O’ZBEKISTON RESPUBLIKASI HALQ TA’LIMI VAZIRLIGI MUQIMIY NOMIDAGI QO’QON DAVLAT PEDAGOGIKA INSTITUTI.

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KLASSIK MEHANIKA MASALALARI VA ULARNI ECHISH USULLARI.

Talabalar va o’qituvchilar uchun uslubiy qo’llanma.

Toshkent - 2006

Ushbu uslubiy qo'llanma Qo'qon Davlat Pedagogika Instituti uslubshunoslik komissiyasining 1609 yig'ilishida ko'rib chiqildi va chop etishga tavsiya enilgan.

O'zbekiston respublikasi xalq ta'limi vazirligi tomonidan chop etishga ruhsat etilgan.

TOSHKENT: O’QTUVCHI NASHRIYOTI 2006.

MUNDARIJA.

KIRISH.

Oliy maktablarning fizika – matematika fakul'tetlarida tahsil oluvchi talabalarning diqqatiga tavsiya etilayotgan ushbu "Klassik mehanikaning kinematika qismidagi ba'zi mavzularga doir masalalar echish metodikasi" qo'llanmasi mualliflarning oliy o'quv yurtlarida shu fandan ko'p yillar davomida nazariy va amaliy mashg'ulotlar olib borish tajribasining natijasidir.

Ushbu qo'llanma oliy pedagogik o'quv yurtlarining fizika – matematika fakul'tetlari talabalari va o'rta maktab o'qituvchilarining fizik masalalar echishda o'z mahoratini yanada takomillashtirishga mo'ljallangan.

Qo'llanmada echimi keltirilgan masalalar (I,II) adabiyotlardan tanlangan.

Masalalarda keltirilgan nomerlarni birinshi raqami qo'llanma oxrida keltirilgan adabiyot raqami, ikkinchisi shu adabiyotdagi vavzuning raqami va oxrgisi masalaning raqamidir. Masalan: (1, 16, 5) I.V. Meshcherskiyning "Nazariy mexanikadan masalalar to'plami" 16-mavzuning 5-masalasidir va hokazo.

Ushbu qo'llanma chiqqanidan keyin uning kamchiliklari haqida o'z fikr mulohazalarini yozib yuboruvchilarga oldindan minnatdorchilik bildiramiz.

Nuqtaning harakat tenglamasi.

Traektoriya.

Fazoda t vaqt davomida harakatlanayotgan moddiy nuqtaning xolati koordinatalar yoki vektor usuli yordamida aniqlanadi. Nuqtaning harakati davomida koordinatalar vaqtga bog'liq ravishda o'zgaradi. Koordinatalarning vaqt bilan bog'lanish ifodasini

$$X=x(t) \quad y=y(t) \quad z=z(t) \qquad (1)$$

Nuqtaning koordinata usulida berilgan harakat tenglamasi deyiladi. Bundan tashqari tuqtaning harakat tenglamasini ham kattalik va ham yo'nalish jihatdan harakterlaydigan radius vektor r orqali ifodalash mumkin. Geometriyadan ma'lum

$r = x\bar{i} + y\bar{j} + z\bar{k} = r(t)$ (2) nuqtaning biror vaqt oralig'ida ketma-ket xolatlarini geometrik o'rniga yoki radius vektorning uchi chizgan egri chizig'iga traektoriya deyiladi. Agar (1) ifodalarning birortasidan vaqtni topib boshqalariga qo'yilsa, nuqtaning traektoriya tenglamasi xosil bo'ladi.

Moddiy nuqtaning xarakat tenglamasi yordamida uning traektoriyasini aniqlashga mansub bo'lgan quyidagi misollarni ko'rish mumkin.

Quyida berilgan nuqtaning harakt tenglamalari uchun xoy tekislikda uning traektoriya tenglamasi va boshlang'ich xolati aniqlansin.

(II. 331. 2.) x=3cost, y=3-5cost

Yechish: Bu tenglamalar yordamida nuqtaning traektoriya tenglamasini aniqlash uchun ularda vaqtni yo'qotish kerak, buning uchun birinchisidan topiladi $\cos t = \frac{x}{3}$ qiymatni ikkinchisiga qo'yish kifoyadir.

$y = 3 - 5\cos t = 3 - 5\frac{x}{3}$ 3y+5x=9

Bu esa yarim tu'g'ri tenglamasidir, chunki t>0

Nuqtaning boshlang'ich xolati esa berilgan xarakat tenglamalariga vaqtning .t = 0 qiymatini qo'yish bilan aniqlanadi. $x(t=0) = x_0 = 3$, $y(t=0) = y_0 =-2$,

(II. 331.5) $x=2\cos2t=2(\cos^2 t - \sin^2 t) = 2(1- \sin^2 t)$ ni topamiz va unga ikkinchi tenglamadan topilgan sint=y/3 qiymatni qo'yib, ya'ni $x = 2(1 - 2\frac{y^2}{9})$ traektoriya tenglamasini topamiz $y^2 = \frac{9}{4}(2 - x)$

Ma'lumki, $x = 2\cos 2t$ tenglamada $-1 \le \cos 2t \le 1$ shuning uchun x ning o'zgarish sohasi $-2 \le x \le 2$ bo'ladi. Nuqtaning boshlang'ich xolati esa t=0 da $x=x_0$ $y=y_0$ shartdan aniqlanadi. Demak, xarakat tenglamalariga t=0 qiymatni qo'ysak, $x_0 =2$, $y_0=0$ bo'ladi.

(II. 331.7) $x = 2tg\frac{t}{2}$ $y = 3\sin t$ $t \prec \pi$

Yechish: bu tenglamalardan vaqt t ni yo'qotib, traektoriya tenglamasini aniqlash uchun avvalo $x = 2tg\frac{t}{2}$ dan $x\cos\frac{t}{2} = 2\sin\frac{t}{2}$ ni aniqlab va $y = 6\sin\frac{t}{2}\cos\frac{t}{2}$ ga qo'yibhosil qilamiz.

$$y = 12\frac{\sin^2\frac{t}{2}}{x} \qquad \text{(a)}$$

Ma'lumki, $x^2\cos^2\frac{t}{2} = 4\sin^2\frac{t}{2}$ yoki $x^2 = (4 + x^2)\sin^2\frac{t}{2}$ oxirgidan $\sin^2\frac{t}{2}$ qiymatini (a) ga qo'yib, traektoriya tenglamasini topamiz

$$y = \frac{12x}{4 + x^2}$$

Vaqtni $t \prec \pi$ o'zgarish chegarasi $0 \le tg\frac{t}{2} \prec \infty$ shuning uchun $0 \le x \prec \infty$ oraliqda o'zgara oladi.

Nuqtaning boshlang'ich xolati esa t=0 da $x_0=0$ $y_0=0$.

Demak, harakat koordinata sistemasining boshlang'ich nuqtasidan boshlanadi.

(II. 331.8) $x = atg\frac{t}{2}$ $y = \cos t$ $t \prec \pi$

Yechish: Traektoriya tenglamasini aniqlash uchun birinchi tenglamaning shaklini ikkinchi tenglamadan foydalanishga moslab o'zgartiramiz.

$$x = atg\frac{t}{2} = a\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}} = a\frac{\sin t}{2\cos^2\frac{t}{2}} = a\frac{\sin t}{1+\cos t} = a\frac{\sqrt{1-\cos^2 t}}{1+\cos t}$$

Ikkinchi tenglamadan foydalanib, $x = a\frac{\sqrt{1-y^2}}{1+y}$ ni hosil qilamiz.

Oxirgi ifodani kvadratga ko'tarib, $x^2 = a^2 \frac{1-y^2}{(1+y)^2} = a^2 \frac{1-y}{1+y}$ ga ega bo'lamiz.

Bu tenglamani y ga nisbatan echib, traektoriya tenglamasini topamiz $y = \frac{a^2 - x^2}{a^2 + x^2}$. Harakat tenglamasidan $t \prec \pi$ bo'lgani uchun x $0 \le x \prec \infty$ sohada bo'ladi.

Nuqtaning boshlang'ich xolati esa t=0 $x=x_0$ $y=y_0$ shartlardan aniqlanadi.

t=0 $x=x_0$ $y=y_0$

Demak, xarakat koordinata o'qiga F(0,1) nuqtadan boshlanadi.

(II. 331. 10) x=a(sint + cost); y=asinwt

<u>Echish:</u> Bu tenglamalarni mos ravishda a va b ga bulib, kvadratga kutarib, va ularning mos ravishda, xadlab kushib, kuyidagiga ega bulamiz.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$

Bu nuktaning traektoriya tenglamasi bulib, u kata va kichik va kichik yarim uklari mos ravishda $\sqrt{2}a$ va $\sqrt{2}b$ bulgan ellips tenglamasidir.

t=0 da $x_0=a$, $y_0=-b$

(I.10.II.I) $x = a\sin 2wt$, $y = a\sin wt$,

Yechish: Traektoriya tenglamasini aniqlash uchun birinchi tenglamaning ikkilamchi burchaklarini trigonometrik qoidasiga asosan ochamiz

$$x=a\sin 2t=2a\sin t\cos t=2a\sin(1-\sin^2 t)^{1/2}$$

Bu ifodaga ikkinchi xarakat tenglamadan $\sin t=\frac{y}{a}$ ni qo'yib, quyidagini xosil qilamiz

$$x = 2y(1-\frac{y^2}{a^2})^{\frac{1}{2}} \quad \text{yoki} \quad ax = 2y(a^2-y^2)^{1/2}$$

Oxirgining ikki tomonini kvadratga ko'tarib, nuqtaning traektoriya tenglamasini quyidagicha yozamiz $a^2x^2=4y^2(a^2-y^2)$

Ma'lumki, $-1 \le \sin^2 \le 1$ shuning uchun $-a \le x \le a$ nuqtaning boshlang'ich holati esa t=0 dan aniqlanadi.

t=0 da $x_0=0$, $y_0=0$

Demak, nuqta boshlang'ich t=0 vaqtda 0(0;0) nuqtada, ya'ni koordinata boshida bo'lgan.

Endi vektor shaklida berilgan nuqtaning xarakat tenglamalari yordamida uning traektoriya tenglamasini aniqlaymiz.

(II.330.3) $r = t^2 i + (5 - 2t^2)\bar{k}$

Yechish. Bu tenglamada nuqtaning radius vektori $\bar{r}$ uning tashkil etuvchilari x, y, z yordamida berilgandir. Xaqiqatda ham ma'lumki, $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ Bu tenglamani berilgan tenglama bilan taqqoslab, quyidagilarni hosil qilamiz

$x=t^2$, $y=0$, $t=5-2t^2$

Birinchi tenglamadagi $t^2=x$ ni uchinchi tenglamaga qo'yib, traektoriya tenglamasini hosil qilamiz: 2x+t=5

Demak, nuqta boshlang'ich holatda M(0;0;5) nuqtada bo'lgan.

t noldan gacha o'zgaradi, shuning uchun $0 \le x < \infty$ bo'ladi.

(II. 330. 1) $r = (2t+1)i + (2-3t)j$

Yechish:Xuddi avvalgi masaladagidek

$x = 2t + 1$ $y = 2 - 3t$ t=0

Birinchi tenglamani 3 – ga ikkichisini 2 - ga ko'paytirib, so'ngra ularni xadlab qo'shib, traektoriya tenglamasini hosil qilamiz.

3x+2y=7. bu erda ham t 0 dan $+\infty$ gacha o'zgaradi, shuning uchun $0 \le x \prec \infty$ bo'ladi. Nuqtaning boshlang'ich xolati t=0 da $x_0=1$, $y_0=2$ $z_0=0$

Demak, nuqta boshlang'ich xolatda M(1;2;0) nuqtada bo'lgan.

(II. 330. 5) $r = (2 + \sin\frac{\pi \cdot t}{3})i + (1 + 2\cos\frac{\pi \cdot t}{3})k$

Yechish: Harakat tenglamasini dekart sistemasida radius – vektorning ifodasidan foydalanib, quyidagicha yozamiz.

$x = 2 + \sin\frac{\pi \cdot t}{3}$ $y = 0$ $t = 1 + 2\cos\frac{\pi \cdot t}{3}$

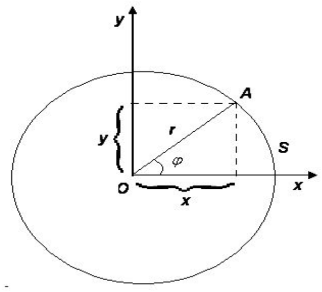
Bulardan traektoriya tenglamasini aniqlash uchun yuqoridagilarni

$x - 2 = \sin\frac{\pi \cdot t}{3}$ $\frac{t-1}{2} = 2\cos\frac{\pi \cdot t}{3}$

Shaklga keltirib, ikkala teglamani kvadratga ko'tarib, so'ngra xadlab qo'shib, traektoriya tenglamasini hosil qilamiz.

$$(x-2)^2 + \left(\frac{t-1}{2}\right)^2 = 1$$

Bu esa katta yarim o'qi $a=1$ kichik yarim o'qi $b=2$ va markazi M(2;0;1) nuqtada bo'lgan ellips tenglamasidir. Nuqtaning harakati esa t=0, $x_0=2$ $y_0=0$, z=3 nuqtadan boshlanadi. x, y, z ning o'zgarish chegarasi esa $1 \le x \le 3$ $y=0$ $-1 \le z \le 3$ bo'ladi.



(II. 330. 10) $\bar{r} = \cos 2ti + \sin t\bar{k}$

Yechish: Ma'lumki $x = \cos 2t$ $y = 0$ $z = \sin t$

Bulardan vaqtni yo'qotish uchun birinchi tenglamani $\cos 2t = 1 - 2\sin^2 t$ ko'rinishga keltirib, so'ngra bunga tenglamadagi $z = \sin t$ qiymatini qo'yib, traektoriya tenglamasini hosil qilamiz.

$x + 2t^2 = 1$, y=0, $-1 \le x \le 1$

Demak, nuqtaning harakati t=0 paytda boshlang'ich M(1;0;0) nuqtadan boshlanadi.

(II. 332)

Nuqta radiusi r bo'lgan aylana bo'ylab soat strelkasiga qarshi yo'nalishda shunday harakat qiladiki, uning o'tadigan yoyi S=kt qonun asosida o'zgaradi. Nuqtaning aylana markaziga joylashgan koordinata sistemasiga nisbatan harakat tenglamasini aniqlang.

Yechish: Masala shartiga asosan nuqtaning o'tgan yoyi S=kt qonun asosida o'zgaradi.

Chizmadan ko'rinib turibdiki $S = 2\varphi$ shuning uchun $\varphi = \dfrac{kt}{r}$ bo'ladi.

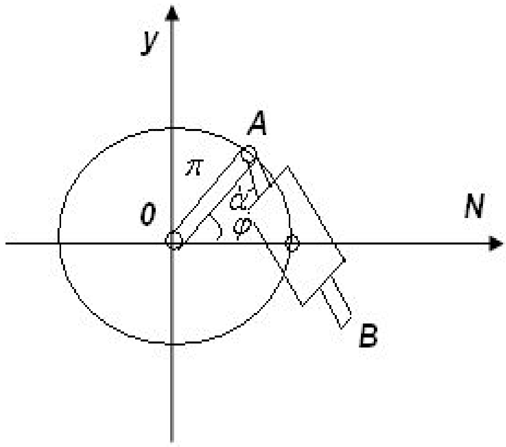
Endi chizmadan nuqta A holatda bo'lganida uning x, y koordinatalarini topamiz.

$x = r\cos\varphi = r\cos\dfrac{kt}{r}$ $y = r\sin\varphi = r\sin\dfrac{kt}{r}$

Bular esa izlanayotgan nuqtaning harakat tenglamalaridir. Nuqtaning harakati x=r, y=0 bo'lgan, ya'ni M(r, 0) nuqtadan boshlanadi.

(II .334)

Uzunligi l bo'lgan AB sterjen shunday harakat qiladiki, uning bir nuqtasi (A) radiusi $r \prec \frac{l}{2}$ aylana chizadi, sterjenning o'zi esa shu aylanada yotgan qo'zg'almas N nuqtadan o'tadi. Agar $\varphi = \omega t$ bo'lsa, sterjenning B nuqtasining harakat tenglamasi tuzilsin.



Yechish: Masala shartiga asosan A va N nuqtalar aylana ustida joylashgan, demak, OA=ON=r va AON uchburchak teng tomonli, shuning uchun $2\alpha + \varphi = \pi$ va $\alpha = \frac{\pi - 4}{2}$

Chizmadan ko'rinib turibdiki

$$x_b = r\cos\varphi + l\cos\alpha = r\cos\omega t + l\sin\frac{\omega t}{2}$$

$$y_b = r\sin\varphi - l\sin\alpha = r\sin\omega t - l\cos\frac{\omega t}{2}$$

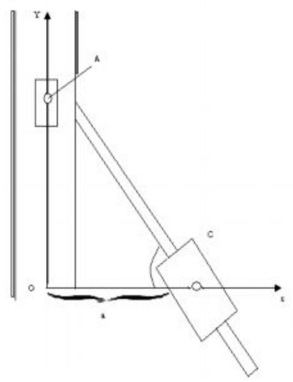
(II.338)

Uzunligi l bo'lgan AB sterjen shunday harakat qiladiki, uning uchi Oy to'g'ri chiziqda siyqanib harakat qiladi.

Sterjenning o'zi esa hamma vaqt y o'qdan OC=a masofada turgan qo'zg'almas C nuqtadan o'tadi. B nuqta qaysi egri chiziq bo'ylab harakat qilishini aniqlang.

Yechish. B nuqtaning x va y o'qlarga proeksiyasini topamiz.

$$x_b = a + BC\cos\varphi, \qquad y_B = BC\sin\varphi \qquad (1)$$



Bu ifodalar φ masala shartida berilmagan bulib, uni aniklash kerak.

CHizmadan va (1) ifodadn kurinib turibdiki,

$$\frac{y_b}{x_b - a} = tg\varphi \quad \text{yoki} \quad y_B = (x_B - a)tg\varphi \qquad (2)$$

va $\frac{a}{l - BC} = \cos\varphi$ yoki $BC = 1 - \frac{a}{\cos\varphi}$

Unda $x = a + (1 - \frac{a}{\cos\varphi})\cos\varphi = l\cos\varphi$ (3) (2) ifodadan

$$y_B^2 = (x_B - a)^2 tg^2\varphi = (x_B - a)^2 \frac{\sin^2\varphi}{\cos^2\varphi} = (x_B - a)^2 \frac{1-\cos^2\varphi}{\cos^2\varphi}$$

Oxirgi (3) ifodadan $\cos\varphi$ - ning qiymatini qo'yib, quyidagilarni hosil qilamiz:

$$y_B^2 = (x_B - a)^2 \frac{(l^2 - x_B^2)}{x_B^2}$$

Bu esa B nuqta harakat qilish zarur bo'lgan traektoriya tenglamasidir.

(II. 342)

CHizmada tasvirlangan mehanizmda AC chizg'ichning oxirlari A va C o'zaro α burchak tashkil qilgan to'g'ri chiziqli yo'naltiruvchilar bo'ylab harakat qiladi. Agar AB=a, BC=b bo'lsa, B nuqtaning traektoriyasi topilsin.

Echish: CHizmadan B nuqtaning kordinatalarini topamiz.

$$x_B = OA\cos\alpha + a\cos\beta \quad (1)$$
$$y_B = b\sin\beta$$

Bunda OA va β aniqlanishi uchun zarur bo'lgan nomaълumlardir. Ularni aniqlash uchun A nuqtadan Ox o'qiga perpendikulyar AK ni tushirib, quyidagi ifodani hosil qilamiz.

$$OA\sin\alpha = AC\sin\beta \quad \text{yoki} \quad OA\sin\alpha = (a+b)\sin\beta \quad (2)$$

Agar (2) ga(1) dan $\sin\beta = \frac{y_b}{b}$ qo'yilsa, OA ni topish imkoniga ega bo'lamiz.

$$OA = \frac{a+b}{b\sin\alpha} y_b$$

$$\cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-\frac{y_b^2}{b^2}}$$

Oxirgilarni (1) ga qo'yib, natijani a ga bo'lib, quyidagi tenglamani hosil qilamiz.

$$\frac{x_b}{a} - \frac{a+b}{ab} y_b ctg\alpha = \sqrt{1-\frac{y_b^2}{b^2}}$$

Bu tenglamani kvadratga ko'tarib, izlanayotgan B nuqtaning traektoriya tenglamasini topamiz.

$$\left[\frac{x_b}{a} - \frac{a+b}{ab} y_b ctg\alpha\right]^2 + \frac{y_b^2}{b^2} = 1$$

(II. 357)

O'zgarmas $\vartheta_1 = 25$ uzel va $\vartheta_2 = 15$ uzel tezliklar bilan harakat qiluvchi ikki A va V kemalarni kursi to'g'ri burchak bilan 0 nuqtada kesishadi.

OA_0=2.2 mil va OV_0=mil bo'lgan xol uchun kemalarning o'zaro masofasi S ni eng yaqinlashish vaqti T_1 ni hamda ularni orasidagi masofa boshlang'ich A_0V_0 ga teng bo'lish vaqti T_2 ni aniqlang.

Yechish: Masalani yechishda qulay bo'lishi uchun quyidagi belgilarni kiritamiz.

OA_0=L=2.2 mil

OV_0=l=2 mil

1) Kemalar OA_0 va OV_0 masofalarning ihtiyoriy nuqtasida bo'lganida ularning orasidagi masofa $S=\sqrt{(L-\vartheta_{1t})^2+(l-\vartheta_{2t})^2}$ bo'ladi, chunki ikkala kemalar A_0 va V_0 nuqtalardan bir vaqtda harakat qila boshlaydilar. Agar kemalarning masofasi yaqinlashsa, unda funkyiyaning minimum shartiga asosan $\frac{dS}{dt}=S=0$ bo'ladi. Bundan $\{\vartheta_1(L-\vartheta_{1t})+\vartheta_2(l-\vartheta_{2t})\}_{t=T}=0$ hosil bo'ladi. Oxirgidan minimum shartini qanoatlantiruvchi vaqtni t=T_1 bilan belgilab, quyidagini topamiz:

$$T_1=\frac{L\vartheta_1+l\vartheta_2}{\vartheta_1^2+\vartheta_2^2}$$

Masala shartidagilani bu ifodaga qo'yib, soatni daqiqalar bilanifodalab, kemalar masofasini minimum bo'lish vaqtini topamiz

$$T_1=\frac{(2.2\cdot 25+2\cdot 15)}{625+225}coam=\frac{85}{850}coam=\frac{1}{10}\cdot 60=6$$ daqiqa

Demak, A_0 va V_0 nuqtalardan boshlangan kemalarning harakati 6 daqiqadan keyin ularning o'zaro masofasi eng yaqin bo'lar ekan.

2) Endi kemalar 0 nuqtani kesib, o'tgandan keyin ularning masofasi yana S_0 ga teng bo'lish vaqti T_2 ni topamiz. Buning uchun yana chizmadan S_0 masofani topamiz.

$S_0=\sqrt{\vartheta_1^2t_1^2+\vartheta_2^2t_2^2}$ 0 nuqtani kesib o'tgandan keyin esa

$S_0'=\sqrt{\vartheta_1^2t_1^2+\vartheta_2^2t_2^2}$. Bu erda t_1 ,t_2 kemalarni mos ravishda OA_0 va OV_0 masofalarni o'tish vaqtlari t_1 ,t_2 esa 0 nuqtani o'tgandan keyin kemalar orasi boshlang'ich masofaga teng bo'lguncha o'tgan vaqtlar.

Masala shartiga asosan S_0 = S_0 , bu erdan

$\vartheta_1^2t_1^2+\vartheta_2^2t_2^2=\vartheta_1^2t_1^2+\vartheta_2^2t_2^2$ yoki $\vartheta_1^2\left(t_1^2t_1'^2\right)=\vartheta_2^2\left(t_2'^2-t_2^2\right)$ (a)

Lekin ikkala kemalar A_0 va V_0 nuqtalardan bir vaqtda harakat boshlagani uchun t=t_1+t_2 va t=t_1+t_2 bo'ladi.

Ohirgidan t_1 va t_2 ularning aniqlangan qiymatlarini (a) ifodani har bir ikki tomoniga qo'yib, quyidagilarni

$t_1^2 - t_1'^2 = (t_1 - t_1')(t_1 + t_1') = (2t_1 - t)(t) \qquad t_2^2 - t_2'^2 = (t_2 - t_2')(t_2 + t_2') = (2t_2 - t)(t)$

Hosil qlamaiz

$\vartheta_1^2(2t_1 - t) = \vartheta_2^2(t - 2t_2)$ yoki $(\vartheta_1^2 + \vartheta_2^2)t = 2(\vartheta_1^2 t_1 + \vartheta_2^2 t_2)$

Masala shartidan $OA_0 = v_1 t_1$ $OV_0 = v_2 t_2$ shuning uchun ikkala kemalarning orasini qayta avvalgi masofaga teng bo'lish vaqti

$$T_2 = t = \frac{2(\vartheta_1 OA_0 + \vartheta_2 OB_0)}{\vartheta_1^2 + \vartheta_2^2} = \frac{2(25\cdot 2.2 + 15\cdot 2)}{625 + 225} soat = \frac{1}{5}\cdot 60 daqiqa = 12 daqiqa$$

Demak kemalarning A_0, V_0 nuqtalardan harakati boshlangandan keyin ularning orasidagi masofa 12 daqiqada yana tenglashar ekan.

(II. 359)

Odamga qisqa vaqt ichida A nuqtadan sohildan 17.3 km masofada joylashgan V orolga borish vazifasi ыoklatildi. Buning uchun u qaysi S joyga tezligi 36 km/soat bo'lgan qayiqqa o'tirish kerak, agar yo'lning As qismida avtomabilning tezligi 72 km/soat bo'lsa?

Yechish: Agar A dan V ga odamning borish vaqti t bilan belgilasak,

$$t = t_1 + t_2 = \frac{AC}{\vartheta_1} + \frac{BC}{\vartheta_2} = \frac{AD - DC}{\vartheta_1} + \frac{\sqrt{BD^2 + DC^2}}{\vartheta_2}$$

Bu erda ϑ_1 , ϑ_2 - mos ravishda avtomabil va qayiq tezligidir. Agar DC ni izlayotgan x bilan ifodalasak, BD=17.3 km.

$$t = \frac{AB - x}{\vartheta_1} + \frac{\sqrt{BD^2 + x^2}}{\vartheta_2}$$

Lekin vaqtning qisqa bo'lishi uchun $\frac{dt}{dx} = 0$ qanoatlantirilishi kerak. SHuning uchun $\frac{1}{\vartheta_1} + \frac{x}{\vartheta_2\sqrt{BD^2 + x^2}} = 0$ $x^2 = \frac{\vartheta_2^2}{\vartheta_1^2}(BD^2 + x^2)$

bu erdan $DC = x = \frac{\vartheta_2 BD}{\sqrt{\vartheta_1^2 - \vartheta_2^2}} = \frac{36\cdot 17.3}{\sqrt{36\cdot 108}}км = \frac{17.3}{\sqrt{3}}км = 10км$

(II.373)

Nuqtaning harakati quyidagi tenglamalar

X=2t $\qquad$ $y=e^{-4t}$ sin4t yordamida berilgan.

1. Traektoriya tenglamasi
2. Nuqta tezligining Ox o'qiga paralel bo'lish vaqtini
3. Nuqta Ox o'qining kesish vaqtlari aniqlansin

Yechish: 1. nuqtani traektoriya tenglamasini aniqlash uchun berilgan tenglamalardan X=2t ni ikkinchi tenglamaga qo'yish kifoyadir.

$y=e^{-4t}\sin 4t$

2. tezlikni tashkil etuvchilarni aniqlaymiz.

$\vartheta_x = x = 2 см/с \qquad \vartheta_и = y = 4e^{-4e} \quad (\cos 4t - \sin 4t)$

Nuqtaning tezligi Ox o'qiga paralel bo'lish uchun $\vartheta = \vartheta_x$ bo'lishi zarur: bu xolda $\vartheta_y = 0$ bo'lishi kerak.

SHuning uchun $(\cos 4t - \sin 4t)$=0 shart bajarilishi kerak. Bunda $4t = \frac{\pi}{4} + \pi n$ yoki

$t_1 = \frac{\pi}{16} + \frac{\pi n}{4}$

YA'ni vaqtni ana shu onlarini nuqtaning tezligi Ox o'qiga paralel bo'ldi.

3. nuqta Ox o'qining kesish nuqtalarida y=0 bo'ladi. Bu xol faqat vaqtning $4t = \pi n$ yoki $t_2 = \frac{n\pi}{4}$ qiymatlari uchun ro'y berishi mumkin.

n= 1,2,3,.... ya'ni nuqta $t_2 = \frac{n\pi}{4}$ vaqtlardagina Ox o'qni kesadi.

(II.376)

Nuqtaning harakat tenglamasi.

$x = \frac{1}{2}t^2 \qquad y = \frac{1}{3}t^3$ berilgan. (x, y –santimetrlarida, t-sekundlarda). SHuning uchun vaqtning onlari aniqlansinki, unda tzlik va tezlanish orasidagi burchak $\alpha = \arccos\frac{3}{\sqrt{10}}$ bo'lsin.

Yechish: Avvalo berilgan tenglamalarga asosan nuqtaning tezlik va tezlanishini aniqlaymiz.

Tezlikni tashkil etuvchilari

$\vartheta_x = x = t$ va $\vartheta_y = y' = t^2$

Tezlanishni tashkil etuvchilari.

$\omega_x = \vartheta'_x = x'' = 1$ va $\omega_y = \vartheta'_y = y'' = 2t$

Bulardan nuqtaning tezlik va tezlanishi

$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2} = t\sqrt{1+t^2} \qquad \omega = \sqrt{\omega_x^2 + \omega_y^2} = \sqrt{1+4t^2}$ bo'ladi.

Ma'lumki, nuqtaning har qanday traektoriya bo'ylab harakatida uning tezligining yo'nalishi urinma tezlanish yo'nalishi bilan mos keladi. SHuning uchun tezlik va tezlanishning orasidagi burchaklar tengdir. SHu sababdan tezlik va to'la tezlanish orasidagi burchak haqida fikr юritish kifoya.

Endi urinma tezlanishni aniqlaymiz.

$$\omega_r = \frac{d\vartheta}{dt} = \sqrt{1+t^2} + \frac{t^2}{\sqrt{1+t^2}} = \frac{1+2t^2}{\sqrt{1+t^2}}$$

Bundan.

$\cos\alpha = \dfrac{\omega_r}{\omega}$ yoki $\dfrac{1+2t^2}{\sqrt{1+t^2}} = \sqrt{1+4t^2} \cdot \dfrac{3}{\sqrt{10}}$

Bu tenglamani kvadratga ko'tarib va soddalashtirib, quyidagi tenglamani topamiz.

$4t^4-5t^2+1=0$

Bu tenglamani echimi izlanayotgan tezlik bilan tezlanish orasidagi burchak.

$\alpha = \arccos\dfrac{3}{\sqrt{10}}$ ga teng bo'lish vaqtlarini beradi.

$t_1=1c$, $t_2=1/2c$

TEZLIK VA TEZLANISH

Nuqta (jism)ning vaqt birligi ichida bosib o'tgan yo'liga uning tezligi deyiladi. Tezlikningtvaqt birligi ichida o'zgarishiga esa tezlanish deyiladi.

Agar nuqtaning harakat tenglamalari to'g'ri burchakli dekart koordinatalar sistemasida berilgan bo'lsa, tezlik va tezlanish ularning proeksiyalari orqali aniqlanadi.

Ta'rifga asosan tezlikning proeksiyalari va tezlik

$\vartheta_x = \dfrac{dx}{dt} = x'$, $\vartheta_y = \dfrac{dy}{dt} = y'$ $\vartheta_z = \dfrac{dz}{dt} = z'$ $\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2 + \vartheta_z^2}$ (1) kabi ifodalanadi.

Tezlikning fazoda yo'nalishi esa yo'naltiruvchi kosinuslar yordamida aniqlanadi.

$\cos(\vartheta x) = \dfrac{\vartheta_x}{\vartheta}$ $\cos(\vartheta y) = \dfrac{\vartheta_y}{\vartheta}$ $\cos(\vartheta z) = \dfrac{\vartheta_z}{\vartheta}$ (2)

Tezlanishning dekart koordinatalar sistemasida proeksiyalari va tezlanish

$\omega_x = \dfrac{d\vartheta_x}{dt} = x''$ $\omega_y = \dfrac{d\vartheta_y}{dt} = y''$ $\omega_z = \dfrac{d\vartheta_z}{dt} = z''$

$\omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2}$ (3) kabi ifodalanadi.

Tezlanishning fazodagi yo'nalishi esa yo'naltiruvchi kosinuslar yordamida aniqlanadi.

$\cos(\omega x) = \dfrac{\omega_x}{\omega}$ $\cos(\omega y) = \dfrac{\omega_y}{\omega}$ $\cos(\omega z) = \dfrac{\omega_z}{\omega}$ (4)

(1.11.3)

Nuqta x=2cost, y=4cos2t (x, y –santimetrlarda, t – sekundlar hisobida), tenglamalarga muvofiq Lissaju figuralarini chizadi. Nuqta Ou o'qda bo'lganda tezligining miqdori bilan yo'nalishi topilsin.

Yechish: nuqta tezligini topami.

$\vartheta_x = x = -2\sin t$ $\quad\quad \vartheta_y = y = -8\sin 2t = -16\sin t\cdot\cos t$

To'la tezlik $\vartheta = \sqrt{x^2 + y^2} = 2\sin t\sqrt{1+8^2\cos^2 t}$

Agar x=0 bo'lsa, cost=0 $\quad t = \frac{\pi}{2}; \frac{3\pi}{2}$

a) $t = \frac{\pi}{2}$ bo'lsa $\vartheta_x = x = -2 см/с$ $\quad \vartheta_y = y = 0$ va $\vartheta = 2$ sm/s

(2)tenglamarlarning birinchisiga asosan

$\cos(\vartheta x) = \frac{\vartheta_x}{\vartheta} = -\frac{2}{2} = -1 \quad\quad \vartheta_x = \pi$

b) $t = \frac{3\pi}{2}$, $\vartheta_x = x = 2 см/с$ $\quad \vartheta_y = y = 0$ $\quad \vartheta = 2 см$

$\cos(\vartheta x) = \frac{\vartheta_x}{\vartheta} = \frac{2}{2} = 1 \quad\quad \vartheta = 0$

(1.10.10)

CHastotasi bir xil lekin amplitudasi va fazalari har xil ikkita garmonik tebranma harakatda bir vaqtda qatnashuvchi nuqtaning traektoriyasi aniqlansin: tebranma harakatlar ikkita o'zaro perpendikulyar o'qlar bo'ylab юzaga keladi:

$x = a\sin(kt+\alpha) \quad\quad y = b\sin(kt+\alpha)$

Yechish: avvalo ikkinchi tenglamaning shaklini quyidagicha o'zgartiramiz.

$$y = b\sin\{(kt+\alpha)+(\beta-\alpha)\} = b\{\sin(kt+\alpha)\cos(\beta-\alpha)+\cos(kt+\alpha)\sin(\beta-\alpha)\}$$

$$\cos(kt+\alpha) = \sqrt{1-\sin^2(kt+\alpha)} = \sqrt{1-\frac{x^2}{a^2}}$$

CHunki $\sin(kt+\alpha) = \frac{x}{a}$ shuning uchun $\frac{y}{b} = \frac{x}{a}\cos(\beta-\alpha)+\sqrt{1-\frac{x^2}{a^2}}\sin(\beta-\alpha)$ yoki $\frac{y}{b} - \frac{x}{a}\cos(\beta-\alpha) = \sqrt{1-\frac{x^2}{a^2}}\sin(\beta-\alpha)$ oxirgini kvadratga ko'tarib, izlanayotgan traektoriyani tenglamasini topamiz:

$$\frac{y^2}{b^2}+\frac{x^2}{a^2}-2\frac{xy}{ab}\cos(\beta-\alpha) = \sin^2(\beta-\alpha)$$

Bu esa ellips tenglamasidir

(I.10.14)

Snaryadning harakati $x = \vartheta_0 \cos \alpha t$ $y = \vartheta_0 \sin \alpha t - \frac{gt^2}{2}$ tenglamalar bilan berilgan, bu erda - ϑ_0-snaryadning boshlang'ich tezligi, α-gorizont bilan ϑ_0 orasidagi burchak, g- og'irlik kuchining tezlanishi. Snaryadning harakat traektoriyasi, H – balandilig, L-uchish uzoqligi va T – uchish vaqtini aniqlaymiz.

$y = xtg\alpha - \frac{gx}{2\vartheta_0^2 \cos^2 \alpha}$ (a) – parabola.

Eng юqori balandlikda $y_{Max} = H$ - va ekstremum shartiga asosan $\frac{dy}{dt} = 0$ shuning uchun ikkinchi tenglamadan $\vartheta_0 \sin \alpha - gT_1 = 0$ snaryad H – balandalikka ko'tarilish vaqti $T_1 = \frac{\vartheta_0}{g} \sin \alpha$ buni ikkinchi tenglamaga qo'ysak, hosil bo'ladi.

Snaryadning uchun uzoqligini aniqlash uchun uning uchish va tushish nuqtalarida y=0 bo'lishini bilish kifoyadir. Haqiqatda ham bu nuqtalar uchun (a) tenglamadan

$x\left(tg\alpha - \frac{gx}{2\vartheta_0^2 \cos^2 \alpha}\right) = 0$ $x_1 = 0$ - snaryadning uchush nuqtasi va

$x_2 = L = 2\frac{\vartheta_0^2}{g} tg\alpha \cos^2 \alpha = \frac{\vartheta_0^2}{g} \sin 2\alpha$ snaryadning uchish uzoqligi.

Snaryadning uchish vaqti esa $T = 2T_1 = 2\frac{\vartheta_0}{g} \sin \alpha$ bo'ladi.

Hosil qilingan ifodadan ma'lumki, snaryadning eng katta uchish uzoqligi $2\alpha = 90^0$ yoki $\alpha = 45^0$ da bo'ladi.

$L = \frac{\vartheta_0^2}{g}$ unga mos balandlik $H = \frac{\vartheta_0^2}{4g}$ va uchish vaqti $T = \sqrt{2}\frac{\vartheta_0}{g}$ bo'ladi.

(I.12.5)

Kopyor to'qmog'i 2.5 m balandlikdan pastga tushadi, uni o'sha balandlikka ko'tarish uchun, shuncha joydan tushishiga qaraganda uch marta ko'proq vaqt ketadi. Agar kopyor to'qmog'i pastga 9.81 m/s^2 tezlanish bilan erkin tushadi deb hisoblansa, u bir daqiqada necha marta uradi?

Yechish: Kopyor to'qmog'ining tushish balandligini h , tushish vaqtining t_1, ko'tarilish vaqti t_2 va ko'tarilib tushish vaqtini esa t bilan belgilaymiz. U xolda to'qmoqning bir ko'tarilib tushish vaqti $t = t_1 + t_2$ bo'ladi.

Masalaning shartiga asosan to'qmoq erkin tushish tezlanishi bilan tushadi. Demak, $h = \frac{gt^2}{2}$ bo'lishi kerak.

Bunday to'qmoqning tushishi uchun $t_1 = \sqrt{\frac{2h}{g}}$ vaqt sarflanadi.

Ko'tarilish vaqti esa $t_2 = 3t_1 = 3\sqrt{\frac{2h}{g}}$

Bir ko'tarilib tushish uchun $t = t_1 + t_2 = 4\sqrt{\frac{2h}{g}}$ vaqt sarflanadi. Kopyor to'qmog'ining 1 daqiqada necha marta urilishini aniqlash uchun t_1=1 daqiqa = 60 s ni t ga bo'lish zarur

$$n = \frac{t_1}{t} = \frac{60}{4\sqrt{\frac{2h}{g}}} = 15\sqrt{\frac{g}{2h}}$$

Bu ifodaga masala shartiga berilgan miqdorlarni qo'yib quyidagini hosil qilamiz.

$n = 15\sqrt{\frac{9.81}{5}} = 15\sqrt{5 \cdot 9.81} \approx 3\sqrt{49} \approx 3 \cdot 7 = 21$ marta. YA'ni kopyor to'qmog'i 1 daqiqada 21 marta zarba uradi.

(I.12. 12)

Nuqta $S = \frac{g}{a^2}\left(at + e^{-at}\right)$ qonuniga muvofiq to'g'ri chiziqli xarakat qiladi, bunga a va g - doimiy miqdorlar. Nuqtaning boshlang'ich tezligi, shuningdek, uning tezlanishi tezlikning funksiyasi sifatida aniqlansin.

Yechish: Nuqtaning tezligi $\vartheta = S = \frac{g}{a}\left(1 - e^{-at}\right)$ qonun asosida o'zgaradi va t→∞ bo'lganda $\vartheta_\infty \to \frac{g}{a}$ bo'ladi. Boshlang'ich (t=0) vaqt uchun $\vartheta_0 = \vartheta_{1=0} = 0$ dir. Tezlanish $\omega = S = ge^{-at}$. YA'ni tezlanish vaqtniig sunuvchi funksiyasidir. Tezlik fomulasidan $ge^{-at} = g - a\vartheta$ va oxirgini tezlanish formulasiga qo'yib, izlanayotgan tezlanishning tezlik funksiyasi sifatida quyidagicha topamiz.

$$W = g - a\vartheta$$

(I. 11. 7)

Tik qirg'okdagi uchta nuqtadan 50, 75, 100 m/s ga teng bo'lgan gorizontal tezlik bilan bir vaqtda otilgan uchta o'q suvga bir vaqtda tushadi. SHu punktning suv sathidan balandliklari h_1, h_2, h_3 aniklasin: birinchi o'k, tushgan nuqtadan qirg'oqgacha bo'lgan masofa 100 m ga teng; faqat og'irlik kuchining tezlanishi g =*9,81* m/s^2 e'tiborga olinsin. SHuningdek, o'klarning uchish vaqti T va ularning suvga tushish vaqtidagi ϑ_1, ϑ_2, ϑ_3 tezliklari aniqlasin.

Yechish: Masala shartiga asosan bir vaqtda otilgan uchta o'q suvga bir vaqtda tushadi, shuning uchun ularning uchish vaqti tengdir. Bundan tashqari birinchi o'qni uchish tezligi 50 m/s bo'lib, u qirg'okdan юz metr uzoqdagi nuqtaga tushgan.

Demak, $x_1 = \vartheta_1 t$ va $T = \frac{x_1}{\vartheta_1} = 2c$. Balandligi $h = \frac{gt^2}{2}$ dan aniqlanadi, lekin uchish vaqti uchchala o'q uchun T,=T_2=T,=T bir xil, demak h_1=h_2= h_3= $\frac{gt^2}{2}$ =19.62 m

Endi ularning tushish paytidagi tezligani aniklaymiz.

Bu tezliklar esa gorizontal va vertikal tashkil etuvchilardan iboratdir. YA'ni $\vartheta_n = \sqrt{\vartheta_n^2 + (gT)^2}$

Birinchi o'q uchun $\vartheta_1 = \sqrt{\vartheta_1^2 + (gT)^2} = \sqrt{50^2 + (9.81 \cdot 2)^2} = 53.71 м/c$

$\vartheta_2 = \sqrt{\vartheta_2^2 + (gT)^2} = 77.552 м/c$ $\vartheta_3 = \sqrt{\vartheta_3^2 + (gT)^2} = 101.95 м/c$

(II. 364)

Nuqtaning harakat tenglamasi berilgan:

$x = R\cos kt + kRt\sin kt$ $y = R\sin kt - kRt\cos kt$

bu erda k- doimiy miqdor.

Nuqtaning tezlik va tezlanishini vaqtning funksiyasi sifatida aniqlang.

Yechish. Tezlik va tezlanish proeksiyalarini aniqlaymiz:

$\vartheta_x = x' = -kR\sin kt + kR\sin kt + k^2 Rt\cos kt = k^2 Rt\cos kt$

$\vartheta_y = y' = kR\cos kt - kR\cos kt + k^2 Rt\sin kt = k^2 Rt\sin kt$

$\omega_x = k^2 R\cos kt - k^3 Rt\sin kt$ $\omega_y = k^2 R\sin kt + k^3 Rt\cos kt$ nuqtaning tezligi

$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2} = k^2 Rt$ tezlanishni $\omega = \sqrt{\omega_x^2 + \omega_y^2} = \sqrt{k^4 R^2 + k^6 R^2 t^2} = k^2 R\sqrt{1 + k^2 t^2}$

(II. 374)

Nuqta vertikal tekislikda harakat qilib, tenglamasi quyidagicha berilgan x=1, u=sint^2 (x, u - santimetrlarda; t - sekundalarda ifodalangan):

1) harakat boshlangandan keyin nuqtaning Ox o'qini eng yaqin kesishganida ttzligining kattaligini;

2) shu vaqt uchun nuqta tezligining kattaligbni;

3) nuqta tezligi kattaligi traektoriyasining eng юqori holatida aniqlansin.

Yechish. Nuqtaning traektoriyasi sinusoida u=sinx^2 bo'ladi.

3) nuqta tezligining tashkil etuvchilarini aniqlaymiz. $\vartheta_x = x' = 0$

$\vartheta_y = y' = 2t\cos t^2$ uning tezligi

$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2} = \sqrt{1 + 4t^2 \cos^2 t^2}$

Nuqtaning Ox o'qini birinchi kesishganida u=sint^2=0 bu erdan $t=\sqrt{\pi}$ vaqtning bu qiymati uchun tezlik

$\vartheta=\sqrt{1+4\pi}=\sqrt{13.56}=3.68 см/с$

Traektoriyaning eng юqori holatida u= sint^2=1 yoki $t=\sqrt{\frac{\pi}{2}}$ va tezligi

ϑ_1=1sm/s, chunki sost^2=0 $t^2=\frac{\pi}{2}$

Tezlanish tashkil etuvchilari

$\omega_x=\frac{d\vartheta_x}{dt}=0$ $\omega_y=\frac{d\vartheta_y}{dt}=2\cos t^2-4t^2\sin t^2$ tezlanish

$\omega=\sqrt{\omega_x^2+\omega_x^2}=\sqrt{0+4(\cos t^2-2t^2\sin t^2)}\Big|_{t=\sqrt{\pi}}=2 м/м^2$

(II. 379)

Ikki nuqtaning harakat tenglamalari mos ravishda quyidagicha berilgan:
1) x=t; u=2t^2 2) x=t^2 u=2t^4 (x, u - santimetrlarda; t - sekundalarda).
1)Nuqtaning traetoriyasi;
2)bir nuqtaning ikkinchisini quvib etish vaqti;
3)shu vaqt uchun nuqtalar tezliklari va tezlanishlarining kattaligi aniklansin.
Yechish: 1) ikkala nuqtaning traektoriyalari u=2x^2 bo'ladi;
2)nuqtalar bir-birini quvib etganda, ularning mos koordinatalari teng bo'ladi, ya'ni t=t^2, t^2=t^4 bu erdan t(t-1)=0, lekin $t\neq 0$ shuning uchun t=T=1s
3)shu vaqt uchun birinchi nuqtaning tezligi

$\vartheta_1=\sqrt{\vartheta_x^2+\vartheta_y^2}=\sqrt{1+(4t)^2}=\sqrt{1+4^2}=\sqrt{17}=4.12 см/с$ ikkinchisiniki

$\vartheta_2=\sqrt{(2\mathrm{t})^2+(8t^3)^2}=\sqrt{4+64}=\sqrt{68}=8.25 см/с$

4) tezlanishlari $\omega_2=\sqrt{\omega_x^2+\omega_x^2}=\sqrt{2^2+24^2}=\sqrt{580}=24.1 см/с^2$

(II. 381)

Boshlang'ich $\vartheta_0=\vartheta_0$i tezlikka ega bo'lgan $\bar{\omega}=-a\bar{i}+a\bar{s}$ tezlanish bilan harakat qiladi. Nuqtaning traektoriya tenlamasi va eng kichik tezligi anqlansin.

Yechish. Nuqtaning tezligini va radius vektorining tashkil etuvchilarini aniqlaymiz.

$\frac{d\vartheta_x}{dt}=-a$ $\vartheta_x=-at+c_1$ t=0 da $\vartheta_x=\vartheta_0$ va $\vartheta_x=\vartheta_0-at$

$\frac{d\vartheta_y}{dt}=-a$ $\vartheta_y=-at+c_2$ t=0 da $\vartheta_y=0$ va $\vartheta_y=at$

tezlikni esa $\vartheta_1 = \sqrt{\vartheta_x^2 + \vartheta_x^2} = \sqrt{(\vartheta_0 - at)^2 + (at)^2}$ $\quad x = \vartheta_0 t - \frac{at^2}{2} \quad y = \frac{at^2}{2}$

Oxirgi tenglamalardan traektoriya tenglamasini aniqlaymiz:

$(x+u)^2 = \vartheta_0^2 t^2$ yoki $(x+y)^2 = \frac{2\vartheta_0^2 y}{a}$

Endi nuqtaning eng kichik tezligini aniqlaymiz;

Funksiyaning ekstremal shartiga asosan $\frac{d\vartheta}{dt} = 0$

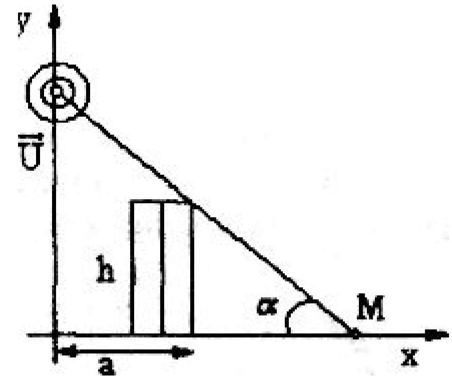
SHuning uchun $\frac{-a(\vartheta_0 - at) + a^2 t}{\vartheta} = 0$ yoki $2at = \vartheta_0$, $at = \vartheta_0/2$

Buni tezlik formulasiga qo'yib, $\vartheta_{мин} = \sqrt{(\vartheta_0 - at)^2 + (at)} = \sqrt{\left(\vartheta_0 - \frac{\vartheta_0}{2}\right)^2 + \left(\frac{\vartheta_0}{2}\right)^2} = \frac{\vartheta_0}{\sqrt{2}}$

Demak, $\vartheta_{мин} = \frac{\vartheta_0}{\sqrt{2}}$

(II. 384)

YOrug'lik manbai A vertikal bo'ylab U=sonst tezlik bilan tushmoqda. Stol ustiga h balandlikka ega bo'lgan ustun vertikaldan *a* masofaga qo'yilgan. M soya oxirini stol bo'ylab xarakatini, tezlik va tezlanishini yorug'lik manbai balandligidan bog'lanishi aniqlasin.



Yechish. CHizmadan ko'rinib turibdiki, $tg\alpha = \frac{y-h}{a} = \frac{h}{x-a}$

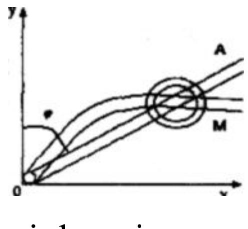
Bundan M soya oxirining koordinatasi $x = a + \frac{ah}{y-h}$

soyaning tezligi esa $\vartheta = x' = \frac{ahU}{(y-h)^2}$ $\quad U = -\frac{dy}{dt} \succ 0$

tezlanishi $\omega = \vartheta' = \frac{2ahU^2}{(y-h)^2}$

(II. 387)

M nuqta tenglamasi $u^2=2Rx$ bo'lgan parabolik yo'naltiruvchi va $\varphi = \omega t$ qonun asosida 0 nuqta atrofida aylanuvchi OA sterjenь yordamida harakatga keltiriladi. M halqa tezligining x koordinata bilan bog'lanishi aniqlansin.



Yechish. CHizmadan $x = ytg\varphi$ ni topamiz. Avvalo tezlikni $\vartheta_x = x'$ ni aniqlaymiz.

$\vartheta_x = x' = y'tg\varphi + \dfrac{y\varphi'}{2}$ lekin $\dfrac{x^2}{y^2} = \dfrac{1-\cos^2\varphi}{\cos^2\varphi}$ bu erdan sos^2 $\varphi = \dfrac{y^2}{x^2+y^2}$

shuning uchun $\vartheta_x = y'\dfrac{x}{y} + \dfrac{y\varphi'}{y^2}\left(x^2+y^2\right) = \dfrac{1}{y}\left\{xy' + \varphi'\left(x^2+y^2\right)\right\}$

oxirgiga u'=2Rx va $y' = \dfrac{1}{2}\sqrt{\dfrac{2P}{x}}x'$ ni qo'yib hosil qilamiz.

$\vartheta_x = x' = \dfrac{1}{\sqrt{2Px}}\left\{\dfrac{1}{2}\sqrt{2Px}x' + \varphi'x(x+2P)\right\}$ yoki $\vartheta_x = 2\dfrac{x\varphi'}{\sqrt{2Px}}(x+2P)$

Endi $\vartheta_x = 2\dfrac{x\varphi'}{\sqrt{2Px}}(x+2P)$ - ni aniklaymiz

$\vartheta_y = y' = \dfrac{1}{2}\sqrt{\dfrac{2P}{x}}x' = \dfrac{1}{2}\sqrt{\dfrac{2P}{x}}\cdot 2\dfrac{x\varphi'}{\sqrt{2Px}}(x+2P) = \varphi'(x+2P)$ tezlik esa

$$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2} = \varphi'\sqrt{\frac{4x}{2P}(x+2P)^2 + (x+2P)^2} = \varphi'\sqrt{\left(\frac{2x}{P}+1\right)(x+2P)^2}$$

Nihoyat, $\vartheta = \dfrac{\varphi'}{\sqrt{P}}\sqrt{(2x+P)}(x+2P)$

(II. 338)

Nuqtaning shunday holatini aniqlangki, uning tezligining kattaligi Ox o'qiga proeksiyasining kattaligidan ikki marta katta bo'lsin. Nuqtaning trasktoriyasi $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ ellips bo'lsin.

Yechish: Traektoriya tenglamasidan u - ni aniqlaymiz.

$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

Tezlikning Ou o'qiga proeksiyasini topamiz:

$\vartheta_y = y' = -\dfrac{b}{a}\dfrac{xx'}{\sqrt{a^2-x^2}}$ tezlik esa $\vartheta = \sqrt{x^2+y^2} = x'\sqrt{1+\dfrac{b^2}{a^2}\dfrac{x^2}{a^2-x^2}} = x'\dfrac{\sqrt{\left(b^2-a^2\right)x^2+a^4}}{a\sqrt{a^2-x^2}}$

Masala shartiga asosan $\dfrac{\vartheta}{\vartheta_x} = \dfrac{\vartheta}{x'} = 2$

SHuning uchun tezlik uchun topilgan ifodaga $\dfrac{\vartheta}{x'} = 2$ - ni ko'yib, natijani kvadratga ko'tarib x -ga nisbatan tenglamani echib quyidagi ifodani topa $x = \pm\dfrac{a^2\sqrt{3}}{\sqrt{b^2+3a^2}}$ miz:

NUQTA HARAKATINI BERILISHINING TABIIY USULI.

Nuqtaning traektoriya bo’ylab harakat tenglamasi

$S=S(t)$ (1)

bu erda S- traektoriya biror nuqtasiga nisbatan hisoblanadigan yoy koordinatasidir.

Nuqtaning yoy bo’ylab harakatida uning tezligining absolют qiymati.

$$\vartheta = \frac{dS}{dt} \quad (2)$$

Agar tezlik vaqtning funksiyasi sifatida berilgan bo’lsa, unda yoy koordinatasi quyidagi formula yordamida aniqlanadi:

$$S = \int_0^t |\vartheta(t)| dt + S_0 \quad (3)$$

bu erda S_0- (3) dan t=0 bo’lgaida $S=S_0$ shartdan aniqlanadi. Agar yoyni hisoblash holati bilan nuqtaning holati mos kelsa, S=0 bo’ladi, unda (3) quydagicha yoziladi:

$$S = \int_0^t |\vartheta(t)| dt \quad (4)$$

Nuqtaning traektoriya bo’ylab harakatida uning harakatining yo’nalishi o’zgarishi mumkin. SHuning uchun (0, t) vaqt oralig’ida uning o’tgan yo’li tezligining alomati o’zgarmay o’tgan yo’li tezliganing alomati o’zgarmay o’tgan yoy bo’laklarining yig’indisiga tengdir. YA’ni

$\Pi = |S_n - S_{n-1}| + |S_{n-1} - S_{n-2}| + ... + |S_1 - S_0|$ (5) Nuqtaning ihtiyoriy harakatining tezlanishi quyidagi formula yordamida anikdanadi.

$$\omega = \sqrt{\omega_\tau^2 + \omega_n^2} \quad (6)$$

Bu erda $\omega_\tau = \frac{d\vartheta}{dt}$ - urinma tezlanish, $\omega_n = \frac{\vartheta^2}{\rho}$ - normal tezlanish

ϑ - tezlik.

Agar nuqtaning harakati tekis tezlanuvchan bo’lsa, $\omega_\tau = const$ bo’ladi va bu holda

$$\vartheta = \vartheta_0 + \omega_\tau t \quad S = S_0 + \vartheta_0 t + \frac{\omega_\tau t^2}{2} \quad S = S_0 + \frac{\vartheta - \vartheta_0}{2} \qquad (7)$$

$S = S_0 + \frac{\vartheta^2 - \vartheta_0^2}{2\omega_0}$ (8) bo’ladi.

(II. 407)

Quyidagi nuqta tezligining tenglamalari uchun uning traektoriya bo'ylab harakat tenglamasini qamda t(0,5) vaqt oralig'ida yoy koordinatining qiymati S va o'tgan yo'li aniqlansin.

1. $\vartheta = (2t+1)$ sm/s

Yechish. Bu tenglamadan ko'rinib turibdiki, t>0 bo'lgani uchun hamisha ϑ>0 bo'ladi, ya'ni tezlik (0,5) vaqt oralig'ida alomatini o'zgartirmaydi.

Nuqtaning traektoriya bo'ylab harakat tenglamasi tekstdagi (4) ga asosan

$S = \int_0^t \vartheta dt = (t^2 + t)$ sm bo'ladi. $S|_{t=5} = 30см$

O'tgan yo'li esa $П|_{t=5} = 30$ sm

2. $\vartheta = (3-t)$ sm/s

Yechish: Bu tezlik tenglamasi vaqt bo'yicha kamaıovchi bo'lib, 0≤ x≤5 da eng katta qiymati 3(t=0) va eng kichik qiymati -2(t=5) ga tengdir va t=3 nuqtada alomatini o'zgartiradi. Nuqtaning traektoriya bo'ylab harakat tenglamasi

$S = \int_0^t (3-t)dt = 3t - \frac{t^2}{2}$ yoy koordinati $S|_{t=5} = 2.5$ sm

$S|_{t=0} = S_0 = 0$ sm $S|_{t=3} = S_1 = 4.5$ sm $S|_{t=5} = S_2 = 2.5$ sm

Nuqtaning 0≤t≤5 vaqt oralig'ida o'tgan yo'li esa

$П = |S_1 - S_0| + |S_2 - S_1| = (4.5-0) + (2.5-4.5) = 4.5 + 2 = 6.5$ 6,5 *sm*

3. $\vartheta = \frac{\pi}{5}\sin\frac{8\pi}{15}t$ sm/s

Yechish: Ma'lumki, bu tezlikni vaqtga bog'lanish funksiyasi. Vaqt quyidagi $\frac{8\pi}{15}t = k\pi$ shartni qanoatlantirganda, o'z alomatini o'zgartiradi,

bu vaqt qiymatlari

$t = \frac{15}{8}k(k = 0,1,2,....)$ ga tengdir.

Vaqtning 0≤ t≤5 oralig'ida esa

$t_0 = 0$ $t_1 = \frac{15}{8}k$ $t_2 = \frac{15}{4}$ va $t_3 = 5$ (1) bo'ladi.

Nuqtaning traektoriya bo'ylab harakat tenglamasi

$S = \int_0^t \vartheta dt = -\frac{3}{8}\cos\frac{8\pi}{15}t\Big|_0^t = \frac{3}{8}(1-\cos\frac{8\pi}{15}t)$ (2)

YOy koordinatining t =5 s dagi qiymati

$S\big|_{t=5} = \frac{3}{8}(1-\cos\frac{8\pi}{15}t)_{t=5} = \frac{9}{16}$ sm ga teng bo'ladi.

Endi nuqtaning traektoriya bo'ylab harakatida o'tgan yo'lini aniqlaymiz. (1) dagi vaqt qiymatlarini ketma-ket (2) ga qo'yib, quyidagini hosil qilamiz.

$t_0 = 0 \quad S_0 = 0 \quad t_1 = \frac{15}{8}c \quad S_1 = \frac{3}{4}cм \quad t_2 = \frac{15}{4}c \quad S_2 = 0 \quad t_3 = 5c \quad S_3 = \frac{9}{16}cм$

$$\Pi = |S_1 - S_0| + |S_2 - S_1| + |S_3 - S_2| = (\frac{3}{4} - 0) + (0 - \frac{3}{4}) + \left|\frac{9}{16} - 0\right| = \frac{3}{4} + \frac{3}{4} + \frac{9}{16} = \frac{33}{16}$$

$$\vartheta = \pi + 2\pi\cos\frac{2\pi}{5}t$$

Yechish. Bu tenglamadan ma'lumki, $\cos\frac{2\pi}{5}t = -\frac{1}{2}$ shartni qanoatlantirganda, (ϑ=0) tezlik alomatini o'zgartiradi. Bu shartni qanoatlantiruvchi vaqt qiymatlari esa

$\frac{2\pi}{5}t = (-1)\frac{k2\pi}{3} + 2k\pi$ yoki $t = (-1)^k\frac{5}{3} + 5k$ (k=0,1,2,......)

$t_1 = \frac{5}{3}c$ (k=0) $t_2 = \frac{10}{3}c$ (k=1)

Demak, tezlik vaqtning shu qiymatlarida alomatini o'zgartiradi. Nuqtaning traektoriya bo'ylab harakat tenglamasi

$S = \pi\int_0^t(1 + 2\cos\frac{2\pi}{5}t)dt = \pi t + 5\sin\frac{2\pi}{5}t$ (2) dan iborat buladi. YOyning koordinatasi esa $S\big|_{t=5} = 5\pi$ sm bo'ladi. Endi vaqtni t= 0, $\frac{5}{3}, \frac{10}{3}, 5s$ qiymatlarini ketma-ket (2) ga qo'yib, nuqtaning trasktoriya bo'ylab harakatida o'tgan yo'lini aniqlaymiz.

$t_0 = 0 \quad S_0 = 0 \quad t_1 = \frac{5}{3}c \quad S_1 = \frac{5\pi}{3} + \frac{5\sqrt{3}}{2} \quad t = \frac{10}{3}c \quad S_2 = \frac{10\pi}{3} - \frac{5\sqrt{3}}{2}$ va $t = 5c$

$S_3 = 5\pi$

$$\Pi = |S_1 - S_0| + |S_2 - S_1| + |S_3 - S_2| = \left|\frac{5\pi}{3} + \frac{5\sqrt{3}}{2} - 0\right| + \left|\frac{10\pi}{3} - \frac{5\sqrt{3}}{2} - \frac{5\pi}{3} - \frac{5\sqrt{3}}{2}\right| + \left|5\pi - \frac{10\pi}{3} + \frac{5\sqrt{3}}{2}\right| =$$

$$= \frac{5\pi}{3} + \frac{5\sqrt{3}}{2} + \left|\frac{5\pi}{3} - 5\sqrt{3}\right| + \left|\frac{5\pi}{3} + \frac{5\sqrt{3}}{2}\right| = \frac{10\pi}{3} + 5\sqrt{3} + 5\sqrt{3} - \frac{5\pi}{3} = \frac{5\pi}{3} + 10\sqrt{3}$$

Demak, nuqtaning traektoriya bo'ylab harakatida o'tgan yo'li P quyidagiga teng bo'ladi: $\Pi = \left(\frac{5\pi}{3} + 10\sqrt{3}\right)$sm

5.

ϑ=(t-3t+2)sm/s

Yechish,

Bu tenglamadan vaqtning $0 \le t \le 5$ oralig'ida tezlik alomatining ikki marta o'zgarishini ko'ramiz. Haqiqatda ham $t_{1,2} = \frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - 2} = \frac{3 \pm 1}{2}$ t_1=1s t_2=2s

Demak, harakatning birinchi va ikkinchi sekundlarida nuqta tezligi yo'nalishini o'zgartirar ekan.

Traektoriya bo'ylab nuqtani harakat tenglamasi

$$S=\int_0^t \vartheta dt=\frac{t^3}{3}-\frac{3t^2}{2}+2t$$

t=5s yoy koordinataning qiymati.

$$S\Big|_{t=5}=\frac{1}{6}\left(2t^3-9t^2+12t\right)\Big|_{t=5}=\frac{1}{6}(250-225+60)=\frac{85}{6}sm$$

Endi nuqtaning o'tgan yo'lini aniqlaymiz:

$t_0=0 \quad S_0=0 \quad t_1=0 \quad S_1=\frac{5}{6}см$ t=2c $S_2=\frac{1}{6}(16-36+24)=\frac{4}{6}$ cm

t=5s $S_3=\frac{1}{6}\left(2t^2-9t^2+12t\right)\Big|_{t=5}=\frac{85}{6}$

$$П=|S_1-S_0|+|S_2-S_1|+|S_3-S_2|=\frac{5}{6}+\left|\frac{4}{6}-\frac{5}{6}\right|+\left|\frac{85}{6}-\frac{4}{6}\right|=\frac{5}{6}+\frac{1}{6}+\frac{81}{16}=\frac{87}{6}=\frac{29}{2}\text{sm}$$

Demak, nuqtaning traektoriya bo'ylab harakatida vaqt oralig'ida o'tgan yo'li 29/2 sm ekan.

(II. 408)

Dekart koordinatalar sistemasida berilgan nuqtaning harakat tenglamalaridan uning traektoriya bo'ylab harakat qonunini aniqlang. YOy koordinatasi S boshlang'ich holatdan hisoblansin.

1. $x=3t^2+5$, $u=4t^2+3$

Yechish: $S=\int_0^t \vartheta(t)dt\int_0^t\sqrt{\vartheta_x^2+\vartheta_y^2}\,dt=\int_0^t\sqrt{dx^2+dy^2}$ berilgan tenglamalardan dx=6tdt, du=8tdt larii hosil qilib va bularni юqoridagi tenglamaga qo'yib. S-ni topamiz. $S=2\int_0^t\sqrt{9+16tdt}=10\int_0^t tdt=St^2$ $S=St^2$

2. x=a+rcos $_\omega$t, u=rsin $_\omega$t

Yechish: dx=- r $_\omega$sin $_\omega$+ d t dy=r $_\omega$cos $_\omega$tdt

$$S=\int_0^t\sqrt{dx^2+dy^2}=r\omega\int_0^t\sqrt{\sin^2\omega t+\cos^2\omega t dt}=r\omega t$$

3. x=4acos^2 $_\omega$t y=3asin^2 $_\omega$t bu erdan harakat qonuni.

<u>Yechish.</u>

$$S=\int_0^t \sqrt{dx^2+dy^2}=a\omega\int_0^t \sqrt{4t+3t}\cos\omega t+\sin tdt=10a\int_0^t \sin\omega td(\sin\omega t)$$ S=asin^2 $_\omega$ t

4. x=e^{t}cost, y=e^{t}sint, t=e^t

Yechish. Harakat qonunini aniqlash uchun bularning differensialini aniqlaymiz: dx=e^t(sost-sint) d t, du=e^x(sint+cost)dt, dz=e^{t}dt harakat qonuni

$$S=\int_0^t e^t\sqrt{\cos^2 t+\sin^2 t-2\cos t\sin t+\sin^2 t+\cos^2 t+2\sin t\cos t+1}dt=\sqrt{3}\int_0^t e^t dt=\sqrt{3}(e^t-1)$$

Demak, bu holda nuqtaning harakat qonuni S= $\sqrt{3}(e^t-1)$ bo'lar ekan.

(II. 415)

Nuqta tinch holatdan radiusi R bo'lgan aylana bo'ylab tekis tezlanuvchan harakat qilmokda va T sekundda bir marta to'la aylanadi. Uning T vaqt oxiridagi tezlik va tezlanishi aniqlansii.

Yechish. Masala shartiga asosan nuqta aylana bo'ylab tekis tezlanuvchan harakat qiladi. SHuning uchun

$\frac{d\vartheta}{dt}=W_t=const$ bundan $\vartheta=W_t t+c_1$

S - integrallash doimiysi bo'lib, boshlang'ich shartdan aniqlanadi. t=0 da nuqta boshlang'ich holatidan qo'zg'almagan edi, shuning uchun t=0, $\vartheta=\vartheta_0=0$ va $s_1=0$ nihoyat,

$$\vartheta=W_\tau t \quad (1)$$

(1) ni yana bir marta integrallasak, $S=\frac{W_\tau t^2}{2}+c_2$ hosil bo'ladi.

t=0 da nuqta boshlang'ich holatda bo'lgani uchun t=0 da $S=S_0=0$ va $s_2=0$ bo'ladi. Nuqtaning traektoriya bo'ylab harakat qonuni esa:

$S=\frac{W_\tau t^2}{2}$ (2) bo'ladi.

Agar t=T bo'lsa, nuqta aylana yoyini to'la aylanib chiqadi. SHuning uchun (2) dan nuqtaning urinma tezlanishi

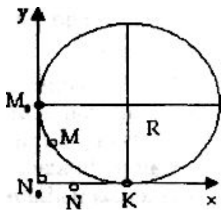
$W_\tau=\frac{2S}{T^2}$ (3) ifodani (1) ga qo'yib, $\vartheta=\frac{2S}{T}$ hosil qilamiz.

Aylama yoyiniig uzunligi. S=2 π R

$W_\tau = \frac{4\pi R}{T^2}$ $\qquad$ $\vartheta = \frac{4\pi R}{T}$ $\qquad$ (4) normal tezlanish esa $W_n = \frac{\vartheta^2}{R} = \frac{16\pi^2 R}{T^2}$

nuqtaning to'la tezlanishi

$$W = \sqrt{W_\tau^2 + W_n^2} = \sqrt{\left(\frac{4\pi R}{T^2}\right)^2 + \left(\frac{4\pi R}{T^2}\right)^2 (4\pi)^2} = \frac{4\pi R}{T^2}\sqrt{1+16\pi}$$



Demak, bu holda nuqtaning tezlik va tezlanishi

$\vartheta = \frac{4\pi R}{T}$ $\qquad$ $W_\tau = \frac{4\pi R}{T^2}\sqrt{1+16\pi^2}$ bo'lar ekan.

(II. 418)

ϑ_0 boshlang'ich tezlik bilan M nuqta $M_0(0;R)$ holatdan soat strelkasiga qarshi yo'nalishda $(x-R)^2+(u-R)^2$ aylana bo'ylab tekis tezlanuvchan harakat qilmokda.

SHu vaqtda boshlang'ich tezliksiz $N_0(0,0)$ nuktadan N nuqta Ox bo'ylab to'g'ri chiziqli tekis tezlanuvchan harakat qilmoqda.

M va N nuqtalar K(R;0) nuqtaga bir xil vaqtda yo'nalishi va kattaligi jihatdan bir xil tezlik bilan kelishi uchun M nuqtaning urinma tezlanishi va N nuqtaning tezlanishi qanday bo'lishini aniqlasin.

<u>Yechish.</u> M nuqta yoy o'zgarmas ω_t, urinma tezlanish bilan va N nukta ω_n tezlanish bilan to'g'ri chiziqli tekis tezlanuvchan harakat qiladi, ya'ni $\frac{d\vartheta_M}{dt} = \omega_t$

va $\frac{d\vartheta_N}{dt} = \omega_N$ Bularni integrallab, quyidagini hosil qilamiz. $\vartheta_M = \vartheta_0 + \omega_t t$ (1)

va $S = \vartheta_0 t + \frac{\omega_t t^2}{2} = \frac{\vartheta_M t + \vartheta_0 t}{2} = \frac{\vartheta_M^2 \vartheta_0^2}{2\omega_t}$ (2) $\qquad$ $\vartheta_N = \omega_N t$ (3)

$x = \frac{\omega_N t^2}{2} = \frac{\vartheta_N t}{2} = \frac{\omega_N^2}{2\omega_N}$ (4)

Ikkala nuqta bir vaqtda bir xil tezlik bilan K(R,0) nuqtaga kelsa. x=R va S aylana uzunligining 1/4 qismiga teng bo'ladi, ya'ni S = πR/2 (2)

chi ifodadan ϑ_tt=2S-ϑ_0t (4) dan ϑ_Nt=2R va masala shartiga asosan ϑ_mt=ϑ_Nt

SHuning uchun $t = \frac{2(S-R)}{\vartheta_0} = \frac{(\pi-2)R}{\vartheta_0}$ (4) ifodadan

$$\omega_N = \frac{2S}{t^2} = \frac{2R\vartheta_0^2}{(\pi-2)^2 R^2} = \frac{2\vartheta_0^2}{(\pi-2)^2 R}$$

(2) ifodadan $\omega_\tau = \frac{\vartheta_0^2 - \vartheta_0^2}{2S}$ lekin, ϑ, shuning uchun (4) dan va $\vartheta_m^2 = 2\omega_N R$

$$\omega_t = \frac{2R\omega_N - \vartheta_0^2}{2S} = \frac{\vartheta_0^2 \left|4R - (\pi - 2)^2 R\right|}{2S(\pi - 2)R} = \frac{(4 - \pi)\vartheta_0^2}{(\pi - 2)^2 R}$$

YA'ni N va M nuktalar bir vaqtda bir xil tezlik bilan K(R,0) nuqtaga kelishi uchun ularning mos ravishda tezlanishi va urinma tezlanishi (5) va (6) formulalar bilan ifodalanishi zarur ekan.

(II. 423)

Ikkita M_1, M_2, nuqtalar aylana bo'ylab bir tomonga mos ravishda $S_1=8+2t^2$; $S_2=t^4$ qonunlar asosida harakat qilmoqda (S-sm, t-sek.), S_1, S_2 holatlari bir nuqtadan hisoblanadi.

Agar aylana radiusi r=16 bo'lsa, ikkala nuqtalarning birinchi uchrashish vaqti va bu vaqt uchun ularning tezlik va tezlanishlarning kattaligi aniqlansin.

Yechish. Berilgai harakat tenglamalaridan ma'lumki, harakat boshlanishi paytida harakat yo'nalishi tomon birinchi nuqta ikkinchi nuqtadan 8 sm oldinda bo'lgan, chunki t=0 bo'lsa, $S_1=8$ sm, $S_2=0$. SHuning uchun harakat davomida ikkinchi nuqta birinchi nuqtani quvib etishi zarur. Nuqtalar uchrashganda $S_1=S_2$ bo'ladi. Bu shartdan $t^4-2t^2-8=0$

Bu tenglamaii echib, t=2 s nn aiiqlaymiz. Endi shu 2 s vaqt uchun nuqtalarni tezlik va tezlanishlarini aniqlaymiz.

Nuqtalar harakat tenglamalarining vaqt bo'yicha birinchi tartibli hosilasi ularning tezligiga teng: $\vartheta_1 = \frac{dS_1}{dt} = 4t \quad \vartheta_2 = 4t^3$ Bularga t=2 s qo'yib, tezliklariing son qiymatini topamiz:

ϑ_1=8 sm/s ϑ_2=32 sm/s

Tezliklardan yana vaqt bo'yicha hosila olsak, nuqtalarning urinma tezlanishlarini aniqlaymiz:

$$\omega_{1\tau} = \frac{d\vartheta_1}{dt} \qquad \omega_2 = \frac{d\vartheta_2}{dt} = 12t^2 = 48 \text{ sm/s}$$

Demak, birinchi nuqtaning urinma tezlanishi o'zgarmas va ikkinchi nuqtaniki esa vaqtning kvadratiga proporsional ravishda oshib boruvchi ekan va t=2s da mos ravishda 4 sm/s^2, 48 sm/s^2 - ga tengdir.

Endi normal tezlanishlarini aniqlaymiz:

$$\omega_{1n} = \frac{\vartheta_1^2}{r} \qquad \omega_{2n} = \frac{\vartheta_2^2}{r}$$

Nuqtalarniig to'la tezlanishlari $\omega = \sqrt{\omega_\tau^2 + \omega_n^2}$ formula yordamida aniqlanadi.

Birinchi nuqtaning tezlanishi:

$$\omega_1 = \sqrt{\omega_{1\tau}^2 + \omega_{1n}^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2} \text{ sm/s}^2$$

Ikkinchi nuqtaning tezlanishi:

$$\omega_2 = \sqrt{\omega_{2\tau}^2 + \omega_{2n}^2} = \sqrt{3^2 \cdot 16^2 + 4^2 \cdot 16^2} = 16\sqrt{3^2 + 4^2} = 80 \text{ sm/s}^2$$

(II. 428)

Parashıot ochilish paytida 40 m/s tezlik bilan vertikal bo'ylab tushayotgan parashıotistni 10 s dan keyingi tezligi 10 m/s ga teng bo'lib qoldi.

Agar parashıotist tezlanishining kattaligi uning tezligi kattaligiga proporsional bo'lsa, 10 s da uning qaysi masofaga tushganini aniqlang.

Yechish: Masala shartiga asosan $\omega \approx \vartheta$ yoki $\frac{d\vartheta}{dt} = d\omega$ bundan $\ln \vartheta = at + c_1$ lekin t=0 da $\vartheta = \vartheta_0$ va $c_1 = \ln \vartheta_0$ s_1- ni o'rniga qo'yib, $\vartheta = \vartheta_0 e^{\alpha t}$ (1) ni hosil qilamiz.

Endi ohirsidan parashıotistning o'tgan masofasini aniqlaymiz:

$$dS = \vartheta_0 e^{\alpha t} dt \quad \text{yoki} \quad S = \vartheta_0 \int e^{\alpha t} dt = \frac{\vartheta_0}{a} \int d\left(e^{at} + c_2\right) = \frac{\vartheta_0}{a}(e^{at} + c_2) \qquad (2)$$

Biz parashıot ochilgandan keyin tezligi 40 m/s dan 10 m/s gacha kamayishiga qadar parashıotistning o'tgan masofasini aniklashimiz kerak. SHuning uchun t=0 da S=0 va s_2=-1 bo'ladi. Oxirgini (2) ga qo'yib, o'tilgan masofani quyidagicha topamiz:

$S = \frac{\vartheta_0}{a}(e^{at} -_2 1)$ (3) (1) dan $at = \ln \frac{\vartheta}{\vartheta_0}$ va $e^{at} = e^{\ln \frac{\vartheta}{\vartheta_0}} = \frac{\vartheta}{\vartheta_0}$ bularni (3) ga qo'yib,

$S = \frac{\vartheta_0}{a}(\frac{\vartheta_0}{\vartheta} - 1) = \frac{(\vartheta_0 - \vartheta)t}{\lg\left(\frac{\vartheta_0}{\vartheta}\right)} \lg l$ masala shartida berilganlarni qo'ysak: $S = \frac{300 \cdot 436}{602} = 216$ m

bo'ladi. YA'ni parashıotist 10 s da 216 m pastga tushar ekan.

(I. 129)

Radiusi R=800 m bo'lgai aylana yoyi bo'ylab, poezd tekis sekinlanuvchan harakat qiladi va S=800 m yo'l bosadi. Uning boshlang'ich tezligi ϑ_0=54 km/soat va oxirgi tezligi ϑ=18 km/soat. Poezdning yoy boshidagi va oxiridagi to'la tezlanishi, shuningdek shu yoy bo'ylab, qancha vaqt harakatlanishi aniqlansin.

Yechish. Poezdning harakati tekis sekinlanuvchan ekanligi sababli uning vaqt bilan tezligining va o'tgan masofasining bog'lanishi $\vartheta = \vartheta_0 - \omega_\tau t \qquad S = \vartheta_0 t - \frac{\omega_\tau t^2}{2}$

bo'ladi. Bu erda ω_g - poezdning urinma tezlanishi bo'lib,

ω_τ>0. Oxirgi ikkala tenglamadan $\omega_\tau = \frac{\vartheta_0^2 - \vartheta^2}{2S}$ (1)

Tezliklarni SI sistemasida ifodalasak, ϑ_0=15 m/s va ϑ=5 m/s bo'ladi va ularni (1) ga qo'ysak,

$\omega_\tau = \frac{225-25}{2\cdot 800} = \frac{1}{8}\, m/s^2$ (2) bo'ladi.

Poezdning harakatlanish vaqti $t = \frac{\vartheta_0 - \vartheta}{\omega_\tau} = 10\cdot 8c = 80c$

Poezdning yoy boshidagi normal tezlanishi $\omega_{1n} = \frac{\vartheta_0^2}{R} = \frac{225}{800} = \frac{2{,}25}{8} = \frac{9}{32}$ m/s^2

yoy oxiridagisi esa quyidagicha bo'ladi:

$\omega_{2n} = \frac{\vartheta^2}{R} = \frac{25}{800} = \frac{1}{32}$ m/s^2

Poezdning yoy boshidagi to'la tezlanishi

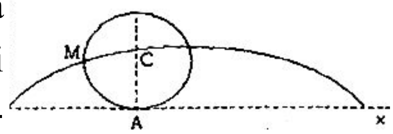
$$\omega_0 = \sqrt{\omega_\tau^2 + \omega_{1n}^2} = \sqrt{\frac{1}{8^2} + \frac{9^2}{32^2}} = \frac{1}{32}\sqrt{16+81} = \frac{\sqrt{97}}{32} = 0.308 \; m/s^2$$

yoy oxiridagisi esa $\omega = \sqrt{\omega_\tau^2 + \omega_{2n}^2} = \sqrt{\frac{1}{8^2} + \frac{1}{32^2}} = \frac{1}{32}\sqrt{16+1} = \frac{\sqrt{17}}{32} = 0.129 \; m/s^2$

bo'ladi.

(I. 12. 17)

Ox gorizontal o'q bo'ylab sirpanmasdan dumalovchi g'ildirak nuqtasi tezlanishining mikdori va yo'nalishi, hamda traektoriyasining efilik radiusi topilsin, nuqta quyidagi tenglamalarga asosan sikloida chizadi: x=20t-sin20t y=1-cos20t (1 – s va x, u - metrlar hisobida).



Yechish. Nuqtaning tezligini aniqlaymiz: ϑ_x~~x~~ '=20-20sos20t=20(1-sos20t) m/s

ϑ_y=20sin20t m/s

$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2} = 20\sqrt{(1-\cos 20t)^2 + \sin^2 20t} = 20\sqrt{2(1-\cos 20t)} = 2 \cdot 20 \sin 10t$ m/s nuqta tezlanishini aniqlaymiz: $\omega_x = \frac{d\vartheta_x}{dt} = 20^2 \sin 20t$ m/s^2 $\omega_y = \frac{d\vartheta_y}{dt} = 20^2 \cos 20t$ m/s^2

$\omega = \sqrt{\omega_\tau^2 + \omega_y^2} = 20^2$ m/s^2 nuqtaning urinma tezlanishi

$\omega_\tau = 2 \cdot 20 \cdot 10 \cos 10t = 20^2 \cos 10t$ m/s^2 normal tezlanishini aniqlaymiz:

$\omega_n = \sqrt{\omega^2 - \omega_\tau^2} = \sqrt{(20^2)^2(1-\cos^2 10t)} = 20^2 \sin 10t$ m/s^2 nuqtaning egrilik radiusi

$\rho = \frac{\vartheta^2}{\omega_\tau} = \frac{4 \cdot 20^2}{20^2} \cdot \frac{\sin^2 10t}{\sin 10t} = 4 \sin 10t$ m t=0 esa $r_0 = r_{t=o} = 0$

Tezlanishning yo'naltiruvchi kosinuslari $\cos(y, w) = \frac{\omega_y}{\omega} = \cos 20t$ y, w=20t, t=0, $y_1, w = 0^0$ $\cos(x, w) = \frac{\omega_x}{\omega} = \sin 20t$ $xw = 20t + \frac{3\pi}{2}$ t=0, $xw = \frac{3\pi}{2}$ ya'ni, t=0, boshlang'ich holatda tezlanish Ou o'qi tomon yo'nalgandir.

(I. 12. 22)

Snaryad x=300t, u=400t-5t^2 (t - sekundlar, x,u - metrlar hisobida), tenglamalarga muvofiq vertikal tekislikda harakat qiladi. Snaryadning:

1) boshlang'ich paytdagi tezlik va tezlanishi;

2) qancha uzoqqa borib tushishi va qancha balandlikka ko'tarilishi;

3) boshlang'ich paytda va eng юqori nuqtada traektoriyasining egrilik radiusi topilsin.

Yechish.

1) Snaryadning boshlang'ich paytdagi tezlik va tezlanishini aniqlaymiz. Buning uchun berilgan tenglamalardan vaqt bo'yicha birinchi va ikkinchi tartibli hosilalarini aniqlaymiz.

$\vartheta_x = \frac{dx}{dt} = 300$ m/s ϑ_y =400-10t (1)

$\omega_x = x'' = 0$ $\omega_y = \vartheta' = -10$ m/s^2 (2)

snaryadning tezligi $\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2} = \sqrt{300^2 + (400 - 10t^2)^2}$

(3)

tezlanish esa $\omega = \sqrt{\omega_x^2 + \omega_y^2} = 10$ m/s^2 (4) bo'ladi.

Snaryadning boshlang'ich tezligi t=0 da, ϑ_0=500 sm/s boshlang'ich tezlanishi

ω_0=10 m/s ga tengdir. (4) dan ma'lumki, snaryadning tezlanishi vaqtga bog'liq emas, ya'ni o'zgarmas bo'lib, 10 m/s - ga tengdir.

2) Snaryadning uchish uzoqligi va ko'tarilish balandligini aniqlaymiz:

a) ma'lumki, snaryadning uchish va tushish nuqtalarida u=0 va u=t(400-5t)=0, bu erdan snaryadning uchish nuqtasi uchun $t_1=0$ va tushish nuqtasi uchun $t_2=80$ s.

Bu vaqtni berilgan tenglamalarning birinchisiga qo'yib, snaryadning uchish uzoqligini topamiz.

$$x=L=300*80=24000m=24\ km$$

b) snaryad eng юqoriga ko'tarilganda, $\vartheta_u=0$ va (1) dan uning eng юqoriga ko'tarilish vaqti t=40 s bo'ladi.

Bu vaqtni berilgan tenglamalarning ikkinchisiga qo'ysak,

$y = H = 400\cdot 40 - 5\cdot 1600 = 8000м = 8км$ bo'ladi.

3) Traektoriyaning egrilik radiusini hisoblash uchun normal tezlanishini aniqlash kerak. Юqorida aniqlangan to'la tezlanishni bilgan holda, (3) dan vaqt bo'yicha hosila olib urinma tezlanishini aniqlaymiz.

$\omega_\tau = \frac{d\vartheta}{dt} = -\frac{10(400-10t)}{\sqrt{300^2+(400-10t)^2}}$ bulardan normal tezlanishni aniqlash mumkin:

$$\omega_n = \sqrt{\omega^2 - \omega_\tau^2} = \sqrt{10^2 - \frac{10^2(400-10t)^2}{300^2+(400-10t)^2}} = \frac{3000}{\sqrt{300^2+(400-10t)^2}} \quad (5)$$

Lekin normal tezlanish egrilik radiusi bilan quyidagi bog'lanishga ega. Bundan egrilik radiusi *r* ni aniklaymiz:

$$\rho = \frac{\vartheta^2}{\omega_n} = \frac{\{300^2+(400-10t)^2\}}{\omega_n} \quad (6)$$

Masalaning shartiga ko'ra snaryad traektoriyasining egrilik radiusini boshlang'ich (t=0 vaqtda) va eng юqori nuqtasidagi (t_3=40 s da) qiymatlarini aniqlash lozim.

t_1=0 (5) dan ma'lumki,

ω_{1n}=3000/500=6 m/s^2, ϑ_0=500l(/s

t_3=40 s esa ω_{1n}=10 m/s^2, ϑ_1=300 m/s

Bularni (6) ga qo'yib, suyidatlarga ega bo'lamiz:

t=0 da $\rho_0 = \frac{\vartheta_0^2}{\omega_{0n}} = \frac{25\cdot 10^4}{6} = 41.67\cdot 10^3 = 41.67$ km

t=40 s $\rho = \frac{\vartheta_1^2}{\omega_{1n}} = \frac{9\cdot 10^4}{10} = 9\cdot 10^3 = 9$ km

(I. 12. 26)

Nuqta harakati qutb koordinatalarida r=ae^{kt} va φ=kt tenglamalar bilan berilgan, bunda a va k berilgan doimiy miqdorlar. Nuqtaning traektoriya tenglamasi, tezligi, tezlanish va traektoriyaning egrilik radiusi uning radius vektori funksiyasi sifatida aniqlansin.

Yechish.

1) g=aeφ -logarifmik spiralь tenglamasining shakliga keladi.

2) nuqtaning tezligini aniqlaymiz. Qutbiy sistemada nuqtaning koordinatalari x=rcos φ, u=rsin φ ko'rinishda beriladi. Bulardan tezlikni tashkil etuvchilari ϑx, ϑy, larni aniqlaymiz va tezlik

$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2} = \sqrt{r^2 + r^2\varphi^2}$ bo'ladi. g=aeφ va φ=kt dan r`=ae$\varphi$ $\varphi$`=r φ` va φ'=k

Budan $\vartheta = \sqrt{r^2\varphi'^2 + r^2\varphi'^2} = \sqrt{2r^2\varphi'^2} = r\varphi'\sqrt{2} = kr\sqrt{2}$

3) nuqtaning tezlanishi $\omega = \sqrt{\omega_2^2 + \omega_\varphi^2} = \sqrt{\left(r'' + r\varphi'^2\right) + \left(2r'\varphi' + r\varphi''\right)^2}$

$\omega_2 = r'' - r\varphi'^2 = \left(ae^4\varphi'^2 + r\varphi''\right) - r\varphi'^2 = r\varphi'^2 + r\varphi'' - r\varphi'^2 = r\varphi''$ lekin φ'=k, φ''=0,

shuning uchun ω2=0 $\omega_\varphi = 2r'\varphi' + r\varphi'' = 2r'\varphi' = 2r\varphi'^2 = 2rk^2$ va

$\omega = \sqrt{\omega_r^2 + \varphi_\varphi^2} = 2k^2 r$

4) nuqta traektoriyasining egrilik radiusini aniklash uchun normal tezlanishini aniqlash zarur. Buning uchun avvalo urinma tezlanishni topamiz.

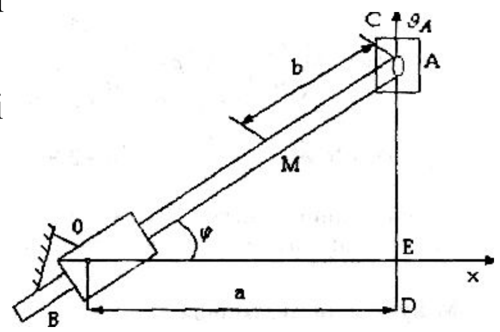
$\omega_\tau = \frac{d\vartheta}{dt} = kr'\sqrt{2} = kr\varphi'\sqrt{2} = k^2 r\sqrt{2}$ normal tezlanish

$\omega_n = \sqrt{\omega^2 - \omega_\tau^2} = \sqrt{4k^4r^2 - 2k^4r^2} = k^2 r\sqrt{2}$

traektoriyani egrilik radiusi esa $\rho = \frac{\vartheta^2}{\omega_n} = \frac{2k^2r^2}{k^2 r\sqrt{2}} = r\sqrt{2}$

(I. 12. 31)

AV sterjenning A uchi SD to'g'i chiziqli yo'naltiruvchi bo'ylab o'zgarmas ϑ_0 tezlik bilan siljiydi. AV stsrjenь doimo SD yo'naltiruvchidan a masofada aylanib, tebranadigan 0 mufta orqali o'tadi. 0 nuqtali qutb deb hisoblab, sterjenning A surilgichdan b masofada turuvchi M nuqtasining tezlik va tezlanishi r, φ qutb koordinatalarida topilsin.



Yechish. CHizmada OM masofani r bilan, SE ni ϑ_A t bilan belgilasak, u holda Pifagor teoremasiga ko'ra

$(r+b)^2 = a^2 + (\vartheta_A t)^2$ yoki $r = a\sqrt{1+\left(\frac{\vartheta_A t}{a}\right)^2} - b$ (1) bo'ladi.

$\frac{\vartheta_A t}{a} = tg\varphi$ yoki $\varphi = arctg\frac{\vartheta_A t}{a}$ (2) qutbiy sistemada tezlik $\vartheta = \sqrt{r'^2 + r^2\varphi'^2}$ (3)

va tezlanish $\omega = \sqrt{\omega_2^2 + \omega_\varphi^2}$ $\omega_2 = r'' - r\varphi'^2$ $\omega_\varphi = r\varphi'' + 2r'\varphi'$ (4)

(2) dan $\frac{\vartheta_A t}{a} = \frac{\varphi'}{\cos^2\varphi}$ yoki $\varphi' = \frac{\vartheta_A t}{a}\cos^2\varphi$ (1) ifodadan

$r = a\sqrt{1+\left(\frac{\vartheta_A t}{a}\right)^2} - b = a\sqrt{1+tg^2\varphi} - b = \frac{a}{\cos^2\varphi} - b$ bundan $r' = a\frac{\sin\varphi}{\cos\varphi}\varphi' = \vartheta_A\sin\varphi$

$\varphi'' = -\left(\frac{\vartheta_A}{a}\right)2\cos\varphi\sin\varphi\varphi'' = -2\left(\frac{\vartheta_A}{a}\right)^2\cos^3\varphi\sin\varphi$ $r'' = \vartheta_A\cos\varphi\varphi'' = a\left(\frac{\vartheta_A}{a}\right)^2\cos^3\varphi$

Endi sterjenь M nuqtasining tezligini aniqlaymiz.

$$\vartheta = \sqrt{r'^2 + r^2\varphi'^2} = \sqrt{\vartheta_A^2\sin^2\varphi + r^2\left(\frac{\vartheta_A}{a}\right)^2\cos^4\varphi} = \frac{\vartheta_A}{a}\sqrt{a\sin^2\varphi + r^2\cos^4\varphi}$$

Tezlanishning tashkil etuvchilari esa

$$\omega_\tau = r'' - r\varphi'^2 = a\left(\frac{\vartheta_A}{a}\right)^2\cos^3\varphi - r\left(\frac{\vartheta_A}{a}\right)^2\cos^4\varphi = \left(\frac{\vartheta_A}{a}\right)^2\cos^3(a - r\cos\varphi)$$

$$\omega_\varphi = r\varphi'' + 2r'\varphi' = -2r\left(\frac{\vartheta_A}{a}\right)^2\cos^3\varphi + 2\vartheta_A\sin\varphi\frac{\vartheta_A}{a}\cos^2\varphi = 2\left(\frac{\vartheta_A}{a}\right)^2\sin\varphi\cos^2(a - r\cos\varphi)$$

M nuqtaning tezlanishi

$$\omega = \sqrt{\omega_2^2 + \omega_\varphi^2} = \left(\frac{\vartheta_A}{a}\right)^2\cos^2\varphi(a - r\cos\varphi)\sqrt{4\sin^2\varphi + \cos^2\varphi} = \frac{\vartheta_A^2 b}{a^2}\cos^3\varphi\sqrt{1+3\sin^2\varphi}$$

chunki, a-rcos φ =bcos φ (chizmadan)

(II. 439)

Nuqtaning harakati tenglamalar

$x = r\cos\omega t + l\sin\frac{\omega t}{2}$ $y = -l\cos\frac{\omega t}{2} + r\sin\omega t$

yordamida berilgan. Bu erda r, l, ω-doimiy mikdorlar. Nuqta traektoriyasining egrilik radiusi $t = \frac{\pi}{\omega}$ vaqtda aniqlansin.

Yechish.

1) Berilgan tenglamalar yordamida nuqtaning tezligani aniqlaymiz

$$\vartheta_x = x' = -r\omega \sin \omega t + \frac{l\omega}{2}\cos\frac{\omega t}{2} \qquad \vartheta_y = y' = \frac{l\omega}{2}\sin\frac{\omega t}{2} + r\omega \sin \omega t \qquad (1)$$

$$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2} = \sqrt{r^2\omega^2 + \left(\frac{l\omega}{2}\right)^2 + rl\omega^2\left(\sin\frac{\omega t}{2}\cos\omega t - \sin\omega t\cos\frac{\omega t}{2}\right)} = \omega\sqrt{r^2 + \frac{l^2}{4} - rl\sin\frac{\omega t}{2}} \qquad (2)$$

2) Endi nuqtaniig tezlanishini aniqlaymiz.

Buiing uchun (1) ifodalardan yana bir marta vaqt bo'yicha hosila olamiz:

$$\omega_y = x\vartheta_y' = \frac{l\omega^2}{4}\cos\frac{\omega t}{2} - r\omega^2\sin\omega t$$

Bulardan foydalanib, to'la tezlanishni aniqlaymiz:

$$\omega = \sqrt{\omega_x^2 + \omega_y^2} = \omega^2\sqrt{r^2 + \left(\frac{l}{4}\right)^2 - \frac{rl}{2}\sin\frac{\omega t}{2}}$$

Nuqtaning egrilik radiusini hisoblash uchun uning normal tezlanishini aniqlash kerak, buning uchun avvalo urinma tezlanishii aniqlash zarur,

(2) ifodadan:

$$\omega_\tau = \frac{d\vartheta}{dt} = \omega^2\frac{-rl\cos\frac{\omega t}{2}}{4\sqrt{r^2 + \frac{l^2}{4} - rl\sin\frac{\omega t}{2}}}$$ normal tezlanish

$$\omega_n = \sqrt{\omega^2 - \omega_\tau^2} = \omega^2\sqrt{r^2 + \frac{l^2}{16} - \frac{rl}{2}\sin\frac{\omega t}{2} + \frac{\left(rl\cos\frac{\omega t}{2}\right)^2}{16\left(r^2 + \frac{l^2}{4} - rl\sin\frac{\omega t}{2}\right)}}$$

Lekin masala shartiga asosan $t = \frac{\pi}{\omega}$ da $\vartheta = \omega\sqrt{r^2 + \frac{l^2}{4} - rl}$

$$\vartheta = \omega\begin{cases} агар, \mathrm{r} \succ \frac{1}{2} бўлсa \mathrm{r} - \frac{1}{2} \\ агар, \mathrm{l} \succ 2\mathrm{r}\ \text{бўлса}\ \frac{1}{2} - \mathrm{r} \end{cases} бўлади$$

$$\omega_m = \omega^2\sqrt{\mathrm{r}^2 + \frac{l^2}{16} - \frac{rl}{2}} = \omega^2\sqrt{\left(r - \frac{l}{4}\right)^2} = \omega^2\begin{cases} агар, \mathrm{r} \succ \frac{1}{4} бўлса\left(\mathrm{r} - \frac{1}{4}\right) \\ агар, \mathrm{l} \succ 4\text{rбrбўл}\ \left(\frac{1}{4 - \mathrm{r}}\right) \end{cases} бўлади$$

Egrilik radiusi $\rho\ \frac{\vartheta^2}{\omega_m}$ dan aniqlanadi.

$$\rho \quad \frac{\vartheta^2}{\omega_m} \quad \frac{\left(r-\frac{1}{2}\right)^2}{r-\frac{l}{4}} = \frac{(2r-l)^2}{4r-l}$$

QATTIQ JISMNING QO'ZG'ALMAS O'Q ATROFIDA AYLANMA HARAKATI.

1. JISMNING BURCHAK TEZLIGI VA BURCHAK TEZLANISHI.

Qattik jismning qo'zg'almas o'q atrofida aylanma harakati qonuni quyidagicha beriladi:

$$\varphi = \varphi(t) \qquad (1)$$

Burchak tezligi ω va tezlanishi mos ravishda

$$\omega = \frac{d\omega}{dt} \quad \text{(rad/s)}; \qquad \varepsilon = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2} \quad (\text{rad/s}^2) \qquad (2)$$

Agar jismning burchak tezligi va tezlanishi alomati bir xil bo'lsa, harakat tezlanuvchan va qarama-qarshi bo'lsa, sekinlanuvchan bo'ladi.

Jismning og'ish burchagi φ uning aylanishlar soni N bilan quyidagicha bog'langan

$$\varphi = 2\pi N \qquad (3)$$

Jismning aylanma harakat burchak tezligi ω - ayl./daq. bilan ham o'lchanadi, uning radian/s bilan bog'lanishi quydagicha

$$\omega = \frac{\pi n}{30} \qquad (4)$$

Tekis aylanma harakat uchun ε=0, ω=const va harakat qonuni

$$\varphi = \varphi_0 + \omega t \qquad (5)$$ bo'ladi.

Agar harakat tekis tezlanuvchan bo'lsa, $\omega = \omega(t)$ ε= const bo'ladi va harakat qonunlari

$$\omega = \omega_0 + \varepsilon t \qquad \varphi = \omega_0 t + \frac{\varepsilon t^2}{2} \qquad (6)$$ bo'ladi.

(I. 13. 3)

AV vertikal o'q atrofida aylanuvchi markazdan qochuvchi regulyatorning mayatnigi daqiqasiga 120 marta aylanadi.

Boshlang'ich paytda aylanish burchagi $\frac{\pi}{6}$ rad. ga teng.

$t=\frac{1}{2}$ s vaqt ichida mayatnikning aylanish burchagi va ko'chish burchagi topilsin.

Yechish. Masala shartiga asosan regulyatorning mayatnigi daqiqasiga 120 marta aylanadi. YA'ni n=120 ayl/daq=2 ayl/s

Demak, tekis aylanma harakat qilar ekan. Bu holda tekstdagi (5) ifodaga asosan mayatnikning burilish burchagi

$\varphi=\varphi_0+\omega t$ (1)

Tekstdagi (4) ifodadan ω - ni aniqlab, masala shargiga asosan $\varphi_0=\frac{\pi}{6}$ rad. ekanligini nazarga olib va natijani (1) ga qo'yib, mayatnikning $t=\frac{1}{2}$ da og'ish burchagini aniqlaymiz.

$$\varphi=\frac{\pi}{6}+4\pi\frac{1}{2}=\frac{13\pi}{6} \quad rad$$

Mayatnikning ko'chish burchagi esa

$\Delta\varphi=\varphi-\varphi_0=2\pi$ rad bo'ladi.

(I. 13. 7)

Qo'zg'almas o'qli g'ildirak 2π rad/s ga teng bo'lgan boshlang'ich burchak tezlik olgan: g'ildirak 10 marta aylangandan keyin podshipniklardagi ishqalanish tufayli to'xtadi. G'ildirakning burchak tezlanishi doimiy deb hisoblab, uning mikdori ε aniqlansin.

Yechish: G'ildirakning harakati podshipniklar ishqalanish tufayli tekis sekinlanuvchan bo'ladi. SHuning uchun echimini tekstdagi (6) tenglamalar asosida aniqlash kerak.

$\omega=\omega_0-\varepsilon t \qquad \varphi=\omega_0^t-\frac{\varepsilon t^2}{2}$ (1)

G'ildirak to'xtaganida ω_0 - εt=0, bu erdan t=ω_0 / t— bo'ladi.

Buni (1) ifodani ikkinchisiga qo'yib,

$$\varphi=\frac{\omega_0^2}{\varepsilon}-\frac{\omega_0^2}{2\varepsilon}=\frac{\omega_0^2}{2\varepsilon}$$

Oxirgi ifodadan g'ildirakning aylanma harakat tezlanishi

$\varepsilon=\frac{\omega_0^2}{2\varphi} \qquad \varphi=2\pi N \qquad \varepsilon=\frac{\omega_0^2}{2\cdot 2\pi N}=\frac{4\pi^2}{4\pi\cdot 10}=0.1\pi рад/с^2$ bo'ladi.

(I. 13. 9)

Jism qo'zg'almas o'q atorofida tebranma harakat qiladi, bunda aylanish burchagi $\varphi = 20^0 \sin\psi$ tenglama bilan berilgan. ψ burchak esa graduslarda ψ =(2t)° (t -s hisobida) munosabat bilan ifodalangan. Jismning t=0 paytdagi burchak tezligi, aylanish yo'nalishi o'zgaradigan eng yaqin t_1 va t_2, vaqtlar, hamda tebranish davri T aniqlansin.

Yechish: Bu masalani yechish uchun avvalo graduslardan radianlarga o'tamiz.

Ma'lumki, $1° = \frac{\pi}{180}$ rad.

Bu holda 20°= $\frac{\pi}{9}$ rad, ψ =(2t)° = $\frac{\pi}{90}t$ va mayatnik $\varphi = \frac{\pi}{9}\sin\frac{\pi}{90}t$ (1) ga og'adi

(1) ifodadan vaqt bo'yicha bir marta hosila olib, mayatnikning burchak tezligini topamiz.

$$\omega = \varphi' = \frac{\pi}{9}\cdot\frac{\pi}{90}\cos\frac{\pi}{90}t = \frac{\pi^2}{810}\cos\frac{\pi}{90}t \qquad (2)$$

Oxirgi tenglama mayatnikning burchak tezligini istalgai vaqtda aniqlashga imkon beradi. Lekin masalaning shartiga ko'ra biz burchak tezlikni t=0 vaqtdagi qiymatini aniqlashimiz kerak.

$$\omega_0 = \omega\Big|_{t=0} = \frac{\pi^2}{810}\ \text{rad / s}$$

Endi aylanish yo'nalishi o'zgaradigan eng yaqin t_1 va t_2 vaqtlarni topamiz. Buning uchun (1) ifodaning ekstremal qiymatlarinini topib, ulardan mos vaqt qiymatlarini aniqlaymiz.

Bu holda (2) nolga teng bo'lish kerak. YA'ni ω=0 bo'lsa, cos= $\frac{\pi}{90}t = 0$ va vaqtni bu shartni qanoatlantiruvchi eng yaqin qiymatlari

$\frac{\pi}{90}t = \frac{\pi}{2}$ $\frac{3\pi}{2}$ bu erdan t_1=45 s, t_2=135 s.

Endi tebranish davrini aniqlaymiz. (1) ifodadan ko'rinib turibdiki, tenglama garmonik tebranma harakag tenglamasidir va uning to'la tebranishda og'ish burchagi, 2 π - ga teng

$\frac{\pi}{90}T = 2\pi$, T=180 s.

(II. 465)

Turbina rotori 3600 ayl/daq. burchak tezligiga ega edi. U 12 s davomida tekis tezlanuvchan aylanib burchak tezligini ikki marta orttirdi. SHu vaqt davomida rotorning necha marta aylanganini aniqlang.

Yechish. Rotorning harakatini tekis tezlanuvchai ekanligini nazarga olib, masalani yechish uchun tekstdagi (6) tenglamalardan foydalanish qulaydir.

$$\omega = \omega_0 + \varepsilon t \qquad \varphi = \omega_0 t + \frac{\varepsilon t^2}{2}$$

Bu tenglamalarning birinchisidan Et= ω - ω_0 ni topib, ikkinchisiga qo'yilsa,

$$\varphi = \frac{\omega + \omega_0}{2} t$$ (1) tenglama hosil bo'ladi.

Bu erda ω =4 πn, oxirgi ω, ω_0 qiymatlarini va t-ni (1) ga qo'yib, quyidagini hosil qilamiz:

$$\varphi = 3\pi n_1 t = 3\pi n_1 \cdot 12 = 36\pi n_1 рад$$

Bu erda φ 2πN va n_1,=3600 ayl/daq=60 ayl/s ekanligini nazarga olsak,

(N- rotorni 12 s da aylanishlar soni) rotorning aylanishlar soni $2\pi N = 36\pi \cdot 60$

yoki N=1080 aylanish bo'ladi.

(II. 468)

Dvigatelning vali $\varepsilon = \pi$ *rad/s²* burchak tezlanish bilan tekis tezlanuvchan aylanib t_2-t_1, vaqt oralig'ida 100 marta aylandi t_1, t_2 vaqtlarda valning aylanishlar sonini ayl/daq. lar bilan aniqlansin.

Yechish. Boshlang'ich holda val harakatsiz bo'lgan, shuning uchun uning boshlang'ich burchak tezligi ω_0 = 0 va istalgan vaqtda burchak tezlikning vaqtga bog'lanishi teksdagi (6) - ning birinchisiga asosan aniqlanadi:

$$\omega = \varepsilon t \qquad \omega_1 = \varepsilon t_1 \qquad \omega_2 = \varepsilon t_2 \qquad (1)$$

Bu vaqtlarda aylanish burchaklari esa (6-ni ikkinchisi) $\varphi_1 = \frac{\varepsilon t_1^2}{2}$ $\varphi_2 = \frac{\varepsilon t_2^2}{2}$

t_2-t_1, vaqt oralig'ida valning og'ish burchagi

$$\Delta\varphi = \varphi_2 - \varphi_1 = \frac{\varepsilon(t_2^2 - t_1^2)}{2} = \frac{\varepsilon(t_2 - t_1)(t_2 - t_1)}{2}$$ bu erdan $t_2 + t_1 = \frac{2\Delta\varphi}{\varepsilon(t_2 - t_1)} = \frac{2 \cdot 2\pi N}{\varepsilon(t_2 - t_1)}$

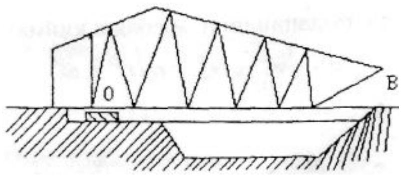
Masala shartiga asosan t_2-t_1=10 s vaqt oralig'ida val 100 marta aylangan, shuning uchun

$$t_2 - t_1 = \frac{4\pi \cdot 100}{\pi \cdot 10} = 40c$$

Endi t_2-t_1=10 s, t_2+t_1=40 s lardan t_2 t_1, larni topamiz: t_1=15 s, t_2=25 s Teksdagi (4) ifodaga asosan (1) dan foydalanib, t_1, t_2 vaqtlar uchun valning izlanayotgan aylanishlar sonini aniqlash mumkin.

$$n_1 = \frac{30\omega_1}{\pi} = \frac{30}{\pi}\varepsilon t_1 = 30\cdot 15 = 450$$

$$n_1 = \frac{30\omega_2}{\pi} = \frac{30}{\pi}\varepsilon t_2 = 30\cdot 25 = 750$$



(P. 471)

Yig'iluvchi ko'prik gorizontal tekislikda 90° ga aylana oladi. Ko'prik aylanishini 0° dan 30° gacha tekis tezlanuvchan, 30° dan 60° gacha tekis va 60° dan 90° gacha tekis sekinlanuvchan hisoblab, uni to'la aylanish vaqtini aniqlang. V nuqtani eng katta tezligi ϑ=1 m/s va OV=50 m deb olinsin.

Yechish: Ko'prikning harakati boshlanish paytida t=0, ω_0=0 bo'ladi. Ko'prikning har bir aylanish qismida uning aylanish burchaklari mos ravishda φ_1, φ_2, φ_3, bilan belgilaymiz. Masala shartiga asosan $\varphi_1 = \varphi_2 = \varphi_3$ dir, chunki har bir qismda 30^0 ga buriladi.

$$\varphi_1 = \frac{\varepsilon_1 t_1^2}{2} \quad \varphi_2 = \omega t_2 \quad \text{va} \quad \varphi_3 = \omega t_3 + \frac{\varepsilon_3 t_3^2}{2} \quad (1)$$

Lekin $\varepsilon_1 = \frac{\omega - \omega_0}{t_1} = \frac{\omega}{t_1}$ $\qquad \varepsilon_3 = \frac{\omega - \omega_0}{t_3} = \frac{\omega}{t_3}$

Bularni hisobga olib, burchak tezlanishining ε (1) tenglamalarga qo'yib, hosil qilamiz:

$$\varphi_1 = \frac{\omega t_1}{2} \qquad \kappa_2 = \omega t_2 \qquad \varphi_3 = \frac{\omega t_3}{2} \quad (2)$$

Bulardan $t_1 = \frac{2\varphi}{\omega}$ $\qquad t_2 = \frac{\varphi}{\omega}$ $\qquad t_3 = \frac{2\varphi}{\omega}$

chunki φ_1=φ_2=φ_3=30°=$\frac{\pi}{6}$ rad

Ko'prikning 90° ga to'la aylanish vaqti. T=t_1+ t_2+ t_3=$\frac{\varphi}{\omega}\cdot 5 = \frac{5\pi}{6\omega}$

Lekin ϑ=ωOV $\qquad \omega = \frac{\vartheta}{OB}$, shuning uchun

T= $\frac{5\pi}{6\vartheta}OB = \frac{5\pi}{3\cdot 1}25 = \frac{125\pi}{3}c = \frac{125\cdot 1.05}{60} = 2$ *daqiqa* 10.8 *s*

2. AYLANUVCHI JISM NUQTALARINING TEZLIGI VA TEZLANISHI.

Qo’zg’almas o’q atrofida aylanma harakat qilayotgan jismning aylanish o’qidan r masofada joylashgan nuqtasining tezligi

$$\vartheta=\omega r \qquad (1)$$

formula yordamida aniqlanadi. Bu erda ϑ - nuqtaning absolıot tezliga, ω - burchak tezligi, r - aylanish o’qidan nuqtagacha bo’lgan masofa.

Jismning ixtiyoriy nuqtasining to’la tezlanishi uning urinma va normal tezlanishlarning geometrik yig’indisiga teng

$$\omega= \omega e+ \omega n \qquad (2)$$

Tezlanishning tashkil etuvchilarining absolıot qiymati

$$\omega = \varepsilon r \qquad (3)$$

To’la tezlanishning absolıot qiymati

$$\omega=\sqrt{\omega_t^2+\omega_n^2}=r\sqrt{\varepsilon^2+\omega^4} \qquad (4)$$

(I. 13. 13)

Radiusi 0,5 m bo’lgan maxovik o’z o’qi atrofida bir tekis aylanadi: g’ildirak gardishida yotgan nuqtalarning tezligi 2 m/s ga teng. G’ildirak bir daqiqada necha marta aylanadi?

Echsh: Masala shartiga ko’ra maxovikni aylanish o’qidan gardishidagi nuqtagacha bo’lgan masofa yoki uning radiusi r=0,5m va uning tezligi ϑ=2m/s dir. Tekstiagi (1) tenglama asosan maxovikning burchak tezligi

$$\omega=\frac{\vartheta}{r}=2\div 0.5=4c^{-1}$$

Endi maxovikning 1 daqiqada necha marta aylanishini aniqlaymiz. Harakat tekis bo’lgani uchun $\varphi=\omega t=2\pi N$ bu erdan $N=\frac{\omega t}{2\pi}=\frac{4\cdot 60}{2\pi}=\frac{120}{\pi}=38.2$ marta

(I. 13. 15)

Radiusi 2 m bo’lgan maxovik tinch holatdan boshlab, tekis tezlanish bilan aylanadi: gardishda yotuvchi nuqtalar t=10 s dan keyin ϑ=100 m/s chiziqli tezlikka ega bo’ladi.

G’ildirak gardishidagi nuqtaning t=15 s bo’lgan vaqtdagi tezligi, urinma va normal tezlanishlari topilsin.

Yechish: Maxovikning harakati boshlang'ich burchak tezligisiz tekis tezlanuvchan bo'lgani uchun uning harakat qonuni $\varphi = \frac{\varepsilon t^2}{2}$ va burchak tezligi $\omega = \varepsilon$ t bo'ladi.

Maxovik gardishidagi nuqtalarning chiziqli tezligi

$\vartheta = \omega$ r= ε tr yoki $\varepsilon = \vartheta$ /rt

Masalaning shartida berilgan ϑ =100 m/s, r=2 m va t=10s ni oxirgiga qo'yib, maxovikning burchak tezlanishini aniqlaymiz *E=5s* $^{-2}$. Endi maxovikning t= 15s uchun burchak tezligini aniqlaymiz

$\omega = \varepsilon$ t=15*5=75s^{-1} t=15s vaqt uchun gardishdagi nuqtalar chiziqli tezligi

ϑ ω r=75*2=150 *m/s* urinma tezlanishi

$\omega_t = \varepsilon$ r=5*2=10 *m/s*^{2} normal tezlanish

$$\omega_n = \frac{\vartheta^2}{r} = \omega^2 r = 75^2 \cdot 2 = 11250$$ *m/s*^{2} bo'ladi

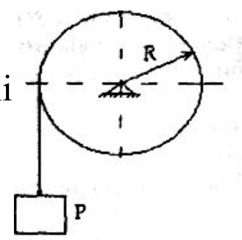
(I. 13. 18)

Radiusi r=10 sm bo'lgan A val unga ipda osilgan R tosh bilan aylantiriladi. Toshning harakati x=100t^2 tenglama bilan ifodalanadi, bunda x-sm bilan, t vaqt-s.lar bilan hisoblanadi.

t vaqtda valning burchak tezligi ω va burchak tezlaiishi ε. SHuningdek, val sirtidagi nuqtalarning to'la tezlanishi w aniqlansin.

Yechish: Masala shartida berilgan tenglama x=100t^2 dan vaqt bo'yicha bir marta hosila olib, valning A nuqtasini t vaqt uchun chiziqli tezlshini aniqlaymiz:

ϑ =x'=200 t sm/s



Bundan tekstdagi (1) ifodadan valning burchak tezligini aniqlaymiz:

$$\omega = \frac{\vartheta}{r} = \frac{200t}{10} = 20tc^{-1}$$

10 Valning burchak tezlanishi

$$\varepsilon = \frac{d\omega}{dt} = 20c^{-2}$$

Oxirgilarni nazarga olib, valning to'la tezlanishini tekstdagi (4) formulaga asosan topamiz:

$$\omega = r\sqrt{\varepsilon^2 + \omega^4} = 10\sqrt{20^2 + (20t)^4} = 200\sqrt{1 + 400t^2} \quad sm/s$$

(I. 14. 2)

I valning aylanishini sekinlashtiradigan va aylanma harakatni II valga uzatadigan tezlik reduktori to'rtta shesternyadan iborat; shesternyalar tishlarining soni: $z_1=10$; $z_2=60$; $z_3=12$; $z_4=70$;. Mexanizmning uzatish soni topilsin.

Yechish. Mexanizmning uzatish soni deb, birinchi valning burchak tezligini ω_1 ikkinchi val burchak tezligiga ω_2 II nisbatining kattaligiga aytiladi. YA'ni $i = \frac{\omega_I}{\omega_{II}}$

Tezlik reduktorining mexanizmida tishlarining soni z_1, bo'lgan shesternya tishlarining soni z_2, bo'lgan shesternya bilan tishlari yordamida o'zaro ulangan va I val harakatga kelishi bilan tishlari z_1, z_2 bo'lgan shesternyalar bir vaqtda harakatga keladi. Bu shesternyalarning teginib turgan nuqtalarining chiziqli tezligi o'zaro teng bo'ladi.

Agar shssternyalar radiuslari mos ravishda z_1, z_2 bo'lsa, chiziqli tezlikni quyidagicha aniqlanadi:

$\vartheta_1 = \omega_1 r_1 = \omega_2 r_2$

Ma'lumki, $2\pi r_1$, $2\pi r_2$ ikkala shesternyalarning aylanasining uzunligidir. Agar shesternyalarda tishlar orasidagi masofa yoki shesternyalarning qadami h bo'lsa, unda aylana uzunligi I=hz bo'ladi. SHuning uchun (2) dan

$$\omega_2 = \frac{r_1}{r_2}\omega_1 = \frac{l_1}{l_2}\omega_1 = \frac{z_1}{z_2}\omega_1 \qquad (3)$$

CHizmadan ko'rinib turibdiki, ikkinchi shesternya uchinchi shesternya bilan 00 umumiy o'q orqali bog'langan, shuning uchun

$$\omega_2 = \omega_3 = \omega_i \qquad (4)$$

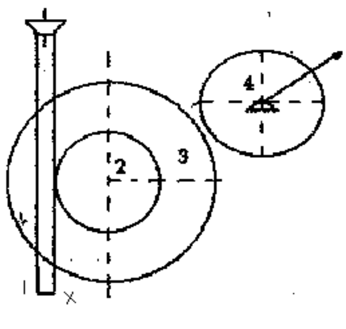
uchinchi shesternya esa to'rtinchi bilan bog'langan va (2) ga asosan

$$\vartheta_2 = \omega_3 r_3 = \omega_p r_4 \qquad (5)$$

Bundan

$$\omega_3 = \frac{r_4}{r_3}\omega_{II} = \frac{z_4}{z_3}\omega_{II} \qquad (6)$$

(4) asosan (3) va (6) dan mexanizmni uzatish soni $i = \frac{\omega_I}{\omega_{II}} = \frac{z_2 z_4}{z_1 z_3} = \frac{60 \cdot 70}{10 \cdot 12} = 35$ YA'ni, mexanizmning uzatish soni 35 ga teng ekan.



(I. 14. 4)

Strelkali indikator mexanizmida harakat o’lchov shtifning 1 reykasidan 2 shesternyaga uzatiladi; 2 shesternyaning o’qiga 3 tishli g’ildirak o’rnatilgan, 3 g’ildirak esa strelka biriktirilgan 4 shesternya bilan tishlashadi. Agar shtiftning harakati x=a sinkt tenglama bilan berilgan bo’lsa, va tishli g’ildiraklarning radiuslari tegishlicha r_2, r_3 va r_4 bo’lsa, strelkaning burchak tezligi aniqlansin.

YECHISH: 1) SHtiftning berilgan x=a sinkt harakat tenglamasidan foydalanib, uniig tezligini aniqlaymiz:

$$\vartheta = x' = ka\cos kt$$

2) 2 shesternyani shtiftga teginib turgan nuqtasinnng tezligi shtift tezligiga tengishligidan $\vartheta = \omega r_2$ (2)

shesternya burchak tezligini aniqlaymiz. (1) va (2) dan $\omega = \frac{\vartheta}{r^2} = \frac{ka}{r^2}\cos kt$

3) 3 tishli g’ildirak shesternya bilan umumiy o’q atrofida aylanadi, shu sababdan uning gardishidagi nuqtalarning tezligi

$$\vartheta = \omega r_3 = \frac{kr_3}{r_2}a\cos kt$$

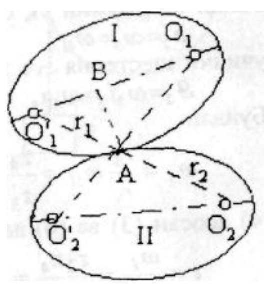
4) 3 tishli g’ildirak gardishidagi nuqtalar 4 tishli g’ildirak bilan tishlashadi, shuning uchun ularni tegingan nuqtalarining tezligi o’zaro teng bo’ladi. $\vartheta_3 = \vartheta_2 = \omega_1 r_4$ (4)

Bu erdan 4 shesternyaning burchak tezligi (3) va (4) dan

$$\omega_1 = \frac{\vartheta_2}{r_4} = \left(\frac{kr_3}{r_2 r_4}\right)a\cos kt \quad (5)$$

Strelka 4 shestsrnyaga biriktirilgan bo’lib, (5) bir vaqtda strelkaning ham izlanayotgan burchak tezligidir.

(I. 14. 7)



YArim o’qlari a va b bo’lgai bir juft elliptik tishli g’ildiraklarning aylanma harakatini uzatish qonuni chiqarilsin. I g’ildirakning burchak tezligi ω_1=*sopst*. O’qlar orasidagi masofa O_1O_2=2a; φ-aylanish o’qlarini tutashtiruvchi to’g’ri chiziq bilan I elliptik g’ildirakning katta o’qi orasidagi burchak. O’qlar ellipslarni fokuslari orqali o’tadi, Yechish: Ellipsning xossasidan ma'lumki, $r_1+r_2=2a$ va $O_1O'_1=2s$. r_1, va r_2 radiuslar oxiridan $O_1O'_1$ o'qga AV perpendikulyar tushiramiz, va AV - ni AVO_1 va AVO'_1 uchburchaklardan aniqlaymiz.

$AB^2=r_1^2-(r_1\cos\varphi)^2=r_2^2-(2c-r_1\cos\varphi)^2$ yoki $(r_1-r_2)(r_1+r_2)=-4c^2+4cr_1\cos\varphi$

Ma'lumki, $r_1+r_2=2a, r_2=2a-r_1$ $\qquad 2a(2r_1-2a)=-4c^2+4cr_1\cos\varphi$

Oxirgini 4 ga bo'lib, r_1 ni aniqlaymiz:

$$r_1=\frac{a^2-c^2}{a-c\cos\varphi} \qquad (1) \qquad r_2=2a-r_1=\frac{2a(a-c\cos\varphi)-a^2+c^2}{a-c\cos\varphi}=\frac{a^2+c^2-2ac\cos\varphi}{a-c\cos\varphi}$$

(2) ikkala g'ildiraklarning o'zaro teginayotgan nuqtalarining chiziqli tezliklari teng bo'ladi:

$\vartheta_1=\vartheta_2$ yoki $\omega_1 r_1=\omega_2 r_2$ $\qquad$ (3)

Bu erda r_1 va r_2 (1) va (2) yordamida aniqlangan, masala shartida berilgan 1 g'ildirakni burchak tezligidir. SHunday qilib, ikinchi g'ildirakning burchak tezligi (3) va (1), (2) ga asosan

$$\omega_3=\frac{r_1}{r_2}\omega_1=\frac{a^2-c^2}{a^2-2ac\cos\varphi+c^2}\omega_1 \qquad (4)$$

Oxirgidan ma'lumki, 2 g'ildirakning eng katta burchak tezligi *(φ=0)*

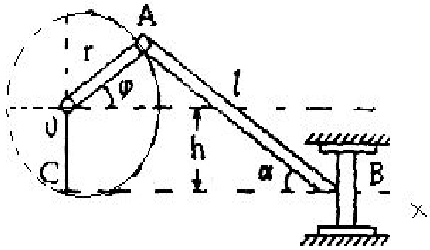
$$\omega_{2\max}=\frac{a^2-c^2}{a^2-2ac+c^2}=\frac{(a-c)(a+c)}{(a-c)^2}=\frac{a+c}{a-c}$$ eng kichik burchak tezligi esa ($\varphi=\pi$)

$$\omega_{2mix}=\frac{a^2-c^2}{a^2+2ac+c^2}=\frac{(a-c)(a+c)}{(a+c)^2}=\frac{a-c}{a+c}$$

(4) ifoda masala shartiga asosan aniqlanishi lozim bo'lgan 2 g'ildirakning aylanma harakatiniig qonunidir.

(I. 14. 14)

Markazlashtirilmagan krivoship surilgichli mexanizm porshenining harakat tenglamasi yozilsin; krivoshipning aylaiish o'qidan yo'naltiruvchi lineykagacha bo'lgan masofa h ga, krivoship uzunligi r ga shatun uzunligi *l* ga teng; s_x o'q surilgich yo'naltiruvchisi bo'ylab yo'nalgan. Masofalar surilgichning chetki



o’ng holatidan boshlab hisoblansin:

$\frac{I}{r}=\lambda$ $\qquad \frac{h}{r}=k$ $\qquad \varphi=\omega_0 t$

Yechish: Surilgich eng chetki o’ng holatda bo’lganida OV_oS Δ dan

$$CB_0=\sqrt{(r+l)^2-h^2}=r\sqrt{(1+\lambda)^2-k^2} \qquad (1)$$

SV esa OA=r va AV=l ning x o’qiga proeksiyasiga teng, ya’ni

SV=rcos φ +$l$cos α (2)

Lekin chizmadan ko’rinib turibdiki,

AD=rsin φ +h=$l$sin α yoki (rsin φ +h)^2=l^2(l-cos^2 α)

bu erdan $l\cos\alpha=\sqrt{l^2-(r\sin\varphi+h)^2}=r\sqrt{\lambda^2-(\sin\varphi+k)^2}$

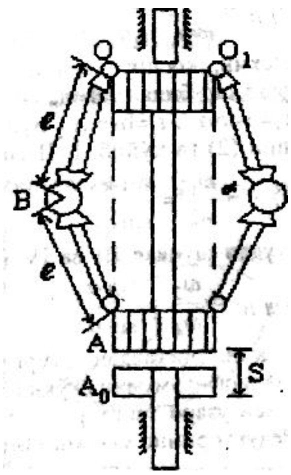
Oxirgini (2) ga qo’yib,

SV=r $\cos\varphi+r\sqrt{\lambda^2-(k+\sin\varphi)^2}=r\left\{\cos\varphi+\sqrt{\lambda^2-(k+\sin\varphi)^2}\right\}$ (3)

Masala shartiga asosan surilgichning holati uning eng chetdagi o’ng holatiga nisbatan aniqlanishi kerak. SHuning uchun (1) va (3) ga asosan

$$x=CB_0-CB=r\left\{\sqrt{(\lambda+l)^2-k}-\sqrt{\lambda^2-(k+\sin\varphi)^2}-\cos\varphi\right\}$$

Bu hosil qilingan ifoda esa masala shartiga ko’ra porshenning aniqlanishi lozim bo’lgan harakat tenglamasidir.



(II. 481)

Agar regulyatorning har bir shari markazining tezlanishi 288 m/s^2 va o’zining burchak tezligi o’zgarmas 60 rad/s ga teng bo’lsa, uning A muftasi qaysi S=AA_0 masofaga siljiydi? A_0 muftani α =0 dagi holati. Sterjenlar uzunligi l=10 sm. OO_1 masofasi nazarga olinmasin.

Yechish: Regulyator o’zgarmas ω =const burchak tezligi bilan aylanadi. SHu sababdan uning urinma tezlanishi nolga teng va to’la tezlanishi normal tezlanishiga tengdir:

$$w=w_n=\frac{\vartheta^2}{r}$$

Bu erda r - shar markazidan sterjengacha bo’lgan masofadir. Tekstdagi (1) formuladan foydalanib,

$w_n=\frac{\omega^2 r^2}{r}=\omega^2 r$ va $r=\frac{w_n}{\omega^2}=\frac{288}{36}sm=8sm$

Endi regulyatorni ω burchak tezligi bilan aylanib turgan paytida uzunligini topamiz; regulyator yarminiig uzunligi

$$l' = \sqrt{l^2 - r^{2=\sqrt{36}}} = \sqrt{100-64} = \sqrt{36} = 6sm$$

Ish paytidagi to'la uzunligi l_I=2l'=12sm

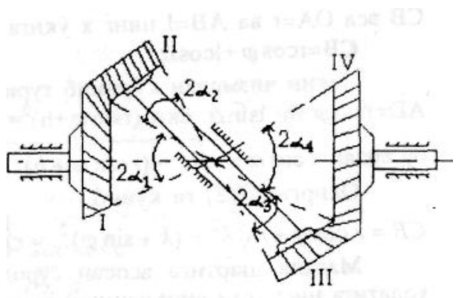
Muftaning siljishi S=AA_0. Regulyatorning boshlang'ich holatidagi uzunligi *2l* va ishlab turgan xolatidagi uzuiligi l_I=2l' ning ayirmasiga teng. YA'ni

S=2l-2l'=2(l-l')=(10-6)=8 sm

Demak, regulyatorning A muftasi boshlang'ich holatiga nisbatan 8 sm ga юqoriga siljir ekan.

(II. 487)

Burchaklari mos ravishda $2\alpha_1$, $2\alpha_2$, $2\alpha_3$, $2\alpha_4$ bo'lgan konik g'ildiraklardan tashkil topgan reduktorning uzatishlar soni aniqlansin.



<u>Yechish.</u> G'ildiraklarning burchak tezligini mos ravishda

ω_1 ω_2 ω_3 va ω_4 bilan belgalaymiz. Unda I va II g'ildiraklarning teginib turgan nuqtalarining chiziqli tezliklari teng bo'ladi:

$\omega_1 r_1 = \omega_2 r_2$ (1) r_1 r_2 I, II g'ildiraklarning asosini radiuslaridir.

I va II g'ildiraklarning uzatish soni deю ω_1/ω_2 nisbatga aytiladi. (1) ifodadan

$$i_{I,II} = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} \qquad (2)$$

Lekin, kosinuslarning asoslarining radiuslari balandligi va α_1 α_2 burchaklar bilan ifodalasak,

$r_1 h \sin\alpha_1, r_2 = h\sin\alpha_2$ bo'ladi. Oxirgini (2) ga qo'yib, I, II g'ildiraklarning uzatish sonini aniqlaymiz:

$$i_{I,II} = \frac{\omega_1}{\omega_2} = \frac{\sin\alpha_2}{\sin\alpha_1} \qquad (3)$$

Xyddi shunday III va IV g'ildiraklarning uzatish soni topiladi

$$i_{III,IV} = \frac{\omega_3}{\omega_4} = \frac{\sin\alpha_4}{\sin\alpha_3}$$

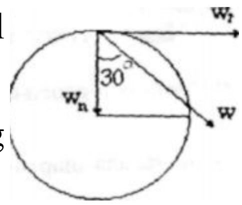
Lekin chizmadan ko'rinib turibdiki, reduktorning II va III g'ildiraklari umumiy o'qqa biriktirilgan bo'lib, ularning burchak tezliklari o'zaro tengdir: $\omega_2 = \omega_3$

Reductornipg uzatish soni $i_{I,IV}=\frac{\omega_1}{\omega_4}=i_{I,II}i_{III,IV}\frac{\sin\alpha_2\sin\alpha_4}{\sin\alpha_1\sin\alpha_3}$ chunki,

$i_{I,IV}=\frac{\omega_1}{\omega_4}=\frac{\omega_1}{\omega_2}\cdot\frac{\omega_2}{\omega_3}\cdot\frac{\omega_3}{\omega_4}=i_{I,II},i_{III,IV}$ ($\omega_3=\omega_2$)

(II. 492)

Turbina tinch holatdan tekis tezlanuvchan shunday harakat qiladiki, aylanish o'qidan 0,4 m masofada turgan M nuqtaning tezlanishi ω, kattaligi jihatidan 40 m/s bo'lib, yo'nalishi radius bilan 30° burchak hosil kiladi. Rotorning aylanma harakat tenglamasi hamda nuqta tezligining kattaligi va normal tezlanishi t=5 s payt uchun aniklansin.



Yechish: 1) Rotorning aylanish qonunini aniqlaymiz, buning uchun

avvalo urinma tezlanishni topish zarur.

$w_\tau=w\cos60^0=20m/s^2$

Lekin, boshqa tomondan $w_\tau=Er$ va burchak tezlanishi $E=\frac{w_\tau}{r}=\frac{20}{0.4}c^{-2}=50c^{-2}$

$E=\frac{d\omega}{dt}$ SHuning uchun ω=50t va φ=25t^2 (1)

Bu esa (1) rotorning aylanish qonunidir.

2) Nuqta tezligani aniqlaymiz:

$\vartheta=\omega$r=50t=50*5*0,4=100 *m!s*

3) Nuqtaning normal tezlanishini aniqlaymiz;

$w_n=\frac{\vartheta^2}{r}=\omega^2r=2500\cdot25\cdot0.4=25000$ m/s^2

QATTIQ JISMNING TEKIS PARALLEL HARAKATI.

Harakatsiz sistemaga nisbatan qattiq jismning tekis harakati Qonuni quyidagi tenglamalar yordamida beriladi:

x=x(t), u=u(t), $\varphi=\varphi$(t) (1)

Bu erda x, u nuqtaning qutb deb ataluvchi koordinatalari va φ xarakatsiz

va harakatli koordinata sistemalari mos o'qlari orasidagi burchakdir.

Jismning harakatida uning ixtiyoriy ikki A va O nuqtalarining tezliklari orasida quyidagi bog'lanish mavjud.

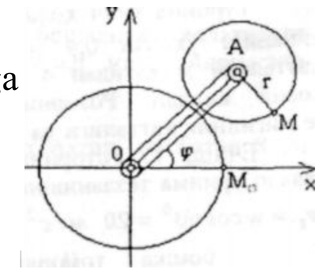
$\overline{\vartheta}_A = \overline{\vartheta}_0 + \overline{\vartheta}_{A0}$ (2) bundan ϑ A A nuqtaning harakatsiz sistemaga nisbatan tezligi, ϑ_0 O nuqtani harakatsiz sistemaga nisbatan tezligi ϑ A0 esa A nuqtani O qutbga nisbatan aylanma harakat tezligadir:

$\vartheta_{Ao} = \omega$ AO (3) ω - jismni aylaima harakat burchak tezligi.

(I. 15. 3)

R radiusli qo'zg'almas tishli g'ildirak bo'ylab dumalovchi r radiusli tishli g'ildirak OA krivoship bilan harakatga keltiriladi: krivoship qo'zg'almas tishli g'ildirakning 0 o'qi atrofida E burchak tezlanish bilan tekis tezlanuvchan aylanma harakat qiladi.

Agar t=0 da krivoshipning burchak tezligi ω_0=0 va boshlang'ich aylanish burchagi $\varphi_0=0$ bo'lsa, qo'zg'aluvchan tishli g'ildirakning harakat tenglamalari tuzilsin: uning A markazi qutb deb olinsin.



Yechish. Tekis tezlanuvchan aylaima harakat tenglamasiga asosan

$$\varphi = \varphi_0 + \omega_0 t + \frac{E_0 t^2}{2}$$

lekin masala shartiga ko'ra ω_0=0, $\varphi_0=0$ SHuning uchun $\varphi = \frac{E_0 t^2}{2}$

1) A qutb nuqtani koordinatalarini aniqlaymiz. CHizmadan ko'rinadiki,

$$x_A = (R + r)\cos\varphi = (R + r)\cos\frac{E_0 t^2}{2} \qquad y_A = (R + r)\sin\frac{E_0 t^2}{2} \quad (1)$$

Bular A nuqtaning dekart koordinat sistemasida harakat tenglamalaridir

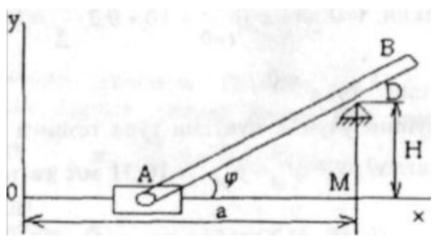
2) Endi tashqi radiusi r bo'lgan tishli g'ildirakning aylanma harakat qonunini topamiz. Buning uchun avvalo (1) dan foydalanib, A nuqtaning tezligani aniqlash zarur.

$$\vartheta_{AX} = x'_A = -(R + r)\frac{E_0 t^2}{2}\sin\frac{E_0 t^2}{2} \qquad \vartheta_{Ay} = y'_A = (R + r)\frac{E_0 t^2}{2}\cos\frac{E_0 t^2}{2} \quad (2)$$

To'la tezlik esa

$$\vartheta_A = \sqrt{x'^2_A + y'^2_\Phi} = (R + r)\ \ E_0 t^2, \quad \vartheta_A = \omega z$$

(2) lekin



ϑ_A - A nuqtaning chiziqli tezligi,

ω - tashqi tishli burchak tezligi.

$$\omega = \left(\frac{R}{r} + 1\right)E_0 t^2 \qquad \omega = \frac{d\varphi_1}{dt}$$

Bundan $\varphi_1 = \left(\frac{R}{r}+1\right)\frac{E_0 t^2}{2}$

Oxirgi esa tashqi g'ildirakning izlanayotgan harakat tenglamasidir.

(I. 15. 7)

AV sterjenning A uchi ϑ o'zgarmas tezlik bilai to'g'ri chiziqli yo'naltiruvchida sirpanadi va bunda sterjen harakat vaqtida V shtiftga tayanadi. Sterjen va uning V uchi harakati tenglamalari yozilsin. Sterjen uzunligi *l* ga teng; shtift to'g'ri chiziqli yo'naltiruvchidan H balandlikda o'rnatilgan.

Harakatning boshlanishida sterjenning A uchi qo'zg'almas koordinatalar sistemasi boshi 0 nuqta bilan ustma-ust tushgan; OM=a. A nuqtani qutb deb olinsin.

Yechish. 1) A nuqtaning koordinatalariii aniqlaymiz. CHizmadan ko'rinib turibdiki, $x_A = \vartheta t$, $u_A = 0$

2) V nuqtaning harakat tenglamalarini aiiklaymiz. CHizmadan

$x_V = \vartheta t + l\cos\varphi$; $u_V = l\sin\varphi$ (1)

CHizmadan $\sin\varphi$ va $\cos\varphi$ larni anislaymiz:

$$\sin\varphi = \frac{H}{AD} = \frac{H}{\sqrt{H^2+(a-\vartheta t)^2}} \qquad \cos = \frac{a-\vartheta t}{\sqrt{H^2+(a-\vartheta t)^2}}$$

(2)

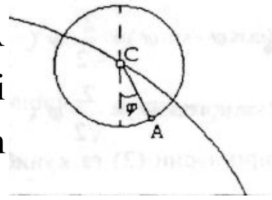
Bundan $tg\varphi = H/a - \vartheta t$ $\quad \varphi = arctg\frac{H}{a-\vartheta t}$ (3) (2) ifodalarni (1) ga qo'yib,

$$x_B = \vartheta t + l\frac{a-\vartheta t}{\sqrt{H^2+(a-\vartheta t)^2}} \qquad y_B = \frac{Hl}{\sqrt{H^2+(a-\vartheta t)^2}} \quad (4)$$

Oxirgilar V nuqtani izlashyotgan harakat tenglamalaridir.

(I. 16. 3)

Radiusi r=20 sm bo'lgan diskning xy vertikal tekisliklagi harakatida uning s markazi $x_s = 10t$ m, $u_s = (100-4{,}9t^2)$ m tenglamalarga asosan harakatlanadi. SHu bilan birga, disk o'zining tekisligiga perpendikulyar bo'lgan s gorizontal o'q atrofida $\omega = \frac{\pi}{2}$ rad/s o'zgarmas burchak tezlik bilan aylanadi. t=0 bo'lgan paytda disk gardishidagi A nuqtaning tezligi aniqlansin. A nuqtaning diskdagi holati vertikalga nisbatan soat strelkasi aylanishga teskari yo'nalishda hisoblanadigan $\varphi = \omega t$ burchak bilan aniqlansin.



Yechish. Avalo A nuqtani xy tekislikda koordinatalarini topamiz. Berilgan s nuqtaning tenglamalari va chizmadan ma’lumki

$X_A = X_c + r\sin\varphi \qquad y_A = Y_c - r\cos\varphi$

Bu tenglamalardan vaqt bo’yicha xosila olib A nuqta tezligining tashkil etuvchilarini x_A', u_A' aniqlaymiz.

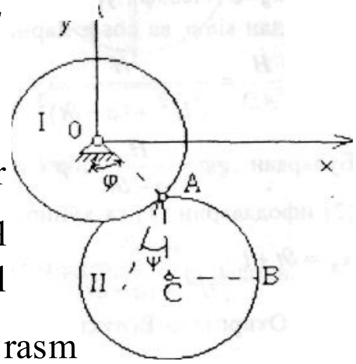
$x_A'=10+r\omega\cos\omega t \quad u_A'=-9.8t+r\omega\sin\omega$

Lekin, t=0 da $\left.x_A'\right|_{t=0} = 10 + 0.2\cdot\frac{\pi}{2} = 10 + 0.1\pi = 10.31 m/s \qquad \left.y_A'\right|_{t=0} = 0$

SHuning uchun A nuqtani to’la tezligi $\vartheta_A = \sqrt{x_A' + y_A'} = 10.31 m/s$

ga teng.

(I. 16. 6)



Har bir radiusi r bo’lgan ikkita bir xil disk A silindrik sharnir vositasida birlashtirilgan. 1 disk 0 qo’zg’almas gorizontal o’q atrofida $\varphi = \varphi(t)$ qonunga binoan aylanadi. 2 disk A gorizontal o’q atrofida $\psi = \psi(t)$ qonunga asosan aylanadi. 0 va A o’qlar rasm tekisligiga perpendikulyar. φ va ψ burchaklar vertikaldan soat strelkasi xarakatiga teskari yo’nalishda hisoblanadi. 2 disk V nuqtasining tezligi topilsin.

agar ASV=$\frac{\pi}{2}$ bo’lsa.

Yechish. Avvalo V nuqtani xy tekisligida koordinatalarini aniqlaymiz. CHizmadan ko’rinib turibdiki,

$X_B = r\sin\varphi + r\sin\psi + r\cos\psi$

$Y_B = -r(\cos\varphi + \cos\psi - \sin\psi) \quad (1)$

(1) ifodalardan vaqt bo’yicha hosila olib, V nuqta tezligining tashkil etuvchilarini aniklaymiz:

$X_B' = r(\varphi'\cos\varphi + \psi'\cos\psi - \psi'\sin\psi)$

$Y_B' = r(\varphi'\sin\varphi + \psi'\sin\psi + \psi'\cos\psi) \quad (2)$

ikkala ifodaning oxirgi ikkitadan a’zolarini quyidagicha soddalashtirish mumkin:

$$\psi'(\cos\psi - \sin\psi) = \frac{2}{\sqrt{2}}\psi'\left(\frac{\sqrt{2}}{2}\cos\psi - \frac{\sqrt{2}}{2}\sin\psi\right) = \sqrt{2}\psi'\cos\left(\psi + \frac{\pi}{4}\right)$$

$$\psi'(\sin\psi + \cos\psi) = \frac{2}{\sqrt{2}}\psi'\left(\frac{\sqrt{2}}{2}\sin\psi - \frac{\sqrt{2}}{2}\cos\psi\right) = \sqrt{2}\psi'\sin\left(\psi + \frac{\pi}{4}\right)$$

Oxirgilarni (2) ga qo'yib, quyidagilarni hosil qilamiz:

$$X'_B = r\left[\varphi'\cos\varphi + \sqrt{2}\psi'\cos\left(\psi + \frac{\pi}{4}\right)\right] \qquad Y'_B = r\left[\varphi'\sin\varphi + \sqrt{2}\psi'\sin\left(\psi + \frac{\pi}{4}\right)\right] \quad (3)$$

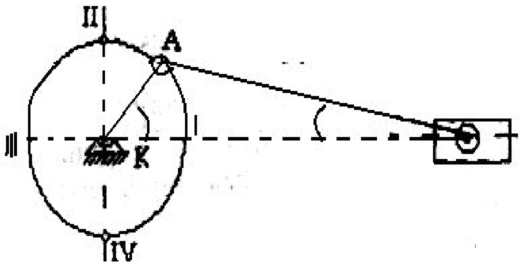
(1) ifoda V nuqtani xy tekisligida harakat tenglamalari, (3) uning tezligining tashkil etuvchilaridir. (3) ifodalardan foydalanib, V nuqtaning to'la tezligini aniqlaymiz:

$$\vartheta_b = \sqrt{x_b'^2 + y_b'^2} = 2\sqrt{\varphi'^2 + 2\varphi'^2 + 2\sqrt{2}\varphi'\psi'\cos\left(\frac{\pi}{4} + \psi - \varphi\right)}$$

Bu masala shartiga ko'ra V nuqtaning izlanayotgan tezligining ifodasidir.

(I. 16. 15)

Krivoship mexanizmida krivoship uzunligi OA=40sm, shatun uzunligi AV=2 m, krivoship 6π rad/s burchak tezlik bilan bir tekis aylanadi. AOV burchagi tegishlicha: 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$ ga teng bo'lgan hollar uchun shatunning burchak tezligi ω va shatun o'rtasidagi M nuqta tezligining qancha bo'lishi topilsin.



Yechish. Masalani yechish uchun avvalo A nuqtani x va u koordinatalarini aniqlaymiz. CHizmadan ma'lumki,

X=OAcosφ+MAcosα, y=MBsinα (1)

Lekin, AK, OAK va ABK uchburchaklarning umumiy tomoni ekanligi uchun AK=OAsinφ=ABsinα (a)

Bu erdan sinα = $\sqrt{1-\cos^2\alpha} = \frac{OA}{AB}\sin\varphi$ yoki cosα = $\sqrt{1-\left(\frac{OA}{AB}\right)^2\sin^2\varphi}$ (2)

Oxirgilarini (1) ga qo'yib,

$$x = OA\cos\varphi + MA\sqrt{1-\left(\frac{OA}{AB}\right)^2\sin^2\varphi}$$

$$Y = \left(\frac{MB}{AB}\right)OA\sin\varphi \qquad (3)$$

Masala shartiga asosan MA=1 m = 100 sm, $\frac{MB}{AB}=\frac{1}{2}$ (3) dan foydalanib, M nuqta tezligining tashkil etuvchilarini aniqlaymiz.

$$\vartheta_x = x' = -OA\omega\sin\varphi - MA\omega\frac{\left(\frac{AO}{AB}\right)^2\sin\varphi\cos\varphi}{\sqrt{1-\left(\frac{AO}{AB}\right)^2\sin^2\varphi}} \qquad \varphi'=\omega$$

$$\vartheta_y = y' = \frac{OA}{2}\omega\cos\varphi \qquad \varphi=\omega t$$

Endi 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$ uchun ϑ_x ϑ_u larni aniqlaymiz:

$\vartheta_{x1}=0$, $\vartheta_{x2}=-OA\omega$, $\vartheta_{x3}=0$ $\vartheta_{x4}=OA\omega$ $\vartheta_{u1}=\frac{OA}{2}\omega$ $\vartheta_{y2}=0$ $\vartheta_{u3}=-\frac{OA}{2}\omega$ $\vartheta_{y4}=0$

To'la tezlikni aniqlaymiz:

$$\vartheta_1=\sqrt{\vartheta_x^2\vartheta_y^2}=\frac{OA}{2}\omega=20\cdot6\pi=120\pi=377sm/s$$

$$\vartheta_2=OA\omega=40\cdot6\pi=240\pi=754sm/s$$

$$\vartheta_3=\frac{OA}{2}\omega=20\cdot6\pi=120\pi=377sm/s$$

$$\vartheta_4=OA\omega=40\cdot6\pi=240\pi=754sm/s$$

Endi shatunning burchak tezligini topamiz;

Buning uchun (a) ifodaning ikki tomonidan vaqt bo'yicha hosila olamiz:

OAcosφ=ABω1cosα $\qquad \omega_1=\frac{d\alpha}{dt}$

bundan (2) dan foydalanib, shatunning burchak tezligini aniqlaymiz:

$$\omega_1=\frac{OA}{AB}\omega\frac{\cos\varphi}{\cos\alpha}=\frac{OA}{AB}\omega\frac{\cos\alpha}{\sqrt{1-\left(\frac{OA}{AB}\right)^2\sin^2\varphi}} \qquad \frac{OA}{AB}=\frac{1}{5}$$

Endn φ=0, $\frac{\pi}{2}$ π, $\frac{3\pi}{2}$, uchui φ_1 ni topamiz:

$$\omega_1(\varphi=0)=\frac{\omega}{5}=\frac{6\pi}{5}rad/s,\omega_1\left(\varphi=\frac{\pi}{2}\right)=0 \qquad \omega_1(\varphi=\pi)=-\frac{\omega}{5}=-\frac{6\pi}{5}rad/s,\omega_1\left(\varphi=\frac{3\pi}{2}\right)=0$$

(1. 16. 24)

G'ildiraklik pressning D porshenini AOVD sharnir richag mexaiizmi vositasida harakatga keltiriladi. Rasmda tasvirlangan holatda OL richag ω=2 rad/s burchak tezlikka ega. Agar OA=15 sm bo’lsa, D porshenning tezligi va AV zvenoning burchak tezligi aniqlansin.

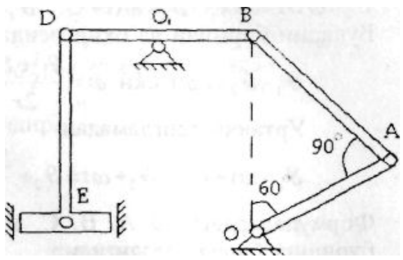
YECHISH: Avvalo A nuktani ϑ_A tezligini aniqlaymiz. Tekstdagi (3) formulaga asosan

$\vartheta_A=\omega$OA=2*15=30 sm/s   (1)

CHizmadan ko’rinib turibdiki, A nuqtaning tezligi AV bo’ylab va V nuqta tezligi VD bo’ylab yo’nalgan, ular orasidagi burchak 30° ga teng. Agar ϑ_A va ϑ_V tezliklarni proeksiyalari haqidagi teoremaga asosan

$\vartheta_A=\vartheta_V\cos 30°$ yoki $\vartheta_V=\frac{2\vartheta_A}{\sqrt{3}}=\frac{60}{\sqrt{3}}=$34,6 sm/s   (2)

Demak, D porshen 34,6 sm/s tezlik bilan harakat qiladi. Endi AV zvenoning burchak tezligini aniqlaymiz. (1) formulaga asosan



(I. 16. 18)

OA krivoship 2 rad/s burchak tezligi bilan bir tskis aylanadi. Agar OA=20 sm, $O_1B=O_1D$ bo’lsa, rasmda ko’rsatilgan holat uchun nasosning uzatmali mexanizm E porshenning tezligi aniqlansin.

Yechish: Avvalo A nuqtani tezligini aniqlaymiz. Ma’lumki, A nuqta O nuqta atrofida 2 rad/s burchak tezlik bilan aylanadi. SHuniig uchun tekstdagi (3) formulaga asosan $\vartheta_A=\omega_0$OA=2*20=40sm/s E porshen yoki V nuqtaning tezligini aniqlash uchun V nuqta tezligini aniqlash kifoyadir, chunki $O_1V=V_1V$ elkalar tengdir va E V nuqtalar tezliklari absolюt jihatdan teng bo’lib, yo’nalishlari V o’zaro qarama-qarshidir.

Agar krivoship OA soat strelkasiga teskari aylanayotgan bo’lsa, A nuktaning tezligi AV bo’ylab yo’naladi va V nuqta tezligi OV bo’ylab юqoriga yo’nalgan bo’ladi.

Agar A, V nuqtalar tezliklarining yo’nalishiga A, V nuktalardan perpendiklyar o’tkazilsa, R nuqtada kesishadi. R nuqtani A, V nuqtalar tezliklarining oniy markazi deyiladi. YA’ni R markaz atrofida A, V nuqtalar biror ω burchak tezligi bilan aylanadi V nuqtaning tezligini aniqlash uchui ω ni aniqlash zarur. CHizmadan ko’rinib turibdiki,

AV=OAtg60°; $BD=\frac{AB}{\cos 60^0}=2AB=2\sqrt{3}OA$

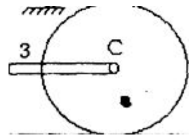
$AD=VPsin60°=\frac{\sqrt{3}}{2}BP = 3\cdot OA$

Tekstdagi (3) formulaga asosan,

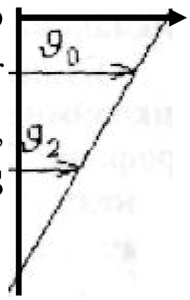
$\vartheta_A = \vartheta_{AP} = \omega AP$ va $\omega = \frac{\vartheta_A}{AP} = \frac{\omega_0 OA}{3\cdot OA} = \frac{\omega_0}{3}$

Endi V nuqtaning tezligini aniqlaymiz: $\vartheta_B = \omega BP = \frac{\omega_0}{3}\cdot 2\sqrt{3}OA = 46.2 sm/s$

Demak, E porshen 46,2 sm/s tezlik bilan harakat qiladi.

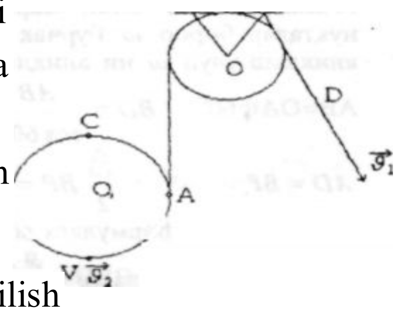


(I. 16. 32)



Rasmda harakatlarni qo'shadigan mexanizm tasvirlangan. O'zaro parallel ikkita 1 va 2 reykalar 5, va ϑ_1 va ϑ_2 o'zgarmas tezliklar bilan bir tomonga xarakatlanishadi. Reykalar orasida r radiusli, reykalar bo'ylab sirpanmay dumalaydigan disk qisilgan. Diskning S o'qiga mahkamlangan 3 Reykaning tezligi 1 va 2 reykalar tezliklari yig'indisiming yarmiga tengligi ko'rsatilsin. SHuningdek, diskning burchak tezliga topilsin.

Yechish: Aniq bo'lish uchun $\vartheta_1 > \vartheta_2$, bo'lsin, unda $\vartheta_1 > \vartheta_s > \vartheta_2$ bo'ladi.



Bu tezliklarni boshlanish va oxirlarini o'zaro ulasak (rasmda ko'rsatilgandek), ularni tutashtiruvchi to'g'ri chiziqlar R nuqtada kesishadi. Bu R nuqta uchala tezliklarning oniy markazi bo'ladi. R nuqtalan V gacha bo'lgan masofani x bilan belgilaymiz. CHizmadan malumki, $\vartheta_2 = \omega BP = \omega x, \vartheta_c = \omega(r+x), \vartheta_1 = \omega(2r+x)$

Formulalardagi ω A, V, S, nuqtalarning nuqta atrofida burilish burchak tezligidir.

(II. 502)

O blokdan o'tkazilgan arqon radiusi R bo'lgan silindrga o'ralgan. Silindrning markazi ϑ_2 tezlik bilan tushish paytida arqon oxiri ϑ_1 tezlik bilan tortiladi. Arqonning silindrdan blokgacha bo'lgan qismini vertikal hisoblab, silindrning burchak tezligini aniqlang. Gorizontal diametrdagi V hamda vertikal diametrdagi S nuqtalar tezligi topilsin.

<u>Yechish.</u> Arqonning blokdan o'tkazilgan qismini ϑ_1 tezlik bilan tortganda, silindrning A nuqtasi ham shunday tezlik bilan юqoriga tortiladi. Silindrning

gorizontal diametrdagi A, V, O nuqtalar tezliganing oniy markazining aniklaymiz. CHizmadan maьlumki, R nuqta ularnin goniy markazidir. R A ni x bilan belgilab, hosil qilamiz:

$$\vartheta_1 = \vartheta_A = \omega PA = \omega x$$
$$\vartheta_0 = \vartheta_2 = \omega(R - x) \qquad (1)$$
$$\vartheta_B = \omega(2R - x)$$

Bu tenglamalarning birinchi va ikkinchisidan

$\vartheta_1 + \vartheta_2 = \omega R$ yoki $\omega = \vartheta_1 + \vartheta_2/R$ (2)

Endi RA=x ni topamiz: $x = \dfrac{\vartheta_1}{\omega} = \dfrac{\vartheta_1}{\vartheta_1 + \vartheta_2} R$

Oxirgini (1) ning uchinchisiga qo'yib, ϑB ni topamiz.

$$\vartheta_B = \omega(2R - x) = \frac{\vartheta_1 + \vartheta_2}{R}\left(2R - \frac{\vartheta_1}{\vartheta_1 + \vartheta_2} R\right) = \vartheta_1 + 2\vartheta_2 \qquad (3)$$

S nuqtaning tezligini aniqlaymiz:

$$PC = \sqrt{O'C^2 + O'P^2} = \sqrt{R^2 (R - x)^2} = \sqrt{R^2 + \left(R - \frac{\vartheta_1}{\vartheta_1 + \vartheta_2} R\right)^2} = \frac{R}{\vartheta_1 + \vartheta_2}\sqrt{\vartheta_2^2 + (\vartheta_1 + \vartheta_2)^2} \qquad (4)$$

$$\vartheta_c = \omega PC = \frac{\vartheta_1 + \vartheta_2}{R} \cdot \frac{R}{\vartheta_1 + \vartheta_2}\sqrt{\vartheta_2^2 + (\vartheta_1 + \vartheta_2)^2} = \sqrt{\vartheta_2^2 + (\vartheta_1^2 + \vartheta_2^2)}$$

(2), (3) va (4) ifodalar masalaning echimi.

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