O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

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«Matritsaviy operatorlarning xos funksiyalari uchun Vaynberg tenglamasi»

mavzusida

BITIRUV MALAKAVIY ISHI

“Ish ko’rildi va himoyaga ruxsat berildi”	**Ilmiy rahbar ________ dots. T.H.Rasulov**
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«Himoya qilishga ruxsat berildi »
Fakultet dekani prof. Sh.M. Mirzayev
“___” _________ 2012 y.

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Mundarija.

KIRISH

> “O’z oldimizga qo’yilgan yuksak vazifa
> va marralarga erishishda zamonaviy bilim
> va tarbiya sohibi bo’lgan, yangicha
> fikrlaydigan yoshlarimiz bizning asosiy
> tayanchimiz va suyanchimizdir”.
>
> I.A.Karimov.

Biz mustaqil O’zbekiston yoshlari har tomonlama yetuk barkamol avlod bo’lib yetishimiz uchun keng imkoniyatlar yaratib Prezidentimiz Islom Abdug’aniyevich Karimov yosh avlod tarbiyasiga juda kata e’tibor berib kelmoqdalar. Jumladan, asosiy vazifamiz Vatanimiz taraqqiyoti va xalqimiz farovonligini yanada yuksaltirish nomli Prezidentimiz I.A.Karimovning 2009-yilning asosiy yakunlari 2010-yilda O’zbekistonning ijtimoiy-iqtisodiy rivojlanishini eng muhim ustuvor yo’nalishlariga bag’ishlangan Vazirlar Mahkamasining majlisidagi ma’ruzasida:

Biz farzandlarimizning nafaqat jismoniy va ma’naviy sog’lom o’sishi, balki ularning eng zamonaviy intelektual bilimlarga ega bo’lgan, uyg’un rivojlangan insonlar bo’lib, XXI asr talablariga to’liq javob beradigan barkamol avlod bo’lib voyaga yetishi uchun zarur barcha imkoniyat va sharoitlarni yaratishini o’z oldimizga maqsad qilib qo’yganmiz.

Shu kunlarda hukumatimiz tomanidan ushbu masala yuzasidan qabul qilingan Davlat dasturi ana shu ezgu maqsadga erishish yo’llariga jami davlat va nodavlat manbalari hisobga olingan holda mavjud barcha resurs va imkoniyatlarning safarbar etishni ko’zda tutadi kabi satrlarni o’qish mumkin.

Shuningdek, “Barkamol avlod” yili davlat dasturi to’g’risida O’zbekiston Respublikasi Prezidenti qarorida “ta’lim jarayonida yangi axborot komunikatsiya va pedagogik texnalogiyalarda, elektron dasturlar, multimediya vositalarini keng joriy etish orqali mamlakatimiz maktablarida, kasb-hunar kollejlari, litseylar va oliy o'quv yurtlariga o'qitish sifatini tubdan yaxshilash, ilm-fanni yanada rivijlantirish, iqtidorli va qobiliyatli yoshlarni ilmiy faoliyatiga keng jalb qilish, ularning o'z ijodiy va intelektual salohiyatini ro'yobga chiqarish uchun sharoit yaratishga doir kompleks chora-tadbirlarni ishlab chiqish "ko'zda tutilgan vazifalardan hisoblanar ekan, darslarda yangi pedagogik texnalogiyalardan foydalanish ta'lim samaradorligini oshirish Prezirentimiz ta'kidlaganidek, "erkin fikrlashga " o'rganish uchun ayni muddao bo'ldi.

Mamlakatimizning istiqlol yo'lidagi birinchi qadamlaridan, buyuk ma'naviyatimizni tiklash va yanada yuksaltirish, milliy ta'lim-tarbiya tizimini takomillashtirish, uning milliy zaminini mustahkamlash, zamon talablari bilan uyg'unlashtirish asosida jahon andozalari va ko'nikmalari darajasiga chiqarish maqsadiga katta ahamiyat berib kalinmoqda. Bu jarayonda oxirgi yillarda qilingan ishlar haqida qisqacha to'xtalib o'tamiz.

Maktab sohasida "ta'lim haqida" qonun qabul qilindi. 1996-1997 o'quv yilidan maktabning birinchi sinflarda o'qish uchun yangi alifboda olib borildi. Yangi imlo, alifboda o'qitish uchun zarur dastur, qo'llanma va darsliklar yaratildi. Joylarda maktablarda o'qituvchi-murabbiylarga, ularning moddiy va ma'naviy rag'batlantirish, yordam berish masalalariga e'tibor ancha kuchaytirildi.

O'rta maxsus ta'lim sohasida viloyatlarda, joylarda biznes maktabi, kichik va o'rta maktab uchun kasb- hunar kurslarining ochilishi, bozor iqtisodoyoti talablaridan kelib chiqqan holda, yangi mutaxassislarning kiritilishi e'tiborga loyiqdir. Toshkent, Samarqand, Urganch, To'rtko'l va Andijonda bank kollejlari hamda Toshkent ayollar kolleji tashkil qilinishi ham shular jumlasidandir.

Oliy maktab sohasida test usuli joriy etilishi, viloyatlar markazlarida pedagogic institutlarning universitetlarga aylantirilishi va joylardagi o'quv yurtlariga yuqori ta’sis nizomi berilishi, tashkil etilgan milliy tashkilot va xalqaro jamoalar hisobidan chet ellarga tajriba almashish va talabalarni o’qishga yuborish yo’lga qo’yilishi, iqtisod va biznes sohasidagi mutaxasis va o’qituvchilarni qayta tayyorlash bo’yicha aniq maqsadga yo’naltirilgan ishlar olib borilishi, o’tish davrida 2 mingdan ortiq talaba va mutaxasisning chet ellarda o’qib kalishi, 100dan ortiq chet el mutaxasisining Respublikamiz o’quv muassasalariga jalb qilinishini qayd etishimiz zarur.

Shuni alohida ta’kidlash lozimki, ta’lim-tarbiya sohasida islohotlar o’tkazish va ularning asosiy yo’nalishlari, talablari va maqsadlarini aniqlash, hamda tegishli xulosalarni chiqarishda, bugungi muhokama qilinadigan hujjatlarda keng jamoatchiligimizning fikr-mulohazalari, tarbiya va izohlari ifoda topgan desak hech qanday mubolag’a bo’lmas. Shuning uchun ham amaldagi ta’lim-tarbiya tizimining zaif tomonlarini, zamon talablari, jamiyatimiz kelajagi va maqsadlariga javob bermaydigan jihatlarini chuqur tasavvur qilish, erkin, badavlat yashayotgan mamlakatlar tajribasini o’rganish, o’z o’lkamizga yuksak malakali, har jihatdan yetuk kadrlar tayyorlash dasturining asosiy sharti bo’lmog’i lozim. Shuning uchun ta’lim-tarbiya sohasida ham belgilanayotgan islohotlarni hayotga tadbiq qilishda islohotlarni bosqichma-bosqich o’tkazish prinsipi qo’yilgan.

Birinchi bosqich-o’tish davri bo’lib, u 1997-2001 yillarda, ya’ni4 yil davomida joriy etiladi. Bunda kadrlar tayyorlash tizimi salohiyatini saqlab qolish, uni rivojlantirish uchun huquqiy-ma’yoriy, ilmiy matodik, maliyaviy moddiy shart- sharoitlar yaratish lozim. Bu bosqichda asosan quyidagi vazifalarni bajarish kerak:

Milliy dasturni amalga oshirishning boshlanishi: yangi talablarga javob bera oladigan pedagog kadrlarni tayyorlash; o’quv shart-sharoitlarini yaratish va yangi o’quv programmalari ustida ishlash umumta’lim maktablarini tuzish jihatidan qayta qurish; uch yillik ta’lim o’rta maxsus va kasb-hunar bilim yurtlari tizimiga zamin tayyorlash; uzluksiz ta’lim tizimiga asos soladigan tadbirlarni amalga oshirish; ijtimoiy himoyani kafolatlash kabi masalalarni yechish bosqichi.

Ikkinchi bosqich- 2001-2005 yillarni o’z ichiga oladi. Bunda milliy dasturni keng miqyosda to’liq amalga oshirishga erishish darkor. Tabiiyki, tizim faoliyatining samaradorligini, mehnat bozorini inobatga olib, ijtimoiy-iqtisodiy sharoitlardan kelib chiqib, dastur g’oyalarini va qoidalarini kerakli o’zgartirishlar kiritilgan.

Uchinchi bosqich-2005 va keying yillarga mo’ljallangan bo’lib, unda tajribalarni tahlil etish va umumlashtirish asosida o’zgaruvchan ijtimoiy-iqtisodiy shart-sharoitlarni e’toborga olgan holda kadrlar tayyorlash tizimini takomillashtirish va yanada rivojlantidish zarur.

Umumlashtirilgan holda kadrlar tayyorlash tizimining shakllanishi va faoliyat ko’rsatishning asosiy tamoyillari quyidagi vazifalarni o’z ichiga oladi:

- barcha xil va turdagi ta’lim muassasalarida yuqori malakali mutaxasislar tayyorlash uchun uzluksiz ta’lim, fan va ishlab chiqarish salohiyatidan samarali foydalanish;
- davlat ta’lim standartlarini joriy etish va ularning faoliyat ko’rsatish mexanizmini ishlab chiqish;
- ixtisosliklar, malaka darajasiga ko’ra mutaxasislarga bo’lgan umumdavlat va mintaqaviy talablarning istiqbolini aniqlash;
- ta’lim tizimini tuzilish va mazmun jihatidan isloh qilish uchun o’qituvchilarni

  va murabbiylarni qayta tayyorlash;
- davlat va ijtimoiy muassasalarning kasbga yo’naltirish bo’yicha faoliyatini takomillashtirish. Bunda kasb tanlashning bozor ehtiyojlari va imkoniyatlarini e’tiborga olish zarur, toki har bir shaxs o’ziga mos kasbni egallay olsin;

- o’quvchi yoshlarni vatanga sadoqat, yuksak axloq, ma’naviyat va ma’rifat, mehnatga vijdonan munosabatda bo’lish ruhida tarbiyalash;

- ta’lim muassasalarini, birinchi navbatda umumta’lim maktablarini davlat tomonidan maliyaviy va maddiy texnikaviy soliq ta’minlash me’yorlarini oshirish va uning mexanizmini takomillashtirish;

- kadrlar tayyorlash tizimi iste’molchilari-korxonalar, muassasalar, firmalar, kadrlar va boshqa tuzilmalarning imkoniyatlaridan, birinchi navdatda, o’rta maxsus kasb-hunar o’quv yurtlarining moddiy va moliyaviy bazasini mustahkamlash uchun mumkin qadar kengroq foydalanish;

- kadrlar tayyorlash va ta’lim sohasida chet el sarmoyalarini, xalqaro donolik tashkilotlari va jamg’armalarining mablag’larini jalb etish;

- qonun doirasida o’quv rejalari, dasturlari va o’qitish yo’riqlari, ta’limiy xizmatlarni belgilashda ta’lim muassasalariga, birinchi navbatda, oliy yurtlariga mustaqillik berish va o’zini-o’zi boshqarish usullarini jaroy etish;

Tabiiyki, bunday murakkab vazifalarni oshirishda davlat va jamiyatning mas’uliyati alohida ahamiyat kasb etadi. Davlat va jamiyat kadrlar tayyorlash tizimini uzluksiz rivojlantirish va takomillashtirish kadili bo’lishi zarur. Ular yuqori malakali rag’batga qodir mutaxasislar tayyorlash bo’yicha ta’lim muassasalarining faoliyatini uyg’unlashtirish lozim.

Kadrlar tayyorlash milliy dasturi “Ta’lim to’g’risida”gi O’zbekiston Respublikasi qonunining qoidalariga muvofiq holda tayyorlangan bo’lib, milliy tajribaning tahlili va ta’lim tizimidagi jahon miqyosidagi ustuvor yutuqlari asasida tayyorlangan hamda yuksak umumiy va kasb –hunar madaniyatiga, ijodiy va ijtimoiy faollikda, ijtimoiy-siyosiy hayotda mustaqil ravishda mo’jalni to’g’ri ola bilish mahoratiga ega bo’lgan, istiqbol vazifalarini ilgari surish va hal qodir kadrlarning yangi avlodini shakllantirishga yo’naltirilgandir.

Dastur kadrlar tayyorlash milliy moselini ro’yobga chiqarishni, har tomonlama kamol topgan, jamiyatda turmushga moslashgan, ta’lim va kasb-hunar dasturini ongli ravishda tiklash va keyinchalik puxta o’zlashtirish uchun ijtimoiy-siyosiy, huquqiy, psixologik-pedagogik va boshqa tarzdagi sharoitlarni yaratishni, jamiyat, davlat va oila oldida o’z javobgarligini his etadigan fuqarolarni tarbiyalashni nazarda tutadi.

Bugun biz tez sur’atlar bilan o’zgarib borayotgan, imkoniyat hozirga qadar boshidan kechirgan davrlardan tubdan farq qiladigan o’ta shiddatli va murakkab bir zamonda yashamoqdamiz. Davlat va siyosat arboblari, faylasuflar va jamiyatshunos olimlar, sharhlovchi va jurnalistlar bu davrni turlicha ta’riflab, har xil nomlar bilan atamoqda. Kimdur uni yuksak texnologiyalar zamoni desa, kimdur tafakkur asri, yana birov yalpi axborotlashuv davri sifatida izohlamoqda. Albatta bu fikrlarni barchasiga ham ma’lum ma’noda haqiqat ratsional mag’iz bor. Chunki ularning har biri o’zida bugungi serqirra va rang-barang hayotning qaysidir belgi alomatini aks ettirishi tabiiy.

Ammo ko’pchilikning ongida bu davr globallashuv davri tariqasida ta’surot uyg’tmoqda. Ana shunday globallashuv fenomini haqida gapirganda, bu atama bugungi kunda ilmiy-falsafiy, hayotiy tushuncha sifatida juda kena ma’noni anglatishni ta’kidlash lozim. Umumiy nuqtai nazardan qaraganda, bu jarayon mutlaqo yangicha ma’no-mazmundagi mo’jalik, ijtimoiy-siyosiy, tabiiy-biologik global muhitning shakllanishi va shu bilan birga, mavjud milliy va mintaqaviy muammolarning jahon miqyosidagi muammolarning jahon miqyosidagi muammolarga aylanib borishini ifoda etmoqda. Globallashuv jarayoni hayotimizga tobora tez va chuqur kirib kelayotganining asosiy omili va sababi xususida gapirganda shuni obyektiv tan olish kerak-bugungi kunda har qaysi davlatning taraqqiyoti va ravnaqi, nafaqat yaqin va uzoq qo’shnilar, balki jahon miqyosida boshqa mintaqa va hududlar bilan shunday chambarchas bog’lanib boryaptiki, biror mamlakatning bu jarayondan chetga turishi ijobiy natijalarga olib kelmasligini tushunish, anglash qiyin emas. Shu ma'noda, globallashuv bu avvalo hayot suratlarining beqiyos darajada tezlashuvi demakdir. Har bir ijtimoiy hodisaning ijobiy va salbiy tomoni bo'lgani singari globallashuv jarayono ham bundan mustasno emas. Hozirgi paytda uning g'oyat o'tkir va keng qamrovli ta'sirini deyarli darcha sohalarda ko'rish, his etish mumkin. Ayniqsa davlatlar va xalqlar o'rtasidagi integratsiya va hamkorlik aloqalarining kuchayishi, xarijiy investitsiyalar kapital va tovarlar ishchi kuchining erkin harakati uchun qulayliklar vujudga kelishi, ko'plab yangi ish o'rinlarining yaratilishi, zamonaviy kommunikatsiya va axborot texnologiyalarning ilm-fan yutuqlarining umuminsoniy negizda uyg'unlashuvi, ekologik ofatlar paytida o'zaro yordam ko'rsatish imkoniyatlarning ortishi tabiiyki, bularning barchasiga globallashuv tufayli erishilmoqda. Ayni paytda hayot haqiqati shuni ko'rsatadiki har qanday taraqqiyot mahsuludan ikki xil maqsadda ezgulik va yovuzlik yo'lida foydalanish mumkin.

Bitiruv malakaviy ish mavzusining dolzarbligi: Ma'lumki matritsaviy operatorlarni xossalarini o'rganish masalasi qattiq jismlar fizikasi, kvant maydon nazariyasi, statistik fizika, kimyoviy reaksiyalar nazariyasi va shunga o'xshash bir qator sohalarda paydo bo'ladi. Xususan, matritsaviy operatorlarning xos funksiyalarga mos Vaynberg tenglamalari bunday operatorlarning diskret spektrining chekliligini isbotlashda muhim o'rin egallaydi. Fok fazosining "qirqilgan uch zarrachali" qism fazosida ta'sir qiluvchi uch turdagi chiziqli, chegaralangan va o'z-o'ziga qo'shma matritsaviy operatorlar qaraladi hamda ularning xos funksiyalariga mos odatdagi Vaynberg tenglamalari quriladi. Bunday tenglamalarning ba'zi xossalari o'rganildi.

Ishning maqsadi va vazifasi: Bitiruv malakaviy ishining mavzusi "Matritsaviy operatorlarning xos funksiyalari uchun Vaynberg tenglamasini qurish" bo'lib, unda qo'yilgan maqsad va vazifalar quyidagilar

1) Fok fazosidagi uch turdagi chiziqli, chegaralangan, o'z-o'ziga qo'shma bo'lgan, 3×3 blok operatorli matritsalarni aniqlash;

2) Aniqlangan uch turdagi blok matritsali operator uchun Vaynberg tenglamasini qurish;

3) Qurilgan Vaynberg tenglamalari va qaralayotgan operatorlarning xos funksiyalari orasidagi bog'lanishlarni aniqlash;

4) Qaralayotgan Vaynberg tenglamalari yordamida qaralgan operatorning diskret spektrining chekliligini isbotlash.

Tadqiqot ob'ekti va predmeti: Bitiruv malakaviy ishining ob'ekti va predmeti bu matritsaviy operatorlar uchun Vaynberg tenglamasi va uning xossalari, Fok fazosi, diskret spektr, muhim spektridir. Bulardan foydalanish jarayonida ham turli xil ta'rif va tushunchlardan foydalaniladi. Bitiruv malakaviy ishini tayyorlash jarayonida matritsaviy operatorlar uchun Vaynberg tenglamasi va uning xossalari o'rganildi. Bunda umumiy natijalar va xulosalar olindi. Ushbu mavzuga oid ilmiy maqolalarni o'rganildi va turli xil adabiyotlardan foydalanildi.

Natijalarning ilmiy yangiligi va amaliy ahamiyati: Ushbu bitiruv malakaviy ishi ilmiy xarakterga ega. Bu ishda olingan natijalarni Fok fazosining qirqilgan uch zarrachali qism fazosida ta'sir qiluvchi matritsaviy operatorlar uchun olingan ba'zi natijalarning Fok fazosining qirqilgan uch zarrachali qism fazosida ta'sir qiluvchi matritsaviy operatorlar uchun umumlashgan analogi sifatida qarash mumkin.

Ushbu bitiruv malakaviy ishidan matematika yo'nalishdagi iqtidorli talabalar va magistrlar operatorlar nazariyasi bo'yicha bilimlarini mustahkamlashda foydalanishlari mumkin.

Tadqiqot usuli va uslubiyoti: Bitiruv malakaviy ishida qattiq jismlar fizikasi, kvant maydon nazariyasi, statistik fizika, kimyoviy reaksiyalar nazariyasi va bir qator sohalarda ba'zi muhim masalalar matritsaviy operatorlarning spektral xossalarini o'rganish masalasiga keltiriladi. O'z navbatida, matritsaviy operatorlar spektral nazariyasida, xususan ularning diskret spektrlarini o'rganishda mos Vaynberg tenglamalari muhim o'rin egallaydi.

Matritsaviy operatorlar xos funksiyalariga mos odatdagi Vaynberg tenglamalarini qurishda Fok fazosining qirqilgan uch zarrachali qism fazosida ta'sir qiluvchi uch turdagi chiziqli, chegaralangan va o'z-o'ziga qo'shma matritsaviy operatorlardan foydalanildi.

Ushbu mavzuga oid ilmiy maqolalarni o'qish, tahlil qilish va o'rganilayotgan matritsaviy operatorlarga tadbiq qilish.

Olingan asosiy natijalar: Funksional analiz tushunchalaridan foydalanib matritsaviy operatorning muhim spektri hamda parametrlarga qo'yilgan ba'zi tabiiy shartlarda ularni joylashuv o'rni o'rganilgan, matritsaviy operatorlarning xos funksiyalari uchun Vaynberg tenglamasi qurilgan, bunda bir necha tasdiqlar to'liq isbotlangan, berilgan operator xos funksiyasi mos Vaynberg tenglamasini qanoatlantirishi ko'rsatilgan va olingan natijalar qat'iy matamatik isbotlangan.

Biz asosan Fok fazosining qirqilgan uch zarrachali qism fazosidagi operatorlarni o'rganganimiz uchun asosiy tushunchalar ham shu fazoda keltirilgan.

Bitiruv malakaviy ishning hajmi va tuzilishi: Bu ish 65 sahifadan iborat bo'lib, kirish, ikkita bob, besh paragrif, ikkita xulosa, xotima va foydalanilgan adabiyotlar ro'yxatidan iborat.

I.BOB. ASOSIY TUSHUNCHALAR.

1.1. Matritsa tipidagi operatorlar uchun umumiy malumotlar

Agar A to’plamning har bir elementiga biror qonun-qoidaga ko’ra B to’plamning yagona elementi mos qo’yilgan bo’lsa, A to’plamda funksiya (akslantirish) aniqlangan deyiladi. Belgilanishi: $f: A \to B$

$f: A \to B$ akslantirish syurektiv (ustiga akslantirish) deyiladi, agar $f(A) = B$ shart bajarilsa.

Inyektiv (o’zaro bir qiymatli) akslantirish deyiladi, agar turli elementlarning tasviri ham turli bo’lsa, ya’ni $a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$

Agar akslantirish ham syurektiv ham inyektiv bo’lsa, bunday akslantirish biyektiv akslantirish deyiladi.

Bizga $L \neq \varnothing$ bo’sh bo’lmagan to’plam berilgan bo’lsin. Uning elementlari $x, y, z, \ldots$ bo’lgan L to’plamda quyidagi ikki amal aniqlangan bo’lsa:

I.Ixtiyoriy ikkita $x, y \in L$ elementlarga ularning yig’indisi deb ataluvchi, aniq bir $(x+y) \in L$ element mos qo’yilgan bo’lib, ixtiyoriy $x, y, z \in L$ elementlar uchun

1) $x + y = y + x$, (kommutativlik)

2) $x + (y + z) = (x + y) + z$ (assosiativlik)

3) L da shunday θ element mavjud bo’lib,

$x + \theta = x$ (nol elementning mavjudligi)

4) $\exists(-x) \in L$ element mavjud bo’lib, $x + (-x) = \theta$ (qarama-qarshi elementning mavjudligi) aksiomalar bajarilsa

II. L to’plamdan olingan ixtiyoriy x element va ixtiyoriy α son $\alpha \in R(C)$ uchun x elementning α songa ko’paytmasi deb ataluvchi aniq bir $\alpha x \in L$ element mos qo’yilgan bo’lib, ixtiyoriy $x, y \in L$ va ixtiyoriy $\alpha, \beta \in R(C)$ sonlar uchun

5) $\alpha(\beta x) = (\alpha\beta)x$

6) $1 \cdot x = x$

7) $(\alpha + \beta)x = \alpha x + \beta x$

8) $\alpha(x + y) = \alpha x + \alpha y$

aksiomalar bajarilsa u holda L to'plam chiziqli fazo deyiladi.

Ta'rifda kiritilgan I va II amallar mos ravishda yig'indi va songa ko'paytirish amallari deb ataladi.

Bizga L va L^* chiziqli fazolar berilgan bo'lsin. Agar bu fazolar o'rtasida o'zaro bir qiymatli moslik o'rnatish mumkin bo'lib,

$$x \leftrightarrow x^* \qquad \text{va} \qquad y \leftrightarrow y^*, \qquad (x, y \in L,\ x^*, y^* \in L^*)$$

ekanligidan

$$x + y \leftrightarrow x^* + y^* \qquad \text{va} \qquad \alpha x \leftrightarrow \alpha y^*, \qquad (\alpha - \text{ixtiyoriy son})$$

ekanligi kelib chiqsa, u holda L va L^* chiziqli fazolar o'zaro izomorf fazolar deyiladi.

Izomorf fazolarni aynan bitta fazoning har xil ko'rinishi deb qarash mumkin.

Agar L chiziqli fazoning $x_1, x_2, ..., x_n$ elementlar sistemasi uchun hech bo'lmaganda birortasi noldan farqli bo'lgan $a_1, a_2, ..., a_n$ sonlar mavjud bo'lib,

$$a_1 x_1 + a_2 x_2 + ... + a_n x_n = 0 \tag{1.1.1}$$

tenglik bajarilsa, u holda $x_1, x_2, ..., x_n$ elementlar sistemasi chiziqli bog'langan deyiladi. Aks holda, ya'ni (1.1.1) tenglikdan

$$a_1 = a_2 = ... = a_n = 0$$

ekanligi kelib chiqsa, $x_1, x_2, ..., x_n$ elementlar sistemasi chiziqli bog'lanmagan yoki chiziqli erkli deyiladi.

Agar $x_1, x_2, ..., x_n, \dots$ cheksiz elementlar sistemasining ixtiyoriy chekli qism sistemasi chiziqli erkli bo'lsa, u holda $\{x_n\}_{n=1}^{\infty}$ sistema chiziqli erkli deyiladi.

Agar L chiziqli fazoda n elementli chiziqli erkli sistema mavjud bo'lib, bu fazoning ixtiyoriy $n+1$ ta elementdan iborat sistemasi chiziqli bog'langan bo'lsa, u holda L n- o'lchamli chiziqli fazo deyiladi va $\dim L = n$ deb yoziladi.

n-o'lchamli L chiziqli fazoning ixtiyoriy n ta elementdan iborat chiziqli erkli sistemasi shu fazoning bazisi deyiladi.

Agar L chiziqli fazoda ixtiyoriy $n \in N$ uchun n elementli chiziqli erkli sistema mavjud bo‘lsa, u holda L cheksiz o‘lchamli chiziqli fazo deyiladi va $\dim L = \infty$ ko‘rinishda yoziladi.

R^n va C^n fazolar n-o‘lchamli chiziqli fazolardir.

L chiziqli fazoda aniqlangan f sonli funksiyaga funksional deyiladi, va quyidagicha ifodalanadi :

$$f : L \to R(C)$$

Agar barcha $x, y \in L$ lar uchun

$$f(x+y) = f(x) + f(y)$$

shart bajarilsa , f additive funksional deyiladi.

Agar barcha $x \in L$ va barcha $\alpha \in R(C)$ sonlar uchun

$$f(\alpha x) = \alpha f(x)$$

bo’lsa, f bir jinsli funksional deyiladi.

Bizga L - chiziqli fazo va unda aniqlangan p funksional berilgan bo‘lsin. Agar p quyidagi uchta shartni qanoatlantirsa, unga norma deyiladi:

1) $p(x) \geq 0, \ \forall x \in L; \quad p(x) = 0 \Leftrightarrow x = \theta$;

2) $p(ax) = |a| p(x), \ \forall a \in C, \ \forall x \in L$;

3) $p(x+y) \leq p(x) + p(y), \ \forall x, y \in L$.

Norma kiritilgan L chiziqli fazo chiziqli normalangan fazo deyiladi va $x \in L$ elementning normasi $\| x \|$ orqali belgilanadi.

X chiziqli normalangan fazoda $\{x_n\}$ ketma-ketlik berilgan bo‘lsin.

Biror $x \in X$ va ixtiyoriy $\varepsilon > 0$ uchun shunday $n_0 = n_0(\varepsilon) > 0$ mavjud bo‘lib, barcha $n > n_0$ larda $\| x_n - x \| < \varepsilon$ tengsizlik bajarilsa, $\{x_n\}$ ketma-ketlik $x \in X$ elementga yaqinlashadi deyiladi.

Agar ixtiyoriy $\varepsilon > 0$ son uchun shunday $n_0 = n_0(\varepsilon) > 0$ mavjud bo‘lib, barcha $n > n_0$ va $p \in N$ larda $\left\| x_{n+p} - x_n \right\| < \varepsilon$ tengsizlik bajarilsa, $\{x_n\}$ - fundamental ketma-ketlik deyiladi.

Agar X chiziqli normalangan fazodagi ixtiyoriy $\{x_n\}$ fundamental ketma-ketlik yaqinlashuvchi bo‘lsa, u holda X to‘la normalangan fazo yoki Banax fazosi deyiladi.

Agar L normalangan fazoning $L_0 \subset L$ qism to‘plamida ixtiyoriy $x, y \in L_0$ elementlari va ixtiyoriy α, β sonlar uchun $\alpha x + \beta y \in L_0$ bo‘lsa L_0 chiziqli ko‘pxillilik deyiladi. Agar $L_0 \subset L$ qism to‘plam yopiq chiziqli ko‘pxillilik bo‘lsa, L_0 qism to‘plam L ning qism fazosi deyiladi.

Agar B to’plamning har bir elementi A to’plamning ham elementi bo’lsa, B to’plam A to’plamning qismi yoki qismiy to’plami (to’plam osti) deyiladi, va $B \subset A$ kabi belgilanadi.

Bizga X va Y chiziqli normalangan fazolar berilgan bo‘lsin.

X fazodan olingan har bir x elementga Y fazoning yagona y elementini mos qo‘yuvchi

$$Ax = y \qquad (x \in X, \ \ y \in Y)$$

akslantirish operator deyiladi.

Umuman A operator X ning hamma yerida aniqlangan bo‘lishi shart emas. Bu holda Ax mavjud va $Ax \in Y$ bo‘lgan barcha $x \in X$ lar to‘plami A operatorning aniqlanish sohasi deyiladi va $D(A)$ bilan belgilanadi, ya’ni:

$$D(A) = \{ \exists x \in X : Ax \quad \text{mavjud va} \quad Ax \in Y \}.$$

Agar chiziqli A operator qaralayotgan bo‘lsa, $D(A)$ ning chiziqli ko‘pxillilik bo‘lishi talab qilinadi, ya’ni agar $x, y \in D(A)$ bo‘lsa, u holda ixtiyoriy $\alpha, \beta \in C$ lar uchun

$$\alpha\, x + \beta\, y \in D(A).$$

Agar ixtiyoriy $x, y \in D(A) \subset X$ elementlar va ixtiyoriy $\alpha, \beta \in C$ sonlar uchun

$$A(\alpha\, x + \beta\, y) = \alpha\, A\, x + \beta\, A\, y$$

tenglik o‘rinli bo‘lsa, A ga chiziqli operator deyiladi.

Bizga $A : X \to Y$ operator va $x_0 \in D(A)$ nuqta berilgan bo‘lsin. Agar $y_0 = Ax_0 \in Y$ ning ixtiyoriy V atrofi uchun, x_0 nuqtaning shunday U atrofi mavjud bo‘lib, ixtiyoriy $x \in U \cap D(A)$ lar uchun $Ax \in V$ bo‘lsa, A operator $x = x_0$ nuqtada uzluksiz deyiladi.

Yuqoridagi ta’rifga teng kuchli quyidagi ta’riflarni keltiramiz.

Agar ixtiyoriy $\varepsilon > 0$ uchun shunday $\delta = \delta(\varepsilon) > 0$ mavjud bo‘lib, $\| x - x_0 \| < \delta$ tengsizlikni qanoatlantiruvchi barcha $x \in D(A)$ lar uchun

$$\| Ax - Ax_0 \| < \varepsilon$$

tengsizlik bajarilsa, A operator $x = x_0$ nuqtada uzluksiz deyiladi.

Agar x_0 nuqtaga yaqinlashuvchi ixtiyoriy x_n ketma-ketlik uchun $\| A\,x_n - A\,x_0 \| \to 0$ bo‘lsa, u holda A operator x_0 nuqtada uzluksiz deyiladi.

Agar A operator ixtiyoriy $x \in D(A)$ nuqtada uzluksiz bo‘lsa, A uzluksiz operator deyiladi.

$Ax = \theta$ tenglikni qanoatlantiruvchi barcha $x \in X$ lar to‘plami A operatorning yadrosi deb ataladi va u $KerA$ bilan belgilanadi.

Biror $x \in D(A)$ uchun $y = A\,x$ bajariladigan $y \in Y$ lar to‘plami A operatorning qiymatlar sohasi yoki tasviri deb ataladi va u $\operatorname{Im} A$ yoki $R(A)$ bilan belgilanadi.

Matematik formulalar yordamida operator yadrosi va qiymatlar sohasini quyidagicha yozish mumkin:

$$KerA = \{\exists x \in D(A) : \ Ax = \theta\},$$

$$R(A) := \operatorname{Im} A = \{\exists\, y \in Y : \text{biror } x \in D(A) \text{ uchun } \quad y = Ax\}.$$

Chiziqli operatorning qiymatlar sohasi va yadrosi chiziqli ko‘pxillik bo‘ladi. Agar $D(A)=X$ bo‘lib, A uzluksiz operator bo‘lsa, u holda $KerA$ yopiq qism fazo bo‘ladi, ya’ni $KerA=[KerA]$. A operator uzluksiz bo‘lgan holda ham $\operatorname{Im} A \subset Y$ yopiq qism fazo bo‘lmasligi mumkin.

Bizga L haqiqiy chiziqli fazo berilgan bo‘lsin. Agar $L \times L$ dekart ko‘paytmada aniqlangan p funksional quyidagi to‘rtta shartni qanoatlantirsa:

1) $p(x,x) \geq 0, \ \ \forall x \in L; \ \ \ p(x,x)=0 \Leftrightarrow x=\theta$;

2) $p(x,y)=p(y,x), \ \ \ \forall x,y \in L$;

3) $p(\alpha\, x,y)=\alpha\, p(x,y), \ \ \forall \alpha \in R, \ \forall x,y \in L$;

4) $p(x_1+x_2,\ y)=p(x_1,y)+p(x_2,y), \ \ \ \forall x_1,x_2,\ y \in L$,

unga skalyar ko‘paytma deyiladi.

Skalyar ko‘paytma kiritilgan chiziqli fazo Evklid fazosi deyiladi va x,y elementlarning skalyar ko‘paytmasi $p(x,y)=(x,y)$ orqali belgilanadi.

Evklid fazosida x elementning normasi

$$\| x \| = \sqrt{(x,x)}$$

formula orqali aniqlanadi. Bu funksional norma aksiomalarini qanoatlantiradi. Skalyar ko‘paytmaning 1-4 shartlaridan normaning 1-2 shartlari bevosita kelib chiqadi. Uchburchak aksiomasining bajarilishi Koshi-Bunyakovskiy tengsizligi deb ataluvchi quyidagi

$$|(x,y)| \leq \| x \| \cdot \| y \|$$

tengsizlikdan kelib chiqadi.

Agar $(x,y)=0$ bo’lsa, u holda x va y vektorlar ortogonal deyiladi.

Agar ixtiyoriy $\alpha \neq \beta$ da $(x_\alpha, x_\beta)=0$ bo’lsa, u holda nolmas $\{x_\alpha\}$ vektorlar sistemasiga ortogonal sistema deyiladi.

Chekli o‘lchamli Evklid fazolari R^n fazoga izomorfdir.

Cheksiz o‘lchamli to‘la Evklid fazosi Hilbert fazosi deyiladi.

Shunday qilib, ixtiyoriy tabiatli $f, g, \varphi, \ldots$ elementlarning H to‘plami Hilbert fazosi bo‘lsa, u quyidagi uchta shartni qanoatlantiradi:

1) H - Evklid fazosi, ya'ni skalyar ko‘paytma kiritilgan chiziqli fazo;

2) $\rho(x,y)=\sqrt{(x-y,x-y)}$ metrika ma'nosida H - to‘la fazo;

3) H fazo - cheksiz o‘lchamli, ya'ni unda cheksiz elementli chiziqli erkli sistema mavjud.

Odatda separabel Hilbert fazolari qaraladi, ya'ni H ning hamma yerida zich bo‘lgan sanoqli to‘plam mavjud.

Agar R va R^* Evklid fazolari o‘rtasida o‘zaro bir qiymatli moslik o‘rnatish mumkin bo‘lib,

$$x \leftrightarrow x^*, \quad y \leftrightarrow y^*, \quad x, y \in R, \quad x^*, y^* \in R^*$$

ekanligidan

$$x+y \leftrightarrow x^*+y^*, \quad \lambda x \leftrightarrow \lambda x^* \quad va \quad (x,y)=(x^*,y^*)$$

munosabatlar kelib chiqsa, R va R^* lar izomorf fazolar deyiladi.

Boshqacha aytganda, Evklid fazolarining izomorfligi shundan iboratki, bu fazolar o‘rtasida o‘zaro bir qiymatli moslik mavjud bo‘lib, bu moslik shu fazolardagi chiziqli amallarni va ulardagi skalyar ko‘paytmani saqlaydi.

Ma'lumki, n - o‘lchamli ixtiyoriy ikkita Evklid fazosi o‘zaro izomorfdir. Cheksiz o‘lchamli Evklid fazolari o‘zaro izomorf bo‘lishi shart emas.

Agar M - H Hilbert fazosining yopiq qism fazosi bo‘lsa, u holda ixtiyoriy $f \in H$ element yagona usul bilan $f = h + h'$, $h \in M_1$, $h' \in M_2$ ko’rinishda tasvirlansa, u holda H fazo o’zaro orthogonal , M_1 va M_2 qism fazolarning to’g’ri yig‘indisiga yoyilgan deyiladi va $H = M_1 \oplus M_2$ ko’rinishda belgilanadi.

$X \subset R^n$ o’lchovli to’plam, $X^n = X \times X \times \ldots \times X$ …… dekart ko’paytma $L_2(X^n) - X^n$ da aniqlangan kvadrati bilan integrallanuvchi funksiyalar Gilbert fazosi. Quyidagi belgilashlarni kiritamiz.

$$H_0 = C\,;$$

$$H_n = L_2(X^n)\,,\ n = 1,2,...;$$

$$H = \mathop{\oplus}_{i=0}^{\infty} H_i\,;$$

$$H^{(N)} = \mathop{\oplus}_{i=0}^{N} H_i\,.$$

U holda

$$H = \{f = (f_0, f_1, ..., f_n, ...):\quad f_i \in H_i,\quad i = 0,1,2,...\ \}\quad ,$$

$$H^{(N)} = \{f = (f_0, f_1, ..., f_N):\quad f_i \in H_i,\quad i = 0,1,2,...N\}$$

bo'ladi.

H Gilbert fazosiga Fok fazosi deyiladi. $H^{(N)}$ Gilbert fazosiga Fok fazosining qirqilgan qism fazosi deyiladi.

Ikki elementning skalyar ko'paytmasi va norma elementi H va $H^{(N)}$ fazolarda aniqlanadi

Ixtiyoriy $f = (f_0, f_1, ...f_n, ...),\quad g = (g_0, g_1, ...g_n, ...) \in H$ elementlarning skalyar ko'paytmasi quyidagicha aniqlanadi:

$$(f, g)_H = \sum_{n=0}^{\infty} (f, g)_{H_n}\quad .$$

Bu yerda $(\cdot,\cdot)_{H_n}$ skalyar ko'paytma H_n Gilbert fazoda aniqlangan bo'ladi.

$$(f_0, g_0)_{H_0} = f_0 \overline{g_0}\,;$$

$$(f_n, g_n)_{Hn} = \int_{X^n} f_n(s)\overline{g_n(s)}ds,\quad n = 1,2,...\ .$$

Ixtiyoriy $f = (f_0, f_1, ...f_N),\quad g = (g_0, g_1, ...g_N) \in H^{(N)}$ elementlar uchun quyidagi tenglik o'rinli bo'ladi.

$$(f, g)_{H^{(N)}} = \sum_{n=0}^{N} (f, g)_{H_n}\,.$$

Istalgan $f = (f_0, f_1, ...f_n, ...) \in H$ elementlarning normasi

$$\|f\|_H = \sqrt{\sum_{n=0}^{\infty} \|f\|_{H_n}^2}$$

kabi belgilanadi.

Bu yerda $\|\cdot\|_{H_n}$ orqali H_n dagi norma belgilangan:

$$\|f\|_{H_0} = |f_0|$$

$$\|f_n\|_{H_n} = \sqrt{\int_{X^n} |f_n(s)|^2 \, ds}, \quad n = 1,2,...$$

Ixtiyoriy $f = (f_0, f_1, ...f_N) \in H^{(N)}$ elementlarning normasi

$$\|f\|_{H^{(N)}} = \sqrt{\sum_{n=0}^{N} \|f\|_{H_n}} \quad N = 1,2,...$$

kabi belgilanadi.

H Gilbert fazosida ta'sir qiluvchi istalgan chiziqli ,chegaralangan A operator

$$A = \left(A_{i,j}\right)_{i,j}^{\infty} = \begin{pmatrix} A_{00} & A_{01} & A_{02} & ... \\ A_{10} & A_{11} & A_{12} & ... \\ A_{20} & A_{21} & A_{22} & ... \\ ... & ... & ... & ... \end{pmatrix}$$

kabi ta'sirlanadi.

Bu yerda $A_{i,j} : H_j \to H_i$, $i, j = 0,1,2,...$ chiziqli, chegaralangan operatorlar.

Zamonaviy matematik fizikada A_{ii} operatorlar qo'zg'almas operatorlar deyiladi.

$i < j$ shart bajarilsa A_{ij} operatorlar "yo'qotish" operatorlari deyiladi.

Agar $i > j$ shart bajarilsa A_{ij} operatorlarga "paydo qilish" operatorlari deyiladi.

Agar biz $A_{\leq N}$ operatorni $H^{(N)}$ Gilbert fazosiga qarasak bu operatorning ko'rinishi quyidagicha bo'ladi:

$$A=\left(A_{i,j}\right)_{i,j}^{N}=\begin{pmatrix} A_{00} & A_{01} & A_{02} & \dots & A_{0N} \\ A_{10} & A_{11} & A_{12} & \dots & A_{1N} \\ A_{20} & A_{21} & A_{22} & \dots & A_{2N} \\ . & & & & \\ & \dots & \dots & \dots & \dots \\ . & & & & \\ A_{N0} & A_{N1} & A_{N2} & \dots & A_{NN} \end{pmatrix}.$$

bu yerda $i,j=0,1,2,...N$ chiziqli chegaralangan operatorlar.

Ixtiyoriy $f=(f_0,f_1,f_2),\quad g=(g_0,g_1,g_2)\in H^{(2)}$ lar uchun

$$A_{\leq 2}(f+g)=A_{\leq 2}f+A_{\leq 2}g$$

bo'lsa, $A_{\leq 2}$ operatorda additive operator deyiladi.

Agar barcha $f=(f_0,f_1,f_2)\in H^{(2)}$ va barcha α - ixtiyoriy sonlar uchun

$$A_{\leq 2}(\alpha f)=\alpha A_{\leq 2}f$$

bo'lsa , $A_{\leq 2}$ operator bir jinsli operator deyiladi.

Additiv operatorlar uchun bir qancha oddiy natijalar olingan.

Masalan, $A_{\leq 2}$ additive operator bo'lsa

$$A_{\leq 2}(\theta)=\theta$$

ixtiyoriy $f=(f_0,f_1,f_2)\in H^{(2)}$ uchun $A_{\leq 2}(-f)=-A_{\leq 2}f$ bo'ladi.

Bu yerda $H^{(2)}$ fazodagi nol element uchun θ belgilash ishlatiladi. $\theta=(\theta;\theta;\theta)\in H^{(2)}$.

$A_{\leq 2}$ operator uchun quyidagi shartlar bajarilsa,

1) $A_{\leq 2}(f+g)=A_{\leq 2}f+A_{\leq 2}g$, ixtiyoriy $f=(f_0,f_1,f_2),\quad g=(g_0,g_1,g_2)\in H^{(2)}$, (additivlik)

2) Ixtiyoriy $f=(f_0,f_1,f_2)\in H^{(2)}$ va α - ixtiyoriy sonlar uchun $A_{\leq 2}(\alpha f)=\alpha A_{\leq 2}f$,(bir jinslilik)

$A_{\leq 2}$ operatorga chiziqli operator deyiladi.

Agar $D(A_{\leq 2})=H^{(2)}$ bo'lib, shunday C musbat soni topilib , ixtiyoriy $f=(f_0,f_1,f_2)\in H^{(2)}$ uchun

$$\left\|A_{\leq 2}f\right\|_{H^{(2)}}\leq C\left\|f\right\|_{H^{(2)}} \tag{1.1.2}$$

bo'lsa , $H^{(2)}$ Gilbert fazoda berilgan $A_{\le 2}$ operatorga chegaralangan operator deyiladi.

(1.1.2) tengsizlikni qanoatlantiruvchi barcha C sonlar to'plamining aniq quyi chegarasiga $A_{\le 2}$ operatorning normasi deyiladi va u $\|A_{\le 2}\|$ kabi belgilanadi, ya'ni

$$\|A_{\le 2}\| = \inf\, C\,.$$

Chiziqli chegaralangan $A_{\le 2}$ operatorning normasi uchun quyidagi formulalar o'rinlidir:

$$\|A_{\le 2}\| = \sup_{\|f\|_{H^{(2)}} \le 1} \|A_{\le 2} f\|_{H^{(2)}}\,;$$

$$\|A_{\le 2}\| = \sup_{\|f\|_{H^{(2)}} = 1} \|A_{\le 2} f\|_{H^{(2)}}\,;$$

$$\|A_{\le 2}\| = \sup_{f \ne 0} \frac{\|A_{\le 2} f\|_{H^{(2)}}}{\|f\|_{H^{(2)}}}\,.$$

Agar ixtiyoriy $f \in H^{(2)}, g \in H_1^{(2)}$ uchun $A_{\le 2} f = g$ tenglama yagona yechimga ega bo'lsa , u holda $A_{\le 2}$ operator teskarilanuvchan operator deyiladi.

Agar $A_{\le 2}$ teskarilanuvchan operator bo'lsa , u holda ixtiyoriy $g \in H_1^{(2)}$ va $A_{\le 2} f = g$ tenglamaning yechimi bo'lgan yagona $f \in H^{(2)}$ element mos keladi. Bu moslikni o'rnatuvchi operator $A_{\le 2}$ operatorga teskari operator deyiladi va $A_{\le 2}^{-1}$ kabi belgilanadi.

$$A_{\le 2}^{-1} : H_1^{(2)} \to H^{(2)}\,.$$

$A_{\le 2}^{-1}$ aniqlanish sohasi $A_{\le 2}$ ning qiymatlar sohasiga teng. Belgilanishi $D(A_{\le 2})$.

$A_{\le 2}^{-1}$ qiymatlar sohasi $A_{\le 2}$ ning aniqlanish sohasiga teng. Belgilanishi $\mathrm{Im}(A_{\le 2})$.

Bundan tashqari teskari operatorning aniqlanishidan

$$A_{\le 2}^{-1} \cdot A_{\le 2} f = f, \quad f \in D(A_{\le 2})\,;$$

$$A_{\le 2} \cdot A_{\le 2}^{-1}\, g = g, \quad g \in D(A_{\le 2}^{-1}) \qquad (1.1.3)$$

tengliklar kelib chiqadi.

Agar $A_{\le 2}$ operator uchun ham chap teskari ham o'ng teskari operator mavjud bo'lsa , u holda ular o'zaro teng.

Agar $A_{\le 2}$ uchun bir vaqtda ham o'ng teskari, ham chap teskari operatorlar mavjud bo'lsa , u holda $A_{\le 2}$ operator teskarilanuvchan operator bo'ladi.

$A_{\le 2}$ chiziqli operatorga teskari bo'lgan $A_{\le 2}^{-1}$ operator ham chiziqlidir.

$A_{\le 2}$ operator X Banax fazosini Y Banax fazosiga biyektiv akslantiruvchi chiziqli, chegaralangan operator bo'lsin. U holda $A_{\le 2}^{-1}$ operator mavjud va chegaralangandir.

$A_{\le 2} : H^{(2)} \to H_1^{(2)}$ chiziqli operator teskarilanuvchan bo'lishi uchun $A_{\le 2} f = 0$ tenglama faqat $f = 0$ yechimga ega bo'lishi zarur va yetarli.

$H^{(2)}$ fazoni $H_1^{(2)}$ fazoga akslantiruvchi chiziqli $A_{\le 2}$ operator berilgan bo'lsin.

$\operatorname{Im} A_{\le 2}$ da chegaralangan $A_{\le 2}^{-1}$ operator mavjud bo'lishi uchun shunday $m > 0$ son mavjud bo'lib, ixtiyoriy $f = (f_0, f_1, f_2) \in H^{(2)}$ lar uchun

$$\|A_{\le 2} f\| \ge m\|f\|$$

tengsizlikning bajarilishi zarur va yetarli.

Chiziqli, chegaralangan $A_{\le 2}^{*}$ operatorning qo'shmasi ham chegaralangandir

$$\|A_{\le 2}^{*}\| = \|A_{\le 2}\|$$

Bizga $H^{(2)}, H_1^{(2)}$ chiziqli normallangan fazolar va $A_{\le 2}^{(1)} : H^{(2)} \to H_1^{(2)}$ chiziqli, chegaralangan operator berilgan bo'lsin. Agar biror $A_{\le 2}^{(1)*} : H_1^{(2)*} \to H^{(2)*}$ va ixtiyoriy $f = (f_0, f_1, f_2), \quad g = (g_0, g_1, g_2) \in H^{(2)}$ lar uchun

$$(A_{\le 2} f, g) = (f, A_{\le 2} g)$$

shart bajarilsa $A_{\le 2}$ operator o'z-o'ziga qo'shma operator deyiladi.

Har bir $g \in H_1^{(2)}$ funksionalga

$$(A_{\le 2} f, g) = (g, A_{\le 2} f)$$

tenglik bilan aniqlanuvchi $A_{\le 2} \in (H^{(2)*})$ funksionalni mos qo'yuvchi $(A_{\le 2}^{(1)})^{*} : (H_1^{(2)})^{*} \to (H^{(2)})^{*}$ operator $A_{\le 2}$ operatorga qo'shma operator deyiladi.

Qo'shma operatorlar quydagi xossalarga ega:

1^o. $(A_{\le 2}^{(1)} + A_{\le 2}^{(2)}) = (A_{\le 2}^{(1)})^* + (A_{\le 2}^{(2)})^*$;

2^o. $(\lambda A_{\le 2})^* = \overline{\lambda} A_{\le 2}^*$;

3^o. $(A_{\le 2}^{(1)} \cdot A_{\le 2}^{(2)}) = (A_{\le 2}^{(1)})^* \cdot (A_{\le 2}^{(2)})^*$.

$H^{(2)}$ Gilbert fazosida chiziqli,chegaralangan $A_{\le 2}$ operator o'z-o'ziga qo'shma deyiladi, agar uning qo'shmasi o'ziga teng bo'lsa, $A_{\le 2}^* = A_{\le 2}$.

Ixtiyoriy chiziqli o'z-o'ziga qo'shma operatorning kombinatsiyasi ham o'z-o'ziga qo'shma operatordan iborat bo'ladi.

Bizga $A_{\le 2} : H^{(2)} \to H^{(2)}$ chiziqli operator va $L \in H^{(2)}$ qism fazo berilgan bo'lsin. Agar ixtiyoriy $f = (f_0, f_1, f_2) \in L$ uchun $A_{\le 2} f \in L$ bo'lsa u holda L qism fazo $A_{\le 2}$ operatorga nisbatan invariant qism fazo deyiladi.

Agar biror $\lambda \in C$ soni va $f = (f_0, f_1, f_2) \in H^{(2)}$ $A_{\le 2} f = \lambda f$ tenglama nolmas f yechimga ega bolsa, λ soni $A_{\le 2}$ operator uchun xos qiymat deyiladi, ushbu nolmas f yechimga λ xos qiymatiga mos keluvchi xos vektor deyiladi.

Har bir $A_{\le 2} : C^n \to C^n$ chiziqli operatorga $A_{ij} - n \times n$ matritsa mos keladi va aksincha, chiziqli algebra kursidan ma'lumki , agar λ son $A_{\le 2}$ operatorning xos qiymati bo'lsa

$$\det(A_{\le 2} - \lambda I) = 0$$

bo'ladi, va aksincha, $n \times n$ matritsa determinantini $\det(A_{\le 2} - \lambda I)$ parametr λ ning n-darajali ko'phadi bo'ladi va $\det(A_{\le 2} - \lambda I) = 0$ tenglama ko'pi bilan n ta ildizga ega, ya'ni

$$A_{\le 2} : C^n \to C^n$$

chiziqli operator ko'pi bilan n ta xos qiymatga ega.

Agar λ soni $A_{\le 2}$ operator uchun xos qiymat bo'lmasa , ya'ni $\det(A_{\le 2} - \lambda I) \ne 0$ bo'lsa , u holda $(A_{\le 2} - \lambda I)$ ga teskari operator manjud va u C^n fazoning hamma yerida aniqlangan bo'ladi.

$A_{\le 2} : C^n \to C^n$ chiziqli operator chegaralangandir.

Chekli o’lchamli fazolardagi chiziqli operatorlar uchun quyidagi ikki holat sodir bo’lishi mumkin.

1) λ soni uchun $A_{\leq 2} f = \lambda f$ tenglama nolmas yechimga ega, ya’ni λ son $A_{\leq 2}$ operator uchun xos qiymat, bu holda $(A_{\leq 2} - \lambda I)$ ga teskari operator mavjud emas.

2) λ soni uchun C^n fazoning hamma yerida aniqlangan $(A_{\leq 2} - \lambda I)^{-1}$ operator mavjud va demak, chegaralangan.

λ soni xos qiymat bo’lishi uchun shunday $f = (f_0, f_1, f_2) \neq 0$ element topilib, $A_{\leq 2} f = \lambda f$ tenglik bajarilishi zarur va yetarlidir.

Agar λ soni $A_{\leq 2}$ chiziqli, chegaralangan operatorning xos qiymati bo’lsa, u holda $L_\lambda(A_{\leq 2}) = \{f \in H^{(2)} : A_{\leq 2} f = \lambda f\}$ to’plamga $A_{\leq 2}$ operatorning λ xos qiymatiga mos keluvchi xos qism fazosi deyiladi.

Ko’rinishiga ko’ra $L_\lambda(A_{\leq 2})$ qism fazo $A_{\leq 2}$ operatorga nisbatan invariant qism fazo bo’ladi.

O’z- o’ziga qo’shma operatorlar uchun xos qism fazo istalgan o’lchamga ega bo’lishi mumkin.

Masalan, $If = f$ birlik operator uchun $\lambda = 1$ xos qiymatga mos xos qism fazo $H^{(2)}$ fazo bilan ustma-ust tushadi, bunda I birlik operator chiziqli, chegaralangan va o’z-o’ziga qo’shma operatordir.

$L_\lambda(A_{\leq 2})$ qism fazoning o’lchamiga $A_{\leq 2}$ operatorning λ xos qiymatining karraligi deyiladi.

Agar $A_{\leq 2}$ operator λ xos qiymatining shunday atrofi topilib, bu atrofda $A_{\leq 2}$ operatorning λ dan farqli xos qiymatlari yotmasa , u holda λ xos qiymat $A_{\leq 2}$ operator uchun yakkalangan xos qiymat deyiladi.

O’z- o’ziga qo’shma bo’lgan $A_{\leq 2}$ operatorning turli λ_1 va λ_2 xos qiymatlariga mos $L_{\lambda_1}(A_{\leq 2})$ va $L_{\lambda_2}(A_{\leq 2})$ xos qism fazolar o’zaro ortogonaldir. Boshqacha aytganda, o’z-o’ziga qo’shma operatorning turli xos qiymatlariga mos xos vektorlari ortogonaldir.

O’z- o’ziga qo’shma $A_{\le 2}$ operatorning barcha xos qiymatlariga uning $m_{A_{\le 2}}$ va $M_{A_{\le 2}}$ chegaralari orasida yotadi, bunda

$$m_{A_{\le 2}} = \inf_{\|f\|=1}(A_{\le 2}f, f);$$

$$M_{A_{\le 2}} = \sup_{\|f\|=1}(A_{\le 2}f, f).$$

O’z-o’ziga qo’shma musbat aniqlangan $A_{\le 2}$ operatorning xos qiymatlari nomanfiydir.

Shunday qilib, $A_{\le 2}$ operatorning xos qiymati tushunchasi

$$(A_{\le 2} - \lambda I)f = g \tag{1.1.4}$$

Tenglamaning yechimga ega yoki ega emasligi bilan uzviy bog’liqdir. Bunda I orqali $H^{(2)}$ fazodagi birlik operator belgilangan.

Shuni ta’kidlab o’tish joizki, agar $A_{\le 2}$ o’z-o’ziga qo’shma operator bo’lsa , u holda barcha haqioqiy λ lar uchun $(A_{\le 2} - \lambda I)$ operator ham o’z-o’ziga qo’shma operatorlarning chiziqli kombinatsiyasi sifatida o’z- o’ziga qo’shmadir.

$(A_{\le 2} - \lambda I)^{-1}$ teskari operator mavjud bo’lishi uchun $(A_{\le 2} - \lambda I)f = \theta$ tenglama yoki unga ekvivalent bo’lgan $A_{\le 2}f = \lambda f$ tenglama faqat nolmas $f = (f_0, f_1, f_2) = \theta$ yechimga ega bo’lishi zarur va yetarlidir, ya’ni λ soni $A_{\le 2}$ operator uchun xos qiymat bo’lmasligi zarur va yetarlidir. Bu holda (1.1.4) tenglama $(A_{\le 2} - \lambda I)$ operatorning qiymatlar sohasidan olingan har bir g uchun yagona f yechimga ega bo’ladi. Bu to’plamni V_λ bilan belgilaymiz.

Faraz qilaylik, $A_{\le 2}$ o’z-o’ziga qo’shma operator bo’lsin. Agar λ soni $A_{\le 2}$ operatorning xos qiymatlari bo’lsa, u holda V_λ to’plam $H^{(2)}$ fazoning hamma yerida zich bo’ladi. Agar λ soni $A_{\le 2}$ operatorning xos qiymati bo’lsa , u holda V_λ to’plam $H^{(2)}$ fazoning hamma yerida zich bo’lmasligi mumkin.

Faraz qilaylik, $A_{\le 2}$ operator o’z- o’ziga qo’shma bo’lsin. Haqiqiy λ sonlar ichidan $A_{\le 2}$ operatorning xos qiymati bo’lmaydigan sonlarni ajratamiz, ya’ni $(A_{\le 2} - \lambda I)$ operatorning qiymatlar sohasi V_λ to’plam $H^{(2)}$ fazo bilan ustma-ust tushadigan λ lar to'plamini ajratamiz, bunda $(A_{\le 2}-\lambda I)^{-1}$ mavjud va chegaralangan deb faraz qilamiz. Boshqacha qilib aytganda $(A_{\le 2}-\lambda I)$ teskarilanuvchan. Bunday λ larga regulyar nuqtalar deyiladi.

Cekli o'lchamli fazolarda $A_{\le 2}$ operatorning barcha xos qiymatlari to'plamiga shu operatorning spektri deyiladi, λ ning qolgan qiymatlariga ega regulyar nuqtalar deyiladi.

Shunday qilib, chekli o'lchamli fazoda quyidagi ikkita hol bo'lishi mumkin.

1) λ son uchun $A_{\le 2}f=\lambda f$ tenglama nolmas yechimga ega, ya'ni λ son $A_{\le 2}$ operator uchun xos qiymat, bu holda $(A_{\le 2}-\lambda I)$ ga teskari operator mavjud emas;

2) λ son uchun $H^{(2)}$ fazoning hamma yerida aniqlangan $(A_{\le 2}-\lambda I)^{-1}$ operator mavjud va demak chegaralangan.

Biz qarayotgan $H^{(2)}$ fazo cheksiz o'lchamli fazo hisoblanadi.

Cheksiz o'lchamli fazoda uchnchi hol ham mavjud.

3) $(A_{\le 2}-\lambda I)^{-1}$ operator mavjud, ya'ni $A_{\le 2}f=\lambda f$ tenglama faqat nol yechimga ega, lekin $(A_{\le 2}-\lambda I)^{-1}$ operator X ning hamma yerida aniqlangan yoki $\overline{\mathrm{Im}(A_{\le 2}-\lambda I)}\ne X$.

$A_{\le 2}$ operatorning barcha chekli karrali yakkalangan xos qiymatlari to'plamiga $A_{\le 2}$ operatorning diskret spektri deyiladi, va $\sigma_{disc}(A_{\le 2})$ kabi belgilanadi.

Spektrning qolgan qismiga, ya'ni $\sigma(A_{\le 2})\setminus\sigma_{ess}(A_{\le 2})$ to'plamga $A_{\le 2}$ operatorning muhim spektri deyiladi va $\sigma_{ess}(A_{\le 2})$ kabi belgilanadi.

Shunday qilib,

$$\sigma(A_{\le 2})=\sigma_{ess}(A_{\le 2})\cup\sigma_{disc}(A_{\le 2})$$

bo'ladi.

Muhim spektrning ikkinchi ta'rifini keltiramiz.

λ soni $A_{\le 2}$ o'z-o'ziga qo'shma operatorning muhim spektriga tegishli bo'lishi uchun shunday $\{f_n\}$ ortonormal vektorlar ketma-ketligi topilib

$$\left\|(A_{\le 2}-\lambda I)f^{(n)}\right\|\to 0, \qquad n\to\infty$$

munosabat bajarilishi zarur va yetarli.

Agar $A_{\leq 2}$ operator $H^{(2)}$ fazoning ixtiyoriy chegaralangan to'plamini nisbiy kompakt to'plamga o'tkazsa, u holda $A_{\leq 2}$ ga kompakt operator deyiladi.

Har qanday kompakt operator chiziqlidir.

Agar $A_{\leq 2}^{(1)}$ va $A_{\leq 2}^{(2)}$ lar $H^{(2)}$ fazoni $H^{(2)}$ fazoga o'tkazuvchi kompakt operatorlar bo'lsa, u holda $\alpha A_{\leq 2}^{(1)} + \beta A_{\leq 2}^{(2)}$ operator ham kompakt operator bo'ladi, bunda α va β lar ixtiyoriy sonlar.

Bizga $H^{(2)}$ ni $H^{(2)}$ ga o'tkazuvchi $\{A_{\leq 2}^{(n)}\}$ kompakt operatorlar ketma-ketligi berilgan bo'lsin. Agar $H^{(2)}$ to'la va $\{A_{\leq 2}^{(n)}\}$ ketma-ketlik biror $A_{\leq 2}$ operatorga tekis yaqinlashuvchi bo'lsa , u holda $A_{\leq 2}$ operator ham kompakt operator bo'ladi. Agar $A_{\leq 2}^{(1)}$ kompakt, $A_{\leq 2}^{(2)}$ esa chegaralangan operator bo'lsa, u holda $A_{\leq 2}^{(1)} \cdot A_{\leq 2}^{(2)}$ va $A_{\leq 2}^{(2)} \cdot A_{\leq 2}^{(1)}$ operator ham kompakt bo'ladi.

Agar $A_{\leq 2}$ operator qiymatlar sohasining o'lchami n ga teng bo'lsa, u holda $A_{\leq 2}$ ga n o'lchamli operator deyiladi.

Ixtiyoriy chiziqli o'lchamli operator kompakdir.

Kompakt operatorning spektriga qarashli istalgan nolmas λ soni shu operratorning xos qiymati bo'ladi. $A_{\leq 2}$ operatorning muhim spektri kompakt qo'zg'alishlarda o'zgarmaydi, ya'ni agar k-kompakt operator bo'lsa, u holda

$$\sigma_{ess}(A_{\leq 2}) = \sigma_{ess}(A_{\leq 2} + k)$$

tenglik o'rinlidir. (Veyl teoremasi)

1.2. Matritsaviy operatorlar va ularning chiziqliligi hamda chegaralanganligi

Faraz qilaylik, $T^{\nu} = [-\pi;\pi]$ - ν-o'lchamli kub, $L_2(T^{\nu}) - T^{\nu}$ da aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatli) funksiyalarning Gilbert fazosi, $L_2^s((T^{\nu})^2) - (T^{\nu})^2$ da aniqlangan kvadrati bilan integrallanuvchi simmetrik (umuman olganda kompleks qiymatli) funksiyalarning Gilbert fazosi, C esa bir o'lchamli kompleks fazo bo'lsin.

Quyidagi belgilashlarni kiritamiz:

$$H_0 = C;$$

$$H_1 = L_2(T^\nu);$$

$$H_2 = L_2^s((T^\nu)^2);$$

$$H = H_0 \oplus H_1 \oplus H_2.$$

H Gilbert fazofa Fok fazosining qirqilgan ikki zarrachali qism fazosi deyiladi.

H Gilbert fazosiga quyidagi

$$A = \begin{pmatrix} A_{00} & A_{01} & 0 \\ A_{10} & A_{11} & A_{12} \\ 0 & A_{21} & A_{22} \end{pmatrix} \tag{1.2.1}$$

3×3 blok matritsali operatorni qaraymiz,
bu yerda $A_{ij} : H_j \to H_i,\ i, j = 0,1,2.$

Operatorlar ushbu tengliklar orqali ta’sir qiladi:

$$(A_{00} f_0)_0 = \omega_0 f_0;$$

$$(A_{01} f_1)_0 = \int_{T^\nu} v_0(s) f_1(s) ds;$$

$$(A_{10} f_0)_1(p) = v_0(p) f_0;$$

$$(A_{11} f_1)_1(p) = \omega_1(p) f_1(p); \tag{1.2.2}$$

$$(A_{12} f_2)_1(p) = \int_{T^\nu} v_1(s) f_2(p,s) ds;$$

$$(A_{21} f_1)_2(p,q) = \frac{1}{2}(v_1(p) f_1(q) + v_1(q) f_1(p));$$

$$(A_{22} f_2)_2(p,q) = \omega_2(p,q) f_2(p,q).$$

Bunda $f_i \in H_i,\ i = 0,1,2.,\ \omega_0$ fiksirlangan haqiqiy son, $\omega_1(\cdot), v_i(\cdot)$, $i = 0,1$ funksiyalar T^ν da aniqlangan haqiqiy qiymatli uzluksiz funksiyalar, $\omega_2(\cdot,\cdot)$ esa $(T^\nu)^2$ da aniqlangan haqiqiy qiymatli simmetrik uzluksiz funksiya.

Parametrlarga qo’yilgan bunday shartlarda (1.2.1) formula bilan H Gilbert fazosida ta’sir qiluvchi A operator chiziqli, chegaralangan va o’z-o’ziga qo’shma operatordir.

A operatorni chiziqlilikka tekshiramiz:

Buning uchun har bir A_{ij} matritsaviy elementlarning chiziqli ekanligini isbotlaymiz,

ya'ni ixtiyoriy $\alpha, \beta \in C$ va ixtiyoriy $f_j, g_j \in H_j$ elementlar uchun

$$A_{ij}(\alpha f_j + \beta g_j) = \alpha A_{ij} f_j + \beta A_{ij} g_j$$

tenglikni ko'rsatish yetarli.

1) $A_{00}(\alpha f_0 + \beta g_0)_0 = \omega_0(\alpha f_0 + \beta g_0) = \alpha \omega_0 f_0 + \beta \omega_0 g_0 = \alpha A_{00} f_0 + \beta A_{00} g_0$,

Demak, A_{00}-chiziqli operator ekan.

2) $A_{01}(\alpha f_1 + \beta g_1)_0 = \int\limits_{T^\nu} v_0(s)(\alpha f_1(s) + \beta g_1(s))ds = \int\limits_{T^\nu} v_0(s)\ \alpha f_1(s) + \int\limits_{T^\nu} v_0(s)\ \beta g_1(s)ds =$

$= \alpha \int\limits_{T^\nu} v_0(s) f_1(s)ds + \beta \int\limits_{T^\nu} v_0(s) g_1(s)ds = \alpha A_{01} f_1 + \beta A_{01} g_1$,

Demak, A_{01}-chiziqli operator ekan.

3) $A_{10}(\alpha f_0 + \beta g_0)_1(p) = v_0(p)(\alpha f_0 + \beta g_0) = \alpha\ v_0(p) f_0 + \beta\ v_0(p) g_0 =$

$= \alpha(A_{10} f_0)(p) + \beta(A_{10} g_0)(p)$.

Bundan kelib chiqadiki, A_{10} -chiziqli operator.

4) $A_{11}(\alpha f_1 + \beta g_1)_1(p) = \omega_1(p)(\alpha\ f_1(p) + \beta\ g_1(p)) = \alpha\, \omega_1(p) f_1(p) + \beta\, \omega_1(p) g_1(p) =$

$= \alpha(A_{11} f_1)(p) + \beta(A_{11} g_1)(p) \Rightarrow A_{00}$-chiziqli operator.

5) $A_{12}(\alpha\, f_2 + \beta\, g_2)_1(p) = \int\limits_{T^\nu} v_1(s)\,(\alpha f_2(p;s) + \beta g_2(p;s))ds = \int\limits_{T^\nu} v_1(s) \alpha f_2(p;s)ds +$

$+ \int\limits_{T^\nu} v_1(s) \beta\, g_2(p;s)ds = \alpha \int\limits_{T^\nu} v_1(s) f_2(p;s)ds + \beta \int\limits_{T^\nu} v_1(s)\, g_2(p;s)ds = \alpha(A_{12} f_2)(p) + \beta(A_{12} g_2)(p)$

Demak, A_{12}-chiziqli operator ekan.

6) $A_{21}(\alpha f_1 + \beta g_1)_2(p;q) = \frac{1}{2} v_1(p)(\alpha f_1(q) + \beta g_1(q)) + \frac{1}{2} v_1(q)(\alpha f_1(p) + \beta g_1(p)) =$

$= \frac{1}{2} v_1(p) \alpha f_1(q) + \frac{1}{2} v_1(p) \beta\, g_1(q) + \frac{1}{2} v_1(q) \alpha f_1(p) + \frac{1}{2} v_1(q) \beta\, g_1(p) =$

$= \alpha \frac{1}{2}(v_1(p) f_1(q) + v_1(q) f_1(p)) + \beta \frac{1}{2}(v_1(p) g_1(q) + v_1(q) g_1(p)) =$

$= \alpha(A_{21} f_1)(p;q) + \beta(A_{21} g_1)(p;q)$

Demak, A_{21} - chiziqli operator ekan.

7) $A_{22}(\alpha f_2+\beta g_2)_2(p;q)=\omega_2(p;q)(\alpha f_2(p;q)+\beta g_2(p;q))=\alpha\omega_2(p;q)f_2(p;q)+$

$+\beta\,\omega_2(p;q)g_2(p;q)=\alpha(A_{22}f_2)(p;q)+\beta(A_{22}g_2)(p;q).\Rightarrow A_{22}-$chiziqli operator.

Yuqorida ko'paytuvchilarni o'rin almashtirish ya'ni kommutativlik shartidan va integral ostidagi ozod hadni integral ostidan chiqarishlik shartlaridan foydalandik.

Endi A_{ij} operatorlarning chiziqli ekanligidan foydalanib , A ning chiziqli operator ekanligini isbotlaymiz.

Buning uchun H Gilbert fazosining

$$f=\begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix} \text{ va } g=\begin{pmatrix} g_0 \\ g_1 \\ g_2 \end{pmatrix}$$

ko'rinishdagi ixtiyoriy vektorlarni H Gilbert fazosidan olamiz. U holda ixtiyoriy $\alpha,\beta\in R(C)$ sonlar uchun

$$A(\alpha f+\beta g)=\alpha Af+\beta Ag$$

tenglik o'rinlidir.

$$A(\alpha f+\beta g)=\begin{pmatrix} A_{00} & A_{01} & 0 \\ A_{10} & A_{11} & A_{12} \\ 0 & A_{21} & A_{22} \end{pmatrix}\cdot\begin{pmatrix} \alpha f_0 & + & \beta g_0 \\ \alpha f_1 & + & \beta g_1 \\ \alpha f_2 & + & \beta g_2 \end{pmatrix}$$

Matritsani vektorga ko'paytirish qoidasiga asosan yuqoridagi ifodani quyidagicha yozamiz:

$$A(\alpha f+\beta g)=\begin{pmatrix} A_{00}(\alpha f_0+\beta g_0) & + & A_{01}(\alpha f_1+\beta g_1) & + & 0 \\ A_{10}(\alpha f_0+\beta g_0) & + & A_{11}(\alpha f_1+\beta g_1) & + & A_{12}(\alpha f_2+\beta g_2) \\ 0 & + & A_{21}(\alpha f_1+\beta g_1) & + & A_{22}(\alpha f_2+\beta g_2) \end{pmatrix}=$$

$$=\begin{pmatrix} A_{00}\alpha f_0+A_{00}\beta g_0+A_{01}\alpha f_1+A_{01}\beta g_1 \\ A_{10}\alpha f_0+A_{10}\beta g_0+A_{11}\alpha f_1+A_{11}\beta g_1+A_{12}\alpha f_2+A_{12}\beta g_2 \\ A_{21}\alpha f_1+A_{21}\beta g_1+A_{22}\alpha f_2+A_{22}\beta g_2 \end{pmatrix}=$$

{Hosil bo'lgan matritsamizni guruhlab uni quyidagicha yozib olamiz}

$$=\begin{pmatrix} \alpha(A_{00}f_0 + A_{01}f_1) + \beta(A_{00}g_0 + A_{01}g_1) \\ \alpha(A_{10}f_0 + A_{11}f_1 + A_{12}f_2) + \beta(A_{10}g_0 + A_{11}g_1 + A_{12}g_2) \\ \alpha(A_{21}f_1 + A_{22}f_2) + \beta(A_{21}g_1 + A_{22}g_2) \end{pmatrix} =$$

Qavslarni ochish qoidasiga asosan ya’ni

birhadni ko’phadga ko’paytirish : $a(b+c) = ab + ac$

Matritsalarning ba’zi xossalari

Bir xil tipli matritsalar yig’indi(ayirma)si deb, elementlari shu matritsalarning mos elementlari yig’indisi(ayirmasi)dan tuzilgan matritsaga aytiladi: $A < m,n >$, $B < m,n >$ tipli matritsalar berilgan.

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \qquad \text{va} \qquad B = \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ b_{21} & \cdots & b_{2n} \\ \vdots & \vdots & \vdots \\ b_{m1} & \cdots & b_{mn} \end{pmatrix}$$

$$A \pm B = \begin{pmatrix} a_{11} \pm b_{11} & \cdots & a_{1n} \pm b_{1n} \\ a_{21} \pm b_{21} & \cdots & a_{2n} \pm b_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} \pm b_{m1} & \cdots & a_{mn} \pm b_{mn} \end{pmatrix}$$

Shulardan foydalanib matritsamizni quyidagi ko’rinishda yozib olamiz.

$$=\begin{pmatrix} \alpha(A_{00}f_0 + A_{01}f_1) \\ \alpha(A_{10}f_0 + A_{11}f_1 + A_{12}f_2) \\ \alpha(A_{21}f_1 + A_{22}f_2) \end{pmatrix} + \begin{pmatrix} \beta(A_{00}g_0 + A_{01}g_1) \\ \beta(A_{10}g_0 + A_{11}g_1 + A_{12}g_2) \\ \beta(A_{21}g_1 + A_{22}g_2) \end{pmatrix}$$

Matritsani songa ko’paytirish deb uning hamma elementlarini shu songa ko’paytirish orqali hosil qilingan matritsaga aytiladi: $A < m,n >$ tipli matritsa va $k = const$ son berilgan.

$$A=\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix},$$

$$k*A=\begin{pmatrix} k\cdot a_{11} & \cdots & k\cdot a_{1n} \\ k\cdot a_{21} & \cdots & k\cdot a_{2n} \\ \vdots & \vdots & \vdots \\ k\cdot a_{m1} & \cdots & k\cdot a_{mn} \end{pmatrix}$$

Shuni hisobga olgan holda oxirgi natijani quyidagicha olamiz:

$$A(\alpha f+\beta g)=\alpha\begin{pmatrix} (A_{00}f_0+A_{01}f_1) \\ (A_{10}f_0+A_{11}f_1+A_{12}f_2) \\ (A_{21}f_1+A_{22}f_2) \end{pmatrix}+\beta\begin{pmatrix} (A_{00}g_0+A_{01}g_1) \\ (A_{10}g_0+A_{11}g_1+A_{12}g_2) \\ (A_{21}g_1+A_{22}g_2) \end{pmatrix}=\alpha Af+\beta Ag\,.$$

Demak, A operator chiziqli operator ekan.

Endi A operatorni chegaralanganlikka tekshiramiz. Ta'rifga asosan shunday musbat $M>0$ soni topilib, ixtiyoriy H Gilbert fazosidan olngan f uchun

$$\|Af\|\leq M\cdot\|f\|$$

tengsizlik bajarilsa, A operator chegaralangan operator deyiladi.

Ixtiyoriy $f=(f_0,f_1,f_2)\in H$ element uchun $\|Af\|$ ni baholaymiz.

$$\|Af\|=\left\|\begin{pmatrix} A_{oo} & A_{01} & 0 \\ A_{10} & A_{11} & A_{12} \\ 0 & A_{21} & A_{22} \end{pmatrix}\cdot\begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix}\right\|=\left\|\begin{matrix} A_{00}f_0+A_{01}f_1 \\ A_{10}f_0+A_{11}f_1+A_{12}f_2 \\ A_{21}f_1+A_{22}f_2 \end{matrix}\right\|=$$

$$=\left\|\begin{matrix} \omega_0 f_0+\int\limits_{T^\nu} v_0(s)f_1(s)ds \\ v_0(p)f_0+\omega_1(p)f_1(p)+\int\limits_{T^\nu} v_1(s)f_2(p;s)ds \\ \frac{1}{2}\big(v_1(p)f_1(q)+v_1(q)f_1(p)\big)+\omega_2(p;q)f_2(p;q) \end{matrix}\right\|$$

Agar $g=\begin{pmatrix} g_0 \\ g_1 \\ g_2 \end{pmatrix}$ bo'lsa, uning normasi $\|g\|$ quyidagi formulaorqalitopiladi

$$\|g\|=\sqrt{\|g_0\|^2+\|g_1\|^2+\|g_2\|^2}\ .$$

bunda

$$\|g_0\|_0 = |\,g_0\,|\ ;$$

$$\|g_1\|_1=\sqrt{\int_{T^\nu}|g_1(s)|^2\,ds}\ ;$$

$$\|g_2\|_2=\sqrt{\int_{T^\nu}\int_{T^\nu}|g_2(s;t)|^2\,dt\,ds}\ .$$

ga teng.

Quyidagi belgilashlar kiritamiz:

$$g_0=\omega_0 f_0+\int_{T^\nu} v_0(s)f_1(s)ds\ ;$$

$$g_1(p)=v_0(p)+\omega_1(p)f_1(p)+\int_{T^\nu} v_1(s)f_2(p;s)ds\ ;$$

$$g_2(p;q)=\frac{1}{2}(v_1(p)f_1(q)+v_1(q)f_1(p))+\omega_2(p;q)f_2(p;q)\,.$$

U holda

$$\|g\|^2=\|g_0\|_0^2+\|g_1\|_1^2+\|g_2\|_2^{\ 2}=\left|\omega_0 f_0+\int_{T^\nu} v_0(s)f_1(s)ds\right|^2+$$

$$+\int_{T^\nu}\left|v_0(p)f_0+\omega_1(p)f_1(p)+\int_{T^\nu} v_1(s)f_2(p;s)ds\right|^2 dp+$$

$$+\int_{T^\nu}\int_{T^\nu}\left|\frac{1}{2}(v_1(p)f_1(q)+v_1(q)f_1(p))+\omega_2(p;q)f_2(p;q)\right|^2 dpdq \qquad (1.2.3)$$

bo'ladi.

(1.2.3) ifodadagi 1-tenglamaning o'ng tomonidagi birinchi qo'shiluvchini baholaymiz.

Buning uchun

$$\left|a+b\right|^2 \le 2\,|\,a\,|^2 + 2\,|\,b\,|^2$$

elementar tengsizlikdan foydalanamiz.

$$\left|\omega_0 f_0 + \int_{T^\nu} v_0(s) f_1(s) ds\right|^2 \le 2\left|\omega_0 f_0\right|^2 + 2\left|\int_{T^\nu} v_0(s) f_1(s) ds\right|^2 \qquad (1.2.4)$$

(1.2.4) tengsizlikning o'ng tomonidagi birinchi qo'shiluvchi uchun

$$\left\|\omega_0 f_0\right\|^2 = \left|\omega_0\right|^2 \cdot \left|f_0\right|^2$$

tenglik o'rinlidir. Shu sababli (1.2.4) tengsizlik

$$\left|\omega_0 f_0 + \int_{T^\nu} v_0(s) f_1(s) ds\right|^2 \le 2\left|\omega_0\right|^2\left|f_0\right|^2 + 2\left|\int_{T^\nu} v_0(s) f_1(s) ds\right|^2$$

kabi yoziladi.

(1.2.4) tengsizlikning o'ng tomonidagi ikkinchi qo'shiluvchi uchun Koshi-Bunyakovskiy tengsizligi, ya'ni

$$\left|\int_{T^\nu} v_0(s) f_1(s) ds\right|^2 \le \int_{T^\nu} \left|v_0(s)\right|^2 ds \cdot \int_{T^\nu} \left|f_1(s)\right|^2 ds$$

tengsizlikdan foydalanamiz va (2) tengsizlikni quyidagicha yozamiz:

$$\left|\omega_0 f_0 + \int_{T^\nu} v_0(s) f_1(s) ds\right|^2 \le 2\left|\omega_0\right|^2 \cdot \left|f_0\right|^2 + 2\int_{T^\nu} \left|v_0(s)\right|^2 ds \cdot \int_{T^\nu} \left|f_1(s)\right|^2 ds \qquad (1.2.5)$$

(1.2.5) tengsizlikning o'ng tomonidagi ikkinchi hadi uchun H_0 va H_1 fazodagi norma formulalaridan foydalanib, (1.2.5) tengsizlikni quyidagicha yozib olamiz.

$$\left|\omega_0 f_0 + \int_{T^\nu} v_0(s) f_1(s) ds\right|^2 \le 2\left\|\omega_0\right\|_0^2 \cdot \left\|f_0\right\|_0^2 + 2\left\|v_0\right\|_1^2 \cdot \left\|f_1\right\|_1^2 \qquad (1.2.6)$$

Endi (1.2.3) tengsizlikning o'ng tomonidagi ikkinchi hadini, ya'ni

$$\int_{T^\nu} \left|v_0(p) f_0 + \omega_1(p) f_1(p) + \int_{T^\nu} v_1(s) f_2(p;s) ds\right|^2 dp$$

ni baholayniz.

Buning uchun quyidagi elementar tengsizlikdan foydalanamiz:

$$\left|a+b+c\right|^2 \le 3\,|\,a\,|^2 + 3\,|\,b\,|^2 + 3\left|c\right|^2$$

Demak,

$$\int_{T^\nu}\left|v_0(p)f_0+\omega_1(p)f_1(p)+\int_{T^{\nu\nu}}v_1(s)f_2(p;s)ds\right|^2 dp \le \int_{T^\nu}3\left|v_0(p)f_0\right|^2 dp+\int_{T^\nu}3\left|\omega_1(p)f_1(p)\right|^2 dp+$$

$$+\int_{T^\nu}3\left|\int_{T^\nu}v_1(s)f_2(p;s)ds\right|^2 dp \qquad (1.2.7)$$

(1.2.7) tengsizlikni hosil qilamiz. Bu tengsizlik uchun integral ostidagi ozod hadni integral belgisi ostidan tashqariga chiqarish mumkinligi to'g'risidagi qoidaga asosan (1.2.7) tengsizlikni quyidagicha yozish mumkin. Shunga ko'ra

$$\int_{T^\nu}\left|v_0(p)f_0+\omega_1(p)f_1(p)+\int_{T^{\nu\nu}}v_1(s)f_2(p;s)ds\right|^2 dp \le 3\int_{T^\nu}\left|v_0(p)f_0\right|^2 dp+3\int_{T^\nu}\left|\omega_1(p)f_1(p)\right|^2 dp+$$

$$+3\int_{T^\nu}\left|\int_{T^\nu}v_1(s)f_2(p;s)ds\right|^2 dp$$

$$(1.2.8)$$

bo'ladi.

(1.2.8) tengsizlikning o'ng tomonidagi uchinchi hadi uchun har bir fiksirlangan $p \in T^\nu$ uchun Koshi-Bunyakovskiy tengsizligini qo'llaymiz:

$$\int_{T^\nu}\left|\int_{T^\nu}v_1(s)f_2(p;s)ds\right|^2 dp \le \int_{T^\nu}\left(\int_{T^\nu}|\,v_1(s)\,|^2\;ds\right)\left(\int_{T^\nu}\left|f_2(p;s)\right|^2 ds\right)dp.$$

Demak,

$$\int_{T^\nu}\left|\int v_1(s)f_2(p;s)\right|^2 dp \le \left\|v_1\right\|_1^2\cdot\left\|f_2\right\|_2^2$$

bo'ladi.

(1.2.8) tengsizlikning o'ng tomonidagi ikkinchi qo'shiluvchi uchun quyidagi belgilashni kiritamiz.

$$3\max_{p;q\in T^\nu}\left|\omega_1(p)\right|^2 = M_1$$

Olingan oxirgi natijalardan (1.2.6) tengsizlikni quyidagi ko'rinishni oladi.

$$\int_{T^\nu}\left|v_0(p)f_0+\omega_1(p)f_1(p)+\int_{T^\nu}v_1(s)f_2(p;s)ds\right|^2dp\le$$

$$\le 3\|v_0\|^2\cdot\|f_0\|^2+M_1\cdot\|f_1\|^2+3\|v_1\|^2\cdot\|f_2\|^2 \qquad (1.2.9)$$

Bulardan so'ng (1.2.3) tengsizlikning o'ng tomonidagi oxirgi hadi, ya'ni

$$\int_{T^\nu}\int_{T^\nu}\left|\frac{1}{2}(v_1(p)f_1(q)+v_1(q)f_1(p))+\omega_2(p;q)f_2(p;q)\right|^2dpdq$$

ni baholaymiz.

Buning uchun

$$|a+b+c|^2\le 3\,|\,a\,|^2\ +3\,|\,b\,|^2\ +3|c|^2$$

elementar tengsizlikdan foydalanamiz:

Integrallash qoidalariga asosan ya'ni yig'indi (ayirma)ning integrali integrallar yig'indisi(ayirmasi) ga teng

$$\int_a^b(U(x)\pm V(x))dx=\int_a^bU(x)dx\pm\int_a^bV(x)dx$$

$$\int_{T^\nu}\int_{T^\nu}\left|\frac{1}{2}(v_1(p)f_1(q)+v_1(q)f_1(p))+\omega_2(p;q)f_2(p;q)\right|^2dpdq=$$

$$=\int_{T^\nu}\int_{T^\nu}\left|\frac{1}{2}v_1(p)f_1(q)+\frac{1}{2}v_1(q)f_1(p)+\omega_2(p;q)f_2(p;q)\right|^2dpdq$$

$$\int_{T^\nu}\int_{T^\nu}\left|\frac{1}{2}v_1(p)f_1(q)+\frac{1}{2}v_1(q)f_1(p)+\omega_2(p;q)f_2(p;q)\right|^2dpdq\le\int_{T^\nu}\int_{T^\nu}\frac{3}{4}|v_1(p)f_1(q)|^2dpdq+$$

$$+\int_{T^\nu}\int_{T^\nu}\frac{3}{4}|v_1(q)f_1(p)|^2dpdq+\int_{T^\nu}\int_{T^\nu}3|\omega_2(p;q)f_2(p;q)|^2dpdq$$

Integral ostidagi ozod sonni integral belgisi ostidan tashqariga chiqarish mumkinligi qoidasiga asosan yuqoridagi tengsizlikni quyidagicha yozish mumkin.

$$+\frac{3}{4}\int_{T^\nu}\int_{T^\nu}|v_1(q)f_1(p)|^2dpdq+3\int_{T^\nu}\int_{T^\nu}|\omega_2(p;q)f_2(p;q)|^2dpdq$$

(1.2.10)

(1.2.10) tengsizlikning o'ng tomonidagi uchinchi hadi uchun

$$3\max_{p'q\in T^{\nu}}\int_{T^{\nu}}\int_{T^{\nu}}|\omega_2(p;q)|^2\,dpdq = M_2$$

belgilashni olamiz.

Bundan so'ng (1.2.10) tengsizlikni ko'rinishini quyidagicha yozib olamiz.

$$\int_{T^{\nu}}\int_{T^{\nu}}\left|\frac{1}{2}(v_1(p)f_1(q)+v_1(q)f_1(p))+\omega_2(p;q)f_2(p;q)\right|^2 dpdq\le\frac{3}{2}\|v_1\|^2\cdot\|f_1\|^2+3M_2\|f_2\|^2 \qquad (1.2.11)$$

Endi (1.2.6), (1.2.9) va (1.2.11) tengsizliklardan foydalanib (1.2.3) tengsizlikni quyidagicha yozib olamiz:

$$\|Af\|^2\le 2\|\omega_0\|_0^2\|f_0\|_0^2+2\|v_0\|_1^2\cdot\|f_1\|_1^2+3\|v_0\|^2\cdot\|f_0\|^2+M_1\|f_1\|^2+3\|v_1\|^2\cdot\|f_2\|^2+\frac{3}{2}\|v_1\|^2\cdot\|f_1\|^2+$$

$$+3M_2\|f_2\|^2=\left(2\|\omega_0\|_0^2+3\|v_0\|^2\right)\cdot\|f_0\|^2+\left(2\|v_0\|^2+M_1+\frac{3}{2}\|v_1\|^2\right)\cdot\|f_1\|^2+\left(3\|v_1\|^2+3M_2\right)\|f_2\|^2\le$$

$$\le\max\{(2\|\omega_0\|^2+3\|v_0\|^2),(2\|v_0\|^2+M_1+\frac{3}{2}\|v_1\|^2).,(3\|v_1\|^2+3M_2)\}\;(\|f_0\|_0^2+\|f_1\|_1^2+\|f_2\|_2^2)=$$

$$=\max\{(2\|\omega_0\|^2+3\|v_0\|^2),(2\|v_0\|^2+M_1+\frac{3}{2}\|v_1\|^2).,(3\|v_1\|^2+3M_2)\}\;\|f\|^2$$

ya'ni

$$\|Af\|\le M\cdot\|f\|$$

bo'ladi.

Bu yerda

$$M=\max\{(2\|\omega_0\|^2+3\|v_0\|^2),(2\|v_0\|^2+M_1+\frac{3}{2}\|v_1\|^2).,(3\|v_1\|^2+3M_2)\}$$

Ta'rifga ko'ra A chegaralangan operator ekan.

1.3. Matritsaviy operatorning o'z-o'ziga qo'shmaligi

Ma'lumki, A operator o'z-o'ziga qo'shma bo'lishi uchun

$$(A_{ii}f_i,g_i)_{H_i}=(f_i,A_{ii}g_i)_{H_i} \qquad i,j=0,1,2$$

$$(A_{ij}f_j,g_i)_{H_i}=(f_j,A_{ji}g_i)_{H_j} \qquad i,j=0,1\;\; i\ne j$$

$$(A_{ij}f_j,g_i)_{H_i}=(f_j,A_{ji}g_i)_{H_j} \qquad i,j=1,2\;\; i\ne j$$

tengliklarning bajarilishi zarur va yetarlidir.

Shu sababli yuqoridagilarni isbotlaymiz.

Avvalo H_0, H_1, H_2 va H fazodagi skalyar ko'paytma formulalarini keltiramiz.

$$(f_0, g_0)_{H_0} = f_0\,\overline{g_0}\,, \text{ ixtiyoriy } f_0, g_0 \in H_0$$

$$(f_1, g_1)_{H_1} = \int_{T^\nu} f_1(s)\,\overline{g_1(s)}\,ds\,, \text{ ixtiyoriy } f_1, g_1 \in H_1$$

$$(f_2, g_2)_{H_2} = \int_{T^\nu}\int_{T^\nu} f_2(s,t)\,\overline{g_2(s,t)}\,dsdt \;, \text{ ixtiyoriy } f_2, g_2 \in H_2$$

$$(f, g) = (f_0, g_0)_{H_0} + (f_1, g_1)_{H_1} + (f_2, g_2)_{H_2}\,,$$

A operatorimiz H Gilbert fazosidan olingan ixtiyoriy $f = \begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix}$ va $g = \begin{pmatrix} g_0 \\ g_1 \\ g_2 \end{pmatrix}$ ko'rinishdagi vektorlar bilan ta'sirini quyidagicha yozamiz.

A operatorning f ga ta'sirini ko'ramiz:

$$Af = \begin{pmatrix} A_{00} & A_{01} & 0 \\ A_{10} & A_{11} & A_{12} \\ 0 & A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} A_{00}f_0 + A_{01}f_1 \\ A_{10}f_0 + A_{11}f_1 + A_{12}f_2 \\ A_{21}f_1 + A_{22}f_2 \end{pmatrix}$$

1) $\left(A_{00}f_0, g_0\right)_{H_0} = \omega_0 f_0\,\overline{g_0} = f_0\,\overline{\omega_0 g_0} = (f_0, A_{00}g_0)_{H_0}$

Bu yerda biz ω_0 ning haqiqiy son ekanligidan foydalandik, ya'ni $\overline{\omega_0} = \omega_0$

2) $(A_{01}\,f_1, g_1)_{H_0} = \int_{T^\nu} v_0(s)\,f_1(s)\,ds\,\overline{g_0} = \int_{T^\nu} f_1(s)\,\overline{v_0(s)g_0}\,ds = (f_1, A_{01}g_0)_{H_1}$

Biz bu yerda $v_0(s)$ funksiyaning T^ν da aniqlangan haqiqiy qiymatli uzluksiz funksiya ekanligidan foydalandik. $\overline{v_0(s)} = v_0(s)$

3) $(A_{10}f_0, g_1)_{H_1} = \int_{T^\nu} v_0(p)\,f_0\,\overline{g_1(p)}\,dp = f_0\int_{T^\nu}\overline{v_0(p)\,g_1(p)}\,dp = (f_0, A_{01}g_1)_{H_0}$

Bu yerda ham biz $v_0(p)$ funksiyaning T^ν da aniqlangan haqiqiy qiymatli uzluksiz funksiya ekanligidan foydalandik, ya'ni $\overline{v_0(p)} = v_0(p)$ bo'ladi.

4) $\left(A_{11}f_1, g_1\right)_{H_1} = \int_{T^\nu}\omega_1(p)\,f_1(p)\,\overline{g_1(p)}\,dp = \int_{T^\nu} f_1(p)\,\overline{\omega_1(p)\,g_1(p)}\,dp = \left(f_1, A_{11}g_1\right)_{H_1}$

$\omega_1(p)$ funksiya T^ν da aniqlangan haqiqiy qiymatli uzluksiz funksiya bo'lgani uchun $\overline{\omega_1(p)} = \omega_1(p)$ ga teng bo'ladi.

$$5)\ (A_{12}f_2,g_1)_{H_1}=\int\limits_{T^\nu}\left(\int\limits_{T^\nu}v_1(s)\,f_2(p;s)\,ds\right)\overline{g_1(p)}\,dp=\int\limits_{T^\nu}\int\limits_{T^\nu}v_1(s)\,\overline{g_1(p)}\,f_2(p;s)\,dpds=$$

$$=\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}v_1(q)\,\overline{g_1(p)}\,f_2(p;q)\,dpdq+\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}v_1(q)\,\overline{g_1(p)}\,f_2(p;q)\,dpdq=$$

$$=\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}v_1(q)\,\overline{g_1(p)}\,f_2(p;q)\,dpdq+\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}v_1(p)\,\overline{g_1(q)}\,f_2(q;p)\,dpdq=$$

$$=\int\limits_{T^\nu}\int\limits_{T^\nu}f_2(p;q)\left(\frac{1}{2}(v_1(p)\,\overline{g_1(q)}+v_1(q)\overline{g_1(p)})\right)dpdq=(f_2,A_{21}g_1)_{H_2}$$

Bu yerda $v_1(q)$ va $v_1(p)$ funksiyalar T^ν da aniqlangan haqiqiy qiymatli uzluksiz funksiya bo’lgani uchun

$$v_1(q)=\overline{v_1(q)}\ \text{va}\ v_1(p)=\overline{v_1(p)}$$

bo’ladi.

$$6)\,(A_{21}f_1,g_2)_{H_2}=\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}(v_1(q)f_1(p)+v_1(p)f_1(q))\,\overline{g_2(p;q)}\,dpdq=$$

$$=\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}v_1(q)f_1(p)\,\overline{g_2(p;q)}\,dpdq+\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}v_1(p)f_1(q)\,\overline{g_2(p;q)}\,dpdq=$$

$$=\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}v_1(q)f_1(p)\,\overline{g_2(p;q)}\,dpdq+\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}v_1(q)f_1(p)\,\overline{g_2(q;p)}\,dpdq=$$

$$=\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}f_1(p)\,v_1(q)\,\overline{g_2(p;q)}\,dpdq+\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}f_1(p)\,v_1(q)\,\overline{g_2(q;p)}\,dpdq=$$

$$=\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}v_1(q)f_1(p)\,\overline{g_2(p;q)}\,dpdq+\frac{1}{2}\int\limits_{T^\nu}\int\limits_{T^\nu}v_1(p)f_1(q)\,\overline{g_2(p;q)}\,dpdq=$$

$$=\int\limits_{T^\nu}\int\limits_{T^\nu}f_1(p)\,v_1(q)\overline{g_2(p;q)}\,dpdq=(f_1,A_{12}g_2)_{H_1}$$

Biz bu yerda $v_1(q)$ va $v_1(p)$ funksiyalar T^ν da aniqlangan haqiqiy qiymatli uzluksiz funksiya bo’lgani uchun

$$v_1(q)=\overline{v_1(q)}\ \text{va}\ v_1(p)=\overline{v_1(p)}$$

shu tengliklar o’rinli bo’lishidan foydalandik.

$$7)\ (A_{22}f_2,g_2)_{H_2}=\int\limits_{T^\nu}\int\limits_{T^\nu}\omega_2(p;q)\,f_2(p;q)\,\overline{g_2(p;q)}\,dpdq=$$

$$=\int\limits_{T^\nu}\int\limits_{T^\nu}f_2(p;q)\,\omega_2(p;q)\,\overline{g_2(p;q)}\,dpdq=\int\limits_{T^\nu}\int\limits_{T^\nu}f_2(p;q)\overline{\omega_2(p;q)g_2(p;q)}\,dpdq=(f_2,A_{22}g_2)_{H_2}$$

Bu yerda $\omega_2(p;q)$ funksiyaning $(T^{\nu})^2$ da aniqlangan haqiqiy qiymatli simmetrik uzluksiz funksiya bo'lgani uchun

$$\omega_2(p;q) = \overline{\omega_2(p;q)}$$

tenglik o'rinli bo'lishidan foydalandik.

Yuqoridagi natijalardan ko'rinib turibdiki, A operator o'z-o'ziga qo'shma operator ekan.

I bob xulosasi

Birinchi bob uchta qismdan iborat. Birinchi qismda matritsa tipidagi operatorlar uchun umumiy ma'lumotlar, matritsaviy operatorlar va ularning chiziqliligi hamda chegaralanganligi va matritsaviy operatorlarning o'z-o'ziga qo'shmaligi va ayrim fazolar haqida ma'lumotlar berilgan. Qaralayotgan matritsaviy operatorning chiziqliligi, chegaralanganligi va o'z-o'ziga qo'shmaligi tekshirilgan. Bunda birinchi qismda keltirilgan ta'riflardan foydalanilgan. Blok operatorli matritsalarning vektor ta'siridagi qiymatlari hamda xossalari o'rganilgan.

II BOB MATRIRSAVIY OPERATORLAR UCHUN VAYNBERG TENGLAMASINI QURISH

2.1. Uch turdagi matritsaviy operatorlar uchun odatdagi Vaynberg tenglamasi

$C\setminus[m(p),M(p)]$ sohada regulyar bo'lgan quyidagi

$$\Delta(p;\lambda_0)=\omega_1(p)-\lambda_0-\frac{1}{2}\int_{T^\nu}\frac{v_1^2(s)ds}{\omega_2(p;s)-\lambda_0}$$

funksiyani qaraymiz, bu yerda $m(p)$ va $M(p)$ sonlari quyidagicha aniqlanadi:

$$m(p)=\min_{q\in T^\nu}\omega_2(p;q)\,;\quad M(p)=\max_{q\in T^\nu}\omega_2(p;q)\,.$$

Quyidagicha belgilashlar kiritamiz: σ orqali biror $p\in T^\nu$ uchun $\Delta(p;\lambda_0)=0$ bo'ladigan $\lambda_0\in C$ lar to'plamini belgilaymiz,

$$m=\min_{p,q\in T^\nu}\omega_2(p;q)\,;\qquad\qquad M=\max_{p,q\in T^\nu}\omega_2(p;q)$$

A operator muhim spektri uchun quyidagi teorema o'rinlidir.

2.1.1-teorema. A operatorning $\sigma_{ess}(A)$ muhim spektri uchun $\sigma_{ess}(A)=[m;M]\cup\sigma$ tenglik o'rinlidir.

Har bir fiksirlangan $\lambda_0\in C\setminus\sigma_{ess}(A)$ uchun H Gilbert fazosida 3×3 blok operatorli matritsa

$$W_A(\lambda_0)=\begin{pmatrix}W_{00}(\lambda_0) & W_{01}(\lambda_0) & 0\\ W_{10}(\lambda_0) & W_{11}(\lambda_0) & 0\\ W_{20}(\lambda_0) & W_{21}(\lambda_0) & 0\end{pmatrix}$$

ni qaraymiz, bunda

$$(W_{00}(\lambda_0)f_0)_0=(\omega_0-\lambda_0+1)f_0$$

$$(W_{01}(\lambda_0)f_1)_0=\int_{T^\nu}v_0(s)f_1(s)ds$$

$$(W_{10}(\lambda_0)f_0)_1=-\frac{v_0(p)f_0}{\Delta(p;\lambda_0)},$$

$$\Delta(p;\lambda_0)=\omega_1(p)-\lambda_0-\frac{1}{2}\int_{T^\nu}\frac{v_1^2(s)ds}{\omega_2(p;s)-\lambda_0}$$

$$(W_{11}(\lambda_0)f_1)_1(p)=\frac{1}{2}\frac{v_1(p)}{\Delta(p;\lambda_0)}\cdot\int_{T^\nu}\frac{v_1(s)f_1(s)}{\omega_2(p;s)-\lambda_0}ds$$

$$(W_{20}(\lambda_0)f_0)_1(p;q)=\frac{v_1(p)}{2(\omega_2(p;q)-\lambda_0)}\frac{v_0(q)f_0}{\Delta(q;\lambda_0)}+\frac{v_1(p)}{2(\omega_2(p;q)-\lambda_0)}\frac{v_0(p)}{\Delta(p;\lambda_0)}f_0$$

$$(W_{21}(\lambda_0)f_0)_2(p;q)=-\frac{v_1(p)}{4(\omega_2(p;q)-\lambda_0)}\cdot\frac{v_1(q)}{\Delta(q;\lambda_0)}\cdot\int_{T^\nu}\frac{v_1(s)f_1(s)}{\omega_2(q;s)-\lambda_0}ds-$$

$$-\frac{v_1(p)}{4(\omega_2(p;q)-\lambda_0)}\cdot\frac{v_1(p)}{\Delta(p;\lambda_0)}\cdot\int_{T^\nu}\frac{v_1(s)f_1(s)}{\omega_2(p;s)-\lambda_0}ds$$

bo'ladi.

2.1.1-ta'rif. $f=W(\lambda_0)f$ tenglamaga Vaynberg tenglamasi deyiladi.

2.1.2-teorema. Agar λ_0 soni A operatorning xos qiymati, f esa bu xos qiymatga mos xos vektor bo'lsa, u holda f vektor $W_A(\lambda_0)f=f$ Vaynberg tenglamasini qanoatlantiradi.

Isbot. Bu teoremani isbotlashda A operator uchun xos qiymatga nisbatan tenglamani qaraymiz:

$$Af=\lambda_0 f\,.$$

H Gilbert fazosidan olingan A operatorimizni f vektorga ta'sirini ko'ramiz:

$$Af=\begin{pmatrix}A_{00} & A_{01} & 0\\ A_{10} & A_{11} & A_{12}\\ 0 & A_{21} & A_{22}\end{pmatrix}\begin{pmatrix}f_0\\ f_1\\ f_2\end{pmatrix}=\begin{pmatrix}A_{00}f_0+A_{01}f_1\\ A_{10}f_0+A_{11}f_1+A_{12}f_2\\ A_{21}f_1+A_{22}f_2\end{pmatrix},$$

o'zgarmas sonni vektorga ko'paytmasi qoidasiga asosan $\lambda_0 f=\begin{pmatrix}\lambda_0 f_0\\ \lambda_0 f_1\\ \lambda_0 f_2\end{pmatrix}$ bo'ladi.

Demak, $Af=\lambda_0 f$ ni

$$\begin{pmatrix}A_{00}f_0+A_{01}f_1\\ A_{10}f_0+A_{11}f_1+A_{12}f_2\\ A_{21}f_1+A_{22}f_2\end{pmatrix}=\begin{pmatrix}\lambda_0 f_0\\ \lambda_0 f_1\\ \lambda_0 f_2\end{pmatrix}$$

ko'rinishda yozib olsa bo'ladi.

Yuqoridagi tenglamani quyidagi chiziqli tenglamalar sistemasi ko'rinishida yozish mumkin:

$$\begin{cases} A_{00}f_0 + A_{01}f_1 = \lambda_0 f_0 \\ A_{10}f_0 + A_{11}f_1 + A_{12}f_2 = \lambda_0 f_1 \\ A_{21}f_1 + A_{22}f_2 = \lambda_0 f_2 \end{cases} \qquad (2.1.1)$$

A_{ij} operatorlar ta'sir qilgan tengliklar orqali (2.1.1) chiziqli tenglamalar sistemasini quyidagicha yozib olamiz:

$$\begin{cases} \omega_0 f_0 + \int\limits_{T^\nu} v_0(s)f_1(s)ds = \lambda_0 f_0 \\ v_0(p)f_0 + \omega_1(p)f_1(p) + \int\limits_{T^\nu} v_1(s)f_2(p;s)ds = \lambda_0 f_1(p) \\ \frac{1}{2}(v_1(p)f_1(q) + v_1(q)f_1(p)) + \omega_2(p;q)f_2(p;q) = \lambda_0 f_2(p;q). \end{cases} \qquad (2.1.2)$$

(2.1.2)- chiziqli tenglamalar sistemasining o'ng tomonidagi ifodalarni chap tomonga o'tkazib soddalashtiramiz:

$$\begin{cases} (\omega_0 - \lambda_0)f_0 + \int\limits_{T^\nu} v_0(s)f_1(s)ds = 0 \\ v_0(p)f_0 + (\omega_1(p) - \lambda_0)f_1(p) + \int\limits_{T^\nu} v_1(s)f_2(p;s)ds = 0 \\ \frac{1}{2}(v_1(p)f_1(q) + v_1(q)f_1(p)) + (\omega_2(p;q) - \lambda_0)f_2(p;q) = 0 \end{cases} \qquad (2.1.3)$$

Ixtiyoriy $p,q \in T^\nu$ lar uchun $\omega_2(p;q) - \lambda_0 \neq 0$ bo'ladi. Shu sababli (2.1.3) tenglamalar sistemasining uchinchi tenglamasidagi $f_2(p;q)$ ni quyidagi ko'rinishda topish mumkin:

$$f_2(p;q) = -\frac{1}{2}\frac{(v_1(p)f_1(q) + v_1(q)f_1(p))}{\omega_2(p;q) - \lambda_0} \qquad (2.1.4)$$

Endi $f_2(p;q)$ uchun topilgan (2.1.4)- ifodani (2.1.3) tenglamalar sistemasining ikkinchi tenglamasiga keltirib qo'yamiz va quyidagicha yozib olamiz:

$$\begin{cases} (\omega_0 - \lambda_0)f_0 + \int\limits_{T^\nu} v_0(s)f_1(s)ds = 0 \\ v_0(p)f_0 + (\omega_1(p) - \lambda_0)f_1(p) + \int\limits_{T^\nu} v_1(s)\left(-\frac{1}{2}\frac{(v_1(p)f_1(s) + v_1(s)f_1(p))}{\omega_2(p;s) - \lambda_0}\right)ds = 0 \end{cases}$$

yoki oxirgi sistemamizni soddalashtirib quyidagicha yozamiz:

$$
\begin{cases}
(\omega_0-\lambda_0)f_0+\int\limits_{T^\nu} v_0(s)f_1(s)ds=0 \\
v_0(p)f_0+(\omega_1(p)-\lambda_0)f_1(p)-\frac{1}{2}\int\limits_{T^\nu}\frac{v_1(s)v_1(p)f_1(s)}{\omega_2(p;s)-\lambda_0}ds-\frac{1}{2}\int\limits_{T^\nu}\frac{v_1^2(s)f_1(p)}{\omega_2(p;s)-\lambda_0}ds=0
\end{cases}
\tag{2.1.5}
$$

(2.1.5) tenglamalar sistemasining ikkinchi tenglamasidagi oxirgi ifodalardagi $f_1(p)$ va $v_1(p)$ funksiyalar T^ν da aniqlangan haqiqiy qiymatli uzluksiz funksiyalar bo'lgani uchun integral belgisi ostidan tashqariga chiqarish mumkin.

Demak,

$$
\begin{cases}
(\omega_0-\lambda_0)f_0+\int\limits_{T^\nu} v_0(s)f_1(s)ds=0 \\
v_0(p)f_0+(\omega_1(p)-\lambda_0)f_1(p)-\frac{1}{2}v_1(p)\int\limits_{T^\nu}\frac{v_1(s)f_1(s)}{\omega_2(p;s)-\lambda_0}ds-\frac{1}{2}f_1(p)\int\limits_{T^\nu}\frac{v_1^2(s)}{\omega_2(p;s)-\lambda_0}ds=0
\end{cases}
\tag{2.1.6}
$$

bo'ladi. (2.1.6) tenglamalar sistemasini soddalashtirib,

$$
\begin{cases}
(\omega_0-\lambda_0)f_0+\int\limits_{T^\nu} v_0(s)f_1(s)ds=0 \\
v_0(p)f_0+(\omega_1(p)-\lambda_0-\frac{1}{2}\int\limits_{T^\nu}\frac{v_1^2(s)}{\omega_2(p;s)-\lambda_0}ds)f_1(p)-\frac{1}{2}v_1(p)\int\limits_{T^\nu}\frac{v_1(s)f_1(s)}{\omega_2(p;s)-\lambda_0}ds=0
\end{cases}
$$

(2.1.7)

ko'rinishda yozib olamiz.

(2.1.7) tenglamalar sistemasining ikkinchi tenglamasidagi $f_1(p)$ ning oldidagi funksiyani quyidagicha belgilab olamiz:

$$
\Delta(p;\lambda_0)=\omega_1(p)-\lambda_0-\frac{1}{2}\int\limits_{T^\nu}\frac{v_1^2(s)ds}{\omega_2(p;s)-\lambda_0}
$$

Ma'lumki, $\lambda_0\notin\sigma$ bo'lganda barcha $p\in T^\nu$ lar uchun $\Delta(p;\lambda_0)\neq 0$ munosabat o'rinli.Shu sababli (2.1.7) tenglamalar sistemasining ikkinchi tenglamasidan $f_1(p)$ ni topib olamiz:

$$
f_1(p)=\frac{1}{2}\frac{v_1(p)}{\Delta(p;\lambda_0)}\cdot\int\limits_{T^\nu}\frac{v_1(s)f_1(s)}{\omega_2(p;s)-\lambda_0}ds-\frac{v_0(p)f_0}{\Delta(p;\lambda_0)}\ . \tag{2.1.8}
$$

$f_1(p)$ uchun topilgan (2.1.8) ifodani ni (2.1.4) – ifodaga etib qo'yamiz:

$$
f_2(p;q)=-\frac{1}{2}\frac{(v_1(p)f_1(q)+v_1(q)f_1(p))}{\omega_2(p;q)-\lambda_0}=-\frac{1}{2}\frac{v_1(p)f_1(q)}{\omega_2(p;q)-\lambda_0}-\frac{1}{2}\frac{v_1(q)f_1(p)}{\omega_2(p;q)-\lambda_0}=
$$

$$= -\frac{v_1(p)}{2(\omega_2(p;q)-\lambda_0)} \cdot \left(\frac{1}{2} \frac{v_1(q)}{\Delta(q;\lambda_0)} \cdot \int_{T^\nu} \frac{v_1(s) f_1(s)}{\omega_2(q;s)-\lambda_0} ds - \frac{v_0(q) f_0}{\Delta(q;\lambda_0)} \right) -$$

$$- \frac{v_1(p)}{2(\omega_2(p;q)-\lambda_0)} \cdot \left(\frac{1}{2} \frac{v_1(p)}{\Delta(p;\lambda_0)} \cdot \int_{T^\nu} \frac{v_1(s) f_1(s)}{\omega_2(p;s)-\lambda_0} ds - \frac{v_0(p) f_0}{\Delta(p;\lambda_0)} \right).$$

(2.1.9)

(2.1.9) ifodani o'ng tominidagi qavslarni ochib chiqamiz,

$$f_2(p;q) = -\frac{v_1(p)}{4(\omega_2(p;q)-\lambda_0)} \cdot \frac{v_1(q)}{\Delta(q;\lambda_0)} \cdot \int_{T^\nu} \frac{v_1(s) f_1(s)}{\omega_2(q;s)-\lambda_0} ds + \frac{v_1(p)}{2(\omega_2(p;q)-\lambda_0)} \frac{v_0(q) f_0}{\Delta(q;\lambda_0)} -$$

$$- \frac{v_1(p)}{4(\omega_2(p;q)-\lambda_0)} \cdot \frac{v_1(p)}{\Delta(p;\lambda_0)} \cdot \int_{T^\nu} \frac{v_1(s) f_1(s)}{\omega_2(p;s)-\lambda_0} ds + \frac{v_1(p)}{2(\omega_2(p;q)-\lambda_0)} \frac{v_0(p) f_0}{\Delta(p;\lambda_0)}. \tag{2.1.10}$$

(2.1.7) tenglamalar sistemasidagi birinchi tenglamaning o'ng va chap tomoniga f_0 ni qo'shamiz. Natijada

$$f_0 + (\omega_0 - \lambda_0) f_0 + \int_{T^\nu} v_0(s) f_1(s) ds = 0 + f_0$$

hosil bo'ladi. Bu tenglamaning chap qismidan f_0 umumiy ko'paytuvchi bo'lgani uchun qavsdan tashqariga chiqaramiz. Shunda

$$(\omega_0 - \lambda_0 + 1) f_0 + \int_{T^\nu} v_0(s) f_1(s) ds = f_0 \tag{2.1.11}$$

bo'ladi.

(2.1.8), (2.1.10) va (2.1.11) tengliklardan foydalanib quyidagi matritsani tuzib olamiz:

$$W_A(\lambda_0) = \begin{pmatrix} W_{00}(\lambda_0) & W_{01}(\lambda_0) & 0 \\ W_{10}(\lambda_0) & W_{11}(\lambda_0) & 0 \\ W_{20}(\lambda_0) & W_{21}(\lambda_0) & 0 \end{pmatrix}$$

Bunda

$$(W_{00}(\lambda_0) f_0)_0 = (\omega_0 - \lambda_0 + 1) f_0$$

$$(W_{01}(\lambda_0) f_1)_0 = \int_{T^\nu} v_0(s) f_1(s) ds$$

$$(W_{10}(\lambda_0) f_0)_1 = -\frac{v_0(p) f_0}{\Delta(p;\lambda_0)},$$

$$\Delta(p;\lambda_0)=\omega_1(p)-\lambda_0-\frac{1}{2}\int_{T^\nu}\frac{v_1^2(s)ds}{\omega_2(p;s)-\lambda_0}$$

$$(W_{11}(\lambda_0)f_1)_1=\frac{1}{2}\frac{v_1(p)}{\Delta(p;\lambda_0)}\cdot\int_{T^\nu}\frac{v_1(s)f_1(s)}{\omega_2(p;s)-\lambda_0}ds$$

$$(W_{20}(\lambda_0)f_0)_1(p;q)=\left(\frac{v_1(p)}{2(\omega_2(p;q)-\lambda_0)}\frac{v_0(q)}{\Delta(q;\lambda_0)}-\frac{v_1(p)}{2(\omega_2(p;q)-\lambda_0)}\frac{v_0(p)}{\Delta(p;\lambda_0)}\right)f_0$$

$$(W_{21}(\lambda_0)f_0)_2(p;q)=-\frac{v_1(p)}{4(\omega_2(p;q)-\lambda_0)}\cdot\frac{v_1(q)}{\Delta(q;\lambda_0)}\cdot\int_{T^\nu}\frac{v_1(s)f_1(s)}{\omega_2(q;s)-\lambda_0}ds-$$

$$-\frac{v_1(p)}{4(\omega_2(p;q)-\lambda_0)}\cdot\frac{v_1(p)}{\Delta(p;\lambda_0)}\cdot\int_{T^\nu}\frac{v_1(s)f_1(s)}{\omega_2(p;s)-\lambda_0}ds$$

bo'ladi.

Shunday qilib, f vektor $W_A(\lambda_0)f=f$ tenglamani qanoatlantirar ekan. Teorema isbot bo'ldi.■

Ikkinchi turdagi matrirsaviy operator

Faraz qilaylik, $L_2((T^\nu)^2)-(T^\nu)^2$ da aniqlangan kvadrati bilan integrallanuvchi simmetrik (umuman olganda kompleks qiymatli) funksiyalarning Gilbert fazosi, bo'lsin.

Quyidagi belgilashlarni kiritamiz:

$$H_2'=L_2((T^\nu)^2);$$

$$H'=H_0\oplus H_1\oplus H_2'.$$

H Gilbert fazosiga quyidagi

$$B=\begin{pmatrix}B_{00} & B_{01} & 0\\ B_{10} & B_{11} & B_{12}\\ 0 & B_{21} & B_{22}\end{pmatrix}$$

3×3 blok matritsali operatorni qaraymiz. Bunda B operator A operator bilan faqat B_{21} elementi bilan farq qiladi va

$$(B_{21}f_1)_2(p,q)=v_1(q)f_1(p)$$

Bunda $f_i \in H_i$, $i = 0,1,2.$, ω_0 fiksirlangan haqiqiy son, $\omega_1(\cdot)$, $v_i(\cdot)$, $i = 0,1$ funksiyalar T^ν da aniqlangan haqiqiy qiymatli uzluksiz funksiyalar, $\omega_2(\cdot,\cdot)$ esa $(T^\nu)^2$ da aniqlangan haqiqiy qiymatli uzluksiz funksiya.

Bizga $f = \begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix}$ vektor va λ_0 son berilgan bo'lsin.

$$\begin{pmatrix} B_{00} f_0 + B_{01} f_1 \\ B_{10} f_0 + B_{11} f_1 + B_{12} f_2 \\ B_{21} f_1 + B_{22} f_2 \end{pmatrix} = \begin{pmatrix} \lambda_0 f_0 \\ \lambda_0 f_1 \\ \lambda_0 f_2 \end{pmatrix}$$

matritsamizni chiziqli tenglamalar sistemasi ko'rinishida yozish mumkin:

$$\begin{cases} B_{00} f_0 + B_{01} f_1 = \lambda_0 f_0 \\ B_{10} f_0 + B_{11} f_1 + B_{12} f_2 = \lambda_0 f_1 \\ B_{21} f_1 + B_{22} f_2 = \lambda_0 f_2 \end{cases} \tag{2.1.12}$$

Yuqorida operatorlar ta'sir qilgan tengliklar orqali (2.1.12) chiziqli tenglamalar sistemasini quyidagicha yozib olamiz:

$$\begin{cases} \omega_0 f_0 + \int\limits_{T^\nu} v_0(s) f_1(s) ds = \lambda_0 f_0 \\ v_0(p) f_0 + \omega_1(p) f_1(p) + \int\limits_{T^\nu} v_1(s) f_2(p;s) ds = \lambda_0 f_1(p) \\ v_1(q) f_1(p) + \omega_2(p;q) f_2(p;q) = \lambda_0 f_2(p;q) \end{cases}$$

tenglamalar sistemasini hosil qilamiz.

Bu yerdan kiritgan f vektor funksiyamizning H_0, H_1 va H_2 fazolardagi mos elementlarini soddalashtiramiz, natijada

$$\begin{cases} (\omega_0 - \lambda_0) f_0 + \int\limits_{T^\nu} v_0(s) f_1(s) ds = 0 \\ v_0(p) f_0 + (\omega_1(p) - \lambda_0) f_1(p) + \int\limits_{T^\nu} v_1(s) f_2(p;s) ds = 0 \\ v_1(q) f_1(p) + (\omega_2(p;q) - \lambda_0) f_2(p;q) = 0 \end{cases} \tag{2.1.13}$$

sistema hosil bo'ladi.

Ixtiyoriy $p,q\in T^{\nu}$ lar uchun $\omega_2(p;q)-\lambda_0\neq 0$ bo'ladi. Shu sababli (2.1.14) tenglamalar sistemasining uchinchi tenglamasidagi $f_2(p;q)$ ni quyidagi ko'rinishda topish mumkin.

Demak,

$$f_2(p;q)=-\frac{v_1(q)f_1(p)}{\omega_2(p;q)-\lambda_0}. \tag{2.1.15}$$

Endi, $f_2(p;q)$ uchun topilgan (2.1.15) ifodani (2.1.14) tenglamalar sistemasining ikkinchi tenglamasiga keltirib qo'yamiz va quyidagini hosil qilamiz:

$$\begin{cases}(\omega_0-\lambda_0)f_0+\int\limits_{T^{\nu}}v_0(s)f_1(s)ds=0\\ v_0(p)f_0+(\omega_1(p)-\lambda_0)f_1(p)-\int\limits_{T^{\nu}}v_1(s)\dfrac{v_1(s)f_1(p)}{\omega_2(p;s)-\lambda_0}ds=0.\end{cases}$$

Bu chiziqli tenglamalar sistemasini quyidagicha soddalashtirib olamiz:

$$\begin{cases}(\omega_0-\lambda_0)f_0+\int\limits_{T^{\nu}}v_0(s)f_1(s)ds=0\\ v_0(p)f_0+(\omega_1(p)-\lambda_0)f_1(p)-\int\limits_{T^{\nu}}\dfrac{v_1^2(s)f_1(p)}{\omega_2(p;s)-\lambda_0}ds=0.\end{cases} \tag{2.1.16}$$

(2.1.16) tenglamalar sistemasining ikkinchi tenglamasidagi integral ostidagi o'zgarmas funksiyani integral belgisi ostidan chiqaramiz.

$$\begin{cases}(\omega_0-\lambda_0)f_0+\int\limits_{T^{\nu}}v_0(s)f_1(s)ds=0\\ v_0(p)f_0+(\omega_1(p)-\lambda_0)f_1(p)-f_1(p)\int\limits_{T^{\nu}}\dfrac{v_1^2(s)}{\omega_2(p;s)-\lambda_0}ds=0\end{cases} \tag{2.1.17}$$

(2.1.17) tenglamalar sistemasining ikkinchi tenglamasidagi ikkinchi va uchinchi hadlaridan umumiy ko'paytuvchi bo'lgan $f_1(p)$ ni qavsdan chiqaramiz:

$$\begin{cases}(\omega_0-\lambda_0)f_0+\int\limits_{T^{\nu}}v_0(s)f_1(s)ds=0\\ v_0(p)f_0+\left((\omega_1(p)-\lambda_0-\int\limits_{T^{\nu}}\dfrac{v_1^2(s)}{\omega_2(p;s)-\lambda_0}ds\right)f_1(p)=0\end{cases} \tag{2.1.18}$$

$z \notin \mathrm{Im}\,\omega_2$ bo'lgani uchun ixtiyoriy $p,s \in T^\nu$ lar uchun $\omega_2(p;s) - \lambda_0 \neq 0$ munosabat o'rinli.

$f_1(p)$ ni oldidagi ifodani

$$\Delta(p;z) = \left(\omega_1(p) - \lambda_0 - \int_{T^\nu} \frac{v_1^2(s)ds}{\omega_2(p;s) - \lambda_0} \right)$$

bilan belgilab olamiz. Bizga ma'lumki, agar $z \notin \sigma$ bo'lsa, u holda ixtiyoriy $p \in T^\nu$ uchun $\Delta(p;z) \neq 0$ munosabat o'rinlidir. Shu sababli (2.1.18) tenglamalar sistemasining ikkinchi tenglamasidan $f_1(p)$ ni topib olamiz:

$$f_1(p) = -\frac{v_0(p) f_0}{\Delta(p;\lambda_0)}. \qquad (2.1.19)$$

(2.1.18) tenglamalar sistemasidan topilgan ikkinchi ifodadagi $f_1(p)$ ning o'rniga (2.1.19) ifodani etib qo'yamiz, ya'ni

$$f_2(p;q) = \frac{v_1(q) v_0(p) f_0}{\Delta(p,\lambda_0)(\omega_2(p;q) - \lambda_0)}$$

bo'ladi.

(2.1.13) tenglamalar sistemasidagi birinchi tenglamaning o'ng va chap tomoniga f_0 ni qo'shamiz.

Natijada

$$f_0 + (\omega_0 - \lambda_0) f_0 + \int_{T^\nu} v_0(s) f_1(s) ds = 0 + f_0$$

hosil bo'ladi.

Hosil bo'lgan tenglamaning chap tomonidan f_0 ni soddalashtiramiz:

$$(\omega_0 - \lambda_0 + 1) f_0 + \int_{T^\nu} v_0(s) f_1(s) ds = f_0.$$

Endi bizga kerakli bo'lgan chiziqli tenglamalar sistemasini tuzib olamiz.

$$\begin{cases} f_0 = (\omega_0 - \lambda_0 + 1) f_0 + \int_{T^\nu} v_0(s) f_1(s) ds \\ f_1(p) = -\dfrac{v_0(p) f_0}{\Delta(p;\lambda_0)} \\ f_2(p;q) = \dfrac{v_1(q) v_0(p) f_0}{\Delta(p;\lambda_0)(\omega_2(p;q) - \lambda_0)}. \end{cases} \qquad (2.1.20)$$

Hosil bo’lgan (8) tenglamalar sistemasidan Vaynberg tenglamasini quramiz;

$$W_B(\lambda_0)=\begin{pmatrix} W_{00}(\lambda_0) & W_{01}(\lambda_0) & 0 \\ W_{10}(\lambda_0) & 0 & 0 \\ W_{20}(\lambda_0) & 0 & 0 \end{pmatrix}$$

Bu yerda

$$(W_{00}(\lambda_0)f_0)_0=(\omega_0-\lambda_0+1)f_0;$$

$$(W_{01}(\lambda_0)f_1)_0=\int_{T^\nu} v_0(s)f_1(s)ds;$$

$$(W_{10}(\lambda_0)f_0)_1(p)=-\frac{v_0(p)f_0}{\Delta(p;\lambda_0)};$$

$$(W_{20}(\lambda_0)f_0)_2(p,q)=\frac{v_1(q)v_0(p)f_0}{\Delta(p;\lambda_0)(\omega_2(p;q)-\lambda_0)}$$

bo’ladi.

Uchinchi tur matritsaviy operator:

H Gilbert fazosiga quyidagi

$$C=\begin{pmatrix} C_{00} & C_{01} & 0 \\ C_{10} & C_{11} & C_{12} \\ 0 & C_{21} & C_{22} \end{pmatrix} \tag{2.1.21}$$

3×3 blok matritsali operatorni qaraymiz,

bu yerda $C_{ij}:H_j\to H_i,\ i,j=0,1,2,$

operatorlar B_{ij} operatorlar bilan faqat C_{22} operator bilan farq qiladi va

$$(C_{22}f_2)_2(p,q)=\omega_2(p,q)f_2(p,q)-\varphi_1(q)\int_{T^\nu}\varphi_1(s)f_2(p;s)ds-\varphi_2(p)\int_{T^\nu}\varphi_2(s)f_2(s;q)ds\,.$$

bo’ladi.

Bizga $f=\begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix}$ vektor va λ_0 son berilgan bo’lsin.

C operator uchun xos qiymatga nisbatan tenglamani qaraymiz:

$$Cf=\lambda_0 f\,.$$

H Gilbert fazosidan C operatorimizni f vektorga ta’sirini qaraymiz.

$$\begin{pmatrix} C_{00}f_0 + C_{01}f_1 \\ C_{10}f_0 + C_{11}f_1 + C_{12}f_2 \\ C_{21}f_1 + C_{22}f_2 \end{pmatrix} = \begin{pmatrix} \lambda_0 f_0 \\ \lambda_0 f_1 \\ \lambda_0 f_2 \end{pmatrix}$$

matritsamizni chiziqli tenglamalar sistemasi ko'rinishida yozish mumkin:

$$\begin{cases} C_{00}f_0 + C_{01}f_1 = \lambda_0 f_0 \\ C_{10}f_0 + C_{11}f_1 + C_{12}f_2 = \lambda_0 f_1 \\ C_{21}f_1 + C_{22}f_2 = \lambda_0 f_2 \end{cases} \tag{2.1.22}$$

Yuqorida operatorlar ta'sir qilgan tengliklar orqali (2.1.22) chiziqli tenglamalar sistemasini quyidagicha yozib olamiz:

$$\begin{cases} \omega_0 f_0 + \int\limits_{T^\nu} v_0(s) f_1(s) ds = \lambda_0 f_0 \\ v_0(p) f_0 + \omega_1(p) f_1(p) + \int\limits_{T^\nu} v_1(s) f_2(p;s) ds = \lambda_0 f_1(p) \\ v_1(q) f_1(p) + \omega_2(p;q) f_2(p;q) - \varphi_1(q)\, \phi_1(p) - \varphi_2(p)\, \phi_2(q) = \lambda_0 f_2(p;q) \end{cases}$$

tenglamalar sistemasini hosil qilamiz, bunda

$$\phi_1(p) = \int\limits_{T^\nu} \varphi_1(s) f_2(p;s) ds;$$

$$\phi_2(q) = \int\limits_{T^\nu} \varphi_2(s) f_2(s;q) ds$$

Bu yerdan kiritgan f vektor funksiyamizning H_0, H_1 va H_2 fazolardagi mos elementlarini soddalashtiramiz, natijada

$$\begin{cases} (\omega_0 - \lambda_0) f_0 + \int\limits_{T^\nu} v_0(s) f_1(s) ds = 0 \\ v_0(p) f_0 + (\omega_1(p) - \lambda_0) f_1(p) + \int\limits_{T^\nu} v_1(s) f_2(p;s) ds = 0 \\ v_1(q) f_1(p) + (\omega_2(p;q) - \lambda_0) f_2(p;q) - \varphi_1(q)\phi_1(p) - \varphi_2(p)\phi_2(q) = 0 \end{cases} \tag{2.1.23}$$

sistema hosil bo'ladi.

Ixtiyoriy $p, q \in T^\nu$ lar uchun $\omega_2(p;q) - \lambda_0 \neq 0$ bo'ladi. Shu sababli (2.1.23) tenglamalar sistemasining uchinchi tenglamasidagi $f_2(p;q)$ ni quyidagi ko'rinishda topish mumkin.

Demak,

$$f_2(p;q)=\frac{\varphi_1(q)\phi_1(p)+\varphi_2(p)\phi_2(q)-\nu_1(q)f_1(p)}{\omega_2(p;q)-\lambda_0} \qquad (2.1.24)$$

bo'ladi.

(2.1.25) tenglikdan foydalanib, $f_2(p;s)$ va $f_2(s;q)$ larni topib olamiz.

$$f_2(p;s)=\frac{\varphi_1(s)\phi_1(p)+\varphi_2(p)\phi_2(s)-\nu_1(s)f_1(p)}{\omega_2(p;s)-\lambda_0} \qquad (2.1.25)$$

$$f_2(s;q)=\frac{\varphi_1(q)\phi_1(s)+\varphi_2(s)\phi_2(q)-\nu_1(q)f_1(s)}{\omega_2(s;q)-\lambda_0}. \qquad (2.1.26)$$

(2.1.25) formuladan foydalanib $\phi_1(p)$ ni topib olamiz. Buning uchun

$$\phi_1(p)=\int_{T^\nu}\varphi_1(s)f_2(p;s)ds$$

tenglikdagi $f_2(p;s)$ ni o'rniga (2.1.26) formulani qo'yamiz. Bundan

$$\phi_1(p)=\int_{T^\nu}\varphi_1(s)\cdot\frac{\varphi_1(s)\phi_1(p)+\varphi_2(p)\phi_2(s)-\nu_1(s)f_1(p)}{\omega_2(p;s)-\lambda_0}ds$$

$\varphi_1(s)$ ni kasr suratiga ko'paytirib chiqamiz. Natijada

$$\phi_1(p)=\int_{T^\nu}\frac{\varphi_1(s)\varphi_1(s)\phi_1(p)+\varphi_1(s)\varphi_2(p)\phi_2(s)-\varphi_1(s)\nu_1(s)f_1(p)}{\omega_2(p;s)-\lambda_0}ds \qquad (2.1.27)$$

tenglik hosil bo'ladi.

Quyidagi qoidadan foydalanamiz.

Ikki yoki bir necha funksiyalar algebraik yig'indisining integrali, shu funksiyalar integrallarining algebraik yig'indisiga teng:

$$\int[f_1(x)+f_2(x)]dx=\int f_1(x)dx+\int f_2(x)dx$$

$$\phi_1(p)=\int_{T^\nu}\frac{\varphi_1(s)\varphi_1(s)\phi_1(p)}{\omega_2(p;s)-\lambda_0}ds+\int_{T^\nu}\frac{\varphi_1(s)\varphi_2(p)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-\int_{T^\nu}\frac{\varphi_1(s)\nu_1(s)f_1(p)}{\omega_2(p;s)-\lambda_0}ds$$

(2.1.28)

O'zgarmas ko'paytuvchini integral ishorasi ostidan chiqarish mumkin, ya'ni $a=const$ bo'lganda

$$\int af(x)dx=a\int f(x)dx$$ bo'ladi.

Shu qoidadan foydalanib (2.1.28) tenglikni quyidagicha yozib olamiz:

$$\phi_1(p)=\phi_1(p)\int_{T^v}\frac{\varphi_1(s)\varphi_1(s)}{\omega_2(p;s)-\lambda_0}ds+\varphi_2(p)\int_{T^v}\frac{\varphi_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-f_1(p)\int_{T^v}\frac{\varphi_1(s)v_1(s)}{\omega_2(p;s)-\lambda_0}ds$$

(2.1.29)

(2.1.29) tenglikdagi $\phi_1(p)$ qatnashgan hadlarni ixchamlaymiz:

$$\phi_1(p)=\phi_1(p)\int_{T^v}\frac{\varphi_1(s)\varphi_1(s)}{\omega_2(p;s)-\lambda_0}ds+\varphi_2(p)\int_{T^v}\frac{\varphi_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-f_1(p)\int_{T^v}\frac{\varphi_1(s)v_1(s)}{\omega_2(p;s)-\lambda_0}ds\,.$$

Umumiy ko'paytuvchi bo'lgan $\phi_1(p)$ni qavsdan tashqariga chiqaramiz. Natijada

$$\phi_1(p)\left(1-\int_{T^v}\frac{\varphi_1(s)\varphi_1(s)}{\omega_2(p;s)-\lambda_0}ds\right)=\varphi_2(p)\int_{T^v}\frac{\varphi_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-f_1(p)\int_{T^v}\frac{\varphi_1(s)v_1(s)}{\omega_2(p;s)-\lambda_0}ds \qquad (2.1.30)$$

bo'ladi.

$\phi_1(p)$ ni oldidagi ifodani $\Delta_1(p;\lambda_0)$ bilan belgilab olamiz ,ya'ni

$$\Delta_1(p;\lambda_0)=\left(1-\int_{T^v}\frac{\varphi_1(s)\varphi_1(s)}{\omega_2(p;s)-\lambda_0}ds\right)$$

bo'ladi.

$\Delta_1(p;\lambda_0)$ deb belgilash kiritganimizdan so'ng (2.1.30) tenglamning ko'rinishi quyidagicha bo'ladi, ya'ni

$$\phi_1(p)\Delta_1(p;\lambda_0)=\varphi_2(p)\int_{T^v}\frac{\varphi_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-f_1(p)\int_{T^v}\frac{\varphi_1(s)v_1(s)}{\omega_2(p;s)-\lambda_0}ds \qquad (2.1.31)$$

Shunga o'xshash $\phi_2(q)$ ni topamiz.

$\phi_2(q)=\int_{T^v}\varphi_2(s)f_2(s;q)ds$ tenglikning o'ng tomonidagi $f_2(s;q)$ ni o'rniga (2.1.26) ifodani qo'yamiz.

$$\phi_2(q)=\int_{T^v}\varphi_2(s)\cdot\frac{\varphi_1(q)\phi_1(s)+\varphi_2(s)\phi_2(q)-v_1(q)f_1(s)}{\omega_2(s;q)-\lambda_0}ds\,.$$

Ko'paytirish qoidalariga asosan

$$\phi_2(q)=\int_{T^v}\frac{\varphi_2(s)\varphi_1(q)\phi_1(s)+\varphi_2(s)\varphi_2(s)\phi_2(q)-\varphi_2(s)v_1(q)f_1(s)}{\omega_2(s;q)-\lambda_0}ds \qquad (2.1.32)$$

tenglikni hosil qilamiz.

Yuqoridagi qoidalarga asosan (2.1.32) ifoda quyidagicha bo'ladi:

$$\phi_2(q)=\int_{T^\nu}\frac{\varphi_2(s)\varphi_1(q)\phi_1(s)}{\omega_2(s;q)-\lambda_0}ds+\int_{T^\nu}\frac{\varphi_2(s)\varphi_2(s)\phi_2(q)}{\omega_2(s;q)-\lambda_0}ds-\int_{T^\nu}\frac{\varphi_2(s)v_1(q)f_1(s)}{\omega_2(s;q)-\lambda_0}ds=$$

$$=\varphi_1(q)\int_{T^\nu}\frac{\varphi_2(s)\phi_1(s)}{\omega_2(s;q)-\lambda_0}ds+\phi_2(q)\int_{T^\nu}\frac{\varphi_2(s)\varphi_2(s)}{\omega_2(s;q)-\lambda_0}ds-v_1(q)\int_{T^\nu}\frac{\varphi_2(s)f_1(s)}{\omega_2(s;q)-\lambda_0}ds$$

(2.1.33)

(2.1.33) tengsizlikning o'ng tomonidagi birinchi hadini tenglikdan chap tomonga o'tkazib, $\phi_2(q)$ umumiy ko'paytuvchi bo'lgani uchun qavsdan tashqariga chiqaramiz.

$$\phi_2(q)\left(1-\int_{T^\nu}\frac{\varphi_2(s)\varphi_2(s)}{\omega_2(s;q)-\lambda_0}ds\right)=\varphi_1(q)\int_{T^\nu}\frac{\varphi_2(s)\phi_1(s)}{\omega_2(s;q)-\lambda_0}ds-v_1(q)\int_{T^\nu}\frac{\varphi_2(s)f_1(s)}{\omega_2(s;q)-\lambda_0}ds$$

(2.1.34)

$\phi_2(q)$ ni oldidagi ifodani $\Delta_2(p;\lambda_0)$ bilan belgilab olamiz ya'ni ,

$$\Delta_2(q;\lambda_0)=\left(1-\int_{T^\nu}\frac{\varphi_2(s)\varphi_2(s)}{\omega_2(s;q)-\lambda_0}ds\right)$$

bo'ladi.

(2.1.34) ifodaning ko'rinishi

$$\phi_2(q)\Delta_2(q;\lambda_0)=\varphi_1(q)\int_{T^\nu}\frac{\varphi_2(s)\phi_1(s)}{\omega_2(s;q)-\lambda_0}ds-v_1(q)\int_{T^\nu}\frac{\varphi_2(s)f_1(s)}{\omega_2(s;q)-\lambda_0}ds \qquad (2.1.35)$$

Endi (2.1.23)- tenglamalar sistemasidagi ikkinchi tenglamaning chap tominidagi integral ostidagi $f_2(p;s)$ o'rniga (2.1.35) ifodani yozamiz. Shunda uning ko'rinishi quyidagicha bo'ladi.

$$v_0(p)f_0+(\omega_1(p)-\lambda_0)f_1(p)+\int_{T^\nu}\frac{v_1(s)\varphi_1(s)\phi_1(p)}{\omega_2(s;p)-\lambda_0}ds+\int_{T^\nu}\frac{v_1(s)\varphi_2(p)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-\int_{T^\nu}\frac{v_1^2(s)f_1(p)}{\omega_2(p;s)-\lambda_0}ds=0$$

Ildiz ostidagi haqiqiy o'zgaruvchili funksiyalarni integral belgisi ostidan chiqaramiz.

$$v_0(p)f_0+(\omega_1(p)-\lambda_0)f_1(p)+\phi_1(p)\int_{T^\nu}\frac{v_1(s)\varphi_1(s)}{\omega_2(s;p)-\lambda_0}ds+$$

$$+\varphi_2(p)\int_{T^\nu}\frac{v_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-f_1(p)\int_{T^\nu}\frac{v_1^2(s)}{\omega_2(p;s)-\lambda_0}ds=0$$

Bu tenglikda umumiy ko'paytuvchi bo'lgan $f_1(p)$ ni qavsdan tashqariga chiqaramiz.

Shunda

$$v_0(p)f_0+\left(\omega_1(p)-\lambda_0-\int_{T^v}\frac{v_1^2(s)}{\omega_2(p;s)-\lambda_0}ds\right)f_1(p)+$$

$$+\phi_1(p)\int_{T^v}\frac{v_1(s)\varphi_1(s)}{\omega_2(s;p)-\lambda_0}ds+\varphi_2(p)\int_{T^v}\frac{v_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds=0 \qquad (2.1.36)$$

bo'ladi.

$f_1(p)$ ni oldidagi ifodani $\Delta(p;\lambda_0)$ bilan belgilaymiz ya'ni ,

$$\Delta(p;\lambda_0)=\left(\omega_1(p)-\lambda_0-\int_{T^v}\frac{v_1^2(s)}{\omega_2(p;s)-\lambda_0}ds\right)$$

bo'ladi.

Demak, (2.1.35) tenglamamizning ko'rinishini quyidagicha yozib olsak bo'ladi.

$$v_0(p)f_0+\Delta(p;\lambda_0)f_1(p)+\phi_1(p)\int_{T^v}\frac{v_1(s)\varphi_1(s)}{\omega_2(s;p)-\lambda_0}ds+\varphi_2(p)\int_{T^v}\frac{v_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds=0 \qquad (2.1.37)$$

Ixtiyoriy ifodaning o'ng va chap qismiga bir xil sonni qo'shsa yoki ayirsa ifoda qiymati o'zgarmaydi . Shu qoidaga asosan (2.123) tenglamalar sistemasining birinchi tenglamasining o'ng va chap qismiga bir xil qilib f_0 ni qo'shamiz.

Demak,

$$f_0+(\omega_0-\lambda_0)f_0+\int_{T^v}v_0(s)f_1(s)ds=f_0 .$$

Endi bu tenglamaning chap qismidagi f_0 ni umumiy ko'paytuvchi bo'lgani uchun bitta qavs qilib yozsa bo'ladi.

Shunda,

$$(\omega_0-\lambda_0+1)f_0+\int_{T^v}v_0(s)f_1(s)ds=f_0 \qquad (2.138)$$

bo'ladi.

(2.1.37) ifodadan $f_1(p)$ ni topib olamiz:

$$f_1(p)=-\frac{\phi_1(p)}{\Delta(p;\lambda_0)}\int_{T^\nu}\frac{\nu_1(s)\varphi_1(s)}{\omega_2(s;p)-\lambda_0}ds-\frac{\varphi_2(p)}{\Delta(p;\lambda_0)}\int_{T^\nu}\frac{\nu_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-\frac{\nu_0(p)f_0}{\Delta(p;\lambda_0)}.$$

Bu tenglamadagi $\phi_1(p)$ va $\phi_2(s)$ lar o'rniga ifodalarini keltirib qo'yamiz, u holda

$$f_1(p)=-\frac{\phi_1(p)}{\Delta(p;\lambda_0)}\int_{T^\nu}\frac{\nu_1(s)\varphi_1(s)}{\omega_2(s;p)-\lambda_0}ds-\frac{\varphi_2(p)}{\Delta(p;\lambda_0)}\int_{T^\nu}\frac{\nu_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-\frac{\nu_0(p)f_0}{\Delta(p;\lambda_0)} \qquad (2.1.39)$$

bo'ladi.

Endi (2.1.34) tenglamani ham soddalashtiramiz, buning uchun kasrlar qoidasidan foydalanamiz:

$$f_2(p;q)=\frac{\varphi_1(q)\phi_1(p)+\varphi_2(p)\phi_2(q)-\nu_1(q)f_1(p)}{\omega_2(p;q)-\lambda_0}$$

$$f_2(p;q)=\frac{\varphi_1(q)\phi_1(p)}{\omega_2(p;q)-\lambda_0}+\frac{\varphi_2(p)\phi_2(q)}{\omega_2(p;q)-\lambda_0}-\frac{\nu_1(q)f_1(p)}{\omega_2(p;q)-\lambda_0}$$

ko'rinishida yozib olamiz. Bu tenglamadagi $\phi_1(p)$ va $\phi_2(q)$ lar o'rniga ifodasini qo'yamiz. Natijada

$$f_2(p;q)=\frac{\varphi_1(q)\phi_1(p)}{\omega_2(p;q)-\lambda_0}+\frac{\varphi_2(p)\phi_2(q)}{\omega_2(p;q)-\lambda_0}-\frac{\nu_1(q)f_1(p)}{\omega_2(p;q)-\lambda_0}=$$

$$=\frac{\varphi_1(q)}{\omega_2(p;q)-\lambda_0}\left(\frac{\phi_2(p)}{\Delta_1(p;\lambda_0)}\int_{T^\nu}\frac{\varphi_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-\frac{f_1(p)}{\Delta_1(p;\lambda_0)}\int_{T^\nu}\frac{\varphi_1(s)\nu_1(s)}{\omega_2(p;s)-\lambda_0}ds\right)+$$

$$+\frac{\varphi_2(p)}{\omega_2(p;q)-\lambda_0}\left(\frac{\varphi_1(q)}{\Delta_2(q;\lambda_0)}\int_{T^\nu}\frac{\varphi_2(s)\phi_1(s)}{\omega_2(s;q)-\lambda_0}ds-\frac{\nu_1(q)}{\Delta_1(q;\lambda_0)}\int_{T^\nu}\frac{\varphi_2(s)f_1(s)}{\omega_2(s;q)-\lambda_0}ds\right)-$$

$$-\frac{\nu_1(q)}{\omega_2(p;q)-\lambda_0}\left(-\frac{\phi_1(q)}{\Delta(p;\lambda_0)}\int_{T^\nu}\frac{\nu_1(s)\varphi_1(s)}{\omega_2(s;p)-\lambda_0}ds-\frac{\varphi_2(p)}{\Delta(p;\lambda_0)}\int_{T^\nu}\frac{\nu_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-\frac{\nu_0(p)f_0}{\Delta(p;\lambda_0)}\right)$$

(2.1.40)

(2.1.40) tenglik hosil bo'ladi. (2.1.40)-tenglikning o'ng tomonidagi birinchi hadini qavsdan qutqaramiz:

$$\frac{\varphi_1(q)}{\omega_2(p;q)-\lambda_0}\left(\frac{\phi_2(p)}{\Delta_1(p;\lambda_0)}\int_{T^\nu}\frac{\varphi_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-\frac{f_1(p)}{\Delta_1(p;\lambda_0)}\int_{T^\nu}\frac{\varphi_1(s)\nu_1(s)}{\omega_2(p;s)-\lambda_0}ds\right)=$$

$$=\frac{\varphi_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{\phi_2(p)}{\Delta_1(p;\lambda_0)}\int_{T^v}\frac{\varphi_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-\frac{\varphi_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{f_1(p)}{\Delta_1(p;\lambda_0)}\int_{T^v}\frac{\varphi_1(s)v_1(s)}{\omega_2(p;s)-\lambda_0}ds=$$

[$\phi_2(p)$ ning o’rniga ifodasini qo’yamiz.]

$$=\frac{\varphi_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{1}{\Delta_1(p;\lambda_0)}\int_{T^v}\varphi_2(t)f_2(t;p)dt\int_{T^v}\frac{\varphi_1(s)}{\omega_2(p;s)-\lambda_0}ds\cdot\int_{T^v}\varphi_2(t)f_2(t;s)dt-$$
$$-\frac{\varphi_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{f_1(p)}{\Delta_1(p;\lambda_0)}\int_{T^v}\frac{\varphi_1(s)v_1(s)}{\omega_2(p;s)-\lambda_0}ds$$

(2.1.41)

Endi (19)- tenglikning o’ng tomonidagi ikkinchi hadini soddalashtiramiz.

$$\frac{\varphi_2(p)}{\omega_2(p;q)-\lambda_0}\left(\frac{\varphi_1(q)}{\Delta_2(q;\lambda_0)}\int_{T^v}\frac{\varphi_2(s)\phi_1(s)}{\omega_2(s;q)-\lambda_0}ds-\frac{v_1(q)}{\Delta_1(q;\lambda_0)}\int_{T^v}\frac{\varphi_2(s)f_1(s)}{\omega_2(s;q)-\lambda_0}ds\right)=$$

$$=\frac{\varphi_2(p)}{\omega_2(p;q)-\lambda_0}\cdot\frac{\varphi_1(q)}{\Delta_2(q;\lambda_0)}\int_{T^v}\frac{\varphi_2(s)\phi_1(s)}{\omega_2(s;q)-\lambda_0}ds-\frac{\varphi_2(p)}{\omega_2(p;q)-\lambda_0}\cdot\frac{v_1(q)}{\Delta_1(q;\lambda_0)}\int_{T^v}\frac{\varphi_2(s)f_1(s)}{\omega_2(s;q)-\lambda_0}ds=$$

[$\phi_1(s)$ ning o’rniga ifodasini qo’yamiz]

$$=\frac{\varphi_2(p)}{\omega_2(p;q)-\lambda_0}\cdot\frac{\varphi_1(q)}{\Delta_2(q;\lambda_0)}\int_{T^v}\frac{\varphi_2(s)}{\omega_2(s;q)-\lambda_0}ds\cdot\int_{T^v}\varphi_1(t)f_2(s;t)dt-$$
$$-\frac{\varphi_2(p)}{\omega_2(p;q)-\lambda_0}\cdot\frac{v_1(q)}{\Delta_1(q;\lambda_0)}\int_{T^v}\frac{\varphi_2(s)f_1(s)}{\omega_2(s;q)-\lambda_0}ds \qquad (2.1.42)$$

(2.1.40) tenglikning o’ng tomonidagi oxirgi, uchinchi hadini ixchamlaymiz. Bu ham yuqoridagilardek soddalashadi:

$$\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\left(-\frac{\phi_1(q)}{\Delta(p;\lambda_0)}\int_{T^v}\frac{v_1(s)\varphi_1(s)}{\omega_2(s;p)-\lambda_0}ds-\frac{\varphi_2(p)}{\Delta(p;\lambda_0)}\int_{T^v}\frac{v_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-\frac{v_0(p)f_0}{\Delta(p;\lambda_0)}\right)=$$

$$=-\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{\phi_1(q)}{\Delta(p;\lambda_0)}\int_{T^v}\frac{v_1(s)\varphi_1(s)}{\omega_2(s;p)-\lambda_0}ds-\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{\varphi_2(p)}{\Delta(p;\lambda_0)}\int_{T^v}\frac{v_1(s)\phi_2(s)}{\omega_2(p;s)-\lambda_0}ds-$$

$$-\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{v_0(p)f_0}{\Delta(p;\lambda_0)}=-\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{1}{\Delta(p;\lambda_0)}\int_{T^v}\varphi_1(t)f_2(q;t)dt\cdot\int_{T^v}\frac{v_1(s)\varphi_1(s)}{\omega_2(s;p)-\lambda_0}ds-$$
$$-\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{\varphi_2(p)}{\Delta(p;\lambda_0)}\int_{T^v}\frac{v_1(s)}{\omega_2(p;s)-\lambda_0}\int_{T^v}\varphi_2(t)f_2(t;s)dtds-\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{v_0(p)f_0}{\Delta(p;\lambda_0)}$$

(2.1.43)

(2.1.41), (2.1.42) va (2.1.43) ifodalardan foydalanib (2.1.20)-tenglikni quyidagicha yozib olamiz.

$$f_2(p;q)=\frac{\varphi_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{1}{\Delta_1(p;\lambda_0)}\int_{T^\nu}\varphi_2(t)f_2(t;p)dt\int_{T^\nu}\frac{\varphi_1(s)}{\omega_2(p;s)-\lambda_0}ds\cdot\int_{T^\nu}\varphi_2(t)f_2(t;s)dt-$$

$$-\frac{\varphi_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{f_1(p)}{\Delta_1(p;\lambda_0)}\int_{T^\nu}\frac{\varphi_1(s)v_1(s)}{\omega_2(p;s)-\lambda_0}ds+\frac{\varphi_2(p)}{\omega_2(p;q)-\lambda_0}\cdot\frac{\varphi_1(q)}{\Delta_2(p;\lambda_0)}\int_{T^\nu}\frac{\varphi_2(s)}{\omega_2(s;q)-\lambda_0}\cdot$$

$$\cdot\int_{T^\nu}\varphi_1(t)f_2(s;t)dt-\frac{\varphi_2(p)}{\omega_2(p;q)-\lambda_0}\cdot\frac{v_1(q)}{\Delta_1(q;\lambda_0)}\int_{T^\nu}\frac{\varphi_2(s)f_1(s)}{\omega_2(s;q)-\lambda_0}ds-$$

$$-\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{1}{\Delta(p;\lambda_0)}\int_{T^\nu}\varphi_1(t)f_2(q;t)dt\cdot\int_{T^\nu}\frac{v_1(s)\varphi_1(s)}{\omega_2(s;p)-\lambda_0}ds-$$

$$-\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{\varphi_2(p)}{\Delta(p;\lambda_0)}\int_{T^\nu}\frac{v_1(s)}{\omega_2(p;s)-\lambda_0}\int_{T^\nu}\varphi_2(t)f_2(t;s)dtds-\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{v_0(p)f_0}{\Delta(p;\lambda_0)}$$

2.1.44)

(2.1.38), (2.1.39) va (2.1.44) ifodalardan foydalanib quyidagi matritsani tuzib olamiz:

$$W_C(\lambda_0)=\begin{pmatrix}W_{00}(\lambda_0) & W_{01}(\lambda_0) & 0\\ W_{10}(\lambda_0) & 0 & W_{12}(\lambda_0)\\ W_{20}(\lambda_0) & W_{21}(\lambda_0) & W_{22}(\lambda_0)\end{pmatrix}$$

Bunda

$$(W_{00}(\lambda_0)f_0)_0=(\omega_0-\lambda_0+1)f_0$$

$$(W_{01}(\lambda_0)f_1)_0=\int_{T^\nu}v_0(s)f_1(s)ds$$

$$(W_{10}(\lambda_0)f_0)_1=-\frac{v_0(p)f_0}{\Delta(p;\lambda_0)},$$

$$\Delta(p;\lambda_0)=\omega_1(p)-\lambda_0-\frac{1}{2}\int_{T^\nu}\frac{v_1^2(s)ds}{\omega_2(p;s)-\lambda_0}$$

$$(W_{12}(\lambda_0)f_2)_1(p)=-\frac{1}{\Delta(p;\lambda_0)}\cdot\int_{T^\nu}\varphi_1(s)f_2(p;s)ds\int_{T^\nu}\frac{v_1(s)\varphi_1(s)}{\omega_2(s;p)-\lambda_0}ds-$$

$$-\frac{\varphi_2(p)}{\Delta(p;\lambda_0)}\int_{T^\nu}\frac{v_1(s)}{\omega_2(p;s)-\lambda_0}\cdot\int_{T^\nu}\varphi_2(t)f_2(t;s)dtds$$

$$(W_{20}(\lambda_0)f_0)_2(p) = -\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{v_0(p)f_0}{\Delta(p;\lambda_0)}$$

$$(W_{21}(\lambda_0)f_1)_2(p) = -\frac{\varphi_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{f_1(p)}{\Delta_1(p;\lambda_0)}\int_{T^\nu}\frac{\varphi_1(s)v_1(s)}{\omega_2(p;s)-\lambda_0}ds -$$

$$-\frac{\varphi_2(p)}{\omega_2(p;q)-\lambda_0}\cdot\frac{v_1(q)}{\Delta_1(q;\lambda_0)}\int_{T^\nu}\frac{\varphi_2(s)f_1(s)}{\omega_2(s;q)-\lambda_0}ds$$

$$\Delta_1(p;\lambda_0) = \left(1-\int_{T^\nu}\frac{\varphi_1(s)\varphi_1(s)}{\omega_2(p;s)-\lambda_0}ds\right)$$

$$(W_{22}(\lambda_0)f_2)_2(p;q) = \frac{\varphi_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{1}{\Delta_1(p;\lambda_0)}\int_{T^\nu}\varphi_2(t)f_2(t;p)dt\int_{T^\nu}\frac{\varphi_1(s)}{\omega_2(p;s)-\lambda_0}ds\cdot\int_{T^\nu}\varphi_2(t)f_2(t;s)dt$$

$$+\frac{\varphi_2(p)}{\omega_2(p;q)-\lambda_0}\cdot\frac{\varphi_1(q)}{\Delta_2(p;\lambda_0)}\int_{T^\nu}\frac{\varphi_2(s)}{\omega_2(s;q)-\lambda_0}\cdot\cdot\int_{T^\nu}\varphi_1(t)f_2(s;t)dt -$$

$$-\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{1}{\Delta(p;\lambda_0)}\int_{T^\nu}\varphi_1(t)f_2(q;t)dt\cdot\int_{T^\nu}\frac{v_1(s)\varphi_1(s)}{\omega_2(s;p)-\lambda_0}ds -$$

$$-\frac{v_1(q)}{\omega_2(p;q)-\lambda_0}\cdot\frac{\varphi_2(p)}{\Delta(p;\lambda_0)}\int_{T^\nu}\frac{v_1(s)}{\omega_2(p;s)-\lambda_0}\int_{T^\nu}\varphi_2(t)f_2(t;s)dtds$$

$$\Delta_2(q;\lambda_0) = \left(1-\int_{T^\nu}\frac{\varphi_2(s)\varphi_2(s)}{\omega_2(s;q)-\lambda_0}ds\right)$$

tengliklar o’rinli.

Bundan kelib chiqadiki, bu matritsamiz ham Vaynberg tenglamasini qanoatlantiradi.

2.2. Vaynberg tenglamasining tadbiqlari

Vaynberg tenglamasi asosan berilgan A operator muhim spektri bo’shliq (lakuna) ga ega bo’lgan holda shu bo’shliqda joylashgan xos qiymatlar sonining chekliligini isbotlashda muhim o’rin egallaydi.

Faraz qilaylik, $\nu = 3$ va $\omega_2(\cdot\,,\cdot)$ analitik funksiya bo’lib, yagona $(0,0)\in(T^3)^2$ nuqtada aynimagan minimumga ega bo’lsin. u holda shunday $C_1, C_2, \delta > 0$ sonlari topilib, barcha $p,q\in U_\delta(0)$ lar uchun

$$C_1(|p|^2+|q|^2)\le\omega_2(p;q)-m\le C_2(|p|^2+|q|^2) \tag{2.2.1}$$

tengsizliklar o’rinlidir, bu yerda $|p|=\sqrt{p_1^2+p_2^2+p_3^2}$, $p=(p_1,p_2,p_3)\in T^3$.

(2.2.1)- tengsizlikka va $v_1(\cdot)$ ning uzluksizligiga ko'ra

$$\int_{T^3} \frac{v_1^2(s)}{\omega_2(p;s)-m} ds$$

chekli integral mavjuddir. Integral belgisi ostida limitga o'tish mumkinligi haqidagi Lebeg teoremasiga ko'ra

$$\lim_{p\to 0} \Delta(p;\omega_2(0;0)) = \Delta(0;m)$$

munosabatlar o'rinlidir. Demak, $\Delta(\cdot;m)$ - T^3 dagi uzluksiz funksiya ekan.

Ma'lumki, agar

$$\max_{p\in T^3} \Delta(p;m) < 0$$

shart bajarilsa, u holda

$$\sigma_{ess}(A) \cap (-\infty;M] = [E_{\min};E_{\max}] \cup [m;M]$$

tenglik o'rinli bo'lib, $E_{\max} < m$. U holda $W(\lambda)$ operator $\lambda \in \left[\frac{E_{\max}+m}{2};m\right]$ lar uchun Gilbert-Shmidt operatori bo'lib, $\left[\frac{E_{\max}+m}{2};m\right]$ kesmada tekis operatorli topologiya ma'nosida uzluksizdir.

Endi A operator $\left[\frac{E_{\max}+m}{2};m\right)$ to'plamda ko'pi bilan cheklita xos qiymatga ega ekanligini isbotlaymiz. Teskarisini faraz qilaylik, ya'ni A operator $\left[\frac{E_{\max}+m}{2};m\right)$ oraliqda yotuvchi va m ga intiluvchi cheksiz ko'p xos qiymatlarga ega bo'lsin. biz ularni

$$\lambda_1, \lambda_2, \ldots \lambda_n, \ldots$$

orqali belgilaymiz, bunda

$$\lim_{n\to\infty} \lambda_n = m$$

$f^{(n)}$ orqali A operatorning z_n xos qiymatiga mos keluvchi ortonormal xos vektori bo'lsin, u holda 2- teoremaga ko'ra

$$W(\lambda_n) f^{(n)} = f^{(n)}$$

tenglik o'rinlidir. Bundan tashqari yuqoridagi mulohazalarga ko'ra $W(m)$ kompakt operator va

$$\|W(\lambda)-W(m)\|\to 0, \qquad \lambda\to m-0.$$

shu sababli

$$1=\|f^{(n)}\|=\|W(\lambda_n)f^{(n)}\|\le\|(W(\lambda_n)-W(m))f^{(n)}\|+\|W(m)f^{(n)}\|\to 0,\; n\to\infty$$

Bu qarama- qarshilikka ko'ra $\lambda=m$ soni $\left[\frac{E_{\max}+m}{2};m\right)$ yotuvchi xos qiymatlarning limitik nuqtasi bo'la olmaydi. Tasdiq to'liq isbotlandi.

II bob. xulosasi.

Ikkinchi bob ikki qismdan iborat, bular uch turdagi matritsaviy operator uchun odatdagi Vaynberg tenglamasining tadbiqlari birinchi qismda uchta turdagi matritsaviy operatorlar uchun Vaynberg tenglamasi qurilgan.

Bular bilan bir qatorda matritsaviy operatorlarning xos qiymatlari, xos funksiyalari ham o'rganilgan.

Ikkinchi bobning birinchi qismida teorema keltirilib isbotlangan.

XOTIMA.

Bu bitiruv malakaviy ishida asosiy tushunchalar, matritsaviy operatorlar, matritsaviy operatorlarning chiziqli, chegaralanganligi, o’z-o’ziga qo’shmaligi o’rganilgan. Shuningdek, uch turdagi matritsaviy operator uchun Vaynberg tenglamalari qurilgan.

Bitiruv malakaviy ishining birinchi bobida matritsa tipidagi operatorlar uchun umumiy ma’lumotlar, matritsaviy operatorlar va ularning chiziqliligi hamda chegaralanganligi va matritsaviy operatorlarning o’z-o’ziga qo’shmaligi va ayrim fazolar haqida ma’lumotlar berilgan. Qaralayotgan matritsaviy operatorning chiziqliligi, chegaralanganligi va o’z-o’ziga qo’shmaligi tekshirilgan. Bunda birinchi qismda keltirilgan ta’riflardan foydalanilgan. Blok operatorli matritsalarning vektor ta’siridagi qiymatlari hamda xossalari o’rganilgan.

Bitiruv malakaviy ishining ikkinchi bobida uch turdagi matritsaviy operator uchun odatdagi Vaynberg tenglamasining tadbiqlari birinchi qismda uchta turdagi matritsaviy operatorlar uchun Vaynberg tenglamasi qurilgan.

Bular bilan bir qatorda matritsaviy operatorlarning xos qiymatlari, xos funksiyalari ham o’rganilgan.

Ikkinchi bobning birinchi qismida teoremalar keltirilib isbotlangan.

Ushbu ishda yoritilgan usullar va tavsiya hamda olingan natijalar iqtidorli talabalarni va magistrlarni ilmiy ishlarga jalb qilish jarayoni samaradorligini oshiradi.

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