)e^{\operatorname{arctg}x}$.

2) Hosilaning ta’rifiga asoslanib, f’(0) ni toping:

$$f(x) = \begin{cases} \frac{2^{\operatorname{tg}x} - 2^{\sin x}}{x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

3) Urinmaning tenglamasini tuzing.

$$y = x + \sqrt{x^3}, \; x_0 = 1.$$

4) dy , d^2y - ?

$$y = e^x\left(\cos 2x + 2\sin 2x\right).$$

5) Differensial yordamida taqribiy hisoblang: $y = \sqrt[3]{x}$, $x = 26{,}46$.

6) Hosilani toping y'_x: $\begin{cases} x = \operatorname{arctg}t \\ y = \ln\frac{\sqrt{1+t^2}}{t+1} \end{cases}$.

7) n -nchi tartibli hosilani toping: $y = \frac{x}{2(3x+2)}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases} x = \sqrt{t} \\ y = \frac{1}{\sqrt{1-t}} \end{cases}$.

9) Funksiyani to’la tekshiring va grafigini yasang: $y = \frac{x^3 - 4}{x^2}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y = -\frac{2\left(x^2 + 3\right)}{x^2 + 2x + 5}, \quad [-5,\ 1].$$

Variant №22.

1) Hosilani toping:

a) $y = 2\sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}$,

b) $y = \dfrac{2}{3}\sqrt{\left(\operatorname{arctg} e^{x}\right)^{3}}$,

c) $y = \ln\left(e^{x} + \sqrt{1+e^{2x}}\right)$,

d) $y = \dfrac{\sin\cos 3\cos^{2} 2x}{4\sin 4x}$,

e) $y = \operatorname{arctg}\dfrac{\operatorname{tg}\left(\dfrac{x}{2}\right)+1}{2}$,

f) $y = \operatorname{arctg}\dfrac{\sqrt{\operatorname{sh}2x}}{\operatorname{ch}x - \operatorname{sh}x}$,

g) $y = x^{e^{\operatorname{arctg}x}}$,

h) $y = \dfrac{x^{4}}{81}\arcsin\dfrac{3}{x} + \dfrac{1}{81}\left(x^{2}+18\right)\sqrt{x^{2}-9}$, $x > 0$

i) $y = \sqrt{(3-x)(2+x)} + 5\arcsin\sqrt{\dfrac{x+2}{5}}$,

j) $y = -\dfrac{1}{3\sin^{3}x} - \dfrac{1}{\sin x} + \dfrac{1}{2}\ln\dfrac{1+\sin x}{1-\sin x}$.

2) Hosilaning ta’rifiga asoslanib, f’(0) ni toping:

$$f(x) = \begin{cases} \operatorname{arctg}\left(\dfrac{3x}{2} - x^{2}\sin\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

3) Urinmaning tenglamasini tuzing::

$$y = \frac{x^{5}+1}{x^{4}+1},\ x_{0} = 1.$$

4) dy , $d^{2}y$ - ?

$$y = x(\sin\ln x - \cos\ln x).$$

5) Differensial yordamida taqribiy hisoblang: $y = \sqrt[3]{x^{2}+2x+5}$, $x = 0{,}97$.

6) Hosilani toping y'_{x} : $\begin{cases} x = \ln\left(1-t^{2}\right) \\ y = \arcsin\sqrt{1-t^{2}} \end{cases}$.

7) n -nchi tartibli hosilani toping: $y = a^{3x}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases} x = \dfrac{1}{t} \\ y = \dfrac{1}{1+t^{2}} \end{cases}$.

9) Funksiyani to’la tekshiring va grafigini yasang: $y = \dfrac{-8x}{x^{2}+4}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y = \sqrt[3]{2(x-1)^{2}(x-4)},\quad [0,\ 4].$$

Variant №23.

1) Hosilani toping:

a) $y=\dfrac{1}{(x+2)\sqrt{x^2+4x+5}}$,

b) $y=2\sqrt{e^x+1}+\ln\dfrac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}$,

c) $y=\ln\dfrac{\ln x}{\sin\left(\frac{1}{x}\right)}$,

d) $y=\dfrac{\cos\sin 5\sin^2 2x}{2\cos 4x}$,

e) $y=\dfrac{2x-1}{4}\sqrt{2+x-x^2}+\dfrac{9}{8}\arcsin\dfrac{2x-1}{3}$,

f) $y=-\dfrac{1}{2}\ln \operatorname{th}\dfrac{x}{2}-\dfrac{\operatorname{ch}x}{2\operatorname{sh}^2x}$,

g) $y=x^{29^x}29^x$,

h) $y=\dfrac{2}{x-1}\sqrt{2x-x^2}+\ln\dfrac{1+\sqrt{2x-x^2}}{x-1}$,

i) $y=x(\arcsin x)^2+2\sqrt{1-x^2}\arcsin x-2x$,

j) $y=\ln\dfrac{\sin x}{\cos x+\sqrt{\cos 2x}}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x)=\begin{cases}\dfrac{\ln(1+2x^2+x^3)}{x}, & x\neq 0\\ 0, & x=0\end{cases}.$$

3) Urinmaning tenglamasini tuzing

$y=x-x^3,\ x_0=-1$.

4) dy , d^2y - ? $\quad y=\cos x\ln \operatorname{tg}x-\ln \operatorname{tg}\dfrac{x}{2}$.

5) Differensial yordamida taqribiy hisoblang: $y=\arcsin x,\ x=0{,}08$.

6) Hosilani toping y'_x : $\begin{cases}x=\operatorname{arctg}\dfrac{t+1}{t-1}\\ y=\arcsin\sqrt{1-t^2}\end{cases}$.

7) n -nchi tartibli hosilani toping: $y=\lg(5x+2)$.

8) Ikkinchi tartibli hosilani toping $\begin{cases}x=\operatorname{arctg}\dfrac{t+1}{t-1}\\ y=\arcsin\sqrt{1-t^2}\end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y=\dfrac{4}{3+2x-x^2}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y=x^2-2x+\frac{16}{x-1}-13,\quad [2,\ 5].$$

Variant №24.

1) Hosilani toping:

a) $y = 3\frac{\sqrt[3]{x^2 + x + 1}}{x + 1}$,

b) $y = \frac{1}{\ln 4}\ln\frac{1 + 2^x}{1 - 2^x}$,

c) $y = \ln\ln\sin\left(1 + \frac{1}{x}\right)$,

d) $y = \mathrm{ctg}\sqrt[3]{5} - \frac{1}{8}\frac{\cos^2 4x}{\sin 8x}$,

e) $y = \frac{x - 3}{2}\sqrt{6x - x^2 - 8} + \arcsin\sqrt{\frac{x}{2} - 1}$,

f) $y = \frac{3}{8\sqrt{2}}\ln\frac{\sqrt{2} + \mathrm{th}x}{\sqrt{2} - \mathrm{th}x} - \frac{\mathrm{th}x}{4\left(2 - \mathrm{th}^2 x\right)}$,

g) $y = x^{e^x} x^9$,

h) $y = \sqrt{9x^2 - 12x + 5}\,\mathrm{arctg}(3x - 2) - \ln\left(3x - 2 - \sqrt{9x^2 - 12x + 5}\right)$,

i) $y = \frac{\sqrt{1 - x^2}}{x} + \mathrm{srcsin}\, x$,

j) $y = \frac{7^x(3\sin 3x + \cos 3x \ln 7)}{9 + \ln^2 7}$.

2) Hosilaning ta'rifiga asoslanib, f'(0) ni toping:

$$f(x) = \begin{cases} \frac{\cos x - \cos 3x}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

3) Urinmaning tenglamasini tuzing::

$$y = 2x + \frac{1}{x},\ x_0 = 1.$$

4) dy , d^2y - ?

$$y = \sqrt{3 + x^2} - x\ln\left|x + \sqrt{3 + x^2}\right|.$$

5) Differensial yordamida taqribiy hisoblang: $y = \sqrt[3]{x}$, $x = 27{,}54$.

6) Hosilani toping y'_x : $\begin{cases} x = \ln \mathrm{tg}t \\ y = \frac{1}{\sin^2 t} \end{cases}$.

7) n -nchi tartibli hosilani toping: $y = \frac{4x + 7}{2x + 3}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases} x = \mathrm{sh}^2 t \\ y = \frac{1}{\mathrm{ch}^2 t} \end{cases}$.

9) Funksiyani to'la tekshiring va grafigini yasang: $y = \frac{4}{x^2 + 2x - 3}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$$y = 2\sqrt{x - 1} - x + 2, \quad [1,\ 5].$$

Variant №25.

1) Hosilani toping:

a) $y = 3\sqrt[3]{\frac{x+1}{(x-1)^2}}$,

b) $y = \frac{1}{2}\operatorname{arctg}\frac{e^x - 3}{2}$,

c) $y = \ln\ln^3 \ln^2 x$,

d) $y = \operatorname{tg}\lg\frac{1}{3} + \frac{1}{4}\frac{\sin^2 4x}{\cos 8x}$,

e) $y = \frac{x}{2\sqrt{1-4x^2}}\arcsin 2x + \frac{1}{8}\ln\left(1-4x^2\right)$,

f) $y = \frac{1}{2}\ln\frac{1+\sqrt{\operatorname{th}x}}{1-\sqrt{\operatorname{th}x}} - \operatorname{arctg}\sqrt{\operatorname{th}x}$,

g) $y = (\operatorname{tg}x)\ln\operatorname{tg}x/4$,

h) $y = 2x - \ln\left(1+\sqrt{1-e^{4x}}\right) - e^{-2x}\arcsin\left(e^{2x}\right)$,

i) $y = x^3\arccos x - \frac{x^2+2}{3}\sqrt{1-x^2}$,

j) $y = 3\frac{\sin x}{\cos^2 x} + 2\frac{\sin x}{\cos^4 x}$.

2) Hosilaning ta’rifiga asoslanib, f’(0) ni toping:

$$f(x) = \begin{cases} 1-\cos\left(x\sin\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

3) Urinmaning tenglamasini tuzing

$y = 2x^2 + 3x - 1,\ x_0 = -2.$

4) $dy,\ d^2y$ -?

$y = \sqrt{x} - (1+x)\operatorname{arctg}\sqrt{x}$.

5) Differensial yordamida taqribiy hisoblang: $y = \frac{x+\sqrt{5-x^2}}{2},\ x = 0{,}98$.

6) Hosilani toping: $\begin{cases} x = \frac{t}{\sqrt{1-t^2}} \\ y = \ln\frac{1+\sqrt{1-t^2}}{t} \end{cases}$.

7) n -nchi tartibli hosilani toping: $y = \sqrt[5]{e^{7x-1}}$.

8) Ikkinchi tartibli hosilani toping $\begin{cases} x = e^t\cos t \\ y = e^t\sin t \end{cases}$.

9) Funksiyani to’la tekshiring va grafigini yasang: $y = \frac{12-3x^2}{x^2+12}$.

10) Berilgan oraliqda y=f(x) funksiyaning ekstremumlari topilsin :

$y = \sqrt[3]{2(x+2)^2(1-x)},\quad [-3,\ 4].$

Variant № 1		Variant № 2	
1.	$\int \frac{\sin x dx}{(1+3\cos x)^3}$	1.	$\int \frac{(2x^2 - x^5)dx}{1+x^6}$
2.	$\int \frac{x^5}{x^{12}-1}dx$	2.	$\int \frac{x^3 dx}{x^2+1}$
3.	$\int \frac{\ln^2 x}{x^2}dx$	3.	$\int x^2 \sin x dx$
4.	$\int \frac{e^x - 1}{3e^x + 1}dx$	4.	$\int \frac{\sqrt{1+x^2}}{x}dx$
5.	$\int \frac{x^4 + 10x^3 + 26x^2 + 22x + 17}{(x^2+8x+7)(x+1)}dx$	5.	$\int \frac{x^3 - 9x + 13}{(x-1)(x^2-3x+2)}dx$
6.	$\int \frac{2x^3 + 5x^2 - 8x + 4}{(x^2-4)(x^2+2)}dx$	6.	$\int \frac{3x^3 + 5x^2 + 28}{x^4 - 16}dx$
7.	$\int \frac{dx}{[(1+x)^2(1+x^2)]}$	7.	$\int \frac{dx}{(x^4-1)^2}$
8.	$\int \frac{dx}{\sin^2 x + tg^2 x}$	8.	$\int \frac{1+\cos x}{\sin^3 x}dx$
9.	$\int \frac{dx}{2\sin x + \sin 2x}$	9.	$\int \frac{dx}{3+\cos x}$
10.	$\int \frac{x+1}{x\sqrt{x-2}}dx$	10.	$\int \frac{(2\sqrt[4]{x}+1)dx}{\sqrt[4]{x^3}(\sqrt{x}+4)}$
11.	$\int \frac{dx}{(2x+1)(1+\sqrt{2x+1})}$	11.	$\int \frac{dx}{\sqrt[3]{(x+1)^2} + 2\sqrt{x+1}}$
12.	$\int \sqrt{x}(1+\sqrt[3]{x})^4 dx$	12.	$\int x^{-1}\left(1+x^{1/3}\right)^{-1} dx$
13.	$\int \frac{dx}{x+\sqrt{x^2-x+1}}$	13.	$\int \frac{dx}{x+\sqrt{x^2+x+1}}$

Individual vazifalar-2

Variant № 3

1. $\int \frac{\cos x}{\sqrt[5]{\sin^2 x}} dx$
2. $\int \frac{x^3 dx}{\sqrt{4-x^3}}$
3. $\int x 3^{\frac{x}{2}} dx$
4. $\int x^2 e^{3x} dx$
5. $\int \frac{x^3 - 2x^2 + x + 2}{x^3 - 2x^2} dx$
6. $\int \frac{2x+1}{x^3 - 1} dx$
7. $\int \frac{dx}{(x^2+1)^4}$
8. $\int \frac{\sin^4 x}{\cos^2 x} dx$
9. $\int \sin 5x \cos 7x dx$
10. $\int \frac{dx}{\sqrt[3]{(2x+1)^2} + \sqrt{2x+1}}$
11. $\int \frac{dx}{(1+\sqrt[4]{x})^3 \sqrt{x}}$
12. $\int \frac{dx}{x\sqrt[3]{x^2+1}}$
13. $\int \frac{dx}{1+\sqrt{1-2x-x^2}}$

Variant № 4

1. $\int \frac{2^{arctg 2x} dx}{1+4x^2}$
2. $\int \frac{\ln^3 x + 2}{x \ln x} dx$
3. $\int \sqrt{4-x^2} dx$
4. $\int x^2 \sin 2x dx$
5. $\int \frac{x^5 - x^4 + 3x - 2}{x^4 - x^3} dx$
6. $\int \frac{3x dx}{x^3 + x^2 + 2x + 2}$
7. $\int \frac{x^4 - 2x^2 + 2}{(x^2 - 2x + 2)^2} dx$
8. $\int tg^4 \frac{2}{3} x dx$
9. $\int \frac{dx}{2\sin x - 3\cos x}$
10. $\int \frac{x dx}{\sqrt{x-1}}$
11. $\int \frac{(\sqrt{x}-1)(\sqrt[6]{x}+1)}{\sqrt[3]{x^2}} dx$
12. $\int x^5 \sqrt[3]{(1+x^3)^2} dx$
13. $\int x\sqrt{x^2 - 2x + 2} dx$

Variant № 5

1. $\int \frac{\sin x dx}{\sqrt{\cos^2 x + 2}}$

2. $\int \frac{dx}{\sqrt{(1-x^2)\arcsin x}}$

3. $\int \sqrt{x} \ln x dx$

4. $\int \frac{dx}{e^x - 1}$

5. $\int \frac{(x^3 - 4x + 1)}{x^3 - 2x^2 + x} dx$

6. $\int \frac{2x^2 + x + 4}{x^3 + x^2 + 4x + 4} dx$

7. $\int \frac{x^5 dx}{(x^3 + 1)(x^3 + 8)}$

8. $\int ctg^4 \frac{x}{2} dx$

9. $\int \frac{dx}{3 + 5\cos x}$

10. $\int \frac{dx}{\sqrt[3]{x} + \sqrt{x}}$

11. $\int \frac{\sqrt[3]{x+2} dx}{(1 + \sqrt{x+2})(\sqrt[6]{x+2})^5}$

12. $\int \frac{dx}{\sqrt[3]{1 + x^3}}$

13. $\int \frac{x - \sqrt{x^2 + 3x + 2}}{x + \sqrt{x^2 + 3x + 2}} dx$

Variant № 6

1. $\int \frac{\sin x dx}{\sqrt{\cos^2 x + 2}}$

2. $\int \frac{dx}{\sqrt{(1-4x^2)\arcsin 2x}}$

3. $\int arctg \frac{1}{x} dx$

4. $\int x \ln |1 + x^2| dx$

5. $\int \frac{x^4 - 3x^2 + 3x + 1}{x^3 - 3x - 2} dx$

6. $\int \frac{(7x - 15) dx}{x^3 - 2x^2 + 5x}$

7. $\int \frac{dx}{x^4 (x^3 + 1)^2}$

8. $\int \frac{dx}{\sin^2 x \cos^4 x}$

9. $\int \frac{dx}{4\sin x + 3\cos x + 1}$

10. $\int \frac{(\sqrt[6]{x} + 1) dx}{x\sqrt[3]{x} + \sqrt[6]{x^5}}$

11. $\int \frac{dx}{\sqrt{x+1} + \sqrt[4]{x+1}}$

12. $\int \frac{dx}{\sqrt[4]{1 + x^4}}$

13. $\int \frac{dx}{\left[1 + \sqrt{x(1+x)}\right]^2}$

Variant № 7

1. $\int \frac{\sqrt[3]{4+\ln(x)}}{x}dx$
2. $\int \frac{\sin x}{\sqrt[3]{3+2*\cos x}}dx$
3. $\int e^{-x}\sin 2xdx$
4. $\int x\ln^2 xdx$
5. $\int \frac{x^3+3x^2+8x+12}{(x^2+4x+4)(x-1)}dx$
6. $\int \frac{1}{x^3+8}dx$
7. $\int \frac{1}{x^8+x^6}dx$
8. $\int \frac{1}{\cos^4 3x}dx$
9. $\int \frac{1}{2+3\cos^2 x}dx$
10. $\int \frac{\sqrt{x}}{x+\sqrt[3]{x^2}}dx$
11. $\int \frac{dx}{\sqrt{x-2}(1+\sqrt[3]{x-2})}$
12. $\int \frac{\sqrt{1-x^4}}{x^5}dx$
13. $\int \frac{x+\sqrt{1+x+x2}}{1+x+\sqrt{1+x+x^2}}dx$

Variant № 8

1. $\int \frac{e^x}{\sqrt{e^x+4}}dx$
2. $\int \frac{\sin 2x}{3\sin^2 x+4}dx$
3. $\int (x^2+1)3^x dx$
4. $\int x^3 e^{-x^2}dx$
5. $\int \frac{2x^4+8x^3+x^2+x-20}{x^3(x+5)}dx$
6. $\int \frac{dx}{x^3+x^2+2x+2}$
7. $\int \frac{dx}{(x^3-1)^2}$
8. $\int \frac{dx}{\sin^3 x}$
9. $\int \sin^2 x\cos^2 xdx$
10. $\int \frac{\sqrt{x}}{x+\sqrt[4]{x^3}}dx$
11. $\int \frac{\sqrt{x+3}}{1+\sqrt[3]{x+3}}dx$
12. $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}dx$
13. $\int \frac{dx}{1+\sqrt{x^2+2x+2}}$

Variant № 9

1. $\int \frac{\sin x}{\sqrt[3]{\cos^2 x}} dx$
2. $\int \frac{x + arctgx}{1 + x^2} dx$
3. $\int \frac{x}{\sin^2 x} dx$
4. $\int x \ln x dx$
5. $\int \frac{2x^3 - 2x^2 - 16x + 32}{(x-2)(x^2-4)} dx$
6. $\int \frac{x^3 - 2x + 5}{x^4 - 1} - 1dx$
7. $\int \frac{x^2}{(x-1)^{10}} dx$
8. $\int \frac{dx}{5 - 3\cos x}$
9. $\int \frac{1 + tgx}{\sin 2x} dx$
10. $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x^2}}$
11. $\int \frac{dx}{\sqrt{2x-1} - \sqrt[4]{2x-1}}$
12. $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x} dx$
13. $\int \frac{dx}{x - \sqrt{x^2 - x + 1}}$

Variant № 10

1. $\int \frac{1 - 2x}{\sqrt{1 - 4x^2}} dx$
2. $\int \frac{e^x}{e^x + e^{-x}} dx$
3. $\int x \ln(x-1) dx$
4. $\int \frac{xcox}{\sin^3 x} dx$
5. $\int \frac{2x^4 + 9x^3 + 4x^2 - 6x - 8}{x^3 + 4x^2} dx$
6. $\int \frac{13x + 26}{(x-2)(x^2 + 6x + 10)} dx$
7. $\int \frac{dx}{(x^2 + 2x + 2)(x^2 + 2x + 5)}$
8. $\int \frac{\sin x}{1 + \sin x} dx$
9. $\int tg^5 3x dx$
10. $\int \frac{dx}{\sqrt[6]{x^5} - \sqrt{x}}$
11. $\int \frac{dx}{\sqrt[3]{x} + \sqrt{x}}$
12. $\int \frac{dx}{x\sqrt[3]{1 + x^5}}$
13. $\int \sqrt{x^2 - 2x - 1} dx$

Variant № 11

1. $\int \frac{dx}{(1-x^2)^{3/2}}$
2. $\int \frac{arctg\sqrt{x}}{\sqrt{x}} \frac{dx}{1+x}$
3. $\int \ln x dx$
4. $\int arctgxdx$
5. $\int \frac{x}{x^3 - 3x + 2} dx$
6. $\int \frac{2x+3}{(x-2)(x+5)} dx$
7. $\int \frac{x}{(x-1)^2 (x+1)^3} dx$
8. $\int \frac{dx}{\sin^4 x \cos^4 x}$
9. $\int \sin^4 x dx$
10. $\int \frac{dx}{(1+\sqrt[4]{x})^2 \sqrt{x}}$
11. $\int \frac{dx}{x(1+2\sqrt{x}+\sqrt[3]{x})}$
12. $\int \sqrt[3]{3x - x^3} dx$
13. $\int \frac{x}{(1+x)\sqrt{1-x-x^2}} dx$

Variant № 12

1. $\int \frac{x^2}{\sqrt{x^2-2}} dx$
2. $\int \frac{dx}{\sqrt{1+e^x}}$
3. $\int x^n \ln x dx$
4. $\int \arcsin x dx$
5. $\int \frac{x^2+1}{(x+1)^2 (x-1)} dx$
6. $\int \frac{2x+1}{(x-2)^3 (x+5)} dx$
7. $\int \frac{dx}{(x^3+1)^2}$
8. $\int \frac{dx}{\sin^3 x \cos^5 x}$
9. $\int \frac{dx}{\sin 2x}$
10. $\int \frac{dx}{x(1+2\sqrt{x}+\sqrt[3]{x})}$
11. $\int \frac{x\sqrt[3]{2+x}}{x+\sqrt[3]{2+x}} dx$
12. $\int \frac{dx}{x^3 \sqrt[5]{1+\frac{1}{x}}}$
13. $\int \frac{\sqrt{x^2+2x+2}}{x} dx$

Variant № 13

1. $\int \sqrt{a^2 - x^2}\,dx$

2. $\int \frac{\sin^2 x}{\cos^6 x}dx$

3. $\int \left(\frac{\ln x}{x}\right)^2 dx$

4. $\int x\,arctgx dx$

5. $\int \left(\frac{x}{x^2 - 3x + 2}\right)^2 dx$

6. $\int \frac{dx}{x^2 - 2x + 2}$

7. $\int \frac{dx}{(x^3 + 1)^3}$

8. $\int \frac{dx}{\sin x \cos^4 x}$

9. $\int \cos^3 x dx$

10. $\int \frac{dx}{\sqrt{x}(1 + \sqrt[4]{x})^3}$

11. $\int \frac{x^3}{\sqrt{1 + 2x - x^2}}dx$

12. $\int \frac{dx}{x^6\sqrt[6]{1 + x^6}}$

13. $\int \frac{dx}{(x^2 + 2)\sqrt{2x^2 - 2x + 5}}$

Variant № 14

1. $\int \frac{dx}{(x^2 + a^2)^{3/2}}$

2. $\int \frac{dx}{1 + e^x}$

3. $\int \sqrt{x}\ln^2 x dx$

4. $\int x^2 \arccos x dx$

5. $\int \frac{dx}{(x + 1)(x + 2)^2(x + 3)^3}$

6. $\int \frac{dx}{x^2 - 2x + 1}$

7. $\int \frac{x^2}{(x^2 + 2x + 1)}dx$

8. $\int tg^5 x dx$

9. $\int \frac{dx}{\cos 2x}$

10. $\int \frac{\sqrt{x + 1} - \sqrt{x - 1}}{\sqrt{x + 1} + \sqrt{x - 1}}dx$

11. $\int x^4\sqrt{a^2 - x^2}dx$

12. $\int \frac{dx}{\sqrt[4]{1 + x^4}}$

13. $\int \frac{(x^2 + 1)}{x\sqrt{x^4 + x^2 + 1}}dx$

Variant № 15

1. $\int\sqrt{\frac{a+x}{a-x}}dx$
2. $\int\frac{(1+e^x)^2}{1+e^{2x}}dx$
3. $\int xe^{-x}dx$
4. $\int\frac{\arcsin x}{x^2}dx$
5. $\int\frac{dx}{x^5+x^4-2x^3-2x^2+x+1}$
6. $\int\frac{2x-2}{x^2-2x+2}dx$
7. $\int\frac{dx}{(x^4+1)^2}$
8. $\int ctg^6 x dx$
9. $\int\frac{dx}{2\sin x-\cos x-1}$
10. $\int\frac{dx}{1+\sqrt{x}+\sqrt{x+1}}$
11. $\int\frac{dx}{x^3\sqrt{x^2+1}}$
12. $\int\frac{dx}{\sqrt[3]{1+x^3}}$
13. $\int\frac{dx}{x+\sqrt{x^2+x+1}}$

Variant № 16

1. $\int\cos^5 x\sqrt{\sin x}dx$
2. $\int ctg^2 x dx$
3. $\int x^2e^{-2x}dx$
4. $\int\ln(x+\sqrt{1+x^2})dx$
5. $\int\frac{x^2+5x+4}{x^4+5x^2+4}dx$
6. $\int\frac{dx}{(x+1)(x^2+1)}$
7. $\int\frac{dx}{(x^4-1)^3}$
8. $\int\frac{\sin^4 x}{\cos^6 x}dx$
9. $\int\frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$
10. $\int\frac{dx}{\sqrt[n]{(x-a)^{n+1}(x-b)^{n-1}}}$
11. $\int\frac{x^{10}}{\sqrt{1+x^2}}dx$
12. $\int\frac{x^5}{\sqrt{1-x^2}}dx$
13. $\int\frac{dx}{1+\sqrt{1-2x-x^2}}$

Variant № 17

1. $\int \frac{\sin x \cos^3 x}{1+\cos^2 x} dx$
2. $\int tg^3 x dx$
3. $\int x^3 e^{-x^2} dx$
4. $\int x \ln \frac{1+x}{1-x} dx$
5. $\int \frac{dx}{(x+1)(x^2+1)}$
6. $\int \frac{dx}{(x^2+1)^2(x+2)}$
7. $\int \frac{x^2}{(x^2+2x+2)^2} dx$
8. $\int \frac{dx}{\sqrt{\sin^3 x \cos^5 x}}$
9. $\int \frac{dx}{1+\cos x}$
10. $\int \frac{x^2}{\sqrt{1+x+x^2}} dx$
11. $\int \frac{x}{(x-1)^2\sqrt{1+2x-x^2}} dx$
12. $\int \frac{x}{\sqrt{1+\sqrt[3]{x^2}}} dx$
13. $\int x\sqrt{x^2-2x+2}dx$

Variant № 18

1. $\int \frac{\ln x}{x\sqrt{1+\ln x}} dx$
2. $\int \frac{dx}{\sin^2 x \cos x}$
3. $\int x \cos x dx$
4. $\int arctg\sqrt{x} dx$
5. $\int \frac{dx}{(x^2-4x+4)(x^2-4x+5)}$
6. $\int \frac{dx}{(x+1)(1-x^2)}$
7. $\int \frac{dx}{(x^4+1)^2}$
8. $\int \frac{dx}{\cos x \sqrt[3]{\sin^2 x}}$
9. $\int \frac{dx}{1-\cos x}$
10. $\int \frac{dx}{(x+1)\sqrt{x^2+x+1}}$
11. $\int \frac{x}{(x^2-1)\sqrt{x^2-x-1}} dx$
12. $\int \frac{\sqrt{x}}{(1+\sqrt[3]{x})^2} dx$
13. $\int \frac{x-\sqrt{x^2+3x+2}}{x+\sqrt{x^2+3x+2}} dx$

Variant № 19

1. $\int \frac{dx}{e^{x/2}+e^{x}}$

2. $\int \frac{dx}{\sin x \cos^3 x}$

3. $\int x^2 \sin 2x dx$

4. $\int \frac{x}{x^4+1} dx$

5. $\int \frac{x}{(x-1)^2 (x^2+2x+2)} dx$

6. $\int \frac{x}{(x+1)(x+2)(x+3)} dx$

7. $\int \frac{dx}{(x^4-1)^3}$

8. $\int \frac{dx}{\sqrt{tgx}}$

9. $\int \frac{dx}{1+\sin x}$

10. $\int \frac{1-\sqrt{x+1}}{1+\sqrt[3]{x+1}} dx$

11. $\int \frac{\sqrt{x^2+x+1}}{(x+1)^2} dx$

12. $\int \sqrt{x^3+x^4} dx$

13. $\int \frac{dx}{(1+\sqrt{x(1+x)})^2}$

Variant № 20

1. $\int x^2 \sqrt{2+x^2} dx$

2. $\int \frac{\cos^5 x}{\sin x} dx$

3. $\int arctg\sqrt{x} dx$

4. $\int \sin x e^x dx$

5. $\int \frac{dx}{x^3+4x}$

6. $\int \frac{x}{x^3-1} dx$

7. $\int \frac{dx}{(x^3+1)^2}$

8. $\int (1+2\cos x)^3 dx$

9. $\int \frac{dx}{\sin^3 x}$

10. $\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}} dx$

11. $\int \frac{x^3}{1+\sqrt[3]{x^4+1}} dx$

12. $\int \frac{\sqrt{x}}{(1+\sqrt[3]{x})^2} dx$

13. $\int \frac{dx}{1+\sqrt{1-2x-x^2}}$

Variant № 21

1. $\int \frac{dx}{x\sqrt{x^2-1}}$

2. $\int \frac{1-\sqrt{x}}{\sqrt{x}(x+1)}dx$

3. $\int arctg\sqrt{x}dx$

4. $\int \sin\ln xdx$

5. $\int \frac{7x^3+9}{x^4-5x^3+6x^3}dx$

6. $\int \frac{x^3-6}{x^4+6x^2+8}dx$

7. $\int \frac{9}{5x^2(3-2x^2)^3}dx$

8. $\int \frac{dx}{\sin^3 x}$

9. $\int \frac{dx}{1+sn^2x}$

10. $\int \frac{1+\sqrt{x}+2\sqrt[3]{x}}{x(1+\sqrt[6]{x})}dx$

11. $\int \frac{x}{2+\sqrt{2x+1}}dx$

12. $\int \sqrt[3]{x(1-x^2)}dx$

13. $\int \frac{dx}{\sqrt{1+e^x+e^{2x}}}$

Variant № 22

1. $\int tgx\ln\cos xdx$

2. $\int \frac{x^3}{x^2+1}dx$

3. $\int \frac{\ln^2 x}{\sqrt{x^5}}dx$

4. $\int e^x\sin xdx$

5. $\int \frac{(x^2-2x+3)}{(x-1)(x^3-4x^2+3x)}dx$

6. $\int \frac{x^5+2x^3+4x+4}{x^4+2x^3+2x^2}dx$

7. $\int \frac{(3x^4+4)dx}{x^2(x^2+1)^3}$

8. $\int \frac{dx}{5-4\sin x+3\cos x}$

9. $\int \cos^6 xdx$

10. $\int \frac{\ln(x+1)}{\sqrt{x+1}}dx$

11. $\int \frac{1-\sqrt[6]{x}}{x+\sqrt[3]{x^4}}dx$

12. $\int \frac{\sqrt[3]{1+x^3}}{x^2}dx$

13. $\int \frac{dx}{x^2(x+\sqrt{1+x^2})}$

Variant № 23

1. $\int \frac{arctgx + x}{1+x^2}dx$

2. $\int \frac{x}{x^4+1}dx$

3. $\int x^2 \cos^2 xdx$

4. $\int \frac{\ln^3 xdx}{x^2}$

5. $\int \frac{x^3-6x^2+9x+7}{(x-2)^3(x-5)}dx$

6. $\int \frac{(3x^2+x+3)dx}{(x-1)^3(x^2+1)}$

7. $\int \frac{(x+2)dx}{(x^2+2x+2)^3}$

8. $\int \frac{\sin^2 xdx}{1-tgx}$

9. $\int \frac{\cos xdx}{(1-\cos x)^2}$

10. $\int \sqrt[3]{\frac{1-x}{1+x}} \bullet \frac{dx}{x}$

11. $\int \frac{\sqrt{2x+1}}{x^2}dx$

12. $\int \frac{dx}{x\sqrt[3]{1+x^5}}$

13. $\int \sqrt{3x^2-3x+1}dx$

Variant № 24

1. $\int \frac{8x - arctg\,2x}{1+4x^2}dx$

2. $\int \frac{x-\frac{1}{x}}{\sqrt{x^2+1}}dx$

3. $\int x^3 \sin xdx$

4. $\int (\arcsin x)^2 dx$

5. $\int \frac{x^2dx}{(x+2)^2(x+4)^2}$

6. $\int \frac{dx}{(x+1)^2(x^2+1)}$

7. $\int \frac{dx}{(x^2+2x+10)^3}$

8. $\int \frac{x^2+\sqrt{1+x}}{\sqrt[3]{1+x}}dx$

9. $\int \sqrt{\frac{2-x}{x-6}}dx$

10. $\int \frac{dx}{5+4\sin x}$

11. $\int \frac{dx}{\sin^4 x\cos^4 x}$

12. $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}dx$

13. $\int \frac{dx}{(2x-3)\sqrt{4x-x^2}}$

Variant № 25

1. $\int \frac{x^2 + \ln x^2}{x} dx$

2. $\int \frac{4arctgx - x}{1 + x^2} dx$

3. $\int x^3 e^x dx$

4. $\int \ln^2 x dx$

5. $\int \frac{x^3 - 6x^2 + 11x - 5}{(x-2)^4} dx$

6. $\int \frac{dx}{(x^2 + 1)(x^2 + x)}$

7. $\int \frac{dx}{x^4 (x^3 + 1)^2}$

8. $\int \frac{dx}{5 - 3\cos x}$

9. $\int \frac{dx}{\cos^3 x \sin^3 x}$

10. $\int \sqrt{\frac{1-x}{1+x}} \bullet \frac{dx}{x}$

11. $\int \frac{(2 + \sqrt[3]{x})dx}{(\sqrt[6]{x} + 2\sqrt[3]{x} + \sqrt{x})\sqrt{x}}$

12. $\int \frac{\sqrt[3]{1 + \sqrt[4]{x}}}{\sqrt{x}} dx$

13. $\int \sqrt{x^2 - 2x - 1} dx$

Variant № 26

1. $\int \frac{1 - \cos x}{(x - \sin x)^2} dx$

2. $\int \frac{x + \frac{1}{x}}{\sqrt{x^2 + 1}} dx$

3. $\int \ln(x^2 + 1) dx$

4. $\int x^2 e^{-x} dx$

5. $\int \frac{x^3 + 1}{x^3 - x^2} dx$

6. $\int \frac{x^2 dx}{1 - x^4}$

7. $\int \frac{(x^2 - 1)^2 dx}{(1 + x)(1 + x^2)^3}$

8. $\int \frac{dx}{1 + tgx}$

9. $\int \frac{\sin^4 x}{\cos^2 x} dx$

10. $\int \sqrt{\frac{1 - \sqrt[3]{x}}{1 + \sqrt[3]{x}}} \bullet \frac{dx}{x}$

11. $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$

12. $\int \frac{\sqrt{1 - x^4}}{x^5} dx$

13. $\int \frac{dx}{(x-1)\sqrt{x^2 + x + 1}}$

Variant № 27

1. $\int \frac{x}{\sqrt[3]{x-1}}dx$
2. $\int \frac{x\cos x+\sin x}{(x\sin x)^2}dx$
3. $\int \frac{xarctgx}{\sqrt{1+x^2}}dx$
4. $\int \frac{\arcsin\sqrt{x}}{\sqrt{1-x}}dx$
5. $\int \frac{x^2}{x^3+5x^2+8x+4}dx$
6. $\int \frac{x^4+1}{x^3-x^2+x-1}dx$
7. $\int \frac{x^6+x^4-4x^2-2}{x^3(x^2+1)^2}dx$
8. $\int \frac{\cos^2 x}{\sin x\cos 3x}dx$
9. $\int \frac{dx}{\cos x\sin^3 x}$
10. $\int \frac{x\sqrt{1+x}}{\sqrt{1-x}}dx$
11. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$
12. $\int \frac{dx}{\sqrt[4]{1+x^4}}$
13. $\int \frac{\sqrt{2x+x^2}}{x^2}dx$

Variant № 28

1. $\int \frac{1-\sqrt{x}}{\sqrt{x}(x+1)}dx$
2. $\int \frac{x^3+1}{x^4+1}dx$
3. $\int x\cos^2 xdx$
4. $\int \frac{\ln x}{x^3}dx$
5. $\int \left(\frac{x+2}{x-1}\right)^2 dx$
6. $\int \frac{2x^2-3x-3}{(x-1)(x^2-2x+5)}dx$
7. $\int \frac{x^2+x+1}{x^5-2x^4+x^3}dx$
8. $\int \frac{dx}{tgx\cos 2x}$
9. $\int \frac{\sin^3 x}{\cos^4 x}dx$
10. $\int \frac{x}{\sqrt{x+1}+\sqrt[3]{x+1}}dx$
11. $\int \frac{dx}{\sqrt{x}(1+\sqrt[4]{x})}$
12. $\int \frac{dx}{\sqrt[3]{1+x^3}}$
13. $\int \frac{dx}{x\sqrt{2+x-x^2}}$

Variant № 29

1. $\int \frac{tg(x+1)}{\cos^2(x+1)}dx$
2. $\int \frac{x^3}{x^2+4}dx$
3. $\int \frac{\arcsin x}{\sqrt{x+1}}dx$
4. $\int xtg^2 xdx$
5. $\int \frac{x^2-3x+2}{x(x^2+2x+1)}dx$
6. $\int \frac{x}{x^3-1}dx$
7. $\int \frac{4x^2-8x}{(x-1)^2(x^2+1)^2}dx$
8. $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}+2\sqrt[4]{x}}$
9. $\int \frac{x\sqrt[3]{2+x}}{x+\sqrt[3]{2+x}}dx$
10. $\int \frac{dx}{\sin x+\cos x}$
11. $\int \sin^3 x\cos^2 xdx$
12. $\int x^5\sqrt[3]{(1+x^3)^2}dx$
13. $\int \frac{dx}{x\sqrt{x^2+4x-4}}$

Variant № 30

1. $\int \frac{dx}{x\sqrt{x^2-1}}$
2. $\int \frac{1+\ln x}{x}dx$
3. $\int arctg\sqrt{4x-1}dx$
4. $\int \frac{x}{\cos^2 x}dx$
5. $\int \frac{4x^4+2x^2-x-3}{x^3-x}dx$
6. $\int \frac{x^3+4x^2+4x+2}{(x+1)^2(x^2+x+1)}dx$
7. $\int \frac{\cos x}{2+\cos x}dx$
8. $\int \sin^6 x\cos^2 xdx$
9. $\int \frac{x^7+2}{x^2+x+1}dx$
10. $\int \frac{dx}{x(\sqrt{x}+\sqrt[5]{x^2})}$
11. $\int \frac{1-\sqrt{x+1}}{1+\sqrt[3]{x^2+1}}dx$
12. $\int \frac{dx}{x\sqrt[3]{x^2+1}}$
13. $\int \frac{dx}{x\sqrt{x^2+2x-1}}$