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Mavzu: Modifitsirlangan Korteveg-de Friz tenglamasining solitonsimon yechimlarini topish.

Ish ko'rib chiqildi va himoyaga ruxsat berildi.

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Har xil turdagi past nochiziqli fizik jarayonlarni yuqori chastotali dispersiyali muhitlarda tekshirishda quyidagi ko`rinishdagi tenglamalar uchraydi:

$$U_t + aU^n U_x + U_{xxx} = 0 \ . \qquad (1)$$

$n = 1$ qiymatda mashhur Korteveg-de Friz (KdF) tenglamasiga, $n = 2$ da modifitsirlangan KdF tenglamasiga ega bo`lamiz. Bu tenglama qattiq jism fizikasida, elektrogidrodinamikada [1] va atmosfera fizikasida [2] uchraydi. Bu ikkala tenglama sochilish nazariyasining teskari masala metodida integrallanadi va N ta haqiqiy soliton yechimi olingan [3,4]. Biroq quyidagi faktga e'tibor qilamiz: agar oddiy KdF tenglamasi uchun nochiziqli had oldidagi a koeffitsientning ishorasi ahamiyatli bo'masada (U ni $-U$ ga almashtirish bilan ishorani o'zgartirish mumkin), lekin modifitsirlangan KdF tenglamasi faqat $a > 0$ bolgandagina haqiqiy N ta soliton yechimga ega.

Aynan shunday farazda bu yechimlar [2]-ishda topilgan. Modifitsirlangan KdF tenglamasining statsionar yechimlari tahlili ko`rsatadiki, $a < 0$ bo'lganda $x \to \pm\infty$ da uning so`nuvchi, hamma o`qda haqiqiy uzluksiz yechimlari mavjud emas. Lekin pog`onada, ya'ni $x \to \pm\infty$ da o`zgarmas $c$ ga intiluvchi solitonlar mavjud bo`ladi,. Bu shuni ko`rsatadiki, $N$ ta soliton yechim ham pog`onada mavjud bo`ladi va u sochilish nazariyasining teskari masalasi metodi bilan yechilisi mumkin. Bunday yechimlarni topish [5]-ish natijalariga asoslanadi.

Biz modifitsirlangan KdF tenglamasini quyidagi ko`rinishda ko`rib chiqamiz.

$$V_t - 6V^2 V_n + V_{xxx} = 0 \qquad (2)$$

[4] ga ko`ra (2) tenglamani L, A juftligi ko'rinishida yozamiz:

$$L_t = i[L, A], \quad L\psi - i\zeta\psi, \quad \psi_t = A\psi;$$

L, A operatorlar quyidagi ko`rinishda aniqlanadi:

$$L = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} D + \begin{pmatrix} 0 & iV \\ iV & 0 \end{pmatrix},$$

$$A=-4\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}D^3+D\begin{pmatrix}-3\left(V^2-V_x\right) & 0\\ 0 & -3\left(V^2+V_x\right)\end{pmatrix}+$$
$$+\begin{pmatrix}-3\left(V^2-V_x\right) & 0\\ 0 & -3\left(V^2+V_x\right)\end{pmatrix}D$$

bu yerda $D=-\frac{i\partial}{\partial x}$, $V(x,t)-$haqiqiy funksiya. Sochilish nazariyasi teskari masalasi yordamida (2) nochiziqli tenglamasini yechish metodi quyidagicha: biz V potensialni L operatorning sochilish berilganlari yordamida topamiz. Sochilish berilganlarining vaqt bo`yicha dinamikasi $x\to\pm\infty$ da A operator asimptotikasini beradi.

1-§. To`g`ri va teskari masala

Sochilish nazariyasining teskari masalasini L operator uchun ko`rib chiqamiz:

$$L\psi = -i\xi\psi, \qquad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \tag{3}$$

Ushbu $\varphi_1 = \psi_1 - i\psi_2$, $\varphi_2 = \psi_1 + i\psi_2$, $q = V - c$ funksiyalarni kiritamiz, bu yerda c o`zgarmas. (3) tenglamadan

$$\varphi_1' + i\xi\varphi_1 = (q + c)\varphi_2, \qquad \varphi_2' - i\xi\varphi_2 = (q + c)\varphi_1 \tag{4}$$

tenglamalar sistemasini keltirib chiqaramiz, bunda $x \to \pm\infty$ da potensial $q \to 0$.

(4) sistemaga mos keluvchi operator uchun sochilish nazariyasining teskari masalasi [5] da yechilgan bo'lib, unda $x \to \pm\infty$ da moduli o`zgarmasga intiladigan nochiziqli Shredinger tenglamasining yechimlari olingan. [5] dan farqli o`laroq bizning holatda potensial q va o`zgarmas c haqiqiy kattalik deb qabul qilinadi. [5] ga asosan, $\zeta = \xi$ haqiqiy qiymatlar uchun quyidagi asiptotikaga ega va (4) sistemani qanoatlantiradiganYost funksiyasini aniqlaymiz:

$$f(x,\xi) = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \to \begin{pmatrix} 1 \\ -i(\gamma - \xi)/c \end{pmatrix} e^{-i\gamma x}, \qquad x \to -\infty,$$

$$g(x,\xi) = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} \to \begin{pmatrix} i(\gamma - \xi)/c \\ 1 \end{pmatrix} e^{i\gamma x}, \qquad x \to +\infty,$$

bunda $\gamma = \sqrt{\xi^2 - c^2}$. Yost funksiyasi $g(x,\xi)$ uchun uchburchakli tasvirlanish quyidagi ko`rinishda bo`ladi:

$$g(x,\xi) = \begin{pmatrix} \dfrac{i(\gamma - \xi)}{c} \\ 1 \end{pmatrix} e^{i\gamma x} - \begin{pmatrix} \int\limits_x^\infty \left[K_1(x,z) \dfrac{i(\gamma - \xi)}{c} + K_2(x,z) \right] e^{i\gamma z} dz \\ \int\limits_x^\infty \left[K_2(x,z) \dfrac{i(\gamma - \xi)}{c} + K_1(x,z) \right] e^{i\gamma z} dz \end{pmatrix}, \tag{5}$$

bunda $K_1(x,z)$ va $K_2(x,z)$ lar ushbu

$$2K_2(x,x)=q(x), \quad K_1, K_2 \xrightarrow{x,y\to\infty} 0$$

chegaraviy shartlarda quyidagi tenglamalar sistemasini qanoatlantiradi:

$$\frac{\partial}{\partial x}K_1(x,z)+\frac{\partial}{\partial z}K_1(x,z)=(q(x)+2c)K_2(x,z), \tag{6}$$

$$\frac{\partial}{\partial x}K_2(x,z)-\frac{\partial}{\partial z}K_2(x,z)=q(x)K_1(x,z).$$

(6) tenglikdan K_1 va K_2 kattaliklar haqiqiy ekanligi kelib chiqadi. Ko`rish mumkinki

$$\bar{g}(x,\xi)=\begin{pmatrix}\bar{g}_2(x,\xi)\\ \bar{g}_1(x,\xi)\end{pmatrix}$$

funksiya (4) sistemaning $g(x,\xi)$ ga chiziqli erkli yechimi bo'ladi va u bilan fundamental yechimlar sistemasini tashkil qiladi. Shu sababli $f(x,\xi)$ Yost funksiyasini g va $\bar{g}$ larni chiziqli kombinatsiyasi ko`rinishida tasvirlash mumkin:

$$f(x,\xi)=a(\xi)\bar{g}(x,\xi)+b(\xi)g(x,\xi). \tag{7}$$

$a(\xi)$ va $b(\xi)$ larni aniqlash uchun (7) tenglamani chiziqli tenglamalar sistemasi deb, quyidagilarni olamiz:

$$a(\xi)=\frac{\begin{vmatrix}f_1(x,\xi) & g_1(x,\xi)\\ f_2(x,\xi) & g_2(x,\xi)\end{vmatrix}}{\begin{vmatrix}\bar{g}_2(x,\xi) & g_1(x,\xi)\\ \bar{g}_1(x,\xi) & g_2(x,\xi)\end{vmatrix}}=\frac{f_1(x,\xi)g_2(x,\xi)-f_2(x,\xi)g_1(x,\xi)}{2\gamma(\xi-\gamma)/c^2}, \tag{8}$$

$$b(\xi)=\frac{\begin{vmatrix}\bar{g}_2(x,\xi) & f_1(x,\xi)\\ \bar{g}_1(x,\xi) & f_2(x,\xi)\end{vmatrix}}{\begin{vmatrix}\bar{g}_2(x,\xi) & g_1(x,\xi)\\ \bar{g}_1(x,\xi) & g_2(x,\xi)\end{vmatrix}}.$$

f funksiya uchun ham ko`rsatish mumkinki, $K_1(x,z)$ va $K_2(x,z)$ funksiyalar yordamida aniqlanuvchi uchburchakli tasvirlash mavjud. Bu tasvirlash $\zeta$ ikki yaproqli Riman tekisligining $\operatorname{Im}\gamma>0$ shartda $(-\infty,-c)\cup(c,\infty)$ kesikka ega yuqori yaprog`ida, $\zeta=\xi$ haqiqiy qiymatlarda aniqlangan f va g funksiyalarning analitik davom qildirilib yozilishini ta'minlaydi. (8) tenglikdan bizning Riman tekisligimizda $a(\zeta)$ funksiyaning analitikligi kelib chiqadi. $a(\zeta)$ funksiyaning nollari oddiy bo`lib, (4) sistemani diskret xos qiymatlari bo`ladilar va ular haqiqiy o`qning $(-c,c)$ oralig`ida joylashgan. q haqiqiy potensial deb qaralayotgan holda $a(\zeta)$ funksiya ξ argumentning juft funksiyasi bo`ladi. Haqiqatan, quyidagi munosabatlar o`rinli:

$$\begin{pmatrix} g_1(-\zeta) \\ g_2(-\zeta) \end{pmatrix} = \begin{pmatrix} g_2(\zeta) \\ g_1(\zeta) \end{pmatrix} \frac{i(\gamma+\zeta)}{c} \;,$$

$$\begin{pmatrix} f_1(-\zeta) \\ f_2(-\zeta) \end{pmatrix} = \begin{pmatrix} f_2(\zeta) \\ f_1(\zeta) \end{pmatrix} \left(-\frac{i(\gamma+\zeta)}{c} \right),$$

bundan

$$a(-\zeta) = \frac{f_1(-\zeta)g_2(-\zeta) - f_2(-\zeta)g_1(-\zeta)}{-2\gamma(\gamma+\xi)/c^2} =$$

$$= \frac{[f_1(\zeta)g_2(\zeta) - f_2(\zeta)g_1(\zeta)]\left[-(\gamma+\zeta)^2/c^2\right]}{-2\gamma(\gamma+\xi)/c^2} =$$

$$= \frac{[f_1(\zeta)g_2(\zeta) - f_2(\zeta)g_1(\zeta)]}{-2\gamma(\gamma-\xi)/c^2} = a(\zeta)\;,$$

shuning uchun

$$(\gamma+\zeta) = -c^2/(\gamma-\zeta).$$

$a(\zeta)$ juft funksiya ekanligidan uning nollari yoki (4) sistemaning diskret xos qiymatlari hamisha $(\zeta_k, -\zeta_k)$ juftlik bo`lib keladi. Diskret spektorning $\zeta_k$ nuqtasida Yost funksiyalari chiziqli bog`liq, hamda (4) sistemaning xos funksiyalari bo`ladilar:

$$f(\zeta_k, x) = \mu(\zeta_k) g(\zeta_k, x).$$

(4) tenglikdan quyidagini olishimiz mumkin:

$$-2if_1(x,\zeta_k)f_2(x,\zeta_k) = \frac{d}{dx}\left[f_1(x,\zeta_k)f_2(x,\zeta_k) - f_1(x,\zeta_k)\dot{f}_2(x,\zeta_k) \right], \qquad (9)$$

bu yerda "$\dot{}$" ζ bo`yicha differensiallashni bildiradi. $f$ Yost funksiyasining ko`rinishidan kelib chiqadiki, $x \to \infty$ da $f_1, f_2, \dot{f}_1 \dot{f}_2 \to 0$. $a(\zeta)$ ifodani ζ bo`yicha ζ_k nuqtada differensiallab, $x \to +\infty$ da quyidagini olamiz:

$$f_1(x,\zeta_k)f_2(x,\zeta_k) - f_1(x,\zeta_k)\dot{f}_2(x,\zeta_k) \to \dot{a}(\zeta_k)\mu(\zeta_k)\left[2\eta_k(\eta_k + i\zeta_k)/c^2\right], \quad (10)$$

bu yerda $\eta_k = \sqrt{c^2 - \zeta_k^{\ 2}}$ haqiqiydir. (9) tenglikni x bo`yicha $-\infty$ dan $+\infty$ gacha integrallab, (10) tenglikdan foydalanib quyidagini olamiz:

$$\eta(\zeta_k)/i\,\dot{a}(\zeta_k) = \eta_k(\eta_k + i\zeta_k)/c^2 \int_{-\infty}^{\infty} g_1(\zeta_k, x)g_2(\zeta_k, x)dx.$$

Tenglikni o`ng tomonidagi ifoda hamisha haqiqiy va musbat ekanligini ko'rsatamiz. Umuman olganda, $g(x,\zeta_k)$ Yost funksiyasining uchburchakli tasviridan

$$g(x,\zeta_k) = \begin{pmatrix} -\dfrac{\eta_k + i\zeta_k}{c}\alpha + \beta \\ \alpha - \dfrac{\eta_k + i\zeta_k}{c}\beta \end{pmatrix}, \quad (11)$$

kelib chiqadi, bunda

$$\alpha = e^{-\eta_k x} - \int_x^{\infty} K_1(x,z)e^{-\eta_k z}dz, \quad \beta = -\int_x^{\infty} K_2(x,z)e^{-\eta_k z}dz,$$

α va β – x ning haqiqiy funksiyalaridir.

Bundan quyidagi

$$\int_{-\infty}^{\infty} g_1(x,\zeta_k)g_2(x,\zeta_k)dx = \frac{\eta_k + i\zeta_k}{c}\int_{-\infty}^{\infty}\left[2\alpha\beta\frac{\eta_k}{c} - \left(\alpha^2 + \beta^2\right)\right]dx. \quad (12)$$

tenglikni olamiz. Integral ostidagi ifodada $\eta_k < c$ bo`lganligi uchun hamisha musbat emas ekanligini ko`rsatish mumkin. (11) va (12) tengliklardan kelib chiqadiki

$$r(\zeta_k) = \frac{i\mu(\zeta_k)}{\dot{a}(\zeta_k)} = -\frac{\eta_k}{c}\frac{1}{\int_{-\infty}^{\infty}\left[2\alpha\beta\eta_k/c - \left(\alpha^2 + \beta^2\right)\right]dx}, \quad (13)$$

bunda $r(\zeta_k)$ musbat. (13) tenglikdan kelib chiqadiki $r(\zeta_k)$ haqiqiy hamda $r(\zeta_k)=r(-\zeta_k)$, shunga asosan (13) tenglikning o`ng tomoni faqat $\eta_k=\sqrt{c^2-\zeta_k{}^2}$ ning funksiyasi hisoblanadi.

$|\xi|>c$ da haqiqiy o`qda aniqlangan holda (7) tenglikka qaytamiz. $b(\xi)\equiv 0$ bo`lganda qaytmaydigan potensial holni ko`rib chiqamiz.

U holda

$$f_1(\xi,x)/a(\xi)=\overline{g}_2(\xi,x), \qquad f_2(\xi,x)/a(\xi)=\overline{g}_1(\xi,x).$$

Bu tengliklarni $\dfrac{e^{-i\gamma z}}{2\pi i\gamma}$, $(z>x)$ ga ko`paytirib, quyidagini olamiz:

$$\begin{aligned}\frac{f_1(\xi,x)e^{-i\gamma z}}{2\pi i\gamma\, a(\xi)}&=\frac{\overline{g}_2(\xi,x)e^{-i\gamma z}}{2\pi i\gamma},\\ \frac{f_2(\xi,x)e^{-i\gamma z}}{2\pi i\gamma\, a(\xi)}&=\frac{\overline{g}_1(\xi,x)e^{-i\gamma z}}{2\pi i\gamma}.\end{aligned} \qquad (14)$$

Rasmda tasvirlangan bizning ikki yaproqli Riman sirtimizning yuqori yaprog`ida yotuvchi kontur bo`yicha (14) tengliklarni integrallaymiz.

Rasm

(14) tenglikning chap tomonini integrallashda uning yuqori yaprog`ining oddiy qutbi hisoblanuvchi $a(\zeta)$ funksiya nollaridan tashqari, barcha yerida analitik ekanligidan foydalanamiz. Shunday ekan, ko`rsatish mumkinki $f_1(\xi,x)$ funksiya cheksizlikda o`zgarmasga intiladi. Cheksizlikda konturni yopiq deb olsak, izlanayotgan integral $a(\zeta)$ funksiyaning nollaridagi chegirmalar yig`indisiga teng bo`ladi. $\zeta_k,-\zeta_k$ nuqtalarda chegirmalarni juft-jufti bilan birlashtirib va o`ng tomondagi integralni hisoblab, natijada ζ_k, $r(\zeta_k)$ berilganlar bo`yicha K_1, K_2 funksiyalarni aniqlash uchun Gelfand-Levitan-Marchenko integral tenglamalar sistemasini olamiz:

$$-cF(x+y)+\int_x^\infty[K_2(x,z)\Phi(y+z)+K_1(x,z)cF(y+z)]dz=K_1(x,y), \qquad (15)$$

$$\Phi(x+y)+\int_x^\infty [K_1(x,z)\Phi(y+z)+K_2(x,z)cF(y+z)]dz=K_2(x,y),$$

bu yerda

$$F(x+y)=-\sum_{k=1}^{N}\left(r(\zeta_k)/\eta_k\right)e^{-\eta_k(x+y)}$$
$$\Phi(x+y)=\sum_{k=1}^{N} r(\zeta_k)e^{-\eta_k(x+y)}. \tag{16}$$

Bu yerda N – (4) sistema uchun xos qiymatlarning diskret spektrini tashkil etuvchi $(\zeta_k,-\zeta_k)$ juftliklar sonini bildiradi. (15) sistemani yechib, r_k, η_k berilgan sonlar yordamida K_1, K_2 funksiyani topamiz va keyin (6) tenglikdan kelib chiquvchi formula yordamida q potensialni tiklaymiz.

$$\left(q^2+2cq\right)/2=(d/dx)K_1(x,x) \tag{17}$$

(15) tenglama yechimini quyidagi ko`rinishda izlaymiz:

$$K_1(x,y)=\sum_{k=1}^{N}\chi_k(x)e^{-\eta_k x}, \qquad K_2(x,y)=\sum_{k=1}^{N}\varphi_k(x)e^{-\eta_k x}.$$

Bu ifodalarni (15) ga qo`yib, $\chi_k(x)$, $\zeta_k(x)$ larni aniqlash uchun quyidagi sistemaga ega bo`lamiz:

$$\begin{cases} r_k e^{-\eta_k x}-\sum\limits_{n=1}^{N}\dfrac{\chi_n r_k}{\eta_n+\eta_k}e^{-(\eta_n+\eta_k)x}+c\sum\limits_{n=1}^{N}\dfrac{\varphi_n r_k}{\eta_k(\eta_n+\eta_k)}e^{-(\eta_n+\eta_k)x}=\varphi_k, \\ -c\dfrac{r_k}{\eta_k}e^{-\eta_k x}-\sum\limits_{n=1}^{N}\dfrac{\varphi_n r_k}{\eta_n+\eta_k}+c\sum\limits_{n=1}^{N}\dfrac{\chi_n r_k}{\eta_k(\eta_n+\eta_k)}=\chi_k. \end{cases} \tag{18}$$

Bu sistema determinanti quyidagiga teng

$$\Delta=\begin{vmatrix} A & B \\ B & A \end{vmatrix},$$

bunda $A,B-$ matritsalar $N-$ tartibli

$$A=\left\|\delta_{nk}-\frac{cr_k e^{-(\eta_n+\eta_k)x}}{\eta_k(\eta_n+\eta_k)}\right\|, \qquad B=\left\|\frac{r_k e^{-(\eta_n+\eta_k)x}}{\eta_n+\eta_k}\right\|.$$

ko’rinishdagi matritsalardir.

(18) sistemadan

$$\chi_k(x) = D_k / \Delta$$

ekanligi kelib chiqadiki, bunda

$$D_k = \begin{vmatrix} A' & B \\ B' & A \end{vmatrix}.$$

A' matritsa A dan farq qilib, undagi k-ustun $(cr_k / \eta_k)e^{-\eta_k x}$ ifodaga almashgan va mos ravishda B' da k-ustun $r_k e^{-\eta_k x}$ ga teng. Ko`rsatish mumkinki,

$$\frac{d}{dx}\Delta(x) = -2\sum \chi_n(x)e^{-\eta_n x} = -2K_1(x,x)$$

va (17) tenglikdan q potensialni aniqlash uchun quyidagi formulaga ega bo`lamiz:

$$q^2 + 2cq + \frac{d^2}{dx^2}\ln\Delta = 0. \tag{19}$$

2-§. Sochilish nazariyasi berilganlarining evolyutsiyasi

q potensialni vaqtga bog\`liqligini aniqlaymiz. Buning uchun $r(\zeta_k)$ son vaqt bo'yicha qanday o\`zgarishini tushintirish zarur. Bu bog\`liqlik berilgan $A$ operator Yost funksiyasining vaqt dinamikasi bo\`yicha aniqlanadi:

$$\psi_t = A\psi . \tag{20}$$

$x \to \pm\infty$ da A operator ushbu

$$A \to -4i\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\frac{\partial^3}{\partial x^3} + i\begin{pmatrix} 6c^2 & 0 \\ 0 & 6c^2 \end{pmatrix}\frac{\partial}{\partial x}$$

asimptotikaga ega. $x \to -\infty$ da $f_k(t,x) = f(\zeta_k, x)$ xos funksiya

$$f_k(t) = \begin{pmatrix} 1 \\ (\eta_k + i\zeta_k)/c \end{pmatrix} e^{\eta_k x + i\omega_k t}$$

ga intiladi. $f_k(t)$ funksiya (20) tenglikni qanoatlantirishidan $\omega_k = 4\eta_k{}^3 - 6\eta_k c^2$ ekanligi kelib chiqadi. $x \to +\infty$ da

$$f_k(t,x) = \mu_k(t) g_k(t,x) \to \mu_k(t)\begin{pmatrix} (-\eta_k + i\zeta_k)/c \\ 1 \end{pmatrix} e^{-\eta_k + i\omega_k t}$$

bo'ladi. Endi bu ifodani (20) tenglikka qo\`yib quyidagiga ega bo\`lamiz:

$$\mu_k(t) = \mu_k(0) e^{-12c^2\eta_k + 8_k{}^3 t} .$$

(20) va (7) tengliklardan $\dfrac{da(\zeta)}{dt}$ ifodani topish mumkin, bundan

$$\eta_k = const, \quad r_k(t) = r_k(0) e^{\left(-12c^2\eta_k + 8\eta_k{}^3\right)t}$$

ekanligi kelib chiqadi. Buni hisobga olsak, A va B matritsalarni quyidagicha yozish mumkin:

$$A = \left\| \delta_{nk} + \frac{c}{\eta_k}\frac{r_k(0)}{\eta_k + \eta_n} e^{-(\eta_k+\eta_n)x + \left(8\eta_k{}^3 - 12c^2\eta_k\right)t} \right\|,$$

$$B = \left\| -\frac{r_k(0)}{\eta_k + \eta_n} e^{-(\eta_k+\eta_n)x + \left(8\eta_k{}^3 - 12c^2\eta_k\right)t} \right\|.$$

(19) dan kelib chiqadiki, Δ determinantning ixtiyoriy qator va ustunini ixtiyoriy eksponensial ko`paytuvchiga ko`paytirish mumkin, bu esa q potensial ifodasiga ta'sir etmaydi. Bu Δ determinantni simmetrik formada yozishga imkon beradi.

$$A = \left\| \delta_{nk} + \frac{c}{\eta_k} \frac{\left(r_n(0) r_k(0)\right)^{\frac{1}{2}}}{\left(\eta_n + \eta_k\right)} e^{-\theta(x,t)} \right\|,$$
$$B = \left\| \frac{\left(r_n(0) r_k(0)\right)^{\frac{1}{2}}}{\left(\eta_n + \eta_k\right)} e^{-\theta(x,t)} \right\|, \tag{21}$$

bu yerda $\theta(x,t) = (\eta_k + \eta_n)x' - 4(\eta_k^3 + \eta_n^3)t, \qquad x' = x - 6c^2 t$.

Natijada (19) tenglikda (2) tenglamaning $N-$solitonsimon yechimi uchun quyidagi ifodani olamiz:

$$V = \pm\sqrt{c^2 - \frac{d^2}{dx^2} \ln\begin{vmatrix} A & B \\ B & A \end{vmatrix}}, \tag{22}$$

bu yerda A, B matritsalar (21) formulalardan aniqlanadi va $0 < \eta_k < c$.

L operatorning bir juft $(\zeta, -\zeta)$ xos qiymatlariga mos keluvchi bir solitonli yechimini ko`rib chiqamiz. Bu holda Δ determinant

$$\Delta = \begin{vmatrix} 1 + \frac{c}{\eta}\frac{r(0)}{2\eta} e^{-\theta} & \frac{-r(0)}{2\eta} e^{-\theta} \\ \frac{-r(0)}{2\eta} e^{-\theta} & 1 + \frac{c}{\eta}\frac{r(0)}{2\eta} e^{-\theta} \end{vmatrix} =$$
$$= 1 + \frac{2c}{\eta}\frac{r(0)}{2\eta} e^{-\theta} + \frac{c^2 - \eta^2}{\eta^2}\frac{r^2(0)}{4\eta^2} e^{-2\theta} =$$
$$= e^{-\theta}\left(\frac{\sqrt{c^2 - \eta^2}}{\eta^2} r(0)\right)\left[ch\theta' + \frac{c}{\sqrt{c^2 - \eta^2}}\right]$$

ga teng, bunda

$$\theta = 2\eta x' - 8\eta^3 t, \qquad \theta' = \theta + \ln\left(\frac{r(0)\sqrt{c^2 - \eta^2}}{2\eta^2}\right).$$

Kvadrat qavsdagi ifodani (22) tenglikka qo\`yib, $\Delta$da eksponensial ko\`paytuvchi q ifoda uchun ta\`sir qilmasligini hisobga olsak, quyidagi ko\`rinishda bir solitonli yechimni hosil qilamiz:

$$V = \pm\left[c - \frac{2\eta^2}{\sqrt{c^2-\eta^2}ch\theta' + c}\right].$$

Soliton amplutudasi $\dfrac{2\eta^2}{\sqrt{c^2-\eta^2}+c}$ ga teng va uning qiymati $2c$ dan oshmaydi, jumladan, agar soliton $x \to \pm\infty$ da c ga intilsa, u holda soliton pasayuvchi to\`lqin hisoblanadi, agarda $-c$ ga intilsa, ko\`tariluvchi sath to'lqini bo\`ladi.

3-§. $N-$ soliton yechimning asimptotikasi

$t\to\pm\infty$ da $N-$soliton yechim uchun asimptotik ifoda keltirib chiqamiz, hamda, uning boshqa solitonlar bir-biri bilan o'zaro ta'sirida fazalar siljishi uchun ifoda olamiz. Faraz qilaylik $\eta_1<\eta_2<...<\eta_N$ bo'lsin. Ushbu $f_k=\sqrt{r_k(0)}\exp\left(-\eta_k x'+4\eta_k^3 t\right)$ belgilashni kiritamiz. Bu belgilashlarda A va B matritsalar quyidagi ko`rinishni oladi:

$$A=\left\|\delta_{n_k}+\frac{c}{\eta_k}f_n f_k\alpha_{nk}\right\|,\qquad B=\left\|-f_n f_k\alpha_{nk}\right\|,\quad \alpha_{nk}=\frac{1}{(\eta_n+\eta_k)}$$

$N-$soliton yechim asimptotikasini ushbu

$$4\eta_m^2 t-x'=const,\quad t\to+\infty \tag{23}$$

chiziqda ko`rib chiqamiz, ya'ni $m-$nomerli solitonni tekshirib ko`ramiz. U holda $i<m$ uchun $f_i\to 0$, $i>m$ uchun $f_i\to\infty$, $i=m$ uchun $f_i\to const$ bo`ladi. Bu holda A va B matritsalar quyidagi matritsalarga intiladi.

$$A\to\begin{Vmatrix}\delta_{ik} & 0\\ 0 & \widetilde{A}\end{Vmatrix},\qquad B\to\begin{Vmatrix}0 & 0\\ 0 & \widetilde{B}\end{Vmatrix},$$

bu yerda $\|\delta_{ik}\|$ – $(m-1)$ tartibli kvadrat birlik matritsa, 0 - nol matritsa. $\widetilde{A},\widetilde{B}$ -lar

$$\widetilde{A}=\left\|\delta_{m-1+l,m}\,\delta_{m,m-1+k}+\frac{c}{\eta_{m-1+l}}f_{m-1+l}f_{m-1+k}\alpha_{n-1+l,m-1+k}\right\|,$$

$$\widetilde{B}=\left\|-f_{m-1+l}f_{m-1+k}\alpha_{n-1+l,m-1+k}\right\|,\quad 1\le l,k\le N-m+1.$$

ko`rinishdagi $N-m+1$ tartibli kvadrat matritsalar.

Quyidagi belgilashlarni kiritamiz:

$$\widetilde{A}(m,N)=\left\|\frac{c}{\eta_{m-1+l}}f_{m-1+l}f_{m-1+k}\alpha_{n-1+l,m-1+k}+\delta_{m-1+l,m}\,\delta_{m,m-1+k}\right\|,\quad 1\le l,\ k\le N-m+1,$$

$$\widetilde{B}(m,N)=\left\|-f_{m-1+l}f_{m-1+k}\alpha_{n-1+l,m-1+k}\right\|,\qquad 1\le l,\ k\le N-m+1.$$

Ko`rinib turibdiki, (23) shart bajarilsa, determinant quyidagiga intiladi:

$$\Delta' = \begin{vmatrix} \delta_{m-1+l}\delta_{m,m-1+k} + \hat{A}(m,N) & \hat{B}(m,N) \\ \hat{B}(m,N) & \delta_{m-1+l}\delta_{m,m-1+k} + \hat{A}(m,N) \end{vmatrix} =$$

$$= \begin{vmatrix} \hat{A}(m+1,N) & \hat{B}(m+1,N) \\ \hat{B}(m+1,N) & \hat{A}(m+1,N) \end{vmatrix} + \begin{vmatrix} A(m,N) & B(m,N) \\ B(m,N) & A(m,N) \end{vmatrix} +$$

$$+2\begin{vmatrix} \frac{c}{\eta_m} f_m^2 \quad \ldots & \frac{c}{\eta_N} f_m f_N a_{mN} & -f_m f_{m+1} a_{mm+1} - \ldots - f_m f_N a_{mN} \\ \frac{c}{\eta_N} f_m f_N a_{mN} & \hat{A}(m+1,N) & \hat{B}(m+1,N) \\ \ldots\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots \\ -f_m f_{m+1} a_{mm+1} & & \\ \ldots\ldots\ldots\ldots & \hat{B}(m+1,N) & \hat{A}(m+1,N) \\ -f_m f_N a_{mN} & & \end{vmatrix},$$

$$\Delta' = \prod_{k=m+1}^{N} f_k^2 \left\{ f_m^4 \prod_{k=m}^{N} \left(\frac{c^2}{\eta_k^2} - 1 \right) |\alpha_{ik}|_{m^2} + 2 f_m^2 \frac{c^2}{\eta_m} \prod_{k=m+1}^{N} \left(\frac{c^2}{\eta_k^2} - 1 \right) |\alpha_{ik}|_m |\alpha_{ik}|_{m+1} + \right.$$

$$\left. + \prod_{k=m+1}^{N} \left(\frac{c^2}{\eta_k^2} - 1 \right) |\alpha_{ik}|_{m+1} \right\}$$

$$|\alpha_{ik}|_m = \left| \frac{1}{\eta_l + \eta_k} \right|, \quad m \le l, \quad k \le N.$$

$|\alpha_{ik}|_m$ determinantni hisoblab, quyidagiga ega bo`lamiz:

$$\Delta' = \prod_{k=m+1}^{N} \left(\frac{c^2}{\eta_k^2} - 1 \right) f_k^2 |\alpha_{ik}|^2_{m+1} \left\{ f_m^4 \left(\frac{c^2}{\eta_m^2} - 1 \right) \left(\frac{1}{2\eta_m} \right)^2 \prod_{l=m+1}^{N} \left(\frac{\eta_l - \eta_n}{\eta_l + \eta_m} \right)^4 + \right.$$

$$\left. + \frac{2c}{\eta_m} \frac{1}{2\eta_m} f_m^2 \prod_{l=m+1}^{N} \left(\frac{\eta_l - \eta_n}{\eta_l + \eta_m} \right)^2 + 1 \right\},$$

bundan kelib chiqadiki, eksponensial ko`paytuvchi aniqligida (23) shart bajarilganda $\Delta$ asimptotikasi quyidagi ko`rinishda bo'ladi:

$$\Delta' = \left[ch\theta'_m + \frac{c}{\sqrt{c^2 - \eta_m^2}} \right],$$

bu yerda

$$\theta_m = 2\eta_m x' - 8\eta_m^3 t + \ln\frac{r_m(0)\sqrt{c^2-\eta_m^2}}{2\eta_m^2} + 2\ln\prod_{i=m+1}^{N}\left(\frac{\eta_i-\eta_m}{\eta_i+\eta_m}\right).$$

(23) to'g'ri chiziq atrofida asimptotik yechim soliton bo`lib, u bir solitonli statsionar yechimdan fazasi bilan farq qiladi. Yuqoridagi analogik tekshirishlar shuni ko`rsatadiki, ushbu

$$t \to \infty, \quad x' - 4\eta_m^2 t = const,$$

shartlar bajarilganda

$$\Delta' \to ch\theta_m'' + \frac{c}{\sqrt{c^2-\eta_m^2}}$$

bo'ladi, bunda

$$\theta_m'' = 2\eta_m x' - 8\eta_m^3 t + \ln\frac{r_m(0)\sqrt{c^2-\eta_m^2}}{2\eta_m^2} + 2\ln\prod_{i=1}^{m-1}\left(\frac{\eta_m-\eta_i}{\eta_m+\eta_i}\right).$$

Bundan kelib chiqadiki $m-$solitonni boshqa solitonlar bilan o'zaro ta'sirida fazalar siljishi δ_m ni

$$\delta_m = 2\prod_{i=m+1}^{N}\ln\left(\frac{\eta_i-\eta_m}{\eta_i+\eta_m}\right) - 2\prod_{i=1}^{m-1}\ln\left(\frac{\eta_m-\eta_i}{\eta_m+\eta_i}\right)$$

ko`rinishda aniqlash mumkin. Bu formula, KdF tenglamasidagi fazalar siljishi va $a$ ning musbat qiymatlarida modifitsirlangan KdF tenglamasi uchun olingan formula bilan aynan ustma-ust tushadi. Farq faqat $0 < \eta < c$ qo`shimcha shartda yakunlanadi. (23) tenglikdan katta vaqtlar uchun $N-$soliton yechim asimptotikasi ham kelib chiqadiki, u quyidagicha bo`ladi:

$$V = \pm\left[C - \sum_{m=1}^{N}\frac{2\eta_m^2}{\sqrt{c^2-\eta_m^2}ch\theta_m' + c}\right].$$

Xulosa

Ushbu bitiruv malakaviy ishida mKdF tenglamasining "chekli zichlik" ka ega bo`lgan funksiyalar sinfida sochilish nazariyasining teskari masalasi usulida solitonsimon yechimlari topilgan.

1) O'z-o'ziga qo'shma bo'lgan Dirak operatori uchun sochilish nazariyasining to'g'ri va teskari teskari masalasi o'rganilgan;

2) Potensiali mKdF tenglamasini qanoatlantiruvchi Dirak operatorining sochilish nazariyasining berilganlari evolyutsiyasi topilgan;

3) N-Soliton yechimlar topilgan;

4) Soliton yechimlarning katta t lardagi asimptotikasi o'rganilgan.

Olingan natijalar solitonlarning o'zaro ta'sirini o'rganishda qo'llaniladi.

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