O’ZBEKISTON RESPUBLIKASI
OLIY VA O’RTA MAXSUS TAHLIM VAZIRLIGI

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O L I Y M A T E M A T I K A
kafedrasi

LABORATORIYa MASHG’ULOTLAR UCHUN

Optimal sonli usullar

fanidan

USLUBIY KO’RSATMA

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Ushbu uslubiy ko'rsatma texnika oliy o'quv yurtlarining kimyo-texnologiya yo'nalishi talabalari uchun 2 kurs 3-semestrda o'qitiladigan “Optimal sonli usullar” fanidan 36 soatli laboratoriya ishlarini bajarishda foydalanish uchun mo'ljallangan.

Ko'rsatmada sonli usullarning amaliyotda ko'p qo'llaniladigan bo'limlari- funktsiyalarni taqribiy xisoblash, algebraik va transtsendent tenglamalarni taqribiy yechish usullari, chiziqli va chiziqsiz tenglamalar sistemalarini yechish usullari, differentsial tenglamalar hamda funltsiyani iterpolyatsiyalash va kichik kvadratlar usul asosida chiziqli va chiziqsiz korrelyatsion – regression tahlil elementlari nazariy bayon qilinib, ularning amaliy masalalarni yechish usullari ko'rsatilgan. Mustaqil ishlar uchun har bir usulga mos laboratoriyalariya topshiriqlari berilgan.

Ushbu uslubiy ko'rsatma “Oliy matematika” kafedrasining uslubiy semenarida muxokama qilingan (№ , . . 2007)

Iqtisod fakulg'tetining uslubiy komissiyasi tomonidan maqullangan. (№ , . . 2007)

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So'z boshi

O'zbekiston mustaqillikka erishgandan so'ng, o'z taraqqiyotining muhim shartlaridan biri bo'lgan xalqning boy maonaviy salohiyati va umuminsoniy qadriyatlariga hamda hozirgi zamon madaniyati, iqtisodiyoti, ilmi, texnikasi va texnologiyasining so'nggi yutuqlariga asoslangan mukammal taolim tizimi barpo etilmoqda.

"Taolim to'g'risida" gi qonun va "Kadrlar tayyorlash milliy dasturi"ning qabul qilinishi natijasida ilmiy-texnika taraqqiyoti yutuqlarini xalq xo'jaligiga tadbiq qilish ijtimoiy-iqtisodiy rivojlanish bilan uzviy bog'liq ekanligining ahamiyati tobora ortib bormoqda.

Oliy o'quv yurtlarining texnika yo'nalishi bo'yicha bakalavrlar tayyorlashning yangi o'quv rejasi va dasturlarida axborot texnologiyalari bilan ishlash, axborotlarga zamonaviy texnik vositalar yordamida ishlov berish va uni tahlil qilish, sonli usullarning amaliy masalalarni yechishga tadbiq qilinishiga katta ehtibor qaratilgan.

Hozirga kunda hisoblash matematikasidan o'zbek tilida yozilgan adabiyotlar mavjud. Lekin ixtisoslikni ehtiborga olgan holda, talabalar uchun tushunarli tilda yozilgan adabiyotlarning yetishmasligi sezilmoqda.

Oliy o'quv yurtlarida tayyorlanayotgan mutaxassislar optimal matematik usullarni o'rganishida eng muhim narsa shuki, ular bu usullarni o'zlashtiribgina qolmay, balki ularni xalq xo'jaligining turli amaliy masalalarini yechishga qo'llay olishlari, olingan yechimni tahlil qila bilishlari hamda yechimga asoslanib qaror qabul qilish muammolarini hal qilishga qadar malakali mutaxassis bo'lib yetishishlari kerak.

KIRISH

Ushbu uslubiy ko’rsatma sonli yechish usullariga bag’ishlangan bo’lib, uni bayon qilishda qathiy matematik asoslashni maqsad qilib qo’yilmagan holda sonli usul algoritmi, uni misol va masalalar yechishga tadbiqlari hamda xatoligini baholashga mo’ljallangan. Uslubiy ko’rsatma 18 ta laboratoriya ishidan iborat bo’lib, 1,2.3,4,5-laboratoriya ishlarida chiziqli tenglamalar sistemasini yechishning aniq va taqribiy usullari keltirilgan. 6-laboratoriya ishida darajali qator yordamida funktsiyalarni taqribiy hisoblash usullari keltirilgan. 7,8.9.10,11- laboratoriya ishlarida algebraik va trantsendent tenglamalarni taqribiy yechish usullari, 12,13- laboratoriya ishlari Lagranj va Ng’yuton usulida interpolyatsiya ko’phadini topish, 14- laboratoriya ishida aniq integrallarni taqribiy hisoblash usullari, 15-laboratoriya ishida oddiy differentsial tnglamalar yechimini taqribiy hisoblashda Eyler va Runge-Kutta usullari keltirilgan. 16-laboratoriya ishida differentsial tnglamalar sistemasini Eyler usulida yechish va ikkinchi tartibli differentsial tnglamani tenglamalar sistemasiga keltirish bilan taqribiy hisoblash usuli ko’rsatilgan. 17- laboratoriya ishi kichik kvadratlar usuli yordamida tajriba natijalarinig chiziqli va parabolik bog’laninshini aniqlash usullariga, 18- laboratoriya ishi kichik kvadratlar usuli yordamida regression bog’lanishining to’g’ri chiziqi va ikkinch darajali tenlamasini aniqlashga bag’ishlangan. **M**ustaqil ishlash uchun topshiriqlar bo’limida har bir laboratoriya ishi uchun variantlar berilgan.

1-laboratoriya ishi

UCHINCHI TARTIBLI DETERMINANTNI HISOBLASH

Maqsad: Uchinchi tartibli determinantni hisoblash usullarini o’rganish va misollarda bajarish o’rganish.

Reja:

1. **Determinantning xossalari.**
2. **Determinantni hisoblash usullari.**

1.1. Determinantning xossalari.

Tavsiya qilingan adabiyotlarga qaralgan determinantning xossalaridan quyidagilarni alohida taokidlab o’tamiz:

1. Determinantning barcha satrlarini(ustunlarini) unga mos nomerli ustunlar(satrlar) qilib yozishdan olingan determinant qiymati berilgan determinant qiymatiga teng bo’ladi

2. Determinantning biror satri (ustuni)ning barcha elementlarini bitta songa ko’paytirib, boshqa satr (ustun) ning mos elementlariga qo’shib yozilsa, determinant qiymati o’zgarmaydi.

3. Determinant ikkn satri(ustuni) barcha elementlarini mos ravishda o’rinlari almashtirilsa uning ishorasi qarama-qarshisiga o’zgaradi.

4. Determinantning nkkn satri (ustuni) bir xil bo’lsa uning qiymati nolga teng.

5. Determinantning biror satri (ustuni)dagi elementlarning umumiy ko’paytuvchisini determinant belgisidan tashqariga chiqarish mumkin.

6. Determinantning biror satri (ustuni)dagi elementlari nolga teng bo’lsa, uning qiymati nolga teng.

Ikkinchi xossadan foydalanib, determinantning qiymatini o’zgartirmasdan determinantda birorta ustunning(satrning) bitta elementidan boshqa barcha elementlarini, nolga aylantirish mumkin. Determinantni shu ustunining (satrining) elementlari bo’yicha yoyib, berilgan uchinchi tartibli determinantni bitta ikkinchi tartibli determinantga keltiramiz. Demak, bu xossaning qo’llanilishi determinantni hisoblashni tezlashtirishi va osonlashtirishi mumkin ekan.

1.2.Uchinchi tartibli determinantni hisoblash usullari.

Uchinchi tartibli determinantni hisoblashda uchburchaklar va Sarryus qoidalaridan foydalanish qulay:

Uchinchi tartibli determinantni hisoblashda uchburchaklar formulasi quyidagi ko’rinishga ega: har bir determinantdagi $*$ lar uchburchaklar qoidasi bilan ko’payuvchi elementlarni bildiradi

$$\begin{vmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{vmatrix} = \begin{vmatrix} * & \bullet & \bullet \\ \bullet & * & \bullet \\ \bullet & \bullet & * \end{vmatrix} + \begin{vmatrix} \bullet & * & \bullet \\ \bullet & \bullet & * \\ * & \bullet & \bullet \end{vmatrix} + \begin{vmatrix} \bullet & \bullet & * \\ * & \bullet & \bullet \\ \bullet & * & \bullet \end{vmatrix} - \begin{vmatrix} \bullet & \bullet & * \\ \bullet & * & \bullet \\ * & \bullet & \bullet \end{vmatrix} - \begin{vmatrix} * & \bullet & \bullet \\ \bullet & \bullet & * \\ \bullet & * & \bullet \end{vmatrix} - \begin{vmatrix} \bullet & * & \bullet \\ * & \bullet & \bullet \\ \bullet & \bullet & * \end{vmatrix}$$

$$\begin{vmatrix} a_{11} a_{12} a_{13} \\ a_{21} a_{22} a_{23} \\ a_{31} a_{32} a_{33} \end{vmatrix} = a_{12}a_{22}a_{33} + a_{12} a_{23} a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21} a_{33} \quad . \quad (3.2)$$

1) Sarryus qoidasida determinantning birinchi va ikkinchi ustunlarini mos ravishda to'rtinchi va beshinchi ustunlar qilib yozib, bosh diagonal va unga parallel qo'shimcha diagonallarni har birida joylashgan elementlarni ko'paytirib, olingan ko'paytmalarni qo'shamiz, so'ngra yuqorida qilingan ishlarni ikkinchi diagonal uchun bajarib, olingan ko'paytmalarni ayiramiz.

$$\begin{vmatrix} a_{11}a_{12}a_{13} \; a_{11}a_{12} \\ a_{21}a_{22}a_{23} \; a_{21}a_{22} \\ a_{31}a_{32}a_{33} \; a_{31}a_{32} \end{vmatrix} = a_{12} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{13} a_{22} a_{31} - a_{12} a_{21} a_{33} - a_{11} a_{23} a_{33}$$

2) Determinantning birinchi va ikkinchi satrlarini mos ravishda to'rtinchi va beshinchi satrlar qilib yozib, yuqorida keltirilgan ishlar bajariladi.

$$\begin{vmatrix} \alpha_{11}\alpha_{12}\ \alpha_{13} \\ \alpha_{21}\alpha_{22}\alpha_{23} \\ \alpha_{21}\alpha_{22}\alpha_{23} \\ \alpha_{21}\alpha_{22}\alpha_{23} \\ \alpha_{21}\alpha_{22}\alpha_{23} \end{vmatrix} = \alpha_{12}\alpha_{22}\alpha_{33} + \alpha_{12}\alpha_{23}\alpha_{31} + \alpha_{13}\alpha_{21}\alpha_{32} - \alpha_{13}\alpha_{22}\alpha_{31} - \alpha_{12}\alpha_{21}\alpha_{33} - \alpha_{11}\alpha_{23}\alpha_{33}$$

Oxirgi ikki bandda keltirilgan qoidalar Sarryus qoidasi deb yuritiladi

1.1-masala.. Uchburchaklar qoidasidan foydalanib, uchinchi tartibli ushbu determinantni hisoblaymiz:

$$\begin{vmatrix} 1 & 3 & -2 \\ 2 & 0 & 3 \\ -2 & 1 & 2 \end{vmatrix} = 1\cdot 0\cdot 2 + 3\cdot 3\cdot(-2) + 2\cdot 2\cdot(-2) - (-2)\cdot 0\cdot(-2) - 2\cdot 3\cdot 2 - 1\cdot 3\cdot 1 = -37$$

Berilgan uchinchi tartibli determinant biror elementining minori deb, undan qaralayotgan element turgan qator va ustun o'chirilgandan so'ng, qolgan elementlardan hosil bo'ladigan ikkinchi tartibli determinantga aytiladi. a_{ij} elementning minori M_{ij} kabi belgilanadi, bu yerda a_{ij} orqali determinantning i-satri va j-ustuniga joylashgan elementi belgilangan. Masalan, a_{22} elementning minori

$$M_{22} = \begin{vmatrix} a_{11}a_{13} \\ a_{31}a_{33} \end{vmatrix}$$

determinant bo'ladi. Ravshanki, uchinchi tartibli har bir determinant o'z elementlarining to'qqizta minoriga ega.

$$\begin{vmatrix} a_{11}\, a_{12}\, a_{13} \\ a_{21}\, a_{22}\, a_{23} \\ a_{31}\, a_{32}\, a_{33} \end{vmatrix} = a_{12}a_{22}a_{33} + a_{12}\, a_{23}\, a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}\, a_{33} - a_{11}a_{23}a_{32} \quad (1.1)$$

formulani qarab chiqamiz va uning o'ng tomonida, masalan, birinchi satrning elementlari bo'lgan sonlarni qavslar tashqarisiga chiqaramiz. Natijada:

$$\begin{vmatrix} a_{11}a_{12}a_{13} \\ a_{21}a_{22}a_{23} \\ a_{31}a_{32}a_{33} \end{vmatrix} = a_{11}(a_{22}a_{33} - a_{23}a_{32}) + a_{12}(a_{23}a_{31} - a_{21}a_{33}) + a_{13}(a_{21}a_{32} - a_{22}a_{31}). \quad (1.2)$$

a_{ij} (i,j=1,2,3) elementlar yonida qavslar ichida turgan ifodalar bu elementlarning algebraik to'ldiruvchisi deyiladi va A_{ij} orqali ifodalanadi.

SHuni ham aytib o'tish kerakki, a_{ij} elementning algebraik to'ldiruvchisi shu elementning minoridan faqat ishorasi bilangina farq qilishi mumkin. Bunda, agar a_{ij} element turgan satr hamda ustunning nomerlari yig'indisi juft bo'lsa, u holda $A_{ij}=M_{ij}$, agar yig'indisi toq bo'lsa, u holda $A_{ij}= -M_{ij}$ bo'ladi, yahni

$$A_{ij} = (-1)^{i+j} M_{ij} \qquad (1.3)$$

(1.3) munosabatdan foydalanib, (1.2) yoyilmani bunday ko'rinishda yozish mumkin:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$= (-1)^{1+1} a_{11}M_{11} + (-1)^{1+2} a_{12}M_{12} + (-1)^{1+3} a_{13}M_{13} = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}.$$

Agar n tartibli determinant berilgan bo'lsa, i-satr bo'yicha yoyib, quyidagicha hisoblaymiz:

$$\begin{vmatrix} a_{11}\ a_{12}\ \ldots a_{1n} \\ a_{21} a_{22} \ldots a_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1} a_{n2} \ldots a_{nn} \end{vmatrix} = a_{i1}A_{i1} + a_{i2}A_{i2} + \ldots. + a_{in}A_{in} \qquad (1.4)$$

Xuddi shunga o'xshash, j – ustun bo'yicha yoyilma

$$\begin{vmatrix} a_{11}\ a_{12}\ \ldots a_{1n} \\ a_{21} a_{22} \ldots a_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1} a_{n2} \ldots a_{nn} \end{vmatrix} = a_{1j}A_{1j} + a_{2j}A_{2j} + \ldots. + a_{nj}A_{nj} \qquad (1.5)$$

ko'rinishda bo'ladi.

1.2-masala.. Ushbu

$$\begin{vmatrix} 1 & -2 & 3 \\ 3 & 5 & -1 \\ 4 & 1 & 2 \end{vmatrix}$$

determinantni ikkinchi satrining elementlari bo'yicha yoyib hisoblang.
Echish.

$$\begin{vmatrix} 1 & -2 & 3 \\ 3 & 5 & -1 \\ 4 & 1 & 2 \end{vmatrix} = (-1)^{2+1} \cdot 3 \begin{vmatrix} -2 & 3 \\ 1 & 2 \end{vmatrix} + (-1)^{2+2} \cdot 5 \cdot \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} + (-1)^{2+3} \cdot (-1) \cdot \begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} =$$

$$= (-3)(-7) + 5(-10) + 1 \cdot 9 = -20$$

2-laboratoriya ishi

MATRITSAGA TESKARI MATRITSA TOPISH

Maqsad: Berilgan matritsaga teskari matritsa topishni o'rganish.

Reja:

1. **Matritsa tahrifi. Xossalari**
2. **Teskari matritsa topish formulasi.**
3. **CHiziqli tenglamalar sistemasini yechishda teskari matritsa topish formulasidan foydalanish.**

2.1-masala. Berilgan matritsaga teskari matritsani toping.

$$A=\begin{pmatrix} 3 & -1 & -1 \\ 2 & 1 & 2 \\ 1 & 3 & -2 \end{pmatrix}$$

Berilgan matritsaga teskari matritsani topish uchun:

1) Berilgan matritsaning **determinantni hisoblaymiz**

$$\Delta=\det A=\begin{vmatrix} 3 & -1 & -1 \\ 2 & 1 & 2 \\ 1 & 3 & -2 \end{vmatrix}=-35$$

2)$\Delta\neq 0$ bo'lgani uchun A matritsa hos bo'lmagan matritsadir va shuning uchun teskari matritsaga ega. A matritsaga teskari matritsani topish uchun Δ ning algebraik to'ldiruvchilarini hisoblaymiz:

$$A_{11}=(-1)^{1+1}\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix}=-8,\quad A_{21}=(-1)^{2+1}\begin{vmatrix} -1 & -1 \\ -3 & -2 \end{vmatrix}=-5,\quad A_{31}=(-1)^{3+1}\begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix}=-1$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix} 2 & 2 \\ 1 & -2 \end{vmatrix}=6,\quad A_{22}=(-1)^{2+2}\begin{vmatrix} 3 & -1 \\ 1 & -2 \end{vmatrix}=-5,\quad A_{32}=(-1)^{3+2}\begin{vmatrix} 3 & -1 \\ -2 & 2 \end{vmatrix}=-8$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}=5,\quad A_{23}=(-1)^{2+3}\begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix}=-10\quad A_{33}=(-1)^{3+3}\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix}=5$$

3)Endi teskari matritsani yozamiz:

$$A^{-1}=\begin{pmatrix} A_{11}/\Delta & A_{21}/\Delta & A_{31}/\Delta \\ A_{12}/\Delta & A_{22}/\Delta & A_{32}/\Delta \\ A_{13}/\Delta & A_{23}/\Delta & A_{33}/\Delta \end{pmatrix}$$

foriulaga asosan teskari matritsani topamiz:

$$A^{-1}=\begin{pmatrix} 8/35 & 1/7 & 1/35 \\ -6/35 & 1/7 & 8/35 \\ -1/7 & 2/7 & -1/7 \end{pmatrix}$$

4) topilgan teskari matritsani to'g'liligini tekshirish uchun $A^{-1}A=E$ ayniyatni bajarilishini tekshiramiz:

$$A^{-1}A=\begin{pmatrix} 8/35 & 1/7 & 1/35 \\ -6/35 & 1/7 & 8/35 \\ -1/7 & 2/7 & -1/7 \end{pmatrix}\begin{pmatrix} 3 & -1 & -1 \\ 2 & 1 & 2 \\ 1 & 3 & -2 \end{pmatrix}=$$

$$=\begin{pmatrix} (8/35)\cdot 3+(1/7)\cdot 2+(1/35)\cdot 1 & (8/35)\cdot(-1)+(1/7)\cdot 1+(1/35)\cdot 3 & (8/35)\cdot(-1)+(1/7)\cdot 2+(1/35)\cdot(-2) \\ (-6/35)\cdot 3+(1/7)\cdot 2+(8/35)\cdot 1 & (-6/35)\cdot(-1)+(1/7)\cdot 1+(8/35)\cdot 3 & (-6/35)\cdot(-1)+(1/7)\cdot 2+(8/35)\cdot(-2) \\ (-1/7)\cdot 3+(2/7)\cdot 2 - (1/7)\cdot 1 & (-1/7)\cdot(-1)+(2/7)\cdot 1 - (1/7)\cdot 3 & (-1/7)\cdot(-1)+(2/7)\cdot 2 - (1/7)\cdot(-2) \end{pmatrix}=$$

$$=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2.3.CHiziqli tenglamalar sistemasini yechishda teskari matritsa topish formulasidan foydalanish.

Faraz qilaylik, determinanti 0 dan farqli bo'lgan n nomahlumli n ta chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = f_1, \\ \alpha_{21}x_1 + \alpha_{22}x_2 + \ldots + \alpha_{2n}x_v = f_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \alpha_{n1}x_1 + \alpha_{n2}x_2 + \ldots + \alpha_{nn}x_n = f_n \end{cases} \qquad (2.1)$$

Matritsalarni ko'paytirish qoidasidan foydalanib, (2.1) sistemani

$$AX=F \qquad (2.2)$$

matritsa ko'rinishda yozamiz, bu yerda

$$A=\begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{10} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

- nomahlumlar oldidagi koeffitsientlardan tuzilgan matritsa(sistema matritsasi):

$$X=\begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}, \qquad F=\begin{pmatrix} f_1 \\ f_2 \\ \ldots \\ f_n \end{pmatrix}$$

- nomahlumlar va ozod hadlardan tuzilgan matritsa ustun-vektorlar.

(2.2) chiziqli tenglamalar sistemasini matritsa shaklida yozilishi (ko'rinishi) deb yuritiladi.

Δ=detA≠ 0 bo'lgani uchun A matritsaga teskari A^{-1} matritsa maijud.
Mahlumki teskari matritsa

$$A^{-1} = \begin{pmatrix} A_{11}/\Delta & A_{21}/\Delta & A_{31}/\Delta \\ A_{12}/\Delta & A_{22}/\Delta & A_{32}/\Delta \\ A_{13}/\Delta & A_{23}/\Delta & A_{33}/\Delta \end{pmatrix}$$

formula bilan hisoblanadi.
(2.2) tenglamaning ikkala qismigiga chapdan A^{-1} matritsani ko'paytirib

$$A^{-1}AX= A^{-1}F$$

yoki

$$X= A^{-1}F \qquad (2.3)$$

echimni olamiz.

Bu yerda, A_{ij} $-\Delta$ determinantning a_{ij} elementiga mos kelgan algebraik to'ldiruvchi. Demak, (2.2) tenglama yechimi (2.3) dan iborat bo'ladi va unda matritsalar usulida amallarni bajarib (2.1) sistemaning yechimini olamiz.

2.2-masala. Quyidagi chiziqli tenglamalar sistemasini teskari matitsa yordamida yeching

$$\begin{cases} x_1 - x_2 + x_3 = 6, \\ 2x_1 + x_2 + x_3 = 3, \\ x_1 + x_2 + 2x_3 = 5. \end{cases}$$

Echish. Berilgan tenglamalar sistemasinirg matitsalari

$$A=\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}, \quad X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad F=\begin{pmatrix} 6 \\ 3 \\ .5 \end{pmatrix}$$

ni yozamiz. A matritsaning determinanti

$$\det A=\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix}=5$$

$$\det A=5 \neq 0.$$

Bu holda A^{-1} matritsaning a_{ij} elementga mos kelgan A_{ij} algebraik to'ldiruvchilarni topamiz.

$$A_{11}=\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}=1, \quad A_{21}=-\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix}=3, \quad A_{31}=\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}=2$$

$$A_{12}=-\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}=-3, \quad A_{22}=\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}=1, \quad A_{32}=-\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}=1$$

$$A_{13}=\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}=1, \quad A_{23}=-\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}=-2, \quad A_{33}=\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}=3$$

A matritsaning determinanti va A_{ij} algebraik to'ldiruvchilarining qiymatlari asosida quyidagi A^{-1} matritsani yozamiz:

$$A^{-1}=\begin{pmatrix} 1/5 & 3/5 & -2/5 \\ -3/5 & 1/5 & 1/5 \\ 1/5 & -2/5 & 3/5 \end{pmatrix}$$

A^{-1} matritsani

$$X=A^{-1}F$$

tenglamaga qo'yib, berilgan sistemaning quyidagi ko'rinishdagi yechimini hosil qilamiz:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 1/5 & 3/5 & -2/5 \\ -3/5 & 1/5 & 1/5 \\ 1/5 & -2/5 & 3/5 \end{pmatrix}\begin{pmatrix} 6 \\ 3 \\ .5 \end{pmatrix}$$

x_1 ni hisoblash uchun A^{-1} matritsaning birinchi satri elementlarini F matritsa – ustun elementlariga ko'paytirish va hosil qilingan ko'paytmalarni qo'shish kerak:

$$x_1=1/5 \bullet 6+3/5 \bullet 3+(-2/5) \bullet 5=1.$$

Xuddi shunga o'xshash x_2=-2, x_3=3 larni topamiz.

3-laboratoriya ishi

CHIZIQLI TENGLAMALAR SISTEMASINI GAUSC USULIDA YeCHISH

CHiziqli algebraik tenglamalar sistemasini yechishda keng tarqalgan Gausc usuli aniq yechish usullari guruhiga mansub bo'lib, uning mohiyati shundan iboratki, nomahlumlarni ketma – ket yo'qotish yo'li bilan berilgan sistema o'ziga ekvivalent bo'lgan pog'onali (uch burchakli) sistemaga keltiriladi. Bu kompg'yuter xotirasidan samarali ravishda foydalanish imkonini beradi .

Ushbu

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + ... + a_{1n}x_n = b_1 \\ \alpha_{22}x_2 + \alpha_{23}x_3 + ... + a_{2n}x_n = b_2 \\ .... \\ \alpha_{\kappa\kappa}x_\kappa + ... + a_{\text{лп}}x_n = b_\kappa \end{cases} \qquad (3.1)$$

ko'rinishdagi chiziqli tenglamalar sistemasi *pog'onali sistema* deyiladi,bu yerda k ≤n, $a_{ij} \neq 0$, i=1, 2, …, k.

Agar k=n bo'lsa, u holda (3.1) sistema *uch burchakli* deyiladi.

Nomahlumlarni ketma – ket yo'qotib borish, asosan, sistemada elementar almashtirishlar qilish yordamida amalga oshiriladi. Bu elementar almashtirishlarga quyidagilar kiradi:

1) sistemaga tegishli istalgan ikkita tenglamaning o'rnini almashtirish;
2) tenglamalardan birining har ikkala qismini noldan farqli istalgan songa ko'paytirish;
3) biror tenglamaning har ikkala qismiga, biror songa ko'paytirilgan ikkinchi tenglamaning mos qismlarini qo'shish.

Elementar almashtirishlar berilgan tenglamalar sistemasini unga ekvivalent sistemaga o'tkazishini isbotlash mumkin.

Oddiylik uchun quyidagi chiziqli tenglamalar sistemasini qaraymiz:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = a_{15}, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = a_{25}, \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = a_{35}, \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = a_{45}. \end{cases}$$

Aytaylik, berilgan sistemada $a_{11} \neq 0$ (etakchi element) bo'lsin, aks holda tenglamalarning o'rinlarini almashtirib, x_1 oldidagi koeffitsienti noldan farqli bo'lgan tenglamani birinchi o'ringa ko'chiramiz.

Sistemaning birinchi tenglamasining barcha koeffitsientlarini a_{11} ga bo'lib,

$$x_1+b_{12}x_2+b_{13}x_3+b_{14}x_4=b_{15} \qquad (3.2)$$

tenglamani hosil qilamiz, bu yerda.

$$b_{ij}=\frac{a_{ij}}{a_{11}},(j=2,3,4,5)]$$

Bu topilgan (3.2) tenglamadan foydalanib, yuqoridagi sistemaning qolgan tenglamalaridagi x_1 qatnashgan hadni yo'qotish mumkin. Buning uchun (3.2) tenglamani ketma-ket a_{21}, a_{31} va a_{41} larga ko'paytirib, mos ravishda sistemaning ikkinchi, uchinchi va to'rtinchi tenglamalaridan ayiramiz.

Natijada quyidagi uchta tenglamalar sistemasini hosil qilamiz.

$$\begin{cases} a_{22}^{(1)}x_2+a_{23}^{(1)}x_3+a_{24}^{(1)}x_4=a_{25}^{(1)} \\ a_{32}^{(1)}x_2+a_{33}^{(1)}x_3+a_{34}^{(1)}x_4=a_{35}^{(1)} \\ a_{42}^{(1)}x_2+a_{43}^{(1)}x_3+a_{44}^{(1)}x_4=a_{45}^{(1)} \end{cases} \quad (3.3)$$

bu sistemadagi $a_{ij}^{(1)}$ koeffitsientlar

$$a_{ij}^{(1)}=a_{ij}-a_{i1}b_{1j} \text{(i=2,3,4; j=2,3,4,5)} \quad (3.4)$$

formula yordamida hisoblanadi. Endi (3.3) sistemaning birinchi tenglamasini $a_{22}^{(1)}$ ga bo'lib,

$$x_2+b_{23}^{(1)}x_3+b_{24}^{(1)}x_4=b_{25}^{(1)} \quad (3.5)$$

tenglamani hosil qilamiz, bu yerda

$$b_{2j}^{(1)}=\frac{a_{2j}^{(1)}}{a_{22}}, \quad (j=3,4,5)$$

(3.5) tenglama yordamida (2.3) sistemaning keyingi tenglamalaridan x_2 ni, yuqoridagidek qoida asosida, yo'qotamiz va quyidagi tenglamalar sistemasini topamiz:

$$\begin{cases} a_{33}^{(2)}x_3 + a_{34}^{(2)}x_4 = a_{35}^{(2)} \\ a_{43}^{(2)}x_3 + a_{44}^{(2)}x_4 = a_{35}^{(2)} \end{cases} \quad (3.6)$$

bu yerda

$$a_{ij}^{(2)} = a_{ij}^{(1)} - a_{i2}^{(1)}b_{2j}^{(1)} \quad (i=3,4; \quad j=3,4,5) \quad (3.7)$$

(3.6) sistemaning birinchi tenglamasini $a_{33}^{(2)}$ ga bo'lib,

$$x_3+b_{34}^{(2)}x_4=b_{35}^{(2)} \quad (3.8)$$

tenglamani hosil qilamiz, bu yerda

$$b_{3j}^{(2)}=\frac{a_{3j}^{(2)}}{a_{33}^{(2)}}, \quad (j=4,5)$$

Bu (3.8) tenglama yordamida (3.6) sistemaning ikkinchi tenglamasidan x_3 ni yo'qotamiz. Natijada

$$a_{44}^{(3)}x_4=a_{45}^{(3)}$$

tenglamani hosil qilamiz, bu yerda

$$a_{4j}^{(3)}=a_{4j}^{(2)}-a_{43}^{(2)}b_{3j}^{(2)} \quad (j=4,5) \quad (3.9)$$

SHunday qilib biz qaralayotgan sistemasini unga ekvivalent bo'lgan quyidagi uchburchakli chiziqli tenglamalar sistemasiga olib keldik.

$$\left.\begin{array}{ccccccccc} x_1 & + & b_{12}x_2 & + & b_{13}x_3 & + & b_{14}x_4 & = & b_{25} \\ & & x_2 & + & b_{23}^{(1)}x_3 & + & b_{24}^{(1)}x_4 & = & b_{25}^{(1)} \\ & & & & x_3 & + & b_{34}^{(2)}x_4 & = & b_{35}^{(2)} \\ & & & & & & a_{44}^{(3)}x_4 & = & b_{45}^{(3)} \end{array}\right\} \qquad (3.10)$$

Bu (3.10) sistemadan foydalanib nomhlumlarni, ketma-ket quyidagicha topamiz:

$$\begin{cases} x_4 = \dfrac{a_{45}^{(3)}}{a_{44}^{(3)}} \\ x_3 = b_{35}^{(2)} - b_{34}^{(2)}x_4 \\ x_2 = b_{25}^{(1)} - b_{24}^{(1)}x_4 - b_{23}^{(1)}x_3 \\ x_1 = b_{15} - b_{14}x_4 - b_{13}x_3 - b_{12}x_2 \end{cases} \qquad (3.11)$$

Demak, yuqorida keltirilgan Gauss usulida sistemaning yechimini topish 2 qismdan iborat bo'lar ekan.

Olg'a borish – (2.1) sistemani uchburchakli (3.10) sistemaga keltirish

Orqaga qaytish- (3.11) formulalar yordamida nomahlumlarni topish.

Gauss usuli bilan nomahlumli n ta chiziqli algebraik tenglamalar sistemasini yechish uchun bajariladigan arifmetik amallarning miqdori quyidagidan iborat:

$(n^3+3n^2-n)/3$ ta ko'paytirish va bo'lish,

$(2n^3+3n^2-5n)/6$ ta qo'shish.

Xususan:

n=2 da $(2^3+3*2^2-2)/3=6$ ko'paytirish va bo'lish

$(2*2^3+3*2^2-5*2)/6=3$ qo'shish,

n=3 da $(3^3+3*3^2-3)/3=17$ ko'paytirish va bo'lish

$(2*3^3+3*3^2-5*3)/6=11$ qo'shish.

3**.1-masala.** Berilgan quyidagi sistemani Gauss usilida yechamiz. Buning uchun nomahlumlarni ketma-ket yo'qotamiz. Yetakchi satr uchun birinchi tenglamani tanlasak bo'ladi, chunki $a_{11} = 2 \neq 0$.

$$\begin{cases} 2x_1 + 7x_2 + 13x_3 = 0 \\ 3x_1 + 14x_2 + 12x_3 = 18 \\ 5x_1 + 25x_2 + 16x_3 = 39 \end{cases} \qquad (3.12)$$

Gauss usili yordamida yechish uchun sistema koeffitsientlarini quyidagicha belgilaymiz:

$a_{11}=2,$	$a_{12}= 7,$	$a_{13}=13$	$b_1 = 0$	[1]	
$a_{21}=3,$	$a_{22}= 14,$	$a_{23}=12$	$b_2=18$	[2]	(3.13)
$a_{31}=5,$	$a_{32}= 25,$	$a_{33}=16$	$b_3=39$	[3]	

Hisoblash jarayoni quyidagicha bo'ladi.

Olg'a b o r i sh

1) (3.6) dagi tenglama koeffitsientlari [1] ni a_{11}= 2 ga bo'lamiz:

$$(1,\ a_{12}/a_{11}\ ,\ a_{13}/a_{11}\ ,\ b_1/a_{11}) = (1,\ 7/2\ ,\ 13/2\ ,\ 0/2) \qquad (3.14)$$

2) (3.12) ning 2- tenglamasidagi x_1 ni yo'qatish uchun (3.14) ni a_{21}=3 ga ko'paytirib, [2] satrdan mos ravishda ayiramiz, yahni [2] –(3.14) a_{21}:

$$a^{(1)}_{21} = a_{21} - a_{21} = 0$$
$$a^{(1)}_{22} = a_{22} - a_{21}a_{12}/a_{11} = 14 - 3(2/2) = 7/2$$
$$a^{(1)}_{23} = a_{23} - a_{21}a_{13}/a_{11} = 12 - 3(6/2) = -15/2$$
$$b^{(1)}_{1} = b_1 - a_{21}b_1/a_{11} = 18 - 3(0/2) = 18$$

Demak, 2- tenglama koeffitsentlari:

$$(\ 0,\ 7/2\ ,\ -15/2\ ,\ 18) \qquad (3.15)$$

bo'ladi.

3) (3.12) ning 3- tenglamasidagi x_1 ni yo'qatish uchun (3.14) ni a_{31}=5 ga ko'paytirib, [3] satrdan mos ravishda ayiramiz, yahni [3] – (3.14) a_{31} :

$$a^{(1)}_{31} = a_{31} - a_{31} = 0$$
$$a^{(1)}_{32} = a_{32} - a_{31}a_{12}/a_{11} = 25 - 5(\ 7/2) = 15/2$$
$$a^{(1)}_{33} = a_{32} - a_{31}a_{13}/a_{11} = 16 - 5(6/2) = -33/2$$
$$b^{(1)}_{3} = b_3 - a_{31}b_1/a_{11} = 39 - 5(\ 0/2\ ) = 39$$

Demak, 3- tenglama koefitsentlari:

$$(\ 0,\ 15/2\ ,\ -33/2\ ,\ 39) \qquad (3.16)$$

bo'ladi.

Natijada topilgan yangi koeffitsientlar asosida quyidagi sistemani hosil qilamiz:

$$\begin{cases} x_1 + (2/2)x_2 + (13/2)x_3 = 0 \\ \qquad (7/2)x_2 - (15/2)x_3 = 18 \\ \qquad (15/2)x_2 - (33/2)x_3 = 39 \end{cases} \qquad (3.17)$$

bu sistemaning 2 va 3-tenglamalaridan x_2 nomahlumni yo'qotish uchun 2- tenglamani $a^{(1)}_{22}$ = 7/2 ga bo'lamiz. Bu tenglama koeffitsentlari:

$$(\ 0,\ 1,\ -15/7\ ,\ 36/7\ ) \qquad (3.11)$$

bo'ladi. Bu (3.11) koefitsentlardan foydalanib (3.17) sistemaning 3- tenglamasidagi x_2 ni yo'qotaimz. Buning uchun (3.11) ni 15/2 ga ko'paytirib 3-tenglama koefitsentlardan mos ravishda ayirib quyidagi koefitsentlar topamiz:

$$(0,\ 0,\ -3/7\ ,\ 3/7\ ) \qquad (3.12)$$

Natijada berilgan sistemani quyidagicha yozamiz:

$$\begin{cases} x_1 + (2/2)x_2 + (13/2)\,x_3 = 0 \\ \qquad\qquad x_2 - (15/7)x_3 = 36/7 \\ \qquad\qquad\quad -\ (3/7)\,x_3 = 3/7 \end{cases}$$

Orqaga qaytish

Bu oxirgi sistemadagi 3- tenglamadan x_3 qiymatini topib bu asosida 2- tenglamadan x_2 ni topamiz. Topilgan x_2 va x_3 asosida 1- tenglamadan x_1 ni topamiz:

x_3= - 1

x_2= 36/7 +(15/7)(-1)= 21/7= 3

x_1= (-7/2)(3) - (6/2)(-1)= -8/2= - 4

Berilgan chiziqli tenglamalar sistemasining yechimi:

$$x_1= - 4, \quad x_2 = 3, \quad x_3 = - 1$$

4-laboratoriya ishi

GAUSS USULIDA DETERMINANTNI HISOBLASH VA MATRITSAGA TESKARI MATRITSA TOPISH

Maqsad: Gauss usulida determinantni hisoblash va matritsaga teskari matritsa topishni o'rganish.

Reja:

1. **Gauss usulida determinantni hisoblash**
2. **Gauss usulida matritsaga teskari matritsa topish**

4.1. Gauss usulida determinantni hisoblash

Determinantlarning tartibi(satr va ustunlar soni) katta bo'lganda yuqoridagi determinantlarni hisoblash qiyin bo'ladi. SHuning uchun bu determinantlarni kompg'terda hisoblash dasturini GAUSS usuli asosida tuzamiz. Bu amalni namuna sifatida quyidagi determinant uchun bajaramiz.

4.1-masala. Ushbu

$$d = \begin{vmatrix} 2 & 7 & 13 \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{vmatrix}$$

asosiy determinantdagi birinchi satrning yetakchi birinchi a_{11}=2≠0 elementini determinant belgisidan tashqariga chiqaramiz:

$$d = 2\begin{vmatrix} 1 & 7/2 & 13/2 \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{vmatrix}$$

hosil bo'lgan determinantda birinchi satr elementlarini ketma-ket 3 va 5 larga ko'paytirib, mosravishda 2- va 3- satrlardan ayiramiz:

$$d = 2\begin{vmatrix} 1 & 7/2 & 13/2 \\ 0 & 7/2 & -15/2 \\ 0 & 15/2 & -33/2 \end{vmatrix}$$

Bu determinantning ikkinchi satridagi yetakchi $a^{(1)}_{22}$=7/2 elementini determinant belgisidan tashqariga chiqaramiz:

$$d = 2 \cdot (7/2) \begin{vmatrix} 1 & 7/2 & 13/2 \\ 0 & 1 & -15/7 \\ 0 & 15/2 & -33/2 \end{vmatrix}$$

hosil bo'lgan determinantda ikkinchi satr elementlarini 15/2 ga ko'paytirib, mosravishda 3- satrdan ayiramiz:

$$d = 2 \cdot (7/2) \begin{vmatrix} 1 & 7/2 & 13/2 \\ 0 & 1 & -15/7 \\ 0 & 0 & -3/7 \end{vmatrix}$$

hosil bo'lgan determinantning oxirgi satridagi yetakchi $a^{(2)}_{33}$= - 3/7 elementini determinant belgisidan tashqariga chiqaramiz:

$$d = 2 \cdot (7/2) \cdot (-3/7) \begin{vmatrix} 1 & 7/2 & 13/2 \\ 0 & 1 & -15/7 \\ 0 & 0 & 1 \end{vmatrix}$$

hosil bo'lgan determinant diagonalp elementlari 1 sonidan va diagonalg' ostidagi elementlari 0 dan iborat bo'lgani uchun uning qiymati 1 ga teng Natijada asosiy determinant qiymati yetakchi elementlar ko'paytmasidan iborat bo'ladi:

$$d = a_{11} a^{(1)}_{22} a^{(2)}_{33} \cdot 1 = 2 \cdot (2/2) \cdot (-3/7) \cdot 1 = -3$$

Huddi shuningdek Gauss usuli bilan qolgan determinantlarni ham hisoblash mumkin.

2. Yuqoridagi Gauss usulini NxN tartibli determinant uchun xisoblash formulasini beramiz:

$$d = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Bu determinant qiymati yetakchi elementlar ko'paytmasidan iborat bo'ladi:

$$\mathrm{d} = \det \mathrm{A} = a_{11} a^{(1)}_{22} a^{(2)}_{33} \cdots a^{(n-1)}_{nn}$$

Bu yetakchi elementlarni quyidagi formulalar asosida topamiz:

i=1,

$$b_{1i} = a_{1\imath} / a_{11}, \qquad j = 2,3,\ldots,n$$
$$a^{(1)}_{ii} = a_{ii} - a_{i1} b_{1j}, \quad \imath = 2,3,\ldots,n$$

i=2,

$$b^{(1)}_{2i} = a^{(1)}_{2i} / a^{(1)}_{22}, \qquad j = 2,3,\ldots,n$$
$$a^{(2)}_{ii} = a^{(1)}_{ii} - a^{(1)}_{i2} b^{(1)}_{2i}, \qquad \imath = 2,3,\ldots,n$$

. . . .

Agar berilgan determinant yetakchi satridagi yetakchi element $a_{11}=0$ bo'lsa, bu satrni yetakchi elementi noldan farqli bo'lga satr bilan almashtiramiz.

4.2. Gauss usulida matritsaga teskari matritsa topish

Berilgan A matritsaga asosan teskari $V=A^{-1}$ matritsani Jordan-Gauss usulida topish uchun quyidagicha kengaytirilgan matritsani tuzamiz.

$$\begin{pmatrix} a_{11} \ a_{12} \ldots a_{1n} & \vdots & b_{11} \ b_{12} \ldots b_{1n} \\ a_{21} \ a_{22} \ \ldots \ a_{2n} & \vdots & b_{21} \ b_{22} \ \ldots b_{2n} \\ \ldots\ldots\ldots\ldots\ldots & & \ldots\ldots\ldots \\ a_{n1} \ a_{n2} \ldots \ a_{nn} & \vdots & b_{n1} \ b_{n2} \ldots \ b_{nn} \end{pmatrix}$$

Bu matritsadagi b_{ij} - $i,j=1,2,3,..,n$ elementlar boshlang'ich holatda birlik matritsa o'rnida bo'lib, A matritsani birlik matritsaga aylantirish bilan teskari matritsa elementlariga aylanadi.

$$\begin{pmatrix} 1 \ \ 0 \ldots 0 \ \ a_{1,\kappa+1}^{(\kappa)} \ldots a_{1n}^{(\kappa)} & \vdots & b_{11}^{(k)} \ b_{12}^{(k)} \ldots b_{1n}^{(k)} \\ 0 \ \ 1 \ \ldots 0 \ \ a_{2,\kappa+1}^{(\kappa)} \ldots a_{2n}^{(\kappa)} & \vdots & b_{21}^{(k)} \ b_{22}^{(k)} \ldots b_{2n}^{(k)} \\ \ldots\ldots\ldots\ldots\ldots\ldots & & \ldots\ldots\ldots\ldots\ldots \\ 0 \ \ 0 \ldots \ 1 \ \ a_{n,\kappa+1}^{(\kappa)} \ldots a_{nn}^{(\kappa)} & \vdots & b_{n1}^{(k)} \ b_{n2}^{(k)} \ldots b_{nn}^{(k)} \end{pmatrix} \Rightarrow \ldots \Rightarrow \begin{pmatrix} 1 \ \ 0 \ldots 0 & \vdots & b_{11}^{(n)} \ b_{12}^{(n)} \ldots b_{1n}^{(n)} \\ 0 \ \ 1 \ \ldots 0 & \vdots & b_{21}^{(n)} \ b_{22}^{(n)} \ldots b_{2n}^{(n)} \\ \ldots\ldots\ldots\ldots & & \ldots\ldots\ldots\ldots \\ 0 \ \ 0 \ldots \ 1 & \vdots & b_{n1}^{(b)} \ b_{n2}^{(n)} \ldots b_{nn}^{(n)} \end{pmatrix}$$

Bu almashtirish elementlarini quyidagicha bog'lash mumkin:

$$a_{ij}^{(k)} = a_{kj}^{(k-1} / a_{kk}^{(k-1}, \qquad \kappa = 1,2,..,n; \quad j = \kappa+1,\ldots,n$$

$$b_{k}^{(k)} = b_{kj}^{(k-1)} / b_{kk}^{(k-1)}, \qquad \kappa = 1,2,..,n; \quad j = 1,2,\ldots,n$$

$$a_{ij}^{(k)} = a_{ij}^{(k\text{-}1)} - a_{kj}^{(k\text{-}1)} a_{ik}^{(k\text{-}1)} / a_{kk}^{(k\text{-}1)}, \qquad \iota = 1,\ldots,\kappa-1.\kappa+1,\ldots,n; \quad j = \kappa+1,\ldots,n$$

$$a_{ik}^{(0)} = a_{ik}$$

$$b_{ij}^{(k)} = b_{ij}^{(k-1)} - b_{kj}^{(k)} a_{ik}^{(k\text{-}1)} / a_{kk}^{(k\text{-}1)}, \qquad \iota = 1,\ldots,\kappa-1.\kappa+1,\ldots,n; \quad j = \kappa+1,\ldots,n$$

$$b_{ii}^{(0)} = a_{ii}$$

4.2-masala. Quyidagi 3- tartibli matritsaga Jardano- Gauss usuli bilan teskari matritsni toping.

$$A=\begin{pmatrix} 4 & 3 & 1 \\ 2 & -1 & -1 \\ 7 & 1 & -3 \end{pmatrix}$$

Echish. Teskari matritsa topish jarayonini matrits yonida ko'rsatib boramiz.
Berilgan matritsaga teskari matritsani Jardano-Gauss usulida topish:

$$AE=\begin{pmatrix} 4 & 3 & 1 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 7 & 1 & -3 & 0 & 0 & 1 \end{pmatrix} \Rightarrow$$

Bu matritsadagi AE matritsaning satrlarini mosravishda [1], [2], [3] kabi belgilab, A matritsani birlik matritsaga, Ye matritsani teskari matritsaga aylantirish uchun quyidagi amallarni **Jardano-Gauss usulida** bajaramiz.

[1]/4

$$\begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 7 & 1 & -3 & 0 & 0 & 1 \end{pmatrix} \Rightarrow$$

[1]

[2]- [1]*2

[3]- [1] *7

$$\begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & -5/2 & -3/2 & -1/2 & 1 & 0 \\ 0 & -17/2 & -19/4 & -7/4 & 0 & 1 \end{pmatrix} \Rightarrow$$

[1]
[2] *(-2/5)

$$\begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 3/5 & 1/5 & -2/5 & 0 \\ 0 & -17/2 & -19/4 & -7/4 & 0 & 1 \end{pmatrix} \Rightarrow$$

[1]
[2]
[3]+ [2]*(17/4)

$$\begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 3/5 & 1/5 & -2/5 & 0 \\ 0 & 0 & -11/5 & -9/10 & -17/10 & 1 \end{pmatrix} \Rightarrow$$

[1]
[2]
[3]*(-5/11)

$$\begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 3/5 & 1/5 & -2/5 & 0 \\ 0 & 0 & 1 & 9/22 & 17/22 & -5/11 \end{pmatrix} \Rightarrow$$

[1]+ [2]*(-3/4)
[2]
[3]

$$\begin{pmatrix} 1 & 0 & -1/5 & 1/10 & 3/10 & 0 \\ 0 & 1 & 3/5 & 1/5 & -2/5 & 0 \\ 0 & 0 & 1 & 9/22 & 17/22 & -5/11 \end{pmatrix} \Rightarrow$$

[1]+ [2]*(-3/4)
[2]+[3]*(-3/4)
[3]

$$\begin{pmatrix} 1 & 0 & 0 & 2/11 & 5/11 & -1/11 \\ 0 & 1 & 0 & -1/22 & -19/22 & 3/11 \\ 0 & 0 & 1 & 9/22 & 17/22 & -5/11 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 2/11 & 5/11 & -1/11 \\ -1/22 & -19/22 & 3/11 \\ 9/22 & 17/22 & -5/11 \end{pmatrix}$$

5-laboratoriya ishi

CHIZIQLI TENGLAMALAR SISTEMASINI YeCHISHDA KETMA-KET YaQINLASHISH (ITERATSIYa) USULI

Maqsad: CHiziqli tenglamalar sistemasini yechishda ketma-ket(iteratsiya) usulidan foydalanish.

Reja:

1. CHiziqli tenglamalar sistemasini yechishda ketma-ket(iteratsiya) usuli.
2. Iteratsiya usuli bilan sistemasini yechishda yaqinlashish shartlari.

Berilgan

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ \alpha_{21}x_1 + \alpha_{22}x_2 + \ldots + \alpha_{2n}x_v = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \alpha_{n1}x_1 + \alpha_{n2}x_2 + \ldots + \alpha_{nn}x_n = b_n \end{cases} \qquad (5.1)$$

sistemadagi A matritsaning dioganal elementlari noldan farqli bo'lsa, yahni

$$a_{ii} \neq 0 \quad (i=1,2, ..., n)$$

(5.1) sistemani quyidagi ko'rinishda yozamiz:

$$\left.\begin{array}{l} x_1 = \dfrac{1}{a_{11}}(b_1 - a_{12}x - \ldots - a_{1n}x_n) \\ x_2 = \dfrac{1}{a_{22}}(b_2 - a_{21}x_1 - a_{22}x_2 - \ldots - a_{2n}x_n) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_n = \dfrac{1}{a_{nn}}(b_n - a_{n1}x_1 - a_{n2}x_2 - \ldots - a_{nn-1}x_{n-1}) \end{array}\right\} \qquad (5.2)$$

Bu holda S matritsaning elementlarini $S_{ij}=-a_{ij} / a_{ii}$ ($i \neq j$), $c_{ii}=0$ kabi aniqlab (5.2) sistemaga iteratsiya shartlarini quyidagicha yozamiz:

$$\sum_{\substack{j=1 \\ i \neq j}}^{n} \left|\frac{a_{ij}}{a_{ii}}\right| \le \alpha < 1 \qquad (i = 1,2,\ldots,n) \qquad (5.3)$$

$$\sum_{\substack{i=1 \\ i \neq j}}^{n} \left|\frac{a_{ij}}{a_{ii}}\right| \le \beta < 1 \qquad (j = 1,2,\ldots,n) \qquad (5.4)$$

A matritsa dioganal elementlari shu sairning elemenilar yig'indisidan katta bo'lsa, yaoni

$$|a_{ii}| > \sum_{j=i} |a_{ij}| \qquad (i = 1,2,\ldots,n) \qquad (5.5)$$

shartni qanoatlantirganda (5.3), (5.4) shartlar bajariladi.

Ikkinchi usul. Bu usulni quyidagi 5.1- misol yordamida tushuntiramiz.

Umuman olganda aosiy matritsasi maxsus bo'lmagan chiziqli tenglamalar sistemasini yechish uchun yaqinlashuvchi iteratsiya usuli mavjud bo'ladi. Lekin bu usullarni amaliy hisoblashlarda doim ham qo'llab bo'lavermaydi.

Agar iteratsiya usuli yaqinlashuvchi bo'lsa, u boshqa usullarga qaraganda quyidagi afzalliklarga ega ekanligini payqash mumkin:

1)Iteratsiya yetarlicha tez yaqinlashuvchi bo'lib, sistemani berilgan aniqlikda yechish uchun n dan kam iteratsiya kerak bo'lgan holda hisoblash vaqtidan yutamiz, chunki bitta iteratsiya uchun zarur bo'lgan arifmetik amallar soni n^2 ga . Gauss usulida esa bu son n^3 ga proportsional bo'ladi.

2) Iteratsiya usulida yaxlitlash xatoligi Gauss usulidan kam bo'ladi. Bundan tashqari iteratsiya usuli o'z-o'zini to'g'rilovchi bo'lib, hisoblashdagi baozi xatolar natijaga taosir qilmaydi. CHunki, iteratsiya jarayonidagi har bir yaqinlashishni yangi boshlang'ich vektor deb qarash mumkin.

3) Iteratsiya usuli bilan mahlum sondagi koeffitsientlari nolga teng bo'lgan sistemalarni yechish qulay bo'ladi.

4) Iteratsiya jarayonini EXM uchun dasturlash juda qulay.

5.1- misol. Oddiy iteratsiya usuli bilan quyidagi chiziqli tenglamalar sistemasini E=10^{-3} aniqlikda yeching.

$$\left.\begin{aligned} 20.91x_1 + 1.2x_2 + 2.1x_3 + 0.9x_4 = 21.70 \\ 1.2x_1 + 21.2x_2 + 1.5x_3 + 2.5x_4 = 27.46 \\ 2.1x_1 + 1.5x_2 + 19.8x_3 + 1.3x_4 = 28.76 \\ 0.9x_1 + 2.5x_2 + 1.3x_3 + 32.1x_4 = 49.72 \end{aligned}\right\} \quad (5.6)$$

Echish. Berilgan (5.6) sistemani (5.2) kabi ko'rinishga keltiramiz:

$$x_1 = \frac{1}{20.9}(21.7 - 1.2x_2 - 2.1x_3 - 0.9x_4)$$
$$x_2 = \frac{1}{21.2}(27.46 - 1.2x_2 - 1.5x_3 - 2.5x_4)$$
$$x_3 = \frac{1}{19.8}(28.76 - 2.1x_1 - 1.5x_2 - 1.3x_4)$$
$$x_4 = \frac{1}{32.1}(49.72 - 0.9x_1 - 2.5x_2 - 1.3x_3)$$

hosil bo'lgan sistemaning koeffitsientlari (5.3) shartni qanoatlantirishini tekshiramiz.

$$\sum_{j \neq 1}\left|C_{1j}\right| \approx 0.20 < 1, \qquad \sum_{j \neq 2}\left|C_{2j}\right| \approx 0.24 < 1,$$
$$\sum_{j \neq 3}\left|C_{3j}\right| \approx 0.25 < 1, \qquad \sum_{j \neq 4}\left|C_{4j}\right| \approx 0.15 < 1,$$

iteratsiya jarayoni yaqinlashuvchi bo'lib, ulardan $\alpha = 0.25 < 1$ ekanligini olamiz, demak,baholashni (6.15) formula bilan bajarilish mumkin, bunda $\frac{\alpha}{1-\alpha} = \frac{1}{3}$ bo'ladi. Sistemaning ozod hadlarini boshlang'ich $\vec{x}^{(0)}$ vektorning mos elementlari uchun qabul qilamiz, yaoni

$$\vec{x}^{(0)} = \begin{pmatrix} 1{,}04 \\ 1{,}03 \\ 1{,}45 \\ 1{,}55 \end{pmatrix}$$

Hisoblash jarayonini

$$\max\left|x_i^{(k)} - x_i^{(k-1)}\right| \leq \frac{0.001}{1/3}, \qquad (i = 1,2,3,4)$$

shart bajarilguncha davom ettiramiz.

Hisoblashni ketma-ket bajara borib, quyidagilarni olamiz:

K=1 da

$$x_1^{(1)} = \frac{1}{20.9}(21.7 - 1.56 - 3.045 - 1.395) = 0.75$$
$$x_2^{(1)} = \frac{1}{21.2}(27.46 - 1.248 - 2.175 - 3.875) = 0.95$$

$$x_3^{(1)} = \frac{1}{19.8}(28.76 - 2.184 - 1.95 - 2.015) = 1.14$$

$$x_4^{(1)} = \frac{1}{32.1}(49.72 - 0.936 - 3.25 - 1.885) = 1.36$$

$$\max\left|x_i^{(1)} - x_i^{(0)}\right| = \max\{0,29; 0,08;\ 0,34;\ 0,19\} = 0,34 > \frac{0.001}{1/3}.$$

K=2 bo'lganda

$$x_1^{(2)} = \frac{16.942}{20.9} = 0.8106, \qquad x_3^{(2)} = \frac{23.992}{19.8} = 1.2117$$

$$x_2^{(2)} = \frac{21.450}{21.2} = 10118, \qquad x_4^{(2)} = \frac{45.1888}{32.1} = 1.4077$$

$$\max\left|x_i^{(2)} - x_i^{(1)}\right| = \max\{0,0606; 0,0618; 0,0717;\ 0,0477\} = 0,0717 > \frac{0,001}{\frac{1}{3}} = 3 \cdot 0,001$$

K=3 bo'lganda

$$x_1^{(3)} = \frac{16.67434}{20.9} = 0.7978, \qquad x_3^{(3)} = \frac{23.71003}{19.8} = 1.1975$$

$$x_2^{(3)} = \frac{21.1503}{21.2} = 0.9977, \qquad x_4^{(3)} = \frac{44.88575}{32.1} = 1.3983$$

$$\max\left|x_i^{(3)} - x_i^{(2)}\right| = \max\{0,0128; 0,0113;\ 0,0112; 0,0072\} = 0,0128 > \frac{0,001}{\frac{1}{3}} = 3 \cdot 0,001$$

K=4 bo'lganda

$$x_1^{(4)} = \frac{16.7295}{20.9} = 0.8104, \qquad x_3^{(4)} = \frac{23.7703}{19.8} = 1.2005$$

$$x_2^{(4)} = \frac{21.2106}{21.2} = 1.0005, \qquad x_4^{(4)} = \frac{44.9510}{32.1} = 1.4003$$

$$\max\left|x_i^{(4)} - x_i^{(3)}\right| = \max\{0,0126;\ 0,0028;\ 0,0030; 0,0020\} = 0,0126 > 3 \cdot 0,001.$$

K=5 bo'lganda

$$x_1^{(5)} = \frac{16.71809}{20.9} = 0.7999, \qquad x_3^{(5)} = \frac{23.7582}{19.8} = 1.1999$$

$$x_2^{(5)} = \frac{21.19802}{21.2} = 0.9999, \qquad x_4^{(5)} = \frac{44.93774}{32.1} = 1.3999$$

$$\max\left|x_i^{(5)} - x_i^{(4)}\right| = \max\ \{0,0105;\ 0,0006;\ 0,0006;\ 0,0004\} = 0,0105 > 3 \cdot 0,001.$$

$$\left|x_1^{(4)} - x_1^{(5)}\right| = 0.0005, \qquad \left|x_3^{(4)} - x_3^{(5)}\right| = 0.0006$$

$$\left|x_2^{(4)} - x_2^{(5)}\right| = 0.0006, \qquad \left|x_4^{(4)} - x_4^{(5)}\right| = 0.0004$$

K=6 uchun hisoblash kerak:

$$x_1^{(6)} = \frac{16,7204}{20,9} = 0,8000; \qquad x_3^{(6)} = \frac{23,7604}{19,8} = 1,2000;$$

$$x_2^{(6)} = \frac{21,2004}{21,2} = 1,0000; \qquad x_4^{(6)} = \frac{44,9404}{32,1} = 1,4000;$$

$$\max\left|x_i^{(6)} - x_i^{(5)}\right| = \max\{0.0001; 0.0001; 0.0001\} = 0.0001 < 3 \cdot 0.001$$

Demak, sistemaning yechimi

x_1=0.8000, x_2=1.0000, x_3=1.2000, x_4=1.4000.

5.2 - misol. Quyidagi tenglamalar sistemasini iteratsiya usuli bilan yeching.

$$\left.\begin{aligned} 1.02x_1 - 0.05x_2 - 0.10x_3 &= -0.795 \\ -0.11x_1 + 0.97x_2 - 0.05x_3 &= 0.849 \\ -0.11x_1 - 0.12x_2 + 1.04x_3 &= 1.398 \end{aligned}\right\} \quad (5.7)$$

Echish. Berilgan sistema matritsasining dioganal elementlari birga yaqin bo'lib, qolganlari modul jixatdan birdan ancha kichik. Iteratsiya usulini qo'llab yechish uchun (5.7) sistemani quyidagi ko'rinishga keltiramiz:

$$\left.\begin{aligned} x_1 &= 0.795 - 0.02x_1 + 0.05x_2 + 0.1x_3 \\ x_2 &= 0.849 + 0.11x_1 + 0.03x_2 + 0.05x_3 \\ x_3 &= 1.398 + 0.11x_1 + 0.12x_2 - 0.04x_3 \end{aligned}\right\}$$

Olingan bu sistema uchun (6.3) yaqinlashish shartini tekshiramiz:

$$\sum_{j=1}^{3}\left|c_{1j}\right| = 0.02+0.05+0.10=0,17<1;$$

$$\sum_{j=1}^{3}\left|c_{2j}\right| = 0.11+0.05+0.03=0.19<1;$$

$$\sum_{j=1}^{3}\left|c_{3j}\right| = 0.11+0.12+0.04=0.27<1.$$

Bulardan, $\alpha=0.27<1$ bo'lib,

$$\frac{\alpha}{1-\alpha} = \frac{0.27}{1-0.27} = 0.369... < 0.37$$

Demak, hosil bo'lgan oxirgi sistemaga qo'llaniladigan iteratsiya yaqinlashuvchi bo'lar ekan.

Boshlang'ich $\vec{x}^0$ vektorning elementlari sifatida ozod hadlarni verguldan so'ng ikki xonagacha aniqlik bilan quyidagicha tanlaymiz:

$$\vec{x}^0 = \begin{pmatrix} 0.80 \\ 0.85 \\ 1.40 \end{pmatrix}$$

Endi hosil bo'lgan sistemaga iteratsiya usulini qo'llash bilan yechimni ketma-ket quyidagicha topamiz:

K=1 bo'lganda

$$x_1^{(1)} = 0.795-0.013+0.0425+0.140=0.9613$$

$$x_2^{(1)} = 0.849+0.088-0.0255+0.070=0.9813$$

$$x_3^{(1)} = 1.398+0.088+0.1020-0.056=1.532$$

K=2 bo'lganda

$$x_1^{(2)} \approx 0.978\,,\ x_2^{(2)} \approx 1.002\,,\ x_3^{(2)} \approx 1.560$$

K=3 bo'lganda

$$x_1^{(3)} \approx 0.980\,,\ x_2^{(3)} \approx 1.004\,,\ x_3^{(3)} \approx 1.563$$

K=2 va K=3 bo'lganda yechim qiymatlarining farqi modul jixatdan $0,37\cdot 10^{-3}$ dan katta emas, shuning uchun taqribiy yechimni quyidagicha olamiz:

$$x_1 \approx 0.980,\ x_2 \approx 1.004,\ x_3 \approx 1.563$$

6-laboratoriya ishi

DARAJALI QATOR YoRDAMIDA BAHZI TRANTSENDENT FUNKTSIYaLARNING QIYMATLARINI HISOBLASH.

Maqsad: Darajali qator yordamida bahzi trantsendent funktsiyalarning qiymatlarini berilgan aniqlikda hisoblashni o'rganish .

Reja:

1. Funktsiyalarni Makloren qatori yoyish formulasi.
2. Funktsiya qiymatini berilgan aniqlikda hisoblash.

Bu yerda shunday transtsendent funktsiyalarni ko'ramizki, ular o'zlarining Makloren qatori yig'indisidan iborat bo'ladi

$$f(x)=\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}x^k \qquad (6.1)$$

Makloren qatorining bir necha hadlari yig'indisini olib, taqribiy formulani hosil qilamiz, yahni

$$f(x)\approx P_n(x)=\sum_{k=0}^{n}\frac{f^{(k)}(0)}{k!}x^k$$

Bu holda, qator qoldig'i $R_n(x)=f(x)-P_n(x)$ ko'rinishda bo'ladi.

Qoldiq had xatoligini baholash qatordan olingan qo'shiluvchilar soniga, yahni $P_n(x)$ ko'phad darajasi n ga bog'liq bo'lishi ravshandir.

6.1. e^x funktsiyaning funktsiyalarining qiymatini hisoblash.

Makloren qatoriga yoyilmasi

$$e^x=\sum_{k=0}^{\infty}\frac{x^k}{k!}, \qquad (-\infty<x<\infty) \qquad (6.2)$$

bo'lishi mahlum. e^x funktsiya qiymatini hisoblashda (6.2) yoyilma asosida quyidagi rekkurrent formuladan foydalanish qulay bo'ladi:

$$e^x=\sum_{k=0}^{\infty}u_k, \qquad u_k=\frac{x}{k}u_{k-1}, \quad S_k=S_{k-1}+u_k \ , \qquad (k=1,2,....)$$

bu yerda $u_0=1$, $S_0=1$. $S_n=\sum_{k=0}^{n}\frac{x^k}{k!}$ bo'lib, bu son e^x ning izlanayotgan taqribiy qiymatlar ketma-ketligining n-yaqinlashishini beradi. Qatorning qoldiq hadini quyidagicha baholash mumkin.:

$$|R_n(x)|<|u_n| \ , (0<2|x|\le n)$$

Qo’shish jarayonini so’nggi qo’shiluvchi u_n oldindan berilgan hisoblash aniqligi, yahni taqribiy qiymat absolyut xatoligi bo’lgan ε sondan modul bo’yicha kichik bo’lganda to’xtatish mumkin:

$$|u_n| < \varepsilon \quad \left(|x| \leq \frac{n}{2}\right)$$

6.1-masala. $\sqrt{e}$ sonini $\varepsilon=10^{-5}$ aniqlik bilan hisoblang.
Echish. Yuqorida aytilganlarga asoslanib

$$e^{\frac{1}{2}} = \sum_{k=0}^{n} u_k + R_n\left(\frac{1}{2}\right) \qquad (6.3)$$

formuladan foydalanib, hisoblaymiz.

Bu yerda $u_0=1$, $u_k=(u_{k-1}) / 2k$ $(k=1,2,...,n)$ bo’lib, quyidagilarni ketma-ket yozishimiz mumkin.

$$u_0=1$$
$$u_1 = \frac{u_0}{2} = 0.5000000$$
$$u_2 = \frac{u_1}{4} = 0.1250000$$
$$u_3 = \frac{u_2}{6} = 0.0208333$$
$$u_4 = \frac{u_3}{8} = 0.0026042$$
$$u_5 = \frac{u_4}{10} = 0.0002604$$
$$u_6 = \frac{u_5}{12} = 0.0000217$$
$$u_7 = \frac{u_6}{14} = 0.0000016$$

$$S_7=1.6487216 \qquad (|u_7| < 10^{-5})$$

Natijani butun qismidan so’ng beshta o’nli raqamgacha yaxlitlab quyidagi natijani yozamiz,

$$\sqrt{e} = 1.64872$$

6.2. Sinus va kosinus funktsiyalarining qiymatini hisoblash.

Sinus va kosinus funktsiyalarining qiymatini hisoblash uchun ularning Makloren qatoriga yoyilmasidan foydalanamiz:

$$Sinx = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}, \quad (-\infty < x < \infty) \qquad (6.4)$$

$$Cosx = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}, \quad (-\infty < x < \infty) \qquad (6.5)$$

(6.4) , (6.5) qatorlar x ning katta qiymatlarida sekin yaqinlashadi. Sinus va kosinus funktsiyalarning davriyligini ehtiborga olgan holda ularni $0 \leq x \leq \pi/4$ oraliqda hisoblash yetarli bo'lib, quyidagi rekkurrent formulalardan foydalanish maqsadga muvofiqdir.

$$Sinx = \sum_{k=1}^{n} U_n + R_n(x)$$

$$u_1 = x, \quad u_{k+1} = -\frac{x^2}{2k(2k+1)} u_k, \quad (k = 1,2,\ldots,n-1) \qquad (6.6)$$

$$Cosx = \sum_{k=1}^{n} V_k + R_n(x)$$

$$v_1 = 1, \quad v_{k+1} = -\frac{x^2}{(2k-1)2k} v_k, \quad (k = 1,2,\ldots,n-1) \qquad (6.7)$$

Yuqoridagi (6.6) qator (0, π/4) oraliqda ishorasi navbatlashuvchi va hadlari modullari bo'yicha monoton kamayuvchi bo'lganligi uchun qatorning qoldiq hadi - R_n ni baholash quyidagicha bo'ladi:

$$|R_n| \le \frac{|x|^{2n+1}}{(2n+1)!} = |u_{n+1}|$$

Xuddi shuningdek, (6.7) qator uchun esa,

$$|R_n| \le |V_{n+1}|$$

Demak, $0 \le x \le \frac{\pi}{4}$ bo'lganda Sinx va Cosx larning qiymatlarini hisoblash jarayonini ko'rsatilgan ε sonidan modul jihatidan kichik bo'ladigan qator hadini olinguncha davom ettirish kifoyadir.

<u>6.2-masala.</u> $Sin23^0 54o$ qiymatini $\varepsilon=10^{-4}$ aniqlik bilan hisoblang.

<u>Echish.</u> Argument qiymatini radianga o'tkazamiz:

$$x = \pi \cdot 23^0 54^o / 180^0 \approx 0.4174$$

(6.6) formulani qo'llash bilan quyidagilarni topamiz:

$u_1 = x = 0.41714$
$u_2 = -x^2 u_1/(2 \cdot 3) = -0.01210$
$u_3 = -x^2 u_2/(4 \cdot 5) = +0.00011$
$u_4 = -x^2 u_3/(6 \cdot 7) = -0.00000$
$(|u_4| < \varepsilon)$
$Sin23^0 54^o = 0.40515 \approx 0.4052$

7-laboratoriya ishi

TRANSTSENDENT VA ALGEBRAIK TENGLAMA ILDIZLARI YoTGAN ORALIQLARNI ANIQLASH.

Maqsad: Transtsendent tenglama ildizi yotgan oraliqni aniqlash usullarini o'rganish.

Reja:

1. Tenglama ildizini ajratish qoidalari
2. Transtsendent tenglama ildizini ajratish
3. Algebraik tenglama ildizlari yotgan oraliqlarni aniqlash.

7.1. Tenglama ildizini ajratish

Amaliyotda, baozi masalalarda

$$f(x)=0 \qquad (7.1)$$

ko'rinishdagi tenglamalarni yechishga to'g'ri keladi. Bunda *f(x)* [*a*,*b*] oraliqda aniqlangan funktsiya bo'lib, *f*(t)=0 bo'lsa, x=t ni (7.1) tenglamaning yechimi deyiladi. Tenglamaning aniq yechimini topish qiyin bo'lgan hollarda uning taqribiy yechimini topish ikki bosqichga bo'linadi.

1) Yechimni ajratish(yakkalash), yahni yagona yechim yotgan intervalni aniqlash;
2) Taqribiy yechimni berilgan aniqlikda topish.

Tenglamaning yagona yechimi yotgan oraliqni aniqlash uchun quyidagi teoremadan foydalaniladi.

7.1-teorema . ***Aytaylik,***

1) f(x) funktsiya [a,v] kesmada uzluksiz va (a,v) irtervalda hosilaga ega bo'lsin;

2) f(a)·f(v)<0, yahni f(x) funktsiya kesmaning chetlarida har xil ishoraga ega bo'lsin;

3) fo(x) hosila (a,v) irtervalda o'z ishorasini saqlasin.

U holda, (7.1) tenglama [a,v] oraliqda yagona yechimga ega bo'ladi.

7.2. Transtsendent tenglama ildizini ajratish

Teoremadagi [***a,b***] kesmani topishda, baozan grafik usuldan foydalaniladi. Bu usulga asosan (7.1) tenglamaning ildizini ajratish uchun y=*f(x)* funktsiyaning [*a,b*] oraliqdagi grafigini chizamiz. Bu grafikning OX o'qi bilan kesishish nuqtasining abtsissasi (7.1) tenglamaning yechimi bo'ladi. u=*f(x)* funktsiyani grafigini chizish qiyin bo'lsa, *f(x)*=0 tenglamani grafigini chizish mumkin bo'lgan funktsiyalarga fjratamiz va

$$f_1(x)=f_2(x) \qquad (7.2)$$

ko'rinishga keltiramiz va u=f_1(x), u=f_2(x) funktsiyalarning grafiklarini chizamiz. Bu grafiklar kesishish nuqtasining abstsissasi *f(x)*=0 tenglamaning yechimi bo'ladi. Bu yechimni o'z ichiga oluvchi [***a,b***] oraliqni yuqoridagi teorema shartlarini tekshirish asosida tanlaymiz.

7.1-Misol. ye^x -10x -2=0 tenglamaning yagona ildizi yotgan oraliq topilsin.

Echish. Tenglamani

$$e^x=10x+2$$

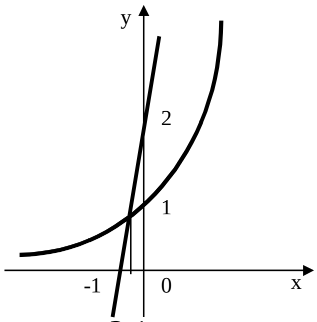
ko'rinishda yozamiz. So'ngra,

$$u=e^x\ ,\ u=10x+2$$

funktsiyalarning grafiklarini chizamiz. 2.1-rasm dan ko'rinadiki,

$$e^x-10x-2=0$$

tenglama yagona yechimini o'z ichiga olgan oraliq [-1,0] bo'ladi.



2.1-расм

[-1,0] oraliqda teorema shartlarini tekshiramiz.

1) *f(x)*=e^x-10x-2 funktsiya [-1,0] oraliqda uzluksiz, (-1,0) intervalda *f*o(x)=e^x-10 hosilaga ega.

2) $f(-1)=e^{-1}-10(-1)-2\approx 3.368>0$, $f(0)=e^0-10\cdot 0-2=-1<0$

bundan: $f(-1)\cdot f(0)<0$

3) $x\in(-1,0)$ bo'lganda *f*o(x)=e^x -10<0

Demak, 7.1-teoremaning barcha shartlari [-1,0] oraliqda bajariladi. Bu [-1,0] oraliqda tenglama yagona yechimga ega ekanligini bildiradi.

7.3. Algebraik tenglama ildizlari yotgan oraliqlarni aniqlash.

Aytaylik, bizga

$$f(x)=\boldsymbol{a}_0\,x^n+\boldsymbol{a}_1\,x^{n-1}+\ldots+\boldsymbol{a}_{n-1}\,x+\boldsymbol{a}_n=0 \qquad (7.3)$$

n-darajali algebraik tenglama berilgan bo'lsin.

1. Algebraik tenglama ildizlarini ajratishda quyidagi teorema va qoidalardan foydalanamiz.

7.2-teorema. ***Agar***

$$\mathbf{A=max\{|a_1/a_0|, |a_2/a_0|,..., |a_n/a_0|\}}$$

$$\mathbf{A1= max\{|a_0/a_n|, |a_1/a_n|,..., |a_{n-1}/a_n|\}}$$

bo'lsa, (7.3) tenglamaning barcha ildizlari

$$\mathbf{r=1/(1+A1)<|x|<1+A=R}$$

halqada yotadi.

Musbat ildizlar chegarasi: $r< x^+ <R$

Manfiy ildizlar chegarasi: $-R< x^- < -r$

Agar (7.3) tenglamani

$$f_1(x)=x^n f(1/x)=0$$

$$f_2(x)= f(- x)=0$$

$$f_3(x)=x^n f(- 1/x)=0$$

ko'rinishga keltirib, mos ravishda topilgan musbat ildizlarining yuqori chegaralari R_1, R_2, R_3 bo'lsa, (7.3) tenglama ildizlarining chegaralari:

$$1/R_1< x^+ <R_2 \quad \text{va} \quad -R_2< x^- < - 1/R_3$$

2. Ishorasi almashinuvchi algebraik tenglamalarning musbat ildizlarining yuqori chegarasini topishda quyidagi Lagranj teoremasidan foydalanamiz:

7.3- teorema. ***(7.3) tenglamada $a_0>0$ va a_k (k≥1 -tartib raqami) - birinchi manfiy koeffitsient bo'lib, B manfiy koeffitsientlar ichida modul bo'yicha eng kattasi bo'lsa, musbat ildizla rining yuqori chegarasi***

$$R = 1 + \sqrt[k]{\frac{B}{a_0}} \qquad (7.4)$$

formula bilan topiladi.

Berilgan (7.3) tenglamaning manfiy ildizlarining quyi chegarasini aniqlash uchun tenglamani

$$f(-x)=0 \qquad (7.5)$$

ko'rinishga keltirib, hosil bo'lgan (7.5) tenglamaga Lagranj teoremasini qo'llab, topilgan musbat ildizlarining yuqori chegarasi ***R1*** bo'lsa, (7.3) tenglama manfiy ildizlarining quyi chegarasi uchun
-R1 bo'lishi ayondir. Demak, berilgan (7.3) tenglamaning barcha haqiqiy ildizlarining chegarasi:

$$R1<x< R\,.$$

7.3-Misol. $2x^3-9x^2-60x+1=0$ tenglama ildizlari yotgan oralikning chegarasini aniqlang.

Echish. Lagranj formulasiga asosan $a_0=2$, V=60, $k_1(a_1=-9)=1$ dan

$$R=1+(V/a_0)^\wedge(1/k)=1+(60/2)^\wedge(1/1)=31$$

Bu musbat ildizlarining yuqori chegarasi R=31.

Manfiy ildizlarining quyi chegarasini topamiz.

$$f(-x)= 2(-x)^3-9(-x)^2-60(-x)+1=0$$

$$f(-x)= 2x^3+9x^2-60x-1=0$$

tenglamadan:

$$a_0=2,\ B2=60,\ k2=2$$

$$R1=1+(B2/a_0)^\wedge(1/k2)=1+(60/2)^\wedge(1/2)\approx 6.77$$

Bundan manfiy ildizlar quyi chegarasini R1=-6.77 bo'ladi

3. Agar berilgan (7.3) tenglamaning barcha koeffitsientlari musbat bo'lsa, ildizlarining chegarasini

$$m<|x|<M$$

tengsizlikka asosan aniqlaymiz, bunda

$$\mathbf{m=min(}\boldsymbol{a}_{\mathbf{k}}\mathbf{/}\,\boldsymbol{a}_{\mathbf{k-1}}\mathbf{),\ M=max(}\boldsymbol{a}_{\mathbf{k}}\mathbf{/}\,\boldsymbol{a}_{\mathbf{k-1}}\mathbf{),\ 1<k<n}$$

SHuningdek, (7.3) tenglamaning barcha koeffitsientlari musbat bo'lganda:

a) $a_0 > a_1 > ... > a_n$ bo'lsa, ildizlar $|x|>1$ doiradan tashqarida yotadi;

b) $a_0 < a_1 < ... < a_n$ bo'lsa, ildizlar $|x|<1$ doira ichida yotadi.

4. Toq darajali algebraik tenglama hech bo'lmaganda bitta ildizga ega bo'ladi.

8-laboratoriya ishi

TENGLAMA ILDIZINI VATARLAR USULIDA HISOBLASH

Maqsad: Transtsendent tenglama ildizi yotgan oraliqda ildizini hisoblash Vatarlar usulini o'rganish.

Reja

1. **Tenglama ildizini Vatarlar usulida hisoblash.**
2. **Tenglama ildizini hisoblashda Vatarlar usulini qo'llash shartlari.**
3. **Vatarlar usulida ildizga yaqinlashish shartlari**

8.1. Tenglama ildizini Vatarlar usulida hisoblash.

Aytaylik, berilgan ***f(x)*=0** tenglamadagi ***f(x)*** funktsiya [*a*,*b*] oraliqda 2.1-teoremaning hamma shartlarini qanoatlantirsin. Bundan tashqari ***f(x)*** funktsiya [*a*,*b*] oraliqda ikkinchi tartibli ***f*oo(x)** uzluksiz hosilaga ega bo'lib, bu hosila shu oraliqda o'z ishorasini saqlasin, yahni quyidagi teorema o'rinli bo'lsin.

8.1- teorema. ***Agar [a,b] kesmada***

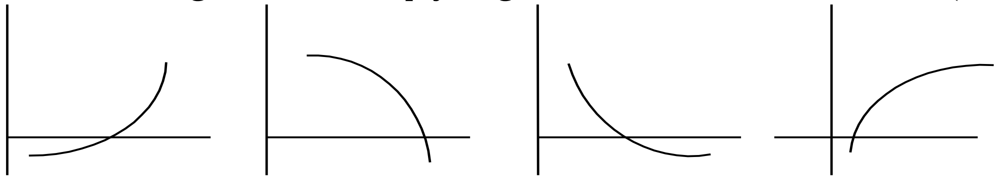
1) f(x), f o (x) funktsiyalar uzluksiz,foo (x) mavjud bo'lsa;

2) f(a)f(v)< 0, yahni f(x) funktsiya kesmaning chetlarida har xil ishoraga ega bo'lsa;

3) fo(x), foo(x) hosilalar [a,v] kesmada o'z ishorasini saqlasa,

f(x)=0 tenglama ildizini aniqlash uchun vatarlar usuli yordamida qurilgan ketma-ketlik ildizga yaqinlashuvchi bo'ladi.

Bu teorema shartlarining mazmunini quyidagi shakllarda ko'rish mumkin (2.2-rasm).



a) *f* o>0, *f*oo>0 b) *f* o<0, *f*oo<0 v) *f* o<0, *f*oo>0 g) *f* o>0, *f*oo<0

2.2-rasm

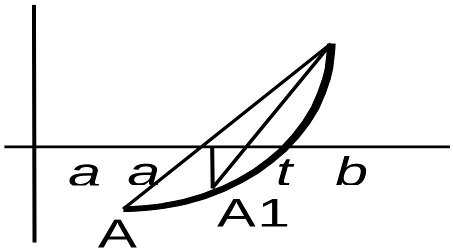
Vatarlar usuli bilan aniqlanadigan taqribiy ildizlar ketma-ketligini qurishda, qaralayotgan oraliqning chetki nuqtalarida

$$f'(x)\,f''(x)<0 \qquad (8.1)$$

shart bajarilishiga qarab, quyidagi ikki holni alohida keltiramiz:

1) Agar [a,b] oraliqning chap uchida $f'(a)f''(a)<0$ shart bajarilsa, vatarlar usulini chap tomondan qo'llaymiz(2.3-rasm):

$$
\begin{aligned}
&a_0= a\\
&a_1= a_0 - (b-a_0)\, f(a_0)/ (f(b)-f(a_0))\\
&. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad\qquad (8.2)\\
&a_n= a_{n-1} - (b -a_{n-1})\, f(a_{n-1})/ (f(b)-f(a_{n-1}))\\
&. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .
\end{aligned}
$$

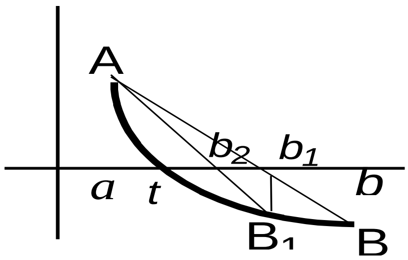


2.3-rasm

Bu ketma-ketlik hadlarini hisoblash jarayonini $|a_n-a_{n-1}|<\varepsilon$ shart bajarilguncha davom ettiramiz va ildiz taqribiy qiymati uchun x≈ a_n ni qabul qilamiz, bu yerda ε taqribiy ildiz aniqligi.

2) Agar [a,b] oraliqning o'ng tomonida $f'(b)\,f''(b)<0$ shart bajarilsa, vatarlar usulini o'ng tomondan qo'llaymiz(2.4-rasm)

$$
\begin{aligned}
&b_0= b,\\
&b_1= b_0 - (a- b_0)\, f(b_0)/ (f(a)-f(b_0)),\\
&. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad\qquad (8.3)\\
&b_n= b_{n-1} - (a- b_{n-1})\, f(b_{n-1})/ (f(a)-f(b_{n-1})),\\
&. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .
\end{aligned}
$$



2.4-rasm

Bu ketma-ketlik hadlarini hisoblash jarayonini $|b_n- b_{n-1}|<\varepsilon$ shart bajarilguncha davom ettiramiz va ildiz taqribiy qiymati uchun x≈b_n ni qabul qilamiz.

8.1-Misol. $ye^x-10x-2=0$ tenglamaning ε=0.01 aniqlikdagi taqribiy ildizi topilsin.

Echisk. $f(x)$=e^x-10x-2 funktsiya [-1,0] oraliqda 8.1-teoremaning barcha shartlarini qanoatlantirishini tekshirib ko'rish qiyinchilik tug'dirmaydi. x$\in$[-1,0] da ikkinchi tartibli hosila *f*oo(x) = ye^x >0. undan tashqari, *f*(0)=-1,
f(-1)=8.368 bo'lganligi uchun, (8.1) shartga asosan *f*o(0)*f*oo(0)<0 bo'lgani uchun {b_n} ketma-ketlik (8.3) jarayon bilan quriladi.

Berilganlar: a=-1, b=0, ε=0. 01

$f(x)$= ye^x-10x-2, $f(-1)=e^{-1}$ -10(-1) -2=8.386, $f(0)=e^0$-10·0-2= -1

(8.3) formulaga asosan:

$b_0= 0$

$b_1= b_0 - (a- b_0) f(b_0)/ (f(a)-f(b_0))= -0.107$

$|b_1 - b_2|>\varepsilon$ bo'lganligi uchun b_2 yaqinlashishni hisoblaymiz. Buning uchun

b_1= -0.107, $f(-0.107)=e^{-0.107}$-10(-0.107)-2=-0.038, $f(a)=f(-1)$=8.386

larga asosan:

$$b_2= b_1 - (a- b_1) f(b_1)/ (f(a)-f(b_1)) = -0,111$$

$$|b_2- b_1|=|- 0.111+0.107|=0.004<\varepsilon=0,01$$

Demak, 0.01 aniqlikdagi taqribiy yechim deb t$\approx b_2$ =-0.11 ni olish mumkin.

9-laboratoriya ishi.

TENGLAMA ILDIZINI URINMALAR (NG'YuTON) USULIDA HISOBLASH.

Maqsad: Transtsendent tenglama ildizi yotgan oraliqda ildizini hisoblashda urinmalar usulini o'rganish.

Reja

1. **Tenglama ildizini urinmalar usulida hisoblash.**
2. **Tenglama ildizini hisoblashda urinmalar usulini qo'llash shartlari.**
3. **Urinmalar usulida ildizga yaqinlashish shartlari**

Aytaylik, $f(x)$ funktsiya [a,b] oraliqda 8.1-teoremaning barcha shartlarini bajarsin. Bu holda, $f(x)$=0 tenglama [a,b] oraliqda yagona x=t yechimga ega bo'ladi. Bu teorema asosida ildizni hisoblash uchun urinmalar usulini *f(x)foo(x)>0* shart oraliqning qaysi chetida bajarilsa, shu tarafdan qo'llash kerakligini ko'ramiz. Bundan:
f(a)foo(a)>0 bo'lganda, boshlang'ich yaqinlashishni chapdan $a_0 = a$, aks holda o'ngdan $b_0=b$ deb olinadi. *f(a) foo(a)>0* bo'lganda x=t yechimning taqribiy qiymatlaridan tuzilgan {a_n} ketma- ketlik quyidagicha topiladi. y=$f(x)$ funktsiya grafigining A(a, $f(a)$) nuqtasiga urinma o'tkazamiz (2.5-rasm), so'ngra bu urinmaning tenglamasini tuzamiz.

u-*f*(a)=*f* o(a)(x- a)

Urinmaning OX o'qi bilan kesishish nuqtasi x=a_1 -desak, bu nuqtada u=0 ekanligidan

0-*f*(a)= *f* o(a)(a_1-a)

ni olamiz. Oxirgidan esa

a_1= a - *f*(a)/*f* o(a)

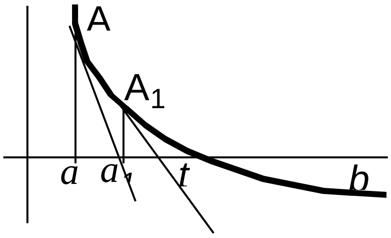
formula topiladi. So'ngra [a_1,b] oraliq uchun yuqoridagi jarayonni takrorlab,

a_2= a_1 - *f*(a_1)/ *f* o(a_1)

formulani olamiz va hokazo, jarayonning n- takrorlanishida (n- qadamda)

$$a_n= a_{n-1} - f(a_{n-1})/ f\,o(a_{n-1}) \qquad (9.1)$$

formulaga ega bo’lamiz. Bu jarayonni cheksiz takrorlash (davom ettirish) natijasida $\{a_n\}$ ketma-ketlikni tuzamiz. Bu urinmalar usulining mohiyatidan iboratdir.



2.5-rasm

Olingan $\{a_n\}$ ketma-ketlik 2.4-teoremaning shartlari bajarilganda aniq yechim x=t ga yaqinlashadi.(9.1) Jarayon $|a_n- a_{n-1}|<\varepsilon$ shart bajarilguncha davom ettiriladi va taqribiy ildiz uchun $x\approx a_n$ ni qabul qilinadi.

Agar $f(b)f''(b)> 0$ bo’lsa, $b_0= b$ deb olib,

$$b_n= b_{n-1} - \frac{f(b_{n-1})}{f'(b_{n-1})}$$

formula asosida $\{b_n\}$ ketma-ketlikni olamiz.

9.1-**Misol.** $ye^x-10x-2=0$ tenglama taqribiy yechimini $\varepsilon=0.01$ aniqlik bilan toping.

Y**echish.** $F(x)=e^x-10x-2$ funktsiya [-1;0] oraliqda 8.1-teoremaning barcha shartlarini qanoatlantiradi.

$$f oo(x)=e^x >0,\ x\in[-1;0] \quad \text{va} \quad f(-1)=8.386>0$$

dan

$$f(-1)\, f oo(-1)>0$$

bo’lgani uchun $a_0=-1$ deb olinadi. $f o(-1)=e^{-1}-10=-9.632$ ni ehtiborga olib, birinchi yaqinlashish a_1 ni hisoblaymiz:

$$a_1=a- f(a)/f o(a)= a- f(-1)/f o(-1)= -1-8.386/(-9.632) = -0.131.$$

Yaqinlashish shartini tekshiramiz:

$$|\, a_1- a_0\, | = |-0.131+1|= 0.869>\varepsilon=0.01$$

bo’lgani uchun ikkinchi yaqinlashish a_2 ni

$$a_2=a_1- f(a_1)/f o(a_1)$$

formula bilan topamiz.

$$f(a_1)=e^{-0.131} + 10(0.131)-2=0.1895,\ \ f o(a_1)= ye^{-0.131} - 10= -9.123$$

lar asosida: $a_2=-0.131- 0.1895/(-9.123) = -0.1104$.

Yana $|a_2- a_1| = 0.0214 > \varepsilon$ bo’lgani uchun a_3 ni topamiz.

$$a_2=-0.1104,\ \ f(a_2)=0.0006\ ,\ \ f o(a_2)=-9.1046$$

lar asosida: $a_3= a_2 - f(a_2)/ f o(a_2)= -0.1104 - 0.\,0006/(-0.1046) =-0.1104;$

yaqinlashish sharti $|a_3-a_2|< \varepsilon=0.01$ bajarilganligi uchun tenglamaning $\varepsilon=0.01$ aniqlikdagi taqribiy yechimi:

$$x\approx a_3= -0.11$$

bo’ladi.

10-laboratoriya ishi.

TENGLAMA ILDIZINI BIRGALASHGAN USULIDA HISOBLASH

Maqsad: Trantsendent tenglama ildizi yotgan oraliqda ildizini hisoblash umumlashgan usulini o'rganish.

Reja

1. **Tenglama ildizini umumlashgan usulida hisoblash.**
2. **Tenglama ildizini hisoblashda umumlashgan usulini qo'llash shartlari.**
3. **Umumlashgan usulida ildizga yaqinlashish shartlari**

Vatarlar va urinmalar usulini bir vaqtda $[a,b]$ oraliqda qo'llab, izlangan t yechimning ikki tomonida yotuvchi a_1 va b_1 birinchi yaqinlashishlarni hisoblaymiz. Oraliqning chetki *a va b* nuqtalaridagi *f(x)foo(x)* ning ishorasiga qarab ildizga yaqinlashish ketma-ketliklarini tuzamiz.

1) $f(a)f\ oo(a\ )>0$ bo'lganda, chapdan urinmalar, o'ngdan esa vatarlar usullarini qo'llash mumkin:

$$a_1 = a - f(a)/ f\ o(a\ )\ ,$$
$$b_1 = b- (a - b\ ) f(b)/(f(a\ )- f(b)) \qquad (10.1)$$

2) $f(b)foo(b)>0$ bo'lganda, chapdan vatarlar, o'ngdan esa urinmalar usullarini qo'llash mumkin:

$$a_{\ 1}= a\ -\ (b - a\ ) f(a\ )/ (f(b\ )-f(a\ ))\ ,$$
$$b_{\ 1}= b - f(b\ )/ fo(b\ ) \qquad \mathbf{(10.2)}$$

Agar $|b_1-a_1|<\varepsilon$ tengsizlik bajarilsa tenglamaning $\varepsilon>0$ aniqlikdagi yechimi deb $t=(a_1+b_1)/2$ olinadi. Aks holda, $[a_1,b_1]$ oraliqda urinmalar va vatarlar usulini qo'llab, aniq yechim t ga yanada yaqinroq bo'lgan a_2 va b_2 nuqtalarni hosil qilamiz.

Agar $|\ b_2- a_2|<\varepsilon$ bo'lsa, taqribiy yechim deb $t=(a_2+b_2)/2$ ni olinadi. Aks holda, yuqoridagi jarayon yana takrorlanadi va hokazo.

10.1- Misol. $ye^x-10x-2=0$ tenglamaning ildizi taqribiy qiymatini $\varepsilon=0.01$ aniqlikda umumlashgan usul asosida hisoblang.

Buning uchun $a=-1\ ,\ f(-1)=e^{-1} -10-2>0$ $\qquad (f(x)=e^{\ x}- 10x - 2)$

$$foo(x)=e^x > 0\ ,\ foo(-1)=e^{-1}>0,\ f(-1)foo(-1)>0$$

larga asosan a_1 va b_1 larni topish uchun (10.1) formulalardan foydalanamiz:

$$a_1=-1 - f(-1)/ fo(-1) =-0.131$$
$$b_1=0- f(0)(-1-0)/ (f(-1)-f(0)) = -0.107$$

$/b_1 -a_1/= 0.024 >\varepsilon$ bo'lganligi uchun a_2 va b_2 larni topamiz.

$$a_1=-0.131\ ,\ f(a_1)=0.1895\ ,\ foo(a_1)>0$$
$$f(a_1)foo(a_1)>0,\ b_1=0.\ 107\ ,\ f(b_1)=-0.\ 038\ ,$$

larga asosan a_2 va b_2 larni hisoblaymiz.

$$a_2= a_1- f(a_1)/ fo(a_1)\ =-0.\ 131- 0.\ 1895/ -0.\ 123 =-0.\ 1107\ ,$$
$$b_2=b_1-f(b_1)(a_1-b_1)/f(a_1)-f(b_1)=$$
$$=-0.107- 0.038(-0.131+0.107)/(0.1895+0.038)=-0.\ 111$$
$$|\ b_2 - a_2\ |= 0.0014<\varepsilon = 0.01$$

Taqribiy yechim deb $t_2= (a_2+v_2)/2 \approx -0.11$ olamiz.

11-laboratoriya ishi.
TENGLAMA ILDIZINI KETMA-KET YaQINLASHISH (ITERATSIYa) USULIDA HISOBLASH

Maqsad: Trantsendent tenglama ildizi yotgan oraliqda ildizini hisoblash iteratsiya **usulini o'rganish.**

Reja

1. **Tenglama ildizini** iteratsiya **usulida hisoblash.**
2. **Tenglama ildizini hisoblashda iteratsiya usulini qo'llash shartlari.**
3. **Iteratsiya usulida ildizga yaqinlashish shartlari**

Berilgan $f(x)=0$ tenglamani unga teng kuchli bo'lgan $x=\varphi(x)$ ko'rinishga keltiramiz.

11.1-teorema. ***Aytaylik,***

1) φ(x) funktsiya [a,v] oraliqda aniqlangan va differentsiallanuvchi bo'lsin;

2) φ(x) funktsiyaning hamma qiymatlari [a,v] oraliqqa tushsin;

3)[a,v] oraliqda $|\varphi'(x)| \le q < 1$ tengsizlik bajarilsin.

Bu holda [a,v] oraliqda x=φ(x) tenglamaning yagona x=t yechimi mavjud va bu yechim $t_0 \in [a;b]$ qanday tanlanishidan qatoiy nazar

$$t_1=\varphi(t_0)\ ,\ t_2=\varphi(t_1)\ ,.\ .\ .\ ,\ t_n=\varphi(t_{n-1}),\ldots$$

formulalar bilan aniqlanadigan { t_n } ketma - ketlikning limitidan iborat bo'ladi.

Bu yerda, t_0 qiymat $[a,v]$ oraliqda yotuvchi ixtiyoriy son bo'lib, yechimning 0-yaqinlashishi, t_i - ni yechimning i - yaqinlashishi deb yuritiladi.

Bu teorema asosida tenglama ildizini quyidagicha aniqlaymiz.

1) $f(x)=0$ tenglamaning yagona ildizi yotgan $[a,b]$ kesmani biror (masalan, grafik) usul bilan aniqlaymiz.

2) $[a, b]$ da $f(x)$ ning uzluksizligi va $f(a) \cdot f(b)<0$ shart bajarilishini tekshiramiz.

3)Tenglamani $x=\varphi(x)$ ko'rinishga keltirib, $\varphi(x) \in [a,b]$ ekanligini hamda $[a;b]$ da $\varphi'(x)$ mavjudligini tekshiramiz va $q = \max\limits_{x \in [a;b]} |\varphi'(x)|$ ni topamiz.

4) Agar $q<1$ bo'lsa, $x_n = \varphi(x_{n-1})$ ketma-ketlikning boshlang'ich yaqinlashishi x_0 ni $[a;b]$ ning ixtiyoriy bitta nuqtasi sifatida olamiz.

5) Ketma-ketlik hadlarini hisoblashni $|x_n - x_{n-1}| < \varepsilon(1-q)/q$ shart bajarilguncha davom ettiramiz.

6) Ildizning taqribiy qiymati uchun x_n ni olamiz.

11.1.-masala. $ye^x -10x -2=0$ tenglamaning $[-1,0]$ oraliqdagi yechimini $\varepsilon=0.01$ aniqlikda toping.

Echish. Berilgan tenglamani unga teng kuchli bo'lgan

$$x=(e^x-2)/10$$

ko'rinishga keltiramiz. Endi $\varphi(x)=(e^x-2)/10$ funktsiya uchun [-1,0] oraliqda 2.5-teoremaning barcha shartlari bajarilishini ko'rsatamiz:

1) [-1,0] da $\varphi(x)$ funktsiya $\varphi o(x)=e^x/10$ uzluksiz hosilaga ega va $\varphi'(x)>0$ ekanligini payqash qiyin emas.

2) [-1,0] da $\varphi(x)$ o'suvchi bo'lganligi sababli, uning qiymatlari uchun $\varphi(-1)=-0.163>-1$,$\varphi(0)=-0.1<0$ ekanligidan

$$-1<\varphi(x)<-0.1, \text{ yahni } \varphi(x)\in(-1;0) \text{ kelib chiqadi.}$$

3) $\varphi''(x)=\dfrac{e^x}{10}$ ikkinchi tartibli hosila [-1,0] oraliqda musbat bo'lgani uchun $\varphi'(x)$-hosila o'suvchi va

$$e^{-1}/10=\varphi o(-1)<\varphi o(0)=e^0/10$$

ekanligidan $q=\max\limits_{x\varepsilon[-1;0]}|\varphi'(x)|=\dfrac{1}{10}=0{,}1$ ni olamiz.

Demak, $x=(e^x-2)/10$ tenglama uchun iteratsiya usulini qo'llash mumkin.

Eslatma. Agar $ye^x-10x-2=0$ ni x ga nisbatan yechib

$$x=\ln(10x+2)$$

ko'rinishga keltirsak, $\varphi(x)=\ln(10x+2)$ bo'lib, bu funktsiya [-1,0] oraliqda 11.1-teorema shartlarini qanoatlantirmaydi. Demak, $x=\ln(10x+2)$ tenglamaga iteratsiya usulini qo'llab bo'lmaydi.

Endi $\varepsilon=0.01$ aniqlik bilan $x=(e^x-2)/10$ tenglamani iteratsiya usuli bilan yechamiz. Dastlabki yaqinlashish sifatida $t_0=0$ ni olamiz. Bu holda

$$t_1=\varphi(t_0)=\varphi(0)=(e^0-2)/10=-0.1;$$

$$t_2=\varphi(t_1)=\varphi(-0.1)=(e^{-0.1}-2)/10=-0{,}1095.$$

$|t_2-t_1|=0.0095<\varepsilon$ bo'lganligi uchun $t=-0.1095\approx-0.11$ ni berilgan tenglamaning 0,01 aniqlikdagi taqribiy ildizi uchun qabul qilamiz.

12-laboratoriya ishi.

LAGRANJ INTERPOLYaTSIYa KO'PHADINI TOPISH

Maqsad: Tajriba natijalarida topilgan qiymatlarning o'zgaruvchilari orasidagi bog'lanishni Lagranj interpolyatsiya ko'phadini yordamida topishni o'rganish.

Reja:

1. Interpolyatsiya masalasini kuyilishi.

2.Lagranj interpolyatsiya ko'phadini topish.

12.1. Interpolyatsiya masalasini kuyilishi.

Agar u=f(x) funktsiya [a,v] kesmaning x_k, k=0,1,2......,n nuktalarda $f(x_k)=u_k$ qiymatlarga ega bo'lsa, kuyidagi jadvalni tuzish mumkin.

$$\begin{cases} x & x_0\ x_1\ x_2 \ldots x_n \\ y & y_0\ y_1\ y_2 \ldots y_n \end{cases} \qquad (12.1)$$

Bu jadvalni asosida berigan funktsiyani polinomi yoki ko'phadini

$$R_n(x)=a_0x^n+ a_1x^{n-1}+ a_2x^{n-2}+\ldots+ a_{n-1}x+ a_n \qquad (12.2)$$

topish uchun quyidagicha shart qo'yamiz: jadvalning har bir x_k, k=0,1,2,...,n nuktada

$$R_n(x_k) \approx f(x_k)=u_k \qquad (12.3)$$

munosabat urinla bo'lsin. Bunday masala interpolyatsiyalash deyiladi. Topilgan ko'phadini interpolyatsiya ko'phadi deyiladi. Topilgan interpolyatsiya ko'phadi asosida biror $[x_k ,x_{k+1}]$ oralikka tegishli x ning taqribiy qiymatini topish masalasini xam yechamiz.

Ikkinchi tartibli

$$R_2(x)=a_0x^2+ a_1x+ a_2 \qquad (12.4)$$

bu ko'phadining koeffitsentlarini

$$R_2(x_i)=y_i ,\ i=0,1,2 \qquad (12.5)$$

shart saosida topish masalasini kuyamiz. Xakikatan xam $x=x_0$, $x= x_1$, $x=x_2$ larda (12.5) shart asosida kuyidagi sistemani tuzamiz:

$$a_0x_0^2+ a_1x_0+ a_2= y_0$$
$$a_0x_1^2+ a_1x_1+ a_2= y_1$$
$$a_0x_2^2+ a_1x_2+ a_2= y_2$$

Bu sistemadagi nomalumlar- koeffitsentlarini

$$D=\begin{vmatrix} X_0^2 & x_0 & 1 \\ x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \end{vmatrix}=(x_1-x_0)\ (x_2-x_0)\ (x_3-x_0)\neq 0$$

bo'lganda topish mumkin. Lyokin yuqori tartibli ko'phadilarni topishda tuziladigan sistemalarni yechish kiyinlashadi. SHuning uchun Lagranj usulidan foydalanamiz.

12.2. Lagranj interpolyatsiyalash formulasi va uning axamiyati.

Yuqoridagi jadval asosida topiladigan ko'phadini quyidagicha tanlaymiz:

$$R_n(x)=a_0(x-x_1)(x-x_2)(x-x_3)\dots(x-x_n)+$$
$$+a_1(x-x_0)(x-x_2)(x-x_3)\dots(x-x_n)+ \qquad (12.6)$$
$$+\dots\dots\dots\dots+$$
$$+a_n(x-x_1)(x-x_2)(x-x_3)\dots(x-x_{n-1})$$

bunda n=2 uchun

$$R_2(x)=a_0(x-x_1)(x-x_2)+a_2(x-x_0)(x-x_2)+a_3(x-x_1)(x-x_2) \qquad (12.7)$$

Bu a_0, a_1, a_2 koeffitsentlarini topish uchun

$$R_2(x_0)=u_0,\ R_2(x_1)=u_1,\ R_2(x_2)=u_2$$

SHartga asosan kuyidagi sistemani topamiz:

$$a_0(x-x_1)(x-x_2)=y_0$$
$$a_1(x-x_0)(x-x_2)=y_1$$
$$a_2(x-x_1)(x-x_2)=y_2$$

bundan:

$$a_0=y_0/(x-x_1)(x-x_2),$$
$$a_1=y_1/(x-x_0)(x-x_2),$$
$$a_2=y_2/(x-x_1)(x-x_2)$$

Endi izlanaetgan interpolyatsiya ko'phadini yozamiz:

$$P_2(x)=y_0\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}+y_1\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}+y_2\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

SHuningdek n=3 bo'lganda:

$$P_3(x)=y_0\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}+y_1\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}+$$
$$+y_2\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}+y_3\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

Demak Lagranj interpolyatsiya ko'phadini umumiy xolda kuyidagicha yozamiz:

$$P_n(x)=\sum_{j=0}^{n} y_j \prod_{i\neq j}\frac{(x-x_i)}{(x_j-x_i)}, \qquad (12.8)$$

12.1-Masala. Quyidagi, y=ln x funktsiya asosida tuzilgan

X	2	3	4	5
U	0.6931	1.0986	1.3863	1.6094

Jadvaldan foydalanib Lagranj interpolyatsiya ko'phadini toping va bu ko'phadilar yerdamida ln 3.5 ni hisoblang.

E CH I SH.

$$L_3(x)=\frac{(x-3)(x-4)(x-5)}{(2-3)(2-4)(1-5)}0.6981+\frac{(x-2)(x-4)(x-5)}{(3-2)(3-4)(3-5)}1.0986+$$
$$+\frac{(x-2)(x-3)(x-5)}{(4-2)(4-3)(4-5)}1.3865+\frac{(x-2)(x-3)(x-4)}{(5-2)(5-3)(5-4)}1.6094=$$

$=0.0089\ X^3-0.1387X^2+0.9305\ X-0.6841$

Hosil bo'lgan ko'phadga asosan

$$\ln 3{,}5\approx L(3{,}5)=0.0089*(3.5)^3-0.1387(3{,}5)^2+0.9805(3.5)-0.684=$$

$$=0.31-1.701+3.2567-0.6841=1.25145$$

bo'ladi.

13-laboratoriya ishi.

NG'YuTON INTERPOLYaTSIYa KO'PHADINI TOPISH.

Maqsad: Tajriba natijalarida topilgan qiymatlarning o'zgaruvchilari orasidagi bog'lanishni **Ng'yuton** interpolyatsiya ko'phadini yordamida topishni o'rganish.

1. CHekli ayirmalar masalasini kuyilishi.

2. Ng'yuton interpolyatsiya ko'phadini topish.

13.1. CHekli ayirmalar masalasini kuyilishi.

Berilgan jadvaldagi x_i, i=1,2,...,n lar bir xil h uzoklikda bo'lsa, ularga mos $y_i=f(x_i)$ i=1,2,...,n lar asosida kuyidagi ayirmalarni tuzamiz:

$$y_1-y_0=f(x_1)-f(x_0)$$
$$y_2-y_1=f(x_2)-f(x_1)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$y_n-y_{n-1}=f(x_n)-f(x_{n-1})$$

Bu ayirmalarni birinchi tartibli chekli ayirmalar deb ataladi.

SHuningdek

1- tartibli	2- tartibli
$\Delta u_0=y_1-y_0$	$\Delta^2 u_0=\Delta y_1-\Delta y_0$
$\Delta u_2=y_2-y_1$	$\Delta^2 u_1=\Delta y_2-\Delta y_1$
....	....
$\Delta u_k=y_{k+1}-y_k$	$\Delta^2 u_k=\Delta y_{k+1}-\Delta y_k$
....	....
$\Delta u_{n-1}=y_n-y_{n-1}$	$\Delta^2 u_{n-1}=\Delta y_n-\Delta y_{n-1}$

3- tartibli: $\Delta^3 u_0=\Delta^2 y_1-\Delta^2 y_0$, $\Delta^3 u_1=\Delta^2 y_2-\Delta^2 y_1$, ...

r- tartibli: $\Delta^p u_k=\Delta^{p-1} y_{k+1}-\Delta^p y_k$, k=1,2,...,n

X	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$
$X_1=x_0+h$	y_1	Δy_1	$\Delta^2 y_1$	$\Delta^3 y_1$	$\Delta^4 y_1$	
$X_2=x_0+2h$	y_2	Δy_2	$\Delta^2 y_2$	$\Delta^3 y_2$		
$X_3=x_0+3h$	y_3	Δy_3	$\Delta^2 y_3$			
$X_4=x_0+4h$	y_4	Δy_4				
$X_5=x_0+5h$	y_5					
....	...					

13.2. Ng'yuton interpolyatsiyalash formulasi.

1. Berilgan jadvalda mos $y_i=f(x_i)$ i=1,2,...,n larga mos x_i, i=1,2,...,n lar bir xil h uzoqlikda bo'lganda, bu bog'lanishni ifodalovchi interpolyatsiya ko'phadini quyidagicha topamiz.

Bu ko'phadini quyidagi ko'rinishda izlaymiz:

$$\begin{aligned} R_n(x) &= A_0+A_1(x-x_0)+ \\ &+A_2(x-x_0)(x-x_1)+ \\ &+A_3(x-x_0)(x-x_1)(x-x_2)+ \\ &+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+ \\ &+A_n(x-x_0)(x-x_1)(x-x_2)\ldots(x-x_{n-1}) \end{aligned} \tag{13.1}$$

Bu yerdagi A_i, i=1,2,..n koeffitsentlarni topish uchun jadvaldagi mos x va u larning qiymatlarini izlanayotgan ko'phadiga kuyamiz.

$x=x_0$ da: $u_0=A_0$; $A_0 = u_0$

$x=x_1$ da: $u_1= A_0+A_1(x_1-x_0)= A_0+A_1h= u_0 +A_1h$;

$u_1= u_0 +A_1h$; $A_1=(u_1- u_0)/h$,

$$A_1 = \frac{y_1 - y_0}{1!h} = \frac{\Delta y_0}{1!h}$$

$x=x_2$ da: $u_2 =A_0+A_1(x_2-x_0)+A_2 (x_2-x_0) (x_2-x_1)$

$u_2 =A_0+A_1 2h+A_2\, 2h\, h$

A_0 va A_1 larning qiymatlarini hisobga olib

$u_2 =u_0+\Delta u 2h/h+A_2\, 2h^2$,

$A_2\, 2h^2= u_2 -u_0-2\Delta u=\Delta u_1-\Delta u_0=\Delta^2 u_0$,

$$A_2 = \frac{\Delta^2 y_0}{2!h^2}$$

SHuningdek ketma-ket koeffitsentlarni topish formulasi: $A_3 = \dfrac{\Delta^3 y_0}{3!h^3}, \ldots\ldots, A_k = \dfrac{\Delta^k y_0}{k!h^k}$

Topilgan koeffitsentlar asosida izlanayotgan interpolyatsiya ko'phadini quyidagicha topamiz:

$$P_n(x) = y_0 + \frac{\Delta\, y_0}{1!h}(x-x_0) + \frac{\Delta^2 y_0}{2!h^2}(x-x_0)(x-x_1) + \frac{\Delta^3 y_0}{3!h^3}(x-x_0)(x-x_1)(x-x_2) + ,\ldots \tag{13.2}$$

Bu Ng'yutonning birinchi interpolyatsiya ko'phadi deyiladi.

1. Ng'yutonning birinchi interpolyatsiya ko'phadida kuyidagicha almashtirish

$$\frac{x-x_0}{h} = t,$$

$$\frac{x-x_1}{h} = \frac{x-(x_0+h)}{h} = \frac{x-x_0}{h} - 1 = t-1,$$

$$\frac{x-x_2}{h} = \frac{x-(x_0+2h)}{h} = \frac{x-x_0}{h} - 2 = t-2$$

va hakazo

$$\frac{x-x_k}{h}==t-k$$

Bu almashtirishlarni hisobga olib (13.2) formulani kuyidagicha yozamiz:

$$P_n(x)=P_n(x_0+ht)=$$

$$=y_0+\frac{\Delta\ y_0}{1!}t+\frac{\Delta^2 y_0}{2!}t(t-1)(x-x_1)+\frac{\Delta^3 y_0}{3!}t(t-1)(t-2)+...+\frac{\Delta^n y_0}{n!}t(t-1)(t-2)+,...+[t-(n-1)],. \qquad (13.3)$$

Bu Ng'yutoning 2- interpolyatsiya ko'phadi deyiladi.

14.2-Masala. Quyidagi, y=ln x funktsiya asosida tuzilgan

X	2	3	4	5
U	0.6931	1.0986	1.3863	1.6094

Jadvaldan foydalanib Ng'yuton interpolyatsiya ko'phadilarini toping va bu ko'phadilar yerdamida ln 3.5 ni hisoblang.

Ng'yutoning interpolyatsiya ko'phadini tuzish uchun chekli ayrimalarning jadvalini tuzamiz:

X	U	ΔU	Δ^2U	Δ^3U
2	0.6931			
3	1.0986	0.1055	-0.1178	0.0532
4	1.3863	0.2877	-0.0646	
5	1.6094	0.2231		

(13.2) formulaga asosan n=3, h=1 bo'lgani uchun

$$P_3(x)=y_0+\frac{\Delta y_0}{h}(x-x_0)+\frac{\Delta^2 y_0}{2!h^2}(x-x_0)(x-x_1)+$$

$$+\frac{\Delta^3 y_0}{3!h^3}(x-x_0)(x-x_1)(x-x_2)=0.6941+0.4055(x-2)-$$

$$-\frac{0.1178}{2}(x-2)(x-3)+\frac{0.0532}{6}(x-2)(x-3)(x-4)=$$

$$= -\ 0.6841-\ 0.9305X-\ 0.1387X^2+0.0089X^3$$

Bu ko'phadidan foydalanib

$$\ln 3{,}5=R_3(3.5)=1.2552$$

ekanligini hisoblab topamiz.

14-laboratoriya ishi.

ANIQ INTEGRALNI TAQRIBIY HISOBLASH

Maqsad: Aniq integralni taqribiy hisoblash usullarini o'rganish

Reja:

1. To'g'ri to'rtburchaklar formulasi.

2. Trapetsiyalar formulasi.

3. Simpson yoki parabola formulasi.

Integrallanuvchi f(x) funktsiyaning boshlang'ichini mahlum funktsiyalar orqali ifodalash mumkin bo'lmaganda, f(x) funktsiya jadval yoki grafik usulda berilganda integrallni taqribiy hisoblashga to'gri keladi.

Aytaylik [a ; v] oraliqda f(x) funktsiya grafigi yordamida x=a, x=v xamda u=0(O_x) chiziqlar bilan chegaralangan yuzani hisoblash kerak bo'lsin.

Quyida bunday yuzalarni taqribiy hisoblash usullarini ko'ramiz.

1.14. To'g'ri to'rtburchaklar formulasi.

Berilgan [*a;b*] oraliqda qadami h=(b-a)/n bo'linish nuqtalari.

$$x_0=a,\ x_i=x_{i-1}+h,\ i=1, 2, 3, 4, 5, \ldots n$$

bo'lganda f(x) funktsiya bo'yicha olingan aniq integralni taqribiy hisoblash formulasi quyidagicha bo'ladi.

$$\int_{в}^{a} f(x)dx = h\sum_{i=1}^{n} f(x_1) = h(y_1 + y_2 + \ldots y_n)$$

14.2. Trapetsiyalar formulasi.

Berilgan [a;v] oraliqda qadami $h=\dfrac{b-a}{n}$ va bo'linish nuqtalari.

$$x_0=0,\ x_k=x_{k-1}+h,\ k=1, 2, \ldots n$$

bo'lganda aniq integralni hisoblash formulasi quyidagicha bo'ladi.

$$\int_{a}^{в} f(x)dx = h\left(\frac{f(a)+f(b)}{2} + \sum_{r=1}^{n-1} f(x_k)\right)$$

14.3. Simpson yoki parabola formulasi.

Berilgan [a;v] oraliqda qadami $h=\dfrac{b-a}{n}$ va bo'linish nuqtalari.

$$x_0=0,\ x_k=x_{k-1}+h,\ k=1, 2, \ldots n$$

bo'lganda aniq integralni taqribiy hisoblash formulasi quyidagicha bo'ladi.

$$\int_{a}^{b} f(x)dx = \frac{h}{3}[f(a)+f(b)+4\sum_{k=1}^{n} f(x_{2x-1}) + 2\sum_{k=1}^{n-1} f(x_{2k})]$$

<u>1-masala.</u>

Quyidagi $\displaystyle\int_{2}^{3,5} \frac{dx}{\sqrt{5+4x-x^2}}$

aniq integralni:

1) To’gri to’rtburchak formulasi bilan.
2) Trapetsiya formulasi bilan.
3) Simpson formulasi bilan.

Bo’linishlar soni 10 bo’lganda Ye=0,001 aniqlikda hisoblang.

Echish.

Integral ostidagi funktsiya qiymatlarini bo’linish qadami

h=(b-a)/n=(3,5-2)/10=0,15

bo’lganda, bo’linish nuqtalari x_i=a+ i h, i =1,10
bo’lsa, nuqtalarni [2;3,5] oraliqda aniqlab, bu nuqtalarda integral ostidagi funktsiya qiymatlarini topamiz.

x_0=2,00 y_0 $y_0 = f(2) = \dfrac{1}{\sqrt{5+42-3^2}} = 0{,}3333$

x_1=2,15 $y_1 = f(2{,}5) = \dfrac{1}{\sqrt{5+4(2{,}5)-(2{,}5)^2}} = 0{,}3388$

x_2=2,30 u_2=f(2,30)=0,3350
x_3=2,45 u_3=f(2,45)=0,3371
x_4=2,60 u_4=f(2,60)=0,3402
x_5=2,75 u_5=f(2,75)=0,3443
x_6=2,90 u_6=f(2,90)=0,3494
x_7=3,05 u_7=f(3,05)=0,3558
x_8=3,20 u_8=f(3,20)=0,3637
x_9=3,35 u_9=f(3,35)=0,3733
x_{10}=3,50 u_{10}=f(3,50)=0,3849

Yukoridagi x va u qiymatlarga asosan to’g’ri **to’rtburchak** formulasiga asosan.

$$\int_2^{3,5}\frac{dx}{\sqrt{5+4x-x^2}} = 0{,}15(0{,}3333+0{,}3388+0{,}3350+0{,}3371+0{,}3402+0{,}3443+0{,}3494+$$
$$+0{,}3858+0{,}3637+0{,}3733+0{,}3849)=0{,}5755$$

Trapetsiya formulasiga asosan.

$$\int_2^{3,5}\frac{dx}{\sqrt{5+4x-x^2}} = 0{,}15\left(\frac{0{,}3333+0{,}3849}{2}+0{,}3388+0{,}3350+0{,}3371+0{,}3402+\right.$$
$$\left.+0{,}3443+0{,}3494+0{,}3858+0{,}3637+0{,}3733\right)=0{,}15*3{,}49178=0{,}52376$$

Simpon formulasiga asosan:

$$\int_2^{3,5}\frac{dx}{\sqrt{5+4x-x^2}} \approx \frac{0{,}15}{3}[0{,}3333-0{,}3849+4(0{,}3338+0{,}3371+0{,}3443+0{,}3558+$$
$$+0{,}3733)+2(0{,}3350+0{,}3402+0{,}3494+0{,}3637)]=0{,}05(1{,}0515+4*1{,}7443+$$
$$+2*1{,}3883)=0{,}05*10{,}8013=0{,}54265.$$

15-laboratoriya ishi.

BIRINCHI TARTIBLI ODDIY DIFFERENTSIAL TENGLAMA UCHUN KOSHI MASALASINI TAQRIBIY YeCHISH.

Maqsad: Birinchi tartibli oddiy differentsial tenglama uchun Koshi masalasini taqribiy yechish usullarini o'rganish.

Reja:

15.1. Eyler usuli
15.2. Runge – kutga usuli

Aytaylik bizga birinchi tartibli

$$\frac{dy}{dx} = f(x;y) \quad \text{yoki} \quad uo = f(x;y) \qquad (15.1)$$

differentsial tenglama berilgan bo'lib, [x;v] kesmada

$$x=x_0,\ u=u_0 \qquad (15.2)$$

boshlang'ich shartni kanoatlantiruvchi yechimni taqribiy hisoblash masalasi quyilgan bo'lsin. Bu masala Koshi masalasi deyiladi. Bu masalani taqribiy yechishning bir necha usullarni ko'ramiz.

15.1. Eyler usuli

Berilgan $[x_0;v]$ kesmani n ta teng bo'lakka bo'lib bo'linish nuqtalari orasidagi qadam

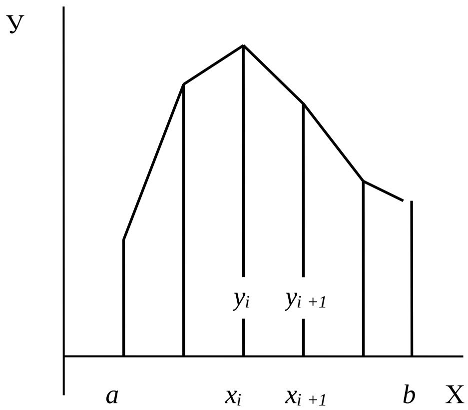
$$h=(b-x_0)/n \qquad (15.3)$$

bo'lganda, bu nuqtalar koordinatalari

$$x_i=x_{i-1}+h, \quad i=1, 2, \ldots n \qquad (15.4)$$

bo'ladi. x_0 va u_0 lardan foydalanib tenglama yechimining qiymatlarini taqriban quyidagicha hisoblaymiz.

$$\begin{aligned} y_1&=y_0+hf(x_0;y_0)\\ y_2&=y_1+hf(x_1;y_1)\\ y_3&=y_2+hf(x_2;y_2) \qquad (15.5)\\ &------------------\\ y_n&=y_{n-1}+hf(x_{n-1};y_{n-1}) \end{aligned}$$



natijada izlanaetgan yechimni kanoatlantiruvchi

$$(x_0;y_0),\ (x_1;y_1),\ (x_2;y_2),\ \ldots\ldots,\ (x_n;y_n)$$

nuqtalarni aniqlaymiz. Bu nuqtalarni tutashtiruvchi sinik chiziq Eyler chiziqi deb ataladi va u tenglama yechimining taqribiy grafigini ifodalydi.

15.2. Runge – Kutta usuli

Birinchi tartibli (15.I) tenglamani (15.2) shartni kanoatlantiruvchi taqribiy yechimni Runge – kutga usuli bilan quyidagicha topamiz.

$$x_i=x_{i-1}+h \qquad (x_i=x_0;\ y_i=y_0)$$

$$Q^{(i)}=h\,f(x_i;y_i)$$

$$
\begin{aligned}
Q_2^{(i)} &= hf(x_i+\frac{h}{2};\ \ y_i+Q_1^{(i)}/2)\\
Q_3^{(i)} &= hf(x_i+\frac{h}{2};\ \ y_i+Q_2^{(i)}/2)\\
Q_4^{(i)} &= hf(x_i+\frac{h}{2};\ \ y_i+Q_3^{(i)}/2)\\
\Delta y_i &= (Q_1^{(i)}+2Q_2^{(i)}+2Q_3^{(i)}+Q_4^{(i)})/6
\end{aligned}
\qquad (15.5)
$$

$$y_{i+1}=y_i+\Delta y_i$$

$$i=0,\ 1,\ 2,\ 3\ldots\ldots n.$$

Bu Runge – Kutta usuli Koshi masalasining yechimni to’rtinchi tartibli aniqlikda hisoblaydi.

15.1-masala.

Quyidagi.

$$y^1=x+\cos(\frac{y}{\sqrt{5}})$$

birinchi tartibli defferitsial tenglamaning

$$x_0=1{,}3 \qquad u_0=2{,}6$$

boshlang’ich shartni kanoatlantiruvchi [1,8;2,8] oraliqda yechimini h=0,1 qadami bilan, Ye=0,001 aniqlikda:

1. Eyler usuli;
2. Runge – kutga usuli;

bilan hisoblang.

Echish.

1. Berilgan differentsial tenglamani Eyler usulida yechamiz, [1,8;2,8] oraliqda

$$n=\frac{b-a}{h}=\frac{2.8-1.8}{0.1}=10$$

bo’lganidan oraliqni n=10 ta bo’lakka ajratamiz. Bo’linish nuqtalarini:

$$x_i=x_{i-1}+h \qquad i=1,2,...,10$$

formulaga asosan topamiz.

$$x_1=x_0+h=1.8+0.1=1.9$$
$$x_2=x_1+h=1.9+0.1=2.0$$

shuningdek

$x_3=2.1$, $x_4=2.2$, $x_5=2.3$, $x_6=2.4$, $x_7=2.5$, $x_8=2.6$, $x_9=2.7$, $x_{10}=2.8$

Berilgan tenglamaning ung tomonidagi

$$F(x;y)=x+\cos(y/\sqrt{5})$$

funktsiyaga asosan, Eyler qoidasi bilan quyidagi

$$y_{i+1}=y_i+h\,f(x_i;y_i), \qquad i=1,2,...,10$$

formulaga asosan berilgan differitsial tenglama yechimining qiymatlarini quyidagicha topamiz.

$$y_1=y_0+hf(x_0;y_0)=y_0+h(x_0+\cos(y_0/\sqrt{5}))=$$
$$=2.6+0.1(1.8+\cos(2.6/\sqrt{5}))=2.6+0.1(1.8+0.3968)=2.81968$$
$$y_2=y_1+h\,f(x_1;y_1)=y_1+h(x_1+\cos(y_1/\sqrt{5}))=$$
$$=2.819+0.1(1.9+\cos(2.819/\sqrt{5}))=2.819+0.1(1.9+0.3968)=3.03948$$

SHuningdek, quyidagilarni topamiz:

$$y_3=3.261,\ y_4=3.4831,\ y_5=3.7045,\ y_6=3.926$$
$$y_7=4.1478,\ y_8=4.3701,\ y_9=4.5931,\ y_{10}=4.8173$$

2. Berilgan tenglamani RUNGE-KUTTA usulida xisoblaymiz.

Runge-Kutta usulida tenglamani yechimini topish uchun quyidagi hisoblash ketma-ketligini bajaramiz.

i=0 bo'lganda $x_{0=}1.3$ $u_0=2.6$ lar uchun yechimning birinchi qiymatini hisoblaymiz.

$$Q_1^0=hf(x,y)=0.1(x_0+\cos(y0/\sqrt{5})=0.1(1.8+\cos(2.6/\sqrt{5})=0.2196$$
$$Q_2^0{}_= hf(x_0+h/2,\ y_0+Q_1^0/2)=0.201245$$
$$Q_3^0{}_= hf(x_0+h/2,\ y_0+Q_2^0/2)=0.2205$$
$$Q_4^0{}_= hf(x_0+h/2,\ y_0+Q_3^0/2)=0.2927$$
$$Y_1=Y_0+(Q_1^0+2Q_2^0+2Q_3^0+Q_4^0)/\ 6=2.02596$$

Demak, berilgan tenglamaning birinchi qiymati

$$u_1=2.02596$$

bo'ladi. Yuqoridagi qoidani

$$i=1,\ x_1=1.9,\ u_1=2.02596$$

lar uchun qo'llab,

$$u_2=3.0408$$

ni topamiz. SHuningdek, i=2,3,…,10 lar uchun tenglama yechimini qolgan qiymatlarini topamiz.

$$u_3=3.2619 \quad u_4=3.4831$$
$$u_5=3.7045 \quad u_6=3.9260$$
$$u_7=4.1478 \quad u_8=4.370$$
$$u_9=4.5931 \quad u_{10}=4.9172$$

16-laboratoriya ishi.

BIRINCHI TARTIBLI DIFFERENTSIAL TNEGLAMALAR SISTEMASI VA IKKINCHI TARTIBLI DIFFERENTSIAL TNEGLAMA UCHUN KOSHI MASALASINI EYLER USULIDA TAQRIBIY YeCHISH

Maqsad: Birinchi tartibli differentsial tneglamalar sistemasi uchun Koshi masalasini takribiy yechishda Eyler usuli va uni ikkinchi tartibli differentsial tneglamalarni yechishda qo'llash.

Reja:

1. Birinchi tartibli differentsial tneglamalar sistemasi uchun Koshi masalasini takribiy yechish.
2. **Ikkinchi** tartibli differentsial tneglamalarni yechish

Quyidagi

$$y' = f_1(x, y, z), \quad z' = f_2(x, y, z,) \qquad (16.1)$$

Birinchi tartibli diffirintsial tenglamalar sistemasini [a, v] oraliqdagi boshlang'ich

U(X0)=U0 , Z(X0)=Z0 (16.2)

SHartlarni kanoatlantiruvchi yechimni kiymatini topish uchun Eyler va Runge-Kutta usullarini kullaymiz.

EYLER USULI.

Berilgan differentsial tenglamalar sistemasining oraliqdagi yechimini topish uchun

Xi=X0+ih . i=0,1,2,…,n

larni topib, har bir tenglama uchun Eyler usulini ko'llaymiz.

Yi+1=Yi+hf1(Xi; Yi; Zi) (16.3)

Zi+1=Yi+hf2(Xi; Yi; Zi)

Natijada differentsial tenglamalar sistemasi yechimining taqribiy qiymatini topamiz.

Y(Xi)=Yi Z(Xi)=Zi i=1,2,3,4,5,…,n

Kuyidagi 16.1-masalani yordamida berilgan ikkinchi tartibli differentsial tenglamani yechimini birinchi tartibli differentsial tenglamalar sistemasiga keltirib uchimni topish qulayligini ko'rsatamiz.

16.1-masala. Kuyidagi

y"+yo/x+y=0

Differentsial tenglamani

y(1)=077 ; yo(1)=-044

Boshlang'ich shartlarni qanoatlantiruvchi, qadami *h=01* **bO'lgan, [1;1.5] oralikdagi yechimi Eyler usuli bilan topilsin.**

ECHISH.

Berilgan differentsial tenglamada

$$y_0=z.\ y''=z_0$$

almashtirish qilib, quyidagi birinchi tartibli differentsial tenglamalar sistemasiga kelamiz.

$$\begin{cases} y'= z \\ z'= -z/x - y \end{cases}$$

boshlangich shartlari esa

$$Y(1)=077 \qquad Z(1)=-044$$

kabi yoziladi

Bu xolda (16.1) differentsial tenglamalar sistemasiga asosan.

$$\begin{cases} f_1(x.y.z) = z = 0.x + 0.y + z \\ f_2(x.y.z) = -z/x - y \end{cases}$$

Endi xosil bulgan(*) differentsial tenglamalar sistemani Eyler va Runge- Kutta usuli bilan yechimini topamiz.

Eyler usulida yechishda (16.3) formulaga asosan

$$X_i=X_0+ih$$
$$Y_{i+1}=Y_i+hf1(X_i,Y_i,Z_i)$$
$$Z_{i+1}=Z_i+hf2(X_i,Y_i,Z_i)$$
$$I=0,1,2,3,...$$

$I=0\ \ X_0=1.0,\ Y_0=0.77,\ Z_0=-0.44$

$$Y_1=Y_0+hf1(X_0,Y_0,Z_0)=0.77+0.1(Z_0)=0.726$$
$$Z_1=Z_0+hf_2(X_0,Y_0,Z_0)=-0.44+0.1(-Z_0/X_0-Y_0)=-0.473$$

$I=1\ X_1=1.1,\ \ Y_1=0.726,\ \ Z_1=-0.473$

$$Y_2=Y_1+hf_1(X_1,Y_1,Z_1)=0.726+0.1(-0.473)=0.679$$
$$Z_2=Z_1+hf_2(X_1,Y_1,Z_1)=-0.473+0.1(-0.473/1.1-0.726)=-0.503$$

Bu koidani $I=2,3,4,5$ lar uchun ketma-ket takrorlab tenglamalar sistemasi yechimining

$$Y_3=0.6284,\ Y_4=0.5756,\ Y_5=0.5205$$

qiymatini xisoblab topamiz.

17-laboratoriya ishi.

KICHIK KVADRATLAR USULI.
TAJRIBA NATIJALARINIG CHIZIQLI VA PARABOLIK BOG'LANINSHINI ANIQLASH.

Maqsad: Kichik kvadratlar usulida tajriba natijalarida topilgan qiymatlar orasidagi chiziqli va parabolik bog'laninshini aniqlash.

Reja:

1. Kichik kvadratlar usuli
2. To'g'ri chiziqi **bog'laninsh** tenlamasini aniqlash.
3. Ikkinch darajali **bog'laninsh** tenglamasini topish.

17.1. Kichik kvadratlar usuli

Aytaylik tajriba natijalari quyidagi jadval asosida berilgan bo'lsin.

X	$x_1, x_2\ x_3 \ldots x_n$
Y	$y_1, y_2\ y_3 \ldots y_n$

Bu ikki o'zgaruvchi orasidagi bog'lanish formulasinianalinik usulda aniqlash masalasini yechamiz. Bu masalani kichik kvadratlar usuli bilan yechamiz. Buning uchun bog'laninshni ifodalovchi funktsiyalar turini tanlaymiz. Masalan:

1) chiziqli bog'laninsh: $y=ax+b$
2) parabolik bog'laninsh: $y=ax^2+bx+s$

Bu bog'lanishlarni aniqlashda ularning koeffitsentlarini aniqlash asosiy masala hisoblanadi. Umumiylik uchun izlanayotgan funktsiyani

$$y=F(x,a,b,c)$$

ko'rinishda oxtaramiz. Bu bog'lanish koeffitsentlarini aniqlash uchun berilgan jadval asocida

$$F(x_i,a,b,c)=y_i\ ,\ i=1,2,...,n$$

shartni yozamiz. Izlanayotgan funktsiya va y_i lar orasidagi farq minimum yoki kichik bo'lish shartini topamiz. Buning uchun quyidagi funktsianalni tuzamiz:

$$F(a,b,c)=\sum[y_i - F(x_i,a,b,c)]^2, \quad i=1,2,...,n$$

Bu F(a,b,c) funktsianing minimumini topish uchun quyidagi zaruriy sharttan foydalanamiz.

$$Fo_a(a,b,c)=0,\ Fo_b(a,b,c)=0.\ Fo_c(a,b,c)=0$$

yaoni

$$\sum[y_i - F(x_i,a,b,c)]\ Fo_a(a,b,c)=0$$
$$\sum[y_i - F(x_i,a,b,c)]\ Fo_b(a,b,c)=0$$
$$\sum[y_i - F(x_i,a,b,c)]\ Fo_c(a,b,c)=0$$

Ushbu sistemeni yechish bilan a,b,c larni topamiz va jadvalni ifodalovchi bog'lanish funktsiasini topamiz.

17.2. To'g'ri chiziqi bog'laninsh tenlamasini aniqlash.

CHiziqli bog'lanish $F(x_i,a,b)=a\,x_i +b$, uchun $Fo_a= x_i$, $Fo_b=1$

$$\sum[y_i - ax_i-b)]^2\ xi=0$$
$$\sum[y_i - ax_i-b)]^2\ 1=0$$

$$\left.\begin{aligned}\left(\sum x_i^{\ 2}\right)a+\left(\sum x_i\right)b=\sum x_i\cdot y_i\\ \left(\sum x_i\right)a+nb=\sum y_i\end{aligned}\right\}$$

Bu sistemani *a.b* larga nisbatan yechamiz:

$$a=\frac{\sum x_i\cdot y_i-\sum x_i\cdot\sum y_i}{\sum x_i^{\,2}-\left(\sum x_i\right)^2}$$

$$b=\frac{\sum y_i\cdot\sum x_i^2-\sum x_i\cdot\sum x_iy_i}{\sum x_i^2+\left(\sum x_i\right)^2}$$

17.3. Ikkinch darajali bog'laninsh tenglamasini topish.

Parabolik bog'laninsh: $F(x_i,a,b,s)=ax_i^2+bx_i+s$, uchun

$$Fo_a=x_i^2,\ Fo_b=x_i,\ Fo_s=1$$
$$\sum[y_i-ax_i^2-bx_i-s)]\,x_i^2=0$$
$$\sum[y_i-ax_i^2-bx_i-s)]\,x_i=0$$
$$\sum[y_i-ax_i^2-bx_i-s)]\,1=0$$

Bu sistemani quyidagicha yozamiz

$$\begin{cases}\left(\sum x_i^4\right)a+\left(\sum x_i^3\right)b+\left(\sum x_i^2\right)c=\sum y_ix_i^2\\ \left(\sum x_i^3\right)a+\left(\sum x_i^2\right)b+\left(\sum x_i\right)c=\sum y_ix_i\\ \left(\sum x_i^2\right)a+\left(\sum x_i\right)b+nc=\sum y_i\end{cases}$$

va uni biror usul bilan yechib *a,b,s* larni topamiz.

18-laboratoriya ishi.

KICHIK KVADRATLAR USULIDA REGRESSIYa BOG'LANISHINING TO'G'RI CHIZIQLI VA IKKINCH DARAJALI TENLAMASINI ANIQLASH

Maqsad: Tajriba natijalarida topilgan qiymatlar orasidagi chiziqli va parabolik bog'laninshini kichik kvadratlar usulida aniqlash.

Reja:

1. Kichik kvadratlar usulida tajriba natijalarida topilgan qiymatlar orasidagi **regressiya** bog'laninshining to'g'ri chiziqli **tenlamasini** aniqlash.
2. Kichik kvadratlar usulida tajriba natijalarida topilgan qiymatlar orasidagi **regressiya** bog'laninshining ikkinch darajali **tenlamasini** aniqlash.

Quyidagi korrelyatsion jadval berilgan bo'lsin:

U/X	92	96	100	104	n_y
158	1	1			2
164	1	4	1		6
170		2	5	1	8
176			1	2	3
182				1	1
n_x	2	7	7	4	N=20
$\bar{y}_x$	161	164,8	170	176	

18.1. To'g'ri chiziqli regressiya tenglamasini topish.

Berilgan jadvaldagi mahlumotlar bo'yicha u ning x ga regressiya to'g'ri chiziqining tanlanma tenglamasini

$$u_x = ax + b \qquad (18.1)$$

ko'rinishda izlaylik

Buning uchun *a*, *b* parametrlarni $u_{xi} - \overline{y}_{xi}$ farqlar kvadratlari minimal bo'ladigan qilib tanlab olish imkonini beruvchi quyidagi tenglamalar sistemasini hosil qilamiz:

$$\left.\begin{aligned} \left(\sum n_x x^2\right)a + \left(\sum n_x x\right)b &= \sum n_x x \cdot \overline{y_x} \\ \left(\sum n_x x\right)a + nb &= \sum n_x \overline{y_x} \end{aligned}\right\} \qquad (18.2)$$

Bu sistemani yechib, a, v - parametrlarni aniqlovchi munosabatlarga ega bo'lamiz.

$$a = \frac{n\sum n_x x \cdot \overline{y_x} - \sum n_x x \cdot \sum n_x \overline{y_x}}{n\sum n_x x^2 - \left(\sum n_x x\right)^2} \qquad (18.3)$$

$$b = \frac{\sum n_x \overline{y_x} \cdot \sum n_x x^2 - \sum n_x x \cdot \sum n_x x\overline{y_x}}{n\sum n_x x^2 - \left(\sum n_x x\right)^2} \qquad (18.4)$$

Berilgan korrelatsion jadvaldagi mahlumotlar asosida quyidagi jadvalni tuzamiz:

2-jadval.

n_x	x	$\overline{y}_x$	$n_x x$	$n_x x^2$	$n_x \overline{y}_x$	$n_x x \overline{y}_x$
2	92	161	164	16928	316	29624
7	96	164,8	672	64512	1154	40746
7	100	170	700	70000	1190	119000
4	104	176	416	43264	704	73216
20			1972	1947004	3370	332586

2-jadvaldagi oxirgi qatorga yozilgan yig'indilarni (16.3) va (16.4) ga qo'yib,

$$a = \frac{20 \cdot 332586 - 1972 \cdot 3370}{20 \cdot 19477004 - 1972^2} = 1{,}3$$

$$b = \frac{3370 \cdot 194704 - 1972 \cdot 332586}{20 \cdot 194704 - 1972^2} = 40{,}8$$

Izlanayotgan regressiya tenglamasi:

$$u_x = 1{,}3x + 40{,}8$$

bu tenglama bo'yicha xisoblanadigan y_{xi} qiymatlar kuzatilgan $\overline{y}_{xi}$ qiymatalarga qanchalik mos kelishini topish uchun, u_{xi} va $\overline{y}_{xi}$ qiymatlari orasidagi farqlarni aniqlash maqsadida quyidagi jadvalni tuzamiz:

x_i	u_{xi}	$\overline{y}_{xi}$	$u_{xi} - \overline{y}_{xi}$
92	160,4	161	-0,6

96	165,4	164,8	0,8
100	170,8	170	0,8
104	176	176	0

Jadvaldagi farqlar bog'lanishining aniqligini ifodalab beradi. Bu jadvaldan ko'rinadiki chetlanishlarning hammasi ham yetarlicha kichik emas. Bu kuzatishlar sonining kamligi bilan izoxlanadi.

18.2. Ikkinch darajali regressiya tenglamasini topish.

Ikkinch darajali regressiya tenglamasini topishni quyidagi misol orqali izohlaymiz. Soddaroq bo'lishi uchun kichikroq jadval, hamda chiziqli bo'lmagan eng ommalashgan holi parabolik hol - kvadrat uchhad ko'rinishi bilan chegaralanamiz.

Masala. Quyidagi korrelyatsion jadvalda keltirilgan mahlumotlar bo'yicha $\bar{y}_x$ =Ax^2+Vx+S regressiya tenglamasini eng kichik kvadratlar usuli yordamida topamiz.

1-jadval

u \ x	2	3	5	n_y
25	20			20
45		30	1	31
110		1	48	49
n_x	20	31	49	N=100

Echish. SHartli o'rta qiymatlarni topamiz.

$$\bar{y}_2 = \frac{25\cdot 20}{20} = 25$$

$$\bar{y}_3 = \frac{45\cdot 30 + 110\cdot 1}{31} = 47{,}1$$

$$\bar{y}_5 = \frac{45\cdot 1 + 110\cdot 48}{49} = 108{,}67$$

2-jadval.

x	n_x	$\bar{y}_x$	$n_x x$	$n_x x^2$	$n_x x^3$	$n_x x^4$
2	20	25	40	80	60	320
3	31	47,1	93	279	837	2511
5	49	108,67	245	12285	6125	30625
$\sum$	100		378	1584	7122	33456

2-jadval(davomi)

x	$n_x \bar{y}_x$	$n_x \bar{y}_x$ x	$n_x \bar{y}_x$ x^2
2	500	1000	2000
3	4380	13140	39420
5	5325	26625	133125
$\sum$	10205	40765	1744545

2-jadval oxirida turgan yig'indilarni ushbu

$$\begin{cases} \left(\sum n_x x^4\right)A + \left(\sum n_x x^3\right)B + \left(\sum n_x x^2\right)C = \sum n_x \bar{y}_x x^2 \\ \left(\sum n_x x^3\right)A + \left(\sum n_x x^2\right)B + \left(\sum n_x x\right)C = \sum n_x \bar{y}_x x \\ \left(\sum n_x x^2\right)A + \left(\sum n_x x\right)B + nc = \sum n_x \bar{y}_x \end{cases}$$

Sistemaga qo'yib, quyidagi sistemani xosil qilamiz:

$$\begin{cases} 33456A + 7122B + 1584C = 174545 \\ 7122A + 1584B + 378C = 40765 \\ 1584A + 378B + 100C = 10205 \end{cases}$$

Cistemani yechib, A=-44,3, V=337,29, S=-472,78 qiymatlarni topamiz va regressiya tenglamasi:

$$\bar{y}_x = Ax^2 + Vx + S$$

ga qo'ysak, uning aniq ko'rinishi:

$$\bar{y}_x = -44{,}2x^2 + 337{,}29x - 472{,}78$$

ni hosil qilamiz.

MUSTAQIL ISHLASH UCHUN TOPSHIRIQLAR

Quyidagi berilgan jadval chiziqli tenglamalar sisitemasining koeffitsentlaridan tuzilgan bo'lib, bu jadval asosida **1,2,3,4-laboratoriya ishlarni** bajarishda foydalaniladi:

Masalan 1-variant quyidagi chiziqli tenglamalar sisitemasining koeffitsentlaridan tuzilgan:

$$\begin{cases} 5x_1 + 8x_2 - x_3 = -7 \\ x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 - 3x_2 + 2x_3 = 9 \end{cases}$$

V=1

a_{ij}	1	2	3	Oz.h
1	**5**	**8**	**-1**	**-7**
2	**1**	**2**	**3**	**1**
3	**2**	**-3**	**2**	**9**

V=2

a_{ij}	1	2	3	Oz.h
1	**1**	**2**	**1**	**4**
2	**3**	**-5**	**3**	**1**
3	**2**	**7**	**-1**	**8**

V=3

a_{ij}	1	2	3	Oz.h
1	**3**	**2**	**1**	**5**
2	**2**	**3**	**2**	**1**
3	**2**	**1**	**3**	**11**

V=4

a_{ij}	1	2	3	Oz.h
1	**1**	**2**	**4**	**31**
2	**5**	**1**	**2**	**29**
3	**3**	**-1**	**1**	**10**

V=5

a_{ij}	1	2	3	Oz.h
1	**4**	**-3**	**2**	**9**
2	**2**	**5**	**-3**	**4**
3	**5**	**6**	**-2**	**18**

V=6

a_{ij}	1	2	3	Oz.h
1	**2**	**-1**	**-1**	**4**
2	**3**	**4**	**-2**	**11**
3	**3**	**-2**	**4**	**11**

V=7

a_{ij}	1	2	3	Oz.h
1	**1**	**1**	**2**	**1**
2	**2**	**-1**	**2**	**4**
3	**4**	**1**	**4**	**-2**

V=8

a_{ij}	1	2	3	Oz.h
1	**3**	**-1**	**0**	**5**
2	**-2**	**1**	**1**	**0**
3	**2**	**-1**	**4**	**15**

V=9

a_{ij}	1	2	3	Oz.h
1	**3**	**-1**	**1**	**4**
2	**2**	**-5**	**-3**	**-17**
3	**1**	**1**	**-1**	**0**

V=10

a_{ij}	1	2	3	Oz.h
1	**1**	**1**	**1**	**2**
2	**2**	**-1**	**-6**	**-1**
3	**3**	**-2**	**0**	**8**

V=11

a_{ij}	1	2	3	Oz.h
1	**2**	**1**	**-1**	**1**
2	**1**	**1**	**1**	**6**
3	**3**	**-1**	**1**	**4**

V=12

a_{ij}	1	2	3	Oz.h
1	**2**	**-1**	**-3**	**3**
2	**3**	**4**	**-5**	**8**
3	**0**	**2**	**7**	**17**

V=13

a_{ij}	1	2	3	Oz.h
1	**1**	**5**	**1**	**-7**
2	**2**	**-1**	**-1**	**0**
3	**1**	**-2**	**-1**	**2**

V=14

a_{ij}	1	2	3	Oz.h
1	**1**	**-2**	**3**	**6**
2	**2**	**3**	**-4**	**16**
3	**3**	**-2**	**-5**	**12**

V=15

a_{ij}	1	2	3	Oz.h
1	**3**	**4**	**2**	**8**
2	**2**	**-1**	**-3**	**-1**
3	**1**	**5**	**1**	**0**

V=16

a_{ij}	1	2	3	Oz.h
1	**2**	**-1**	**3**	**7**
2	**1**	**3**	**-2**	**0**
3	**0**	**2**	**-1**	**2**

V=17

a_{ij}	1	2	3	Oz.h
1	**2**	**1**	**4**	**20**
2	**2**	**-1**	**-3**	**3**
3	**3**	**4**	**-5**	**-8**

V=18

a_{ij}	1	2	3	Oz.h
1	**1**	**-1**	**0**	**4**
2	**2**	**3**	**1**	**1**
3	**2**	**1**	**3**	**11**

V=19

a_{ij}	1	2	3	Oz.h
1	**1**	**5**	**-1**	**7**
2	**2**	**-1**	**-1**	**4**
3	**3**	**-2**	**4**	**11**

V=20

a_{ij}	1	2	3	Oz.h
1	**11**	**3**	**-1**	**2**
2	**2**	**5**	**-5**	**0**
3	**1**	**1**	**1**	**2**

V=21

a_{ij}	1	2	3	Oz.h
1	**7**	**5**	**2**	**18**
2	**1**	**-1**	**-1**	**3**
3	**1**	**1**	**2**	**-2**

V=22

a_{ij}	1	2	3	Oz.h
1	**2**	**3**	**1**	**1**
2	**1**	**0**	**1**	**0**
3	**1**	**-1**	**-1**	**2**

V=23

a_{ij}	1	2	3	Oz.h
1	**1**	**-2**	**-2**	**3**
2	**1**	**1**	**-2**	**0**
3	**1**	**-1**	**-1**	**1**

V=24

a_{ij}	1	2	3	Oz.h
1	**3**	**1**	**-5**	**-7**
2	**2**	**-3**	**4**	**-1**
3	**5**	**-1**	**3**	**0**

V=25

a_{ij}	1	2	3	Oz.h
1	**1**	**-2**	**1**	**15**
2	**2**	**1**	**3**	**9**
3	**2**	**3**	**2**	**-2**

V=26

a_{ij}	1	2	3	Oz.h
1	**2**	**-1**	**-2**	**1**
2	**3**	**2**	**1**	**1**
3	**2**	**3**	**3**	**0**

V=27

a_{ij}	1	2	3	Oz.h
1	**2**	**3**	**4**	**5**
2	**3**	**4**	**-1**	**3**
3	**4**	**5**	**-2**	**3**

V=28

a_{ij}	1	2	3	Oz.h
1	**2**	**-1**	**-3**	**-9**
2	**1**	**2**	**1**	**3**
3	**3**	**1**	**-1**	**-1**

V=29

a_{ij}	1	2	3	Oz.h
1	**3**	**1**	**-2**	**4**
2	**2**	**-3**	**1**	**9**
3	**5**	**1**	**3**	**-4**

V=30

a_{ij}	1	2	3	Oz.h
1	**2**	**-1**	**3**	**-4**
2	**1**	**3**	**-1**	**2**
3	**5**	**2**	**1**	**5**

1-laboratoriya ishi bo’yicha mustaqil ishlash uchun topshiriqlar

Yuqoridagi berilgan jadvalda ozod hadlarini hisobga olmay tuziladigan uchinchi tartibli determinant variatlari(V) ustida quyidagi amallarni bajaring:

1) uchburchaklar va Sarryus usulida hisoblang
2) minorlarga yoyish usuli bilan hisoblang

2-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

1-laboratoriya ishida berilgan jadvalda ozod hadlarini hisobga olmay tuziladigan uchinchi tartibli matritsaga teskari matritsni topish amalini bajaring

3-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

1-laboratoriya ishida berilgan jadval asosida ozod hadlarini hisobga olib tuziladigan uchinchi nomahlumli tenglamalar sistemasining yechimini Gauss usulida topish amalini bajaring.

4-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

1-laboratoriya ishida berilgan jadvalda ozod hadlarini hisobga olmay tuziladigan uchinchi tartibli determinantni va matritsaga teskari matritsni Gauss usulida topish amalini bajaring

5-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

Quyidagi tenglamalar sistemasining yechimini oddiy iteratsiya usulida toping.

1. $$\begin{cases} x_1 = 0{,}23x_1 - 0{,}04x_2 + 0{,}21x_3 - 0{,}18x_4 + 1{,}24; \\ x_2 = 0{,}45x_1 - 0{,}23x_2 + 0{,}06x_3 - 0{,}88; \\ x_3 = 0{,}26x_1 + 0{,}34x_2 - 0{,}11x_3 + 0{,}62; \\ x_4 = 0{,}05x_1 - 0{,}26x_2 + 0{,}34x_3 - 0{,}12x_4 - 1{,}17. \end{cases}$$

2. $$\begin{cases} x_1 = 0{,}21x_1 + 0{,}12x_2 - 0{,}34x_3 - 0{,}16x_4 - 0{,}64; \\ x_2 = 0{,}34x_1 - 0{,}08x_2 + 0{,}17x_3 - 0{,}18x_4 + 1{,}42; \\ x_3 = 0{,}16x_1 + 0{,}34x_2 + 0{,}15x_3 - 0{,}31x_4 - 0{,}42; \\ x_4 = 0{,}12x_1 - 0{,}26x_2 - 0{,}08x_3 + 0{,}25x_4 + 0{,}83. \end{cases}$$

3. $$\begin{cases} x_1 = 0{,}32x_1 - 0{,}18x_2 + 0{,}02x_3 + 0{,}21x_4 + 1{,}83; \\ x_2 = 0{,}16x_1 + 0{,}12x_2 - 0{,}14x_3 + 0{,}27x4 - 0{,}65; \\ x_3 = 0{,}37x_1 + 0{,}27x_2 - 0{,}02x_3 - 0{,}24x_4 + 2{,}23; \\ x_4 = 0{,}12x_1 + 0{,}21x_2 - 0{,}18x_3 + 0{,}25x_4 - 1{,}13. \end{cases}$$

4. $$\begin{cases} x_1 = 0{,}42x_1 - 0{,}32x_2 + 0{,}03x_3 + 0{,}44; \\ x_2 = 0{,}11x_1 - 0{,}26x_2 - 0{,}36x_3 + 1{,}42; \\ x_3 = 0{,}12x_1 + 0{,}08x_2 - 0{,}14x_3 - 0{,}24x_4 - 0{,}83; \\ x_4 = 0{,}15x_1 - 0{,}35x_2 - 0{,}18x_3 - 1{,}42. \end{cases}$$

5. $$\begin{cases} x_1 = 0{,}18x_1 - 0{,}34x_2 - 0{,}12x_3 + 0{,}15x_4 - 1{,}33; \\ x_2 = 0{,}11x_1 + 0{,}23x_2 - 0{,}15x_3 + 0{,}32 + 0{,}84; \\ x_3 = 0{,}05x_1 - 0{,}12x_2 + 0{,}14x_3 - 0{,}18x_4 - 1{,}16; \\ x_4 = 0{,}12x_1 + 0{,}08x_2 + 0{,}06x_3 + 0{,}57. \end{cases}$$

6. $$\begin{cases} x_1 = 0{,}13x_1 + 0{,}23x_2 - 0{,}44x_3 - 0{,}05x_4 + 2{,}13; \\ x_2 = 0{,}24x_1 - 0{,}31x_2 + 0{,}15x_3 - 0{,}18; \\ x_3 = 0{,}06x_1 + 0{,}15x_2 - 0{,}23x_3 + 1{,}44; \\ x_4 = 0{,}72x_1 - 0{,}08x_2 - 0{,}05x_3 + 2{,}42. \end{cases}$$

7. $$\begin{cases} x_1 = \quad 0{,}17x_1 + 0{,}31x_2 - 0{,}18x_3 + 0{,}22x_4 - 1{,}71; \\ x_2 = -0{,}21x_1 + 0{,}33x_3 + 0{,}22x_4 + 0{,}62; \\ x_3 = \quad 0{,}32x_1 - 0{,}18x_2 + 0{,}05x_3 - 0{,}19x_4 - 0{,}89; \\ x_4 = \quad 0{,}12x_1 + 0{,}28x_2 + 0{,}14x_3 + 0{,}94. \end{cases}$$

8. $$\begin{cases} x_1 = \quad 0{,}13x_1 + 0{,}27x_2 - 0{,}22x_3 - 0{,}18x_4 + 1{,}21; \\ x_2 = -0{,}21x_1 - 0{,}45x_3 + 0{,}18x_4 - 0{,}33; \\ x3 = \quad 0{,}12x_1 + 0{,}13x_2 - 0{,}33x_3 + 0{,}18x_4 - 0{,}48; \\ x_4 = \quad 0{,}33x_1 - 0{,}05x_2 - 0{,}06x_3 - 0{,}28x_4 - 0{,}17. \end{cases}$$

9. $\begin{cases} x_1 = 0{,}19x_1 - 0{,}07x_2 + 0{,}38x_3 - 0{,}21x_4 - 0{,}81; \\ x_2 = -0{,}22x_1 - 0{,}08x_2 + 0{,}11x_3 + 0{,}33x_4 - 0{,}64; \\ x_3 = 0{,}51x_1 - 0{,}07x_2 + 0{,}09x_3 - 0{,}11x_4 + 1{,}71; \\ x_4 = 0{,}33x_1 - 0{,}41x_2 - 1{,}21. \end{cases}$

10. $\begin{cases} x_1 = 0{,}22x_2 - 0{,}11x_3 + 0{,}31x_4 + 2{,}7; \\ x_2 = 0{,}38x_1 - 0{,}12x_3 + 0{,}22x_4 - 1{,}5; \\ x_3 = 0{,}11x_1 - 0{,}23x_2 - 0{,}51x_4 + 1{,}2; \\ x_4 = 0{,}17x_1 - 0{,}21x_2 + 0{,}31x_3 - 0{,}17. \end{cases}$ 2

11. $\begin{cases} x_1 = 0{,}07x_1 - 0{,}08x_2 + 0{,}11x_3 - 0{,}18x_4 - 0{,}51; \\ x_2 = 0{,}18x_1 + 0{,}52x_2 + 0{,}21x_4 + 1{,}17; \\ x_3 = 0{,}13x_1 + 0{,}31x_2 - 0{,}21x_4 - 1{,}02; \\ x_4 = 0{,}08x_1 - 0{,}33x_3 + 0{,}28x_4 - 0{,}28. \end{cases}$

12. $\begin{cases} x_1 = 0{,}05x_1 - 0{,}06x_2 - 0{,}12x_3 + 0{,}14x_4 - 2{,}17; \\ x_2 = 0{,}04x_1 - 0{,}12x_2 + 0{,}08x_3 + 0{,}11x_4 + 1{,}4; \\ x_3 = 0{,}34x_1 - 0{,}08x_2 - 0{,}06x_3 + 0{,}14x_4 - 2{,}1; \\ x_4 = 0{,}11x_1 + 0{,}12x_2 - 0{,}03x_4 - 0{,}8. \end{cases}$

13. $\begin{cases} x_1 = 0{,}08x_1 - 0{,}03x_2 - 0{,}04x_4 - 1{,}2; \\ x_2 = 0{,}31x_2 + 0{,}27x_3 - 0{,}08x_4 + 0{,}81; \\ x_3 = 0{,}33x_1 - 0{,}07x_3 + 0{,}21x_4 - 0{,}92; \\ x_4 = 0{,}11x_1 - 0{,}03x_3 - 0{,}58x_4 + 0{,}17. \end{cases}$

14. $\begin{cases} x_1 = 0{,}12x_1 - 0{,}23x_2 + 0{,}25x_3 - 0{,}16x_4 + 1{,}24; \\ x_2 = 0{,}14x_1 + 0{,}34x_2 - 0{,}18x_3 + 0{,}24x_4 - 0{,}89; \\ x_3 = 0{,}33x_1 + 0{,}03x_2 + 0{,}16x_3 - 0{,}32x_4 + 1{,}15; \\ x_4 = 0{,}12x_1 - 0{,}05x_2 + 0{,}15x_4 - 0{,}57. \end{cases}$

15. $\begin{cases} x_1 = 0{,}23x_1 - 0{,}14x_2 + 0{,}06x_3 - 0{,}12x_4 + 1{,}21; \\ x_2 = 0{,}12x_1 + 0{,}32x_3 - 0{,}18x_4 - 0{,}72; \\ x_3 = 0{,}08x_1 - 0{,}21x_2 + 0{,}23x_3 + 0{,}32x_4 - 0{,}58; \\ x_4 = 0{,}25x_1 + 0{,}22x_2 + 0{,}14x_3 + 1{,}56. \end{cases}$

16. $\begin{cases} x_1 = 0{,}14x_1 + 0{,}23x_2 + 0{,}18x_3 - 0{,}17x_4 - 1{,}42; \\ x_2 = 0{,}12x_1 - 0{,}14x_2 + 0{,}08x_3 + 0{,}09x_4 - 0{,}83; \\ x_3 = 0{,}16x_1 - 0{,}24x_2 + 0{,}35x_4 + 1{,}21; \\ x_4 = 0{,}23x_1 - 0{,}08x_2 + 0{,}05x_3 + 0{,}25x_4 + 0{,}65. \end{cases}$

17. $\begin{cases} x_1 = 0{,}24x_1 + 0{,}21x_2 + 0{,}06x_3 - 0{,}34x_4 + 1{,}42; \\ x_2 = 0{,}05x_1 + 0{,}32x_3 + 0{,}12x_4 - 0{,}57; \\ x_3 = 0{,}35x_1 - 0{,}27x_2 - 0{,}05x_4 + 0{,}68; \\ x_4 = 0{,}12x_1 - 0{,}43x_2 + 0{,}04x_3 - 0{,}21x_4 - 2{,}14. \end{cases}$

18. $\begin{cases} x_1 = 0{,}17x_1 + 0{,}27x_2 - 0{,}13x_3 - 0{,}11x_4 - 1{,}42; \\ x_2 = 0{,}13x_1 - 0{,}12x_2 + 0{,}09x_3 - 0{,}06x_4 + 0{,}48; \\ x_3 = 0{,}11x_1 + 0{,}05x_2 - 0{,}02x_3 + 0{,}12x_4 - 2{,}34; \\ x_4 = 0{,}13x_1 + 0{,}18x_2 + 0{,}24x_3 + 0{,}43x_4 + 0{,}72. \end{cases}$

19. $\begin{cases} x_1 = 0{,}15x_1 + 0{,}05x_2 - 0{,}08x_3 + 0{,}14x_4 - 0{,}48; \\ x_2 = 0{,}32x_1 - 0{,}13x_2 - 0{,}12x_3 + 0{,}11x_4 + 1{,}24; \\ x_3 = 0{,}17x_1 + 0{,}06x_2 - 0{,}08x_3 + 0{,}12x_4 + 1{,}15; \\ x_4 = 0{,}21x_1 - 0{,}16x_2 + 0{,}36x_3 - 0{,}88. \end{cases}$

20. $\begin{cases} x_1 = 0{,}28x_2 - 0{,}17x_3 + 0{,}06x_4 + 0{,}21; \\ x_2 = 0{,}52x_1 + 0{,}12x_3 + 0{,}17x_4 - 1{,}17; \\ x_3 = 0{,}17x_1 - 0{,}18x_2 + 0{,}21x_3 - 0{,}81; \\ x_4 = 0{,}11x_1 + 0{,}22x_2 + 0{,}03x_3 + 0{,}05x_4 + 0{,}72. \end{cases}$

6-laboratoriya ishi bo’yicha mustaqil ishlash uchun topshiriqlar

Darajali qator yordamida bahzi trantsendent funktsiyalarning qiymatlarini hisoblash.

x\y	e^x	Sinx	Cosx
x_1	0.716+0.043V	0.232+0.012V	
x_2	2.834-0.02813v	0.747-0.014V	

7,8,9,10,11-laboratoriya ishi bo’yicha mustaqil ishlash uchun topshiriqlar

Quyidagi tenglamalarning:

1) Ildizlarning qisqa atrofini analitik yoki grafik usulda aniqlang;
2) Aniqlangan oraliqda ildizni vatarlar usulida hisoblang;
3) Aniqlangan oraliqda ildizni urinmalar usulida hisoblang;
4) Aniqlangan oraliqda ildizni umumlashgan usulida hisoblang

1	1) $2^x+5x-3=0$ 2) $3x^4-4x^3-12x^2-5=0$ 3) $0.5^x+1=(x-2)^2$ 4) $(x-3)\mathrm{Cos}x=1$, $(-2\pi\leq x\leq 2\pi)$	2	1) $\mathrm{arctg}x-1/(3x^3)=0$ 2) $2x^3-9x^2-60x+1=0$ 3) $[\log_2(-x)](x+2)=-1$ 4) $\mathrm{Sin}(x+\pi/3)-0.5x=0$
3	1) $5^x+3x=0$ 2) $x^4-x-1=0$ 3) $0.5^x+x^2=2$ 4) $(x-1)^2\mathrm{Ln}(x+1)=1$	4	1) $2e^x=2+5x$ 2) $2x^4-x^2-10=0$ 3) $x\ \mathrm{Log}_3(x+1)=1$ 4) $\mathrm{Cos}(x+0.5)=x^3$
5.	1) $3^{x-1}-2-x=0$ 2) $3x^4+8x^3+6x^2-10=0$ 3) $(x-4)^2\log_{0.5}(x-3)=-1$ 4) $5\mathrm{Sin}x=x$	6.	1) $\mathrm{arctg}x-1/(2x^3)=0$ 2) $x^4-18x^2+6=0$ 3) $x^2 2^x=1$ 4) $\mathrm{tg}x=x+1$, $(-\pi/2\leq x\leq\pi/2)$
7.	1) $e^{-2x}-2x+1=0$ 2) $x^4+4x^3-8x^2-17=0$ 3) $0.5^x-1=(x+2)^2$ 4) $x^2\mathrm{Cos}2=-1$	8.	1) $5^x-6x-3=0$ 2) $x^4-x^3-2x^2+3x-3=0$ 3) $0.5^x-2x^2-3=0$ 4) $x\ \mathrm{Log}(x+1)=1$
9.	1) $\mathrm{arctg}(x-1)+2x=0$ 2) $3x^4+4x^3-12x^2+1=0$ 3) $(x-2)^2 2^x=1$ 4) $x^2-20\mathrm{Sin}x=0$	10.	1) $2\mathrm{arctg}x-x+3=0$ 2) $3x^4-8x^3-18x^2+3=0$ 3) $2\mathrm{Sin}(x+\pi/3)=0.5x^2-1$ 4) $2\mathrm{Log}x-x/2+1=0$
11	1) $3^x+2x-2=0$ 2) $2x^4-8x^3+8x^2-1=0$ 3) $[(x-2)^2-1]2^x=1$ 4) $(x-2)\mathrm{Cos}x=1$	12.	1) $2\mathrm{arctg}x-3x+2=0$ 2) $2x^4+8x^3+8x^2-1=0$ 3) $\mathrm{Sin}(x-0.5)-x+0.8=0$ 4) $(x-1)\mathrm{Log}_2(x+2)=1$
13.	1) $3^x+2x-5=0$ 2) $x^4-4x^3-8x^2+1=0$	14.	1) $2e^x+3x+3x+1=0$ 2) $3x^4+4x^3-12x^2-5=0$

	3) $0.5^x + x^2 - 3=0$ 4) $(x-2)^2 Lg(x+1)=1$		3) $Cos\ (x+0.3)=x^2$ 4) $x\ Log_3(x+1)=2$
15.	1) $3^{x-1}-4-x=0$ 2) $2x^3-9x^2-60x+1=0$ 3) $(x-3)^2 Log_{0.5}(x-2)=-1$ 4) $Sinx=x-1$	16.	1) $arctgx-1/(3x^3)=0$ 2) $x^4-x--1=0$ 3) $(x-1)^2 2^x=1$ 4) $tg^3x=x-1$
17.	1) $e^x+x+1=0$ 2) $2x^4-x^2-1=0$ 3) $0.5^x - 3=(x+2)^2$ 4) $x^2Cos2x=-1$, $(-2\pi \le x \le 2\pi)$	18.	1) $3^x-2x+5=0$ 2) $3x^4+8x^3+6x^2-10=0$ 3) $2x^2-0.5^x=0$ 4) $x\ lg(x+1)=1$
19	1) $arctg(x-1)+3x-2=0$ 2) $x^4-18x^2+6=0$ 3) $x^2-20Sinx=0$ 4) $(x-2)^2 2^x=1$	20.	1) $2arctgx-x+3=0$ 2) $x^4+4x^3-8x^2-17=0$ 3) $2\ Sin(x+\pi/2)=x^2-0.8$ 4) $2\ lgx-x/2+1=0$
21	1) $2^x-3x-2=0.$ 2)$x^4-x^3-2x^2+3x-3=0;$ 3)$(0.5)^x+1=(x-2)^2$ 4)$(x-3)sosx = -1, -2p<x<2p.$	22	1) $arctgx+2x-1=0$ 2) $3x^4+4x^3-12x^2+1=0$ 3)$(x+2)Log_2(x)=1$ 4) $Sin(x+1)=0.5x$
23	1) $3^x+2x-3=0.$ 2) $3x^4-8x^3-18x^2+2=0;$ 3) $(0.5)^x=4-x^2$ 4) $(x+2)^2Lg(x+11)=1$	24	1) $2e^x-2x-3=0.$ 2) $3x^4+4x^3-12x^2-5=0;$ 3) $xLog_2(x+1)=1$ 4) $cos(x+0.5)= x^3$
25	1) $3^x+2+x=0.$ 2) $2x^3-9x^2-60x+1=0;$ 3) $(x-4)^2Log_{0.5}(x-3)=-1$ 4) $5Sinx=x-0.5$	26	1) $arctg(x-1)+2x-3=0$ 2) $x^4x -1=0;$ 3) $(x-1)^2 2^x=1$ 4) $tg^3x=x-1, (-\pi/2 \le x \le \pi/2)$
27	1) $2e^x-2x-3=0.$ 2) $2x^4-x^2-10=0;$ 3) $(0.5)^x-3=-(x+1)^2$ 4) $x^2sos2x = 1$	28	1) $3^x-2x-5=0.$ 2) $3x^4+8x^3+6x^2-10=0;$ 3) $2x^2-0.5^x - 3=0$ 4) $xLg(x+1)=1$
29	1) $arctg(x-1)+2x=0$ 2) $x^4-18x^2+6=0$ 3) $(x-2)^2 2^x=1$ 4) $x^2-10sinx = 0$	30	1) $3^x+5x-2=0.$ 2) $3x^4+4x^3-12x^2+1=0;$ 3) $(x-2)^2=0.5^x+1$ 4)$(x+3)sosx = 1, -2p<x<2p.$

12,13-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

Jadvalda berilgan (x_i, y_i) nuqtalar yordamida x qiymatlari teng uzoqlikda bo'lmagan 1-jadval uchun LAGRANJ, x qiymatlari teng uzoqlikda bo'lgan 2- jadval uchun Ng'yuton interpolyatsion ko'phadini tuzing.

Variant 1

Jadval 1

X	0,43	0,48	0,55	0,62	0,70	0,75
U	1,63597	1,73234	1,87686	2,03345	2,22846	2,35973

Jadval 2

X	1	7	13	19	25
U	0,702	0,512	0,645	0,736	0,608

Variant 2

Jadval 1

X	0,02	0,08	0,12	0,17	0,23	0,30
U	1,02316	1,09590	1,14725	1,21483	1,30120	1,40976

Jadval 2

X	2	8	14	20	26
U	0,102	0,114	0,125	0,203	0,154

Variant 3

Jadval 1

X	0,35	0,41	0,47	0,51	0,56	0,64
U	2,739	2,300	1,968	1,787	1,595	1,345

Jadval 2

X	3	9	15	21	27
U	0,526	0,453	0,482	0,552	0,436

Variant 4

4 Jadval 1

X	0,41	0,46	0,52	0,60	0,65	0,72
U	2,574	2,325	2,093	1,862	1,749	1,620

Jadval 2

X	4	10	16	22	28
U	0,616	0, 478	0,665	0,537	0,673

Variant 5

5 Jadval 1

X	0,68	0,73	0,80	0,88	0,93	0,99
U	0,808	0,894	1,029	1,209	1,340	1,523

Jadval 2

X	5	11	17	23	29
U	0,896	0,812	0,774	0,955	0,715

Variant 6

Jadval 1

X	0,11	0,15	0,21	0,29	0,35	0,40

U	9,054	6,616	4,691	3,351	2,739	2,365

Jadval 2

X	6	12	18	24	30
U	0,314	0,235	0,332	0,275	0,186

Variant 7

Jadval 1

X	1,375	1,380	1,385	1,390	1,395	1,400
U	5,041	5,177	5,320	5,470	5,629	5,797

Jadval 2

X	1	7	13	19	25
U	1,3832	1,3926	1,3862	1,3934	1,3866

Variant 8

Jadval 1

X	0,115	0,120	0,125	0,130	0,135	0,140
U	8,657	8,293	7,958	7,648	7,362	7,096

Jadval 2

X	2	8	14	20	16
U	0,1264	0,1315	0,1232	0,1334	0,1285

Variant 9

Jadval 1

X	0,150	0,155	0,160	0,165	0,170	0,175
U	6,616	6,399	6,196	6,005	5,825	5,655

Jadval 2

X	3	9	15	21	27
U	0,1521	0,1611	0,1662	0,1542	0,1625

Variant 10

Jadval 1

X	0,180	0,185	0,190	0,195	0,200	0,205
U	5,615	5,466	5,326	5,193	5,066	4,946

Jadval 2

X	4	10	16	22	28
U	0,1838	0,1875	0,1944	0,1976	0,2038

Variant 11

Jadval 2

X	0,210	0,215	0,220	0,225	0,230	0,235
U	4,831	4,722	4,618	4,519	4,424	4,333

Jadval 2

X	5	11	17	23	29
U	0,2121	0,2165	0,2232	0,2263	0,2244

Variant 12

Jadval 1

X	1,415	1,420	1,425	0,430	0,435	0,440
U	0,888	0,889	0,890	0,891	0,892	0,893

Jadval 2

X	6	12	18	24	30
U	1,4179	1,4258	1,4396	1,4236	1,4315

Variant 13

Jadval 1

X	0,33	0,38	0,45	0,52	0,60	0,65
U	1,63597	1,73234	1,87686	2,03345	2,22846	2,35973

Jadval 2

X	1	5	9	14	18
U	0,702	0,512	0,645	0,736	0,608

Variant 14

Jadval 1

X	0,03	0,09	0,13	0,18	0,24	0,31
U	1,02316	1,09590	1,14725	1,21483	1,30120	1,40976

Jadval 2

X	2	6	10	14	18
U	0,102	0,114	0,125	0,203	0,154

Variant 15

Jadval 1

X	0,25	0,31	0,37	0,41	0,46	0,54
U	2,739	2,300	1,968	1,787	1,595	1,345

Jadval 2

X	3	6	9	12	15
U	0,526	0,453	0,482	0,552	0,436

Variant 16

4 Jadval 1

X	0,21	0,26	0,32	0,40	0,45	0,52
U	2,574	2,325	2,093	1,862	1,749	1,620

Jadval 2

X	4	7	10	13	16
U	0,616	0, 478	0,665	0,537	0,673

Variant 17

5 Jadval 1

X	0,38	0,43	0,50	0,58	0,63	0,69
U	0,808	0,894	1,029	1,209	1,340	1,523

Jadval 2

X	5	11	17	23	29
U	0,896	0,812	0,774	0,955	0,715

Variant 18

Jadval 1

X	0,31	0,35	0,41	0,49	0,55	0,60
U	9,054	6,616	4,691	3,351	2,739	2,365

Jadval 2

X	6	7	8	9	10
U	0,314	0,235	0,332	0,275	0,186

Variant 19

Jadval 1

X	1,175	1,180	1,185	1,190	1,195	1,200
U	5,041	5,177	5,320	5,470	5,629	5,797

Jadval 2

X	1	6	10	14	18
U	1,3832	1,3926	1,3862	1,3934	1,3866

Variant 20

Jadval 1

X	0,215	0,220	0,225	0,230	0,235	0,240
U	8,657	8,293	7,958	7,648	7,362	7,096

Jadval 2

X	2	7	12	17	22
U	0,1264	0,1315	0,1232	0,1334	0,1285

Variant 21

Jadval 1

X	0,250	0,255	0,260	0,265	0,270	0,275
U	6,616	6,399	6,196	6,005	5,825	5,655

Jadval 2

X	3	9	15	21	27
U	0,1521	0,1611	0,1662	0,1542	0,1625

Variant 22

Jadval 1

X	0,280	0,285	0,290	0,295	0,300	0,305
U	5,615	5,466	5,326	5,193	5,066	4,946

Jadval 2

X	4	10	16	22	28
U	0,1838	0,1875	0,1944	0,1976	0,2038

Variant 23

Jadval 1

X	0,310	0,315	0,320	0,325	0,330	0,335
U	4,831	4,722	4,618	4,519	4,424	4,333

Jadval 2

X	5	11	17	23	29
U	0,2121	0,2165	0,2232	0,2263	0,2244

Variant 24

Jadval 1

X	1,315	1,320	1,325	0,330	0,335	0,340
U	0,888	0,889	0,890	0,891	0,892	0,893

Jadval 2

X	6	12	18	24	30
U	1,4179	1,4258	1,4396	1,4236	1,4315

Variant 25

Jadval 1

X	0,315	0,320	0,325	0,330	0,335	0,340
U	8,657	8,293	7,958	7,648	7,362	7,096

Jadval 2

X	2	4	6	8	10
U	0,1264	0,1315	0,1232	0,1334	0,1285

Variant 26

Jadval 1

X	0,450	0,455	0,460	0,465	0,470	0,475
U	6,616	6,399	6,196	6,005	5,825	5,655

Jadval 2

X	3	7	11	15	19
U	0,1521	0,1611	0,1662	0,1542	0,1625

Variant 27

Jadval 1

X	0,580	0,585	0,590	0,595	0,600	0,605
U	5,615	5,466	5,326	5,193	5,066	4,946

Jadval 2

X	4	9	14	19	24
U	0,1838	0,1875	0,1944	0,1976	0,2038

Variant 28

Jadval 1

X	0,410	0,415	0,420	0,425	0,430	0,435
U	4,831	4,722	4,618	4,519	4,424	4,333

Jadval 2

X	3	10	17	24	31
U	0,2121	0,2165	0,2232	0,2263	0,2244

Variant 29

Jadval 1

X	0,315	0,320	0,325	0,330	0,335	0,340
U	0,888	0,889	0,890	0,891	0,892	0,893

Jadval 2

X	6	11	16	21	26
U	1,4179	1,4258	1,4396	1,4236	1,4315

Variant 30

Jadval 1

X	2,315	2,320	2,325	2,330	2,335	2,340
U	0,888	0,889	0,890	0,891	0,892	0,893

Jadval 2

X	3	7	11	15	19
U	1,4179	1,4258	1,4396	1,4236	1,4315

14-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

Quyidagi integrallarni:

1)to'rtburchak; 2)trapetsiya; 3) Simpson usulida hisoblang.

1. $\int_{1}^{3,5} \frac{\ln x}{x\sqrt{1+\ln x}} dx$ **2 .** $\int_{\pi/6}^{\pi/3} (tg^2 x + ctg^2 x) dx$

3. $\int_{1}^{4} \frac{1}{x} \ln^2 x dx$ **4.** $\int_{2}^{3} \frac{1}{x \lg x} dx$

5. $\int_{0}^{\ln 2} \sqrt{e^x - 1} dx$ **6.** $\int_{0}^{1} x e^x \sin x dx$

7. $\int_{0}^{2} \frac{1}{\sqrt{9 + x^3}} dx$ **8.** $\int_{1}^{2.5} \frac{1}{x^2} \sin \frac{1}{x} dx$

9. $\int_{0}^{\sqrt{3}} x \operatorname{arctg} x dx$ **10.** $\int_{0}^{3} \arcsin \sqrt{\frac{x}{1+x}} dx$

11. $\int_{1}^{3} x^x (1 + \ln x) dx$ **12.** $\int_{0}^{1} \frac{dx}{\sqrt{1 + 3x + 2x^2}}$

13. $\int_{1}^{2} \frac{1}{x} \sqrt{x^2 + 0.16} dx$ **14.** $\int_{0}^{1} \frac{x \operatorname{arctg} x}{\sqrt{1 + x^2}} dx$

15. $\int_{0}^{2} \frac{e^{3x} + 1}{e^x + 1} dx$ **16.** $\int_{0}^{1.99} x^2 \sqrt{4 - x^2} dx$

17. $\int_{0}^{\pi} e^x \cos^{-2} x dx$ **18.** $\int_{1}^{e} (x \ln x)^2 dx$

19. $\int_{-1}^{2} \arccos \sqrt{\frac{x}{1+x}} dx$ **20.** $\int_{0}^{1} \frac{(x^2 + 4) dx}{(x^2 + 1)\sqrt{x^4 + 1}}$

21. $\int_{1}^{1.5} \sin x \ln(tgx) dx$ **22.** $\int_{0}^{1.5} \frac{e^x (1 + \sin x)}{1 + \cos x} dx$

23. $\int_{0}^{3/4} (x + 1) / \sqrt{x^2 + 1} dx$ **24.** $\int_{0}^{1} \frac{dx}{(3 \sin x + 2 \cos x)^2}$

25. $\int_{1}^{2} (\frac{\ln x}{x})^3 dx$ **26.** $\int_{1}^{2} \frac{x^3}{\sqrt{x} + 3} dx$

27. $\int_{1}^{2} \frac{x}{x^4 + 3x^2 + 2} dx$ **28.** $\int_{0}^{\pi/2} \sqrt{1 - \frac{1}{4} \sin 2x} dx$

29. $\int_{0}^{\pi/2} \sqrt{2 + \cos x} dx$ **30.** $\int_{\ln 2}^{\ln 3} \frac{e^{2x}}{e^x - e^{-x}} dx$

15-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

Quyidagi birinchi tartibli differietsial tenglamalar uchun Koshi masalasinining taqribiy yechimini h=0.1 qadam bilan Eyler va Runge-Kutta usulida toping.

1)	uo=x/(x+y)	u(0)=1	[0,1]
2)	uo-2u=3e^x	u(0,3)=1,415	[0,1;0,5]
3)	uo=x+y^2	u(0)=0	[0;0,3]
4)	uo=y^2-x^2	u(1)=1	[1;2]
5)	uo=x^2+y^2	u(0)=0.27	[0;1]
6)	uo+xy(1-y^2)=0	u(0)=0.5	[0;1]
7)	uo=x^2-xy+y^2	u(0)=0.1	[0;1]
8)	uo=(2y-x)/y	u(1)=2	[1;2]
9)	uo=x^2+xy+y^2+1	u(0)=0	[0;1]
10)	uo+y=x^3	u(1)=-1	[1;2]
11)	uo=xy+e^y	u(0)=0	[0;0.1]
12)	uo=2xy+x^2	u(0)=0	[0;0.5]
13)	uo=$x+\sin\frac{y}{3}$	u(0)=1	[0;1]
14)	uo=e^x-y^2	u(0)=0	[0;0.4]
15)	uo=2x+cosy	u(0)=0	[0;0.1]
16)	uo=x^3+y^2	u(0)=0.5	[0;0.5]
17)	uo=xy^3-y	u(0)=1	[0;1]
18)	uo=y^2e^x-2y	u(0)=1	[0;1]
19)	uo=$\frac{1}{y^2-x}$	u(1)=0	[1;2]
20)	uo=$\frac{x^2+1}{e^x}$	u(1)=1	[1;2]
21)	uo=e^xcosy/x	u(1)=1	[1;2]
22)	uo=e^xsiny/x	u(1)=1	[1;2]
23)	uocosx-ysinx=2x	u(0)=0	[0;1]
24)	uo=ytgx-$\frac{1}{\cos^3 x}$	u(0)=0	[0;1]
25)	uo+ycosx=cosx	u(0)=0	[0;1]
26)	uo=$\frac{y}{x}+\mathrm{tg}\frac{x}{y}$	u(0)=0	[0;1]
27)	uo=$(1+\frac{y-1}{2x})^2$	u(1)=1	[1;2]
28)	xuo-$\frac{y}{x+1}$-x=0	u(1)=1/2	[1;2]
29)	uo=$\frac{y}{x}$(1+lny-lnx)	u(1)=e	[1;2]
30)	u^3xdx=(x^2y+2)dy	u(0.348)=2	[0;1]

16-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

1. Quyidagi birinchi tartibli differentsial tenglamalar sistemasi uchun KOSHI masalasini Eyler usulida takribiy yechimini toping.

1. $\begin{cases} y' = \cos(y+2z)+3, \\ z' = 2/(x+3x^2)+y+x, \end{cases}$ y(0)=1, z(0)=0.05

2. $\begin{cases} x' = \sin(2x^2)+t+y \\ y' = t+x-3y^2+1 \end{cases}$ x(0)=1 y(0)=0.5

3. $\begin{cases} x' = \sqrt{(t^2+2x^2)+y} \\ y' = \cos(3y+x), \end{cases}$ x0)=0.5 y(0)=1

4. $\begin{cases} x' = \ln(6t+\sqrt{2t^2+y^2}) \\ y' = (2t^2+x^2) \end{cases}$ x(0)=1, y(0)=0.5

5. $\begin{cases} x' = e^{-(x^2+y^2)}+0.15t \\ y' = 6x^2+y \end{cases}$ x(0)=0.5 y(0)=1

6. $\begin{cases} y' = z/x+\sqrt{(x+y)} \\ x' = 2z^2/(x(y-1))+z/x \end{cases}$ x(1)=1/3 y(1)=0

7. $\begin{cases} y' = (z-y)x \\ z' = (z+y)x, \end{cases}$ y(0)=1 z(0)=1

8. $\begin{cases} y' = \cos(y+2z)+2 \\ z' = 2/(x+2y^2)+x+1 \end{cases}$ y(0)=1 z(0)=1

9. $\begin{cases} y' = e^{-(y^2+z^2)}+2x \\ z' = 2y^2+z, \end{cases}$ y(0)=0.5 z(0)1

10. $\begin{cases} y' = y+2z-\sin z^2 \\ z' = -y-3z+x(e^{(x^2/2)}-1) \end{cases}$ y(0)=0 z(0)=0

11. $\begin{cases} y' = -z + xy \\ z' = z^2/y \end{cases}$ y(1)=1 z(1)= -0.5

12. $\begin{cases} y' = (z-1)/z \\ z' = 1/(y-x), \end{cases}$ y(0)=-1 z(0)=1

13. $\begin{cases} y' = 2xy/(x^2 - y^2 - z^2) \\ `z' = 2xz/(x^2 - y^2 - z^2), \end{cases}$ y(1)=2 z(1)=1

14. $\begin{cases} y' = z/(z-y)^2 \\ z' = y/(z-y)^2 \end{cases}$ y(0)=1 z(0)=2

15. $\begin{cases} y' = -y/x + xz \\ z' = -2y/x^3 + z/x \end{cases}$ y(1)=1 z(1)=2

16. $\begin{cases} dx/dt = x - 2y \\ dy/dt = x - y, \end{cases}$ x(0)=1 z(0)=1

17. $\begin{cases} dy/dx = z - y \\ dz/dx = -y - z \end{cases}$ y(0)=2.23 z(0)=1.05

18. $\begin{cases} dy/dx = 1 - 1/z \\ dz/dx = 1/(y-x) \end{cases}$ y(0)=2.12 z(0)=1,13

19. $\begin{cases} dy/dx = x/yz \\ dz/dx = x/y^2 \end{cases}$ y(0)=1 z(0)=2

20. $\begin{cases} dy/dx = -z \\ dz/dx + 4y = 0, \end{cases}$ y(0)=1.2 z(0)=-2

21. $\begin{cases} y' = z/x \\ z' = 2z^2/(x(y-1) + z/x) \end{cases}$ u(1)=0 z(1)=1/3

22. $\begin{cases} y' = (z-y)x \\ z' = (z+y)x \end{cases}$ y(0)=1 z(0)=1

23. $\begin{cases} y' = \cos(y + 2z) + 2 \\ z' = 2/(x + 2y^2) + x + 1 \end{cases}$ y(0)=1 z(0)=005

24. $\begin{cases} y' = e^{-(y^2+z^2)} + 2x \\ z' = 2y^2 + z \end{cases}$ y(0)=05 z(0)=1

25. $\begin{cases} y' = (z-y)y \\ z' = (z+y)z \end{cases}$ y(0)=1.05 z(0)=2

2.Quydagi ikkinchi tartibli differentsial tenglamalar uchun Koshi masalasining yechimini topishda, ikkinchi tartibli differentsial tenglamani birinchi tartibli differentsial tenglamalar sistemasiga keltirib Eyler usulida takribiy yechimini toping.

1. $y''=1/cosx-y;$	$y(0)=1;$	$yo(0)=0,$	[0;0.5],	$h=0.1$
2. $(1+x\text{^}2)y''+(yo)^2+1=0,$	$y(0)=1,$	$yo(0)=1,$	[0; 0.5],	$h=0.05$
3. $y''+2yo+2y=2e^{-x}cosx,$	$y(0)=1,$	$yo(0)=0,$	[0; 0.5],	$h=0.05$
4. $y''+4y=e^{3x}(13x-7),$	$y(0)=0,$	$yo(0)=-1,$	[0.1],	$h=01$
5. $y''+4yo+4y=0,$	$y(0)=1,$	$yo(0)=-1,$	[0,1],	$h=01$
6. $y''-y=sinx+cos3x,$	$y(0)=1.8,$	$yo(0)=-05,$	[0;2],	$h=02$
7. $y''-3yo=e^{5x},$	$y(0)=2.2,$	$yo(0)=0.8,$	[0;02],	$h=002$
8. $y''+y=cosx,$	$y(0)=08,$	$yo(0)=2,$	[0;1],	$h=08$
9. $y''-yo-6y=2e^{4x},$	$y(0)=1.433,$	$yo(0)=-0.367,$	[0;1],	$h=01$
10. $y-2yo+y=5xe^{x},$	$y(0)=1,$	$yo(0)=2,$	[0;1],	$h=0.1$
11. $y''+yo-6y=3x^{2-x}-x,$	$y(0)=-0.9,$	$yo(0)=3.2$	[0;1],	$h=0.1$
12. $8yoo+2yo-3y=x+5,$	$y(0)=1/9,$	$yo(0)=-7/12,$	[0;1],	$h=0.1$
13. $y''-4yo+5y=3x,$	$y(0)=1.48,$	$yo(0)=3.6,$	[0;0.5],	$h=005$
14. $y''-5yo+6y=e^{x},$	$y(0)=0,$	$yo(0)=0,$	[0;0.2],	$h=0.02$
15. $y''-3yo+2y=x^2+3x,$	$y(0)=5.1,$	$yo(0)=4.2,$	[0;1],	$h=01$
16. $y''+(1/x)yo-(1/x)y=8x,$	$y(1)=4,$	$yo(1)=4,$	[1, 1.5],	$h=005$
17. $x^2y''+xyo=0,$	$y(1)=5,$	$yo(1)=-1,$	[1; 1.5],	$h=005$
18. $y''-2yo+y=xe^{x},$	$y(0)=1,$	$yo(0)=2,$	[0;05],	$h=005$
19. $y''-3yo+2y=2sinx,$	$y(0)=2,$	$yo(0)=3.2,$	[0;1],	$h=0.1$
20. $x\text{^}2y''+2.5yox-y=0,$	$y(0)=2,$	$yo(1)=3.5,$	[0;1],	$h=0.1$
21. $4xy''+2yo+y=0,$	$y(1)=1.3817,$	$yo(1)=-0.1505,$	[1;2],	$h=0.1$
22. $x^2y''-4xyo+6y=2,$	$y(1)=1.43,$	$yo(1)=2.3,$	[1,2],	$h=0.1$
23. $y''-y=e^{2x}(x-1),$	$y(0)=11/9,$	$yo(0)=-11/9,$	[0;1],	$h=01$
24. $y''-3yo-2y=cos2x,$	$y(0)=1.95,$	$y(0)=2.7,$	[0;05],	$h=005$
25. $y''-05yo-05y=3e^{x/2},$	$y(0)=-4,$	$yo(0)=-2.5,$	[0;1],	$h=01$
26. $y''+4yo=sinx+sin2x,$	$y(0)=1,$	$yo(0)=-23/12,$	[0;1],	$h=01$
27. $y''+y=x^2-x+2,$	$y(0)=1,$	$yo(0)=0,$	[0;1],	$h=0.1$
28. $x^2y''-2y=0,$	$y(1)=5/6,$	$yo(1)=2/3,$	[1;2],	$h=01$
29. $y''+4yo+4y=2x-3,$	$y(0)=-1/4,$	$yo(0)=-1/2,$	[0;05],	$h=005$
30. $y''+y=x^{\text{^}2}-x+2,$	$y(0)=1,$	$yo(0)=0,$	[0;1],	$h=01$

17-laboratoriya ishi bo’yicha mustaqil ishlash uchun topshiriqlar

Kichik kvadratlar usuli. Tajriba natijalarinig chiziqli va parabolik bog’laninshini aniqlash. **12-laboratoriya ishida berilgan topshiriqlardagi variantlarning 1-jadvallaridan foydalanamiz.**

18-laboratoriya ishi bo’yicha mustaqil ishlash uchun topshiriqlar

Quyidagi korrelyatsion jadval asosida kichik kvadratlar usulida regression bog’lanishining to’g’ri chiziqi va ikkinch darajali tenlamasini aniqlang.

1. 1-15 variantlar uchun 1-korrelyatsion jadval (o’rta qavs ichidagi sonning butun qismi, v-talaba varianti)

U/X	92	96	100	104	n_y
158	[(30-v)/10]	[(30-v)/7]	0	0	
164	[(30-v)/5]	[(30-v)/6]	[(30-v)/15]	0	
170	[(30-v)/8]	[(30-v)/5]	[(30-v)/7]	[(30-v)/9]	
176	0	[(30-v)/7]	[(30-v)/4]	[(30-v)/3]	
182	0	0	[(30-v)/3]	[(30-v)/6]	
n_x					N=
$\bar{y}_x$	161	164,8	170	176	

2. 16-30 variantlar uchun 2-korrelyatsion jadval (o’rta qavs ichidagi sonning butun qismi)

U/X	92	96	100	104	n_y
158	[(35-v)/2]	[(35-v)/7]	0	0	
164	[(35-v)/5]	[(35-v)/2]	[(35-v)/3]	0	
170	[(35-v)/3]	[(35-v)/5]	[(35-v)/4]	[(35-v)/2]	
176	0	[(35-v)/7]	[(35-v)/3]	[(35-v)/3]	
182	0	0	[(35-v)/2]	[(35-v)/6]	
n_x					N=
$\bar{y}_x$	161	164,8	170	176	

Masalan, korrelyatsion jadvalni hosil qilish. V=1 bo’lsa,

U/X	92	96	100	104	n_y
158	[(30-1)/10]	[(30-1)/7]	0	0	
164	[(30-1)/5]	[(30-1)/6]	[(30-1)/15]	0	
170	[(30-1)/8]	[(30-1)/5]	[(30-1)/7]	[(30-1)/9]	
176	0	[(30-1)/7]	[(30-1)/4]	[(30-1)/3]	
182	0	0	[(30-1)/3]	[(30-1)/6]	
n_x					N=
$\bar{y}_x$	161	164,8	170	176	

Bu jadval quyidagicha hosil bo’ladi:

U/X	92	96	100	104	n_y
158	2	4			6
164	5	4	1		10
170	3	5	4	3	15
176		4	7	9	20
182			9	4	13
n_x	10	17	21	16	N=64
$\bar{y}_x$	161	164,8	170	176	

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