O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

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Qo’lyozma huquqi

SATIMOV RAHMONALI USMONALIYEVICHNING

EHTIMOLLAR NAZARIYASINING CHEGARAVIY MASALALARIDAGI BA’ZI FUNKSIONALLARNING TAQSIMOTI HAQIDA

5A460104-“Ehtimollar nazariyasi va matematik sitatistika” mutaxassisligi bo’yicha magistr darajasini olish uchun

MAGISTRLIK DISSERTATSIYASI

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MUNDARIJA

K I R I S H

“Har qanday millatning rivoji umumbashariyat tarixida tutgan o’rni, mavqeyi va shuhrati bevosita o’z farzandlarining aqliy va jismoniy yetukligiga bog’liqdir”

I.A.Karimov.

Ma'lumki, O‘zbekiston Respublikasi mustaqillikka erishgan kunlardan boshlaboq Respublikamiz Prezidenti Islom Abdug’aniyevich Karimov mamlakatimizda kadrlar tayyorlash masalasiga asosiy e'tiborni qaratib kelmoqda. Xususan, 1997 yilning 29 avgustida O‘zbekiston Respublikasi Oliy Majlisining 9 sessiyasida “Ta'lim to‘g‘risida” gi qonunga asoslangan “Kadrlar tayyorlash milliy dasturi” qabul qilindi. Bu dastur uch bosqichdan iborat:

1-bosqich: 1997-2001 yillar. Bunda kadrlar tayyorlash milliy dasturining amalga oshirish uchun zarur shart sharoitlar yaratish.

2-bosqich: 2001-2005 yillar. Bunda xuquqiy me'yoriy hujjatlar asosida barkamol avlodni Davlat ta'lim standartlariga javob beradigan qilib tayyorlash maqsad qilib qo‘yilgan.

3-bosqich: 2005 yil va undan keyingi yillar. Bunda davr talabiga mos bo‘lgan barkamol avlodni jamiyatimizning rivojlanishi darajasida tarbiyalab voyaga yetkazish maqsad qilib qo‘yilgan.

Hozirgi paytda Respublikamizda “Kadrlar tayyorlash milliy dasturi”da nazarda tutilgan barcha rejalar muvaffaqiyat bilan amalga oshirildi va davom ettirilmoqda.

Hurmatli Prezidentimiz I.A.Karimovning **“Jahon moliyaviy iqtisodiy inqirozi, O’zbekiston sharoitida uni bartaraf etishning yo’llari va choralari”** nomli asarida hozirgi davrdagi jahon moliyaviy iqtisodiy inqirozining mazmuni, kelib chiqish sabablari va salbiy ta’siri oqibatlari, O’zbekistonda inqiroz ta’siri oqibatlarining oldini olish va yumshatishga asos bo’luvchi omillar, mamlakatimiz iqtisodiyotini modernizatsiyalash va yangilashni izchil davom ettirish borasida qo’lga kiritilayotgan yutuqlar, ustuvor yo’nalishlar, mavjud muammolar va ularni hal etish yo’llari to’g’risida talabalarga bilim berish maqsadida asosiy masalalar atroflicha yoritib berilgan.Biz oliy o’quv yurti talabalari va professor o’qituvchilari prezidentimiz tomonidan yozilgan ushbu asarni atroflicha o’rganib, dars berish jarayonida talabalarga uni mohiyatini ochib berish va ulardan kelib chiqadigan muammolarni kundalik tarbiyaviy ta’limiy ishlarimizdagi elementar muammolarni hal qilishga dasturil amal qilishimiz lozim bo’ladi.[1] Yana biz o’quvchi talabalarga shuni o’rgatishimiz va har bir qiladigan ishlarimizda Prezidentimizning O’zbeksiton Respublikasi Oliy majlisi qonunchilik palatasi va Senatining qo’shma majlisidagi 2010-yil 12-noyabrda qilgan “Mamlakatimizda demokratik islohotlarni yanada chuqurlashtirish va fuqarolik jamiyatini rivojlantirish konsepsiyasi” nomli ma’ruzalarida davlat hokimiyati va uni boshqaruvini demokratlashtirish, sud huquq tizimini isloh etish,axborot sohasini isloh qilish va so’z erkinligini ta’minlash,O’zbekistonda saylov erkinligini ta’minlash va saylov qonunchiligini rivojlantirish, fuqarolik jamiyati insitutini shakllantirish va rivojlantirish,demokratik bozor islohotlarini va iqtisodiyotini liberallashtirish va yanada chuqurlashtirish kabi masalalar to’g’risida atroflicha ma’ruza qildilar.[2] Hurmatli Prezidentimizning **2010-yilni “Barkamol avlod yili”** deb e’lon qildilar. Ana shu munosabat bilan Hurmatli Prezidentimiz tomonidan 2010-yilda davlat byudjetining 50% dan ko’prog’i mamlakatimizda faqat ta’lim tarbiya va sog’liqni saqlash sohalariga yo’naltirildi. Jumladan, davlat ta’lim standartlarini o’quv darsturlari va o’quv adabiyotlarini takomillashtirish, oliy o’rta maxsus ta’lim tizimida talim yonalishlari va mutahassisliklarini bugungi kun talablari nuqtai nazaridan qayta ko’rib chiqish zarur ekanligi takidlab o’tildi.Shuningdek o’quv jarayoniga yangi axborot va pedagogik texnologiyalarni keng joriy etish, bolalarimizni komil insonlar etib tarbiyalashda jonbozlik ko’rsatadigan o’qituvchi va domlalarga e’tiborimizni yanada oshirish, qisqacha aytganda, ta’lim tarbiya tizimini sifat

[1] I.A.Karimov “Jahon moliyaviy iqtisodiy-inqirozi, O’zbekiston sharoitida uni bartaraf etish yo’llari va choralari” Toshkent.Iqtisodiyot 2009

[2] I.A.Karimov “Mamlakatimizda demokratik islohotlarni yanada chuqurlashtirish va fuqarolik jamiyatini rivojlantirish konsepsiyasi”. Toshkent.,O’zbekiston 2010-yil

jihatdan butunlay yangi bosqichga ko'tarish diqqatimiz markazida bo'lishi darkor.[3] Yuqorida aytib o'tilgan fikrlar mamlakatimizda uzluksiz ta'lim tizimini joriy qilinganligidan dalolat berib turibdi.Har qanday ta'lim o'z oldiga ikkita maqsadni qo'yadi:

a) Ma'lum bir dastur asosida o'quvchi talabalarga elementar matematik bilimlar sistemasini berish.

b) Ana shu berilgan bilimlar orqali o'quvchilarning mantiqiy fikrlash qobiliyatlarini o'stirishdan iboratdir.

Hozirgi davrda axborot – kommunikatsiya texnologiyalari orqali uzatiladigan axborot jamiyat rivojining eng muhim shartlaridan biri bo'lib qoldi. U ishlab chiqarish resursi, insonlar orasidagi aloqani ta`minlovchi qudratli vositaga aylandi. Mazkur sohada samarali huquqiy mexanizmlarning yaratilishi hamda joylardagi sohaga oid qonun hujjatlari ijrosini ta`minlash iste'molchilarning barcha qatlamlari hamda jadallik bilan rivojlanib borayotgan zamonaviy axborot – kommunikatsiya texnologiyalariga mos tushadigan, tubdan yangilangan axborot – kommunikatsiya tarmoqlari vujudga kelishiga zamin yaratib, mamlakatimizda axborotlashtirish jarayonlarini jadallashtirish hamda O'zbekistonning butun jahon axborot hamjamiyatida munosib o'rin egallashiga hizmat qiladi.

"Bugun O'zbekistonimiz bosib o'tgan mustaqil taraqqiyot yo'lini holisona baholar ekanmiz, o'tgan davr mobaynida biz erishgan, dunyo jamiyatchiligi tan olgan olamshumul yutuq va marralar, avvalambor, izchil rivojlanayotgan iqtisodiyotimiz va uning barqaror o'sish sur'atlari, aholi farovonligining yildan –yilga oshib, jahon hamjamiyatida mamlakatimiz obro'-e'tiborining tobora yuksalib borayotgani –bularning barchasi Konstitutsiyamizning asosiga qo'yilgan , chuqur va puxta o'ylangan maqsad, prinsip va normalarning hayotbaxsh samarasi desak, hech qanday mubolag'a bo'lmaydi[4]". Muhtaram Prezidentimiz Islom Abdug'aniyevich Karimov, muntazam ravishda yosh avlodning barkamol bo'lib yetishishi va mustaqil O'zbekistonimizning fidoiy kadrlari bo'lmog'i uchun sharoitlar yaratib bermoqdalar, o'zlarining har bir nutqida yoshlarning ta'lim tarbiyasi, bilim saviyasi, dunyoqarashini shaklantirish kabi dolzarb muammolarga alohida e'tibor qaratmoqdalar.

Kichik biznes va xususiy tadbirkorlik bugungi kunda jamiyatimizdagi ijtimoiy va siyosiy barqarorklikning kafolati va tayanchiga, yurtimizni taraqqiyot yo'lida faol harakatlantiradigan kuchga aylanib bormoqda. Shuning uchun Prezidentimiz Islom Abdug'aniyevich Karimov 2011 yilni **" Kichik biznes va xususiy tadbirkorlik yili"** deb e'lon qildilar va bu borada Prezidentimizning quyidagi fikrlarini keltirib o'tishimiz mumkin: "Ilg'or ilm – fan yutuqlariga asoslangan kichik biznes va xususiy tadbirkorlikni rivojlantirishga keng yo'l ochib berishimiz zarur… Kichik biznes va xususiy tadbirkorlik – jamiyatimizning, bugungi va kelajak taraqqiyotimiz, farovon hayotimizning mustahkam tayanchi bo'lishi shart".

2011 yil O'zbekiston Respublikasi mustaqilligining 20 – yilligini nishonlandi. Prezidentimiz I.A.Karimovning 2011 yil 6 apreldagi "O'zbekiston Respublikasining 20 – yilligiga bag'ishlangan" qarorida ham mamlakatimizning 20 yilligini munosib ravishda kutib olishda vatanimiz yoshlarining o'rni beqiyosligini, ularni aqlan va jismonan rivojlanishi uchun sharoitlar yaratib berish kerakligini aytib o'tganlar. Bunga misol qilib "Barkamol avlod" sport o'yinlarini, mamlakatimiz yoshlari ishtirokida "Biz – yigirma yoshdamiz" forumini, mehnat va o'quv jamoalari o'rtasida sport turlari bo'yicha "Mustaqillik kubogi" musobaqalari, an'anaviy "Sharq taronalari" xalqaro musiqa festivalini, yosh ijrochilar o'rtasida "Nihol", iste'dodli yoshlar o'rtasida "Kelajak ovozi", jurnalistika sohasida "Eng ulug', eng aziz", "Oltin qalam", "Ozod yurt to'lqinlari", shuningdek, yangitdan tashkil etilayotgan "Istiqlol tarix ko'zgusida" (mustaqillik tarixini yoritishga bag'ishlangan eng yaxshi ilmiy va ilmiy – ommobop ishlar uchun). "Istiqlol

[3] I.A.Karimov "Barkamol avlod yili davlat dasturi" Toshkent O'zbekiston 2010-yil

[4] I.A.Karimov "Mamlakatimizni modernizatsiya qilish yo'lini izchil davomi – taraqqiyotimizning muhim omilidir" "Ishonch" gazetasi: 2010 yil, 8 dekabr.

solnomasi” (kino san’ati sohasidagi eng yaxshi asarlar uchun) kabi tanlovlarini yuqori darajada o’tkazish, tashkillashtirishni ta’kidlab o’tganlar.

2010 – yil “Barkamol avlod yili”da amalga oshirilgan ishlar, 2011 yil “Kichik biznez va xususiy tadbirkorlik yili”da bajarilgan ishlar ko’lami ham juda katta bo’lib, rejalashtirilgan vazifalarni amalga oshirishda ilm – fan yutuqlaridan, ilg’or ishlab chiqarish texnologiyalaridan foydalanish bosh vazifa bo’lib qoladi, bunday ulkan ishlarni muvaffaqiyat bilan uddalash uchun aniq hisob – kitoblar, xullas matematika fanining yutuqlaridan ustalik, izchillik bilan foydalanish lozim bo’ladi.

Tabiiyki, bu maqsadlarni yuqori saviyada amalga oshirish uchun oliygohlar, kollej va litseylarning bitiruvchilari har tomonlama yetuk kadrlar bo‘lib yetishmog‘i – zamon talabidir. Oliygohlarda magistrlar tayyorlashni yuqori saviyada tashkil etish ham shu programmaning bir qismidir.

Muhtaram Prezidentimiz I.A.Karimov, muntazam ravishda yosh avlodning bilimli, barkamol bo‘lib yetishishi va mustaqil O‘zbekistonimizning fidoyi kadrlari bo‘lmog‘i uchun sharoitlar yaratib bermoqdalar, o‘zlarining har bir nutqlarida yoshlarning ta'lim tarbiyasi, bilim saviyasi, dunyoqarashini shakllantirish kabi dolzarb muammolarga alohida e'tibor qaratmoqdalar. Biz quyida Prezidentimiz I.A.Karimovning turli paytlarda so‘zlagan nutqlaridan iqtiboslar keltiramiz.

O‘zbekiston mustaqillikka erishgach butun ta’lim tizimini tubdan isloh qilish rejalari ishlab chiqildi, bunda asosiy e'tibor Respublika uchun ilmiy kadrlar tayyorlashni kengaytirishga qaratilgandir. Bu haqda Prezidentimiz I.A.Karimovning quyidagi fikrlarini keltirish joizdair:

- “Yoshlarni zamonaviy fan – tehnikaning, umuman ilm – fanning yutuqlaridan bahramand qilmasdan turib, ularga yuqori malakali mutaxassislar bo‘lib yetishishga sharoit tug‘dirmay turib, biz respublikamiz xalq xo‘jaligini, sanoat ishlab chiqarish sohalarini tubdan o‘zgartira olmaymiz, buni har doim yodda tutish kerak. Chunki ishsizlik, maosh kamligi, mutaxassislar yetishmasligi va boshqa ko‘p yetishmovchiliklar ana shundan deb o‘ylayman. ...Demak, xalq saylab qo‘ygan deputatlarning birinchi navbatdagi vazifalaridan biri – yoshlarimiz tarbiyasi. Milliy siyosatni, harakat dasturini ishlab chiqish masalasidir.”

- “Ayniqsa, o‘sib kelayotgan avlod taqdiriga hyech kim befarq qaray olmaydi. Bunda oliy o‘quv yurtlarining ahamiyati kattadir. Yoshlarni qay usulda o‘qitish, ularni tarbiyalash, mustaqil mamlakatning yetuk mutaxassislari bo‘lishiga qayg‘urish har birimizning muqaddas burchimizdir. Bunda oliy va o‘rta maxsus ta'lim tizimi saviyasini jahon andozalari darajasiga yetkazish, xalq xo‘jaligida kadrlarga bo‘lgan talab va ehtiyojlarni ilmiy tahlil asosida aniqlash, xorijiy mamlakatlar tajribasidan oqilona foydalanish shu kunning dolzarb vazifalaridandir...” – deb ta'kidlagan muhtaram Prezidentimiz I.A.Karimov. ([2]. “Ilm – ziyo salohiyati – yurt boyligi” nutqidan. “Ma'rifat” gazetasi. 1993 yil, 21 iyul).

Kasb – hunar kollejlari, litsey talabalari, oliy o‘quv yurtini tamomlayotgan har bir yosh mutaxassisni o‘z kasbining bilimdoni va jonkuyar qilib tayyorlash shu kunning eng birinchi talabidir. Bu o‘rinda Prezidentimiz I.A.Karimovning ushbu so‘zlarini keltirish joizdir:

- “**Hukumatimiz fan, ma'rifat va madaniyat sohasiga jiddiy e'tibor bilan qaramoqda. O‘tish davrining eng qiyin va murakkab paytida ham davlat maorifni takomillashtirish, bilimli va har tomonlama barkamol avlodni voyaga yetkazish uchun hech narsani ayamayapti.**

Axir, o‘ylab ko‘raylik, birodarlar, biz bugun ne – ne mashaqqatlar bilan qurayotgan davlatimiz, mustaqil va har jihatdan mustahkam mamlakatimiz ertaga kimning qo‘lida qoladi?

Bugungi katta islohotlar evaziga qo‘lga kiritayotgan yutuqlarimizni ertaga farzandlarimiz davom ettirishga qodir bo‘ladimi – yo‘qmi?...

Islohotning taqdiri, uning qay darajada amalga oshirish, birinchi navbatda kadrlarning malakasiga, ularning o‘z ishlarini qay darajada o‘zlashtirib olganligiga, yurtparvarligi va fidoiyligiga bog‘liq”. (I.A.Karimovning xalq deputatlari Samarqand viloyati kengashi sessiyasida so‘zlagan nutqidan, “Ma'rifat” gazetasi. 1995 yil 29 noyabr).

O’zbekiston Respublikasida inson huquqlari va erkinligiga rioya etilishini, jamiyatning ma’naviy yangilanishni, ijtimoiy yo’naltirilgan bozor iqtisodiyotini shakillantirishni, jahon hamjamiyatiga qo’shilishni ta’minlaydigan demokratik xuquqiy davlat va ochiq fuqorolik jamiyati qutilmoqda.

Inson, uning har tomonlama uyg’un kamol topishi va farovonligi, shaxs manfa’tlarni ro’yobga chiqarishning sharoitlarini yaratish, eskirgan tafakkur va ijtimoiy xulq atvorining andozalarini o’zgartirish respublikada amalga oshirilayotgan islohotlarning asosiy maqsadi va harakatlantiruvchi kuchidan xalqning boy intelektual merosi va umumbashariy qadriyatlar asosida, zamonaviy madaniyati, iqtisodiyot, fan, tehnika va tehnologiyalarning yutuqlari asosida kadrlar tayyorlashning mukammal tizimini shakillantirish O’zbekiston taraqqiyotining muhim shartidir.

“Kadrlar tayyorlash milliy dasturi”, “Ta’lim to’g;risida”gi O’zbekiston Respublikasi Qonuning qoidalariga muvofiq holda tayyorlangan bo’lib, milliy tajribaning tahlili va ta’lim tizimidagi jahon miqyosidagi yutuqlar asosida tayyorlangan hamda yuksak umumiy va kasb-hunar madaniyatiga, ijodiy va ijtimoiy faollikka, ijtimoiy siyosiy hayotda mustaqil ravishda mo’ljalni to’g’ri ola bilish mahoratiga ega bo’lgan istiqbol vazifalarini ilgari surish va hal etishga qodir kadrlarning yangi avlodni shakillantirishga yo’naltirilgan.

O’zbekiston Respublikasi davlat mustaqilligiga erishish, iqtisodiy va ijtimoiy rivojlanishning o’ziga hos yo’lini tanlashi kadrlar tayyorlash tuzilmasi va mazmunini qayta tashkil etishni zarur qilib qo’ydi va qator chora tadbirlar ko’rishni **“Ta’lim to’g’risida”**gi Qonuni joriy etishni (1992-yil), yangi o’quv rejalari, dasturlari, darsliklarini joriy etishni, zamonaviy didaktik ta’minotni ishlab chiqishni; o’quv yurtlarini atestatsiyadan o’tkazishni va akkreditatsiyalashni yangi tipdagi ta’lim muassasalarini tashkil etishni taqozo etadi.

Hammamizga ma’lumki, hayotimizni barcha sohalarida islohotlarni o’tkazish, o’zbek modeli deb nom olgan islohatlar siyosatning asoslari bo’lmish besh tamoyilning birida islohatlarni bosqichma-bosqich o’tkazish vazifasi qo’yilgan.

O’zbekiston Respublikasi Prezidentining Respublika Oliy Majlisi IX sessiyasida so’zlagan nutqi (1997 yil 29 avgust) va **“Ta’lim tarbiya kadrlar tayyorlash tizimining tubdan isloh qilish barkamol avlodni voyaga etkazish to’g’risida”** gi formasi muhim ahamiyatga ega bo’ladi. Bu esa oliy maktabda o’qitish sistemasini isloh qilishni taqozo etadi.

Shuning uchun zamon talabiga mos o’quv qo’llanmalar yaratish milliy dasturni amalga oshirish vazifalaridan biri hisoblanadi.

Xalq ta’limi rivojlanishidagi xozirgi davr – muhim davrlardan bo’lib, bu “Ta’lim to’g’risida” gi qonun va **“Kadrlar tayyorlash milliy dasturi”** Vazirlar Maxkamasining 1998 yil 13 may **“Umumiy o’rta ta’limni tashkil etish va singdirishdan iboratdir”**. Bu hujjatlar ta’lim sohasida jamiyat ehtiyojini to’la qondirishni nazarda tutadi. Bu vazifalarni amalga oshirish uchun mo’ljallangan tadbirlar tizimida matematika ta’limining davlat standartlarini joriy etish katta ahamiyat kasb etadi.

Ma’lumki, matematika moddiy dunyoning ob’ektlarini o’rganadi. Lekin boshqa fanlardan farqli ravishda uning miqdoriy munosabatlari va fazoviy shakllari asosiy ob’ekt sifatida qaraladi. Matematika o’sib kelayotgan yosh avlodni kamol toptirishda o’quv fani sifatida keng imkoniyatlarga ega. U o’quv tafakkurini rivojlantirib, ularning aqlini charxlaydi, uni tartibga soladi. O’quvchilarga maqsadga yo’nalganlik, mantiqiy fikrlash, topqirlik xislatlarini shakllantiradi. Shu bilan bir qatorda teoremani isbotlash va mulohazalarning to'g'ri, go'zal tuzilganligi o'quvchilarni didi, go'zallikka ehtiyotli qilib tarbiyalab boradi.

Prezidentimizni O'zbekiston Respublikasi Oliy Majlisi qonunchilik palatasi va senatining 2010 yil 27 yanvarda bo'lib o'tgan qo'shma majlisidagi **"Mamlakatimizni modernizastiya qilish va kuchli fuqarolik jamiyati barpo etish ustivor maqsadimiz"** ma'ruzalari har qanday fanni rivojlantirishda uni zamonga mos va xos qolipga tushirish ko'rsatib o'tilgan.

Prezidentimiz I.A.Karimovning **"Yuksak ma'naviyat engilmas kuch"** asaridan. Bugun biz tarixiy bir davrda — xalqimiz o'z oldiga ezgu va ulug' maqsadlar qo'yib, tinch-osoyishta hayot kechirayotgan, avvalambor o'z kuch va imkoniyatlariga tayanib, demokratik davlat va fuqarolik jamiyati qurish yo'lida ulkan natijalarni qo'lga kiritayotgan bir zamonda yashamoqdamiz.

Biz o'z taqdirimizni o'z qo'limizga olib, azaliy qadriyatlarimizga suyanib, shu bilan birga, taraqqiy topgan davlatlar tajribasini hisobga olgan holda, mana shunday olijanob intilishlar bilan yashayotganimiz, xalqimiz asrlar davomida orziqib kutgan ozod, erkin va farovon hayotni barpo etayotganimiz, bu yo'lda erishayotgan yutuqlarimizni xalqaro hamjamiyat tan olgani — bunday imkoniyatlarning barchasini aynan mustaqillik berganini bugun hammamiz chuqur anglaymiz.

Ana shu haqiqatni xalqimiz har tomonlama to'g'ri tushunib, tanlagan taraqqiyot yo'limizni ongli ravishda qabul qilgani va qo'llab-quvvatlayotgani oldimizga qo'ygan maqsadlarga erishishning asosiy manbai va garovi ekanini hayotning o'zi tasdiqlamoqda.

Biz xalqimizning dunyoda hech kimdan kam bo'lmasligi, farzandlarimizning bizdan ko'ra kuchli, bilimli, dono va albatta baxtli bo'lib yashashi uchun bor kuch va imkoniyatlarimizni safarbar etayotgan ekanmiz, bu borada ma'naviy tarbiya masalasi, hech shubhasiz, beqiyos ahamiyat kasb etadi. Agar biz bu masalada hushyorlik va sezgirligimizni, qat'iyat va mas'uliyatimizni yo'qotsak, bu o'ta muhim ishni o'z holiga, o'zbilarchilikka tashlab qo'yadigan bo'lsak, muqaddas qadriyatlarimizga yo'g'rilgan va ulardan oziqlangan ma'naviyatimizdan, tarixiy xotiramizdan ayrilib, oxir-oqibatda o'zimiz intilgan umumbashariy taraqqiyot yo'lidan chetga chiqib qolishimiz mumkin.

El-yurtimiz o'zining ko'p asrlik tarixi davomida bunday mash'um xatolarni necha bor ko'rgan, ularning jabrini tortgan. Shunday asoratlar tufayli tilimiz, dinimiz va ma'naviyatimiz bir paytlar qanday xavf ostida qolganini barchamiz yaxshi bilamiz. Ana shu fojiali o'tmish, bosib o'tgan mashaqqatli yo'limiz barchamizga saboq bo'lishi, bugungi voqelikni teran taxlil qilib, mavjud tahdidlarga nisbatan doimo ogoh bo'lib yashashga da'vat etishi lozim. O'z tarixini bilmaydigan, kechagi kunini unutgan millatning kelajagi yo'q. Bu haqiqat kishilik tarixida ko'p bora o'z isbotini topgan. Ma'lumki, biz boshimizdan kechirgan sobiq mustabid tuzum davrida milliy ma'naviyatni rivojlantirishga mutlaqo yo'l qo'yilmagan. Aksincha, xalqimizning tabiati, yashash tarziga yot bo'lgan kommunistik mafkurani har qanday yo'llar va zo'ravonlik bilan joriy etishga harakat qilingan. Shuning uchun ham istiklolning dastlabki kunlaridanoq bu sohadagi ahvolni tubdan o'zgartirish yurtimizda eng dolzarb va hal qiluvchi vazifalardan biriga aylandi.

Zaminimizda yashab o'tgan buyuk allomalarimiz, mutafakkir bo-bolarimizning ibratli hayoti va faoliyati, bemisl ilmiy-ijodiy kashfiyotlari bugun ham ja-hon ahlini hayratga solayotganini g'urur bilan ta'kidlash lozim.

Masalan, Muhammad Muso Xorazmiyning o'nlik sanoq sistemasini, algoritm va algebra tushunchalarini dunyoda birinchi bo'lib ilm-fan sohasiga joriy etgani va shu asosda aniq fanlar rivoji uchun o'z vaqtida mustahkam asos yaratgani umuminsoniy taraqqiyot rivojida qanday katta ahamiyatga ega bo'lganini barchamiz yaxshi bilamiz. Bugungi kunda odamzotning ilm-fan va zamonaviy texnologiyalar borasida erishayotgan ulkan yutuqlarini ko'z oldimizga keltirar ekanmiz, beixtiyor ana shu buyuk bobomiz misolida bunday yuksak marralarga erishishda o’zbek xalqining ham munosib hissasi borligidan qalbimiz iftixorga to’ladi.

Shuni alohida ta’kidlash joizki, xalqimiz tarixning har qanday to’fon va suronlariga qaramasdan, milliy o’zligi va azaliy qadriyatlarini saqlab olib, bugungi dorilomon zamonlarga bezavol etib kelishida uning qon-qoni, suyak-suyagida bo’lgan ma’naviy jasorat tuyg’usi, hech shubhasiz, hal qiluvchi ta’sir o’tkazib kelmokda.

Chunki, o’zbek xalqi dunyo maydonida kuni kecha tasodifan paydo bo’lib qolgani yo’q. Biz — boy tarix, yuksak madaniyat, buyuk ma’naviyat vorislarimiz. Qadimiy tariximizni har tomonlama o’rganib, shunday xulosaga kelish mumkinki, eng qaltis va taxlikali davrlarda millatimizga umid va ishonch bergan, uni yovlarga qarshi kurashga undagan, ajdodlarimizni ilmiy kashfiyotlar, harbiy zafarlarga, ma’rifat mash’alini baland ko’tarib jaholatga qarshi chiqishga chorlagan beqiyos kuch ham aynan ma’naviy jasorat tuyg’usidir.

Manaviyatning hayotimizdagi o’rni va ahamiyati haqidagi fikr-mulohazalarimizga yakun yasar ekanmiz, avvalambor, shuni chuqur anglab olishimiz darkorki, ma’naviy yuksalishga erishish — bu bir yillik yoki besh-o’n yillik ish emas. Xalq, millat o’z milliy ma’naviyatini yillar, asrlar davomida kjsaltirib, boyitib boradi. Chunki ma’naviyat qotib qolgan aqidalar yig’indisi emas, aksincha, doimiy harakatdagi uzluksiz jarayon bo’lib, taraqqiyot davom etar ekan, uning shiddatli yurishi tufayli ma’naviy hayot oldiga qo’yiladigan talablar ham muttasil paydo bo’laveradi.

Ya’ni, mamlakatimiz taraqqiyot pillapoyasidan yangi-yangi cho’qqilar sari qadam qo’yar ekan, biz yashayotgan zamon sur’atlari shildat bilan tezlashib, oldimizda yana qancha muammo va mashaqqatlar paydo bo’lar ekan, tabiiyki, ma’naviy hayotimiz ham ana shu sinovlarda toblanib, yuksalib, jamiyatimiz, millatimizning yorug’ va sog’lom kelajagini har qanday taxdid va to’fonlardan – davr o’zgarishi bilan ularning ko’rinishi va shakli o’zgarishiga qaramasdan – bezavol saqlash va asrab qolishga qodir bo’lishi darkor.

Ana shu fikr va xulosalardan kelib chiqqan holda, «Biz kimmiz?» degan savolga javob bermasdan turib, eng muhimi, ma’naviy boylikni, ma’naviyatni yuksaltirishga doimiy intilmasdan turib biz o’z oldimizga qo’ygan ezgu maqsadlarga erishish mumkin emas, desam, qalbimdan chuqur joy olgan fikrni aytgan bo’laman.

Bugungi kunda ma’naviyat dunyosiga nisbatan mavjud bo’lgan xavf-xatarlardan albatta ko’z yumib bo’lmaydi, lekin bizning ishonchimiz komilki, xalqimiz tarixning murakkab jarayonlarida irodasi chiniqib, har qanday hujum va tazyiqlarga qaramasdan, ma’naviy olami kuchayib va yuksalib borayotgani, bizni ko’rolmaydigan kuchlar ham bu haqiqatni tan olayotganini mamnuniyat bilan qayd etamiz.

Chunki xalq – bamisoli ulug’ va sharafli yo’ldan ilgarilab borayotgan ulkan karvon. Uni yo’ldan chalg’itishga urinuvchilar, payt poylab orqasidan hamla qiluvchilar hamisha bo’lgan, bundan keyin ham bo’lishi mumkin. Karvon bexatar bo’lmas, degan gap bejiz aytilmagan. Ammo xalq karvonini hech qanday kuch ortga qaytarolmaydi. Nega deganda, xalqning qalbida ne-ne avlodlardan meros engilmas kuch – ma’naviyat bor.

Barchamizga g’urur va iftixor bag’ishlaydigan tomoni shundaki, bizning ezgu intilishlarimiz zamirida ham ana shunday pok niyat, buyuk ishonch mujassam. Aniq maqsad yo’lidagi bunyodkorlik ishlarimiz amaliy natijalar bera boshlagan, ko’zlagan rejalarimiz bosqichma-bosqich ro’yobga chiqib, dunyo hamjamiyatidan o’zimizga munosib o’rin egallab borayotgan hozirgi kunda xalqimiz, millatimiz qalbidagi ana shu ishonch yanada mustahkamlanmoqda.

Shu bois men shonli tarixiga sadoqat bilan qarab, bugungi ozod hayotini qadrlab, o’z kelajagiga katta umid bilan intilayotgan xalqimizning donishmandligi va matonati, uning iymon-e’tiqodi, mustahkam irodasi va yuksak ma’naviy ruhi doimo barqaror yashaydi, deb ishonaman.

Mavzuning dolzarbligi. Tasodifiy jarayonlar, xususan, tasodifiy daydishlar uchun chegaraviy masalalar ehtimollar nazariyasida juda muhim o'rin tutadi. Buning sabablaridan biri ularning matematik tomondan ma'nodorligi bo'lsa, keyingisi ular ehtimollar nazariyasining amaliy sohalarida juda ko'p tadbiqlariga ega. Tarixan ko'p chegaraviy masalalar amaliyotdan kelib chiqqan. Biz bu yerda juda ko'p chegaraviy masalalar ichidan tasodifiy daydish uchun ikki chegara bilan bog'liq bo'lgan ba'zi masalalarni qaraymiz. Bular tasodifiy daydishning yo'lakdan (полоса) birinchi bora chiqish vaqti, yo'lakdan chiqishdagi holati kabi tasodifiy miqdorlarning taqsimot qonunlari va ularning sonli harakteristikalarini o'rganish masalasidir.

Mavzuning maqsad va vazifalari. Ushbu magistrlik dissertatsiyasining assosiy maqsad va vazifalari ehtimollar nazariyasining chegaraviy masalalaridagi ba'zi bir funksionallarning taqsimot qonunlarini, ularning sonli harakteristikalarini o'rganishdan iborat.

Amaliy ahamiyati. Bu turdagi masalalarga ommaviy xizmat tizimlarini o'rganish, zahiralarini saqlash, ketma-ket tahlil (последователъны анализ), optimal boshqaruv nazariyalari masalalari olib keladi. Bu masalalarni hal qilish, odatda, juda nozik analitik usullarni qo'llashni talab qiladi.

Ikki chegarali masalalar bilan ilmiy izlanishlar birinchi bo'lib Lagranj, Muavr, Ya. Bernulli kabi buyuk matematiklar tomonidan olib borilgan. Bu avtorlarning ilmiy ishlarida juda sodda tasodifiy daydishlar uchun chegaraviy funksionallarning harakteristikalari topilgan. Chegaraviy funksionallarning, ayniqsa, ikki chegara bilan bog'liq bo'lgan holda, taqsimot qonuni va sonli harakteristikalari uchun aniq formulalarni ba'zi bir xususiy hollardagina olish imkoniyati bo'lar ekan. Shuning uchun keyingi paytlarda asimptotik izlanishlarga ko'proq e'tibor berilmoqda.

Chegaraviy masalalar sohasida buyuk kashfiyotlarni qilgan olimlar A.N.Kolmogorov, N.V.Smirnov, B.V.Gnedenko, A.A.Borovkov, V.S.Korolyuk, S.V.Nagayev, A.A.Mogulskiy, V.M.Shurenkov, V.I.Lotov va boshqalardir.

Ushbu magistrlik desertatsiyasi yuqorida bayon qilingan ikki yo'nalishdagi tadqiqotlarga, ya'ni chegaraviy funksionallar taqsimot qonunlari uchun aniq va asimptotik formullar topishga bag'ishlangan. Bunda „vaqt", ya'ni tasodifiy daydishda qo'shiluvchilar soni ortib borishi bilan yo'lak kengligi ortib borishi talab qilinadi.

Ilmiy yangiligi. Birinchi bobda tasodifiy daydish izlarining yo'lakdan birinchi marta chiqish momenti va shu momentdagi uning holati uchun aniq formulalar topilgan. Bu yerda tasodifiy daydishni hosil qiluvchi tasodiviy miqdor ikki tomonlama geometrik taqsimot qonuniga ega, deb faraz qilinadi.

Ikkinchi bobda Kramer tipidagi shart bajarilganda ba'zi lokal va integral shartli ehtimolliklar uchun asimptotik yoyilmalarning birinchi hadlari topilgan. Bunda A.A.Borovkov tomonidan [1] da ishlab chiqilgan va chegaraviy masalalarni hal etishda keng qo'llaniladigan faktorizatsion usuldan foydalaniladi.

Uchunchi bobda to'g'ri chiziqli yo'lak (полоса) faqat bitta chegara hisobiga kengayadigan hol qaraladi. Ushbu bobda ikki chegarali masalalarni bir nechta bir chegarali masalalarga keltirish usuli qo'llaniladi.

Tadqiqot obekti. Ehtimollar nazariyasining chegaraviy masalalaridagi ba'zi funksionallarning taqsimoti haqida.

Magistrlik dissertatsiya ishining asosiy natijalari. NamDU fizika-matematika fakulteti Ilmiy konferansiyalarida, seminarlarda ma'ruza qilingan. Shu bilan birga, ilmiy ish bo'yicha bitta ilmiy maqola chop etilgan.

Dissertatsiyaning tuzilishi. Disseratatsiya kirish, uchta bob va adabiyotlar ro'yxatidan iborat. Dissertatsiyaning umumiy hajmi betdan iborat.

I bob. **BA’ZI CHEGARAVIY FUNKSIONALLAR TAQSIMOT QONUNLARI UCHUN ANIQ FORMULALAR.**

1-§. Masalaning qo’yilishi

O’zaro bog’liqsiz, bir xil taqsimlangan butun qiymatlarni qabul qiladigan $\xi_1,\ \xi_2$...... tasodifiy miqdorlar ketma-ketligini qaraymiz. Matematik kutilma $M\xi_1 = 0$ va ξ_1 tasodifiy miqdor qabul qiladigan qiymatlarining mumkin bo’lgan barcha ayirmalari eng katta umumiy bo’luvchisi 1 ga teng, ya’ni ushbu ketma-ketlikdan hosil bo’lgan tasodifiy daydish davriy bo’lmasin, deb faraz qilamiz. $S_0 = 0,\quad S_n = \xi_1 + \xi_2 + ... + \xi_n,\quad n \ge 1$. Undan tashqari ξ_1 tasodifiy miqdor taqsimot qonuniga Kramer tipidagi ushbu shartni qo’yamiz: $h(\lambda) = M\lambda^{\xi_1}$ hosil etuvchi funksiya biror $\varepsilon > 0$ da $1-\varepsilon < |\lambda| < 1+\varepsilon$ halqada analitik bo’lsin, $h(\lambda) \ne 1$. Butun $a > 0$ va $b > 0$ sonlar uchun qaralayotgan tasodifiy daydishning $(-a,b)$ intervaldan birinchi matra chiqish momenti bo’lgan *N* tasodifiy miqdorni aniqlaymiz: $N = N(a,b) = \min\{k : S_k \notin (-a,b)\}$. Agar barcha k larda $S_k \in (-a,b)$ bo’lsa, $N = \infty$ deb hisoblaymiz.

[2], [3], [4] maqolalarda

$P(S_N = k,\quad N = n) \quad k \notin (-a,b), \qquad P(S_n = k,\ N > n) \qquad k \in (-a,b)$

ehtimolliklar uchun $a = a(n) \to \infty,\ b = b(n) \to \infty,\quad n \to \infty$ shartlar bajariladigan har xil hollarda to’la asimptotik yoyilmalar topilgan.

$P(N = n)$ ehtimollik uchun aniq formula ξ_1 tasodifiy miqdor -1 va 1 qiymatlarni *p* va *q*, $p + q = 1$ ehtimolliklar bilan qabul qiladigan hol uchun [6] ishda keltirilgan. Ushbu ishning 3-paragrafida yuqoridagiga o’xshash formula ξ_1 tasodifiy miqdor ikki tomonlama geometrik taqsimot qonuniga ega bo’lgan holda topilgan.

$a = c_1\sqrt{n},\quad b = c_2\sqrt{n},\quad k = c\sqrt{n},\quad c_2 > c > c_1 \quad (c_1, c_2, c$ - musbat o’zgarmas sonlar) bo’lgan holda

$$P\left(S_n = k \big/ N > n\right) \text{ va } P\left(S_n < k \big/ N > n\right)$$

ehtimolliklar uchun asimptotik yoyilmalarning birinchi hadlari 4, 5-paragraflarda keltirilgan.

a va *b* sonlardan faqat bittasi n ga mos ravishda o’sib boradigan holda $P(S_n = k,\quad N = n),\quad k \notin (-a,b)$ ehtimollik uchun $1/\sqrt{n}$ ning darajalari bo’yicha to’la asimptotik yoyilmalar 7-paragrafda keltirib chiqarilgan.

§ 2 Zarur ma'lumotlar.

Quyidagi belgilarni kiritamiz.

$$Q_o(z,\lambda)=\sum_{n=0}^{\infty} z^n \sum_{k=-a+1}^{b-1} \lambda^k p(S_n=k,\ N>n),\ |z|<1,\ |\lambda|=1,$$

$$Q_1(z,\lambda)=\sum_{n=1}^{\infty} z^n \sum_{k=-\infty}^{-a} \lambda^k p(S_N=k,\ N=n),\ |z|\le 1,\ |\lambda|=1,$$

$$Q_2(z,\lambda)=\sum_{n=1}^{\infty} z^n \sum_{k=b}^{b-1} \lambda^k p(S_N=k,\ N>n),\ |z|\le 1,\ |\lambda|=1,$$

$$r_z(\lambda)=1-zh(\lambda).$$

$r_z(\lambda)=r_{z+}(\lambda)\cdot r_{z-}(\lambda)$ - $|\lambda|=1,\ (|z|<1)$ aylanadagi faktorizatsiya bo'lsin. $r_{z+}^{\pm 1}(\lambda)$ funksiyalar $|\lambda|<1$ sohada analitik va chegarada uzluksiz, $r_{z-}^{\pm 1}(\lambda)$ funksiyalar esa $|\lambda|>1$ sohada analitik va chegarada uzluksiz bo'ladi.

$|z|<1,\ |\lambda|=1$ bo'lganda quyidagi tenglama o'rinlidir ([5]):

$$r_z(\lambda)Q_o(z,\lambda)=1-Q_1(z,\lambda)-Q_2(z,\lambda) \tag{2.1}$$

Bu tenglamadan $Q_1(z,\lambda)$ va $Q_2(z,\lambda)$ funksiyalarni topish uchun quyidagi tenglamalar hosil qilinadi:

$$Q_1(z,\lambda)=r_z(\lambda)\left[r_{z-}^{-1}(\lambda)\right]^{(-\infty,-a)}-r_{z-}^{(\lambda)}\left[r_z^{-1}(\lambda)r_{z+}(\lambda)\left[r_{z+}^{-1}(\lambda)\right]^{[b,\infty)}\right]^{(-\infty,-a]}+$$

$$+r_{z-}(\lambda)\left[r_z^{-1}(\lambda)r_{z+}(\lambda)\left[r_{z+}^{-1}(\lambda)Q_1(z,\lambda)\right]^{[b,\infty)}\right]^{(-\infty,-a]} \tag{2.2}$$

$$Q_2(z,\lambda)=r_{z+}(\lambda)\left[r_{z+}^{-1}(\lambda)\right]^{[b,\infty)}-r_{z+}^{(\lambda)}\left[r_{z+}^{-1}(\lambda)\left[r_{z-}(\lambda)\right]^{(-\infty,-a]}\right]^{[b,\infty)}+$$

$$+r_{z-}(\lambda)\left[r_{z+}^{-1}(\lambda)r_{z-}(\lambda)\left[r_{z-}^{-1}(\lambda)Q_2(z,\lambda)\right]^{(-\infty,-a]}\right]^{[b,\infty)}. \tag{2.3}$$

Bu yerda tarif bo'yicha

$$\left[\sum_{k=-\infty}^{\infty}\lambda^k fk\right]^A=\sum_{k\in A}\lambda^k f_k,\quad |\lambda|=1$$

Faktorizatysiya komponentlari $r_{z\pm}(\lambda)$ funksiyalarning [1] dan ma'lum bo'lgan ba'zi xossalarni keltiramiz.

$\lambda_1(z),\ \lambda(z)$ $r_z(\lambda)$ funksiyaning $z=1$ nuqtaning qandaydir δ - atrofida aniqlangan nollari bo'lsin, $\lambda_1(1)=\lambda(1)=1$ va $z\in(1-\delta,1)$ da $\lambda_1(z)<1<\lambda(z)$. Shunday $\delta_1>0$ mavjudki, musbat komponenta $r_{z+}(\lambda)$ $|\lambda|<1+\delta_1$ da analitik, chegarada uzluksiz va $|\lambda|<1+\delta_1$ sohada

$\lambda(z)$ uning yagona noli bo'ladi. $\vartheta_z^{\pm 1}(\lambda)=\left(\dfrac{r_z+(\lambda)}{\lambda-\lambda(z)}\right)^{\pm 1}$ funksiyalar $|\lambda|<1+\delta_1$ sohada analitik, chegarada uzluksiz bo'ladi, $|z-1|<\delta$. Manfiy komponenta $r_{z-}(\lambda)$ $|\lambda|>1-\delta_1$ da analitik, chegarada uzluksiz, $u_z^{\pm 1}(\lambda)=\left(\dfrac{\lambda r_{z-}(\lambda)}{\lambda-\lambda_1(z)}\right)^{\pm 1}$ funksiyalar $|\lambda|>1-\delta_1$ da analitik, chegarada uzluksiz bo'ladi, $|z-1|<\delta$.

$z\in D\delta=\{|z-1|<\delta\}-\{z\geq 1\}$ sohada quyidagi yoyilmalar o'rinlidir:

$$\lambda_1(z)=1+1i(z-1)^{1/2}+\psi_2 i^2(z-1)+.....,$$

$$\lambda(z)=1-\psi_1 i(z-1)^{1/2}+\psi_2 i^2(z-1)-.....\,, \qquad \psi_1=\sqrt{\frac{2}{D\xi_1}}>0\,.$$

Lemma 2.1. $z\in D_\delta=\{|z-1|<\delta\}-\{z\geq 1\},\ |\lambda|=1$ *da*

$G_z(\lambda)=\sum\limits_{k=-\infty}^{\infty}\lambda^k g_k(z)$ *va* $G_z(\lambda)(\lambda-\lambda(z))(\lambda-\lambda_1(z))$ *funksiya* $1-\delta_1<|\lambda|<1+\delta_1,\ \ |z-1|<\delta$ *sohada analitik bo'lsin. U holda qandaydir* $\gamma>0$ *da* $z\in D_\delta$ *bo'yicha tekis*

$$g_k(z)=\lambda^{-k-1}(z)\left[(\lambda(z)-\lambda)G_z(\lambda)\right]_{\lambda=\lambda(z)}+O\left(e^{-\gamma k}\right),\ \ k\to\infty,$$

$$g_k(z)=\lambda_1^{-k-1}(z)\left[(\lambda-\lambda_1(z))G_z(\lambda)\right]_{\lambda=\lambda(z)}+O\left(e^{\gamma k}\right),\ \ k\to-\infty,$$

tengliklar o'rinli va $g_k(z)$ *funksiyalar uchun* D_δ *da*

$$g_k(z)=\sum_{\gamma=-1}^{\infty}g_k^{(j)}i^j(z-1)^{j/2},\qquad k=0,1,2,....$$ *yoyilmalar o'rinli bo'ladi.*

Lemma 2.2. *([3]).* $|\lambda|=1$ *da* $f(\lambda)=\sum\limits_{k=-\infty}^{\infty}f_k\lambda^k$ *bo'lsin. Agar* $f(\lambda)$ *funksiya* $1<|\lambda|<1+\delta_1$ *da analitik va chegarada uzluksiz bo'lsa,*

$$|f_k|\leq \sup_{|\lambda|=1+\delta}{}^{|f(\lambda)|}\Big/_{(1-\delta_1)^{k+1}},\qquad k\geq 0$$ *bo'ladi.*

Agar $f(\lambda)$ funksiya $1-\delta_1<|\lambda|<1$ da analitik va chegarada uzluksiz bo'lsa,

$$|f_k|\leq \sup_{|\lambda|=1-\delta_1}{}^{|f(\lambda)|}\Big/_{(1-\delta_1)^{k-1}},\qquad k\leq 0$$ bo'ladi.

Bu lemmalarning isboti [2] da keltirilgan. Quyidagi belgilashlarni kiritamiz:

$$T_k(z)=\sum_{n=1}^{\infty}z^nP(S_N=k,\ N=n)\qquad k\notin(-a,b),$$

$$T_k(z)=\sum_{n=1}^{\infty}z^nP(S_n=k,\ N>n)\qquad k\in(-a,b).$$

Teorema: 2.1. *([3] ga qarang)* $k\in[a,b)$ *o'zgarmas son bo'lsin. U holda yetarli katta a va b lar uchun* $z\in\lambda_\delta=\{|z-1|<\delta,\ \ |z|<1\}$ *da*

$$T_k(z)= fk(z)-\frac{\mu^b(z)\cdot\left(1-H_1(z)\mu^a(z)\right)}{\left(\lambda_1(z)-\lambda(z)\right)u_z\left(\lambda_1(z)\right)\vartheta_z\left(\lambda(z)\right)\lambda_1^k(z)\left(1-H_2(z)\mu^{b+a}(z)\right)}-$$

$$-\frac{\mu^a(z)\left(1-H_3(z)\mu^b(z)\right)}{\left(\lambda_1(z)-\lambda(z)\right)u_z\left(\lambda_1(z)\right)\vartheta_z\left(\lambda(z)\right)\lambda^k(z)\left(1-H_2(z)\mu^{b+a}(z)\right)}+\frac{\varepsilon(z)}{(z-1)^{1/2}}$$

tenglik o'rinlidir. Bu yerda $\varepsilon(z)$ *funksiya* λ_δ *da analitik,* $|\varepsilon(z)|\le c\left(e^{-\gamma a}+e^{-\gamma b}\right)$,

$$r_z^{-1}(\lambda)=\sum_{k=-\infty}^{\infty}\lambda^k f_k(z),\qquad (\lambda)=1,\qquad \mu(z)=\frac{\lambda_1(z)}{\lambda(z)},\qquad H_1(z)=\frac{u_z(\lambda(z))}{u_z(\lambda_1(z))},$$

$$H_2(z)=\frac{v_z(\lambda_1(z))H_1(z)}{v_z(\lambda(z))},\qquad H_3(z)=\frac{\vartheta_z(\lambda_1(z))}{\vartheta_z(\lambda(z))}.$$

$|z-1|\ge\delta,\ |z|=1$ *bo'lganda* $\left|(z-1)^{1/2}\left(T_k(z)-f_k(z)\right)\right|c\left(e^{-\gamma a}+e^{-\gamma b}\right),\quad \gamma>0$ *bo'ladi.*

Endi qator koiffitsentlarini hisoblash uchun zarur va [2], [3] dan ma'lum bo'lgan dovon usuli haqida ba'zi ma'lumotlarni keltiramiz.

Qandaydir $\delta>0$ son uchun $z\in\lambda_\delta$ da

$$u(z)=\frac{F(z)\lambda_1^{k_1a+k_2b}(z)\left(1-H_3(z)\mu^b(z)\right)}{\lambda^{k_3a+k_4b}(z)\left(1-H_2(z)\mu^{b+a}(z)\right)}+\frac{\varepsilon(z)}{(z-1)^{1/2}}\qquad (2.4)$$ bo'lsin.

Bu yerda

$$F(z)=\sum_{k=-1}^{\infty}f_k i^k(z-1)^{k/2},\qquad \varepsilon(z)=\sum_{k=0}^{\infty}\varepsilon_k i^k(z-1)^{k/2},$$

$k_i,\ i=1,2,3,4$ - o'zgarmas sonlar.

$$J(n)=\frac{1}{2\pi i}\int_\Gamma u(z)z^{-z-1}dz$$

belgilash kiritamiz. Bu yerda $\Gamma-|z|=1,\ |z-1|<\delta$ yoyning $z=1$nuqta atrofida λ_δ uchiga egilishidan hosil bo'lgan kontur.

$$h_{1s}\left(i(z-1)^{1/2}\right)=(\tau_1 s+\tau_2 s)\ln\mu(z)+(k_1\tau_1+k_2\tau_2)\ln\lambda_1(z)-$$

$$-(k_3\tau_1+k_4\tau_2)\ln\lambda(z)+s\tau_3\ln H_2(z)-\ln z,$$

$$h_{2s}\left(i(z-1)^{1/2}\right)=h_{1s}\left(i(z-1)^{1/2}\right)+\tau_2\ln\mu(z),\qquad 0\le s\le M,$$

$$M-1=\left[\tau_0/\max(\tau_1,\tau_2)\right],\quad \tau_0-\textit{yetarli kichik son}.$$

Nol nuqta atrofida

$$h_{js}(t)=h_{js}(t,\tau_1,\tau_2,\tau_3)=\psi_1 t\left[(25+k_1+k_3)\tau_1+\left(2(s+j-1)+k_2+k_4\right)\tau_2+s\eta_1\tau_3\right]+$$

$$+\frac{\tau^2}{2}\left[1+D\left(s(\tau_1+\tau_1)\right)\right]+O\left(t^3\right),\qquad j=1,2.$$

$t_{js} - h_{js}^1(t) = 0$ tenglamaning $0 \le s \le M$ da τ_1, τ_2 *va* τ_3 darajalari bo'yicha nol nuqta atrofida yoyilmalarga ega bo'lgan ildizlari:

$$t_{js} = -\psi_1\left(s + \frac{k_1 + k_3}{2}\right)\tau_1 - \psi_1\left(s + j - 1 + \frac{k_2 + k_4}{2}\right)\tau_2 - \frac{s\eta_1}{2}\tau_3 + O\left(s^2(\tau_1 + \tau_2)^2\right)$$

$$h_{js}(t_{js}) = -\Big|\psi_1\left(s + \frac{k_1 + k_3}{2}\right)\tau_1 - \psi_1\left(s + j - 1 + \frac{k_2 + k_4}{2}\right)\tau_2 -$$

$$-\frac{s\eta_1}{2}\tau_3 + O\left(s^3(\tau_1 + \tau_2)^3\right)\Big|,$$

$$h_{js}(t_{js}) = 2 + O(s(\tau_1 + \tau_2))$$

$$H_2(z) = 1 + \sum_{k=1}^{\infty} \eta_k i^k (z-1)^{k/2}, \qquad \frac{a}{n} = \tau_1, \ \frac{b}{n} = \tau_2, \qquad \frac{1}{n} = \tau_3,$$

$$Q_{jsi} = \frac{(-1)^{1/2}}{2\pi} \sum_{r=0}^{2i} q_{2i,r}^{j,s} \left(-h_{js}^{(0)}\right)^{-r-i-1/2} \Gamma\left(r + i + 1/2\right),$$

$q_{i,r}^{j,s}$ - quyidagi yoyilmada z^i oldidagi koeffitsenti:

$$\frac{1}{r!}\left(a_{js}^{(0)} + a_{js}^{(1)}(z) + \ldots\right)\left(\overset{(1)}{h}_{js}(z) + \overset{(2)}{h}_{js} Z^2 + \ldots\right)^2,$$

$$a_{js}^{(k)} = \frac{a_i^{(k)}(t_{js})}{k!}, \qquad h_{js}^{(k)} = \frac{1}{(k+1)!} h_{js}^{(k+2)}(t_{js}), \qquad j = 1,2 \qquad k = 0,1,2,\ldots,$$

$$a_1(t) = -2tF(1-t^2)(1-t^2)^{-1}, \qquad a_2(t) = a_1(t)H_3(z), \qquad (z-1)^{1/2} \qquad i = t$$

Teorema 2.2. *([2]).* $a = c_1\sqrt{n}, \ b = c_2\sqrt{n}, \ k \in (-a, b]$ *bo'lsin. U holda ixtiyoriy* $q \ge 1$ *uchun*

$$J(n) = \sum_{i=0}^{q-1} \frac{1}{n^{i+1/2}} \sum_{r=0}^{\infty} \frac{1}{n^{r/2}} P_{ri}(k_1, k_2, k_3, k_4, F) + O\left(\frac{1}{n^{q+1/2}}\right),$$

bunda

$$P_{ri}(k_1, k_2, k_3, k_4, F) = \sum_{S=0}^{\infty} \left(q_r^{1,i}(s) lxp\right)\left\{-\psi_1^2\left[c_1\left(s + \frac{k_1 + k_3}{2}\right)c_2\left(s + \frac{k_2 + k_4}{2}\right)\right]^2\right\} -$$

$$-q_2^{2,i}(s)\exp\left\{-\psi_1^2\left[c_1\left(s + \frac{k_1 + k_3}{2}\right) + c_2\left(s + 1 + \frac{k_2 + k_4}{2}\right)\right]\right\}\Bigg),$$

$q_r^{j,i}(s)$ *koeffitsentlar quyidagi yoyilmadan aniqlanadi:*

$$Q_{jsi} \exp\left\{nh_{js}(t_{js}) + \psi_1^2\left[c_1\left(s + \frac{k_1 + k_3}{2}\right) + c_2\left(s + j - 1 + \frac{k_2 + k_4}{2}\right)\right]^2\right\} = \sum_{r=0}^{\infty} \frac{1}{n^{r/2}} q_r^{j,i}(s).$$

Agar $U(z) = \dfrac{F(z)\lambda_1^{k_1 a + k_2 b}(z)}{\lambda^{k_3 a + k_4 b}(z)}$ *bo'lsa, ixtiyoriy* $q \ge 1$ *uchun*

$$J(n)=\sum_{i=0}^{q-1}\frac{1}{n^{i+1/2}}\quad\sum_{r=0}^{\infty}\frac{1}{n^{r/2}}\,Qri(k_1,k_2,k_3,k_4)+O\left(\frac{1}{n^{q+1/2}}\right)\ \textit{bo'ladi.}$$

Bu yerda $t_0=-\frac{\psi_1}{2}(k_1+k_3)\tau_1-\frac{\psi_1}{2}(k_2+k_4)\tau_2+O\left((\tau_1+\tau_2)^2\right),$

$$h_0(t_0)=-\left[\frac{\psi_1}{2}(k_1+k_3)\tau_1-\frac{\psi_1}{2}(k_2+k_4)\tau_1\right]+O\left((\tau_1+\tau_2)^3\right),$$

$$e^{nh_0(t_0)}Q_{10i}=\sum_{r=0}^{\infty}\frac{1}{n^{r/2}}Qri(k_1,k_2,k_3,k_4,F),\ \ i=0,2,3....\ .$$

Agar (2.4) da $H_3(z)$ o'rniga $H_1(z)$ ni qo'ysak, $a_2\left(i(z-1)^{1/2}\right)=a_1\left(i(z-1)^{1/2}\right)H_1(z)$ bo'ladi.

Agar

$$u(z)=\frac{F(z)\cdot\lambda_1^{k_1a_1+k_2b}(z)\left(1-\mu^a(z)\cdot H_3(z)\right)}{\lambda^{k_3a+k_4b}(z)\left(1-\mu^{b+a}(z)H_2(z)\right)}$$

bo'lsa, u holda

$$t_{js}=-\psi_1\left(s+j-1+\frac{k_1+k_3}{2}\right)\tau_1-\psi_1\left(s+\frac{k_1+k_3}{2}\right)\tau_2\,\frac{s\eta_1}{2}\tau_3+O\left(s^2(\tau_1+\tau_2)^2\right)$$

bo'ladi.

Endi $\phi(z)=\sum_{k=0}^{\infty}\varphi_k i^k(z-1)^{k/2}$ bo'lganda

$$J_1(n)=\frac{1}{2\pi i}\int_{\Gamma}\frac{\phi(z)\cdot\lambda_1^{k_1a+k_2b}(z)\cdot\left(1-\mu^b(z)\right)dz}{\lambda^{k_3a+k_4b}(z)Z^{n+1}\left(1-\mu^{b+a}(z)H_2(z)\right)(1-z)}$$

integralni qaraymiz.

Teorema 2.3 *([2])*

$$J_1(n)=-2\Phi(1)\left[\sum_{S=0}^{M-1}\left(\Phi\left(|t_{2s}|\sqrt{2n\overset{(0)}{h_{2s}}}\right)-\left(|tis|\sqrt{2n\overset{(0)}{h_{1s}}}\right)\right)\right]+O\left(\frac{1}{\sqrt{n}}\right)$$

$$\frac{1}{2\pi i}\int_{\Gamma}\frac{F_1(z)\cdot\lambda_1^{k_1a+k_2b}(z)dz}{\lambda^{k_3a+k_4b}(z)z^{n+1}(1-z)}=2F_1(1)\Phi\left(|t_0|\sqrt{2n\overset{(0)}{h_o}}\right)+O\left(\frac{1}{\sqrt{n}}\right).$$

Bu yerda ham $F_1(z)=\sum_{k=0}^{\infty}f_k^{(1)}i^k(z-1)^{k/2}$, $\qquad\Phi(t)=\frac{1}{\sqrt{2\pi}}\int_t^{\infty}e^{-\frac{x^2}{2}}dx.$

Eslatib o'tamiz

$$\left|\int_{\Gamma}\frac{\varepsilon(z)dz}{i(z-1)^{1/2}z^{n+1}}\right|=O\left(e^{-\gamma k(n)}\right),\quad k(n)\to\infty,\ \ n\to\infty\,.$$

§ 3. Aniq formulalar.

$$P(\xi_1=-k)=q_2p_2^{k-1},\quad P(\xi_1=0)=q,\quad P(\xi_1=k)=q_1p_1^{k-1},\ k=1,2,...,$$

ya'ni ξ_1 tasodifiy miqdor ikki tomonlama geometrik taqsimotga ega bo'lsin. $p_2<|\lambda|<1/p_1$ bo'lganda

$$h(\lambda)=M\lambda^{\xi_1}=\frac{q_2}{\lambda-p_2}+q+\frac{\lambda q_1}{1-\lambda p_1},\quad M\left(\lambda^{\xi_1},\xi_1<0\right)=\frac{q_2}{\lambda-p_2}.$$

$0<|z|<1,\ |\lambda|=1$ da $|zh(\lambda)|<1$ bo'lganligi uchun $r_z(\lambda)=1-zh(\lambda)$ funksiya argumentining $|\lambda|=1$ dagi orttirmasi nolga teng va $|\lambda|<1$ da $r_z(\lambda)$ yagona nolga ega bo'ladi.

$$r_z(\lambda)=1-z\left(\frac{q_2}{\lambda-p_2}+q+\frac{\lambda q_1}{1-\lambda p_1}\right)=\frac{a(z)(\lambda-\lambda_1(z))(\lambda-\lambda(z))}{(\lambda-p_2)(1-\lambda p_1)},$$

bu yerda $a(z)=z(q_1-p_1q)+p_1$,

$$\lambda_{\cdot,1}(z)=\frac{\left(z\left(pq_2-q-qp_1p_2+p_2q_1\right)+p_1p_2+1\right]\pm\sqrt{[\ldots]^2-4(zq_1-zp_1q+b)(zq_2-zp_2q+p_2)}}{2(zq_1-zp_1q+p_1)}$$

$r_{z+}(\lambda)=\dfrac{-a(z)(\lambda-\lambda(z))}{1-p_1\lambda}$ funksiya $r_z(\lambda)$ funksiyaning $|\lambda|=1$ dagi faktorizatsiyaning musbat komponentasi, $r_z(-\lambda)=\dfrac{\lambda-\lambda_1(z)}{\lambda-p_2}$ funksiya esa manfiy komponentasi bo'ladi.

Endi § 2 dagi (2.1) tenglamaga qaytamiz. $|\lambda|=1,\ |z|<1$ bo'lganda

$$\frac{1}{\lambda-\lambda_1(z)}=\sum_{k=0}^{\infty}\frac{\lambda_1^{\,k}(z)}{\lambda^{k+1}},\quad \frac{1}{\lambda(z)-\lambda}=\sum_{k=0}^{\infty}\frac{\lambda^k}{\lambda^{k+1}(z)}.$$

Oddiy amallar yordamida quydagilarni osongina tekshirish mumkin:

$$r_{z-}(\lambda)\left[r_{z-}^{-1}\lambda\right]^{(-\infty,-a]}=\frac{\lambda_1^{a-1}(z)(\lambda_1(z)-p_2)}{\lambda^{a-1}(\lambda-p_2)}\qquad(3.1),$$

$$r_{z+}(\lambda)\left[r_{z+}^{-1}\lambda\right]^{(b,\infty]}=\frac{\lambda^b(1-p\lambda(z))}{\lambda^b(z)(1-p_1\lambda)},$$

$$\left[r_{z^+}^{-1}(\lambda)Q_1(z,\lambda)\right]^{[b,\infty)} = \frac{1}{a(z)} \cdot \left[\frac{1-p_1\lambda}{\lambda-\lambda(z)}Q_1(z,\lambda)\right]^{[b,\infty)} =$$

$$= -\frac{1}{a(z)\lambda(z)}\left[\sum_{k=0}^{\infty}\frac{\lambda^k}{\lambda^k(z)}\sum_{k=-\infty}^{-a}\lambda^k T_k(z)\lambda^k\right]^{[b,\infty)} +$$

$$+\frac{P_1\lambda}{\lambda(z)}\left[\sum_{k=o}^{\infty}\frac{\lambda^k}{\lambda^k(z)}\sum_{k=-\infty}^{-a}\lambda^k T_k(z)\right]^{[b,\infty)}$$

$$= -\frac{1}{a(z)\lambda(z)}\left\{T_{-a}(z)\lambda^{-a}\sum_{k=b+a}^{\infty}\frac{\lambda^k}{\lambda^k(z)} + T_{-a-1}(z)\lambda^{-a-1}\sum_{k=b+a+1}^{\infty}\frac{\lambda^k}{\lambda^k(z)} + \ldots.\right\} +$$

$$+\frac{P_1\lambda}{a(z)\lambda(z)}\left\{T_{-a}(z)\lambda^{-a}\sum_{k=b+a-1}^{\infty}\frac{\lambda^k}{\lambda^k(z)} + T_{-a-1}(z)\lambda^{-a-1}\sum_{k=b+a}^{\infty}\frac{\lambda^k}{\lambda^k(z)} + \ldots..\right\} +$$

$$= \frac{\lambda^b Q_1(z,\lambda(z))(1-p_1\lambda)z)}{a(z)\cdot\lambda^b(z)(\lambda(z)-\lambda)}.$$

$\left[\lambda^n f(\lambda)\right]^{(-\infty,-a]} = \lambda^n\left[f(\lambda)\right]^{(-\infty,-a-n]}$ bo'lgani uchun

$$r_{z-}(\lambda)\left[r_z^{-1}(\lambda)z_z\left[{}_{z+}^{-1}(\lambda)Q_1(z,\lambda)\right]^{[b,\infty)}\right]^{(-\infty,-a]} =$$

$$= \frac{\lambda-\lambda_1(z)}{\lambda-P_2}\cdot\left[\frac{\lambda-p_2}{\lambda-\lambda_1(z)}\frac{\lambda^b Q_1(z,\lambda(z))(1-p_1\lambda(z))}{\lambda^b(z)\cdot(1-p_1\lambda)}\right]^{(-\infty,-a]} =$$

$$= \frac{(\lambda-\lambda_1(z))\lambda^b(1-p_1\lambda(z))Q_1\cdot(z,\lambda(z))}{(\lambda-P_2)\lambda^b(z)}\cdot\left[\frac{\lambda-p_2}{(\lambda-\lambda_1(z))(1-p_1\lambda)}\right]^{(-\infty,-a-b]} =$$

$$= \frac{\lambda-\lambda_1(z)}{\lambda-p_2}\cdot\frac{(1-p_1\lambda(z))Q_1(z,\lambda(z))}{\lambda^b(z)}\frac{\lambda_1(z)-p_2}{1-p_1\lambda_1(z)}\left[\frac{1}{\lambda-\lambda_1(z)}\right]^{(-\infty,-b-a)} =$$

$$= \frac{\lambda_1^{\,b+a-1}(z)(1-p_1\lambda(z))(\lambda_1(z)-p_2)}{\lambda^{a-1}\lambda^b(1-P_1\lambda_1(z))(\lambda-P_2)}\cdot Q_1(z,\lambda(z)) \qquad (3.2)$$

Huddi shunga o'xshash

$$r_{z-}(\lambda)\left[r_{z-}^{1-}(\lambda)\cdot r_{z+}(\lambda)\left[r_{z+}^{-1}(\lambda)\right]^{[b,\infty)}\right]^{(-\infty,-a)} = \frac{\lambda_1^{b+a-1}(z)(1-p_1\lambda(z))(\lambda_1(z)-p_2)}{\lambda^{a-1}\lambda^b(z)(1-p_1\lambda_1(z))(\lambda-p_2)} \qquad (3.3)$$

(3.2) va (3.3) larni (2.1) ga qo'yib $Q_1(z,\lambda)$ ni topish uchun tenglama hosil qilamiz.

Quyidagi belgilashlarni kiritsak,

$$\frac{\lambda_1(z)-p_2}{\lambda(z)-p_2}=H_1(z),\quad \frac{1-p_1\lambda(z)}{1-p_1\lambda_1(z)}=H_{3z},\quad H_1(z)\cdot H_3(z)=H_2(z),\ \mu(z)=\frac{\lambda_1(z)}{\lambda(z)},$$

$\lambda=\lambda(z)$ da

$$Q_2(z,\lambda(z))=\frac{\mu^{a-1}(z)H_1(z)-\mu^{b-a-1}(z)H_2(z)}{1-\mu^{b+a-1}(z)H_2(z)}$$ tenglikni hosil qilamiz.

Natijada

$$Q_2(z_1\lambda)=\frac{\lambda_1{}^{a-1}(z)\cdot(\lambda_1(z)-p_1)}{\lambda^{a-1}(\lambda-p_2)}\left[\frac{1-\mu^b(z)+H_3(z)}{1-^{b+a-1}(z)H_2(z)}\right]$$

tenglik hosil bo'ladi. Quyidagi teorema isbot bo'ldi.

Teorema 3.1. $|z|<1,\quad k\le 0$ *bo'lganda*

$$T_{k-a}(z)=\sum_{n=1}^{\infty}z^nP(S_N=k-a,\ N=n)=\frac{1}{p_2^k}\frac{\lambda_1^{a-1}(\lambda_1(z)-p_2)(1-\mu^b(z)H_3(z))}{1-\mu^{b+a-1}(z)H_2(z)}$$

bo'ladi.

Huddi shu usul bilan $M(\lambda^{\xi_1},\xi_1<0)$ va $M(\lambda^{\xi_1},\xi_1>0)$ ratsional funksiyalar bo'lganda $T_k(z)$ uchun aniq ifodalar topish mumkin. Lekin bunda o'ta murakkab belgilashlar va hisoblashlar qilishga to'g'ri keladi.

Lemma 3.1 $q_1=q_2,\ p_1=p_2=p$ *bo'lganda* $T_{k-a}(z)$ *funksiya z ning ratsional funksiyasi bo'ladi.*

Isboti: Bu holda

$$\lambda_{\cdot,1}(z)=\frac{2zpq_1-zq-zp^2q+1+p^2}{2(zq_1-zpq+p)}\pm\sqrt{\frac{(2zpq_1-zq-zp^2q+1+p^2)^2}{4(zq_1-zpq+p)^2}-1}$$

$$\lambda_1(z)\cdot\lambda(z)=1,$$

$$T_{k-a}(z)=\frac{1}{p^k}\frac{[1+p^2-p(\lambda_1(z)+\lambda(z))][\lambda^2(z)-\lambda_1^2(z)-p(\lambda^{b-1}(z)-\lambda_1^{b-1}(z))]}{\lambda^{b+a}(z)-\lambda_1^{b+a}(z)-2p(\lambda^{b+a-1}(z))+p^2(\lambda^{b+a-2}(z)-\lambda^{b+a-2}(z))},$$

$\lambda_{\cdot,1}(z)$ *quyidagi ko'rinishiga ega bo'ladi:*

$$\lambda_1(z)=\frac{\chi(z)-\sqrt{Y(z)}}{Z(z)},\qquad \lambda(z)=\frac{\chi(z)+\sqrt{Y(z)}}{Z(z)}$$

va bu yerda $X(z),Z(z)$ - z ning chiziqli funksiyalari, $Y(z)$ esa kvadrat uchxaddir.

U holda

$$\lambda^n(z)-\lambda_1^n(z)=\frac{1}{Z_1^n(z)}\Big[\chi^n(z)+c_n^1\chi^{n-1}(z)\sqrt{Y(z)}+c_n^2\chi^{n-2}(z)Y(z)+\ldots+c_n^nY^{n/2}(z)-$$

$$-\chi^{-n}(z)+_n^1\chi^{n-1}\sqrt{Y(z)}-_n^2\chi^{n-2}(z)\cdot Y(z)+\ldots+(-1)^{n-1}Y^{\frac{n}{2}}(z)\Big]=\frac{\sqrt{Y(z)}Q_{n-1}(z)}{Z_1^n(n)},$$

$$T_{k-a}(z)=\frac{Q_{b+a-1}^1(z)}{Q_{b+a-1}(z)}.$$

Bu yerda $Q_m^1(z),Q_m(z)$ - m – darajali ko'phad.

Lemma isbotlanadi.

Quyidagi almashtirishlarni bajaramiz ([6]):

$$\frac{2zpq_1 - zq - zp^2q + 1 + p^2}{2(zq_1 - zpq + p)} = \cos\varphi, \quad z = \frac{1 + p^2 - 2p\cos\varphi}{2\cos\varphi(q_1 - pq) - (2pq_1 - q - p^2q)}, \qquad 3.4$$

Bu yerda $\cos\varphi$ funksiya kompleks o'zgaruvchining funksiyasi sifatida qaraladi. U holda

$$\lambda_1(z) = \cos\varphi - i\sin\varphi, \quad \lambda(z) = \cos\varphi + i\sin\varphi,$$

$$T_{k-a}(z) = \frac{1}{p^k}\frac{(1 - 2p\cos\varphi + p^2)(\sin b\varphi - p\sin(b-1)\varphi)}{\sin(b+a)\varphi - 2p\sin(b+a-1)\varphi + p^2\sin(b+a-2)\varphi}.$$

$\varphi_1, \varphi_2, ..., \varphi_m$ lar $T_{k-a}(z)$ funksiyaning qutblari bo'lsin, $0 < \varphi_1 < \pi$. Bu qutblar sodda qutblar ekanligiga ishonch hosil qilish qiyin emas va agar

$$A = \{\varphi : \sin(\varphi b) - p\sin(b-1)\varphi = 0, \qquad 0 < \varphi < \pi\}$$

$$B = \{\varphi : \sin(b+a)\varphi - 2p\sin(b+a-1)\varphi + p^2\sin(b+a-2)\varphi = 0, \qquad 0 < \varphi < \pi\}$$

bo'lsa, u holda

$$C = \{\varphi_1, \varphi_2, ..., \varphi_m\} = B \setminus A, \qquad m \le b + a - 1 \text{ bo'ladi.}$$

(3.4) akslantirishlar o'zaro bir qiymatli bo'lgani uchun $0 \le \varphi < \pi$ da har bir φ_i ga bittadan z_i mos keladi.

$T_{k-a}(z)$ ratsional funksiya bo'lgani uchun, $|z| < 1$ da

$$T_{k-a}(z) = \frac{1}{p^k}\frac{(1 - 2p\cos\varphi + p^2)(\sin b\varphi - p\sin(b-1)\varphi)}{\sin(b+a)\varphi - 2p\sin(b+a-1)\varphi + p^2\sin(b+a-2)\varphi} =$$

$$= \rho_0 + \frac{\rho_1}{z_1 - z} + ... + \frac{\rho_m}{z_m - z}$$

yoyilma o'rinli bo'ladi. U holda

$$\rho_i = \lim_{z \to zi}(t_i - z)T_{k-a}(z) = \frac{2}{p^k[2(q_1 - pq)\cos\varphi_i - (2pq_1 - q - p^2q)]^2} *$$

$$* \frac{[p(2pq_1 - q - p^2q)\sin\varphi_i + (q_1 - pq)(1 + p^2)\sin\varphi_i](1 - 2p\cos\varphi_i + p^2)[\sin b\varphi_i - p\sin(b-1)\varphi_i]}{(b+a)\cos(b+a)\varphi_i - 2p(b+a-1)\cos(b+a-1)\varphi_i + p^2(b+a-2)\cos(b+a-2)\varphi_i}.$$

bundan quyidagi teorema isboti kelib chiqadi.

Teorema 3.2. *$k \le 0$ bo'lganda*

$$P(S_N = k-a, \quad N=n)=\sum_{i=1}^{m}\left\{\frac{\left[2(q_1-pq)\cos\varphi_i-\left(2pq_1-q-p^2q\right)\right]^{n-1}}{\left[1-2p\cos\varphi_i+p^2\right]^n}*\right.$$

$$\left.*\frac{\left[p\left(2pq_1-q-p^2q\right)\sin\varphi_i+(q_1-pq)\left(1+p^2\right)\sin\varphi_i\right]\left[\sin b\varphi_i-p\sin(b-1)\varphi_i\right]}{\left[(b+a)\cos(b+a)\varphi_i-2(b+a-1)p\cos(b+a-1)\varphi_i+p^2(b+a-2)\cos(b+a-2)\varphi_i\right]}\right\}.$$

Natija 3.1. $k \le 0$ *bo'lganda*

$$P(S_n = k-a)\frac{1-p}{p^k}\frac{b(1-p)+p}{(b+a-1)(1-p)+p+1},$$

$$P(S_N \le -a)\frac{1-p}{p^k}\frac{b(1-p)+p}{(b+a-1)(1-p)+p+1},$$

$$P(S_N \ge b)\frac{1-p}{p^k}\frac{b(1-p)+p}{(b+a-1)(1-p)+p+1}.$$

II bob. **SHARTLI LOKAL VA INTEGRAL TURDAGI EHTIMOLLIKLAR UCHUN LIMIT TEOREMALAR.**

§ 4. **Lokal turdagi shartli ehtimolliklar uchun limit teoremalar.**

Bu paragrafda $k=c\sqrt{n},\ a=c_1\sqrt{n},\ b=c_2\sqrt{n},\ c_2>c$ holda $P\left(s_n=k \middle/ N>n\right)$ ehtimollik uchun $\frac{1}{\sqrt{n}}$ ning darajalari bo'yicha asimptotik yoyilmaning birinchi hadi topiladi. Soddalik uchun $D\xi_1=1$ deb faraz qilamiz.

Shartli ehtimollikning ta'rifiga ko'ra

$$P\left(s_n=k \middle/ N-n\right)=\frac{P(s_n=k,\quad N>n)}{P(N>n)}.$$

$P(N>n)$ ehtimollik uchun asimptotik yoyilmaning birinchi hadi [7] dan ma'lum va u quyidagicha bo'ladi:

$$P(N>n)=\frac{1}{\sqrt{2\pi}}\sum_{S=-\infty}^{\infty}(-1)^S\int_{sc_2+(s-1)c_1}^{(s+1)c_2+sc_1}e^{-\frac{x^2}{2}}dx+O\left(\frac{1}{\sqrt{n}}\right).$$

Teorema 2.1. dagi $T_k(z)$ uchun lemma 2.1 ni qo'llab, $k=c\sqrt{n}\to\infty,\ n\to\infty$ hol uchun hosil etuvchi funksiyani topamiz.

Teorema 4.1 $z\in Z\delta^1=\{|z|<1,\ |z-1|<\delta\}$ *bo'lganda*

$$T_k(z)=\sum_{n=0}^{\infty} z^n P(S_n=k,\ N>n)=-\frac{1}{\lambda^k(z)}\left\{\frac{1}{\lambda(z) r_z^1(\lambda(z))}\right\}-$$

$$-\frac{\lambda_1^b(z)\left(1-H_1(z)\mu^a(z)\right)}{\lambda^{k+b}(z)\left(1-H_2(z)\mu^{b+a}(z)\right)}\left\{\frac{1}{(\lambda_1(z)-\lambda(z))u_z(\lambda_1(z))\vartheta_z(\lambda(z))}\right\}-$$

$$-\frac{\lambda_1^a(z)\left(1-H_3(z)\mu^b(z)\right)}{\lambda^{k+a}(z)\left(1-H_2(z)\mu^{b+a}(z)\right)}\left\{\frac{1}{(\lambda_1(z)-\lambda(z))u_z(\lambda_1(z))\vartheta_z(\lambda(z))}\right\}+\frac{\varepsilon(z)}{\sqrt{z-1}}\ ,$$

$|z|=1,\ |z-1|\ge\delta$ *bo'lganda esa,*

$$\left|T_k(z)\sqrt{1-z}\right|\le O\left(e^{-\gamma\sqrt{n}}\right),\quad \gamma>0\,.$$

Quyidagi belgilashlarni kiritamiz:

$$\Phi_1(z)=\frac{1}{\lambda(z) r_z^1(\lambda(z))}=\sum_{k=-1}^{\infty}\varphi_k^{(1)}t^k,\ \ t=i(z-1)^{1/2},$$

$$\Phi_2(z)=\left[(\lambda_1(z)-\lambda(z))u_z(\lambda_1(z))\vartheta_z(\lambda(z))\right]^{-1}=\sum_{k=1-}^{\infty}\varphi_k^{(2)}t^k\,.$$

Endi qator koeffisentlarini topish uchun davon usulidan foydalanamiz. Teorema 2.2 dan quyidagilarni hosil qilamiz:

$$J_1\ \frac{1}{2\pi i}\int_{\Gamma}\frac{\Phi_1(z)dz}{\lambda^k(z)^{n+1}}=-\frac{\varphi_{-1}^{(1)}}{\sqrt{\pi}}e^{-\frac{c^2}{2}}\frac{1}{\sqrt{n}}+O\left(\frac{1}{n}\right),$$

bu holda

$$k_1=k_2=k_4=0,\quad k_3=1,\quad t_0=-\frac{\psi_1 c}{2\sqrt{n}}+O\left(\frac{1}{n}\right),$$

$$h_0(t_0)=-\frac{\psi_1^2c^2}{2n}+O\left(\frac{1}{n^{3/2}}\right),\qquad h_0^{11}(t_0)=2+O\left(\frac{1}{\sqrt{n}}\right),$$

$$Q_{00}=Q_{00}(0,0,1,0,\Phi_1)=-\frac{\varphi_{-1}^{(1)}}{\sqrt{\pi}}e-\frac{\psi_1^2c_2}{4},\quad \psi_1=\sqrt{2}\,.$$

Lemma 4.1

$$J_2=\frac{1}{2\pi i}\int_{\Gamma}\frac{\lambda_1^b(z)\left(1-H_1(z)\mu^a(z)\right)\Phi_2(z)}{\lambda^{b+k}(z)\left(1-H_2(z)\mu^{b+a}(z)\right)z^{n+1}}dz=P_{00}^1\cdot\frac{1}{\sqrt{n}}+O\left(\frac{1}{n}\right),$$

$$P_{00}^{1}=-\frac{\varphi_{-1}^{(2)}}{\sqrt{\pi}}\sum_{S=0}^{\infty}\left(e^{-2\left[sc_2s+c_1s+c_2-\frac{c}{2}\right]^2}-e^{-2\left[c_2s+c_1s+c_2-\frac{c}{2}\right]^2}\right)$$

Lemmaning isboti Teorema 2.2 dan kelib chiqadi.

Bunda

$$k_1-1-\frac{c}{c_2},\quad k_2=k_4=0,\quad k_3=1,\quad t_{1S}=-\psi\left(s+\frac{2c_2-c}{2c_2}\right)\frac{c_2}{\sqrt{n}}-\psi_1 s\frac{c_1}{\sqrt{n}}-\frac{s\eta_1}{2n}+O\left(\frac{s^2}{n}\right),$$

$$t_{2S}=-\psi_1\left(s+\frac{2c_2-c}{2c_2}\right)\frac{c_2}{\sqrt{n}}-\psi_2(s+1)\frac{c_1}{\sqrt{n}}-\frac{s\eta_1}{2n}+O\left(\frac{s^2}{n}\right),\quad \psi_1=\sqrt{2}\,,\quad h_{js}^{11}(t_{js})=2+O\left(\frac{s}{\sqrt{n}}\right),$$

$$Q_{1S_0}=-\frac{1}{\sqrt{\pi}}\sum_{k=0}^{\infty}\varphi_{k-1}^{(2)}t_{iS}^{k}\cdot\sum_{k=0}^{\infty}t_{1S}^{2k}+O\left(\frac{s}{\sqrt{n}}\right);\quad q_0^{j,0}(S)=-\frac{\varphi_{-1}^{(1)}}{\sqrt{n}},\quad j=1,2\,.$$

Bu qiymatlarni Teorema 2.2 ning tasdiq qismida qo'yib, talab qilingan natijaga keltiramiz.

Huddi shunga o'xshash ushbu lemma isbot qilinadi.

Lemma 4.2

$$J_3=\frac{2}{2\pi i}\int_{\Gamma}\frac{\lambda_1^{a}(z)\left(1-H_3(z)\mu^{b}(z)\right)\Phi_2(z)}{\lambda^{k+a}(z)\left(1-H_2(z)\mu^{b+a}(z)\right)z^{n+1}}\,dz=P_{00}\frac{1}{\sqrt{n}}+O\left(\frac{1}{n}\right),$$

$$P_{00}=P_{00}\left(1,0,0,1+\frac{c}{c_2},\Phi_2\right)=-\frac{\varphi_{-1}^{(1)}}{\sqrt{\pi}}\cdot\sum_{S=0}^{\infty}\left[e^{-2\left[c_1s+c_2s+\frac{c_1}{2}+\frac{c_2}{2}+\frac{c}{2}\right]^2}-e^{-2\left[c_1s+c_2s+\frac{c_1}{2}+\frac{3c_2}{2}+\frac{c}{2}\right]^2}\right]$$

Teorema 4.2 $\quad a=c_1\sqrt{n},\ b=c_2\sqrt{n},\ k=c\sqrt{n},\ D\xi_1=1$

bo'lganda

$$P(S_n=K/N>n)=C_1\frac{1}{\sqrt{n}}+O\left(\frac{1}{n}\right),$$

$$C_1=\left\{\frac{\varphi_{-1}^{(1)}}{\sqrt{\pi}}e^{-C^2}+\frac{\varphi_{-1}^{(2)}}{\sqrt{\pi}}\sum_{S=0}^{\infty}\left[e^{-2\left[c_1s+c_2s+\frac{c_1}{2}+\frac{c_2}{2}+\frac{c}{2}\right]^2}-e^{-2\left[c_1s+c_2s+\frac{c_1}{2}+\frac{3c_2}{2}+\frac{c}{2}\right]^2}e^{-2\left[c_1s+c_2s+c_2-\frac{c}{2}\right]^2}e^{-2\left[c_1s+c_2s+c_2+c_1\right]^2}\right]\right\}*$$

$$*\left\{\frac{1}{\sqrt{\pi\cdot 2}}\sum_{S=-\infty}^{\infty}(-1)^{s}\int_{sc_2+(s-1)c_1}^{(s+1)c_2+sc_1}e^{-\frac{x^2}{2}}dx\right\}^{-1}.$$

Bu yerda c_3, c_4 o'zgarmas sonlar bo'lsa,

$$\frac{\frac{c_3}{\sqrt{n}}+O\left(\frac{1}{n}\right)}{c_4+O\left(\frac{1}{\sqrt{n}}\right)}=\frac{c_3}{c_4\sqrt{n}}=+O\left(\frac{1}{n}\right)$$

munosabat o'rinli bo'lishidan foydalanildi.

§ 5. Integral turdagi shartli ehtimolliklar uchun limit teorema.

Bu paragrafdagi maqsadimiz $a=c_1\sqrt{n},\ b=c_2\sqrt{n},\ x=c\sqrt{n},\ c_2>c$ bo'lganda $P\left(S_n<x \big/ N>n\right)$ ehtimollik $\frac{1}{\sqrt{n}}$ bo'yicha yoyilmasining birinchi hadini topishdan iborat.

$$P\left(S_n<x \big/ N>n\right)=\frac{P(S_n<x,N>n)}{P(N>n)}$$

va $P(N>n)$ ehtimollik uchun asimptotik yoyilma birinchi hadi [2] dan ma'lum bo'lgani uchun bizga $P(S_n<x,\ N>n)$ ehtimollik asimptotik yoyilmasining birinchi hadini topish kifoya. Bu masalani hal etish ikki qismdan iborat: 1) hosil etuvchi funksiya uchun asimptotik formula topish. Bu yerda Teorema 2.1 va Lemma 2.1 asosiy hisoblanadi. 2) Dovon usuli yordamida [2], [3] dagi natijalar yordamida yoyilma topish.

1. $$T_k(z)=\sum_{n=0}^{\infty}z^nP(S_n=k,\ N>n)\quad k\in(-a,b)$$

bo'lsin. U holda

$$G_x(z)=\sum_{n=0}^{\infty}z^nP(S_n<x,\ N>n)=\sum_{k=-a+1}^{x-1}T_k(z).$$

Teorema 2.1 dan quyidagilarni hosil qilamiz.

$$G_x(z)=\sum_{k=-a+1}^{x-1}f_k(z)-\sum_{k=-a+1}^{x-1}\lambda_1^{b-k}(z)\frac{1-H_1(z)\cdot\mu^a(z)}{(\lambda_1(z)-\lambda(z))u_z(\lambda_1(z))\vartheta_z(\lambda(z))\lambda^b(z)\left(1-H_2(z)\mu^{b+a}(z)\right)}-$$

$$-\sum_{k=-a+1}^{x-1}\frac{1}{\lambda^{a+k}(z)}\frac{\lambda_1^a(z)\left(1-H_3(z)\mu^b(z)\right)}{(\lambda_1(z)-\lambda(z))u_z(\lambda_1(z))\vartheta_z(\lambda(z))\lambda^b(z)\left(1-H_2(z)\mu^{b+a}(z)\right)}+\sum_{k=-a+1}^{x-1}\frac{\varepsilon(z)}{\sqrt{z-1}}\qquad(5.1)$$

Eslatib o'tamiz,

$$r_z^{-1}(\lambda)=\sum_{k=-\infty}^{\infty}\lambda^kf_k(z)\qquad|\lambda|=1,\qquad\sum_{k=-\infty}^{\infty}f_k(z)=\frac{1}{1-z}.$$

$$\sum_{k=-a+1}^{x-1} f_k(z) = \frac{1}{1-z} - \sum_{k=-\infty}^{-a} f_k(z) - \sum_{k=x}^{-a} f_k(z).$$

$r_z^{-1}(\lambda)$ funksiyaga Lemma 2.1 ni qo'llaymiz. U holda

$$\sum_{k=x}^{\infty} f_r(z) = \sum_{k=x}^{\infty} \frac{1}{\lambda^{k+1} r_z^1(\lambda(z))} = \frac{1}{\lambda^x(z)} \cdot \frac{1}{(\lambda(z)-1) r_z^1(\lambda(z))}, \tag{5.2}$$

$$\sum_{k=-\infty}^{-a} f_r(z) = \sum_{k=-\infty}^{-a} \frac{1}{\lambda^{k+1} r_z^1(\lambda_1(z))} = \frac{\lambda_1^{a-1}(z)}{(1-\lambda_1(z)) r_z^1(\lambda_1(z))}, \tag{5.3}$$

$$\sum_{k=-a+1}^{x-1} \lambda^{b-k}(z) = \sum_{k=b-x+1}^{b+a-1} \lambda^k(z) = \frac{\lambda_1^{\,b-x+1}(z) - \lambda_1^{\,b+1}(z)}{1-\lambda_1(z)}, \tag{5.4}$$

$$\sum_{k=-a+1}^{x-1} \frac{1}{\lambda^{a+k}(z)} = \sum_{k=1}^{a+x-1} \frac{1}{\lambda^k(z)} = \frac{1}{\lambda(z)-1} \frac{1}{\lambda^{a+x-1}(z)(\lambda(z)-1)}, \tag{5.5}$$

$$\sum_{k=-a+1}^{x-1} \left| \frac{\varepsilon(z)}{\sqrt{z-1}} \right| = \sum_{k=-a+1}^{x-1} \frac{O\left(e^{-\gamma\sqrt{n}}\right)}{\left|\sqrt{z-1}\right|} = (x+a-2) \frac{O\left(e^{-\gamma\sqrt{n}}\right)}{\left|\sqrt{z-1}\right|} = \frac{O\left(e^{-\gamma\sqrt{n}}\right)}{\left|\sqrt{z-1}\right|} \tag{5.6}$$

$G_x(z)$ ni hosil bo'lish uchun (5.2) – (5.6) musbatlarni (5.1) ga qo'yish qoldi xolos.

Teorema 5.1 Yetarli katta a va b uchun $x \in (-a, b)$ *va* $z \in \lambda_\delta$ *bo'lganda*

$$G_x(z) = \frac{1}{1-z}\left(1 + \frac{1}{\lambda^x(x)}\left\{\frac{1-z}{(\lambda(z)-1) r_z^1(\lambda(z))}\right\} - \lambda_1^a(z)\left\{\frac{(1-z)\cdot\lambda_1 z}{(\lambda(z)-1) r_z^1(\lambda(z))}\right\} -\right.$$

$$- \frac{\lambda_1^{b-x}(z)\cdot(1-H_1(z)\mu^a(z))}{\lambda^b(z)(1-H_2(z)\mu^{b+a}(z))}\left\{\frac{(1-z)\cdot\lambda_1(z)}{(\lambda_1(z)-\lambda(z))u_z(\lambda_1(z))\vartheta_z(\lambda(z))(1-\lambda_1(z))}\right\} +$$

$$+ \frac{\lambda_1^{b+a}(z)\cdot(1-H_1(z)\mu^a(z))}{\lambda^b(z)(1-H_2(z)\mu^{b+a}(z))}\left\{\frac{1-t}{(\lambda_1(z)-\lambda(z))(1-\lambda_1(z))u_z(\lambda_1(z))\vartheta_z(\lambda(z))}\right\} -$$

$$- \frac{\lambda_1^a(z)(1-H_3(z)\mu^a(z))}{1-H_2(z)\mu^{b+a}(z)}\left\{\frac{1-t}{(\lambda_1(z)-\lambda(z))(\lambda(z)-1)u_z(\lambda_1(z))\vartheta_z(\lambda(z))}\right\} +$$

$$+ \frac{\lambda_1^a(z)(1-H_3(z)\mu^a(z))}{\lambda^{a+x}(z)(1-H_2(z)\mu^{b+a}(z))}\left\{\frac{1-t}{(\lambda_1(z)-\lambda(z))(\lambda(z)-1)u_z(\lambda_1(z))\vartheta_z(\lambda(z))}\right\} + \frac{O\left(e^{-\gamma\sqrt{n}}\right)}{\sqrt{z-1}}.$$

Quyidagi begilashlarni kiritamiz:

$$F_1(z)=\frac{1-z}{(\lambda(z)-1)r_z^1(\lambda(z))},\quad F_2(z)=\lambda_1(z)\frac{1-z}{(1-\lambda_1(z))r_z^1(\lambda_1(z))},$$

$$F_3(z)=\frac{(1-z)\lambda_1(z)v_z^{-1}(\lambda(z))}{(\lambda_1(z)-\lambda(z))u_z(\lambda_1(z))(1-\lambda_1(z))},\qquad F_4(z)=\frac{1}{\lambda_1(z)}F_3(z),$$

$$F_6=\lambda_1(z)F_5(z)=\frac{(1-z)\lambda_1(z)}{(\lambda_1(z)-\lambda(z))(\lambda(z)-1)u_z(\lambda_1(z))\vartheta_z(\lambda(z))}.$$

Ishonch hosil qilish qiyin emaski,

$$F_i(z)=\sum_{k=0}^{\infty} f_k^{(i)}t^k,\quad t=i(z-1)^{1/2},$$

$$-F_1(1)=F_2(1)=F_3(1)=F_4(1)=F_5(1)=F_6(1).$$

$|z|=1,\ |z-1|\ge\delta$ bo’lganda $|G_x(x)|\le\dfrac{O\left(e^{-\gamma\sqrt{n}}\right)}{\left|(z-1)^{1/2}\right|}$.

Shu bilan birinchi qism yakunlanadi.

2. Teskarilash formulasiga asosan

$$P(S_n<x,\quad N>n)=\frac{1}{2\pi i}\int_{|z|=1}\frac{G_x(z)}{z^{n+1}}dz$$

Teorema 2.3 va 2.4 larga asosan

$$1)\ J_1=\frac{1}{2\pi i}\int_{\Gamma}\frac{F_1(z)dz}{\lambda^x(z)(1-z)z^{n+1}}.$$

Bu holda

$$k_1=k_2=k_4=0,\quad k_3=1$$

$$t_0=-\frac{c}{\sqrt{2n}}+O\left(\frac{1}{n}\right),\quad h_0(t_0)=-\frac{c^2}{2n}+O\left(\frac{1}{n^{3/2}}\right),\quad h_0^{11}(t_0)=2+O\left(\frac{1}{\sqrt{n}}\right),$$

$$h_0^{(0)}=1+O\left(\frac{1}{\sqrt{n}}\right),\quad |t_0|\sqrt{2nh_0^{(0)}}=c+O\left(\frac{1}{\sqrt{n}}\right),$$

$$J_1 = 2F_1(1)\Phi\left(c + O\left(\frac{1}{\sqrt{n}}\right)\right) = 2F_1(1)\Phi(c) + O\left(\frac{1}{\sqrt{n}}\right)$$

$$\Phi(t) = \frac{1}{\sqrt{2\pi}} \cdot \int_t^{\infty} e^{-x^2/2}\, dx\,.$$

$$2)\ J_2 = \frac{1}{2\pi i}\int_\Gamma \frac{\lambda_1^a(z)F_2(z)}{(1-z)\cdot z^{n+1}}dz, \quad k_1 = 1,\ k_2 = k_3 = k_4 = 0\,,$$

$$t_0 = -\frac{c_1}{\sqrt{2n}} + O\left(\frac{1}{n}\right), \quad h_0(t_0) = -\frac{c^2}{2n} + O\left(\frac{1}{n^{3/2}}\right), \quad h_0^{11}(t_0) = 2 + 0\left(\frac{1}{\sqrt{n}}\right)$$

$$h_0^{(0)} = 1 + O\left(\frac{1}{\sqrt{n}}\right); \quad |t_0|\sqrt{2nh_0^{(0)}} = c_1 + O\left(\frac{1}{\sqrt{n}}\right)$$

$$J_2 = 2F_2(1)\Phi(c_1) + O\left(\frac{1}{\sqrt{n}}\right).$$

$$3)\ J_3 = \frac{1}{2\pi C}\int_\Gamma \frac{\lambda_1^{b-x}(z)\left(1 - H_1(z)\mu^a(z)\right)F_3(z)dx}{\lambda^b(z)\left(1 - H_2(z)\mu^{b+a}(z)\right)(1-z)z^{n+1}}\,,$$

$$k_1 = 1 - \frac{c}{c_2}, \quad k_2 = k_4 = 0, \quad k_3 = 1,$$

$$t_{1S} = -\sqrt{2}\left(s + \frac{2c_2 - c}{2c_2}\right)\frac{c_2}{\sqrt{n}} - \sqrt{2}\,\frac{sc}{\sqrt{n}} - \frac{s\eta_1}{2n} + O\left(\frac{s^2}{n}\right),$$

$$t_{2S} = -\sqrt{2}\left(s + \frac{2c_2 - c}{2c_2}\right)\frac{c_2}{\sqrt{n}} - \sqrt{2}\,\frac{sc}{\sqrt{n}} - \frac{s\eta_1}{2n} + O\left(\frac{s^2}{n}\right),$$

$$h_{js}^{(0)} = 1 + O\left(\frac{s}{\sqrt{n}}\right); \quad |t_{1S}|\sqrt{2nh_{iS}^{(0)}} = 2\left(sc_2 + sc_1 + c_2 - \frac{c}{2}\right) + O\left(\frac{s^2}{\sqrt{n}}\right),$$

$$|t_{2S}|\sqrt{2nh_{2s}^{(0)}} = 2\left(sc_1 + sc_2 + c_1 + c_2 - \frac{c}{2}\right) + O\left(\frac{s^2}{\sqrt{n}}\right)$$

$$J_3 = 2F_3(1)\cdot\left[\sum_{S=0}^{M-1}\left(\Phi\left(2\left(sc_2 + sc_1 + c_2 + c_1 - \frac{c}{2}\right) + O\left(\frac{s^2}{\sqrt{n}}\right)\right) - \Phi\left(2\left(sc_2 + sc_1 + c_2 + c_1 - \frac{c}{2}\right) + O\left(\frac{s^2}{\sqrt{n}}\right)\right)\right)\right] +$$

$$+ O\left(\frac{1}{\sqrt{n}}\right) = 2F_3(1)\cdot\sum_{S=0}^{\infty}\frac{1}{\sqrt{2\pi}}\int_{2\mathrm{csc}_1 + sc_2 + c_1c_2 - \frac{C}{2}}^{2\mathrm{csc}_1 + c_2 s + c_2 - \frac{c}{2}} e^{-\frac{x^2}{2}}dx + O\left(\frac{1}{\sqrt{n}}\right).$$

$$4)\ J_4=\frac{1}{2\pi i}\int_\Gamma \frac{\lambda_1^{b+a}(z)\left(1-H_1(z)\mu^a(z)\right)F_4(z)dz}{\lambda^b(z)\left(1-H_2(z)\mu^{b+a}(z)\cdot(1-z)z^{n+1}\right)},$$

$$k_1=k_2=k_3=1,\quad k_4=0,$$

$$t_{js}=-\sqrt{2}(s+1)\frac{c_1}{\sqrt{n}}-\sqrt{2}(s+j-1)\frac{c_1}{\sqrt{n}}-\frac{s\eta_1}{2n}+O\left(\frac{s^2}{n}\right)$$

$$h_{js}^{(0)}=1+O\left(\frac{s}{\sqrt{n}}\right);\qquad |t_{jS}|\sqrt{2nh_{js}^{(0)}}=2\left(c_2s+sc_1+c_2+\left(j-\frac{1}{2}\right)c_1+a\frac{s^2}{\sqrt{n}}\right)$$

$$J_4=2F_4(1)\left[\sum_{S=0}^{\infty}\left(\Phi\left(2\left(sc_2+sc_1+c_2+\frac{c_1}{2}\right)\right)-\Phi\left(2\left(sc_2+sc_1+c_2+\frac{c_1}{2}\right)\right)\right)\right]+O\left(\frac{1}{\sqrt{n}}\right)=$$

$$=2F_4(1)\sum_{S=0}^{\infty}\frac{1}{\sqrt{2\pi}}\int_{2\left(sc_2+sc_1+c_2+\frac{c_1}{2}\right)}^{2\left(sc_2+sc_1+c_2+\frac{3}{2}c_1\right)}e^{-\frac{x^2}{2}}dx+O\left(\frac{1}{\sqrt{n}}\right).$$

$$5)\ J_5=\frac{1}{\sqrt{2\pi}}\int_\pi \frac{\lambda_1^{a}(z)\left(1-H_3(z)\mu^b(z)\right)F_S(z)dz}{\left(1-H_2(z)\mu^{b+a}(z)\right)z^{n+1}(1-z)},$$

$$k_1=1,\ k_2=k_3=k_4=0,\quad t_{js}=-\sqrt{2^1}(s+1)\frac{c_1}{\sqrt{n}}\sqrt{2}(s+j-1)\frac{c_2}{\sqrt{n}}-\frac{s\eta_1}{2n}+O\left(\frac{s^2}{\sqrt{n}}\right),$$

$$h_{jS}^{(0)}=1+O\left(\frac{s}{\sqrt{n}}\right),$$

$$J_5=2F_5(1)\sum_{S=0}^{\infty}\left[\Phi\left(2\left(sc_2+sc_1+c_2+\frac{c_1}{2}\right)\right)-\Phi\left(2\left(sc_2+sc_1+c_2+\frac{c_1}{2}\right)\right)\right]+O\left(\frac{1}{\sqrt{n}}\right)=$$

$$=2F_5(1)\sum_{S=0}^{\infty}\frac{1}{\sqrt{2\pi}}\int_{2\left(sc_2+sc_1+c_2+\frac{c_1}{2}\right)}^{2\left(sc_2+sc_1+c_2+\frac{3}{2}c_1\right)}e^{-\frac{x^2}{2}}dx+O\left(\frac{1}{\sqrt{n}}\right).$$

$$6)\ J_6=\frac{1}{2\pi i}\int_\Gamma \frac{\lambda_1^{a}(z)\left(1-H_3(z)\mu^b(z)\right)F_6dz}{\lambda^{a+x}(z)\left(1-H_2(z)\mu^{b+a}(z)z^{n+1}(1-z)\right)},$$

$$k_1=1,\ k_2=k_4=0,\quad k_3=1+\frac{c}{c_2},$$

$$t_{jS} = -\sqrt{2}\left(s + \frac{2c_1 + c}{2c_1} \right)\frac{c_1}{\sqrt{n}} - \sqrt{2}(s + j - 1)\frac{c_2}{\sqrt{n}} - \frac{s\eta_1}{2n} + O\left(\frac{s^2}{n} \right),$$

$$h_{jS}^{(0)} = 1 + O\left(\frac{s}{\sqrt{n}} \right),$$

$$J_6 = 2F_6(1)\sum_{S=0}^{\infty}\left[\Phi\left(2\left(sc_1 + sc_2 + c_2 + \frac{c}{2} \right) \right) - \Phi\left(2\left(sc_1 + sc_2 + c_1 + c_1\right)\right) \right] + O\left(\frac{1}{\sqrt{n}} \right) =$$

$$= 2F_6(1)\sum_{S=0}^{\infty}\frac{1}{\sqrt{2\pi}}\int_{2\left(sc_1+sc_2+c_1+\frac{c}{2} \right)}^{2\left(sc_1+sc_2+c_1+c_2+\frac{c}{2} \right)} e^{-\frac{x^2}{2}}dx + O\left(\frac{1}{\sqrt{n}} \right),$$

$$P(Sn < x,\ \ N > n) = 1 + J_1 - J_2 - J_3 + J_4 - J_5 + J_6 .$$

Yuqoridagi 1) -6) natijalarni jamlab, quyidagi tasdiqni hosil qilamiz.

Teorema 5.2 $a = c_1\sqrt{n},\quad b = c_2\sqrt{n};\quad x = c\sqrt{n};\quad c_2 > c,\quad D\xi_1 = 1$ bo’lganda

$$P\left(S_n < x \big/ N > n \right) = C_2(x) + O\left(\frac{1}{\sqrt{n}} \right)$$

$$C_2(x) = \{1 + 2F_1(1)[\Phi(c) + \Phi(c_1)] - 2F_3(1)\frac{1}{\sqrt{2\pi}}\sum_{S=0}^{\infty}\left[\int_{2sc_1+2(s+1)c_2-c}^{2(s+1)c_1+2(s+1)c_2-c} e^{-\frac{x^2}{2}}dx - \right.$$

$$\left. - \int_{(2s+1)c_1+2(s+1)c_2}^{(2s+3)c_1+2(s+1)c_2} e^{-\frac{x^2}{2}}dx + \int_{(2s+1)c_1+2sc_2}^{(2s+3)c_1+2(s+1)c_2} e^{-\frac{x^2}{2}} - \int_{2(s+1)c_1+2sc_2+c}^{2(s+1)c_1+2(s+1)c_2+c} e^{-\frac{x^2}{2}} \right]\Bigg\}\left\{ \frac{1}{2\pi i}\sum_{2\pi i}^{\infty}\int_{sc_2+(s-1)c_1}^{(s+1)c_2+sc_1} e^{-\frac{x^2}{2}} \right\}^{-1}.$$

III bob. **FAQAT BIR CHEGARA O'SUVCHI BO'LGAN HOLDA ASIMPTOTIK FORMULALAR.**

§ 6 Hosil etuvchi funksiyalar uchun asimptotik formulalar.

$n \to \infty$ da $a = a(n) \to \infty$, b ixtiyoriy musbat butun son bo'lsin. [3] dan ma'lumki,

$$Q_2(z,\lambda) = r_{z+}(\lambda)\left[r_{z+}^{-1}(\lambda)\right]^{[b,\infty)} - r_{z+}(\lambda)\left[r_{z+}^{-1}(\lambda) r_{z-}(\lambda)\left[r_{z-}^{-1}(\lambda)\right]^{(-\infty,-a]}\right]^{[b,\infty)} +$$

$$+ r_{z+}(\lambda)\left[r_{z+}^{-1}(z) r_{z-}(\lambda)\left[r_{z-}^{-1}(z) Q_2(z,\lambda)\right]^{(-\infty,-a]}\right]^{[b,\infty)} .$$

$\eta(b) = \min\ \{k : S_k \geq b\},\quad \chi(b) = S_{\eta(b)}$ belgilashlar kiritamiz.

Teorema 6.1. *Shunday* $\delta > 0,\ \gamma > 0,\ c > 0$ *sonlar mavjudki,*

$z \varepsilon Z_\delta = \{|z| < 1, \ , |z-1| < \delta\};\ |\lambda| = 1$ *bo'lganda*

$$Q_2(z,\lambda) = \sum_{n=1}^{\infty} z^n \sum_{k=b}^{\infty} \lambda^k P(\chi(b) = k,\ \eta(b) = n) - \frac{\lambda^b v_z(\lambda) H_1(z) \mu^a(z)(1 - G(z, \lambda_1(z)))}{\lambda^b v_z(\lambda(z))(1 - H_2(z)\mu^{b+a}(z))} +$$

$$\sum_{k=b}^{\infty} \lambda^k \rho_{k+a}(z), \quad |\rho_k(z)| \leq c e^{-\gamma k},$$

bo'ladi. Bu yerda

$$G(z,\lambda) = r_{z+}(\lambda)\left[r_{z+}^{-1}(\lambda)\right]^{[b,\infty)}.$$

Lemma 6.1 $|z| < 1,\ \ |\lambda| = 1,\ k \geq b$ *bo'lganda*

$$G(z_1 \lambda) = \sum_{n=1}^{\infty} z^n \sum_{k=b}^{\infty} \lambda^k P(\chi(b) = k_1 \eta(b) = n)$$

bo'ladi.

Lemmaning isboti (2.1) tenglikdan $a = \infty$ da kelib chiqadi.

Lemma 6.2. *$f(x)$ funksiya $1-\delta_1<|\lambda|<1+\delta_1$, $\delta_1>0$ halqada analitik, chegarada uzluksiz bo'lsin. U holda shunday $\gamma>0$, $c>0$ sonlar mavjudki,*

$$r_{z+}(\lambda)\left[r_{z+}^{-1}(\lambda)r_{z-}(\lambda)\left[r_{z-}^{-1}(\lambda)f(\lambda)\right]^{(-\infty,-a)}\right]^{(b,\infty)}=$$

$$=\frac{\lambda^b v_z(\lambda)u_z(\lambda(z))\lambda_1^a(z)f(\lambda_1(z))}{\lambda^{b+a}(z)\upsilon_z(\lambda(z))u_z(\lambda_1(z))}+(\lambda(z)-\lambda_1(z))\psi(z,\lambda)-(\lambda-\lambda(z))\varphi(z,\lambda)$$

bo'ladi. Bu yerda

$$\varphi(z_1\lambda)=\sum_{k=b}^{\infty}\lambda^b\varphi^{(k)}(z),\quad \left|\varphi^{(k)}(z)\right|\le ce^{-\gamma(k+a)},$$

$$\psi(z_1\lambda)=\sum_{k=b}^{\infty}\lambda^k\psi^{(k)}(z),\quad \left|\psi^{(k)}(z)\right|\le ce^{-\gamma(k-b+a)}.$$

Isboti. *$r_{z-}^{-1}(\lambda)f(\lambda)$ funksiya $1-\delta_1<|\lambda|<1+\delta_1$, $|z-1|<\delta$ da analitik, chegarada uzluksiz bo'lgani uchun unga Lemma 2.1 ni qo'llash mumkin. Natijada $a=a(n)\to\infty$, $n\to\infty$ bo'lganda*

$$r_{z-}(\lambda)\cdot\left[r_{z-}^{-1}(\lambda)f(\lambda)\right]^{(-\infty,-a]}=\frac{u_z(\lambda)\cdot\lambda_1^a(z)f(\lambda_1(z))}{\lambda^a u_z(\lambda_1(z))}+(\lambda-\lambda_1(z))\theta_1(z,\lambda)$$

$$\theta_1(z,\lambda)=\sum_{k=-\infty}^{-a-1}\lambda^k\theta_1^{(k)}(z)\qquad \left|\theta_1^{(k)}(z)\right|\le ce^{\gamma k}$$

bo'ladi.

Demak,

$$r_{z+}(\lambda)\cdot\left[r_{z+}^{-1}(\lambda)\left[r_{z-}^{-1}(\lambda)f(\lambda)\right]^{(-\infty,-a]}\right]^{[b,\infty)}=r_{z+}(\lambda)\cdot\left[\frac{r_{z+}^{-1}(\lambda)\cdot u_z(\lambda)\cdot\lambda_{z-}^{-1}(z)f(\lambda_1(z))}{\lambda^a u_z(\lambda_1(z))}\right]^{[b;\infty)}+$$

$$+r_{z+}(\lambda)\left[r_{z+}^{-1}(\lambda)\cdot(\lambda-\lambda_1(z))Q_1(z,\lambda)\right]^{[b,\infty)}.$$

funksiya $r_{z+}^{-1}(\lambda)\cdot u_z(\lambda)(\lambda-\lambda(z))(\lambda-\lambda(z))$ $|z-1|<\delta$, $1-\delta_1<|\lambda|<1+\delta_1$, da analitik. $r_{z+}^{-1}(\lambda)u_z(\lambda)$ funksiyaga lemma 2.1 ni qo'llasak,

$$r_{z+}(\lambda)\left[r_{z+}^{-1}(\lambda)\frac{u_z(\lambda)\lambda_1^a(z)f(\lambda_1(z))}{\lambda^a u_z(\lambda_1(z))}\right]^{[b,\infty)}=\frac{r_{z+}(\lambda)\lambda_1^a(z)f(\lambda_1(z))}{\lambda^a u_z(\lambda_1(z))}$$

$$\left[r_{z+}^{-1}(\lambda)\cdot u_z(\lambda)\right]^{[b+a,\infty)}=\frac{\lambda^b v_z(\lambda)u_z(\lambda(z))\lambda_1^a(z)f(\lambda_1(z))}{\lambda^{b+a}(z)u_z(\lambda_1(z))\vartheta_z(\lambda(z))}+$$

$$+\frac{(\lambda-\lambda(z))\vartheta_z(\lambda)}{\lambda^a}\cdot\sum_{k=b+a}^{\infty}\lambda^k\cdot O\left(e^{-\gamma k}\right)=\frac{\lambda^b\vartheta_z(\lambda)u_z(\lambda(z))\lambda_1^a(z)f(\lambda_1(z))}{\lambda^{b+a}(z)u_z(\lambda_1(z))\vartheta_z(\lambda(z))}+$$

$$+(\lambda-\lambda(z))\varphi(z,\lambda)$$

hosil bo'ladi.

Bu yerda

$$\left[\frac{g(\lambda)}{\lambda^a}\right]^{[b,\infty)}=\frac{1}{\lambda^a}[g(\lambda)]^{[b+a,\infty)}$$

munosabatdan foydalanildi.

Endi $r_{z+}(\lambda)\left[r_{z+}^{-1}(\lambda)(\lambda-\lambda_1(z))Q_1(z_1\lambda)\right]^{[b,\infty)}$ funksiyani qaraymiz.

$$\theta_1(z_1)=\sum_{k=-\infty}^{-a-1}\lambda^k\theta_1^{(k)}(z)=\frac{1}{\lambda^a}\sum_{k=-\infty}^{-a-1}\lambda^{k+a}\theta_1^{(k)}(z)=\frac{1}{\lambda^a}\cdot\sum_{k=-\infty}^{-1}\lambda^k\theta_1^{(k-a)}(z)$$

$$\left|\sum_{k=-\infty}^{-1}\lambda^k\theta_1^{(k-a)}(z)\right|\le c\left|\sum_{k=-\infty}^{-1}\lambda^k e^{\gamma(k-a)}\right|=ce^{-\gamma a}\sum_{k=-\infty}^{-1}\left|\lambda e^{\gamma}\right|^k .$$

Bu qator $\left|\lambda e^{\gamma}\right|<1$ bo'lganda ya'ni $|\lambda|>e^{-\gamma}$ da yaqinlashadi. $\gamma<-\ln(1-\delta_1)$ bo'lganda $r_{z+}(\lambda)(\lambda-\lambda_1(z))\theta_1(z,\lambda)\lambda^a$ funksiyaga Lemma 2.1 ni qo'llash mumkin:

$$r_{z+}(\lambda)\left[r_{z+}^{-1}(\lambda)(\lambda-\lambda_1(z))\theta_1(z_1\lambda)\right]^{[b,\infty)}=\frac{r_{z+}(\lambda)}{\lambda^a}\left[r_{z+}^{-1}(\lambda)(\lambda-\lambda_1(z))\sum_{k=-\infty}^{-1}\lambda^k\theta_1^{(k-a)}(z)\right]^{[b+a,\infty)}=$$

$$=\frac{\lambda^b v_z(\lambda)\sum\limits_{k=-\infty}^{-1}\lambda^{k-a}\theta_1^{(k-a)}(z)}{\lambda^b(z)\vartheta_z(\lambda(z))}=\sum_{k=b}^{\infty}\lambda^b\psi^{(k)}(z)=\psi(z,\lambda) .$$

Lemma 2.2 ga asosan esa

$$\left|\psi^{(k)}(z)\right|=\left|\frac{b_{k-b}(z)\sum\limits_{k=-\infty}^{-1}\lambda^{k-a}(z)\theta_1^{(k-a)}(z)}{\lambda^b(z)\vartheta_z(\lambda(z))}\right|\le\frac{c_1}{(1-\delta_1)^{k-b}}\sum_{k=1}^{\infty}\left|\theta_1^{-(k+a)}(z)\right|\le ce^{-\gamma(k-b+a)},\qquad \gamma<\ln(1+\delta_1).$$

$1-\delta_1<|\lambda|<1+\delta_1,\quad |z-1|<\delta$ bo'lganda

$$\frac{1}{\lambda^a}\left(\lambda-\lambda(z)v_z(\lambda)\cdot\sum_{k=b+a}^{\infty}\lambda^k\theta\left(e^{-\gamma k}\right)\right)=(\lambda-\lambda(z))\varphi(z_1\lambda)$$

bo'lishini eslatib o'tamiz. Lemma isbotlandi.

Teoremaning isboti. Lemma 6.2 ga asosan

$$Q_2(\lambda_1\lambda)=G(z_1\lambda)-\frac{\lambda^b v_z(\lambda)\cdot u_z(\lambda(z))\lambda_1^a(z)}{\lambda^{b+a}(z)v_z(\lambda(z))u_z(\lambda_1(z))}(1-Q_2 z,\lambda_1(z))-$$

$$-(\lambda(z)-\lambda_1(z))\varphi_1(z,\lambda)+(\lambda(z)-\lambda_1(z))\cdot\psi_2(z,\lambda)-$$

$$-(\lambda-\lambda(z))\varphi_2(z,\lambda)+(\lambda-\lambda(z))\varphi_3(z,\lambda)\,, \qquad (6.1)$$

bu yerda

$$\psi_i(z,\lambda)=\sum_{n=b}^{\infty}\lambda^b\psi_i^{(k)}(z), \qquad \left|\psi_i^{(k)}(z)\right|\le ce^{-\gamma(k-b+a)},$$

$$\varphi_i(z,\lambda)=\sum_{n=b}^{\infty}\lambda^b\varphi_i^{(k)}(z), \qquad \left|\varphi_i^{(k)}(z)\right|\le ce^{-\gamma(k+a)}.$$

(6.1) tenglikda $\lambda=\lambda_1(z)$ deb olamiz va hosil bo'lgan tenglamadan $Q_2(z,\lambda_1(z))$ ni topamiz:

$$Q_2(z,\lambda_1(z))=\frac{G(z,\lambda_1(z))-\mu^{b+a}(z)H_2(z)}{1-\mu^{b+a}(z)H_2(z)}-\frac{\lambda(z)-\lambda_1(z)}{1-\mu^{b+a}(z)H_2(z)}$$

$$\left(\varphi_2(z,\lambda_1(z))+\varphi_3(z,\lambda_1(z))-\psi_1(z,\lambda_1(z))-\psi_2(z,\lambda_1(z))\right). \qquad (6.2)$$

Qandaydir $\delta>0$ sonda $\dfrac{\lambda(z)-\lambda_1(z)}{1-\mu^{b+a}(z)H_2(z)}$ funksiya λ_δ da tekis chegaralangan $\delta>0$ sonni shunday tanlanganda

$$\left|Q_2(z,\lambda_1(z))-\frac{G(z,\lambda_1(z)-\mu^{b+a}(z)H_2(z))}{1-\mu^{b+a}(z)H_2(z)}\right|\le ce^{-\gamma a} \qquad (6.3)$$

(6.1), (6.2) musbatlar (6.3) baho bilan birgalikda teorema isbotini yakunlaydi.

Natija 6.1.1 Shunday $\delta>0,\ \ \gamma>0,\ \ c>0$ sonlar mavjudki $z\in\lambda_\delta$ da

$$Q_1(z,\lambda)=\frac{u_z(\lambda)\lambda_1^a(z)(1-G(z,\lambda_1(z)))}{\lambda^a u_z(\lambda_1(z))(1-\mu^{b+a}(z)H_2(z))}+\sum_{k=-\infty}^{-a}\lambda^k\varepsilon_k(z),\ \ |\varepsilon_k(z)|\le ce^{\gamma k}$$

bo'ladi.

Isboti: [3] dan ma'lumki,

$$Q_1(z,\lambda)=r_{z-}(\lambda)\left[r_z^{-1}(\lambda)(1-Q_2(z,\lambda))\right]^{(-\infty,-a]}, \qquad |\lambda|=1.$$

U holda Lemma 2.1 ni $r_{z-}^{-1}(\lambda)(1-Q_2(z,\lambda))$ funksiyaga qo'llash qoladi xolos.

Natija 6.1.2. Shunday $\delta>0,\ \ \gamma>0,\ \ c>0$ sonlar mavjudki, $z\in\lambda_\delta,\ \ k\ge 0$ da etarli katta a uchun

$$T_{k+b}(z)=\sum_{n=1}^{\infty}z^nP(\lambda(b)=k+b,\ \ \eta(b)=n)-\frac{b_k(z)(1-G(z,\lambda_1(z))\mu^a(z)H_1(z))}{\lambda^b(z)\vartheta_z(\lambda(z))(1-\mu^{b+a}(z)H_2(z))}+\rho_{k+a+b}(z),$$

$$|\rho_{k+a+b}(z)|\le ce^{-\gamma(k+a+b)}$$

bo'ladi.

Natija 6.1.3. Shunday $\delta>0,\ \ \gamma>0,\ \ c>0$ sonlar mavjudki, $z\in\lambda_\delta,\ \ k\ge 0$ da yetarli katta a uchun

$$T_{k-a}(z)=-\frac{\beta_k(z)\lambda_1^a(z)(1-G(z,\lambda_1(z)))}{u_z(\lambda_1(z))(1-\mu^{b+a}(z)H_2(z))}+\varepsilon_{k-a}(z), \quad |\varepsilon_{k-a}(z)|\le ce^{\gamma(k-a)}.$$

Bu yerda

$$\vartheta_z(z)=\frac{r_z+(\lambda)}{\lambda-\lambda(z)}=\sum_{k=0}^{\infty}b_k(z)\lambda^k, \qquad u_z(\lambda)=\frac{\lambda r_z-(\lambda)}{\lambda-\lambda_1(z)}=\sum_{k=-\infty}^{0}\beta_k(\lambda)\lambda^k, \quad |\lambda|=1.$$

$$H_1(z)=1+\sum_{k=1}^{\infty}v_k i^k(z-1)^{k/2}, \qquad H_2(z)=1+\sum_{k=1}^{\infty}\eta_n i^k(z-1)^{k/2}.$$

Natijada 6.1.4.

$$(i)\quad P\cdot(S_N=k+b)=P(\chi(b)=k+b)-\frac{\psi_1 b_k(1)f(1)}{(b+a)\cdot\psi_1+\eta_1/2}+O\left(e^{-\gamma(k+z+b)}\right),\quad k\ge 0,$$

bu yerda

$$f(1)=\sum_{k=1}^{b}kb_{k-b}^{(1)}(1),\qquad \vartheta_z^{-1}(\lambda)=\sum_{k=0}^{\infty}\lambda^k b_k^{(1)}(z),\qquad |\lambda|=1.$$

$$(ii)\quad P(S_N\ge b)=1-\frac{\psi_1\vartheta_1(1)f(1)}{(b+a)\psi_1+\eta_1/2}+O\left(e^{-\gamma(a+b)}\right),$$

$$(iii)\quad P(S_N=k-a)=\frac{\beta_k(1)\cdot\vartheta_1(1)\cdot f(1)}{u_1(1)}\cdot\frac{\psi_1}{(b+a)\psi_1+\eta_1/2}+O\left(e^{\gamma(k-b)}\right),\quad k<0$$

$$(iv)\quad P(S_N\le -a)=\frac{\psi_1\vartheta_1(1)\cdot f(1)}{(b+a)\cdot\psi_1+\eta_1/2}+O\left(e^{-\gamma a}\right),\quad k\ge 0.$$

Isbot quyidagi munosabatlardan kelib chiqadi.

$$1-G(z,\lambda_1(z))=(\lambda(z)-\lambda_1(z))\vartheta_z(\lambda_1(z))\left[\frac{\vartheta_z^{-1}(z)}{(\lambda(z))-\lambda}\right]_{\lambda=\lambda_1(z)}^{[0,b)},$$

$$f(z)=\left[\frac{\vartheta_z^{-1}(\lambda)}{\lambda(z)-\lambda}\right]_{\lambda=\lambda_1(z)}^{[0,b)}=\left[\sum_{k=0}^{\infty}\lambda^k b_k^{(1)}(z)\sum_{k=0}^{\infty}\frac{\lambda^k}{\lambda^{k+1}(z)}\right]_{\lambda=\lambda_1(z)}^{[0,b)}=$$

$$=b_0^{(1)}(z)\frac{\lambda^b(z)-\lambda_1^b(z)}{\lambda^b(z)(\lambda(z)-\lambda_1(z))}+b_1^{(1)}(z)\frac{\lambda^{b-1}(z)-\lambda_1^{b-1}(z)}{\lambda^{b-1}(z)(\lambda(z)-\lambda_1(z))}+\ldots$$

$$+b_{b-1}^{(1)}(z)\frac{\lambda(z)-\lambda_1(z)}{\lambda(z)(\lambda(z)-\lambda_1(z))}.$$

Lemma 6.3 $|z-1|\ge\delta,\quad |z|=1$ *bo'lganda*

$$|T_{k-a}(z)|\le ce^{-\gamma(k+a)},\left|T_{k+b}(z)-\sum_{n=1}^{\infty}z^nP(\chi(b)=k+b,\eta(b)=n)\right|\le ce^{-\gamma(k+a)},\qquad k\ge 0.$$

Lemmaning isboti $r_{z+}^{-1}(\lambda)$ funksiyani $|z-1|\ge\delta,\ |z|=1$ da $|\lambda|<1+\delta_1$ doiraga analitikning davom ettirish mumkinligidan, (2.2), (2.3) tengliklar Lemma 2.2 dan kelib chiqadi. $r_{z-}^{-1}(\lambda)$ funksiya esa $|\lambda|>1-\delta_1$ analitik davom ettrilishi mumkin.

§ 7 Ehtimolliklar uchun asimptotik formulalar.

Quyidagi belgilarni kiritamiz:

$$F_1(z)=\frac{b_k(z)\cdot(1-G(z,\lambda_1(z)))H_1(z)}{\vartheta_z(\lambda(z))},\qquad F_2(z)=\frac{\beta_k(z)(1-G(z,\lambda_1(z)))}{u_z(\lambda_1(z))}.$$

$z\in D\delta=\{|z-1|<\delta\}\setminus\{z\geq 1\}$ bo'ganda

$$1-G(z,\lambda_1(z))=(\lambda(z)-\lambda_1(z))\vartheta_z(\lambda_1(z))\left[\frac{\vartheta_z^{-1}(z)}{\lambda(z)-\lambda}\right]_{\lambda=\lambda_1(z)}^{[0,b)}=$$

$$=\sum_{k=1}^{\infty}\varphi_k i^k(z-1)^{k/2},$$

va $v_z^{\pm1}(\lambda)$ funksiya $-1-\delta_1<|\lambda|<1+\delta_1,\qquad |z-1|<\delta$ analitik bo’lgani uchun

$$F_j(z)=\sum_{k=1}^{\infty}g_k^{(j)}i^k\cdot(z-1)^{k/2},\qquad z\in D\delta,\qquad j=1,2,$$

bo'ladi.

[2] da keltirilgan davon usuli yordamida $P(S_n=k,\ \ N=n),\ k\notin(-a,b)$ ehtimollik uchun to’la asimptotik yoyilma topamiz.

$$J_1=\frac{1}{2\pi i}\int_{\Gamma}\frac{F_1(z)\lambda_1^a(a)dz}{\lambda^{b+a}(z)(1-\mu^{b+a}(z)H_2(z)z^{n+1})}$$

bo’lsin. Bu yerda $\Gamma-|z|=1,\ |z|<\delta$ yoyning $z=1$ nuqta atrofida ichkariga egilishidan hosil bo’lgan kontur,

$$\frac{a}{n}=\tau_1,\quad \frac{b}{n}=\tau_2,\quad \frac{1}{n}=\tau_3,\quad \tilde{a}_1(z)=F_1(z)z^{-1},\quad a_1(t)=-2t\cdot\tilde{a}_1(1-t^2)$$

$$t=i(z-1)^{1/2},\quad h_S(t)=(\tau_1 s+\tau_2 s)\ln\mu(z)+\tau_1\ln-\lambda_1(z)-(\tau_1+\tau_2)\ln\lambda(z)-\ln z,\quad 0\leq s\leq M.$$

Nol nuqta atrofida

$$h_s(z)=\psi_1 t[(2s+2)\tau_1+(2s+1)\tau_2+s\eta_1\tau_3]+\frac{t^2}{2}\left[1+O\left(\frac{s}{\sqrt{n}}\right)\right]+O(t^3),$$

t_S $\quad h_S^1(t)=0$ tenglamaning yechimlari. Yetarli katta n lar uchun

$$t_S=-\psi_1(s+1)\tau_1-\psi_1\left(s+1/2\right)\tau_2-\frac{s\eta_1}{2}\tau_3+O\left(\frac{s^2n}{}\right),$$

$$h_s(t_s)=-\left[\psi_1(s+1)\tau_1+\psi_1\left(s+1/2\right)\tau_2+\frac{s\eta_1}{2}\tau_3\right]^2+O\left(s^3(\tau_1+\tau_2)^3\right),$$

$$h_s''(t_s)=2+O\left(\frac{s}{\sqrt{n}}\right).$$

Teorema 7.1 Har qanday butun $q\geq 1$ *uchun,* $a=a(n)\to\infty,\quad n\to\infty$ *bo’lganda*

$$J_1(n)=\sum_{S=0}^{M-1}\left\{e^{nhS(tj)}\left[\sum_{i=0}^{a-1}\frac{Q_{1Si}}{n^{i+1/2}}+\frac{R_{isa}}{n^{a+1/2}}\right]\right\}+O\left(e^{\gamma\sqrt{n}}\right),$$

$$Q_{1Si}=\frac{(-1)^{1/2}}{2\pi}\sum_{r=0}^{2i}q_{2i,r}^{i,S}\left(-h_S^{(0)}\right)^{-r-i-1/2}\Gamma\left(r+i+\frac{1}{2}\right),$$

$q_{1,r}^{i,S}$ z^i *- quyidagi ko'paytmadagi* z^i *oldidagi koeffisent*

$$\frac{1}{r!}\left(a_S^{(0)}+a_{Sz}^{(1)}+\ldots\right)\left(h_S^{(k)}t+h_S^{(k^2)}z^2+\ldots\right)^r,\ M-1=\left[\tau_0/e_1\right],\tau_0$$ - yetarli kichik son,

$$a_S^{(k)}=\frac{a_1^{(k)}(t_S)}{k!},\ h_S^{(k)}=\frac{1}{(k+2)!}h_S^{(k+2)}(t_S),\quad k=0,1,2,\ldots,$$

R_{1sq} - s bo'yicha tekis chegaralangan.

Ushbu teoremaning isboti [2] dagi teorema 3.4 isbotining aynan o'zi bo'lgani uchun uni bu erda keltirmaymiz.

$a=c_1\sqrt{n}$ bo'lsin. U holda

$$Q_{1Si}\exp\left\{nhs(t_S)+\psi_1^2c_1^2(s+1)^2\right\}=\sum_{r=0}^{\infty}\frac{1}{n^{r/2}}q_r^i(s),$$

$$\sum_{S=0}^{M-1}e^{nhS(tS)}Q_{1Si}=\sum_{S=0}^{M-1}\exp\left\{-\psi_1^2c_1^2(s+1)^2\right\}\sum_{r=0}^{\infty}\frac{1}{n^{r/2}}q_r^i(s)=\sum_{r=0}^{\infty}\frac{1}{n^{r/2}}\sum_{S=0}^{M-1}q_e^i(s)\exp\left\{-\psi_1^2c_1^2(s+1)^2\right\}=$$

$$=\sum_{r=0}^{\infty}\frac{1}{n^{r/2}}\sum_{S=0}^{\infty}q_r^i(s)*\exp\left\{-\psi_1^2c_1^2(s+1)^2\right\}+O\left(e^{-\gamma\sqrt{n}}\right),$$

$$P_{ri}=\sum_{S=0}^{\infty}q_r^i(s)\exp\left\{-\psi_1^2c_1^2(s+1)^2\right\}.$$

Natijada 7.1.1 $a=c_1\sqrt{n},\ c_1>0$ *bo'lganda ixtiyoriy butun* $q\geq 0$ *va* $k\geq 0$ *uchun*

$$P(S_N=k+b,N=n)=P(\chi(b)=k+b,\ \eta(b)=n)-$$

$$-\sum_{i=0}^{q-1}\frac{1}{n^{i+\frac{1}{2}}}\left(\sum_{r=0}^{\infty}\frac{1}{n^{\frac{r}{2}}}P_{ri}\right)+O\left(\frac{1}{n^{q+\frac{1}{2}}}\right),$$

$$P_{00}=0,\quad p_{10}=0,\qquad P_{20}+P_{01}=\frac{g_1^{(1)}}{2\sqrt{\pi}}\cdot\sum_{S=0}^{\infty}\exp\left\{-\psi_1^2c^2(s+1)^2\right\}-$$

$$-\frac{3g_k^{(1)}\cdot\psi_1^2c^2}{4\sqrt{\pi}}\sum_{S=0}^{\infty}(s+1)^2\exp\left\{-\psi_1^2c^2(s+1)^2\right\}.$$

Natija 7.1.2 Ixtiyoriy butun $q\geq 1$ *uchun* $\frac{a}{\sqrt{n}}\to\infty,\qquad k\geq 0$ *bo'lganda*

$$P\cdot(S_N=k+b,N=n)=P\cdot(\chi(b)=k+b,\eta(b)=n)-$$

$$-\left[\sum_{i=0}^{a-1}\frac{1}{n^{i+\frac{1}{2}}}\cdot e^{nh_0(t_0)}Q_{10i}+\frac{1}{n^{q+\frac{1}{2}}}\cdot O\left(e^{-\gamma n\tau_1^2}\right)\right]*\left(1+O\left(e^{-\gamma n\tau_1^2}\right)\right)+O\left(e^{-\gamma\sqrt{n}}\right).$$

$J_2 = \frac{1}{2\pi i} \cdot \int\limits_{\Gamma} \frac{\lambda_1^a(z) \cdot F_2(z) \cdot dz}{z^{n+1}\left(1 - \mu^{b+a}(z) H_2(z)\right)}$ *interval uchun ham yuqoridagiga o'xshash tasdiq o'rnili bo'ladi.*

Teorema 7.2. butun $q \geq 1$ *uchun*

$$J_2(n) = \sum_{i=0}^{q-1} \frac{1}{n^{i+\frac{1}{2}}} \cdot \sum_{S=0}^{M-1} e^{n h_s(t_s)} \cdot Q_{1Si} + \frac{1}{n^{q+\frac{1}{2}}} \sum_{S=0}^{M-1} e^{n h_s(t_s)} R_{isq} + O\left(e^{-\gamma\sqrt{n}}\right) +$$

$$+ O\left(e^{-\gamma k(n)}\right), \quad k(n) \to \infty, \quad n \to \infty.$$

Bu yerda

$$t_s = -\psi_1 \left(s + \frac{1}{2} \right) \tau_1 - \psi_1 s \tau_2 - s \eta_1 \frac{\tau_3}{2} + O\left(\frac{s^2}{n} \right),$$

$$h_s(t_s) = -\left[\psi_1 \left(s + \frac{1}{2} \right) \tau_1 + \psi_1 s \tau_2 + s \eta_1 \tau_{\frac{3}{2}} \right]^2 + O\left(\frac{s^3}{n^{\frac{3}{2}}} \right),$$

$$Q_1 si = \frac{(-1)^{\frac{1}{2}}}{2\pi} \sum_{\eta=0}^{2i} q_{2i,\eta}^s \left(- h_s^{(0)} \right)^{-r-i-\frac{1}{2}} \cdot \Gamma\left(r + i + \frac{1}{2} \right),$$

$q_{i,r}^S$ *- quyidagi ko'paytmadagi* z^i *oldidagi koeffisent.*

$$\frac{1}{r!}\left(a_s^{(0)} + a_s^{(1)} z + \ldots\right) \cdot \left(h_s^{(1)} z + h_s^{(2)} z^2 + \ldots\right)^r, \qquad a_s^{(k)} = \frac{a_2^{(k)}(t_s)}{k!},$$

$$h_s^{(k)} = \frac{1}{(k+2)!} h_s^{(k+2)}(t_s), \quad k = 0,1,2,\ldots, a_2(t) = -2t \cdot \frac{F_2\left(1 - t^2\right)}{4 - t^2}$$

Natija 7.2.1 $a = c\sqrt{n}, c > 0,\ q \geq 1\ \ k \leq 0$ *bo'lganda*

$$P(S_n = k - a, N = n) = \sum_{i=0}^{q-1} \frac{1}{n^{i+\frac{1}{2}}} \left(\sum_{r=0}^{\infty} \frac{1}{n^{\frac{r}{2}}} P_{ri} \right) + O\left(\frac{1}{n^{q+\frac{1}{2}}} \right), \qquad P_{00} = P_{10} = 0,$$

$$P_{20} + P_{01} = \frac{g_1(2)}{2\sqrt{\pi}} \sum_{S=0}^{\infty} \exp\left\{ -\psi_1^2 c^2 \left(s + \frac{1}{2} \right) \right\} - \frac{3 g_1^{(2)} \psi_1^2 c^2}{4\sqrt{\pi}} \sum_{S=0}^{\infty} \left(s + \frac{1}{2} \right)^2 \exp\left\{ -\psi_1^2 c^2 \left(s + \frac{1}{2} \right)^2 \right\}.$$

Natija 7.3.2 $\frac{a}{\sqrt{n}} \to \infty, \quad q \geq 1\ \ k \leq 0$ *bo'lganda*

$$P(S_N = k - a, N = n) = \sum_{i=0}^{q-1} \frac{1}{n^{i+\frac{1}{2}}} e^{n h_0(t_0)} Q_{10i} + \frac{1}{n^{q+\frac{1}{2}}} O\left(e^{-\gamma n \tau_1^2}\right) *$$

$$* \left(1 + O\left(e^{-\gamma n \tau_1^2}\right)\right) + O\left(e^{-\gamma k(n)}\right) + O\left(e^{-\gamma a}\right),$$

$$k(n) \to \infty, n \to \infty.$$

XULOSA

Dissertatsiya ishida bog’liqsiz bir xil taqsimlangan tasodifiy miqdorlar yig’indisining intervaldan birinchi marta chiqish momenti va shu momentdagi qiymati taqsimotlari o’rganilgan.

Qo’shiluvchi tasodifiy miqdorlar faqat butun qiymatlar qabul qiladigan, ya’ni diskret hol qaralgan ikki tomonlama geometrik taqsimot holi suhun qaralayotgan chegaraviy funksionallar taqsimoti uchun aniq formulalar topilgan.

Tasodifiy daydishning yo’lakdan biror marta ham chiqmagan sharti ostidagi local va integral ehtimolliklar uchun limit teoremalar keltirilgan va qoldiq had baholangan.

Odatda Ehtimollar nazariyasining ikki chegarali masalalarda asimptotik izlanishlar ikkala chegara cheksiz izohlashib borishi sharti ostida olib boriladi. Bu yerda o’rganilayotgan tasodifiy miqdorlar taqsimoti uchun asimptotik formullar faqat bir chegara izohlashib borgan holda o’rganilgan. Bunda ikki chegarali masala bir chegarali masalalarga keltirilgan.

Dissertatsiya ishida XX asrning 60-yillarida A.A. Borovkov tomonidan yaratilgan va so’ngi yillarda chegaraviy masalalarni hal etishda keng qo’llanib kelinayotgan faktorizatsion usuldan foydalanilgan.

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ANNOTATSIYA

Ushbu Dissertatsiya ishida bog'liqsiz bir xil taqsimlangan tasodifiy miqdorlar yig'indisining intervaldan birinchi marta chiqish momenti va shu momentdagi qiymati taqsimotlari o'rganilgan.