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DIFFERENSIAL TENGLAMALAR KAFEDRASI

HAITOVA FOTIMANING

"TO'LQIN TENGLAMASINI MAPLE PAKETI YORDAMIDA YECHISH"

5130100 - matematika ta'lim yo'nalishi bakalavr akademik daraja olish uchun

BITIRUV MALAKAVIY ISH

Ilmiy rahbar: **dots. Z.H. Ochilov**

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MUNDARIJA

Kirish

1. Masalaning qo'yilishi: To'lqin tenglamalarini o'rganish mobaynida to'g'ri to'rtburchakli va doiraviy membranalarning tebranish tenglamalariga duch kelamiz.

Bitiruv malakaviy ishida Maple paketi orqali kvadratli va doiraviy membranalarning tebranish tenglamalarini Fur*'ye usuli ya'ni o'zgaruvchilarni ajratish usuli yordamida yechishni* analitik, sonli usularini qo'llash hamda natijalarni grafik tarzda ifodalash vositalarini qo'llash borasida amalga oshiriladigan vazifalarni namoyish qilishdan iborat.

2. Mavzuning dolzarbligi: Giperbolik tipdagi tenglamalar uchun qo'yilgan chegaraviy va aralash masalalarni Fur'ye usulida ya'ni o'zgaruvchilarni almashtirish usuli yordamida yechiladi.

Bitiruv malakaviy ishida, Maple matematik paketidan foydalanib kvadratli va doiraviy membranalarning tebranish tenglamalarini Fur'ye usuli ya'ni o'zgaruvchilarni ajratish usuli yordamida yechish keltirilgan. Maple paketi orqali to'lqin tenglamalarni yechish jarayoni qoidaga mos ta'lim berish uchun qiziqarli misollar yordamida tasvirlangan. Maple paketini har bir turdagi masalani yechishga qo'llanilishi ketma-ket tarzda keltirilgan, ya'ni to'lqin tenglamalarni yechishda misollarga quyidagicha tavsif berilgan: hisoblash formulasi, analitik va sonli yechimi, shuningdek, yechimning ikki o'lchovli animasiyali grafigi tasvirlangan, bundan tashqari ba'zi misollar uchun bir qancha vaqt momentlarini ikki o'lchovli grafigi tasvirlangan.

3. **Ishning maqsadi va vazifalari:** Bitiruv malakaviy ishining maqsadi *kvadratli va doiraviy membranalarning tebranish tenglamalarini Fur'ye usuli ya'ni o'zgaruvchilarni ajratish usuli yordamida yechishning* nazariy asoslarini muhim jixatlarini aniqlash, tanlangan to'lqin tenglamalarni yechishning Maple tizimidagi vositalarini aniqlash, yechimning ikki o'lchovli animasiyali grafigi tasvirlash, bundan tashqari ba'zi misollar uchun bir qancha vaqt momentlarini ikki

o’lchovli grafigi tasvirlash va amaliy jixatdan qo`llash uslublarini ko`rsatishdan iborat.

4. **Ilmiy tadqiqot usullari:** Bitiruv malakaviy ishining maqsad va vazifalarini bajarish maqsadida “Matematik fizika tenglamalari”, “Differensial tenglamalar”, “Kompyuter algebrasi tizimlari” fanlarining tadqiqot usullaridan foydalanildi.

5. **Ishning ilmiy ahamiyati:** Bu ish ilmiy tadqiqotlarni bajarish uchun Maple paketidan foydalanishga oid fundamental va amaliy ko’rsatmalar berilgan. Ularning barchasi matematika mutaxassislariga, shuningdek turli darajadagi ta’lim tizimiga ham tegishli.

6. **Ishning amaliy ahamiyati:** Bitiruv malakaviy ishdagi ma’lumotlar matematik xarakterdagi masalalarni aniq Maple tizimida ifodalashda keng doiradagi mutaxassislarga, magistrlar, talabalarga foydalanish uchun qo’shimcha uslubiy qo’llanma sifatida xizmat qilishi mumkin. Ma’lumotlarni ifodalashda yetarlicha qiziqarli misollar keltirilgan.

7. **Ishning tuzilishi:** Bitiruv malakaviy ishi kirish, uchta bob, o’nta paragraf, xulosa va adabiyotlar ro’yxatidan iborat.

Birinchi bob to’rtta paragrafdan iborat. Bu bobda tebranish tenglamasi uchun masalaning qo’yilishi, ikkinchi tartibli xususiy hosilali tenglamalarning xarakteristikalari, to’lqin tenglamasi uchun chegaraviy va aralash masalalarni yechishning Fur’ye usuli, torning majburiy tebranish tenglamasiga qo’yilgan bir jinsli chegaraviy masalani yechishning Fur’ye usuli to’g’risida ma’lumotlar keltirilgan.

Ikkinchi bobda to’g’ri to’rtburchak membranasining tebranishlari, Maple paketi orqali kvadratli membrananing tebranish tenglamasini Fur’ye usuli (o’zgaruvchilarni ajratish usuli) yordamida yechish va kvadratli membrana tebranish tenglamalariga misollar qaralgan, shuningdek, yechimning ikki o’lchovli animasiyali grafigi tasvirlangan, bundan tashqari ba’zi misollar uchun bir qancha vaqt momentlarini ikki o’lchovli grafigi tasvirlangan.

Uchinchi bobda doiraviy membrananing tebranishlari, Maple paketi orqali doiraviy membrananing tebranish tenglamasini Fur'ye usuli yordamida misollar yechilgan, shuningdek, yechimning ikki o'lchovli animasiyali grafigi tasvirlangan, bundan tashqari ba'zi misollar uchun bir qancha vaqt momentlarini ikki o'lchovli grafigi tasvirlangan.

Doiraviy memrana tenglamalari va unga qo'yilgan chegaraviy shartlarni Fur'ye usulida Maple paketida yechish qaralgan va unga oid misollar keltirilgan.

8. Olingan natijalarining qisqacha mazmuni: Bitiruv malakaviy ishida, Maple matematik paketidan foydalanib, kvadratli va doiraviy membranalarning tebranish tenglamalarini Fur'ye usuli ya'ni o'zgaruvchilarni ajratish usuli yordamida yechish keltirilgan. Maple paketi orqali to'lqin tenglamalarni yechish jarayoni qoidaga mos ta'lim berish uchun qiziqarli misollar yordamida tasvirlangan. Maple paketini har bir turdagi masalani yechishga qo'llanilishi ketma-ket tarzda keltirilgan, ya'ni to'lqin tenglamalarni yechishda misollarga quyidagicha tavsif berilgan: hisoblash formulasi, analitik va sonli yechimi, shuningdek, yechimning ikki o'lchovli animasiyali grafigi tasvirlangan, bundan tashqari ba'zi misollar uchun bir qancha vaqt momentlarini ikki o'lchovli grafigi tasvirlangan.

1- Bob. Giperbolik tipdagi tenglamalar

1.1- §. Tebranish tenglamasi uchun masalaning qo'yilishi

Giperbolik tipdagi tenglamani ko'rib chiqamiz [1].

Faraz qilaylik $u(x,t) \in C^2 \ \left((x,t): 0 < x < l, t > 0\right)$ bo'lsin, shunda

$$u_{tt} = a^2 u_{xx}, \ \left((x,t): 0 < x < l, t > 0\right) \tag{1.1.1}$$

Tenglama ideal torning tebranish tenglamasi deyiladi.

Ikki fazoviy o'zgaruvchilarning funksiyasi $u(x,y,t)$ holida:

$$u_{tt} = a^2 \Delta u, (x,y) \in D, t > 0 \tag{1.1.2}$$

bu elastik membrananing tebranish tenglamasi.

(1.1.2) tenglamani qaraymiz. Biz quyidagi boshlang'ich shartlarni berishimiz mumkin:

$$\begin{cases} u \ (x,0) = \phi(x), \ 0 < x < l; \\ u_t \ (x,0) = \varphi(x), \ 0 < x < l, \end{cases} - \tag{1.1.3}$$

torninng muvozanat holatidan chetlanishini izohlaydi; va chegaraviy shartlarni:

$$\begin{cases} u \ (l,t) = \mu \ (t), \ t > 0; \ (\max \textit{kamlanlangan} \quad \textit{holda} \ \ \mu \equiv 0) \\ u_x \ (l,t) = \nu \ (t), \ t > 0; \\ u \ (l,t) + \alpha \ u_x(l,t) = \theta \ (t). \ t > 0 \end{cases} \tag{1.1.4}$$

odatda bizlar ulardan ba'zilarini olamiz.

Giperbolik yoki tebranish tenglamalar uchun chegaraviy masalalar tuzamiz [1-4].

Birinchi chegaraviy masala.

$$\begin{cases} u_{tt} = a^2 u_{xx}, & 0 < x < l, \ 0 < t \le T; \\ u(x,0) = \phi(x), & 0 \le x \le l; \\ u_t(x,0) = \varphi(x), & 0 \le x \le l; \\ u(0,t) = \mu_1(t), & 0 \le t \le T; \\ u(l,t) = \mu_2(t), & 0 \le t \le T; \end{cases} \tag{1.1.5}$$

Xuddi shuni o'zi yarim to'g'ri chiziq uchun :

$$\begin{cases} u_{tt} = a^2 u_{xx}, & x > 0, \ 0 < t \le T; \\ u(x,0) = \phi(x), & x \ge 0; \\ u_t(x,0) = \varphi(x), & x \ge 0; \\ u(0,t) = \mu(t), & 0 \le t \le T; \end{cases} \tag{1.1.6}$$

Shuningdek oddiy Koshi masalasini qarash mumkin:

$$\begin{cases} u_{tt} = a^2 u_{xx}, & -\infty < x < +\infty, \ 0 < t \le T; \\ u(x,0) = \phi(x), & -\infty < x < +\infty; \\ u_t(x,0) = \varphi(x), & -\infty < x < +\infty. \end{cases} \tag{1.1.7}$$

Bizlar tebranish tenglamasi uchun Koshi masalasini qaraymiz.

$$\begin{cases} (1) \quad u_{tt} = a^2 u_{xx}, & -\infty < x < +\infty, \ 0 < t \le T; \\ (2) \quad u(x,0) = \phi(x), & -\infty < x < +\infty; \\ (3) \quad u_t(x,0) = \varphi(x), & -\infty < x < +\infty. \end{cases}$$

Faraz qilaylik, $u \in C^2\left(R \times R^+\right)$ funksiya bo'lib, u (1.1.7) Koshi masalasining yechimi bo'lsin. Yangi $\xi, \ \eta$ o'zgaruvchilarni quyidagicha aniqlaymiz:

$$\begin{cases} \xi = x + at; \\ \eta = x - at; \end{cases} \Rightarrow \begin{cases} x = \dfrac{\xi + \eta}{2}; \\ t = \dfrac{\xi - \eta}{2a}. \end{cases}$$

Yangi funksiyani aniqlaymiz:

$$v(\xi, \eta) = u\left(\frac{\xi + \eta}{2}, \frac{\xi - \eta}{2a}\right).$$

Bu funksiyaning xususiy hosilalarini topamiz.

$$v_\xi = u_x\left(\frac{\xi + \eta}{2}, \frac{\xi - \eta}{2a}\right)\frac{1}{2} + u_t\left(\frac{\xi + \eta}{2}, \frac{\xi - \eta}{2a}\right)\frac{1}{2a};$$

$$v_{\xi\eta}=u_{xx}\left(\frac{\xi+\eta}{2},\frac{\xi-\eta}{2a}\right)\frac{1}{4}+u_{xt}\left(\frac{\xi+\eta}{2},\frac{\xi-\eta}{2a}\right)\left(-\frac{1}{4a}\right)$$
$$+u_{tx}\left(\frac{\xi+\eta}{2},\frac{\xi-\eta}{2a}\right)\frac{1}{4a}+u_{tt}\left(\frac{\xi+\eta}{2},\frac{\xi-\eta}{2a}\right)\left(-\frac{1}{4a}\right)=$$
$$=u_{xx}\left(\frac{\xi+\eta}{2},\frac{\xi-\eta}{2a}\right)\frac{1}{4}-\frac{1}{4a}u_{tx}\left(\frac{\xi+\eta}{2},\frac{\xi-\eta}{2a}\right)=$$
$$\{tebranish\quad tenglamasi\}=0;$$

Endi teskari integrallashni amalga oshiramiz:

$$v_{\xi\eta}(\xi,\eta)=0,\quad \overset{\xi\quad bo'yicha\ \text{int}egral}{\Longrightarrow}\quad v_{\eta}(\xi,\eta)=\tilde{f}_1(\eta)\quad\Rightarrow$$
$$\overset{\eta\quad bo'yicha\ \text{int}egral}{\Longrightarrow}\quad v(\xi,\eta)=\int\tilde{f}_1(\eta)d\eta+f_2(\xi)$$
$$\Rightarrow v(\xi,\eta)=f_1(\eta)+f_2(\xi)\Rightarrow\{u(x,t)=v(x+at,\,x-at)\}\Rightarrow$$
$$u(x,t)=f_1(x-at)+f_1(\,x+at), \tag{1.3.3}$$

bu yerda $\tilde{f}_1,\ f_1,\ f_2$- lar integrallash davomida hosil bo'ladigan funksiyalar. Shunday qilib biz tebranish tenglamasi yechimi bo'lgan u funksiyaning umumiy ko'rinishini hosil qildik. Boshlang'ich shartlardan foydalanib f_1, f_2-larni aniqlaymiz:

$$\begin{cases} u\ \ (x,\ 0)=\ f_1(x)+f_2(x)=\phi(x); \\ u_t(x,\ 0)=-af_1'\ (x)+af_1'\ (x)=\varphi(x); \end{cases}\Rightarrow$$
$$\Rightarrow\begin{cases} -f_1(x)+f_2(x)=\dfrac{1}{a}\displaystyle\int_{x_0}^{x}\varphi(\xi)d\xi+C \\ f_1(x)+f_2(x)=\phi(x). \end{cases}$$

Sistemadagi tenglamalarni qo'shib va biridan birini ayirib quyidagini hosil qilamiz.

$$\begin{cases} f_2(x)=\dfrac{\phi(x)}{2}+\dfrac{1}{2a}\displaystyle\int_{x_0}^{x}\varphi(\xi)d\xi+\dfrac{c}{2}; \\ f_1(x)=\dfrac{\phi(x)}{2}-\dfrac{1}{2a}\displaystyle\int_{x_0}^{x}\varphi(\xi)d\xi-\dfrac{c}{2}. \end{cases}\Rightarrow$$
$$\Rightarrow\{u\ \ (x,\ t)=\ f_1(x-at)+f_2(x+at)\}\Rightarrow$$

$$u\ (x,\ t)=\frac{\phi(x-at)+\phi(x+at)}{2}+\frac{1}{2a}\int_{x-at}^{x+at}\varphi(\xi)d\xi \qquad (1.3.4)$$

(1.3.4) formula Dalamber formulasi deyiladi.

Teorema 1.1 (Koshi masalasi yechimining mavjudligi va yagonaligi).

Faraz qilaylik $\phi\,(x)\in C^2(R),\ \varphi\,(x)\in C^1(R)$. (1.1.7) Koshi masalasining yechimidan iborat shunday $u\,(x,y)$ funksiya mavjud va yagonadirki, bunda $u\in C^2\left(R\times\overline{R}^+\right)$. Bu yerda $\phi\,(x)$, $\varphi\ (x)$ funksiyalar boshlang'ich shartlarni aniqlaydi.

Teorema 1.2 (Turg'unlik teoremasi).

Faraz qilaylik $\phi_1,\phi_2\ (x)\in C^2(R),\ \varphi_1,\varphi_1\ (x)\in C^1(R)$ va ular R fazoda cheagralangan bo'lsin. Agar $u_1,u_2(x,t)$ funksiyalar (1.1.7) tipdagi masalaning yechimlari vam mos ravishda $\phi_1,\phi_2,\varphi_1,\varphi_2$ boshlang'ich shartlar bilan berilgan yechimlari bo'lsa, shunda

$$\sup_{x\in R,\,0\le t\le T}\left|u_1(x,t)-u_2(x,t)\right|\le\sup_{x\in R}\left|\ \phi_1(x)-\phi_2(x)\right|+T\sup_{x\in R}\left|\ \varphi_1(x)-\varphi_2(x)\right|$$

bo'ladi.

1.2 - §. Ikkinchi tartibli xususiy hosilali tenglamalarning xarakteristikalari.

Ikkinchi tartibli xususiy hosilali klassik tenglama quyidagi ko'rinishga ega [1-4]:

$$a_{11}(x,y)U_{xx}+2a_{12}(x,y)U_{xy}+a_{22}(x,y)U_{yy}=F(x,y,u,u_x,u_y) \qquad (1.4.1)$$

Unga bir qiymatli moslik bilan quyidagi oddiy differensial tenglamani qo'yamiz:

$$a_{11}(dy)^2-2a_{12}dxdy+a_{22}(dx)^2=0 \qquad (1.4.2)$$

Shunda (1.4.2) ning yechimlari bo'lgan funksiyalar (egri chiziqlar) (1.4.1) tenglamaning xarakteristikalari deyiladi. Masalan

$a^2U_{xx} - U_{tt} = 0$ tebranish tenglamasi uchun xarakteristikalar hosil qilinadigan tenglama

$$a^2(dt)^2 - (dx)^2 = 0$$

ko'rinishga ega.

Undan quyidagini hosil qilamiz:

$$\begin{bmatrix} a\ dt + dx = 0; \\ a\ dt - dx = 0. \end{bmatrix} \Rightarrow \begin{bmatrix} x + at = const; \\ x - at = const. \end{bmatrix}$$

Bular giperbolik tipdagi tenglamalarning xarakteristikalaridan iborat ikki to'g'ri chiziqdir.

Faraz qilaylik u (x, t) funksiya ma'lum bir Koshi masalasining yechimi bo'lsin. *Oxy* tekisligining birinchi choragida ixtiyoriy (x_0, y_0) nuqta olamiz. Bu nuqtadan faqat ikkita xarakteristika o'tadi:

$$x - at = x_0 - at_0,\ x + at = x_0 + at_0$$

Ular O*x* o'qini $(x_0 + at_0, 0),\ (x_0 - at_0, 0)$ nuqtalar orqali kesib o'tib, bunda xarakteristik uchburchakni hosil qiladi.

u (x, t) funksiya uchun $u(x_0, t_0)$ nuqtada (1.3.4) Dalamber formulasini yozib

$$u(x_0, t_0) = \frac{\phi(x_0 - t_0) + \phi(x_0 + t_0)}{2} + \frac{1}{2a}\int_{x_0 - at_0}^{x_0 + at_0} \varphi(\xi) d\xi$$

hosil qilamizki, u (x, t) funksiyaning qiymati faqat xarakteristik uchburchakning asosidagi $\phi(x),\ \varphi(x)$ qiymatlari bilan aniqlanadi.

Bu giperbolik tipdagi tenglamalarning muxim o'ziga xos xususiyat. Uni quyidagi misolda tushinib olish mumkin.

Faraz qilaylik $\phi(x), \varphi(x)$ funksiyalar biror [a, b] kesmaning tashqarisida 0 ga teng bo'lsin. Shunda II, III sohalarda u (*x, t)* funksiya ham 0 ga aynan teng bo'ladi. Bu Dalamber formulasidan osongina ko'rish mumkin. Ushbu fakt (dalil) giperbolik tenglamadagi u (x, t) signal (xabar)ni tarqalishining (x o'qi bo'yicha) (t vaqt mobaynidagi) oxiridagi tezligini ko'rsatadi.

Aksincha issiqlik o'tkazuvchanlik tenglamasi uchun berilgan Koshi masalasida

$$\begin{cases} u_t = a^2 u_{xx}, & -\infty < x < \infty, \quad t > 0 \\ u(x,0) = \phi(x) & -\infty > x > \infty, \end{cases}$$

yechim, keyinchlik ko'rsatadiganidek, quyidagi ko'rinishga ega bo'ladi:

$$u(x,t) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{4\pi a^2 t}} \exp\left(-\frac{(x-s)^2}{4a^2} \right) \phi(s) ds$$

Ko'rinib tipibdiki, agar $\phi(s)$ funksiya uzluksiz, manfiy bo'lmagan va biror nuqtada 0 dan farqli bo'lsa, unda

$$u(x,t) > 0, \quad \forall t > 0$$

bo'ladi.

Ya'ni biz shuni hosil qildikki issiqlik o'kazuvchanlik tenglamasi holida signal (xabar) amalda darhol tarqaladi.

Birinchi chegaraviy masala

Yarim to'g'ri chiziqdagi bir jinsli shartga ega bo'lgan tebranish tenglamasi uchun birinchi chegaraviy masala quyidagi ko'rinishga ega:

$$\begin{cases} (1) \quad u_{tt} = a^2 u_{xx}, & x > 0, t < 0; \\ (2) \quad u(x,t) = 0, & t < 0; \\ (3) \quad u(x,0) = \phi(x), & x \geq 0; \\ (4) \quad u_t(x,0) = \varphi(x), & x \geq 0; \end{cases}$$

u (x, t) va u_t *(x, t)* funksiyalarning 0 da uzluksizligini ta'minlash uchun

$$\begin{cases} \phi(0) = 0; \\ \varphi(0) = 0. \end{cases}$$

bog'lanish shartlarini qo'shamiz (usloviya sopryajeniya).

Ushbu chegaraviy masalaning yechimini topish uchun, uni to'liq to'g'ri chiziq holigacha kengaytirish asosida aniqlaymiz. Yangi Φ, Ψ fuknsiyalarni kiritgan xolda $\phi(x), \varphi(x)$ funksiyalarni butun to'g'ri chiziqda toq tarzada qo'shimcha aniqlaymiz.

$$\Phi(x) = \begin{cases} \phi(x), & x \geq 0; \\ -\phi(-x), & x < 0. \end{cases}$$

$$\Psi(x) = \begin{cases} \varphi(x), & x \geq 0; \\ -\varphi(-x), & x < 0. \end{cases}$$

Modifikasiyalangan Koshi masalasini qaraymiz.

$$\begin{cases} u_{tt}(x,t) = a^2\, u_{xx}(x,t), & -\infty < x < \infty,\ t > 0; \\ u\,(x,0) = \Phi(x), \\ u\,(x,0) = \Psi(x). \end{cases}$$

Bu holda U(x,t) ni topish uchun biz Dalamber formulasidan foydalanamiz.

$$\mathrm{U}\,(\mathrm{x},\mathrm{t}\,) = \frac{\Phi(\mathrm{x}-\mathrm{at}) + \Phi(\mathrm{x}+\mathrm{at})}{2} + \frac{1}{2a}\int_{x-at}^{x+at} \Psi(\xi)\,d\xi.$$

$x,t \geq 0$ bizga kerakli u (x, t) funksiya sifatida U (x, t) funksiyani olamiz. Ko'rinib tipibdiki (1), (3) va (4) shartlar $x,t \geq 0$ bo'lganda birdaniga bajariladi, bu $\Psi(x),\Phi(x)$ larni tarifidan kelib chiqadi. (2) shartning bajrilishi quyidagi almashtirishlardan kelib chiqadi.

$$u(0,t) \overset{def}{=} U(0,t) = \frac{\Phi(-at) + \Phi(at)}{2} + \frac{1}{2a}\int_{-at}^{at} \Psi(\xi)\,d\xi.$$

1-chi va 2-chi qo'shiluvchilar tegishli funksiyalarning toqligi sabali nolga aylanadi. Bu esa 2chi shartning bajarilishini ko'rsatadi. Shunday qilib bizlar tuzgan u (x, t) funksiya birinchi chegaraviy masalalarning yechimi ekanligini isbotladik. $\Psi(x),\Phi(x)$ funksiyalarni mos ravishda isxodnыye funksiyalar $\phi(x), \varphi(x)$ orqali ifodalaymiz:

$$Agar \quad x \geq at \quad bo'lsa \begin{cases} \Phi(x+at) = \phi(x+at); \\ \Phi(x-at) = \phi(x-at); \\ \quad \Psi(\xi) = \varphi(\xi), \quad agar \quad \xi \in [x-at; x+at] \quad bo'lsa \end{cases}$$

$$Agar \quad x < at \quad bo'lsa \begin{cases} \Phi(x+at) = \phi(x+at); \\ \Phi(x-at) = -\phi(x-at); \end{cases}$$

Birinchi chegaraviy masalani yechish uchun quyidagi yordamchi formulani yozamiz.

$$\grave{A}g\grave{a}r \;\; x < at \;\; bo'lsa, \; unda \int_{x-at}^{x+at} \Psi(\xi)d\xi = \int_{x-at}^{0} \Psi(\xi)d\xi + \int_{0t}^{x-a} \Psi(\xi)d\xi =$$

$$= \int_{x-at}^{0} -\varphi(-\xi)d\xi + \int_{0}^{x+at} \varphi(\xi)d\xi =$$

$$= \{-\xi = \xi \;\; deb \;\; olamiz\} = \int_{at-\div}^{0} \varphi(\xi)d\xi + \int_{0}^{x+at} \varphi(\xi)d\xi = \int_{at-x}^{at+x} \varphi(\xi)d\xi$$

Shunda umumiy formula quyidagicha bo'ladi:

$$u(x,t) = \begin{cases} \dfrac{\phi(x+at)+\phi(x-at)}{2} + \dfrac{1}{2a}\displaystyle\int_{x-at}^{x+at} \varphi(\xi)d\xi, \;\; x \geq at; \\ \dfrac{\phi(at+x)-\phi(at-x)}{2} + \dfrac{1}{2a}\displaystyle\int_{at-xt}^{at+xt} \varphi(\xi)d\xi, \;\; x < at; \end{cases}$$

1.3- §. To'lqin tenglamasi uchun chegaraviy va aralash masalalarni yechishning Fur'ye usuli

Xususiy hosilali differensial tenglamalar nazariyasida chegaraviy yoki aralash masalalrni yechishda eng ko'p qo'llaniladigan usullardan biri bu o'zgaruvchilarni ajratish yoki Fur'ye usuli hisoblanadi [1-4]. Biz bu mavzuda ushbu usulni uchlari mustahkam mahkamlangan torning erkin tebranish tenglamasiga qo'yilgan ch1-chegaraviy masala misolida tanishib chiqamiz. Qolgan chegaraviy masalalar va aralash masalalar xuddi shunga o'xshash tarzda yechiladi.

Masalaning qo'yilishi. Biz yuqorida to'lqin tenglamasi uchun uchta turdagi chegaraviy masalalar va aralash turdagi chegaraviy masalalarning qo'yilishi bilan tanishgan edik. Hozir biz bir jinsli chegaraviy shartli 1-chegaraviy masala yechimini, ya'ni

$$u_{tt} = a^2 u_{xx},\ 0 < x < \ell,\ t > 0 \qquad (1.3.1)$$

to'lqin tenglamasining

$$\left.\begin{aligned} u(x,0) &= \varphi(x) \\ u_t(x,0) &= \psi(x),\ 0 < x < \ell \end{aligned}\right\} \qquad (1.3.2)$$

boshlang'ich shartni hamda uchlari mustahkamlanishga mos (uchlari siljishi yo'q)

$$\left.\begin{aligned} u(0,t) &= 0 \\ u(\ell,t) &= 0,\ t > 0 \end{aligned}\right\} \qquad (1.3.3)$$

1-tur chegaraviy shartni qanoatlantiruvchi yechimini topish bilan tanishamiz.

Xuddi shu kabi bir jinsli 2-tur va 3-tur chegaraviy masalalarni hamda ular yordamida tuzuladigan aralash tipdagi bir jinsli chegaraviy masalalrni ham ta'riflashimiz mumkin (Oldingi mavzularda mavjudligi uchun ularni keltirishni o'quvchiga havola qilamiz).

Bizga yagonalik teoremasidan ma'lumki qoyiladigan bu chegaraviy masalalr yagona yechimga ega. Quyida biz ushbu yechimni topish masalasi bilan tanishamiz.

Hozircha (1.3.1)-(1.3.3) masalaning

$$u(x,t) = X(x)T(t),\ 0 < x < \ell,\ t > 0 \qquad (1.3.4)$$

ko'rinishdagi nolmas yechimi mavjud deb faraz qlamiz va uninig ko'rinishini topamiz. Buning uchun (1.3.4) dan kerakli xususiy hosilalarni olamiz

$$u_{xx}(x,t) = X''(x)T(t),\ u_{tt}(x,t) = X(x)T''(t)$$

hamda ularni (1.3.1) ga qo'yamiz:

$$X(x)T''(t) = a^2 X''(x)T(t).$$

Ushbu tenglamani shartga ko'ra aynan nolga teng bo'lmagan $a^2 X(x)T(t)$ ifodaga bo'lib, unga teng kuchli bolgan tenglamaga kelamiz:

$$\frac{X''(x)}{X(x)} = \frac{T''(t)}{a^2 T(t)}.$$

Bu tenglamaning chap qismi faqat $x \in (0, \ell)$ o'zgaruvchining funksiyasi bo'lsa, uning o'ng tomoni faqat $t \in (0, +\infty)$ o'zgaruvchining funksiyasidan iboratdir. Demak u nolmas yechimga ega bo'lishi uchun har ikkala kasrlar aynan bir o'zgarmas songa teng bo'lishi kerak. Hisoblashda qulaylik bo'lsishi uchun uni $-\lambda$ deb belgilaymiz:

$$\frac{X''(x)}{X(x)} = \frac{T''(t)}{a^2 T(t)} = -\lambda . \qquad (1.3.5)$$

Tabiiyki (1.3.5) tenglamalar sistemasi aynan nolga teng bo'lmagan yechimga ega bo'lishi lozim bo'lgan quyidagi ikkita oddiy diferensial tenglamalarga ajraladi:

$$X''(x) + \lambda X(x) = 0, \quad X(x) \neq 0$$

$$T''(t) + \lambda a^2 T(t) = 0, \; T(t) \neq 0 .$$

(1.3.3) chegaraviy shartlarni qaraymiz:

$$\left.\begin{array}{l} u(0,t) = X(0)T(t) = 0 \\ u(\ell,t) = X(\ell)T(t) = 0 \end{array}\right\}$$

Shartga ko'ra $T(t) \neq 0$, aks holda $u(x,t) \equiv 0$ bo'lar edi. Shuning uchun yuqoridagi chegaraviy shartlardan

$$X(0) = X(\ell) = 0$$

shartlarni hosil qilamiz. Shunday qilib biz qo'yilgan chegaraviy masalani yechish jarayonida $X(x)$ funksiya uchun Stuurm-Liuvill masalasi deb ataluvchi quyidagi masalaga keldik.

Ta'rif. λ ning

$$\left.\begin{array}{l} X''(x) + \lambda X(x) = 0 \\ X(0) = X(\ell) = 0 \end{array}\right\} \qquad (1.3.6)$$

masala nolmas yechimga ega bo'ladigan qiymatiga shu masalaning xos qiymati va unga mos nolmas yechimga esa λ xos qiymatga mos xos funksiya deyiladi. λ xos qiymatni va unga mos xos funksiyani topish masalasiga odatda Sturm-Liuvill masalasi deb yuritiladi.

Ushbu masalaninig yechimini topish maqsadida λ ning manfiy, nolga teng va musbat qiymatli hollarini alohida – alohida qaraymiz.

1-hol. Faraz qilaylik $\lambda < 0$ bo'lsin. Bu holda differensial tenglamalar kursidan bizga ma'lumki, (1.3.6) dagi ikkinchi tartibli oddiy differensial tenglamaning umumiy yechimi

$$X(x) = C_1 e^{\sqrt{-\lambda}\,x} + C_2 e^{-\sqrt{-\lambda}\,x} \tag{1.3.7}$$

ko'rinishda bo'ladi. Bunda C_1, C_2- ixtiyoriy haqiqiy sonlar. Ularni shunday tanlaymizki, (1.3.6) dagi chegaraviy shartlar o'rinli bo'lsin:

$$\left.\begin{aligned} &X(0) = C_1 + C_2 = 0 \\ &X(\ell) = C_1 e^{\ell\sqrt{-\lambda}} + C_2 e^{-\ell\sqrt{-\lambda}} = 0 \end{aligned}\right\} \text{ yoki } \left.\begin{aligned} &C_2 = -C_1 \\ &C_1(e^{\ell\sqrt{-\lambda}} - e^{-\ell\sqrt{-\lambda}}) = 0 \end{aligned}\right\}.$$

Qaralayotgan holda $\lambda < 0$ va $\ell > 0$ haqiqiy sonlar bo'lganligi uchun $e^{\ell\sqrt{-\lambda}} - e^{-\ell\sqrt{-\lambda}} \neq 0$. Demak ikkinchi tenglamadan $C_1 = 0$ va birinchisidan esa $C_2 = C_1 = 0$ hosil bo'ladi.

Demak (1.3.7) ga asosan $\lambda < 0$ bo'lganda (1.3.6) masala faqat nol yechimga ega bo'lar ekan, ya'ni bu holda Shtuurm-Liuvill masalasi xos qiymat va xos funksiyaga ega emas ekan.

2-hol. Faraz qilaylik $\lambda = 0$ bo'lsin. Bu holda (1.3.6) dagi ikkinchi tartibli oddiy differensial tenglama $X''(x) = 0$ bo'lib, ununig umumiy yechimi

$$X(x) = C_1 x + C_2 \tag{1.3.8}$$

ko'rinishda bo'ladi. Bunda C_1, C_2- ixtiyoriy haqiqiy sonlar. (1.3.6) dagi chegaraviy shartlardan ularni tanlaymiz:

$$\left.\begin{aligned} &X(0) = C_2 = 0 \\ &X(\ell) = C_1\ell + C_2 = 0 \end{aligned}\right\} \quad \text{yoki} \quad \left.\begin{aligned} &C_2 = 0 \\ &C_1 = 0 \end{aligned}\right\}.$$

Demak (1.3.8) ga asosan $\lambda = 0$ holda ham (1.3.6) masala faqat nol yechimga ega bo'lib, Shtuurm-Liuvill masalasi xos qiymat va xos funksiyaga ega bo'lmas ekan.

3-hol. Faraz qilaylik $\lambda > 0$ bo'lsin. Bu holda differensial tenglamalar kursidan bizga ma'lumki, (1.3.6) dagi ikkinchi tartibli oddiy differensial tenglama ikkita qo'shma kompleks xarakteristik ildizlarga ega bo'lib, uning umumiy yechimi

$$X(x) = C_1 \cos\sqrt{\lambda}\, x + C_2 \sin\sqrt{\lambda}\, x \qquad (1.3.9)$$

ko'rinishda bo'ladi. Bunda C_1, C_2- ixtiyoriy haqiqiy sonlar. Ularni shunday tanlaymizki, (1.3.6) dagi chegaraviy shartlar o'rinli bo'lsin:

$$\left.\begin{array}{l} X(0) = C_1 = 0 \\ X(\ell) = C_2 \sin\sqrt{\lambda}\ell = 0 \end{array}\right\}.$$

$X(x) \neq 0$ ekanligidan $C_2 \neq 0$ bo'ladi. Demak bu sistemadan

$$\sin\sqrt{\lambda}\ell = 0$$

ekanligini olmiz. Bu sodda trigonometrik tenglamaning yechimi

$$\lambda = \lambda_n = \left(\frac{n\pi}{\ell}\right)^2, \ n \in Z.$$

Shunday qilib, (1.3.6) masala faqat $\lambda = \lambda_n = \left(\frac{n\pi}{\ell}\right)^2, \ n \in Z$ bo'lgan holda aynan nolga teng bo'lmagan

$$X_n(x) = C_n \sin\frac{n\pi}{\ell} x$$

yechimlarga ega bo'lar ekan. Bunda C_n - ixtiyoriy doimiy.

Demak (1.3.6) Shtuurm-Liuvill masalasi uchun $\lambda = \lambda_n = \left(\frac{n\pi}{\ell}\right)^2 > 0$ sonlar xos qiymatlar va

$$X_n(x) = \sin\frac{n\pi}{\ell} x \qquad (1.3.10)$$

funksiyalar esa o'zgarmas ko'paytuvchi aniqligida olingan xos funksiyalar bo'ladi. Bu xos funkiyalar skalyar ko'paytmasi uchun

$$(X_n, X_m) = \int_0^\ell X_n(x) X_m(x) dx = \int_0^\ell \sin\frac{n\pi}{\ell} x \sin\frac{m\pi}{\ell} x dx =$$

$$=\frac{1}{2}\left\{\int_0^{\ell}\cos\frac{(n-m)\pi}{\ell}x-\cos\frac{(n+m)\pi}{\ell}xdx\right\}=\begin{cases}\frac{\ell}{2}, \text{ agar } n=m \text{ bo'lsa}\\ 0, \text{ agar } n=m \text{ bo'lsa}\end{cases}$$

tenglik o'rinli.

Shunday qilib biz quyidagi tasdiqni isbotladik .

Teorema. (1.3.6) Shtuurm-Liuvill masalasi faqat $\lambda=\lambda_n=\left(\frac{n\pi}{\ell}\right)^2>0$ bo'lgandagina nolmas yechimga ega bo'lib, barcha xos qiymatlar musbat va har xil xos qiymatga mos keluvchi xos funksiyalar o'zaro ortogonaldir.

$\lambda=\lambda_n=\left(\frac{n\pi}{\ell}\right)^2>0$ bo'lganda

$$T''(t)+\lambda a^2T(t)=0,\ T(t)\neq 0$$

differensial tenglama (1.3.6) dagi differensial tenglamaga o'xshash bo'lganligi ($X(x)$ o'rnida $T(t)$ va λ o'rnida esa $a^2\lambda$ keladi) uchun uning umumiy yechimi quyidagi ko'rinishda bo'ladi:

$$T_n(t)=A_n\cos\frac{n\pi}{\ell}at+B_n\sin\frac{n\pi}{\ell}at. \qquad (1.3.11)$$

U holda $\lambda=\lambda_n=\left(\frac{n\pi}{\ell}\right)^2>0$ bo'ganda (1.3.1) to'lqin tenglamining (1.3.3) bir jinsli chegaraviy shartni qanoatlantiruvchi xususiy yechimi (1.3.4), (1.3.10) va (1.3.11) ga asosan quydagi ko'rinishda bo'ladi:

$$u_n(x,t)=X_n(x)T_n(t)=\left(A_n\cos\frac{n\pi}{\ell}at+B_n\sin\frac{n\pi}{\ell}at\right)\sin\frac{n\pi}{\ell}x.$$

Berilgan (1.3.1) tenglama chiziqli va bir jinsli ikkinchi tartibli xususiy hosilali differensial tenglama bo'lganligi uchun ushbu xususiy yechimlarnig yig'indisi ham (1.3.1) tenglamani va (1.3.3) chegaraviy shartni qanoatlantiradi:

$$u(x,t)=\sum_{n=1}^{\infty}u_n(x,t)=\sum_{n=1}^{\infty}\left(A_n\cos\frac{n\pi}{\ell}at+B_n\sin\frac{n\pi}{\ell}at\right)\sin\frac{n\pi}{\ell}x. \qquad (1.3.12)$$

Bu yechimdagi A_n va B_n koeffisientlarni shunday tanlaymizki, (1.3.2) boshlang'ich shartlar ham bajarilsin, ya'ni:

$$\left.\begin{aligned} u(x,0) &= \sum_{n=1}^{\infty} A_n \sin\frac{n\pi}{\ell}x = \varphi(x) \\ u_t(x,0) &= \sum_{n=1}^{\infty} \frac{n\pi}{\ell} aB_n \sin\frac{n\pi}{\ell}x = \psi(x) \end{aligned}\right\} \tag{1.3.13}$$

Bu sistemadan A_n va B_n koeffisientlarni topish uchun $0 \le x \le \ell$ oraliqda aniqlangan har qanday uzluksiz differensiallanuvchi $f(x)$ funksiyani sinuslar (yoki kosinuslar) bo'yicha Fur'e qatori deb ataluvchi trigonometrik qatorga yoyish mumkinligidan foydalanamiz:

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{\ell}x.$$

Bunda $b_n, n \in N$ koeffisientlarga $f(x)$ funksiyaning Fur'e koeffisientlari deb aytiladi va

$$b_n = \frac{2}{\ell}\int_0^{\ell} f(s)\sin\frac{n\pi}{\ell}s\,ds.$$

Bundan foydalanib (1.3.13) sistemani A_n va B_n larga nisbatan yechish maqsadida uzluksiz differensiallanuvchi $\varphi(x)$ va $\psi(x)$ funksiyalarni Fur'e qatoriga yoyamiz va mos ravishda Fur'e koeffisientlarini yozamiz

$$\varphi(x) = \sum_{n=1}^{\infty} \varphi_n \sin\frac{n\pi}{\ell}x, \qquad \varphi_n = \frac{2}{\ell}\int_0^{\ell} \varphi(s)\sin\frac{n\pi}{\ell}s\,ds \tag{1.3.14}$$

$$\psi(x) = \sum_{n=1}^{\infty} \psi_n \sin\frac{n\pi}{\ell}x, \qquad \psi_n = \frac{2}{\ell}\int_0^{\ell} \psi(s)\sin\frac{n\pi}{\ell}s\,ds. \tag{1.3.15}$$

(1.3.14) va (1.3.15) ni (1.3.13) ga qo'yib, mos koeffisientlarni tenglashtirish bilan A_n va B_n lar uchun quyidagi ifodalarni hosil qilamiz:

$$A_n = \varphi_n = \frac{2}{\ell}\int_0^{\ell} \varphi(s)\sin\frac{n\pi}{\ell}s\,ds \quad \text{va} \quad B_n = \frac{\ell}{n\pi a}\psi_n = \frac{2}{\ell}\int_0^{\ell} \psi(s)\sin\frac{n\pi}{\ell}s\,ds. \tag{1.3.16}$$

A_n va B_n larning (1.3.16) formulalar bo'yicha topilgan bu qiymatlarini (1.3.12) ga qo'yib, (1.3.1)-(1.3.3) bir jinsli 1-tur chegaraviy masalaning formal ko'rinishda yozilgan yechimini hosil qilamiz. Chunki bu ko'rinishda yozilgan (1.3.12) yechim cheksiz hadli qator bo'lib, bu qator uzoqlashuvchi bo'lishi yoki uning yig'indisi differensiallanuvchan bo'lmasligi mumkin. Bu holda (1.3.12) orqali qurilgan funksiyani biz qaralayotgan masala yechimi deya olmaymiz. Shu maqsadda koeffisientlari (1.3.16) formulalar bilan aniqlanuvchi (1.3.12) funksional qator yig'indisining uzluksizligini va uni differensiallash natijasida hosil bo'lgan qatorning tekis yaqinlashuvchanligini ko'rsatishimiz yetarli.

(1.3.12) qatorning ko'rinishidan uni k-marta differensiallash natijasida hosil bo'lgan qatorning umumiy hadi

$$n^k(|\varphi_n|+|\psi_n|)$$

bilan yuqoridan baholanadi. Fur'e qatorlari nazariyasidan esa

$$\sum_{n=1}^{\infty} n^k(|\varphi_n|+|\psi_n|)$$

qator yaqinlashuvchan bo'ladi. Bundan esa funksional qatrolarning tekis yaqinlashuvchanligi haqidagi Veyershtrass teoremasiga asosan (1.3.12) qator tekis yaqinlashuvchi bo'ladi, yani uning yigindisi uzluksiz funksiya bo'lib, qatorni istalgan marta differensiallash mumkin.

Shu bilan (1.3.12) ko'rinishdagi yechim biz qaragan masalaning haqiqiy yechimi ekanligini isbotladik.

1.4-§. Torning majburiy tebranish tenglamasiga qo'yilgan bir jinsli chegaraviy masalani yechishning Fur'e usuli.

Ushbu usulni faqat torning erkin tebranish tenglamasiga emas, balki uning majburiy tebranish tenglamasiga qo'yilgan chegaraviy masalani yechishga ham tatbiq etilishi mumkin [1,4].

Faraz qilaylik bizga torning majburiy tebranish tenglamasi

$$u_{tt} = a^2 u_{xx} + f(x,t),\ 0 < x < \ell,\ t > 0 \qquad (1.4.1)$$

ning

$$\left.\begin{aligned} &u(x,0)=\varphi(x) \\ &u_t(x,0)=\psi(x),\ 0\le x\le \ell \end{aligned}\right\} \qquad (1.4.2)$$

boshlang'ich shartlarni hamda

$$\left.\begin{aligned} u(0,t) &= 0 \\ u(\ell,t) &= 0,\ t>0 \end{aligned}\right\} \qquad (1.4.3)$$

bir jinsli chegaraviy shartni qanoatlantiruvchi yechimini topish talab qilinsin.

Ushbu masala yechimini $u(x,t)$ funksiyning x bo'yicha Fur'ye qatori ko'rinishida izlaymiz:

$$u(x,t)=\sum_{n=1}^{\infty}u_n(t)\sin\frac{n\pi}{\ell}x.$$

Xuddi shu kabi $f(x,t), \varphi(x)$ va $\psi(x)$ funksiyalarni ham Fur'e qatoriga yoyamiz va bu qatorlarni va yechimning tanlangan Fur'e qatorini (1.4.1) ga qo'yib, quyidagi tenglamaga kelamiz:

$$u_n''(t)+\left(\frac{n\pi}{\ell}a\right)^2 u_n(t)=f_n(t). \qquad (1.4.4)$$

Bunda $u_n(t)$ va $f_n(t)$ bilan mos ravishda $u(x,t)$ va $f(x,t)$ funksiyalarning har bir tayinlangan t da Fur'e koeffisientlari belgilangan.

(1.4.2) boshlang'ich shartlar fur'e qatorlariga yoyilgandan keyin

$$u_n(0)=\varphi_n,\ \ u_m'(0)=\psi_m. \qquad (1.4.5)$$

(1.4.5) shartlar asosida (1.4.4) differensial tenglama bir qiymatli yechiladi. Uning yechimi bir jinsli tenglamaning umumiy yechimi bilan bir jinsli bo'lmagan tenglamaning bitta xususiy yechimidan iborat bo'lib, quyidagi ko'rinishda yoziladi:

$$u(x,t)=\sum_{n=1}^{\infty}\left(\varphi_n\cos\frac{n\pi}{\ell}at+\frac{\ell}{n\pi a}\psi_n\sin\frac{n\pi}{\ell}at\right)\sin\frac{n\pi}{\ell}x+$$

$$+\sum_{n=1}^{\infty}\frac{\ell}{n\pi a}\int_0^{\ell}\sin\frac{n\pi}{\ell}a(t-\tau)\sin\frac{n\pi}{\ell}x\, f_n(\tau)d\tau.$$

Ba'zan **bir jinsli bo'lmagan chegaraviy masala** ham qaraladi [2,5]:

$$u_{tt}=a^2u_{xx}+f(x),\ 0<x<\ell,\ t>0$$

tenglamaning

$$\left.\begin{aligned} u(x,0) &= \varphi(x) \\ u_t(x,0) &= \psi(x),\ 0\le x\le \ell \end{aligned}\right\}$$

boshlang'ich shartlarni hamda

$$\left.\begin{aligned} u(0,t) = A \\ u(\ell,t) = B \end{aligned}\right\}$$

chegaraviy shartni qanoatlantiruvchi yechimini topish lozim.

Ushbu masala noma'lum funksiyani

$$u(x,t) = w(x) + v(x,t)$$

ko'rinishda izlash bilan oldin o'rganilgan bir jinsli chegaraviy masalaga keltirish mumkin. Haqiqatan ham yechimning izlangan shaklini berilgan tenglamaga qo'yib

$$v_{tt} = a^2 v_{xx} + a^2 w''(x) + f(x).$$

Bunda $w(x)$ funksiyani quyidagicha tanlaymiz:

$$a^2 w''(x) + f(x) = 0, \quad w(0) = A, \quad w(\ell) = B.$$

Bu oddiy differensial tenglamaga qo'yilgan Koshi masalasini yechib, topamiz:

$$w(x) = A + (B - A)\frac{x}{\ell} + \frac{x}{\ell a^2}\int_0^{\ell}\int_0^{s} f(\xi)d\xi ds - \frac{1}{a^2}\int_0^{x}\int_0^{s} f(\xi)d\xi ds.$$

U holda $v(x,t)$ funksiya biz yuqorida qarab chiqqan quyidagi masalani qanoatlantirishi kerak bo'ladi:

$$v_{tt} = a^2 v_{xx}, \; 0 < x < \ell, \; t > 0$$

$$\left.\begin{aligned} v(x,0) = \varphi(x) - w(x) \\ v_t(x,0) = \psi(x), \; 0 \le x \le \ell \end{aligned}\right\}$$

$$v(0,t) = v(\ell,t) = 0.$$

So'ngi masalani yechishni esa biz to'laligicha qarab chiqdik. Uni yechib, dastlabki qo'yilgan masalaning yechimini ham topishimiz mumkin bo'ladi.

Yechimning ko'rinishidan uning masalaning boshlang'ich shartlaridan uzluksiz bog'liqligi ko'rinib turibdi. Bu esa chegaraviy masala yechimining turgun ekanligini isbotlaydi.

Eslatma. Ta'kidlash lozimki, tor tebranish tenglamasiga qo'yilgan 1-tur chegaraviy maslani yechish jarayonida qo'llanilgan ushbu usulni 2-tur , 3-tur va aralash masalalarni yechishga ham to'g'ridan-to'g'ri qo'llash mumkin. Bunda asosiy farq Shturm-Liuvill masalasi chegaraviy shartlari o'zgarishi bilan uning xos qiymatlari va xos funksiyalari o'zgarishi mumkin. Hosil bo'lgan yechim ko'rinishi va unga mos Fur'e ko'rinishlari ham biroz o'zgarishi mumkin.

Biror $[-\ell,\ell]$ oraliqda uzluksiz differensiallanuvchi $f(x)$ funksiyaning Fur'e qatori umumiy holda

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi}{\ell}x+b_n\sin\frac{n\pi}{\ell}x\right)$$

bo'lib, uning $a_0,\, a_n, b_n,\, n\in N$ Fur'e koeffisientlari mos ravishda quyidagi formulalar bilan aniqlanadi.

$$a_n=\frac{1}{\ell}\int_{-\ell}^{\ell}f(s)\cos\frac{n\pi}{\ell}s\,ds,\quad b_n=\frac{1}{\ell}\int_{-\ell}^{\ell}f(s)\sin\frac{n\pi}{\ell}s\,ds.$$

Berilgan chegaraviy shartga mos Shtuurm – Liuvill masalsi xos funksiyalariga mos ravishda fur'e qatori tanlanadi.

Ikkinchi chegaraviy masala

Yarim to'g'ri chiziqdagi bir jinsli chegaraviy shart bilan berilgan ikkinchi chegaraviy masala quyidagi ko'rinishga ega:

$$\begin{cases}(1)\quad u_{tt}=a^2U_{xx},\, x>0, t>0\\ (2)\quad u_x(0,t)=0, t\ge 0;\\ (3)\quad u(x,0)=\phi(x), x\ge 0;\\ (4)\quad u_t(x,0)=\psi(x), x\ge 0.\end{cases}$$

Oldingi holdagiday harakat qilamiz. Lekin bizlarni faqat juft davom ettirish qanoatlantiradi:

$$\Phi(x)=\begin{cases}\phi(x), x\ge 0;\\ \phi(-x), x<0.\end{cases}\quad \Psi(x)=\begin{cases}\psi(x), x\ge 0;\\ \psi(x), x<0.\end{cases}$$

Yangi Koshi masalasi va uning uchun Dalamber formulasi bo'yicha yechimi 2-chi ma'ruzada ko'rsatganimzdek bo'ladi:

$$U(x,t)=\frac{\Phi(x-at)+\Phi(x+at)}{2}+\frac{1}{2a}\int_{x-at}^{x+at}\Psi(\xi)d\xi.$$

Odlingi holdagidek, $u(x,t)=U(x,t), x, y>0$ bo'lsin.

U holda (1), (3), (4) shartlarning bajarilishi ayon.

(2) shartni tekshiramiz. Dalamber formulasini differensiallasak va $\Psi(t)$ juft funksiyaning hosilasi toq funksiya bo'lishini inobatga olib, quyidagi tenglikni hosil qilamiz:

$$u_x(0,t) = U_x(0,t) = \frac{\Phi'(at) + \Phi'(-at)}{2} + \frac{1}{2a}[\Psi(at) - \Psi(-at)]$$

$\Phi'(t)$ toqligidan va $\Psi(t)$ juftligidan ko'rinadiki ikkala had ham nolga teng.

$u(x,t)$ uchun umumiy formula shunga o'xshash olinadi.

$[0,l]$ kesmada ortonormallashgan funksiyalar sistemalarini qaraymiz.

$$\left\{\sqrt{\frac{2}{l}}\sin\left(\frac{\pi n}{l}x\right)\right\}, n = 1,2,3,... \quad \left\{\frac{1}{\sqrt{l}}\sqrt{\frac{2}{l}}\cos\left(\frac{\pi n}{l}x\right)\right\}, n = 1,2,3,...$$

Fur'ye koeffisiyentlarini

$$\phi_n = \int_0^l \phi(s)\sin\left(\frac{\pi n}{l}s\right)ds; \quad \tilde{\phi}_n = \int_0^l \phi(s)\sin\left(\frac{\pi n}{l}s\right)ds.$$

kabi aniqlaymiz.

U holda matematik analiz kursidan ma'lumki, agar $\phi(x) \in C[a;b]$ bo'lsa, u holda

$$\sum_{n=1}^{\infty}\phi_n^2 \quad , \quad \sum_{n=1}^{\infty}\tilde{\phi}_n^2$$

qatorlar yaqinlashadi. Buni eslab qolamiz va bir jinsli chegaraviy shartlar bilan berilgan bir jinsli tebranish tenglamasi uchun birinchi chegaraviy masalaga o'tamiz:

$$\begin{cases} (1).u_{tt} = a^2 u_{xx}, 0 < x < l, t > 0; \\ (2).u(0,t) = u(l,t) = 0, t \geq 0; \\ (3).u(x,0) = \phi(x), 0 \geq x \geq l; \\ (4).u_t(x,0) = \psi(x), 0 \geq x \geq l. \end{cases} \tag{1.4.6}$$

Uning yechimini quyidagi usul bilan topamiz: biror $u(x,t)$ funksiyaga keltiruvchi almashtirishlarni bajaramiz, so'ngra, ma'lum bir shartlarni qanoatlantiruvchi $\phi(x)$ va $\psi(x)$ funksiyalar uchun bu funksiya mavjud bo'lishini va berilgan masala yechimi ekanligini isbotlaymiz.

Yechimni $v(x,t) = X(x)T(t)$ ko'rinishda izlaymiz. Bu nolga aynan teng bo'lmagan funksiya bo'lsin. $v(x,t)$ ni tebranish tenglamasiga qo'yib, quyidagini hosil qilamiz:

$$T''(t)X(x) = a^2 X''(x)T(t) \Rightarrow \frac{X''(x)}{X(x)} = \frac{T''(t)}{a^2 T(t)} = -\lambda$$

bu yerda λ qandaydir o'zgarmas son.

Bu ayniyatlardan ikkita tenglama kelib chiqadi:

$$\begin{cases} X''(x) + \lambda X(x) = 0, 0 < x < l; \\ T''(t) + \lambda a^2 T(t) = 0, t > 0. \end{cases}$$

$X(0) = X(l) = 0$ da $v(x,t)$ funsiya (2) shartni qanoatlantiradi.

Quyidagi masalani qaraymiz.

$$\begin{cases} X''(x) + \lambda X(x) = 0, 0 \le x \le l; \\ X(0) = X(l) = 0. \end{cases}$$

Shturm – Liuvill masalasining trivial bo'lmagan yechimlarni topamiz.

Issiqlik o'tkazuvchanlik tenglamasi uchun yechimni chiqarishda, quyidagi xos qiymatlar va ularga mos xos funksiyalar to'g'ri keladi (buni bizlar keyinchalik ko'rsatamiz):

$$\lambda_n = (\frac{\pi n}{l})^2; \quad X_n(x) = \sin(\frac{\pi n}{l} x), n = 1, 2, ...$$

Topilgan λ_n larni $T(t)$ uchun tenglamaga qo'yamiz:

$$T_n''(t) + (\frac{\pi n}{l} a)^2 T_n(t) = 0 \Rightarrow T_n(t) = a_n \cos(\frac{\pi n}{l} at) + b_n \sin(\frac{\pi n}{l} at),$$

bu yerda a_n va b_n lar qandaydir o'zgarmaslar.

Shunday qilib (1), (2) shartlar qanoatlantiradigan $X_n(x), T_n(t)$ funksiyalarni topdik.

$v_n(x,t) = X_n(x)T_n(t)$ deb olamiz. Ravshanki, bu funksiya uchun ham (1), (2) shartlar bajariladi.

(3), (4) shartlardan a_n, b_n konstantalarni

$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t)$ deb olamiz;

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} \sin(\frac{\pi n}{l}x)[a_n \cos(\frac{\pi n}{l}at) + b_n \sin(\frac{\pi n}{l}at)];$$

$$\phi(x) = u(x,0) = \sum_{n=1}^{\infty} a_n \sin(\frac{\pi n}{l}x) \Rightarrow a_n = \frac{2}{l}\int_0^l \phi(s)\sin(\frac{\pi n}{l}s)ds;$$

$$\psi(x) = u_t(x,0) = \sum_{n=1}^{\infty} (b_n \frac{\pi na}{l})\sin(\frac{\pi n}{l}x) \Rightarrow \frac{\pi na}{l} b_n = \frac{2}{l}\int_0^l \psi(s)\sin(\frac{\pi n}{l}s)ds \Rightarrow$$

$$b_n = \frac{2}{\pi na}\int_0^l \psi(s)\sin(\frac{\pi n}{l}s)ds.$$

Natijada, konstantalarni topdik, endi to'la formulani yozamiz;

$$u(x,t) = \sum_{n=1}^{\infty} [\frac{2}{l}\int_0^l \cos(\frac{\pi n}{l}s)\phi(s)\sin(\frac{\pi n}{l}s)ds +$$

$$+\frac{2}{\pi na}\int_0^l \sin(\frac{\pi n}{l}at)\psi(s)\sin(\frac{\pi n}{l}s)ds]\sin(\frac{\pi n}{l}x).$$

Endi bu formula korrekt bo'ladigan shartlarni ifodalaymiz.

2- Bob. Kvadratli membrananing tebranish tenglamasini Maple paketi yordamida yechish

2.1 - §. To'g'ri to'rtburchak membranasining tebranishlari

Membrana deb bizlar juda yupqa plenkani tushunamiz, u torga o'xshab faqat cho'zilishga ishlaydi, bukilishga ishlamaydi [4]. Agarda membrana tekis tortilishi kuchi ta'sirda bo'lsa, muvozanat holatida *(x, y)* tekislikda joylashgan bo'lsa, (bizlar faqat *Oz* o'qiga parallel siljishlarni o'rganamiz) shunda *(x, y)* membrana nuqtasini siljishi *x, y, t* o'zgaruvchilarni u funksiyasi bo'ladi va bu funksiya tor tenglamasiga o'xshab analogik quyidagi differensial tenglamani qanoatlantiradi

$$u_{tt} = a^2\left(u_{xx} + u_{yy}\right) + f(x, y, t), \qquad (2.1.1)$$

Bu yerda

$$a = \sqrt{\frac{T_0}{p}},$$

P- membrana sirtining zichligi, ρ- f tashqi kuch yoki yuklama.

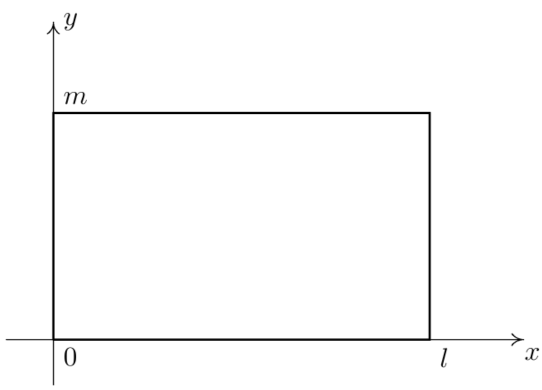
(1) differensial tenglamadan tashqari bizlar chegaraviy shartlarni inobatga olish kerak. $u$funksiya S kontorda ya'ni membrana chegarasida qonoatlantiradigan shartlar

$$u = 0. \qquad (2.1.2)$$

Holatni qaraymiz. Bundan tashqari boshlang'ich shartlar berish kerak ya'ni boshlang'ich momentlarni barcha nuqtalarini siljishi

$$u\big|_{t=0} = \varphi 1(x, y),\ u_t\big|_{t=0} = \varphi 2(x, y) \qquad (2.1.3)$$

To'rtburchakli membrana ozod tebranishlari kuzatamiz



Kontur *(x, y)* tekislikda

$$x = 0, x = l, y = 0, y = m$$

ko'rinishdagi to'g'rito'rtburchakni xosil qiladi. Tashqi kuch yo'q deb hisoblaymiz ya'ni $f = 0$

$$u_{tt} = a^2 (u_{xx} + u_{yy}), \quad (2.1.4)$$

(4) tenglamani (2) (3) shartlarni qanoatlantiruvchi yechimini topamiz.

Furye usulidan foydalanib (4) tenglamani yechimini qo'yidagi shaklda izlaymiz

$$(\alpha \cos \omega t + \beta \sin \omega t) U(x, y) \quad (2.1.5)$$

Bu esa bizlarga quyidagini beradi.

$$-\omega^2 (\alpha \cos \omega t + \beta \sin \omega t) U(x, y) = a^2 (U_{xx} + U_{yy})(\alpha \cos \omega t + \beta \sin \omega t),$$

$\frac{\omega^2}{a^2} = k^2$, belgilash keritamiz.

U funksiya uchun qo'yidagi tenglamani olamiz

$$U_{xx} + U_{yy} + k^2 U = 0,$$

bu tenglamani xususiy yechimini quyidagi ko'rinishda axtaramiz

$$U(x, y) = X(x) Y(y),$$

bu yerda

$$X''(x) Y(y) + X(x) Y''(y) + k^2 X(x) Y(y) = 0,$$

$$\frac{X''(x)}{X(x)} = -\frac{Y''(y) + k^2 Y(y)}{Y(y)} = -\lambda^2, \lambda = const.$$

Shunday qilib quyidagi sistemani hosil qilamiz

$$\begin{cases} X''(x) + \lambda^2 X(x) = 0 \\ Y''(y) + \mu^2 Y(y) = 0; \end{cases} \qquad \mu^2 = k^2 - \lambda^2. \quad (2.1.6)$$

(6) tenglamalar bizlarga $X(x)$ va $Y(y)$ funksiyalarni quyidagi umumiy ko'rinishlari beradi

$$X(x) = C_1 \sin \lambda x + C_2 \cos \lambda x,$$

$$\mathrm{Y(y)} = \mathrm{C}_3 \sin \mu x + C_4 \cos \mu x.$$

$u = 0,$ shartdan bizlar $U(x, y) = 0$ xosil qilamiz. Bu shart esa qo'yidagi shartlarga ajralib ketadi:

$$\mathrm{X(0)} = 0, X(l) = 0, \mathrm{Y}(0) = 0, Y(m) = 0.$$

Bu yerda aniq ko'rinib turibdiki $C_2 = C_4 = 0,$ va agarda bizlar $C_1 \neq C_3$ ligidan va doimiy ko'paytuvchilarni olib tashqariga chiqarsak

$$X(x) = \sin \lambda l,$$

$$Y(y) = \sin \mu m,$$

bu yerda

$$\sin \lambda l = 0, \quad \sin \mu m = 0. \tag{2.1.7}$$

(7) tenglamadan $\lambda = \frac{\pi n_1}{l}, \quad \mu = \frac{\pi n_2}{m}, \quad n_1, n_2 \in N$ larni topamiz so'ngra esa k doimiy qiymatlarini topish mumkin,

$$\mathrm{k}^2 \mathrm{n}_1 \mathrm{n}_2 = \lambda^2 \mathrm{n}_1 + \mu^2 \mathrm{n}_2 = \pi^2 \left(\frac{n_1^2}{l^2} + \frac{n_2^2}{m^2} \right)$$

va ω chastotani topilgan

$$\omega^2{}_{n_1 n_2} = a^2 k^2{}_{n_1 n_2} = a^2 \pi^2 \left(\frac{n^2}{l^2} + \frac{n_2^2}{m^2} \right)$$

(5) tenglamaga qo'yib quyidagini hosil qilamiz

$$\left(\alpha_{n_1 n_2} \cos \omega_{n_1 n_2} t + \beta_{n_1 n_2} \sin \omega_{n_1 n_2} t \right) \sin \frac{\pi n_1 x}{l} \sin \frac{\pi n_2 y}{m}.$$

α, β doimiylar boshlang'ich shartlarda aniqlanadi

$$u = \sum_{n_1, n_2 = 1}^{\infty} \left(\alpha_{n_1 n_2} \cos \omega_{nqn_2} t + \beta_{n_1 n_2} t \right) \sin \frac{\pi n_1 x}{l} \sin \frac{\pi n_2 y}{m},$$

$$u_t = \sum_{n_1, n_2 = 1}^{\infty} \omega_{n_1 n_2} \left(\beta_{n_1 n_2} \cos \omega_{n_1 n_2} t - \alpha_{n_1 n_2} \sin \omega_{n_1 n_2} t \right) \sin \frac{\pi n_1 x}{l} \sin \frac{\pi n_2 y}{m},$$

yuqoridagi formulada $t = 0$ deb olsak va (3) formulaga asoslanib quyidagilarni hosil qilamiz:

$$u_t\big|_{t=0} = \varphi 1(x,y) = \sum_{n_1,n_2=1}^{\infty} \alpha_{n_1 n_2} \sin\frac{\pi n_1 x}{l}\sin\frac{\pi n_2 y}{m},$$

$$u_t\big|_{t=0} = \varphi 2(x,y) = \sum_{n_1,n_2=1}^{\infty} \beta_{n_1 n_2}\,\omega_{n1n2} \sin\frac{\pi n_1 x}{l}\sin\frac{\pi n_2 y}{m}.$$

Bu formulada $\varphi_1\,\varphi_2$ funksiyalarni ikkinchi tur Furye qatori yoyilmasi bo'ladi. Va $\alpha\ \beta$ kaeffesiyentlar esa qo'yidagi formula bo'yicha aniqlanadi

$$\alpha_{n_1 n_2} = \frac{4}{lm}\int_0^l \int_0^m \varphi_{1(\xi,\eta)} \sin\frac{\pi n_1 \xi}{l}\sin\frac{\pi n_2 \eta}{m}\, d\xi dn,$$

$$\beta_{n_1 n_2} = \frac{4}{\omega_{n_1 n_2} lm}\int_0^l \int_0^m \varphi_{2(\xi,\eta)} \sin\frac{\pi n_1 \xi}{l}\sin\frac{\pi n_2 \eta}{m}\, d\xi dn,$$

Bularni hammasi qo'yilgan masalani yechimini beradi. Bizlar qaragan membrana tor bilan nimasi bilan farq qiladi. Torda har qanday chastotada o'zini tebranishlariga mos bo'ladigan vabirnechta qismga ajraladigan va va tugun yordamida bir necha qismlarga ajraladi toning o'zini formasi mos keladi.

Membrana uchun bitta chastotaga membrana bir necha formasi har xil tugunlar bilan formasiga mos kelishi mumkin. Boshqacha aytganda membrana tebranish chastotasi nolga aylanadi.

2.2-§. Maple paketi orqali kvadratli membrananing tebranish tenglamasini Fur'ye usuli (o'zgaruvchilarni ajratish usuli) yordamida yechish

Kvadratli membrananing erkin tebranishini qaraymiz [6-8]. Buning uchun quyidagi

$$\frac{\partial^2}{\partial t^2}\mathrm{u}(t,x,y) = a^2\left(\left(\frac{\partial^2}{\partial x^2}\mathrm{u}(t,x,y)\right)+\left(\frac{\partial^2}{\partial y^2}\mathrm{u}(t,x,y)\right)\right)$$

bir jinsli tenglamani, ushbu

$$\mathrm{u}(0,x,y) = \mathrm{F}(x,y),$$

$$\frac{\partial}{\partial t}\mathrm{u}(0,x,y) = \mathrm{f}(x,y).$$

boshlang'ich shartlar va quyidagi

$$u(t,0,0)=0,$$

$$u(t,0,L)=0,$$

$$u(t,L,0)=0,$$

$$u(t,L,L)=0.$$

chegaraviy shartlar (ya'ni membrannaning ikki uchi mahkamlangan) bilan berilgan masalani yechamiz.

```
> restart;
```

Bir jinsli tenglamani o'zgaruvchilarni ajratish usuli yordamida yechamiz:

```
> PDE:=diff(u(t,x,y),t,t)=a^2*(diff(u(t,x,y),x,x)+diff(u(t,x,y),y,y));
struc:=pdsolve(PDE,HINT=T(t)*X(x)*Y(y));
```

$$PDE := \frac{\partial^2}{\partial t^2}u(t,x,y)=a^2\left(\left(\frac{\partial^2}{\partial x^2}u(t,x,y)\right)+\left(\frac{\partial^2}{\partial y^2}u(t,x,y)\right)\right)$$

$$struc := (u(t,x,y)=T(t)\,X(x)\,Y(y))\ \&where$$
$$\left[\left\{\frac{d^2}{dy^2}Y(y)=-Y(y)\,\_c_2+\frac{\_c_1\,Y(y)}{a^2},\ \frac{d^2}{dx^2}X(x)=\_c_2\,X(x),\ \frac{d^2}{dt^2}T(t)=\_c_1\,T(t)\right\}\right]$$

```
> dsolve(diff(T(t),`$`(t,2)) = _c[1]*T(t));
dsolve(diff(X(x),`$`(x,2)) = _c[2]*X(x));
dsolve(diff(Y(y),`$`(y,2)) = Y(y)/a^2*_c[1]-_c[2]*Y(y));
```

$$T(t)=\_C1\,\mathbf{e}^{(\sqrt{\_c_1}\,t)}+\_C2\,\mathbf{e}^{(-\sqrt{\_c_1}\,t)}$$

$$X(x)=\_C1\,\mathbf{e}^{(\sqrt{\_c_2}\,x)}+\_C2\,\mathbf{e}^{(-\sqrt{\_c_2}\,x)}$$

$$Y(y)=\_C1\sin\left(\frac{\sqrt{\_c_2\,a^2-\_c_1}\,y}{a}\right)+\_C2\cos\left(\frac{\sqrt{\_c_2\,a^2-\_c_1}\,y}{a}\right)$$

O'zgaruvchilarni ajratish bo'yicha quyidagi almashtirishlarni olamiz:

$$\_c_1 == -\ \lambda^2$$

$$\_c_2 == -\ \frac{\kappa^2}{a^2}$$

```
> dsolve(diff(T(t),`$`(t,2)) = -lambda^2*T(t));
dsolve(diff(X(x),`$`(x,2)) = -kappa^2*X(x)/a^2);
dsolve(diff(Y(y),`$`(y,2)) = -Y(y)/a^2*lambda^2+kappa^2*Y(y)/a^2);
```

$$T(t)=\_C1\sin(\lambda\,t)+\_C2\cos(\lambda\,t)$$

$$X(x) = \_C1 \sin\left(\frac{\kappa x}{a}\right) + \_C2 \cos\left(\frac{\kappa x}{a}\right)$$

$$Y(y) = \_C1 \sin\left(\frac{\sqrt{\lambda^2 - \kappa^2}\, y}{a}\right) + \_C2 \cos\left(\frac{\sqrt{\lambda^2 - \kappa^2}\, y}{a}\right)$$

Birinchi chegaraviy shartni hisobga olib, ikkinchi tenglamani yechamiz:

$$X(0) = 0$$

```
> dsolve({diff(X(x),`$`(x,2)) = -kappa^2*X(x)/a^2, X(0)=0}, X(x));
```

$$X(x) = \_C1 \sin\left(\frac{\kappa x}{a}\right)$$

Endi ikkinchi chegaraviy shartdan foydalanamiz:

$$X(L) = 0$$

```
>  _EnvAllSolutions := true:
solve(sin(kappa/a*L)=0,kappa);
```

$$\frac{\pi\, \_Z1{\sim}\, a}{L}$$

yoki, oddiy ko'rinishda,

```
> kappa:=Pi*nx*a/L;
```

$$\kappa := \frac{\pi\, nx\, a}{L}$$

Birinchi chegaraviy shartni hisobga olib, uchinchi tenglamani yechamiz:

$$Y(0) = 0$$

```
> dsolve({diff(Y(y),`$`(y,2)) = -Y(y)/a^2*lambda^2+kappa^2*Y(y)/a^2, Y(0)=0}, Y(y));
```

$$Y(y) = \_C1 \sin\left(\frac{\sqrt{\lambda^2 L^2 - \pi^2 nx^2 a^2}\, y}{a L}\right)$$

Endi ikkinchi chegaraviy shartdan foydalanamiz:

$$Y(Ly) = 0$$

```
>  _EnvAllSolutions := true:
solve(sin(1/a/L*(lambda^2*L^2-Pi^2*nx^2*a^2)^(1/2)*L)=0,lambda);
```

$$\frac{\sqrt{\pi^2\, nx^2 + \pi^2\, \_Z2{\sim}^2}\; a}{L},\; -\frac{\sqrt{\pi^2\, nx^2 + \pi^2\, \_Z2{\sim}^2}\; a}{L}$$

yoki, oddiy ko'rinishda,

```
> lambda:=simplify((Pi^2*nx^2+Pi^2*ny^2)^(1/2)*a/L);
lambda^2;
```

$$\lambda := \frac{\pi\sqrt{nx^2 + ny^2}\; a}{L}$$

$$\frac{\pi^2\,(nx^2 + ny^2)\, a^2}{L^2}$$

Shuning uchun har bir *n* uchun quyidagilarni olamiz:

```
> T[nx,ny](t):=C1[nx,ny]*cos(lambda*t)+C2[nx,ny]*sin(lambda*t);
X[nx,ny](x):=sin(kappa/a*x);
Y[nx,ny](y):=simplify(sin(1/a/L*(lambda^2*L^2-Pi^2*nx^2*a^2)^(1/2)*y))
assuming a::posint, ny::posint;
```

$$T_{nx,\, ny}(t) := C1_{nx,\, ny} \cos\left(\frac{\pi\sqrt{nx^2 + ny^2}\; a\, t}{L}\right) + C2_{nx,\, ny} \sin\left(\frac{\pi\sqrt{nx^2 + ny^2}\; a\, t}{L}\right)$$

$$X_{nx,\, ny}(x) := \sin\left(\frac{\pi\, nx\, x}{L}\right)$$

$$Y_{nx,\, ny}(y) := \sin\left(\frac{\pi\, ny\, y}{L}\right)$$

```
> u[nx,ny](t,x,y):=T[nx,ny](t)*X[nx,ny](x)*Y[nx,ny](y);
```

$$u_{nx,\, ny}(t, x, y) := \left(C1_{nx,\, ny} \cos\left(\frac{\pi\sqrt{nx^2 + ny^2}\; a\, t}{L}\right) + C2_{nx,\, ny} \sin\left(\frac{\pi\sqrt{nx^2 + ny^2}\; a\, t}{L}\right)\right) \sin\left(\frac{\pi\, nx\, x}{L}\right) \sin\left(\frac{\pi\, ny\, y}{L}\right)$$

Natijada umumiy yechim quyidagi ko'rinishga ega bo'ladi:

```
> u(t,x,y):=Sum(Sum(u[nx,ny](t,x,y), nx=1..infinity), ny=1..infinity);
```

$$u(t, x, y) := \sum_{ny=1}^{\infty}\left(\sum_{nx=1}^{\infty}\left(\left(C1_{nx,\, ny} \cos\left(\frac{\pi\sqrt{nx^2 + ny^2}\; a\, t}{L}\right) + C2_{nx,\, ny} \sin\left(\frac{\pi\sqrt{nx^2 + ny^2}\; a\, t}{L}\right)\right) \sin\left(\frac{\pi\, nx\, x}{L}\right) \sin\left(\frac{\pi\, ny\, y}{L}\right)\right)\right)$$

$C1_n$ va $C2_n$ koeffisentlarni aniqlash uchun boshlang'ich shartlardan foydalanamiz:

$$u(0,x,y)=F(x,y)$$

$$\frac{\partial}{\partial t}u(0,x,y)=f(x,y)$$

```
> simplify(subs(t=0,u(t,x,y))=F(x,y));
simplify(subs(t=0,diff(u(t,x,y),t))=f(x,y));
```

$$\sum_{ny=1}^{\infty}\left(\sum_{nx=1}^{\infty} C1_{nx,ny}\sin\left(\frac{\pi\, nx\, x}{L}\right)\sin\left(\frac{\pi\, ny\, y}{L}\right)\right)=F(x,y)$$

$$\sum_{ny=1}^{\infty}\left(\sum_{nx=1}^{\infty}\frac{C2_{nx,ny}\,\pi\sqrt{nx^2+ny^2}\,a\sin\left(\frac{\pi\, nx\, x}{L}\right)\sin\left(\frac{\pi\, ny\, y}{L}\right)}{L}\right)=f(x,y)$$

Olingan munosabatlarni har ikkala qismini quyidagi ifodaga ko'paytiramiz

$$\sin\left(\frac{\pi\, kx\, x}{L}\right)\sin\left(\frac{1\,\pi\, ky\, y}{L}\right)$$

```
>
Sum(Sum(C1[nx,ny]*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y)*sin(Pi*kx/L*x)*sin(1/L*
Pi*ky*y),nx=1..infinity),ny=1..infinity)=F(x,y)*sin(Pi*kx/L*x)*sin(1/L*Pi*ky*y);
Sum(Sum(C2[nx,ny]*Pi*(nx^2+ny^2)^(1/2)*a/L*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*
y)*sin(Pi*kx/L*x)*sin(1/L*Pi*ky*y),nx = 1 .. infinity),ny = 1 .. infinity) =
f(x,y)*sin(Pi*kx/L*x)*sin(1/L*Pi*ky*y);
```

$$\sum_{ny=1}^{\infty}\left(\sum_{nx=1}^{\infty} C1_{nx,ny}\sin\left(\frac{\pi\, nx\, x}{L}\right)\sin\left(\frac{\pi\, ny\, y}{L}\right)\sin\left(\frac{\pi\, kx\, x}{L}\right)\sin\left(\frac{\pi\, ky\, y}{L}\right)\right)= F(x,y)\sin\left(\frac{\pi\, kx\, x}{L}\right)\sin\left(\frac{\pi\, ky\, y}{L}\right)$$

$$\sum_{ny=1}^{\infty}\left(\sum_{nx=1}^{\infty}\frac{C2_{nx,ny}\,\pi\sqrt{nx^2+ny^2}\,a\sin\left(\frac{\pi\, nx\, x}{L}\right)\sin\left(\frac{\pi\, ny\, y}{L}\right)\sin\left(\frac{\pi\, kx\, x}{L}\right)\sin\left(\frac{\pi\, ky\, y}{L}\right)}{L}\right)= f(x,y)\sin\left(\frac{\pi\, kx\, x}{L}\right)\sin\left(\frac{\pi\, ky\, y}{L}\right)$$

$C1_n$ va $C2_n$ koeffisentlarni aniqlash uchun bu munosabatlarni x va y bo'yicha integrallash zarur.

Dastlab

$$\sin\left(\frac{\pi\, nx\, x}{L}\right)$$

$$\sin\left(\frac{1\,\pi\, ny\, y}{L}\right)$$

funksiyalar sistemasi orthogonal funksiyalar sistemasi ekanligidan foydalanamiz:

```
> simplify(int(sin(Pi*nx/L*x)*sin(Pi*kx/L*x), x=0..L)) assuming nx::integer, kx::integer;
simplify(int(sin(Pi*nx/L*x)*sin(Pi*kx/L*x), x=0..L)) assuming nx::integer, kx::integer;

simplify(int(sin(Pi*nx/L*y)*sin(Pi*nx/L*y), y=0..L)) assuming nx::integer;
simplify(int(sin(Pi*nx/L*y)*sin(Pi*nx/L*y), y=0..L)) assuming nx::integer;
```

$$0$$

$$0$$

$$\frac{L}{2}$$

$$\frac{L}{2}$$

Shuning uchun integrallashdan quyidagiga ega bo'lamiz:

```
>
int(int(C1[nx,ny]*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y)*sin(Pi*kx/L*x)*sin(1/L*Pi*ky*y), x=0..L), y=0..L) assuming nx::integer, kx::integer, ny::integer, ky::integer;
int(int(C2[nx,ny]*Pi*(nx^2+ny^2)^(1/2)*a/L*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y)*sin(Pi*kx/L*x)*sin(1/L*Pi*ky*y), x=0..L), y=0..L) assuming nx::integer, kx::integer, ny::integer, ky::integer;

Int(Int(C1[nx,ny]*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y)*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y), x=0..L), y=0..L)=simplify(int(int(C1[nx,ny]*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y)*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y), x=0..L), y=0..L)) assuming nx::integer, ny::integer;
Int(Int(C2[nx,ny]*Pi*(nx^2+ny^2)^(1/2)*a/L*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y)*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y), x=0..L),
```

y=0..L)=simplify(int(int(C2[nx,ny]*Pi*(nx^2+ny^2)^(1/2)*a/L*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y)*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y), x=0..L), y=0..L)) assuming nx::integer, ny::integer;

$$0$$

$$0$$

$$\int_0^L\int_0^L C1_{nx,\,ny}\sin\left(\frac{\pi\, nx\, x}{L}\right)^2\sin\left(\frac{\pi\, ny\, y}{L}\right)^2 dx\, dy = \frac{1}{4}L^2\, C1_{nx,\,ny}$$

$$\int_0^L\int_0^L \frac{C2_{nx,\,ny}\,\pi\sqrt{nx^2+ny^2}\;a\sin\left(\frac{\pi\, nx\, x}{L}\right)^2\sin\left(\frac{\pi\, ny\, y}{L}\right)^2}{L}\, dx\, dy = \frac{1}{4}\pi\, L\, C2_{nx,\,ny}\sqrt{nx^2+ny^2}\; a$$

Bu natija quyidagilar uchun yig'indi ostida yagona qo'shiluvchilarligicha qoladi:

$$nx = kx$$

$$ny = ky$$

Shuning uchun:

> 1/4*L^2*C1[nx,ny]=int(int(F(x,y)*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y),x = 0 .. L),y = 0 .. L);

1/4*Pi*L*C2[nx,ny]*(nx^2+ny^2)^(1/2)*a=int(int(f(x,y)*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y),x = 0 .. L),y = 0 .. L);

$$\frac{1}{4}L^2\, C1_{nx,\,ny} = \int_0^L\int_0^L \mathrm{F}(x,y)\sin\left(\frac{\pi\, nx\, x}{L}\right)\sin\left(\frac{\pi\, ny\, y}{L}\right) dx\, dy$$

$$\frac{1}{4}\pi\, L\, C2_{nx,\,ny}\sqrt{nx^2+ny^2}\; a = \int_0^L\int_0^L \mathrm{f}(x,y)\sin\left(\frac{\pi\, nx\, x}{L}\right)\sin\left(\frac{\pi\, ny\, y}{L}\right) dx\, dy$$

Bu yerdan $C1_n$ va $C2_n$ koeffisentlar uchun quyidagi ifodalarni olamiz:

> C1[nx,ny]:=solve(1/4*L^2*C1[nx,ny] = int(int(F(x,y)*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y),x = 0 .. L),y = 0 .. L),C1[nx,ny]);

C2[nx,ny]:=solve(1/4*Pi*L*C2[nx,ny]*(nx^2+ny^2)^(1/2)*a = int(int(f(x,y)*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y),x = 0 .. L),y = 0 .. L),C2[nx,ny]);

$$C1_{nx,\,ny} := \frac{4}{L^2}\int_0^L\int_0^L \mathrm{F}(x,y)\,\sin\left(\frac{\pi\, nx\, x}{L}\right)\sin\left(\frac{\pi\, ny\, y}{L}\right)dx\,dy$$

$$C2_{nx,\,ny} := \frac{4}{\pi\, L\,\sqrt{nx^2+ny^2}\; a}\int_0^L\int_0^L \mathrm{f}(x,y)\,\sin\left(\frac{\pi\, nx\, x}{L}\right)\sin\left(\frac{\pi\, ny\, y}{L}\right)dx\,dy$$

Natijada umumiy yechimni quyidagi ko'rinishda yozamiz:

```
> u(t,x,y) :=
Sum(Sum((C1[nx,ny]*cos(Pi*(nx^2+ny^2)^(1/2)*a/L*t)+C2[nx,ny]*sin(Pi*(nx^2
+ny^2)^(1/2)*a/L*t))*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y),nx = 1 .. infinity),ny = 1 ..
infinity);
```

$$\mathrm{u}(t,x,y) := \sum_{ny=1}^{\infty}\left(\sum_{nx=1}^{\infty}\left(\left(\frac{4\cos\left(\frac{\pi\sqrt{nx^2+ny^2}\; a\, t}{L}\right)}{L^2}\int_0^L\int_0^L \mathrm{F}(x,y)\,\sin\left(\frac{\pi\, nx\, x}{L}\right)\sin\left(\frac{\pi\, ny\, y}{L}\right)dx\,dy + \frac{4\sin\left(\frac{\pi\sqrt{nx^2+ny^2}\; a\, t}{L}\right)}{\pi\, L\sqrt{nx^2+ny^2}\; a}\int_0^L\int_0^L \mathrm{f}(x,y)\,\sin\left(\frac{\pi\, nx\, x}{L}\right)\sin\left(\frac{\pi\, ny\, y}{L}\right)dx\,dy\right)\sin\left(\frac{\pi\, nx\, x}{L}\right)\sin\left(\frac{\pi\, ny\, y}{L}\right)\right)\right)$$

2.3-§. Kvadratli membrana tebranish tenglamalariga misollar

1 – Misol.

```
> restart;
```

$$\frac{\partial^2}{\partial t^2}\mathrm{u}(t,x,y) = a^2\left(\left(\frac{\partial^2}{\partial x^2}\mathrm{u}(t,x,y)\right)+\left(\frac{\partial^2}{\partial y^2}\mathrm{u}(t,x,y)\right)\right)$$

bir jinsli tenglamani quyidagi bir jinsli chegaraviy va boshlang'ich shartlat bilan yeching

$$u(0, x) = F(x),$$

$$\frac{\partial}{\partial t} u(0, x, y) = f(x, y).$$

```
> a:=1;L:=1;alpha:=1;
F(x,y):=4*alpha*x^2*(L-x)*y*(L-y)/L^2;
f(x,y):=0;
```

$$a := 1$$

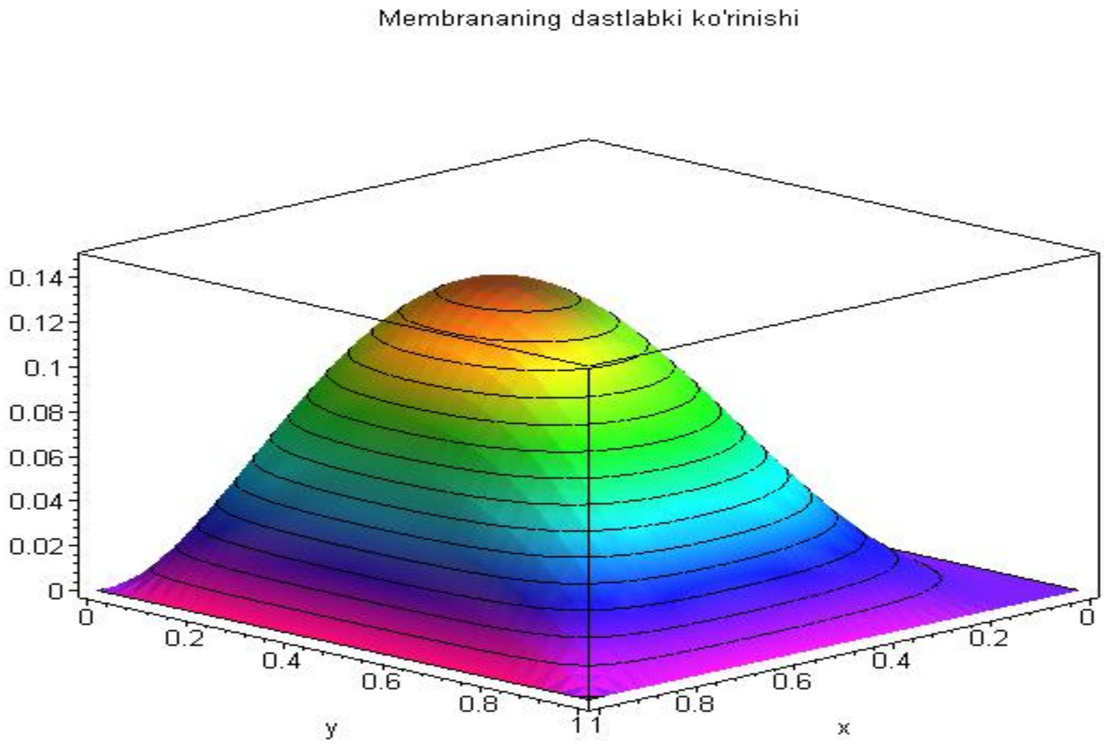
$$L := 1$$

$$\alpha := 1$$

$$F(x, y) := 4 x^2 (1 - x) y (1 - y)$$

$$f(x, y) := 0$$

```
> plot3d(F(x,y),x=0..1,y=0..1,title="Membrananing dastlabki ko'rinishi",axes=BOXED,lightmodel=light3,numpoints=1000,orientation=[45,65],shading=ZHUE,style=PATCHCONTOUR);
```




```
> C1[nx,ny]:=(4/L^2)*int(int(F(x,y)*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y),x=0..L),y=0..L);
```

C2[nx,ny]:=(4/(L^2*lambda))*int(int(f(x,y)*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y),x=0..L), y=0..L);

$$C1_{nx,\,ny} := 16\,(-4\,\pi\,nx - 2\,\pi^2\,nx^2\,\sin(\pi\,nx) - 8\,\cos(\pi\,nx)\,\pi\,nx + 12\,\sin(\pi\,nx) + 4\,\cos(\pi\,nx)\,\pi^2\,nx\,ny\,\sin(\pi\,ny) + 2\,\pi^2\,nx^2\,\sin(\pi\,nx)\,\cos(\pi\,ny) + 4\,\pi\,nx\,\cos(\pi\,ny) + 8\,\cos(\pi\,nx)\,\pi\,nx\,\cos(\pi\,ny) - 12\,\sin(\pi\,nx)\,\cos(\pi\,ny) - 6\,\sin(\pi\,nx)\,\pi\,ny\,\sin(\pi\,ny) + \pi^3\,nx^2\,\sin(\pi\,nx)\,ny\,\sin(\pi\,ny) + 2\,\pi^2\,nx\,ny\,\sin(\pi\,ny))\,/\,(\pi^7\,nx^4\,ny^3)$$

$$C2_{nx,\,ny} := 0$$

Tenglamani yechimi:

>

u(t,x,y):=Sum(Sum((C1[nx,ny]*cos(Pi*(nx^2+ny^2)^(1/2)*a/L*t)+C2[nx,ny]*sin(Pi*(nx^2+ny^2)^(1/2)*a/L*t))*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y),nx=1..12),ny=1..12);

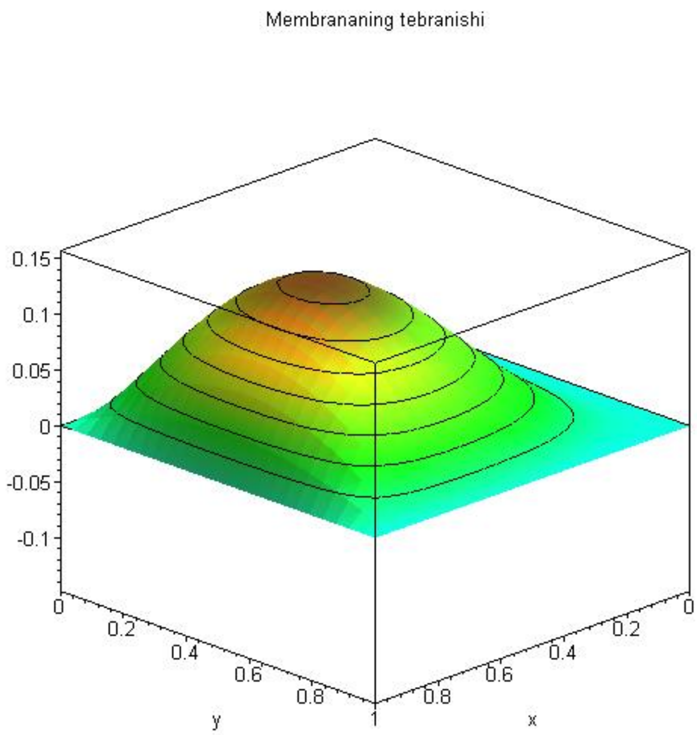
$$u(t, x, y) := \sum_{ny=1}^{12}\left(\sum_{nx=1}^{12} (16\,(-4\,\pi\,nx - 2\,\pi^2\,nx^2\,\sin(\pi\,nx) - 8\,\cos(\pi\,nx)\,\pi\,nx + 12\,\sin(\pi\,nx) + 4\,\cos(\pi\,nx)\,\pi^2\,nx\,ny\,\sin(\pi\,ny) + 2\,\pi^2\,nx^2\,\sin(\pi\,nx)\,\cos(\pi\,ny) + 4\,\pi\,nx\,\cos(\pi\,ny) + 8\,\cos(\pi\,nx)\,\pi\,nx\,\cos(\pi\,ny) - 12\,\sin(\pi\,nx)\,\cos(\pi\,ny) - 6\,\sin(\pi\,nx)\,\pi\,ny\,\sin(\pi\,ny) + \pi^3\,nx^2\,\sin(\pi\,nx)\,ny\,\sin(\pi\,ny) + 2\,\pi^2\,nx\,ny\,\sin(\pi\,ny))\,\cos(\pi\sqrt{nx^2+ny^2}\,t)\,\sin(\pi\,nx\,x)\,\sin(\pi\,ny\,y)\,/\,(\pi^7\,nx^4\,ny^3))\right)$$

Olingan yechimlarni animasiyali grafik ko'rinishida tasvirlaymiz:

> with(plots):

animate(plot3d,[u(t,x,y),x=0..1,y=0..1], t=0..2, frames=30, title="Membrananing tebranishi",axes=BOXED,lightmodel=light3,numpoints=1000,orientation=[45,65],shading=ZHUE,style=PATCHCONTOUR);

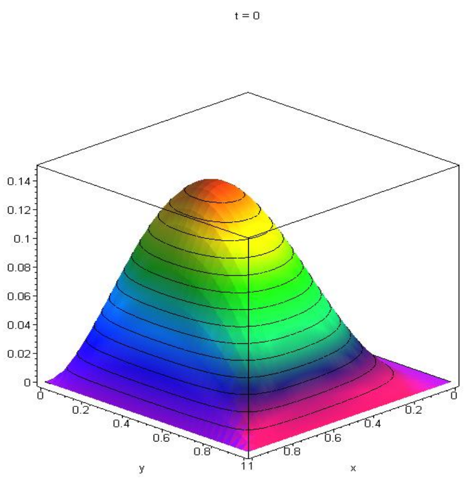
Warning, the name changecoords has been redefined

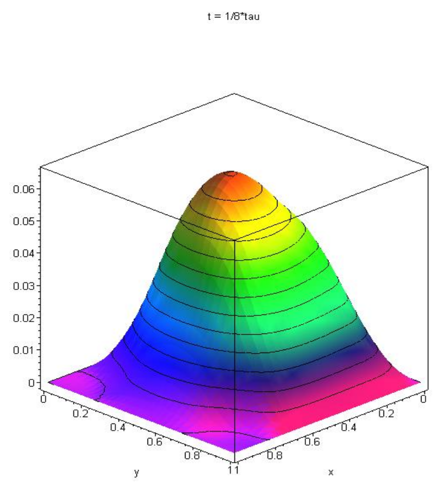


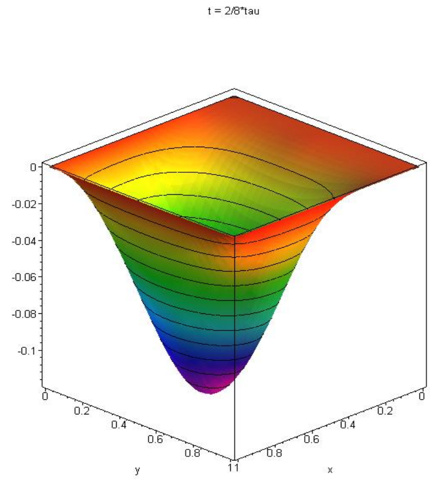
Bir qancha vaqt momentlari uchun olingan yechimni grafik ko’rinishida tasvirlaymiz:

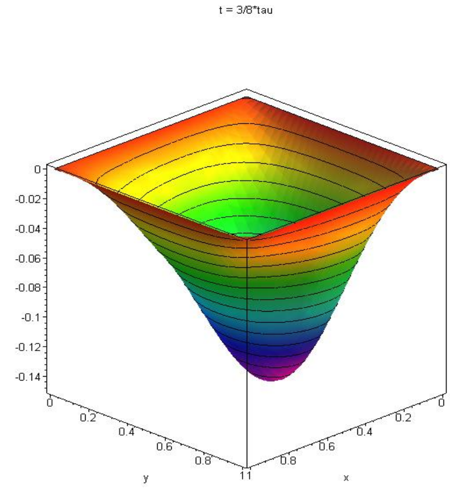
```
> tau:=2*Pi/(Pi*a/L):

u_1(x,y):=subs(t=tau*0,u(t,x,y)):
u_2(x,y):=subs(t=tau*(1/8),u(t,x,y)):
u_3(x,y):=subs(t=tau*(2/8),u(t,x,y)):
u_4(x,y):=subs(t=tau*(3/8),u(t,x,y)):
u_5(x,y):=subs(t=tau*(4/8),u(t,x,y)):
u_6(x,y):=subs(t=tau*(5/8),u(t,x,y)):
u_7(x,y):=subs(t=tau*(6/8),u(t,x,y)):
u_8(x,y):=subs(t=tau*(7/8),u(t,x,y)):

plot3d(u_1(x,y),x=0..1,y=0..1, title="t = 0",
axes=BOXED,lightmodel=light2,numpoints=1000,orientation=[45,
65],shading=ZHUE,style=PATCHCONTOUR);
```

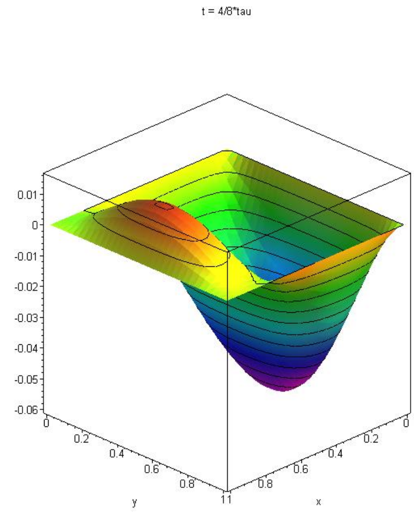
```
plot3d(u_2(x,y),x=0..1,y=0..1, title="t = 1/8*tau",
axes=BOXED,lightmodel=light2,numpoints=1000,orientation=[45,
65],shading=ZHUE,style=PATCHCONTOUR);
plot3d(u_3(x,y),x=0..1,y=0..1, title="t = 2/8*tau",
axes=BOXED,lightmodel=light2,numpoints=1000,orientation=[45,
65],shading=ZHUE,style=PATCHCONTOUR);
plot3d(u_4(x,y),x=0..1,y=0..1, title="t = 3/8*tau",
axes=BOXED,lightmodel=light2,numpoints=1000,orientation=[45,
65],shading=ZHUE,style=PATCHCONTOUR);
plot3d(u_5(x,y),x=0..1,y=0..1, title="t = 4/8*tau",
axes=BOXED,lightmodel=light2,numpoints=1000,orientation=[45,
65],shading=ZHUE,style=PATCHCONTOUR);
plot3d(u_6(x,y),x=0..1,y=0..1, title="t = 5/8*tau",
axes=BOXED,lightmodel=light2,numpoints=1000,orientation=[45,
65],shading=ZHUE,style=PATCHCONTOUR);
plot3d(u_7(x,y),x=0..1,y=0..1, title="t = 6/8*tau",
axes=BOXED,lightmodel=light2,numpoints=1000,orientation=[45,
65],shading=ZHUE,style=PATCHCONTOUR);
plot3d(u_8(x,y),x=0..1,y=0..1, title="t = 7/8*tau",
axes=BOXED,lightmodel=light2,numpoints=1000,orientation=[45,
65],shading=ZHUE,style=PATCHCONTOUR);
```

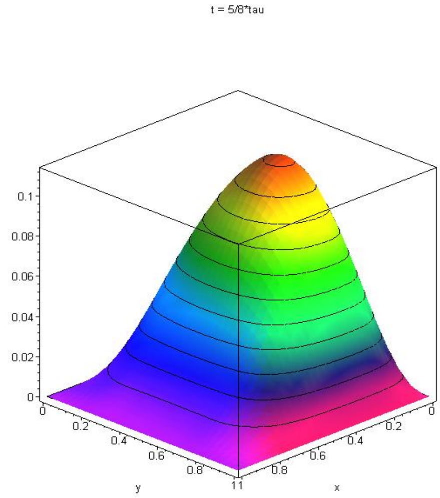


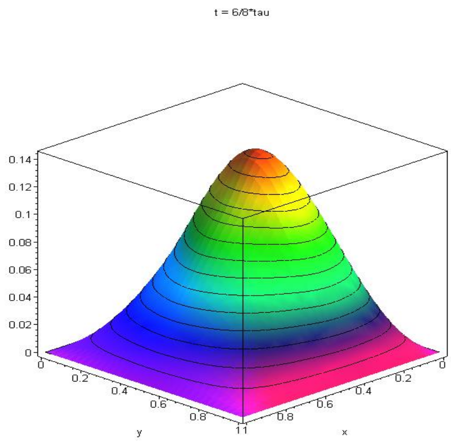


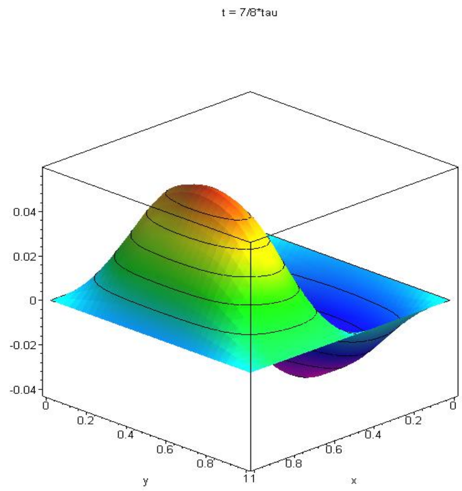












2 – Misol.

> restart;

$$\frac{\partial^2}{\partial t^2} \mathrm{u}(t, x, y) = a^2\left(\left(\frac{\partial^2}{\partial x^2} \mathrm{u}(t, x, y)\right)+\left(\frac{\partial^2}{\partial y^2} \mathrm{u}(t, x, y)\right)\right)$$

bir jinsli tenglamani quyidagi bir jinsli chegaraviy va boshlang'ich shartlat bilan yeching

$$u(0, x) = F(x),$$

$$\frac{\partial}{\partial t} u(0, x, y) = f(x, y).$$

```
> a:=1;L:=1;alpha:=1;
F(x,y):=4*alpha*x*(L-x)/L^2*sin(y)^5*sin(L-y);
f(x,y):=0;
```

$$a := 1$$

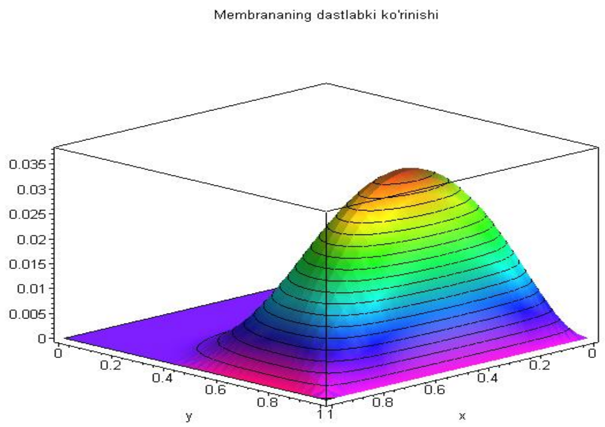
$$L := 1$$

$$\alpha := 1$$

$$F(x, y) := -4\, x\, (1 - x) \sin(y)^5 \sin(-1 + y)$$

$$f(x, y) := 0$$

```
> plot3d(F(x,y),x=0..1,y=0..1,title="Membrananing dastlabki
ko'rinishi",axes=BOXED,lightmodel=light3,numpoints=1000,orientation=[45,
65],shading=ZHUE,style=PATCHCONTOUR);
```




```
> C1[nx,ny]:=(4/L^2)*int(int(F(x,y)*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y),x=0..L),
y=0..L);
C2[nx,ny]:=(4/(L^2*lambda))*int(int(f(x,y)*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y),x=
0..L), y=0..L);
```

$$C1_{nx,\,ny} := \big(-5\,\pi^6\,nx\,ny^5 \sin(\pi\,ny)\sin(3)\sin(\pi\,nx)$$
$$-30\,\pi^5\,nx\,ny^4\cos(\pi\,ny)\cos(3)\sin(\pi\,nx)+1440\,\pi^2\,nx\,ny\sin(\pi\,ny)\sin(3)\sin(\pi\,nx)$$
$$+11520\,\pi\,nx\cos(\pi\,ny)\cos(1)\sin(\pi\,nx)+\pi^6\,nx\,ny^5\sin(\pi\,ny)\sin(5)\sin(\pi\,nx)$$
$$+20\,\pi^4\,nx\,ny^3\sin(\pi\,ny)\sin(5)\sin(\pi\,nx)+5760\,\pi^2\,nx\,ny\sin(\pi\,ny)\sin(1)\sin(\pi\,nx)$$
$$-40\,\pi^3\,nx\,ny^2\cos(\pi\,ny)\cos(5)\sin(\pi\,nx)-96\,\pi^2\,nx\,ny\sin(\pi\,ny)\sin(5)\sin(\pi\,nx)$$
$$+1080\,\pi^3\,nx\,ny^2\cos(\pi\,ny)\cos(3)\sin(\pi\,nx)-11520\,\pi\,ny\sin(1)\sin(\pi\,ny)$$
$$+10\,\pi^6\,nx\,ny^5\sin(\pi\,ny)\sin(1)\sin(\pi\,nx)+140\,\pi^4\,nx\,ny^3\sin(\pi\,ny)\sin(3)\sin(\pi\,nx)$$
$$-520\,\pi^4\,nx\,ny^3\sin(\pi\,ny)\sin(1)\sin(\pi\,nx)+10\,\pi^5\,nx\,ny^4\cos(\pi\,ny)\cos(5)\sin(\pi\,nx)$$
$$+40\,ny^4\,\pi^4\cos(\pi\,ny)\cos(1)\cos(\pi\,nx)+20\,\pi^5\,nx\,ny^4\cos(\pi\,ny)\cos(1)\sin(\pi\,nx)$$
$$+23040\cos(1)-2160\,\pi^2\,ny^2\cos(3)\cos(\pi\,ny)+2\,ny^5\,\pi^5\sin(\pi\,ny)\sin(5)\cos(\pi\,nx)$$
$$+2080\,\pi^2\,ny^2\cos(1)\cos(\pi\,ny)+23040\cos(\pi\,ny)\cos(1)\cos(\pi\,nx)$$
$$+1040\,ny^3\,\pi^3\sin(1)\sin(\pi\,ny)-23040\cos(1)\cos(\pi\,nx)$$
$$-40\,ny^4\,\pi^4\cos(1)\cos(\pi\,ny)-20\,ny^5\,\pi^5\sin(1)\sin(\pi\,ny)-23040\cos(1)\cos(\pi\,ny)$$
$$-10\,ny^5\,\pi^5\sin(\pi\,ny)\sin(3)\cos(\pi\,nx)+2880\,\pi\,ny\sin(\pi\,ny)\sin(3)\cos(\pi\,nx)$$
$$-11520\,\pi\,nx\cos(1)\sin(\pi\,nx)-280\,ny^3\,\pi^3\sin(3)\sin(\pi\,ny)$$
$$-40\,ny^3\,\pi^3\sin(5)\sin(\pi\,ny)-1040\,\pi^3\,nx\,ny^2\cos(\pi\,ny)\cos(1)\sin(\pi\,nx)$$
$$+80\,\pi^2\,ny^2\cos(5)\cos(\pi\,ny)-80\,\pi^2\,ny^2\cos(\pi\,ny)\cos(5)\cos(\pi\,nx)$$
$$-20\,ny^4\,\pi^4\cos(5)\cos(\pi\,ny)+192\,\pi\,ny\sin(5)\sin(\pi\,ny)-2\,ny^5\,\pi^5\sin(5)\sin(\pi\,ny)$$
$$-192\,\pi\,ny\sin(\pi\,ny)\sin(5)\cos(\pi\,nx)-2080\,\pi^2\,ny^2\cos(\pi\,ny)\cos(1)\cos(\pi\,nx)$$
$$+20\,ny^5\,\pi^5\sin(\pi\,ny)\sin(1)\cos(\pi\,nx)+11520\,\pi\,ny\sin(\pi\,ny)\sin(1)\cos(\pi\,nx)$$
$$+2160\,\pi^2\,ny^2\cos(\pi\,ny)\cos(3)\cos(\pi\,nx)+280\,ny^3\,\pi^3\sin(\pi\,ny)\sin(3)\cos(\pi\,nx)$$
$$-60\,ny^4\,\pi^4\cos(\pi\,ny)\cos(3)\cos(\pi\,nx)-2880\,\pi\,ny\sin(3)\sin(\pi\,ny)$$
$$-1040\,ny^3\,\pi^3\sin(\pi\,ny)\sin(1)\cos(\pi\,nx)+10\,ny^5\,\pi^5\sin(3)\sin(\pi\,ny)$$
$$+60\,ny^4\,\pi^4\cos(3)\cos(\pi\,ny)+20\,ny^4\,\pi^4\cos(\pi\,ny)\cos(5)\cos(\pi\,nx)$$
$$+40\,ny^3\,\pi^3\sin(\pi\,ny)\sin(5)\cos(\pi\,nx)\big)\Big/\big(ny\,nx^3\,\pi^4$$
$$(ny^6\,\pi^6-56\,ny^4\,\pi^4+784\,\pi^2\,ny^2-2304)\big)$$

$$C2_{nx,\,ny} := 0$$

Tenglamani yechimi:

>

u(t,x,y):=Sum(Sum((C1[nx,ny]*cos(Pi*(nx^2+ny^2)^(1/2)*a/L*t)+C2[nx,ny]*sin(Pi*(nx^2+ny^2)^(1/2)*a/L*t))*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y),nx=1..12),ny=1..12);

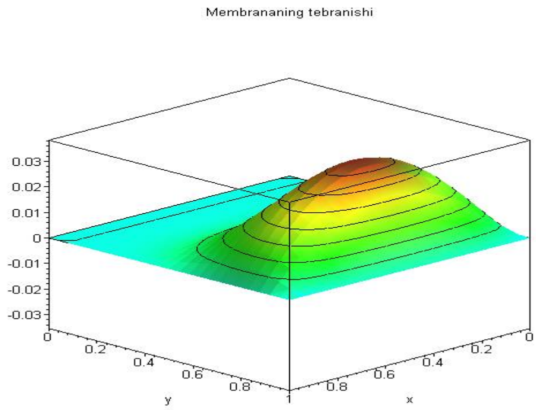
$$\mathrm{u}(t,x,y) := \sum_{ny=1}^{12}\left(\sum_{nx=1}^{12}\big(-5\,\pi^6\,nx\,ny^5\sin(\pi\,ny)\sin(3)\sin(\pi\,nx)\right.$$
$$-30\,\pi^5\,nx\,ny^4\cos(\pi\,ny)\cos(3)\sin(\pi\,nx)+1440\,\pi^2\,nx\,ny\sin(\pi\,ny)\sin(3)\sin(\pi\,nx)$$
$$+11520\,\pi\,nx\cos(\pi\,ny)\cos(1)\sin(\pi\,nx)+\pi^6\,nx\,ny^5\sin(\pi\,ny)\sin(5)\sin(\pi\,nx)$$

$$
\begin{aligned}
&+20\,\pi^4\,nx\,ny^3\sin(\pi\,ny)\sin(5)\sin(\pi\,nx)+5760\,\pi^2\,nx\,ny\sin(\pi\,ny)\sin(1)\sin(\pi\,nx)\\
&-40\,\pi^3\,nx\,ny^2\cos(\pi\,ny)\cos(5)\sin(\pi\,nx)-96\,\pi^2\,nx\,ny\sin(\pi\,ny)\sin(5)\sin(\pi\,nx)\\
&+1080\,\pi^3\,nx\,ny^2\cos(\pi\,ny)\cos(3)\sin(\pi\,nx)-11520\,\pi\,ny\sin(1)\sin(\pi\,ny)\\
&+10\,\pi^6\,nx\,ny^5\sin(\pi\,ny)\sin(1)\sin(\pi\,nx)+140\,\pi^4\,nx\,ny^3\sin(\pi\,ny)\sin(3)\sin(\pi\,nx)\\
&-520\,\pi^4\,nx\,ny^3\sin(\pi\,ny)\sin(1)\sin(\pi\,nx)+10\,\pi^5\,nx\,ny^4\cos(\pi\,ny)\cos(5)\sin(\pi\,nx)\\
&+40\,ny^4\,\pi^4\cos(\pi\,ny)\cos(1)\cos(\pi\,nx)+20\,\pi^5\,nx\,ny^4\cos(\pi\,ny)\cos(1)\sin(\pi\,nx)\\
&+23040\cos(1)-2160\,\pi^2\,ny^2\cos(3)\cos(\pi\,ny)+2\,ny^5\,\pi^5\sin(\pi\,ny)\sin(5)\cos(\pi\,nx)\\
&+2080\,\pi^2\,ny^2\cos(1)\cos(\pi\,ny)+23040\cos(\pi\,ny)\cos(1)\cos(\pi\,nx)\\
&+1040\,ny^3\,\pi^3\sin(1)\sin(\pi\,ny)-23040\cos(1)\cos(\pi\,nx)\\
&-40\,ny^4\,\pi^4\cos(1)\cos(\pi\,ny)-20\,ny^5\,\pi^5\sin(1)\sin(\pi\,ny)-23040\cos(1)\cos(\pi\,ny)\\
&-10\,ny^5\,\pi^5\sin(\pi\,ny)\sin(3)\cos(\pi\,nx)+2880\,\pi\,ny\sin(\pi\,ny)\sin(3)\cos(\pi\,nx)\\
&-11520\,\pi\,nx\cos(1)\sin(\pi\,nx)-280\,ny^3\,\pi^3\sin(3)\sin(\pi\,ny)\\
&-40\,ny^3\,\pi^3\sin(5)\sin(\pi\,ny)-1040\,\pi^3\,nx\,ny^2\cos(\pi\,ny)\cos(1)\sin(\pi\,nx)\\
&+80\,\pi^2\,ny^2\cos(5)\cos(\pi\,ny)-80\,\pi^2\,ny^2\cos(\pi\,ny)\cos(5)\cos(\pi\,nx)\\
&-20\,ny^4\,\pi^4\cos(5)\cos(\pi\,ny)+192\,\pi\,ny\sin(5)\sin(\pi\,ny)-2\,ny^5\,\pi^5\sin(5)\sin(\pi\,ny)\\
&-192\,\pi\,ny\sin(\pi\,ny)\sin(5)\cos(\pi\,nx)-2080\,\pi^2\,ny^2\cos(\pi\,ny)\cos(1)\cos(\pi\,nx)\\
&+20\,ny^5\,\pi^5\sin(\pi\,ny)\sin(1)\cos(\pi\,nx)+11520\,\pi\,ny\sin(\pi\,ny)\sin(1)\cos(\pi\,nx)\\
&+2160\,\pi^2\,ny^2\cos(\pi\,ny)\cos(3)\cos(\pi\,nx)+280\,ny^3\,\pi^3\sin(\pi\,ny)\sin(3)\cos(\pi\,nx)\\
&-60\,ny^4\,\pi^4\cos(\pi\,ny)\cos(3)\cos(\pi\,nx)-2880\,\pi\,ny\sin(3)\sin(\pi\,ny)\\
&-1040\,ny^3\,\pi^3\sin(\pi\,ny)\sin(1)\cos(\pi\,nx)+10\,ny^5\,\pi^5\sin(3)\sin(\pi\,ny)\\
&+60\,ny^4\,\pi^4\cos(3)\cos(\pi\,ny)+20\,ny^4\,\pi^4\cos(\pi\,ny)\cos(5)\cos(\pi\,nx)\\
&+40\,ny^3\,\pi^3\sin(\pi\,ny)\sin(5)\cos(\pi\,nx))\cos(\pi\sqrt{nx^2+ny^2}\,t)\sin(\pi\,nx\,x)\sin(\pi\,ny\,y)\\
&\Big/\left(ny\,nx^3\,\pi^4\,(ny^6\,\pi^6-56\,ny^4\,\pi^4+784\,\pi^2\,ny^2-2304)\right)\Big)
\end{aligned}
$$

Olingan yechimlarni animasiyali grafik ko'rinishida tasvirlaymiz:

```
> with(plots):
animate(plot3d,[u(t,x,y),x=0..1,y=0..1], t=0..2, frames=30, title="Membrananing tebranishi",axes=BOXED,lightmodel=light3,numpoints=1000,orientation=[45, 65],shading=ZHUE,style=PATCHCONTOUR);
```

Warning, the name changecoords has been redefined



3 – Misol.

> restart;

$$\frac{\partial^2}{\partial t^2} u(t, x, y) = a^2 \left(\left(\frac{\partial^2}{\partial x^2} u(t, x, y) \right) + \left(\frac{\partial^2}{\partial y^2} u(t, x, y) \right) \right)$$

bir jinsli tenglamani quyidagi bir jinsli chegaraviy va boshlang'ich shartlat bilan yeching

$$u(0, x) = F(x) \text{,}$$

$$\frac{\partial}{\partial t} u(0, x, y) = f(x, y) \text{.}$$

> a:=0.4;L:=1;beta:=500;l1:=L/4-L/100;l2:=L/4+L/100;
F(x,y):=0;
f(x,y):=beta*x^3*(L-x)*y*(L-y)^2;

$$a := 0.4$$

$$L := 1$$

$$\beta := 500$$

$$l1 := \frac{6}{25}$$

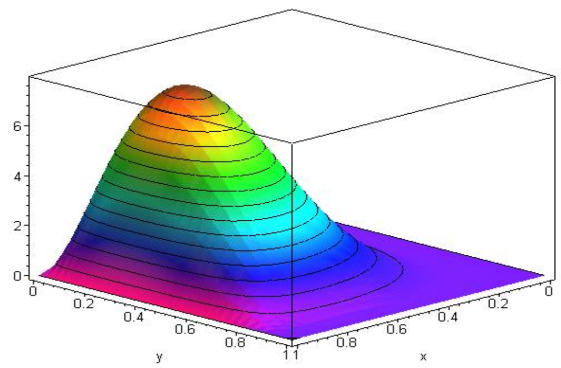
$$l2 := \frac{13}{50}$$

$$F(x, y) := 0$$

$$f(x, y) := 500\, x^3 (1 - x)\, y\, (1 - y)^2$$

> plot3d(f(x,y),x=0..1,y=0..1,title="Membrana tezligining dastlabki ko'rinishi",axes=BOXED,lightmodel=light3,numpoints=1000,orientation=[45, 65],shading=ZHUE,style=PATCHCONTOUR);

Membrana tezligining dastlabki ko'rinishi




```
> lambda:=simplify((Pi^2*nx^2+Pi^2*ny^2)^(1/2)*a/L);
```

$$\lambda := 1.256637062\sqrt{nx^2+ny^2}$$

$$C1_{nx,\,ny} := 0$$

$$C2_{nx,\,ny} := -3183.098860(-48\cos(\pi\,nx)\,\pi\,ny + 2\,\pi^4\,nx^3\sin(\pi\,nx)\,ny - 36\,nx\sin(\pi\,nx)\,\pi^2\,ny + 12\,\pi^3\,nx^2\cos(\pi\,nx)\,ny + 48\,\pi\,ny + 6\,\pi^3\,nx^2\cos(\pi\,nx)\,ny\cos(\pi\,ny) - 18\,\pi^2\,nx^2\cos(\pi\,nx)\sin(\pi\,ny) - 24\cos(\pi\,nx)\cos(\pi\,ny)\,ny\,\pi - 3\,\pi^3\,nx^3\sin(\pi\,nx)\sin(\pi\,ny) + 72\cos(\pi\,nx)\sin(\pi\,ny) + \pi^4\,nx^3\sin(\pi\,nx)\cos(\pi\,ny)\,ny - 72\sin(\pi\,ny) + 24\cos(\pi\,ny)\,ny\,\pi + 54\,\pi\,nx\sin(\pi\,nx)\sin(\pi\,ny) - 18\,nx\sin(\pi\,nx)\,\pi^2\cos(\pi\,ny)\,ny) \Big/ (\sqrt{nx^2+ny^2}\,\pi^9\,nx^5\,ny^4)$$

Tenglamani yechimi:

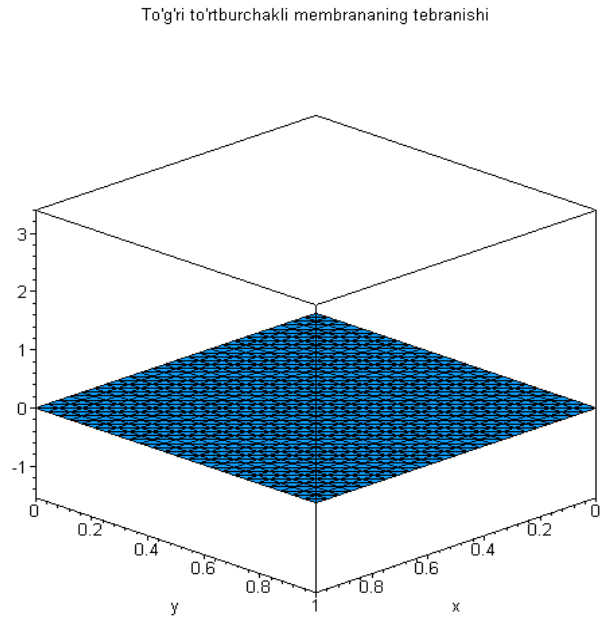
```
>
u(t,x,y):=Sum(Sum((C1[nx,ny]*cos(Pi*(nx^2+ny^2)^(1/2)*a/L*t)+C2[nx,ny]*sin(
Pi*(nx^2+ny^2)^(1/2)*a/L*t))*sin(Pi*nx/L*x)*sin(1/L*Pi*ny*y),nx=1..12),ny=1..
12);
```

$$u(t,x,y) := \sum_{ny=1}^{12}\left(\sum_{nx=1}^{12}(-3183.098860(-48\cos(\pi\,nx)\,\pi\,ny + 2\,\pi^4\,nx^3\sin(\pi\,nx)\,ny - 36\,nx\sin(\pi\,nx)\,\pi^2\,ny + 12\,\pi^3\,nx^2\cos(\pi\,nx)\,ny + 48\,\pi\,ny + 6\,\pi^3\,nx^2\cos(\pi\,nx)\,ny\cos(\pi\,ny) - 18\,\pi^2\,nx^2\cos(\pi\,nx)\sin(\pi\,ny) - 24\cos(\pi\,nx)\cos(\pi\,ny)\,ny\,\pi - 3\,\pi^3\,nx^3\sin(\pi\,nx)\sin(\pi\,ny) + 72\cos(\pi\,nx)\sin(\pi\,ny) + \pi^4\,nx^3\sin(\pi\,nx)\cos(\pi\,ny)\,ny - 72\sin(\pi\,ny) + 24\cos(\pi\,ny)\,ny\,\pi + 54\,\pi\,nx\sin(\pi\,nx)\sin(\pi\,ny) - 18\,nx\sin(\pi\,nx)\,\pi^2\cos(\pi\,ny)\,ny)\sin(0.4\,\pi\sqrt{nx^2+ny^2}\,t)\sin(\pi\,nx\,x)\sin(\pi\,ny\,y) \Big/ (\sqrt{nx^2+ny^2}\,\pi^9\,nx^5\,ny^4))\right)$$

Olingan yechimlarni animasiyali grafik ko’rinishida tasvirlaymiz:

> with(plots):
animate(plot3d,[u(t,x,y),x=0..1,y=0..1], t=0..2, frames=20, title="To'g'ri to'rtburchakli membrananing tebranishi",axes=BOXED,lightmodel=light3,numpoints=1000,orientation=[45, 65],shading=ZHUE,style=PATCHCONTOUR);

Warning, the name changecoords has been redefined



3- bob. Doiraviy membrananing tebranish tenglamasi

3.1-§. Doiraviy membrananing tebranishlari

Doira membranasini erkin tebranishlarini tekshirish markazi koordinata boshida bo'lgan radiusi l Doira kontorida membranasini siljitmasdan hisoblaymiz [4]. To'g'ri burchakli koordinatalarga (x, y) mos (r, θ) qutb koordinatalarini kiritamiz.

$$u\big|_{r=l} = 0.$$

$$u_{tt} = a^2(u_{xx} + u_{yy})$$

Tenglamani xususiy yechimini

$$(\alpha\cos\omega t + \beta\sin\omega t)U(r, \theta).$$

ko'rinishda izlaymiz.

$U(r, \theta)$ funksiya uchun qo'yidagi differensial tenglamani olamiz.

$$U_{xx} + U_{yy} + k^2U = 0 \tag{3.1.1}$$

Faqat yangi (r, θ) o'zgaruvchiga almashtirish kerak, buni uchun Laplas operatorini ifodalash yetarli.

$$\Delta U = U_{xx} + U_{yy}. \tag{3.1.2}$$

Uch o'zgaruvchili Laplas operatori quyidagicha bo'ladi

$$\Delta U = U_{xx} + U_{yy} + U_{zz}$$

Silindrik koordinatalarda ifodalaymiz

$$\begin{cases} x = \rho\cos\varphi, \\ y = \rho\sin\varphi, \\ z = z; \end{cases}$$

Shu ko'rinishda

$$\Delta U = \frac{1}{\rho}\left(\rho U_{\rho\rho} + \frac{1}{\rho}U_{\varphi\varphi} + \rho U_{zz}\right)$$

U ni z dan bog'liqmas hisoblab (3.1.2) ni qutb koordinatalar orqali ifodalaymiz. Bundan keyin radius vektor uzunligini r harfi bilan belgilaymiz. Qutbiy burchakni θ harfi bilan belgilaymiz.

$$U_{xx} + U_{yy} = U_{rr} + \frac{1}{r}U_r + \frac{1}{r^2}U_{\theta\theta}$$

Xuddi shunday (2.1.13) ni quyidagi ko'rinishda yozamiz.

$$U_{rr} + \frac{1}{r}U_r + \frac{1}{r^2}U_{\theta\theta} + k^2U = 0$$

Uning xususiy yechimini quyidagi ko'rinishda topamiz.

$$U(r,\theta) = T(\theta)R(r),$$

Bulardan quyidagilar keladi.

$$T(\theta)\left[R''(r) + \frac{1}{r}R'(r) + k^2R(r)\right] + \frac{1}{r^2}T''(\theta)R(r) = 0,$$

$$\frac{T''(\theta)}{T(\theta)} = -\frac{r^2R''(r) + rR'(r) + k^2r^2R(r)}{R(r)} = -\lambda^2, \lambda = const.$$

Ikkita tenglamani olamiz.

$$T''(\theta) + \lambda^2T(\theta) = 0 \tag{3.1.3}$$

$$R''(r) + \frac{1}{r}R'(r) + \left(k^2 - \frac{\lambda^2}{r^2}\right)R(r) = 0 \tag{3.1.4}$$

(3.1.3) tenglamani umumiy yechimini ko'rinishi quyidagicha bo'ladi.

$$T(\theta) = C\cos\lambda\theta + D\sin\lambda\theta.$$

U funksiya bir qiymatli davriy va davri 2π ga teng bo'lsin, u holda $T(\theta)$ funksiya xuddi yuqoridagidek xossalarga ega, λ butun son, λ faqat musbat qiymatlarni qabul qiladi, $\lambda = 0,1,2,...,n,...$ ga mos $T(\theta)$ va $R(r)$ funksiyalarni quyidagicha ifodalaymiz.

$$T_0(\theta), T_1(\theta), T_2(\theta), \ldots, T_n(\theta), \ldots, R_0(0), R_1(r), R_2(r), \ldots, R_n(r), \ldots$$

Shu yo'l bilan cheksiz to'plamda (3.1.1) tenglamaniyechimini qo'yidagi ko'rinishini olamiz.

$$(\alpha\cos\omega t+\beta\sin\omega t)(C\cos\theta+D\sin n\theta)R_n(r),\ \omega=ak.$$

$R_n(r)$ funksiya (20) tenglamani qanoatlantiradi, agar λ ni n ga almashtirsak

$$R''_n(r)+\frac{1}{r}R'_n(r)+\left(k^2-\frac{n^2}{r^2}\right)R_n(r)=0$$

Integralning umumiy tenglamasi quyidagicha bo'ladi.

$$R_n(r)=C_1J_n(kr)+C_2K_n(kr),$$

Bu yerda $J_n(x)$ - birinchi tur Bessel funksiyasi va $K_n(x)$ - ikkinchi tur Bessel funksiyasi.

3.2-§. Maple paketi orqali doiraviy membrananing tebranish tenglamasini Furye usuli (o'zgaruvchilarni ajratish usuli) yordamida yechish

Doiraviy membrananing erkin tebranishini qaraymiz [6-8]. Buning uchun quyidagi

$$\frac{\partial^2}{\partial t^2}\mathrm{u}(t,r)=a^2\left(\left(\frac{\partial^2}{\partial r^2}\mathrm{u}(t,r)\right)+\frac{1\left(\frac{\partial}{\partial r}\mathrm{u}(t,r)\right)}{r}\right)$$

bir jinsli tenglamani, ushbu

$$\mathrm{u}(0,r)=\mathrm{F}(r)\,,$$

$$\frac{\partial}{\partial t}\mathrm{u}(0,r)=\mathrm{f}(r)\,.$$

boshlang'ich shartlar va quyidagi

$$\mathrm{u}(t,r0)=0\,,$$

chegaraviy shartlar (ya'ni membrannaning uchlari mahkamlangan) bilan berilgan masalani yechamiz.

```
> restart;
```

Bir jinsli tenglamani o'zgaruvchilarni ajratish usuli yordamida yechamiz:

> **PDE:=diff(u(t,r),t,t)=a^2*(diff(u(t,r),r,r)+(1/r)*diff(u(t,r),r));**
struc:=pdsolve(PDE,HINT=T(t)*R(r));

$$PDE := \frac{\partial^2}{\partial t^2} u(t, r) = a^2 \left(\left(\frac{\partial^2}{\partial r^2} u(t, r) \right) + \frac{\frac{\partial}{\partial r} u(t, r)}{r} \right)$$

$struc := (u(t, r) = T(t)\, R(r))$ &where

$$\left[\left\{ \frac{d^2}{dt^2} T(t) = \_c_1\, T(t),\ \frac{d^2}{dr^2} R(r) = \frac{R(r)\, \_c_1}{a^2} - \frac{\frac{d}{dr} R(r)}{r} \right\} \right]$$

> **dsolve(diff(T(t),`$`(t,2)) = _c[1]*T(t));**
dsolve(diff(R(r),`$`(r,2)) = R(r)/a^2*_c[1]-diff(R(r),r)/r);

$$T(t) = \_C1\, \mathbf{e}^{(\sqrt{\_c_1}\, t)} + \_C2\, \mathbf{e}^{(-\sqrt{\_c_1}\, t)}$$

$$R(r) = \_C1\, \mathrm{BesselJ}\left(0, \frac{\sqrt{-\_c_1}\, r}{a}\right) + \_C2\, \mathrm{BesselY}\left(0, \frac{\sqrt{-\_c_1}\, r}{a}\right)$$

O'zgaruvchilarni ajratish bo'yicha quyidagi almashtirishlarni olamiz:

$$\_c_1 \ == -\ \lambda^2 a^2$$

> **dsolve(diff(T(t),`$`(t,2)) = -lambda^2*a^2*T(t));**
dsolve(diff(R(r),`$`(r,2)) = -lambda^2*R(r)-diff(R(r),r)/r);

$$T(t) = \_C1 \sin(a\, \lambda\, t) + \_C2 \cos(a\, \lambda\, t)$$

$$R(r) = \_C1\, \mathrm{BesselJ}(0, \lambda\, r) + \_C2\, \mathrm{BesselY}(0, \lambda\, r)$$

Ikkinchi tenglamani yechimi

$$R(r) = \_C1\, \mathrm{BesselJ}(0, \lambda\, r) + \_C2\, \mathrm{BesselY}(0, \lambda\, r)$$

Membrane markazidagi regulyarlik shartini qanoatlantirishi kerak.

BesselJ funksiya nolda regulyar:

> **BesselJ(0,lambda*r)=series(BesselJ(0,lambda*r), r=0,4);**

$$\mathrm{BesselJ}(0, \lambda\, r) = 1 - \frac{\lambda^2}{4} r^2 + O(r^4)$$

BesselY funksiya nolda singulyar:

> **BesselY(0,lambda*r)=series(BesselY(0, lambda*r), r=0,4);**

$$\mathrm{BesselY}(0, \lambda r) = \left(\frac{2\left(\ln\left(\frac{\lambda}{2}\right)+\ln(r)\right)}{\pi}+\frac{2\gamma}{\pi}\right)+\left(-\frac{1}{2}\,\frac{\left(\ln\left(\frac{\lambda}{2}\right)+\ln(r)\right)\lambda^2}{\pi}+\frac{(2-2\gamma)\lambda^2}{4\pi}\right)r^2+\mathrm{O}(r^4)$$

Natijada, radial tenglamaning umumiy yechimiga quyidagini qo'yamiz:

$$\_C2 = 0$$

Shuning uchun radial tenglamaning yechimi quyidagi ko'rinishga ega bo'ladi:

$$\mathrm{R}(r) = \mathrm{BesselJ}(0, \lambda r).$$

Membrananing (r = r0) chegarasi uchun chegaraviy shartni hisobga olamiz:

$$\mathrm{R}(r0) = 0$$

Bu shartni quyidagi funksiya qanoatlantiradi

> **R[n](r):=BesselJ(0,lambda[n]*r);**

$$R_n(r) := \mathrm{BesselJ}(0, \lambda_n r)$$

Bu yerda λ_n -Bessel funksiyasining μ_n nollarini tanlashdan aniqlanadigan o'zgaruvchilarni ajratuvchi xos qiymat :

> **BesselJ(0,mu[n])=0;**

mu:=BesselJZeros:mu(0,n);

$$\mathrm{BesselJ}(0, \mu_n) = 0$$

$$\mathrm{BesselJZeros}(0, n)$$

bunda:

> **lambda[n]:=mu(0,n)/r0;**

$$\lambda_n := \frac{\mathrm{BesselJZeros}(0, n)}{r0}$$

Shuning uchun radial tenglamaning yechimi quyidagi ko'rinishga ega bo'ladi:

> **R[n](r):=BesselJ(0,r*lambda[n]);**

$$R_n(r) := \mathrm{BesselJ}\left(0, \frac{\mathrm{BesselJZeros}(0, n)\, r}{r0}\right)$$

Vaqt tenglamasi uchun quyidagiga ega bo'lamiz:

> **T[n](t):= C1[n]*sin(a*lambda[n]*t)+C2[n]*cos(a*lambda[n]*t);**

$$T_n(t) := C1_n \sin\left(\frac{a\, \mathrm{BesselJZeros}(0, n)\, t}{r0}\right) + C2_n \cos\left(\frac{a\, \mathrm{BesselJZeros}(0, n)\, t}{r0}\right)$$

> **u[n](t,r):=T[n](t)*R[n](r);**

$$u_n(t,r) := \left(C1_n \sin\left(\frac{a\ \mathrm{BesselJZeros}(0,n)\ t}{r0}\right) + C2_n \cos\left(\frac{a\ \mathrm{BesselJZeros}(0,n)\ t}{r0}\right)\right) \mathrm{BesselJ}\left(0, \frac{\mathrm{BesselJZeros}(0,n)\ r}{r0}\right)$$

Natijada umumiy yechim quyidagi ko'rinishga ega bo'ladi:

> **u(t,r):=Sum(u[n](t,r), n=1..infinity);**

$$u(t,r) := \sum_{n=1}^{\infty} \left(C1_n \sin\left(\frac{a\ \mathrm{BesselJZeros}(0,n)\ t}{r0}\right) + C2_n \cos\left(\frac{a\ \mathrm{BesselJZeros}(0,n)\ t}{r0}\right)\right) \mathrm{BesselJ}\left(0, \frac{\mathrm{BesselJZeros}(0,n)\ r}{r0}\right)$$

Quyidagicha almashtirish olamiz:

$$r \ \text{-->}\ \rho = \frac{r}{r0}\ ,\ r = \rho\ r0$$

ρ - o'zgaruvchining o'zgarish intervali [0, 1].

> **u(t,rho):=subs(r=rho*r0,u(t,r));**

$$u(t,\rho) := \sum_{n=1}^{\infty} \left(C1_n \sin\left(\frac{a\ \mathrm{BesselJZeros}(0,n)\ t}{r0}\right) + C2_n \cos\left(\frac{a\ \mathrm{BesselJZeros}(0,n)\ t}{r0}\right)\right) \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0,n)\ \rho)$$

$C1_n$ va $C2_n$ koeffisentlarni aniqlash uchun boshlang'ich shartlardan foydalanamiz:

$$u(0,r) = F(r)$$

$$\frac{\partial}{\partial t} u(0,r) = f(r)$$

> **simplify(subs(t=0,u(t,rho))=F(rho*r0));**
simplify(subs(t=0,diff(u(t,rho),t))=f(rho*r0));

$$\sum_{n=1}^{\infty} C2_n\ \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0,n)\ \rho) = F(\rho\ r0)$$

$$\sum_{n=1}^{\infty} \frac{C1_n\ a\ \mathrm{BesselJZeros}(0,n)\ \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0,n)\ \rho)}{r0} = f(\rho\ r0)$$

Olingan munosabatlarni har ikkala qismini quyidagiga ko'paytiramiz

$$\rho\ \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0,k)\ \rho)$$

>

Sum(C2[n]*rho*BesselJ(0,BesselJZeros(0,n)*rho)*BesselJ(0,BesselJZeros(0,k)*rho),n=1..infinity)=rho*BesselJ(0,BesselJZeros(0,k)*rho)*F(rho*r0);
Sum(C1[n]*rho*BesselJ(0,BesselJZeros(0,n)*rho)*a*BesselJZeros(0,k)/r0*BesselJ(0,BesselJZeros(0,n)*rho),n = 1 .. infinity)=rho*BesselJ(0,BesselJZeros(0,k)*rho)*f(rho*r0);

$$\sum_{n=1}^{\infty} C2_n\, \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, k)\, \rho) = \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, k)\, \rho)\, \mathrm{F}(\rho\, r0)$$

$$\sum_{n=1}^{\infty} \frac{C1_n\, \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)^2\, a\, \mathrm{BesselJZeros}(0, k)}{r0} = \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, k)\, \rho)\, \mathrm{f}(\rho\, r0)$$

Endi olingan munosabatlarni ρ bo'yicha integrallaymiz:

>

int(Sum(C2[n]*rho*BesselJ(0,BesselJZeros(0,n)*rho)^2,n=1..infinity),rho=0..1)=int(rho*BesselJ(0,BesselJZeros(0,n)*rho)*F(rho*r0),rho=0..1);
int(Sum(C1[n]*rho*BesselJ(0,BesselJZeros(0,n)*rho)^2*a*BesselJZeros(0,n)/r0,n=1..infinity),rho=0..1)=int(rho*BesselJ(0,BesselJZeros(0,n)*rho)*f(rho*r0),rho=0..1);

$$\frac{1}{2}\left(\sum_{n=1}^{\infty} C2_n\, \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))^2\right) = \int_0^1 \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)\, \mathrm{F}(\rho\, r0)\, d\rho$$

$$\frac{1}{2}\,\frac{a\left(\sum_{n=1}^{\infty} C1_n\, \mathrm{BesselJZeros}(0, n)\, \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))^2\right)}{r0} = \int_0^1 \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)\, \mathrm{f}(\rho\, r0)\, d\rho$$

Bessel funksiyasining ortogonallik shartidan quyidagiga kelamiz:

>

Int(rho*BesselJ(0,BesselJZeros(0,n)*rho)^2,rho=0..1)=simplify(int(rho*BesselJ(0,BesselJZeros(0,n)*rho)^2,rho=0..1));
Int(rho*BesselJ(0,BesselJZeros(0,n)*rho)*BesselJ(0,BesselJZeros(0,k)*rho),r

ho=0..1)=0;

$$\int_0^1 \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)^2\, d\rho = \frac{1}{2} \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))^2$$

$$\int_0^1 \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, k)\, \rho)\, d\rho = 0$$

Oxirgi munosabatdan ,

$$\int_0^1 \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, k)\, \rho)\, d\rho$$

integral faqat n = k da noldan farqli, qolgan holler uchun nolga teng. Shunda qatorda faqat bitta qo’shiluvchi qoladi.

Natijada quyidagiga ega bo’lamiz:

>

1/2*C2[n]*BesselJ(1,BesselJZeros(0,n))^2=int(rho*BesselJ(0,BesselJZeros(0,n)*rho)*F(rho*r0),rho=0 .. 1);

1/2*C1[n]*a/r0*BesselJZeros(0,n)*BesselJ(1,BesselJZeros(0,n))^2=int(rho*BesselJ(0,BesselJZeros(0,n)*rho)*f(rho*r0),rho=0..1);

$$\frac{1}{2} C2_n\, \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))^2 = \int_0^1 \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)\, \mathrm{F}(\rho\, r0)\, d\rho$$

$$\frac{1}{2} \frac{C1_n\, a\, \mathrm{BesselJZeros}(0, n)\, \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))^2}{r0} = \int_0^1 \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)\, \mathrm{f}(\rho\, r0)\, d\rho$$

>

C1[n]:=solve(1/2*C1[n]*a/r0*BesselJZeros(0,n)*BesselJ(1,BesselJZeros(0,n))^2 = int(rho*BesselJ(0,BesselJZeros(0,n)*rho)*f(rho*r0),rho = 0 .. 1),C1[n]);

C2[n]:=solve(1/2*C2[n]*BesselJ(1,BesselJZeros(0,n))^2 = int(rho*BesselJ(0,BesselJZeros(0,n)*rho)*F(rho*r0),rho = 0 .. 1),C2[n]);

$$C1_n := \frac{2\, r0}{a\, \mathrm{BesselJZeros}(0, n)\, \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))^2} \int_0^1 \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)\, \mathrm{f}(\rho\, r0)\, d\rho$$

$$C2_n := \frac{2}{\mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))^2} \int_0^1 \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)\, \mathrm{F}(\rho\, r0)\, d\rho$$

Natijada umumiy yechimni quyidagi ko’rinishda yozamiz:

> **u(t,rho) := Sum((C1[n]*sin(a*BesselJZeros(0,n)/r0*t)+C2[n]*cos(a*BesselJZeros(0,n)/r0*t))*BesselJ(0,BesselJZeros(0,n)*rho),n = 1 .. infinity);**

$$u(t, \rho) := \sum_{n=1}^{\infty} \left(\frac{2\, r0 \sin\left(\frac{a\, \mathrm{BesselJZeros}(0, n)\, t}{r0}\right)}{a\, \mathrm{BesselJZeros}(0, n)\, \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))^2} \int_0^1 \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)\, \mathrm{f}(\rho\, r0)\, d\rho + \frac{2 \cos\left(\frac{a\, \mathrm{BesselJZeros}(0, n)\, t}{r0}\right)}{\mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))^2} \int_0^1 \rho\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)\, \mathrm{F}(\rho\, r0)\, d\rho \right) \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\, \rho)$$

3.3-§. Doiraviy membrana tebranish tenglamalariga misollar

1 – Misol.

> **restart;**

$$\frac{\partial^2}{\partial t^2} u(t, r) = a^2 \left(\left(\frac{\partial^2}{\partial r^2} u(t, r) \right) + \frac{1 \left(\frac{\partial}{\partial r} u(t, r) \right)}{r} \right)$$

bir jinsli tenglamani quyidagi bir jinsli chegaraviy va boshlang’ich shartlar bilan yeching

$$u(0, r) = F(r),$$

$$\frac{\partial}{\partial t} u(0, r) = f(r).$$

Bu yerda

> **r0:=1; H:=1;a:=1;**
F(rho):=-(rho*r0)^2+H;
f(rho):=0;

$$r0 := 1$$

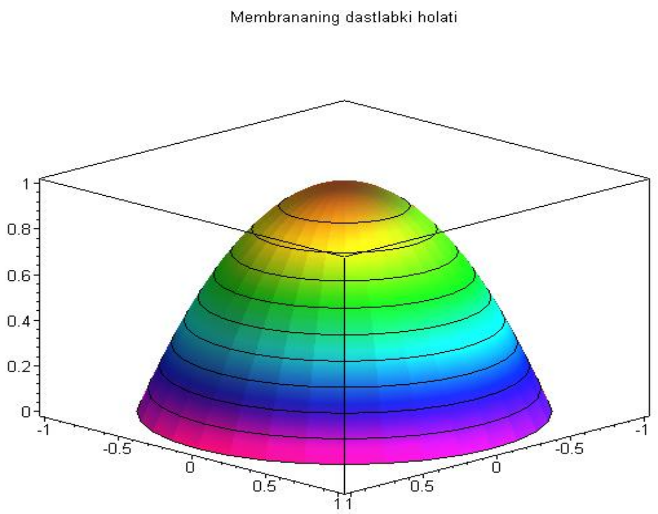
$$H := 1$$

$$a := 1$$

$$F(\rho) := -\rho^2 + 1$$

$$f(\rho) := 0$$

> **addcoords(M_cylindrical,[z,rho,theta],[rho*cos(theta),rho*sin(theta),z]);**
plot3d(F(rho),rho=0..r0,theta=0..2*Pi,coords=M_cylindrical,title="Membrananing dastlabki holati",axes=BOXED,lightmodel=light3,numpoints=1000,orientation=[45,65],shading=ZHUE,style=PATCHCONTOUR);



>
C1[n]:=2*r0/a/BesselJZeros(0,n)/BesselJ(1,BesselJZeros(0,n))^2*int(rho*BesselJ(0,BesselJZeros(0,n)*rho)*f(rho*r0),rho=0..1);
C2[n]:=simplify(2/BesselJ(1,BesselJZeros(0,n))^2*int(rho*BesselJ(0,BesselJZeros(0,n)*rho)*F(rho*r0),rho=0..1));

$$C1_n := 0$$

$$C2_n := -\frac{4\,(\mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n))\, \mathrm{BesselJZeros}(0, n) - 2\, \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(}{\mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))^2\, \mathrm{BesselJZeros}(0, n)^3}$$

Tenglamani yechimi:

>
u(t,rho):=Sum((C1[n]*sin(a*BesselJZeros(0,n)/r0*t)+C2[n]*cos(a*BesselJZer

os(0,n)/r0*t))*BesselJ(0,BesselJZeros(0,n)*rho),n = 1 .. 24);

$$u(t,\rho) := \sum_{n=1}^{24} \left(\frac{8\cos(\text{BesselJZeros}(0,n)\,t)\,\text{BesselJ}(0,\text{BesselJZeros}(0,n)\,\rho)}{\text{BesselJ}(1,\text{BesselJZeros}(0,n))\,\text{BesselJZeros}(0,n)^3} \right)$$

Olingan yechimni animasiyali grafik ko'rinishida tasvirlaymiz:

> **with(plots):**

addcoords(M_cylindrical,[z,rho,theta],[rho*cos(theta),rho*sin(theta),z]);

animate(plot3d,[u(t,rho),rho=0..r0,theta=0..2*Pi,coords=M_cylindrical],

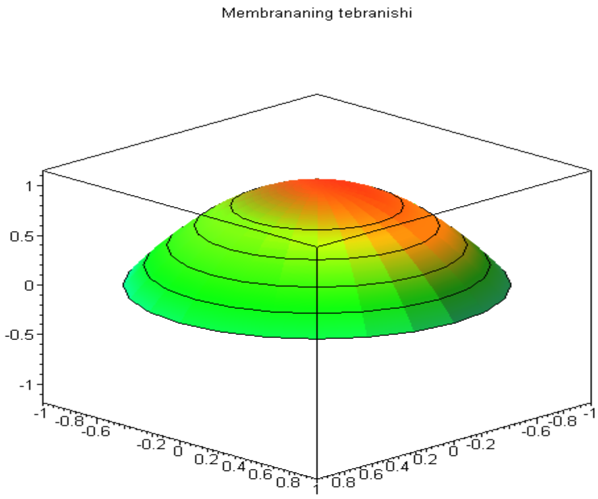
t=0..4, frames=20, title="Membrananing

tebranishi",axes=BOXED,lightmodel=light1,numpoints=1000,orientation=[45

, 65],shading=ZHUE,style=PATCHCONTOUR);

Warning, the name changecoords has been redefined

Warning, coordinates already exists, system redefined.



2 – Misol.

> **restart;**

$$\frac{\partial^2}{\partial t^2} u(t,r) = a^2 \left(\left(\frac{\partial^2}{\partial r^2} u(t,r) \right) + \frac{1 \left(\frac{\partial}{\partial r} u(t,r) \right)}{r} \right)$$

bir jinsli tenglamani quyidagi bir jinsli chegaraviy va boshlang'ich shartlar bilan yeching

$$u(0, r) = F(r),$$

$$\frac{\partial}{\partial t} u(0, r) = f(r).$$

Bu yerda

> **r0:=1; H:=1;a:=1;**

F(rho):=-(rho*r0)+H;

f(rho):=0;

$$r0 := 1$$

$$H := 1$$

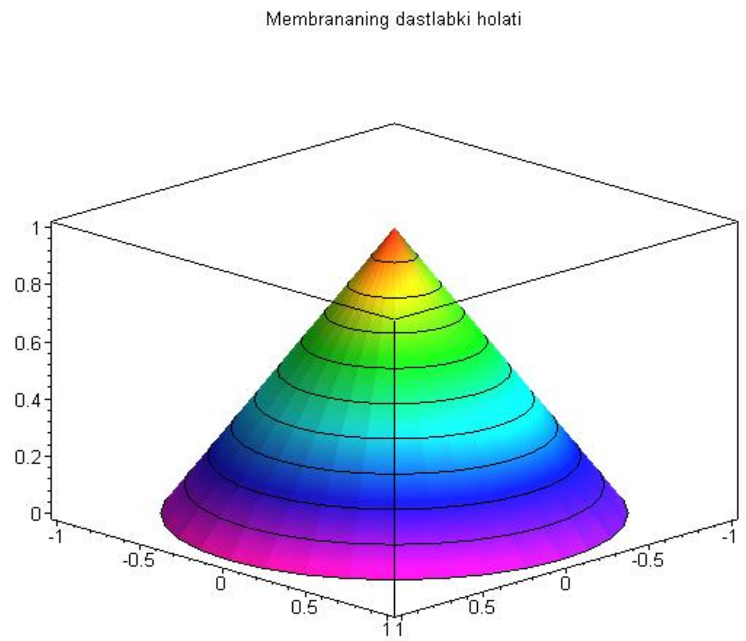
$$a := 1$$

$$F(\rho) := -\rho + 1$$

$$f(\rho) := 0$$

> **addcoords(M_cylindrical,[z,rho,theta],[rho*cos(theta),rho*sin(theta),z]);**

plot3d(F(rho),rho=0..r0,theta=0..2*Pi,coords=M_cylindrical,title="Membran aning dastlabki holati",axes=BOXED,lightmodel=light3,numpoints=1000,orientation=[45,65], shading=ZHUE,style=PATCHCONTOUR);



>

C1[n]:=2*r0/a/BesselJZeros(0,n)/BesselJ(1,BesselJZeros(0,n))^2*int(rho*Bess

elJ(0,BesselJZeros(0,n)*rho)*f(rho*r0),rho=0..1);

C2[n]:=simplify(2/BesselJ(1,BesselJZeros(0,n))^2*int(rho*BesselJ(0,BesselJZeros(0,n)*rho)*F(rho*r0),rho=0..1));

$$C1_n := 0$$

$$C2_n := \pi\,(-\mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n))\,\mathrm{StruveH}(1, \mathrm{BesselJZeros}(0, n)) + \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))\,\mathrm{StruveH}(0, \mathrm{BesselJZeros}(0, n)))/(\mathrm{BesselJZeros}(0, n)^2\,\mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))^2)$$

Tenglamani yechimi:

>

u(t,rho):=Sum((C1[n]*sin(a*BesselJZeros(0,n)/r0*t)+C2[n]*cos(a*BesselJZeros(0,n)/r0*t))*BesselJ(0,BesselJZeros(0,n)*rho),n = 1 .. 64);

$$\mathrm{u}(t, \rho) := \sum_{n=1}^{64} \pi\,\mathrm{StruveH}(0, \mathrm{BesselJZeros}(0, n))\cos(\mathrm{BesselJZeros}(0, n)\,t)\,\mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n)\,\rho)/(\mathrm{BesselJZeros}(0, n)^2\,\mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n)))$$

Olingan yechimni animasiyali grafik ko'rinishida tasvirlaymiz:

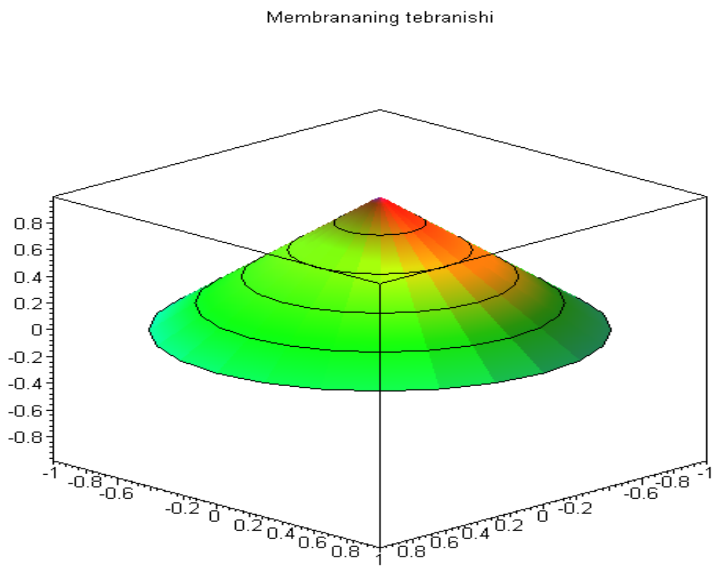
> **with(plots):**

addcoords(M_cylindrical,[z,rho,theta],[rho*cos(theta),rho*sin(theta),z]);

animate(plot3d,[u(t,rho),rho=0..r0,theta=0..2*Pi,coords=M_cylindrical], t=0..4, frames=20, title="Membrananing tebranishi",axes=BOXED,lightmodel=light1,numpoints=1000,orientation=[45, 65],shading=ZHUE,style=PATCHCONTOUR);

Warning, the name changecoords has been redefined

Warning, coordinates already exists, system redefined.



3 – Misol.

> **restart;**

$$\frac{\partial^2}{\partial t^2}u(t,r)=a^2\left(\left(\frac{\partial^2}{\partial r^2}u(t,r)\right)+\frac{1\left(\frac{\partial}{\partial r}u(t,r)\right)}{r}\right)$$

bir jinsli tenglamani quyidagi bir jinsli chegaraviy va boshlang'ich shartlar bilan yeching

$$u(0,r)=F(r),$$

$$\frac{\partial}{\partial t}u(0,r)=f(r).$$

Bu yerda

> **r0:=1; H:=1;a:=1;**

F(rho):=0;

f(rho):=-100*(rho*r0)+H;

$$r0 := 1$$

$$H := 1$$

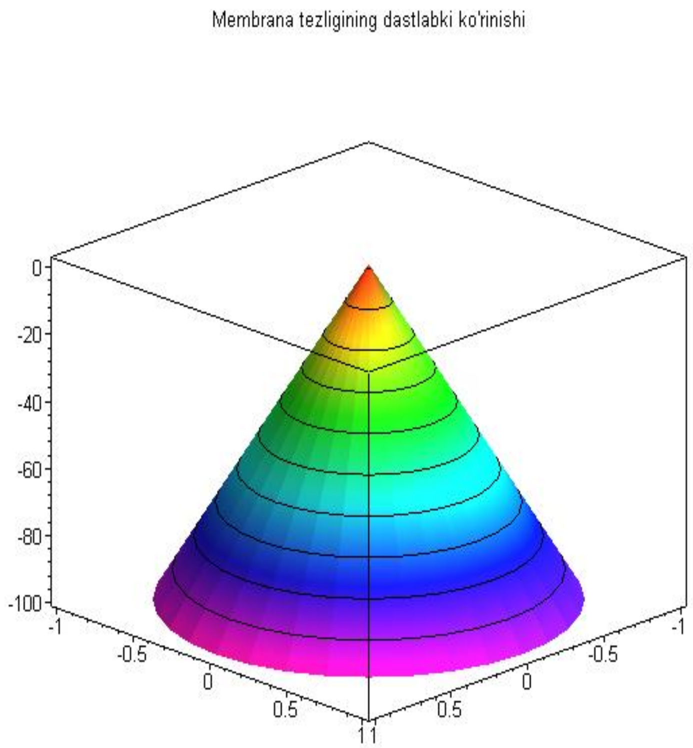
$$a := 1$$

$$F(\rho) := 0$$

$$f(\rho) := -100\,\rho+1$$

> **addcoords(M_cylindrical,[z,rho,theta],[rho*cos(theta),rho*sin(theta),z]);**

plot3d(f(rho),rho=0..r0,theta=0..2*Pi,coords=M_cylindrical,title="Membrana tezligining dastlabki ko'rinishi",axes=BOXED,lightmodel=light3,numpoints=1000,orientation=[45,65],shading=ZHUE,style=PATCHCONTOUR);



>

C1[n]:=2*r0/a/BesselJZeros(0,n)/BesselJ(1,BesselJZeros(0,n))^2*int(rho*BesselJ(0,BesselJZeros(0,n)*rho)*f(rho*r0),rho=0..1);

C2[n]:=simplify(2/BesselJ(1,BesselJZeros(0,n))^2*int(rho*BesselJ(0,BesselJZeros(0,n)*rho)*F(rho*r0),rho=0..1));

$$C1_n := 2\Big(-100\Big(\mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n)) + \frac{1}{2}\pi\,(\mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0, n))\ \mathrm{StruveH}(1, \mathrm{BesselJZeros}(0, n)) - \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))\ \mathrm{StruveH}(0, \mathrm{BesselJZeros}(0, n)))/\mathrm{BesselJZeros}(0, n)\Big)/\mathrm{BesselJZeros}(0, n) + \frac{\mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))}{\mathrm{BesselJZeros}(0, n)}\Big) \Big/ (\mathrm{BesselJZeros}(0, n)\ \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0, n))^2)$$

$$C2_n := 0$$

Tenglamani yechimi:

```
>
u(t,rho):=Sum((C1[n]*sin(a*BesselJZeros(0,n)/r0*t)+C2[n]*cos(a*BesselJZeros(0,n)/r0*t))*BesselJ(0,BesselJZeros(0,n)*rho),n = 1 .. 24);
```

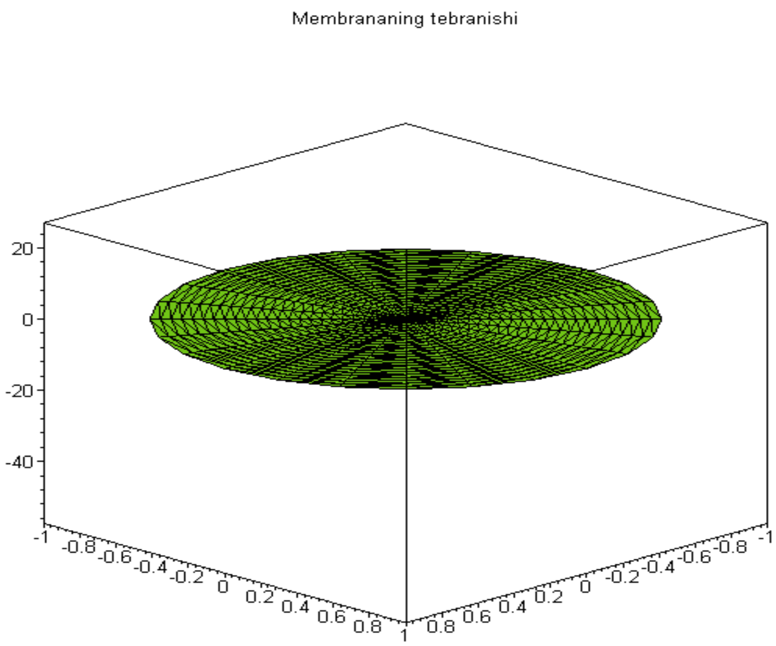
$$u(t,\rho) := \sum_{n=1}^{24} \left(2\left(-100\left(\mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0,n)) - \frac{1}{2}\,\frac{\pi\, \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0,n))\, \mathrm{StruveH}(0, \mathrm{BesselJZeros}(0,n))}{\mathrm{BesselJZeros}(0,n)}\right)\Big/ \mathrm{BesselJZeros}(0,n) + \frac{\mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0,n))}{\mathrm{BesselJZeros}(0,n)}\right) \sin(\mathrm{BesselJZeros}(0,n)\, t)\, \mathrm{BesselJ}(0, \mathrm{BesselJZeros}(0,n)\, \rho) \Big/ \left(\mathrm{BesselJZeros}(0,n)\, \mathrm{BesselJ}(1, \mathrm{BesselJZeros}(0,n))^2\right)\right)$$

Olingan yechimni animasiyali grafik ko'rinishida tasvirlaymiz:

```
> with(plots):
addcoords(M_cylindrical,[z,rho,theta],[rho*cos(theta),rho*sin(theta),z]);
animate(plot3d,[u(t,rho),rho=0..r0,theta=0..2*Pi,coords=M_cylindrical],
t=0..4, frames=20, title="Membrananing
tebranishi",axes=BOXED,lightmodel=light1,numpoints=1000,orientation=[45
, 65],shading=ZHUE,style=PATCHCONTOUR);
```

Warning, the name changecoords has been redefined

Warning, coordinates already exists, system redefined.



Xulosa

Turmush hayotimizda muhim ahamiyatga ega bo’lgan to’lqin tarqalish jarayoni, shuningdek to’g’ri to’rtburchakli ya’ni kvadratli va doiraviy membranalarning tebranish tenglamalari giperbolik tipli tenglamalar orqali o’rganiladi. Bu tenglamalar uchun ham chegaraviy va Koshi masalari tenglama yechimini bir qiymatli ajratib olishga imkon yaratadi va ular belgilangan rejimga asosan tanlab olinadi.

Bitiruv malakaviy ishida, Maple matematik paketidan foydalanib, kvadratli va doiraviy membranalarning tebranish tenglamalarini Fur’ye usuli ya’ni o’zgaruvchilarni ajratish usuli yordamida yechish keltirilgan. Maple paketi orqali to’lqin tenglamalarni yechish jarayoni qoidaga mos ta’lim berish uchun qiziqarli misollar yordamida tasvirlangan. Maple paketini har bir turdagi masalani yechishga qo’llanilishi ketma-ket tarzda keltirilgan, ya’ni kvadratli va doiraviy membranalarning tebranish tenglamalari yechishda misollarga quyidagicha tavsif berilgan: hisoblash formulasi, analitik va sonli yechimi, shuningdek, yechimning ikki o’lchovli animasiyali grafigi tasvirlangan, bundan tashqari ba’zi misollar uchun bir qancha vaqt momentlarini ikki o’lchovli grafigi tasvirlangan.

Foydalanilgan adabiyotlar ro'yxati.

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