O`ZBEKISTON RESPUBLIKASI OLIY VA O`RTA MAXSUS TA`LIM VAZIRLIGI

AL-XORAZMIY NOMLI URGANCH DAVLAT UNIVERSITETI

FIZIKA-MATEMATIKA FAKULTETI

«Amaliy matematika va matematik fizika» kafedrasi

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Mavzu: Chekli zonali potensiallar holida Shturm-Liuvill operatori uchun teskari spektral masala.

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Butun o'qda berilgan

$$Hy \equiv -y''+q(x)y = \lambda y, x \in R',$$

Shturm-Liuvill operatorining ushbu

$$R(\lambda)=\begin{pmatrix} \xi(\lambda) & \eta(\lambda) \\ \eta(\lambda) & \zeta(\lambda) \end{pmatrix}$$

ko'rinishdagi spektral matritsa-funksiyasning mavjudligi ilk bor 1910-yildan boshlab, G.Veyl, keyinchalik E.Titchmarsh, B.M.Levitan, N.Levinson va K.Iosidalar tomondan har xil usullarda ko'rsatildi.

Shturm- Liuvull operatorining $q(x), x \in R'$patensiali berilgan holda, uning $R(\lambda)$ spektral matritsa-funksiyasini topish va xossalarini o'rganish masalasiga to'g'ri spektral masala deyiladi. Aksincha, $R(\lambda)$ spektral matritsa-funksiya berilgan holda $q(x), x \in R'$ patensialni topish masalasiga teskari masala deyiladi. $R(\lambda)$ spektral matritsa funksiya bo`yicha teskari masala ilk bor M.Blox tomonidan 1953-yilda qaralgan, ammo bu ishda ayrim nuqsonlar bo`lgani bois qo`yilgan teskari masala o`z yechimini topmagan. 1967-yilda bu masala F.S.Rofe-Beketov tomonidan mukammal yechildi.

1-§. Zaruriy ma'lumotlar .

Ushbu

$$-y''+q(x)y=\lambda y, \quad -\infty<x<\infty \qquad (1.1)$$

Shturm- Liuvill tenglamasini qaraylik. Bu yerda $q(x)\in C(-\infty,\infty)$ haqiqiy uzluksiz funksiya, λ esa ixtiyoriy parametr.

$\theta(x,\lambda)$ va $\varphi(x,\lambda)$ orqali (1.1) tenglamaning ushbu

$$\theta(0,\lambda)=1, \quad \theta'(0,\lambda)=0; \quad \varphi(0,\lambda)=0, \quad \varphi'(0,\lambda)=1 \qquad (1.2)$$

boshlang`ich shartlarni qanoatlantiruvchi yechimlarini belgilaymiz. Quyidagi tasdiqlarni isbotsiz keltiramiz.

Teorema 1.1. (1.1) tenglamaning (1.2) boshlang`ich shartlarni qanoatlantiruvchi $\theta(x,\lambda)$ va $\varphi(x,\lambda)$ yechimlari uchun quyidagi

$$\theta(x,\lambda)=\cos\sqrt{\lambda}x+\int_{-x}^{x}K(x,t)\cos\sqrt{\lambda}t\,dt \qquad (1.3)$$

$$\varphi(x,\lambda)=\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}+\int_{-x}^{x}K(x,t)\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}dt \qquad (1.4)$$

integral tasvir o`rinli. Bunda $K(x,t)$ yadro $\lambda$ ga bog`liq emas va quyidagi

$$\frac{\partial^2 K}{\partial t^2}=\frac{\partial^2 K}{\partial x^2}-q(x)K \qquad (1.5)$$

$$K(x,x)=\frac{1}{2}\int_{0}^{x}q(s)ds \qquad (1.6)$$

$$K(x,-x)=0 \qquad (1.7)$$

Gursa masalasining yechimidan iborat bo`ladi.

Ayrim adabiyotlarda (1.3) va (1.4)integral tasvirlarni almashtirish operatorlari deb ham yuritiladi.

Quyidagi

$$\begin{aligned} &-y''+q(x)y=\lambda y, \qquad (-\infty<x<0)\\ &y(0)=0\\ &-y''+q(x)y=\lambda y, \qquad (0<x<\infty)\\ &y(0)=0 \end{aligned}$$

chegaraviy masalalarning Veyl yechimlarini mos ravishda

$$\psi_j(x,y)=\theta(x,\lambda)+m_j(\lambda)\varphi(x,\lambda)$$

orqali belgilaymiz. Bu yerda $m_j(\lambda),\ j=1,2,$ -Veyl-Titchmarsh funksiyalari bo`lib, ular uchun ushbu

$$m_j(\lambda)=\pm i\sqrt{\lambda}+\overline{\overline{O}}(1),\ j=1,2, \quad \lambda\to\infty \tag{1.8}$$

asimptotikalar o`rinli.

Teorema 1.2. (G.Veyl).(1.1) ko`rinishdagi ixtiyoriy Shturm- Liuvill tenglamasi uchun kamida bitta monoton kamaymaydigan ushbu

$$R(\lambda)=\begin{pmatrix}\xi(\lambda) & \eta(\lambda)\\ \eta(\lambda) & \zeta(\lambda)\end{pmatrix} \tag{1.9}$$

ko`rininshda matritsa-funksiya (spektral matritsa-funksiya) mavjud bo`lib, ixtiyoriy λ^{2N+1} cheksiz marta differensiallanuvchi finit funksiya uchun quyidagi

$$f(x)=\int_{-\infty}^{\infty}\{F(\lambda)\theta(x,\lambda)d\xi(\lambda)+[F(\lambda)\varphi(x,\lambda)+G(\lambda)\theta)(x,\lambda)]d\eta(\lambda)+G(\lambda)\varphi(x,\lambda)d\zeta(\lambda)\}.$$

spektral yoyilma o`rinli bo`ladi. Bu yerda

$$F(\lambda)=\int_{-\infty}^{\infty}f(x)\theta(x,\lambda)dx, \qquad G(\lambda)=\int_{-\infty}^{\infty}f(x)\varphi(x,\lambda)dx \tag{1.11}$$

funksiyaning ixtiyoriyligidan foydalansak, ushbu

$$\int_{-\infty}^{\infty}\{\theta(x,\lambda)\theta(y,\lambda)d\xi(\lambda)+[\varphi(x,\lambda)\theta(y,\lambda)+\theta(x,\lambda)\varphi(y,\lambda)]d\eta(\lambda)+U(x,\lambda)\varphi(y,\lambda)d\zeta(\lambda)\}=\delta(x-y) \tag{1.12}$$

simvolik ko`rinishdagi Parseval tengligiga ega bo`lamiz. Bu yerda $\delta(x)$-delta Dirak funksiyasi. Agar (1.1) tenglamada $q(x)\equiv 0$ bo`lsa, u holda (1.12) tenglik quyidagi

$$\frac{1}{2\pi}\int_0^{\infty}\left[\cos\sqrt{\lambda}x\cos\sqrt{\lambda}y+\sin\sqrt{\lambda}x\sin\sqrt{\lambda}y\right]\frac{d\lambda}{\sqrt{\lambda}}=\delta(x-y) \tag{1.13}$$

ko`rinishni oladi.

Izoh. (1.10) tenglikka butun o'qda berilgan Shturm-Liuvill operatorining spectral yoyilmasi deyiladi. Quyidagi matritsa- funksiyaga esa

$$R(\lambda)=\begin{pmatrix}\xi(\lambda) & \eta(\lambda)\\ \eta(\lambda) & \zeta(\lambda)\end{pmatrix} \qquad \lambda\in(-\infty,\infty)$$

butun o'qda berilgan Shturm-Liuvill masalasining spektral matritsa-funksiyasi deyiladi. Agar $\lambda=\lambda_0$ nuqtaning biror atrofida spektral matritsa-funksiya o'zgarmas bo'lsa, λ parametrning bu qiymatiga Shturm-Liuvill masalasining regulyar qiymati deyiladi. Haqiqiy o'qning regulyar bo'lmagan barcha nuqtalaridan tuzilgan to'plamga Shturm-Liuvill masalasining spektri deyiladi. Agar spektrning $\lambda=\lambda_0$ nuqtasida spektral matritsa-funksiya uzilishga ega bo'lsa, bu qiymatga Shturm-Liuvill masalasining xos qiymati deyiladi. Spektrning spektral matritsa-funksiyasi uzluksiz bo'ladigan nuqtalardan tuzilgan to'plamga Shturm-Liuvill masalasining uzluksiz spektri deyiladi. Yechim tilida aytadigan bo'lsak, bu quyidagilarni bildiradi: agar $\lambda=\lambda_0$ bo'lganda Shturm-Liuvill tenglamasi noldan farqli, $L^2(-\infty,\infty)$ fazoga qarashli $y(x,\lambda_0)$ yechimga ega bo'lsa, bu $\lambda=\lambda_0$ qiymat xos qiymat bo'ladi, $y(x,\lambda_0)$ yechim esa xos funksiya bo'ladi; agar $\lambda=\lambda_0$ bo'lganda Shturm-Liuvill tenglamasi noldan farqli, $L^2(-\infty,\infty)$ fazoga qarashli bo'lmagan, chegaralangan yechimga ega bo'lsa, bu qiymat uzluksiz spektrga tegishli bo'ladi.

Agar $\lambda=\lambda_0$ xos qiymatga ikkita chiziqli erkli xos funksiya mos kelsa, bu xos funksiyalar mos kelgan qiymat ikki karrali deyiladi. Agar $\lambda=\lambda_0$ uzluksiz spektrning nuqtasi bo'lib, unga ikkita chiziqli erkli, nodan farqli, chegaralangan yechimlar mos kelsa, bu qiymat uzluksiz spektrning ikki karrali nuqtasi deyiladi. Xos qiymat, uzluksiz spektrning ichida ham, chetida ham, tashqarisida ham joylashishi mumkin. Spektr faqat diskret, ya'ni faqat xos qiymatlardan iborat bo'lishi, yoki aksincha faqat uzluksiz bo'lishi, yoki aralash bo'lishi mumkin. Bularning hammasi $q(x)$ patensialning xossalariga bog'liq.

Ta'rif 1.1. Berilgan $q(x)$ patensial bo'yicha Shturm-Liuvill operatorining spektral matritsa-funksiyasini topish masalasiga to'g'ri spektral masala deyiladi, aksincha spektral matritsa-funksiya bo'yicha $q(x)$ patensialni tiklash masalasiga teskari spektral masala deyiladi.

Teorema 1.3. $R(\lambda)$ spektral matritsa-funksiyaning elementlari uchun quyidagi

$$\xi(\lambda)=-\lim_{y\to0}\frac{1}{\pi}\int_0^\lambda \operatorname{Im}\left\{\frac{1}{m_2(x+iy)-m_1(x+iy)}\right\}dx$$

$$\eta(\lambda)=\lim_{y\to0}\frac{1}{\pi}\int_0^\lambda \operatorname{Im}\left\{\frac{1}{2}\frac{m_2(x+iy)+m_1(x-iy)}{m_2(x+iy)-m_1(x+iy)}\right\}dx \qquad (1.14)$$

$$\zeta(\lambda)==-\lim_{y\to0}\frac{1}{\pi}\int_0^\lambda \operatorname{Im}\left\{\frac{1}{2}\frac{m_2(x+iy)m_1(x-iy)}{m_2(x+iy)-m_1(x+iy)}\right\}dx$$

Titchmarsh-Kodoira formulalari o`rinli.

Teorema 1.4. (Gelfand-Levitan-Blox) (1.1) tenglamaning (1.2) boshlang`ich shartlarini qanoatlantiruvchi $\theta(x,\lambda),\varphi(x,\lambda)$ yechimlarining $(1.3)-(1.4)$ integral tasvirlaridagi $K(x,t)$ yadro quyidagi

$$K(x,t)signx+F(x,t)+\int_{-x}^{x}K(x,s)F(s.t)ds=0,\ \ (|z|<|x|) \qquad (1.15)$$

inegral tenglamani qanoatlantiradi. Bu yerda

$$F(x,t)=\int_{-\infty}^{\infty}\cos\sqrt{\lambda}x\cos\sqrt{\lambda}td\sigma(\lambda)+\int_{-\infty}^{\infty}\frac{\sin\sqrt{\lambda}(x+t)}{\sqrt{\lambda}}d\eta(\lambda)+$$
$$+\int_{-\infty}^{\infty}\frac{\sin\sqrt{\lambda}x\sin\sqrt{\lambda}t}{\lambda}d\tau(\lambda),$$

$$\sigma(\lambda)=\xi(\lambda)-\xi_0(\lambda),\quad \tau(\lambda)=\zeta(\lambda)-\zeta_0(\lambda)$$

$$\xi_0(\lambda)=\begin{cases}0, \ \lambda\le 0\\ \dfrac{1}{\pi}\sqrt{\lambda}, \ \lambda>0\end{cases},\qquad \zeta_0(\lambda)=\begin{cases}0, \ \lambda\le 0\\ \dfrac{1}{3\pi}\sqrt{\lambda^3}, \ \lambda>0\end{cases}$$

Teoremaning isbotini keltirishdan oldin quyidagi asosiy lemmaning isbotini keltiramiz. Agar (1.3) va (1.4)tengliklarni mos ravishda $\cos\sqrt{\lambda}t$ va $\dfrac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}$ funksiyalarga nisbatan qarasak, u Volterraning ikkinchi tur integral tenglamalari bo`ladi. Bunda $\theta(x,t),\varphi(x,t)$ va $K(x,t)$ funksiyalar berilgan deb hisoblaymiz. Bu tenglamalarning birinchisini $\cos\sqrt{\lambda}t$ ga, ikkinchisini esa $\dfrac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}$ ga nisbatan yechsak, quyidagilar kelib chiqadi:

$$\begin{aligned}\cos\sqrt{\lambda}t&=\theta(t,\lambda)+\int_{-t}^{t}L(t,s)\theta(s,\lambda)ds,\\ \frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}&=\varphi(t,\lambda)+\int_{-t}^{t}L(t,s)\varphi(s,\lambda)ds\end{aligned}\qquad(1.16)$$

Lemma 1.1. $\theta(x,t),\varphi(x,t)$ yechimlar va $R(\lambda)$ spektral matritsa-funksiyaning elementlati quyidagi tengliklarni qanoatlantiradi:

$$\int_{-\infty}^{\infty}\theta(x,\lambda)\cos\sqrt{\lambda}t\,d\xi(\lambda)+\int_{-\infty}^{\infty}\varphi(x,\lambda)\cos\sqrt{\lambda}t\,d\eta(\lambda)+\int_{-\infty}^{\infty}\theta(x,\lambda)\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}d\eta(\lambda)+$$

$$+\int_{-\infty}^{\infty}\varphi(x,\lambda)\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}d\zeta(\lambda)=0,\qquad (|z|<|x|).\qquad(1.17)$$

Isbot. (1.17)yenglikning chap tomoniga (1.16)tenglikni qo'yib, integrallash tartibini almhtiramiz, hamda Parseval tengligining (1.12)simvolik ko'rinishidan foydalanamiz:

$$\int_{-\infty}^{\infty}\{\theta(x,\lambda)\theta(t,\lambda)d\xi(\lambda)+\varphi(x,\lambda)\theta(t,\lambda)d\eta(\lambda)+\theta(x,\lambda)\varphi(t,\lambda)d\eta(\lambda)+\varphi(x,\lambda)\varphi(t,\lambda)d\zeta(\lambda)\}+$$
$$+\int_{-t}^{t}L(t,s)\int_{-\infty}^{\infty}\{\theta(x,\lambda)\theta(s,\lambda)d\xi(\lambda)+\varphi(x,\lambda)\theta(s,\lambda)d\eta(\lambda)+\theta(x,\lambda)\varphi(s,\lambda)d\eta(\lambda)+$$
$$+\varphi(x,\lambda)\varphi(s,\lambda)d\zeta(\lambda)\}ds=\delta(x-t)+\int_{-t}^{t}L(t,s)\delta(x-s)ds=\delta(x-t)+L(t,x)sign(t)=0$$

Bu yerda $|t|<|x|$ bo’lganda $L(t,x)=0$ ekanligidan foydalanildi.

Endi teorema.1.4 ning isbotini keltiramiz. (1.17) tenglikka (1.3) va (1.4) integral tasvirlarni qo`yamiz:

$$\int_{-\infty}^{\infty}\left\{\cos\sqrt{\lambda}x\cos\sqrt{\lambda}t+\int_{-x}^{x}K(x,s)\cos\sqrt{\lambda}s\cos\sqrt{\lambda}tds\right\}d\sigma(\lambda)+$$
$$+\int_{-\infty}^{\infty}\left\{\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}\cos\sqrt{\lambda}t+\int_{-x}^{x}K(x,s)\frac{\sin\sqrt{\lambda}s}{\sqrt{\lambda}}\cos\sqrt{\lambda}tds\right\}d\eta(\lambda)+$$
$$+\int_{-\infty}^{\infty}\left\{\cos\sqrt{\lambda}x\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}+\int_{-x}^{x}K(x,s)\frac{\sin\sqrt{\lambda}s}{\sqrt{\lambda}}\cos\sqrt{\lambda}sds\right\}d\eta(\lambda)+$$
$$+\int_{-\infty}^{\infty}\left\{\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}+\int_{-x}^{x}K(x,s)\frac{\sin\sqrt{\lambda}s}{\sqrt{\lambda}}\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}ds\right\}d\tau(\lambda)+$$
$$+\frac{1}{2\pi}\int_{0}^{\infty}\left\{\cos\sqrt{\lambda}x\cos\sqrt{\lambda}t+\sin\sqrt{\lambda}x\sin\sqrt{\lambda}t\right\}\frac{d\lambda}{\sqrt{\lambda}}+$$
$$+\int_{-x}^{x}K(x,s)\frac{1}{2\pi}\int_{0}^{\infty}\left\{\cos\sqrt{\lambda}x\cos\sqrt{\lambda}t+\sin\sqrt{\lambda}x\sin\sqrt{\lambda}t\right\}\frac{d\lambda}{\sqrt{\lambda}}ds=0$$

Bu yerda integrallash tartibini almashtirib, $F(x,t)$belgilashdan hamda $q(x)\equiv 0$ bo`lgandagi parseval tengligining simvolik yozuvidan foydalansak, quyidagi ayniyatni keltirib chiqaramiz:

$$F(x,t)\int_{-x}^{x}K(x,s)F(s,t)ds+\delta(x-t)+\int_{-x}^{x}K(x,s)\delta(s-t)ds=0$$

Agar $|t|<|x|$ ekanligini hisobga olsak, ushbu

$$K(x,t)signx+F(x,t)+\int_{-x}^{x}K(x,s)F(s,t)ds=0$$

Gelfand-Levitan-Blox integral tenglamasi kelib chiqadi. Teorema isbot bo`ldi.

Teorema1.5. Biror $R(\lambda)$ matritsa-funksiya musbat aniqlangan bo`lib, o`sish nuqtalar to`plami chekli limitik nuqtaga ega bo`lsa, (1.15)integral tenglamaning yechimi mavjud va yagona bo`ladi.

Izoh. Gelfand-Levitan-Blox integral tenglamasining $K(x,t)$ yechimi $K(x,-x)=0$ shartni qanoatlantirgandagina u yordamida tuzilgan $\theta(x,\lambda)$ va $\varphi(x,\lambda)$ funksiyalar potensiali bo`lgan

Shturm- Liuvill tenglamasining yechimi bo`ladi. Aks holda, $R(\lambda)$ matritsa funksiya hech bir Shturm- Liuvill masalasining spektral matritsa-funksiyasi bo'la olmaydi.

Izoh. $K(x,-x)=0$ shart hozirgi kunga qadar $R(\lambda)$ spektral matritsa-funksiya tilida ifodalanmagan.

$R(\lambda)$matritsa-funksiya qanday shartlarni qanoatlantirganda u biror Shturm-Liuvill masalasining spektral matritsa-funksiyasi bo‘ladi degan tabiiy savol tug‘iladi. Bu muammoning yechimi Ukrainalik matematik Rofe-Beketov tomonidan berilgan. Bunda, butun o‘qdagi teskari masala $(-\infty,0]$ va $[0,\infty)$ oraliqlardagi teskari masalalarga keltiriladi.

Rofe-Beketov teoremasini bayon qilishdan oldin, quyidagi funksiyalarni kiritib olamiz:

$$M_{11}(z)=\frac{1}{m^+(z)-m^-(z)}, \qquad M_{12}(z)=\frac{1}{2}\cdot\frac{m^+(z)+m^-(z)}{m^+(z)-m^-(z)},$$

$$M_{22}(z)=\frac{m^+(z)m^-(z)}{m^+(z)-m^-(z)}. \tag{1.17}$$

Bu funksiyalar ushbu

$$M_{11}(z)M_{22}(z)-M_{12}^2(z)=-\frac{1}{4}, \tag{1.18}$$

Rofe-Beketov ayniyatni qanoatlantirishi ravshan. (5.6.1) tengliklardan quyidagi formulalar osongina kelib chiqadi

$$m^+(z)=\frac{-M_{22}(z)}{\frac{1}{2}-M_{12}(z)}=\frac{\frac{1}{2}+M_{12}(z)}{M_{11}(z)}, \tag{1.19}$$

$$m^-(z)=\frac{M_{22}(z)}{\frac{1}{2}+M_{12}(z)}=\frac{-\frac{1}{2}+M_{12}(z)}{M_{11}(z)}. \tag{1.20}$$

Titchmarsh-Kodaira formulalarini (1.17) funksiyalar yordamida yozadigan bo‘lsak, ular quyidagi ko‘rinishni oladi:

$$\xi(\lambda) = -\lim_{y\to 0}\frac{1}{\pi}\int_0^{\lambda} \mathrm{Im}\{M_{11}(x+iy)\}dx,$$

$$\eta(\lambda) = \lim_{y\to 0}\frac{1}{\pi}\int_0^{\lambda} \mathrm{Im}\{M_{12}(x+iy)\}dx, \qquad (1.21)$$

$$\zeta(\lambda) = -\lim_{y\to 0}\frac{1}{\pi}\int_0^{\lambda} \mathrm{Im}\{M_{22}(x+iy)\}dx$$

Oxirgi formulalarga Stiltes almshtirishini qo’llasak,quyidagi integral tasvirlar kelib chiqadi:

$$M_{11}(z) = \int_{-\infty}^{\infty}\frac{d\xi(\lambda)}{z-\lambda}, \qquad M_{12}(z) = \int_{-\infty}^{\infty}\frac{d\eta(\lambda)}{z-\lambda},$$

$$M_{22}(z) = \int_{-\infty}^{\infty}\left(\frac{1}{z-\lambda}+\frac{\lambda}{\lambda^2+1}\right)d\zeta(\lambda)+a \qquad (1.22)$$

Bu yerda a o’zgarmas son ushbu

$$M_{22}(z) = -\frac{i}{2}\cdot\sqrt{z}+\overline{\overline{o}}(1), \quad (z\to i\infty),$$

asimptotikadan anqlanadi.

Torema 1.6.(Rofe-Beketov) $R(\lambda)$matritsa-funksiya biror

$$-y''+q(x)y=\lambda y, \quad x\in(-\infty,\infty)$$

Shturm-Liuvill masalasining spektral matritsa-funksiyasi bo’lishi uchun quyidagi shartlar bajarilishi zarur va yetarli:

1) $\mathrm{Im}\, z>0$ bo’lganda (1.22)-(1.23) tengliklar yordamida aniqlanadigan $M_{ij}(\lambda),\ j=1,2$ funksiyalar (1.18) Rofe-Beketov ayniyatini qanoatlantiradi.

2) Ushbu

$$m^{+}(z) = \frac{\frac{1}{2}+M_{12}(z)}{M_{11}(z)}, \quad m^{-}(z) = \frac{-\frac{1}{2}+M_{12}(z)}{M_{11}(z)}, \qquad (1.24)$$

funksiyalar mos ravishda

$$\begin{cases} -y'' + q_+(x)y = \lambda y, & 0 \le x < \infty, \\ y(0) = 0, \end{cases} \quad \text{va} \quad \begin{cases} -y'' + q_-(x)y = \lambda y, & -\infty < x \le 0, \\ y(0) = 0, \end{cases}$$

ko'rinishdagi biror Shturm-Liuvill masalasining Veyl-Titchmarsh funksiyalari bo'ladi.

Izoh. Rofe-Beketov teoremasiga ko'ra teskari masala yechsak, umuman olganda $q(x)$ patensial $x = 0$ nuqtada uzilishga ega bo'lishi mumkin.

Izoh. Rofe-Beketov teoremasining shartlari bajarilsa, $q(x)$ patensialnibirdaniga butun o'qda Gelfand-Levitan-Blox integral tenglamasidan foydalanib topish mumkin.

A,B. Xasanovning "Shturm- Liuvill chegaraviy masalalari nazaryasiga kirish " kitobining V bobining 7-§ da Rofe-Beketov teoremasining shartlarini tekshirishga doir bir qancha misollar keltirilgan.

Navbtdagi paragrafda Rofe-Beketov teoremasidan foydalanib, uzluksiz spektri

$$E = R^1 \setminus \left\{ (-\infty, \lambda_0) \cup \bigcup_{j=1}^{N} (\lambda_j, \mu_j) \right\}$$

to'plamdan iborat bo'lgan (1.1) ko'rinishdagi Shturm- Liuvill operatorini qurish bilan shug'ullanamiz. Bunda ushbu

$$(-\infty, 0), (\lambda_1, \mu_1), \ldots (\lambda_N, \mu_N)$$

integrallarga lakunalar deyiladi.

2-§ Chekli zonali patensiallar holida teskari spektral masala.

Butun o'qda ixtiyoriy $2N + 1$ ta nuqta berilgan bo'lsin:

$$\lambda_0 < \lambda_1 < \mu_1 < \ldots < \lambda_N < \mu_N < \infty$$

Umumiylikka xilof qilmagan holda $\lambda_0 = 0$ deb hisoblymiz. Chunki spectral parametrni siljitish natijasida bunga erishish mumkin. Bu paragrafda spektri ushbu

$$E = R^1 \setminus \left\{ (-\infty, \lambda_0) \cup \bigcup_{k=1}^{N} (\lambda_k, \mu_k) \right\}$$

to'plam bilan ustma-ust tushadigan Shturm-Liuvill operatorini tuzish bilan shug'ullanamiz.

Quyidagi ko'phadlarni ko'rib chiqamiz:

$$R(\lambda) = \lambda(\lambda - \lambda_1)(\lambda - \mu_1)\ldots(\lambda - \lambda_N)(\lambda - \mu_N) \qquad (2.1)$$

$$P(\lambda) = (\lambda - \xi_1)(\lambda - \xi_2)\ldots(\lambda - \xi_N) \qquad (2.2)$$

$$Q(\lambda) = P(\lambda)\sum_{k=1}^{N} \frac{a_k \sqrt{-R(\xi_k)}}{(\lambda - \xi_k)P(\xi_k)} \qquad (2.3)$$

Bu yerda $\xi_k \in [\lambda_k, \mu_k],\ k = \overline{1,N}$ ixtiyoriy sonlar va $a_k = \pm 1,\ k = \overline{1,N}$ ixtiyoriy ishoralar.

Lemma 2.1. Ixtiyoriy sonlar $\xi_k \in [\lambda_k, \mu_k],\ k = \overline{1,N}$ va ixtiyoriy ishoralar $a_k = \pm 1,\ k = \overline{1,N}$ uchun

$$S(\lambda) = \frac{R(\lambda) + Q^2(\lambda)}{P(\lambda)} \qquad (2.4)$$

ratsional funksiya bosh kaeffitsienti 1 ga teng bo'lgan $N+1$ darajali ko'phad bo'lib, har bir $[\lambda_k \mu_k], k = \overline{1,N}$ kesmada $S(\lambda)$ bittadan nolga ega.

Isbot. $S(\lambda) + Q^2(\lambda)$ ko'phad $P(\lambda)$ ko'phadga bo'linishini isbotlaymiz. Buning uchun $R(\xi_j) + Q(\xi_j) = 0,\ j = \overline{1,N}$ tenglik bajarilishini ko'rsatish zarur va yetarli.

$$Q(\xi_j) = P(\lambda)\frac{a_j\sqrt{-R(\xi_j)}}{(\lambda - \xi_j)P(\xi_j)}\Bigg|_{\lambda=\xi_j} + P(\xi_j)\sum_{k=1}^{N}\frac{a_k\sqrt{-R(\xi_k)}}{(\xi_j - \xi_k)P(\xi_k)} = a_j\sqrt{-R(\xi_j)} + 0 = a_j\sqrt{-R(\xi_j)}$$

. Demak, haqiqatan ham $R(\xi_j) + Q(\xi_j) = 0,\ j = \overline{1,N}$ tengliklar bajariladi. $S(\lambda)$ ko'phad ekanligi isbotlandi. $S(\lambda) + Q^2(\lambda)$ va $P(\lambda)$ ko'phadlarning bosh hadlar mos ravishda λ^{2N+1} va λ^N bo'lgani uchun $S(\lambda)$ ning bosh hadi λ^{N+1} ga teng bo'ladi.

$\lambda \in \overset{0}{E}$ bo'lsa, $S(\lambda)P(\lambda) = R(\lambda) + Q^2(\lambda) > 0$ bo'lishi ravshan. Spektrning har bir bo'lagida $P(\lambda)$ ning ishorasi o'zgarmas bo'lib, keyingi bo'lakka o'tganda bu ishora qarama-qarshi ishoraga o'zgaradi.

$S(\lambda)P(\lambda) > 0$ ekanligidan ushbu xossa $S(\lambda)$ uchun ham o'rinli bo'ladi. Bundan $S(\lambda)$ ko'phad har bir yopiq lakunada kamida bitta nolga ega ekanligi kelib chiqadi. Lakunalar soni $N+1$ ta bo'lgani uchun va $S(\lambda)$ning darajasi ham $N+1$ bo'lgani uchun har bir lakunada bittadan ildiz bo'ladi. Lemma 2.1. isbotlandi.

Endi $R(\lambda), P(\lambda) Q(\lambda), S(\lambda)$ ko'phadlar bo'yicha

$$R(\lambda) = \begin{pmatrix} \xi(\lambda) & \eta(\lambda) \\ \eta(\lambda) & \zeta(\lambda) \end{pmatrix} \qquad \lambda \in (-\infty, \infty) \qquad (2.5)$$

matritsa-funksiya tuzib olamiz. Bu yerda

$$\frac{d\xi(\lambda)}{d\lambda} = \begin{cases} \dfrac{1}{2\pi} \dfrac{P(\lambda)}{\pm\sqrt{R(\lambda)}}, & \lambda \in E \\ 0, & \lambda \notin E \end{cases}$$

$$\frac{d\eta(\lambda)}{d\lambda} = \begin{cases} \dfrac{1}{2\pi} \dfrac{Q(\lambda)}{\pm\sqrt{R(\lambda)}}, & \lambda \in E \\ 0, & \lambda \notin E \end{cases} \qquad (2.6)$$

$$\frac{d\zeta(\lambda)}{d\lambda} = \begin{cases} \dfrac{1}{2\pi} \dfrac{S(\lambda)}{\pm\sqrt{R(\lambda)}}, & \lambda \in E \\ 0, & \lambda \notin E \end{cases}$$

Teorema 2.1. $R(\lambda)$ matritsa-funksiya biror Shturm-Liuvill operatorining spektral matritsa-funksiyasi bo'ladi.

Ushbu teoremani isbotlash uchun Rofe-Beketov teoremasining shartlari bajarilishini ko'rsatish yetarli. Buning uchun avvalo quyidagi lemmalarni isbotlaymiz.

Lemma 2.2. $R(\lambda)$ matritsa-funksiya uchun Rofe-Beketov teoremasidagi $M_{11}(z), M_{12}(z), M_{22}(z)$funksiyalar quyidagi ko'rinishda bo'ladi:

$$M_{11}(z) = -\frac{1}{2}\frac{P(z)}{\sqrt{R(z)}},\ M_{12}(z) = -\frac{1}{2}\frac{Q(z)}{\sqrt{R(z)}},\ M_{22}(z) = -\frac{1}{2}\frac{S(z)}{\sqrt{R(z)}}$$

(2.7)

Isbot. $M_{11}(z), M_{12}(z), M_{22}(z)$ funksiyalar ta'rifiga ko'ra

$$M_{11}(z) = \int_{-\infty}^{\infty}\frac{d\xi(\lambda)}{z-\lambda},\qquad M_{12}(z) = \int_{-\infty}^{\infty}\frac{d\eta(\lambda)}{z-\lambda},\qquad M_{22}(z) = \int_{-\infty}^{\infty}\left(\frac{1}{(z-\lambda)} + \frac{\lambda}{1+\lambda^2}\right)d\xi(\lambda) + a$$

Bu yerda a_1 o'zgarmas son ushbu

$$M_{22}(z) = -\frac{1}{2}\sqrt{z} + O(1),\ (z \to i\infty)$$

munosabatdan aniqlanadi.

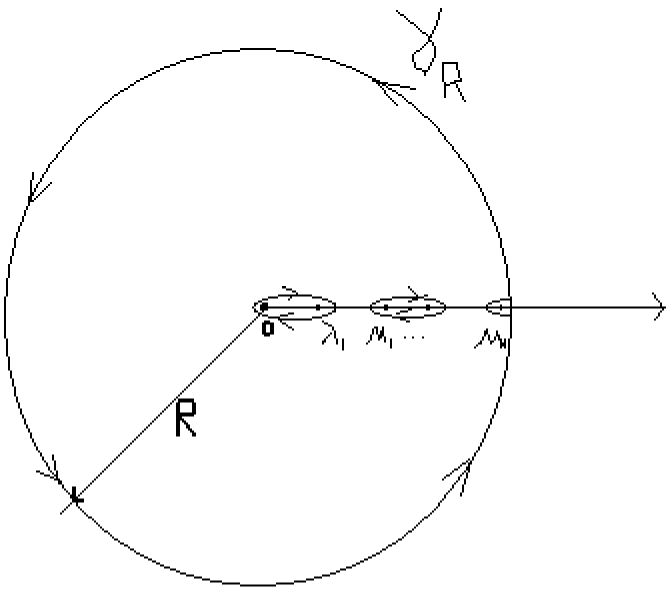
(2.6) ga ko'ra

$$M_{11}(z) = \frac{1}{2\pi}\int_E \frac{1}{z-\lambda}\frac{P(\lambda)}{\sqrt{R(\lambda)}}d\lambda$$

Quyidagi funksiyani

$$f(\lambda) = \frac{1}{2\pi}\frac{1}{z-\lambda}\frac{P(\lambda)}{\sqrt{R(\lambda)}}$$

1-rasmdagi kontur bo'yicha integrallaymiz.



Qoldiqlar haqidagi Koshi teoremasiga binoan

$$\int\limits_{\gamma_R} f(\lambda)d\lambda + \int\limits_{E_\varepsilon^+} f(\lambda)d\lambda + \int\limits_{E_\varepsilon^-} f(\lambda)d\lambda = 2\pi i res f(\lambda)$$

Ushbu tenglikda $\varepsilon \to +0$ va $R \to +\infty$ limitga o'tamiz. Buning uchun quyidagilarni hisobga olish kerak:

$$f(\lambda) \sim \frac{\lambda^N}{\lambda\sqrt{\lambda^{2N+1}}} = \frac{\lambda^N}{\lambda\lambda^{N+\frac{1}{2}}} = \frac{1}{\lambda\sqrt{\lambda}}, \quad |\lambda| \to \infty,$$

$$\int\limits_{\gamma_R} f(\lambda)d\lambda = \int_0^\pi f(\mathrm{Re}^{i\varphi})Rie^{i\varphi}d\varphi = 0\left(\frac{1}{\sqrt{R}}\right) \to 0, \quad R \to +\infty$$

$$f(\lambda)\big|_{E^-} = -f(\lambda)\big|_{E^+}$$

ikkinchi va uchinchi integrallarda integrallash tartibi qarama-qarshi

$$2\int\limits_E f(\lambda)d\lambda = 2\pi i\left(-\frac{1}{\pi}\frac{P(z)}{\sqrt{R(z)}}\right), \qquad \int\limits_E f(\lambda)d\lambda = -\frac{1}{2}\frac{P(z)}{\sqrt{R(z)}}$$

Demak,

$$M_{11}(z) = -\frac{1}{2}\frac{P(z)}{\sqrt{R(z)}}$$

Qolgan tengliklar hamxuddi shunday isbotlanadi. Lemma 2.2. isbotlandi.

Natija. $R(\lambda)$ matritsa-funksiya uchun Rofe-Beketov ayniyati

$$M_{11}(z)M_{22}(z) - M_{12}^2 = -\frac{1}{4}$$

bajariladi.Haqiqatan ham lemma 2.2 ga ko'ra

$$M_{11}(z)M_{22}(z) - M_{12}^2 = -\frac{1}{2}\frac{P(z)}{\sqrt{R(z)}}\left(-\frac{1}{2}\frac{S(z)}{\sqrt{R(z)}}\right) - \left(-\frac{1}{2}\frac{Q(z)}{\sqrt{R(z)}}\right)^2 =$$

$$= -\frac{1}{4}\frac{P(z)S(z) - Q^2(z)}{R(z)} = -\frac{1}{4}$$

Demak, qaralayotgan holda Rofe-Beketov ayniyati

$$P(z)S(z) - Q^2(z) = R(z)$$

ko’rinishda bo’lar ekan.

Lemma 2.3. $R(\lambda)$ matritsa-funksiya uchun Rofe-Beketov teoremasidagi $m_1(z)$ va $m_2(z)$ funksiyalar quyidagicha tasvirlanadi:

$$m_1(z)=\frac{Q(z)}{P(z)}+i\frac{\sqrt{R(z)}}{P(z)}, \qquad m_2(z)=\frac{Q(z)}{P(z)}-i\frac{\sqrt{R(z)}}{P(z)} \tag{2.8}$$

Isbot. $m_1(z)$ funksiya tuzilishiga ko’ra

$$m_1(z)=-\frac{M_{22}(z)}{\frac{1}{2}-M_{12}(z)}$$

(2.6) tenglikdan

$$m_1(z)=\frac{S(z)}{Q(z)-i\sqrt{R(z)}}=\frac{S(z)[Q(z)+i\sqrt{R(z)}]}{Q^2(z)+R(z)}=\frac{S(z)[Q(z)+i\sqrt{R(z)}]}{S(z)P(z)}=\frac{Q(z)}{P(z)}+i\frac{\sqrt{R(z)}}{P(z)}$$

kelib chiqadi. $m_2(z)$ uchun ham xuddi shunday isbotlanadi. Lemma 2.3. isbotlandi\.

Lemma 2.4. $R(\lambda)$ matritsa-funksiya uchun Rofe-Beketov teoremasidagi $m_1(z)$ va $m_2(z)$ funksiyalar quyidagicha tasvirlanadi:

$$\begin{aligned} m_1(z)&=-\int_E\left(\frac{1}{z-\lambda}+\frac{\lambda}{1+\lambda^2}\right)\frac{\sqrt{R(\lambda)}}{\pi P(\lambda)}d\lambda+2\sum_{a_k=1}\frac{\sqrt{-R(\xi_k)}}{(z-\xi_k)P(\xi_k)}+\gamma,\\ m_2(z)&=\int_E\left(\frac{1}{z-\lambda}+\frac{\lambda}{1+\lambda^2}\right)\frac{\sqrt{R(\lambda)}}{\pi P(\lambda)}d\lambda-2\sum_{a_k=-1}\frac{\sqrt{-R(\xi_k)}}{(z-\xi_k)P(\xi_k)}-\gamma \end{aligned} \tag{2.9}$$

bu yerda $\gamma - z$ ga bog’liq bo’lmagan son.

Isbot. Avvalo $i\frac{\sqrt{R(z)}}{P(z)}$ uchun integral tasvir olamiz. Buning uchun 1-rasmdagi kontur bo’yicha

$$f(\lambda)=\frac{1}{2\pi}\left(\frac{1}{z-\lambda}+\frac{\lambda}{1+\lambda^2}\right)\frac{P(\lambda)}{\sqrt{R(\lambda)}}$$

funksiyani integrallaymiz. Koshi teoremasidan foydalanib, lemma.2.2 isbotidek amallarni bajarib,

$$i\frac{\sqrt{R(z)}}{P(z)}=-\int_E\left(\frac{1}{z-\lambda}+\frac{\lambda}{1+\lambda^2}\right)\frac{\sqrt{R(\lambda)}}{\pi P(\lambda)}d\lambda+\sum_{k=1}^{N}\frac{\sqrt{-R(\xi_k)}}{(z-\xi_k)P(\xi_k)}+\gamma$$

tenglikni hosil qilamiz.

$Q(z)$ ko'phading tuzilishi (2.3) ga ko'ra

$$\frac{Q(z)}{P(z)}=\sum_{k=1}^{N}\frac{a_k\sqrt{-R(z)}}{(z-\xi_k)P(\xi_k)}=\sum_{a_k=1}\frac{\sqrt{-R(\xi_k)}}{(z-\xi_k)P(\xi_k)}-\sum_{a_k=-1}\frac{\sqrt{-R(\xi_k)}}{(z-\xi_k)P(\xi_{\backslash k})}$$

Lemma 2.3 dan (2.9) tengliklar kelib chiqadi. Lemma.2.4 isbotlandi.

Izoh. Lemma.2.4.dagi tasvirdan foydalanib Rofe-Beketov teoremasidagi $\rho_1(\lambda)$va $\rho_2(\lambda)$ funksiyalarni tuzib olishimiz mumkin. $\rho_1(\lambda)$va $\rho_2(\lambda)$ funksiyalar $(-\infty,0]$ va $[0.\infty)$ yarim o'qlarda $y(0)=0$chegaraviy shartda biror Shturm-Liuvill operatorining spektral funksiyalari ekanligini ko'rsatish mumkin. Shuning bilan Rofe-Beketov teoremasining shartlari bajarilishi to'la ko'rsatildi. Demak, teorema.2.1 isbotlandi.

Shunday qilib, $m_1(z)$ funksiya bo'yicha $q(x)$ patensial $x\le 0$ uchun, $x\ge 0$ da esa $m_2(z)$ yordamida aniqlanadi. Biz $q(x)$ funksiyani butun o'qda $\left(x\in R^1\right)$ ushbu

$$sign\, xK(x,y)+F(x,y)+\int_{-x}^{x}K(x,t)F(t,y)dt=0, \qquad (2.10)$$

Gelfand-Levitan-Blox integral tenglamasidan aniqlaymiz.

$$\begin{aligned}
&F(x,y)=F_1(x,y)+F_2(x,y)+F_3(x,y)\\
&F_1(x,y)=\int_{-\infty}^{\infty}\cos\sqrt{\lambda}x\cos\sqrt{\lambda}y\,d\sigma(\lambda)\\
&F_2(x,y)=\int_{-\infty}^{\infty}\frac{\sin\sqrt{\lambda}(x+y)}{\sqrt{\lambda}}d\eta(\lambda)\\
&F_3(x,y)\int_{-\infty}^{\infty}\frac{\sin\sqrt{\lambda}x\sin\sqrt{\lambda}y}{\lambda}d\tau(\lambda)
\end{aligned} \qquad (2.11)$$

Bu yerda

$$\sigma(\lambda)=\begin{cases}\xi(\lambda)-\frac{1}{\pi}\sqrt{\lambda}, & \lambda\ge 0\\ \xi(\lambda), & \lambda<0\end{cases}$$
$$\tau(\lambda)=\begin{cases}\zeta(\lambda)-\frac{1}{3\pi}\lambda^{\frac{3}{2}}, & \lambda\ge 0\\ \zeta(\lambda), & \lambda<0\end{cases} \tag{2.12}$$

$$q(x)=2\frac{dK(x,x)}{d(x)} \tag{2.13}$$

Integral tenglamada qatnashayotgan $F_1(x), F_2(x)$ va $F_3(x)$ funksiyalarni lakunalar bo'yicha integrallashga keltirish ancha foydali bo'ladi. Biz $F_1(x)$ funksiya uchun bu hisoblashlarni batafsil ko'rib chiqamiz:

$$F_1(x,y)=\frac{1}{2\pi}\int_E\left[\frac{P(\lambda)}{\sqrt{R(\lambda)}}-\frac{1}{\sqrt{\lambda}}\right]\cos\sqrt{\lambda}x\cos\sqrt{\lambda}y\,d\lambda-\frac{1}{2\pi}\int_G\cos\sqrt{\lambda}x\cos\sqrt{\lambda}y\frac{d\lambda}{\sqrt{\lambda}},$$

bu yerda

$$G=\bigcup_{j=1}^{N}(\lambda_j,\mu_j)$$

Ushbu

$$f_1(x)=\frac{1}{2\pi}\int_E\left[\frac{P(\lambda)}{\sqrt{R(\lambda)}}-\frac{1}{\sqrt{\lambda}}\right]\cos\sqrt{\lambda}x\,d\lambda-\frac{1}{2\pi}\int_G\frac{\cos\sqrt{\lambda}x}{\sqrt{\lambda}}d\lambda$$

funksiyalni qaraylik. U holda

$$F_1(x,y)=\frac{1}{2}\left[f_1(x+y)+f_1(x-y)\right]$$

Quyidagi integralni $\left(x\ge 0,\ \operatorname{Im}\sqrt{\lambda}>0\right)$ katta yarim aylana bo'yicha qaraymiz.

$$\int_C e^{i\sqrt{\lambda}x}\left[\frac{P(\lambda)}{\sqrt{R(\lambda)}}-\frac{1}{\sqrt{\lambda}}\right]d\lambda=0$$

Bu yerda $R \to \infty, \quad \rho \to 0$ bo'lganda limitga o'tib

$$\int_{-\infty}^{\infty} e^{i\sqrt{\lambda}\,x}\left[\frac{P(\lambda)}{\sqrt{R(\lambda)}} - \frac{1}{\sqrt{\lambda}}\right]d\lambda = 0$$

topamiz. Bu tenglikning haqiqiy qismini ajratib

$$\int_E \cos\sqrt{\lambda}x\left[\frac{P(\lambda)}{\sqrt{R(\lambda)}} - \frac{1}{\sqrt{\lambda}}\right]d\sqrt{\lambda} - \int_G \frac{\cos\sqrt{\lambda}x}{\sqrt{\lambda}}d\lambda + i\int_G \sin\sqrt{\lambda}x\frac{P(\lambda)}{\sqrt{R(\lambda)}}d\lambda = 0$$

hosil qilamiz. Shuning uchun

$$f_1(x) = \frac{1}{2\pi}\int_G \frac{P(\lambda)}{\sqrt{R(\lambda)}}\sin\sqrt{\lambda}|x|dx$$

Xuddi shunday hisoblashlar natijasida

$$F_2(x) = -\frac{1}{2\pi}\int_G \frac{Q(\lambda)}{\sqrt{-R(\lambda)}}\cos\sqrt{\lambda}xd\lambda + \frac{1}{2}\frac{Q(0)}{\sqrt{\lambda_1\mu_1...\lambda_n\mu_n}}, \quad F_2(-x) = -F_2(x), \quad x > 0$$

$$F_3(x,y) = \frac{1}{2}\left[f_3(x-y) - f_3(x+y)\right],$$

$$f_3(x) = \frac{1}{2\pi}\int_G \frac{S(\lambda)}{\sqrt{-R(\lambda)}}\sin\sqrt{\lambda}|x|d\lambda - \frac{1}{2}|x|\frac{S(0)}{\sqrt{\lambda_1\mu_1...\lambda_n\mu_n}}$$

topamiz.

Natija. Shunday qilib, berilgan

$$E = R^1 \setminus \bigcup_{k=1}^{N}(\lambda_k, \mu_k)$$

to'plam va $\xi_k \in [\lambda_k, \mu_k],\ k = \overline{1,N}$ ixtiyoriy sonlar hamda $a_k = \pm 1,\ k = \overline{1,N}$ ishoralar uchun yagona Shturm-Liuvill operatori mavjud bo'lib, uning spektral matritsasi $R(\lambda)$bilan ustma-ust tushadi. Ushbu operatorniong kaeffitsienti $q(x)$ ga N zonali patensial deyiladi, u ko'pincha $q_N(x)$ kabi belgilanadi. $\xi_k \in [\lambda_k, \mu_k],\ k = \overline{1,N}$ sonlar va $a_k = \pm 1,\ k = \overline{1, N}$ ishoralarga N zonali patensial $q_N(x)$ ning spektral parametrlari deyiladi.

Xulosa

Shunday qilib, biz butun o’qda berilgan chekli zonali Shturm-Liuvill operatorining

$q(x)$ patensialini qurishning quyidagi algaritmini keltiramiz:

1.Berilgan

$$\lambda_0 < \lambda_1 < \mu_1 < \lambda_2 < \mu_2 < \ldots < \lambda_N < \mu_N$$

haqiqiy sonlar va $\xi_k \in [\lambda_k, \mu_k],\ k = \overline{1,N}$ hamda $a_k = \pm 1,\ k = \overline{1,N}$ spektral parametrlar yordamida $R(\lambda), P(\lambda), Q(\lambda), S(\lambda)$ kophadlarni ushbu

$$P(\lambda)S(\lambda) - Q^2(\lambda) = R(\lambda)$$

Rofe-Beketov ayniyatini qanoatlantiradigan qilib tanlaymiz.

2. Qurilgan ko’phadlar yordamida $\xi'(\lambda), \eta'(\lambda)$ va $\zeta'(\lambda)$ spektral matritsa-funksiyaning elementlarini (2.6) formula yordamida aniqlaymiz.

3. Gelfand-Levitan-Blox integral tenglamasining

$$F(x,y) = F_1(x,y) + F_2(x,y) + F_3(x,y)$$

yadrosini (2.11)-(2.12) formulalardan foydalanib aniqlaymiz.

4. Gelfand-Levitan-Blox integral tenglamasini yechib, $K(x,y)$ ni topamiz.

5. Chekli zonali Shturm-Liuvill operatorining $q(x)$ patensialini

$$q(x) = 2\frac{d}{dx}K(x,x)$$

formula bo’yicha hisoblaymiz.

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