

**O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA
MAXSUS TA'LIM VAZIRLIGI
ALISHER NAVOIY NOMIDAGI
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**Mexanika-matematika fakulteti
"Nazariy va amaliy mexanika kafedrası"**

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**TUTASH MUHIT MEXANIKASI FANIDAN MUSTAQIL
ISHLAR TOPSHIRIQLARI VA ULARNI BAJARISH
BO'YICHA
USLUBIY QO'LLANMA**

Samarqand 2015

Tutash muhit mexanikasi fanidandan mustaqil ishlar topshiriqlari va ularni bajarish bo'yicha uslubiy qo'llanma. – Samarqand: SamDU nashri, 2013.

Mazkur uslubiy qo'llanmada mexanika ta'lim yo'nalishida tahsil olayotgan talabalar mustaqil ishlar topshiriqlari va ularni bajarish bo'yicha nazariy ma'lumotlar berilgan.

1-MUSTAQIL ISH

Mavzu: Koordinatalarni almashtirish

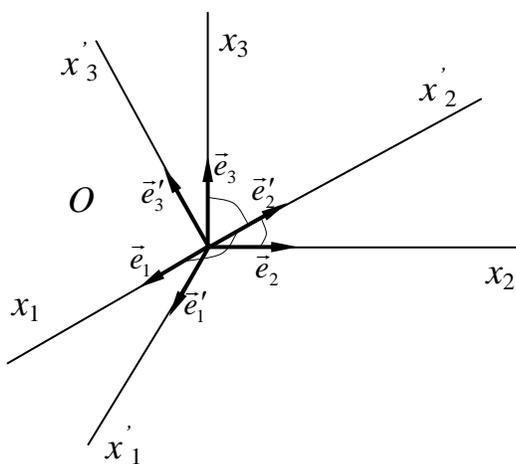
Maqsad: Talabalarga koordinatalarni almashtirish va ular orasidagi munosabatlarni chuqurroq o'rgatish. Topshiriqlar orqali ularning bilimni sinash.

REJA:

1. Koordinatalar sistemasi, bazis vektorlari, radius vektor;
2. Koordinatalarni burish;
3. Yangi va eski koordinatalar orasidagi o'zaro munosabat.

Asosiy tushunchalar

Koordinatalar sistemasini biror burchakka burish natijasida yangi koordinatalar sistemasi hosil bo'ladi (1-chizma). Hosil bo'lgan koordinatalar sistemasining \vec{e}'_i bazis



1-chizma

vektorlarini eski \vec{e}_j bazis vektorlari orqali quyidagicha ifodalash mumkin

$$\vec{e}'_i = \alpha_{ij} \vec{e}_j, \quad (1)$$

bu yerda α_{ij} yangi bazis vektorlarining eski bazis bilan tashkil qilgan burchaklari kosinuslari. Xuddi shunday eski bazislarni ham yangi bazislar orqali ifodalash mumkin

$$\vec{e}_i = \alpha'_{ij} \vec{e}'_j, \quad (2)$$

bu yerda α'_{ij} eski bazis vektorlarining yangi bazis bilan tashkil qilgan burchaklari kosinuslari.

Masala. $Ox'_1x'_2x'_3$ va $Ox_1x_2x_3$ koordinatalar sistemasini orasidagi bog'lanish munosabat bog'lovchi koordinat almashtirishlar quyidagi jadval shaklida berilgan:

1. Ortogonallik sharti bajarilishini ko'rsating.
2. Berilgan $A(1,2,4)$ nuqtaning shtrixli koordinatalar

	X_1	X_2	X_3
X'_1	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$
X'_2	$\frac{4}{5}$	0	$-\frac{3}{5}$
X'_3	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$

sistemasidagi koordinatalarini aniqlang.

3. $a(a_1, a_2, a_3)$ vektorni shtrixli koordinatalar sistemasida ifodalang.
4. $Ax + By + Cz + D = 0$ tekislik tenglamasini shtrixli koordinatalar sistemasida ifodalang.

Yechish

1. Ortogonallikni ixtiyoriy satr(ustun)ning komponentalarini boshqa ixtiyoriy satr(ustun)ning mos komponentalariga ko'paytmalari yig'indisi nolga tengligidan topamiz

$$\begin{aligned}
 -\frac{3}{5\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{4}{5} \cdot 0 + \frac{3}{5\sqrt{2}} \cdot \frac{1}{\sqrt{2}} &= 0; \\
 -\frac{3}{5\sqrt{2}} \cdot \left(-\frac{4}{5\sqrt{2}}\right) + \frac{4}{5} \cdot \left(-\frac{3}{5}\right) + \frac{3}{5\sqrt{2}} \cdot \frac{4}{5\sqrt{2}} &= 0; \\
 -\frac{3}{5\sqrt{2}} \cdot \left(-\frac{4}{5}\right) + \frac{4}{5} \cdot 0 + \frac{3}{5\sqrt{2}} \cdot \left(-\frac{3}{5}\right) &= 0; \\
 -\frac{3}{5\sqrt{2}} \cdot \frac{3}{5\sqrt{2}} + \left(\frac{1}{\sqrt{2}}\right)^2 - \frac{4}{5\sqrt{2}} \cdot \frac{4}{5\sqrt{2}} &= 0.
 \end{aligned}$$

Demak ortogonallik sharti bajarilar ekan.

2. Shtrixli koordinatalar sistemasida A nuqtaning koordinatalarini topamiz

$$\begin{aligned}
 x'_1 &= -\frac{3}{2\sqrt{5}} \cdot 1 + \frac{1}{\sqrt{2}} \cdot 2 - \frac{4}{5\sqrt{2}} \cdot 4 = -\frac{23}{5\sqrt{2}}; \\
 x'_2 &= -\frac{4}{5} \cdot 1 + 0 \cdot 2 - \frac{3}{5} \cdot 4 = -\frac{16}{5}; \\
 x'_3 &= \frac{3}{2\sqrt{5}} \cdot 1 + \frac{1}{\sqrt{2}} \cdot 2 + \frac{4}{5\sqrt{2}} \cdot 4 = \frac{19 + \sqrt{5}}{5\sqrt{2}}.
 \end{aligned}$$

Demak, shtrixli koordinatalar sistemasida $A\left(-\frac{23}{5\sqrt{2}}, -\frac{16}{5}, \frac{19 + \sqrt{5}}{5\sqrt{2}}\right)$.

3. Tekislik tenglamasini shtrixli koordinatalarda ifodalaymiz

$$\begin{aligned}
 Ax_1 + Bx_2 + Cx_3 + D &= A\left(-\frac{3}{5\sqrt{2}}x'_1 + \frac{1}{\sqrt{2}}x'_2 - \frac{4}{5\sqrt{2}}x'_3\right) + B\left(\frac{4}{5}x'_1 - \frac{3}{5}x'_3\right) + \\
 &+ C\left(\frac{3}{5\sqrt{2}}x'_1 + \frac{1}{\sqrt{2}}x'_2 + \frac{4}{5\sqrt{2}}x'_3\right) + D = 0 \Rightarrow \\
 \left(-\frac{3A}{5\sqrt{2}} + \frac{4B}{5} + \frac{3C}{5\sqrt{2}}\right)x'_1 &+ \left(\frac{A}{\sqrt{2}} + \frac{C}{\sqrt{2}}\right)x'_2 + \left(-\frac{4A}{5\sqrt{2}} - \frac{3B}{5} + \frac{4C}{5\sqrt{2}}\right)x'_3 + D = 0
 \end{aligned}$$

1-MUSTAQIL ISH TOPSHIRIQLARI

$Ox_1x_2x_3$ va $Ox_1x_2x_3$ koordinatalar sistemasini bog'lovchi koordinat almashtirishlar quyidagi jadval shaklida berilgan:

1. Ortogonallik sharti bajarilishini ko'rsating.
2. Berilgan $A(x_0, y_0, z_0)$ nuqtaning har ikkala koordinatalar sistemasida radius vektorini aniqlang.
3. $a(a_1, a_2, a_3)$ vektorni shtrixli koordinatalar sistemasida ifodalang.
4. $Ax + By + Cz + D = 0$ tekislik tenglamasini shtrixli koordinatalar sistemasida ifodalang.

1-variant

	x_1	x_2	x_3
x_1'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$

3-variant

	x_1	x_2	x_3
x_1'	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
x_2'	$-\frac{1}{3}$	$-\frac{2}{3}$	0
x_3'	0	0	1

2-variant

	x_1	x_2	x_3
x_1'	$\frac{3}{5}$	$-\frac{4}{5}$	0
x_2'	0	0	1
x_3'	$\frac{4}{5}$	$\frac{3}{5}$	0

4-variant

	x_1	x_2	x_3
x_1'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$

5-variant

	x_1	x_2	x_3
x_1'	0	$-\frac{4}{5}$	$\frac{3}{5}$
x_2'	1	0	0
x_3'	0	$\frac{3}{5}$	$\frac{4}{5}$

6-variant

	x_1	x_2	x_3
x_1'	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
x_2'	0	0	1
x_3'	$-\frac{1}{3}$	$-\frac{2}{3}$	0

7-variant

	x_1	x_2	x_3
x_1'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$

8-variant

	x_1	x_2	x_3
x_1'	$\frac{3}{5}$	$-\frac{4}{5}$	0
x_2'	0	0	1
x_3'	$\frac{4}{5}$	$\frac{3}{5}$	0

9-variant

	x_1	x_2	x_3
x_1'	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
x_2'	$-\frac{1}{3}$	$-\frac{2}{3}$	0
x_3'	0	0	1

10-variant

	x_1	x_2	x_3
x_1'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$

11-variant

	x_1	x_2	x_3
x_1'	0	$-\frac{4}{5}$	$\frac{3}{5}$
x_2'	1	0	0
x_3'	0	$\frac{3}{5}$	$\frac{4}{5}$

12-variant

	x_1	x_2	x_3
x_1'	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
x_2'	0	0	1
x_3'	$-\frac{1}{3}$	$-\frac{2}{3}$	0

13-variant

	x_1	x_2	x_3
x_1'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$

14-variant

	x_1	x_2	x_3
x_1'	$\frac{3}{5}$	$-\frac{4}{5}$	0
x_2'	0	0	1
x_3'	$\frac{4}{5}$	$\frac{3}{5}$	0

15-variant

	x_1	x_2	x_3
x_1'	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
x_2'	$-\frac{1}{3}$	$-\frac{2}{3}$	0
x_3'	0	0	1

16-variant

	x_1	x_2	x_3
x_1'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$

17-variant

	x_1	x_2	x_3
x_1'	0	$-\frac{4}{5}$	$\frac{3}{5}$
x_2'	1	0	0
x_3'	0	$\frac{3}{5}$	$\frac{4}{5}$

18-variant

	x_1	x_2	x_3
x_1'	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
x_2'	0	0	1
x_3'	$-\frac{1}{3}$	$-\frac{2}{3}$	0

19-variant

	x_1	x_2	x_3
x_1'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$

20-variant

	x_1	x_2	x_3
x_1'	$\frac{3}{5}$	$-\frac{4}{5}$	0
x_2'	0	0	1
x_3'	$\frac{4}{5}$	$\frac{3}{5}$	0

21-variant

	x_1	x_2	x_3
x_1'	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
x_2'	$-\frac{1}{3}$	$-\frac{2}{3}$	0
x_3'	0	0	1

22-variant

	x_1	x_2	x_3
x_1'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$

23-variant

	x_1	x_2	x_3
x_1'	0	$-\frac{4}{5}$	$\frac{3}{5}$
x_2'	1	0	0
x_3'	0	$\frac{3}{5}$	$\frac{4}{5}$

24-variant

	x_1	x_2	x_3
x_1'	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
x_2'	0	0	1
x_3'	$-\frac{1}{3}$	$-\frac{2}{3}$	0

25-variant

	x_1	x_2	x_3
x_1'	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$
x_2'	$-\frac{2}{3}$	0	0
x_3'	0	$-\frac{1}{3}$	1

26-variant

	x_1	x_2	x_3
x_1'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$

Adabiyotlar

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2 – MUSTAQIL ISH

Mavzu: Tenzorlar va ular ustida amallar

Maqsad: Talabalarda tenzorlar ustida amallarni bajarish ko'nikmalarini shakllantirish. Mavzu bo'yicha masala yechish orqali ularning bilimini mustahkamlash va bilimini sinash.

REJA:

1. Tenzor va uning komponentalari;
2. Tenzorni simmetrik va antisimmetrik qismlarga ajratish;
3. Tenzorning sharsimon va deviator qismlari;
4. Tenzorning bosh qiymatlari va bosh yo'nalishlari;
5. Masala.

Asosiy tushunchalar

Komponentalari quyidagicha aniqlanuvchi ob'ektga tenzor deyiladi

$$T = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix} = T_{11}\bar{e}^1\bar{e}^1 + T_{12}\bar{e}^1\bar{e}^2 + T_{13}\bar{e}^1\bar{e}^3 + T_{21}\bar{e}^2\bar{e}^1 + T_{22}\bar{e}^2\bar{e}^2 + T_{23}\bar{e}^2\bar{e}^3 + T_{31}\bar{e}^3\bar{e}^1 + T_{32}\bar{e}^3\bar{e}^2 + T_{33}\bar{e}^3\bar{e}^3. \quad (1)$$

Agar bu tenzor uchun $T_{ij} = T_{ji}$ shart bajarilsa simmetrik, $T_{ij} = -T_{ji}$ shart bajarilsa antisimmetrik deyiladi.

Tenzor simmetrik va antisimmetrik qismlarga quyidagicha ajratiladi

$$T_{ij} = T_{ij}^{sim} + T_{ij}^{antisim}, \quad T_{ij}^{sim} = \frac{1}{2}(T_{ij} + T_{ji}), \quad T_{ij}^{antisim} = \frac{1}{2}(T_{ij} - T_{ji}). \quad (2)$$

Umumiy holda tenzor sharsimon va deviator qismlarga quyidagi formula bo'yicha ajratiladi

$$T_{ij} = T_{ij}^{shar} + T_{ij}^{dev}, \quad T_{ij}^{shar} = \frac{1}{3}(T_{11} + T_{22} + T_{33})\delta_{ij}, \quad T_{ij}^{dev} = T_{ij} - T_{ij}^{shar}. \quad (3)$$

Tenzorning bosh qiymatlari (komponentalari) quyidagi tenglamadan topiladi

$$\begin{vmatrix} T_{11} - \lambda & T_{12} & T_{13} \\ T_{21} & T_{22} - \lambda & T_{23} \\ T_{31} & T_{32} & T_{33} - \lambda \end{vmatrix} = 0. \quad (4)$$

Bosh qiymatlarga mos bosh yo'nalishlar esa quyidagi tenglamalardan topiladi

$$\begin{aligned}(T_{11} - \lambda)n_1 + T_{12}n_2 + T_{13}n_3 &= 0; \\ T_{21}n_1 + (T_{22} - \lambda)n_2 + T_{23}n_3 &= 0; \\ T_{31}n_1 + T_{32}n_2 + (T_{33} - \lambda)n_3 &= 0; \\ n_1^2 + n_2^2 + n_3^2 &= 1.\end{aligned}\tag{5}$$

Tenzor sirti esa quyidagi tenglamadan aniqlanadi

$$T_{ij}x_i x_j = \pm c^2.\tag{6}$$

Masala. $T = \begin{pmatrix} 4 & 6 & 3 \\ 3 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix}$ tenzor uchun quyidagilarni aniqlang

1. Tenzorni matritsa shaklida bo'lsa, uni vektor shaklida va aksincha yozing;
2. Tenzorni simmetrik va antisimmetrik tenzorlarga ajrating va ularning yig'indisidan iborat ekanligini ko'rsating;
3. Tenzorning bosh komponentalarini toping;
4. Tenzorning bosh o'qlarini toping;
5. Tenzorni sharsimon va deviator qismlariga ajrating;
6. Tenzor sirti tenglamasini va shaklini toping.

Yechish.

1. Tenzorni vektor ko'rinishga o'tkazamiz

$$T = 4\bar{e}^1\bar{e}^1 + 6\bar{e}^1\bar{e}^2 + 3\bar{e}^1\bar{e}^3 + 3\bar{e}^2\bar{e}^1 + 4\bar{e}^2\bar{e}^2 + \bar{e}^3\bar{e}^1 + 4\bar{e}^3\bar{e}^3;$$

2. Simmetrik va antisimmetrik qismlarga ajratamiz va ularning yig'indisi sifatida ifodalaymiz

$$T = \begin{pmatrix} 4 & 6 & 3 \\ 3 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix} = \underbrace{\begin{pmatrix} 4 & 4,5 & 2 \\ 4,5 & 4 & 0 \\ 2 & 0 & 4 \end{pmatrix}}_{\text{simmetrik}} + \underbrace{\begin{pmatrix} 0 & 1,5 & 1 \\ -1,5 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}}_{\text{antisimmetrik}};$$

3. Tenzorning sharsimon va deviator qismlari

$$T = \begin{pmatrix} 4 & 6 & 3 \\ 3 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix} = \underbrace{\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\text{sharsimon}} + \underbrace{\begin{pmatrix} 0 & 6 & 3 \\ 3 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}}_{\text{deviator}}.$$

4. Tenzorning bosh komponentalarini topish uchun

$$\begin{vmatrix} 4-\lambda & 6 & 3 \\ 3 & 4-\lambda & 0 \\ 1 & 0 & 4-\lambda \end{vmatrix} = 0$$

tenglamani yechamiz. Bu kubik tenglamaga keladi

$$(4-\lambda)^3 - 3(4-\lambda) - 18(4-\lambda) = 0,$$

$$(4-\lambda)((4-\lambda)^2 - 21) = 0,$$

Uning ildizlari $\lambda_1 = 4$, $\lambda_2 = 4 + \sqrt{21}$, $\lambda_3 = 4 - \sqrt{21}$.

5. Bosh qiymatlarga mos bosh yo'nalishlarni topish uchun esa quyidagi tenglamalarni yechamiz

$\lambda = 4$ uchun

$\lambda = 4 + \sqrt{21}$ uchun

$\lambda = 4 - \sqrt{21}$ uchun

$$\begin{cases} 6n_2 + 3n_3 = 0; \\ 3n_1 = 0; \\ n_1 = 0; \\ n_1^2 + n_2^2 + n_3^2 = 1. \end{cases} \Rightarrow \begin{cases} n_1 = 0; \\ n_2 = -\frac{1}{\sqrt{5}}; \\ n_3 = \frac{2}{\sqrt{5}}. \end{cases} \quad \begin{cases} -\sqrt{21}n_1 + 6n_2 + 3n_3 = 0; \\ 3n_1 - \sqrt{21}n_2 = 0; \\ n_1 - \sqrt{21}n_3 = 0; \\ n_1^2 + n_2^2 + n_3^2 = 1, \end{cases} \Rightarrow \begin{cases} n_1 = \pm\sqrt{\frac{21}{31}}; \\ n_2 = \pm\frac{3}{\sqrt{31}}; \\ n_3 = \pm\frac{1}{\sqrt{31}}. \end{cases} \quad \begin{cases} \sqrt{21}n_1 + 6n_2 + 3n_3 = 0; \\ 3n_1 + \sqrt{21}n_2 = 0; \\ n_1 + \sqrt{21}n_3 = 0; \\ n_1^2 + n_2^2 + n_3^2 = 1, \end{cases} \Rightarrow \begin{cases} n_1 = \pm\sqrt{\frac{21}{31}}; \\ n_2 = \mp\frac{3}{\sqrt{31}}; \\ n_3 = \mp\frac{1}{\sqrt{31}}. \end{cases}$$

6. Tensor sirtini topamiz

$$T_{ij}x_i x_j = 4x_1^2 + 6x_1x_2 + 3x_1x_3 + 3x_2x_1 + 4x_2^2 + 0 \cdot x_1x_3 + x_3x_1 + 0 \cdot x_3x_2 + 4x_3^2 = \pm c^2$$

yoki $4x_1^2 + 4x_2^2 + 4x_3^2 + 9x_1x_2 + 4x_1x_3 = \pm c^2$.

Demak tensor sirti yuqoridagi tenglama bilan ifodalanuvchi ellipsoiddan iborat ekan.

2-MUSTAQIL ISH TOPSHIRIQLARI

1. Tenzorni matritsa shaklida bo'lsa, uni vektor shaklida va aksincha yozing.
2. Tenzorni simmetrik va antisimmetrik tenzorlarga ajrating.
3. Tenzorning simmetrik va antisimmetrik tenzorlari yig'indisidan iborat ekanligini ko'rsating.
4. Tenzorning bosh komponentalarini toping.
5. Tenzorning bosh o'qlarini toping.
6. Tenzorni sharsimon va deviator qismlariga ajrating.
7. Tensor sirtini va shaklini aniqlang.

1-variant

$$(T^{ij}) = \begin{pmatrix} 5 & -3 & 2 \\ -3 & 4 & 1 \\ 2 & 1 & 4 \end{pmatrix}$$

2-variant

$$(T^{ij}) = \begin{pmatrix} 2 & -3 & 0 \\ 3 & 6 & 0 \\ 8 & 0 & 1 \end{pmatrix}$$

3-variant

$$(T^{ij}) = \begin{pmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{pmatrix}$$

4-variant

$$(T^{ij}) = \begin{pmatrix} 1 & 2 & 4 \\ 6 & 7 & 8 \\ 10 & 12 & 13 \end{pmatrix}$$

5-variant

$$A = 2 \bar{\omega}^1 \bar{\omega}^1 + 2 \bar{\omega}^1 \bar{\omega}^3 + 3 \bar{\omega}^2 \bar{\omega}^3 + 5 \bar{\omega}^3 \bar{\omega}^1 + \\ + 6 \bar{\omega}^3 \bar{\omega}^2 + 4 \bar{\omega}^3 \bar{\omega}^3$$

6-variant

$$A = 2 \bar{\omega}^1 \bar{\omega}^1 + 3 \bar{\omega}^1 \bar{\omega}^2 + 5 \bar{\omega}^2 \bar{\omega}^2 + 6 \bar{\omega}^3 \bar{\omega}^1 + \\ + 4 \bar{\omega}^3 \bar{\omega}^2 + \bar{\omega}^3 \bar{\omega}^3$$

7-variant

$$A = 4 \bar{\omega}^1 \bar{\omega}^1 + 2 \bar{\omega}^1 \bar{\omega}^2 + 5 \bar{\omega}^2 \bar{\omega}^1 + 6 \bar{\omega}^2 \bar{\omega}^2 + \\ + 4 \bar{\omega}^3 \bar{\omega}^1 + 7 \bar{\omega}^3 \bar{\omega}^2$$

8-variant

$$A = \bar{\omega}^1 \bar{\omega}^1 + 2 \bar{\omega}^1 \bar{\omega}^2 + 5 \bar{\omega}^1 \bar{\omega}^3 + 6 \bar{\omega}^2 \bar{\omega}^1 + \\ + 4 \bar{\omega}^2 \bar{\omega}^3 + 7 \bar{\omega}^3 \bar{\omega}^1 + 4 \bar{\omega}^3 \bar{\omega}^2 + 6 \bar{\omega}^3 \bar{\omega}^3$$

9-variant

$$(T^{ij}) = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 4 & 5 \\ 0 & 7 & 6 \end{pmatrix}$$

10-variant

$$(T^{ij}) = \begin{pmatrix} 5 & 6 & 3 \\ 3 & 4 & 5 \\ 1 & 3 & 2 \end{pmatrix}$$

11-variant

$$(T^{ij}) = \begin{pmatrix} -4 & 1 & 7 \\ 9 & 4 & 5 \\ 6 & 5 & 8 \end{pmatrix}$$

12-variant

$$(T^{ij}) = \begin{pmatrix} 5 & 8 & 2 \\ 3 & 8 & 5 \\ 7 & 7 & 4 \end{pmatrix}$$

13-variant

$$A = 2 \bar{\omega}^1 \bar{\omega}^1 + 2 \bar{\omega}^1 \bar{\omega}^3 + 3 \bar{\omega}^2 \bar{\omega}^3 + 5 \bar{\omega}^3 \bar{\omega}^1 + \\ + 6 \bar{\omega}^3 \bar{\omega}^2 + 4 \bar{\omega}^3 \bar{\omega}^3$$

14-variant

$$A = 2 \bar{\omega}^1 \bar{\omega}^1 + 3 \bar{\omega}^1 \bar{\omega}^2 + 5 \bar{\omega}^2 \bar{\omega}^2 + 6 \bar{\omega}^3 \bar{\omega}^1 + \\ + 4 \bar{\omega}^3 \bar{\omega}^2 + \bar{\omega}^3 \bar{\omega}^3$$

15-variant

$$A = 4 \bar{\omega}^1 \bar{\omega}^1 + 2 \bar{\omega}^1 \bar{\omega}^2 + 5 \bar{\omega}^2 \bar{\omega}^1 + 6 \bar{\omega}^2 \bar{\omega}^2 + \\ + 4 \bar{\omega}^3 \bar{\omega}^1 + 7 \bar{\omega}^3 \bar{\omega}^2$$

16-variant

$$A = \bar{\omega}^1 \bar{\omega}^1 + 2 \bar{\omega}^1 \bar{\omega}^2 + 5 \bar{\omega}^1 \bar{\omega}^3 + 6 \bar{\omega}^2 \bar{\omega}^1 + \\ + 4 \bar{\omega}^2 \bar{\omega}^2 + 2 \bar{\omega}^2 \bar{\omega}^3 + 3 \bar{\omega}^3 \bar{\omega}^3 + 6 \bar{\omega}^3 \bar{\omega}^1 + 4 \bar{\omega}^3 \bar{\omega}^3$$

17-variant

$$(T^{ij}) = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 4 & 5 \\ 0 & 7 & 6 \end{pmatrix}$$

18-variant

$$A = 2 \bar{\omega}^1 \bar{\omega}^1 + 2 \bar{\omega}^1 \bar{\omega}^3 + 3 \bar{\omega}^2 \bar{\omega}^3 + 5 \bar{\omega}^3 \bar{\omega}^1 + \\ + 6 \bar{\omega}^3 \bar{\omega}^2 + 4 \bar{\omega}^3 \bar{\omega}^3$$

19-variant

$$A = 2 \bar{\varepsilon}^1 \bar{\varepsilon}^1 + 3 \bar{\varepsilon}^1 \bar{\varepsilon}^2 + 5 \bar{\varepsilon}^2 \bar{\varepsilon}^2 + 6 \bar{\varepsilon}^3 \bar{\varepsilon}^1 + 4 \bar{\varepsilon}^3 \bar{\varepsilon}^2 + \bar{\varepsilon}^3 \bar{\varepsilon}^3$$

20-variant

$$A = 4 \bar{\varepsilon}^1 \bar{\varepsilon}^1 + 2 \bar{\varepsilon}^1 \bar{\varepsilon}^2 + 5 \bar{\varepsilon}^2 \bar{\varepsilon}^1 + 6 \bar{\varepsilon}^2 \bar{\varepsilon}^2 + 4 \bar{\varepsilon}^3 \bar{\varepsilon}^1 + 7 \bar{\varepsilon}^3 \bar{\varepsilon}^2$$

21-variant

$$A = \bar{\varepsilon}^1 \bar{\varepsilon}^1 + 2 \bar{\varepsilon}^1 \bar{\varepsilon}^2 + 5 \bar{\varepsilon}^1 \bar{\varepsilon}^3 + 6 \bar{\varepsilon}^2 \bar{\varepsilon}^1 + 4 \bar{\varepsilon}^2 \bar{\varepsilon}^3 + 7 \bar{\varepsilon}^3 \bar{\varepsilon}^1 + 4 \bar{\varepsilon}^3 \bar{\varepsilon}^2 + 6 \bar{\varepsilon}^3 \bar{\varepsilon}^3$$

22-variant

$$(T^{ij}) = \begin{pmatrix} 2 & -3 & 0 \\ 3 & 6 & 0 \\ 8 & 0 & 1 \end{pmatrix}$$

23-variant

$$(T^{ij}) = \begin{pmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{pmatrix}$$

24-variant

$$(T^{ij}) = \begin{pmatrix} 1 & 2 & 4 \\ 6 & 7 & 8 \\ 10 & 12 & 13 \end{pmatrix}$$

25-variant

$$(T^{ij}) = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 4 & 5 \\ 0 & 7 & 6 \end{pmatrix}$$

26-variant

$$A = \bar{\varepsilon}^1 \bar{\varepsilon}^1 + 2 \bar{\varepsilon}^1 \bar{\varepsilon}^2 + 5 \bar{\varepsilon}^1 \bar{\varepsilon}^3 + 6 \bar{\varepsilon}^2 \bar{\varepsilon}^1 + 4 \bar{\varepsilon}^2 \bar{\varepsilon}^2 + 2 \bar{\varepsilon}^2 \bar{\varepsilon}^3 + 3 \bar{\varepsilon}^3 \bar{\varepsilon}^3 + 6 \bar{\varepsilon}^3 \bar{\varepsilon}^1 + 4 \bar{\varepsilon}^3 \bar{\varepsilon}^3$$

Adabiyotlar

1. «Механика сплошной среды в примерах и задачах» . Учебное пособие. У.Г.У. Свердловск, 1979 г.
2. Ильюшин А.А., Ломакин В.А., Шмаков А.П. Задачи и упражнения по механике сплошной среды. - М. : Изд. МГУ, 1973 г.
3. Мейз. Дж. Теория и задачи механики сплошной среды.- М.: Мир, 1974 г.
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3-MUSTAQIL ISH

Mavzu: Tutash mihit kinematikasiga doir masalalar

Мақсад: Tutash muhit harakat qonunining berilish usullari, tezliklar va tezlanishlar maydonlari, hamda deformatsiyasi haqid tasavvurlarni yanada kengaytirish. Mavzu bo'yicha masala yechish orqali ularning bilimini mustahkamlash va sinash.

REJA:

1. Tutash muhit harakatining Lagranj va Eyler ko'rinishlari, ularning biridan ikkinchisiga o'tish;
2. Ko'chish vektori, tezlik va tezlanishlar maydoni;
3. Deformatsiya va deformatsiya tezliklari.

Ma'lumki tutash muhit harakatini o'rganishda Lagranj va Eyler nuqtai nazarlari mavjud. Agar harakat $x_i = x_i(\xi_1, \xi_2, \xi_3, t)$ ko'rinishda berilgan bo'lsa, Lagranj ko'rinishida, Agar harakat $\xi_i = \xi_i(x_1, x_2, x_3, t)$ ko'rinishida berilgan bo'lsa, Eyler ko'rinishida berilgan deyiladi. Ular bir ko'rinishda berilgan bo'lsa uni ikkinchi ko'rinishga o'tkazish uchun dastlab bir qiymatli moslik bajarilishini tekshirishimiz kerak, ya'ni yakobian noldan farqli bo'lishi kerak

Lagranjdan Eylerga otish uchun

Lagranjdan Eylerga otish

$$\left| \frac{\partial x_i}{\partial \xi_j} \right| \neq 0$$

$$\left| \frac{\partial \xi_i}{\partial x_j} \right| \neq 0$$

(1)

Agar boshlang'ich holatda Lagranj va Eyler koordinatalari ustma-ust tushsa, u holda ko'chish vektori

$$w_i = x_i - \xi_i \quad (2)$$

kabi aniqlanadi.

Tezlik vektori komponentalari

Lagranj koordinatalarida

$$v_i(\xi_1, \xi_2, \xi_3, t) = \frac{\partial w_i(\xi_1, \xi_2, \xi_3, t)}{\partial t};$$

(3)

$$v_i(x_1, x_2, x_3, t) = \frac{\partial w_i(x_1, x_2, x_3, t)}{\partial t} + v_k(x_1, x_2, x_3, t) \frac{\partial w_i(x_1, x_2, x_3, t)}{\partial x_k}. \quad (4)$$

(4) da tezlik vektori komponentalari oshkormas ko'rinishda berilgan.

Tezlanish vektori komponentalari

Lagranj koordinatalarida

$$a_i(\xi_1, \xi_2, \xi_3, t) = \frac{\partial v_i(\xi_1, \xi_2, \xi_3, t)}{\partial t}; \quad (5)$$

Eyler koordinatalarida

$$a_i(x_1, x_2, x_3, t) = \frac{\partial v_i(x_1, x_2, x_3, t)}{\partial t} + v_k(x_1, x_2, x_3, t) \frac{\partial v_i(x_1, x_2, x_3, t)}{\partial x_k}. \quad (6)$$

Deformatsiya tenzori komponentalari

Lagranj koordinatalarida

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial w_i}{\partial \xi_j} + \frac{\partial w_j}{\partial \xi_i} + \frac{\partial w_k}{\partial \xi_i} \frac{\partial w_k}{\partial \xi_j} \right); \quad (7)$$

Eyler koordinatalarida

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i} - \frac{\partial w_k}{\partial x_i} \frac{\partial w_k}{\partial x_j} \right). \quad (8)$$

Cheksiz kichik deformatsiya holida (5) va (6) dagi kvadratik hadlar tashlab yuborilishi mumkin.

Deformatsiya tezliklari tenzori komponentalari

$$e_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial \xi_j} + \frac{\partial v_j}{\partial \xi_i} \right). \quad (8)$$

Masala. Tutash muhit harakati Lagranj o'zgaruvchilarida $x_1 = \xi_1 e^t + \xi_3 (e^t - 1)$; $x_2 = \xi_2 + \xi_3 (e^t - e^{-t})$; $x_3 = \xi_3$ ko'rinishda berilgan. Uni Eyler ko'rinishga o'tkazing. Lagrang va Eyler koordinatalarida ko'chish vektori komponentalarini toping. Lagrang koordinatalarida tezlik, tezlanish, deformatsiya va deformatsiya tezliklari tenzori komponentalarini toping.

Yechish. Harakatni Eyler ko'rinishida ifodalash uchun dastlab yakobianning noldan farqliligini tekshiramiz

$$J = \left| \frac{\partial x_i}{\partial \xi_j} \right| = \begin{vmatrix} e^t & 0 & e^t - 1 \\ 0 & 1 & e^t - e^{-t} \\ 0 & 0 & 1 \end{vmatrix} = e^t \neq 0$$

Demak, Eyler koordinatalariga o'tish mumkin.

$x_1 = \xi_1 e^t + \xi_3 (e^t - 1)$; $x_2 = \xi_2 + \xi_3 (e^t - e^{-t})$; $x_3 = \xi_3$ tenglamalarni ξ_j larga nisbatan yechamiz.

$$\begin{aligned} \xi_1 &= x_1 e^{-t} - x_3 (1 - e^{-t}); \\ \xi_2 &= x_2 - x_3 (e^t - e^{-t}); \\ \xi_3 &= x_3. \end{aligned}$$

Endi ko'chish vektori komponentalarini topamiz. $w_i = x_i - \xi_i$ formulaga ko'ra

Lagranj koordinatalarida

$$\begin{aligned} w_1 &= \xi_1 (e^t - 1) + \xi_3 (e^t - 1); \\ w_2 &= \xi_3 (e^t - e^{-t}); \\ w_3 &= 0. \end{aligned}$$

Eyler koordinatalarida

$$\begin{aligned} w_1 &= x_1 (1 - e^{-t}) + x_3 (1 - e^{-t}); \\ w_2 &= x_3 (e^t - e^{-t}); \\ w_3 &= 0. \end{aligned}$$

Tezlik vektori komponentalari

$$\begin{aligned} v_1 &= \xi_1 (e^t + 1) + \xi_3 (e^t - 1); \\ v_2 &= \xi_3 (e^t + e^{-t}); \\ v_3 &= 0. \end{aligned}$$

Tezlanish vektori komponentalari

$$\begin{aligned} a_1 &= \xi_1 (e^t + 1) + \xi_3 (e^t - 1); \\ a_2 &= x_3 (e^t - e^{-t}); \\ a_3 &= 0. \end{aligned}$$

Deformatsiya tenzori komponentalari

$$\varepsilon_{11} = (e^t - 1), \quad \varepsilon_{22} = 0, \quad \varepsilon_{33} = 0, \quad \varepsilon_{12} = \varepsilon_{21} = 0, \quad \varepsilon_{13} = \varepsilon_{31} = (e^t - 1), \quad \varepsilon_{23} = \varepsilon_{32} = (e^t - e^{-t}).$$

Deformatsiya tezliklari tenzori komponentalari

$$e_{11} = (e^t + 1), \quad e_{22} = 0, \quad e_{33} = 0, \quad e_{12} = e_{21} = 0, \quad e_{13} = e_{31} = (e^t - 1), \quad e_{23} = e_{32} = (e^t + e^{-t}).$$

3-MUSTAQIL ISH TOPSHIRIQLARI

Muhit harakati $x = x(\xi_1, \xi_2, \xi_3)$ qonuniyat bilan berilgan. Lagrang va Eyler koordinatalarida:

1. ko'chish vektori komponentalarini;
2. tezlik vektori komponentalarini;
3. tezlanish vektori komponentalarini;
4. deformatsiya tezliklari tenzori komponentalarini

toping.

1. $x_1 = \xi_1$; $x_2 = \xi_2 e^t + A\xi_3$; $x_3 = \xi_3 e^t + A\xi_2$,
2. $x_1 = \xi_1 A + \xi_3 e^{-t}$; $x_2 = \xi_2 e^{kt}$; $x_3 = \xi_3 + A\xi_1$,
3. $x_1 = \xi_1 e^t + A\xi_2$; $x_2 = A\xi_1$; $x_3 = \xi_3 e^{-t}$,
4. $x_1 = \xi_1 + \xi_3 (e^t - 1)$; $x_2 = \xi_2 + \xi_3 (e^{2t} - e^{-2t})$; $x_3 = e^t \xi_3 + A\xi_2$,
5. $x_1 = 3\xi_1 \xi_3 e^{-t^2}$; $x_2 = 2\xi_1 \xi_2 e^{-t^2}$; $x_3 = 5\xi_2 \xi_3 e^{-t}$,
6. $x_1 = (3\xi_2 - 4\xi_3) e^{-t}$; $x_2 = (2\xi_1 - \xi_3) e^{-t}$; $x_3 = (4\xi_2 - \xi_1) e^{-t}$,
7. $x_1 = (\xi_1 A + \xi_3) e^{-t}$; $x_2 = \xi_2 e^{-t^2}$; $x_3 = (\xi_3 + A\xi_1) e^t$,
8. $x_1 = (\xi_1 + A\xi_2) e^t$; $x_2 = A\xi_1 e^{-t^2}$; $x_3 = \xi_3$,
9. $x_1 = \xi_1 e^t + \xi_3 (e^t - 1)$; $x_2 = \xi_2 + \xi_3 (e^t - e^{-t})$; $x_3 = \xi_3$,
10. $x_1 = k\xi_1 \xi_3 e^{-t}$; $x_2 = k\xi_1 \xi_2 e^t$; $x_3 = k\xi_2 \xi_3$,
11. $x_1 = \xi_1 e^{-t}$; $x_2 = (\xi_2 + A\xi_3) e^t$; $x_3 = \xi_3 + A\xi_2$,
12. $x_1 = (\xi_1 A + \xi_3) e^{-t}$; $x_2 = \xi_2$; $x_3 = (\xi_3 + A\xi_1) e^{-t}$,
13. $x_1 = \xi_1 + A\xi_2 e^t$; $x_2 = A\xi_1 e^t$; $x_3 = \xi_3$,
14. $x_1 = \xi_1 + \xi_3 (e^2 - 1)$; $x_2 = \xi_2 + \xi_3 (e^2 - e^{-2})$; $x_3 = \xi_3$,
15. $x_1 = e^{-t} \xi_3$; $x_2 = k\xi_1$; $x_3 = k\xi_3 (e^{-t} + e^t)$,
16. $x_1 = 3\xi_1 \xi_3 e^{-t^2}$; $x_2 = 2\xi_1 \xi_2 e^{-t^2}$; $x_3 = 5\xi_2 \xi_3 e^{-t}$,
17. $x_1 = \xi_2 e^{-t}$, $x_2 = (\xi_2 + A\xi_3) e^t$, $x_3 = \xi_3$,
18. $x_1 = (3\xi_2 - 4\xi_3) e^{-t}$, $x_2 = (2\xi_1 - \xi_3) e^{-2t}$, $x_3 = 4\xi_2 e^{-t}$,
19. $x_1 = \xi_1 + \xi_3 (e^t - 1)$; $x_2 = \xi_2 + \xi_3 (e^{2t} - e^{-2t})$; $x_3 = e^t \xi_3 + A\xi_2$,

20. $x_1 = (\xi_1 A + \xi_3)e^{-t}$; $x_2 = \xi_2 e^{-t^2}$; $x_3 = (\xi_3 + A\xi_1)e^t$,
21. $x_1 = k\xi_1\xi_3 e^{-t}$; $x_2 = \xi_1\xi_2 e^t$; $x_3 = k\xi_2\xi_3$,
22. $x_1 = \xi_1 + \xi_2 e^t$; $x_2 = A\xi_1 e^t$; $x_3 = k\xi_3$,
23. $x_1 = \xi_1 + \xi_3 e^{-t}$; $x_2 = \xi_2 e^{kt}$; $x_3 = \xi_3 + A\xi_1$,
24. $x_1 = (\xi_1 A + \xi_3)e^{-kt}$; $x_2 = \xi_2$; $x_3 = (\xi_3 + A\xi_1)e^{-kt}$,

Adabiyotlar

1. «Механика сплошной среды в примерах и задачах» . Учебное пособие. У.Г.У. Свердловск, 1979 г.
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3. Мейз. Дж. Теория и задачи механики сплошной среды.- М.: Мир, 1974 г.
4. Седов Л.И. Механика сплошной среды. - М.: Наука, 1973 г. В I,II-т.

4 – MUSTAQIL ISH

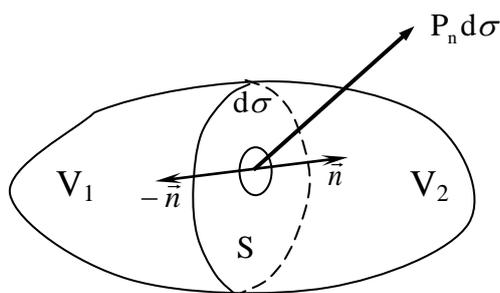
MAVZU: KUCHLANISH TENZORI VA KUCHLANISH VEKTORI

I. NAZARIY QISM

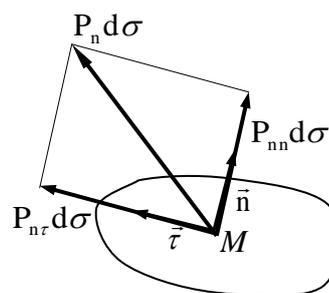
Tutash muhit mexanikasi fani bo'yicha ma'ruzalardan ma'lumki 1-chizmada tasvirlangan \vec{P}_n vektor kuchlanish vektori bo'ladi. Uni $d\sigma$ elementar yuzachaning normal \vec{n} ($|\vec{n}|=1$) va urinmasi $\vec{\tau}$ ($|\vec{\tau}|=1$) bo'yicha tuzuvchilarga ajratish mumkin (2-chizma).

$$\vec{P}_n = P_{nn}\vec{n} + P_{n\tau}\vec{\tau} \quad (1)$$

bu yerda \vec{P}_n - M nuqtaga qo'yilgan normal \vec{n} bo'lgan yuzachadagi kuchlanish vektori, P_{nn} va $P_{n\tau}$ lar mos ravishda normal va urinma kuchlanishlar deyiladi.



1-chizma



2-chizma

Kuchlanish vektori uchun quyidagi Koshi formulasi o'rinli

$$\vec{P}_n = \vec{P}_1 n_1 + \vec{P}_2 n_2 + \vec{P}_3 n_3 \quad (2)$$

bunda n_i - berilgan yuza normal \vec{n} yo'naltiruvchi kosinuslari; \vec{P}_i - M nuqtaga qo'yilgan va koordinata o'qlariga parallel yo'nalgan kuchlanish vektorlari. \vec{P}_i vektorlarni \vec{e}_j bazislar bo'yicha yoyib chiqamiz.

$$\vec{P}_i = \sigma_{ij} \vec{e}_j$$

Bundan

$$\begin{aligned} P_{n1} &= \sigma_{11}n_1 + \sigma_{12}n_2 + \sigma_{13}n_3; \\ P_{n2} &= \sigma_{21}n_1 + \sigma_{22}n_2 + \sigma_{23}n_3; \\ P_{n3} &= \sigma_{31}n_1 + \sigma_{32}n_2 + \sigma_{33}n_3. \end{aligned} \quad (3)$$

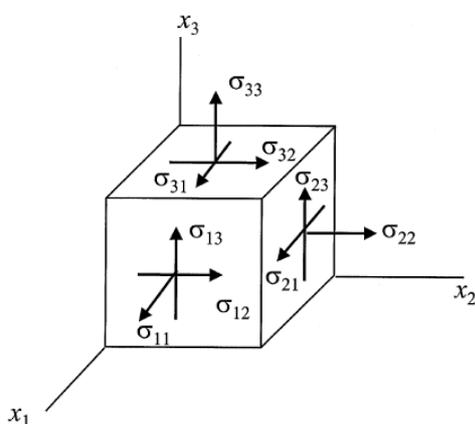
Koshi formulasiidan foydalanib, kuchlanishning normal va urinma tuzuvchilarini topamiz

$$P_{nn} = (\bar{P}_n \vec{n}) = \sigma_{ij} n_j n_i = (n_1, n_2, n_3) (\sigma_{ij}) \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}, \quad (4)$$

$$P_{n\tau} = \left((P_{n1})^2 + (P_{n2})^2 + (P_{n3})^2 - (P_{nn})^2 \right)^{\frac{1}{2}} \quad (5)$$

$Ox_1x_2x_3$ Dekart koordinatalar sistemasida berilgan (σ_{ij}) miqdorlar majmuasi ikkinchi rang tenzorni tashkil qiladi va kuchlanish tenzori deb ataladi

$$(\sigma_{ij}) = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} \quad (6)$$



3-chizma

Ichki harakat miqdori momentlar va juftlar mavjud bo'lmagan holda kuchlanish tenzori simmetrik bo'ladi va $\sigma_{11}, \sigma_{22}, \sigma_{33}$ lar x_1, x_2, x_3 koordinata o'qlariga perpendikulyar yuzalardagi normal kuchlanishlar, $\sigma_{12} = \sigma_{21}, \sigma_{13} = \sigma_{31}, \sigma_{23} = \sigma_{32}$ esa urinma kuchlanishlar deyiladi.

Kuchlanish tenzorining koordinatalarni almashtirishga nisbatan invariantlari 3 ta bo'lib, ular quyidagicha aniqlanadi

$$\begin{aligned} I_1 &= \sigma_1 + \sigma_2 + \sigma_3; \\ I_2 &= \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{13} \\ \sigma_{31} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{32} & \sigma_{33} \end{vmatrix}; \\ I_3 &= \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix}. \end{aligned} \quad (7)$$

Kuchlanish tenzorining bosh qiymatlari

$$\begin{vmatrix} \sigma_{11} - \sigma & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \sigma & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \sigma \end{vmatrix} = 0. \quad (8)$$

yoki

$$\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0$$

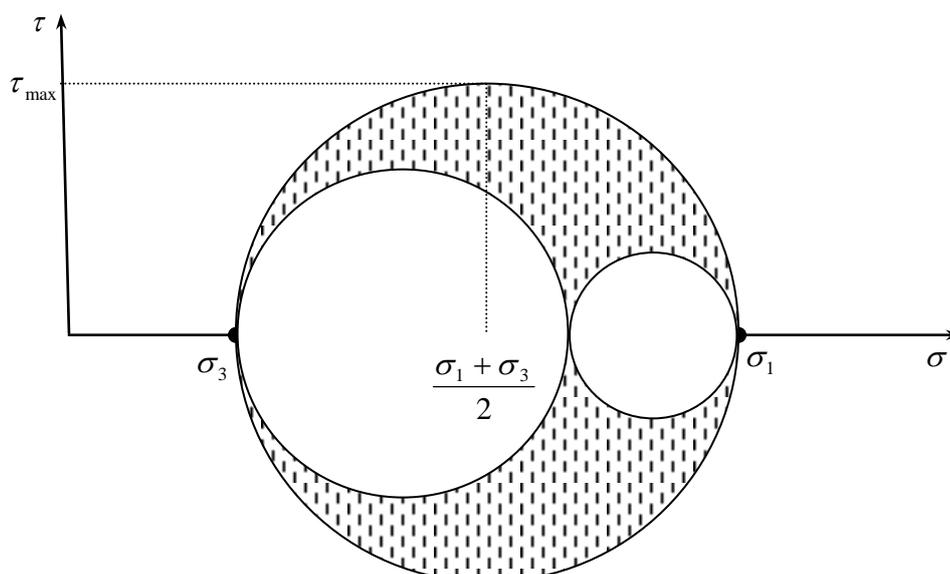
Tenglamaning yechimlari bo'ladi va $\sigma_1, \sigma_2, \sigma_3$ kabi belgilanadi. Topilgan bosh qiymatlarga mos bosh yo'nalishlar esa quyidagi tenglamalardan topiladi

$$\begin{aligned}(\sigma_{11} - \sigma)n_1 + \sigma_{12}n_2 + \sigma_{13}n_3 &= 0; \\ \sigma_{21}n_1 + (\sigma_{22} - \sigma)n_2 + \sigma_{23}n_3 &= 0; \\ \sigma_{31}n_1 + \sigma_{32}n_2 + (\sigma_{33} - \sigma)n_3 &= 0; \\ n_1^2 + n_2^2 + n_3^2 &= 1.\end{aligned}\tag{9}$$

Demak koordinatalarni $A = \begin{pmatrix} n_1^{(1)} & n_2^{(1)} & n_3^{(1)} \\ n_1^{(2)} & n_2^{(2)} & n_3^{(2)} \\ n_1^{(3)} & n_2^{(3)} & n_3^{(3)} \end{pmatrix}$ kabi almashtirganda kuchlanish

tenzori $(\sigma'_{ij}) = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$ ko'rinishga keladi.

Bosh kuchlanishlarni kamayish tartibida raqamlab $\sigma_1 > \sigma_2 > \sigma_3$ va ular yordamida radiuslari $\frac{\sigma_1 - \sigma_3}{2}$; $\frac{\sigma_1 - \sigma_2}{2}$; $\frac{\sigma_2 - \sigma_3}{2}$ ga teng bo'lgan doiralarni chizamiz

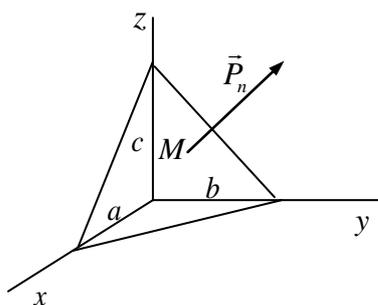


4-chizma

Chizmada σ normal, τ urinma kuchlanishni bildiradi. Shtrixlangan sohadagi qiymatlarni urinma kuchlanish qabul qilishi mumkin. Ko'rinib turibdiki, normal kuchlanish $\frac{\sigma_1 + \sigma_3}{2}$ ga teng bo'lganda, urinma kuchlanish $\frac{\sigma_1 - \sigma_3}{2}$ ga teng bo'lgan maksimal qiymatga erishadi.

II. TOPSHIRIQNI BAJARISH NAMUNASI

Masala. $Oxyz$ Dekart koordinatalar sistemasida tutash muhitning M nuqtasidagi (σ_{ij}) kuchlanish tenzori berilgan. Koordinata o'qlarini koordinata boshidan mos ravishda a, b, c masofalarda kesib o'tuvchi yuzachadagi kuchlanish vektorini, normal va urinma kuchlanishlarni, shuningdek kuchlanish tenzorining invariantlarini, bosh qiymatlari va unga mos bosh yo'nalishlarini toping. Mor doirasi



5-chizma

yordamida maksimal va minimal urinma kuchlanishlarni aniqlang.

$$(\sigma_{ij}) = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad a = 4, \quad b = 6, \quad c = 2$$

Yechish. Analitik geometriya kursidan ma'lumki, koordinata o'qlarini kesib o'tuvchi tekislik tenglamasi quyidagi ko'rinishda bo'ladi

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Ushbu tenglamaga berilganlarni qo'yib, uni kanonik ko'rinishga keltiramiz

$$3x + 2y + 6z - 12 = 0$$

Ko'rinib turibdiki, berilgan tekislikka $\vec{N}(3,2,6)$ vektor perpendikulyar bo'ladi. Unga mos birlik vektori quyidagicha aniqlanadi

$$\vec{n} = \left(\frac{N_1}{N}, \frac{N_2}{N}, \frac{N_3}{N} \right)$$

va demak $\vec{n} = \left(\frac{3}{7}, \frac{2}{7}, \frac{6}{7} \right)$

Endi (3) formulalar yordamida kuchlanish vektori komponentalarini topamiz

$$P_{n1} = 3 \cdot \frac{3}{7} - 1 \cdot \frac{2}{7} + 0 \cdot \frac{6}{7} = 1,$$

$$P_{n2} = -1 \cdot \frac{3}{7} + 3 \cdot \frac{2}{7} + 0 \cdot \frac{6}{7} = \frac{3}{7},$$

$$P_{n3} = 0 \cdot \frac{3}{7} + 0 \cdot \frac{2}{7} + 1 \cdot \frac{6}{7} = \frac{6}{7}.$$

$$\vec{P}_n = \left(1, \frac{3}{7}, \frac{6}{7} \right)$$

(4) va (5) formulalarga ko'ra normal va urinma kuchlanishlarni topamiz

$$P_{nn} = \left(\frac{3}{7}, \frac{2}{7}, \frac{6}{7} \right) \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{3}{7} \\ \frac{2}{7} \\ \frac{6}{7} \end{pmatrix} = \frac{9}{7} \quad P_{n\tau} = \left(1^2 + \left(\frac{3}{7} \right)^2 + \left(\frac{6}{7} \right)^2 - \left(\frac{9}{7} \right)^2 \right)^{\frac{1}{2}} = \frac{\sqrt{13}}{7}$$

(7) formula yordamida berilgan kuchlanish tenzorining bosh qiymatlarini topamiz

$$\begin{vmatrix} 3-\lambda & -1 & 0 \\ -1 & 3-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

Bu tenglama yechimlari $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 4$

Topilgan bosh qiymatlarga mos bosh yo'nalishlar (o'qlar) ni topamiz

$\lambda_1 = 4$ uchun

$$\begin{cases} -n_1 - n_2 = 0 \\ -n_1 - n_2 = 0 \\ -3n_3 = 0 \\ n_1^2 + n_2^2 + n_3^2 = 1 \end{cases} \Rightarrow \vec{n} = \left(\pm \frac{\sqrt{2}}{2}, \mp \frac{\sqrt{2}}{2}, 0 \right)$$

$\lambda_2 = 2$ uchun

$$\begin{cases} n_1 - n_2 = 0 \\ -n_1 + n_2 = 0 \\ -n_3 = 0 \\ n_1^2 + n_2^2 + n_3^2 = 1 \end{cases} \Rightarrow \vec{n} = \left(\pm \frac{\sqrt{2}}{2}, \pm \frac{\sqrt{2}}{2}, 0 \right)$$

$\lambda_3 = 1$ uchun

$$\begin{cases} 2n_1 - n_2 = 0 \\ -n_1 + 2n_2 = 0 \\ n_1^2 + n_2^2 + n_3^2 = 1 \end{cases} \Rightarrow \vec{n} = (0, 0, \pm 1)$$

Natijaning to'g'riligini bitta holda koordinatalarni almashtirish yordamida tekshiramiz

$$\begin{aligned} (p'_{ij}) &= \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \\ &= \begin{pmatrix} 2\sqrt{2} & -2\sqrt{2} & 0 \\ \sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned}$$

Demak koordinatalarni $A = \begin{pmatrix} \pm \frac{1}{\sqrt{2}} & \mp \frac{1}{\sqrt{2}} & 0 \\ \pm \frac{1}{\sqrt{2}} & \pm \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & \pm 1 \end{pmatrix}$ kabi almshtirganimizda

kuchlanish tenzori

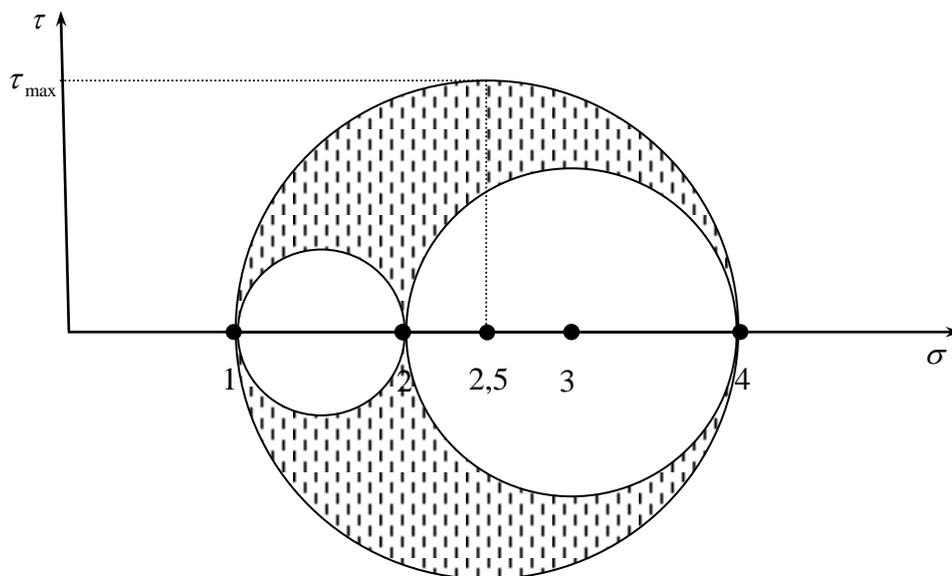
$$(p'_{ij}) = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ko'rinishni olar ekan.

Endi bosh kuchlanishlar yordamida maksimal urinma kuchlanishlarni aniqlaymiz. Buning uchun radiuslari

$$\frac{p_1 - p_3}{2} = \frac{4 - 1}{2} = 1,5; \quad \frac{p_1 - p_2}{2} = \frac{4 - 2}{2} = 1; \quad \frac{p_2 - p_3}{2} = \frac{2 - 1}{2} = 0,5$$

bo'lgan doiralarni chizamiz



Chizmadan ko'rinadiki, urinma kuchlanish $\sigma = 2,5$ bo'lganda $\tau_{\max} = 1,5$ maksimal qiymatga erishadi. Shuningdek, normal kuchlanish bosh kuchlanishlarga teng bo'lganda urinma kuchlanishlar nolga teng bo'ladi.

III. 4-MUSTAQIL ISH TOPSHIRIQLARI

Masala Tutash muhitning M nuqtasida P kuchlanish tenzori berilgan x, y, z koordinata o'qlarini koordinata boshidan mos ravishda a, b, c masofalarda kesib o'tuvchi yuzada kuchlanish vektorini, normal va urinma kuchlanishlarni, shuningdek kuchlanish tenzorining bosh qiymatlari va unga mos bosh yo'nalishlarini toping.

$$1. P = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad a=4, b=6, c=2$$

$$2. P = \begin{pmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \quad a=3, b=3, c=2$$

$$3. P = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad a=-4, b=3, c=2$$

$$4. P = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad a=5, b=4, c=2$$

$$5. P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 1 & 3 \end{pmatrix}, \quad a=4, b=4, c=3$$

$$6. P = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \quad a=4, b=1, c=1$$

$$7. P = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}, \quad a=4, b=4, c=4$$

$$8. P = \begin{pmatrix} 1 & 0 & -2 \\ -0 & 5 & 0 \\ -2 & 0 & 1 \end{pmatrix}, \quad a=3, b=5, c=5$$

$$9. P = \begin{pmatrix} 2 & -1 & -2 \\ -1 & 0 & 0 \\ -2 & 0 & 2 \end{pmatrix}, \quad a=3, b=3, c=2$$

$$10. P = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad a=2, b=4, c=2$$

$$11. P = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix}, \quad a=4, b=6, c=2$$

$$12. P = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad a=4, b=-6, c=-2$$

$$13. P = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}, \quad a=4, b=6, c=2$$

$$14. P = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad a=4, b=6, c=2$$

$$15. P = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad a=-4, b=-6, c=2$$

$$16. P = \begin{pmatrix} 3 & -1 & -1 \\ -1 & 0 & -2 \\ -1 & -2 & 0 \end{pmatrix}, \quad a=-3, b=6, c=-2$$

$$17. P = \begin{pmatrix} 5 & 0 & 0 \\ 0 & -6 & -12 \\ 0 & -12 & 8 \end{pmatrix}, \quad a=-1, b=-6, c=1$$

$$18. P = \begin{pmatrix} 3 & -3 & 0 \\ -3 & 0 & 1 \\ 0 & 1 & 8 \end{pmatrix}, \quad a=4, b=-6, c=2$$

$$19. P = \begin{pmatrix} 3 & 2 & 0 \\ 2 & 3 & 4 \\ 0 & 4 & 3 \end{pmatrix}, \quad a=4, b=6, c=2$$

$$20. P = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad a=3, b=-6, c=-2$$

$$21. P = \begin{pmatrix} 3 & -5 & 2 \\ -5 & 2 & 0 \\ 2 & 0 & 0 \end{pmatrix}, \quad a=4, b=-4, c=4$$

$$22. P = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}, \quad a=1, b=1, c=1$$

$$23. P = \begin{pmatrix} 4 & 1 & 4 \\ 1 & 4 & 0 \\ 4 & 0 & 4 \end{pmatrix}, \quad a=4, b=-3, c=2$$

$$24. P = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad a=-4, b=1, c=2$$

$$25. P = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -3 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \quad a = -1, \quad b = -1, \quad c = 1$$

$$26. P = \begin{pmatrix} 5 & 1 & 0 \\ 1 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}, \quad a = 4, \quad b = 8, \quad c = 2$$

$$27. P = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad a = 2, \quad b = -2, \quad c = 2$$

$$28. P = \begin{pmatrix} 7 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 5 \end{pmatrix}, \quad a = 3, \quad b = 1, \quad c = 1$$

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5 – MUSTAQIL ISH

MAVZU: ELASTIK JISM UCHUN ASOSIY MUNOSABATLAR

Maqsad: Mavzu bo'yicha talabalar bilimini sinash, ko'chish va deformatsiya, deformatsiya va kuchlanish tenzori komponentalari orasidagi munosabatlarni topish davomida ortogonal egri chiziqli koordinatalarda ishlash ko'nikmalarini hosil qilish, ularning bilimni mustahkamlash.

I. NAZARIY QISM

REJA

1. Dekart koordinatalarida ko'chish va deformatsiya orasidagi munosabatlar.
2. Silindrik koordinatalarda ko'chish va deformatsiya orasidagi munosabatlar.
3. Sferik koordinatalarda ko'chish va deformatsiya orasidagi munosabatlar.
4. Deformatsiya va kuchlanish orasidagi munosabatlar.

Tutash muhitlar mexanikasi ma'ruzalaridan ma'lumki, deformatsiyalar kichik bo'lganda chiziqlimas hadlarni hisoblamalik mumkin. Tutash muhit ko'chish vektori komponentalari va deformatsiya tenzori komponentalari orasida quyidagi geometrik munosabatlar (Koshi munosabatlari) o'rinli:

Dekart koordinatalarida

$$\begin{aligned} \varepsilon_{xx} = \frac{\partial U_x}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial U_y}{\partial y}, \quad \varepsilon_{zz} = \frac{\partial U_z}{\partial z}, \quad \varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2} \left(\frac{\partial U_x}{\partial y} + \frac{\partial U_y}{\partial x} \right), \\ \varepsilon_{xz} = \varepsilon_{zx} = \frac{1}{2} \left(\frac{\partial U_x}{\partial z} + \frac{\partial U_z}{\partial x} \right), \quad \varepsilon_{yz} = \varepsilon_{zy} = \frac{1}{2} \left(\frac{\partial U_y}{\partial z} + \frac{\partial U_z}{\partial y} \right) \end{aligned} \quad ; \quad (1)$$

Silindrik koordinatalarda

$$\begin{aligned} \varepsilon_{rr} = \frac{\partial U_r}{\partial r}, \quad \varepsilon_{\varphi\varphi} = \frac{1}{r} \frac{\partial U_\varphi}{\partial \varphi} + \frac{U_r}{r}, \quad \varepsilon_{zz} = \frac{\partial U_z}{\partial z}, \quad 2\varepsilon_{r\varphi} = \frac{1}{r} \frac{\partial U_r}{\partial \varphi} + r \frac{\partial}{\partial r} \left(\frac{U_\varphi}{r} \right), \\ 2\varepsilon_{\varphi z} = \frac{\partial U_\varphi}{\partial z} + \frac{1}{r} \frac{\partial U_z}{\partial \varphi}, \quad 2\varepsilon_{rz} = \frac{\partial U_z}{\partial r} + \frac{\partial U_r}{\partial z} \end{aligned} \quad (2)$$

Sferik koordinatalarda

$$\begin{aligned} \varepsilon_{rr} = \frac{\partial U_r}{\partial r}, \quad \varepsilon_{\varphi\varphi} = \frac{1}{r} \frac{\partial U_\varphi}{\partial \varphi} + \frac{U_r}{r}, \quad \varepsilon_{\theta\theta} = \frac{1}{r \sin \varphi} \frac{\partial U_\theta}{\partial \theta} + \frac{U_r}{r} + \frac{U_\varphi \operatorname{ctg} \varphi}{r}, \quad 2\varepsilon_{r\varphi} = \frac{1}{r} \frac{\partial U_r}{\partial \varphi} + \frac{\partial U_\varphi}{\partial r} - \frac{U_\varphi}{r}, \\ 2\varepsilon_{\theta r} = \frac{\partial U_\theta}{\partial r} + \frac{1}{r \sin \varphi} \frac{\partial U_r}{\partial \varphi} - \frac{U_\theta}{r}, \quad 2\varepsilon_{\theta\varphi} = \frac{1}{r \sin \varphi} \frac{\partial U_\varphi}{\partial \theta} + \frac{1}{r} \frac{\partial U_\theta}{\partial \varphi} - \frac{U_\theta \operatorname{ctg} \varphi}{r}. \end{aligned} \quad (3)$$

Deformasiya va kuchlanish tenzori komponentalari orasida quyidagi fizik munosabatlar (Guk qonuni o'rinli):

$$\sigma_{ii} = \lambda I_1(\varepsilon) + 2\mu\varepsilon_{ij}, \quad \sigma_{ij} = 2\mu\varepsilon_{ij} \quad (i \neq j) \quad (4)$$

bunda dekart koordinatalarida $I_1(\varepsilon) = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$, silindrik koordinatalarda $I_1(\varepsilon) = \varepsilon_{rr} + \varepsilon_{\varphi\varphi} + \varepsilon_{zz}$, sferik koordinatalarda $I_1(\varepsilon) = \varepsilon_{rr} + \varepsilon_{\varphi\varphi} + \varepsilon_{\theta\theta}$, λ, μ - Lamé koefitsiyentlari.

II. TORSHIRIQNI BAJARISH NAMUNASI

Masala Tutash muhit ko'chish vektori komponentalari U_i Dekart, silindrik va sferik koordinatalar sistemasida berilgan. Deformasiya va kuchlanish tenzori komponentalarini toping.

Dekart koordinatalarida
$$U_x = \frac{x^2}{y}, \quad U_y = \frac{y^2}{z}, \quad U_z = \frac{z^2}{x};$$

Silindrik koordinatalarda
$$U_r = r \sin \varphi, \quad U_\varphi = \frac{z^2}{r} \cos \varphi, \quad U_z = \frac{r^2}{z} \cos \varphi;$$

Sferik koordinatalarda
$$U_r = r \sin \varphi \cos \theta, \quad U_\varphi = r \cos \varphi \sin \theta, \quad U_\theta = r \sin \theta.$$

Yechish Masalani yechishda dekart koordinatalari uchun

$$U_x = x^2 y, \quad U_y = yz, \quad U_z = xyz$$

(1) ga ko'ra deformasiya tenzori komponentalarini topamiz

$$\varepsilon_{xx} = \frac{2x}{y}, \quad \varepsilon_{yy} = \frac{2y}{z}, \quad \varepsilon_{zz} = \frac{2z}{x}, \quad \varepsilon_{xy} = -\frac{x^2}{2y^2}, \quad \varepsilon_{yz} = -\frac{y^2}{2z^2}, \quad \varepsilon_{xz} = -\frac{z^2}{2x^2}$$

(4) formulalardan foydalanib kuchlanish tenzori komponentalarini topamiz

$$\sigma_{xx} = \lambda \left(\frac{2x}{y} + \frac{2y}{z} + \frac{2z}{x} \right) + 4\mu \frac{x}{y}, \quad \sigma_{yy} = \lambda \left(\frac{2x}{y} + \frac{2y}{z} + \frac{2z}{x} \right) + 4\mu \frac{y}{z},$$

$$\sigma_{zz} = \lambda \left(\frac{2x}{y} + \frac{2y}{z} + \frac{2z}{x} \right) + 4\mu \frac{z}{x}, \quad \sigma_{xy} = -\mu \frac{x^2}{2y^2}, \quad \sigma_{yz} = -\mu \frac{y^2}{2z^2}, \quad \sigma_{xz} = -\mu \frac{z^2}{2x^2}$$

Silindrik koordinatalarda

$$U_r = r \sin \varphi, \quad U_\varphi = \frac{z^2}{r} \cos \varphi, \quad U_z = \frac{r^2}{z} \cos \varphi$$

(2) ga ko'ra deformasiya tenzori komponentalarini topamiz

$$\begin{aligned}\varepsilon_{rr} &= \sin \varphi, & \varepsilon_{\varphi\varphi} &= -\frac{z^2}{r^2} \sin \varphi, & \varepsilon_{zz} &= -\frac{r^2}{z^2} \cos \varphi, & \varepsilon_{r\varphi} &= \frac{1}{2} \left(1 - \frac{2z^2}{r^2} \right) \cos \varphi, \\ \varepsilon_{\varphi z} &= \frac{1}{2} \left(\frac{2z}{r} \cos \varphi - \frac{r}{z} \sin \varphi \right), & \varepsilon_{zr} &= \frac{r}{z} \cos \varphi\end{aligned}$$

(4) formulalardan foydalanib kuchlanish tenzori komponentalarini topamiz

$$\begin{aligned}\sigma_{rr} &= \lambda \left(\sin \varphi - \frac{z^2}{r^2} \sin \varphi - \frac{r^2}{z^2} \cos \varphi \right) + 2\mu \sin \varphi, & \sigma_{r\varphi} &= \mu \left(1 - \frac{2z^2}{r^2} \right) \cos \varphi, \\ \sigma_{\varphi\varphi} &= \lambda \left(\sin \varphi - \frac{z^2}{r^2} \sin \varphi - \frac{r^2}{z^2} \cos \varphi \right) - 2\mu \frac{z^2}{r^2} \sin \varphi, & \sigma_{\varphi z} &= \mu \left(\frac{2z}{r} \cos \varphi - \frac{r}{z} \sin \varphi \right), \\ \sigma_{zz} &= \lambda \left(\sin \varphi - \frac{z^2}{r^2} \sin \varphi - \frac{r^2}{z^2} \cos \varphi \right) - 2\mu \frac{r^2}{z^2} \sin \varphi, & \sigma_{zr} &= \mu \frac{r}{z} \cos \varphi.\end{aligned}$$

Sferik koordinatalarda

$$U_r = r \sin \varphi \cos \theta, \quad U_\varphi = r \cos \varphi \sin \theta, \quad U_\theta = r \sin \theta$$

(2) ga ko'ra deformatsiya tenzori komponentalarini topamiz

$$\begin{aligned}\varepsilon_{rr} &= \sin \varphi \cos \theta, \quad \varepsilon_{\varphi\varphi} = 0, \quad \varepsilon_{\theta\theta} = \frac{\cos \theta}{\sin \varphi} + \sin \varphi \cos \theta + \frac{\cos^2 \varphi \sin \theta}{\sin \varphi} \\ \varepsilon_{r\varphi} &= \frac{1}{2} \cos \varphi \cos \theta, \quad \varepsilon_{\theta r} = \frac{1}{2} (\sin \theta + \operatorname{ctg} \varphi - \cos \theta), \quad \varepsilon_{\theta\varphi} = \frac{1}{2} (\operatorname{ctg} \varphi \cos \theta - \sin \theta \operatorname{ctg} \varphi)\end{aligned}$$

(4) formulalardan foydalanib kuchlanish tenzori komponentalarini topamiz

$$\begin{aligned}\sigma_{rr} &= \lambda \left(2 \sin \varphi \cos \theta + \frac{\cos \theta}{\sin \varphi} + \frac{\cos^2 \varphi \sin \theta}{\sin \varphi} \right) + 2\mu \sin \varphi \cos \theta, \\ \sigma_{\varphi\varphi} &= \lambda \left(2 \sin \varphi \cos \theta + \frac{\cos \theta}{\sin \varphi} + \frac{\cos^2 \varphi \sin \theta}{\sin \varphi} \right), \\ \sigma_{\theta\theta} &= \lambda \left(2 \sin \varphi \cos \theta + \frac{\cos \theta}{\sin \varphi} + \frac{\cos^2 \varphi \sin \theta}{\sin \varphi} \right) + 2\mu \left(\frac{\cos \theta}{\sin \varphi} + \sin \varphi \cos \theta + \frac{\cos^2 \varphi \sin \theta}{\sin \varphi} \right), \\ \sigma_{r\varphi} &= \mu \cos \varphi \cos \theta, \quad \sigma_{\theta r} = \mu (\sin \theta + \operatorname{ctg} \varphi - \cos \theta), \quad \sigma_{\theta\varphi} = \mu (\operatorname{ctg} \varphi \cos \theta - \sin \theta \operatorname{ctg} \varphi)\end{aligned}$$

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5 – MUSTAQIL ISH VARIANTLARI

Dekart koordinatalarida

1. $U_x = \frac{x^2}{y+z}, U_y = -\frac{y^2}{x+z}, U_z = \frac{z^2}{x+y};$
2. $U_x = \frac{y^2}{x}, U_y = \frac{z^2}{x+z}, U_z = \frac{x^2}{z+y};$
3. $U_x = \frac{x^3}{y^2+z^2}, U_y = -\frac{y^2}{z}, U_z = \frac{z^2}{y};$
4. $U_x = \frac{x^2}{y+z}, U_y = \frac{z^2}{x+y}, U_z = \frac{y^2}{x+z};$
5. $U_x = \frac{x^3}{y^2}, U_y = \frac{y^2}{x+z}, U_z = \frac{z^3}{x^2+y^2};$
6. $U_x = \frac{yx^2}{z^2}, U_y = \frac{zy^2}{x^2}, U_z = \frac{z^2}{x+y};$
7. $U_x = \frac{yz}{x+z}, U_y = \frac{y^2}{x+z}, U_z = \frac{x^2+y^2}{x+y};$
8. $U_x = \frac{y^2}{x}, U_y = \frac{yz}{x}, U_z = \frac{xy}{z};$
9. $U_x = \frac{xy}{z}, U_y = \frac{xz}{y}, U_z = \frac{xy}{z};$
10. $U_x = \frac{x^2}{z}, U_y = \frac{xy}{z}, U_z = \frac{xz}{y};$
11. $U_x = \frac{xz}{y}, U_y = -\frac{yx}{z}, U_z = \frac{yz}{x};$
12. $U_x = \frac{x^2y}{z^2}, U_y = \frac{xy^2}{z^2}, U_z = \frac{yz^2}{x^2};$
13. $U_x = \frac{xy^2}{z^2}, U_y = \frac{x^2y}{z^2}, U_z = \frac{yz^2}{x^2};$
14. $U_x = \frac{yz^2}{x^2}, U_y = \frac{x^2z}{y^2}, U_z = \frac{yx^2}{z^2};$
15. $U_x = \frac{xy}{z}, U_y = \frac{zy}{x}, U_z = \frac{xz}{y};$

Silindrik koordinatalarda

- $$U_r = r \cos \varphi, U_\varphi = r \sin \varphi, U_z = \frac{r^2}{z} \cos \varphi$$
- $$U_r = r \sin \varphi, U_\varphi = r \cos \varphi, U_z = -r \cos \varphi$$
- $$U_r = r \operatorname{sh} \varphi, U_\varphi = r \cos \varphi, U_z = \frac{r^2}{z} \cos \varphi$$
- $$U_r = r \operatorname{ch} \varphi, U_\varphi = r \sin \varphi, U_z = r \cos \varphi$$
- $$U_r = r \sin \varphi, U_\varphi = -r \operatorname{ch} \varphi, U_z = z \sin \varphi$$
- $$U_r = r \sin^2 \varphi, U_\varphi = r \sin^2 \varphi, U_z = z \cos^2 \varphi$$
- $$U_r = r \operatorname{ch} \varphi, U_\varphi = \frac{z^2}{r} \cos \varphi, U_z = r \cos \varphi$$
- $$U_r = \frac{r^2}{z} \sin^2 \varphi, U_\varphi = \frac{z^2}{r} \cos \varphi, U_z = z \cos^2 \varphi$$
- $$U_r = r \sin^2 \varphi, U_\varphi = \frac{z^2}{r} \cos \varphi, U_z = \frac{z^2}{r} \cos \varphi$$
- $$U_r = \frac{r^2}{z} \sin \varphi, U_\varphi = r \cos \varphi, U_z = z \cos^2 \varphi$$
- $$U_r = r \cos^2 \varphi, U_\varphi = (r+z) \cos \varphi, U_z = \frac{r^2}{z} \sin \varphi$$
- $$U_r = (r+z) \sin \varphi, U_\varphi = r \cos^2 \varphi, U_z = r \cos \varphi$$
- $$U_r = \left(r + \frac{z^2}{r}\right) \sin \varphi, U_\varphi = r \cos \varphi, U_z = \frac{r^2}{z} \sin \varphi$$
- $$U_r = r \sin^2 \varphi, U_\varphi = \frac{r^2}{z} \cos \varphi, U_z = -z \sin^2 \varphi$$
- $$U_r = \frac{r^2}{z} \cos \varphi, U_\varphi = r \sin^2 \varphi, U_z = (z+r) \cos \varphi$$

Sferik koordinatalarda

- $$U_r = r \sin \varphi \sin \theta, U_\varphi = r \sin \varphi \cos \theta, U_\theta = r \sin \theta$$
- $$U_r = r \sin \varphi \cos \theta, U_\varphi = r \cos \varphi \sin \theta, U_\theta = r \cos \varphi$$
- $$U_r = r \sin \varphi \cos \theta, U_\varphi = r \cos \varphi \sin \theta, U_\theta = r \sin \theta \cos \varphi$$
- $$U_r = -r \sin \varphi \cos \theta, U_\varphi = r \cos^2 \varphi, U_\theta = r \sin \theta \cos \varphi$$
- $$U_r = r \sin^2 \varphi, U_\varphi = r \cos \varphi \sin \theta, U_\theta = r \operatorname{sh} \theta \cos \varphi$$
- $$U_r = r \cos \varphi \cos \theta, U_\varphi = -r \cos \varphi \sin \theta, U_\theta = r \cos^2 \varphi$$
- $$U_r = r \cos^2 \varphi, U_\varphi = r \cos \varphi \sin \theta, U_\theta = -r \sin^2 \theta$$
- $$U_r = -r \cos^2 \theta, U_\varphi = -r \cos \varphi \sin \theta, U_\theta = r \cos \theta \cos \varphi$$
- $$U_r = r \sin \varphi \cos \theta, U_\varphi = r \cos^2 \varphi \sin \theta, U_\theta = -r \sin \theta \cos \varphi$$
- $$U_r = r \sin^2 \varphi, U_\varphi = r \cos^2 \varphi, U_\theta = -r \cos \theta \cos \varphi$$
- $$U_r = r \operatorname{sh} \varphi \cos \theta, U_\varphi = r \cos \varphi \sin \theta, U_\theta = r \sin \theta \operatorname{sh} \varphi$$
- $$U_r = r \cos^2 \theta, U_\varphi = -r \cos \varphi \operatorname{sh} \theta, U_\theta = -r \sin \theta \cos \varphi$$
- $$U_r = -r \sin^2 \theta, U_\varphi = r \cos \varphi \operatorname{sh} \theta, U_\theta = -r \sin \theta \cos \varphi$$
- $$U_r = r \operatorname{sh} \varphi \cos \theta, U_\varphi = r \operatorname{ch} \varphi \sin \theta, U_\theta = r \operatorname{sh} \theta \cos \varphi$$
- $$U_r = -r \cos^2 \varphi, U_\varphi = r \cos \varphi \operatorname{sh} \theta, U_\theta = r \sin^2 \theta$$

$$16. U_x = \frac{yz^2}{x^2}, U_y = \frac{x^2y}{z^2}, U_z = \frac{zx^2}{y^2};$$

$$17. U_x = \frac{zy^2}{x^2}, U_y = \frac{yz^2}{x^2}, U_z = \frac{xy^2}{z^2};$$

$$18. U_x = \frac{zy^2}{x^2}, U_y = \frac{yx^2}{z^2}, U_z = \frac{xz^2}{y^2};$$

$$19. U_x = \frac{x^2}{y}, U_y = \frac{z^3}{xy}, U_z = \frac{x^3}{zy};$$

$$20. U_x = \frac{x^3}{yz}, U_y = \frac{y^2}{z}, U_z = \frac{z^3}{xy};$$

$$21. U_x = \frac{yx^2}{z^2}, U_y = \frac{y^2z}{x^2}, U_z = \frac{x^2y^2}{z^3};$$

$$22. U_x = \frac{y^2}{x}, U_y = \frac{yz}{x}, U_z = -\frac{xy}{z};$$

$$23. U_x = \frac{x^2y}{z^2}, U_y = -\frac{x^2y^2}{z^3}, U_z = \frac{y^2z^2}{x^3};$$

$$24. U_x = \frac{xy}{z+y}, U_y = \frac{yz}{x}, U_z = \frac{y^2}{z+x};$$

$$25. U_x = \frac{x^2y}{z^2}, U_y = \frac{x^3}{y^2+z^2}, U_z = \frac{z^2}{x+y};$$

$$26. U_x = -\frac{xy}{z}, U_y = \frac{x^3}{y^2+z^2}, U_z = \frac{y^2}{x+z};$$

$$27. U_x = \frac{x^3}{y^2+z^2}, U_y = \frac{y^2}{x+z}, U_z = \frac{x^2}{z+y};$$

$$28. U_x = \frac{x^2}{y+z}, U_y = \frac{z^2}{x+y}, U_z = \frac{y^2}{x+z};$$

$$29. U_x = \frac{xy}{z}, U_y = \frac{x^2+y^2}{z}, U_z = \frac{y^2+z^2}{x};$$

$$30. U_x = \frac{x^2+z^2}{y}, U_y = \frac{y^2+z^2}{x}, U_z = \frac{x^2}{z};$$

$$U_r = r \operatorname{ch}^2 \varphi, U_\varphi = \frac{z^2}{r} \cos \varphi, U_z = \frac{r^2}{z} \operatorname{sh} \varphi$$

$$U_r = \frac{r^2}{z} \sin \varphi, U_\varphi = r \sin^2 \varphi, U_z = z \sin \varphi$$

$$U_r = r \sin^2 \varphi, U_\varphi = \frac{r^2}{z} \cos \varphi, U_z = \frac{z^2}{r} \sin \varphi$$

$$U_r = r \operatorname{ch} \varphi, U_\varphi = -\frac{z^2}{r} \cos \varphi, U_z = \frac{r^2}{z} \operatorname{sh} \varphi$$

$$U_r = \left(r - \frac{r^2}{z} \right) \sin \varphi, U_\varphi = \frac{z^2}{r} \cos \varphi, U_z = \frac{r^2}{z} \operatorname{ch} \varphi$$

$$U_r = \frac{r^2}{z} \sin \varphi, U_\varphi = z \cos \varphi, U_z = \frac{r^2}{z} \cos \varphi$$

$$U_r = r \sin^2 \varphi, U_\varphi = r \cos^2 \varphi, U_z = \frac{r^2}{z} \cos \varphi$$

$$U_r = \frac{r^2}{z} \sin \varphi, U_\varphi = \frac{z^2}{r} \operatorname{ch} \varphi, U_z = \frac{r^2}{z} \cos \varphi$$

$$U_r = r \operatorname{sh} \varphi, U_\varphi = \frac{z^2}{r} \cos \varphi, U_z = \frac{r^2}{z} \cos^2 \varphi$$

$$U_r = \frac{z^2}{r} \sin \varphi, U_\varphi = r \cos^2 \varphi, U_z = -z \cos \varphi$$

$$U_r = z \sin^2 \varphi, U_\varphi = r \cos \varphi, U_z = r(\cos \varphi - \sin \varphi)$$

$$U_r = \frac{r^2}{z} \sin \varphi, U_\varphi = -r \sin^2 \varphi, U_z = z \sin \varphi$$

$$U_r = r \sin^2 \varphi, U_\varphi = \frac{r^2}{z} \sin \varphi, U_z = \frac{z^2}{r} \operatorname{ch} \varphi$$

$$U_r = \frac{r^2}{z} \cos \varphi, U_\varphi = r \cos^2 \varphi, U_z = \frac{r^2}{z} \sin \varphi$$

$$U_r = -r \sin \varphi, U_\varphi = \frac{r^2}{r} \cos^2 \varphi, U_z = \frac{r^2}{z} \operatorname{ch} \varphi$$

$$U_r = -r \sin \varphi \cos \theta, U_\varphi = r \cos^2 \varphi \sin \theta, U_\theta = r \sin^2 \theta \cos \varphi$$

$$U_r = r \operatorname{sh} \varphi \cos \theta, U_\varphi = -r \cos \varphi, U_\theta = -r \sin \theta \cos \varphi$$

$$U_r = r \sin \varphi \cos \theta, U_\varphi = -r \cos \varphi \sin \theta, U_\theta = r \sin^2 \theta$$

$$U_r = r \operatorname{sh} \varphi \operatorname{ch} \theta, U_\varphi = r \operatorname{ch}^2 \theta, U_\theta = r \sin \theta \cos \varphi$$

$$U_r = r \cos^2 \theta, U_\varphi = -r \cos^2 \varphi, U_\theta = -r \sin \theta \cos \varphi$$

$$U_r = -r \sin \varphi \cos \theta, U_\varphi = -r \cos \varphi \sin \theta, U_\theta = -r \sin^2 \theta$$

$$U_r = r \sin \varphi \cos \theta, U_\varphi = r \cos \varphi \sin \theta, U_\theta = r \sin \theta \cos \varphi$$

$$U_r = r \sin \varphi \cos \theta, U_\varphi = r \cos \varphi \sin \theta, U_\theta = r \sin \theta \cos \varphi$$

$$U_r = r \sin^2 \theta, U_\varphi = -r \cos^2 \varphi, U_\theta = r \sin^2 \theta$$

$$U_r = r \sin \varphi \cos \theta, U_\varphi = r \operatorname{ch}^2 \varphi \sin \theta, U_\theta = r \operatorname{sh} \theta \cos \varphi$$

$$U_r = -r \sin^2 \varphi, U_\varphi = r \operatorname{sh}^2 \theta, U_\theta = r \operatorname{sh} \theta \operatorname{ch} \varphi$$

$$U_r = r \cos^2 \theta, U_\varphi = r \cos \varphi \sin \theta, U_\theta = r \sin^2 \theta$$

$$U_r = r \sin \varphi \operatorname{sh} \theta, U_\varphi = -r \cos^2 \varphi \sin \theta, U_\theta = r \operatorname{sh} \theta \cos \varphi$$

$$U_r = r \sin \varphi \cos \theta, U_\varphi = r(\operatorname{ch} \varphi - \operatorname{sh} \theta), U_\theta = r \sin \theta \cos \varphi$$

$$U_r = r \sin \varphi \cos \theta, U_\varphi = r(\cos \varphi - \sin \theta), U_\theta = r \sin \theta \cos \varphi$$

6 – MUSTAQIL ISH

MAVZU: SUYUQLIKLAR UCHUN ASOSIY MUNOSABATLARGA DOIR

MASALALAR

Reja

1. Tezliklar maydoni;
2. Uzviylik tenglamasi;
3. Tezliklar maydoni uyurmasi;
4. Tezlanishlar maydoni;
5. Navye-Stoks qonuni;

Asosiy tushunchalar

Tezliklar maydoni koordinatalar va vaqtdan bog'liq bo'lsin. Tutash muhitning berilgan tezliklar maydoni uchun uzviylik tenglamasi quyidagicha bo'ladi

$$\frac{d\rho}{dt} + \rho \operatorname{div}\vec{V} = 0 \quad (1)$$

Siqilmaydigan suyuqlik uchun esa

$$\operatorname{div}\vec{V} = 0 \quad (2)$$

Bu tenglama Dekart koordinatalar sistemasida ushbu ko'rinishni oladi

$$\frac{\partial\rho}{\partial t} + v_x \frac{\partial\rho}{\partial x} + v_y \frac{\partial\rho}{\partial y} + v_z \frac{\partial\rho}{\partial z} + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0 \quad (3)$$

Siqilmaydigan suyuqlik uchun

$$\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0 \quad (4)$$

Silindrik koordinatalarda

$$r \frac{\partial\rho}{\partial t} + \frac{\partial(\rho U_r r)}{\partial r} + \frac{\partial(\rho U_\varphi)}{\partial\varphi} + r \frac{\partial(\rho U_z)}{\partial z} = 0 \quad (5)$$

Siqilmaydigan suyuqlik uchun

$$\frac{1}{r} \frac{\partial(U_r r)}{\partial r} + \frac{1}{r} \frac{\partial U_\varphi}{\partial\varphi} + \frac{\partial U_z}{\partial z} = 0 \quad (6)$$

Sferik koordinatalarda

$$r^2 \sin \varphi \frac{\partial \rho}{\partial t} + \sin \varphi \frac{\partial (\rho U_r r^2)}{\partial r} + r \frac{\partial (\rho U_\varphi \sin \varphi)}{\partial \varphi} + r \frac{\partial \rho U_\theta}{\partial \theta} = 0 \quad (7)$$

Siqilmaydigan suyuqlik uchun

$$\frac{1}{r^2} \frac{\partial (U_r r^2)}{\partial r} + \frac{1}{r \sin \varphi} \frac{\partial (U_\varphi \sin \varphi)}{\partial \varphi} + \frac{1}{r \sin \varphi} \frac{\partial \rho U_\theta}{\partial \theta} = 0 \quad (8)$$

Tezlik vektori uyurmasi Dekart koordinatalarida

$$\omega = \frac{1}{2} \text{rot} \vec{V} = \frac{1}{2} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} \quad (9)$$

Silindrik koordinatalarda

$$\omega = \frac{1}{2} \text{rot} \vec{V} = \frac{1}{2} \left[\left(\frac{1}{r} \frac{\partial V_z}{\partial \varphi} - \frac{\partial V_\varphi}{\partial z} \right) \vec{e}_r + \left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r} \right) \vec{e}_\varphi + \frac{1}{r} \left(\frac{\partial (r V_\varphi)}{\partial r} - \frac{\partial V_r}{\partial \varphi} \right) \vec{e}_z \right] \quad (10)$$

Sferik koordinatalarda

$$\omega = \frac{1}{2} \text{rot} \vec{V} = \frac{1}{2r \sin \varphi} \left[\left(\frac{\partial}{\partial \varphi} (V_\theta \sin \varphi) - \frac{\partial V_\varphi}{\partial \theta} \right) \vec{e}_r + \frac{1}{r} \left(\frac{\partial V_r}{\sin \varphi} - \frac{\partial (r V_\theta)}{\partial r} \right) \vec{e}_\varphi + \frac{1}{r} \left(\frac{\partial (r V_\varphi)}{\partial r} - \frac{\partial V_r}{\partial \varphi} \right) \vec{e}_\theta \right] \quad (11)$$

Tezlanish vektori komponentalari Dekart koordinatalarida

$$a_j = \frac{\partial V_j}{\partial t} + V_i \frac{\partial V_j}{\partial x_i} \quad (12)$$

Silindrik koordinatalarda

$$\begin{aligned} a_r &= \frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\varphi}{r} \frac{\partial V_r}{\partial \varphi} + V_z \frac{\partial V_r}{\partial z} - \frac{V_\varphi^2}{r}, \\ a_\varphi &= \frac{\partial V_\varphi}{\partial t} + V_r \frac{\partial V_\varphi}{\partial r} + \frac{V_\varphi}{r} \frac{\partial V_\varphi}{\partial \varphi} + V_z \frac{\partial V_\varphi}{\partial z} + \frac{V_r V_\varphi}{r}, \\ a_z &= \frac{\partial V_z}{\partial t} + V_r \frac{\partial V_z}{\partial r} + \frac{V_\varphi}{r} \frac{\partial V_z}{\partial \varphi} + V_z \frac{\partial V_z}{\partial z}; \end{aligned} \quad (13)$$

Sferik koordinatalarda

$$\begin{aligned} a_r &= \frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\varphi}{r} \frac{\partial V_r}{\partial \varphi} + \frac{V_\theta}{r \sin \varphi} \frac{\partial V_r}{\partial \theta} - \frac{V_\varphi^2 + V_\theta^2}{r}, \\ a_\varphi &= \frac{\partial V_\varphi}{\partial t} + V_r \frac{\partial V_\varphi}{\partial r} + \frac{V_\varphi}{r} \frac{\partial V_\varphi}{\partial \varphi} + \frac{V_\theta}{r \sin \varphi} \frac{\partial V_\varphi}{\partial \theta} + \frac{V_r V_\varphi - V_\theta^2 \text{ctg} \varphi}{r}, \\ a_\theta &= \frac{\partial V_\theta}{\partial t} + V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\varphi}{r} \frac{\partial V_\theta}{\partial \varphi} + \frac{V_\theta}{r \sin \varphi} \frac{\partial V_\theta}{\partial \theta} + \frac{V_r V_\theta + \text{ctg} \varphi V_\varphi V_\theta}{r}; \end{aligned} \quad (14)$$

Deformasiya tezliklari tenzori Dekart koordinatalarida

$$\begin{aligned}
e_{xx} &= \frac{\partial V_x}{\partial x}, \quad e_{yy} = \frac{\partial V_y}{\partial y}, \quad e_{zz} = \frac{\partial V_z}{\partial z}, \quad e_{xy} = e_{yx} = \frac{1}{2} \left(\frac{\partial V_r}{\partial y} + \frac{\partial V_y}{\partial x} \right), \\
e_{xz} &= e_{zx} = \frac{1}{2} \left(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial x} \right), \quad e_{yz} = e_{zy} = \frac{1}{2} \left(\frac{\partial V_y}{\partial z} + \frac{\partial V_z}{\partial y} \right)
\end{aligned} \tag{15}$$

Silindrik koordinatalarda

$$\begin{aligned}
e_{rr} &= \frac{\partial V_r}{\partial r}, \quad e_{\varphi\varphi} = \frac{1}{r} \frac{\partial V_\varphi}{\partial \varphi} + \frac{V_r}{r}, \quad e_{zz} = \frac{\partial V_z}{\partial z}, \quad 2e_{r\varphi} = \frac{1}{r} \frac{\partial V_r}{\partial \varphi} + r \frac{\partial}{\partial r} \left(\frac{V_\varphi}{r} \right), \\
2e_{\varphi z} &= \frac{\partial V_\varphi}{\partial z} + \frac{1}{r} \frac{\partial V_z}{\partial \varphi}, \quad 2e_{rz} = \frac{\partial V_z}{\partial r} + \frac{\partial V_r}{\partial z}
\end{aligned} \tag{16}$$

Sferik koordinatalarda

$$\begin{aligned}
e_{rr} &= \frac{\partial V_r}{\partial r}, \quad e_{\varphi\varphi} = \frac{1}{r} \frac{\partial V_\varphi}{\partial \varphi} + \frac{V_r}{r}, \quad e_{\theta\theta} = \frac{1}{r \sin \varphi} \frac{\partial V_\theta}{\partial \theta} + \frac{V_r}{r} + \frac{V_\varphi \operatorname{ctg} \varphi}{r}, \quad 2e_{r\varphi} = \frac{1}{r} \frac{\partial V_r}{\partial \varphi} + \frac{\partial V_\varphi}{\partial r} - \frac{V_\varphi}{r}, \\
2e_{\theta r} &= \frac{\partial V_\theta}{\partial r} + \frac{1}{r \sin \varphi} \frac{\partial V_r}{\partial \varphi} - \frac{V_\theta}{r}, \quad 2e_{\theta\varphi} = \frac{1}{r \sin \varphi} \frac{\partial V_\varphi}{\partial \theta} + \frac{1}{r} \frac{\partial V_\theta}{\partial \varphi} - \frac{V_\theta \operatorname{ctg} \varphi}{r}.
\end{aligned} \tag{17}$$

Navye-Stoks qonuni

$$p_{ij} = -pg_{ij} + \lambda g_{ij} \operatorname{div} \vec{V} + 2\mu e_{ij} \tag{18}$$

Masala Berilgan tezliklar maydoni $\vec{V} = \left(\frac{Ax^2}{y} \sin \omega t, \frac{By^2}{z} \cos \omega t, \frac{Az^2}{x} \sin \omega t \right)$ uchun

tutash muhit zichligi o'zgarishini aniqlang, uyurma vektorini, tezlanishni va kuchlanish tenzori komponentalarini aniqlang.

Yechish Zichlikning o'zgarishini topish uchun uzviylik tenglamasidan foydalanamiz. Tezliklar maydoni divergensiya topamiz

$$\operatorname{div} \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = \frac{2Ax}{y} \sin \omega t + \frac{2By}{z} \cos \omega t + \frac{2Az}{x} \sin \omega t \neq 0$$

Demak, muhit siqilmas ekan.

(9) ga ko'ra tezliklar maydoni uyurmasini topamiz

$$\omega = \frac{1}{2} \operatorname{rot} \vec{V} = \frac{1}{2} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{Ax^2}{y} \sin \omega t & \frac{By^2}{z} \cos \omega t & \frac{Az^2}{x} \sin \omega t \end{vmatrix} = \frac{1}{2} \left(\frac{By^2}{z^2} \cos \omega t \vec{i} + \frac{Az^2}{x^2} \sin \omega t \vec{j} + \frac{Bx^2}{y^2} \sin \omega t \vec{k} \right)$$

tezlanish vektori komponentalari

$$a_x = \frac{Ax^2\omega}{y} \cos \omega t + \frac{2A^2x^3}{y} \sin^2 \omega t - \frac{ABx}{2z} \sin 2\omega t$$

$$a_y = -\frac{By^2\omega}{z} \sin \omega t + \frac{2B^2y^3}{z^2} \cos^2 \omega t - \frac{ABy^2}{2x} \sin 2\omega t$$

$$a_z = \frac{Az^2\omega}{x} \cos \omega t - \frac{A^2z^2}{y} \sin^2 \omega t + \frac{2A^2z^2}{x^2} \sin^2 \omega t$$

Deformasiya tezliklari tenzori komponentalari

$$e_{xx} = \frac{2Ax}{y} \sin \omega t, \quad e_{yy} = \frac{2Ay}{z} \cos \omega t, \quad e_{zz} = \frac{2Az}{x} \sin \omega t, \quad e_{xy} = e_{yx} = -\frac{Ax^2}{2y^2} \sin \omega t,$$

$$e_{xz} = e_{zx} = -\frac{Az^2}{2x^2} \sin \omega t, \quad e_{yz} = e_{zy} = -\frac{By^2}{2z^2} \cos \omega t$$

Kuchlanish tenzori komponentalari

$$p_{xx} = -p + \lambda \left(\frac{2Ax}{y} \sin \omega t + \frac{2By}{z} \cos \omega t + \frac{2Az}{x} \sin \omega t \right) + \frac{4\mu Ax}{y} \sin \omega t,$$

$$p_{yy} = -p + \lambda \left(\frac{2Ax}{y} \sin \omega t + \frac{2By}{z} \cos \omega t + \frac{2Az}{x} \sin \omega t \right) + \frac{4\mu Ay}{z} \cos \omega t,$$

$$p_{zz} = -p + \lambda \left(\frac{2Ax}{y} \sin \omega t + \frac{2By}{z} \cos \omega t + \frac{2Az}{x} \sin \omega t \right) + \frac{4\mu Az}{x} \sin \omega t,$$

$$p_{xy} = -\frac{\mu Ax^2}{y^2} \sin \omega t, \quad p_{xz} = -\frac{\mu Az^2}{x^2} \cos \omega t, \quad p_{yz} = -\frac{\mu Ay^2}{z^2} \sin \omega t,$$

6 - MUSTAQIL ISH VARIANTLARI

Berilgan tezliklar maydoni $\vec{V} = (V_i)$ uchun Dekart, silindrik va sferik koordinatalarda tutash muhit zichligi o'zgarishini aniqlang, uyurma vektorini, tezlanishni va kuchlanish tenzori komponentalarini aniqlang.

$$1. \quad V_x = A \frac{y^2}{x} \sin \omega t, \quad V_y = A \frac{yz}{x} \sin \omega t, \quad V_z = A \frac{xy}{z} \sin \omega t;$$

$$V_r = -Ar \sin \varphi \sin \omega t, \quad V_\varphi = A \frac{r^2}{z} \cos^2 \varphi \sin \omega t, \quad V_z = A \frac{r^2}{z} \sin \varphi \sin \omega t$$

$$V_r = Ar \sin \varphi \cos \theta \sin \omega t, \quad V_\varphi = Ar (\cos \varphi - \sin \theta) \sin \omega t, \quad V_\theta = Ar \sin \theta \cos \varphi \sin \omega t$$

$$2. \quad V_x = \frac{xy}{tz}, \quad V_y = \frac{x^2 + y^2}{tz}, \quad V_z = \frac{y^2 + z^2}{tx};$$

$$V_r = \frac{r^2}{tz} \cos \varphi, \quad V_\varphi = \frac{r}{t} \cos^2 \varphi, \quad V_z = \frac{r^2}{tz} \sin \varphi$$

$$V_r = Ar \sin \varphi \cos \theta \sin \omega t, \quad V_\varphi = Ar(ch\varphi - sh\theta) \sin \omega t, \quad V_\theta = Ar \sin \theta \cos \varphi \sin \omega t$$

$$3. \quad V_x = \frac{x^2}{(y+z)t}, \quad V_y = \frac{z^2}{(x+y)t}, \quad V_z = \frac{y^2}{(x+z)t};$$

$$V_r = Ar \sin^2 \varphi \sin \omega t, \quad V_\varphi = A \frac{r^2}{z} \sin \varphi \cos \omega t, \quad V_z = A \frac{z^2}{r} ch\varphi \sin \omega t$$

$$V_r = Ar \sin \varphi sh\theta \cos \omega t, \quad V_\varphi = -Ar \cos^2 \varphi \sin \theta \sin \omega t, \quad V_\theta = Arsh\theta \cos \varphi \cos \omega t$$

$$4. \quad V_x = A \frac{x^3}{y^2 + z^2} \sin \omega t, \quad V_y = A \frac{y^2}{x+z} \sin \omega t, \quad V_z = \frac{Ax^2}{z+y} \sin \omega t;$$

$$V_r = \frac{r^2}{tz} \sin \varphi, \quad V_\varphi = -\frac{r}{t} \sin^2 \varphi, \quad V_z = \frac{z}{t} \sin \varphi$$

$$V_r = Ar \cos^2 \theta \sin \omega t, \quad U_\varphi = Ar \cos \varphi \sin \theta \sin \omega t, \quad U_\theta = Ar \sin^2 \theta \sin \omega t$$

$$5. \quad V_x = -\frac{xy}{tz}, \quad V_y = \frac{x^3}{(y^2 + z^2)t}, \quad V_z = \frac{y^2}{(x+z)t};$$

$$V_r = Az \sin^2 \varphi \sin \omega t, \quad V_\varphi = Ar \cos \varphi \sin \omega t, \quad V_z = Ar(\cos \varphi - \sin \varphi) \sin \omega t$$

$$V_r = -Ar \sin^2 \varphi \sin \omega t, \quad V_\varphi = Arsh^2\theta \sin \omega t, \quad V_\theta = Arsh\theta ch\varphi \sin \omega t$$

$$6. \quad V_x = \frac{x^2 y}{tz^2}, \quad V_y = \frac{x^3}{(y^2 + z^2)t}, \quad V_z = \frac{z^2}{(x+y)t};$$

$$V_r = \frac{Az^2}{r} \sin \varphi \cos \omega t, \quad V_\varphi = Ar \cos^2 \varphi \cos \omega t, \quad V_z = -Az \cos \varphi \cos \omega t$$

$$V_r = Ar \sin \varphi \cos \theta \sin \omega t, \quad V_\varphi = Arch^2\varphi \sin \theta \sin \omega t, \quad V_\theta = Arsh\theta \cos \varphi \sin \omega t$$

$$7. \quad V_x = \frac{xy}{(z+y)t}, \quad V_y = \frac{yz}{tx}, \quad V_z = \frac{y^2}{t(z+x)};$$

$$V_r = Ar sh\varphi \sin \omega t, \quad V_\varphi = A \frac{z^2}{r} \cos \varphi \sin \omega t, \quad V_z = \frac{Ar^2}{z} \cos^2 \varphi \sin \omega t$$

$$V_r = Ar \sin^2 \theta \sin \omega t, \quad V_\varphi = -Ar \cos^2 \varphi \sin \omega t, \quad V_\theta = Ar \sin^2 \theta \sin \omega t$$

$$8. \quad V_x = \frac{x^2 y}{tz^2}, \quad V_y = -\frac{x^2 y^2}{tz^3}, \quad V_z = \frac{y^2 z^2}{tx^3};$$

$$V_r = \frac{r^2}{tz} \sin \varphi, \quad V_\varphi = \frac{z^2}{tr} ch\varphi, \quad V_z = \frac{r^2}{tz} \cos \varphi$$

$$V_r = Ar \sin \varphi \cos \theta \sin \omega t, \quad V_\varphi = Ar \cos \varphi \sin \theta \sin \omega t, \quad V_\theta = Ar \sin \theta \cos \varphi \sin \omega t$$

$$9. \quad V_x = \frac{x^2}{ty}, \quad V_y = \frac{z^3}{txy}, \quad V_z = \frac{x^3}{tzy};$$

$$V_r = \frac{r}{t} \operatorname{ch} \varphi, \quad V_\varphi = -\frac{z^2}{tr} \cos \varphi, \quad V_z = \frac{r^2}{tz} \operatorname{sh} \varphi$$

$$V_r = Ar \sin \varphi \cos \theta \sin \omega t, \quad V_\varphi = Ar \cos \varphi \sin \theta \sin \omega t, \quad V_\theta = Ar \sin \theta \cos \varphi \sin \omega t$$

$$10. \quad V_x = \frac{x^3}{tyz}, \quad V_y = \frac{y^2}{tz}, \quad V_z = \frac{z^3}{txy};$$

$$V_r = A \left(r - \frac{r^2}{z} \right) \sin \varphi \sin \omega t, \quad V_\varphi = \frac{Az^2}{r} \cos \varphi \sin \omega t, \quad V_z = \frac{Ar^2}{z} \operatorname{ch} \varphi \sin \omega t$$

$$V_r = Ar \cos^2 \theta \cos \omega t, \quad V_\varphi = -Ar \cos^2 \varphi \cos \omega t, \quad V_\theta = -Ar \sin \theta \cos \varphi \cos \omega t$$

$$11. \quad V_x = \frac{yx^2}{tz^2}, \quad V_y = \frac{y^2z}{tx^2}, \quad V_z = \frac{x^2y^2}{tz^3};$$

$$V_r = \frac{r^2}{tz} \sin \varphi, \quad V_\varphi = Az \cos \varphi \sin \omega t, \quad V_z = \frac{r^2}{tz} \cos \varphi$$

$$V_r = -Ar \sin \varphi \cos \theta \sin \omega t, \quad V_\varphi = -Ar \cos \varphi \sin \theta \sin \omega t, \quad V_\theta = -Ar \sin^2 \theta \sin \omega t$$

$$12. \quad V_x = \frac{y^2}{xt}, \quad V_y = \frac{yz}{xt}, \quad V_z = \frac{xy}{zt}; \quad V_r = Ar \sin^2 \varphi \sin \omega t, \quad V_\varphi = Ar \cos^2 \varphi \sin \omega t, \quad V_z = A \frac{r^2}{z} \cos \varphi \sin \omega t$$

$$V_r = Ar \sin \varphi \cos \theta \cos \omega t, \quad V_\varphi = Ar \cos \varphi \sin \theta \cos \omega t, \quad V_\theta = Ar \sin \theta \cos \varphi \cos \omega t$$

$$13. \quad V_x = \frac{xz}{ty}, \quad V_y = -\frac{yx}{tz}, \quad V_z = \frac{yz}{tx};$$

$$V_r = Ar \cos^2 \varphi \sin \omega t, \quad V_\varphi = A(r+z) \cos \varphi \sin \omega t, \quad V_z = \frac{Ar^2}{z} \sin \varphi \sin \omega t$$

$$V_r = Ar \operatorname{sh} \varphi \cos \theta \cos \omega t, \quad V_\varphi = Ar \cos \varphi \sin \theta \cos \omega t, \quad V_\theta = Ar \sin \theta \operatorname{sh} \varphi \cos \omega t$$

$$14. \quad V_x = \frac{x^2y}{tz^2}, \quad V_y = \frac{xy^2}{tz^2}, \quad V_z = \frac{yz^2}{tx^2};$$

$$V_r = A(r+z) \sin \varphi \cos \omega t, \quad V_\varphi = Ar \cos^2 \varphi \cos \omega t, \quad V_z = Ar \cos \varphi \cos \omega t$$

$$V_r = Ar \sin \varphi \cos \theta \sin \omega t, \quad V_\varphi = Ar \cos \varphi \sin \theta \sin \omega t, \quad V_\theta = Ar \sin \theta \cos \varphi \sin \omega t$$

$$15. \quad V_x = \frac{xy^2}{tz^2}, \quad V_y = \frac{x^2y}{tz^2}, \quad V_z = \frac{yz^2}{tx^2};$$

$$V_r = A \left(r + \frac{z^2}{r} \right) \sin \varphi \sin \omega t, \quad V_\varphi = Ar \cos \varphi \sin \omega t, \quad V_z = \frac{Ar^2}{z} \sin \varphi \sin \omega t$$

$$V_r = -Ar \sin^2 \theta \sin \omega t, \quad V_\varphi = Ar \cos \varphi \operatorname{sh} \theta \sin \omega t, \quad V_\theta = -Ar \sin \theta \cos \varphi \sin \omega t$$

$$16. \quad V_x = \frac{yz^2}{tx^2}, \quad V_y = \frac{x^2z}{ty^2}, \quad V_z = \frac{yx^2}{tz^2};$$

$$V_r = Ar \sin^2 \varphi \cos \omega t, \quad V_\varphi = \frac{Ar^2}{z} \cos \varphi \cos \omega t, \quad V_z = -Az \sin^2 \varphi \cos \omega t$$

$$V_r = Arsh \varphi \cos \theta \sin \omega t, \quad V_\varphi = Arch \varphi \sin \theta \sin \omega t, \quad V_\theta = Arsh \theta \cos \varphi \sin \omega t$$

$$17. \quad V_x = \frac{xy}{tz}, \quad V_y = \frac{zy}{tx}, \quad V_z = \frac{xz}{ty};$$

$$V_r = \frac{r^2}{tz} \cos \varphi, \quad V_\varphi = Ar \sin^2 \varphi \sin \omega t, \quad V_z = A(z+r) \cos \varphi \sin \omega t$$

$$V_r = -Ar \cos^2 \varphi \sin \omega t, \quad V_\varphi = Ar \cos \varphi sh \theta \sin \omega t, \quad V_\theta = Ar \sin^2 \theta \sin \omega t$$

$$18. \quad V_x = \frac{y^2}{tx}, \quad V_y = \frac{yz}{tx}, \quad V_z = \frac{xy}{tz};$$

$$V_r = Ar ch^2 \varphi \cos \omega t, \quad V_\varphi = \frac{Az^2}{r} \cos \varphi \cos \omega t, \quad V_z = \frac{Ar^2}{z} sh \varphi \cos \omega t$$

$$V_r = -Ar \sin \varphi \cos \theta \sin \omega t, \quad V_\varphi = Ar \cos^2 \varphi \sin \theta \sin \omega t, \quad V_\theta = Ar \sin^2 \theta \cos \varphi \sin \omega t$$

$$19. \quad V_x = \frac{zy^2}{tx^2}, \quad V_y = \frac{yz^2}{tx^2}, \quad V_z = \frac{xy^2}{tz^2}; \quad V_r = \frac{Ar^2}{z} \sin \varphi \cos \omega t, \quad V_\varphi = Ar \sin^2 \varphi \cos \omega t, \quad V_z = Az \sin \varphi \cos \omega t$$

$$V_r = Arsh \varphi \cos \theta \sin \omega t, \quad V_\varphi = -Ar \cos \varphi \sin \omega t, \quad V_\theta = -Ar \sin \theta \cos \varphi \sin \omega t$$

$$20. \quad V_x = \frac{zy^2}{tx^2}, \quad V_y = \frac{yx^2}{tz^2}, \quad V_z = \frac{xz^2}{ty^2}; \quad V_r = Ar \sin^2 \varphi \sin \omega t, \quad V_\varphi = \frac{Ar^2}{z} \cos \varphi \sin \omega t, \quad V_z = \frac{Az^2}{r} \sin \varphi \sin \omega t$$

$$V_r = Ar \sin \varphi \cos \theta \cos \omega t, \quad V_\varphi = -Ar \cos \varphi \sin \theta \sin \omega t, \quad V_\theta = Ar \sin \theta \sin \omega t$$

$$21. \quad V_x = \frac{x^2}{(y+z)t}, \quad V_y = -\frac{y^2}{(x+z)t}, \quad V_z = \frac{z^2}{(x+y)t};$$

$$V_r = Ar \cos \varphi \sin \omega t, \quad V_\varphi = Ar \sin \varphi \sin \omega t, \quad V_z = \frac{Ar^2}{z} \cos \varphi \sin \omega t$$

$$V_r = Ar \sin \varphi \sin \theta \cos \omega t, \quad V_\varphi = Ar \sin \varphi \cos \theta \sin \omega t, \quad V_\theta = Ar \sin \theta \cos \omega t$$

$$22. \quad V_x = \frac{y^2}{xt}, \quad V_y = \frac{z^2}{t(x+z)}, \quad V_z = \frac{x^2}{(z+y)t};$$

$$V_r = Ar \sin \varphi \cos \omega t, \quad V_\varphi = Ar \cos \varphi \cos \omega t, \quad V_z = -Ar \cos \varphi \sin \omega t$$

$$V_r = Ar \sin \varphi \cos \theta \sin \omega t, \quad V_\varphi = Ar \cos \varphi \sin \theta \cos \omega t, \quad V_\theta = Ar \cos \varphi \sin \omega t$$

$$23. \quad V_x = \frac{x^3}{(y^2+z^2)t}, \quad V_y = -\frac{y^2}{tz}, \quad V_z = \frac{z^2}{ty};$$

$$V_r = Ar sh \varphi \cos \omega t, \quad V_\varphi = Ar \cos \varphi \cos \omega t, \quad V_z = \frac{Ar^2}{z} \cos \varphi \cos \omega t$$

$$V_r = Ar \sin \varphi \cos \theta \sin \omega t, V_\varphi = Ar \cos \varphi \sin \theta \sin \omega t, V_\theta = Ar \sin \theta \cos \varphi \sin \omega t$$

$$24. V_x = \frac{x^2}{(y+z)t}, V_y = \frac{z^2}{(x+y)t}, V_z = \frac{y^2}{(x+z)t};$$

$$V_r = Ar \operatorname{ch} \varphi \sin \omega t, V_\varphi = Ar \sin \varphi \sin \omega t, V_z = Ar \cos \varphi \sin \omega t;$$

$$V_r = -Ar \sin \varphi \cos \theta \cos \omega t, V_\varphi = Ar \cos^2 \varphi \cos \omega t, V_\theta = Ar \sin \theta \cos \varphi \cos \omega t.$$

$$25. V_x = \frac{x^3}{ty^2}, V_y = \frac{y^2}{(x+z)t}, V_z = \frac{z^3}{(x^2+y^2)t};$$

$$V_r = Ar \sin \varphi \cos \omega t, V_\varphi = -Ar \operatorname{ch} \varphi \cos \omega t, V_z = Az \sin \varphi \cos \omega t;$$

$$V_r = Ar \sin^2 \varphi \sin \omega t, V_\varphi = Ar \cos \varphi \sin \theta \sin \omega t, V_\theta = Ar \operatorname{sh} \theta \cos \varphi \sin \omega t;$$

$$26. V_x = \frac{yx^2}{tz^2}, V_y = \frac{zy^2}{tx^2}, V_z = \frac{z^2}{(x+y)t};$$

$$V_r = Ar \sin^2 \varphi e^{-\omega t}, V_\varphi = Ar \sin^2 \varphi e^{-\omega t}, V_z = Az \cos^2 \varphi e^{-\omega t};$$

$$V_r = Ar \cos \varphi \cos \theta e^{-\omega t}, V_\varphi = -Ar \cos \varphi \sin \theta e^{-\omega t}, V_\theta = Ar \cos^2 \varphi e^{-\omega t}$$

$$27. V_x = \frac{yz}{(x+z)t}, V_y = \frac{y^2}{(x+z)t}, V_z = \frac{x^2+y^2}{(x+y)t};$$

$$V_r = Ar \operatorname{ch} \varphi \cos \omega t, V_\varphi = \frac{Az^2}{r} \cos \varphi \cos \omega t, V_z = Ar \cos \varphi \cos \omega t$$

$$V_r = Ar \cos^2 \varphi \sin \omega t, V_\varphi = Ar \cos \varphi \sin \theta \sin \omega t, V_\theta = -Ar \sin^2 \theta \sin \omega t.$$

Adabiyotlar

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