

**O`ZBEKISTON RESPUBLIKASI OLIY VA O`RTA MAXSUS
TA`LIM VAZIRLIGI**

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**«Ommaviy xizmat ko'rsatish sistemasidagi ba'zi limit
teoremlarining yaqinlashish tezligini baholash »**

(Bakalavr akademik daraja olish uchun bajarilgan bitiruv malakaviy ishi)

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KIRISH

Masalaning qo'yilishi.

Bir kanalli $M|G|1|\infty$ modeli ommaviy xizmat ko'rsatish sistemasi qaraladi. Bu sistemaning ba'zi xarakteristikalari (sistemaning bandlik davrida va sistemadagi talablarning kutish vaqti) o'rganiladi. $M|G|1|\infty$ sistemaning tavsifi quyida keltiriladi. Bu sistemada qaralayotgan uchta limit teoramalar ya'ni sistemaning bandlik davri va statsionar kutish vaqti (sistemadagi talablarga xizmat ko'rsatish tartibi «FIFO» va «RS» bo'lgan hollar, bu hollar quyida keltirilgan) uchun kritik zagruzkada isbot qilingan limit teoremlar o'rganiladi. Bu limit teoremlarning yaqinlashish tezliklari baholanadi.

Mavzuning dolzarbligi.

Keyingi yillarda $M|G|1|\infty$ modeli ommaviy xizmat ko'rsatish sistemasining limit teoremlari va bu limit teoremlarning yaqinlashish tezligini baholash bo'limi rivojlanmoqda.

Ommaviy xizmat ko'rsatish sistemalarining xarakteristikalarini amalga tatbiq qilishda ularning ba'zi shartlar asosida limit teoremlari va bu limit teoramalar yaqinlashish tezligining baholari ahamiyatlidir. Shu maqsadda ushbu malakaviy bitiruv ishida $M|G|1|\infty$ ommaviy xizmat ko'rsatish sistemasining ba'zi xarakteristikalari uchun olingan limit teoremlarning yaqinlashish tezligini baholash mavzusining dolzarbligidan dalolatdir.

Ishning maqsadi.

Bir kanalli ommaviy xizmat ko'rsatish sistemasidagi sistemaning bandlik davri va talablarning statsionar kutish vaqti xarakteristikalari uchun olingan limit teoremlarning yaqinlashish tezligini kritik zagruzkada baholash bitiruv ishining maqsadidir.

Ilmiy tadqiqot usuli.

Bitiruv ishida keltirilgan limit teoremlarning yaqinlashish tezligini baholashda sistemaning talablarga xizmat ko'rsatish vaqtining uzunligi uchun

Laplas-Stelts almashtirishining asimptotok yoyilmasidan foydalaniladi. Bundan tashqari yaqinlashish tezligini baholashda Berri-Eseen tengsizligidan foydalanilgan.

Ishning tuzilishi.

Bir kanalli $M|G|1|^\infty$ modeli ommaviy xizmat ko'rsatish sistemasi quyidagicha ifodalaniladi:

Sistemaga a parametrli Puasson taqsimotli bog'liqsiz talablar kelib tushadi. Talablarga xizmat ko'rsatish vaqtining uzunligi ixtiyoriy $B(x)$, $B(+0) = 0$ taqsimot funksiyaga ega.

Limit teoremlar $B(x)$ taqsimot funksiyaning birinchi ikkita β_1 va β_2 momentlarning chekliligi shartida ya'ni $S \rightarrow 0$, $(\operatorname{Re} S \geq 0)$ da

$$\beta(x) = \int_0^\infty e^{-sx} dB(x) = 1 - \beta_1 \cdot s + \frac{1}{2} \beta_2 \cdot s^2 (1 + o_s(1)) \quad (1)$$

yoyilma o'rinli bo'lgan sharti ostida isbot qilingan. Shu bilna birga bu limit teoremlarning yaqinlashish tezligini baholashda limit teoremlarga qo'yilganshertlar qanoatlantiriladi.

Mazkur birtiruv malakaviy ishda ba'zi qo'shimcha shartlar bajarilganda limit teoremlarning yaqinlashish tezligi baholanadi. Yuqorida keltirilgan (1) shart o'rniga quyidagi kuchliroq shart bajarilishi faraz qilinadi: $S \rightarrow 0$, $(\operatorname{Re} S \geq 0)$ da

$$\beta(s) = 1 - \beta_1 \cdot s + \frac{1}{2} \beta_2 \cdot s^2 - \varepsilon s^\delta (1 + o_s(1)) \quad (2)$$

yoyilma o'rinli bo'lsin, bu yerda $2 < \delta \leq 3$, ε - musbat o'zgarmas son.

Biz ko'rib o'tayotgan $M|G|1|^\infty$ ommaviy xizmat ko'rsatish sistemasida quyidagi xarakteristkalar asosiy rol o'ynaydi.

π -, $M|G|1|^\infty$ sistemaning bandlik davri, $\pi(u)$, $(u \geq 0)$ - u zagruzkali badlik davri, ω - talablarning sistemada statsionar kutish vaqti, $\rho = 1 - a \cdot \beta$ - sistemadagi talablarning zagruzkasi, ya'ni bir birlik vaqt ichida sistemaga kelib tushgan talablarga o'rtacha xizmat ko'rsatish vaqti.

Statsionar kutish vaqti sistemada qabul qilingan xizmat ko'rsatish tartibiga bog'liq bo'ladi:

“FIFO” - birinchi kelgan talablarga birinchi xizmat ko'rsatiladi.

“RS” – xizmat ko'rsatish uchun talablar tasodifiy tanlanadi.

Mazkur bitiruv malakavi ishida ba'zi limit teoremlar o'rganiladi va bu limit teoremlarning yaqinlashish tezligi kritik zagrulkada baholanadi.

Bitiruv malakaviy ishi kirish qismi, uchta paragraf, xulosa va foydalanilgan adabiyotlardan iborat.

Kirish qismida quyida keltirilgan natijalar uchun qo'yilgan shartlar, $M|G|1|\infty$ sistemaning tavsifi va bitiruv ishida qo'llaniladigan ba'zi belgilashlar keltirilgan.

Birinchi paragrafda $M|G|1|\infty$ sistemaning u - zagrulkali bandlik davri uchun limit teorema keltirilgan va shu limit teoremaning yaqinlashish tezligini kritik zagrulkadagi bahosi olingan.

Ikkinchi va uchunchi paragraflarda mos ravishda “FIFO” va “RS” tartibli $M|G|1|\infty$ modeli ommaviy xizmat ko'rsatish sistemasidagi talablarning statsionar kutish vaqti uchun olingan limit teoremlar keltirilgan va bu teoremlarning kritik zagrulkada yaqinlashish tezligi baholangan.

1-§. Sistemaning bandlik davri uchun yaqinlashish tezligining tahlili

Bu paragrafda $M|G|1|^\infty$ sistemaning u zagruzkali bandlik davri uchun olingan limit teoremaning yaqinlashish tezligining (1) va (2) shartlar bajarilgandagi bahosi olingan.

Keyinchalik qulaylik uchun quyidagicha belgilashlar kiritamiz. $\pi(u)$, $(u \geq 0)$ - $M|G|1|^\infty$ sistemaning u zaderjkali bandlik davri, $\pi - M|G|1|^\infty$ sistemaning bandlik davri,

$$\rho_1 = a\beta_1, \quad \rho_2 = \frac{a\beta_2}{2}, \quad \beta_k = \int_0^\infty x^k dB(x),$$

$$\Pi^{(u)}(x) = P\{\pi(u) < x\}, \quad (x \geq 0), \quad \Pi(x) = P\{\pi < x\}, \quad (x \geq 0),$$

$$\pi^{(u)}(s) = \int_0^\infty e^{-sx} d\Pi^{(u)}(x), \quad (s \geq 0), \quad \pi(s) = \int_0^\infty e^{-sx} d\Pi(x), \quad (s \geq 0)$$

$\Gamma(\cdot)$ - Eylerning gamma funksiyasi,

$$\mu(s) = s + a - a\pi(s), \quad (s \geq 0),$$

$$\nu(s) = s - a + a\beta(s), \quad (s \geq 0).$$

deb, olamiz.

Teorema 1.1. ([4]) (1) va $u \rightarrow +\infty$, $\rho \downarrow 0$ da $\frac{\rho u}{\rho_2} = \tau \rightarrow \tau^0$, $(0 < \tau^0 < +\infty)$

shartlar bajarilsin. U holda

$$\lim P\left\{\frac{\rho^2 \pi(u)}{\rho_2} < x\right\} = F(x), \quad (x \geq 0) \quad (1.1)$$

limit mavjud, bu yerda

$$\left. \begin{aligned} F'(x) &= \frac{\tau^0}{2x\sqrt{\pi x}} \exp\left\{-\frac{(\tau^0 - x)^2}{4x}\right\}, \\ f(s) &= \int_0^\infty e^{-sx} dF(x) = \exp\left\{-\tau^0 \Delta(s)\right\}, (s \geq 0), \\ \Delta(s) &= \sqrt{s + 1/4} - 1/2 \end{aligned} \right\} \quad (1.2)$$

Keyinchalik $M|G|_1^\infty$ sistemadagi kritik zagrzkada (1.1) limit teoramaning yaqinlashish tezligi baholanadi.

Quyidagi tasdiqlarni keltiramiz:

Lemma 1.1. Yuqoridagi (2) shart bajarilsin. U holda $s \geq 0$ uchun quyidagi asimptotik yoyilma o'rinli

$$\mu\left(\frac{\rho^2 s}{\rho_2}\right) = \frac{\rho}{\rho_2} \left[\Delta(s) + A \frac{\Delta^\delta(s)}{1 + 2\Delta(s)} \rho^{\delta-2} (1 + o(1)) \right] \quad (1.3)$$

bu yerda $A = 2\varepsilon / \beta_2 \rho_2^{\delta-2}$

Lemma 1.2. (2) va $u \rightarrow +\infty$, $\rho \downarrow 0$ da $\frac{\rho u}{\rho_2} = \tau \rightarrow \tau^0$, ($0 < \tau^0 < +\infty$) shartlar

bajarilsin. U holda $s \geq 0$ uchun quyidagi asimptotik yoyilma o'rinli

$$\begin{aligned} \pi^{(u)}(\rho^2 \rho_2^{-1} s) = & \exp\left\{-\tau^0 \Delta(s)\right\} + \frac{\Delta(s)}{\exp\left\{\tau^0 \Delta(s)\right\}} \varepsilon_1 (1 + o(1)) - \\ & \tau^0 A \frac{\Delta^\delta(s)}{(1 + 2\Delta(s)) \exp\left\{\tau^0 \Delta(s)\right\}} \cdot \rho^{\delta-2} (1 + o(1)) \end{aligned} \quad (1.4)$$

bu yerda $\varepsilon_1 = |\tau^0 - \tau|$.

Teorema 1.2 (2) va $\rho \downarrow 0$, $u \rightarrow +\infty$ da $\frac{\rho u}{\rho_2} = \tau \rightarrow \tau^0$ ($0 < \tau^0 < +\infty$) shartlar

bajarilsin. Agar $\lim \left[|\varepsilon_1| / \rho^{\delta-2} \right] = d$, ($\varepsilon_1 = \tau^0 - \tau$) limit mavjud bo'lsa, u holda

$$\begin{aligned} \sup_{x \geq 0} \left| \Pi^{(u)}(\rho_2 x / \rho^2) - F(x) \right| \leq & C_1 \cdot \chi(d = +\infty) + \\ & + C_2 \chi(0 \leq d < +\infty) \rho^{\delta-2} \end{aligned} \quad (1.5)$$

tenglik o'rinli bo'ladi, bu yerda

$$C_2 = C_1 \cdot d + \sqrt{2} A (\tau^0)^{-(\delta-2)/2} \cdot b \cdot \Gamma(\delta/2) \cdot \left(1 + \frac{2\delta}{\tau^0} + \frac{\delta(\delta+2)}{(\tau^0)^2}\right),$$

$$b > (2\pi)^{-1} - o'zgarmas \text{ son} \quad \chi(A) = \begin{cases} 1, \text{ agar } A \text{ bajarilsa} \\ 0, \text{ agar } A \text{ bajarilmasa} \end{cases}$$

Lemma 1.1 isboti:

Berilgan $\delta(2 < \delta \leq 3)$ va $\theta \geq 0$ lar uchun quyidagi

$$F_{\theta}(z, s) = 0 \quad (1.6)$$

tenglamani qaraymiz, bu yerda

$$F_{\theta}(z, s) = z + z^2 - \theta z^{\delta} - s.$$

$F_{\theta}(z, s)$ funksiya $O(0,0)$ nuqta atrofida analitik funksiya bo'lib quyidagi

$$F_{\theta}(0,0) = 0, \quad \left. \frac{\partial F(z, s)}{\partial z} \right|_{\substack{z=0 \\ s=0}} = 1$$

shartlarni qanoatlantiradi. Shuning uchun oshkormas funksiya haqidagi teorema ko'ra ([11]) $z = D_{\theta}(s)$ funksiya yuqorida qarayotgan (1.6) tenglamaning yechimi bo'lib, $s = 0$ nuqta atrofida $D_{\theta}(0) = 0$ boshlang'ich shartni qanoatlantiruvchi yagona analitik funksiya bo'ladi. Bundan quyidagi ayniyatga ega bo'lamiz.

$$s \equiv D_{\theta}(s) + [D_{\theta}(s)]^2 - \theta [D_{\theta}(s)]^{\delta} \quad (1.7)$$

$s > 0$, $\theta \downarrow 0$ bo'lsin. Quyidagi

$$E(s, \theta) = D_{\theta}(s) - \Delta(s)$$

belgilashni kiritamiz.

$$D_{\theta}(s) = E(s, 0) + \Delta(s)$$

Ifodani (1.7) tenglikka qo'ysak quyidagini hosil qilamiz

$$s \equiv \Delta(s) + E(s, \theta) + [\Delta(s) + E(s, \theta)]^2 - \theta [\Delta(s) + E(s, \theta)]^{\delta}. \quad (1.8)$$

Chekli θ va s larda $\Delta(s)$ va $D_{\theta}(s)$ lar chegaralangan bo'lganligi uchun $E(s, \theta)$ ham chegaralanganligi kelib chiqadi. U holda $\theta \downarrow 0$ da (1.7) tenglikdan $s \equiv D_0(s) + [D_0(s)]^2$ tenglik o'rinli. Yoki xuddi shuningdek

$$D_0(s) = \lim_{\theta \downarrow 0} D_{\theta}(s) = \Delta(s)$$

shuning uchun

$$\lim_{\theta \downarrow 0} E(s, \theta) = 0.$$

Endi $\theta \downarrow 0$ da (1.8) tenglikni quyidagi ko'rinishda yozamiz:

$$E(s, \theta)(1 + 2\Delta(s)) = \theta \Delta^\delta(s) + o_\theta(E(s, \theta)).$$

Bundan $\theta \downarrow 0$ da osongina ko'rsatish mumkinki

$$E(s, \theta) = \theta \cdot \frac{\Delta^\delta(s)}{1 + 2\Delta(s)} (1 + o_\theta(1)) \quad (1.9)$$

$s^* \downarrow 0$ da quyidagi $v(s^*) = s^* - a + a\beta(s^*)$ funksiyani qaraymiz. (2) munosabatni e'tiborga olsak

$$v(s^*) = \rho s^* + \rho_2 (s^*)^2 - \frac{2\varepsilon}{\beta_2} \cdot \rho_2 (s^*)^\delta (1 + o_{s^*}(1))$$

ega bo'lamiz.

Yuqoridagi tenglikdan $s^* = \rho \cdot \rho_2^{-1} s$ deb olsak, $\rho \downarrow 0$ da quyidagi

$$v(\rho \rho_2^{-1} s) = \rho^2 \rho_2^{-1} [s + s^2 - \frac{2\varepsilon}{\beta_2 \rho_2^{\delta-2}} \rho^{\delta-2} \cdot s^\delta (1 + \varphi)]$$

munosabatni hosil qilamiz, bu yerda $\rho \downarrow 0$ da $\varphi \rightarrow 0$. Agar $A_\rho = \frac{2\varepsilon \rho^{\delta-2}}{\beta_2 \rho_2^{\delta-2}}$ deb

belgilasak u holda quyidagi asimptotik munosabatni hosil qilamiz:

$$v(\rho \rho_2^{-1} s) = \rho^2 \rho_2^{-1} [s + s^2 - A_\rho s^\delta (1 + \varphi)] \quad (1.10)$$

[4] da $\rho_1 \leq 1$ bo'lganda $\mu(s)$ va $v(s)$ funksiyalar uchun quyidagi

$$\mu(v(s)) = s \quad (1.11)$$

Ifoda o'rinli ekanligi isbot qilingan. (1.10) va (1.11) larga asosan

$$s^* = \mu(\rho^2 \rho_2^{-1} (s + s^2 - A_\rho s^\delta (1 + \varphi))) \quad (1.12)$$

hosil qilamiz.

O'rta qiymat haqidagi teorema ko'ra ([10])

$$|\mu(u + v) - \mu(u)| = |\mu'(u + mv)| \cdot |v|$$

tenglik o'rinli, bu yerda $u + v > 0$, $u > 0$ $0 < m < 1$.

$\mu'(s)$ ($s > 0$) funksiya barcha musbat u va v lar uchun monoton kamayuvchi bo'lganligi uchun.

$$\mu'(u + mv) \leq \mu'(0) = \rho^{-1}$$

Bundan

$$\begin{aligned} & \left| \mu(\rho^2 \rho_2^{-1}(s + s^2 - A_\rho s^\delta (1 + \varphi))) - \mu(\rho^2 \rho_2^{-1}(s + s^2 - A_\rho s^\delta)) \right| \leq \\ & \leq \rho \rho_2^{-1} A_\rho s^\delta |\varphi|. \end{aligned}$$

Bu tengsizlikdan (1.12) tenglikni e'tiborga olsak

$$\left| 1 - \frac{\mu(\rho^2 \rho_2^{-1}(s + s^2 - A_\rho s^\delta))}{\mu(\rho^2 \rho_2^{-1}(s + s^2 - A_\rho s^\delta (1 + \varphi)))} \right| \leq A_\rho s^{\delta-1} |\varphi|$$

tengsizlikni hosil qilamiz.

Bundan quyidagini hosil qilamiz.

$$\mu(\rho^2 \rho_2^{-1}(s + s^2 - A_\rho s^\delta)) = \mu(\rho^2 \rho_2^{-1}(s + s^2 - A_\rho s^\delta (1 + \psi)))(1 + \psi) \quad (1.13)$$

bu yerda ψ , $s > 0$ da quyidagi

$$\lim_{\rho \downarrow 0} \frac{|\psi|}{|\varphi| A_\rho s^\delta} \leq 1 \quad (1.14)$$

shartni qanoatlantiradi.

Quyidagi

$$s + s^2 - A_\rho s^\delta = \tau \quad (\tau \geq 0)$$

tenglamani qaraymiz. Bu tenglamaning yechimi $s = D_{A_\rho}(\tau)$ funksiyadan iborat.

(1.13) tenglikdagi s o'rniga $D_{A_\rho}(\tau)$ ni qo'ysak va (1.14) shartni e'tiborga olsak

$$\mu(\rho^2 \rho_2^{-1}) = \rho \rho_2^{-1} (\Delta(\tau) + A_\rho \cdot \frac{\Delta^\delta(\tau)}{1 + 2\Delta(\tau)} (1 + O_\rho(1)))$$

munosabatni hosil qilamiz. Bu 1.2. Lemmaning isbotidir.

1.2. Lemmaning isboti

Ma'lumki

$$\pi^{(u)}(\rho^2 s / \rho_2) = \exp \left\{ -u\mu(\rho^2 s / \rho_2) \right\} \quad (1.15)$$

(1.3) ifodani (1.15) ga qo'ysak

$$\pi^{(u)}(\rho^2 s / \rho_2) = \exp \left\{ -u\rho\rho_2^{-1} \left[\Delta(s) + A \frac{\Delta^\delta(s)}{1+2\Delta(s)} \rho^{\delta-2} (1+0(1)) \right] \right\} \quad (1.16)$$

(1.16) –munosabatda $\tau = \frac{u\rho}{\rho_2}$, $u \rightarrow +\infty$ va $\rho \downarrow 0$ da $\tau \rightarrow \tau^0$, $\varepsilon_1 = \tau^0 - \tau$ larni

e'tiborga olsak

$$\begin{aligned} \pi^{(u)}(\rho^2 s / \rho_2) &= \exp \{ -\tau\Delta(s) \} \cdot \exp \left\{ -\tau A \frac{\Delta^\delta(s)}{1+2\Delta(s)} \cdot \rho^{\delta-2} (1+0(1)) \right\} = \\ &= \exp \{ -\tau\Delta(s) \} \cdot \left[1 - \tau A \frac{\Delta^\delta(s)}{1+2\Delta(s)} \cdot \rho^{\delta-2} (1+0(1)) \right] = \\ &= \exp \{ \varepsilon_1 - \tau^0 \Delta(s) \} \left[1 - \tau^0 A \frac{\Delta^\delta(s)}{1+2\Delta(s)} \cdot \rho^{\delta-2} (1+0(1)) \right] = \\ &= \exp \{ -\tau^0 \Delta(s) \} [1 + \varepsilon_1 \Delta(s) \cdot (1+0(1))] \cdot \left[1 - \tau^0 A \frac{\Delta^\delta(s)}{1+2\Delta(s)} \cdot \rho^{\delta-2} (1+0(1)) \right] \end{aligned}$$

Bundan

$$\begin{aligned} \pi^{(u)}(\rho^2 s / \rho_2) &= \exp \{ -\tau\Delta(s) \} + \frac{\Delta(s)}{\exp \{ \tau^0 \Delta(s) \}} \varepsilon_1 (1+0(1)) - \\ &- \tau^0 A \frac{\Delta^\delta(s)}{(1+2\Delta(s)) \exp \{ \tau^0 \Delta(s) \}} \cdot \rho^{\delta-2} (1+0(1)) \end{aligned}$$

1.2 Lemma isbot bo'ldi.

1.2. Teoremaning isboti. 1.1 Teoremda aniqlangan

$\Pi^{(u)}(\frac{\rho_2^x}{\rho^2})$ va $F(x)$ funksiyalar uchun bizga ma'lum bo'lgan Berri- Esseen

tengsizligini ([5]) qo'llasak

$$\begin{aligned} & \nabla \stackrel{\text{def}}{=} \sup_{x \geq 0} \left| \Pi^{(u)}\left(\frac{\rho_2 x}{\rho^2}\right) - F(x) \right| \leq \\ & \leq b_0 \int_{-T}^T \left| \frac{\pi^{(u)}(-\rho^2 ti / \rho_2) - f(-ti)}{ti} \right| dt + r(b_0) C \frac{1}{T}, \end{aligned} \quad (1.17)$$

bu yerda $b_0 > (2\pi)^{-1}$, T – ixtiyoriy musbat son, $r(b_0)$ – faqat b_0 ga bog'liq bo'lgan musbat o'zgarmas, i – mavhum birlik, $\pi^{(u)}(-\rho^2 ti / \rho_2)$ va $f(-ti)$ lar mos ravishda $\Pi^{(u)}(\rho_2 x / \rho^2)$ va $F(x)$ taqsimot funksiyalarning mos xarakteristik funksiyalari, o'zgarmas C esa $\sup_{x \geq 0} |F'(x)| \leq C$ shartdan aniqlanadi.

Shuni takidlab, o'tamizki $F'(x)$ funksiya musbat yarim o'qda uzluksiz va $x_0 = \sqrt{9 + (\tau^0)^2} - 3$ nuqtada eng katta qiymatga erishadi. Shuning uchun C o'zgarmas sifatida

$$C = \frac{\tau^0}{2\sqrt{\pi}} \exp \left\{ -\frac{(\tau^0 - x_0)}{4x_0} \right\} \cdot \frac{1}{x_0 \sqrt{x_0}}$$

qiymatni olish mumkin.

1.2 Lemmaning tasdig'ini va $f(-ti) = \exp\{-\tau^0 \Delta(-ti)\}$ ni (1.17) ga qo'ysak

$$\begin{aligned} \nabla \leq & b \left[\int_{-T}^T \frac{|\Delta(-ti)|}{|t| \exp\{\tau^0 \Delta(-ti)\}} dt \cdot |\varepsilon_1| \cdot (1 + 0(1)) + \right. \\ & \left. + \tau^0 A \int_{-T}^T \frac{|\Delta(-ti)|^\delta}{|t| \cdot |1 + 2\Delta(-ti)| \cdot \exp\{\tau^0 \Delta(-ti)\}} dt \cdot \rho^{\delta-2} (1 + 0(1)) \right] + r(b) C T^{-1} \end{aligned} \quad (1.18)$$

tengsizlikni hosil qilamiz.

$$\text{Quyidagi } (t \in [-T, +T], \quad v(t)) = \sqrt{1 + 16t^2})$$

$$\begin{aligned}
|\Delta(-ti)| &= \frac{1}{2} \sqrt{1+v(t) - \sqrt{2(1+v(t))}}, \\
|1 + 2\Delta(-ti)| &= \sqrt{v(t)}, \\
\left| \exp \left\{ \tau^0 \Delta(-ti) \right\} \right| &= \exp \left\{ -\frac{\tau^0}{2} \left[1 - \sqrt{\frac{1+v(t)}{2}} \right] \right\},
\end{aligned}$$

tengliklarni e'tiborga olsak (1.18) tengsizlikni

$$\begin{aligned}
\nabla \leq r(b_0) \frac{C}{T} + b_0 e^{\tau^0/2} [R(T) \cdot |\varepsilon_1| (1 + 0(1)) + \\
+ \frac{A\tau^0}{2^{\delta-1}} \cdot L(T) \rho^{\delta-2} (1 + 0(1))],
\end{aligned} \tag{1.19}$$

ko'rinishida yozish mumkin, bu yerda

$$R(T) = \int_0^T \frac{\sqrt{1+v(t) - \sqrt{2(1+v(t))}}}{t \exp \left\{ \frac{\tau^0}{2\sqrt{2}} \cdot \sqrt{1+v(t)} \right\}} dt \tag{1.20}$$

$$L(T) = \int_0^T \frac{[1+v(t) - \sqrt{2(1+v(t))}]^{\delta/2}}{t \cdot \sqrt{v(t)} \cdot \exp \left\{ \frac{\tau^0}{2\sqrt{2}} \cdot \sqrt{1+v(t)} \right\}} dt \tag{1.21}$$

Quyidagi $x = \sqrt{1+v(t)}$ yangi o'zgaruvchini kiritib va ba'zi almashtirishlarni bajarsak (1.20) ifoda

$$R(T) = 2 \left[R_1 - R_2(t) \cdot e^{\frac{\tau^0}{2\sqrt{2}} \cdot \sqrt{1+v(t)}} \right], \tag{1.22}$$

ko'rinishini oladi, bu yerda

$$R_1 = \int_{\sqrt{2}}^{\infty} \frac{x^2 - 1}{x^2 - 2} \sqrt{\frac{x - \sqrt{2}}{x}} \cdot e^{-\frac{\tau^0 x}{2\sqrt{2}}} dx \tag{1.23}$$

$$R_2(T) = \int_0^{\infty} \frac{\omega^2(T, x - \frac{1}{T})}{\omega^2(T, x) - \frac{2}{T}} \cdot \sqrt{\frac{\omega(T, x) - \frac{2}{T}}{\omega(T, x)}} \cdot e^{-\frac{\tau^0}{2\sqrt{2}} x} dx \tag{1.24}$$

$$\omega(T, x) = \frac{x}{\sqrt{T}} + \sqrt{\frac{1}{T} + \sqrt{\frac{1}{T} + 16}}, \quad (x \in (0, +\infty)).$$

Xuddi shunga o'xshash (1.21) munosabatdan

$$L(T) = 2 \left[L_1 - L_2(T) T^{\frac{\delta-2}{2}} \cdot e^{\frac{\tau^0}{2\sqrt{2}} \cdot \sqrt{1+v(t)}} \right] \quad (1.25)$$

ni hosil qilamiz, bu yerda

$$L_1 = \int_{\sqrt{2}}^{\infty} \frac{[x(x - \sqrt{2})]^{(\delta-2)/2}}{x + \sqrt{2}} \cdot \sqrt{x^2 - 1} \cdot e^{-\frac{\tau^0}{2\sqrt{2}}} dx \quad (1.26)$$

$$L_2(T) = \int_0^{\infty} \frac{[\omega(T, x)]^{(\delta-2)/2} \cdot [\omega(T, x) - \sqrt{2/T}]^{\delta/2}}{\omega^2(T, x) - (2/T)} \cdot \sqrt{\omega^2(T, x) - (1/T)} \cdot e^{-\frac{\tau^0}{2\sqrt{2}}} dx \quad (1.27)$$

Veyershtass teoremasiga asosan (1.24) va (1.27) munosabatlardan $R_2(\rho^{-1})$ va $L_2(\rho^{-1})$ funksiyalar $\rho \in [0, 1]$ da tekis yaqinlashadi. U holda $\rho \downarrow 0$ (1.22) va (1.25) larga ko'ra mos ravishda

$$R(\rho^{-1}) = 2R_1(1 + o_{\rho}(1)) \quad (1.28)$$

$$L(\rho^{-1}) = 2L_1(1 + o_{\rho}(1)) \quad (1.29)$$

munosabatlar o'rinli.

Shuning uchun $d \neq +\infty$ hol uchun (1.19), (1.28) va (1.29) lardan $\rho \downarrow 0$ da quyidagi ($b > b_0 > (2\pi)^{-1}$)

$$\nabla \leq 2be^{\tau^0/2} \left[R_1 d + \frac{A\tau^0}{2^{\delta-1}} \cdot L_1 \right] \rho^{\delta-2} \quad (1.30)$$

baho kelib chiqadi, $d = +\infty$ hol uchun (1.19), (1.28) va (1.29) larga ko'ra

$$\nabla \leq 2be^{\tau^0/2} R_1 \cdot |\varepsilon_1| \quad (1.31)$$

tengsizlik o'rinli .

(1.30) va (1.31) - tengsizliklar shuni ko'rsatadiki ∇ baho uchun (1.23) va (1.26)– integrallarni qarab chiqishga to'g'ri keladi. R_1 uchun quyidagi yuqori integral bahoni yozish mumkin.

$$R_1 \leq \frac{1}{2^{7/4}} \int_{\sqrt{2}}^{\infty} x^2 (x - \sqrt{2})^{-1/2} e^{-\tau^0 x / 2\sqrt{2}} dx$$

Bu tengsizlikning o'ng tomonidagi integral jadval integralidir ([12]) ya'ni uning uchun quyidagi munosabat o'rinli

$$R_1 \leq \sqrt{\frac{\pi}{2}} \cdot e^{-\tau^0/2} \cdot \psi\left(\frac{1}{2}; \frac{7}{2}; \frac{\tau^0}{2}\right) \quad (1.32)$$

Bu yerda $\psi(x; y; z)$ funksiya umumlashgan giper geometrik funksiya

$${}_1F_1(x; y; z) = \sum_{k \geq 0} \frac{(x)_k}{(y)_k k!} \cdot z^k,$$

orqali ifodalanadi, bu yerda $(x)_k = x(x+1)\dots(x+k-1)$. Ya'ni bizning hol uchun

$$\psi\left(\frac{1}{2}; \frac{7}{2}; \frac{\tau^0}{2}\right) = \frac{3}{4} \left(\frac{2}{\tau^0}\right)^{5/2} {}_1F_1\left(-2; -\frac{3}{2}; \frac{\tau^0}{2}\right)$$

ko'rinishni oladi. Ba'zi hisoblardan so'ng

$$\psi\left(\frac{1}{2}; \frac{7}{2}; \frac{\tau^0}{2}\right) = \sqrt{\frac{2}{\tau^0}} \left(1 + \frac{2}{\tau^0} + \frac{3}{(\tau^0)^2}\right) \quad (1.33)$$

qiymatni hosil qilamiz.

(1.33) ni (1.32) qo'ysak R_1 uchun quyidagi bahoni topamiz.

$$R_1 \leq \sqrt{\frac{\pi}{\tau^0}} \cdot e^{-\frac{\tau^0}{2}} \cdot \left(1 + \frac{2}{\tau^0} + \frac{3}{(\tau^0)^2}\right) \quad (1.34)$$

Huddi shunday jadval integralidan foydalanib L_1 uchun quyidagi yuqori bahoni topamiz.

$$L_1 \leq \frac{1}{2^{(10-\delta)/4}} \int_{\sqrt{2}}^{\infty} x^2 (x - \sqrt{2})^{(\delta/2)-1} e^{-\tau^0 x / 2\sqrt{2}} dx,$$

Ya'ni ([12] ga qarang)

$$L_1 \leq \frac{(\sqrt{2})^{\delta-3} \Gamma(\delta/2)}{\exp\{\tau^0/2\}} \psi\left(\frac{\delta}{2}; 3 + \frac{\delta}{2}; \frac{\tau^0}{2}\right) \quad (1.35)$$

Keyingi hisoblashlar hatijasida

$$\psi\left(\frac{\delta}{2}, 3 + \frac{\delta}{2}; \frac{\tau^0}{2}\right) = \left(\sqrt{\frac{2}{\tau^0}}\right)^{\delta} \left(1 + \frac{2\delta}{\tau^0} + \frac{\delta(\delta+2)}{(\tau^0)^2}\right),$$

ifodani (1.35) qo'ysak L_1 uchun quyidagi

$$L_1 \leq \frac{\Gamma(\delta/2) 2^{\delta-(3/2)}}{(\tau^0)^{\delta/2} \exp\{\tau^0/2\}} \cdot \left[1 + \frac{2\delta}{\tau^0} + \frac{\delta(\delta+2)}{(\tau^0)^2}\right] \quad (1.36)$$

bahoni hosil qilamiz. (1.30), (1.31), (1.34) va (1.36) tengsizliklar 1.2 Teoremaning isbotini tasdiqlaydi.

2-§. « FIFO » tartibli kutish vaqti uchun yaqinlashish tezligining tahlili.

Mazkur paragrafda « FIFO » tartibli $M|G|1|_{\infty}$ sistemada talablarning statsionar kutish vaqti uchun olingan limit teoremaning yaqinlashish tezligi (1) va (2) – shartlar bajarilganda bahosi olingan.

Statsionar kutish vaqti ω sistemaning talablarga xizmat qilish tartibiga bog'liq.

Faraz qilaylik, talablarga « FIFO » tartibli xizmat ko'rsatilayotgan bo'lsin, ya'ni birinchi kelgan talabga birinchi xizmat ko'rsatilsin.

Qulaylik uchun quyidagicha belgilashlar kiritaylik.

$$W^F(x) = P\{\omega^F < x\} (x \geq 0), \quad \omega^F(s) = \int_0^{\infty} e^{-sx} dW^F(x) (s \geq 0),$$

bu yerda yuqorgi F indeks « FIFO » tartibli xizmat ko'rsatilayotganligini bildiradi.

Teorema 2.1 [8]. (1) -munosabat o'rinli bo'lsin. U holda $\rho \downarrow 0$ da quyidagi

$$\lim P\{(\rho \cdot \omega^F / \rho_2) < x\} = E(x), \quad (x \geq 0) \quad (2.1)$$

limit mavjud, bu yerda

$$E(x) = 1 - e^{-x} \quad (x \geq 0), \quad e(s) = \int_0^{\infty} e^{-sx} dE(x) = \frac{1}{1+s} \quad (s \geq 0)$$

Bu paragrafda $M|G|1|_{\infty}$ sistemadagi kritik zagruzkada 2.1-limit teoremaning yaqinlashish tezligi baholanadi.

Quyidagi tasdiqlarni keltiramiz:

Lemma 2.1 Yuqoridagi (2) munosabat o'rinli bo'lsin. U holda $\rho \downarrow 0$ da $s \geq 0$ uchun quyidagi asimptotik yoyilma o'rinli.

$$\omega(\rho s / \rho_2) = \frac{1}{1+s} + A \frac{s^{\delta-1}}{(1+s)^2} \rho^{\delta-2} (1 + o_{\rho}(1)) \quad (2.2)$$

Teorema 2.2. (2)- munosabat o'rinli bo'lsin. U holda $\rho \downarrow 0$ da

$$\sup_{x \geq 0} |W^F(\rho_2 x / \rho) - E(x)| \leq C_3 \chi(\delta < 3) \rho^{\delta-2} + C_4 \chi(\delta = 3) \rho \ln \rho \quad (2.3)$$

tengsizlik o'rinli bo'ladi, bu yerda

$$C_3 = \frac{b\pi A}{\sin \frac{\delta-1}{2}\pi}, \quad C_4 = \frac{2b\beta_3}{3\beta_2\rho_2}, \quad b > (2\pi)^{-1}.$$

2.1. Lemmaning isboti.

Ma'lumki [1],

$$\omega^F(s^*) = \frac{\rho s^*}{s^* - a(1 - \beta(s^*))} \quad (2.4)$$

(2.4) da $s^* = \rho \cdot \rho_2^{-1} s$ deb olsak

$$\omega^F(\rho s / \rho_2) = \frac{\rho^2 s / \rho_2}{(\rho s / \rho_2) - a + a\beta(\rho s / \rho_2)}. \quad (2.5)$$

hosil qilamiz.

(2) –munosabatni e'tiborga olsak quyidagi

$$\beta(\rho s / \rho_2) = 1 - \beta_1 \rho_2^{-1} \rho s + \beta_2 \rho^2 s^2 / (2\rho^2) - \varepsilon \rho_2^{-\delta} s^\delta (1 + o_\rho(1)) \quad (2.6)$$

asimptotik yoyilmani hosil qilamiz. (2.6) munosabatni (2.5) ga qo'ysak

$$\begin{aligned} \omega^F(\rho s / \rho_2) &= \\ &= \frac{\rho^2 s / \rho_2}{(\rho s / \rho_2) + (a\beta_1 s \rho / \rho_2) + (a\beta_2 s^2 \rho^2 / (2\rho_2^2)) - (a\varepsilon \rho^\delta / \rho_2^\delta)(1 + o_\rho(1))} = \\ &= \frac{\rho^2 s / \rho_2}{((1 - a\beta_1)\rho s / \rho_2) + (\rho^2 s^2 / \rho_2) - (2a\varepsilon \beta_2 \rho^\delta s / (2\rho_2^\delta \beta_2))(1 + o_\rho(1))} = \\ &= \frac{\rho^2 s / \rho_2}{(\rho^2 s / \rho_2) + (\rho^2 s^2 / \rho_2) - (2\varepsilon \rho^\delta / (\beta_2 \rho_2^{\delta-1}))(1 + o_\rho(1))} = \\ &= \frac{\rho^2 s / \rho_2}{(\rho^2 (s + s^2) / \rho_2) + [1 - (2\varepsilon \rho^{\delta-2} / \beta_2 \rho_2^{\delta-2} (s + s^2))(1 + o_\rho(1))]} \end{aligned}$$

Bundan

$$\omega^F(\rho s / \rho_2) = \frac{1}{1+s} + \frac{2\varepsilon}{\beta_2 \rho_2^{\delta-2}} \cdot \frac{s^{\delta-1}}{(1+s)^2} \rho^\delta (1 + o_\rho(1)).$$

Bu yerda agar $A = 2\varepsilon / \beta_2 \rho_2^{\delta-2}$ deb belgilasak (2.2) ni hosil qilamiz. 2.1. lemma isbot bo'ldi.

2.2 Teoremaning isboti. 2.1 Teoramada aniqlangan $W^F(\frac{\rho_2 x}{\rho})$ va $E(x)$

Funksiyalar uchun bizga ma'lum bo'lgan Berri-Esseen tengsizligini ([5]) qo'llasak

$$\begin{aligned} \nabla^F & \stackrel{\text{def}}{=} \sup_{x \geq 0} |W^F(\rho_2 x / \rho) - E(x)| \leq \\ & \leq b_0 \int_{-T}^T \left| \frac{\omega^F(-\rho t i / \rho_2) - e(-ti)}{ti} \right| dt + r(b_0) C^{(1)} T^{-1} \end{aligned} \quad (2.7)$$

bu yerda $b_0 > (2\pi)^{-1}$, T -ixtiyoriy musbat son, $r(b_0)$ -faqat b_0 ga bog'liq bo'lgan musbat o'zgarmas, i -mavhum birlik, $\omega^F(-\rho t i / \rho_2)$ va $e(-ti)$ lar mos ravishda $W(\rho_2 x / \rho)$ va $E(x)$ taqsimot funksiyalarning mos xarakteristik funksiyalari, o'zgarmas $C^{(1)}$ esa $\sup_{x \geq 0} |E'(x)| \leq C^{(1)}$ shartdan aniqlanadi. Shuni takidlab o'tamizki $E'(x)$ funksiya musbat yarim o'qda uzluksiz va $x_0 = 0$ nuqtada eng katta qiymatga erishadi. Shuning uchun $C^{(1)}$ o'zgarmas sifatida $C^{(1)} = 1$ qiymatni olishi mumkin.

$$s = -ti \text{ ni hisobga lsak, 2.1.Lemmaning tasdig'ini va } \ell(-ti) = \frac{1}{1-ti} \text{ ni (2.7)}$$

ga qo'ysak

$$\nabla^F \leq b_0 A \int_{-T}^T \frac{|t|^{\delta-2}}{|1-ti|^2} dt \rho^{\delta-2} (1 + o_\rho(1)) + r(b_0) T^{-1},$$

yoki

$$\nabla^F \leq 2b_0 A \Phi(T) \rho^{\delta-2} (1 + o_\rho(1)) + r(b_0) \cdot T^{-1} \quad (2.8)$$

tengsizlikni hosil qilamiz, bu yerda

$$\Phi(T) = \int_0^T \frac{t^{\delta-2}}{1+t^2} dt.$$

Endi bu integralni hisoblaymiz.

$$\begin{aligned} \Phi(T) &= \int_0^T \frac{t^{\delta-2}}{1+t^2} dt = \frac{1}{2} \int_0^{T^2} \frac{x^{(\delta-3)/2}}{1+x} dx = \\ &= \frac{1}{2} \left[\chi(\delta < 3) \int_0^{T^2} \frac{x^{(\delta-3)/2}}{1+x} dx + \chi(\delta = 3) \int_0^{T^2} \frac{dx}{1+x} \right] = \\ &= \frac{1}{2} \left[\chi(\delta = 3) \ln(1+T^2) + \chi(\delta < 3) \right. \\ &\quad \left. \left(\int_0^\infty \frac{y^{(\delta-3)/2}}{y+T^{-2}} dy - \int_1^\infty \frac{y^{(\delta-3)/2}}{y+T^{-2}} dy \right) \right] \end{aligned}$$

Bundan

$$\begin{aligned} \Phi(T) &= \frac{\chi(\delta < 3)\pi}{2 \sin \frac{\delta-1}{2} \pi} + \frac{1}{2} \chi(\delta = 3) \ln(1+T^2) - \\ &\quad - \frac{1}{2} \chi(\delta < 3) \Phi_1(T) T^{\delta-3} \end{aligned} \quad (2.9)$$

bu yerda

$$\Phi_1(T) = \int_1^\infty \frac{y^{(\delta-3)/2}}{y+T^{-2}} dy \quad (2.10)$$

(2.9) ni (2.8) tengsizlikka qo'ysak

$$\begin{aligned} \nabla^F &\leq b_0 A \left[\chi(\delta < 3) \left(\frac{\pi}{\sin \frac{\delta-1}{2} \pi} - T^{\delta-3} \Phi_1(T) \right) + \right. \\ &\quad \left. + \chi(\delta = 3) \ln(1+T^2) \right] \rho^{\delta-2} (1 + 0_\rho(1)) + r(b_0) T^{-1} \end{aligned} \quad (2.11)$$

hosil qilamiz.

Veyershtass teoremasiga asosan (2.10) munosabatdan $\Phi_1(\rho^{-1})$ funksiya $\rho \in [0,1]$ da tekis yaqinlashadi. U holda $T = \rho^{-1}$ deb olsak, $\rho \downarrow 0$ da (2.11) dan quyidagi ($b > b_0 > (2\pi)^{-1}$)

$$\nabla^F \leq \frac{\chi(\delta < 3)}{\sin \frac{\delta-1}{2} \pi} \rho^{\delta-2} + \chi(\delta = 3) 2Ab \rho \ln \rho^{-1}$$

Baho kelib chiqadi. Bu tasdiq 2.2 teoremaning isbotidir.

§3. “RS” tartibli kutish vaqti uchun yaqinlashish tezligining tahlili.

Bu paragrafda “RS” tartibli $M | G | 1 | \infty$ sistemada talablarning statsionar kutish vaqti uchun olingan limit teoremaning yaqinlashish tezligini (1)- va (2) – shartlar bajarilganda bahosi olingan.

Statsionar kutish vaqti ω , sistemaning talablarga xizmat qilish tartibiga bog’liqdir.

Faraz qilaylik, talablarda “RS” tartibli xizmat ko’rsatilayotgan bo’lsin, ya’ni talablarni tasodifiy ravishda tanlab xizmat ko’rsatilsin.

$$W^R = P\{\omega^R < x\}(x \geq 0) \omega^R(s) = \int_0^{\infty} e^{-Sx} dW^R(x), \quad (S \geq 0),$$

bu yerda yuqori R indeks “RS” tartibli xizmat ko’rsatilayotganligini bildiradi.

Teorema 3.1([3]).

(1) – munosabat o’rinli bo’lsin. U holda $\rho \downarrow 0$ da quyidagi

$$\lim P\{(\rho \cdot \omega^R / \rho_2) < x\} = R(x), (x \geq 0) \quad (3.1)$$

limit mavjud. Bu yerda

$$R(x) = 1 - \int_0^{\infty} \exp\{-\sqrt{t^2 + x}\} dt$$

$$r(S) = \int_0^{\infty} e^{-Sx} dR(x) = \int_0^{\infty} \frac{e^{-z}}{1 + Sz} dz. (\operatorname{Re} S \geq 0)$$

Bu paragrafda yuqoridagi 3.1 limit teoremaning yaqinlashish tezligi kritik zagrutzkada baholanadi.

Quyidagi tasdiqlarni keltiramiz:

Lemma 3.1

Yuqoridagi (2) munosabat o’rinli bo’lsin. U holda $\rho \downarrow 0$ da $S \geq 0$ uchun quyidagi asimptotik yoyilma o’rinli.

$$\begin{aligned}\omega^R(\rho S / \rho_2) &= \int_0^\infty \varphi(S, Z) dz + \\ &+ \rho^{\delta-2} \int_0^\infty \varphi(S, Z) dz (1 + o_\rho(1)),\end{aligned}\quad (3.2)$$

bu yerda $\varphi(S, Z) = \frac{e^{-Z}}{1 + SZ}$;

$$\begin{aligned}\varphi(s, z) &= \chi(\delta = 3) \left[\frac{1}{6} \cdot \frac{s}{1 + sz} \cdot z^3 e^{-z} - \right. \\ &\left. - \frac{s}{1 + sz} z e^{-z} \right] + \frac{2\varepsilon}{\rho_2 \rho_2^{\delta-2}} \cdot \frac{(sz)^{\delta-1}}{(1 + sz)^2} e^{-z}\end{aligned}\quad (3.3)$$

Teorema 3.2.

(2)- munosabat o'rinli bo'lsin. U holda $\rho \downarrow 0$ da

$$\begin{aligned}\sup_{x \geq 0} |W^R(\rho_2 x / \rho) - R_0(x)| &\leq C_5 \chi(\delta < 3) \cdot \\ &\cdot \rho^{\delta-2} + C_6 \chi(\delta = 3) \rho \ln \rho\end{aligned}\quad (3.4)$$

bu yerda $C_5 = \frac{b\pi A}{\sin \frac{\delta-1}{2}} \pi$, $C_6 = -\frac{2b\beta_3}{3\beta_2 \rho_2} - \frac{8}{3}b$

$$A = 2\varepsilon / \beta_2 \rho_2^{\delta-2}, \quad b > (2\pi)^{-1}$$

3.1. Lemmaning isboti.

Ma'lumki ([6])

$$\begin{aligned}\omega^R(S) &= \rho \cdot \int_0^{a(1-\pi(s))} \left[\frac{SZ}{zs - a(1 - \beta(zs))} - \right. \\ &\left. - \frac{(1+z)s}{zs - a(1 - \beta(1+z)s)} \right] \exp\left(\int_0^z \frac{sdv}{sv - a(1 - \beta(s(1+v)))}\right) dz\end{aligned}$$

(2) –munosabatga ko'ra osongina keltirish mumkin.

$$\begin{aligned}a(1 - \beta(\frac{z\rho s}{\rho_2})) &= \frac{zs}{\rho_2} \rho - (\frac{zs}{\rho_2} - \frac{(zs)^2}{\rho_2}) \rho^2 + \\ &+ \frac{2\varepsilon}{\beta_2} \cdot \frac{(zs)^\delta}{\rho_2^{\delta-1}} \rho^\delta (1 + o(1))\end{aligned}\quad (3.6)$$

U holda (3.6) ga ko'ra quyidagini hosil qilamiz.

$$\begin{aligned}
& \frac{sz\rho^2 / \rho_2}{(zs\rho / \rho_2) - a(1 - \beta(zs\rho / \rho_2))} = \\
& = \frac{sz\rho^2 / \rho_2}{[(zs / \rho_2) + (z^2\rho^2 / \rho_2)]\rho^2 - (2\varepsilon z^\delta s^\delta / \beta_2\rho_2^{\delta-1})\rho^\delta(1 + 0_\rho(1))} = \\
& = \frac{sz\rho^2 / \rho_2}{1 + zs - (2\varepsilon(zs)^{\delta-1} / \beta_2\rho_2^{\delta-2})\rho^{\delta-2}(1 + 0_\rho(1))} = \frac{1}{1 + zs} \cdot \\
& \cdot [1 + (2\varepsilon(zs)^{\delta-1} / \beta_2\rho_2^{\delta-2}(1 + z))\rho^{\delta-2}(1 + 0_\rho(1))].
\end{aligned}$$

bundan $\rho \downarrow 0$ da

$$\frac{zs\rho^2 / \rho_2}{(zs\rho / \rho_2) - a(1 - \beta(zs\rho / \rho_2))} = \frac{1}{1 + zs} + A \frac{(zs)^{\delta-1}}{(1 + zs)^2} \rho^{\delta-2}(1 + 0_\rho(1))$$

bu yerda $A = 2\varepsilon / \beta_2\rho_2^{\delta=2}$ ni hosil qilamiz.

Huddi shunday o'xshash (3.6) munosabatni qo'llab $\rho \downarrow 0$ da osongina quyidagilarni hosil qilamiz.

$$\begin{aligned}
& \frac{s\rho / \rho_2}{(zs\rho / \rho_2) - a[1 - \beta((1 + v)s\rho / \rho_2)]} = \\
& = -[1 + (1 + v + (1 + v)^2 s(\rho(1 + 0_\rho(1))))]
\end{aligned} \tag{3.8}$$

yoki

$$\frac{(1 + z)s\rho^2 / \rho_2}{(zs\rho / \rho_2) - a[1 - \beta((1 + z)s\rho / \rho_2)]} = (1 + z)\rho(1 + 0_\rho(1)) \tag{3.9}$$

(3.5) formuladagi s o'rniga $s^* = \rho s / \rho_2$ ifodani qo'ysak, kritik zagruzkada (3.7)-(3.9) munosabatlardan osongina quyidagilarni keltirib chiqaramiz.

$$\begin{aligned}
\omega^R(\rho s / \rho_2) &= \int_0^{a(1-\pi(s^*)) / s^*} \left[\frac{1}{1+zs} + A_1 \frac{(zs)^{\delta-1}}{(1+zs)^2} \rho^{\delta-2} (1+0_\rho(1)) + \right. \\
&+ (1+z)\rho(1+0_\rho(1)) \left. \right] \cdot \exp \left\{ -z - \int_0^z (1+v + (1+v)^2 s) dv \cdot \rho(1+0_\rho(1)) \right\} dt = \\
&= \int_0^{a(1-\pi(s^*)) / s^*} \left[\frac{1}{1+zs} + (A_1 \frac{(zs)^{\delta-1}}{(1+zs)^2} + (1+z)\chi(\delta=3)\rho^{\delta-2}(1+0_\rho(1)) \right] \cdot \\
&\cdot \exp \left\{ -z - [(1+s)z + (\frac{1}{2} + s)z^2 + \frac{1}{3}sz^3] \rho^{\delta-2}(1+0_\rho(1)) \right\} dz = \\
&= \int_0^{a(1-\pi(s^*)) / s^*} \left[\frac{1}{1+sz} + (\chi(\delta=3)(1+z) + A_1 \frac{(zs)^{\delta-1}}{(1+zs)^2}) \rho^{\delta-2}(1+0(1)) \right] \cdot \\
&\cdot [e^{-z} - ((1+s)ze^{-z} + (\frac{1}{2} + s)z^2 e^{-z} + \frac{1}{3}sz^3 e^{-z}) \cdot \rho(1+0_\rho(1))] dz = \\
&= \int_0^{a(1-\pi(s^*)) / s^*} \left\{ \frac{e^{-z}}{1+zs} - [\chi(\delta=3)(\frac{1+s}{1+sz} ze^{-z} + \frac{1}{2} \cdot \frac{1+2s}{1+zs} z^2 e^{-z} + \frac{1}{3} \cdot \frac{s}{1+zs} \cdot \right. \\
&z^3 e^{-z} - (1+z)e^{-z} - A_1 \frac{(zs)^{\delta-1} e^{-z}}{(1+zs)^2}] \rho^{\delta-2}(1+0_\rho(1)) \left. \right\} dz.
\end{aligned}$$

Bundan $\rho \downarrow 0$ da

$$\omega^R(\rho s / \rho_2) = \int_0^{a(1-\pi(s^*)) / s^*} \varphi(zs) dz - \int_0^{a(1-\pi(s^*)) / s^*} \varphi^{(1)}(zs) dz \cdot \rho^{\delta-2}(1+0_\rho(1))$$

yoki

$$\begin{aligned}
\omega^R(\rho s / \rho_2) &= \int_0^\infty \varphi(zs) dz - \int_{a(1-\pi(s^*)) / s^*}^\infty \varphi(z, z) dz - \left[\int_0^\infty \varphi^{(1)}(s, z) dz - \right. \\
&- \left. \int_{a(1-\pi(s^*)) / s^*}^\infty \varphi^{(1)}(s, z) dz \right] \cdot \rho^{\delta-2}(1+0_\rho(1))
\end{aligned} \tag{3.10}$$

bu yerda $\varphi(s, z) = \frac{e^{-z}}{1+sz}$,

$$\begin{aligned}
\varphi^{(1)}(s, z) &= \chi(\delta=3) \left[\frac{1}{3} \cdot \frac{s}{1+zs} z^3 e^{-z} + \frac{1}{2} \cdot \frac{1+2s}{1+zs} z^2 e^{-z} + \right. \\
&+ \left. \frac{1+s}{1+zs} ze^{-z} - (1+z)e^{-z} \right] - A_1 \frac{(zs)^{\delta-1}}{(1+zs)^2} e^{-z}
\end{aligned}$$

Bundan keyin quyidagini ko'rsatamiz.

$$\lim_{\rho \downarrow 0} \frac{a(1 - \pi(s^*))}{s^*} = \infty \quad (3.11)$$

(2)– munosabatni e'tiborga olib va $\pi(s) = \beta(s + a - a\pi(s^*))$ ni qo'llab quyidagiga ega bo'lamiz.

$$1 - \pi(s^*) = \beta_1 \cdot (s^* + a - a\pi(s^*)) - \frac{1}{2} \beta_2 (s^* + a - a\pi(s^*))^2 + \varepsilon(s^* + a - a\pi(s^*))^\delta (1 + 0(1))$$

Bundan

$$\begin{aligned} \frac{a(1 - \pi(s^*))}{s^*} &= 1 - \rho + \frac{a(1 - \pi(s^*))}{s^*} (1 - \rho) - \\ &- \rho_2 s^* (1 + \frac{a(1 - \pi(s^*))^2}{s^*}) + a\varepsilon(s^*)^{\delta-1} (1 + \frac{a(1 - \pi(s^*))}{s^*})^\delta (1 + 0(1)), \end{aligned}$$

yoki

$$\begin{aligned} \frac{a(1 - \pi(s^*))}{s^*} \rho &= 1 - \rho - \rho_2 s^* [1 + 2 \frac{a(1 - \pi(s^*))^2}{s^*} + \\ &+ (\frac{a(1 - \pi(s^*))}{s^*})^\delta]^\delta (1 + 0(1)) \end{aligned} \quad (3.12)$$

Quyidagi

$$L(s^*) = a(1 - \pi(s^*)) / s^*$$

belgilash kiritsak, (3.12) tenglikdan

$$\begin{aligned} (2\rho_2 s^* + \rho)L(s^*) &= 1 - \rho - \rho_2 s^* - \rho_2 s^* L^2(s^*) + \\ &+ a\varepsilon(s^*)^{\delta-1} (1 + L(s^*))^\delta (1 + 0(1)) \end{aligned} \quad (3.13)$$

hosil qilamiz.

(3.11) munosabatning to'g'ligini isbotlash uchun

$$\lim_{\rho \downarrow 0} L(s^*) = +\infty$$

ni ko'rsatish yetarlidir.

$$\lim_{\rho \downarrow 0} L(s^*) = A < +\infty.$$

U holda shunday $\{\rho_n\}_{n \geq 1}$ lar mavjudki $\lim_{n \rightarrow \infty} \rho_n = 0$ va $\lim_{n \rightarrow \infty} L^{(n)} = A$ bu yerda

$$L^{(n)} = L(s^*)|_{\rho=\rho_n}$$

(3.13) munosabatda ρ o'rniga ρ_n ni qo'ysak va s^* munosabatni e'tiborga olsak

$$\begin{aligned} \rho_n(2s+1)L^{(n)} &= 1 - \rho_n - \rho_n s - \rho_n s \cdot [L^{(n)}]^2 + \frac{\varepsilon}{\rho_2 \beta_1} \cdot \\ &\cdot (1 - \rho_n) \rho_n^{\delta-1} s^{\delta-1} (1 + L^{(n)})^\delta \cdot (1 + o(1)) \end{aligned}$$

ni hosil qilamiz.

Bunday ziddiyatga kelib qoldik. Haqiqatdan ham oxirgi tenglikning chap tomoni nolga intiladi, o'ng tomoni esa birga intiladi.

Quyidagi

$$\lim_{\rho \downarrow 0} \frac{1}{\rho^{\delta-2}} \cdot \int_{a(1-\pi(s^*)) / s^*}^{\infty} \varphi(s, z) dz = 0 \quad (3.14)$$

tenglik o'rinli ekanini ko'rsatamiz.

Haqiqatdan

$$\begin{aligned} 0 &\leq \frac{1}{\rho^{\delta-2}} \int_{\frac{a(1-\pi(s^*))}{s^*}}^{\infty} \varphi(s, z) dz = \frac{1}{\rho^{\delta-2}} \int_{\frac{a(1-\pi(s^*))}{s^*}}^{\infty} \frac{e^z dz}{1 + sz} = \frac{1}{\rho^{\delta-2}} \cdot \\ &\cdot e^{-\frac{a(1-\pi(s^*))}{s^*}} \cdot \int_0^{\infty} \frac{e^{-x}}{1 + \frac{as(1-\pi(s^*))}{s^*} + sx} dx = \frac{1}{\rho^{\delta-2}} e^{-\frac{a(1-\pi(s^*))}{s^*}} \cdot \\ &\cdot \int_0^{\infty} \frac{e^{-x}}{1 + \frac{a\rho_2(1-\pi(\rho s / \rho_2))}{\rho}} dx \leq \frac{1}{\rho^{\delta-2}} \cdot e^{-\frac{a(1-\pi(s^*))}{s^*}} \cdot \\ &\cdot \int_0^{\infty} \frac{e^{-x}}{1 + sx} dx \leq \frac{1}{\rho^{\delta-2}} \cdot e^{-\frac{a(1-\pi(s^*))}{s^*}}. \end{aligned}$$

Bundan

$$0 \leq \frac{1}{\rho^{\delta-2}} \int_{\frac{a(1-\pi(s^*))}{s^*}}^{\infty} \frac{e^{-z} dz}{1+sz} \leq \frac{1}{\rho^{\delta-2}} \cdot e^{-\frac{a(1-\pi(s^*))}{s^*}} \quad (3.15)$$

osongina keltirib chiqarish munkinki

$$\lim_{\rho \downarrow 0} \frac{1}{\rho^{\delta-2}} \cdot e^{-\frac{a(1-\pi(s^*))}{s^*}} = 0.$$

U holda (3.15) dan (3.14) kelib chiqadi. Bundan keyin (3.11) ni e'tiborga olsak quyidagini hosil qilamiz.

$$\lim_{\rho \downarrow 0} \frac{\int_0^{\infty} \varphi^{(1)}(s, z) dz}{\int_0^{\infty} \varphi^{(1)}(s, z) dz} = 0 \quad (3.16)$$

(3.10), (3.14), (3.16) ladan va murakkab bo'lmagan almashtirishlar ni bajarsak (3.2) ni hosil qilamiz.

3.1. Lemma isbot bo'ldi.

3.2. Teoremaning isboti.

Berri Essen tengsizligini ([5]) 3.1. teoremda aniqlangan $W^R(\rho_2 x / \rho)$ va $R(x)$ taqsimot funksiyalar uchun qo'llab quyidagini hosil qilamiz.

$$\begin{aligned} & \nabla_1^R \underline{\sup}_{x \geq 0} |W^R(\rho_2 x / \rho) - R_0(x)| \leq \\ & \leq b_0 \int_{-T}^T \left| \frac{\omega^R(-\rho t i / \rho_2) - \tau_0(-t i)}{t i} \right| dt + \quad (3.17) \\ & + \tau(b_0) c^{(3)} T^{-1} \end{aligned}$$

bu yerda $b_0 > (2\pi)^{-1}$, T – ixtoriy musbat son, $T(b_0)$ - faqat b_0 ga bog'liq bo'lgan musbat o'zgarmas, i – mavhum birlik, $\omega^R(-\rho t i / \rho_2)$ va $r(-t i)$ lar mos ravishda $\omega^R(\rho_2 x / \rho)$ va $R(x)$ taqsimot funksiyalarga mos xarakteristik funksiyalar, o'zgarmas $C^{(3)}$ esa $\sup_{x \geq 0} |R'(x)| C^{(3)}$ shartdan aniqlanadi.

Shuni ta'kidlab o'tamizki, $R'(x)$ funksiya musbat yarim o'qda musbat, uzluksiz va $R'(0)=0$, $R'(+\infty)=0$. Shuning uchun $R'(x)$ funksiya o'zining eng katta qiymatiga erishadi.

$s = -ti$ ni hisobga olsak, 3.1. lemmaning tasdig'ini va

$r(-ti) = \int_0^\infty \frac{e^{-z}}{1-tzi} dz$ ni (3.17) ga qo'ysak, quyidagini hosil qilamiz.

$$\begin{aligned} \nabla_1^R &\leq b_0 \int_{-T}^T \left[\int_0^\infty \frac{|\varphi - (ti, z)|}{|t| \wedge 1} dz \right] dt \cdot \\ &\cdot \rho^{\delta-2} (1 + 0\rho(1)) + \tau(b_0) C^{(3)} T^{-1} \end{aligned} \quad (3.18)$$

(3.3) ni

$$bR_1^{(1)}(T) = \int_{-T}^T \left[\int_0^\infty \frac{|\varphi(-ti, z)|}{|t|} dz \right] dt$$

ga qo'ysak osongina quyidagini hosil qilamiz.

$$\begin{aligned} R_1^{(1)}(T) &\leq 2\chi(\delta=3) \left\{ \frac{1}{6} \int_0^T \left[\int_0^\infty \frac{z^3 e^{-z}}{\sqrt{1+(zt)^2}} dz \right] dt + \right. \\ &+ \left. \int_0^T \left[\int_0^\infty \frac{z^3 e^{-z}}{\sqrt{1+(zt)^2}} dt \right] dz \right\} + \frac{4\varepsilon}{\beta_2 \rho_2^{\delta-2}} \cdot \\ &\cdot \int_0^T \left[\int_0^\infty \frac{t^{\delta-2} z^{\delta-1} e^{-z}}{1+(zt)^2} dz \right] dt. \end{aligned} \quad (3.19)$$

Shuni ko'rsatamizki,

$$\int_0^\infty \frac{t^{\delta-2} z^{\delta-1} e^{-z}}{1+(zt)^2} dz$$

Integral $[0, T]$ kemada t – lar bo'yicha tekis yaqinlashuvchi.

Haqiqatdan

$$0 \leq \int_0^\infty \frac{t^{\delta-2} z^{\delta-1} e^{-z}}{1+(zt)^2} dz \leq T^{\delta-2} \int_0^\infty z^{\delta-1} e^{-z} dz = T^{\delta-2} \Gamma(\delta)$$

Huddi shunday (3.19) tengsizlikning o'ng tomonidagi qolgan integrallar ham $[0, T]$ kesmada t lar bo'yicha tekis yaqinlashishini osongina ko'rsatish mumkin.

Bularni e'tiborga olsak, integralning tartibini almashtirish teoremasiga ko'ra ([6])

(3.19) tengsizliklarni quyidagicha ifodalash mumkin.

$$\begin{aligned}
 R_1^{(1)}(T) \leq & 2\chi(\delta=3) \left\{ \frac{1}{6} \int_0^\infty \int_0^T \frac{dt}{\sqrt{1+(zt)^2}} \cdot \right. \\
 & \cdot z^3 e^{-z} dz + \int_0^\infty \int_0^T \frac{t^{\delta-2} dt}{\sqrt{1+(zt)^2}} \left. \right] z e^{-z} dz \Big\} + \\
 & + \frac{4\varepsilon}{\beta_2 \rho_2^{\delta-2}} \int_0^\infty \int_0^T \frac{dt}{1+(zt)^2} \left. \right] z^{\delta-2} e^{-z} dt
 \end{aligned} \tag{3.20}$$

Quyidagi integrallarni hisoblaymiz.

$$R_2^{(1)}(T, z) = \int_0^T \frac{dt}{\sqrt{1+(zt)^2}}$$

va

$$R_3^{(1)}(T, z) = \int_0^T \frac{t^{\delta-2}}{1+(zt)^2} dt$$

$$R_2^{(1)}(T, z) = \frac{1}{2} \int_0^{zt} \frac{du}{\sqrt{1+u^2}} = \frac{1}{z} \ln |u + \sqrt{1+u^2}| \Big|_0^{zt}$$

Bundan,

$$R_2^{(1)}(T, z) = \frac{1}{z} \ln(2T + \sqrt{1+(zT)^2}) \tag{3.21}$$

$$\begin{aligned}
R_3^{(1)}(T, z) &= \frac{1}{2z^{\delta-1}} \int_0^{(zt)^2} \frac{u^{(\delta-3)/2} du}{1+u} = \\
&= \frac{1}{2z^{\delta-1}} [\chi(\delta < 3) \int_0^{(zt)^2} \frac{u^{(\delta-3)/2} du}{1+u} + \\
&+ \chi(\delta = 3) \int_0^{(zt)^2} \frac{du}{1+u} +] = \frac{\chi(\delta < 3) T^{\delta-3}}{2z^2} \cdot \\
&\cdot \int_0^1 \frac{x^{(\delta-3)/2}}{x + (zt)^2} + \frac{\chi(\delta = 3)}{2z^2} \ln(1 + (zT)^2) = \\
&= \frac{\chi(\delta < 3)}{2z^2} T^{\delta-3} \left[\int_0^\infty \frac{x^{(\delta-3)/2} dx}{x + (zt)^{-2}} - \right. \\
&\left. - \int_1^\infty \frac{x^{(\delta-3)/2} dx}{x + (zt)^{-2}} \right] + \frac{\chi(\delta = 3)}{2z^2} \ln(1 + (zt)^2) .
\end{aligned}$$

Bundan quyidagi osongina lelib chiqadi.

$$\begin{aligned}
R_3^{(1)}(T, z) &= \frac{\chi(\delta = 3)}{2z^2} \ln(1 + (zt)^2) + \\
&+ \frac{\chi(\delta < 3)}{2z^{\delta-1}} \cdot \frac{\pi}{\sin \frac{\delta-1}{2} \pi} - R_4^{(1)}(T, z) \frac{\chi(\delta < 3)}{2z^2} T^{\delta-3}
\end{aligned} \tag{3.22}$$

bu yerda

$$R_4^{(1)}(T, z) = \int_1^\infty \frac{x^{(\delta-3)/2} dx}{x + (zT)^{-2}}$$

Agar $T = \rho^{-1}$ deb olsak va

$$\begin{aligned}
\ln(zt + \sqrt{1 + (z\pi)^2}) &= \ln(z + \sqrt{\rho^2 + z^2}) - \ln \rho \\
\ln(1 + (zt)^2) &= \ln(\rho^2 + z^2) - 2 \ln \rho
\end{aligned}$$

larni hisobga olsak (3.21) va (3.22) lardan quyidagilarni hosil qilamiz.

$$R_2^{(1)}(\rho^{-1}, z) = \frac{1}{z} [\ln(z + \sqrt{\rho^2 + z^2}) - \ln \rho] \tag{3.23}$$

$$\begin{aligned}
R_3^{(1)}(\rho^{-1}, z) &= \frac{\chi(\delta=3)}{2z^2} [\ln(\rho^2 + z^2) - 2\ln \rho] + \\
&+ \frac{\chi(\delta < 3)}{2z^{\delta-1}} \cdot \frac{\pi}{\sin \frac{\delta-1}{2}} - R_4^{(1)}(\rho^{-1}, z) \cdot \frac{\chi(\delta < 3)}{2z^2} \cdot \rho^{-(\delta-3)}
\end{aligned} \tag{2.24}$$

U holda (3.23) va (3.21) larni (3.20) ga qo'ysak quyidagini hosil qilamiz.

$$\begin{aligned}
R_3^{(1)}(\rho^{-1}, z) &\leq \frac{1}{3} \chi(\delta=3) [R_5^{(1)}(\rho) + \\
&+ \frac{\beta_3}{\beta_2 \rho_2} R_6^{(1)}(\rho) - (8 + \frac{\beta_2}{\beta_2 \rho_2}) \ln \rho] + \\
&+ \frac{2\epsilon \chi(\delta < 3)}{\beta_2 \rho_2^{\delta-2}} \left[\frac{\pi}{\sin \frac{\delta-1}{2} \pi} - R_7^{(1)}(\rho) \rho^{3-\delta} \right]
\end{aligned} \tag{3.25}$$

bu yerda

$$\begin{aligned}
R_5^{(1)}(\rho) &= \int_0^\infty (z^2 + 6) e^{-z} \ln(z + \sqrt{\rho^2 + z^2}) dz \\
R_6^{(1)}(\rho) &= \int_0^\infty e^{-z} \ln(\rho^2 + z^2) dz \\
R_7^{(1)}(\rho) &= \int_0^\infty R_4(\rho^{-1}, z) z^{\delta-3} e^{-z} dz
\end{aligned}$$

Yuqoridagi $R_5^{(1)}, R_6^{(1)}(\rho)$ va $R_7^{(1)}(\rho)$ integrallarning $0 \leq \rho \leq 1$ kesmada tekis yaqinlashishini ko'rsatamiz.

Haqiqatdan

$$\begin{aligned}
0 &\leq \int_0^\infty (z^2 + 6) e^{-z} \ln(z + \sqrt{\rho^2 + z^2}) dz \leq \\
&\leq \int_0^\infty (z^2 + 6) e^{-z} \ln(z + \sqrt{1 + z^2}) dz = \\
&= \int_0^\infty (z^2 + 6) e^{-z} \left[\int_0^z \frac{dx}{\sqrt{1 + x^2}} \right] dz \leq \\
&\leq \int_0^\infty z(z^2 + 6) e^{-z} dz = 12
\end{aligned}$$

Huddi shunday $R_6^{(1)}(\rho)$ va $R_7^{(1)}(\rho)$ intervallar ham $0 \leq \rho \leq 1$ kesmada tekis yaqinlashadi.

U holda (3.25) ni e'tiborga olsak quyidagini hosil qilamiz.

$$\begin{aligned}
 R_1^{(1)}(\rho^{-1})\rho^{\delta-2}(1+0_\rho(1)) &\leq \frac{2\varepsilon\chi(\delta < 3)}{\beta_2\rho_2^{\delta-2}} \cdot \\
 &\cdot \frac{\pi}{\sin \frac{\delta-1}{2}\pi} \rho^{\delta-2}(1+0_\rho(1)) - \left(\frac{8}{3} + \frac{2\beta_3}{3\beta_2\rho_2}\right) \cdot \\
 &\cdot \chi(\delta = 3)\rho \ln(1+0_\rho(1))
 \end{aligned} \tag{3.26}$$

(3.26) ni (3.18) ga qo'ysak 3.2 teoremaning tasdigini ($b < b_0 > (2\pi)^{-1}$) uchun hosil qilamiz.

XULOSA

Bitiruv malakaviy ishida “FIFO” va “RS” tartibli $M|G|1|^\infty$ sistemaning bandlik davri va statsionar kutish vaqti uchun kritik zagrutzkada olingan limit teoremlarning yaqinlashish tezligi baholangan.

Yuqorida keltirilgan 1.2., va 3.2. teoremlarning isboti davomida baholar olishga erishdim.

Bir kanalli $M|G|1|^\infty$ modelli ommaviy xizmat ko’rsatish sistemasi quyidagicha ifodalaniladi:

Sistemaga a parametrli Puasson taqsimotli bog’liqsiz talablar kelib tushadi. Talablarga xizmat ko’rsatish vaqtining uzunligi ixtiyoriy $B(x)$, $B(+0) = 0$ taqsimot funksiyaga ega.

Limit teoremlar $B(x)$ taqsimot funksiyaning birinchi ikkita β_1 va β_2 momentlarning chekliligi shartida ya’ni $S \rightarrow 0$, $(\operatorname{Re} S \geq 0)$ da

$$\beta(s) = \int_0^\infty e^{-sx} dB(x) = 1 - \beta_1 \cdot s + \frac{1}{2} \beta_2 \cdot s^2 (1 + o_s(1)) \quad (1)$$

yoyilma o’rinli bo’lgan sharti ostida isbot qilingan. Shu bilna birga bu limit teoremlarning yaqinlashish tezligini baholashda limit teoremlarga qo’yilganshertlar qanoatlantiriladi.

Mazkur birtiruv malakaviy ishda ba’zi qo’shimcha shartlar bajarilganda limit teoremlarning yaqinlashish tezligi baholanadi. Yuqorida keltirilgan (1) shart o’rniga quyidagi kuchliroq shart bajarilishi faraz qilinadi: $S \rightarrow 0$, $(\operatorname{Re} S \geq 0)$ da

$$\beta(s) = 1 - \beta_1 \cdot s + \frac{1}{2} \beta_2 \cdot s^2 - \varepsilon s^\delta (1 + o_s(1)) \quad (2)$$

yoyilma o’rinli bo’lsin, bu yerda $2 < \delta \leq 3$, ε - musbat o’zgarmas son.

Biz ko’rib o’tayotgan $M|G|1|^\infty$ ommaviy xizmat ko’rsatish sistemasida quyidagi xarakteristkalar asosiy rol o’ynaydi.

π -, $M|G|1|^\infty$ sistemaning bandlik davri, $\pi(u)$, $(u \geq 0)$ - u zagrutzkali badlik davri, ω - talablarning sistemada statsionar kutish vaqti, $\rho = 1 - a \cdot \beta$ -

sistemadagi talablarning zagruzkasi, ya'ni bir birlik vaqt ichida sistemaga kelib tushgan talablarga o'rtacha xizmat ko'rsatish vaqti.

Statsionar kutish vaqti sistemada qabul qilingan xizmat ko'rsatish tartibiga bog'liq bo'ladi:

“FIFO ”- birinchi kelgan talablarga birinchi xizmat ko'rsatiladi.

“RS” – xizmat ko'rsatish uchun talablar tasodifiy tanlanadi.

Mazkur bitiruv malakavi ishida ba'zi limit teoremlar o'rganiladi va bu limit teoremlarning yaqinlashish tezligi kritik zagruzkada baholanadi.

Bitiruv malakaviy ishi kirish qismi, uchta paragraf, xulosa va foydalanilgan adabiyotlardan iborat.

Bitiruv malakaviy ishida $M|G|1|\infty$ sistemaning u - zagruzkali bandlik davri uchun limit teorema keltirilgan va shu limit teoremaning yaqinlashish tezligini kritik zagruzkadagi bahosi olingan.

Ushbu ishda mos ravishda “ FIFO” va “RS” tartibli $M|G|1|\infty$ modeli ommaviy xizmat ko'rsatish sistemasidagi talablarning statsionar kutish vaqti uchun olingan limit teoremlar keltirilgan va bu teoremlarning kritik zagruzkada yaqinlashish tezligi baholangan.

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