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Mavzu: Izospektral Shturm-Liuvill chegaraviy masalalari.

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Mundarija

Kirish.

Teskari spectral masalaning zamonaviy nazariyasi T.Borgning birinchi maqolasidan boshlangan.Borgning maqolasidagi ikkita muhim natijalar bizning ishimizga judayam aloqador. Borg ko'rsatadiki, agar $\{\lambda_n\}$ spektr berilgan bo'lsa, u holda $\{\lambda_n\}$ spektrga ega bo'lgan

$$-y''+q(x)y=\lambda y, \qquad 0\le x\le\pi \tag{0.1}$$

$$y'(0)-hy(0)=0 \tag{0.2}$$

$$y'(\pi)+Hy(\pi)=0 \tag{0.3}$$

masalalar cheksiz ko'p ya'ni juda ko'p $q(x)$ potensial va (h,H) juftlik sonlari mavjud. (0.1)-(0.2)-(0.3) masalaning spektrni $\lambda_0<\lambda_1<...<\lambda_n<...$ orqali (0.1) tenglamaga (0.2) chegaraviy shart va ushbu

$$y'(\pi)+H_1y(\pi)=0 \qquad (0.3)'$$

cheraviy shartdan iborat yangi masalani spektrni $\mu_0<\mu_1<...<\mu_n<...$ orqali belgilaymiz, bu yerda $H_1\ne H$. U holda $\{\lambda_n\}$ va $\{\mu_n\}$, $n\ge0$ spektrlar, $q(x)$ potensialni va h, H va H_1 sonlarni yagona ravishda aniqlaydi. Klassik Shturm-Liuvill teskari masalasining rivojlanishdagi keyingi muhim qadam N.Levinson tomonidan tashlangan.

Tarif. (0.1)-(0.2)-(0.3) Shturm –Liuvill chegaraviy masalasi toq deyiladi, agarda

1) $q(x)\equiv q(\pi-x)$ va

2) $H=h$

shartlar bajarilsa.

Teorema. (Levinson [6]). Toq Shturm-Liuvill masalalar sinfida $\{\lambda_n\}_{n=0}^{\infty}$ spektr, $q(x)$ potensalni va h sonni yagona ravishda aniqlaydi. 1950-yilda B.Marchenko ([8]) almashtirish operatorlari yordamida yarim o'qda berilgan Shturm-Liuvill masalalari

$$-y''+q_1(x)y=\lambda y \qquad y'(0)=h_1 y(0);$$
$$-y''+q_2(x)y=\lambda y \qquad y'(0)=h_2 y(0);$$

bir xil spektral funksiyaga ega bo'lsa, u holda $q_2(x)\equiv q_1(x)$ va $h_2=h_1$ bo'lishini isbotladi. T.Borg, N.Levinson va B.Marchenkolarning natijalari muhim va ular keyingi izlanishlarga yo'l ko'rsatadi. Keyinchalik chegaraviy masalalarda $q(x)$ potensalni h,H sonlarni effektiv qurish masalasi muhim o'rin egallaydi. Bu masala ilk bor I.M.Gelfand va B.M.Leviton ([9]) tamonidan yechilgan. Bu ikki fikr bizga spektrol funksiyalar bilan, teskari masalani to'liq o'rganish imkonini beruvchi integral tenglamani tuzish imkonini bergan. Bu integral tenglama ham bu yerda asosiy ro'lni o'ynaydi. Teskari Shturm-Liuvill masalasini yechishni boshqa usuli M.Keyn tamonidan ([4],[5]) o'rganilgan.

Bizning ishimiz kirish va beshta bo'limdan iborat. Boshlang'ich birinchi bo'limda Gelfand-Levitan integral tenglamasini yangi isboti hamda almashtirish operatorini mavjutligini isbotlari keltirilgan. Ikkinchi bo'limda birinchi bo'limni natijalarini izaspertallik masalasini yechishga qo'llaymiz. Yanada aniqroq aytadigan bo'lsak, (0.1)-(0.2)-(0.3) Shturm-Liuvill masalasining $\{\lambda_n\}_{n=0}^{\infty}$ spetri berilgan bo'lsa, u holda nolga yetarli darajada tez intiluvchi va teorema 1.1 ni shartlarini qanoatlantiruvchi har bir haqiqiy C_n, sonlar ketma-ketligi uchun $\lambda_n, n\geq 0$ spektrga ega bo'lgan (0.1)-(0.2)-(0.3) chegaravi masalaning uchligini $(q(x),h,H)$ni qurib olamiz. Biz h va H sonlar uchun aniq formulani keltiramiz, lekin $q(x)$potensialni aniqlash uchun faqat (1.9) formulani beramiz. Uchinchi bo'limda biz to'g'ridan to'g'ri Gelfand-Levitan integral tenglamasini qo'lashga asoslangan izospektral masalasini o'rganishni boshqa usilini qaraymiz. Bu usul hatto chegaraviy masalalar har xil spektrga ega bo'lgan holda ham o'rinlidir. To'rtinchi bo'limda $h+H<2h_0$ ochiq yarim fazo ustiga (3.9)-(3.10) akislantirishni isbotlaymiz. Oxirgi beshinchi bo'limda biz Drexli chegaraviy shartlar holatini qisqacha qaraymiz.

1-§. Boshlang'ich natijalar.

$q_0(x),\ 0 \le x \le \pi$ haqiqiy differensiallanuvchi funksiya va h_0 va H_0 fikserlangan haqiqiy sonlar berilgan bo'lsin. Quyidagi klassik Shturm-Liuvill chegaraviy masalasini qaraymiz.

$$-y''+q_0(x)y=\lambda y, \qquad 0\le x\le \pi \tag{1.1}$$

$$y'(0)-h_0y(0)=0, \qquad y'(\pi)+H_0y(\pi)=0 \tag{1.2}$$

(1.1) tenglamaning

$$\varphi_0(0,\lambda)=1, \qquad \varphi_0'(0,\lambda)=h_0$$

boshlang'ich shartlarni qanoatlantiruvchi yechimi $\varphi_0(x,\lambda)$ bo'lsin.U holda $\varphi_0(x,\lambda)$ har bir λ uchun (1.2) ning birinchi chegaraviy shartini qanoatlantiradi.(1.1)-(1.2) masalaning $\lambda_0<\lambda_1<\lambda_2<\ldots$ xos qiymatlari ushbu

$$\varphi_0'(\pi,\lambda)+H_0\varphi_0(\pi,\lambda)=0 \tag{1.3}$$

tenglmaning ildizlaridan iborat bo'ladi.

Unga mos bo'lgan xos funksiyalar $\varphi_0(x,\lambda_n),n\ge 0$ ko'rinishda bo'ladi. $L^2(0,\pi)$ fazoda,bu funksiyalarni normallashtirmaymiz.Biz integral tenglamani yadrosini quyidagicha tuzamiz.C_n ,$n=0,1,2...$ ketma-ketlik $n\to\infty$ da nolga tez intiluvchi ixtiyporiy haqiqiy sonlar ketma-ketligi bo'lsin va ushbu

$$F(x,y)=\sum_{n=0}^{\infty}C_n\varphi_0(x,\lambda_n)\varphi_0(y,\lambda_n) \tag{1.4}$$

funksiya ikkinchi tartibli uzliksiz hosilaga ega bo'lsin.Bu yerda ushbu

$$K(x,y)+F(x,y)+\int_0^x K(x,t)F(t,y)dt=0, \quad 0\le y\le x\le\pi \tag{1.5}$$

integral tenglama muhim ro'l o'ynaydi.

Teorema.1.1. Barcha $n\ge 0$ lar uchun $\quad 1+C_n\alpha_{n,0}^2>0 \qquad (1.6)$

o'rinli bo'lsin deb faraz qilamiz.U holda (1.5) integral tenglama har bir, $x,\ 0<x\le\pi$ uchun yagona yechimga ega. Bu yerda $\alpha_{n,0}^2=\int_0^\pi\varphi_0^2(x,\lambda_n)dx.$

Isbot. Buning uchun

$$h_x(y)+\int_0^x F(t,y)h_x(t)dt=0 \qquad (1.7)$$

bir jinsli tenglamani faqat nol yechimga egaligini isbotlash yetarli. $h_x(y)$ funksiya (1.7) tenglamaning no'l yechimi bo'lmasin deb faraz qilaylik.(1.7) tenglamani $h_x(y)$ ga ko'paytiramiz va (o,x) oraliqda y o'zgaruvchi bo'yicha integrallaymiz. U holda quyidagiga ega bo'lamiz.

$$\int_0^x h_x^2(y)dy=\sum_{n=0}^{\infty}\frac{1}{\alpha_{n,0}^2}\left(\int_0^x h_x(y)\varphi_0(y,\lambda_n)dy\right)^2$$

Bundan ushbu

$$\int_0^x h_x^2(y)dy+\int_0^x\int_0^x F(t,y)h_x(t)h_x(y)dtdy=\sum_{n=0}^{\infty}\frac{1}{\alpha_{n,0}^2}\left[1+C_n\alpha_{n,0}^2\right]\left[\int_0^x h_x(y)\varphi_0(y,\lambda_n dy)\right]=0$$

kelib chiqadi. Oxirgi tenglikdan va (1.6) shartdan

$$\int_0^x h_x(y)\varphi_0(y,\lambda_n)dy=0, \quad n=0,1,2,...$$

o'rinli bo'lishi kelib chiqadi.

$\{\varphi_n(x,\lambda_n);n\geq 0\}$

funksiyalar sistemasining to'laligidan,biz $h_x(y)\equiv 0$ tenglikka ega bo'lamiz.

Teorema.1.2. (1.5) integral tenglamaning $K(x,y)$ yechimi ushbu

$$\frac{\partial^2 K}{\partial x^2}-q(x)K=\frac{\partial^2 K}{\partial y^2}-q_0(x)K, \quad 0\leq y\leq x\leq \pi \qquad (1.8)$$

husۘusiy hosilali differensial tenglamani qanoatlantiradi, bu yerda

$$q(x)=q_0(x)+2\frac{d}{dx}K(x,x) \qquad (1.9)$$

$K(x,y)$ quyidagi chegaraviy shartlarni ham qanoatlantiradi:

$$K(x,x)=\frac{1}{2}\int_0^x[q(t)-q_0(t)]dt-F(0,0) \qquad (1.10)$$

$$\left.\left(\frac{\partial K}{\partial y}-h_0K\right)\right|_{y=0}=0 \qquad (1.11)$$

Isbot. Isboti oddiy.Biz uni asosiy qisimlarini ko'rsatamiz.Quyidagi belgilashni kiritib olamiz:

$$J = K(x,y) + F(x,y) + \int_0^x K(x,t)F(t,y)dt = 0 \qquad (0 \le y \le x \le \pi)$$

U holda

$$I = J_{xx} - J_{yy} - q(x)J + q_0(y)J = 0 \qquad (1.12)$$

o'rinli bo'ladi, bu yerda $q(x)$ (1.9) ko'rinishdagi ifodaga ega va $q_0(x)$ (1.1) tenglamaning potensiali.Elemetar hisoblashlardan keyin biz (1.12) ifodadan quyidagiga ega bo'lamiz:

1)

$$\left[\frac{\partial^2 K}{\partial x^2} - \frac{\partial^2 K}{\partial y^2} - q(x)K + q_0(y)K\right] + \int_0^x \left[\frac{\partial^2 K}{\partial x^2} - \frac{\partial^2 K}{\partial t^2} - q(x)K + q_0(t)K\right] F(t,y)dt = 0$$

(1.12.1)

2) (1.10) va (1.11) chegaraviy shartlarga ega bo'lamiz.(1.12.1) tenglama ko'rsatadiki

$$h_x(y) = \frac{\partial^2 K}{\partial x^2} - \frac{\partial^2 K}{\partial y^2} - q(x)K + q_0(y)K$$

funksiya (1.7) bir jinsli integral tenglamaning yechimi ekanligini va undan $h_x(y) \equiv 0$ aynan no'lga tengligini topamiz.Bu esa (1.8) differensial tenglamani o'rinli bo'lishini isbotlaydi.

Teorema.1.3. Agar $K(x,y)$ (1.5) integral teng lamaning yechimi bo'lsa,u holda

1) Har bir ko'mpleks λ uchun ushbu funksiya

$$\varphi(x,\lambda) = \varphi_0(x,\lambda) + \int_0^x K(x,t)\varphi_0(t,\lambda)dt \qquad (1.13)$$

quyidagi

$$-y'' + q(x)y = \lambda y, \qquad 0 \le x \le \pi \qquad (1.14)$$

fferensial tenglamaning yechimi bo'ladi,bu yerda

$$(x) = q_0(x) + 2\frac{d}{dx}K(x,x);$$

2) $\varphi(x,\lambda)$ funksiya ushbu boshlang'ich shartlarni qanoatlantiradi.

$$\varphi(0,\lambda), \qquad \varphi'(0,\lambda) = h_0 - \sum_{k=0}^{\infty} C_k = h \tag{1.15}$$

Isbot. (1.15) shartlar (1.13) tenglikda $x = y = 0$ bo'lganda va (1.5) integral tenglamadan hamda $K(0,0) = -F(0,0)$ ifodadan kelib chiqadi. (1.14) differensial tenglamani isbotlash uchun ushbu ayniyatlarni o'rinliligini ko'rsatamiz:

$$\varphi''(x,\lambda) = (q_0 - \lambda)\varphi_0 + \frac{d}{dx}K(x,x)\varphi_0 + K(x,x)\varphi_0' + \left.\frac{\partial K}{\partial x}\right|_{t=0} + \\ + \int_0^x \frac{\partial^2 K}{\partial x^2}\varphi_0(t,\lambda)dt \tag{1.16}$$

$$\lambda\varphi(x,\lambda) = \lambda\varphi_0 - K(x,x)\varphi_0' + h_0 K(x,t)\big|_{t=0} - \left.\frac{\partial K}{\partial t}\right|_{t=0} + \left.\frac{\partial K}{\partial t}\right|_{t=x}\varphi_0 - \\ - \int_0^x \frac{\partial^2 K}{\partial t^2}\varphi_0(t,\lambda)dt + \int_0^x q_0(x)K(x,t)\varphi_0(t,\lambda)dt \tag{1.17}$$

$$q(x)\varphi(x,\lambda) = q(x)\varphi_0(x,\lambda) + \int_0^x q(x)K(x,t)\varphi_0(t,\lambda)dt \tag{1.18}$$

(1.16),(1.17) va (1.18) tenglamalarini qo'shish orqali va (1.8) tenglamadan foydalanib,ushbu

$$\varphi'' - q(x)\varphi + \lambda\varphi = \int_0^x \left[\frac{\partial^2 K}{\partial x^2} - \frac{\partial^2 K}{\partial t^2} - q(x)K + q_0(t)K\right]\varphi_0(t,\lambda)dt$$

tenglamaga ega bo'lamiz.

Teorema.1.4. (1.1)-(1.2) qo'zg'almas masalaning xos qiymatlari λ_n, $n \geq 0$ bo'lsin, u holda (1.13) tenglamadan aniqlangan va λ o'rniga λ_n qo'yilgan holdagi $\varphi(x,\lambda_n)$ funksiyalar ushbu formula bilan ifodalaniladi:

$$\varphi(x,\lambda_n) = \varphi_0(x,\lambda_n) - \sum_{k=0}^{\infty} C_n(x,\lambda_k) \int_0^x \varphi_0(t,\lambda_n)\varphi_0(t,\lambda_k)dt \qquad (1.19)$$

Isbot. (1.4) dan va (1.5) integral tenglamadan quyidagini yozib olamiz.

$$K(x,t) = -F(x,t) - \int_0^x K(x,s)F(s,t)ds =$$

$$= -\sum_{k=0}^{\infty} C_k\varphi_0(t,\lambda_k)\left[\varphi_0(x,\lambda_k) + \int_0^x K(x,s)\varphi_0(s,\lambda_k)ds\right] = -\sum_{k=0}^{\infty} C_k\varphi_0(t,\lambda_k)\varphi(x,\lambda_k)$$

(1.20)

(1.19) formula, endi (1.13) dagi va $K(x,t)$ uchun yozilgan (1.20) ifodadan kelib chiqadi.

Eslatma. Biz bu bo'limda ruxsat etilgan har bir $\{C_n\}_{n=0}^{\infty}$ ketma-ketlik uchun berilgan $\{\lambda_n\}$, $n \geq 0$ spektrga va $q(x)$ potensiolga ega bo'lamiz.

2-§. Izospektrallik masalasi.

1) (1.13) va (1.19) formulalar bizga izaspektrallik masalasini qarash imkonini beradi. Biz h_0 va H_0 chekli sonlar deb faraz qilamiz.Birinchi bo'limda teorema.1.3 da ko'rsatilganidek $\varphi(x,\lambda_n)$, $n \geq 0$ funksiyalar (1.14) differensial tenglamalarni qanotlantiradi va shuning uchun ushbu

$$\varphi'(0,\lambda_n) - h\varphi(0,\lambda_n) = 0$$

chegaraviy shartlarini ham qanoatlantiradi, bu yerda

$$h = h_0 - \sum_{n=0}^{\infty} C_n \tag{2.1}$$

(1.1)-(1.2) operator bilan izospektral operatorni qurish uchun, haqiqiy H uchun va $x=\pi$ nuqtada $\varphi(x,\lambda_n)$ ni ushbu

$$\varphi'(x,\lambda_n) + H\varphi(x,\lambda_n) = 0$$

chegaraviy shartni qanoatlantirishini ko'satish kerak.Bu bilan bir vaqtda H uchun ham formula olamiz.(1.19) formulaga $x=\pi$ ni qo'yamiz va quyidagiga ega bo'lamiz:

$$\begin{aligned}\varphi(\pi,\lambda_n) &= \varphi_0(\pi,\lambda_n) - \sum_{k=0}^{\infty} C_k \varphi(\pi,\lambda_k) \int_0^{\pi} \varphi_0(t,\lambda_k)\varphi_0(t,\lambda_n)dt = \\ &= \varphi_0(\pi,\lambda_n) - C_n\alpha_{n,0}^2\varphi(\pi,\lambda_n)\end{aligned} \tag{2.2}$$

Bu yerda $\alpha_{n,0}^2 = \int_0^{\pi} \varphi_0^2(x,\lambda_n)dx$

(2.2) formuladan

$$\varphi(\pi,\lambda_n) = \frac{\varphi_0(\pi,\lambda_n)}{1 + C_n\alpha_{n,0}^2} \tag{2.3}$$

ekanligi kelib chiqadi.(1.19) differensiallaymiz

$$\varphi'(x,\lambda_n) = \varphi_0(x,\lambda_n) - \sum_{k=0}^{\infty} C_k \varphi'(x,\lambda_n) \int_0^{x} \varphi_0(t,\lambda_k)\varphi_0(t,\lambda_k)dt -$$

$$- \sum_{k=0}^{\infty} C_k \varphi(x,\lambda_k)\varphi_0(x,\lambda_k)\varphi_0(x,\lambda_n)$$

$x=\pi$ uchun (2.3) va(1.2) ning ikkinchi chegaraviy shartdan foydalanib biz quyidagiga ega bo'lamiz:

$$\varphi'(\pi,\lambda_n)(1+C_n\alpha_{n,0}^2)=-H_0\varphi_0(\pi,\lambda_n)-\varphi_0(\pi,\lambda_n)\sum_{k=0}^{\infty}C_k\varphi(\pi,\lambda_k)\varphi_0(\pi,\lambda_k)=$$

$$=-H_0\varphi(\pi,\lambda_n)(1+C_n\alpha_{n,0}^2)-(1+C_n\alpha_{k,0}^2)\varphi(\pi,\lambda_n)\sum_{k=0}^{\infty}\frac{\varphi_0^2(\pi,\lambda_k)}{1+C_k\alpha_{k,0}^2}$$

Oxirgi tenglamadan esa

$$\varphi'(\pi,\lambda_n)=-H\varphi(\pi,\lambda_n)$$

bo'lishi kelib chiqadi,bu yerda

$$H=H_0+\sum_{k=0}^{\infty}C_k\frac{\varphi_0^2(\pi,\lambda_k)}{1+C_k\alpha_{k,0}^2} \tag{2.4}$$

3) Endi barcha $n\geq 0$ uchun

$$\varphi_0(\pi,\lambda_n)=(-1)^n \tag{2.5}$$

deb faraz qilamiz.Keyingi bo'limda biz (1.1)-(1.2) Shturm-Liuvill masalasini toq bo'lishi uchun (2.5) shartni bajarishi zarur va yetarli.Toq masala uchun (2.4) formula ushbu ko'rinishni oladi.

$$H=h_0+\sum_{k=0}^{\infty}\frac{C_k}{1+C_k\alpha_{k,0}^2} \tag{2.6}$$

(2.1) va (2.6) larni qo'shish orqali

$$h+H=2h-\sum_{k=0}^{\infty}\frac{C_k^2\alpha_{k,0}^2}{1+C_k\alpha_{k,0}^2} \tag{2.7}$$

ifodani olamiz.Keyingi teoremada (2.7) ni qo'llash yordamida h va H larni fazodagi nuqtaning koordinatalari sifatida qaraymiz.

Teorema.2.1. 1) (h,H) nuqtalar $h=H<2h_0$ va ochiq yarim fazoda yotadilar:

2)Agar $h+H=2h_0$ bo'lsa,u holda $h=H=h_0$ va $q(x)=q_0(x)$ o'rinli bo'ladi.

Isbot. 1-hol (2.7) tenglamadan darrov kelib chiqadi. 2-holni ko'rsatish uchun esa biz $h+H=2h_0$ deb faraz qilamiz. U holda (2.7) tenglamadan

$$\sum_{k=0}^{\infty}\frac{C_k^2\alpha_{k,0}^2}{1+C_k\alpha_{k,0}^2}=0$$

bo'lishi kelib chiqadi chiqadi.Barcha k uchun $1+C_k\alpha_{k,0}^2>0$ bo'lishidan biz $C_k=0,\ k\geq 0$ bo'lishini ko'ramiz.

Eslatma.Teorema.2.1. dagi 2-hol Amborsiumyan klassik teoremasini isbotlash uchun foydalaniladi .

Teorema.2.2. (B.A.Amborsiumyan [13])

Ushbu

$$\begin{aligned}&-y''+q(x)y=\lambda y,\quad 0\leq x\leq\pi\\&y'(0)=y'(\pi)=0\end{aligned}\qquad(2.8)$$

masalani hos qiymatlari $\lambda_n=n^2,\ n\geq 0$ bo'lsa, u holda $q(x)\equiv 0$ bo'ladi.

Isbot. (2.8) qo'zg'algan va ushbu

$$\begin{aligned}&-y''=\lambda y,\quad 0\leq x\leq\pi\\&-y'(0)=y'(\pi)=0\end{aligned}$$

qo'zg'almagan masalani qaraymiz.Bu ikki masala bir xil $\lambda_n=n^2,\ n\geq 0$ spektrga ega. (Bu bizga (2.7) formulani korrektligi uchun zarur).Qo'zg'almagan masala uchun $h=H=0$ o'rinli.Qo'zg'almagan masala toq va uning uchun $h_0=H_0=0$ o'rinli. Endi biz teorema.2.1 ni 2-holini qo'llasak teoremani isbotlagan bo'lamiz.

3-§.Alternativ usul.

Bu bo'liomda biz (2.1) va (2.4) formulani alternativ isbotlarini beramiz. Bu isbot Gelfand-Levitan integral tenglamasiga asoslangan va uni umumlashgan holi quyida ko'rsatiladi.Ushbu

$$\begin{cases} -y''+q(x)y=\lambda y \\ y'(0)-hy(0)=0 \\ y'(\pi)+Hy(\pi)=0 \end{cases} \tag{3.1}$$

masalani qaraymiz. Quyidagi

$$\varphi(0,\lambda)=0$$
$$\varphi'(0,\lambda)=h$$

boshlangich shartlarni qanoatlantiruvchi (3.1) tenglamani yechimi $\varphi(x,\lambda)$ bo'lsin.(3.1) masalani $\lambda_0<\lambda_1<\lambda_2<...$ xos qiymatlari (3.1) dagi ikkinchi chegaraviy shartdan kelib chiqqan ushbu

$$D(\lambda)\equiv\varphi'(\pi,\lambda)+H\varphi(\pi,\lambda)=0$$

tenglamani ildizlaridir. Ushbu

$$\alpha_n^2=\int_0^\pi\varphi^2(x,\lambda_n)dx\ ,\quad n\geq 0$$

belgilashni kiritib olamiz.

1)Biz boshida quyidagi ikkita lemmeni keltiramiz.

Lemma 3.1. Quyidagi ifoda o'rinli

$$\alpha_n^2=\varphi(\pi,\lambda_n)\cdot\dot{D}(\lambda_n)=|\varphi(\pi,\lambda_n)||\dot{D}(\lambda_n)|\ ,\quad n=0,1,2... \tag{3.2}$$

Bu yerda $\dot{D}(\lambda)-D(\lambda)$ ni λ ga nisbatan hosilasini bildiradi.

Isbot. Ixtiyoriy λ va μ uchun $y=\varphi(x,\lambda)$ $z=\varphi(x\mu)$ bo'lsin. Grin ayniyatidan

$$\int_0^\pi yzdx=\frac{\varphi'(\pi,\lambda)\varphi(\pi,\mu)-\varphi(\pi,\lambda)\varphi'(\pi,\mu)}{\mu-\lambda}$$

O'rinli bo'lishi kelib chiqadi. Bundan biz ushbu

$$\int_0^\pi\varphi^2(x,\lambda)dx=\varphi'(\pi,\lambda)\dot{\varphi}(\pi,\lambda)-\varphi(\pi,\lambda)\dot{\varphi}'(\pi,\lambda)$$

ifodani olamiz. Agar $\lambda = \lambda_n$ bo'lsa

$\varphi'(\pi,\lambda_n) = -H\varphi(\pi,\lambda)$

ekanligidan

$\alpha_n^2 = -\varphi'(\pi,\lambda_n)[\dot{\varphi}'(\pi,\lambda_n) + H\dot{\varphi}(\pi,\lambda_n)] = -\varphi(\pi,\lambda_n)\dot{D}(\lambda_n)$

bo'lishi kelib chiqadi.

Eslatma. 1. $\varphi(\pi,\lambda_n) = 0$ bo'lgan holda

$$\alpha_n^2 = \int_0^\pi \varphi^2(x,\lambda_n)dx = \varphi'(\pi,\lambda_n)\dot{\varphi}(\pi,\lambda) \tag{3.3}$$

bo'ladi.

Eslatma. 2. $\{\lambda_n\}$, $n \geq 0$ spektr berilgan bo'lsin. U holda biz $D(\lambda)$ va bundan $\dot{D}(\lambda)$, $n \geq 0$ larni aniqlab bilamiz. Agarda $\varphi(\pi,\lambda_n)$, $n \geq 0$ qiymatlar ham berilgan bo'lsa, u holda (3.2) va (3.3) formulalardan foydalangan holda α_n^2 normallovchi o'zgarmaslarni topib olib bilamiz. Spektr va normallovchi o'zgarmaslardan esa potensial va chegaraviy shartlarni yagona ravishda aniqlovchi spectral funksiyani aniqlab bilamiz.

Lemma.3.2. (N.Levinson[6]).

(3.1) Shturm-Liuvill masalasini toq bo'lishi uchun

$$\varphi(\pi,\lambda_n) = (-1)^n, \quad n = 0,1,2,.... \tag{3.4}$$

o'rinli bo'lishi zarur va yetarli. Bu teoremaning isboti A.B.Hasanovning [1] kitobida keltirilgan.

2)(3.2) formulani qo'llagan holda (2.1) formulaning va (2.4) formulaning o'ng tomonlaridagi $\{C_n\}$ ketma-ketliklarni xos funksiyalarning chegaraviy hollari yordamida ifodalangan formulalari bilan shakl almashtiramiz. (1.1)-(1.2) va (3.1) masalalarni qaraymiz.

$$\alpha_{n,0}^2 = \int_0^x \varphi_0^2(x,\lambda_n)dx, \quad \alpha_n^2 = \int_0^x \varphi^2(x,\lambda_n)dx$$

o'rinli bo'lsin. Bu ikki masalani birlashtiruvchi integral tenglama quyidagicha bo'ladi.

$$K(x,y)+F(x,y)+\int_0^x K(x,t)F(t,y)dt=0,\ 0\le y\le x\le\pi \qquad (3.5)$$

bu yerda

$$F(x,y)=\sum_{n=0}^{\infty}\left(\frac{1}{\alpha_n^2}-\frac{1}{\alpha_{n,0}^2}\right)\varphi_0(x,\lambda_n)\varphi_0(y,\lambda_n) \qquad (3.6)$$

Agar (3.5)-(3.6) tenglamalarni (1.4)-(1.5) tenglamalar bilan solishtirsak, ushbu $C_n=\frac{1}{\alpha_n^2}-\frac{1}{\alpha_{n,0}^2}$ tenglikka ega bo'lamiz. Bunadan esa (3.2) formula yordamida biz

$$C_n=\frac{1}{|\dot{D}(\lambda_n)|}\left(\frac{1}{|\varphi(\pi,\lambda_n)|}-\frac{1}{|\varphi_0(\pi,\lambda_n)|}\right) \qquad (3.7)$$

ifodani olamiz va (2.1) dan esa biz birinchi almashtirish formulasini olamiz.

$$h=h_0-\sum_{n=0}^{\infty}\frac{1}{|\dot{D}(\lambda_n)|}\left(\frac{1}{|\varphi(\pi,\lambda_n)|}-\frac{1}{|\varphi_0(\pi,\lambda_n)|}\right) \qquad (3.8)$$

Agar qo'zg'almagan masala toq bo'lsa, u holda $\varphi_0(\pi,\lambda_n)=(-1)^n$ o'rinli bo'ladi va (3.8) formula quyidagi sodda formulani oladi:

$$h=h_0-\sum_{n=0}^{\infty}\frac{1}{|\dot{D}(\lambda_n)|}\left(\frac{1}{|\varphi(\pi,\lambda_n)|}-1\right) \qquad (3.9)$$

(2.4) formula uchun xuddi shunday ifodani olish uchun, biz birinchi $C_n\frac{\varphi_0^2(\pi,\lambda_n)}{1+C_n\alpha_{n,0}^2}$ ifodali shakl bilan almashtiramiz C_n uchun bizda (3.7) ko'rinishdagi ifoda bor. (3.2) ifodadan foydalanib biz ushbu

$$1+C_n\alpha_{n,0}^2=1+\left(\frac{1}{\alpha_n^2}-\frac{1}{\alpha_{n,0}^2}\right)\alpha_{n,0}^2=\frac{\alpha_{n,0}^2}{\alpha_n^2}=\frac{\varphi_0(\pi,\lambda_n)}{\varphi(\pi,\lambda_n)}$$

tenglikka ega bo'lamiz. Oddiy hisoblashlar orqali biz

$$C_n\frac{\varphi_0^2(\pi,\lambda_n)}{1+C_n\alpha_{n,0}^2}=\frac{1}{|\dot{D}(\lambda_n)|}\left(|\varphi_0(\pi,\lambda_n)|-|\varphi(\pi,\lambda_n)|\right)$$

ifodaga ega bo'lamiz. Oxirgi natija (2.4) ni yangi ko'rinishini ya'ni ikkinchi shakl almashtirish formulasini yozamiz.

$$H=h_0-\sum_{n=0}^{\infty}\frac{1}{|\dot{D}(\lambda_n)|}(\varphi(\pi,\lambda_n)-1) \tag{3.10}$$

(agarda qo'zg'almagan masala toq bo'lsa).

4) (3.9) va (3.10) formulalar qo'zg'algan va qo'zg'almagan masalalarning $\{\mu_n\}$ va $\{\lambda_n\}$ spektrlari har xil bo'lgan holni umumlashtirish imkonini beradi. Bu holda integral tenglama xuddi (3.5) ni ko'rinishida bo'ladi, lekin $F(x,y)$ funksiya yangi murakkabroq ifodaga ega bo'ladi.

$$F(x,y)=\sum_{n=0}^{\infty}\left[\frac{1}{\alpha_n^2}\varphi_0(x,\mu_n)\varphi_0(y,\mu_n)-\frac{1}{\alpha_{n,0}^2}\varphi_0(x,\lambda_n)\varphi_0(y,\lambda_n)\right] \tag{3.11}$$

Oldingidek (3.5) integral tenglamadan

$$(3.11)\rightarrow h=h_0-\sum_{n=0}^{\infty}\frac{1}{\alpha_n^2}-\frac{1}{\alpha_{n,0}^2}=$$

$$=h_0-\sum_{n=0}^{\infty}\left\{\frac{1}{|\dot{\varepsilon}(\mu_n)|}\cdot\frac{1}{|\varphi(\pi,\mu_n)|}-\frac{1}{|\dot{D}(\lambda_n)|}\cdot\frac{1}{|\varphi_0(\pi,\lambda_n)|}\right\}$$

bo'lishi kelib chiqadi, bu yerda esa

$$D(\lambda)=\varphi_0'(\pi,\lambda)+H_0\varphi_0(\pi,\lambda),$$
$$\varepsilon(\lambda)=\varphi'(\pi,\lambda)+H\varphi(\pi,\lambda)$$

(3.10) formulaning umumlashmasini olish uchun ushbu

$$\begin{aligned}&-y''+q(\pi-x)y=\lambda_n y\\&-y''+q(\pi-x)y=\mu_n y,\quad 0\le x\le\pi\end{aligned} \tag{3.12}$$

tenglamalarni qaraymiz. Bu tenglamarning yechimlari quyidagicha bo'ladi

$$\psi_0(x,\lambda_n)=\frac{\varphi_0(\pi-x,\lambda_n)}{\varphi_0(\pi,\lambda_n)}$$

$$\psi(x,\mu_n)=\frac{\varphi(\pi-x,\mu_n)}{\varphi(\pi,\mu_n)}$$

keyingi bo'limni natijalaridan foydalanib ixtiyoriy haqiqiy h va H $(h+H<2h_0)$ va berilgan spektr bilan Shutrm-Liuvill masalasini mavjudligini isbotlay olamiz.

Eslatma.2. Ushbu ifodani eslatib o'tamiz:

$$\alpha_n^2=\int_0^{\pi}\varphi^2(x,\mu_n)dx=-\varphi(\pi,\mu_n)\dot{\varepsilon}(\mu_n)$$

Demak, $\{\mu_n\}$ spektr va $\varphi(\pi,\mu_n)$ sonlar spektral funksiyani aniqlaydi, demak Shturm-Liuvill masalasi ham yagona ravishda aniqlanadi.

4-§. (3.9)-(3.10) akslantirishni $h+H<2h_0$ yarim fazo ustidaligini isbotlash.

Eslatib o'tamiz (3.10) va (3.9) tenglamalar mos ravishda (3.1) qo'zg'algan masalaning chegaraviy shartlaridag h va H parametrlarini ifodalaydi, bu yerda qo'zg'almagan masalani $p_0(x)$ potensiali toq deb olinadi:

$$H=h_0-\sum_{n=0}^{\infty}\frac{1}{\left|\dot{D}(\lambda_n)\right|}\left(\left|\varphi(\pi,\lambda_n)\right|-1\right) \qquad (4.1)$$

$$h=h_0-\sum_{n=0}^{\infty}\frac{1}{\left|\dot{D}(\lambda_n)\right|}\left(\frac{1}{\left|\varphi(\pi,\lambda_n)\right|}-1\right) \qquad (4.2)$$

(2.1) teoremaga ko'ra $h+H<2h_0$ bo'ladi.

Bu bo'limda biz $h+H<2h_0$ yarim fazoda har bir (h,H) nuqta (3.1) Shturm-Liuvill masalasining (h,H) parametrlari sifatida qaraladi. Bu Shturm-Liuvill maslasi yagona bir xil $\{\lambda_n\}$ spektr va yagona aniqlangan $(h_0,H_0)=(h_0,h_0)$ parametr bilan berilgan yagona toq masalaning qo'zg'alishidir. Bu shuni anglatadiki, biz shunday $q(x)$ patensiyalni topishimiz kerakki $\left|\varphi(\pi,\lambda_n)\right|$ sonlar (4.1) va (4.2) tenglamalarni qanoatlantiradi.

Teorema4.1. (3.1) Shturm-Liuvill masalasining spektri $\{\lambda_n\}$ bo'lsin, u yerdagi chegaraviy shartlardan paydo bo'ladigan h_0 esa yagon aravishda aniqlangan parametr bo'lsin. U holda $h+H<2h_0$ yarim fazodagi barcha (h,H) nuqtalar uchun shunday $q(x)$ potensial mavjudki, $q(x)$ potensial va chegaraviy (h,H) parametrlar bilan berilgan Shturm-Liuvill masalasini $\varphi(x,\lambda_n)$ xos funksiyalari bor va bunda (4.1) va (4.2) tenglamalar qanoatlantiriladi.

Isbot. Biz buni ikki qadam bilan ko'rsatamiz. Birinchi qadam bu algebraik, ikkinchi qadamda esa biz mos potensialni qurishni isbotlaymiz. Birinchi qadam: bu qadamni cheksiz ko'p nomalumlar bilan berilgan ikkita algebraic tenglamalar sistemasi bilan boshlaymiz. (4.1) va (4.2) tenglamalar quyidagi ko'rinishlarga ega:

$$\sum_{n=0}^{\infty} \omega_n (x_n - 1) = a, \qquad \sum_{n=0}^{\infty} \omega_n \left(\frac{1}{x_n} - 1 \right) = b \tag{4.3}$$

bu yerda ω_n berilgan musbat sonlar a va b sonlari berilgan haqiqiy sonlar, shuningdek musbat $\{x_n\}$ sonlar ketma-ketligi (4.3) tenglamalarni qanoatlantiradigan etib topiladi. Barcha n lar uchun $x_n = 1$ bo'lmaganda $a + b > 0$ bo'lishi zarur, chunki $x = 1$ dan tashqari barcha musbat x lar uchun $x + \frac{1}{x} - 2 > 0$ bo'ladi. Biz (4.3) tenglamani aynan 1 ga teng yechimga ega ekanligini ko'rsatamiz. Buni ayrim kvadratik tenglamalarni yechish orqali ettiramiz. Bu xususiy holdagi masala bilan boshlanadi. Bundan keyin shu fikrlarni umumiy holda yechimlarni toppish uchun ishlatamiz. Bundan tashqari xususiy hol ayrim tadbiqlarda yetarli bo'lishi ham mumkin. bu yerda ushbu masalani keltiramiz. Mayli $\omega_n \equiv 1, x_1 = x, x_2 = y$ va $x_n = 1$ bo'lsin barcha $n > 2$ da. Biz shunday barcha a va b sonlarni izlaymizki, ular $a + b > 0$ barcha musbat x va y lar uchun $x + y - 2 = 0$ va $\frac{1}{x} - \frac{1}{y} - 2 = b$ o'rinli bo'lsin. Oddiylik uchun ushbu belgilashlarni kiritamiz. $\alpha = a + 2, \beta = b + 2$ demak $x + y = \alpha > 0, \frac{1}{x} + \frac{1}{y} = \beta > 0, x > 0, y > 0, \alpha + \beta > 4$ tengliklarni yechimlarini izlaymiz. Bu yerda $\alpha > 0, \beta > 0$ (yoki $a > -2, b > -2$) bo'lishi kerak, chunki $x > 0, y > 0$. Natijada biz

$$\beta xy = x + y = \alpha, \; y = \alpha - x, \; \beta x(\alpha - x) = \alpha, \beta x^2 - \alpha\beta x + \alpha$$

deb yozishimiz mumkin. U holda

$$x_{\pm} = \frac{(\alpha\beta \pm \sqrt{(\alpha\beta)^2 - 4\beta\alpha})}{2\beta}$$

bo'ladi. $x_\pm$ yechimlar musbat bo'lishi uchun $\alpha\beta > 4$ bo'lishi zarur va yetarli. Agar shu shart bajarilsa $x_\pm > 0$ bo'ladi. Bundan tashqari

$$y_\pm = \alpha - x_\pm = \frac{\alpha\beta \pm \sqrt{(\alpha\beta)^2 - 4\beta a}}{2\beta} > 0$$

bo'ladi. Demak berilgan tenglamalar musbat (x, y) yechimga ega bo'lishi uchun $a > -2, b > -2, a + b > 0$ va $(a+2)(b+2) \geq 4$ bo'lishi zarur va yetarli ekan. Geometrik manoda esa, $(a+2)(b+2) = 4$ giperbolaning yuqori shohidagi (0,0) dan tashqari barcha (a,b) nuqtalar ushbu ko'rinishda ifodalanishi mumkin.

$$(a,b) = \left(x + y - 2, \frac{1}{x} + \frac{1}{y} - 2\right), x > 0, y > 0.$$

(0,0) ni chiqarib tashlashni asosiy sababi shuki, koordinatalarining yig'indisi musbat bo'lmay qoladi. Bu misol bizni masalamizni yecha olmaydi, chunki $a + \mathrm{b} > 0$ yarim fazodagi hamma nuqtalar talab qilingan ko'rinishda ifodalana olmaydilar.

1. $\omega_n \equiv 1$ bo'lgandagi xususiy hol. (4.3) tenglamalarni xususiy holi sifatida quyidagilarni qaraymiz:

$$x_n = \begin{cases} x, agar\ 0 \leq n < k \\ y, agar\ k \leq n < 2k \\ 1,\ agar\ n \geq 2k \end{cases}$$

(4.3) tenglamalar ushbu masalaga aylanadi: shunday barcha a va b larni topish kerakki, $a + b > 0$ bo'lsin va barcha musbat x va y lar uchun

$$kx + ky - 2k = a, \frac{k}{x} + \frac{k}{y} - 2k = b$$

bo'lsin. Ikkala tenglamani k ga bo'lish orqali biz yuqoridagi qaralayotgan masalamizga qaytamiz, lekin a va b ni o'rnida $\frac{a}{k}$ va $\frac{b}{k}$ turadi. Bunga asosan tenglamalar musbat (x, y) juftlikka ega ega bo'lishi uchun $a > -2k, b > -2k$ va $a + b > 0$ va $(a+2k)(b+2k) \geq 4k^2$ bo'lishi zarur va yetarli. Geometric jihatdan

$(a+2k)(b+2k)\geq 4k^2$ giperbolaning yuqori shohidagi $(0,0)$ dan tashqari barcha (a,b) nuqtalar

$$(a,b)=\left(kx+ky-2k,\frac{k}{x}+\frac{k}{y}-2k\right),x>0\,,y>0$$

ko'rinishda ifodalashi mumkin. Agar $a+b>0$ bo'lsa, u holda k yetarlicha katta qiymatlarida

$$(a+2k)(b+2k)-4k^2=ab+2k(a+b)\geq 0$$

bo'ladi.

2.Umumiy hol. Endi $\sum_{n=0}^{\infty}\omega_n=+\infty$ deb olamiz. Bizning tadbiqimizda x_n ni quyidagicha qilib olamiz:

$$x_n=\begin{cases}x, agar\ 0\leq n<k\\ y, agar\ n=k\\ 1,\ agar\ n>k\end{cases}$$

(4.3) tenglama ushbu ko'rinishda bo'ladi:

$$\begin{aligned}&\sum_{n=0}^{k-1}\omega_n x+\omega_k y-\sum_{n=0}^{k}\omega_n=a\,,\\&\sum_{n=0}^{k-1}\frac{\omega}{x}+\frac{\omega_k}{y}-\sum_{n=0}^{k}\omega_n=b\,.\end{aligned}\qquad(4.4)$$

Ifodani soddalashtirish uchun quyidagicha belgilash kiritamiz.

$$v=\sum_{n=0}^{k-1}\omega_n\quad \text{va}\quad \omega=\omega_k$$

(4.4) tenglamalar bilan bog'langan masala ushbu ko'rinishni oladi.

$a+b>0$ shartni qanoatlantiruvchi barcha a,b larni toppish kerakki musbat x va y lar uchun

$$vx+\omega y-(v+\omega)=a\quad \text{va}\quad \frac{v}{x}+\frac{\omega}{y}-(v+\omega)=b\,.$$

Yana oddiylik uchun $\alpha=a+v+\omega$ va $\beta=b+v+\omega$ belgilashlar kiritamiz va bundan xuddi oldingidek musbat (x,y) juftlik yechimlarni izlaymiz:

$$\nu x+\omega y=\alpha>0\ , \qquad \frac{\nu}{x}+\frac{\omega}{y}=\beta>0$$

Quyidagilarni yozishimiz mumkin:

$y=(\alpha-\nu x)/\omega\ , \quad \beta xy=\nu y+\omega x,\ \beta x(\alpha-\nu x)=\omega^2 x+\nu(\alpha-\nu x),$

$-\nu\beta x^2+(\alpha\beta+\nu^2-\omega^2)x+\nu d=0,\ \nu\beta x^2-(\alpha\beta+\nu^2-\omega^2)x+\nu d=0$

bu x ning ikkita qiymatini beradi.

$$x=\frac{(\alpha\beta+\nu^2-\omega^2)\pm\sqrt{(\alpha\beta+\nu^2-\omega^2)^2-4(\nu\beta)\nu\alpha}}{2(\nu\beta)} \qquad (4.5)$$

Bizga suratni va diskriminantni musbat bo'lishi muhim. Agar diskiriminantni

$(\alpha\beta+\nu^2-\omega^2)^2-4(\alpha\beta)\nu^2$

ko'rinishida yozsak, unda u ushbu

$$(A+B-C)^2-4AB=(A-B+C)^2-4AC$$

ko'rinishga ega ekanligini ko'rsatamiz, bu yerda A,B va C lar musbat. Bu ifoda nomanfiy bo'ladi, agarda faqat va faqatgina

$$A^2-2(B+C)A+(B-C)^2\geq 0$$

o'rinli bo'lsa, boshqacha aytganda shunday intervaldan tashqarida bo'lishi kerakki uning oxirgi $A_\pm$ nuqtalari ushbu formula bilan aniqlanadi.

$$A_\pm=\frac{2(B+C)\pm\sqrt{(2(B+C))^2-4(B-C)^2}}{2}=B+C\pm\sqrt{(B+C)^2-(B-C)^2}=$$
$$=(\sqrt{B}\pm\sqrt{C})^2$$

ya'ni

$$A\leq(\sqrt{B}-\sqrt{C})^2 \text{ yoki } A\geq(\sqrt{B}+\sqrt{C})^2 \qquad (4.6)$$

Biz $A=\alpha\beta$, $B=\nu^2$ va $C=\omega^2$ deb olsak.U holda (4.6) bizga diskriminantning nomanfiy bo'lish shartini beradi:

$$\alpha\beta\leq(\nu-\omega)^2 \text{ yoki } \alpha\beta\geq(\nu+\omega)^2 \qquad (4.7)$$

(4.5) bilan berilgan x ning qiymatlari musbat bo'lishiga ishonch hosil qilish uchun

$$\alpha\beta+\nu^2-\omega^2>0$$

bo'lishini bilishimiz zarur.

Ya'ni biz

$$\alpha\beta > \omega^2 - v^2 \tag{4.8}$$

deb olishimiz kerak.Biz bu qiymatning ham musbatligiga ishonch hosil qilishimiz kerak.Bizda ushbu

$$y = (\alpha - vx)/\omega$$

ifoda bor va bu

$$y = \frac{(\alpha\beta - v^2 + \omega^2) \pm \sqrt{(\alpha\beta - v^2 + \omega^2)^2 - 4\alpha\beta\omega^2}}{2\omega\beta} \tag{4.9}$$

beradi.Diskriminant xuddi x nikidan ozgina boshqacha ifodalangan. Demak, agar (4.7) o'rinli bo'lsa diskriminant nomanfiy bo'ladi.Biz yana

$$\alpha\beta > v^2 - \omega^2 \tag{4.10}$$

bo'lishini bilishimiz zarur va --------- (4.11)

(4.11) shart (4.8) va (4.10) larni o'rinli bo'lishini keltirib chiqaradi. Bunga asosan (4.4) tenglamani musbat juft yechimga ega bo'ladigan zaruriy va yetarlilik shartlari quyidagicha bo'ladi:

$$a > -(v+\omega), \quad b > -(v+\omega), \; a+b > 0$$

va (4.12)

$$(a+\omega+v)(b+v+\omega) \geq (v+\omega)^2$$

geometrik jihatdan ushbu

$$(x+v+\omega)(y+v+\omega) = (v+\omega)^2$$

giperbolaning (0,0) nuqtadan boshqa yuqori shoxidagi barcha (a,b) nuqtalari ushbu ko'rinishda ifodalanishi mumkin.

$$(a,b) = \left(vx + \omega y - (v+\omega), \frac{v}{x} + \frac{\omega}{y} - (v+\omega) \right)$$

(4.5) orqali berilgan $x > 0$ va (4.9) orqali berilgan

$$y = (\alpha - vx)/\omega > 0$$

Biz $\sum_{n=0}^{\infty} \omega_n$ qatorning ixtiyoriy chekli yig'indilarini

$$v=\sum_{n=0}^{k-1}\omega_n$$

topishimiz bilan, biz ko'ramizki bu turdagi giperbolaning yuqori soxasi $a+b>0$ yarim fazoni to'ldiradi, huddi hususiy holdagidek.Bu yerdan $a+b>0$ qanoatlantiruvchi barcha a va b uchun (4.3) tenglamalarni yechimini mavjudligini bilishimiz uchun, biz faqat qatorni uzoqlashuvchi deb hisoblashimiz kerak. Hususan, $a+b>0$ qanoatlantiruvchi a va b berilgan bo'lsin. k shunday katta qilib tanlashimiz kerakki

$$v+\omega>-a,\ v+\omega>-b$$

va (4.13)

$$v+\omega\geq-(ab)/(a+b)$$

o'rinli bo'ladi, (4.4) tenglamalarni o'rinli bo'lishi maqsadida

$$v+\omega=\sum_{n=0}^{k}\omega_n$$

bo'lishini eslagan holda (4.13) ni quyidagicha yozamiz: $(a+b)>0$ qanoatlantiruvchi haqiqiy a,b sonlar berilgan bo'lib, k>0 ni shunday katta qilib tanlanadiki

$$\sum_{n=0}^{k}\omega_n>-a,\sum_{n=0}^{k}\omega_n>-b \text{ va } \sum_{n=0}^{k}\omega_n\geq\frac{-ab}{a+b} \tag{4.14}$$

U holda (4.5) bilan berilgan x va (4.9) bilan berilgan y lar orqali $\{x_n\}$ ketma-ketlik

$$x_1=x_2=...=x_{k-1}=x,x_k=y \quad \text{va} \quad x_n=1$$

orqali aniqlanadi, aks holda (4.3) tenglamalarni qanoatlantiradi. Ikkinchi qadam (3.8) tenglamani eslaymi, ya'ni

$$C_n=\frac{1}{|\dot{D}(\lambda_n)|}\cdot\left(\frac{1}{|\varphi(\pi,\lambda_n)|}-\frac{1}{|\varphi_0(\pi,\lambda_n)|}\right)$$

chunki bizning qo'zg'almagan masalamiz toq va bu

$$C_n=\frac{1}{|\dot{D}(\lambda_n)|}\cdot\left(\frac{1}{|\varphi(\pi,\lambda_n)|}-1\right) \tag{4.15}$$

bo'ladi. Biz ushbu belgilashni kiritamiz.

$$\omega_n = \frac{1}{|\dot{D}(\lambda_n)|}$$

va (4.3) tenglamalarni yechamiz,

$$a = h_0 - H \quad \text{va} \quad b = h_0 - h$$

bo'lganda, keyin (4.15) dagi $|\varphi(\pi, \lambda_n)|$ o'rniga x_n qiymatlarni qo'yamiz. x_n aynan 1 ga tegligi bizga nolga teng bo'lmagan C_n sonlarning chekli sonini beradi va biz ularni $F(x, y)$ funksiyani qurishda foydalanamiz va $q_0(x)$ malum deb faraz qilib, xos funksiyalari (3.7) tenglamani qanoatlantiruvchi $q(x)$ potensialni qurib olamiz. Bu (4.1) teoremani isbotini yakunlaydi.

5-§. Dirixle chegaraviy shartlari.

1) Dirixle chegaraviy shartlarida har bir mos C_n sonlar to'plami uchun, qo'zg'almagan operator uchun chegaraviy shartlar Dirixleniki bo'lib qoladi.

Isbot. Qo'zg'almagan $\varphi_0(x,\lambda_n)$ xos funksiyalar $n \geq 1$ uchun biz ushbu

$$\varphi_0(0,\lambda_n) = \varphi_0(\pi,\lambda_n) = 0$$

ifodaga egamiz. Qozg'almagan $\varphi(x,\lambda_n)$ xos funksiyalar uchun (1.13) tenglama ushbu

$$\varphi(0,\lambda_n) = 0,\ n \geq 1$$

ifodani beradi va (1.19) tenglamadan biz

$$\varphi(\pi,\lambda_n) = \varphi_0(\pi,\lambda_n) - C_n \alpha_{n,0}^2 \varphi(\pi,\lambda_n)$$

$$\varphi(\pi,\lambda_n) = \frac{\varphi_0(\pi,\lambda_n)}{1 + C_n \alpha_{n,0}^2} = 0,\ n \geq 1$$

tengliklarni olamiz.

2) $h_0 = \infty, H = \infty$ bo'lgan Dirixle chegaraviy shartlari.Quyidagi oraliq hollarni qaraymiz:

$$h_0 \neq \infty, H_0 = \infty \text{ yoki } h_0 = \infty, H_0 \neq \infty.$$

Xuddi oldingidek, biz birinchi hol uchun

$$\varphi(0,\lambda_n) = 1, \quad \varphi'(0,\lambda_n) = h = h_0 - \sum_{n=0}^{\infty} C_n, \quad \varphi(\pi,\lambda_n)$$

ifodalarga ega bo'lamiz.

Ikkinchi hol uchun

$$\varphi(0,\lambda_n) = 0, \quad \varphi'(\pi,\lambda_n) + H\varphi(\pi,\lambda_n) = 0$$

ifodaga ega bo'lamiz, bu yerda

$$H = H_0 + \sum_{n=0}^{\infty} C_n \frac{\varphi_0^2(\pi,\lambda_n)}{1 + C_n \alpha_{n,0}^2}.$$

Xulosa.

Mazkur bitiruv ishi Izospektral Shturm-Liuvill chegaraviy masalalariga bag'ishlangan.

Xulosa qilib shuni aytish mumkinki quyidagi:

$$q(x) = q_0(x) + 2\frac{d}{dx}K(x,x) \ ;$$

$$h = h_0 - \sum_{n=0}^{\infty} C_n \ ;$$

$$H = H_0 + \sum_{n=0}^{\infty} C_n \frac{\varphi_0^2(\pi, \lambda_n)}{1 + C_n \alpha_{n,0}^2} \ ;$$

formulalar yodamida $\{\lambda_n\}_{n=0}^{\infty}$ spektrga ega bo'lgan

$$-y'' + q(x)y = \lambda y, \qquad 0 \le x \le \pi$$
$$y'(0) - hy(0) = 0$$
$$y'(\pi) + Hy(\pi) = 0$$

Shturm-Liuvill chegaraviy masalalarini cheksiz ko'p ya'ni juda ko'p $q(x)$ potensial va (h, H) juftlik sonlari tuzishimiz mumkin.

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