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MEXANIKA-MATEMATIKA fakulteti “MATEMATIK-FIZIKA VA FUNKSIYALAR NAZARIYASI” Kafedrasi

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Firidrixs modeli va uning spektrial xossalari

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Kirish.

Masalaning qo'yilishi. $H=L_2(T^1)$ Hilbert fazosida aniqlangan h_λ Fridrixs model operatorlar oilasining spektrial xossalarini o'rganish.

Mavzuning dolzarbligi. Qattiq jismlar fizikasi kvant mexanikasi statistik fizika matematik fizikaning zamonaviy masalalarini yechishda Fridrixs modelining spektrial masalalarini yechishga to'g'ri keladi. Ishda qaralayotgan operatorning muhim spektri o'rni va muhim spektridan tashqarida yagona xos qiymatga ega ekanligi ko'rsatilgan.

Ishning maqsadi va vazifalari. Bitiruv malakaviy ishining maqsadi h_λ Fridrixs modelining muhim spektrdan chapda yagona xos qiymatga ega ekanligini ko'rsatishdan iborat.

Ilmiy tadqiqot usullari. Funksiyaning nollari chiziqli o'z-o'ziga qo'shma operatorlar nazariyasi, h_λ Fridrixs modeliga mos Fredgolm determinant.

Ishning ilmiy ahamiyati. Bitiruv malakaviy ishiga olingan natijalar referativ xarakterga ega.

Ishning amaliy ahamiyati. Bitiruv malakaviy ishida olingan natijalar nazariy ahamiyatga ega.

Ishning tuzilishi. Bitiruv malakaviy ishi kirish, II bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan iborat. Har bir bob bo'limlarga ajratilgan va ular o'zining raqamlanish va belgilanishlariga ega. Ishning I bobida asosiy natijani bayon qilish uchun kerakli tushunchalar keltirilgan. II bobida esa masalaning qo'yilishi, olingan natija va uning isboti bayon qilingan.

Olingan natijalar. Ushbu ishda h_λ operatorning muhim spektridan chapda yagona xos qiymatga ega ekanligi ko'rsatilgan va bu xos qiymatga mos xos funksiyaning ko'rinishi topilgan. Bundan tashqari h_λ operatorni $z=0$ nuqtada virtual sathga ega ekanligi va virtual holat ko'rinishi ko'rsatilgan.

I. Bob boshlang'ich tushunchalar

Bu bobda biz ushbu malakaviy bitiruv ishini bayon qilishda kerak bo'ladigan eng asosiy tushunchalar teorema ta'riflarini keltiramiz.

1§.Chiziqli fazo.

Ta'rif 1.1.1. Bo'sh bo'lmagan L to'plam berilgan bo'lsin. Uning elemetlari $x,y,z,\ldots$ bilan belgilaymiz. Agar L to'plamda elementlarni qo'shish va ko'paytirishga amallar kiritilgan bo'lib, ular quyidagi shartlarni qanoatlantirsa L ga chiziqli fazo deyiladi.

I. Ixtiyoriy ikkita $x,y,\in L$ elementlar uchun ularning yig'indisi deb ataluvchi va x+y bilan belgilanuvchi uchinchi $z \in L$ element mavjud bo'lib, quyidagi shartlar bajarilsa:

1) $x+y = y+x$.

2) $x+(y+z) = (x+y)+z$.

3) L da shunaqangi o element mavjudki, ixtiyoriy $x\in L$ uchun $x+o = x$ bo'lib (nol element mavjudligi)

4) ixtiyoriy $x \in L$ uchun shunday -x element topilib, $x+(-x) = o$ bo'lsa (qarama-qarshi element mavjudligi).

II. Ixtiyoriy $\alpha \in C\ (R)$ son va ixtiyoriy $x \in L$ uchun $\alpha \cdot x \in L$ element aniqlangan
bo'lib (x elementni α songa ko'paytirish)

1) $\alpha\ (\beta x) = (\alpha \cdot \beta)x$.

2) $1 \cdot x = x$ bo'lsa, qo'shish va ko'paytirish amallari distributivlik qonunu.

3) $(\alpha + \beta)\, x = \alpha x + \beta x \qquad \alpha,\ \beta \in C\,(R) \quad \forall\ x \in L$

4) $\alpha\,(x + y) = \alpha x + \alpha y \qquad \alpha \in C\,(R)$

$\forall\, x,y \in L$

Agar to'plam elementlari uchun faqat haqiqiy songa ko'paytirish amali aniqlangan bo'lsa, u holda L haqiqiy chiziqli fazo deyiladi.

Misol 1.1.2. $L = l_2$ Fazoni qaraymiz. Elementlari hadlarining modullari kvadratlaridan tuzilgan qator yaqinlashuvchi bo'lgan barcha ketma-ketliklar to'plami ya'ni

$$l_2 = \left\{ x = \{x_i\}_{i=1}^{\infty},\ \sum_{i=1}^{\infty} |x_i|^2 < \infty \right\}$$

Bu fazoda qo'shish va songa ko'paytirish amallarini quyidagicha kiritamiz.

$$x+y = \{x_i\}_{i=1}^{\infty} + \{y_i\}_{i=1}^{\infty} = \{x_i + y_i\}_{i=1}^{\infty}$$

$\alpha x = \{\alpha \cdot x_i\}_{i=1}^{\infty}$ $x \in l_2$, $\alpha \in$ C (R)

Haqiqatdan ham $x,y \in l_2$ bo'lsa

$$\sum_{i=1}^{\infty} |x_i + y_i|^2 \le \sum_{i=1}^{\infty} 2\left(|x_i|^2 + |y_i|^2\right) = 2\left(\sum_{i=1}^{\infty} |x_i|^2 + \sum_{i=1}^{\infty} |y_i|^2\right) < \infty$$

va
$$\sum_{i=1}^{\infty} |x_i|^2 = \sum_{i=1}^{\infty} \alpha^2 |x_i|^2 = \alpha^2 \sum_{i=1}^{\infty} |x_i|^2 < \infty$$

Demak kiritilgan qo'shish va songa ko'paytirish amallari to'g'ri ekan. Endi chiziqli fazo shartlarini qanoatlantirishini tekshiramiz.

1) $\forall x,y \in l_2$ uchun $x+y = \{x_i\}_{i=1}^{\infty} + \{y_i\}_{i=1}^{\infty} = \{y_i\}_{i=1}^{\infty} + \{x_i\}_{i=1}^{\infty} = y+x$

2) $\forall x,y, z \in l_2$ uchun $x+(y+z) = \{x_i\}_{i=1}^{\infty} + (\{y_i\}_{i=1}^{\infty} + \{z_i\}_{i=1}^{\infty}) =$

$(\{x_i\}_{i=1}^{\infty} + \{y_i\}_{i=1}^{\infty}) + \{z_i\}_{i=1}^{\infty} = (x+y) + z.$

3) $\theta = (0,0, . \_, 0, \_) \in l_2$ va ixtiyoriy $x \in l_2$ uchun $(x+0) = (x_1, x_2, \dots, x_n, \dots) +$
$+ (0,0, . \_, 0, \_) = (x_1+0, x_2+0, \dots, x_n+0, \dots) = (x_1, x_2, . \_, x_n, . \_) = x.$

4) Ixtiyoriy $x \in l_2$ uchun -x $\{x_i\}_{i=1}^{\infty}$ mavjud va $x+(-x) = \{x_i\}_{i=1}^{\infty} + \{-x_i\}_{i=1}^{\infty} =$

$= \{x_i - x_i\}_{i=1}^{\infty} = \theta$.

5) Ixtiyoriy $x \in l_2$ element va ixtiyoriy $\alpha + \beta \in$ C (R) sonlar uchun $\alpha(\beta x) =$

$= \alpha \{\beta x_i\}_{i=1}^{\infty} = \{\alpha \beta x_i\}_{i=1}^{\infty} = (\alpha\beta) \{x_i\}_{i=1}^{\infty} = (\alpha\beta)$ x.

6) Ixtiyoriy $x \in l_2$ va ixtiyoriy element uchun $1x = 1 \cdot \{x_i\}_{i=1}^{\infty} = 1 = \{1 \cdot x_i\}_{i=1}^{\infty} = x.$

7) $\forall x \in l_2$ va $\forall \alpha, \beta \in C(R)$ sonlar uchun $(\alpha+\beta)x = (\alpha+\beta)\{x_i\}_{i=1}^{\infty} =$

$= \{(\alpha+\beta) x_i\}_{i=1}^{\infty} = \{\alpha x_i + \beta x_i\}_{i=1}^{\infty} = \alpha \{x_i\}_{i=1}^{\infty} + \beta \{x_i\}_{i=1}^{\infty} = \alpha x + \beta x.$

8) Ixtiyoriy $x, y \in l_2$ va ixtiyoriy $\alpha \in C(R)$ uchun $\alpha(x+y) = \alpha(\{x_i\}_{i=1}^{\infty} +$

$+ \{y_i\}_{i=1}^{\infty}) = \alpha \{x_i\}_{i=1}^{\infty} + \alpha \{y_i\}_{i=1}^{\infty} = \alpha x + \alpha y.$

Demak biz ko'rdikki l_2 fazoning elementlari chiziqli fazo ya'rifi barcha shartlarni qanoatlantiradi. Shuning uchun l_2 chiziqli fazo bo'ladi.

Ta'rif: 1.1.3. L va L * chiziqli fazolar izomarf deyiladi, agar ular elementlari o'rtasida. Quyidagi shartlarni qanoatlantiruvchi o'zaro bir qiymatli moslik o'rnatish mumkin bo'lsa.

$x \longleftrightarrow x^*$ va $y \longleftrightarrow y^*$

$(x,y \in L, \quad x,^* y^* \in L)$ ekanligidan $x + y \longleftrightarrow x^* + y^*$ va

$\alpha x \longleftrightarrow \alpha x^*$ (α ixtiyoriy son) ekanligi kelib chiqsa

Misol 1.1.5. Ixtiyoriy ikkita n o'lchamli haqiqiy chiziqli fazolar izomorfdir.

Haqiqatdan ham L va E. n o'lchamli fazolar , $\ell_1, \ell_2, \_ \ell_n$ va $\ell_1^1, \ell_2^1, \_ \ell_n^1$ - mos ravishda L va E chiziqli fazolarda bazislar bo'lsin.

Ushbu $\varphi(\sum_{i=1}^{n} a_i l_i) = \sum_{i=1}^{n} a_i l_i$

Formula izomorfizmni aniqlaydi, chunki L fazoning ixtiyoriy elementni bu fazo bazisi bilan ifodalash mumkin, ya'ni

$$x = \sum_{i=1}^{n} a_i l_i$$

Xuddi shunday E da ham Demak ixtiyoriy $x,y \in L$ $x = \sum_{i=1}^{n} a_i \ \ell_i$; $y = \sum_{i=1}^{n} b_i \ \ell_i$

$\alpha \in R$ bo'lsa, $x+y = \sum_{i=1}^{n} a_i \ \ell_i + \sum_{i=1}^{n} b_i \ell_i = \sum_{i=1}^{n} a_i \ \ell_i + b_i \cdot \ell_i = \sum_{i=1}^{n} (a_i + b_i) \ell_i$

$\alpha x = \sum_{i=1}^{n} (\alpha a_i) \ \ell_i$ bo'ladi shu sababli $\varphi \ (x+y) = \sum_{i=1}^{n} (a_i + b_i) \ \ell_i = \sum_{i=1}^{n} a_i \ \ell_i +$

$+ \sum_{i=1}^{n} b_i \ \ell_i = \varphi \ (x) \ \varphi(y)$

$\varphi \ (\alpha \text{x}) = \sum_{i=1}^{n} (\alpha a_i) \ \ell_i = \alpha \sum_{i=1}^{n} a_i \ \ell_i = \alpha \ \varphi$ (x) demak φ chiziqli akslantirish

bo'ladi. Teskari akslantirishni esa $\varphi^{-1} \ (\sum_{i=1}^{n} a_i \ \ell_i) = \sum_{i=1}^{n} a_i \ \ell_i$ formula aniqlaydi. Shuning uchun φ biyektivdir.

Ta'rif: 1.1.5. Agar L chiziqli fazoning $x,y, \dots , \omega$ elementlari uchun hech bo'lmaganda bittasi noldan farqli bo'lgan $\alpha , \beta , \dots , \lambda$ sonlar topilib

$$\alpha x + \beta y + ... + \lambda \ \omega = 0. \qquad (2)$$

bo'lsa, $x,y, \dots , \omega$ elementlar chiziqli bog'langan deyiladi.

Agar (2) tenglik faqat $\alpha = \beta = \dots = \lambda = 0$ bo'lganda bajarilsa x,y, … , ω elementlar chiziqli erkli yoki chiziqli bog'lanmagan deyiladi.

Agar x, y, … cheksiz elementlar sistemasining ixtiyoriy qism sistemasi chizikli erkli bo'lsa u halda x, y, … sistema chiziqli erkli deyiladi.

Misol 1.1.6. $C[0, \pi]$ fazoda $x(t) = cos^2 2t$, $1 + cos\, 4t$ ayniyatga

ko'ra $\alpha_1 = 2$, $\alpha_2 = \alpha_3 = -1$ $\alpha_1 x(t) + \alpha_2 y(t) + \alpha_3 z(t) = 0$ munosabat o'rinli bo'ladi. $C[0, \pi]$ fazoda $x(t) = t^2$ $y(t) = sin^2 t$ $z(t)\ cos\, t$ funksiyalar chiziqli bog'lanmagan $\alpha_1 t^2 + \alpha_2 y(t) + \alpha_3 z(t) = 0$ tenglik bajarilsin desak ya'ni barcha $t \in [0, \pi]$ uchun $\alpha_1 t^2 + \alpha_2 sin^2 t + \alpha_3 cos\, t = 0$ tenglik o'rinli bo'lsin. Agar $t = 0$, $t = \frac{\pi}{2}$, $t = \pi$ deb olsak quyidagi $\alpha_3 = 0$,

$\frac{\pi^2}{4}\alpha_1 + \alpha_2 = 0$, $\pi^2 \alpha_1 - \alpha_3 = 0$ tengliklarga ega bo'lamiz. Bu tengliklardan esa $\alpha_1 = \alpha_2 = \alpha_3 = 0$ ekanligi kelib chiqadi.

Ta'rif 1.1.7. Agar L chiziqli fazoda n ta chiziqli bog'lanmagan element topilib, bu fazoning ixtiyoriy n+1 ta elementi chiziqli bog'langan bo'lsa, L fazo n o'lchovli chiziqli dazo deyiladi va dim L = n kabi belgilanadi. n o'lchovli L fazoning bazisi deb bu fazoning bog'lanmagan elementlariga aytiladi.

Misol 1.1.8. $R^n = \{x = (x_1, x_2, \dots x_n) : x_k \in R\}$ chiziqli fazo n o'lchamli, chunki (fazo) n $\ell_1 = (1, 0, \dots, 0)$, $\ell_2 = (0, 1, 0, \dots, 0)$, $\ell_3 = (0,0,1, \dots, 0)$ $\dots$, $\ell_n = (0, 0, \dots, 1)$ vektorlar uchun $\alpha_1 \ell_1 + \alpha_2 \ell_2 + \dots + \alpha_n \ell_n = \theta$ Tenglik faqat $\alpha_1 = \alpha_2 = \dots \alpha_n = 0$ bo'lganda bajariladi. Bu esa $\ell_1, \ell_2, \dots, \ell_n$ vektorlar chiziqli bog'lanmaganligini ko'rsatadi.

Har qanday n+1 ta vektordan iborat $\ell_1 = (1, 0, \dots, 0)$, $\ell_2 = (0, 1, 0, \dots, 0)$, $\dots$, $\ell_n = (0, 0, \dots, 1)$, $\ell_{n+1} = (0, \dots, 1)$ vektorlar sistemasi uchun $\alpha_1 \ell_1 + \alpha_2 \ell_2 + \dots + \alpha_n \ell_n + \alpha_{n+1} \ell_{n+1} = \theta$ tenlik $\alpha_1 = \alpha_2 = \dots \alpha_{n-1} = 0$ va $\alpha_n = -\alpha_{n+1}$ bo'lganda bajariladi. Demak $\ell_1, \ell_2, \dots, \ell_n, \ell_{n+1}$ vektorlar sistemasi chiziqli bog'langan. R^n n o'lchamli chiziqli fazo ekan.

Ta'rif 1.1.9. Agar chiziqli fazoda ixtiyoriy $n \in N$ uchun n ta chiziqli erkli element mavjud bo'lsa, u holda L cheksiz o'lchamli chiziqli fazo deyiladi va dim L = ∞ kabi belgilanadi.

Misol 1.1.10. $l_2 = \{x = \{x_i\}_{i=1}^{\infty},\ \sum_{i=1}^{\infty} |x_i|^2 < \infty\}$

Bu fazo cheksiz o'lchamli, chunki ixtiyoriy $n \in N$ uchun bu fazoda

$$\ell_1 = (1, 0, \dots, 0, \dots)$$
$$\ell_2 = (0, 1, \dots, 0, \dots)$$
$$\ell_n = (0, 0, \dots, 1, 0, \dots 0)$$

elementlar chiziqli bog'lanmagan sistemani tashlik qiladi. Buning uchun bu sistemaning ixtiyoriy chekli qismi $\ell_{n_1}, \ell_{n_2}, ..., \ell_{n_k}$ larni chiziqli erkliligini ko'rsatamiz. $\ell_{n_1}, \ell_{n_2}, ..., \ell_{n_k}$, O'zgarmaslar uchun $x = \alpha_{n_1}\ell_{n_1} + \alpha_{n_2}\ell_{n_2} + ... + \alpha_{n_k}\ell_{n_k}$, elementning $n_1, n_2, ..., n_k$, nomerli koordinatalari mos ravishda $\alpha_{n_1}, \alpha_{n_2}, ..., \alpha_{n_k}$ ga teng bo'ladi. Demak, $x = 0$ dan $\alpha_{n_1}, \alpha_{n_2}, ..., \alpha_{n_k} = 0$ kelib chiqadi.

Ta'rif 1.1.11. L to'plam L chiziqli fazoning bo'sh bo'lmagan qism to'plami bo'lsin. Agar L to'plam L da kiritilgan amallarga nisbatan chiziqli fazoni tashkil qilsa L ga L chiziqli fazoning qism fazosi deyiladi.

Misol 1.1.12. $L = \{\varphi : \varphi(2) = 0, \varphi \in C_{[0,2]}\}$ to'plam $C_{[0,2]}$ - $[0,2]$ da aniqlangan uzluksiz funksiyalar fazosining qism fazosi ekanligini ko'rsatamiz. L to'plam $C_{[0,2]}$ fazoning qism to'plami ixtiyoriy φ, g $\in L$ uchun
$(\varphi + g)(2) = \varphi(2) + g(2) = 0.$
Bundan $\varphi + g \in L$. Ixtiyoriy α son va ixtiyoriy $\varphi \in L$ uchun
$(\alpha\varphi)(2) = \alpha\varphi(2) = 0,$ bundan $\alpha\varphi \in L$. Bu esa L to'plam $C_{[0,2]}$ fazoning qism fazosi ekanligini ko'rsatadi.

Ta'rif 1.1.13. L chiziqli fazo, L^1 uning qism fazosi bo'lsin. Agar x,y$\in L$ elementlar uchun $x - y$ element L^1 ga tegishli bo'lsa, u halda x va y elementlar bitta qo'shni sinfga tegishli deyiladi. Barcha qo'shi sinflar to'plamini L ning L^1 bo'yicha faktor fazosi deb ataladi va L/L^1 ko'rinishda yoziladi. Agar L n o'lchamli chiziqli fazo va uning L qism fazosi m (m$\angle$n) o'lchamga ega bo'lsa, u halda L/L^1 faktor fazo $n - m$ o'lchamga ega bo'ladi.

Misol 1.1.14. $L = R^3$. $L_0 = \{x \in R^3 : x\} = (0, x_2, 0)$ bo'lsin. Agar $a = (a_1, a_2, a_3)$, $b(b_1, b_2, b_3)$ elementlarni a-$b \in L_0$ bo'lish uchun $a_1 = b_1$,

$a_3 = b_3$ shartning bajarilishi zarur va yetarli, a vektor L/L_0 fazoning qaysi sinfiga tegishli bo'lishi $a_1 = a_3$ ga bog'liq ya'ni $\begin{cases} x = x_0 \\ z = z_0 \end{cases}$ to'g'ri chiziqning hamma nuqtalari bir sinfga tegishlidir va aksinchα a har bir sinf R^3 fazoda biror

$$\begin{cases} x = x_0 \\ z = z_0 \end{cases}$$

to'g'ri chiziqni aniqlaydi. Shuning uchun L/L_0 fazoni elementlari $0Y$ o'qqa parallel bo'lgan to'g'ri chiziqlar deb qarash mumkin.

Ta'rif 1.1.15. L chiziqli fazoning har bir elementiga aniq bir sonni mos qo'yuvchi φ akslantirish funksional deyiladi. φ funksional additiv deyiladi. Agar ixtiyoriy $x,y \in L$ $\varphi(x+y) = \varphi(x) + \varphi(y)$ bo'lsa, φ funksional bir jinsli deyiladi. Agar ixtiyoriy $x \in L$ va ixtiyoriy $\alpha \in R$ son uchun $\varphi(\alpha x) = \alpha\varphi(x)$ bo'lsa additiv bir jinsli φ funksional chiziqli funksional deb ataladi.

Misol 1.1.16. ℓ_p fazoda aniqlangan $\varphi(x) = \sum_{n\in N} \frac{x_n}{2^n}$, $x = (x_1, x_2, \ldots) \in \ell_p$, p>1 funksionalni chiziqli ekanligini ko'rsatamiz. A) $\varphi(x+y) = \sum_{n\in N} \frac{x_n + y_n}{2^n} =$

$= \sum_{n\in N} \left(\frac{x_n}{2^n} + \frac{y_n}{2^n}\right) = \sum \frac{x_n}{2^n} + \sum_{n\in N} \frac{y_n}{2^n} = \varphi(x) + \varphi(y)$ ixtiyoriy $x,y \in \ell_p$,

B) $\forall x \in \ell_p$ va ixtiyoriy $\alpha \in R$ uchun, $\varphi(\alpha x) = \sum_{n\in N} \frac{\alpha x_n}{2^n} = \alpha \sum_{n\in N} \frac{x_n}{2^n} = \alpha\varphi(x)$.

2 §. Normalangan fazo.

Ta'rif 1.2.1. L chiziqli fazo va P unda aniqlangan funksional bo'lsin. Agar P quyidagi 3ta shartni qanoatlantirsa P ga norma deyiladi.

1) $P(x) \geq 0$. $\forall x \in L$ faqat $x=0$ uchun $P(x) = 0$.

2) $P(\alpha x) = |\alpha|\, P(x)$, $\alpha \in C(R)$

3) $P(x+y) \leq P(x) + P(y)$, $x, y, \in L$

Norma kiritilgan L chiziqli fazo normalangan fazo deyiladi va x elementning P (x) normasi odatda $\|x\|$ bilan belgilanadi.

Misol 1.2.2. R^n fazoda normani $\|x\| = \sqrt{\sum_{i=1}^{n} |x_i|^2}$, $x = (x_1, x_2, \ldots x_n)$ ko'rinishda kiritamiz. Norma shartlarini tekshiramiz.

1) $\|x\| = \sqrt{\sum_{i=1}^{n} |x_i|^2} \geq 0$. $\forall x \in R^n$.

$\|x\| = \sqrt{\sum_{i=1}^{n} |x_i|^2} = 0$ $\sum_{i=1}^{n} |x_i|^2 = 0$. $\Rightarrow$ $|x_i|^2 = 0$, i= $\overline{1,n}$ bundan har bir $x_i = 0$, i= $\overline{1,n}$

bundan x=0 kelib chiqadi. Agar x=0 bo'lsa $x_i = 0$ i= $\overline{1,n}$ $|x_i|^2 = 0$, i= $\overline{1,n}$

$\Rightarrow \sum_{i=1}^{n} |x_i|^2 = 0 \Rightarrow \sqrt{\sum_{i=1}^{n} |x_i|^2} = 0 \Rightarrow \|x\| = 0.$

2) $\forall x \in R^n$

$$\|x\| = \sqrt{\sum_{i=1}^{n}|\alpha x_i|^2} = \sqrt{\sum_{i=1}^{n}\alpha^2|x_i|^2} = \sqrt{\alpha^2\sum_{i=1}^{n}|x_i|^2} = |\alpha|\sqrt{\sum_{i=1}^{n}|x_i|^2} = |\alpha|\ \|x\|.$$

3) $\forall x, y \in R^n$

$$\|x\| = \sqrt{\sum_{i=1}^{n}|x_i + y_i|^2} \le \sqrt{\sum_{i=1}^{n}|x_i|^2} + \sqrt{\sum_{i=1}^{n}|y_i|^2} = \|x\| + \|y\|$$

$\|x\| = \sqrt{\sum_{i=1}^{n}|x_i|^2}$ uchun normaning uchala sharti ham bajariladi. Demak, R^n chiziqli normalangan fazo bo'ladi.

Misol 1.2.3. $P : C^{(1)}_{[a,b]} \to R$ $P(x) = |x(b) - x(a)| + \max\limits_{a\le t\le b} |x^i(t)|$ bunda $C^{(1)}_{[a,b]} \longrightarrow [a,b]$ da aniqlangan 1 marta differensiallanuvchi uzluksiz funksiyalar fazosi P(x) ni norma aniqlash yoki aniqlamasligini ko'rsatamiz. Norma shartlarini tekshiramiz.

1) P(x) ≥ 0.

$P(x) = |x(b) - x(a)| + \max\limits_{a\le t\le b} |x^i(t)| \ge 0$. $P(x) = 0$ bo'lsa ya'ni $|x(b) - x(a)| + \max\limits_{a\le t\le b} |x^i(t)| = 0$.

Bu tenglik bajarilishi uchun $|x(b) - x(a)| = 0$ va $\max\limits_{a\le t\le b} |x^i(t)| = 0$ bo'lishi kerak . Chunki musbat sonlar yig'indisi faqat vq faqat ularning o'zlari nolga teng bo'lganda nolga teng bo'ladi. Demak, $x(b) - x(a)$ va $|x^i(t)| = 0$ ekan.

Bundan x(t) = C bunda C – ixtiyoriy o'zgarmas son $x(b) - x(a)$ va $x(t) = C$ tengliklardan ko'rinib turibdiki $x(t) = C$ ekanligi. Bu esa $P(x) = 0 \Leftrightarrow x = 0$ shartga ziddir. Demak, yuqorida P(x) funksional normani aniqlamaydi.

Ta'rif 1.2.4. L chiziqli normalangan fazo va shu fazo elementlari ketma-ketliki $\{x_n\}$ berilgan bo'lsin. Agar ixtiyoriy $x \in L$ element va $\forall \varepsilon > 0$ son uchun shunday $n_0 = n_0(\varepsilon)$ nomer topilib, $\forall\ n > n_0$ natural sonlar uchun $\|x_n - x\| < \varepsilon$ bo'lsa, $\{x_n\}$ ketma- ketlik x elementga yaqinlashadi deyiladi.

Misol 1.2.5. $C[-\pi, \pi]$ fazoda $x_n(t) = \frac{\sin nt}{n}$ ketma-ketligi x(t) = 0 funlsiyaga yaqinlashadi. Haqiqatdan ham $\|x_n - x\| = \|x_n\| = \max\limits_{t\in[v_1, v_1]} \left|\frac{\sin nt}{n}\right| = \frac{1}{n} \xrightarrow[n\to\infty]{} 0$.

Ta'rif 1.2.6. Agar ixtiyoriy $\varepsilon>0$ son uchun shunday n_0 (ε) nomer topilib, barcha $n_0 = n_0$ (ε) va barcha P natural solar uchun $\left\|x_{n+p}-x_n\right\|$ > ε tengsizlik bajarilsa $\{x_n\}$ ketma- ketlik fundamental ketma-ketlik deyiladi.

Misol 1.2.7. C $[-\pi,\pi]$ fazoda $f_n(t) = \left\{\dfrac{\sin^{2n} t}{2n}\right\}$ ketma-ketlikni qaraymiz.

$$\| f_{2n}(t) - f_n(t)\| = \max \cdot \left|\frac{\sin^{4n} t}{4n} - \frac{\sin^{2n} t}{2n}\right| \le -\pi \le t \le \pi \le \left|\frac{\sin^{4n} t}{4n}\right| \le \le$$

$$\frac{1}{4n} \underset{n\to\infty}{\to} 0 .$$

$\| f_{2n} - f_n\| \to 0$ $n\to\infty$. Demak, $\{f_n(t)\}$ ketma – ketlik C $[-\pi,\pi]$ fazoda fundamental ketma-ketlik ekan.

Ta'rif 1.2.8. Agar L chiziqli fazodagi (normalangan) ixtiyoriy fundamental ketma – ketlik v yaqinlashuvchi bo'lsa, bu fazoga to'la normalangan fazo yoki Banox fazosi deyiladi.

Misol 1.2.9. C $[-\pi,\pi]$ fazoda $f_n(t) = \left\{\dfrac{\sin^{2n} t}{2n}\right\}$ fundamental ketma-ketlikni qaraymiz. $f_n(t) \to f_0(t) = 0$ bo'lishini ko'rsatamiz.

$$\left\|f_n(t) - f_0(t)\right\| = \|f_n(t)\| = \max_{t\in[-\pi,\pi]} \left\|f_n(t)\right\| = \max_{t\in[-\pi,\pi]} \left\{\frac{\sin^{2n} t}{2n}\right\} =. \frac{1}{2n}$$

$$\left\|f_n - f_0\right\| \underset{n\to\infty}{\to} 0 \quad f_n \to f_0.$$

Bundan C $[-\pi,\pi]$ fazo to'la normalangan fazp bo'ladi.

Ta'rif 1.2.10. Normalangan fazoning qism fazosi deganda yopiq qism fazotushuniladi.

3§. Evklid fazosi.

Ta'rif 1.3.1. Bizga L haqiqiy chiziqli fazo berilgan bo'lsin. Agar $L_1 x L$ dekart ko'paytmada aniqlangan P funksional quyidagi to'rtta shartni qanoatlantirsa.

1) $P(x,x) \ge 0 \quad \forall x \in L$, $\quad P(x,x) = 0 \quad \Leftrightarrow \quad x=0.$

2) $P(x,y) = P(y,x)$, $\quad \forall x, y \in L$

3) $P(\alpha x,y) = \alpha P(y,x)$, $\quad \forall x, y \in L$, $\quad \forall \alpha \in R$

4) $P(x_1+x_2, y) = P(x_1, y) + P(x_2, y) \quad \forall x_1, x_2, \quad y \in L$

Skalyar ko'paytma kiritilgan chiziqli fazo. Evklid fazosi deyiladi va x,y elementning skalyar ko'paytmasi (x,y) orqali belgilanadi. Evklid fazosida norma $\|x\| = \sqrt{(x,x)}$ formula orqali aniqlanadi.

Misol 1.3.2. $\ell_2 = \{x = (x_1, x_2, \dots x_n) : \sum_{k=1}^{\infty} |x_k|^2 < \infty \}$ kvadrati bilan jamlanuvchi ketma-ketliklar fazosi. Bu yerda skalyar ko'paytma quyidagicha kiritiladi. $(x,y) = \sum_{i=1}^{\infty} x_i y_i$

1) Ixtiyoriy $x \in \ell_2$ uchun $(x,y) = \sum_{i=1}^{\infty} x_i y_i = \sum_{i=1}^{\infty} x_i^2 \geq 0$

$(x,y) = \sum_{i=1}^{\infty} x_i^2 = 0 \Rightarrow x_i^2 = 0 \Rightarrow x_i = 0, \ n = 1,2,\dots \Rightarrow x = 0 = x = 0 \Rightarrow x_i = 0.$

$i = 1.\overline{\infty} \Rightarrow x_i^2 = 0. \quad i = 1.\overline{\infty} \Rightarrow \sum_{i=1}^{\infty} x_i^2 = 0 \Rightarrow (x,x) = 0.$

2) *Ixtiyoriy* $x,y \in \ell_2$. $(x,y) = \sum_{i=1}^{\infty} x_i y_i = \sum_{i=1}^{\infty} y_i x_i = (y,x).$

3) $\forall \alpha \in R$ *va ixtiyoriy* $x \in \ell_2$. $(\alpha x, y) = \sum_{i=1}^{\infty} \alpha x_i y_i = \alpha \sum_{i=1}^{\infty} y_i x_i = \alpha \ (y,x).$

4) $x,y,z \in \ell_2$. $(x+y,z) = \sum_{i=1}^{\infty} (x_i y_i) z_i = \sum_{i=1}^{\infty} x_i z_i + \sum_{i=1}^{\infty} y_i z_i = (x,z)\ (y,z).$

Skalyar ko'paytmaning barcha shartlari bajariladi. Demak, ℓ_2 Evklid fazosiga misol bo'ladi.

Ta'rif 1.3.3. Cheksiz o'lchamli to'la Evklid fazosi Gilbert fazosi deyiladi. Shunday qilib ixtiyoriy tabiatli *f,g,...* elementlarining H to'plami Gilbert fazosi bo'lib, u quyidagi uchta shartni qanoatlatiradi.

1) *H* Evklid fazosi, ya'ni skalyar ko'paytma kiritilgan chiziqli fazo.

2) $\rho(x,y) = \sqrt{(x-y, x-y)}$ metrika ma'nosida H to'la fazo.

3) H cheksiz o'lchamli ya'ni unda cheksiz ko'p chiziqli erkli elementlar mavjud.

Misol 1.3.4. $l_2 = \{x = \{x_i\}_{i=1}^{\infty}, \sum_{i=1}^{\infty} |x_i|^2 < \infty \}$ fazoni Gilbert fazosi ekanligini ko'rsatamiz. Gilbert fazosining birinchi va uchunchi shartlari mos ravishda 1.3.2. va 1.1.10 misollarda ko'rsatilgan, shu sababli metrika ma'nosida ko'rsatish kifoya. $\{x^{(n)}\}$ - l_2 fazodagi fundamental ketma-ketlik bo'lsin. Bundan ixtiyoriy $\varepsilon > 0$ uchun shunday N topilib ixtiyoriy n,m > N lar uchun

$$\rho\ (x^{(n)}, x^{(m)}) = \sqrt{\sum_{n=1}^{\infty} (x_k^{(n)} - x_k^{(m)})^2} \leq \varepsilon$$

$$\rho^2 (x^{(n)}, x^{(m)}) = \sum_{i=1}^{\infty} (x_k^{(n)} - x_k^{(m)})^2 < \varepsilon$$

bo'ladi. Bu yerda $x^{(n)} = (x_1^{(n)}, x_2^{(n)}, \dots x_k^{(n)}, \dots$ 1) dan ixtiyoriy κ uchun $(x_k^{(n)} - x_k^{(m)})^2 < \varepsilon$ ya'ni har bir κ uchun $\{x_k^{(n)}\}$ ketma-ketlik fundamental, shuning uchun yaqinlashadi: $x_k = \lim\limits_{n\to\infty} x_k^{(n)}$ bo'lsin x ketma-ketlikni $(x_1, x_2, \dots x_n, \dots)$ bilan belgilaymiz. Quyidagi shartlar bajarilishini ko'rsatish yetarli.

a) $\sum\limits_{k=1}^{\infty} x_k^2 < \infty$, $x \in \ell_2$.

b) $\lim\limits_{n\to\infty} \rho(x^n, x) = 0.$

(1) dan ixtiyoriy fiksirlangan. M uchun $\sum\limits_{k=1}^{M} (x_k^{(n)} - x_k^{(m)})^2 < \varepsilon$ Yig'indida faqat chekli hadlar qatnashganligi uchun n ni fiksirlab m $\to\infty$ limitga o'tamiz.

$\sum\limits_{k=1}^{M} (x_k^{(n)} - x_k^{(m)})^2 < \varepsilon$ tengsizlik ixtiyoriy M uchun o'rinli bo'ladi. m $\to \infty$ da

quyidagiga ega bo'lamiz. $\sum\limits_{k=1}^{\infty} (x_k^{(n)} - x_k^{(m)})^2 \leq \varepsilon$ (2)

$\sum\limits_{k=1}^{\infty} (x_k^{(n)})^2$ va $\sum\limits_{k=1}^{\infty} (x_k^{(n)} - x_k)^2$ qatorlar yaqinlashuvchiligidan $\sum\limits_{k=1}^{\infty} x_k^2$ qatorning ham yaqinlashuvchiligi kelib chiqadi. ε ning yetarli kichikligi va (2) tengsizlikdan $\lim\limits_{n\to\infty} \rho(x^{(k)}, x) = \lim\limits_{n\to\infty} \sum\limits_{k=1}^{\infty} (x_k^{(n)} - x_k) = 0$ yani $x_k^{(n)}$-x.

Bundan ℓ_2 fazoning to'laligi kelib chiqadi.

Ta'rif 1.3.5. $M \subset H$ gilbert fazosining qism fazosi bp'lsin. Barcha $f \in M$ elementlarga ortogonal bo'lgan $g \in H$ elementlar toplamni M^+=HOM orqali belgilaymiz ya'ni M^+=HOM = $\{ g \in H\ (f,g) = 0, \forall f \in M \}$. M^+ ham H ning qism fazosi ekanligini isbotlaymiz. Bu to'plamning qo'shish va songa ko'paytirish amallariga nisbatan yaqinligini ko'rsatamiz. Agar $g_1, g_2 \in M^+$ bo'lsa, u holda

$(\alpha_1 g_1 + \alpha_2 g_2, f) = \alpha_1 (g_1, f) + \alpha_2 (g_2, f) = 0$. Endi M^+ to'plamning yaqinligini ko'rsatamiz. Faraz qilaylik $g_n \in M^+$ elementlar ketma – ketligi $g \in H$ element yaqinlashsin . U holda skalyar ko'paytmaning uzunligiga ko'ra

$(g,f) = (\lim g_n, f) = \lim\limits_{n\to\infty} (g_n, f) = 0$.

Demak, $g \in M^+$, ya'ni M^+ yopiq qism fazo bo'lar ekan. M^+ qism fazo M qism fazoning ortogonal to'ldiruvchisi deyiladi.

Ta'rif 1.3.6. Agar H gilbert fazosining ixtiyoriy $a \in H$ elementini $f = h + h^1$,
$h \in M$, $h^1 \in M^+$ ko'rinishda tasvirlansa, u holda H fazo o'zaro ortogonal M va M^+ fazolarning to'g'ri yig'indisiga yoyilgan deyiladi va $H = M \oplus M^+$ ko'rinishida yoziladi. Sonoqli sondagi $H_1, H_2, ..., H_n$ ... gilbert fazosining to'g'ri yig'indisi $H = \sum\limits_{k=1}^{\infty} \oplus H_n$ quyigidagicha aniqlanadi.

$H = \{ h = (h_1, h_2, \ldots, h_n), h_n \in H_n \ \sum\limits_{n=1}^{\infty} \|h_n\|^2 < +\infty \}$. Skalyar ko'paytma quyidagicha kiritiladi. $(h,g) = \sum\limits_{n=1}^{\infty} (h_n, g_n)$, $g = (g_1, g_2, \ldots, g_n \ldots)$, $g_n \in H_n$.

4§.Hilbert fazolari.

Ta'rif: Cheksiz o'lchamli to'la Evklid fazosi Hilbert fazosi deyiladi.

Shunday qilib ihtiyoriy tabiatli $f, g, \varphi ...$ elementlarning H to'plami Hilbert fazosi bo'lsa u quyidagi uchta shartni qanoatlantiradi

1) H Evklid fazosi, yani skalyar ko'paytma kiritilgan chiziqli fazo.
2) $p(x,y) = \sqrt{(x-y, x-y)}$ metrik ma'nosida H–to'la fazo.
3) H – fazo cheksiz o'lchamli yani unda cheksiz elementli chiziqli erkli sistema mavjud

   Odatda separabel Helbert fazolari qaraladi, ya'ni H ning hamma yerida zich bo'lgan sanoqli to'plam mavjud. Bundan keyin biz faqat separabel Helbert fazolarini qaraymiz .

   1- misol $C_2[a,b]$ Evklid fazosi tola emas shuning uchun $C_2[a,b]$ Helbert fazosi bo'la olmaydi.

2- Misol l_2 va $L_2[a,b]$ lar cheksiz o'lchamli to'la Separabel Evklid fazolaridir.Shuning uchun ular Helbert fazolari bo'ladi.

Kompleks Evklid fazolari.

Haqiyqiy evklid fazolarida skalyar ko'paytmaning 1-4 aksiomalari quydagicha edi.

1) $(x,x)\geq 0,\forall x\in E,(x,x)=0\Leftrightarrow x=\theta$

2) $(x,y)=(y,x),\forall x,y\in E$

3) $(\lambda x,y)=\lambda(x,y),\forall\lambda\in R,\forall x,y\in E$

4) $(x_1+x_2,y)=(x_1,y)+(x_2y),\forall x_1,x_2,y\in E$

Biz 2 va 3 dan quydagiga ega bo'lamiz.

$(\lambda x,\lambda x)=\lambda(x,\lambda x)=\lambda(\lambda x,x)=\lambda^2(x,x)$.

Agar λ kompleks son bo'lsa, u holda $\lambda=i$ bo'lganda, $(ix,ix)=-(x,x)$ ya'ni x va *ix* vektorlarning skalyar ko'paytmasi bir vaqtda musbat bo'la olmaydi, bu esa 1-shartga zid ya'ni kompleks chiziqli fazoda holida 1,2, va 3 shartlar bir vaqtda bajarishi mumkin emas demak kompleks chiziqli fazolarda skalyar ko'paytmaning shartlarini biroz o'zgartirish kerak.

Konpleks chiziqli fazoda skalyar ko'paytmaning shartlarini keltiramiz

1) $(x,x)\geq 0,\forall x\in E,(x,x)=0\Leftrightarrow x=\theta$

2) $(x,y)=(\overline{y,x}),\forall x,y\in E$

3) $(\lambda x,y)=\lambda(x,y),\forall\lambda\in C,\forall x,y\in E$

4) $(x_1+x_2,y)=(x_1,y)+(x_2y),\forall x_1,x_2,y\in E$

2 va 3 da $(x,\lambda y)=\overline{\lambda}(x,y)$ kelib chiqadi haqiqatan ham.

$(x,\lambda y)=(\overline{\lambda y,x})=\overline{\lambda(y,x)}=\overline{\lambda}(\overline{y,x})=\overline{\lambda}(x,y).$

Misol $E=C^n$ - kompleks chiziqli fazo. Bu fazoda skalyar ko'paytma quydagicha kiritiladi.

$$(x,y)=\sum_{k=1}^{n}x_k,\bar{y}_k$$

5§. Chiziqli operatorlar.

Ta'rif 1.5.1. E fazoning har bir elementiga E_1 fazoning yagona elementini mos qo'yubchi $y = Ax$ $(x\in E,\ y\in E_1)$ akslantirish operator deyiladi.

Ta'rif 1.5.2. Agar ixtiyoriy $x,y \in E$ elementlar va $\forall \alpha\beta \in C$ sonlar uchun $A(\alpha x + \beta y) = \alpha Ax + \beta Ay$ tenglik o'rinli bo'lsa, A ga chiziqli operator deyiladi. A operator aniqlangan barcha $x \in E$ lar to'plamini D_A bilan belgilaymiz va uni A operatorning aniqlanish sohasi deb ataymiz. Umuman aytganda $D_A = E$ bo'lishi shart emas lekin D_A ning chiziqli ko'p xillik bo'lishi talab qilinadi ya'ni agar $x,y \in D_A$ bo'lsa, u holda $\alpha,\beta \in C$ uchun $\alpha x + \beta y \in D_A$.

Misol 1.5.3. A: $C_{[0,1]} \to C_{[0,1]}$. $(Ax)(t) = x(0)\, t + x(\frac{1}{2}) + \frac{1}{2} x(1)$ operatorning chiziqli bo'lishini ko'rsating. Ixtiyoriy $x,y \in C_{[0,1]}$ va ixtiyoriy $\alpha,\beta \in R$ sonlar uchun $(A(\alpha x + \beta y))(t) = (\alpha\, x(0) + \beta y(0))\, t + \alpha\, x(\frac{1}{2}) + \beta y(\frac{1}{2}) +$

$$+\frac{1}{2}(\alpha\, x(1) + \beta y(1)) = \alpha\, x(0)t + \beta y(0)t + x(\frac{1}{2}) + \beta y(\frac{1}{2}) + \frac{1}{2}\alpha\, x(1) + \beta y(1) =$$

$$\alpha\, (x(0)t + x(\frac{1}{2}) + \frac{1}{2}(1)) + \beta (y(0)t + y(\frac{1}{2}) + \frac{1}{2}\, y(1)) = \alpha\, (Ax)(t) + \beta\, (Ay)(t).$$

Bundan A operatorning chiziqli ekanligini ko'rsatadi.

Misol 1.5.4. $A: L_{2\,[0,1]} \to C$ $A(x) = \int_2^1 const\, x t dt$

operatorni chiziqlikga tekshiramiz.

$\forall x,y \in L_{2\,[0,1]}$ va $\forall \alpha,\beta \in C$ sonlar uchun

$$A(\alpha x + \beta y) = \int_2^1 const\,(\alpha\, x(t) + \beta\, y(t))\, dt = \int_2^1 \alpha const\, x(t)\, dt + \int_2^1 \beta const\, x(t)\, dt =$$

$= \alpha A(x) + \beta\, A(y)$.

Demak, berilgan operator chiziqli ekan.

Ta'rif 1.5.5. $A{:}E \to E_1$ va $x_0\ D_A$. $A x_0 = y_0$ bo'lsin. Agar $y_0 = A x_0 \in E_1$ ning ixtiyoriy V atrofi uchun x_0 nuqtsning shunday U atrofi topilib, $\forall x \in U$, $V \in D_A$

lar uchun $Ax \in V$ bo'lsa, A operator $x = x_0$ nuqtada uzluksiz deyiladi. Agar $\forall\ \varepsilon > 0$ sin uchun shunday $\delta = \delta(\varepsilon) > 0$ son mavjud bo'lib, $\|x - x_0\| < \delta$ tengsizlikni qanoatlantiruvchi barcha $x \in D_A$ lar uchun $\|Ax - Ax_0\| < \varepsilon$ tengsizlik bajarilsa A operator $x = x_0$ nuqtada uzluksiz deyiladi.

Ta'rif 1.5.7. $Ax = 0$ tenglikni qanoaylantiruvchi $x \in E$ lar to'plamini A operatorning yadrosi deb ataymiz va uni $Ker\, A$ bilan belgilaymiz. Birorta $x \in D_A$ uchun $y = Ax$ tenglik bajariladigan $y \in E_1$ lar to'plamini A operatorning qiymatlar shohasi yoki obrazi deb ataymiz va uni ImA yoki R(A) bilan belgilaymiz.

Ta'rif 1.5.8. $A{:}E \to E_1$ chiziqli operator bo'lsin. Agar shunday $C>0$ son topilib, ixtiyoriy $x \in D_A = E$ uchun $\|Ax\|_{E_1} \le C\,\|x\|_E$ tengsizlik bajarilsa A chegaralangan operator deyiladi. Agar chiziqli $A{:}E \to E_1$ operator chegaralangan bo'lsa, $\|A\| = \inf\{c>0{:}\ \|Ax\|_{E_1} \le C\,\|x\|_E\ ,\ \forall x \in D_A\}$ son A operatorning normasi deyiladi.

Teorema 1.5.9. $A{:}E \to E_1$ chiziqli operator uzluksiz bo'lishi uchun chegaralangan bo'lishi zarur va yetarli. Bunda E va E_1 lar normalangan fazolar.

Misol 1.5.10. $A{:}\quad \ell_2 \to \ell_2 .\quad Ax = (\frac{x_1}{2},\ \frac{1}{4}x_2,\ \frac{1}{8}x_3,\ \dots,\ \frac{1}{2^n}x_n, \dots)$ operatorni uzluksizlik va chegaralanglikka tekshiramiz. A operatorning chiziqliligini ko'rsatamiz. $A(\alpha x + \beta y) = (\frac{\alpha x_1 + \beta y_1}{2},)\ \frac{1}{4}(\alpha x_2 + \beta y_2\ ,\ \dots,\ \frac{1}{2^n}$

$(\alpha x_n + \beta y_n), \dots) = \frac{1}{2}\alpha x_1 + \frac{1}{2}\beta y_1\ ,$

$\frac{1}{4}\alpha x_2 + \frac{1}{4}\beta y_2,\ \frac{1}{2^n}\alpha x_n + \frac{1}{2^n}\beta y_n = (\frac{1}{2}\alpha x_1,\ \frac{1}{4}\alpha x_2, \dots, \frac{1}{2^n}\alpha x_n, \dots) +$

$+ (\frac{1}{2}\beta y_1,\ \frac{1}{4}\beta y_2, \dots, \frac{1}{2^n}\beta y_n, \dots) = \alpha Ax + \beta Ay.$

Demak, A chiziqli $\|Ax\|_{\ell_2} = \sqrt{\sum_{n=1}^{\infty}(\frac{1}{2^n}x_n)^2} = \sqrt{\sum_{n=1}^{\infty}\frac{1}{2^n}x_n^{\ 2}} \le \frac{1}{2}$

$\sqrt{\sum_{n=1}^{\infty} x_n^{\ 2}} = \frac{1}{2}\ \|x\|_{\ell_2} \Rightarrow C = \frac{1}{2}.$

Bundan A operatorning chegaralanganligi ko'rinadi. Teorema 1.5.9. ga ko'ra A operator uzluksiz bo'ladi.

Ta'rif 1.5.11. Agar ixtiyoriy $y \in Im\,A$ uchun $Ax=y$ tenglama yagona yechimga ega bo'lsa, u holda A operator teskarilanuvchan operator deyiladi. Agar A teskarilanuvchan operator bo'lsa, u holda ixtiyoriy $y \in Im\,A$ ga $Ax=y$ tenglamaning yechimi bo'lgan yagona $x \in D_A$ element mos keladi. Bu moslikni o'rnatuvchi operator A operatorga teskari operator deyiladi va A^{-1} bilan belgilanadi. $A^{-1} : E_1 \to E,\ D_{A^{-1}} = Im\,A\ ,\ Im\,A^{-1} = D_A$.

Misol 1.5.12. $A : R^3 \to R^3,\ Ax = (x_1 - x_3\ , x_2\ , x_2 + x_3)$ operatorning teskarilanuvchanligini ko'rsatib teskarisini toping. $Ax = y.\quad (x_1 - x_3\ , x_2\ , x_2 + x_3) =$
$= (y_1, y_2, y_3).$

$$\begin{cases} x_1 - x_2 = y_1 \\ x_2 = y_2 \\ x_2 + x_3 = y_3 \end{cases} \qquad \begin{vmatrix} 1, & 0, & -1 \\ 0, & 1, & 0 \\ 0, & 1, & 1 \end{vmatrix} = 1 \neq 0.$$

Sistemaga yagona yechimga ega shuning uchun A teskarilanuvchi $y_2 = x_2$ $x_3 = y_3 - y_2$, $x_1 = y_1 + y_3 - y_2$, $A^{-1} = (y_1 + y_3 - y_2,\ y_2,\ y_3 - y_2)$.

Teorema 1.5.13. A chiziqli operatorga teskari bo'lgan A^{-1} operator ham chiziqlidir.

Teorema 1.5.14. (Teskari operator haqida Banax teoremasi). A E banax fazosini E_1 Banax fazoga biyektiv akslantiruvchi chiziqli chegaralangan operator

bo'lsin. U holda A^{-1} operator mavjud va chegaralangan.

Ta'rif 1.5.15. E evklid fazosida A chiziqli operator berilgan bo'lsin. Agar biror $A^*: E \to E$ operator uchun va ixtiyoriy $x, y \in E$ lar uchun $(Ax,y) = (x, A^*y)$ tenglik o'rinli bo'lsa, A^* operator A operatorga qo'shma operator deyiladi.

Ta'rif 1.5.16. H gilbert fazosi va $A: H \to H$ chegaralangan operator bo'lsin. $\forall x, y \in H$ uchun $(Ax,y) = (x, Ay)$ tenglik o'rinli bo'lsa, A operator o'z-o'ziga qo'shma operator deyiladi.

Misol 1.5.17. $A: \ \ell_2 \to \ell_2$. $(a_1 x_1, a_2 x, \ldots, a_n x_n, \ldots)$ operatorga qo'shma operatorni toping $(Ax,y) = \sum\limits_{n=1}^{\infty} a_n x_n \bar{y}_n = \sum\limits_{n=1}^{\infty} x_n \bar{\bar{a}}_n y_n = \sum\limits_{n=1}^{\infty} x_n (A^*y)_n =$
$= (x, A^*y)$. $A^*y = (\bar{a}_1 y_1, \bar{a}_2 y_2, \ldots, \bar{a}_n y_n, \ldots)$.

Ta'rif 1.5.18. Bizga $A_n \in L(X,Y)$ ($L(X,Y)$ – X ni Y ga akslanturuvchi chiziqli chegaralangan operatarlar fazosi) lar berilgan bo'lsin. Agar $\lim\limits_{n\to\infty} \|A_n - A\| = 0$ bo'lsa, A_n operatorlar ketma-katligi A operatorga tekis yoki norma bo'yicha yaqinlashadi deyiladi.

Misol 1.5.19. $B_n: \ell_2 \to \ell_2$. $B_n x = (\frac{1}{n} x_1, \ldots, \frac{1}{n} x_n, \ldots)$ operatorlar ketma – ketligi $B_0 = \theta$ operatorga tekis yaqinlashadi.

$$\|B_n x\| = \sqrt{\sum_{k=1}^{\infty} (\frac{1}{n} x_k)^2} = \frac{1}{n} \sqrt{\sum_{n=1}^{\infty} \| x_k \|^2} = \frac{1}{n} \|x\|. \ \|B_n - B_0\| = \|B_n\| = \sup_{\|x\|=1}$$
$$\|B_n x\| = \sup_{\|x\|=1} \frac{1}{n} \|x\| = \frac{1}{n}.$$

$$\lim_{n\to\infty} \|B_n - B_0\| = \lim_{n\to\infty} \frac{1}{n} = 0.$$

Ta'rif 1.5.20. H – gilbert fazosi va A unda aniqlangan chiziqli operator bo'lsin. Agar $\forall x \in D(A)$ uchun $(Ax,x) \geq 0$ tengsizlik o'rinli bo'lsa, u holda A ga mudbat operator deyiladi. Odatda agar A musbat operator bo'lsa, $A \geq 0$ va agar

A-$B \geq 0$ bo'lsa, $A \geq B$ kabi yoziladi.

Ta'rif 1.5.21. Agar ixtiyoriy $x \in E$ uchun $A_n x \to Ax$ *bo'lsa* $\{A_n\}$ operatorlar ketma – ketligi A operatorga kuchli yoki nuqtali yaqinlashadi deyiladi.

Misol 1.5.22. $C_n : \ell_2 \to \ell_2$. $C_n x = (0,0, \ldots, x_n, x_{n+1}, \ldots)$ operatorlar ketma – ketligi $C_0 = \theta$ operatorga kuchli yqinlashadi.

$$\|C_n x - Cx\| = \|C_n x\| = \sqrt{\sum_{k=1}^{\infty} | x_k |^2} \xrightarrow[n\to\infty]{} 0.$$

Ta'rif 1.5.23. Agar $\forall f \in Y^*$ va $\forall f \in E$ uchun $f(A_n x) \to f(Ax)$ bo'lsa, $\{A_n\}$ operatorlar ketma – ketligi A operatorga kuchsiz yaqinlashuvchi deyiladi.

1.5.24. $B_n : \ell_2 \to \ell_2$. $B_n x = (0,0, \ldots, 0, x_1, x_2, \ldots)$ operatorlar ketma – ketligi $A = \theta$ operatorga kuchsiz yaqinlashadi. Ixtiyoriy $y \in \ell_2$ uchun

$$|(B_n x, y)| = | \sum_{k=1}^{\infty} x_k \cdot y_n + \leq k|^2 \quad \|x\|^2 \sum_{k=n+1}^{\infty} |y_k|^2 \to 0,$$ n$\to\infty$. Demak,

B$_n$ operatorlar ketma – ketligi θ nol operatorga kuchsiz ma'noda yaqinlashar ekan.

Teorema 1.5.25. Agar X va Y lar Banax fazolari bo'lsa, u holda $L(X,Y)$ operatorlar fazosi kuchli yaqinlashishga nisbatan to'ladir.

Ta'rif 1.5.26. Agar $\lambda \in C$ soni uchun A- λI operator teskarilanuvchan bo'lib, teskari operator E ning hamma yerida aniqlangan bo'lsa, λ soni A operatorning regulyar nuqtasi deyiladi.

Barcha regulyar nuqtalar to'plami A operatorning rezolventa to'plami deyiladi va $\rho(A)$ orqali belgilanadi.

Ta'rif 1.5.27. $C / \rho(A)$ to'plamga esa A operatorning spetkri deyiladi va $T(A)$ orqali belgilanadi.

Ta'rif 1.5.28. Agar biror λ son uchun $(A$-$\lambda I)\ x = 0$ tenglama nolmas $(x \neq 0)$ yechimga ega bo’lsa λ son A operatorning xos qiymati deyiladi. $A = A^*$ operatorning barcha xos qiymatlari to’plamiga shu operatorning diskret spektri deyiladi va $T_{disc}\ (A)$ kabi belgilanadi. Barcha xos qiymatlar to’plami skektrda yotadi, chunki λ xos qiymat bo’lsa, $A\text{-}\lambda I$ operatorning teskarisi mavjud emas. Barcha xos qiymatlar to’plami nuqtali spektr deyiladi va $T_{pp}(A)$ orqali belgilanadi.

Spektrning qolgan qismi ya’ni $(A\text{-}\lambda I)^{-1}$ operator mavjud ammo chegaralanmagan $\lambda \in$ T(A) lar to’plami A operatorning uzluksiz spektri deyiladi va $T_{const}(A)$ kabi belgilanadi. Chegaralangan o’z – o’ziga qo’shma A operator uchun $T(A)/\ T_{disc}\ (A)$ to’plamga A operatorning muhim spetri deyiladi va $Tess\ (A)$ bilan belgilanadi.

Misol 1.5.29. $A:\ L_2\,[a,b] \rightarrow\ L_2\,[a,b],\quad Ax(t) = tx(t)$ operatorning spetri va rezolventasini toping

$$(A\text{-}\lambda I)\ x(t) = (t\text{-}\lambda)\ x(t) = y(t),\quad R_\lambda(A) = (A\text{-}\lambda I)^{-1}x(y) = \frac{x(t)}{t-\lambda}$$

$\lambda \in [a,b]$ *da* $T = [a,b]$ $\qquad \rho(A) = C/\,[a,b]$.

Teorema 1.5.30. Agar $A \in L(E)$ bo’lib $\|\lambda\| > \|A\|$ tengsizlik bajarilsa, u holda λ regulyar nuqta bo’ladi ya’ni $\lambda \in \rho(A)$

Teorema 1.5.31. X to’la normalangan fazo va $M\ c\ X$ bo’lsin. Agar shunday $k>0$ topilib barcha $x \in M$ lar uchun $\|x\| \le k$ bajarilsa, u holda M to’plamga chegaralangan to’plam deyiladi.

Ta’rif 1.5.32. Agar M to’plamning ixtiyoriy ketma – ketligidan yaqinlashuvchi qismiy ketma – ketlik ajratish mumkin bo’lsa M to’plamga nisbiy kompakt to’plam deyiladi.

Ta’rif 1.5.33. Agar $A \in L\ (X,Y)$ operator X dagi ixtiyoriy chegaralangan to’plamni Y dagi nisbiy kompakt operator deyiladi.

Teorema 1.5.34. $A : C^n \rightarrow C^n$ chiziqli operator kompaktdir.

Teorema 1.5.35. Agar $A \in L\ (X,Y)$ va dim $R(A)$ Б ∞ bo’lsa, u holda A kompakt bo’ladi.

$$A:\ L_2\,[a,b] \rightarrow\ L_2\,[a,b],\quad (Af)\,(x) = \int_0^1 (\lambda + \mu x)\ f(x)$$

operatorni kompaktlikka tekshiramiz A ikki o’lchamli bo’lganligi uchun teorema 1.5.35. ga asosan kompaktdir.

Teorema 1.5.37. Agar $A_n \in L\ (X,Y)$ bo’lib, A_n kompakt va $\lim\limits_{n\to\infty} \|A_n - A\| = 0$ bo’lsa, u hoda limitik operator A kompakt bo’ladi.

Teorema 1.5.38. Agar A kompakt operator bo’lsa, u holda $A \cdot B$ va $B \cdot A$ operatorlar ham kompakt bo’ladi.

Teorema 1.5.39. A operator kompakt bo'lishi uchun uning qo'shmasi A^* kompakt bo'lishi zarur va yetarli.

Ta'rif 1.5.40. Agar H gilbert fazosida aniqlangan A operator har qanday kuchsiz yaqinlashuvchi ketma – ketlikni kuchli yaqinlashuvchi ketma – ketlikga o'tqazsa, u holda A ga kompakt operator deyiladi.

Tasdiq 1.5.41. H komleks gilbert fazosidagi o'z – o'ziga qo'shma chegaralangan A operatorning barcha xos qiymatlari haqiqiydir.

Teorema 1.5.42. (Мунд kriteriyasi) Bizga $A = A^* \in L\ (H)$ operator berilgan bo'lsin $\lambda \in Tess\ (A)$ bo'lishi uchun shunday $\{\varphi_n\}$ ortonormal sistema topilib,

$$\|(A-\lambda)\varphi_n\| \xrightarrow[n\to\infty]{} 0$$

bo'lishi zarur va yetarli.

Teorema 1.5.43. (Veyl teoremasi) A, K o'z - o'ziga qo'shma operatorlar va K kompakt operator bo'lsa, u holda A operatorning muhim spetri , $A + K$ operatorning muhim spetriga

teng ya'ni Tess (A) = Tess (A + K)

Teorema 1.5.43 (Fredgolmning analatik teoremasi) D C komleks tekisligida soha

bo'lsin . $F:\ D \to L\ (H)$ funksiya shunday analatik operatorni ifodalovchi funksiya bo'lsinki $f(z)$ $\forall z \in D$ uchun kompakt operator bo'lsin, u holda

a) $(I\text{-}f(z))^{-1}$ hech qanday $z \in D$ uchun mavjud emas;

yoki

b) $(I\text{-}f(z))^{-1}$ hamma $z \in D\ /\ S$ uchun mavjud. Bunda $S\ c\ D$ diskret to'plamining D da limitik

nuqtalari mavjud emas. Bu holda $(I\text{-}f(z))^{-1} D$ da meramorf D/S da analitik uning qutblaridagi qoldiqlari chekli o'lchamli operatorlardir va agar $x \in S$ bo'lsa, u holda f(z) $\varphi = \varphi$ tenglama H da nolmas yechimga ega.

Natija 1.5.44. (Fredgolm afternativasi) . Agar A operator H da kompakt operator bol'lsa, $(I\text{-}A)^{-1}$ mavjud yoki $A\varphi = \varphi$ tenglama yachimga ega.

II bob

Fridrixs modeli va unng spektrial xossalari.

$T^{d}=(-\pi;\pi]^{d}$ orqali T=(-π,π]torning d marta dekart ko'paytmasini belgilaymiz, ya'ni $T^{d}=\underbrace{T\times T\times\ldots\times T}_{d\ ta}$. $L_2(T^d)$ orqali T^d da aniqlangan modulining kvadrati bilan integrallanuvchi funksiyalarning Hilbert fazosini belgilaymiz, ya'ni

$$L_2(T^d)=\left\{f\in T^d,\ \int_{T^d}|f(q)|^2dq<\infty\right\}.$$

C^1 - bir o'lchamli kompleks Hilbert fazosi.

Misol: $f(x)=\sum_{i=1}^{d}(1-\cos x)$ $\quad f(x)=\sum_{i=1}^{d}\cos x_i$

$L_2(T^d)$ da quyidagi formula bilan aniqlangan Fridrexs moduli deb ataluvchi operatorni qaraymiz.

(1) $\qquad h_\lambda=h_0-\lambda\Phi^*\Phi$

Bunda

$$(h_0f)(q)=(1-\cos q)f(q)$$
$$\Phi^*:C^1\to L_2(T^d)$$
$$\Phi^*f_0=\sin qf_0$$
$$\Phi:L_2(T^d)\to C^1$$
$$\Phi f=(\sin q;f)$$

Ya'ni $h_\lambda(q)$ operator $L_2(T^d)$ fazoda quyidagi ko'rinishga ega: λ>0

$$h_\lambda f(q)=(1-\cos q)f(q)-\lambda\sin q\int_{T^d}\sin tf(t)dt$$

Faraz qilamiz d=1 bo'lsin.

Lemma-1: $\forall\lambda>0$ va $\forall q\in(-\pi,\pi]$ uchun h_λ operator chiziqli, chegaralangan va o'z-o'ziga qo'shma operator bo'ladi.

Isbot $\boldsymbol{h_\lambda}$ operatorning chegaralanganligini ko'rsatamiz.

a) $h_\lambda(\alpha_1f_1+\alpha_2f_2=\alpha_1h_\lambda(f_1)+\alpha_2h_\lambda f_2$

tenglikni tekshiramiz: Opertatorni aniqlanishidan

$$h_\lambda(\alpha_1f_1+\alpha_2f_2)(q)=(1-cosq)(\alpha_1f_1(q)+\alpha_2f_2(q))$$

$$-\lambda sinq\int_{T_1}sint\big(\alpha_1f_1(t)+\alpha_2f_2(t)\big)dt=$$

$$\alpha_1(1-cosq)f_1(q)-\alpha_1\lambda sinq\int_{T_1}sintf_1(t)dt+\alpha_2(1-cosq)f_2(q)$$

$$-\alpha_2\lambda sinq\int_{T_1} sint f_2(t)dt=\alpha_1[(1-cosq)f_1(q)-$$

$$-\lambda sinq\int_{T_1} sint f_1(t)dt]+\alpha_2[(1-cosq)f_2(q)-\lambda sinq\int_{T_1} sint f_2(t)dt]=$$

$$=\alpha_1 h_\lambda(f_1)+\alpha_2 h_\lambda f_2$$

h_λ operator chiziqli ekan.

b) $$\|h_\lambda f\|^2=\int_{T^1}\left|(1-\cos q)f(q)-\lambda\sin q\int\sin tf(t)dt\right|^2 dq\le$$

$|a-b|^2\le 2|a|^2+2|b|^2$ tengsizligiga ko'ra

$$\le 2\int_{T^1}|(1-\cos q)f(q)|^2+\lambda\int_{T^1}|\sin qf(t)|^2$$

$$\le 8\int_{T^1}|f(q)|^2 dq+\lambda\int_{T^1}\left|\int f(t)dt\right|^2 dq=$$

$$8\|f\|^2+\lambda\int_{T^1}\left[\int|f(t)|^2 dt\right]dq=$$

$$8\|f\|^2+\lambda\|f\|^2\int_{T^1}dq=(8+\lambda 2\pi)\|f\|^2$$

$$\|h_\lambda f\|\le M\|f\|$$

$$M=\sqrt{8+2\pi\lambda}$$

h_λ operator chegaralangan ekan .

c) h_λ operatorning o'z –o'ziga qo'shmaligini ta'rif bo'yicha ko'rsatamiz ya'ni $\forall f,g\in L_2(-\pi;\pi]$ lar uchun

$(h_\lambda f,g)_{T^1}=(f,h_\lambda,g)_{T^1}$ tenglik bajarilishini ko'rsatamiz.

$$(h_\lambda f,g)=(f,h_\lambda g)$$

$$(h_\lambda f,g)=\int_{T^1}(h_\lambda f)(q)\overline{g(q)}dq=$$

$$=\int_{T^1}\left[(1-\cos q)f(q)-\lambda\sin q\int\sin tf(t)dt\right]\overline{g(q)}dq=$$

$$=\int_{T^1}(1-\cos q)f(q)\overline{g(q)}dq-\int_{T^1}\left[\lambda\sin q\int\sin t(f(t)dt\right]\overline{g(q)}dq=$$

$$=\int_{T^1}f(q)\overline{(1-\cos q)g(q)}dq-\lambda\int_{T^1}\sin tf(t)dt\int_{T^1}\sin q\overline{g(q)}dq=$$

$$\int_{T^1}\left[f(q)\overline{(1-\cos q)g(q)}-\lambda\sin qf(q)\overline{\int\sin tg(q)dt}\right]dq=$$

$$\int_{T^1}f(q)\left[\overline{(1-\cos q)g(q)-\lambda\sin q\int\sin tg(t)dt}\right]=(f,h_\lambda g)$$

$$(h_\lambda f,g)=(f,h_\lambda g)$$

Lemma isbot bo'ldi. Δ

Lemma-2: $\Phi^*\Phi$ operator kompakt operator.

Isbot:

Buning uchun $\Phi^*\Phi$ chegaralangan ekanligini ko'rsatamiz.

$$\left|\Phi^*\Phi f\right|^2 dq=\int_{T^1}\left|\Phi^*\Phi f\right|^2 dq=\int_{T^1}\left[\sin q\int_{T^1}\sin tf(t)dt\right]^2 dq\le$$

$$\le\int_{T^1}\left|\int_{T^1}f(t)dt\right|^2 dq=2\pi\int_{T^1}|f(t)|^2 dt=2\pi\|f\|^2$$

Bundan kelib chiqadiki $\left\|\Phi^*\Phi\right\|\le\sqrt{2\pi}\|f\|$

$\Phi^*\Phi$ bir o'lchamli operator ekanligidan uning kompakt ekanligini hosil qilamiz. Lemma isbot bo'ldi. Δ

Tasdiq-1: $(\Phi^*\Phi f,f)=(\Phi f,\Phi^* f)=\|\Phi f\|^2\ge 0$ Ya'ni, $\Phi^*\Phi$ operator nomanfiy operator.

Lemma-3: h_λ operatorning muhim spektri [0,2] kesmadan iborat

Isbot. Muhim spektr o'rni haqidagi Veyl kriteriyasiga asosan $z\in\sigma_{ess}(h_\lambda)$ munosabat bajarilishi uchun shunday $\{f_n\}$ ortonormal sistema topilib

$$\|(h_\lambda-z)f_n\|\underset{k\to\infty}{\longrightarrow}0$$

bo'lihsi zarur va yetarli . Faraz qilaylik z soni [0,2] kesmadagi ihtiyoriy son bo'lsin $\{f_n\}$ ortonormal sistema sifatida quyidagi funksiyalar sistemasini olamiz.

$$f_n(q)=\frac{x_{\vartheta_n}(q)}{\sqrt{M\vartheta_n(q)}}$$

Bu yerda

$$\vartheta_n(q)=\{q\in\pi\frac{1}{n+1}\le|1-\cos q-z|\le\frac{1}{n}\}$$

x_{v_n} esa v_n ning xarakteristik funksiyasi $\|(h_\lambda - z)f_n\| \xrightarrow[k\to\infty]{} 0$ (1) munosabatni o'rinli ekanini ko'rsatamiz.

Buning uchun $(h_\lambda - z)f_n$ ning normasini quydagicha baholaymiz.

$$\|(h_\lambda - z)f_n\|^2 = \|(1-\cos q - z)f_n(q)\|^2 =$$

$$= \int_{T^1} |1-\cos q - z|^2 |f_n(q)|^2 dq =$$

$$\int_{T^1} |1-\cos q - z|^2 \left| \frac{x_{\vartheta_n}(q)}{\sqrt{M(\vartheta_n(q))}} \right|^2 dq =$$

$$\frac{1}{M(\vartheta_n(q))} \cdot \int_{T^1} |1-\cos q - z|^2 \cdot |x_{\vartheta_n}(q)|^2 dq \le$$

$$\le \frac{1}{n^2} \cdot \frac{1}{M(\vartheta_n(q))} \cdot \int_{T^1} |x_{\vartheta_n}(q)|^2 dq = \frac{1}{n^2} \cdot \frac{M(\vartheta_n(q))}{M(\vartheta_n(q))} =$$

$$= \frac{1}{n^2} \xrightarrow[n\to\infty]{} 0$$

Shunday qilib biz $\forall n > N_0$ uchun

$$\|(h_\lambda - z)f_n\| \le \frac{1}{n}$$

tengsizlikni hosil qildik . Bu tengsizlik $z \in \sigma_{ess}(h_\lambda)$ munosabat o'rinli ekanligini ko'rsatadi. $z \in \sigma_{ess}(h_\lambda)$ sonining ixtiyorligidan $[0,2] \in \sigma_{ess}(h_\lambda)$ munosabat o'rinli ekanligi kelib chiqadi. Bundan esa $\sigma_{ess}(h_0)=[0,2]$ ekanligini xosil qilamiz.

Lemma isbot bo'ldi. Δ

Tasdiq-2: Veyl teoremasiga ko'ra h_λ operatorni muhim spektri h_λ operatorning spektri bilan ustma ust tushadi ya'ni:

$$\sigma_{ess}(h_\lambda) = \sigma_{ess}(h_0) = [0,2]$$

Lemma-4:

h_λ operator 2 dan o'ngda xos qiymatga ega emas.

Isbot:

h_λ operatorning o'z-o'ziga qo'shmaligidan uning spektri haqiqiy sonlar to'plamining qism to'plami bo'ladi.

$\Phi^*\Phi$ operatorning musbatligidan $h_\lambda \le h_0$ kelib chiqadi.

va bundan esa $\sup t(h_\lambda) \le 2$ kelib chiqadi. Bu esa $\tau_{disk}(h_\lambda \cap (2+\infty)) = \emptyset$ ekanligini isbotlaydi.Δ

Demak biz 0 sonidan chapda joylashgan xos qiymatlari bilan qiziqamiz.

$C \setminus [0,2]$ da aniqlangan quyidagi funksiya (Fredgolm determinanti) ni kiritamiz.

$$\Delta_\lambda(z) = 1 - \lambda \int_{T^1} \frac{\sin^2 q}{1-\cos q - z} dq$$

Lemma isbot bo'ldi. Δ

Lemma-5: $z < 0$ soni h_λ operatorning xos qiymati bo'lishi uchun

$\Delta_\lambda(z) = 0$ bo'lishi zarur va yetarli va z xos qiymatga mos xos funksiyaning ko'rinishi quyidagicha bo'ladi:

$$f(q) = \frac{\lambda \sin qC}{1-\cos q - z}$$

Isbot:

Zarurligi: Faraz qilaylik $z < 0$ soni h_λ operatorning xos qiymati bo'lsin u holda $h_\lambda f = zf$ nolmas $f \in L_2(T^1)$ yechimga ega bo'ladi.

Ya'ni

$$(1-\cos q) f(q) - \lambda \sin q \int_{T^1} \sin t f(t) dt = z f(q)$$

bo'ladi. Bunda

$$(1-\cos q - z) f(q) = \lambda \sin q C_f$$

tenglikni xosil qilamiz, bunda

$$f(q) = \frac{\lambda \sin qC}{1-\cos q - z}$$

$$C_f = \int_{T^1} \sin t f(t) dt \qquad (1)$$

(1) ga ko'ra-

$$C_f = \lambda \int_{T^1} \sin t \frac{\sin t C_f}{1-\cos t - z} dt$$

$$C_f = C_f \lambda \int_{T^1} \sin t \frac{\sin^2 t}{1-\cos t - z} dt$$

$$C_f \cdot (1 - \lambda \int_{T^1} \sin t \frac{\sin^2 t}{1-\cos t - z} dt) = 0 \qquad (2)$$

$C_f \neq 0$ chunki $C_f = 0$ deb faraz qilsak $f(q) = 0$ ekanligi hosil bo'ladi bu esa zidlikka olib keladi.

(2) tenglikdan

$$1 - \lambda \int_{T^1} \sin t \frac{\sin^2 t}{1 - \cos t - z} dt = 0,$$

ya'ni $\Delta_\lambda(z) = 0$ ekanligi kelib chiqadi.

Yetarliligi:

Faraz qilaylik $\Delta_\lambda(z) = 0$ bo'lsin u holda $z < 0$ soni h_λ operator uchun xos qiymat ekanligini ko'rsatamiz.

$f_0 = \dfrac{\lambda \sin tC}{1 - \cos q - z}$ funksiyani aniqlaymiz.

$h_\lambda f_0 - zf_0$ ayirmani qaraymiz.

$$(1 - \cos q) \frac{\sin q}{1 - \cos q - z} - \lambda \sin q \int_{T^1} \sin t \frac{\sin t}{1 - \cos z - z} dt =$$

$$= \sin q - \lambda \sin q \int_{T^1} \frac{\sin^2 t}{1 - \cos z - z} dt = \sin q (1 - \lambda \int_{T^1} \frac{\sin^2 t}{1 - \cos t - z}) = 0$$

$h_\lambda f_0 = zf_0$ tenglikni xosil qilamiz, Bu esa $z < 0$ soni h_λ uchun xos qiymat ekanligini bildiradi.

Lemma isbot bo'ldi. Δ

Lemma-6: $\forall \lambda > 0$ uchun quyidagi munosabatlar o'rinli.

a) $\Delta_\lambda(z) = 0$ funksiya $(-\infty, 0)$ oraliqda z bo'yicha uzluksiz va monoton kamayuvchi.

b) $\lim\limits_{z \to -\infty} \Delta_\lambda(z) = 1$

c) $\lim\limits_{z \to 0} \Delta_\lambda(z) = 1 - 2\lambda\pi$

Isbot:

a) $z \in (-\infty, 0)$ bo'lgani uchun $\Delta_\lambda(z)$ funksiya $(-\infty, 0)$,yarim o'qda z bo'yicha uzluksiz $(-\infty, 0)$ da $\Delta_\lambda(z)$ funksiyanini monoton o'suvchiligini ko'rsatish uchun $z_1 < z_2$ munosabat bajariladigan $\forall z_1, z_2 \in (-\infty, 0)$ larda $\Delta_\lambda(z_1) < \Delta_\lambda(z_2)$ tengsizlik bajarilishini ko'rsatish yetarli. Faraz qilamiz $z_1 < z_2 < 0$ bo'lsin u holda

$$\frac{1}{\omega(q)-z_1}<\frac{1}{\omega(q)-z_2}$$

$$\int_{T^1}\frac{\varphi^2(q)}{\omega(q)-z_1}dq<\int_{T^1}\frac{\varphi^2(q)}{\omega(q)-z_2}dq$$

$$\lambda\int_{T^1}\frac{\varphi^2(q)}{\omega(q)-z_1}dq<\lambda\int_{T^1}\frac{\varphi^2(q)}{\omega(q)-z_2}dq$$

$$-\lambda\int_{T^1}\frac{\varphi^2(q)}{\omega(q)-z_1}dq>-\lambda\int_{T^1}\frac{\varphi^2(q)}{\omega(q)-z_2}dq$$

$$1-\lambda\int_{T^1}\frac{\varphi^2(q)}{\omega(q)-z_1}dq>1-\lambda\int_{T^1}\frac{\varphi^2(q)}{\omega(q)-z_2}dq$$

Ya'ni $\Delta_\lambda(z_1)>\Delta_\lambda(z_2)$ kelib chiqadi bu esa $\Delta_\lambda(z)$ funksiyaning $(-\infty,0)$ da monoton kamayuvchi ekanligini bildiradi.

c) $\lim\limits_{z\to 0}\Delta_\lambda(z)=\lim\limits_{z\to 0}(1-\lambda\int_{T^1}\frac{\sin^2 q}{1-\cos q-z}dq)=$ $\qquad 0<\lambda<\frac{1}{2\pi}$

$$1-\lambda\int_{T^1}\frac{(1-cosq)(1+cosq)}{1-cosq}=1-\lambda\int_{T^1}(1+cosq)dq=1-2\pi\lambda$$

Lemma isbot bo'ldi. Δ

Lemma-7: Ushbu

$$1-\lambda\int_{T^1}\frac{sin^2q}{1-cosq-z}dq=0$$

tenglama $z_\lambda=(1-\pi\lambda-\frac{1}{4\pi\lambda})$ ildizga ega.

Isbot:

$$I=\int_{T^1}\frac{sin^2q}{1-cosq-z}dq$$

integralni xisoblaymiz.

$z=-a<0$ belgilash olamiz va quyidagi almashtirishlarni kiritamiz:

$$cos=\frac{1-tg^2\frac{q}{2}}{1+tg^2\frac{q}{2}}=\frac{1-t^2}{1+t^2}$$

$$tg\frac{q}{2}=t \qquad dx=\frac{2dt}{1+t^2}$$

$$I=\int_{-\infty}^{\infty}\frac{1-\left(\frac{1-t^2}{1+t^2}\right)^2}{1-\frac{1-t^2}{1+t^2}+a}\cdot\frac{2dt}{1+t^2}=$$

$$=2\int_{-\infty}^{\infty}\frac{(1+t^2)^2-(1-t^2)^2}{(1+t^2)^3\left[1-\frac{1-t^2}{1+t^2}+a\right]}dt=$$

$$=2\int_{-\infty}^{\infty}\frac{(1+t^2-1+t^2)(1+t^2+1-t^2)}{(1+t^2)^2\left[1+t^2-1+t^2+a(1+t^2)\right]}dt=$$

$$=2\int_{-\infty}^{\infty}\frac{4t^2}{(1+t^2)^2\left[2t^2+a(1+t^2)\right]}dt=$$

$$=8\int_{-\infty}^{\infty}\frac{t^2}{(1+t^2)^2\left[2t^2+a(1+t^2)\right]}dt=$$

Ushbu integralni hisoblashda noma’lum koefitsientlar metodidan foydalanamiz.

$$\frac{t^2}{(1+t^2)^2[(2+a)t^2+a]}=\frac{At+B}{(1+t^2)^2}+\frac{Ct+D}{1+t^2}+\frac{Et+F}{(2+a)t^2+a}$$

$$t^2=(At+B)[(2+a)t^2+a)]+(Ct+D)(1+t^2)((2+a)t^2+a)+(Et+F)(1+t^2)$$

$$\begin{cases} C(2+a)+E=0\\ D(2+a)+F=0\\ A(2+a)+(2+a)C+Ca+2E=0\\ B(a+2)+D(2+a)+aD+2F=1\\ Aa+Ca+E=0\\ Ba+Da+F=0 \end{cases}$$

$$\begin{cases} 2D+aD+F=0\\ Ba+Da+F=0\\ Ca+2C+E=0\\ Aa+Ca+E=0\\ Ba+2D+2aD+2F=0\\ 2Ca+2C+aA+2E=0 \end{cases}$$

$$2A+aA+2C+aC+aC-2aC-2C-aA=0$$

$$aB+2B+2D+aD+aD-aB-2D-2aD=1$$

$$2A=0 \quad\Rightarrow\quad A=0$$

$$2B = 1 \quad \Rightarrow \quad B = \frac{1}{2}$$

$$\begin{cases} 2D + aD + F = 0 \\ aB + aD + F = 0 \end{cases} \quad \Rightarrow \quad 2D\text{-}aB=0 \quad 2D=aB \quad D = \frac{1}{4}a$$

$$\begin{cases} aC + 2C + E = 0 \\ aA + aC + E = 0 \end{cases} \quad \Rightarrow \quad \begin{cases} 2C - aA = 0 \\ C = \frac{-aA}{2} = 0 \end{cases} \quad \Rightarrow \quad E=0$$

$$F = -2D - aD = -\frac{1}{2}a - \frac{1}{4}a^2 = -\frac{1}{2}a(\frac{1}{2}a + 1)$$

$$A = 0, \quad B = \frac{1}{2}, \quad C = 0, \quad D = \frac{1}{4}a, \quad E = 0, \quad F = -\frac{1}{2}a(\frac{1}{2}a + 1)$$

$$\frac{t^2}{(1+t^2)^2[(2+a)t^2 + a]} = \frac{1}{2(1+t^2)^2} + \frac{1}{4}\cdot\frac{a}{1+t^2} - \frac{1}{2}\cdot\frac{a(\frac{1}{2}a+1)}{(2+a)t^2 + a}$$

$$I = 4\int_{-\infty}^{\infty}\frac{1}{(1+t^2)^2}dt + 2a\int_{-\infty}^{\infty}\frac{dt}{1+t^2} - 4a(\frac{1}{2}a+1)\int_{-\infty}^{\infty}\frac{dt}{(2+a)t^2+a} =$$

$$= 4\left(\frac{1}{2}\cdot\frac{t}{1+t^2} + \frac{1}{2}arctgt\right)|_{-\infty}^{\infty} + 2arctgt|_{-\infty}^{\infty} - \frac{(2a^2+4a)}{2+a}\int_{-\infty}^{\infty}\frac{dt}{t^2 + \frac{a}{a+2}} =$$

$$2\frac{\pi}{2} + 2\frac{\pi}{2} + 2a\frac{\pi}{2} + 2a\frac{\pi}{2} - \frac{2a(a+2)}{a+2}\int_{-\infty}^{\infty}\frac{dt}{t^2 + \sqrt{(\frac{a}{a+2})^2}} =$$

$$= 2\pi + 2a\pi + \frac{\sqrt{(a+2)}}{\sqrt{a}}\cdot arctgt\frac{\sqrt{(a+2)}}{\sqrt{a}}|_{-\infty}^{\infty} = 2\pi(a+1) - 2a\pi\frac{\sqrt{(a+2)}}{\sqrt{a}} =$$

$$= 2\pi(a+1) - 2\pi\sqrt{a^2+2a} = 2\pi(a+1-\sqrt{a^2+2a} = 2\pi\left(1 - z - \sqrt{z^2-2z}\right)$$

$$I = 2\pi\left(1 - z - \sqrt{z^2-2z}\right)$$

$$1 - 2\pi\lambda\left(1 - z - \sqrt{z^2-2z}\right) = 0$$

$$1 - z - \sqrt{z^2-2z} = \frac{1}{2\pi\lambda}$$

$$1 - z - \frac{1}{2\pi\lambda} = \sqrt{z^2-2z}$$

Endi tenglikning har ikkala tomonini kvadratga ko'taramiz.

$$1 - 2z + z^2 - \frac{1}{\pi\lambda}(1-z) + \frac{1}{4\pi^2\lambda^2} = z^2 - 2z$$

$$1-\frac{1}{\pi\lambda}+\frac{1}{4\pi^2\lambda^2}+\frac{z}{\pi\lambda}=0$$
$$z=\pi\lambda(\frac{1}{\pi\lambda}-1-\frac{1}{4\pi^2\lambda^2})$$
$$z_\lambda=\left(1-\pi\lambda-\frac{1}{4\pi\lambda}\right)\ \lambda>\frac{1}{2\pi}$$
Lemma isbot bo'ldi. Δ

Ta'rif: $h_\lambda f=0$ tenglama nolmas $f\notin L_2(T^1)$ yechimga ega bo'lsa $z=0$ virtual sath deyiladi.

Lemma-8: $h_\lambda f=0$ tenglama nolmas yechimga ega faqat va faqat $\Delta(\lambda,0)=0$ tenglama o'rinli bo'lsa va $f(q)=\frac{\lambda C sinq}{1-cosq}$ funksiya (1) tenglamani nolmas yechimi bo'ladi.

Isbot: **Zaruriyligi:**

$$(1-cosq)f=\lambda sinq\cdot C \qquad C=\int\limits_{T^1} sint f(t)dt$$
$$f(q)=\frac{\lambda sinq\cdot C}{1-cosq}$$
$$C=\int\limits_{T^1} sint\cdot\frac{\lambda sint\cdot C}{1-cosq}dt$$
$$C(1-\lambda\int_{T^1}\frac{sin^2t}{1-cost}dt)=0 \quad \Delta(\lambda,0)=0$$

Yetarliligi: $\Delta(\lambda,0)=0$tenglik o'rinli bo'lsin u holda $f(q)=\frac{\lambda sinq\cdot C}{1-cosq}$ $h_\lambda f=0$ tenglamaning nolmas yechimi ekanligini ko'rsatamiz.

$$h_\lambda f=(1-cosq)\cdot\frac{\lambda sinq\cdot C}{1-cosq}-\lambda sinq\int\limits_{T^1} sint\cdot\frac{\lambda sint\cdot C}{1-cost}dt=$$
$$\lambda sinq\cdot C-\lambda^2 sinq\int\limits_{T^1}\frac{sin^2t}{1-cost}=\lambda sinq\cdot C\cdot\Delta(\lambda,0)=0$$
Lemma isbot bo'ldi. Δ

Tasdiq: $\Delta(\lambda,0)=0$ ekanligidan $\lambda=\frac{1}{2\pi}$ ekanligi kelib chiqadi va aksincha.

Lemma-9: $f(q) = \frac{\lambda sinq \cdot C}{1-cosq}$ funksiya $L_2(T^1)$ fazoga tegishli emas.

Isbot:

$$\|f\|^2_{L_2(T^1)} = \int_{-\pi}^{\pi} \frac{sin^2q}{1-cosq} = \int_{-\pi}^{\pi} \frac{1+cosq}{1-cosq} dq =$$

$$\int_{-\delta}^{\delta} \frac{1+cosq}{1-cosq} dq + \int_{-\pi}^{-\delta} \frac{1+cosq}{1-cosq} dq + \int_{\delta}^{\pi} \frac{1+cosq}{1-cosq} dq$$

$$I_1 = \int_{-\delta}^{\delta} \frac{1+cosq}{1-cosq} dq$$

$$I_2 = \int_{-\pi}^{-\delta} \frac{1+cosq}{1-cosq} dq$$

$$I_3 = \int_{\delta}^{\pi} \frac{1+cosq}{1-cosq} dq$$

$$I_1 = \int_{-\delta}^{\delta} \frac{1+cosq}{1-cosq} dq = 2\int_{0}^{\delta} \frac{1+cosq}{1-cosq} dq$$

$$\int_{0}^{\delta} \frac{1}{1-cosq} dq = \infty$$

va $\int_0^\delta \left(\frac{1+cosq}{1-cosq} : \frac{1}{1-cosq}\right) dq = \delta + sin\delta < \infty$

ekanligidan

$$\|f\|^2_{L_2(T^1)} = \infty$$

ekanligi xosil bo'ladi.

Lemma isbot bo'ldi.Δ

Teorema:

a) $\lambda > \frac{1}{2\pi}$ da h_λ operator yagona $z_\lambda = 1 - \pi\lambda - \frac{1}{4\pi\lambda} < 0$ xos qiymatga ega va bu xos qiymatga mos xos funksiya

$f(q) = \frac{\lambda sinq \cdot C}{1-cosq-z_\lambda}$ ko'rinishga ega.

b) $0 < \lambda < \frac{1}{2\pi}$ da h_λ operator xos qiymatga ega emas.

c) $\lambda = \frac{1}{2\pi}$ da $z = 0$ nuqta h_λ operator uchun virtual sath bo'ladi va virtual holat ko'rinishi quyidagicha bo'ladi.

$$f(q) = \frac{\lambda sinq \cdot C}{1 - cosq} \notin L_2(T^1).$$

Isbot:

a) Lemma-1 ga asosan

$$\lim_{z\to\infty} \Delta_\lambda = 1$$

$$\lim_{z\to 0} \Delta_\lambda = 1 - 2\lambda\pi < 0$$

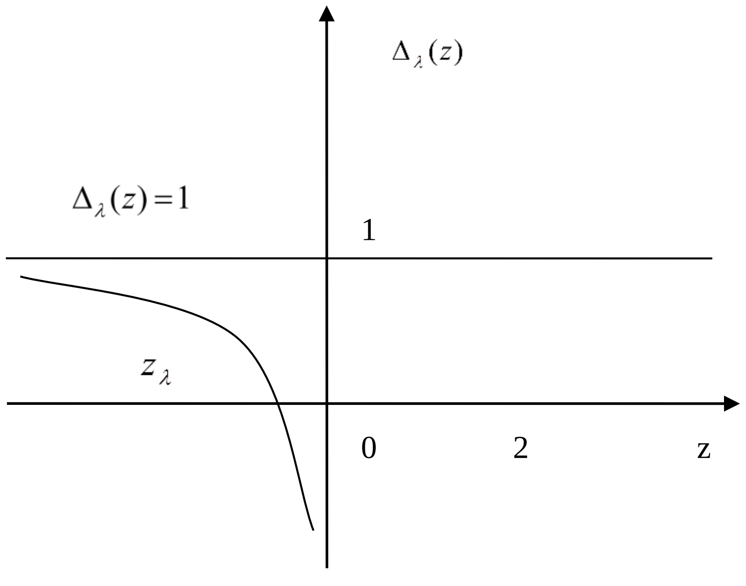
va $\Delta_\lambda(z)$ $z \in (-\infty, 0)$ da monoton kamayuvchi va uzluksiz shunga asosan Boltsano Koshining 1-teoremasiga ko'ra $\Delta_\lambda(z)$ $(-\infty, 0)$ da yagona xos qiymatga ega bo'ladi, h_λ operator $(-\infty, 0)$ da yagona xos qiymatga ega va Lemma-7 ga ko'ra

$$z_\lambda = (1 - \pi\lambda - \frac{1}{4\pi\lambda})$$

va lemma-5 ga ko'ra bu xos qiymatga mos xos funksiyaning ko'rinishi quyidagicha bo'ladi, $f(q) = \frac{\lambda sinq \cdot C}{1 - cosq - z_\lambda}$

b) $\Delta_\lambda(z) = 1 - \lambda \int_{T^1} \frac{\sin^2 q}{1 - \cos q - z} dq$ $\lim_{z\to-\infty} \Delta_\lambda = 1$ $\lim_{z\to 0} \Delta_\lambda = 1 - 2\lambda\pi < 0$

ekanidan va $\Delta_\lambda(z)$ ni $(-\infty, 0)$ da monotonligidan $\Delta_\lambda(z) = 0$ $(-\infty, 0)$ da yechimga ega emasligi kelib chiqadi.



Bundan esa lemma 5 ga ko'ra h_λ operator xos qiymatga ega emasligi kelib chiqadi.

c) $\lambda = \frac{1}{2\pi}$ bo'lsa *hf=0* tenglama lemma 5 ga ko'ra $f(q) = \frac{\lambda sinq \cdot C}{1-cosq}$ xos funksiyaga ega va bu xos funksiya Lemma-9 ga ko'ra $f \notin L_2(T^1)$ bo'lgani uchun ta'rifga asosan h_λ operator uchun $z = 0$ nuqtada virtual sathga ega ekanligi kelib chiqadi.
Teorema isbot bo'ldi. Δ

Xulosa

Bitiruv malakaviy ishi h_λ Fridrixs modelining spektrial xossalarini o'rganishga bag'ishlangan.

Ishda $L_2(T^1)$ Hilbert fazosida aniqlangan h_λ Fridrixs modeli qaralgan. h_λ operatorning muhim spektri o'rni va undan tashqari muhim spektrdan chapda yagona xos qiymatga ega bo'lishi va uning ko'rinishi teorema sifatida keltirilgan.

Olingan natija kvant mexanikasi orqali qattiq jismlar fizikasi, matematik fizika masalalari o'z-o'ziga qo'shma chiziqli chegaralangan operatorlarning spektrial nazariyasi fanlarida muhim o'rin kasb etadi.

Bu ish ijodiy xarakterga ega.

Foydalanil gan adabiyotlar.

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