O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

ZAHIRIDDIN MUHAMMAD BOBUR NOMIDAGI

ANDIJON DAVLAT UNIVERSITETI

FIZIKA-MATEMATIKA FAKULTETI

MATEMATIKA KAFEDRASI

Qo’lyozma xuquqida

HAYDAROVA DILRABOXON

“Trigonometrik funksiyalarni tizimli o’rgatish”

5400100- matematika yo’nalishi bakalavr akademik darajasini olish uchun yozilgan

BITIRUV MALAKAVIY ISH

ILMIY RAHBAR: **dotsent. A. Axlimirzayev**

ANDIJON-2015

Kirish

Respulikamiz mustaqillikka erishgandan so'ng barcha sohalarda bo'lgani kabi ta'lim tizimida ham chuqur islohotlar amalga oshirildi.

Ular:

- kasb-hunar kollejlari va akademik litseylarning tashkil qilinishi, oliy o'quv yurtlarida esa bakalavrlar va magistrlar tayyorlashga o'tilganligi;
- o'quvchilar va talabalarning fan asoslaridan olgan bilim saviyalarini aniqlashda yangi pedagogik tizim-reyting tizimidan foydalanishga otilganligi;
- talabalikka qabul qilishda uzoq yillardan beri qo'llanilib kelingan kirish imtixonlarini yangi bir usul-test usuli bilan almashtirilishi;
- barcha ta'lim muassasalarida o'qitiladigan fanlar bo'yicha yangi Davlat Ta'lim standartlarini qabul qilinishi va hokazolardir.

Tal'im tizimida amalga oshirilgan islohotlarni maqsadi "Ta'lim to'g'risidagi qonun" va "Kadrlar tayyorlash milliy dasturida" belgilangan vazifalarni hayotga joriy qilish va respublikamizda jaxon andozalariga mos keladigan raqobatbardosh mutaxasislarni tayyorlashga qaratilgandir. Bunday mutaxasislarni tayyorrlashda matematika fanining o'rni salmoqlidir.

Yangi qabul qilingan Davlat Ta'lim standartlarida uzluksiz ta'lim tizimining barcha bo'g'inlarida, jumladan umumiy o'rta ta'lim maktablari, akademik litsey va kasb-hunar kollejlarida matematika kursining mazmuni va maqsadi aniq bayon qilingan. Navbatdagi asosiy vazifalardan biri aniqlangan mazmunni o'quvchilarga mukammal yetkazishdan iboratdir. Buning uchun esa matematika o'qituvchisi o'qitish jarayonida zamonaviy pedagogik va axborot kommunikatsion texnologiyalardan to'laqonli foydalanishi kerak. Bunda eng avvalo o'qituvchining o'zi matematikaning mazmunini mukammal egallagan bo'lishi kerak.

Ma'lumki Trigonometrik funksiyalar umumiy o'rta ta'lim maktablarining 9-sinfida o'rganila boshlanib keyinchalik akademik litsey va kasb-xunar kollejlarida davom ettiriladi. 9-sinfda o'qitiladigan trigonometrik funksiyalar trgonometriya elementlari deb ataladi. "trigonometriya elementlari" o'rta maktab matematikasining an'anaviy boblaridan xisoblanadi. Uni o'rganish trigonometriyaning paydo bo'lishi, rivojlanishiga, ayniqsa uning tadbiklariga xissa qo'shgan allomalarimiz Muxammad Muso al-Xorazimiy, Axmad Farg'oniy, Abu Rayxon Beruniy, Mirzo Ulug'beklarni faoliyatlarini o'rganishdan boshlanadi. Keyinchalik burchakning radian o'lchovi nuqtani kordinatalar boshi atrofida burish, burchakning sinusi, kosinusi, tangensi va kotangensi tariflari "sin α, cosα va tgα ishoralari'' Ayni bir burchak sinusi, kosinusi, tangensi va kotangensi orasidagi munosabat, "trigonometrik ayniyatlar" α va $-\alpha$ burchaklarining sinusi, kosinusi, tangensi va kotangensi "qo'shish formulalari", "sinuslar yig'indisi va ayrmasini ko'paytmaga almashtirish" mavzulari o'rganiladi.

Trigonometrik funksiyalarni o'rganish akademik litsey va kasb-hunar kollejlarida davom ettiriladi xamda trigonometrik tenglama va tengsizliklarni yechish bilan yakunlanadi. Xulosa qilib aytganda trigonometrik funksiyalarni o'rganish o'tkir burchakning trigonometrik funksiyalarini o'rganishdan boshlanadi va ixtiyoriy burchak uchun umumlashtiriladi. Shuning uchun xam trigonometrik funksiyalarni tizimli o'rganish masalasi **dolzarb** masalalardan xisoblanadi.

Bitiruv malakaviy ishning predmeti umumiy o'rta ta'lim maktablari, akademik litsey va kasb-xunar kollejlari matematika kursining muxim bo'limlaridan biri xisoblangan trigonometrik funksiyalarini tizimi o'rgatish masalalarini yoritishdan iborat.

Bitiruv malakaviy ishning metodologik asosini O'zbekiston respublikasi prezidentining mamlakatning ta'lim tizimini yanada takomillashtirishga qaratilgan qarorlari, Ta'im to'g'risidagi O'zbekiston respublikasi qonuni, "kadrlar tayyorlash milliy dasturi" , umumiy o'rta ta'lim va o'rta maxsus kasb-hunar ta'limi uchun matematikadan Davlat Talim standartlari tashkil etadi.

Bitiruv malakaviy ishini tayyorlashda akademik litsey va kasb-hunar kollejlarining matematika fani darsliklari va o'quv qo'llanmalari, internetdan olingan ma'lumotlar hamda akademik litsey va kasb hunar kollejlaridagi matematika darslarini kuzatishdan olingan natija, ko'nikmalar va xulosalar e'tiborga olingan.

Bitiruv malakaviy ishdan olingan natijalardan akademik litsey va kasb-hunar kollejlarida matematikani o'qitish jarayonida, o'quvchilar bilan mustaqil ishlarni tashkil qilishda, matematikadan o'quv-uslubiy qo'llanmalarni tayyorlashda va iqdidorli o'quvchilar bilan ishlashda foydalanish mumkun.

Bitiruv malakaviy ishdan olingan natijalar malakaviy amaliyot va bitiruv oldi amaliyoti davrida to'plangan materiallardan iborat.

Bitiruv malakaviy ish kirish, 2 ta bob va 12 ta paragraf, xulosa va foydalanilgan adbiyotlar ro'yhatidan iborat.

I. BOB. TRIGONOMETRIK FUNKSIYALAR.

§1. O’tkir burchakning trigonometrik funksiyalari

Bu bobda yangi funksiyalar sinfi – trigonometrik funksiyalar o’rganiladi. Bu funksiya tabiatshunoslikda uchrab turadigan turli davriy jarayonlarni tavsiflash uchun ishlatiladi. Davriy ravishda takrorlanib turuvchi jarayonlarga insonlar har qadamda duch keladilar. Bularga misol sifatida astronomik hodisalarni keltirish mumkin. masalan, quyoshning chiqishi va botishi, yil vaqtlarini takrorlanishi, osmondagi yulduzlarning holati, qorong’ulikni tushishi, planeta harakati va hokazo.

Yurakning urishi, inson organizmining hayot faoliyati, g’ildirakning aylanishi, gripp epidemiyasining tarqalishi va hokazo jarayonlardagi umumiylik ularning davriyligidir. Bu davriy jarayonlar esa trigonometrik funksiyalar bilan ifodalanadi.

Trigonometriya so’zi lug’oviy jihatdan uchburchaklarni o’lchash demakdir. Trigonometriya matematika kursining muxim bo’limlaridan iborat bo’lib, u trigonometrik funksiyalar va ularning xossalari, trigonometrik Funksiyalar qatnashgan tenglamalar va tensizliklarni yechishni o’rganadi.

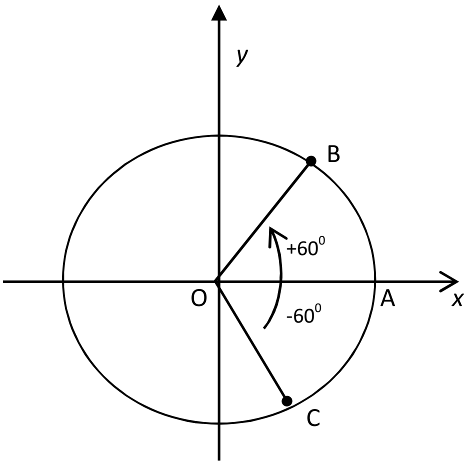
Burchak tushunchasi geometriyaning muhim tushunchalaridan biri hisoblanadi. Bu tushuncha trigonometriyani o’rganishda ham muhimdir.

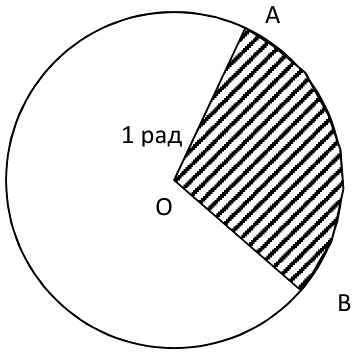
Bir nuqtadan chiquvchi ikkita nur va tekislikning ular bilan chegaralangan bo’lagiga burchak deb ataladi. Burchak graduslarda va radianlarda o’lchanadi.

Agar burchakning tomonlari to’g’ri chiziq hosil qilsa, u holda burchakni yoyiq burchak deb ataladi. Yoyiq burchakni qiymati 180^0 ga teng.

Markazi koordinata boshida bo’lgan R radiusli aylana olamiz. 0x o’qida yotgan radiusni oxirini A nuqta bilan belgilaymiz. 0A radiusni boshlang’ich radius deb ataymiz va quyidagi kelishuvlarni hosil qilamiz:

0A radius soat strelkasi yo’nalishida burilsa, u holda hosil bo’lgan burchakni manfiy, aks holda musbat deb olamiz.





Burchak va yoyni o'lchov birligi sifatida 1 gradusli burchak va 1 gradusli yoy qabul qilingan. Uni 1^0 kabi yoziladi.

1^0-li burchak, bu boshlang'ich radusni to'la aylananing soat strelkasiga qarama-qarshi yo'nalishda $\frac{1}{360}$ bir bo'lagiga burilgandagi burchagidir.

1^0 ning $\frac{1}{60}$ bo'lagiga minut deyiladi va 1^1 kabi belgilanadi. 1^1 ning $\frac{1}{60}$ bo'lagi esa sekund deb ataladi va u 1^{11} kabi yoziladi.

Burchakning ikkinchi xil o'lchovi radiandir.

1 radian burchak, uzunligi radiusni uzunligiga teng bo'lgan yoyga tiralgan markaziy burchakdir.

Agar boshlang'ich radius bir marta to'la aylansa, u holda 360^0 ga yoki 2π radianga teng burchak hosil bo'ladi.

1^0 radian o'lchovida $\frac{2\pi}{360} = \frac{\pi}{180}$ ga teng.

A^0 li burchak $\alpha = \frac{A\pi}{180}$ radianga teng.

α radianli burchakda $\frac{\alpha \bullet 180}{\pi}$ gradus bor.

α radianli yoyni uzunligi $C = \alpha \bullet R$ ga teng.

A^0 li yoyni uzunligi $C = \frac{\pi R A^0}{180^0}$ ga teng.

$\pi = 180^0$ dan $2\pi = 360^0$, $\frac{\pi}{2} = 90^0$, $\frac{\pi}{3} = 60^0$, $\frac{\pi}{6} = 30^0$ lar kelib chiqadi.

$1^0 = \frac{\pi}{180} =$0,017 rad. 1 rad $= \frac{180^0}{\pi} =$57,296^0 yoki 1radq57^0 17^{1}45^{11}.

Misollar:

1. α=150^0 ni radianda ifodalang.

Yechish: $150^0 = 150 \bullet \frac{\pi}{180}$ rad $= \frac{5\pi}{6}$ rad

2. $\alpha = 4{,}5$ rad ni gradusda ifodalang.

Yechish: 4,5 rad $= 4{,}5 \bullet \frac{180^0}{\pi} = 258^0$.

3. Radiusi 16 sm ga teng bo’lgan aylananing $\frac{\pi}{4}$ radianli yoyi uzunligini toping.

Yechish: $C = \alpha R$ formuladan foydalanamiz. Bizda $\alpha = \frac{\pi}{4}$ va R=16. $C = \frac{\pi}{4} \bullet 16 = 4\pi$ sm.

4. 240^0 ning radian o’lchovini toping.

Yechish: $240^0 = 240 \bullet 1^0 = 240 \bullet \frac{\pi}{180} = \frac{4\pi}{3}$ radian.

5. 72^{0}ning radian o’lchovini toping.

Yechish: $72^0 = 72 \bullet 1^0 = 72 \bullet \frac{\pi}{180} = \frac{2\pi}{5}$ radian.

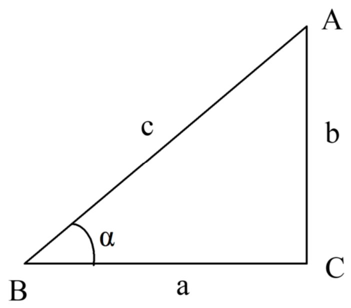
6. $\frac{5\pi}{4}$ radian necha radus bo’ladi?

Yechish: $\frac{5\pi}{4}$radian $= \frac{5\pi}{4} \bullet \frac{180^0}{\pi} = 225^0$

7. $\frac{4\pi}{3}$ radian necha gradusga teng?

Yechish: $\frac{4\pi}{3}$radian $= \frac{4\pi}{3} \bullet \frac{180^0}{\pi} = 240^0$

O’tkir burchaklaridan biri α bo’lgan ABC to’g’ri burchakli uchburchakni qaraymiz:



AB=c-gipotenuza, BC=a-o’tkir burchakkka yopishgan katet va AC=b o’tkir burchak qarshisida yotgan katet.

ABC uchburakning a,b va c tomonlaridan foydalanib, $\frac{b}{c}$, $\frac{a}{c}$, $\frac{b}{a}$,$\frac{a}{b}$, $\frac{c}{a}$ va $\frac{c}{b}$ nisbatlarni yozishimiz mumkin. Bu nisbatlarni o’zgarishi α burchakni ham o’zgarishiga olib keladi. Bu nisbatlarning har biri bilan α burchak orasidagi bog’lanishlarni alohida nomlaymiz.

α burchak qarshisida yotgan katet uzunligini gipotenuza uzunligiga nisbati α burchakning <u>sinusi</u> deyiladi va uni sin α deb yoziladi. Demak, $\sin\alpha = \frac{b}{c}$.

α burchakka yopishgan katet uzunligini gipotenuza uzunligiga nisbati, α burchakning <u>kosinusi</u> deyiladi va u cos α deb yoziladi. Demak, $\cos\alpha = \frac{a}{c}$.

α burchak qarshisida yotgan katet uzunligini, α burchakka yopishgan katet uzunligiga nisbati, α burchakning <u>tangensi</u> deyiladi va u tg α deb yoziladi. Demak, $tg\alpha = \frac{b}{a}$.

α burchakka yopishgan katet uzunligini, α burchak qarshisida yotgan katet uzunligiga nisbati, α burchakning <u>kotangensi</u> deyiladi va u ctg α deb yoziladi. Demak, $ctg\alpha = \frac{b}{a}$.

Gipotenuza uzunligini α burchakka yopishgan katet uzunligiga nisbati α burchakning sekansi deyiladi va u sec α deb yoziladi. Demak, $\sec\alpha = \frac{c}{a}$.

Gipotenuza uzunligini α burchak qarshisidagi katet uzunligiga nisbati, α burchakning kosekansi deyiladi va cosec α kabi yoziladi. Demak, $\operatorname{cosec}\alpha = \frac{c}{b}$.

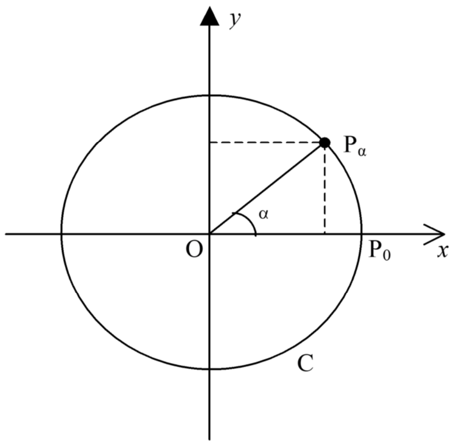
sin α, cos α, tg α, ctg α, sec α va cosec α lar trigonometrik funksiyalar deb ataladi.

Bulardan oxirgi ikkitasi, dastlabki ikkitasiga nisbatan teskari bo'lganligi uchun ularni alohida o'rganish qiziqish uyg'otmaydi. Shuning uchun ham biz bundan buyon dastlabki 4 ta funksiyani ko'proq o'rganamiz.

Agar biz $a^2+b^2=c^2$ (Pifagor teoremasi) tenglikni har ikkala qismini hadma-had c^2 ga bo'lsak, u holda $(\frac{a}{c})^2 + (\frac{b}{c})^2 = 1$ yoki $\sin^2\alpha + \cos^2\alpha = 1$ ni hosil qilamiz. Bu tenglik Pifagor teoremasiga ekvivalent bo'lib, uni asosiy trigonometrik ayniyat deb ataladi.

§2. Ixtiyoriy burchakning trigonometrik funksiyalari

Bundan oldingi paragrafda biz o'tkir burchak trigonometrik funksiyalarining ta'riflarini keltirdik. Bu ta'riflarni ixtiyoriy burchak uchun ham umumlashtirish mumkin. Buning uchun markazi koordinata boshida va radiusi 1 ga teng bo'lgan aylanani olamiz.



Aylanadagi (1;0) koordinatali nuqtani R_0 bilan belgilaymiz va uni boshlang'ich nuqta deb ataymiz. Ixtiyoriy α sonni olamiz va boshlang'ich nuqtani α burchakka buramiz. Natijada Rα nuqtani hosil qilamiz.

R α nuqtaning koordinatalarini x_α va y_α deb belgilaymiz. OP α =R=1 ekanligini e'tiborga olsak, $\sin\alpha = \frac{y_\alpha}{R} = y_\alpha$ va $\cos\alpha = \frac{x_\alpha}{R} = 1$larni yoki $\sin\alpha = y_\alpha$ va $\cos\alpha = x_\alpha$ larni hosil qilamiz. Bulardan esa sinus va kosinuslar uchun boshqacha ta'riflar berish mumkinligi kelib chiqadi.

R α nuqtaning ordinatasiga α burchakning sinusi va abstsissasiga α burchakning kosinusi deyiladi.

Demak, $\sin\alpha = y_\alpha$ va $\cos\alpha = x_\alpha$.

R α $(x_\alpha\ ;\ y_\alpha)$ nuqtaning koordinatalari uchun $x_\alpha^2 + y_\alpha^2 = 1$ ya'ni $\sin^2\alpha + \cos^2\alpha = 1$ tenglik o'rinlidir. Bu asosiy trigonometrik ayniyatdir. Undan quyidagilarni yozish mumkin: $\sin^2\alpha = 1 - \cos^2\alpha$ va $\cos^2\alpha = 1 - \sin^2\alpha$.

Agar biz $tg\alpha = \frac{y_\alpha}{x_\alpha}$ va $ctg\alpha = \frac{x_\alpha}{y_\alpha}$ hamda $x_\alpha = \cos\alpha$ va $y_\alpha = \sin\alpha$ ekanligini e'tiborga olsak, u holda $tg\alpha = \frac{\sin\alpha}{\cos\alpha}$ va $ctg\alpha = \frac{\cos\alpha}{\sin\alpha}$ larni hosil qilamiz. Demak, α burchak sinusini uning kosinusiga nisbatiga α burchakning tangensi deyiladi.

α burchak kosinusini uning sinusiga nisbatiga esa α burchakning kotangensi deyiladi.

Demak, ta'riflarga asosan $tg\alpha = \frac{\sin\alpha}{\cos\alpha}$ va $ctg\alpha = \frac{\cos\alpha}{\sin\alpha}$.

Bu yerda tangens $\cos\alpha \neq 0$ va kotangens $\sin\alpha \neq 0$ hollarda aniqlangan.

Agar biz $\sec\alpha = \frac{R}{x_\alpha} = \frac{1}{x_\alpha}$ va $\operatorname{cosec}\alpha = \frac{R}{y_\alpha} = \frac{1}{y_\alpha}$ ekanligini e'tiborga olsak, u holda biz $\sec\alpha = \frac{1}{\cos\alpha}$ va $\operatorname{cosec}\alpha = \frac{1}{\sin\alpha}$ larni hosil qilamiz. Demak,

α burchakning kosinusiga teskari kattalikka shu burchakning sekansi deyiladi.

α burchakning sinusiga teskari kattalikka shu burchakning kosekansi deyiladi.

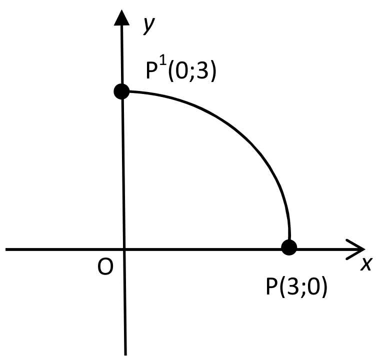
$tg\alpha = \dfrac{\sin\alpha}{\cos\alpha}$ va $ctg\alpha = \dfrac{\cos\alpha}{\sin\alpha}$ lardan tg α va ctg α larni o'zaro teskari sonlar ekanligini ko'ramiz. O'zaro teskari sonlar ko'paytmasi esa 1 ga teng. Demak, $tg\alpha \bullet ctg\alpha = 1$.

Bundan esa $tg\alpha = \dfrac{1}{ctg\alpha}$ va $ctg\alpha = \dfrac{1}{tg\alpha}$ lar kelib chiqadi.

Misollar:

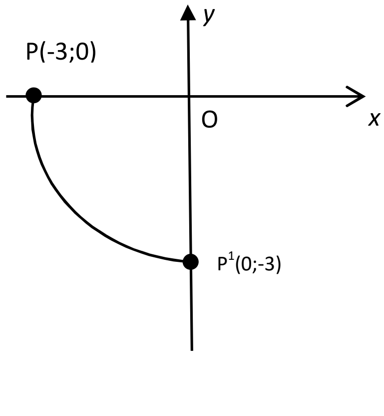
1. R(3;0) nuqtani koordinata boshi atrofida 90^0ga burganda u qaysi nuqtaga o'tadi?

Yechish: R(3;0) nuqtani soat strelkasi yo'nalishiga qarama-qarshi yo'nalishda 90^0 ga burganda u $R^1(0;3)$ nuqtaga o'tadi.

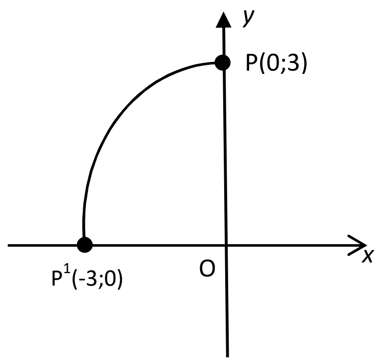


2. R(-3;0) nuqtani koordinata boshi atrofida 90^0 ga burganda hosil bo'ladigan nuqtaning koordinatalarini toping.

Yechish: R(-3;0) nuqtani koordinata boshi atrofida soat strelkasi yo'nalishiga qarama-qarshi yo'nalishda 90^0 ga burganda u R^1 (0;-3) nuqtaga o'tadi.



3. R(0;3) nuqtani koordinata boshi atrofida 90^0 ga burganda hosil bo'ladigan nuqtaning koordinatalarini toping.



Yechish: R(0, 3) nuqtani koordinata boshi atrofida soat streklasi yo'nalishiga qarama-qarshi yo'nalishda 90^0 ga burganda u $R^1(-3;0)$ nuqtaga o'tadi.

4. $(\sin\alpha+\cos\alpha)^2+(\sin\alpha-\cos\alpha)^2-2$ ni soddalashtiring.

Yechish: $(\sin\alpha+\cos\alpha)^2+(\sin\alpha-\cos\alpha)^2-2=\sin^2\alpha+2\sin\alpha\bullet\cos\alpha+\cos^2\alpha+$ $+\sin^2\alpha-2\sin\alpha\bullet\cos\alpha+\cos^2\alpha-2=2\sin^2\alpha+2\cos^2\alpha-2=2(\sin^2\alpha+\cos^2\alpha)-2=$ $=2\bullet1-2=2-2=0$

Javob: 0

5. $\sin^6\alpha+\cos^6\alpha+3\sin^2\alpha\bullet\cos^2\alpha$ ni soddalashtiring.

Yechish: $\sin^6\alpha+\cos^6\alpha+3\sin^2\alpha\bullet\cos^2\alpha=(\sin^2\alpha)^3+(\cos^2\alpha)^3+3\sin^2\alpha\bullet$ $\cdot\cos^2\alpha=(\sin^2\alpha+\cos^2\alpha)(\sin^4\alpha-\sin^2\alpha\bullet\cos^2\alpha+\cos^4\alpha)+3\sin^2\alpha\bullet\cos^2\alpha=$ $=\sin^4\alpha-\sin^2\alpha\bullet\cos^2\alpha+\cos^4\alpha+3\sin^2\alpha\bullet\cos^2\alpha=\sin^4\alpha+2\sin^2\alpha\bullet\cos^2\alpha+$ $+\cos^4\alpha=(\sin^2\alpha+\cos^2\alpha)^2=1^2=1$.

Javob: 1

6. $\sin^2x+\cos^2x+tg^2x$ ni soddalashtiring.

Yechish: $\sin^2x+\cos^2x+tg^2x=1+tg^2x=1+\frac{\sin^2 x}{\cos^2 x}=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}$.

Javob: $\frac{1}{\cos^2 x}$

7. Agar $\sin x+\cos x=0,5$ bo'lsa, $16(\sin^3x+\cos^3x)$ ni toping

Yechish: sinx+cosx=0,5 tenglikni har ikkala qismini hadma-had kubga ko'taramiz.

$(\sin x+\cos x)^3=0{,}5^3$, $\sin^3x+3\sin^2x\bullet\cos x+3\sin x\bullet\cos^2x+\cos^3x=\frac{1}{8}$

$\sin^3x+\cos^3x+3\sin x\cos x(\sin x+\cos x)=\frac{1}{8}$, $\sin^3x+\cos^3x+\frac{3}{2}\sin x\bullet\cos x=\frac{1}{8}$

$\sin^3x+\cos^3x=\frac{1}{8}-\frac{3}{2}\sin x\bullet\cos x$

$\sin x\bullet\cos x$ ni topish uchun sinx+cosx=0,5 tenglikni har ikkala qismini hadma-had kvadratga ko'taramiz.

$(\sin x+\cos x)^2=0{,}5^2$, $\sin^2x+2\sin x\bullet\cos x+\cos^2x=\frac{1}{4}$, $1+2\sin x\bullet\cos x=\frac{1}{4}$,

$2\sin x\bullet\cos x=\frac{1}{4}-1$, $2\sin x\bullet\cos x=-\frac{3}{4}$, $\sin x\bullet\cos x=-\frac{3}{8}$.

Buni e'tiborga olsak, $\sin^3x+\cos^3x=\frac{1}{8}-\frac{3}{2}\bullet(-\frac{3}{8})$, $\sin^3x+\cos^3x=\frac{11}{16}$.

$16(\sin^3x+\cos^3x)=16\bullet\frac{11}{16}=11$.

Javob: 11

8. $\frac{\cos^2\alpha-ctg^2\alpha}{tg^2\alpha-\sin^2\alpha}$ ni soddalashtiring.

Yechish: $\frac{\cos^2\alpha-ctg^2\alpha}{tg^2\alpha-\sin^2\alpha}=\frac{\cos^2\alpha-\frac{\cos^2\alpha}{\sin^2\alpha}}{\frac{\sin^2\alpha}{\cos^2\alpha}-\sin^2\alpha}=\frac{\frac{\sin^2\alpha\bullet\cos^2\alpha-\cos^2\alpha}{\sin^2\alpha}}{\frac{\sin^2\alpha-\sin^2\alpha\bullet\cos^2\alpha}{\cos^2\alpha}}=$

$=\frac{\cos^2\alpha(\sin^2\alpha-1)\bullet\cos^2\alpha}{\sin^2\alpha(1-\cos^2\alpha)\bullet\sin^2\alpha}=-\frac{\cos^4\alpha(1-\sin^2\alpha)}{\sin^4\alpha\bullet\sin^2\alpha}=-\frac{\cos^6\alpha}{\sin^6\alpha}=-ctg^6\alpha$

Javob: $-ctg^6\alpha$

9. $\frac{3\sin^2\alpha+\cos^4\alpha}{1+\sin^2\alpha+\sin^4\alpha}$ ni soddalashtiring

Yechish: $\frac{3\sin^2\alpha+\cos^4\alpha}{1+\sin^2\alpha+\sin^4\alpha}=\frac{3\sin^2\alpha+(\cos^2\alpha)^2}{1+\sin^2\alpha+\sin^4\alpha}=\frac{3\sin^2\alpha+(1-\sin^2\alpha)^2}{1+\sin^2\alpha+\sin^4\alpha}=$

$=\frac{3\sin^2\alpha+1-2\sin^2\alpha+\sin^4\alpha}{1+\sin^2\alpha+\sin^4\alpha}=\frac{1+\sin^2\alpha+\sin^4\alpha}{1+\sin^2\alpha+\sin^4\alpha}=1$.

Javob: 1

10. $\dfrac{1+\cos^2\alpha+\cos^4\alpha}{3\cos^2\alpha+\sin^4\alpha}$ ni soddalashtiring

Yechish: $\dfrac{1+\cos^2\alpha+\cos^4\alpha}{3\cos^2\alpha+\sin^4\alpha}=\dfrac{1+\cos^2\alpha+\cos^4\alpha}{3\cos^2\alpha+(1-\cos^2\alpha)^2}=$

$$=\frac{1+\cos^2\alpha+\cos^4\alpha}{3\cos^2\alpha+1-2\cos^2\alpha+\cos^4\alpha}=\frac{1+\cos^2\alpha+\cos^4\alpha}{1+\cos^2\alpha+\cos^4\alpha}=1.$$

Javob: 1

11. $\dfrac{1-\sin^4\alpha-\cos^4\alpha}{\cos^4\alpha}$ ni soddalashtiring.

Yechish: $\dfrac{1-\sin^4\alpha-\cos^4\alpha}{\cos^4\alpha}=\dfrac{(1-\sin^2\alpha)(1+\sin^2\alpha)-\cos^4\alpha}{\cos^4\alpha}=$

$$=\frac{\cos^2\alpha(1+\sin^2\alpha)-\cos^4\alpha}{\cos^4\alpha}=\frac{\cos^2\alpha+\cos^2\alpha\bullet\sin^2\alpha-\cos^4\alpha}{\cos^4\alpha}=$$

$$\frac{\cos^2\alpha(1-\cos^2\alpha)+\cos^2\alpha\bullet\sin^2\alpha}{\cos^4\alpha}=\frac{\cos^2\alpha\bullet\sin^2\alpha+\cos^2\alpha\bullet\sin^2\alpha}{\cos^4\alpha}=\frac{2\sin^2\alpha\bullet\cos^2\alpha}{\cos^4\alpha}=$$

$$=2\bullet\frac{\sin^2\alpha}{\cos^2\alpha}=2\mathrm{tg}^2\,\alpha$$

Javob: $2\mathrm{tg}^2\,\alpha$

12. $\dfrac{\sin^2\alpha-\cos^2\alpha+\cos^4\alpha}{\cos^2\alpha-\sin^2\alpha+\sin^4\alpha}$ ni soddalashtiring.

Yechish: $\dfrac{\sin^2\alpha-\cos^2\alpha+\cos^4\alpha}{\cos^2\alpha-\sin^2\alpha+\sin^4\alpha}=\dfrac{\sin^2\alpha-\cos^2\alpha(1-\cos^2\alpha)}{\cos^2\alpha-\sin^2\alpha(1-\sin^2\alpha)}=$

$$\frac{\sin^2\alpha-\cos^2\alpha\bullet\sin^2\alpha}{\cos^2\alpha-\sin^2\alpha\bullet\cos^2\alpha}=\frac{\sin^2\alpha(1-\cos^2\alpha)}{\cos^2\alpha(1-\sin^2\alpha)}=\frac{\sin^2\alpha\bullet\sin^2\alpha}{\cos^2\alpha\bullet\cos^2\alpha}=\frac{\sin^4\alpha}{\cos^4\alpha}=\mathrm{tg}^4\,\alpha$$

Javob: $\mathrm{tg}^4\,\alpha$

§3. Trigonometrik funksiyalarning ishoralari va qiymatlari

Trigonometrik funksiyalarning ishoralari qaralayotgan burchakning qaysi chorakda yotishiga qarab aniqlanadi.

α burchakning sinusi R α nuqtaning ordinatasidan iborat bo'lganligi uchun u I va II choraklarda musbat, III va IV choraklarda esa manfiy bo'ladi.

α burchakning kosinusi R α nuqtaning abstsissasidan iborat bo'lgani uchun u I va IV choraklarda musbat, II va III choraklarda esa manfiy bo'ladi.

α burchakning tangensi va kotangensi R α nuqta koordinatalarining nisbatlari bo'lganligi uchun, ular R α nuqtaning koordinatalari bir xil ishorali bo'lgan (I va III) choraklarda musbat va har xil ishorali bo'lgan (II va IV) choraklarda manfiy bo'ladi.

Bularni umumlashtirib quyidagi jadvalni hosil qilamiz:

Choraklar Funksiyalar	I	II	III	IV
Sin α	+	+	-	-
Cos α	+	-	-	+
tg α	+	-	+	-
Ctg α	+	-	+	-

Misollar:

1. Agar sin α $\bullet$ cos α >0 bo'lsa, α burchak qaysi chorakka tegishli?

Yechish: sin α va cos α ning ko'paytmasi musbat bo'lishi uchun ularning ishoralari bir xil bo'lishi kerak. Ular I va III choraklarda bir xil ishorali bo'ladi.

Javob: I yoki III.

2. Agar tg α $\bullet$ ctg α >0 bo'lsa, α burchak qaysi chorakka tegishli?

Yechish tg α va ctg α ning ko'paytmasi musbat bo'lishi uchun ular bir xil ishorali bo'lishi kerak. Ular I va II choraklarda bir xil ishoralidir.

Javob: I yoki II

3. Agar sin α $\bullet$ cos α <0 bo'lsa, α burchak qaysi chorakka tegishli?

Yechish: sin α va cos α ning ko'paytmasi manfiy bo'lishi uchun ular turli ishorali bo'lishi kerak. Ular II va IV choraklarda turli ishoralidir.

Javob: II yoki IV.

4. cos3, sin4, sin2, tg2 va cos9 lardan qaysi biri musbat?

Yechish: 3 radian II-chorakdagi burchak bo'lganligi uchun cos3<0. 4 radian III-chorakdagi burchak bo'lganligi uchun sin4<0. 2 radian II-chorakdagi burchak bo'lganligi uchun sin2>0 va tg2<0. 9 radian II-chorakdagi burchak bo'lganligi uchun cos9<0.

Javob: sin2.

5. $\frac{ctg187^0}{\sin 316^0}$, $\frac{\cos 340^0}{\sin 185^0}$, $\frac{\sin 148^0}{\cos 317^0}$, $\frac{ctg105^0}{tg185^0}$ va $\frac{tg215^0}{\cos 125^0}$ sonlardan qaysi biri musbat?

Yechish: $\frac{ctg187^0}{\sin 316^0}<0$, Chunki ctg$187^0>0$ va sin$316^0<0$.

$\frac{\cos 340^0}{\sin 185^0}<0$, Chunki cos $340^0>0$ va sin$185^0<0$.

$\frac{\sin 148^0}{\cos 317^0}>0$, Chunki sin$148^0>0$ va cos$317^0>0$.

$\frac{ctg105^0}{tg185^0}<0$, Chunki ctg $105^0<0$ va tg$185^0>0$.

$\frac{tg215^0}{\cos 125^0}<0$, Chunki tg$215^0>0$ va cos$125^0<0$.

Javob: $\frac{\sin 148^0}{\cos 317^0}$

6. $\sin 122^0 \bullet \cos 322^0$, $\cos 148^0 \bullet \cos 289^0$, $tg196^0 \bullet ctg189^0$, $tg220^0 \bullet \sin 100^0$ va $ctg320^0 \bullet \cos 186^0$ lardan qaysi biri manfiy?

Yechish: $\sin 122^0 \bullet \cos 322^0 >0$,Chunki sin$122^0>0$ va cos$322^0>0$.

$\cos 148^0 \bullet \cos 289^0<0$,Chunki cos $148^0<0$ va cos$289^0>0$.

$tg196^0 \bullet ctg189^0>0$,Chunki tg$196^0>0$ va ctg$189^0>0$.

$tg220^0 \bullet \sin 100^0 >0$,Chunki tg$220^0>0$ va sin$100^0>0$.

$ctg320^0 \bullet \cos 186^0>0$,Chunki ctg$320^0<0$ va cos$186^0<0$.

Javob: $\cos 148^0 \bullet \cos 289^0$

Amaliyotda ko'pincha trigonometrik funksiyalarning qiymatlari bilan ish ko'riladi. α burchakning trigonometrik funksiyalarini qiymatlari Rα nuqtaning koordinatalari bilan bog'liq. Ya'ni, $y_\alpha=\sin\alpha$, $x_\alpha=\cos\alpha$, tg $\alpha=\frac{y_\alpha}{x_\alpha}$ va ctg $\alpha=\frac{x_\alpha}{y_\alpha}$

α burchak 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$ va 2π qiymatlarni qabul qilganda Rα nuqtaning koordinatalarini osongina topiladi. α burchak 0^0, 30^0, 45^0 va 60^0 qiymatlarni qabul qilganda Rα nuqtaning koordinatalarini o'tkir burchagi α bo'lgan to'g'ri burchakli uchburchakdan topiladi.

quyidagi jadvalda trigonometrik funksiyalarning ba'zi bir burchaklardagi qiymatlari keltirilgan.

Burchaklar / Funksiyalar	0	30^0	45^0	60^0	90^0	180^0	270^0	360^0
sin α	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos α	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
tg α	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	-	0	-	0
ctg α	-	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	-	0	-

Jadvaldagi «-» belgi ko'rsatilgan burchakda berilgan funksiyaning qiymati mavjud emasligini bildiradi.

Misollar:

1. $5\sin90^0+2\cos0^0-2\sin270^0+10\cos180^0$ ni hisoblang.

Yechish: $5\sin90^0+2\cos0^0-2\sin270^0+10\cos180^0=5\cdot1+2\bullet1-2\cdot(-1)+10\cdot(-1)=$ $=5+2+2-10=-1$.

Javob: -1

2. $\sin180^0+\sin270^0-\text{ctg}90^0+\text{tg}180^0-\cos90^0$ ni qiymatini hisoblang.

Yechish: $\sin180^0+\sin270^0-\text{ctg}90^0+\text{tg}180^0-\cos90^0=0-1-0+0-0=-1$.

Javob: -1.

3. $3\text{tg}0^0+2\cos90^0+3\sin270^0-3\cos180^0$ ni hisoblang.

Yechish: $3\text{tg}0^0+2\cos90^0+3\sin270^0-3\cos180^0=3\bullet0+2\bullet0+3\bullet(-1)-3\bullet$ $\bullet(-1)=-3+3=0$.

Javob: 0

§4. Trigonometrik funksiyalarning davriyligi, juft-toqligi va keltirish formulalari

Rα nuqtaning ordinatasi sinα va abstsissasi cosα ni bildirishi ma’lum. Agar biz 0Rα radiusni butun son marta burchakka bursak ham Rα nuqtaning koordinatalari o’zgarmaydi. Bu esa ixtiyoriy burchakning sinusi va kosinusini hisoblashni 360^0 dan kichik musbat burchakning sinusi va kosinusini hisoblashga keltirish mumkinligini bildiradi. Masalan:

$\sin 785^0=\sin(2\bullet 360^0+65^0)=\sin 65^0$; $\cos 400^0=\cos(360^0+40^0)=\cos 40^0$

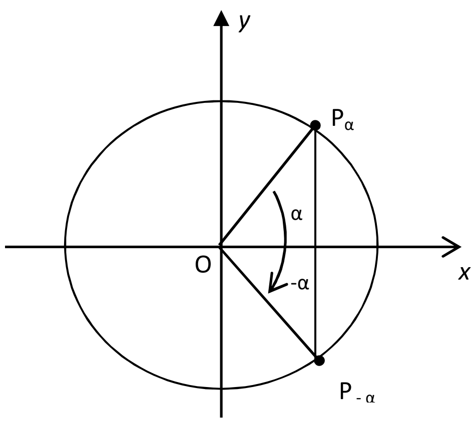
360^0 yoki 2π ni sinα va cosα larning <u>eng kichik musbat davri</u> deyiladi. Demak, $\sin(2\pi+\alpha)=\cos\alpha$; $\cos(2\pi+\alpha)=\sin\alpha$

R α nuqtaning ordinatasini uning abstsissasiga nisbati α burchakning tangensi va R α nuqtaning abstsissasini uning ordinatasiga nisbati α burchakning kotangensi ekanligi ma’lum. Agar Rα nuqtani musbat yo’nalishi bo’yicha π ga teng burchakka burchak $P_{\alpha+\pi}$ nuqtani hosil qilamiz. $P_{\alpha+\pi}$ nuqta koordinatalarining o’zaro nisbatlari R α nuqta koordinatalarining o’zaro nisbatlari bilan bir xil ekanligini ko’ramiz. Demak, $tg(\pi+\alpha)=tg\,\alpha$ va $ctg(\pi+\alpha)=ctg\,\alpha$

Bulardan esa tangens va kotangenslarning eng kichik musbat davri π ga ya’ni, 180^0 ga tengligi kelib chiqadi.

Masalan: $tg215^0=tg(180^0+35^0)=tg35^0$; $ctg205^0=ctg25^0$.

Rα va R$_-\alpha$ nuqtalarning koordinatalarini qaraymiz.



Rα nuqtaning koordinatalari x_α va y_α lardan iborat.

$R_{-\alpha}$ nuqtaning koordinatalari esa x_α va $-y_\alpha$ lardan iborat. Ya'ni, $\sin(-\alpha)=$
$=-\sin\alpha$ va $\cos(-\alpha)=\cos\alpha$ ekan. Bu tengliklardan esa $tg(-\alpha)=\frac{\sin(-\alpha)}{\cos(-\alpha)}=\frac{-\sin\alpha}{\cos\alpha}=-tg\alpha$ va $ctg(-\alpha)=\frac{\cos(-\alpha)}{\sin(-\alpha)}=\frac{\cos\alpha}{-\sin\alpha}=-ctg\alpha$ bo'lishi kelib chiqadi.

Demak, trigonometrik funksiyalardan kosinus juft, sinus, tangens va kotangenslar toq ekan.

Misollar:

1. $2tg(-765^0)$ ni hisoblang.

Yechish: $2tg(-765^0)= -2tg(4\bullet180^0+45^0)= -2tg45^0=-2\bullet1=-2$

Javob: -2

2. $\sin(-45^0)+\cos405^0+tg(-945^0)$ ni hisoblang.

Yechish: $\sin(-45^0)+\cos405^0+tg(-945^0)=-\sin45^0+\cos(360^0+45^0)-$
$-tg(5\bullet180^0+45^0)=-\frac{\sqrt{2}}{2}+\cos45^0-tg45^0=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}-1=-1.$

Javob: -1

3. $\cos(-45^0)+\sin315^0+tg(-855^0)$ ni hisoblang.

Yechish: $\cos(-45^0)+\sin315^0+tg(-855^0)=\cos45^0+\sin(360^0+(-45^0))-$
$-tg(5\bullet180^0+(-45^0)) = \frac{\sqrt{2}}{2}+\sin(-45^0)+ tg(-45^0)= \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}-1=-1$

Javob: -1

4. $tg\frac{\pi}{6}\bullet\sin\frac{\pi}{3}\bullet ctg\frac{5\pi}{4}$ ni hisoblang.

Yechish: $tg\frac{\pi}{6}\bullet\sin\frac{\pi}{3}\bullet ctg\frac{5\pi}{4}=\frac{\sqrt{3}}{3}\bullet\frac{\sqrt{3}}{2}\bullet ctg(\pi+\frac{\pi}{4})=\frac{1}{2}\bullet ctg\frac{\pi}{4}=\frac{1}{2}\bullet1=\frac{1}{2}.$

Javob: $\frac{1}{2}$.

O'tkir burchakning trigonometrik funksiyalarini jadval yoki to'g'ri burchakli uchburchakdan foydalanib hisoblash mumkin. Ixtiyoriy burchakni trigonometrik funksiyalarini qiymatlarini hisoblashni doimo o'tkir bo'rchak trigonometrik funksiyalari qiymatlarini hisoblashga keltirish mumkin. Bunday formulalarni keltirish formulalari deyiladi. Ularni quyidagi jadvalda keltiramiz:

Argument funk-lar	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$	$2\pi+\alpha$
sin α	cos α	cos α	sin α	-sin α	-cos α	-cos α	-sin α	sin α
cos α	sin α	-sin α	-cos α	-cos α	-sin α	sin α	cos α	cos α
tg α	ctg α	-ctg α	-tg α	tg α	ctg α	-ctg α	-tg α	tg α
ctg α	tg α	-tg α	-ctg α	ctg α	tg α	-tg α	-ctg α	ctg α

Misollar:

1. $\cos 870^0$ ni hisoblang

Yechish: $\cos 870^0=\cos(2\bullet 360^0+150^0)=\cos 150^0=\cos(90^0+60^0)=-\sin 60^0=-\frac{\sqrt{3}}{2}$

Javob: $-\frac{\sqrt{3}}{2}$

2. $\sin 2010^0$ ni hisoblang.

Yechish: $\sin 2010^0=\sin(5\bullet 360^0+210^0)=\sin 210^0=\sin(180^0+30^0)=-\sin 30^0=-\frac{1}{2}$.

Javob: $-\frac{1}{2}$.

3. $\sin^2(3570^0)$ ni hisoblang.

Yechish: $\sin^2(3570^0)=(\sin(10\bullet 360^0+(-30^0))^2=(\sin(-30^0))^2=(-\sin 30^0)^2=$ $=(-\frac{1}{2})^2==\frac{1}{4}=0,25$.

Javob: 0,25

4. $\dfrac{\sin(2\pi-\alpha)}{ctg(\frac{3\pi}{2}-\beta)}$ ni soddalashtiring.

Yechish: $\dfrac{\sin(2\pi-\alpha)}{ctg(\frac{3\pi}{2}-\beta)}=\dfrac{-\sin\alpha}{tg\beta}=-\dfrac{\sin\alpha}{tg\beta}$.

Javob: $-\dfrac{\sin\alpha}{tg\beta}$.

5. $\cos(\frac{3\pi}{2}-\alpha)\bullet tg(\pi-\alpha)$ ni soddalashtiring

Yechish: $\cos(\frac{3\pi}{2}-\alpha)\bullet tg(\pi-\alpha)$=-sin α $\bullet$(-tg α)=sin α $\bullet$tg α .

Javob: sin α $\bullet$tg α

6. $\cos^2(\pi+x)+\cos^2(\frac{\pi}{2}+x)$ ni soddalashtiring.

Yechish: $\cos^2(\pi+x)+\cos^2(\frac{\pi}{2}+x)$ =$(-\cos x)^2+(-\sin x)^2=\cos^2x+\sin^2x=1$.

Javob: 1

7. $tg1^0 \bullet tg2^0 \ldots \bullet tg88^0 \bullet tg89^0$ hisoblang.

Yechish: $tg46^0=ctg44^0$, $tg47^0=ctg43^0$, …. , $tg88^0=ctg2^0$ va $tg89^0=ctg1^0$ bo'lishini hisobga olsak.

$tg1^0 \bullet tg2^0 \ldots \bullet tg88^0 \bullet tg89^0=tg1^0 \bullet tg2^0 \bullet \ldots \bullet ctg2^0 \bullet ctg1^0=$
$=tg1^0 \bullet ctg1^0 \bullet tg2^0 \bullet ctg2^0 \ldots tg45^0=1 \bullet 1 \bullet \ldots \bullet 1=1$

Javob: 1

8. $\dfrac{\sin(\frac{\pi}{2}-\alpha)\bullet\cos(\pi+\alpha)}{ctg(\pi+\alpha)\bullet tg(\frac{3\pi}{2}-\alpha)}$ ni soddalashtiring.

Yechish: $\dfrac{\sin(\frac{\pi}{2}-\alpha)\bullet\cos(\pi+\alpha)}{ctg(\pi+\alpha)\bullet tg(\frac{3\pi}{2}-\alpha)}=\dfrac{\cos\alpha\bullet(-\cos\alpha)}{ctg\alpha\bullet ctg\alpha}=-\dfrac{\cos^2\alpha}{ctg^2\alpha}=-\dfrac{\cos^2\alpha}{\frac{\cos^2\alpha}{\sin^2\alpha}}=$

$=-\sin^2\alpha$.

§5. Ikki burchak yig'indisi va ayirmasining trigonometrik funksiyalari. Ikkilangan va uchlangan burchakning trigonometrik funksiyalari

Ba'zi hollarda berilgan burchakning trigonometrik funksiyalari qiymatlarini bevosita hisoblab bo'lmaydi. Lekin bu burchakni ikkita burchak yig'indisi yoki ayirmasi sifatida qaralib quyidagi formulalardan foydalanilsa ularni bevosita hisoblash mumkin bo'ladi. Ular:

$\sin(\alpha+\beta)=\sin\alpha\bullet\cos\beta+\cos\alpha\bullet\sin\beta \qquad \sin(\alpha-\beta)=\sin\alpha\bullet\cos\beta-\cos\alpha\bullet\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha\bullet\cos\beta-\sin\alpha\bullet\sin\beta$ $\cos(\alpha+\beta)=\cos\alpha\bullet\cos\beta-\sin\alpha\bullet\sin\beta$

$tg(\alpha+\beta)=\dfrac{tg\alpha+tg\beta}{1-tg\alpha\bullet tg\beta}$ $tg(\alpha-\beta)=\dfrac{tg\alpha-tg\beta}{1+tg\alpha\bullet tg\beta}$

$ctg(\alpha+\beta)=\dfrac{1-tg\alpha\bullet tg\beta}{tg\alpha+tg\beta}$ $ctg(\alpha-\beta)=\dfrac{1+tg\alpha\bullet tg\beta}{tg\alpha-tg\beta}$

Misollar:

1. $\cos45^0\bullet\cos15^0+\sin45^0\bullet\sin15^0$ ni hisoblang.

Yechish: $\cos45^0\bullet\cos15^0+\sin45^0\bullet\sin15^0=\cos(40^0-15^0)=\cos30^0=\dfrac{\sqrt{3}}{3}$

Javob: $\dfrac{\sqrt{3}}{3}$

2. Agar $\alpha=-45^0$ va $\beta=15^0$ bo'lsa, $\cos(\alpha+\beta)+2\sin\alpha\bullet\sin\beta$ ning qiymatini toping.

Yechish: $\cos(\alpha+\beta)+2\sin\alpha\bullet\sin\beta=\cos\alpha\bullet\cos\beta-\sin\alpha\bullet\sin\beta+2\sin\alpha\bullet\sin\beta$ $=\cos\alpha\bullet\cos\beta+\sin\alpha\bullet\sin\beta=\cos(\alpha-\beta)=\cos(-45^0-15^0)=\cos(-60^0)=\cos60^0=\dfrac{1}{2}$

Javob: $\dfrac{1}{2}$

3. $\dfrac{\cos(\alpha+\beta)+2\sin\alpha\bullet\sin\beta}{\sin(\alpha+\beta)-2\sin\alpha\bullet\cos\beta}$ ni soddalashtiring

Yechish: $\dfrac{\cos(\alpha+\beta)+2\sin\alpha\bullet\sin\beta}{\sin(\alpha+\beta)-2\sin\alpha\bullet\cos\beta}=\dfrac{\cos\alpha\bullet\cos\beta-\sin\alpha\bullet\sin\beta+2\sin\alpha\bullet\sin\beta}{\sin\alpha\bullet\cos\beta+\cos\alpha\bullet\sin\beta-2\sin\alpha\bullet\cos\beta}=$

$=\dfrac{\cos\alpha\bullet\cos\beta+\sin\alpha\bullet\sin\beta}{\sin\alpha\bullet\cos\beta+\cos\alpha\bullet\sin\beta}=\dfrac{\cos(\alpha-\beta)}{\sin(\beta-\alpha)}=\dfrac{\cos(\beta-\alpha)}{\sin(\beta-\alpha)}=ctg(\beta-\alpha)$.

4. $\cos15^0+\sqrt{3}\sin15^0$ ni hisoblang.

Yechish: $\cos15^0+\sqrt{3}\sin15^0=2(\frac{1}{2}\cos15^0+\frac{\sqrt{3}}{2}\sin15^0)=2(\sin30^0\bullet\cos15^0+$ $+\cos30^0\bullet\sin15^0)=2\bullet\sin(30^0+15^0)=2\sin45^0=2\bullet\dfrac{\sqrt{2}}{2}=\sqrt{2}$.

5. $\sin105^0+\sin75^0=?$

Yechish: $\sin 105^0+\sin 75^0=\sin(60^0+45^0)+\sin(45^0+30^0)=\sin 60^0 \bullet \cos 45^0+$
$+\cos 60^0 \bullet \sin 45^0+\sin 45^0 \bullet \cos 30^0+\cos 45^0 \bullet \sin 30^0=\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}=$

$$=2 \bullet \frac{\sqrt{6}}{4}+2 \bullet \frac{\sqrt{2}}{4}=\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}=\frac{\sqrt{6}+\sqrt{2}}{2}=\frac{\sqrt{2}(\sqrt{3}+1)}{2}=\frac{\sqrt{3}+1}{\sqrt{2}}=$$

$$=\sqrt{\frac{(\sqrt{3}+1)^2}{2}}=\sqrt{\frac{3+2\sqrt{3}+1}{2}=}=\sqrt{\frac{4+2\sqrt{3}}{2}}=\sqrt{\frac{2(2+\sqrt{3})}{2}}=\sqrt{2+\sqrt{3}}.$$

6. $\frac{\cos 18^0 \bullet \cos 28^0+\cos 108^0 \bullet \sin 208^0}{\sin 18^0 \bullet \sin 78^0+\sin 108^0 \bullet \sin 168^0}$ ni soddalashtiring.

Yechish: $\frac{\cos 18^0 \bullet \cos 28^0+\cos 108^0 \bullet \sin 208^0}{\sin 18^0 \bullet \sin 78^0+\sin 108^0 \bullet \sin 168^0}=$

$$=\frac{\cos 18^0 \bullet \cos 28^0+\cos(90^0+18^0) \bullet \sin(180^0+28^0)}{\sin 18^0 \bullet \sin(90^0-12^0)+\sin(90^0+18^0) \bullet \sin(180^0-12^0)}=$$

$$=\frac{\cos 28^0 \bullet \cos 18^0+\sin 28^0 \bullet \sin 18^0}{\sin(18^0+12^0)}=\frac{\cos(28^0-18^0)}{\sin 30^0}=\frac{\cos 10^0}{\frac{1}{2}}=2\cos 10^0.$$

7. $\mathrm{tg}105^0$ ni hisoblang.

Yechish: $\mathrm{tg}105^0=\mathrm{tg}(60^0+45^0)=\frac{tg60^0+tg45^0}{1-tg60^0 \bullet tg45^0}=\frac{\sqrt{3}+1}{1-\sqrt{3}}=$

$$=\frac{1+\sqrt{3}}{1-\sqrt{3}}=\frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}=\frac{1+2\sqrt{3}+3}{1-3}=-\frac{4+2\sqrt{3}}{2}=-(2+\sqrt{3})=-2-\sqrt{3}.$$

8. Agar $\sin\alpha \bullet \cos\beta=1$ va $\sin\beta \bullet \cos\alpha=\frac{1}{2}$ bo'lsa, $\alpha-\beta$ ning qiymatlarini toping.

Yechish: $\sin(\alpha-\beta)=\sin\alpha \bullet \cos\beta-\cos\alpha \bullet \sin\beta=1-\frac{1}{2}=\frac{1}{2}$, $\sin(\alpha-\beta)=\frac{1}{2}$,

$$\alpha-\beta=(-1)^k \bullet \frac{\pi}{6}+k\pi.$$

9. $tg(\frac{\pi}{4}-\alpha)=2$ bo'lsa, tg α ning qiymatini toping.

Yechish: $tg(\frac{\pi}{4}-\alpha)=\dfrac{tg\frac{\pi}{4}-tg\alpha}{1+tg\frac{\pi}{4}\bullet tg\alpha}=\dfrac{1-tg\alpha}{1+tg\alpha}$; $\dfrac{1-tg\alpha}{1+tg\alpha}=2$, 1-tg α =2+2tg α ;

3tg α=-1; tg $\alpha=-\frac{1}{3}$.

Ba'zi hollarda α burchak trigonometrik funksiyalarining qiymatlarini bilgan holda 2 α va 3 α burchakning trigonometrik funksiyalari qiymatlarini hisoblashga to'g'ri keladi. U quyidagi formulalar yordamida amalga oshiriladi.

sin2 α =2sin α $\bullet$ cos α ; cos2 α =cos^2 α -sin^2 α ; tg2 $\alpha=\dfrac{2tg\alpha}{1-tg^2\alpha}$; ctg $\alpha=\dfrac{1-tg^2\alpha}{2tg\alpha}$

sin3 α =3sin α -4sin^3 α ; cos3 α =4cos^3 α -3cos α ; tg3 $\alpha=\dfrac{3tg\alpha-tg^3\alpha}{1-3tg^2\alpha}$

ctg3 $\alpha=\dfrac{1-3tg^3\alpha}{3tg\alpha-tg^3\alpha}$

Bu formulalarni barchasini ikki burchak yig'indisining trigonometrik funksiyalarini berilgan burchaklar trigonometrik funksiyalari orqali ifodalash formulalaridan foydalanib keltirib chiqariladi.

cos2 α =cos^2 α -sin^2 α formulaning o'ng tomonidagi cos^2 α ni sin^2 α orqali yoki sin^2 α ni cos^2 α orqali ifodalab quyidagi formulalarni hosil qilish mumkin.

cos2 α =1-2sin^2 α yoki cos2 α =2cos^2-1.

Bu formulalardan esa $\sin^2\alpha=\dfrac{1-\cos2\alpha}{2}$ yoki $\cos^2\alpha=\dfrac{1+\cos2\alpha}{2}$ formulalarni hosil qilamiz.

Bu formulalarni trigonometrik Funksiyalarni darajalarini pasaytirish formulalari deb ham ataladi.

Yuqorida ko'rib o'tilgan formulalardan 2 α ni α bilan almashtirib yana bir qancha formulalarni hosil qilish mumkin.

$$\sin\alpha=2\sin\frac{\alpha}{2}\bullet\cos\frac{\alpha}{2},\ \cos\alpha=\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2},\ \ tg\alpha=\frac{2tg\frac{\alpha}{2}}{1-tg^2\frac{\alpha}{2}},\ \ ctg\alpha=\frac{1-tg^2\frac{\alpha}{2}}{2tg\frac{\alpha}{2}}$$

Bu formulalar trigonometrik ifodalarni soddalashtirishda, trigonometrik tenglamalar va tengsizliklarni yechishda ko'p qo'llaniladi.

Misollar:

1. $14\sqrt{2}(\sin^4\frac{3\pi}{8}-\cos^4\frac{3\pi}{8})$ ni hisoblang

Yechish: $14\sqrt{2}(\sin^4\frac{3\pi}{8}-\cos^4\frac{3\pi}{8}) = 14\sqrt{2}(\sin^2\frac{3\pi}{8}+\cos^2\frac{3\pi}{8})\ (\sin^2\frac{3\pi}{8}-\cos^2\frac{3\pi}{8}) =$

$= 14\sqrt{2}\bullet 1\bullet(-(\cos^2\frac{3\pi}{8}-\sin^2\frac{3\pi}{8})) = \quad -14\sqrt{2}\bullet\cos 2\bullet\frac{3\pi}{8} = \quad -14\sqrt{2}\cos\frac{3\pi}{4} =$

$= -14\sqrt{2}\bullet\cos(\frac{\pi}{2}+\frac{\pi}{4}) = -14\sqrt{2}\bullet(-\sin\frac{\pi}{4}) = 14\sqrt{2}\bullet\frac{\sqrt{2}}{2} = 14$

2. $8\sin^2\frac{7\pi}{8}\bullet\cos^2\frac{9\pi}{8}$ ni hisoblang.

Yechish: $8\sin^2\frac{7\pi}{8}\bullet\cos^2\frac{9\pi}{8} = 8\sin^2(\pi-\frac{\pi}{8})\bullet\cos^2(\pi+\frac{\pi}{8}) = 8\bullet\sin^2\frac{\pi}{8}\bullet\cos^2\frac{\pi}{8} =$

$= 2\bullet(2\sin\frac{\pi}{8}\bullet\cos\frac{\pi}{8})^2 = 2\sin^2\frac{\pi}{4} = 2\bullet(\frac{\sqrt{2}}{2})^2 = 2\bullet\frac{2}{4} = 1$

3. $\frac{1+\sin 2\alpha}{\sin\alpha+\cos\alpha}-\sin\alpha$ ni soddalashtiring.

Yechish: $\frac{1+\sin 2\alpha}{\sin\alpha+\cos\alpha}-\sin\alpha = \frac{\sin^2\alpha+2\sin\alpha\bullet\cos\alpha+\cos^2\alpha}{\sin\alpha+\cos\alpha}-\sin\alpha =$

$= \frac{(\sin\alpha+\cos\alpha)^2}{\sin\alpha+\cos\alpha}-\sin\alpha = \sin\alpha + \cos\alpha - \sin\alpha = \cos\alpha$

4. $\frac{4\sin 40^0\bullet\sin 50^0}{\cos 10^0} = ?$

Yechish: $\frac{4\sin 40^0\bullet\sin 50^0}{\cos 10^0} = \frac{2\bullet 2\sin 40^0\bullet\cos 40^0}{\cos 10^0} = \frac{2\bullet\sin 80^0}{\cos 10^0} = \frac{2\cos 10^0}{\cos 10^0} = 2$

5. $\cos 92^0\bullet\cos 2^0+0,5\sin 4^0+1$ ni hisoblang.

Yechish: $\cos 92^0\bullet\cos 2^0+0,5\sin 4^0+1=\cos(90^0+2^0)\bullet\cos 2^0+0,5\sin 4^0+1=$

$=-\sin 2^0\bullet\cos 2^0+\frac{1}{2}\bullet 2\sin 2^0\bullet \mathrm{sos} 2^0+1=-\sin 2^0\bullet\cos 2+\sin 2^0\bullet\cos 2^0+1=1.$

6. $\frac{1+\cos 2\alpha+\cos^2\alpha}{\sin^2\alpha}$ ni soddalashtiring

Yechish: $\frac{1+\cos 2\alpha+\cos^2\alpha}{\sin^2\alpha} = \frac{2\cos^2\alpha+\cos^2\alpha}{\sin^2\alpha} = \frac{3\cos^2\alpha}{\sin^2\alpha} = 3\mathrm{ctg}^2\alpha$.

7. $\frac{1-\cos 2\alpha}{1+\cos 2\alpha}+1$ ni soddalashtiring.

Yechish: $\frac{1-\cos 2\alpha}{1+\cos 2\alpha}+1=\frac{2\sin^2\alpha}{2\cos^2\alpha}+1=1+tg^2\alpha=\frac{1}{\cos^2\alpha}=\cos^{-2}\alpha$

8. $\frac{1}{\sin 10^0}-\frac{\sqrt{3}}{\cos 10^0}$ ni hisoblang.

Yechish: $\frac{1}{\sin 10^0}-\frac{\sqrt{3}}{\cos 10^0}=\frac{\cos 10^0-\sqrt{3}\sin 10^0}{\sin 10^0\bullet\cos 10^0}=\frac{2(\frac{1}{2}\cos 10^0-\frac{\sqrt{3}}{2}\sin 10^0)}{\sin 10^0\bullet\cos 10^0}=$

$$=\frac{2(\sin 30^0\bullet\cos 10^0-\cos 30^0\bullet\sin 10^0)}{\frac{1}{2}\bullet 2\sin 10^0\bullet\cos 10^0}=\frac{4\sin(30^0-10^0)}{\sin 20^0}=\frac{4\sin 20^0}{\sin 20^0}=4.$$

9. $\frac{\sin 36^0}{\sin 12^0}-\frac{\cos 36^0}{\cos 12^0}$ ni qiymatini toping.

Yechish: $\frac{\sin 36^0}{\sin 12^0}-\frac{\cos 36^0}{\cos 12^0}=\frac{\sin 36^0\bullet\cos 12^0-\cos 36^0\bullet\sin 12^0}{\sin 12^0\bullet\cos 12^0}=$

$$=\frac{\sin(36^0-12^0)}{\frac{1}{2}\bullet 2\sin 12^0\bullet\cos 12^0}=\frac{2\sin 24^0}{\sin 24^0}=2$$

10. $tg\alpha=\frac{1}{2}$ bo’lsa, tg2 α ni qiymatini hisoblang.

Yechish: $tg\alpha=\frac{2tg\alpha}{1-tg^2\alpha}=\frac{2\bullet\frac{1}{2}}{1-\frac{1}{4}}=\frac{1}{\frac{3}{4}}=\frac{4}{3}.$

§6. Trigonometrik funksiyalar ko’paytmasini yig’indiga va aksincha almashtirish formulalari

Trigonometrik ifodalarni soddalashtirishda, ba’zi bir burchaklardagi trigonometrik funksiyalar qiymatlarini hisoblashda va trigonometrik tenglamalarni yechishda ko’pincha trigonometrik funksiyalar ko’paytmasini yig’indiga almashtirish formulalaridan foydalanish qulay bo’ladi. Bu formulalarni ikki burchak yig’indisi va ayirmasining trigonometrik fukntsiyalari uchun yozilgan formulalardan osongina keltirib chiqariladi. Masalan,

$\sin(\alpha+\beta)=\sin\alpha\bullet\cos\beta+\cos\alpha\bullet\sin\beta$ va $\sin(\alpha-\beta)=\sin\alpha\bullet\cos\beta-\cos\alpha\bullet\sin\beta$

formulalarni hadma-had qo’shib, so’ngra 2 ga bo’lib

$\sin\alpha \bullet \cos\beta = \dfrac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{2}$ formulani hosil qilamiz.

Xuddi shu kabi $\cos(\alpha+\beta)$ va $\cos(\alpha-\beta)$ lar uchun yozilgan formulalardan $\cos\alpha \bullet \cos\beta = \dfrac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2}$, $\sin\alpha \bullet \sin\beta = \dfrac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$ formulalarni keltirib chiqaramiz.

Ba'zi hollarda $tg\alpha \bullet tg\beta = \dfrac{tg\alpha + tg\beta}{ctg\alpha + ctg\beta}$, $ctg\alpha \bullet ctg\beta = \dfrac{ctg\alpha + ctg\beta}{tg\alpha + tg\beta}$ formulalardan ham foydalaniladi.

Misollar:

1. $\cos^2 5+\cos^2 1-\cos 6 \bullet \cos 4$ ni hisoblang.

Yechish: $\cos 6 \bullet \cos 4 = \dfrac{\cos 10+\cos 2}{2} = \dfrac{\cos^2 5-\sin^2 5+\cos^2 1-\sin^2 1}{2}$ ekanligini e'tiborga olamiz.

$$\cos^2 5+\cos^2 1 - \frac{\cos^2 5-\sin^2 5+\cos^2 1-\sin^2 1}{2} =$$

$$= \frac{2\cos^2 5+2\cos^2 1-\cos^2 5+\sin^2 5-\cos^2 1+\sin^2 1}{2} = \frac{\cos^2 5+\cos^2 1+\sin^2 5+\sin^2 1}{2} =$$

$$= \frac{\cos^2 5+\sin^2 5+\cos^2 1+\sin^2 1}{2} = \frac{1+1}{2} = 1.$$

Javob: 1

2. $\sin^4 105^0 \bullet \cos^4 75^0$ ni hisoblang.

Yechish: $\sin^4 105^0 \bullet \cos^4 75^0 = (\sin 105^0 \bullet \cos 75^0)^4 = \left(\dfrac{\sin 180^0 + \sin 30^0}{2}\right)^4 =$

$$= \left(\frac{\frac{1}{2}}{2}\right)^4 = \left(\frac{1}{4}\right)^4 = \frac{1}{256}.$$

Javob: $\dfrac{1}{256}$

3. $8\sin^2\dfrac{7\pi}{8} \bullet \cos^2\dfrac{9\pi}{8}$ ni hisoblang.

Yechish: $8\sin^2\frac{7\pi}{8}\bullet\cos^2\frac{9\pi}{8}=8(\sin\frac{7\pi}{8}\bullet\cos\frac{9\pi}{8})=8\bullet\left(\frac{\sin 2\pi-\sin\frac{\pi}{4}}{2}\right)^2=$

$$=8\bullet\frac{(0-\frac{1}{\sqrt{2}})^2}{4}=2\bullet\frac{1}{2}=1.$$

Javob: 1

4. $\cos 92^0\bullet\cos 2^0+0{,}5\sin 4^0+1$ ni hisoblang.

Yechish: $\cos 92^0\bullet\cos 2^0+0{,}5\sin 4^0+1=\frac{\cos 94^0+\cos 90^0}{2}+\frac{1}{2}\sin 4^0+1=$

$$=\frac{-\sin 4^0+0}{2}+\frac{1}{2}\sin 4^0+1=1.$$

Javob: 1

Trigonometrik ifodalarni soddalashtirishda, ba’zi bir burchaklardagi trigonometrik funksiyalarni qiymatlarini hisoblashda, trigonometrik tenglamalarni va tengsizliklarni yechishda bir xil nomdagi trigonometrik funksiyalar yig’indisi va ayirmasini ko’paytmaga almashtirish formulalaridan foydalanish qulay bo’ladi.

Bu formulalar bundan oldingi paragrafdagi formulalardan osongina keltirib chiqariladi. Masalan: $\sin\alpha\bullet\cos\beta=\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{2}$ yoki $\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\bullet\cos\beta$ formuladagi $\alpha+\beta$ ni x bilan $\alpha-\beta$ ni y bilan almashtiramiz. So’ngra α va β larni x va y lar o rqali ifodalaymiz. Buning uchun $\begin{cases}\alpha+\beta=x\\ \alpha-\beta=y\end{cases}$ sistemani dastlab hadma-had qo’shib, so’ngra hadma-had ayrib $\alpha=\frac{x+y}{2}$ va $\beta=\frac{x-y}{2}$ larni hosil qilamiz. α va β larning bu ifodalarini $\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\bullet\cos\beta$ ga qo’yamiz: Natijada:

$\sin x+\sin y=2\sin\frac{x+y}{2}\bullet\cos\frac{x-y}{2}$ formulani hosil qilamiz. Xuddi, shu kabi

$\sin x-\sin y=2\sin\frac{x-y}{2}\bullet\cos\frac{x+y}{2}$, $\cos x+\cos y=2\cos\frac{x+y}{2}\bullet\cos\frac{x-y}{2}$,

$\cos x - \cos y = -2\sin\frac{\alpha+\beta}{2} \bullet \sin\frac{\alpha-\beta}{2}$ formulalarni hosil qilamiz.

$tg\alpha + tg\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha \bullet \cos\beta} \quad (\alpha \neq \frac{\pi}{2} + k\pi, \;\; \beta \neq \frac{\pi}{2} + k\pi)$.

$tg\alpha - tg\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha \bullet \cos\beta} \quad (\alpha \neq \frac{\pi}{2} + k\pi, \;\; \beta \neq \frac{\pi}{2} + k\pi)$.

$ctg\alpha + ctg\beta = \frac{\sin(\alpha+\beta)}{\sin\alpha \bullet \sin\beta} \quad (\alpha \neq k\pi, \;\; \beta \neq k\pi)$.

$ctg\alpha - ctg\beta = \frac{\sin(\alpha-\beta)}{\sin\alpha \bullet \sin\beta} \quad (\alpha \neq k\pi, \;\; \beta \neq k\pi)$ formulalarni esa tg α va ctg α lar o'rniga ularni sin α va cos α lar orqali ifodalarini qo'yib keltirib chiqariladi.

Ba'zi hollarda asin α +bcos α =r sin($\alpha+\beta$) formuladan foydalanish qulay bo'ladi. Bu yerda $r = \sqrt{a^2+b^2}$, $\cos\varphi = \frac{a}{\sqrt{a^2+b^2}}$, $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$

Misollar:

1. $\sin 10^0 + \sin 50^0 - \cos 20^0$ ni hisoblang.

Yechish: $\sin 10^0 + \sin 50^0 - \cos 20^0 = 2\sin\frac{10^0+50^0}{2} \bullet \cos\frac{10^0-50^0}{2} - \cos 20^0 =$

$= 2\sin 30^0 \bullet \cos 20^0 - \cos 20^0 = 2 \bullet \frac{1}{2} \bullet \cos 20^0 - \cos 20^0 = \cos 20^0 - \cos 20^0 = 0$.

Javob: 0

2. $\frac{\sin 35^0 + \cos 65^0}{2\cos 5^0}$ ni hisoblang.

Yechish: $\frac{\sin 35^0 + \cos 65^0}{2\cos 5^0} = \frac{\cos 55^0 + \cos 65^0}{2\cos 5^0} = \frac{2\cos\frac{55^0+65^0}{2} \bullet \cos\frac{55^0-65^0}{2}}{2\cos 5^0} =$

$= \frac{2\cos 60^0 \bullet \cos 5^0}{2\cos 5^0} = \cos 60^0 = \frac{1}{2}$.

Javob: $\frac{1}{2}$

3. $\left(\frac{\sin 100^0 + \sin 20^0}{\sin 50^0}\right)^2$ ni hisoblang.

Yechish: $\left(\dfrac{\sin 100^0+\sin 20^0}{\sin 50^0}\right)^2=\left(\dfrac{2\sin\dfrac{100^0+20^0}{2}\bullet\cos\dfrac{100^0-20^0}{2}}{\sin 50^0}\right)^2=$

$=\left(\dfrac{2\sin 60^0\bullet\cos 40^0}{\sin 50^0}\right)^2=\left(\dfrac{2\bullet\dfrac{\sqrt{3}}{2}\bullet\sin 50^0}{\sin 50^0}\right)^2=(\sqrt{3})^2=3.$

Javob: 3

4. $\dfrac{\sin 2\alpha+2\sin\alpha\bullet\cos 2\alpha}{1+\cos\alpha+\cos 2\alpha+\cos 3\alpha}$ ni soddalashtiring.

Yechish: $\dfrac{\sin 2\alpha+2\sin\alpha\bullet\cos 2\alpha}{1+\cos\alpha+\cos 2\alpha+\cos 3\alpha}=\dfrac{2\sin\alpha\bullet\cos\alpha+2\sin\alpha\bullet\cos 2\alpha}{1+\cos 2\alpha+\cos\alpha+\cos 3\alpha}=$

$=\dfrac{2\sin\alpha(\cos\alpha+\cos 2\alpha)}{2\cos^2\alpha+2\cos 2\alpha\bullet\cos\alpha}=\dfrac{2\sin\alpha(\cos\alpha+\cos 2\alpha)}{2\cos\alpha(\cos\alpha+\cos 2\alpha)}=\text{tg}\,\alpha$.

Javob: tg α

5. tg15^0-ctg15^0 ni hisoblang.

Yechish: tg15^0-ctg15^0=tg15^0-tg75^0= $\dfrac{\sin(15^0-75^0)}{\cos 15^0\bullet\cos 75^0}=\dfrac{-\sin 60^0}{\dfrac{1}{2}(\cos 90^0+\cos 60^0)}=$

$=\dfrac{-\dfrac{\sqrt{3}}{2}}{\dfrac{1}{2}(0+\dfrac{1}{2})}=-\dfrac{\sqrt{3}}{\dfrac{1}{2}}=-2\sqrt{3}$.

Javob: $-2\sqrt{3}$

6. ctg2 α -ctg α ni soddalashtiring.

Yechish: ctg2 α -ctg α $=\dfrac{\sin(\alpha-2\alpha)}{\sin 2\alpha\bullet\sin\alpha}=-\dfrac{\sin\alpha}{\sin 2\alpha\bullet\sin\alpha}=-\dfrac{1}{\sin 2\alpha}.$

Javob: $-\dfrac{1}{\sin 2\alpha}$

§7. Yarim burchak va butun burchak trigonometrik funksiyalari orasidagi bog'lanish. Trigonometrik funksiyalarni birini qolganlari orqali ifodalash

Ko'p hollarda α burchakning trigonometrik funksiyalarini bilgan holda $\frac{\alpha}{2}$ burchakning trigonometrik funksiyalarini aniqlashga to'g'ri keladi. Bunda quyidagi formulalardan foydalaniladi: $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$; $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$;

$tg\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$; $ctg\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$; $tg\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$; $tg\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$.

Bulardan dastlabki ikkitasi cos2 α =1-2sin^2 α va cos2 α =2cos^2 α -1 formulalardan keltirib chiqariladi. Keyingi ikkitasi esa tangens va kotangenslarni sinus va kosinuslar orqali ifodalaridan keltirib chiqariladi.

Oxirgi formulalarni keltirib chiqaramiz:

$$tg\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{2\sin\frac{\alpha}{2}\bullet\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}} = \frac{\sin\alpha}{1+\cos\alpha};$$

$$tg\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{2\sin\frac{\alpha}{2}\bullet\cos\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\bullet\cos\frac{\alpha}{2}} = \frac{2\sin^2\frac{\alpha}{2}}{\sin\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$$

Dastlabki to'rtta formulalardagi ± ishoralardan qaysi birini olinishi $\frac{\alpha}{2}$ burchakni qaysi chorakda yotishiga bog'liq bo'ladi.

Misollar:

1. $\sin\frac{5\pi}{12}$ ni hisoblang.

Yechish: $\sin\frac{5\pi}{12} = \sqrt{\frac{1-\cos 2\bullet\frac{5\pi}{12}}{2}} = \sqrt{\frac{1-\cos\frac{5\pi}{6}}{2}} = \sqrt{\frac{1-\cos(\pi-\frac{\pi}{6})}{2}} =$

$$= \sqrt{\frac{1+\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{1}{2}\sqrt{2+\sqrt{3}}\,.$$

Javob: $\frac{1}{2}\sqrt{2+\sqrt{3}}$

2. $\cos 2227^0\ 30^1$ ni hisoblang.

Yechish: $\cos 2227^0\ 30^1=\cos(6\bullet 360^0+67^0 30^1)=\cos 67^0 30^1=\sqrt{\frac{1+\cos 135^0}{2}}=$

$$=\sqrt{\frac{1+\cos(90^0+45^0)}{2}}=\sqrt{\frac{1-\sin 45^0}{2}}=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}=\sqrt{\frac{2-\sqrt{2}}{4}}=\frac{1}{2}\sqrt{2-\sqrt{2}}.$$

Javob: $\frac{1}{2}\sqrt{2-\sqrt{2}}$

3. $\sin^4 15^0+\cos^4 15^0$ ni hisoblang.

Yechish: $\sin^4 15^0+\cos^4 15^0=\left(\sqrt{\frac{1-\cos 30^0}{2}}\right)^4+\left(\sqrt{\frac{1+\cos 30^0}{2}}\right)^4=$

$$=\left(\frac{1-\cos 30^0}{2}\right)^2+\left(\frac{1+\cos 30^0}{2}\right)^2=\left(\frac{1-\frac{\sqrt{3}}{2}}{2}\right)^2+\left(\frac{1+\frac{\sqrt{3}}{2}}{2}\right)^2=\left(\frac{2-\sqrt{3}}{4}\right)^2+\left(\frac{2+\sqrt{3}}{4}\right)^2=$$

$$=\frac{(2-\sqrt{3})^2+(2+\sqrt{3})^2}{16}=\frac{4-4\sqrt{3}+3+4+4\sqrt{3}+3}{16}=\frac{14}{16}=\frac{7}{8}.$$

Javob: $\frac{7}{8}$

4. $\dfrac{tg\alpha+\sin\alpha}{2\cos^2\frac{\alpha}{2}}$ ni soddalashtiring.

Yechish: $\dfrac{tg\alpha+\sin\alpha}{2\cos^2\frac{\alpha}{2}}=\dfrac{\frac{\sin\alpha}{\cos\alpha}+\sin\alpha}{2\cos^2\frac{\alpha}{2}}=\dfrac{\frac{\sin\alpha+\sin\alpha\bullet\cos\alpha}{\cos\alpha}}{2\cos^2\frac{\alpha}{2}}=\dfrac{\sin\alpha(1+\cos\alpha)}{2\cos^2\frac{\alpha}{2}\bullet\cos\alpha}=$

$$=\frac{\sin\alpha\bullet 2\cos^2\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}\bullet\cos\alpha}=\frac{\sin\alpha}{\cos\alpha}=\mathrm{tg}\ \alpha.$$

Javob: tg α

Trigonometrik ifodalarni soddalashtirishda, trigonometrik funksiyalarni qiymatlarini hisoblashda, trigonometrik tenglamalar va tengsizliklarni yechishda ko'pincha sin α , cos α , tg α va ctg α larni $tg\frac{\alpha}{2}$ orqali ifodalaridan, ya'ni quyidagi formulalardan foydalanish qulay bo'ladi:

$$\sin\alpha=\frac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}};\ \cos\alpha=\frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}};\ tg\alpha=\frac{2tg\frac{\alpha}{2}}{1-tg^2\frac{\alpha}{2}};\ ctg\alpha=\frac{1-tg^2\frac{\alpha}{2}}{2tg\frac{\alpha}{2}}.$$

Bu formulalarni keltirib chiqarishda ikkilangan burchak trigonometrik funksiyalari uchun hosil qilingan formulalardan foydalaniladi.

Masalan, $\sin\alpha=\frac{\sin\alpha}{1}=\frac{2\sin\frac{\alpha}{2}\bullet\cos\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}}=\frac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}.$

$$\cos\alpha=\frac{\cos\alpha}{1}=\frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}}=\frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}.$$

Agar yuqoridagi formulalarda α ni 2 α bilan almashtirsak, quyidagi formulalar hosil bo'ladi: $\sin 2\alpha=\frac{2tg\alpha}{1+tg^2\alpha}$; $\cos 2\alpha=\frac{1-tg^2\alpha}{1+tg^2\alpha}$; $tg2\alpha=\frac{2tg\alpha}{1-tg^2\alpha}$;

$ctg2\alpha=\frac{1-tg^2\alpha}{2tg\alpha}$.

Misollar:

1. Agar tg α =0,2 bo'lsa, $\frac{2}{3+4\cos 2\alpha}$ ning qiymatini toping.

Yechish: cos2 α ni tg α orqali ifodalash formulasidan foydalanamiz.

$\cos 2\alpha=\frac{1-tg^2\alpha}{1+tg^2\alpha}=\frac{1-0{,}04}{1+0{,}04}=\frac{0{,}96}{1{,}04}=\frac{96}{104}=\frac{12}{13}$; $\frac{2}{3+4\cos 2\alpha}=\frac{2}{3+4\bullet\frac{12}{13}}=\frac{2}{3+\frac{48}{13}}=$

$\frac{2}{\frac{39+48}{13}}=\frac{26}{87}.$

Javob: $\frac{26}{87}$

2. Agar $ctg\alpha=\sqrt{3}$ bo'lsa, $\frac{9}{\sin^4\alpha+\cos^4\alpha}$ ni hisoblang.

Yechish: $\sin^4\alpha+\cos^4\alpha=(\sin^2\alpha+\cos^2\alpha)^2-2\sin^2\alpha\bullet\cos^2\alpha=1-\frac{1}{2}\sin^2 2\alpha$.

$tg\alpha=\frac{1}{ctg\alpha}=\frac{1}{\sqrt{3}}$. $\sin 2\alpha$ ni $tg\,\alpha$ orqali ifodasidan foydalanamiz va $\sin 2\alpha$ ni qiymatini topamiz.

$$\sin 2\alpha=\frac{2tg\alpha}{1+tg^2\alpha}=\frac{2\bullet\frac{1}{\sqrt{3}}}{1+(\frac{1}{\sqrt{3}})^2}=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{6}{4\sqrt{3}}=\frac{3}{2\sqrt{3}}=\frac{\sqrt{3}}{2}.$$

$$\frac{9}{\sin^4\alpha+\cos^4\alpha}=\frac{9}{1-\frac{1}{2}\sin^2 2\alpha}=\frac{9}{1-\frac{1}{2}\bullet\frac{3}{4}}=\frac{9}{1-\frac{3}{8}}=\frac{9}{\frac{5}{8}}=\frac{72}{5}=14{,}4.$$

Javob: 14,4

Ko’p hollarda trigonometrik Funksiyalardan birini qolganlari orqali ifodalash formulalaridan foydalaniladi. Bu formulalarni asosiy trigonometrik ayniyatlardan keltirib chiqariladi. Masalan, $\sin\alpha$ ni $\cos\alpha$ orqali va $\cos\alpha$ ni $\sin\alpha$ orqali ifodasi $\sin^2\alpha+\cos^2\alpha=1$ dan keltirib chiqariladi. Ya’ni, $\sin\alpha=\pm\sqrt{1-\cos^2\alpha}$ va $\cos\alpha=\pm\sqrt{1-\sin^2\alpha}$.

$\sin\alpha$ ni $tg\,\alpha$ orqali ifodasi $tg\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\sin\alpha}{\pm\sqrt{1-\sin^2\alpha}}$ ni $\sin\alpha$ ga nisbatan yechib keltirib chiqariladi. Ularni barchasini quyidagi jadvalda keltiramiz:

Ifodalanishi kerak bo’lgan funksiya / Funksiya	$\sin\alpha$	$\cos\alpha$	$tg\,\alpha$	$ctg\,\alpha$
$\sin\alpha$	$\sin\alpha$	$\pm\sqrt{1-\cos^2\alpha}$	$\pm\frac{tg\alpha}{\sqrt{1+tg^2\alpha}}$	$\pm\frac{1}{\sqrt{1+ctg^2\alpha}}$
$\cos\alpha$	$\pm\sqrt{1-\sin^2\alpha}$	$\cos\alpha$	$\pm\frac{1}{\sqrt{1+tg^2\alpha}}$	$\pm\frac{ctg\alpha}{\sqrt{1+ctg^2\alpha}}$
$tg\,\alpha$	$\pm\frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}$	$\pm\frac{\sqrt{1-\cos^2\alpha}}{\cos\alpha}$	$tg\,\alpha$	$\frac{1}{ctg\alpha}$
$ctg\,\alpha$	$\pm\frac{\sqrt{1-\sin^2\alpha}}{\sin\alpha}$	$\pm\frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$	$\frac{1}{tg\alpha}$	$ctg\,\alpha$

Misollar:

1. Agar ctg α =2 bo'lsa, $\dfrac{\sin^2\alpha - 2\cos^2\alpha}{3\sin\alpha \bullet \cos\alpha + \cos^2\alpha}$ ni qiymatini toping.

Yechish: sin α va cos α ning stg α orqali ifodasidan foydalanamiz.

$$\frac{\sin^2\alpha - 2\cos^2\alpha}{3\sin\alpha \bullet \cos\alpha + \cos^2\alpha} = \frac{\dfrac{1}{1+ctg^2\alpha} - 2 \bullet \dfrac{ctg^2\alpha}{1+ctg^2\alpha}}{3 \bullet \dfrac{1}{\sqrt{1+ctg^2\alpha}} \bullet \dfrac{ctg\alpha}{\sqrt{1+ctg^2\alpha}} + \dfrac{ctg^2\alpha}{1+ctg^2\alpha}} = \frac{1-2ctg^2\alpha}{3ctg\alpha + ctg^2\alpha} =$$

$$= \frac{1-8}{3 \bullet 2 + 4} = -\frac{7}{10} = -0{,}7 .$$

Javob: -0,7

2. Agar $0 < \alpha < \dfrac{\pi}{2}$ va tg α =2 bo'lsa, cos α ni hisoblang.

Yechish: cos α ni 1-chorakda musbatligini e'tiborga olamiz va uni tg α orqali ifodasidan foydalanamiz. $\cos\alpha = \pm\dfrac{1}{\sqrt{1+tg^2\alpha}} = \dfrac{1}{\sqrt{1+4}} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$.

Javob: $\dfrac{\sqrt{5}}{5}$

3. Agar $tg\alpha = -\dfrac{3}{4}$ va $\dfrac{\pi}{2} < \alpha < \pi$ bo'lsa, sin α -cos α ning qiymatini toping.

Yechish: sin α va cos α larni tg α orqali ifodalaridan foydalanamiz hamda sin α ni 2-chorakda musbat va cos α ni manfiy bo'lishini hisobga olamiz.

$$\sin\alpha = \frac{tg\alpha}{\sqrt{1+tg^2\alpha}} = \frac{-\dfrac{3}{4}}{\sqrt{1+\dfrac{9}{16}}} = -\frac{\dfrac{3}{4}}{\dfrac{5}{4}} = -\frac{3}{5} = -0{,}6 .$$

$$\cos\alpha = -\frac{1}{\sqrt{1+tg^2\alpha}} = -\frac{1}{\sqrt{1+\dfrac{9}{16}}} = -\frac{1}{\dfrac{5}{4}} = -\frac{4}{5} = -0{,}8 .$$

sin α -cos α =-0,6+0,8=0,2=$\dfrac{1}{5}$.

Javob: $\dfrac{1}{5}$

II. BOB. TRIGONOMETRIK FUNKSIYALARNING XOSSALARI

§1. $y = sinx$ funksiyaning xossalari

Trigonometrik tenglamalar va tengsizliklarni yechishda va Funksiyalarni tekshirishda trigonometrik Funksiyalarning xossalarini bilish muhim ahamiyatga ega.

1. Funksiyaning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iborat.

2. Funksiyaning qiymatlar sohasi [-1;1] kesmadan iborat. Demak, y=sinx Funksiya chegaralangan.

3. Funksiya toq. Chunki sin(-x)=-sinx $(x \in R)$.

4. Funksiya davriy bo'lib, uning eng kichik musbat davri 2π ga teng. Ya'ni $x \in R$ lar uchun sin(x+2 π)=sinx.

5. x=k π, $k \in Z$ da sinx=0.

6. $x \in (2k\pi; \pi + 2k\pi)$, $k \in Z$ da sinx>0.

7. $x \in (\pi + 2k\pi; 2\pi + 2k\pi)$, $k \in Z$ da sinx<0.

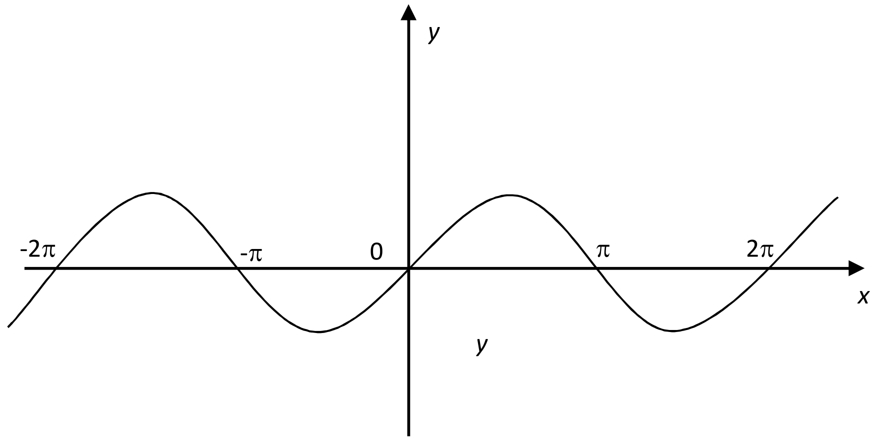
8. Funksiya $\left[-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right]$, $k \in Z$ kesmada –1 dan 1 gacha o'sadi.

9. Funksiya $\left[\frac{\pi}{2} + 2k\pi; \frac{3\pi}{2} + 2k\pi\right]$, $k \in Z$ kesmada 1 dan -1 gacha kamayadi.

10. Funksiya $x = \frac{\pi}{2} + k\pi$, $k \in Z$ nuqtalarda 1 ga teng eng katta qiymatga erishadi.

11. Funksiya $x = \frac{3\pi}{2} + 2k\pi$, $k \in Z$ nuqtalarda -1 ga teng eng kichik qiymatga erishadi.

y=sinx funksiyaning yuqoridagi xossalariga asoslanib $[-\pi; \pi]$ kesmada, ya'ni uzunligi 2π ga teng kesmada uni davriyligini e'tiborga olib esa butun sonlar to'g'ri chizig'ida grafigini yasash mumkin.



Misollar:

1. $2\sin^2x+\cos^2x$ ning eng katta qiymatini toping.

Yechish: $2\sin^2x+\cos^2x=\sin^2x+\cos^2x+\sin^2x=1+\sin^2x \le 2$. sinx ning eng katta qiymati 1 ga teng bo'lgani uchun $\sin^2x$ ning eng katta qiymati ham 1 ga teng bo'ladi.

Javob: 2

2. $2\sin^2\beta+\cos^2\beta$ ning eng kichik qiymatini toping.

Yechish: $2\sin^2\beta+\cos^2\beta \ =\sin^2\beta+\cos^2\beta+\sin^2\beta= \ 1+\sin^2\beta\ge 1$. Bu holda $\sin^2\beta$ ning eng kichik qiymati nolga teng.

Javob: 1

3. $\sin\frac{x}{2}\bullet\cos^3\frac{x}{2}-\sin^3\frac{x}{2}\bullet\cos\frac{x}{2}$ ifodaning eng katta qiymatini aniqlang.

Yechish: $\sin\frac{x}{2}\bullet\cos^3\frac{x}{2}-\sin^3\frac{x}{2}\bullet\cos\frac{x}{2} \ =\sin\frac{x}{2}\bullet\cos\frac{x}{2}(\cos^2\frac{x}{2}-\sin^2\frac{x}{2})=$

$=\frac{1}{2}\sin x\bullet\cos x=\frac{1}{4}\sin 2x\le\frac{1}{4}$. Chunki sin2x ning eng katta qiymati 1 ga teng.

Javob: $\frac{1}{4}$

4. $\sin^4\alpha+\cos^4\alpha$ ning eng kichik qiymatini toping.

Yechish: $\sin^4\alpha+\cos^4\alpha=(\sin^2\alpha+\cos^2\alpha)-2\sin^2\alpha\bullet\cos^2\alpha=1-\frac{1}{2}\sin^2 2\alpha$. Ayirma eng kichik qiymatga ega bo'lishi uchun ayriluvchi eng katta qiymatga ega bo’lishi kerak. Ayriluvchining eng katta qiymati esa $\frac{1}{2}$ ga teng. Demak

$1-\frac{1}{2}\sin^2 2\alpha \geq \frac{1}{2}$

Javob: $\frac{1}{2}$

5. Agar α -o’zgaruvchi miqdor bo’lsa 4 $(\sqrt{3}\cos\alpha+\sin\alpha)$ ning eng katta qiymati qancha ga teng bo’ladi.

Yechish: 4 $(\sqrt{3}\cos\alpha+\sin\alpha)$ $=8\left(\frac{\sqrt{3}}{2}\cos\alpha+\frac{1}{2}\sin\alpha\right)=8(\sin60^0 \bullet \cos\alpha+\cos60^0 \bullet$

$\bullet \sin\alpha)=8\sin(60^0+\alpha)\leq 8$. Chunki, $\sin(60^0+\alpha)$ ning eng katta qiymati 1 ga teng.

Javob: 8

6. $y=\sqrt{\sin x}+\sqrt{16-x^2}$ funksiyaning aniqlanish sohasiga tegishli x ning butun qiymatlari nechta?

Yechish: $\begin{cases} \sin x \geq 0 \\ 16-x^2 \geq 0 \end{cases}$; $\begin{cases} 2k\pi \leq x \leq \pi+2k\pi \\ x^2 \leq 16 \end{cases}$; $\begin{cases} 2k\pi \leq x \leq \pi+2k\pi \\ -4 \leq x \leq 4 \end{cases}$ oxirgi sistemaning birinchi tengsizligidan k=0 da $0 \leq x \leq \pi$ kelib chiqadi va u $-4 \leq x \leq 4$ ga tegishli bo’ladi. k ning noldan farq li qiymatlarida birinchi tengsizlikdan hosil bo’lgan kesma [-4;4] ga tegishli bo’lmaydi. Demak, k=0.

Javob: 1 ta

7. $y=\sqrt{2\sin x-1}$ funksiyaning aniqlanish sohasini toping

Yechish: $2\sin x-1 \geq 0$, $\sin x \geq \frac{1}{2}$, $\frac{\pi}{6}+2k\pi \leq x < \frac{5\pi}{6}+2k\pi$, $k \in Z$.

Javob: $\left[\frac{\pi}{6}+2k\pi;\frac{5\pi}{6}+2k\pi\right]$, $k \in Z$.

8. y=sin(3x+1) funksiyani davrini toping.

Yechish: y=sin(3x+1)=sin[3(x+$\frac{2\pi}{3}$)+1]

Javob: $\frac{2\pi}{3}$

9. $f(x)=\frac{\sin 2x}{\cos x}$ funksiyaning qiymatlar sohasini toping.

Yechish: $f(x)=\frac{\sin 2x}{\cos x}=\frac{2\sin x\bullet\cos x}{\cos x}=2\sin x$ sinx ning qiymatlar sohasi [-1;1] kesmadan iborat bo'lgani uchun 2sinx ning qiymatlar sohasi [-2;2] dan iborat bo'ladi. Lekin sinx -1 va 1 qiymatlarni qabul qil ganda cosx nolga aylanadi. Shuning uchun $f(x)=\frac{\sin 2x}{\cos x}$ ning qiymatlar sohasi (-2;2) dan iborat bo'ladi.

Javob: (-2;2)

§2. $y=cosx$ funksiyaning xossalari

1. Funksiyaning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iborat.

2. Funksiyaning qiymatlar sohasi [-1;1] kesmadan iborat. Demak, Funksiya chegaralangan.

3. Funksiya juft, Chunki cos(-x)=cosx

4. Funksiya davriy bo'lib, uning eng kichik musbat davri 2 π ga teng. Chunki, cos(x+2 π)=cosx

5. Barcha $x=\frac{\pi}{2}+k\pi$, $k\in Z$ larda cosx=0.

6. x ning $(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi)$, $k\in Z$ dagi barcha qiymatlarida cosx>0.

7. x ning $(\frac{\pi}{2}+2k\pi;\ \frac{3\pi}{2}+2k\pi)$, $k\in Z$ dagi barcha qiymatlarida cosx<0.

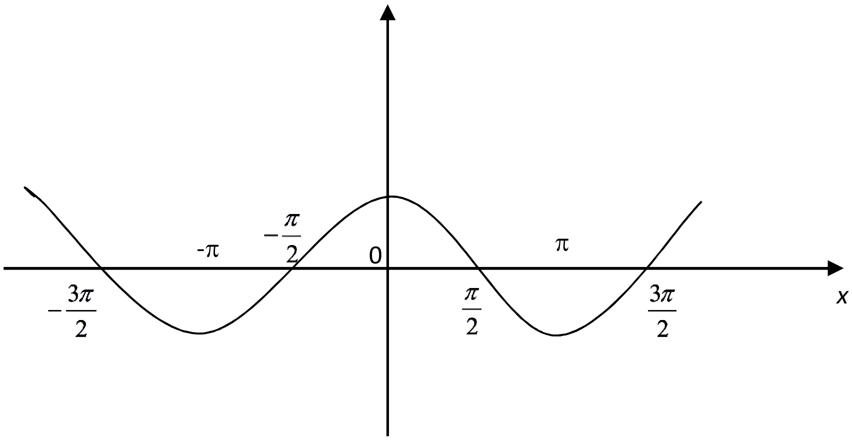
8. Funksiya $(2k\pi;\ \pi+2k\pi)$, $k\in Z$ da 1 dan -1 gacha kamayadi.

9. Funksiya $(-\pi+2k\pi;\ 2k\pi)$, $k\in Z$ da -1 dan 1 gacha o'sadi.

10. Funksiya $x=2k\pi$, $k\in Z$ nuqtalarda 1 ga teng eng katta qiymatni qabul qiladi .

11. Funksiya $x=\pi+2k\pi$, $k\in Z$ nuqtalarda -1 ga teng eng kichik qiymatni qabul qiladi.

y=cosx ning bu xossalaridan foydalanib dastlab uni grafigini $[-\pi;\pi]$ da so'ngra butun sonlar to'g'ri chizig'ida yasash mumkin.



Misollar:

1. $\sin^2\alpha+2\cos^2\alpha$ ning eng katta qiymatini toping.

Yechish: $\sin^2\alpha+2\cos^2\alpha=\sin^2\alpha+\cos^2\alpha+\cos^2\alpha=1+\cos^2\alpha\le 2$. Chunki $\cos^2\alpha$ ning eng katta qiymati 1 ga teng.

Javob: 2

2. $13\sin^2 5x+17\cos^2 5x$ ifodaning eng kichik qiymatini toping.

Yechish: $13\sin^2 5x+17\cos^2 5x=13\sin^2 5x+13\cos^2 5x+4\cos^2 5x=$ $=13(\sin^2 5x+\cos^2 5x)+4\cos^2 5x=13+4\cos^2 5x\ge 13$. Chunki $4\cos^2 5x$ ning eng kichik qiymati 0 ga teng.

Javob: 13

3. $\frac{10}{x^2+8x+41}+\cos 5y$ ning eng katta qiymati nechaga teng bo'lishi mumkin?

Yechish: Yig'indi eng katta qiymatga ega bo'lishi uchun har bir qo'shiluvchi eng katta qiymatga ega bo'lishi kerak. Birinchi qo'shiluvchi kasr bo'lganligi uchun u maxrajining eng kichik qiymatida eng katta bo'ladi. $x^2+8x+41$ uchhadning eng kichik qiymatini aniqlaymiz. $x^2+8x+41=(x+4)^2+25\ge 25$.

Demak, birinchi qo'shiluvchining eng katta qiymati $\frac{10}{25}=\frac{2}{5}$ ga teng ekan.

Ikkinchi qo'shiluvchining eng katta qiymati esa 1 ga teng. Shunday qilib berilgan ifodaning eng katta qiymati $1+\frac{2}{5}=1\frac{2}{5}=1{,}4$ ga teng .

Javob: 1,4

4. $y=\sqrt{1-2\cos^2 x}$ funksiyaning aniqlanish sohasini toping.

Yechish: 1-2cos^{2}x ≥ 0, 1-(1+cos2x) ≥ 0, 1-1-cos2x ≥ 0, -cos2x ≥ 0, cos2x ≤ 0. Bundan $\frac{\pi}{2}+2k\pi \leq 2x \leq \frac{3\pi}{2}+2k\pi$ yoki $\frac{\pi}{4}+k\pi \leq x \leq \frac{3\pi}{4}+k\pi$, $k \in Z$ kelib chiqadi.

Javob: $[\frac{\pi}{4}+k\pi; \frac{3\pi}{4}+k\pi]$, $k \in Z$.

5. $y=\cos(\frac{5}{2}x-\frac{5}{2})$ funksiyaning eng kichik musbat davrini toping.

Yechish: $y=\cos(\frac{5}{2}x-\frac{5}{2}) = \cos\left[\frac{5}{2}(x+\frac{4}{5}\pi)-\frac{5}{2}\right]$

Demak, berilgan funksiyaning eng kichik musbat davri $\frac{4\pi}{5}$ ga teng ekan.

Javob: $\frac{4\pi}{5}$

6. $f(x)=2\cos\frac{x}{2}+3$ funksiyaning qiymatlar sohasini toping.

Yechish: $-1 \leq \cos\frac{x}{2} \leq 1$, $- \leq 2\cos\frac{x}{2} \leq 2$ bo'lgani uchun $-2+3 \leq 2\cos\frac{x}{2}+3 \leq 2+3$ yoki $1 \leq 2\cos\frac{x}{2}+3 \leq 5$

Javob: [1;5]

7. $y=2\cos^2\frac{x}{2}+tgx \bullet ctgx$ funksiyaning qiymatlari to'plamini toping.

Yechish: $y=2\cos^2\frac{x}{2}+tgx \bullet ctgx$ =1+cosx+1=2+cosx. $-1 \leq \cos x \leq 1$ bo'lgani uchun $-1+2 \leq 2+\cos x \leq 1+2$ yoki $1 \leq 2+\cos x \leq 3$

Javob: [1;3]

8. f(x)=6cosx-7 funksiyaning eng katta qiymatini toping.

Yechish: f(x)=6cosx-7 eng katta qiymatga ega bo'lishi uchun 6cosx eng katta qiymatga ega bo'lishi kerak. 6cosx ning eng katta qiymati 6 ga teng. Demak, f(x)=6cosx-7 ning eng katta qiymati 6-7=-1 ga teng.

Javob: -1

§3. $y = tgx$ funksiyaning xossalari

1. Funksiyaning aniqlanish sohasi $x = \frac{\pi}{2} + k\pi$, $k \in Z$ dan farqli barcha haqiqiy sonlar to'plamidan iborat.

2. Funksiyaning qiymatlar to'plami barcha haqiqiy sonlar to'plamidan iborat.

3. Funksiya toq, chunki aniqlanish sohasidan olingan barcha x lar uchun tg(-x)=-tgx.

4. Funksiya davriy bo'lib, uni eng kichik musbat davri π ga teng. Chunki tg(x+ π)=tgx.

5. Barcha x=k π , $k \in Z$ nuqtalarda tgx=0.

6. $(k\pi; \frac{\pi}{2} + k\pi)$, $k \in Z$ dan olingan barcha nuqtalarda tgx>0.

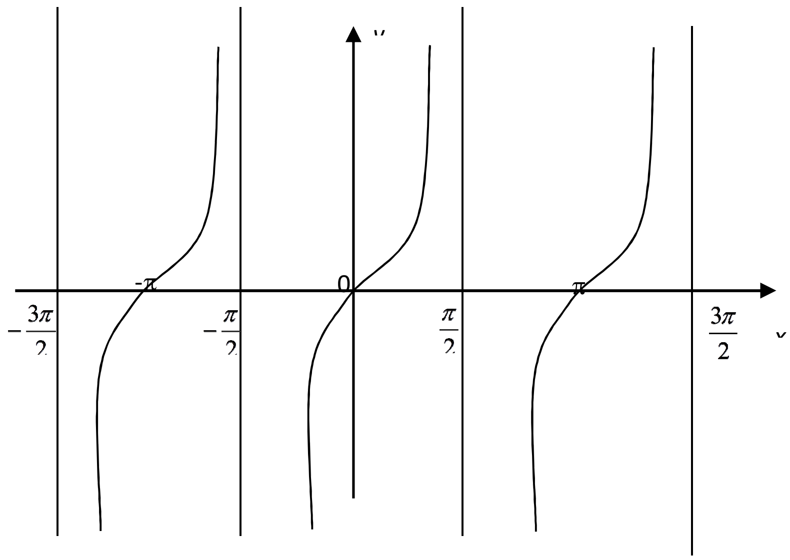
7. $(-\frac{\pi}{2} + k\pi, k\pi)$, $k \in Z$ dan olingan barcha nuqtalarda tgx<0.

8. Funksiya $(-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi)$, $k \in Z$ oraliqda o'suvchidir. Yuqoridagi xossalarga asoslanib dastlab, $(-\frac{\pi}{2}; \frac{\pi}{2})$ oraliqda, so'ngra butun sonlar o'qida y=tgx funksiyani grafigini yasash mumkin.

Misollar:

1. $\frac{5}{tg^2\alpha + ctg^2\alpha} + \frac{5\sin 2\alpha - \cos\gamma}{5 + \cos t}$ ifodaning eng katta qiymatini toping.

Yechish: Yig'indi eng katta qiymatga ega bo'lishi uchun har bir qo'shiluvchi eng katta qiymatga ega bo'lishi kerak. Birinchi qo'shiluvchi eng katta qiymatga tg^2 α +ctg^2 α ning eng kichik qiymatida erishadi. Uning eng kichik qiymati esa 2 ga teng. Chunki tg^2 α +ctg^2 α =tg^2 $\alpha + \frac{1}{tg^2\alpha} \geq 2$. (Har qanday o'zaro teskari sonlar yig'indisi 2 dan kichik emas).



Demak, birinchi qo'shiluvchining eng katta qiymati $\frac{5}{2}=2{,}5$ ga teng.

Ikkinchi qo'shiluvchi sin2α ning 1 ga teng eng katta, cosγ ning –1 ga teng eng kichik va cost ning –1 ga teng eng kichik qiymatlarida eng katta bo'ladi. Demak, $\frac{5\sin 2\alpha-\cos\gamma}{5+\cos t}=\frac{5\bullet 1-(-1)}{5+(-1)}=\frac{5+1}{5-1}=\frac{6}{4}=1{,}5$

Shunday qil ib berilgan ifodaning eng katta qiymati 2,5+1,5=4 ga teng.

Javob: 4.

2. $tg^{100}x+ctg^{100}x$ yig'indining eng kichik qiymatini toping.

Yechish: $tg^{100}x+ctg^{100}x=tg^{100}x+\frac{1}{tg^{100}x}$

Bu o'zaro teskari sonlar yig'indisidir. O'zaro teskari sonlar yig'indisi esa har doim 2 dan kichik emas. Demak, berilgan yig'indining eng kichik qiymati 2 ga teng.

Javob: 2

3. $y=\sqrt{tgx+1}$ funksiyaning aniqlanish sohasini toping.

Yechish: $tgx+1\geq 0$, $tgx\geq -1$, $-\frac{\pi}{4}+k\pi\leq x<\frac{\pi}{2}+k\pi$, $k\in Z$

Javob: $\left[-\frac{\pi}{4}+k\pi;\frac{\pi}{2}+k\pi\right)$, $k\in Z$

4. $y=tg\frac{x}{2}-2\sin\frac{x}{2}+3\cos\frac{2}{3}x$ funksiyaning eng kichik davrini toping.

Yechish: Berilgan funksiyaning eng kichik davri $y_1 = tg\frac{x}{2}$, $y_2 = 2\sin\frac{x}{2}$ va $y_3 = 3\cos\frac{2}{3}x$ funksiyalar eng kichik davrlarini eng kichik umumiy karralisidan iborat. Har bir Funksiyaning eng kichik davrini topamiz:

$y_1 = tg\frac{x}{2} = tg\left[\frac{1}{2}(x+2k)\right]$, Demak, $T_1 = 2\pi$.

$y_2 = 2\sin\frac{x}{2} = 2\sin\left[\frac{1}{2}(x+4\pi)\right]$, demak, $T_2 = 4\pi$.

$y_3 = 3\cos\frac{2}{3}x = 3\cos\left[\frac{2}{3}(x+3\pi)\right]$, demak $T_3 = 3\pi$.

$T_1 = 2\pi$, $T_2 = 4\pi$ va $T_3 = 3\pi$ larning eng kichik umumiy karralisi 12 π ga teng

Javob: 12 π

5. u=2^{tgx} funksiya grafigining 0y o'qi bilan kesishish nuqtasi ordinatasini toping.

Yechish: Berilgan Funksiya grafigining 0u o'qi bilan kesishish nuqtasining abstsissasi nolga teng bo'lishi kerak. Demak, y=2^{tg0}=2^0=1

Javob: 1

§4. $y = ctgx$ funksiyaning xossalari

1. Funksiyaning aniqlanish sohasi $x = k\pi$, $k \in Z$ dan farqli barcha haqiqiy sonlar to'plamidan iborat.

2. Funksiyaning qiymatlar sohasi sonlar o'qining barcha nuqtalari to'plamidan iborat. Ya'ni funksiya chegaralanmagan.

3. Funksiya toq, Chunki aniqlanish sohasidan olingan barcha x larda ctg(-x)=-ctgx

4. Funksiya π davrli davriy Funksiyadir. Chunki aniqlanish sohasidan olingan barcha x larda ctg(x+ π)=ctgx

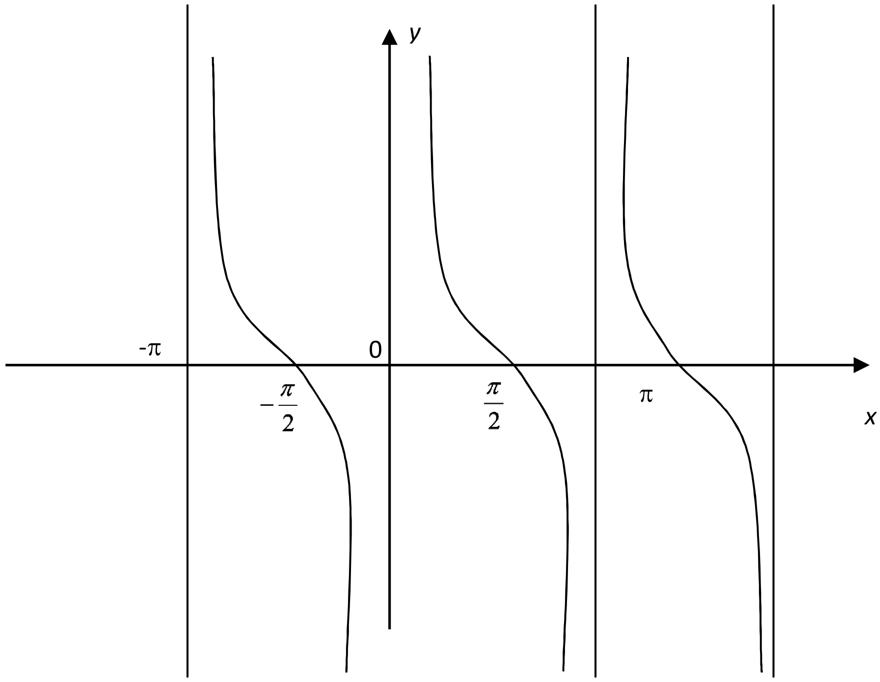
5. $x = \frac{\pi}{2} + k\pi$, $k \in Z$ nuqtalarda ctgx=0.

6. $x \in (k\pi; \frac{\pi}{2} + k\pi)$, $k \in Z$ nuqtalarda ctgx>0.

7. $x \in (-\frac{\pi}{2} + k\pi; k\pi)$, $k \in Z$ nuqtalarda ctgx<0.

8. Funksiya $(k\pi; \pi + k\pi)$ oraliqlarda kamayuvchidir.

y=ctgx Funksiyaning yuqoridagi xossalaridan foydalanib dastlab $(0; \pi)$ oraliqda so'ngra butun koordinatalar to'g'ri chizig'ida kotangensni grafigini yasash mumkin.



Misollar:

1. y=tg3x+ctg2x funksiya x ning qanday qiymatlarida aniqlanmagan.

Yechish: Dastlab y_1=tg3x va y_2=ctg2x funksiyalar aniqlanmagan nuqtalarni topamiz. y_1=tg3x Funksiya $3x = \frac{\pi}{2} + k\pi$, $x = \frac{\pi}{6} + \frac{k\pi}{3}$, $k \in Z$ nuqtalarda aniqlanmagan.

y_2=ctg2x funksiya esa $2x = k\pi$, $k \in Z$, $x = \frac{k\pi}{2}$, $k \in Z$ nuqtalarda aniqlanmagan.

$x = \frac{\pi}{6} + \frac{k\pi}{3}$, $k \in Z$ va $x = \frac{k\pi}{2}$, $k \in Z$ lardan birinchisi ikkinchisini o'z ichiga oladi. Demak berilgan Funksiya $x = \frac{\pi}{6} + \frac{k\pi}{3}$, $k \in Z$ nuqtalarda aniqlanmagan.

Javob: $x = \frac{\pi}{6} + \frac{k\pi}{3}$, $k \in Z$

2. $y=\sqrt{ctgx-1}$ funksiyaning aniqlanish sohasini toping.

Yechish: ctgx-1 ≥ 0, ctgx ≥ 1, $k\pi < x \leq \frac{\pi}{4}+k\pi$, $k \in Z$

Javob: $(k\pi;\frac{\pi}{4}+k\pi]$, $k \in Z$

3. $y=ctg\frac{x}{3}+tg\frac{x}{2}$ funksiyaning eng kichik davrini toping.

Yechish: Berilgan funksiyaning davri $y_1=ctg\frac{x}{3}$ va $y_2=tg\frac{x}{2}$ funksiyalar eng kichik davrlarining eng kichik umumiy karralisiga teng. Ularni topamiz:

$y_1=ctg\frac{x}{3}=ctg\left[\frac{1}{3}(x+3\pi)\right]$, demak $T_1=3\pi$. $y_2=tg\frac{x}{2}=tg\left[\frac{1}{2}(x+2\pi)\right]$, demak $T_2=2\pi$.

$T_1=3\pi$ va $T_2=2\pi$ larning eng kichik umumiy karralisi $T=6\pi$ ga teng.

Javob: 6π

§5. Teskari trigonometrik funksiyalar.

Shu vaqtga qadar biz α burchakning berilgan qiymatlariga asosan sin α, cos α, tg α va ctg α larning qiymatlarini topish bilan shug'ullandik. Endi bunga teskari masalani ya'ni sin α, cos α, tg α va ctg α larning qiymatlariga asosan α burchakning qiymatlarini aniqlash masalasini ham qo'yish mumkin. Bu masala teskari trigonometrik funksiya tushunchasini kiritishga olib keladi. Teskari trigonometrik funksiya tushunchasini kiritish uchun esa dastlab teskari funksiya tushunchasini kiritish kerak bo'ladi.

Aniqlanish sohasi D va qiymatlar sohasi E dan iborat bo'lgan y=f(x) funksiya o'zining aniqlanish sohasida monoton bo'lsin. U holda x ning D dan olingan har bir qiymatiga y ning E dagi bitta qiymati mos keladi va aksincha. y ning E dan olingan har bir qiymatiga x ning D dagi bitta qiymati mos keladi. Demak, bu holda E da aniqlangan shunday yangi funksiyani tuzish mumkinki, unda E dan olingan har bir y ga D da y=f(x) tenglamani qanoatlantiruvchi bitta x ni mos qo'yish mumkin. Hosil qil ingan bu yangi funksiya y=f(x) funksiyaga teskari funksiya deyiladi.

y=f(x) funksiyaga teskari funksiyani topish uchun x ni y orqali ifodalab so'ngra x va y larni o'rinlarini o'zaro almashtirish kerak. y=f(x) funksiyaga teskari Funksiyani y=g(x) ko'rinishda yoziladi.

Agar y=f(x) va y=g(x) funksiyalar o'zaro teskari funksiyalar bo'lsa, u holda y=f(x) ning aniqlanish sohasi y=g(x) uchun qiymatlar sohasi, qiymatlar sohasi esa y=g(x) uchun aniqlanish sohasi bo'ladi. Ya'ni D(f)=E(g) va D(g)=E(f).

O'zaro teskari funksiyalar grafiklari y=x to'g'ri chiziqqa nisbatan simmetrik bo'ladi.

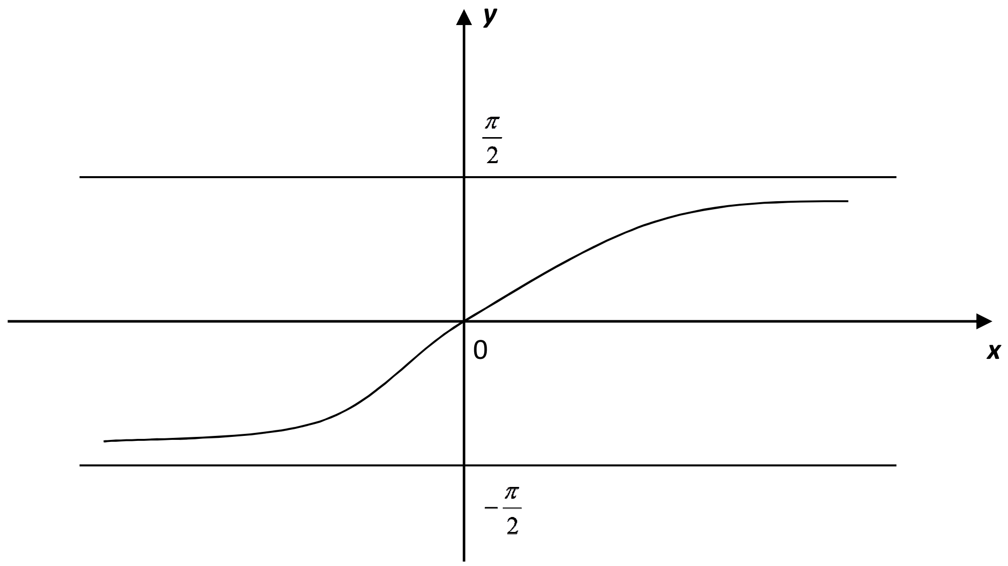
y=sinx funksiyaga teskari funksiyani topish masalasi bilan shug'allanamiz. Bu funksiya $(-\infty;+\infty)$ oraliqda monoton emas. Demak, bu oraliqda y=sinx funksiyaga teskari funksiya mavjud emas. y=sinx funksiya $[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmada monoton bo'lganligi uchun, bu kesmada unga teskari bo'lgan funksiyaga o'tish mumkin.

$[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmada y=sinx funksiya –1 dan 1 gacha o'sadi. Demak, x va u ning qiymatlari o'zaro bir qiymatli moslik orqali bog'langan. Moslik o'zaro bir qiymatli bo'lgani sababli, u ning [-1;1] kesmadagi har bir qiymatiga x ning $[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmadagi bitta qiymati mos keladi. Demak, bu holda yangi funksiya tuzish mumkin.

Ta'rif: $[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmada qaralayotgan y=sinx funksiyaga teskari bo'lgan funksiya arksinus deyiladi. Bu funksiya y=arcsinx kabi yoziladi

arcsinx ifoda $[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmada olingan yoydan iborat bo'lib, uning sinusi x ga teng, ya'ni sin(arcsinx)=x

Masalan: arcsin(-1)= $-\frac{\pi}{2}$; arcsin $(-\frac{\sqrt{3}}{2}) = -\frac{\pi}{3}$; arcsin $(-\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$; arcsin $(-\frac{1}{2}) = -\frac{\pi}{6}$; arcsin0=0; arcsin$\frac{1}{2} = \frac{\pi}{6}$; arcsin $\frac{\sqrt{3}}{2} = \frac{\pi}{3}$; arcsin$\frac{\sqrt{2}}{2} = \frac{\pi}{4}$; arcsin$\frac{1}{2} = \frac{\pi}{6}$; arcsin1= $\frac{\pi}{2}$.



y=arcsinx funksiya [-1;1] kesmada $-\frac{\pi}{2}$ dan $\frac{\pi}{2}$ gacha o'sadi.

y=arcsinx toq funksiyadir.

x ning $[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmadagi barcha qiymatlarida arcsin(sinx)=x.

Misollar:

1. $\frac{12\arcsin(-\frac{1}{2})}{\pi}$ ni hisoblang.

Yechish: $\frac{12\arcsin(-\frac{1}{2})}{\pi} = \frac{12 \bullet (-\frac{\pi}{6})}{\pi} = -2$.

Javob: -2

2 $\frac{\sin(\pi + \arcsin\frac{\sqrt{3}}{2})}{\cos(0{,}5\pi + \arcsin\frac{1}{2})}$ ni hisoblang.

Yechish: $\dfrac{\sin(\pi+\arcsin\frac{\sqrt{3}}{2})}{\cos(0{,}5\pi+\arcsin\frac{1}{2})}=\dfrac{\sin(\pi+\frac{\pi}{3})}{\cos(0{,}5\pi+\frac{\pi}{6})}=\dfrac{-\sin\frac{\pi}{3}}{-\sin\frac{\pi}{6}}=\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$.

Javob: $\sqrt{3}$

y=cosx funksiya $(-\infty;+\infty)$ oraliqda monoton emas. Demak, bu oraliqda y=cosx ga teskari Funksiya mavjud emas. y=cosx $[0;\pi]$ kesmada monoton bo'lgani uchun bu kesmada unga teskari bo'lgan Funksiyaga o'tish mumkin.

$[0;\pi]$ kesmada y=cosx funksiya 1 dan –1 gacha kamayadi. Ya'ni, bu kesmada x va u ning qiymatlari o'zaro bir qiymatli moslikda. Demak, bu holda yangi Funksiya tuzish mumkin.

Ta'rif: $[0;\pi]$ kesmada qaralayotgan y=cosx ga teskari bo'lgan funksiyani arkkosinus deyiladi.

Bu funksiya y=arccosx kabi yoziladi.

arccosx 0 dan π gacha bo'lgan kesmada olingan yoy ya'ni: $0\le \arccos x\le\pi$ bo'lib, bu yoyning kosinusi x ga teng: cos(arccosx)=x, bunda $-1\le x\le 1$.

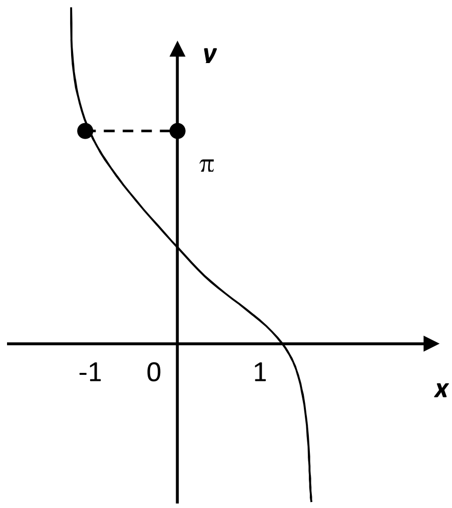
Masalan, arccos(-1)= π ; arccos $(-\frac{\sqrt{3}}{2})=\frac{5\pi}{6}$; arccos $(-\frac{\sqrt{2}}{2})=\frac{3\pi}{4}$; arccos $(-\frac{1}{2})=\frac{2\pi}{3}$;

$\arccos 0=\frac{\pi}{2}$; $\arccos\frac{1}{2}=\frac{\pi}{3}$; $\arccos\frac{\sqrt{2}}{2}=\frac{\pi}{4}$; $\arccos\frac{\sqrt{3}}{2}=\frac{\pi}{6}$; arccos1=0.

y=arccosx funksiya quyidagi xossalarga ega:

1^0. y=arccosx funksiya [-1;1] kesmada π dan 0 gacha kamayadi.

2^0. arccos(-x)= π-arccosx tenglik o'rinlidir.



Misollar:

1. $\arccos(-\frac{1}{2})-\arcsin(-\frac{\sqrt{2}}{2})$ ni hisoblang.

Yechish: $\arccos(-\frac{1}{2})-\arcsin(-\frac{\sqrt{2}}{2})=\frac{2\pi}{3}-(-\frac{\pi}{4})=\frac{2\pi}{3}+\frac{\pi}{4}=\frac{11\pi}{12}$.

Javob: $\frac{11\pi}{12}$.

2. $\arccos(-\frac{\sqrt{2}}{2})-\arcsin(-\frac{\sqrt{3}}{2})$ ni hisoblang.

Yechish: $\arccos(-\frac{\sqrt{2}}{2})-\arcsin(-\frac{\sqrt{3}}{2})=\frac{3\pi}{4}-(-\frac{\pi}{3})=\frac{3\pi}{4}+\frac{\pi}{3}=\frac{13\pi}{12}$.

Javob: $\frac{13\pi}{12}$.

3. $2\arcsin(-\frac{1}{2})+\frac{1}{2}\arccos\frac{\sqrt{3}}{2}$ ni hisoblang.

Yechish: $2\arcsin(-\frac{1}{2})+\frac{1}{2}\arccos\frac{\sqrt{3}}{2}=2\bullet(-\frac{\pi}{6})+\frac{1}{2}\bullet\frac{\pi}{6}=$

$=-\frac{\pi}{3}+\frac{\pi}{12}=\frac{-4\pi+\pi}{12}=-\frac{3\pi}{12}=-\frac{\pi}{4}$.

Javob: $-\frac{\pi}{4}$.

y=tgx funksiya $(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi)$ oraliqlarning har birida $-\infty$ dan $+\infty$ gacha o’sadi. Shuning uchun bu oraliqlarning har birida y=tgx ga teskari funksiyaga o’tsa bo’ladi.

Ta’rif: $(-\frac{\pi}{2};\frac{\pi}{2})$ oraliqda y=tgx ga nisbatan teskari bo’lgan funksiya arktangens deyiladi.

Bu funksiya y=arctgx kabi yoziladi.

y=arctgx $(-\frac{\pi}{2};\frac{\pi}{2})$ oraliqda olingan yoy, ya’ni $-\frac{\pi}{2} < arctgx < \frac{\pi}{2}$ bo’lib, uning tangensi x ga teng. Bu yerda x-istalgan haqiqiy son.

Masalan, arctgx(-1)= $-\frac{\pi}{4}$; arctg $(-\frac{\sqrt{3}}{3}) = -\frac{\pi}{6}$; arctg $(-\sqrt{3}) = -\frac{\pi}{3}$; arctg0=0; arctg$\frac{\sqrt{3}}{3} = \frac{\pi}{6}$; arctg$\sqrt{3} = \frac{\pi}{3}$; arctg. $1 = \frac{\pi}{4}$

y=arctgx quyidagi xossalarga ega.

1^0. y=arctgx x ning barcha qiymatlarida aniqlangan, o’suvchi Funksiyadir.

2^0. y=arctgx toq funksiyadir: arctg(-x)=-arctgx

y=tgx funksiyani grafigini yasash uchun x=tgy tangensoidaning tarmog’ini yasash kifoyadir.

Misollar:

1. $\arccos(-\frac{\sqrt{2}}{2}) - arctg\frac{1}{\sqrt{3}}$ ni hisoblang .

Yechish: $\arccos(-\frac{\sqrt{2}}{2}) - arctg\frac{1}{\sqrt{3}} = \frac{3\pi}{4} - \frac{\pi}{6} = \frac{9\pi - 2\pi}{12} = \frac{7\pi}{12} = 105^0.$

Javob: 105^0

2. m=arcsin $\frac{\sqrt{3}}{2}$, n=arcos $(-\frac{1}{2})$ va p=arctg1 sonlarni kamayish tartibida joylashtiring.

Yechish: m=arcsin$\frac{\sqrt{3}}{2}$=60^0, n=arcos$(-\frac{1}{2})$=120^0 va p=arctg1=45^0. Demak, $120^0>60^0>45^0$ bo’lgani uchun n>m>p

Javob: n>m>p

3. $\cos(arctg\sqrt{3}+\arccos\frac{\sqrt{3}}{2})$ ni hisoblang.

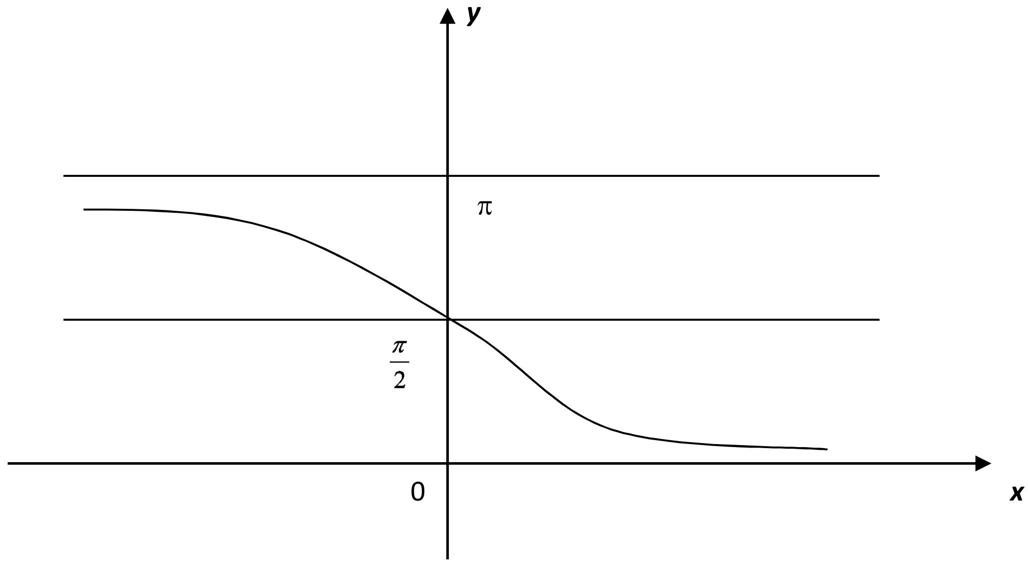
Yechish: $\cos(arctg\sqrt{3}+\arccos\frac{\sqrt{3}}{2})=\cos(60^0+30^0)=\cos 90^0=0$.

Javob: 0

4. $tg(\arcsin\frac{\sqrt{3}}{2}+arctg\sqrt{3})$ ni hisoblang.

Yechish: $tg(\arcsin\frac{\sqrt{3}}{2}+arctg\sqrt{3})=tg(60^0+60^0)=tg120^0=-ctg30^0=-\sqrt{3}$.

Javob: $-\sqrt{3}$



Ta'rif (0; π) oraliqda y=ctgx ga nisbatan teskari bo'lgan funksiyani arkkotangens deyiladi.

Bu funksiya y=arcctgx kabi yoziladi.

y=arcctgx, (0; π) oraliqda olingan yoy, ya'ni 0<arcctgx< π bo'lib, uning kotangensi x ga tengdir: ya'ni ctg(arcctgx)=x.

Bu yerda x-istalgan haqiqiy sondir.

y=arcctgx quyidagi xossalarga ega:

1^0. y=arcctgx funksiya x ning hamma haqiqiy qiymatlarida aniqlangan kamayuvchi funksiyadir.

2^0. arcctg(-x)= π-arcctgx.

Misollar:

1. (00.03.54)arcctg(tg(-37^0)) ni hisoblang.

Yechish: arcctg(tg(-37^0))=arcctg(-ctg53^0)= π-arcctg(ctg53^0)= π-53^0= =180^0-53^0= 27^0.

Javob: 127^0

2. arcctg(ctg(-3)) ni hisoblang.

Yechish: arcctg(ctg(-3))=arcctg(-ctg3)= π-arcctg(ctg3)= π-3.

Javob: π-3

3. arctg(tg$(-\frac{3\pi}{5})$)+arcctg(ctg$(-\frac{3\pi}{5})$)=?

Yechish:arctg(tg$(-\frac{3\pi}{5})$)+arcctg(ctg$(-\frac{3\pi}{5})$)=arctg(-tg$(\pi-\frac{2\pi}{5})$)+

+arcctg(-ctg$\frac{3\pi}{5}$)=arctg(tg$\frac{2\pi}{5}$)+ π-arcctg(ctg$\frac{3\pi}{5}$)=$\frac{2\pi}{5}$+ π-$\frac{3\pi}{5}$= π-$\frac{\pi}{5}$=$\frac{4\pi}{5}$.

Javob: $\frac{4}{5}\pi$

Birgina argumentga bog'liq bo'lgan trigonometrik funksiyalar biri ikkinchisi orqali algebraik ifoda qil inadi. Shuning uchun istalgan arkFunksiya ustida biror trigonometrik amalni bajarish natijasida algebraik ifoda hosil bo'ladi.

1^0. Teskari trigonometrik funksiyalarning ta'rifiga muvofiq, [-1;1] kesmada sin(arcsinx)=x va cos(arccosx)=x ekanligi ma'lum. Shuningdek $(-\infty;+\infty)$ oraliqda tg(arctgx)=x va ctg(arcctgx)=x.

2. $\cos\alpha=\pm\sqrt{1-\sin^2\alpha}$ formulada α =arcsinx deb olib quyidagi formulani hosil qilamiz:

$\cos(\arcsin x)=\pm\sqrt{1-\sin^2(\arcsin x)}=\pm\sqrt{1-x^2}$.

α =arcsinx yoy kosinus manfiy bo'lmaydigan $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ kesmada joylashgan bo'lgani uchun radikal oldidagi musbat ishorani olamiz. Shunday qilib, cos(arcsinx)=$\sqrt{1-x^2}$

3^0. Shunga o'xshash: sin(arccosx)= $\sqrt{1-x^2}$.

4^0. tg(arcsinx)=$\frac{\sin(arcsinx)}{\cos(arcsinx)} = \frac{x}{\sqrt{1-x^2}}$.

5^0. $tg\alpha = \frac{1}{ctg\alpha}$ va $ctg\alpha = \frac{1}{tg\alpha}$ munosabatlardan foydalanib $tg(arcctgx) = \frac{1}{x}$ va $ctg(arctgx) = \frac{1}{x}$ larni hosil qilamiz.

6^0. Sinusni tangens orqali ifodalovchi $\sin\alpha = \pm\frac{tg\alpha}{\sqrt{1+tg^2\alpha}}$ formulada $\alpha = arctgx$ deb olib $\sin(arctgx) = \frac{x}{\sqrt{1+x^2}}$ ni hosil qilamiz.

7^0. sin(arcsinx+arcsiny)=x$\sqrt{1-y^2}$ +y$\sqrt{1-y^2}$.

8^0. x=y deb olib quyidagini hosil qilamiz: sin(2arcsinx)=2x$\sqrt{1-x^2}$.

Misollar:

1. $\sin(\frac{\pi}{2} - \arccos\frac{3}{5})$ ni hisoblang.

Yechish: $\sin(\frac{\pi}{2} - \arccos\frac{3}{5}) = \cos(\arccos\frac{3}{5}) = \frac{3}{5} = 0{,}6$.

Javob: 0,6

2. $\sin(2\arcsin\frac{1}{3})$ ni hisoblang.

Yechish: $\sin(2\arcsin\frac{1}{3}) = 2\sin(\arcsin\frac{1}{3}) \bullet \cos(\arcsin\frac{1}{3}) = 2 \bullet \frac{1}{3} \bullet \sqrt{1-\frac{1}{9}} =$

$= \frac{2}{3} \bullet \sqrt{\frac{8}{9}} = \frac{2}{3} \bullet \frac{2\sqrt{2}}{3} = \frac{4\sqrt{2}}{9}$.

Javob: $\frac{4\sqrt{2}}{9}$

3. cos(2arccos $\frac{1}{3}$) ni hisoblang.

Yechish: cos(2arccos $\frac{1}{3}$)=cos^2(arccos $\frac{1}{3}$)-cin^2(arccos $\frac{1}{3}$)=[cos(arccos $\frac{1}{3}$)]^2-

-[sin(arccos $\frac{1}{3}$)]^2=$\frac{1}{9}$-(1-$\frac{1}{9}$)=$\frac{1}{9}$-1+$\frac{1}{9}$=-$\frac{7}{9}$

Javob: -$\frac{7}{9}$

4. sin (2arctg3) ni qiymatini toping.

Yechish: sin(2arctg3)=2sin(arctg3) $\bullet$ cos(arctg3)=

$$=2\bullet\frac{3}{\sqrt{1+9}}\bullet\frac{1}{\sqrt{1+9}}=\frac{6}{\sqrt{10}\bullet\sqrt{10}}=\frac{6}{10}=0{,}6.$$

Javob: 0,6

5. tg(2arcsin$\frac{3}{4}$) ni hisoblang.

Yechish: $\text{tg}(2\arcsin\frac{3}{4})=\dfrac{2tg(\arcsin\frac{3}{4})}{1-tg^2(\arcsin\frac{3}{4})}=\dfrac{2\bullet\dfrac{\frac{3}{4}}{\sqrt{1-\frac{9}{16}}}}{1-\dfrac{\frac{9}{16}}{1-\frac{9}{16}}}=\dfrac{2\bullet\frac{3}{\sqrt{7}}}{1-\frac{9}{7}}=$

$$=\frac{\frac{6}{\sqrt{7}}}{-\frac{2}{7}}=-\frac{6}{\sqrt{7}}\bullet\frac{7}{2}=-3\sqrt{7}$$

Javob: $-3\sqrt{7}$

6. $\cos(\arcsin\frac{40}{41}-\arcsin\frac{4}{5})$ni hisoblang

Yechish: $\cos(\arcsin\frac{40}{41}-\arcsin\frac{4}{5})=\cos(\arcsin\frac{40}{41})\bullet\cos(\arcsin\frac{4}{5})+\sin(\arcsin\frac{40}{41})\bullet$

$$\bullet\sin(\arcsin\frac{4}{5})=\sqrt{1-\frac{1600}{1681}}\bullet\sqrt{1-\frac{16}{25}}+\frac{40}{41}\bullet\frac{4}{5}=\frac{9}{41}\bullet\frac{3}{5}+\frac{32}{41}=\frac{27}{205}+\frac{32}{41}=\frac{27+160}{205}=\frac{187}{205}.$$

Javob: $\frac{187}{205}$

To’ldiruvchi yoylarning trigonometrik funksiyalari orasidagi munosabatlar, o’xshash (nomlari bo’yicha) arkfunksiyalarning (arksinus va arkkosinus, arktangens va arkkotangens) birini ikkinchisi orqali ifodalashga imkon beradi.

Teorema. x ga berilishi mumkin bo’lgan hamma qiymatlar uchun arcsinx+arccosx=$\frac{\pi}{2}$, arctgx+arcctgx=$\frac{\pi}{2}$ munosabatlar o’rinlidir.

Bulardan tashqari quyidagi munosabatlar ham o’rinlidir:

arctgx=arcsin$\frac{x}{\sqrt{1+x^2}}$; arcsinx=arctg$\frac{x}{\sqrt{1-x^2}}$; arccosx=arcctg$\frac{x}{\sqrt{1-x^2}}$;

arcctgx=arccos$\frac{x}{\sqrt{1+x^2}}$; $\arcsin x = \begin{cases} \arccos\sqrt{1-x^2}, & 0 \le x \le 1 \\ -\arccos\sqrt{1-x^2}, & -1 \le x < 0 \end{cases}$

$$\arccos x = \begin{cases} \arcsin\sqrt{1-x^2}, & 0 \le x \le 1 \\ \pi - \arcsin\sqrt{1-x^2}, & -1 \le x < 0 \end{cases}$$

$$\arccos x = \begin{cases} arctg\frac{\sqrt{1-x^2}}{x}, & 0 < x \le 1 \\ \pi + arctg\frac{\sqrt{1-x^2}}{x}, & -1 \le x < 0 \end{cases}$$

$$arctgx = \begin{cases} arcctg\frac{1}{x}, & x > 0 \\ arcctg\frac{1}{x} - \pi, & x < 0 \end{cases}$$

Misollar:

1. Agar 3arccosx+2arcsinx=$\frac{3\pi}{2}$ bo'lsa, $|x+3|^3$ ning qiymati nechaga teng bo'ladi?

Yechish: 3arccosx+2arcsinx=$\frac{3\pi}{2}$, 2(arccosx+arcsinx)+arcosx=$\frac{3\pi}{2}$,

$2 \bullet \frac{\pi}{2}$+arccosx=$\frac{3\pi}{2}$, π+arccosx=$\frac{3\pi}{2}$, arccosx=$\frac{\pi}{2}$, x=0.

Demak, $|x+3|^3=|0+3|^3=3^3=27$

Javob: 27

2. Agar 4arcsinx+arccosx=π bo'lsa, $3x^2$ ning qiymatini hisoblang.

Yechish: 4arcsinx+arccosx=π, 3arcsinx+arcsinx+arccosx=π,

3arcsinx+$\frac{\pi}{2}=\pi$, 3arcsinx=$\frac{\pi}{2}$, arcsinx=$\frac{\pi}{6}$, x=$\frac{1}{2}$.

Demak, $3x^2=3 \bullet (\frac{1}{2})^2=\frac{3}{4}$=0,75.

Javob: 0,75

Xulosa

Ushbu bitiruv malakaviy ishda umumiy o'rta ta'lim maktablari, akdemik litsey va kasb-hunar kollejlari matmatika fani o'quv dasturi va ular uchun mo'ljallangan darslik, o'quv qo'llanmalar chuqur tahlil qilinib "Trigonometrik funksiyalar" bo'limining matematika kursidagi o'rni va ahamiyati yoritilgan.

Trigonometrik funksiyalarni o'rganish asosan umumiy o'rta ta'lim maktablarining 9-sinfidan boshlangani bois ularni o'quvchilarga o'rgatishni qanday hajmda tashkil etish kerakligi haqida tavsiyalar berilgan. Bundan tashqari bitiruv malakaviy ishda trigonometrik funksiyalar bo'yicha asosiy tushunchalar tigonometrik tenglama va tengsizliklarni yechishda muhim ekanligi ta'kidlangan hamda asosiy tushunchalar yetarlicha misollar yordamida mustaxkamlangan.

Trigonometrik tenglama va tengsizliklarni yechishda ularning xossalari muhim ekanligi bois bitiruv malakaviy ishda barcha trigonometrik funksiyalarning xossalari mukammal yoritilgan va ularni grafiklari keltirilgan.

Bitiruv malakaviy ish teskari trigonometrik funksiyalarni qanday o'rganish mumkinligi bo'yicha tavsiyalar bilan yakunlangan.

Foydalanilgan adabiyotlar ro'yxati

1. I.A.Karimov "Yuksak ma'naviyat – yengilmas kuch" Toshkent 2009 y.
2. I.A.Karimov "Barkamol avlod - O'zbekiston taraqqiyotining poydevori" Toshkent 1998 y.
3. I.A.Karimov "O'zbekiston XXI asrga intilmoqda" Toshkent 2000 y.
4. O'zbekiston Respublikasining "Ta'lim to'g'risida"gi qonuni. Kadrlar tayyorlash milliy dasturi. T.:SHarq. 1997.
5. Alimov SH.A., Xolmuxamedov O.R., Mirzaahmedov M.A. "Algebra–7". T.: "O'qituvchi" – 2009.
6. Alimov SH.A., Xolmuxamedov O.R., Mirzaahmedov M.A. "Algebra–8". T.: "O'qituvchi" – 2010.
7. Alimov SH.A., Xolmuxamedov O.R., Mirzaahmedov M.A. "Algebra–9". T.: "O'qituvchi" – 2010.
8. A.U.Abduxamidov, X.A.Nasimov, U.M.Nosirov, J.X.Xusanov. Algebra va matematik analiz asoslari. 1-qism. "O'qituvchi". T.: 2008.
9. A.U.Abduxamidov, X.A.Nasimov, U.M.Nosirov, J.X.Xusanov. Algebra va matematik analiz asoslari. 2-qism. "O'qituvchi". T.: 2010.
10. Farberman B. L. Ilgor pedagogik texnologiyalar. T.: Fan. 2000
11. Ishmuxamedov R.J. Innovatsion texnologiyalar yordamida ta'lim samaradorligini oshirish yo'llari. TDPU. T.: 2004 .
12. J.G.Yo'ldoshev, S. A. Usmonov. Pedagogik texnologiya asoslari. T.O'kituvchi. 2004.
13. Mamedov K. va b. Pedagogik texnologiyalar va pedagogik maxorat. T.2003
14. Farberman. B. L. Oliy o'quv yurtlarida o'qitishning zamonaviy usullari.T. 2002.
15. А.В.Норин и другие. Сборник задач по математике для поступающих в вузы. М., 2005.
16. А.Н.Рурукин. Математика. М., 2004.
17. С.И.Новоселов. Обратные тригонометрические функции. М., 1950.
18. В.С.Крамор. Повторяем и систематизируем школьный курс алгебры и начал анализа. М., 1990.
19. A.Axlimirzaev. Matematika. I-qism. Andijon, 2005.

Davlat test markazi axborotnomalari. 1996-2003 yillar.

MUNDARIJA