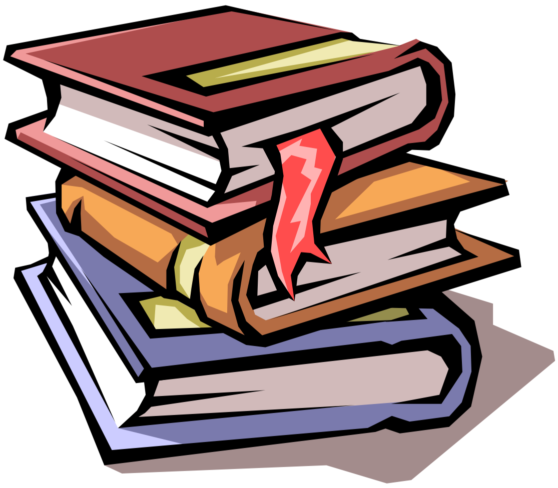
O’ZBEKISTON RESPUBLIKASI OLIY VA
O’RTA MAXSUS TA’LIM VAZIRLIGI

AL-XORAZMIY NOMIDAGI
URGANCH DAVLAT UNIVERSITETI

Saparbayeva D.R.

«Analitik geometriya va chiziqli algebra» fanidan laboratoriya ishlari



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Saparbayeva D.R.

“Analitik geometriya va chiziqli algebra” fanidan laboratoriya ishlari O’zbekiston Respublikasi oliy va o'rta maxsus Ta’lim Vazirligining Oliy ta’lim bo’yicha o’quv-uslubiy boshqarmasi tasdiqlagan o’quv dastur asosida yozilgan.

“Analitik geometriya va chiziqli algebra” fanidan laboratoriya ishlari mualliflar, talabalar shuningdek “Analitik geometriya va chiziqli algebra” fanini mustaqil o’rganuvchilar uchun ham foydali bo’ladi.

Ushbu uslubiy tavsiyanoma universitet uslubiy kengashi qaroriga asosan tasdiqlangan.

So`z boshi

Respublikamizda ta’lim sohasida olib borilayotgan keng islohatlar, talabalarni bilimini oshirish va mustaqil fikr yuritadigan mutaxassis bo`lib etishishlari maqsadida, o`qitilayotgan har bir fandan darslik, o`quv qo`llanma, uslubiy qo`llanma, masalalar to`plami, laboratoriya ishlaridan topshiriqlar va ko`rsatmalar majmuasi, tarqatma materiallar hamda ularning elektron variantlarini yoratishni taqozo qiladi.

Bitkazuvchi talaba o`z ixtisosligi bo`yicha fanlarning zamonaviy yutuqlaridan foydalana olishi uchun u o`qishning boshidanoq bu fanlarning asoslarini mustahkam egallamog`i zarur. Analitik geometriya va chiziqli algebra ana shunday asosiy fanlardan ekanligiga shak-shubha yo`q.

Mazkur majmua « Analitik geometriya va chiziqli algebra » fanidan o`tkaziladigan laboratoriya ishlariga bag`ishlangan bo`lib, u «Amaliy matematika va informatika» yo`nalishida tahsil olayotgan talabalarga mo`ljallangan. Unda har bir laboratoriya uchun kerakli nazariy ma’lumotlar keltirilgan, laboratoriyani bajarish namunalari ko`rsatilgan, har bir topshirik 20 dan ortiq variantga bo`lingan. Bundan tashqari bu majmuadan, ishni bajarish tartibi, nazorat uchun savollar va foydalanilgan adabiyotlar ro`yxati o`rin olgan.

Ushbu majmuani yaxshilashga qaratilgan har qanday tanqidiy fikr, mulohaza va tavsiyalarni muallif mamnuniyat bilan qa’bul qiladi va oldindan o`z minnatdorchiligini bildiradi.

1- laboratoriya ishi.
Mavzu: Chiziqli tenglamalar sistemalari. Teskari matritsani topish.

Ishdan maqsad: Chiziqli tenglamalar sistemasini Gauss usulidan, matritsa usulidan va Kramer formulasidan foydalanib yechishni o’rganish, yana teskari matrisani topishni ham o’rganishdan iborat.

Nazariy qism:

Uch noma’lumli uchta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1, \\ a_{21}x + a_{22}y + a_{23}z = b_2, \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

ning bosh determinanti

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

Bo’lganda yagona yechimga ega bo’lib, bu yechim Kramer formulalari bilan hisoblanadi:

$$x = \frac{\Delta_x}{\Delta}, \quad y = \frac{\Delta_y}{\Delta}, \quad z = \frac{\Delta_z}{\Delta},$$

bunda

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}, \quad \Delta_y = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}, \quad \Delta_z = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}.$$

Agar $\Delta = 0$ va $\Delta_x, \ \Delta_y, \ \Delta_z$ determinantlardan aqalli bittasi noldan farqli bo’lsa, u holda berilgan sistema yechimga ega bo’lmaydi va bu sistema *birgalikda bo’lmagan sistema* deb ataladi. Kamida bitta yechimga ega bo’lgan sistema *birgalikdagi sistema* deb ataladi.

n ta noma’lumli n ta chiziqli tenglamalar sistemasini n ning katta $(n \geq 4)$ qiymatlarida Kramer qoidasi bilan yechish bir nechta yuqori tartibli determinantlarni hisoblashni talab etadi. Shu sababli, bunday sistemalarni yechishda Gauss usulidan foydalanish maqsadga muvofiq. Bu usulning mohiyati shundan iboratki, unda noma’lumlar ketma-ket yo’qotilib, sistema uchburchaksimon shaklga keltiriladi. Agar sistema uchburchaksimon shaklga kelsa, u yagona yechimga ega bo’ladi va uning noma’lumlari oxirgi tenglamadan boshlab topib boriladi. (Sistema cheksiz ko’p yechimga ega bo’lsa, noma’lumlar ketma-ket yo’qotilgach, u trapetsiyasimon shaklga keladi.)

Teskari matritsa. Agar kvadrat matritsa maxsusmas bo'lsa, u holda $AA^{-1} = A^{-1}A = E$ tenglikni qanoatlantiruvchi yagona A^{-1} matritsa mavjud bo'ladi va u A matrisaga *teskari matritsa* deyiladi. A matrisaning A^{-1} teskari matrisasi quyidagicha aniqlanadi:

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} & . & . & . & A_{n1} \\ A_{12} & A_{22} & . & . & . & A_{n2} \\ . & . & . & . & . & . \\ A_{1n} & A_{2n} & . & . & . & A_{nn} \end{pmatrix}.$$

Bu yerda A_{ik} A matritsa determinanti a_{ik} elementining algebraik to'ldiruvchisi.

n ta noma'lumli n ta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n = b_n \end{cases}$$

ni matritsa ko'rinishda

$$AX = B$$

kabi yozish mumkin, bunda

$$A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{pmatrix},$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix}.$$

Agar A maxsusmas matritsa, ya'ni $\det A \neq 0$ bo'lsa, u holda bu sistemaning matritsa shaklidagi yechimi Ushbu ko'rinishga ega bo'ladi:

$$X = A^{-1}B.$$

Masalaning qo'yilishi.

1) Ushbu

$$\begin{cases} x - 2y + z = -4, \\ 3x + 2y - z = 8, \\ 2x - 3y + 2z = -6 \end{cases}$$

chiziqli tenglamalar sistemasini yeching.

2) Ushbu

$$\begin{cases} x+y+5z+2t=1, \\ x+y+3z+4t=-3, \\ 2x+3y+11z+5t=2, \\ 2x+y+3z+2t=-3 \end{cases}$$

chiziqli tenglamalar sistemasini Gauss usuli bilan yeching.

3) Berilgan

$$A=\begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & -2 & 5 \end{pmatrix}$$

matritsaga teskari matritsani toping.

Yechilishi: 1) Determinantlarni topamiz:

$$\Delta=\begin{vmatrix} 1 & -2 & 1 \\ 3 & 2 & -1 \\ 2 & -3 & 2 \end{vmatrix}=1\cdot 2\cdot 2+3\cdot(-3)\cdot 1+2\cdot(-2)\cdot(-1)-[2\cdot 2\cdot 1+1\cdot(-3)\cdot(-1)+2\cdot 3\cdot(-2)]=-1-(-5)=4.$$

Determinant $\Delta=4\neq 0$ bo'lgani uchun sistema yagona yechimga ega va Kramer formulasini qo'llab, uni topamiz:

$$\Delta_x=\begin{vmatrix} -4 & -2 & 1 \\ 8 & 2 & -1 \\ -6 & -3 & 2 \end{vmatrix}=-4\cdot 2\cdot 2+8\cdot(-3)\cdot 1+(-6)\cdot(-2)\cdot(-1)-[(-6)\cdot 2\cdot 1+(-4)\cdot(-3)\cdot(-1)+2\cdot 8\cdot(-2)]=$$
$$=-52-(-56)=4;$$

$$\Delta_y=\begin{vmatrix} 1 & -4 & 1 \\ 3 & 8 & -1 \\ 2 & -6 & 2 \end{vmatrix}=1\cdot 8\cdot 2+3\cdot(-6)\cdot 1+2\cdot(-4)\cdot(-1)-[2\cdot 8\cdot 1+1\cdot(-6)\cdot(-1)+2\cdot 3\cdot(-4)]=$$
$$=6-(-2)=8;$$

$$\Delta_z=\begin{vmatrix} 1 & -2 & -4 \\ 3 & 2 & 8 \\ 2 & -3 & -6 \end{vmatrix}=1\cdot 2\cdot(-6)+3\cdot(-3)\cdot(-4)+2\cdot(-2)\cdot 8-[2\cdot 2\cdot(-4)+1\cdot(-3)\cdot 8+(-6)\cdot 3\cdot(-2)]=$$
$$=-8-(-4)=-4.$$

$$x=\frac{\Delta_x}{\Delta}=\frac{4}{4}=1,\quad y=\frac{\Delta_y}{\Delta}=\frac{8}{4}=2,\quad z=\frac{\Delta_z}{\Delta}=\frac{-4}{4}=-1$$

J: $x=1,\quad y=2,\quad z=-1.$

2) Ikkinchi, uchinchi, to'rtinchi tenlamalardan *x* larni yo'qotamiz. Buning uchun birinchi tenglamani ketma-ket -1, -2, -2 ga ko'paytiramiz va mos ravishda ikkinchi, uchinchi, to'rtinchi tenglamalar bilan qo'shamiz. Natijada ushbu sistemaga ega bo'lamiz:

$$\begin{cases} x+y+5z+2t=1, \\ 2z-2t=4, \\ y+z+t=0, \\ -y-7z-2t=-5, \end{cases}$$

yoki

$$\begin{cases} x+y+5z+2t=1, \\ y+z+t=0, \\ y+7z+2t=5, \\ z-t=2. \end{cases}$$

Uchinchi tenglamadan ikkinchi tenglamani ayiramiz:

$$\begin{cases} x+y+5z+2t=1, \\ y+z+t=0, \\ 6z+t=5, \\ z-t=2, \end{cases}$$

so'ngra to'rtinchi tenglamani -6 ga ko'paytirib, uchinchi tenglamaga qo'shsak, uchburchakli sistema hosil bo'ladi:

$$\begin{cases} x+y+5z+2t=1, \\ y+z+t=0, \\ z-t=2, \\ 7t=-7. \end{cases}$$

Bundan,

$$\begin{aligned} &t=-1, \\ &z=2+t=1, \\ &y=-z-t=0, \\ &x=1-y-5z-2t=-2. \end{aligned}$$

J: $x=-2,\quad y=0,\quad z=1,\quad t=-1$.

3) Matritsaning determinantini hisoblaymiz:

$$\det A=\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & -2 & 5 \end{vmatrix}=6+16+4-30=-4\neq 0.$$

Demak, A matritsa maxsusmas matritsa ekan. Endi A_{ik} algebraik to'ldiruvchilarni hisoblaymiz:

$$A_{11}=\begin{vmatrix}0 & 2\\ -2 & 5\end{vmatrix}=4; \qquad A_{21}=-\begin{vmatrix}2 & -1\\ -2 & 5\end{vmatrix}=-8; \qquad A_{31}=\begin{vmatrix}2 & -1\\ 0 & 2\end{vmatrix}=4;$$

$$A_{12}=-\begin{vmatrix}3 & 2\\ 4 & 5\end{vmatrix}=-7; \qquad A_{22}=\begin{vmatrix}1 & -1\\ 4 & 5\end{vmatrix}=9; \qquad A_{32}=-\begin{vmatrix}1 & -1\\ 3 & 2\end{vmatrix}=-5;$$

$$A_{13}=\begin{vmatrix}3 & 0\\ 4 & -2\end{vmatrix}=-6; \qquad A_{23}=-\begin{vmatrix}1 & 2\\ 4 & -2\end{vmatrix}=10; \qquad A_{33}=\begin{vmatrix}1 & 2\\ 3 & 0\end{vmatrix}=-6.$$

Teskari matritsani tuzamiz:

$$A^{-1}=-\frac{1}{4}\begin{pmatrix}4 & -8 & 4\\ -7 & 9 & -5\\ -6 & 10 & -6\end{pmatrix}=\begin{pmatrix}-1 & 2 & -1\\ \frac{7}{4} & -\frac{9}{4} & \frac{5}{4}\\ \frac{3}{2} & -\frac{5}{2} & \frac{3}{2}\end{pmatrix}.$$

$AA^{-1}=A^{-1}A=E$ ekanini tekshirish mumkin.

V A R I A N T L A R

1-topshiriq. Berilgan tenglamalar sistemasining birgalikda ekanligini tekshiring, agar birgalikda bo'lsa, ularni:

A) Kramer qoidasidan foydalanib,

B) Gauss usuli bilan,

C) matritsa usuli bilan yeching.

1. $\begin{cases}3x+2y+z=5,\\ 2x+3y+z=1,\\ 2x+y+3z=11.\end{cases}$ **2.** $\begin{cases}4x-3y+2z=9,\\ 2x+5y-3z=4,\\ 5x+6y+2z=18.\end{cases}$ **3.** $\begin{cases}2x-y-z=4,\\ 3x+4y-2z=11,\\ 3x-2y+4z=11.\end{cases}$

4. $\begin{cases}x+y-z=1,\\ 8x+3y-6z=2,\\ -4x-y+3z=-3.\end{cases}$ **5.** $\begin{cases}7x-5y=31,\\ 4x+11z=-43,\\ 2x+3y+4z=-20.\end{cases}$ **6.** $\begin{cases}x-2y+3z=6,\\ 2x+3y-4z=20,\\ 3x-2y-5z=6.\end{cases}$

7. $\begin{cases}x+y+3z=-1,\\ 2x-y+2z=-4,\\ 4x+y+4z=-2.\end{cases}$ **8.** $\begin{cases}3x+4y+2z=8,\\ 2x-y-3z=-1,\\ x+5y+z=-7.\end{cases}$ **9.** $\begin{cases}x-4y-2z=-7,\\ 3x+y-z=5,\\ -3x+5y+6z=7.\end{cases}$

10. $\begin{cases} x+2y+4z=31, \\ 5x+y+2z=20, \\ 3x-y+z=0. \end{cases}$ **11.** $\begin{cases} x+5y+z=-2, \\ 2x-4y-3z=0, \\ 3x+4y+2z=3. \end{cases}$ **12.** $\begin{cases} 2x-3y+2z=-6, \\ 5x+8y-z=0, \\ x+2y+3z=6. \end{cases}$

13. $\begin{cases} x-4y-2z=0, \\ 3x-5y-6z=7, \\ 3x+y+z=6. \end{cases}$ **14.** $\begin{cases} 2x-y+5z=10, \\ 5x+2y-13z=21, \\ 3x-y+5z=12. \end{cases}$ **15.** $\begin{cases} 2x+y-5z=-1, \\ x+y-z=-2, \\ 4x-3y+z=13. \end{cases}$

16. $\begin{cases} 2x+3y+4z=-10, \\ 4x+11z=-29, \\ 7x-5y=7. \end{cases}$ **17.** $\begin{cases} 2x+7y-z=10, \\ 3x-5y+3z=-14, \\ x+2y+z=-1. \end{cases}$ **18.** $\begin{cases} 4x+y-3z=-6, \\ 8x+3y-6z=-15, \\ x+y-z=-4. \end{cases}$

19. $\begin{cases} 3x-2y-5z=-14, \\ x-2y+3z=0, \\ 2x+3y-4z=-10. \end{cases}$ **20.** $\begin{cases} 5x+6y-2z=-9, \\ 2x+5y-3z=-1, \\ 4x-3y+2z=-15. \end{cases}$ **21.** $\begin{cases} 2x+y-z=9, \\ 2x-3y=0, \\ 5x-4y-2z=9. \end{cases}$

22. $\begin{cases} 2x-y+z=3, \\ x+y-z=0, \\ 3x-y+2z=7. \end{cases}$ **23.** $\begin{cases} 2x-3y+2z=-2, \\ 3x+2y-2z=5, \\ 5x-4y+5z=2. \end{cases}$ **24.** $\begin{cases} 3x+2y-2z=3, \\ x+y-z=0, \\ 4x+y-5z=1. \end{cases}$

25. $\begin{cases} x-3y-z=6, \\ 2x+5y-8z=3, \\ x+4y+z=11. \end{cases}$ **26.** $\begin{cases} 4x+2y+z=4, \\ x-3y+z=3, \\ 3x+5y+2z=8. \end{cases}$

2-topshiriq. Bir jinsli tenglamalar sistemasini yeching.

1. $\begin{cases} 3x+4y+2z=0, \\ x-y+4z=0, \\ 5x+2y+10z=0. \end{cases}$ **2.** $\begin{cases} 2x-3y+z=0, \\ x+y+z=0, \\ 3x-2y+2z=0. \end{cases}$ **3.** $\begin{cases} 3x+2y-z=0, \\ 2x-y+3z=0, \\ x+3y-4z=0. \end{cases}$

4. $\begin{cases} 2x-y+3z=0, \\ x+2y-5z=0, \\ 3x+y-2z=0. \end{cases}$ **5.** $\begin{cases} 3x-2y+z=0, \\ 5x-14y+15z=0, \\ x+2y-3z=0. \end{cases}$ **6.** $\begin{cases} 3x+4y+2z=0, \\ x-y+4z=0, \\ 5x+2y+10z=0. \end{cases}$

7. $\begin{cases} 2x-3y+z=0, \\ x+y+z=0, \\ 3x-2y+2z=0. \end{cases}$ **8.** $\begin{cases} 3x+2y-z=0, \\ 2x-y+3z=0, \\ x+3y-4z=0. \end{cases}$ **9.** $\begin{cases} 2x-y+3z=0, \\ x+2y-5z=0, \\ 3x+y-2z=0. \end{cases}$

10. $\begin{cases} 3x-2y+z=0, \\ 4x+3y-5z=0, \\ x+5y-6z=0. \end{cases}$ **11.** $\begin{cases} 2x+2y-3z=0, \\ 4x-2y-9z=0, \\ x-2y+7z=0. \end{cases}$ **12.** $\begin{cases} 3x-3y+2z=0, \\ 4x+3y-5z=0, \\ x+5y-6z=0. \end{cases}$

13. $\begin{cases} 5x-2y+3z=0, \\ 2x+3y-4z=0, \\ 3x-5y+7z=0. \end{cases}$ **14.** $\begin{cases} 3x+y-z=0, \\ 5x+3y-z=0, \\ 4x+2y-z=0. \end{cases}$ **15.** $\begin{cases} x-2y-2z=0, \\ x+3y-5z=0, \\ 2x-y-7z=0. \end{cases}$

16. $\begin{cases} 2x-y-5z=0, \\ 5x+3y-z=0, \\ 4x+2y-z=0. \end{cases}$ **17.** $\begin{cases} 2x-y+4z=0, \\ 2x+3y-5z=0, \\ 4x+2y-z=0. \end{cases}$ **18.** $\begin{cases} 4x-3y-2z=0, \\ 2x-y+2z=0, \\ x-y-2z=0. \end{cases}$

19. $\begin{cases} 2x-3y-3z=0, \\ 2x+5y-z=0, \\ 2x+y-2z=0. \end{cases}$ **20.** $\begin{cases} 7x-y+2z=0, \\ x+3y-4z=0, \\ 3x-2y+3z=0. \end{cases}$ **21.** $\begin{cases} x-3y-2z=0, \\ 3x+y+4z=0, \\ 5x-y+z=0. \end{cases}$

22. $\begin{cases} x-2y+z=0, \\ x+2y-2z=0, \\ 3x+2y-3z=0. \end{cases}$ **23.** $\begin{cases} 3x+y+4z=0, \\ 5x+y+6z=0, \\ -x-7y-8z=0. \end{cases}$ **24.** $\begin{cases} 3x+2y+3z=0, \\ x-4y-z=0, \\ 2x-y+z=0. \end{cases}$

25. $\begin{cases} 2x+4y-z=0, \\ 4x-3y-3z=0, \\ 2x+5y-5z=0. \end{cases}$ **26.** $\begin{cases} 2x+y+7z=0, \\ 2x-3y-5z=0, \\ 2x-y+z=0. \end{cases}$

3-topshiriq. A matrisa berilgan. A^{-1} teskari matrisani toping va $AA^{-1}=A^{-1}A=E$ ekanini tekshiring.

1. $\begin{pmatrix} 1 & 2 & -1 \\ 1 & -2 & 3 \\ 4 & 1 & -4 \end{pmatrix}$ **2.** $\begin{pmatrix} 3 & 2 & -1 \\ 7 & 3 & 0 \\ 1 & 2 & 2 \end{pmatrix}$ **3.** $\begin{pmatrix} 1 & -3 & 5 \\ 2 & 4 & 0 \\ 3 & -3 & -1 \end{pmatrix}$

4. $\begin{pmatrix} -2 & 3 & 3 \\ 4 & 5 & 1 \\ -3 & 4 & 0 \end{pmatrix}$ **5.** $\begin{pmatrix} 0 & 1 & -3 \\ 1 & -5 & 4 \\ 2 & 3 & 2 \end{pmatrix}$ **6.** $\begin{pmatrix} 1 & 1 & 3 \\ 2 & -2 & -5 \\ 1 & 4 & 3 \end{pmatrix}$

7. $\begin{pmatrix} -5 & 7 & -4 \\ 8 & 0 & -1 \\ 4 & -5 & 0 \end{pmatrix}$ **8.** $\begin{pmatrix} -1 & 8 & 1 \\ -1 & 5 & 5 \\ 0 & -1 & 3 \end{pmatrix}$ **9.** $\begin{pmatrix} 4 & -2 & 1 \\ -3 & 4 & 1 \\ 1 & -1 & 1 \end{pmatrix}$

10. $\begin{pmatrix} 1 & 2 & 1 \\ 1 & 9 & 7 \\ 4 & -3 & 1 \end{pmatrix}$ **11.** $\begin{pmatrix} 3 & -3 & 4 \\ -1 & -5 & -7 \\ 0 & -1 & 5 \end{pmatrix}$ **12.** $\begin{pmatrix} 3 & -1 & 4 \\ 7 & 8 & -2 \\ 2 & -3 & 3 \end{pmatrix}$

13. $\begin{pmatrix} 1 & -1 & 8 \\ 1 & -5 & 5 \\ -2 & 3 & 10 \end{pmatrix}$ **14.** $\begin{pmatrix} 2 & 3 & 4 \\ 2 & 1 & 3 \\ -7 & 0 & 2 \end{pmatrix}$ **15.** $\begin{pmatrix} 5 & -1 & 3 \\ 4 & -2 & 0 \\ 2 & -4 & 5 \end{pmatrix}$

16. $\begin{pmatrix} 1 & -3 & -2 \\ -2 & 1 & 3 \\ -2 & 4 & 4 \end{pmatrix}$ **17.** $\begin{pmatrix} 5 & 6 & 4 \\ 2 & 0 & -3 \\ 1 & 3 & 4 \end{pmatrix}$ **18.** $\begin{pmatrix} 3 & 1 & 0 \\ 2 & 2 & 1 \\ 6 & 3 & 7 \end{pmatrix}$

19. $\begin{pmatrix} 4 & 1 & 2 \\ 3 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$ **20.** $\begin{pmatrix} 4 & 2 & 1 \\ 1 & 3 & 3 \\ 3 & 2 & -1 \end{pmatrix}$ **21.** $\begin{pmatrix} 2 & 1 & 5 \\ 1 & 3 & 1 \\ 1 & 4 & 8 \end{pmatrix}$

22. $\begin{pmatrix} 1 & -1 & 3 \\ 2 & 0 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ **23.** $\begin{pmatrix} 8 & 7 & 3 \\ 1 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix}$ **24.** $\begin{pmatrix} -2 & 3 & 0 \\ 1 & 2 & 3 \\ 11 & 5 & 7 \end{pmatrix}$

25. $\begin{pmatrix} 2 & 6 & 3 \\ 4 & 7 & 1 \\ -3 & -8 & -2 \end{pmatrix}$ **26.** $\begin{pmatrix} 4 & 1 & 7 \\ 9 & -1 & 1 \\ 6 & -1 & 10 \end{pmatrix}$

4-topshiriq. Quyidagi matrisaviy tenglamani yeching:

1. $X\begin{pmatrix} 2 & 2 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$ **2.** $X\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 7 \\ 3 & 5 \end{pmatrix}$

3. $X\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$ **4.** $\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}X = \begin{pmatrix} 5 & -2 & 1 \\ 1 & 3 & 0 \end{pmatrix}$

5. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix}X = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 3 & 2 \\ 0 & -1 & 1 \end{pmatrix}$ **6.** $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}X = \begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}$

7. $\begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}X = \begin{pmatrix} 3 & 1 \\ 1 & 5 \end{pmatrix}$ **8.** $\begin{pmatrix} 4 & 2 \\ -3 & -1 \end{pmatrix}X = \begin{pmatrix} 0 & 0 \\ 1 & 5 \end{pmatrix}$

9. $\begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}X = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$ **10.** $\begin{pmatrix} -2 & 1 \\ 3 & -2 \end{pmatrix}X = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

11. $\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}X = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ **12.** $X\begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$

13. $\begin{pmatrix} 3 & 1 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & 4 \end{pmatrix}X = \begin{pmatrix} 0 & -1 & 3 \\ 10 & 0 & 25 \\ -5 & 5 & 0 \end{pmatrix}$ **14.** $\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}X = \begin{pmatrix} -1 & 7 \\ 3 & 5 \end{pmatrix}$

15. $X\begin{pmatrix} -4 & -2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$

16. $\begin{pmatrix} 1 & 2 & -1 \\ 1 & -2 & 3 \\ 4 & 1 & -4 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & -3 & 5 \\ 2 & 4 & 0 \\ 3 & -3 & -1 \end{pmatrix}$

17. $X \cdot \begin{pmatrix} 1 & 2 & 1 \\ 1 & 9 & 7 \\ 4 & -3 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 0 \\ 3 & 4 & -5 \\ 1 & -1 & 2 \end{pmatrix}$

18. $X \cdot \begin{pmatrix} -2 & 3 & 0 \\ 1 & 2 & 3 \\ 11 & 5 & 7 \end{pmatrix} = \begin{pmatrix} 0 & 2 & -3 \\ -1 & 2 & 3 \\ 1 & 0 & -1 \end{pmatrix}$

19. $\begin{pmatrix} 3 & 1 & 0 \\ 2 & 2 & 1 \\ 6 & 3 & 7 \end{pmatrix} \cdot X = \begin{pmatrix} 4 & 3 & -3 \\ 6 & -5 & 4 \\ 2 & 3 & -2 \end{pmatrix}$

20. $X \cdot \begin{pmatrix} 3 & 1 & 0 \\ 2 & 2 & 1 \\ 6 & 3 & 7 \end{pmatrix} = \begin{pmatrix} 2 & -3 & 0 \\ 7 & -4 & 3 \\ -2 & 0 & 1 \end{pmatrix}$

21. $X \cdot \begin{pmatrix} 4 & 2 & 1 \\ 1 & 3 & 3 \\ 3 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 6 & 0 & 6 \\ -2 & 5 & -3 \\ 1 & 8 & -1 \end{pmatrix}$

22. $X \cdot \begin{pmatrix} 3 & -1 & 4 \\ 7 & 8 & -2 \\ 2 & -3 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 5 \\ -2 & 6 & 3 \\ 1 & -5 & 0 \end{pmatrix}$

23. $\begin{pmatrix} -5 & 7 & -4 \\ 8 & 0 & -1 \\ 4 & -5 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 4 & 3 & 5 \\ 6 & 7 & 1 \\ 9 & 1 & 8 \end{pmatrix}$

24. $\begin{pmatrix} 1 & 1 & 3 \\ 2 & -2 & -5 \\ 1 & 4 & 3 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & -4 & 3 \\ 0 & 5 & 6 \\ 8 & 7 & -4 \end{pmatrix}$

25. $X \cdot \begin{pmatrix} 5 & 6 & 4 \\ 2 & 0 & -3 \\ 1 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 6 & 7 \\ 3 & -5 & 9 \\ -2 & 4 & 3 \end{pmatrix}$

26. $\begin{pmatrix} -2 & 3 & 0 \\ 1 & 2 & 3 \\ 11 & 5 & 7 \end{pmatrix} \cdot X = \begin{pmatrix} 6 & 2 & 3 \\ 7 & -1 & -4 \\ -3 & 0 & 5 \end{pmatrix}$

Ishni bajarish tartibi.

1. Talaba laboratoriya ishi bilan tanishadi.
2. Laboratoriya ishi bo’yicha hisobot tayyorlaydi.
3. Nazorat savollariga javob beradi.

Nazorat savollari.

1. Chiziqli tenglamalar sistemasi deb nimaga aytiladi?
2. Yechiladigan sistema deb nimaga aytiladi?
3. Yechilmaydigan sistema deb nimaga aytiladi?
4. Qanday sistemaga bir jinsli sistema deyiladi?
5. Qanday tenglamalarga mustaqil tenglamalar deyiladi?
6. Kramer formulasini yozib bering?
7. Gauss usuli qanday usul?
8. Maxsus matritsa nima?
9. Maxsus matritsa uchun teskari matritsa mavjudmi?
10. Qanday matritsaga biriktirilgan matritsa deyiladi?
11. Kvadratmas matritsani teskarisini topish mumkinmi?

Foydalanilgan adabiyotlar.

1. A.G.Kurosh. «Oliy algebra kursi». Toshkent, O'qituvchi, 1976.
2. R.Iskandarov. «Oliy algebra», I-qism. Toshkent, 1963.
3. D.K.Fadeev, I.S.Sominskiy. «Sbornik zadach po vыsshey algebre». Moskva, Nauka, 1977.
4. I.V.Proskuryakov. «Sbornik zadach po lineynoy algebre». Moskva, Nauka, 1984.
5. Yo.U.Soatov. «Oliy matematika». III-qism. Toshkent, O'zbekiston, 1996.

2 - laboratoriya ishi.
Mavzu: Kompleks sonlarning algebraik va trigonometrik shakli.
Kompleks sonlardan ildiz chiqarish

Ishdan maqsad: Kompleks sonlarning algebraik, trigonometrik shaklini hamda kompleks sondan ildiz chiqarishni o'rganish.

Nazariy qism:

Kompleks sonning algebraik formasi. Ushbu

$$a+bi$$

ko'rinishdagi sonlar *kompleks sonlar* deyiladi, bu yerda a, b lar haqiqiy sonlardir. Kompleks sonlarni bitta harf bilan ham belgilash mumkin, masalan:

$$\alpha = a+bi,$$

bu erda a son α ning *haqiqiy*, b esa *mavhum* qismi deb ataladi va mos ravishda

$$\operatorname{Re}\alpha = a \text{ va } \operatorname{Im}\alpha = b$$

ko'rinishda yoziladi (latincha *reabis* – haqiqiy, *imaginarius* – mavhum demakdir). Masalan:

$$\operatorname{Re}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = -\frac{\sqrt{3}}{2}; \qquad \operatorname{Im}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = \frac{1}{2}.$$

$a+bi$ son kompleks sonning *algebraik formasi (shakli)* deyiladi.

Kompleks sonlar ustida to'rt amal. Agar berilgan

$$\alpha = a+bi \qquad \text{va} \qquad \beta = c+di$$

kompleks sonlarda

$$a=c \text{ va } b=d$$

bo'lsa, bu kompleks sonlar teng deyiladi. Bu $\alpha=\beta$ kabi yoziladi.

Har qanday ikkita kompleks soyni quyidagicha qo'shish va ayirish qabul qilingan:

$$\alpha \pm \beta = (a+bi) \pm (c+di) = (a \pm c) + (b \pm d)i$$

Masalan:

$$(5-2i)+(8-7i) = (5+8)+i(-2-7) = 13-9i;$$

$$(2+i\sqrt{3})-(6-i) = (2-6)+i(\sqrt{3}+1) = -4+i(1+\sqrt{3}).$$

Har qanday ikkita kompleks sonni ko'paytirish ko'phadlarni ko'paytirish qoidasiga binoan bajariladi, ya'ni

$$\alpha \cdot \beta = (a+bi)(c+di) = ac+adi+bci+bdi^2.$$

Ma'lumki, $i^2=-1$, shuning uchun

$$\alpha\beta = (a+bi)(c+di) = (ac-bd)+(ad+bc)i.$$

Quyidagi

$$\alpha = a + bi \quad \text{va} \quad \overline{\alpha} = a - bi$$

kompleks sonlar o'zaro *qo'shma* sonlar deyiladi. Qo'shma sonlarning yig'indisi ham, ko'paytmasi ham haqiqiy son bo'ladi:

$$\alpha + \overline{\alpha} = 2a \quad \text{va} \quad \alpha\overline{\alpha} = a^2 + b^2.$$

Masalan:

$$(3+4i)+(3-4i)=6,$$

$$(3+4i)(3-4i)=9+16=25.$$

Ikkita $\alpha = a+bi$ va $\beta = c+di$ kompleks sonning birini ikkinchisiga bo'lish uchun kasrning surat va maxrajini $\overline{\beta} = c-di$ qo'shma kompleks songa ko'paytiramiz. U holda

$$\frac{\alpha}{\beta} = \frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{(ac+bd)+i(bc-ad)}{c^2+d^2} = \frac{ac+bd}{c^2+d^2} + i\frac{bc-ad}{c^2+d^2}.$$

Masalan:

$$\frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{1+2i+i^2}{1^2+1^2} = \frac{2i}{2} = i;$$

$$\frac{1}{i} = \frac{-i}{i(-i)} = \frac{-i}{1} = -i.$$

Kompleks sonning trigonometrik shakli. $\alpha = a+bi$ kompleks sonning ikki xil geometrik ma'nosi bor:

a) u tekislikda (a, b) nuqtani tasvirlaydi;

b) u (0, 0) nuqta bilan (a, b) nuqtani tutashtiruvchi vektorni tasvirlaydi (1-shakl).

Ma'lumki, tekislikda har bir kompleks songa bitta nuqta va aksincha, har bir nuqtaga bitta kompleks son mos keladi. 1-shakldan quyidagilarni ko'rish mumkin:

$$a = r\cos\varphi, \; b = r\sin\varphi, \; r = \sqrt{a^2+b^2}, \qquad (2.1)$$

$$tg\varphi = \frac{b}{a}, \quad \varphi = arctg\frac{b}{a}.$$

Bu yerda ishtirok etayotgan r va $\alpha = a + bi$ ning *moduli*, φ esa uning *argumenti* (yoki fazasi) deyilib, quyidagicha yoziladi:

$$r = |\alpha| = |a + bi| = \sqrt{a^2 + b^2} \quad \text{va} \quad \varphi = \arg \alpha.$$

Shakldan ko'rinadiki,

$$0 \le r < +\infty \quad \text{va} \quad 0 \le \varphi < 2\pi;$$

ba'zan $-\pi < \varphi < \pi$ chegaralari ham ishlatilib, ikala chegara ham bir maqsadga olib keladi. (2.1) ga muvofiq α kompleks son ushbu

$$\alpha = a + bi = r(\cos\varphi + i\sin\varphi) \qquad (2.2)$$

ko'rinishga ega bo'lib, u kompleks sonning *trigonometrik shakli* deyiladi. Kompleks sonlar ustida amallar bajarishda asosan (2.2) forma ishlatiladi.

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$$e^{ix} = \cos x + i\sin x$$

formulasidan foydalanib, (2.2) ni

$$\alpha = re^{i\varphi} \qquad (2.3)$$

ko'rinishda yozish mumkin. (2.3) kompleks sonning *ko'rsatkichli shakli* deyiladi.

Ta'rif. α kompleks son deb ma'lum bir tartibda berilgan a va b haqiqiy sonlar juftiga aytiladi:

$$\alpha = (a,\, b).$$

Shunday qilib, bitta kompleks sonni quyidagicha

$$\alpha = a + bi, \qquad \alpha = r(\cos\varphi + i\sin\varphi), \qquad \alpha = re^{i\varphi}$$

uch xil shaklda yozish mumkin bo'lib, so'nggi ikkita ifoda ko'pincha qo'l keladi. Har qanday kompleks sonni bir shakldan ikkinchi shaklga o'tkazish mumkin. Buning uchun (8.1) munosabatlardan foydalanishga to'g'ri keladi. Ko'pincha kompleks sonni trigonometrik shaklga keltirish talab qilinadi, shunday vaqtlarda 1-shaklga murojaat qilinib, kompleks songa tegishli vektorning qaysi chorakda yotishiga e'tibor qilish tavsiya etiladi.

Izoh. φ burchakka uning davrini ham qo'shib, quyidagicha yoziladi:

$$Arg\alpha = \arg\alpha + 2\pi k = \varphi + 2\pi k, \qquad k = 0,\ \pm 1,\ \pm 2, \ldots$$

Kompleks sonni ko'paytirish va darajaga ko'tarish. Trigonometrik shaklda berilgan kompleks sonlarni o'zaro ko'paytirish va darajaga ko'tarish quyidagicha bajariladi:

$$\alpha \cdot \beta = r(\cos\varphi + i\sin\varphi) \cdot \rho(\cos\theta + i\sin\theta) = r\rho[\cos(\varphi + \theta) + i\sin(\varphi + \theta)],$$

ya'ni ikki kompleks sonning ko'paytmasi yana kompleks son bo'lib, uning moduli ko'paytuvchilar modullarining ko'paytmasiga, argumenti ko'paytuvchilar argumentlarining yig'indisiga tengdir.

Agar $\alpha = \beta$ bo'lsa,

$$\alpha^2 = [r(\cos\varphi + i\sin\varphi)]^2 = r^2(\cos 2\varphi + i\sin 2\varphi).$$

Umuman

$$\alpha^n = [r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi). \tag{2.4}$$

Xususan α ning moduli $r = 1$ bo'lsa, (8.4) dan:

$$(\cos\varphi + i\sin\varphi)^n = \cos n\varphi + i\sin n\varphi. \tag{2.5}$$

Bu tenglik *Muavr formulasi* deb ataladi. Buning chap tomonidagi qavslarni Nyuton binomi bo'yicha ochib, so'ngra tenglikning ikki tomonidagi haqiqiy qismlarni o'zaro hamda mavhum qismlarni o'zaro tenglashtirish natijasida trigonometriyaga doir turli formulalarni keltirib chiqarish mumkin.Misol uchun (2.5) da $n = 2$ bo'lsin, u holda

$$\cos^2\varphi + 2i\sin\varphi\cos\varphi - \sin^2\varphi = \cos 2\varphi + i\sin 2\varphi,$$

bu erdan

$$\cos 2\varphi = \cos^2\varphi - \sin^2\varphi \quad \text{va} \quad \sin 2\varphi = 2\sin\varphi\cos\varphi$$

va h.k.

Agar algebraik shaklda berilgan kompleks sonni darajaga ko'tarish talab qilinsa, dastlab uni trigonometrik shaklga keltirib olish maqsadga muvofiqdir.

Misol. $(1+i)^{25}$ ni hisoblang.

Yechilishi: Dastlab $1+i$ ni trigonometrik shaklga keltirib olamiz:

$$1+i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right).$$

Endi (8.4) formulaga muvofiq:

$$(1+i)^{25} = \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{25} = \left(\sqrt{2}\right)^{25}\left(\cos\frac{25\pi}{4} + i\sin\frac{25\pi}{4}\right) =$$

$$= \left(\sqrt{2}\right)^{25}\left(\cos\left(6\pi + \frac{\pi}{4}\right) + i\sin\left(6\pi + \frac{\pi}{4}\right)\right) = \left(\sqrt{2}\right)^{25}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) =$$

$$= \left(\sqrt{2}\right)^{25}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = 2^{12}(1+i).$$

Demak,

$$(1+i)^{25} = 2^{12}(1+i).$$

Kompleks sonlardan ildiz chiqarish. Agar $a+bi$ kompleks sonning n – darajali ildizlarini topish lozim bo'lsa, dastlab uni trigonometrik shaklga keltirib olish zarur.

Kompleks sondan n – darajali ildiz chiqarish formulasi quyidagidan iborat:

$$\alpha_k = \sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \sqrt[n]{r}\left(\cos\frac{\varphi+2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n}\right), \qquad (2.6)$$

$k = 0, 1, 2, ..., n-1.$ Ma'lumki, $\cos\frac{\varphi+2k\pi}{n} = \cos\frac{\varphi+2(k+n)\pi}{n}$, shuning uchun $k = n, n+1, ...,$ deb faraz qilinsa, oldingi ildiz kelib chiqaveradi. Shunday qilib,

$\alpha_0, \alpha_1, \alpha_2, ..., \alpha_{n-1}$

ildizlar hosil bo'ladi, ya'ni $a+bi$ sonning n – darajali ildizlari faqat n ta bo'ladi $(n = 2, 3, 4, ...)$.

Xususiy holda (2.6) formuladan 1 ning ildizlari kelib chiqadi:

$$\beta_k = \sqrt[n]{1} = \sqrt[n]{\cos(0+2k\pi) + i\sin(0+2k\pi)} = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}, \qquad (2.7)$$

$k = 0, 1, 2, ..., n-1.$ Masalan:

a) $\beta_k = \sqrt{1} = \cos k\pi + i\sin k\pi,\ \beta_0 = \cos 0 + i\sin 0 = 1,\ \beta_1 = \cos\pi + i\sin\pi = -1.$

b) $\beta_k = \sqrt[3]{1} = \cos\frac{2k\pi}{3} + i\sin\frac{2k\pi}{3},\ \beta_0 = 1,\ \beta_1 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2},$

$\beta_2 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}.$

V A R I A N T L A R

1-topshiriq. Quyida berilgan kompleks sonlarni trigonometrik shaklga keltiring.

1. $(2+2i\sqrt{3})^{10}$ **2.** $(5-12i)^{12}$ **3.** $(2+\sqrt{3}+i)^5$ **4.** $(-3-3i)^6$

5. $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^{18}$ **6.** $(1+i\sqrt{2})^{14}$ **7.** $\left(\frac{2}{3} + \frac{2}{3}i\right)^8$ **8.** $\left(-\frac{1}{2} + \frac{1}{2}i\right)^{16}$

9. $(\sqrt{3}-i)^{10}$ **10.** $(12i)^{20}$ **11.** $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^8$ **12.** $(-1+i\sqrt{3})^8$

13. $(12+5i)^6$ **14.** $(3i+4)^4$ **15.** $(-2-2i\sqrt{3})^9$ **16.** $\left(\frac{1}{3} - i\frac{1}{3}\right)^6$

17. $(8+6i)^5$ **18.** $(-15+8i)^7$ **19.** $(-3-4i)^{10}$ **20.** $(-1+i\sqrt{3})^{15}$

21. $(1+i)^{20}$ **22.** $(2-2i)^{15}$ **23.** $\left(\frac{2}{1+i}\right)^{4}$ **24.** $(-\sqrt{3}-i)^{10}$

25. $(-5+5i)^{15}$ **26.** $\frac{(1+i)^{12}}{(1-i)^{10}}$

2-topshiriq. Quyidagi kompleks sonlarni algebraik shaklda tasvirlang.

1. $12(\cos 330^0 + i\sin 330^0)^{15}$ **2.** $21(\cos 750^0 + i\sin 750^0)^{12}$

3. $15\left(\cos\left(-\frac{2\pi}{30}\right) + i\sin\left(-\frac{2\pi}{30}\right)\right)^5$ **4.** $3\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)^{15}$

5. $7\left(\cos\frac{19\pi}{24} + i\sin\frac{19\pi}{24}\right)^6$ **6.** $6\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)^3$

7. $4\left(\cos\frac{19\pi}{15} + i\sin\frac{19\pi}{15}\right)^5$ **8.** $16\left(\cos\frac{5\pi}{14} + i\sin\frac{5\pi}{14}\right)^7$

9. $2(\cos 945^0 + i\sin 945^0)^6$ **10.** $3(\cos(-1005^0) + i\sin(-1005^0))^3$

11. $4\left(\cos\left(-\frac{9\pi}{40}\right) + i\sin\left(-\frac{9\pi}{40}\right)\right)^5$ **12.** $5(\cos 1395^0 + i\sin 1395^0)^4$

13. $(\cos 675^0 + i\sin 675^0)^{10}$ **14.** $\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)^{100}$

15. $12\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right)^{124}$ **16.** $6\left(\cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}\right)^{90}$

17. $(\cos(-2295^0) + i\sin(-2295^0))^4$ **18.** $(\cos 765^0 + i\sin 765^0)^{10}$

19. $\left(3\left(\cos\frac{17\pi}{18} + i\sin\frac{17\pi}{18}\right)\right)^6$ **20.** $\left(2\left(\cos\left(-\frac{13\pi}{16}\right) + i\sin\left(-\frac{13\pi}{16}\right)\right)\right)^8$

21. $12\left(\cos\left(-\frac{211\pi}{144}\right) + i\sin\left(-\frac{211\pi}{144}\right)\right)^{24}$ **22.** $20\left(\cos\left(-\frac{15\pi}{34}\right) + i\sin\left(-\frac{15\pi}{34}\right)\right)^{51}$

23. $15(\cos 495^0 + i\sin 495^0)^{17}$ **24.** $16\left(\cos\left(-\frac{9\pi}{28}\right) + i\sin\left(-\frac{9\pi}{28}\right)\right)^{14}$

25. $6(\cos(-1005^0) + i\sin(-1005^0))^{12}$ **26.** $120(\cos 675^0 + i\sin 675^0)^{16}$

3-topshiriq. Hisoblang.

1. $\sqrt{-2i}$ **2.** $\sqrt{8i}$ **3.** $\sqrt{3+4i}$ **4.** $\sqrt{15-8i}$

5. $\sqrt{-3+4i}$ **6.** $\sqrt{4+3i}$ **7.** $\sqrt{2-3i}$ **8.** $\sqrt{-8-6i}$

9. $\sqrt[4]{-1}$ **10.** $\sqrt{1-i}$ **11.** $\sqrt{1+i\sqrt{3}}$ **12.** $\sqrt{\sqrt{3}-i}$

13. $\sqrt[3]{1-i\sqrt{3}}$ **14.** $\sqrt[4]{1+i}$ **15.** $\sqrt[3]{-1-i\sqrt{3}}$ **16.** $\sqrt{\dfrac{-1-i\sqrt{3}}{1+i}}$

17. $\sqrt[3]{-1}$ **18.** $\sqrt{\dfrac{-1+i\sqrt{3}}{1-i}}$ **19.** $\sqrt{\dfrac{1}{2}-i\dfrac{\sqrt{3}}{2}}$ **20.** $\sqrt{3+4i}$

21. $\sqrt{\dfrac{1}{3}+i\dfrac{1}{3}}$ **22.** $\sqrt{2-2i}$ **23.** $\sqrt[3]{i}$

24. $\sqrt[4]{1-\sqrt{3}i}$ **25.** $\sqrt[4]{\dfrac{1}{2}-i\dfrac{\sqrt{3}}{2}}$ **26.** $\sqrt{-2-2i}$

Ishni bajarish tartibi.

1. Talaba laboratoriya ishi bilan tanishadi.
2. Laboratoriya ishi bo’yicha hisobot tayyorlaydi.
3. Nazorat savollariga javob beradi.

Nazorat savollari.

1. Qanday conga kompleks son deyiladi?
2. Mavhum birlik nima?
3. Kompleks sonning haqiqiy qismi qanday belgilanadi?
4. Kompleks sonning mavhum qismi qanday belgilanadi?
5. Kompleks sonning qo’shmasi?
6. Kompleks sonning trigonometrik shakli qanday?
7. Muavr formulasini yozib bering?

Foydalanilgan adabiyotlar.

1. A.G.Kurosh. «Oliy algebra kursi». Toshkent, O’qituvchi, 1976.
2. R.Iskandarov. «Oliy algebra», I-qism. Toshkent, 1963.
3. D.K.Fadeev, I.S.Sominskiy. «Sbornik zadach po visshey algebre». Moskva, Nauka, 1977.
4. Maqsudov Sh.T. «Analitik funksiyalar nazariyasidan mashqlar». Toshkent, O’qituvchi, 1978.

3-laboratoriya ishi.
Mavzu: Chiziqli operatorning xos vektori va xos qiymati. Xarakteristik ko`phadi.

Ishdan maqsad: Chiziqli operatorning xos vektori va xos qiymatlarini topishni o'rganish.

Nazariy qism:

1.Vektor fazolarning chiziqli akslantirishi.

U vektor fazoni V vektor fazoga akslantiruvchi φ akslantirish berilgan bo'lsin. Agar shunday akslantirish mavjud bo'lsa, biz uni $\varphi: U \to V$ orqali belgilaymiz.

Mazkur akslantirishda U ning barcha vektorlari V ning vektorlariga akslanadi (barchasiga bo'lishi shart emas). U vektor fazoning ixtiyoriy $\overline{x}$ elementiga φ akslantirish yordamida V vektor fazodan mos keluvchi vektorni $\overline{y}$ deb belgilaymiz. Bu moslik $\varphi: \overline{x} \to \overline{y};\ \overline{x} \xrightarrow{\varphi} \overline{y};\ \ \varphi\overline{x} = \overline{y};\ \overline{y} = \varphi(\overline{x})$ ko'rinishlarda belgilanadi.

Ta'rif-1. P sonlar maydonida aniqlangan U vektor fazoni V vektor fazoga akslantiruvchi φ akslantirish uchun quyidagi ikkita shart

$$1)\ \varphi(\overline{x_1} + \overline{x_2}) = \varphi(\overline{x_1}) + \varphi(\overline{x_2});$$

$$2)\ \varphi(\lambda\ \overline{x}) = \lambda\ \varphi(\overline{x})\ (\lambda \in P)$$

bajarilsa, U ***vektor fazoni*** V ***vektor fazoga chiziqli akslanadi*** deyiladi.

U fazoni V fazoga chiziqli akslantirishlar to'plami $Hom(U, V)$ orqali belgilanadi.

U vektor fazoni o'z-o'ziga akslantirish U fazoda aniqlangan ***operator*** deyiladi.

Operatorlar $f, \varphi, \psi, \ldots$ harflar orqali belgilanib, ular chiziqli akslantirishlarning xususiy holidan iborat, ya'ni 1-ta'rifda $U = V$ bo'ladi. SHuning uchun V fazoning barcha operatorlari to'plami ham $Hom(V, V)$ bo'ladi.

Ta'rif-2. U vektor fazoni o'z-o'ziga chiziqli akslantirish U ***fazoda aniqlangan chiziqli operator*** deyiladi.

φ gomomorfizm (chiziqli akslantirish) ta'sirida $\varphi\overline{x} = \overline{y}$ bo'lsa, $\overline{y}$ vektor $\overline{x}$ vektorning ***obrazi*** (tasviri), $\overline{x}$ esa $\overline{y}$ vektorning ***proobrazi*** (asli) deb yuritiladi. $\overline{x} \in U$ bo'lganda $\varphi\overline{x} \in V$ vektorlar to'plami odatda φ akslantirishning ***obrazi*** deb yuritiladi va $im\varphi$ yoki φU orqali belgilanadi.

Shuni alohida qayd qilamizki, $\varphi\overline{x}$ simvol ikki ma'noga ega:

1) bu simvol $\overline{x}$ vektorga φ akslantirishni qo'llash jarayonidir;
2) mazkur akslantirishning natijasini, ya'ni $\overline{x}$ vektorning obrazini bildiradi.

2.Chiziqli akslantirishlar matritsasi.

Faraz qilaylik, biror φ chiziqli akslantirish berilgan bo'lib, u n o'lchovli U_n vektor fazoni m o'lchovli V_m vektor fazoga o'tkazsin. U_n va V_m fazolarning bazislari mos ravishda $\overline{e_1}, \overline{e_2}, \ldots, \overline{e_n}$ va $\overline{f_1}, \overline{f_2}, \ldots, \overline{f_m}$ bo'lsin.

Agar $\overline{e_i} \in U_n (i = \overline{1, n})$ vektorlarga akslantirishni tatbiq etganda hosil bo'lgan vektorni $\varphi\overline{e_i} \in V_m$ orqali belgilasak, bu vektorlarni V_m ning bazis vektorlari orqali chiziqli ifodalash mumkin, ya'ni

$$\varphi\overline{e_i} = a_{1i}\overline{f_1} + a_{2i}\overline{f_2} + \ldots + a_{mi}\overline{f_m}$$

yoki

$$\varphi\overline{e_i} = \sum_{k=1}^{m} a_{ki}\overline{f_k} \quad (i = \overline{1, n}). \tag{1}$$

(1) tengliklardagi a_{ki} koeffitsientlardan

$$A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{pmatrix}$$

matritsani tuzamiz. Ana shu matritsaga φ akslantirishning U_n fazo bazisini V_m fazo bazisiga akslantirgandagi matritsa deb yuritiladi.

Endi masalani quyidagicha qo'yamiz. U_n fazoning ixtiyoriy $\overline{x}$ vektori koordinatalari bilan uning $\varphi: U_n \to V_m$ akslantirish natijasida hosil qilingan $\overline{y} = \varphi(\overline{x})$ proobrazi koordinatalari orasida qanday bog'lanish mavjud? Bu savolga javob berish uchun $\overline{x} = \sum_{i=1}^{n} \alpha_i \overline{e_i}$ va $\overline{y} = \varphi(\overline{x}) = \sum_{k=1}^{m} \beta_k \overline{f_k}$

vektorlarni olamiz. U holda

$$\overline{y} = \sum_{k=1}^{m} \beta_k \overline{f_k} = \varphi(\overline{x}) = \varphi\left(\sum_{i=1}^{n} \alpha_i \overline{e_i}\right) = \sum_{i=1}^{m} \alpha_i \varphi\overline{e_i} = \sum_{i=1}^{n} \alpha_i \left(\sum_{k=1}^{m} a_{ki}\overline{f_k}\right) = \sum_{k=1}^{m}\left(\sum_{i=1}^{n} a_{ki}\alpha_i\right)\overline{f_k},$$

$$\overline{y} = \sum_{k=1}^{m}\left(\sum_{i=1}^{n} a_{ki}\alpha_i\right)\overline{f_k} \tag{2}$$

bo'lib, bu yerda α_i, β_i va a_{ki} lar qandaydir P sonlar maydoni elementlaridir.

Bulardan tashqari, V_m fazoning ixtiyoriy $\overline{y}$ vektorini $\overline{f_1}, \overline{f_2}, \ldots, \overline{f_m}$ bazis orqali quyidagicha yozish mumkin:

$$\overline{y} = \beta_1\overline{f_1} + \beta_2\overline{f_2} + \ldots + \beta_m\overline{f_m} \tag{3}$$

(2) va (3) tengliklarda $\overline{f_1}, \overline{f_2}, \dots, \overline{f_m}$ larning mos koeffitsientlarini tenglashtirib (ikkita vektorning tengligi shartidan) quyidagilarga ega bo'lamiz:

$$\begin{cases} \beta_1 = a_{11}\alpha_1 + a_{12}\alpha_2 + \dots + a_{1n}\alpha_n \\ \beta_2 = a_{21}\alpha_1 + a_{22}\alpha_2 + \dots + a_{2n}\alpha_n \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \beta_m = a_{m1}\alpha_1 + a_{m2}\alpha_2 + \dots + a_{mn}\alpha_n \end{cases} \tag{4}$$

Demak, $\varphi : U_n \to V_m$ akslantirish berilgan bo'lsa, uni ixtiyoriy $\overline{x} = \sum_{i=1}^{n} \alpha_i \overline{e_i} \in U_n$ vektorga tatbiq qilishdan hosil bo'lgan har qanday $\overline{y} = \varphi(\overline{x}) \in V_m$ vektorni aniqlash mumkin ekan. (4) tengliklar $\overline{x} \in U_n$ va probrazi bo'lgan $\varphi(\overline{x}) \in V_m$ larning mos ravishda $\overline{e_1}, \overline{e_2}, \dots, \overline{e_n}$ va $\overline{f_1}, \overline{f_2}, \dots, \overline{f_m}$ bazislarning koordinatalari orasidagi bog'lanishni ifodalaydi. Agar (4) tengliklarni matritsa ko'rinishida yozadigan bo'lsak,

$$Y = AX \tag{5}$$

hosil bo'ladi.

Shunday qilib, biz yuqorida ko'rib o'tganimizga binoan har bir $\varphi : U_n \to V_m$ chiziqli akslantirishga bitta (m, n) turli matritsa kelar ekan.

Endi masalani aksincha qo'yamiz.

Har bir (m, n) turli $\|a_k\| (k = \overline{1,m};\ i = \overline{1,n})$ matritsaga mos keluvchi biror $\varphi : U_n \to V_m$ chiziqli akslantirish mavjudmi? Bu savol ijobiy javobga ega.

Haqiqatan, agar $\|a_{ki}\|$ matritsa berilgan bo'lsa, ixtiyoriy $\overline{x} = \sum_{i=1}^{n} \alpha_i \overline{e_i} \in U_n$ vector uchun $\overline{y} = \varphi(\overline{x}) \in V_m$ vektorni (4) formulalar yordamida aniqlay olamiz. Endi $\varphi : \overline{x} \to \overline{y}$ akslantirishni kiritamiz. Bu akslantirish chiziqli bo'ladi. Haqiqatan:

(1) agar $\varphi(\overline{x}) = \overline{y}$ bo'lsa, $\varphi(\alpha\overline{x}) = \sum_{k=1}^{m} \alpha\beta_k \overline{f_k} = \alpha \sum_{k=1}^{m} \beta_k \overline{f_k} = \alpha\varphi(\overline{x}),\ \varphi\left(\alpha\overline{x}\right) = \alpha\varphi\overline{x}$ o'rinli;

(2) $\overline{x_1} = \sum_{i=1}^{m} \alpha_i \overline{e_i},\ \overline{x_1}' = \sum_{i=1}^{m} \alpha_i' \overline{e_i},\ \overline{x_1} + \overline{x_1}' = \sum_{i=1}^{m} (\alpha_i + \alpha_i')\overline{e_i},\ \varphi\overline{x_1} =$
$= \sum_{k=1}^{m} \beta_k \overline{f_k},\ \varphi\overline{x_1}' = \sum_{k=1}^{m} \beta_k' \overline{f_k}$

larga binoan,

$$\varphi(\overline{x_1} + \overline{x_1}') = \sum_{k=1}^{m} (\beta_k + \beta_k')\overline{f_k} = \sum_{k=1}^{m} \beta_k \overline{f_k} + \sum_{k=1}^{m} \beta_k' \overline{f_k} = \varphi\overline{x_1} + \varphi\overline{x_1}',$$

$$\varphi(\overline{x_1} + \overline{x_1}') = \varphi\overline{x_1} + \overline{x_1}'$$

hosil bo'ladi.

Shunday qilib, $\varphi:\overline{x}\to\overline{y}$ akslantirish chiziqli akslantirishning ikkala shartini ham qanoatlantirgani tufayli bu akslantirish chiziqlidir. Demak, *n* o'lchovli U_n fazoni *m* o'lchovli *V* fazoga o'tkazuvchi chiziqli akslantirishlar to'plami bilan *(m, n)* turli matritsalar to'plami orasida o'zaro bir qiymatli akslantirish mavjud ekan.

Natija. Bitta chiziqli operatorga bitta kvadrat matritsa mos keladi va aksincha.

3. Chiziqli operatorlar ustida amallar.

1-ta'rif. Agar U_n fazoning ikkita φ va ψ chiziqli operatorlari uchun $\varphi\overline{x}=\psi\overline{x}$ tenglik U_n fazoning istalgan $\overline{x}$ vektori uchun bajarilsa, φ va ψ *operatorlari o'zaro* teng deyiladi.

Biror U_n fazoda ikkitadan kam bo'lmagan chiziqli operatorlar aniqlangan bo'lsa, bu operatorlarning yig'indisi, ayirmasi haqida gapirish mumkin.

U_n fazoda φ va ψ chiziqli operatorlar berilgan bo'lsin.

2-ta'rif. Agar U_n fazoning istalgan $\overline{x}$ vektori uchun $\mathrm{f}\,\overline{\mathrm{x}}=\varphi\overline{x}+\psi\overline{x}$ tenglik bajarilsa, *f* operator φ va ψ operatorlar yig'indisi deyiladi va $\mathrm{f}=\varphi+\psi$ orqali yoziladi.

Chiziqli operatorlar yig'indisi yana chiziqli operator bo'ladi.

Haqiqatan, agar φ operatorga mos keluvchi matritsani A,ψ operatorga mos keluvchi matritsani B, *f* operatorga operator mos keluvchi matritsa *C* orqali belgilasak, u holda $\mathrm{C}=A+B$ tenglik o'rinli bo'ladi. Chiziqli operatorlar uchun

1) $\varphi+\psi=\psi+\varphi$;

2) $\varphi+(\psi+f)=(\varphi+\psi)+f$;

3) $\varphi+0=\varphi$.

tengliklar o'rinlidir. $\varphi-\psi$ ayirma ham xuddi shu usulda aniqlanadi (tekshirib ko'ring).

3-ta'rif. $\alpha\in P$ bo'lib, U_n fazoda berilgan operatorlar uchun $(\alpha\varphi)\overline{x}=\alpha\varphi\overline{x}$ tenglik U_n fazoning istalgan $\overline{x}$ elementi uchun bajarilsa, u holda $\alpha\varphi$ ga φ *operatorning* α *skalyar miqdorga ko'paytmasi* deyiladi.

Natija. *P* sonlar maydoni ustida berilgan chiziqli operatorlar to'plami chiziqli fazo bo'ladi.

4-ta'rif. Agar φ operatorga biror $\overline{e_1}, \overline{e_2}, \ldots, \overline{e_n}$ bazisga nisbatan A kvadrat matritsa mos kelsa, A matritsaning rangi φ *chiziqli operatorning ham rangi* deyiladi.

Chiziqli operatorlar orasida $\forall \bar{x} \in U_n$ uchun $\varphi \bar{x} = \bar{x}$ va $\varphi \bar{x} = 0$ kabi operatorlar mavjud bo’lsa, ular mos ravishdaayni (birlik) va nol operatorlar deb ataladi. Birlik operator ε, nol operator esa $\bar{0}$ orqali belgilanib, ularga mos ravishda birlik, ya’ni *E* va nol $\|0_{ij}\|$ matritsalar to’g’ri keladi.

Ba’zi hollarda U_n fazoning nolmas vektorlari φ operator ta’sirida nol vektorga akslanishi mumkin.

5-ta’rif. U_n fazoning φ operator yordamida nolga akslanuvchi barcha elementlari to’plamiga φ ***operatorning yadrosi*** deyiladi va u $Ker\,\varphi$ orqali belgilanadi.

1-teorema. φ chiziqli operatorlar yadrosi shu operator qaralayotgan fazoning qism fazosi bo’ladi.

Isbot. $\bar{x}_1 \in Ker\,\varphi,\ \bar{x}_2 \in Ker\,\varphi$ bo’lganda $\varphi \bar{x}_1 = \bar{0}$ va $\varphi \bar{x}_2 = \bar{0}$ hamda φ chiziqli operator bo’lgani uchun

1) $\varphi(\bar{x}_1 - \bar{x}_2) = \varphi \bar{x}_1 - \varphi \bar{x}_2 = \bar{0} - \bar{0} = \bar{0};$

2) $\varphi(\lambda \bar{x}) = \lambda \varphi \bar{x} = \lambda \cdot \bar{0} = \bar{0},\ \ \varphi(\lambda \bar{x}) = \bar{0}$ *ekanligidan* $\bar{x}_1 = -\bar{x}_2 \in Ker\,\varphi, \lambda \bar{x} \in \in Ker\,\varphi$ bo’ladi. Demak, $Ker\,\varphi$ U_n fazoning qisim fazosidir.

6-ta’rif. φ chiziqli operator yadrosining o’lchoviga shu ***operatorning defekti*** deyiladi.

2-teorema. Agar U_n fazoda aniqlangan φ chiziqli operator matritsaning rangi r ga teng bo’lsa, Kerφ yadroning o’lchovi $n-r$ ga teng bo’ladi.

Isbot. Faraz qilaylik $\bar{x} = \sum_{i=1}^{n} \alpha_i \bar{e}_i \in Ker\,\varphi$ bo’lsin. $Ker\,\varphi$ ning barcha vektorlari nolga akslanganidan (4) tengliklar sistemasi

$$a_{i1}\alpha_1 + a_{i2}\alpha_2 + \ldots + a_{in}\alpha_n = 0 \ \ (i = \overline{1,m}) \qquad (6)$$

ko’rinishni oladi.

Aksincha, koordinatalar bir jinsli chiziqli tenglamalar sistemasining nolmas yechimini ifodalovchi barcha vektorlar $Ker\,\varphi$ ga tegishli bo’ladi. Shunday qilib, $Ker\,\varphi$ yadroning o’lchovi (6) sistemaning chiziqli bog’lanmagan yechimlari soniga (ya’ni fundamental sistema yechimlari soniga) teng ekan. Ma’lumki, bunday yechimlar soni *n-r* ga tengdir. Bu erda *r* soni φ operatorga mos keluvchi *A* matritsa rangini bildiradi.

3-teorema. Agar $\bar{e}_1, \bar{e}_2, \ldots, \bar{e}_n$ vektorlar sistemasi fazoning bazisi va $\bar{f}_1, \bar{f}_2, \ldots, \bar{f}_n$ lar shu fazoning ixtiyoriy vektorlari bo’lsa, unda shunday yagona φ operator mavjudki, u $\bar{e}_1, \bar{e}_2, \ldots, \bar{e}_n$ bazis sistemani $\bar{f}_1, \bar{f}_2, \ldots, \bar{f}_n$ larga o’tkazadi.

4. Chiziqli operatorning turli bazislardagi matritsalari orasidagi bog'lanish. Fazoning ikkita

$$\overline{e_1}, \overline{e_2}, \ldots, \overline{e_n} \tag{7}$$

$$\overline{f_1}, \overline{f_2}, \ldots, \overline{f_n} \tag{8}$$

bazisi va bitta φ operatorini olamiz. U φ operatorning (7) va (8) bazislardagi matritsalari

$$A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{pmatrix} \quad \text{va} \quad B = \begin{pmatrix} b_{11} & b_{12} & . & . & . & b_{1n} \\ b_{21} & b_{22} & . & . & . & b_{2n} \\ . & . & . & . & . & . \\ b_{n1} & b_{n2} & . & . & . & b_{nn} \end{pmatrix}$$

bo'lsin. Bu matritsalarni aniqlovchi tengliklar qisqacha bunday yoziladi:

$$\begin{cases} \varphi\overline{e_k} = \sum\limits_{i=1}^{n} a_{ik}\overline{e_i} (k = \overline{1,n}) \\ \varphi\overline{f_k} = \sum\limits_{i=1}^{n} b_{ik}\overline{f_i} (k = \overline{1,n}). \end{cases} \tag{9}$$

(8) bazisni (7) bazis orqali chiziqli ifodalaymiz:

$$\begin{cases} \overline{f_1} = c_{11}\overline{e_1} + c_{21}\overline{e_2} + \ldots + c_{n1}\overline{e_n}, \\ \overline{f_2} = c_{12}\overline{e_1} + c_{22}\overline{e_2} + \ldots + c_{n2}\overline{e_n}, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \overline{f_n} = c_{1n}\overline{e_1} + c_{2n}\overline{e_2} + \ldots + c_{nn}\overline{e_n}. \end{cases} \tag{10}$$

(10) sistemaning

$$C = \begin{pmatrix} c_{11} & c_{12} & . & . & . & c_{1n} \\ c_{21} & c_{22} & . & . & . & c_{2n} \\ . & . & . & . & . & . \\ c_{n1} & c_{n2} & . & . & . & c_{nn} \end{pmatrix}$$

matritsa xosmasdir. Yuqoridagi 3-teoremaga asosan yagona ν chiziqli operator mavjud bo'lib, u (7) bazis vektorlarini (10) vektorlariga akslantiradi:

$$\nu\,\overline{e_i} = \overline{f_i} \quad (i = \overline{1, n}). \tag{11}$$

(11) ning ikala tomoniga φ operatorni tatbiq etamiz. Natijada $\varphi\nu\,\overline{e_i} = \varphi\overline{f_i}$ hosil bo'ladi.

Oxirgi tengliklarning o'ng tomonidagi $\varphi\overline{f_i}$ $(i = \overline{1, n})$ larni (9) Bilan almashtirsak, $\varphi\nu\,\overline{e_k} = \sum\limits_{i=1}^{n} b_{ik}\overline{f_i}$ kelib chiqadi. Agar $\overline{f_i}$ $(i = \overline{1, n})$ larning o'rniga (4) ni qo'ysak, natijada quyidagiga ega bo'lamiz:

$$\varphi\nu\,\overline{e_k} = \sum_{i=1}^{n} b_{ik}\nu\overline{e_i} \quad . \tag{12}$$

ν ning $|C|$ determinanti 0 dan farqli bo'lgani sababli, ν ga teskari ν^{-1} operator mavjud bo'lib, uni (12) vektorga tatbiq etamiz:

$$\nu^{-1}\varphi\overline{e_k} = \nu^{-1}\sum_{i=1}^{n} b_{ik}\nu\overline{e_i} = \sum_{i=1}^{n}\nu^{-1}b_{ik}\nu\overline{e_i} = \sum_{i=1}^{n} b_{ik}\nu^{-1}\nu\overline{e_i} = \sum_{i=1}^{n} b_{ik}\varepsilon\overline{e_i} = \sum_{i=1}^{n} b_{ik}\,\overline{e_i}\,, \tag{13}$$

$\nu^{-1}\varphi\overline{e_k} = \nu^{-1}\sum_{i=1}^{n} b_{ik}\,\overline{e_i} =$ ($\varepsilon -$birlik operator).

Birlik tomondan $\nu^{-1}\varphi\nu$ operatorning (7) bazisdagi matritsasi $C^{-1}AC$ bo'lib (chunki $\nu^{-1} \to C^{-1}$, $\varphi \to A$ va $\nu \to C$), ikkinchi tomondan, (13) ga muvofiq, bu operatorning (7) bazisidagi matritsasi B bo'lganligi sababli

$$B = C^{-1}AC \tag{14}$$

bo'ladi. Bunda C ni ***(8) bazisdan (7) bazisga o'tish matritsasi*** deyiladi.

Ta'rif. (14) tenglik Bilan bog'langan A va B matritsalar ***o'xshash matritsalar*** deyiladi.

Misol. Uch o'lchovli arifmetik V fazoning

$$\overline{e_1} = (1,\,0,\,0),\ \overline{e_2} = (0,1,\,0),\ \overline{e_3} = (0,\,0,\,1),$$
$$\overline{f_1} = (1,1,1),\ \overline{f_2} = (1,\,2,\,1),\ \overline{f_3} = (2,\,-1,\,1)$$

bazislarini va $\varphi(a_1,\,a_2,\,a_3) = (a_1,\,2a_2,\,3a_3)$ operatorlarni olamiz. Bu operatorning birinchi bazisdagi matritsasi

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

bo'lib, ikkinchi bazisning birinchi bazis orqali chiziqli ifodasi quyidagidan iborat:

$$\begin{cases} \overline{f_1} = \overline{e_1} + \overline{e_2} + \overline{e_3}, \\ \overline{f_2} = \overline{e_1} + 2\overline{e_2} + \overline{e_3}, \\ \overline{f_3} = 2\overline{e_1} - \overline{e_2} + \overline{e_3}. \end{cases}$$

Demak,

$$C = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix} \text{ va } C^{-1} = \begin{pmatrix} -3 & -1 & 5 \\ 2 & 1 & -3 \\ 1 & 0 & -1 \end{pmatrix}$$

lardan iborat bo'lgani uchun φ operatorning ikkinchi bazisdagi matritsasi

$$B = C^{-1}AC = \begin{pmatrix} 10 & 8 & 11 \\ -5 & -3 & -7 \\ -2 & -2 & -1 \end{pmatrix}$$

bo'ladi.

5. O'zaro teskari chiziqli operatorlar. P maydon ustidagi V_n fazo va uning

$$\overline{e_1},\ \overline{e_2},\ \ldots,\ \overline{e_n} \tag{15}$$

bazisi berilgan bo'lsin. φ chiziqli operatorni va uning (15) bazisdagi

$$A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{pmatrix}$$

matritsani olamiz. Bu matritsaning

$$|A| = \begin{vmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{vmatrix}$$

determinanti A ga mos φ operatorning ham ***determinanti*** deyiladi.

Ta'rif. $|A|$ determinant noldan farqli bo'lganda φ chiziqli operator ***xosmas operator***, $|A| = 0$ bo'lsa, φ ***xos operator*** deb ataladi.

Ta'rif. φ chiziqli operator uchun shunday ψ chiziqli operator mavjud bo'lib,

$$\varphi\psi = \psi\varphi = \varepsilon \tag{16}$$

tenglik bajarilsa, ψ ni φ ga ***teskari operator*** deyiladi.

(16) tenglikdan quyidagini topamiz: $\overline{x} \in V_n$ vektor uchun $(\varphi\psi)\overline{x} = \varepsilon\overline{x} = \overline{x}$. Endi $\psi\overline{x} = \overline{y}$ bo'lsa, u holda $(\varphi\psi)\overline{x} = \varphi(\psi\overline{x}) = \varphi\overline{y} = \overline{x}$ bo'ladi, ya'ni φ operator $\overline{y}$ ni $\overline{x}$ ga akslantirsa, teskari ψ operator, aksincha, $\overline{x}$ ni $\overline{y}$ ga akslantiradi.

Teorema. Chiziqli operatorga teskari operator mavjud bo'lishi uchun uning xosmas operator bo'lishi zarur va etarli.

Natija. Xosmas chiziqli operatorlar to'plami operatorlar kompozitsiyasi (ko'paytirish amali) ga nisbatan gruppa tashkil qiladi.

Misol. Uch o'lchovli arifmetik V_3 fazoning

$$\overline{e_1} = (1,\ 0,\ 0),\ \overline{e_2} = (0,\ 1,\ 0),\ \overline{e_3} = (0,\ 0,\ 1) \tag{17}$$

bazisi va

$$\varphi(a_1,\ a_2,\ a_3) = (a_1 + a_2,\ a_2 + a_3,\ a_3 + a_1),$$

$$\varphi(a_1,\ a_2,\ a_3) = (a_1,\ a_2,\ a_3)$$

operatorlari berilgan. φ xosmas operator, chunki

$$\varphi\overline{e_1} = (1,\ 0,\ 1) = 1\cdot\overline{e_1} + 0\cdot\overline{e_2} + 1\cdot\overline{e_3},$$

$$\varphi\overline{e_2} = (1,\ 1,\ 0) = 1\cdot\overline{e_1} + 1\cdot\overline{e_2} + 0\cdot\overline{e_3},$$

$$\varphi\overline{e_3} = (0,\ 1,\ 1) = 0\cdot\overline{e_1} + 1\cdot\overline{e_2} + 1\cdot\overline{e_3}.$$

Demak,

$$|A| = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 2, \quad |A| \neq 0.$$

Shunday qilib, φ ga teskari operator mavjud bo’lgani holda uning (17) bazisdagi matritsasi ushbudan iborat:

$$A^{-1} = \frac{1}{2}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix}.$$

Lekin ψ operator xosdir, chunki uning matritsasi

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

bo’lib, bu xos matritsadir.

V A R I A N T L A R

1-topshiriq. Quyidagi akslantirish chiziqli operator bo`la oladimi?

1. $\varphi: R^3 \to R^3, \qquad \varphi(\overline{x}) = (x_1 + x_2,\ x_2^2 + 1,\ x_1 - x_3)$

2. $\varphi: R^3 \to R^3, \qquad \varphi(\overline{x}) = (x_1 + x_2 - x_3,\ 2x_1 - x_3,\ x_1 + 2x_3)$

3. $\varphi: R^3 \to R^3, \qquad \varphi(\overline{x}) = (x_1^2,\ x_2 + 1,\ x_3 + 4)$

4. $\varphi: R^3 \to R^3, \qquad \varphi(\overline{x}) = (x_1 - x_3,\ x_1 + x_2 + x_3,\ x_1)$

5. $\varphi: R^3 \to R^3, \qquad \varphi(\overline{x}) = (2x_1 + x_2^2,\ x_3^2 + 1,\ x_1)$

6. $\varphi: R^3 \to R^3, \qquad \varphi(\overline{x}) = (3x_1 + x_2 + x_3,\ x_2^2,\ x_1 + x_3)$

7. $\varphi: R^3 \to R^3, \qquad \varphi(\overline{x}) = (x_1 + 2x_2,\ x_2 + 4,\ x_3 + 2x_1)$

8. $\varphi: R^3 \to R^3, \qquad \varphi(\overline{x}) = (x_1 + x_2 + x_3^2,\ x_1 - 1,\ x_3 + x_2)$

9. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (x_1^2 + x_2^2 + x_3^2,\ x_1,\ x_1 + x_2)$

10. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (x_1 + x_2 - 3x_3,\ 2x_1 - x_2,\ x_3)$

11. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (x_1 + x_2 - x_3^2,\ 2x_1 - x_3,\ x_3^2)$

12. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (x_1 + x_2 - 2x_3,\ x_1 + x_3^2,\ x_3)$

13. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (x_1,\ x_2, x_3^2 + 5)$

14. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (x_1 + x_2,\ x_3 + x_2 - x_1,\ x_3 + 2x_1)$

15. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (x_1 + x_2 - 3x_3,\ x_2^2 + 2,\ x_3 + 4x_1)$

16. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (-x_1 + x_2, -x_2, -x_3)$

17. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (x_1 + x_2^2 - 3,\ x_1^2 - 2,\ x_3)$

18. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (x_1 + x_2, -x_2 - x_3 + 5,\ x_1^2 + x_2^2)$

19. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (x_1 + x_2 - x_3, -x_1 + x_3,\ 2x_1 - x_2 + 3x_3)$

20. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (x_1 + 2x_2 - x_3, -x_1, -x_2)$

21. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = (x_1^2 + 2,\ x_3^2 + 4,\ x_2 + x_3)$

22. $\varphi: R^3 \to R^3, \qquad \varphi(\bar{x}) = \left(\dfrac{x_1 + x_2}{x_1}, \dfrac{x_2^2}{x_1}, \dfrac{x_3^2}{x_1}\right)$

2-topshiriq. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi quyidagi A matritsa bilan berilgan. Uni xos vektori va xos qiymatlarini toping.

1. $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 2 & -1 & 0 \end{pmatrix}$ **2.** $A = \begin{pmatrix} 1 & -2 & -1 \\ -1 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$ **3.** $A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$

4. $A = \begin{pmatrix} 3 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & -1 & 4 \end{pmatrix}$ **5.** $A = \begin{pmatrix} 4 & -2 & -3 \\ 1 & 0 & 1 \\ 6 & -6 & 5 \end{pmatrix}$ **6.** $A = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$

7. $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \\ -1 & 2 & 3 \end{pmatrix}$ **8.** $A = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 0 & 2 \\ -2 & 1 & -1 \end{pmatrix}$ **9.** $A = \begin{pmatrix} 4 & -1 & -1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$

10. $A = \begin{pmatrix} 2 & -1 & -1 \\ 3 & -2 & -3 \\ -1 & 1 & 2 \end{pmatrix}$ **11.** $A = \begin{pmatrix} -2 & 1 & -2 \\ 1 & -2 & 2 \\ 3 & -3 & 5 \end{pmatrix}$ **12.** $A = \begin{pmatrix} 3 & -2 & -1 \\ 3 & -4 & -3 \\ 2 & -4 & 0 \end{pmatrix}$

13. $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ **14.** $A = \begin{pmatrix} -1 & 1 & -2 \\ 4 & 1 & 0 \\ 2 & 1 & -1 \end{pmatrix}$ **15.** $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 4 \\ 1 & 0 & -1 \end{pmatrix}$

16. $A=\begin{pmatrix} 2 & -1 & -1 \\ 2 & -1 & -1 \\ -1 & 1 & 2 \end{pmatrix}$ **17.** $A=\begin{pmatrix} 4 & -1 & 0 \\ 3 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$ **18.** $A=\begin{pmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix}$

19. $A=\begin{pmatrix} 1 & -3 & 4 \\ 4 & -7 & 8 \\ 6 & -7 & 7 \end{pmatrix}$ **20.** $A=\begin{pmatrix} 1 & -3 & 3 \\ -2 & -6 & 13 \\ -1 & -4 & 8 \end{pmatrix}$ **21.** $A=\begin{pmatrix} 2 & 5 & -6 \\ 4 & 6 & -9 \\ 3 & 6 & -8 \end{pmatrix}$

22. $A=\begin{pmatrix} 0 & 2 & 1 \\ -2 & 0 & 3 \\ -1 & -3 & 0 \end{pmatrix}$

3-topshiriq.

1. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A=\begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}$ matritsa bilan berilgan. $e_1' = e_1 - e_2,\ \ e_2' = -3e_1 + 4e_2$ bazisdagi matritsasini toping.

2. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A=\begin{pmatrix} 3 & 1 \\ -2 & 0 \end{pmatrix}$ matritsa bilan berilgan. $e_1' = e_1 + e_2,\ \ e_2' = -2e_1 + 3e_2$ bazisdagi matritsasini toping.

3. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A=\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ matritsa bilan berilgan. $\bar{x} = 2\bar{e}_1 - 3\bar{e}_2$ vektor obrazining $\bar{e}_1, \bar{e}_2$ bazisdagi koordinatalarini toping.

4. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A=\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ matritsa bilan berilgan. $\bar{x} = 2\bar{e} - 3\bar{e}_2$ vektor obrazining $\bar{e}_1, \bar{e}_2$ bazisdagi koordinatalarini toping.

5. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A=\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning rangini toping.

6. Chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisda $A=\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning determinantini toping.

7. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A=\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ matritsa bilan berilgan. Uni xarakteristik ko'phadini toping.

8. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 0 \end{pmatrix}$ matritsa bilan berilgan. Uni matritsaviy va spektral izlarini toping.

9. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 2 & 0 \\ -1 & 2 & 3 \end{pmatrix}$ matritsa bilan berilgan. Uni matritsaviy va spektral izlarini toping.

10. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & 0 \\ -3 & 1 & 0 \end{pmatrix}$ matritsa bilan berilgan. Uni matritsaviy va spektral izlarini toping.

11. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ matritsa bilan berilgan. $\bar{y} = e_1 + 3e_2$ vektor qaysi vektorni obrazi bo'ladi.

12. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & -2 \\ 0 & 1 & 0 \end{pmatrix}$ matritsa bilan berilgan. $\bar{y} = 2e_1 - e_2 + e_3$ vektor qaysi vektorni obrazi bo'ladi.

13. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$ matritsa bilan berilgan. Bu operatorga teskari operator toping.

14. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ -3 & 0 & -1 \end{pmatrix}$ matritsa bilan berilgan. Bu operatorga teskari operator toping.

15. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ matritsa bilan berilgan. Bu operatorga teskari operator toping.

16. $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ bazisda φ chiziqli operator $A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ matritsa bilan berilgan. $e_1' = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, e_2' = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ bazisda ψ chiziqli operator $A' = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$ matritsa bilan berilgan. $\varphi + \psi$ ni $\bar{e}_1', \bar{e}_2'$ dagi matritsasini toping.

17. $e_1=\begin{pmatrix}1\\2\end{pmatrix}, e_2=\begin{pmatrix}2\\0\end{pmatrix}$ bazisda φ chiziqli operator $A=\begin{pmatrix}1&2\\0&2\end{pmatrix}$ matritsa bilan berilgan. $e_1'=\begin{pmatrix}1\\0\end{pmatrix}, e_2'=\begin{pmatrix}3\\2\end{pmatrix}$ bazisda ψ chiziqli operator $A'=\begin{pmatrix}2&3\\1&3\end{pmatrix}$ matritsa bilan berilgan. $\varphi\cdot\psi$ ni $\bar{e}_1', \bar{e}_2'$ dagi matritsasini toping.

Ishni bajarish tartibi.

1. Talaba laboratoriya ishi bilan tanishadi.
2. Laboratoriya ishi bo’yicha hisobot tayyorlaydi.
3. Nazorat savollariga javob beradi.

Nazorat savollari.

1. Chiziqli akslantirish deb qanday akslantirishga aytiladi?
2. Operator deb nimaga aytiladi?
3. Chiziqli operator deb nimaga aytiladi?
4. Chiziqli operatorning rangi deb nimaga aytiladi?
5. O’tish matritsasi deb qanday matritsaga aytiladi?
6. Xos operator deb nimaga aytiladi?
7. Xosmas operator deb nimaga aytiladi?
8. Teskari operator deb nimaga aytiladi?

Foydalanilgan adabiyotlar.

1. A.G.Kurosh. «Oliy algebra kursi». Toshkent, O’qituvchi, 1976.
2. J.Hojiyev? A.S.Faynleyb. «Algebra va sonlar nazariyasi kursi».Toshkent, O’zbekiston, 2001.
3. R.N.Nazarov, B.T.Toshpo’latov, A.D.Do’sumbetov. «Algebra va sonlar nazariyasi». 1-qism. Toshkent, O’qituvchi, 1993.
4. R.Iskandarov. «Oliy algebra», I-qism. Toshkent, 1963.
5. R.I.Iskandarov. «Algebra va sonlar nazariyasi». 1-qism. Toshkent, O’qituvchi, 1977.
6. I.V.Proskuryakov. «Sbornik zadach polineynoy algebre». Moskva, Nauka, 1984.

4-laboratoriya ishi.
Mavzu: Kvadratik formalar va ularni kanonik ko'rinishga keltirish(Lagranj usuli)

Ishdan maqsad: Kvadratik formani kanonik ko'rinishga Lagranj usulidan foydalanib keltirish hamda kvadratik formani musbat aniqlanganligini tekshirishdan iborat.

Nazariy qism:

Quyidagi funktsiyaga kvadratik forma deyiladi:

$$f=\sum_{i,j=1}^{n} a_{ij}x_i x_j \ . \tag{1}$$

Bu yerda $a_{ij}=a_{ji}$ haqiqiy sonlar va $x_1, x_2, ..., x_n$ haqiqiy o'zgaruvchilar. $A=(a_{ij})$ matritsaga (1) kvadratik formaning matritsasi deyiladi. Bu matritsa simmetrik matritsa bo'ladi.

(1) kvadratik formani matritsalar ko\`paytmasi ko\`rinishida ifodalaymiz:

$$(x_1, x_2, \dots, x_n)\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}=$$

$$=(x_1, x_2, \dots, x_n)\begin{pmatrix} \sum_{j=1}^{n} a_{1j}x_j \\ \sum_{j=1}^{n} a_{2j}x_j \\ \dots \\ \sum_{j=1}^{n} a_{nj}x_j \end{pmatrix}=\sum_{i=1}^{n}\left(\sum_{j=1}^{n} a_{ij}x_j\right)x_i=\sum_{i,j=1}^{n} a_{ij}x_i x_j=f\ ,$$

ya'ni

$$f=\bar{x}^T\cdot A\cdot \bar{x} \tag{2}$$

tenglik o\`rinli ekan. Bu yerda

$$\bar{x} = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}.$$

Xosmas chiziqli almashtirish bajarganda, ya'ni

$$\bar{x} = C \cdot \bar{y}, \qquad (\det C \neq 0) \tag{3}$$

almashtirish bajarganda (1) kvadratik formaning matritsasi qanday qonuniyat bo`yicha o`zgarishini ko`ramiz:

$$f = \bar{x}^T \cdot A \cdot \bar{x} = (C \cdot \bar{y})^T \cdot A \cdot C \cdot \bar{y} = \bar{y}^T \cdot C^T \cdot A \cdot C \cdot \bar{y} = \bar{y}^T \cdot (C^T AC) \cdot \bar{y}.$$

Demak, $\bar{x} = C \cdot \bar{y}$ xosmas chiziqli almashtirish bajarsak,

$$f = \bar{y}^T \cdot B \cdot \bar{y}, \tag{4}$$

bo`lib,

$$B = C^T AC \tag{5}$$

bo`ladi.

Natija 1. Matritsani xosmas matritsaga ko`paytirganda uning rangi o`zgarmasligini bilamiz. Shuning uchun $\det C \neq 0$ va (5) ga ko`ra

$$rang(B) = rang(A)$$

bo`ladi. Boshqacha qilib aytganda, kvadratik forma matritsasining rangi xosmas chiziqli almashtirishga bog`liq emas ekan. Shuning uchun $rang(A)$ ga kvadratik formaning rangi deyiladi.

Natija 2. (5) tenglikdan $\det B$ va $\det A$ lar bir xil ishorali ekani kelib chiqadi, chunki

$$\det B = \det(C^T AC) = \det C^T \cdot \det A \cdot \det C = \det A \cdot (\det C)^2.$$

Teorema 1. Ixtiyoriy kvadratik formani $\bar{x} = C \cdot \bar{y}$, $(\det C \neq 0)$ xosmas chiziqli almashtirish yordamida quyidagi ko'rinishga keltirish mumkin:

$$f = \lambda_1 y_1^2 + \lambda_2 y_2^2 + \ldots + \lambda_n y_n^2. \tag{6}$$

Bu yerda $\lambda_1, \lambda_2, \ldots, \lambda_n$ haqiqiy sonlar. (6) ko'rinishga f kvadratik formaning kanonik ko'rinishi deyiladi.

Isbot. (Lagranj usuli). Induksiya usulida isbotlaymiz. Erkin o’zgaruvchilar soni 1 ga teng bo’lgan holda

$$f = a_{11}x_1^2$$

bo’lib, uning o’zi kanonik ko’rinishda bo’lib, almashtirish bajarishning hojati yo’q.

Erkin o’zgaruvchilar soni $n-1$ ga teng bo’lgan holda keltirilgan fikr o’rinli deb olib, erkin o’zgaruvchilar soni n ga teng bolgan holda isbotlaymiz.

1-hol. $a_{11} \neq 0$ bo’lsin. Bu holda f ning tarkibidan x_1 o’zgaruvchi qatnashgan hadlarni ajratib olib, quyidagi tarzda yozib olamiz:

$$f = a_{11}x_1^2 + 2x_1(a_{12}x_2 + \ldots + a_{1n}x_n) + g(x_2, \ldots, x_n)\,. \tag{7}$$

Bu yerda $g(x_2, \ldots, x_n)$ kvadratik forma tarkibida x_1 o’zgaruvchi qatnashmagan.

$a_{11} \neq 0$ bo’lgani uchun (7) ifodani quyidagi ko’rinishda yozib olamiz va to’rtburchak qavs ichida to’liq kvadrat ajratamiz:

$$f = \frac{1}{a_{11}}\left[a_{11}^2x_1^2 + 2a_{11}x_1(a_{12}x_2 + \ldots + a_{1n}x_n)\right] + g(x_2, \ldots, x_n) =$$

$$= \frac{1}{a_{11}}\left[(a_{11}x_1 + \ldots + a_{1n}x_n)^2 - (a_{12}x_2 + \ldots + a_{1n}x_n)^2\right] + g(x_2, \ldots, x_n) =$$

$$= \frac{1}{a_{11}}(a_{11}x_1 + \ldots + a_{1n}x_n)^2 + g(x_2, \ldots, x_n) - \frac{1}{a_{11}}(a_{12}x_2 + \ldots + a_{1n}x_n)^2\,,$$

ya’ni

$$f = \frac{1}{a_{11}}(a_{11}x_1 + \ldots + a_{1n}x_n)^2 + g_1(x_2, \ldots, x_n)\,. \tag{8}$$

Induksiya faraziga ko’ra $g_1(x_2, \ldots, x_n)$ kvadratik formani xosmas

$$\begin{cases} y_2 = c_{22}x_2 + \ldots + c_{2n}x_n \\ y_3 = c_{32}x_2 + \ldots + c_{3n}x_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_n = c_{n2}x_2 + \ldots + c_{nn}x_n \end{cases} \tag{9}$$

chiziqli almashtirish yordamida

$$g_1 = \lambda_2 y_2^2 + \ldots + \lambda_n y_n^2 \tag{10}$$

kanonik ko’rinishga keltirish mumkin.

Demak, (8), (9) va (10) ga ko'ra ushbu

$$\begin{cases} y_1 = a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \\ y_2 = \qquad c_{22}x_2 + \ldots + c_{2n}x_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_n = \qquad c_{n2}x_2 + \ldots + c_{nn}x_n \end{cases} \tag{11}$$

chiziqli almashtirishdan so'ng f kvadratik forma quyidagi kanonik ko'rinishni oladi:

$$f = \frac{1}{a_{11}} y_1^2 + \lambda_2 y_2^2 + \ldots + \lambda_n y_n^2 .$$

(11) almashtirish xosmas almashtirish, chunki

$$\begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ 0 & c_{22} & \ldots & c_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ 0 & c_{n2} & \ldots & c_{nn} \end{vmatrix} = a_{11} \cdot \begin{vmatrix} c_{22} & \ldots & c_{2n} \\ \ldots & \ldots & \ldots \\ c_{n2} & \ldots & c_{nn} \end{vmatrix} \neq 0 .$$

2-hol. Agar $a_{11} = 0$ bo'lib, biror $a_{kk} \neq 0$ bo'lsa, u holda f kvadratik forma ushbu

$$\begin{cases} z_1 = x_k \\ z_k = x_1 \\ z_i = x_i , \quad i \neq 1, i \neq k \end{cases}$$

almashtirishdan so'ng, 1-holga keltiriladi.

3-hol. $a_{11} = 0$, $a_{22} = 0$, …, $a_{nn} = 0$ bo'lsa, ya'ni f kvadratik formaning tarkibida kvadratlar qatnashmagan bo'lsa, u holda agar $a_{12} \neq 0$ bo'lsa, ushbu

$$x_1 = z_1 + z_2 ,\ \ x_2 = z_1 - z_2 ,\ \ x_3 = z_3 ,\ \ \ldots ,\ x_n = z_n ,$$

almashtirishdan so'ng

$$f = 2a_{12}(z_1^2 - z_2^2) + \varphi(z_1, \ldots, z_n)$$

ko'rinishga keladi. Bunga 1-holni qo'llash mumkin. Biror $a_{ij} \neq 0$ bo'lgan hol ham shunga o'xshash almashtirish yordamida 1-holga keltiriladi.

Teorema isbotlandi.

Izoh. f kvadratik formani $\bar{x} = C \cdot \bar{y}$, $(\det C \neq 0)$ xosmas chiziqli almashtirish yordamida

$$f = \lambda_1 y_1^2 + \lambda_2 y_2^2 + \ldots + \lambda_n y_n^2. \qquad (6)$$

kanonik ko'rinishga keltirganda nol bo'lmagan koeffitsientlar soni $r = rang(A)$ ta bo'ladi. Chunki oxirgi kvadratik formaning matritsasi

$$B = \begin{pmatrix} \lambda_1 & 0 & \ldots & 0 \\ 0 & \lambda_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \lambda_n \end{pmatrix}$$

bo'lib, $rang(B) = rang(A) = r$ bo'lgani uchun nol bo'lmagan r ta ustun bo'ladi, ya'ni $\lambda_1, \lambda_2, \ldots, \lambda_n$ sonlar orasida nolga teng bo'lmagani r ta.

Izoh. Kerak bo'lsa qayta almashtirish bajarib, f kvadratik formani quyidagi ko'rinishga keltirish mumkin:

$$f = \lambda_1 y_1^2 + \lambda_2 y_2^2 + \ldots + \lambda_k y_k^2 - \lambda_{k+1} y_{k+1}^2 - \ldots - \lambda_r y_r^2.$$

Bu yerda $\lambda_1 > 0, \lambda_2, \ldots, \lambda_r > 0$.

Agar biz yanada

$$z_1 = \sqrt{\lambda_1} y_1,\ \ldots,\ z_r = \sqrt{\lambda_r} y_r,\ z_{r+1} = y_{r+1},\ \ldots,\ z_n = y_n$$

almashtirishni bajarsak, kvadratik formaning ushbu

$$f = z_1^2 + z_2^2 + \ldots + z_k^2 - z_{k+1}^2 - \ldots - z_r^2$$

ko'rinishini olamiz. Bu ko'rinishga kvadratik formaning normal ko'rinishi deyiladi.

Teorema (Inertsiya qonuni). Kvadratik formani turli xil xosmas almashtirish yordamida normal ko'rinishga keltirganda musbat (manfiy) koeffitsientlar soni bir xil bo'ladi, ya'ni normal ko'rinishdagi musbat va manfiy koeffitsientlar soni almashtirishga bog'liq bo'lmaydi.

Isbot. Rangi r bo'lgan f kvadratik forma

$$\bar{y} = B \cdot \bar{x}, \qquad (\det B \neq 0) \qquad (7)$$

xosmas almashtirish yordamida

$$f = y_1^2 + y_2^2 + \ldots + y_p^2 - y_{p+1}^2 - \ldots - y_r^2 \qquad (8)$$

normal ko'rinishga va

$$\bar{z} = C \cdot \bar{x}, \qquad (\det C \neq 0) \tag{9}$$

xosmas almashtirish yordamida

$$f = z_1^2 + z_2^2 + \ldots + z_q^2 - z_{q+1}^2 - \ldots - z_r^2 \tag{10}$$

normal ko'rinishga keltiriladigan bo'lsin. $p < q$ deb faraz qilaylik. U holda quyidagi tengliklarni ko'rib chiqamiz:

$$y_1 = 0, \ldots, y_p = 0, \; z_{q+1} = 0, \ldots, z_r = 0, \ldots, z_n = 0. \tag{11}$$

Bu tengliklarning chap tomonlarini (7) va (9) almashtirishdan foydalanib, yozsak, $x_1, x_2, \ldots, x_n$ no'ma'lumlarga nisbatan chiziqli bir jinsli tenglamalar sistemasi hosil bo'ladi. Bu sistemadagi tenglamalar soni $p + (n - q) = n - (q - p)$ bo'lib, u no'ma'lumlar soni n dan kichik. Demak, bu sistema noldan farqli $\alpha_1, \alpha_2, \ldots, \alpha_n$ yechimga ega. Ushbu

$$\begin{aligned} & y_1^2(\bar{x}) + y_2^2(\bar{x}) + \ldots + y_p^2(\bar{x}) - y_{p+1}^2(\bar{x}) - \ldots - y_r^2(\bar{x}) = \\ & = z_1^2(\bar{x}) + z_2^2(\bar{x}) + \ldots + z_q^2(\bar{x}) - z_{q+1}^2(\bar{x}) - \ldots - z_r^2(\bar{x}) \end{aligned}$$

ayniyatda $x_1, x_2, \ldots, x_n$ lar o'rniga $\alpha_1, \alpha_2, \ldots, \alpha_n$ sonlarni qo'ysak, quyidagi tenglik kelib chiqadi:

$$y_{p+1}^2(\alpha) + \ldots + y_r^2(\alpha) + z_1^2(\alpha) + z_2^2(\alpha) + \ldots + z_q^2(\alpha) = 0. \tag{12}$$

Bu tenglikdan, xususan,

$$z_1(\alpha) = 0, z_2(\alpha) = 0, \ldots, z_q(\alpha) = 0 \tag{13}$$

hosil bo'ladi. (11) va (13) tengliklardan

$$z_1(\alpha) = 0, z_2(\alpha) = 0, \ldots, z_n(\alpha) = 0 \tag{14}$$

kelib chiqadi. (14) ga ko'ra

$$C \cdot \bar{x} = 0 \tag{15}$$

tenglama noldan farqli yechimga ega. Ziddiyat, chunki $\det C \neq 0$ bo'lgani uchun (15) tenglama faqat nol yechimga ega bo'ladi.

$p > q$ deb faraz qilsak, ham shu tarzda ziddiyatga kelamiz. Demak, $p = q$ bo'lar ekan. **Teorema isbotlandi.**

V A R I A N T L A R

1-topshiriq. Quyida berilgan kvadratik formani kanonik shaklga Lagranj usulidan foydalanib keltirilsin.

1) $f = 2x_1x_2 + 2x_1x_3 - 6x_2x_3$
2) $f = x_1x_3 - 4x_2x_3 + x_2x_4$
3) $f = x_1x_3 - 4x_2x_3 + x_2x_4$
4) $f = x_1^2 + 2x_1x_2 + 2x_2^2 - 2x_2x_3$
5) $f = x_1^2 + 2x_1x_2 + 2x_2^2 + 4x_2x_3 + 5x_3^2$
6) $f = x_1^2 - 4x_1x_2 + 2x_1x_3 + 4x_2^2 + x_3^2$
7) $f = x_1x_2 + x_1x_3 + x_2x_3$
8) $f = x_1^2 + x_1x_2 + x_3x_4$
9) $f = 2x_1x_2 + 2x_1x_3 - 6x_2x_3$
10) $f = x_1^2 - 2x_1x_2 + 2x_1x_3 - 2x_1x_4 + x_2^2 + 2x_2x_3 - 4x_2x_4 + x_3^2 - 2x_4^2$
11) $f = 2x_1^2 + x_2^2 - 4x_1x_2 - 4x_2x_3$
12) $f = 2x_1^2 + 2x_2^2 + 3x_3^2 - 4x_1x_2 - 4x_2x_3$
13) $f = 3x_1^2 + 4x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_2x_3$
14) $f = 2x_1^2 + 5x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_1x_3 - 8x_2x_3$
15) $f = x_1^2 - 2x_2^2 - 2x_3^2 - 4x_1x_2 + 4x_1x_3 + 8x_2x_3$
16) $f = 5x_1^2 + 6x_2^2 + 4x_3^2 - 4x_1x_2 - 4x_1x_3$
17) $f = 3x_1^2 + 6x_2^2 + 3x_3^2 - 4x_1x_2 - 8x_1x_3 - 4x_2x_3$
18) $f = 7x_1^2 + 5x_2^2 + 3x_3^2 - 8x_1x_2 + 8x_2x_3$
19) $f = 2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_4^2 - 4x_1x_2 + 2x_1x_4 + 2x_2x_3 - 4x_3x_4$
20) $f = x_1^2 + x_2^2 + x_3^2 + x_4^2 + 2x_1x_2 - 2x_1x_4 - 2x_2x_3 + 2x_3x_4$
21) $f = 8x_1x_3 + 2x_1x_4 + 2x_2x_3 + 8x_2x_4$
22) $f = 2x_1x_2 + 2x_3x_4$

2-topshiriq. Quyidagi kvadratik formalarni musbat aniqlanganlikka tekshiring.

1) $f = 2x_1x_2 + 2x_1x_3 - 2x_1x_4 - 2x_2x_3 + 2x_2x_4 + 2x_3x_4$
2) $f = x_1^2 + x_2^2 + x_3^2 + x_4^2 - 2x_1x_2 + 6x_1x_3 - 4x_1x_4 - 4x_2x_3 + 6x_2x_4 - 2x_3x_4$

3) $f = 2x_1^2 + x_2^2 - 4x_1x_2 - 4x_2x_3$

4) $f = 2x_1^2 + 5x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_1x_3 - 8x_2x_3$

5) $f = (x_1 + x_2)(2x_1 - 4x_3) + 10x_1(x_1 - x_3)$

6) $f = x_1^2 + 2x_2^2 + 3x_3^2 - 4x_1x_2 - 4x_2x_3$

7) $f = x_1^2 - 2x_2^2 - 2x_3^2 - 4x_1x_2 + 4x_1x_3 + 8x_2x_3$

8) $f = 3x_1^2 + 4x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_2x_3$

9) $f = x_1^2 - 2x_1x_2 + x_2x_3 + 4x_3^2$

10) $f = 4x_1x_2 - 6x_1x_3 + 8x_2x_3$

11) $f = 2x_1^2 + 2x_1x_2 + 2x_2^2 + 2x_1x_3 + 2x_2x_3 + 2x_3^2$

12) $f = -x_1^2 + 2x_1x_2 - 2x_2^2 + 2x_2x_3 - 2x_3^2$

13) $f = 4x_1^2 - 4x_1x_2 - x_3^2$

14) $f = 2x_1x_2 - 6x_2x_3 + 2x_3x_1$

15) $f = 4x_1^2 + x_2^2 + 5x_3^2 + 2x_1x_2 - 8x_1x_3 - 4x_2x_3$

16) $f = 2x_1^2 + x_2^2 + 4x_3^2 + 2x_1x_2 - 6x_1x_3 + 2x_2x_3$

17) $f = 2x_1^2 + 3x_2^2 + 4x_3^2 - 2x_1x_2 + 4x_1x_3 - 3x_2x_3$

18) $f = x_1^2 - 2x_2^2 + x_3^2 + 2x_1x_2 + 4x_1x_3 + 2x_2x_3$

19) $f = x_1^2 + x_2^2 + 3x_3^2 + 4x_1x_2 + 2x_1x_3 + 2x_2x_3$

20) $f = x_1^2 + 5x_2^2 - 4x_3^2 + 2x_1x_2 - 4x_1x_3$

21) $f = 4x_1^2 + x_2^2 + x_3^2 - 4x_1x_2 + 4x_1x_3 - 3x_2x_3$

22) $f = 3x_1^2 + 2x_2^2 - x_3^2 - 2x_4^2 + 2x_1x_2 - 4x_2x_3 + 2x_2x_4$

Ishni bajarish tartibi.

1. Talaba laboratoriya ishi bilan tanishadi.
2. Laboratoriya ishi bo'yicha hisobot tayyorlaydi.
3. Nazorat savollariga javob beradi.

Nazorat savollari.

1. Kvadratik forma deb nimaga aytiladi?
2. Kvadratik formaning normal ko'rinishi deb nimaga aytiladi?
3. Kvadratik formaning kanonik ko'rinishi deb nimaga aytiladi?
4. Kvadratik forma qachon musbat aniqlangan deyiladi?
5. Kvadratik formaning rangi deb nimaga aytiladi?
6. Kvadratik forma uchun Lagranj usulini tushuntiring.

7. Kvadratik forma uchun Inertsiya qonunini aytib bering.

Foydalanilgan adabiyotlar.

1. A.G.Kurosh. «Oliy algebra kursi». Toshkent, O’qituvchi, 1976.
2. J.Hojiyev, A.S.Faynleyb. «Algebra va sonlar nazariyasi kursi».Toshkent, O’zbekiston, 2001.
3. R.N.Nazarov, B.T.Toshpo’latov, A.D.Do’sumbetov. «Algebra va sonlar nazariyasi». 1-qism. Toshkent, O’qituvchi, 1993.
4. R.Iskandarov. «Oliy algebra», I-qism. Toshkent, 1963.
5. R.I.Iskandarov. «Algebra va sonlar nazariyasi». 1-qism. Toshkent, O’qituvchi, 1977.
6. D.K.Fadeev, I.S.Sominskiy. «Sbornik zadach po vыsshey algebre». Moskva, Nauka, 1977.
7. I.V.Proskuryakov. «Sbornik zadach po lineynoy algebre». Moskva, Nauka, 1984.

5-laboratoriya ishi.
Mavzu: Karrali ko'paytuvchilarga ajratish. Viyet formulalari

Ishdan maqsad: Ko'phadlarni karrali ko'paytuvchilarga ajratishni va viyet formulalarini qo'llashni o'rganish.

Nazariy qism:

Ta'rif. Agar $f(x)$ ko'phad $\varphi^{\alpha}(x)$ ga bo'linib, lekin $\varphi^{\alpha+1}(x)$ ga bo'linmasa, $\varphi(x)$ ko'phad $f(x)$ ning α *karrali ko'paytuvchisi* deyiladi.

Bu ta'rifga asosan $f(x)$ ko'phadni

$$f(x) = \varphi^{a}(x)q(x) \qquad (1)$$

shaklda tasvirlash mumkin, bunda $q(x)$ ko'phad $\varphi(x)$ ga bo'linmaydi, chunki, aks holda $q(x) = \varphi(x)h(x)$ ifodani (1) ga qo'yib, ushbuni hosil qilamiz:

$$f(x) = \varphi^{\alpha+1}(x)h(x),$$

bu esa $f(x)$ ning $\varphi^{\alpha+1}(x)$ ga bo'linishini ko'rsatadi.

Masalan, $f(x) = x^5 + x^4 + x^3 - x^2 - x - 1$ ko'phad uchun $\varphi(x) = x^2 + x + 1$ ikki karrali ko'paytuvchidir. Chunki $f(x)$ ko'phad $(x^2 + x + 1)^2$ ga bo'linadi, lekin $(x^2 + x + 1)^3$ ga bo'linmaydi. Biz $f(x) = (x^2 + x + 1)^2(x - 1)$ ga egamiz. $f(x) = x^4 + 2x^3 + 2x^2 + 3x - 2$ ko'phad uchun $\varphi(x) = x^3 + 2x - 1$ bir karrali ko'paytuvchi, chunki $f(x) = (x^3 + 2x - 1)(x + 2)$.

$$f(x) = 5(x^2 - 4)^4(2x^3 + x - 1)^3(x + 1)(x^4 - 3x^3 + 1)^5$$

ko'phad uchun $\varphi_1(x) = x^2 - 4$ to'rt karrali ko'paytuvchi, $\varphi_2(x) = 2x^3 + x - 1$ uch karrali ko'paytuvchi, $\varphi_3(x) = x + 1$ bir karrali ko'paytuvchi va $\varphi_4(x) = x^4 - 3x^3 + 1$ besh karrali ko'paytuvchi ekanligi ravshan.

Teorema. Agar keltirilmaydigan $p(x)$ ko'phad $f(x)$ ko'phad uchun α karrali ko'paytuvchi bo'lsa, uning $f(x)$ hosilasi uchun bu $p(x)$ ko'phad $(\alpha - 1)$ karrali ko'paytuvchi bo'ladi.

Endi $f(x)$ ko'phadning karrali ko'paytuvchilarini ajratish metodi bilan tanishamiz. Faraz qilaylik, $f(x)$ ko'phad keltirilmaydigan ko'phadlar ko'paytmasiga quyidagicha yoyilgan bo'lsin:

$$f(x) = a \cdot p_1^{\alpha_1}(x) \cdot p_2^{\alpha_2}(x) \cdot \ldots \cdot p_r^{\alpha_r}(x). \qquad (2)$$

Bu yoyilmadagi hamma bir karrali keltirilmaydigan ko'phadlar ko'paytmasini X_1 orqali, bittadan (ya'ni birinchi darajali qilib) olingan hamma ikki karrali keltirilmaydigan ko'phadlarning ko'paytmasini X_2 orqali, bittadan olingan hamma uch karrali keltirilmaydigan ko'phadlarning ko'paytmasini X_3 orqali belgilaymiz va h. k., nihoyat, keltirilmaydigan ko'phadlar orasida eng yuqori s

karrali ko’phadlarning bittadan olib tuzilgan ko’paytmasini X_s orqali belgilaymiz. Agar yoyilmada biror k karrali ko’phadlar bo’lmasa $X_k = 1$ deb hisoblaymiz. Shunday qilib, yuqoridagi yoyilma

$$f(x) = a \cdot X_1 \cdot X_2^2 \cdot X_3^3 \cdot \ldots \cdot X_s^s$$

ko’rinishni oladi.

Masalan, $f(x)$ ko’phadning keltirilmaydigan ko’phadlarga yoyilmasi

$$f(x) = 4(x^2-3)^3(x-1)(x-2)(3x^3+1)(2x^2+1)(x+7)^8$$

bo’lsa, bunda

$$X_1(x-1)(x-2), \quad X_2 = 1, \quad X_3 = x^2-3, \quad X_4 = 1,$$

$$X_5 = (3x^3+1)(2x^2+1), \quad X_6 = 1, \quad X_7 = 1, \quad X_8 = x+7$$

bo’ladi. Demak, bu misolda

$$f(x) = 4 \cdot X_1 \cdot X_2^2 \cdot X_3^3 \cdot X_4^4 \cdot X_5^5 \cdot X_6^6 \cdot X_7^7 \cdot X_8^8.$$

$f(x)$ning (2) yoyilmadagi har bir $p_l(x)$ ko’paytuvchi $f'(x)$ hosila uchun bitta kam karrali ko’paytuvchi bo’ladi. Shu sababli $f'(x)$ uchun X_1 ko’paytuvchi bo’lmaydi, X_2 esa bir karrali ko’paytuvchi, X_3 uch karrali ko’paytuvchi bo’ladi va h. k. Demak,

$$f(x) = a \cdot X_2 \cdot X_3^2 \cdot X_4^3 \cdot \ldots \cdot X_s^{s-1}\varphi_1(x)$$

bo’lib, bunda $\varphi_1(x)$ bilan hosilagagina xos, ya’ni $f(x)$ga kirmaydigan ko’paytuvchilarning ko’paytmasini belgiladik. $f(x)$ va $f'(x)$ ning eng katta umumiy bo’luvchisi $d_1(x)$ bu ikki ko’phad uchun umumiy bo’lgan ko’paytuvchilardangina tuziladi. Shu sababli

$$d_1(x) = a \cdot X_2 \cdot X_3^2 \cdot \ldots \cdot X_s^{s-1}.$$

Xuddi yuqoridagi mulohazani takrorlab, $d_1(x)$ning hosilasi

$$d_1'(x) = a \cdot X_3 \cdot X_4^2 \cdot \ldots \cdot X_s^{s-2}\varphi_2(x)$$

ko’rinishga ega degan natijaga kelamiz. $d_1(x)$ va $d_1'(x)$ning eng katta umumiy bo’luvchisi esa ushbudan iborat:

$$d_2(x) = a \cdot X_3 \cdot X_4^2 \cdot \ldots \cdot X_s^{s-2}.$$

So’ngra $d_1(x)$ va uning

$$d_2'(x) = a \cdot X_4 \cdot X_5^2 \cdot \ldots \cdot X_s^{s-3}\varphi_3(x)$$

hosilasi uchun eng katta umumiy bo’luvchi

$$d_3(x) = a \cdot X_4 \cdot X_5^2 \cdot \ldots \cdot X_s^{s-3}$$

ekanini topamiz va hokazo. Shu yo’l bilan eng oxirda

$$d_{s-1}(x) = a \cdot X_s \text{ va } d_s(x) = 1$$

ko’phadlarga kelamiz. Endi ushbu nisbatlarni tuzamiz;

$$E_1 = \frac{f(x)}{d_1(x)} = a \cdot X_1 \cdot X_2 \cdot \cdot \, . \, . \, . \, . \cdot X_s,$$

$$E_2 = \frac{d_1(x)}{d_2(x)} = a \cdot X_2 \cdot X_3 \cdot \cdot \, . \, . \, . \, . \cdot X_{s-1} \cdot X_s,$$

$$E_3 = \frac{d_2(x)}{d_3(x)} = a \cdot X_3 \cdot \cdot \, . \, . \, . \, . \cdot X_{s-1} \cdot X_s,$$

$$. \, . \, . \, . \, . \, . \, . \quad .$$

$$E_{s-1} = \frac{d_{s-2}(x)}{d_{s-1}(x)} = a \cdot X_{s-1} \cdot X_s,$$

$$E_s = \frac{d_{s-1}(x)}{d_s(x)} = a \cdot X_s.$$

Natijada karrali ko'paytuvchilar quyidagicha ajraladi:

$$\frac{E_1}{E_2} = X_1, \quad \frac{E_2}{E_3} = X_2, \, . \, . \, . \, , \frac{E_{s-1}}{E_s} = X_{s-1}, \quad E_s = aX_s.$$

Misol. $f(x) = x^4 + x^3 - 3x^2 - 5x - 2$ ko'phadning karrali ko'paytuvchilarini ajrating.

Yechilishi. Ko'phadni karrali ko'paytuvchilarga ajratish uchun avvalo ko'phadning hosilasini topamiz:

$$f'(x) = 4x^3 + 3x^2 - 6x - 5.$$

Endi Evklid algoritmi yordami bilan $f(x)$ va $f'(x)$ ning eng katta umumiy bo'luvchini topamiz:

$$\begin{array}{l|l}
4x^4 + 4x^3 - 12x^2 - 20x - 8 & 4x^3 + 3x^2 - 6x - 5 \\ \cline{2-2}
\underline{4x^4 + 3x^3 - 6x^2 - 5x \qquad\quad} & \qquad x + 1 \\
\qquad x^3 - 6x^2 - 15x - 8 & \\
\quad 4x^3 - 24x^2 - 60x - 32 & \\
\quad \underline{4x^3 + 3x^2 - 6x - 5 \quad} & \\
\qquad -27x^2 - 54x - 27 & \\
\qquad\qquad x^2 + 2x + 1. &
\end{array}$$

Demak, $d_1(x) = x^2 + x + 1$. Endi $d_1(x)$ va $d_1'(x) = 2x + 2$ hosilaning eng katta umumiy bo'luvchisini izlaymiz:

$$\begin{array}{l|l}
-\begin{matrix} x^2 + 2x + 1 \\ x^2 + x \end{matrix} & \begin{matrix} x + 1 \\ \hline x + 1 \end{matrix} \\ \hline
\qquad -\begin{matrix} x + 1 \\ \underline{x + 1} \end{matrix} & \\
\qquad\quad 0 &
\end{array}$$

Bundan $d_2(x)=x+1$. Nihoyat, $d_2(x)=x+1$ va $d_2'(x)=1$ ning eng katta umumiy bo'luvchisi $d_3(x)=1$. Bularga asosan

$$E_1=\frac{f(x)}{d_1(x)}=x^2-x-2,\quad E_2=\frac{d_1(x)}{d_2(x)}=x+1,\quad E_3=\frac{d_2(x)}{d_3(x)}=x+1$$

va eng oxirgi

$$X_1=\frac{E_1}{E_2}=x-2,\quad X_2=\frac{E_2}{E_3}=1,\quad X_3=E_3=x+1.$$

Shunday qilib,

$$f(x)=X_1\cdot X_2^2\cdot X_3^3=(x-2)(x+1)^3.$$

Viyet formulalari. Bosh koeffitsenti 1 bo'lgan n- darajali $f(x)$ ko'phad berilgan bo'lsin:

$$f(x)=x^n+a_1x^{n-1}+a_2x^{n-2}+...+a_{n-1}x+a_n \tag{3}$$

va $\alpha_1,\alpha_2,...,\alpha_n$ uning ildizlari bo'lsin. U holda $f(x)$ quyidagi yoyilmaga ega bo'ladi:

$f(x)=(x-\alpha_1)(x-\alpha_2)...(x-\alpha_n)$.

O'ng tarafdagi qavslarni ko'paytirib, so'ngra o'xshash hadlarni ixchamlab va hosil bo'lgan koeffisentlarni (3) dagi koeffisentlar bilan taqqoslab, *Viyet formulalari* deb ataluvchi va ko'phad koeffisentlarini uning ildizlari orqali ifodalovchi quyidagi tengliklarni hosil qilamiz:

$$a_1=-(\alpha_1+\alpha_2+\ldots+\alpha_n),$$
$$a_2=\alpha_1\alpha_2+\alpha_1\alpha_3+\ldots+\alpha_1\alpha_n+\alpha_2\alpha_3+...+\alpha_{n-1}\alpha_n,$$
$$a_3=-(\alpha_1\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_4+\ldots+\alpha_{n-2}\alpha_{n-1}\alpha_n),$$
$$\ldots\ldots\quad\ldots\ldots\ldots\ldots\quad\ldots\ldots$$
$$a_{n-1}=(-1)^{n-1}(\alpha_1\alpha_2\,\ldots\,\alpha_{n-1}+\alpha_1\alpha_2\,\ldots\,\alpha_{n-2}\alpha_n+\ldots+\alpha_2\alpha_3\,\ldots\,\alpha_n),$$
$$a_{n-1}=(-1)^n\alpha_1\alpha_2\,\ldots\,\alpha_n.$$

Shunday qilib, $k-$ tenglikning $(k=1,2,\ldots,n)$ o'ng tomonida k ta ildizning mumkin bo'lgan barcha ko'paytmalarining k ning juft yoki toqligiga qarab musbat yoki manfiy ishora bilan olingan yig'indisi turibdi.

$n=2$ da bu formulalar kvadrat ko'phadning ildizlari va koeffitsientlari orasidagi elementlar algebradan ma'lum bo'lgan munosabatga aylanadi. $n=3$ da, ya'ni uchinchi darajali ko'phadlar uchun bu formulalar quyidagi ko'rinishda bo'ladi:

$$a_1=-(\alpha_1+\alpha_2+\alpha_3),\quad a_2=\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3,\quad a_3=-\alpha_1\alpha_2\alpha_3.$$

Viyet formulalari ko'phadni uning berilgan ildizlari bo'yicha yozishni osonlashtiradi. Masalan, oddiy ildizlari 5 va –2 hamda ikki karrali ildizi 3 bo'lgan to'rtinchi darajali $f(x)$ ko'phadni topaylik. Quyidagilarni hosil qilamiz:

$$a_1 = -(5-2+3+3) = -9,$$
$$a_2 = 5\cdot(-2)+5\cdot 3+5\cdot 3+(-2)\cdot 3+(-2)\cdot 3+3\cdot 3 = 17,$$
$$a_3 = -[5\cdot(-2)\cdot 3+5\cdot(-2)\cdot 3+5\cdot 3\cdot 3+(-2)\cdot 3\cdot 3] = 33,$$
$$a_4 = 5\cdot(-2)\cdot 3\cdot 3 = -90,$$

shuning uchun

$$f(x) = x^4 - 9x^3 + 17x^2 + 33x - 90.$$

Agar $f(x)$ ko'phadning bosh koeffitsienti a_0 birdan farqli bo'lsa, Viyet formulasini tatbiq qilish uchun dastlab barcha koeffitsientlarni a_0 ga bo'lish kerak, bu ko'phad ildizlariga ta'sir qilmaydi. Shunday qilib, bu holda Viyet formulalari barcha koeffitsientlarning bosh koeffitsientga nisbati uchun ifodani beradi.

V A R I A N T L A R

1-topshiriq.

1. $f(x) = x^4 + x^3 - 3x^2 - 5x - 2$ ko'phadni karrali ko'paytuvchiga ajrating.

2. $f(x)=x^3+4x^2+5x+2$ ko'phad karrali ildizga egami?

3. $f(x) = x^5 - 3x^4 + 4x^3 - 4x^2 + 3x - 1$ ko'phadni karrali ko'paytuvchiga ajrating.

4. $f(x)=x^3-4x^2+5x-2$ ko'phad karrali ildizga egami?

5. $f(x) = x^7 - 3x^6 + 5x^5 + 7x^4 + 7x^3 - 5x^2 + 3x - 1$ ko'phadni karrali ko'paytuvchiga ajrating.

6. $f(x)=x^3+3x^2-4$ ko'phad karrali ildizga egami?

7. $f(x) = x^6 - 4x^4 - 16x^2 + 16$ ko'phadni karrali ko'paytuvchiga ajrating.

8. $f(x)=x^3-3x^2+4$ ko'phad karrali ildizga egami?

9. $f(x) = x^8 + 2x^7 + 5x^6 + 6x^5 + 8x^4 + 6x^3 + 5x^2 + 2x + 1$ ko'phadni karrali ko'paytuvchiga ajrating.

10. $f(x)=x^3+2x^2-4x-8$ ko'phad karrali ildizga egami?

11. $f(x) = x^7 - 3x^6 + 5x^5 - 7x^4 + 7x^3 - 5x^2 + 3x - 1$ ko'phadni karrali ko'paytuvchiga ajrating.

12. $f(x)=x^3-3x+\lambda$ ko'phad λ ning qanday qiymatlarida karrali ildizga ega?

13. $f(x)=x^7-3x^5-10x^4+16x^3+24x^2-32x-32$ ko'phadni karrali ko'paytuvchiga ajrating.

14. *f(x)=x³-12x+*λ ko'phad λ ning qanday qiymatlarida karrali ildizga ega?

15. $f(x)=x^5+3x^4-6x^3-10x^2+21x-9$ ko'phadni karrali ko'paytuvchiga ajrating.

16. $f(x)=2x^3-9x^2+12x+\lambda$ ko'phad λ ning qanday qiymatlarida karrali ildizga ega?

17. $f(x)=x^6-2x^5-x^4-2x^3+5x^2+4x+4$ ko'phadni karrali ko'paytuvchiga ajrating.

18. $f(x)=2x^3-9x^2-12x+\lambda$ ko'phad λ ning qanday qiymatlarida karrali ildizga ega?

19. $f(x)=x^5+8x^4+25x^3+38x^2+28x+8$ ko'phadni karrali ko'paytuvchiga ajrating.

20. $f(x)=x^6-2x^5+4x^3-3x^2-2x+2$ ko'phadni karrali ko'paytuvchiga ajrating.

21. $f(x)=x^5-2x^4-9x^3+22x^2+4x-24$ ko'phadni karrali ko'paytuvchiga ajrating.

2-topshiriq.

1. Viyet teoremasidan foydalanib, ildizlari -3, 2, -1 bo'ladigan kubik ko'phad tuzing.
2. Viyet teoremasidan foydalanib, ildizlari 6, -5, 2 bo'ladigan kubik ko'phad tuzing.
3. Viyet teoremasidan foydalanib, ildizlari $\frac{1}{2}$, 1, -2 bo'ladigan kubik ko'phad tuzing.
4. Viyet teoremasidan foydalanib, ildizlari 6, 3, -4 bo'ladigan kubik ko'phad tuzing.
5. Viyet teoremasidan foydalanib, ildizlari $-\frac{1}{3}, -\frac{1}{2}, -\frac{1}{4}$ bo'ladigan kubik ko'phad tuzing.

6. Viyet teoremasidan foydalanib, ildizlari -1, 2, 4 bo'ladigan kubik ko'phad tuzing.
7. Viyet teoremasidan foydalanib, ildizlari 2, 3, -4 bo'ladigan kubik ko'phad tuzing.
8. Viyet teoremasidan foydalanib, ildizlari 0, -1, 2 bo'ladigan kubik ko'phad tuzing.
9. Viyet teoremasidan foydalanib, ildizlari 3, 4, -5 bo'ladigan kubik ko'phad tuzing.
10. Viyet teoremasidan foydalanib, ildizlari 1, 0, 7 bo'ladigan kubik ko'phad tuzing.
11. Viyet teoremasidan foydalanib, ildizlari 3, 3, -2 bo'ladigan kubik ko'phad tuzing.
12. Viyet teoremasidan foydalanib, ildizlari 6, 4, -1 bo'ladigan kubik ko'phad tuzing.
13. Viyet teoremasidan foydalanib, ildizlari 2, -5, 7 bo'ladigan kubik ko'phad tuzing.
14. Viyet teoremasidan foydalanib, ildizlari 8, -1, -3 bo'ladigan kubik ko'phad tuzing.
15. Viyet teoremasidan foydalanib, ildizlari $\frac{1}{8}$, 1, -8 bo'ladigan kubik ko'phad tuzing.
16. Viyet teoremasidan foydalanib, ildizlari 6, 0, -7 bo'ladigan kubik ko'phad tuzing.
17. Viyet teoremasidan foydalanib, ildizlari -3, -5, -1 bo'ladigan kubik ko'phad tuzing.
18. Viyet teoremasidan foydalanib, ildizlari 2, -4, -6 bo'ladigan kubik ko'phad tuzing.
19. Viyet teoremasidan foydalanib, ildizlari 11, -1, 0 bo'ladigan kubik ko'phad tuzing.
20. Viyet teoremasidan foydalanib, ildizlari 3, -5, -11 bo'ladigan kubik ko'phad tuzing.
21. Viyet teoremasidan foydalanib, ildizlari 0, -2, 3 bo'ladigan kubik ko'phad tuzing.
22. Viyet teoremasidan foydalanib, ildizlari 4, -5, -1 bo'ladigan kubik ko'phad tuzing.
23. Viyet teoremasidan foydalanib, ildizlari 11, -12, -3 bo'ladigan kubik ko'phad tuzing.

3-topshiriq. Quyidagi $f(x)$ ko'phadning barcha ildizlari kvadratlari yig'indisini toping:
1. $f(x) = x^4 + x^3 - 3x^2 - 5x - 2$

2. $f(x)=x^6-2x^5+4x^3-3x^2-2x+2$
3. $f(x)=x^5+x^4-10x^3-3x^2-3x-1$
4. $f(x)=x^5-x^4+x^3+7x^2-12x+10$
5. $f(x)=x^4+7x^3+19x^2+23x+10$
6. $f(x)=3x^4+2x^3+x^2+2x-2$
7. $f(x)=3x^4-6x^3+5x^2+2x-2$
8. $f(x)=x^4+7x^3+18x^2+22x+12$
9. $f(x)=x^5+3x^4-12x^3-52x^2-52x-12$
10. $f(x)=x^4+6x^3-7x^2+8x+9$
11. $f(x)=x^4-9x^2+6x-5$
12. $f(x)=2x^5-8x^4+12x^3-10x+2$
13. $f(x)=3x^5-6x^4+9x^3+7x-15$
14. $f(x)=x^4+6x^3+8x-5$
15. $f(x)=x^4-2x^3-7x^2+8x+1$
16. $f(x)=3x^4+12x^3+9x^2-1$
17. $f(x)=9x^4-126x^3-252x-140$
18. $f(x)=4x^4-12x^2+8x-1$
19. $f(x)=2x^5-10x^3+10x-3$
20. $f(x)=x^4-2x^3+3x^2-9x+1$
21. $f(x)=x^3-2x^2+4x+8$
22. $f(x)=x^3-3x^2+3x-11$
23. $f(x)=2x^5+8x^4-4x^3+6x^2-4$
24. $f(x)=x^5-x^4+4x^3-5x^2+2x-7$
25. $f(x)=x^4+5x^3-3x^2+2x+6$

4-topshiriq.

1. $f(x)=x^3+3x^2-5x+\lambda$ ko'phadni ikkita ildizini yig'indisi 4 ga teng bo'lsa, λ ni toping.

2. $f(x)=x^3-4x^2+5x+\lambda$ ko'phadni ikkita ildizini yig'indisi -2 ga teng bo'lsa, λ ni toping.

3. $f(x)=x^3-2x^2-3x-\lambda$ ko'phadni ikkita ildizini yig'indisi -3 ga teng bo'lsa, λ ni toping.

4. $f(x)=x^3-2x^2-2x+\lambda$ ko'phadni ikkita ildizini yig'indisi 4 ga teng bo'lsa, λ ni toping.

5. $f(x)=x^3-5x^2+2x+\lambda$ ko'phadni ikkita ildizini ko'paytmasi 2 ga teng bo'lsa, λ ni toping.

6. $f(x)=x^3-\lambda x^2+2x+4$ ko'phadni ikkita ildizini ko'paytmasi 2 ga teng bo'lsa, λ ni toping.

7. $f(x)=x^3-\lambda x^2-2x-3$ ko'phadni ikkita ildizini ko'paytmasi 1 ga teng bo'lsa, λ ni toping.

8. $f(x)=x^3-\lambda x^2+2x+4$ ko'phadni ikkita ildizini ko'paytmasi 2 ga teng bo'lsa, λ ni toping.

9. $f(x)=x^3-5x^2+\lambda x-3$ ko'phadni ikkita ildizini ko'paytmasi 1 ga teng bo'lsa, λ ni toping.

10. $f(x)=x^3+\lambda x^2+5x4-2$ ko'pxad ildizlari kvadratlarining yig'indisi 20 ga teng bo'lsa, λ ni toping.

11. x_1, x_2, x_3 lar $f(x)=x^3+3x^2+4x+7$ ning ildizlari bo'lsa,

$q=\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}$ ni hisoblang.

12. x_1, x_2, x_3 lar $f(x)=x^3-4x^2+7x-5$ ning ildizlari bo'lsa,

$q=\frac{1}{x_1x_2}+\frac{1}{x_2x_3}+\frac{1}{x_1x_3}$ ni hisoblang.

13. x_1, x_2, x_3 lar $f(x)=x^3-5x^2+2x-7$ ning ildizlari bo'lsa, $q=\frac{1}{x_1^2}+\frac{1}{x_2^2}+\frac{1}{x_3^2}$ ni hisoblang.

14. x_1, x_2, x_3 lar $f(x)=x^3-x^2+3x+11$ ning ildizlari bo'lsa,

$q=\frac{1}{x_1^2x_2x_3}+\frac{1}{x_1x_2^2x_3}+\frac{1}{x_1x_2x_3^2}$ ni hisoblang.

15. x_1, x_2, x_3 lar $f(x)=x^3+x^{2-}x-2$ ning ildizlari bo'lsa,

$q=\frac{1}{x_1^3x_2^4x_3^3}+\frac{1}{x_1^4x_2^3x_3^3}+\frac{1}{x_1^3x_2^3x_3^4}$ ni hisoblang.

16. x_1, x_2, x_3, x_4 lar $f(x)=x^4-x^3+3x^2-5x+4$ ning ildizlari bo'lsa,

$q=\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\frac{1}{x_4}$ ni hisoblang.

17. x_1, x_2, x_3, x_4 lar $f(x)=x^4+5x^3-4x^2+x-7$ ning ildizlari bo'lsa,

$q=\frac{1}{x_1x_2x_3}+\frac{1}{x_2x_3x_4}+\frac{1}{x_1x_3x_4}+\frac{1}{x_1x_2x_4}$ ni hisoblang.

18. x_1, x_2, x_3 lar $f(x)=x^3-2x^2+5x-2$ ning ildizlari bo'lsa,

$q=\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}$ ni hisoblang.

19. x_1, x_2, x_3 lar $f(x)=x^3-2x^2+5x-2$ ning ildizlari bo'lsa,

$q=\frac{1}{x_1^2}+\frac{1}{x_2^2}+\frac{1}{x_3^2}$ ni hisoblang.

20. x_1, x_2, x_3 lar $f(x)=x^3+11x^2-13x+5$ ning ildizlari bo'lsa,

$q=\frac{1}{x_1x_2}+\frac{1}{x_2x_3}+\frac{1}{x_1x_3}$ ni hisoblang.

21. x_1, x_2, x_3 lar $f(x)=x^3-7x^2+15x+10$ ning ildizlari bo'lsa,

$q=\frac{1}{x_1x_2^2}+\frac{1}{x_2x_3^2}+\frac{1}{x_1^2x_3}$ ni hisoblang.

22. x_1, x_2, x_3 lar $f(x)=x^3+2x^2-6x+4$ ning ildizlari bo'lsa,

$q=\frac{1}{x_1x_2}+\frac{1}{x_2x_3}+\frac{1}{x_1x_3}$ ni hisoblang.

23. x_1, x_2, x_3 lar $f(x)=x^3+9x^2-5x+12$ ning ildizlari bo'lsa,

$q=\frac{1}{x_1^3x_2^4x_3^3}+\frac{1}{x_1^4x_2^3x_3^3}+\frac{1}{x_1^3x_2^3x_3^4}$ ni hisoblang.

Ishni bajarish tartibi.

1. Talaba laboratoriya ishi bilan tanishadi.
2. Laboratoriya ishi bo'yicha hisobot tayyorlaydi.
3. Nazorat savollariga javob beradi.

Nazorat savollari.

1. Evklid algoritmi deb nimaga aytiladi?

2. Ko’phadlarning umumiy bo’luvchisi deb nimaga aytiladi?
3. Ko’phadlarning eng katta umumiy bo’luvchisi deb nimaga aytiladi?
4. O’zaro tub ko’phadlar deb qanday ko’phadlarga aytiladi?
5. Karrali ko’paytuvchi deb nimaga aytiladi?
6. Viyet formulasini yozib bering?

Foydalanilgan adabiyotlar.

1. A.G.Kurosh. «Oliy algebra kursi». Toshkent, O’qituvchi, 1976.
2. R.Iskandarov. «Oliy algebra», I-qism. Toshkent, 1963.
3. D.K.Fadeev, I.S.Sominskiy. «Sbornik zadach po vыsshey algebre». Moskva, Nauka, 1977.
4. I.V.Proskuryakov. «Sbornik zadach po lineynoy algebre». Moskva, Nauka, 1984.

6-laboratoriya ishi.
Mavzu: Tekislikda to'g'ri chiziq tenglamalari.

Ishdan maqsad: To'g'ri chiziqning qanday usullarda berilishini o'rganish va shularga oid misollar echish.

Nazariy qism:

Tekislikda chiziq berilgan bo'lishi uchun uning nuqtalari holatini aniqlab beruvchi biror qoida maolum bo'lishi kerak. To'g'ri burchakli dekart koordinatalar sistemasida chiziqning tenglamasi

$$F(x,\ y) = 0 \qquad (6.1)$$

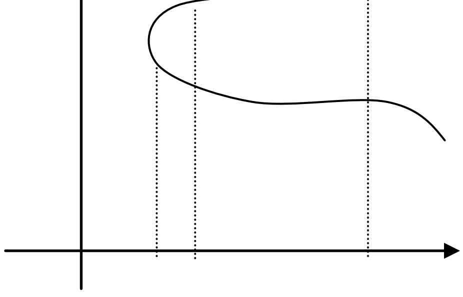
yoki

$$y = f(x), \qquad a \le x \le b \qquad (6.2)$$

ko'rinishda bo'ladi. (2) funktsiya x argument [a,b] kesmada o'zgarganda $f(x)$ funktsiya uzluksiz o'zgaradi deb, faraz qilamiz. Ko'pincha $f(x)$ funktsiyani bir qiymatli funktsiya deb, x va y larni esa, dekart koordinatalar tekisligidagi biror M nuqta koordinatalari deb faraz qilinadi. U holda x ning har bir qiymati uchun (2) tenglamadan u ning yagona qiymati aniqlanadi.

Demak, x ning har bir qiymatiga tekislikning (koordinatalari x, $f(x)$ bo'lgan) bir dona nuqtasi mos keladi. Agar x uzluksiz o'zgarib, turli qiymatlar qabul qilsa, u holda M nuqta koordinatalar tekisligida biror nuqtalar to'plamini tasvirlaydi. Bu nuqtalar to'plami esa, biror chiziqni ifodalaydi. Agar $f(x)$ funktsiya ko'p qiymatli bo'lsa, yaoni x ning har bir qiymatiga koordinatalar tekisligida $M_1, M_2, \ldots, M_s$ nuqtalar mos keladi.

Masalan, $y = f(x)$ funktsiya ikki qiymatli bo'lsin. Bu holda x ning har bir x_1 qiymatiga y ning $y_1 = f(x_1)$ va $y_2 = f(x_2)$ qiymatlari mos kelib, koordinatalar tekisligida ikkita $M_1(x_1,\ y_1)$ va $M_2(x_2, y_2)$ nuqtalar aniqlanadi. x o'zgaruvchi [a,b] kesmada uzluksiz o'zgarganda, M_1 va M_2 nuqtalar ham o'rinlarini uzluksiz o'zgartiradi va bu nuqtalar chiziqni tasvirlaydi.



Ta'rif. Agar chiziq ixtiyoriy nuqtasining x va y koordinatalari (6.1) tenglamani qanoatlantirsa, va aksincha, bu tenglamani qanoatlantiradigan har bir juft (x, y) qiymat chiziq nuqtasini tasvirlasa, (6.1) tenglama *chiziqning oshkormas tenglamasi* deyiladi.

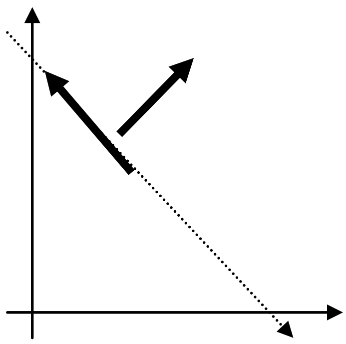
Analitik geometriyada asosan ikkita masala bilan shug'ullaniladi:

1) chiziq nuqtalarning geometrik o'rni sifatida berilgan bo'lib, uning tenglamasini tuzish talab qilinadi;

2) tenglama berilgan, uning grafigini yasash talab qilinadi.

Tekislikdagi chiziqlardan eng soddasi to'g'ri chiziq bo'lib, uning tenglamalari bilan tanishib chiqaylik.

Dekart koordinatalar tekisligida biror l to'g'ri chiziq berilgan bo'lsin. Shuningdek l to'g'ri chiziqda yotuvchi $M_1(x_1,y_1)$ nuqta va l to'g'ri chiziqka perpendikular $\vec{n}$ ={A, B} vektor yoki unga parallel $\vec{S}$ ={m,n} vektorlar berilgan bo'lsa, l to'g'ri chiziqning tekislikdagi turli ko'rinshdagi tenglamalarini keltirib chiqaramiz.



Oxy tekislikda l to'g'ri chiziq berilgan bo'lsin. Unda yotuvchi $M_1(x_1, y_1)$ nuqta va l to'g'ri chiziqqa perpendikular $\vec{n}$ $(l \perp \vec{n})$ vektor olamiz. $\vec{n}$ vektorni l to'g'ri chiziqning *normal vektori* deyiladi.

$M_1(x_1, y_1)$ nuqta va $\vec{n}$ normal vektor l to'g'ri chiziqning Oxy tekislikdagi holatini to'la aniqlaydi. $M(x, y)$ nuqta l to'g'ri chiziqning ixtiyoriy nuqtasi bulsin, u holda $\overrightarrow{M_1M} = \{x - x_1,\ y - y_1\}$ vektor l to'g'ri chiziq ustida yotadi va $\overrightarrow{M_1M} \perp \vec{n}$ bo'ladi. Shuning uchun $\overrightarrow{M_1M}$ va $\vec{n}$ vektorlarning skalyar ko'paytmasi 0 ga teng.

$$\overrightarrow{M_1M} \cdot \bar{n} = 0$$

yoki

$$A(x - x_1) + B(y - y_1) = 0 \qquad (6.3)$$

(6.3) tenglama M_1 nuqta orqali o'tib, $\vec{n}$ vektorga perpendikular bo'lgan to'g'ri chiziqning tenglamasini ifodalaydi.

x va y koordinatalarga nisbatan istalgan birinchi darajali

$$Ax + By + C = 0 \qquad (6.4)$$

ko'rinishdagi tenglama Oxy tekislikda yotuvchi to'g'ri chiziqning umumiy tenglamasi ekanligini ko'rsataylik.

Haqiqatan (6.3) tenglamada A yoki V koeffitsientlardan biri 0 dan farqli deb faraz qilamiz (aks holda A=V=0 bo'lib, S=0 bo'lib qoladi). Masalan, $B \neq 0$ bo'lsin. U holda (6.4) tenglamani quyidagi ko'rinishda yozib olamiz: $A(x-0) + B\left(y + \frac{C}{B}\right) = 0$.

(2.2) tenglamaga teng kuchli bo'lgan tenglamaga ega bo'lamiz. Bu tenglama $M_1\left(0; -\frac{C}{B}\right)$ nuqtadan o'tuvchi va $\vec{n}$ vektorga perpendikular to'g'ri chiziq tenglamasidir.

Demak, (6.4) tenglama to'g'ri chiziqning *umumiy tenglamasi*_ bo'ladi.

Bu to'g'ri chiziqning koordinata o'qlariga nisbatan joylashuvini tekshiraylik.

a) agar $C = 0,\ A \neq 0,\ B \neq 0$ bo'lganda, to'g'ri chiziq tenglamasi $Ax + By = 0$ bo'lib, u koordinatalar boshidan o'tadi;

b) $A = 0,\ C \neq 0,\ B \neq 0$ bo'lganda, tenglama $By + C = 0$ bo'lib, to'g'ri chiziq Ox o'qiga parallel bo'ladi;

v) $B = 0,\ A \neq 0,\ C \neq 0$ bo'lganda, tenglama $Ax + C = 0$ bo'lib, to'g'ri chiziq Oy o'qiga parallel bo'ladi;

g) $B = 0,\ C = 0,\ A \neq 0$ bo'lganda, tenglama $Ax = 0$ bo'lib, Oy o'qni ifodalaydi;

d) $A = 0,\ C = 0,\ B \neq 0$ bo'lganda, tenglama $By = 0$ bo'lib, to'g'ri chiziq Ox o'qni ifodalaydi.

Ta'rif. To'g'ri chiziqqa parallel yoki shu to'g'ri chiziqda yotuvchi har qanday $\overline{S}$ ={m, n} vektor bu to'g'ri chiziqning *yo'naltiruvchi vektori* deyiladi.

Parametrik tenglama *l* to'g'ri chiziq va shu to'g'ri chiziqqa tegishli $M_0(x_0, y_0)$ nuqta va yo'naltiruvchi $\overline{S}$ ={m, n} vektor bilan to'la aniqlanadi. Bu berilganlarga ko'ra *l* to'g'ri chiziq tenglamasini chiqaramiz.

M_0 nuqtadan boshqa *l* to'g'ri chiziqda ixtiyoriy $M(x, y)$ nuqta olamiz. U holda $\overrightarrow{M_0M}$ vektor $\vec{S}$ vektor bilan kollinear bo'ladi. Demak, shunday *t* son topiladiki,

$$\overline{M_0M} = t\overline{S}, \quad (t \in R) \qquad (6.5)$$

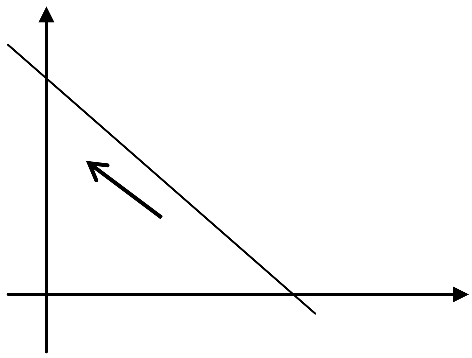
bo'ladi. M, M_0 nuqtalarning radius vektorlarini mos ravishda $\vec{r}$, $\vec{r_0}$ orqali belgilasak, yaoni $\vec{r} = \overrightarrow{OM}$, $\vec{r_0} = \overrightarrow{OM_0}$, bo'lsa, u holda $\overline{M_0M} = \bar{r} - \bar{r_0}$. (6.5)tenglikdan

$$\bar{r} = \bar{r_0} + t\bar{S}. \tag{6.6}$$

(6.6) tenglama *l* to'g'ri chiziqning *vektorli tenglamasi* deyiladi. (6.6) tenglamadagi *t* ga turli qiymatlar berib, *l* to'g'ri chiziqka tegishli nuqtalarning radius-vektorlarini topamiz. (6.6) tenglamadagi *t* o'zgaruvchi *parametr* deyiladi. Endi (6.6) ni koordinatalarda yozamiz: $\vec{r} = \overrightarrow{OM} = x\vec{i} + y\vec{j}$,

$$\vec{r_0} = \overrightarrow{OM_0} = x_0\vec{i} + y_0\vec{j}$$

$$\overrightarrow{M_0M} = \mathrm{t}\vec{S} = \mathrm{t}(m\vec{i} + n\vec{j})$$



Bularni (6.6) ga qo'yib, so'ngra ikki vektorning tengligiga ko'ra ushbu tenglamalarga ega bo'lamiz:

$$\begin{cases} x = x_0 + mt \\ y = y_0 + nt \end{cases} \tag{6.7}$$

(6.7) tenglamani *l* to'g'ri chiziqning *parametrik tenglamasi* deyiladi. Agar *l* to'g'ri chiziq koordinata o'qlaridan birortasiga ham parallel bo'lmasa, (yaoni $\vec{m}, \vec{n} \neq 0$ shart bajarilsa) u holda (6.7) dan ushbu

$$\begin{cases} x = x_0 + mt \\ y = y_0 + nt \end{cases} \Rightarrow \begin{cases} t = \dfrac{x - x_0}{m} \\ t = \dfrac{y - y_0}{n} \end{cases}$$

yoki

$$\frac{x - x_0}{m} = \frac{y - y_0}{n} \tag{6.8}$$

tenglamani hosil qilamiz. (6.8) tenglamani esa to'g'ri chiziqning *kanonik tenglamasi* deyiladi.

Bizga maolumki, M_1 va M_2 nuqtalardan yagona to'g'ri chiziq o'tadi. M_1 va M_2 nuqtalarning koordinatalari maolum deb faraz qilib, shu nuqtalar orqali o'tuvchi *l* to'g'ri chiziq tenglamasini tuzaylik.

$M_1(x_1,y_1)$, $M_2(x_2,y_2)$ nuqtalar izlanayotgan l to'g'ri chiziqqa tegishli bo'lsin. Shuningdek bu to'g'ri chiziqda $M(x, y)$ ixtiyoriy nuqtani olamiz. Natijada bu nuqtalar yordamida to'g'ri chiziqda o'zaro kollinear bo'lgan $\overrightarrow{M_1M_2} = \{x_2 - x_1,\ y_2 - y_1\}$ va $\overrightarrow{M_1M}$ vektorlarga ega bo'lamiz. Bu vektorlar kollinear bo'lganligi uchun $\overrightarrow{M_1M} = t\,\overrightarrow{M_1M_2}$ tenglik o'rinli bo'ladi. Bundan vektorlarning tengligiga asosan

$$x - x_1 = t(x_2 - x_1) \quad \text{va} \quad y - y_1 = t(y_2 - y_1)$$

ga ega bo'lamiz. Oxirgidan

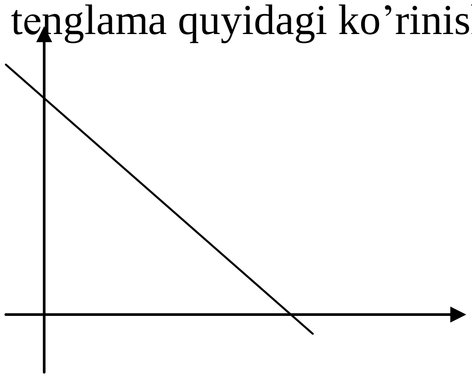
$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}\,. \tag{6.9}$$

(6.9) tenglama berilgan ikki nuqta orqali o'tuvchi to'g'ri chiziq tenglamasi bo'ladi.

Ikki nuqta orqali o'tuvchi to'g'ri chiziq tenglamasini uchinchi tartibli determinant yordamida ham yozish mumkin:

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

l to'g'ri chiziqni aniqlovchi M_1 va M_2 nuqtalar koordinata o'qlari Ox va Oy da yotsin. Aniqrog'i $M_1(a;0)$ Ox o'qda, $M_2(0;b)$ esa Oy o'qda yotsin. Bu holda (6.9) tenglama quyidagi ko'rinishda bo'ladi:



$$\frac{x - a}{-a} = \frac{y - 0}{b} \Rightarrow \frac{x}{a} + \frac{y}{b} = 1 \tag{6.10}$$

(6.10) tenglama to'g'ri chiziqning *kesmalar bo'yicha tenglamasi* deyiladi.

Ta'rif. $\vec{S}$ vektor $\{\vec{i}, \vec{j}\}$ bazisda m,n koordinatalarga ega bo'lib, $m \neq 0$ bo'lsa, $\frac{n}{m} = k$ son $\vec{S}$ vektorning *burchak koeffitsienti* deyiladi.

Agar l to'g'ri chiziq Oy o'qiga parallel bo'lsa, u holda bunday to'g'ri chiziqning burchak koeffitsientli tenglamasi mavjud emas. Shuning uchun Oy o'qqa parallel bo'lgan l to'g'ri chiziq $M_0(x_0, y_0)$ nuqtadan o'tsin va k burchak koeffitsientiga ega bo'lsin deb faraz qilamiz. (2.6) dan $m \neq 0$ deb quyidagiga ega bo'lamiz:

$$\frac{x - x_0}{m} = \frac{y - y_0}{n} \Rightarrow y - y_0 = \frac{n}{m}(x - x_0) \Rightarrow y - y_0 = k(x - x_0) \ \text{(chunki } \frac{n}{m} = k\text{)}$$

yoki

$$y = kx + b \tag{6.11}$$

bunda

$$b = y_0 - kx_0$$

(6.11) ni to'g'ri chiziqning *burchak koeffitsientli tenglamasi* deyiladi.

Bundan tashqari to'g'ri chiziq tenglamalariga oid quyidagi maolumotlarni keltirib chiqaramiz:

1) $A_1x + B_1y + C_1 = 0$ va $A_2x + B_2y + C_2 = 0$ to'g'ri chiziqlar orasidagi burchakni $tg\alpha = \frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}$ formula bilan topiladi.

$\frac{A_1}{A_2} = \frac{B_1}{B_2}$ parallellik sharti, $A_1A_2 + B_1B_2 = 0$ esa perpendikulyarlik sharti bo'ladi.

2) Uch nuqtaning bir to'g'ri chiziqda yotish sharti:

$$\frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1} \qquad \text{yoki} \qquad \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

bo'ladi.

3) $\rho = \frac{p}{\cos(\varphi - \alpha)}$ to'g'ri chiziqning qutb koordinatalar sistemasidagi tenglamasidir.

4)Nuqtadan to'g'ri chiziqqacha bo'lgan masofa $d = \rho(M_0, l) = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$ formula bilan topiladi.

V A R I A N T L A R

1-topshiriq.

1. To'g'ri chiziqning umumiy tenglamasi $2x - 5y - 10 = 0$ berilgan. Uning parametrik tenglamalarini yozing.
2. To'g'ri chiziqning $x = 2 - t,\ y = 3 + 2t$ parametrik tenglamasi bo'yicha uning umumiy tenglamasini yozing.
3. A(-3, 5) nuqtadan o'tib, $\overline{p} = \{1, -2\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.
4. Koordinata o'qlaridan mos ravishda $a = 2,\ b = -5$ kesmalarni kesib o'tuvchi to'g'ri chiziq tenglamasini tuzing.
5. A(7, -5) nuqtadan o'tib, Ox o'qqa parallel bo'lgan to'g'ri chiziq tenglamasini tuzing.

6. $N(-8,\ 1)$ nuqtadan o'tib, Oy o'qqa parallel bo'lgan to'g'ri chiziq tenglamasini tuzing.
7. $M(0,\ -2)$ va $N(3,-4)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini tuzing.
8. Quyidagi to'g'ri chiziqlarning yo'naltiruvchi vektorlarini toping:

1) $6x-7y+5=0$; 2) $3x+7y+8=0$; 3) $x-9=0$;
4) $-2x+3y-4=0$; 5) $-x+3y-7=0$.

9. $A(3,-4)$ nuqtadan o'tib, $2x-3y+5=0$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziq tenglamasini tuzing.
10. Uchburchakning uchlari berilgan: *A(4;6)*, *B(-4;0)* va *C(-1;-4)*.Uning *C* uchidan *AB* tomoniga tushirilgan balandligining tenglamasi tuzilsin.
11. Koordinatalar boshidan o'tib, 2x-3y+4=0 to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziq tenglamasini tuzing.
12. Uchburchakning uchlari bilan berilgan: A(3;-2), V(4;-1) va S(0;-4). Uchlarning har biridan unga qarshi yotgan tomonga parallel to'g'ri chiziq o'tkazilsin.
13. Uchlari $A(-3,-2),\ B(1,2),\ C(4,-5)$ nuqtalarda bo'lgan uchburchakning tomonlarining tenglamasini tuzing.
14. Uchlari $A(-3,-2),\ B(1,2),\ C(4,-5)$ nuqtalarda bo'lgan uchburchakning medianalarining tenglamasini tuzing.
15. Uchlari $A(0,1),\ B(6,\ 9),\ C(3,-3)$ nuqtalarda bo'lgan uchburchak *A* ichki burchagi bissektrisasining tenglamasini tuzing.
16. Quyidagi to'g'ri chiziqlarning koordinatalar o'qiga nisbatan qanday joylashishini tekshiring va bu to'g'ri chiziqlarni yasang:

1) $2x+y=0$ 2) $6x-2y+7=0$ 3) $3x-8=0$ 4) $3y+1=0$ 5) $7y=0$

17. Quyidagi to'g'ri chiziqlarning kesishgan nuqtasini toping.

1) $3x-5y-21=0$ va $2x-y-7=0$ 2) $x+3y-54=0$ va $3x+9y+7=0$

18. Quyidagi to'g'ri chiziqlarning o'zaro joylashishini tekshiring, agar kesishsa, ularning kesishish nuqtalarini koordinatalarini toping:

1) $8x-3y-1=0,\quad 4x+y-13=0$ 2) $x+y-6=0,\ \ 2x+2y-5=0$
3) $5x-2y+13=0,\ x+3y-11=0$ 4) $x+y-3=0,\ 2x+2y-6=0$
5) $x=-2,\ y-3=0$ 6) $\sqrt{5x}-3y+1=0,\ \frac{5}{3}x-\sqrt{5y}+\frac{\sqrt{5}}{3}=0$

19. t ning qanday qiymatlarida $3x-8y+1=0$ va $(t+8)x-2ty=0$ to'g'ri chiziqlar parallel bo'ladi?
20. 2x+3y+5=0 tenglama bilan berilgan to'g'ri chiziqning normal vektorini toping.
21. Quyida keltirilgan to'g'ri chiziqlar ichidan parallellarini ko'rsating.

1) x+2y-3=0 2) 2x-y+5=0 3) –4x+2y+3=0 4) x-3y-4=0

2-topshiriq.

1. Koordinatalar boshidan $4x+y-5=0$ to’g’ri chiziqqa parallel to’g’ri chiziq o’tkazing.
2. *a* va *b* larning qanday qiymatlarida quyidagi ikkita to’g’ri chiziq

$$ax-2y-1=0, \qquad 6x-4y-b=0$$

bitta umumiy nuqtaga ega bo’ladi?
3. Uchburchakning ikkita tomonining tenglamasi: $3x-y+8=0,\ 3x+5y-1=0$ medianalarining kesishgan nuqtasi $M\left(-\frac{7}{3},-1\right)$ ni bilgan holda, uning uchinchi tomonining tenglamasini toping.
4. Koordinatalar boshidan $3x-2y+17=0,\ 2x+3y-6=0$ to’g’ri chiziqlarning kesishgan nuqtasigacha bo’lgan masofani toping.
5. Parallelogramm ikki tomonining $8x-3y+1=0,\ 2x+y-1=0$ tenglamalari va bitta diagonalining $3x+2y+3=0$ tenglamasi berilgan. Uning uchlarining koordinatalarini toping.
6. Quyidagi uchta to’g’ri chiziqning o’zaro joylashishini tekshiring:

A) $\begin{cases} 3x-y-1=0 \\ 2x-y+3=0 \\ x-y+7=0 \end{cases}$ B) $\begin{cases} y=3 \\ x-y+5=0 \\ 2y-5=0 \end{cases}$ C) $\begin{cases} 3x-y+6=0 \\ 4x+3y-5=0 \\ 2x-y+5=0 \end{cases}$

D) $\begin{cases} 2x-y+5=0 \\ x+y-3=0 \\ x-y=0 \end{cases}$ E) $\begin{cases} x-y+3=0 \\ \frac{1}{2}x-\frac{1}{2}y+\frac{3}{2}=0 \\ \sqrt{3x}+\sqrt{3y}-3\sqrt{3}=0 \end{cases}$ F) $\begin{cases} x-y=0 \\ 2x-2y+3=0 \\ -x+y+1=0 \end{cases}$

7. Uchburchakning uchlari $A(4,\ 6),\ B(-4,\ 0),\ C(-1,\ -4)$ nuqtalarda bo’lsa, uning (BN) ichki bissektrisasining tenglamasini tuzing.
8. $\lambda x+\mu y+1=0,\ 2x-3y+5=0,\ x-1=0$ to’g’ri chiziqlarning bir nuqtadan o’tishi uchun λ, μ lar qanday shartni qanoatlantirishi kerak?
9. $x+2=0,\ y+3=0,\ x+y=0$ to’g’ri chiziqlar uchburchak hosil qiladimi?
10. Uchburchakning uchlari berilgan: *A(-1;2)*, *B(3;-1)* va *C(0;4)*. Uning *B* uchidan *AC* tomoniga tushirilgan balandligining tenglamasi tuzilsin.
11. $B(4,-2)$ nuqtadan o’tib, $5x+2y-3=0$ to’g’ri chiziqqa perpendikular bo’lgan to’g’ri chiziqning tenglamasini tuzing.
12. $M(3,\ 2)$ nuqtaning $2x-3y+6=0$ to’g’ri chiziqdagi proektsiyasini toping.
13. $x+4y+3=0$ to’g’ri chiziqqa nisbatan $M(2,\ 3)$ nuqtaga simmetrik bo’lgan nuqtani toping.
14. *a* va *b* larning qanday qiymatlarida quyidagi ikkita to’g’ri chiziq

$$ax-2y-1=0, \qquad 6x-4y-b=0$$

ustma-ust tushadi?
15. $P(-8, 12)$ nuqtaning $A(2, -3)$, $B(-5, 1)$ nuqtalardan o'tuvchi to'g'ri chiziqdagi proektsiyasini toping.
16. Agar to'rtburchak tomonlarining tenglamasi mos ravishda $x=4,\ \ y=5,\ \ y=x,\ \ y=2x$ bo'lsa, uning diagonallarining tenglamasini tuzing.
17. Agar uchburchakning uchlari $A(0,\ 1)$, $B(2,\ 0)$, $C(3,\ -4)$ nuqtalarda bo'lsa, uning balandliklarining tenglamasini tuzing.
18. M(-3, 2) nuqta hamda $5x-6y+3=0$ va $x-4y-1=0$ to'g'ri chiziqlarning kesishish nuqtasidan o'tuvchi to'g'ri chiziq tenglamasini tuzing.
19. *a* va *b* larning qanday qiymatlarida quyidagi ikkita to'g'ri chiziq:

$$ax-2y-1=0, \qquad 6x-4y-b=0$$

kesishmaydi?
20. Markazi (1,-6) nuqtada bo'lgan to'g'ri chiziqlar dastasini tenglamasini yozing.
21. $B(-2,\ 7)$ nuqtadan o'tib, $-3x+6y-2=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

Ishni bajarish tartibi.

1. Talaba laboratoriya ishi bilan tanishadi.
2. Laboratoriya ishi bo'yicha hisobot tayyorlaydi.
3. Nazorat savollariga javob beradi.

Nazorat savollari.

1. Tekislikda chiziq tenglamasi nimani anglatadi?
2. To'g'ri chiziqning oshkormas tenglamani taoriflang.
3. To'g'ri chiziqning normal vektori deb nimaga aytiladi?
4. To'g'ri chiziqning umumiy tenglamasi qanday ifodalanadi?
5. To'g'ri chiziqning vektorli tenglama deb nimaga aytiladi?
6. To'g'ri chiziqning parametrik tenglamalarini ko'rsating?
7. To'g'ri chiziqning kanonik tenglamasi qanday ifodalanadi?
8. Ikki nuqta orqali o'tuvchi to'g'ri chiziq tenglamasini ko'rsating?
9.To'g'ri chiziqning kesmalar bo'yicha va burchak koeffitsientli tenglamalari qanday ko'rinishda bo'ladi?

Foydalanilgan adabiyotlar.

1. Suberbiller O.N.«Analitik geometriyadan masalalar va mashqlar». Toshkent-1960 y.
2. Bakelman I.YA. Analitik geometriya va chiziqli algebra. Toshkent, O’qituvchi,1978 y.
3. X.X.Nazarov, X.O.Ochilova, E.G.Podgarnova. «Geometriyadan masalalar to’plami».I-kism. Toshkent-1983 y.
4. L.S.Atanasyan, V.A.Atanasyan. «Sbornik zadach po geometrii». CHast-I. Moskva-1973 g.
5. N.D.Dodajonov, M.Sh.Jo’rayeva. Geometriya.1-qism.Toshkent-1996 y.
6.X.R.Latipov, F.U.Nosirov, Sh.I. Tojiyev. Analitik geometriya va chiziqli algebradan masalalar yechish bo’yicha qo’llanma. Toshkent-1999y.

7-laboratoriya ishi.
Mavzu: Fazoda tekislik tenglamalari

Ishdan maqsad: Tekislik tenglamalarini qanday usullarda berilishini o'ganish.

Nazariy qism:

1.Nokollinear ikki $\overline{p}_1, \overline{p}_2$ vektor va bitta M_0 nuqta P tekislikning vaziyatini to`la aniqlaydi.

$\forall\ M \in \Pi$ nuqtani olaylik . U holda $\overline{M_0M}$ vektor va $\overline{p}_1, \overline{p}_2$ vektorlar bilan komplanar bo`ladi, demak, bu vektorlar chiziqli bog`liq bo`lib, bundan ularning koordinatalaridan tuzilgan uchinchi tenglamaning determenanti nolga teng bo`lishi kelib chiqadi. Shuni koordinatalarda yozaylik.

$$M_0(x_0, y_0, z_0),\ \ \overline{p}_1\{a_1, b_1, c_1\},\ \ \overline{p}_2\{a_2, b_2, c_2\} \qquad (7.1)$$

bo`lsin. $M$ ning koordinatalarini $x, y, z$ deb olaylik . $\overline{M_0M}\{x - x_0,\ y - y_0,\ z - z_0\}$ bo`lib, quyidagi tenglama hosil bo`ladi:

$$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0. \qquad (7.2)$$

Aksincha, (7.2) shart bajarilsa, M nuqta albatta P tekislikka tegishli bo`ladi. Demak, (7.2) P ning tenglamasi. Bu tenglama berilgan nuqtadan o`tib, berilgan (nokkollinear) ikki vektorga parallel bo`lgan tekislikning *tenglamasi* deb yuritiladi.

Bundan tashqari $\overline{M_0M}$, $\overline{p}_1, \overline{p}_2$ vektorlar bir tekislikda yotgani uchun ular chiziqli bog`liqdir, ya'ni :

$$\overline{M_0M} = u\,\overline{p}_1 + \vartheta\,\overline{p}_2,\ u,\ \vartheta \in R, \qquad (7.3)$$

bu yerda $u,\ \vartheta$ sonlar parametrlardir. (7.3) dan

$$\begin{cases} x - x_0 = u\,a_1 + \vartheta a_2, \\ y - y_0 = u\,b_1 + \vartheta b_2, \\ z - z_0 = u\,c_1 + \vartheta c_2. \end{cases} \Rightarrow \begin{cases} x = u\,a_1 + \vartheta a_2 + x_0, \\ y = u\,b_1 + \vartheta b_2 + y_0, \\ z = u\,c_1 + \vartheta c_2 + z_0. \end{cases} \qquad (7.4)$$

(7.4) *tekislikning parametrik tenglamalari* deb ataladi.(u va ϑ ga istalgan qiymatlar berib, tekislikning shu parametrlarga mos nuqtalarini topish mumkin).

Endi (7.2) tenglamani quyidagicha yozaylik:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0 \qquad (7.5)$$

$$A = \begin{vmatrix} b_1 c_1 \\ b_2 c_2 \end{vmatrix}, B = \begin{vmatrix} c_1 a_1 \\ c_2 a_2 \end{vmatrix}, C = \begin{vmatrix} a_1 b_1 \\ a_2 b_2 \end{vmatrix} \qquad (7.6)$$

$(7.5) \Rightarrow Ax + By + Cz - (Ax_0 + By_0 + Cz_0) = 0$ bunda $-(Ax_0 + By_0 + Cz_0) = D$ demak,

$$Ax + By + Cz + D = 0 \qquad (7)$$

tenglama hosil bo`ladi.( 7.2) bilan (7.7) teng kuchli bo`lgani uchun (7.7) ham tekislik tenglamasi bo`ladi.( 7.6) da *A, B ,C* larning kamida bittasi noldan farqli, aks holda *A=B=C=0* bo`lsa, (7.6) dan: $a_1 = \lambda a_2, b_1 = \lambda b_2$, $c_1 = \lambda c_2 \Rightarrow \bar{p}_1 \parallel \bar{p}_2$, bu esa $\vec{p}_1, \vec{p}_2$ larning berilishiga zid. Shunday qilib tekislikning affin reperida teskarisi ham o`rinlidir. Ya'ni (7.7) ko`rinishdagi har qanday chiziqli tenglama fazodagi biror affin reperga nisbatan tekislikni aniqlaydi.

Haqiqatan ham, $Ax + By + Cz + D = 0$ $(A^2 + B^2 + C^2 \neq 0)$ tenglama biror affin reperda biror nuqtalar to`plamini aniqlasin.Uch o`zgaruvchini bog`lagan bu tenglamani yechimi cheksiz ko`pdir. ularning biri (x_0, y_0, z_0) bo`lsa, u holda $Ax_0 + Bx_0 + Cz_0 + D = 0$ bundan va (7.7) dan : $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$ tekislik tenglamasi bo`ladi.

(7.7) tenglama tekislikning umumiy tenglamasi deyiladi.

2. Bir to`g`ri chiziqda yotmaydigan uchta nuqta tekislikning vaziyatini aniqlaydi. Shu ma'lumotlarga ko'ra uning tenglamasini tuzaylik. Berilgan nuqtalar $M_1(x_1, y_1, z_1), M_2(x_2, y_2, z_2), M_3(x_3, y_3, z_3)$, bo`lsin. Biz $M_0 = M_1$, $\bar{p}_1 = \overline{M}_1\overline{M}_2$, $\bar{p}_2 = \overline{M}_1\overline{M}_3$ desak, hamda

$$\overline{M_1M_2} = \{x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1\}, \qquad \overline{M_1M_3} = \{x_3 - x_1,\ y_3 - y_1,\ z_3 - z_1\}$$

ni etiborga olsak, (7.2) tenglama quyidagi ko'rinishni oladi:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \qquad (7.8)$$

Uch nuqtadan o`tgan tekislik tenglamasi shudir.

Agar tekislikga koordinatalar boshidan o`tmasa, u $(O_x), (O_y), (O_z)$ o`qlarni uchta $M_1(a, 0, 0)$, $M_2(0, b, 0)$, $M_3(0, 0, c)$ nuqtada kesadi, bu yerda *a, b, c* tekislikning shu o`qlarning ajratgan kesmalari bo`ladi.

Bunga (7.8) ko`rinishli tenglamani tatbiq qilamiz:

$$\begin{vmatrix} x - a & y - b & z - c \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = 0,$$

bundan

$$\frac{x}{a} + \frac{y}{b} + \frac{c}{z} = 1, \qquad (7.9)$$

bu tenglama *tekislikning koordinata o`qlarining ajratgan kesmalari bo`yicha tenglamasi* deyiladi.

Biz bu mavzuda tekislikning 6 xil ko`rinishidagi (7.2), (7.3), (7.4), (7.7), (7.8), (7.9) tenglamalarini ko`rdik.

$\bar{n}(A,B,C)$ vektor ushbu $Ax+By+Cz+D=0$ tekislikning *normal vektori* deyiladi. Normal vektor tekislikga perpendikulyar bo`ladi.

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$$

Bu tenglama $M_0(x_0,y_0,z_0)$ nuqtadan o`tib, $\bar{n}(A,B,C)$ vektorga perpendikulyar tekisliknig tenglamasi deyiladi.

***Ta`rif.*** Berilgan $M_1$ nuqtaning berilgan $\Pi$ tekislikkacha bo`lgan masofa deb, shu nuqtaning tekislikga tushirilgan perpendikulyar to`g`ri chiziqlarini tekislik bilan kesishgan nuqtasi orasidagi masofaga aytiladi.

$$\rho(M_1,\Pi)=\frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$$

Ikki tekislik orasidagi burchak:

$$\cos\varphi=\cos(\bar{n}_1;\bar{n}_2)=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A^2{}_1+B^2{}_1+C^2{}_1}\sqrt{A^2{}_2+B^2{}_2+C^2{}_2}}$$

V A R I A N T L A R

1-topshiriq.

1. Oy o'qdan va $x-y=0,\ x+y-2z-1=0,\ 2x+z-4=0$ tekisliklarning umumiy M_0 nuqtasidan o'tuvchi tekislik tenglamasini toping.
2. z o'qidan va (4, -7, 5) nuqtadan o'tuvchi tekislikning tenglamasi yozilsin.
3. x o'qiga parallel va (4, 0, -2), (5, 1, 7) nuqtalardan o'tuvchi tekislikning tenglamasi yozilsin.
4. Ushbu

$$3x-5y+z+15=0$$

tekislikning koordinata o'qlaridan kesgan kesmalari hisoblansin.

5. (-2, 7, 3) nuqtadan o'tuvchi va $x-4y+5z-1=0$ tekislikka parallel bo'lgan tekislik tenglamasini tuzing.
6. $15x-10y+6z-190=0$ tekislikning koordinatalar boshidan masofasi topilsin.
7. Koordinatalar boshidan 6 birlik masofada o'tib, koordinata o'qlaridan kesgan kesmalari $a:b:c=1:3:2$ munosabat bilan bog'langan tekislikning tenglamasi tuzilsin.
8. $2x-y+2z+9=0$ tekislikka perpendikular to'g'ri chiziqning yo'naltiruvchi kosinuslari topilsin.
9. Ushbu $3x-y+2z+15=0$ va $5x+9y-3z-1=0$ tekisliklar orasidagi burchak topilsin.

10. Koordinata tekisliklarining $2x-3y+5z-2=0$ tekislik bilan kesishish chiziqlari yasalsin.
11. $x+3y+2z+5=0$ tekislikning normal vektori koordinatalarini yozing.
12. Quyidagi tekisliklarning o'zaro vaziyatini aniqlang:

$$3x+5y+z-5=0; \qquad 8x+7y+4z-1=0.$$

13. $M_0(3,-2,5)$ nuqtadan o'tib, $x-2y+7z-5=0$ tekislikka parallel bo'lgan tekislik tenglamasini toping.
14. $x-2y+4z-3=0$ va $2x+y-4z+3=0$ tekisliklarning kesishish chizig'iga tegishli birorta nuqtaning koordinatalarini toping.
15. $3y+2z+6=0$ va $2x+5y+6z+4=0$ tekisliklarning kesishish chizig'idan va koordinatalar boshidan o'tgan tekislik tenglamasini tuzing.
16. $2x-y+z-4=0,\ x+y-z-2=0,\ 2x-y+3z-6=0$ tekisliklar bir nuqtada kesishishini ko'rsating va bu nuqtaning koordinatalarini toping.
17. $x-y=0,\ x+y-2z+1=0,\ 2x+z-4=0$ tekisliklarning kesishgan nuqtasi hamda $M(2, 1, 7)$ va $O(0, 0, 0)$ nuqtalardan o'tgan tekislikning tenglamasini toping.
18. (xz) tekislikka parallel va (-3, 2, -5) nuqtadan o'tuvchi tekislikning tenglamasi yozilsin.
19. O'zaro parallel bo'lgan $2x+6y-3z-3=0$ va $4x+12y-6z-7=0$ tekisliklar orasidagi masofani hisoblang.
20. $M_0(3,-4,-2)$ nuqtadan va Oz o'qdan o'tuvchi tekislik tenglamasini toping.
21. Ushbu $2x+3y-z+12=0$ tekislikning tenglamasini normal shaklga keltirilsin.

2-topshiriq.

1. $ABCDA_1B_1C_1D_1$ parallelepipedning $A(4,\ 0,\ 2),\ B(0, 5, 1),\ C(4, -1, 3),\ A_1(3, -1, 5)$ uchlari berilgan. Parallelepiped yoqlari orqali o'tuvchi tekisliklarning tenglamalarini tuzing.
2. $2x-y+3z=0$ va $x+4y-6z=0$ tekisliklar orasidagi burchakni hisoblang.
3. $M_0(1, 0,-2)$ nuqtadan va Oy o'qdan o'tuvchi tekislik tenglamasini toping.
4. $x-4y-8z+5=0$ tekislikdan 4 birlik masofada yotuvchi, unga parallel tekislik tenglamasini toping.
5. $x+y-z+2=0,\ 4x-3y-3z-1=0,\ 2x+y+1=0$ tekisliklarning kesishgan M nuqtasidan o'tib, (xOz) tekislikka parallel bo'lgan tekislik tenglamasini toping.

6. $M_0(-2,\,3,\,1)$ nuqtadan o’tib, $2x - y + z + 1 = 0$ tekislikka parallel bo’lgan tekislik tenglamasini toping.
7. $x + y + z + 1 = 0,\ x + 2y + 3z + 4 = 0,\ x - y + \lambda z - 1 = 0$ tekisliklar λ ning qanday qiymatlarida yagona nuqtada kesishadi?
8. $M_0(-2, 5, -3)$ nuqtadan o’tib, $2x - 5y - z - 8 = 0$ tekislikka parallel bo’lgan tekislik tenglamasini toping.
9. O’zaro parallel bo’lgan $x - 3y + 2z + 5 = 0$ va $2x - 6y + 4z + 3 = 0$ tekisliklar orasidagi masofani hisoblang.
10. $x - 4y + 3z - 1 = 0$ va $-3x - y + 4z = 0$ tekisliklar orasidagi burchakni hisoblang.
11. $M_0(-2,\,5,\,-3)$ nuqtadan o’tib, $\bar{n}(4,\,-3,\,5)$ vektorga perpendikular bo’lgan tekislik tenglamasini toping.
12. $M_0(-3,\,1,\,6)$ nuqtadan o’tib, Ox o’qqa perpendikular bo’lgan tekislik tenglamasini toping.
13. $M_0(-3,\,1,\,6)$ nuqtadan o’tib, Oy o’qqa perpendikular bo’lgan tekislik tenglamasini toping.
14. $M_0(-3,\,1,\,6)$ nuqtadan o’tib, Oz o’qqa perpendikular bo’lgan tekislik tenglamasini toping.
15. $A(1,\,-3,\,1)$ va $B(0,\,2,\,4)$ nuqtalardan bir xil uzoqlikda yotgan nuqtalar to’plamining tenglamasini tuzing.
16. Ordinatasi 3 bo’lgan nuqta yOz tekisligiga va $x + y + z - 1 = 0$ tekislikka tegishli ekani maolum bo’lsa, uning abstsissa va applikatasini toping.
17. $6x - 3y + 2z - 14 = 0$ tekislikdan 3 birlik masofada yotuvchi nuqtalar to’plamining tenglamasini toping.
18. $x + 4y - 2z + 3 = 0$ va $2x - 3y + 5z - 9 = 0$ tekisliklar orasidagi burchakni hisoblang.
19. $M(-3,\,1,\,0)$ nuqtadan va $x + 2y - z + 4 = 0$, $3x - y + 2z - 1 = 0$ tekisliklarning kesishish chizig’idan o’tgan tekislik tenglamasini toping.
20. $M_0(-2,\,5,-1)$ nuqtadan o’tib, $3x - 5y + 2z + 1 = 0$ tekislikka parallel bo’lgan tekislik tenglamasini toping.
21. Koordinatalar boshidan o’tib, $-2x + 5y + 6z - 4 = 0$ tekislikka parallel bo’lgan tekislik tenglamasini toping.

Ishni bajarish tartibi.

1. Talaba laboratoriya ishi bilan tanishadi.
2. Laboratoriya ishi bo’yicha hisobot tayyorlaydi.
3. Nazorat savollariga javob beradi.

Nazorat savollari.

1. Tekislik qanday usullar bilan berilishi mumkin?
2. Tekislikning normal vektori deb nimaga aytiladi?
3. Tekislikning umumiy tenglamasi qanday ifodalanadi?
4. Tekislikning parametrik tenglamasini ko'rsating.
5. Uch nuqtadan o'tuvchi tekislik tenglamasi qanday ko'rinishda bo'ladi?
6. Tekislikning vektor ko'rinishdagi tenglamasini yozing.
7. Tekisliklarning paralellik va perpendikularlik shartlarini aytib bering.
8. Nuqtadan tekislikkacha bo'lgan masofani topish formulasini ko'rsating.

Foydalanilgan adabiyotlar.

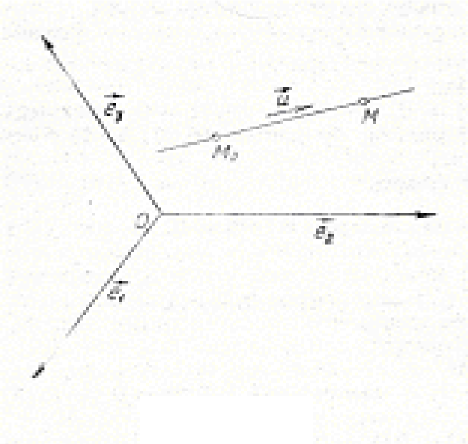
1. Suberbiller O.N.«Analitik geometriyadan masalalar va mashqlar». Toshkent-1960 y.
2. Bakelman I.YA. Analitik geometriya va chiziqli algebra. Toshkent, O'qituvchi,1978 y.
3. X.X.Nazarov, X.O.Ochilova, E.G.Podgarnova. «Geometriyadan masalalar to'plami».I-kism. Toshkent-1983 y.
4. L.S.Atanasyan, V.A.Atanasyan. «Sbornik zadach po geometrii». CHast-I. Moskva-1973 g.
5. N.D.Dodajonov, M.Sh.Jo'rayeva. Geometriya.1-qism.Toshkent-1996 y.
6. X.R.Latipov, F.U.Nosirov, Sh.I. Tojiyev. Analitik geometriya va chiziqli algebradan masalalar yechish bo'yicha qo'llanma. Toshkent-1999y.

8-laboratoriya ishi.
Mavzu: Fazoda to'g'ri chiziq tenglamalari

Ishdan maqsad: Fazoda berilgan to'g'ri chiziq tenglamalarini, to'g'ri chiziq va tekislikning vaziyatini o'rganish.

Nazariy qism:

I. Fazoda to'g'ri chiziq tenglamalari.



1-chizma

1. Fazodagi to'g`ri chiziq o'zining nuqtasi va shu chiziqqa parallel biror $\vec{u} \neq \vec{0}$ vektor bilan to'la aniqlanadi (1-chizma).
$(O,\vec{e}_1, \vec{e}_2, \vec{e}_3)$ reperda $M_o(x_0, y_0, z_0)$, $\vec{u}(l,m,n)$ bo'lsin. To'g`ri chiziqning ihtiyoriy *M(x, u, z)* nuqtasini olaylik:

$$\overrightarrow{M_0M} \,//\, \vec{u} \Rightarrow \overrightarrow{M_0M} = t\overline{u} \ (t \in R). \tag{1}$$

$\overrightarrow{OM_0} = \vec{r}_0$, $\overrightarrow{OM} = \vec{r}$ desak hamda

$\overrightarrow{M_0M} = \overrightarrow{OM} - \overrightarrow{OM_0}$ ni hisobga olsak, (1) ni quyidagicha yozish mumkin:

$$\vec{r} = \vec{r}_0 + t\overline{u}. \tag{2}$$

(2) tenglama *to`g`ri chiziqning vektorli tenglamasi* deb ataladi, t ga har xil qiymatlar berish bilan to'g`ri chiziqqa tegishli nuqtaning radius-vektori topiladi.

$\overrightarrow{MM_0}(x - x_0,\ y - y_0,\ z - z_0)$ va (1) dan

$$\begin{cases} x - x_0 = t \cdot l, \\ y - y_0 = t \cdot m, \\ z - z_0 = t \cdot n, \end{cases} \quad \text{yoki} \quad \begin{cases} x = x_0 + t \cdot l, \\ y = y_0 + t \cdot m, \\ z = z_0 + t \cdot n. \end{cases} \tag{3}$$

Bu (3) tenglamalar sistemasi to'g`ri chiziqning *parametrik* tenglamalari deb yuritiladi. M_o - berilgan nuqta, $\vec{u}$ esa u ning *yo'naltiruvchi vektori* deb ataladi.

Agar $l \cdot m \cdot n \neq 0$ bo'lsa, u holda (3)$\Rightarrow t = \dfrac{x - x_0}{l}$, $t = \dfrac{y - y_0}{m}$, $t = \dfrac{z - z_0}{n}$,

bulardan

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n} \tag{4}$$

Bu tenglamalar *to'g`ri chiziqning kanonik tenglamalari* deb ataladi.

2. To'g`ri chiziqning ikki nuqtasi uning fazodagi vaziyatini to'la aniqlaydi: faraz etaylik, $M_1(x_1, \; u_1 \; z_1)$, $M_2(x_2, \; u_2, \; z_2)$ nuqtalardan $u$ to'g`ri chiziq o'tsin $(M_1 \neq M_2)$. Oldingi banddagi M_o nuqta o'rniga M_1 va $\vec{u} = \overrightarrow{M_1 M_2}$ olinsa, (4) ga asosan:

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}. \tag{5}$$

Berilgan ikki nuqtadan o'tgan to'g`ri chiziqning tenglamalari (5) dir.

3. Fazodagi har bir to'g`ri chiziqni ikki tekislikning kesishish chizig`i deb qarash mumkin. Shungamuvofiq.

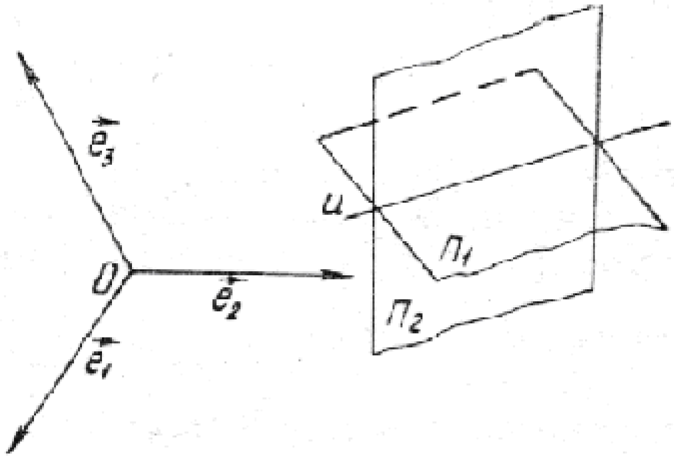
$$\begin{aligned} \Pi_1 &: A_1 x + B_1 y + C_1 z + D_1 = 0, \\ \Pi_2 &: A_2 x + B_2 y + C_2 z + D_2 = 0 \end{aligned} \tag{6}$$

tenglamalar sistemasi $\Pi_1 // \Pi_2 \Rightarrow A_1 : B_1 : C_1 \neq A_2 : B_2 : C_2$ shart bajarilganda to'g`ri chiziqni aniqlaydi(2-chizma) .

To'g`ri chiziqning Yuqorida ko'rilgan (2)-(5) tenglamalarining biridan qolganlariga o'tish mumkin. Lekin u (6) ko'rinishdagi tenglamalari bilan berilsa, kanonik ko'rinishga bevosita o'tish mumkin ekanligi ochiqdan-ochiq ravshan emas. Biz hozir shu masalaga to'xtalamiz. Kanonik tenglamalarni yozish uchun to'g`ri chiziqning bitta nuqtasi va yo'naltiruvchi vektorini bilish kerak. (6) uch noma'lumli ikki tenglama, demak, o'zgaruvchilardan biriga, masalan, z ga $z = z_0$ qiymat berib va hosil qilingan ikki noma'lumli ikkita tenglamani echib, $x = x_0$, $u = u_0$ qiymatlarni topamiz (bunda biz $\begin{vmatrix} A_1 & A_2 \\ B_1 & B_2 \end{vmatrix} \neq 0$ deb faraz qildik). Natijada (x_0, u_0, z_0) nuqta (6) to'g`ri chiziqqa tegishli bo'ladi, u holda (6) ni quyidagicha yozib olsak bo'ladi.

$$\begin{aligned} A_1(x - x_0) + B_1(y - y_0) + C_1(z - z_0) = 0, \\ A_2(x - x_0) + B_2(y - y_0) + C_2(z - z_0) = 0. \end{aligned}$$

Bu sistemadan quyidagilarni topamiz:



$$x - x_0 = \begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix} t,$$

$$y - y_0 = \begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix} t,$$

$$z - z_0 = \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} t.$$

2-chizma

Bulardan

$$\frac{x - x_0}{\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix}} = \frac{y - y_0}{\begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix}} = \frac{z - z_0}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}. \qquad (7)$$

Agar (6) tenglamalarni dekart reperida qarasak, $\vec{n}_1 = (A_1, B_1, C_1)$ vektor P_1 tekislikning $\vec{n}_2 = (A_2, B_2, C_2)$ vektor P_2 tekislikning normal vektori bo'ladi. (7) tenglamalardagi maxrajlarda turgan ifodalar P_1 , P_2 tekisliklar normal vektorlarining vektor ko'paytmasining mos koordinatalaridan iborat, ya'ni $\vec{u} = [\vec{n}_1, \vec{n}_2]$.

1-misol. $M_0(1, 0, -4)$ nuqtadan o'tadigan va $\vec{u} = \{1, -3, 2\}$ vektorga parallel to'g`ri chiziqning parametrik va kanonik tenglamalarini yozib, uning uchta nuqtasini toping.

Yechish. Bu yerda $x_0 = 1,\ y_0 = 0,\ z_0 = -4,$ va $l = 1, m = -3,\ n = 2;$
tegishli tenglamalar quyidagi ko'rinishni oladi:

$$x = 1 + t, \quad y = -3t, \quad z = -4 + 2t; \qquad \frac{x-1}{1} = \frac{y}{-3} = \frac{z+4}{2}.$$

Endi shu to'g`ri chiziqning M_o dan tashqari yana ikki nuqtasini topish uchun t ga ikkita qiymat beramiz:

$$t = 1 \Rightarrow x = 2,\ y = -3,\ z = -2, \quad M_1(2, -3, -2),$$

$$t = -1 \Rightarrow x = 0,\ y = 3,\ z = -6, \quad M_2(0, 3, -6).$$

2-misol.

$$\begin{cases} 2x - y + z - 4 = 0, \\ x + y + 5z - 2 = 0 \end{cases}$$

to'g`ri chiziqning kanonik tenglamalarini yozing.

Yechish. Bu to'g`ri chiziqning biror nuqtasini topamiz, $z = 0$ deb faraz qilish bilan hosil qilingan

$$\begin{cases} 2x - y - 4 = 0, \\ x + y - 2 = 0 \end{cases}$$

sistemadan $x = 2,\ y = 0 \Rightarrow M_0(2, 0, 0)$.

Endi yo'naltiruvchi vektorning koordinatalarini topamiz. Bu yerda

$$A_1 = 2,\ B_1 = -1,\ C_1 = 1,\ \ A_2 = 1,\ \ B_2 = 1,\ C_2 = 5 \Rightarrow$$

$$\Rightarrow l = \begin{vmatrix} -1 & 1 \\ 1 & 5 \end{vmatrix} = -6, \quad m = \begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = -9, \qquad n = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 3.$$

Bu qiymatlarni (32) ga qo'yamiz:

$$\frac{x-2}{-6} = \frac{y}{-9} = \frac{z}{3} \qquad \text{yoki} \qquad \frac{x-2}{2} = \frac{y}{3} = \frac{z}{-1}.$$

II. Fazoda tekislik bilan to'g'ri chiziqning o'zaro vaziyati.

Dekart reperida u to'g`ri chiziq parametrik tenglamalari bilan, Π tekislik umumiy tenglamasi bilan berilgan bo'lsin:

$$u: \begin{cases} x = x_0 + lt, \\ y = y_0 + mt, \\ z = z_0 + nt, \end{cases} \quad \vec{u}(l, m, n), \tag{8}$$

$$\Pi:\ Ax + By + Cz + D = 0 \qquad \vec{n}(A, B, C). \tag{9}$$

Avvalo, to'g`ri chiziq bilan tekislikning kesishish nuqtasini topish masalasiga to'xtalaylik: buning uchun berilgan tenglamalarni sistema deb qarash kerak. (8) va (9) dan

$$Ax_0 + By_0 + Cz_0 + D + t(Al + Bm + Cn) = 0. \tag{10}$$

$Al + Bm + Cn \neq 0$ shartda

$$t = -\frac{Ax_0 + By_0 + Cz_0 + D}{Al + Bm + Cn} \tag{11}$$

bo'ladi. t ning bu qiymatini (8) ga qo'ysak, izlangan nuqta topiladi. Lekin

$$Al + Bm + Cn = 0 \tag{11'}$$

shart bajarilsa, ya'ni $\vec{u} \perp \vec{n}$ bo'lsa, u to'g`ri chiziq Π ga parallel bo'ladi. Aksincha, $u \,//\, \Pi \Rightarrow \vec{u} \perp \vec{n} \Rightarrow \vec{u} \cdot \vec{n} = 0$.

Demak, $\vec{u} \cdot \vec{n} = Al + Bm + Cn = 0$ shart to'g`ri chiziq bilan tekislikning parallelligini bildiradi.

$$u \perp \Pi \Rightarrow \vec{u} \,//\, \vec{n} \Rightarrow \frac{l}{A} = \frac{m}{B} = \frac{n}{C}, \tag{12}$$

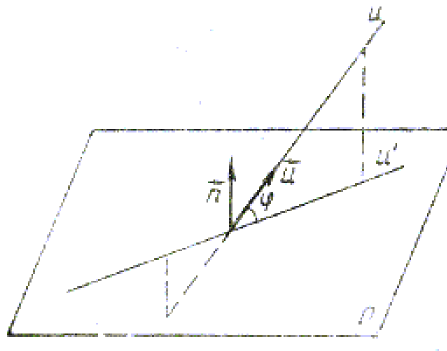
bu (12) shart to'g`ri chiziqning tekislikka perpendikularligini bildiradi.

$u \in \Pi$ bo'lgan hol uchun to'g`ri chiziq bilan tekislik o'zaro vaziyatining xususiy holidir. Bu vaqtda (11') shart bajarilib, undan tashqari $M_0 \in \Pi$ bo'lishi lozim, ya'ni

$$Ax_0 + By_0 + Cz_0 + D = 0. \tag{13}$$

Demak, $u \subset \Pi \Leftrightarrow$ (11'), (13).

Endi to’g`ri chiziq bilan tekislik orasidagi burchakni topish formulasini beramiz.



3-chizma

Ta’rif. To’g`ri chiziq bilan tekislik orasidagi burchak deb, to’g`ri chiziq bilan uning shu tekislikdagi ortogonal proektsiyasi orasidagi burchakka aytiladi (3-chizma). Biz $0 \le \varphi \le \dfrac{\pi}{2}$ deb faraz qilamiz.

3-chizmadan ko’rinadiki, φ ning o’rniga $(\vec{n}, \vec{u})$ burchakni qabul qilish mumkin. Bu burchak $\dfrac{\pi}{2} - \varphi$ ga yoki $\dfrac{\pi}{2} + \varphi$ ga teng. Demak, $\cos\left(\dfrac{\pi}{2} - \varphi\right) = \sin\varphi$, $\cos\left(\dfrac{\pi}{2} + \varphi\right) = -\sin\varphi$, shuning uchun

$$\sin\varphi = \left|\cos(\vec{n}, \vec{u})\right| = \frac{|Al + Bm + Cn|}{\sqrt{A^2 + B^2 + C^2} \cdot \sqrt{l^2 + m^2 + m^2}}. \qquad (14)$$

1-misol. Ushbu

$$\begin{cases} x = 1 + 2t, \\ y = 3t, \\ z = -2 + t \end{cases}$$

to’g`ri chiziq bilan $2x - y + z + 1 = 0$ tekislikning kesishish nuqtasini toping.

Yechish. To’g`ri chiziq tenglamalaridagi x, y, z ning qiymatlarini tekislik tenglamasiga qo’yamiz:

$$2(1 + 2t) - 3t + (-2 + t) = 0 \quad \text{yoki} \quad t = \frac{1}{2}.$$

U holda $x = 1 + 2(-\frac{1}{2}) = 0$, $y = -\frac{3}{2}$, $z = -2 - \frac{1}{2} = -\frac{5}{2}$;;

izlangan nuqta $\left(0, -\frac{3}{2}, -\frac{5}{2}\right)$.

2-misol. $\frac{x+3}{1}=\frac{y-2}{-2}=\frac{z+1}{2}$ to'g`ri chiziq bilan $4x+2y+2z-5=0$ tekislik orasidagi burchakni toping.

Yechish. $\vec{u}(1,-2,\ 2),\ \ \vec{n}(4,\ 2,\ 2),$

$$\sin\varphi=\frac{|4\cdot 1+(-2)\cdot 2+2\cdot 2|}{\sqrt{4^2+2^2+2^2}\cdot\sqrt{1^2+(-2)^2+2^2}}=\frac{\sqrt{6}}{9}\Rightarrow\varphi=\arcsin\frac{\sqrt{6}}{9}.$$

3-misol. *R (7, 9, 7)* nuqtadan $\frac{x-2}{4}=\frac{y-1}{3}=\frac{z}{2}$ to'g`ri chiziqqacha bo'lgan masofani toping.

Yechish. Nuqtadan to'g`ri chiziqqacha bo'lgan masofani topish uchun biz formula berganimiz yo'q, bunday masala quyidagicha oson hal qilinadi:

a) berilgan nuqtadan o'tib, berilgan to'g`ri chiziqqa perpendikular tekislik tenglamasi tuziladi;

b) shu tekislik bilan berilgan to'g`ri chiziqning kesishgan nuqtasi topiladi;

v) bu topilgan nuqta bilan berilgan nuqta orasidagi masofa topiladi.

Shu yo'sinda masalani echishga kirishamiz.

a) P(7,9,7) nuqtadan utib, $\vec{n}=\vec{u}(4,3,2)$ vektorga perpendikulyar tekislikning tenglamasini tuzamiz: $4(x-7)+3(y-9)+2(z-7)=0$ yoki

$$4x+3y+2z-69=0 \qquad (*)$$

b) berilgan eugri chiziq tenglamasini parametrik ko`rinishda yozamiz: $x=2+4t,\ \ y=(1+3t),\ \ z=2t$ va bularni (*) tenglamaga kuyamiz:

$$4(2+4t)+3(1+3t)+2\cdot 2t\_69=0,$$

$$29t-58=0,$$

$$t=2$$

$\left.\begin{array}{l}x=2+4\cdot 2=10,\\ y=1+3\cdot 2=7,\\ z=2\cdot 2=4\end{array}\right\}\Rightarrow Q(10,\ 7,4)$ nuqta to`g`ri chiziq bilan tekislikning kesishgan nuqtasidir.

VARIANTLAR

1-topshiriq.

1. *a)* $M_0(-4,-2,\ 3)$ nuqtadan o'tib, $\vec{u}(-5,4,-7)$ vektorga parallel bo'lgan to'g'ri chiziqning parametrik va kanonik tenglamalarini tuzing.

b) Quyidagi to'g'ri chiziqlar orasidagi burchakni hisoblang:

$$\begin{cases}3x-4y-2z=0\\ 2x+y-2z=0\end{cases}\quad va\quad\begin{cases}4x+y-6z-2=0\\ y-3z+2=0\end{cases}$$

2. *a)* A(7, -2, 0) nuqtadan o’tib, *Oz* o’qqa parallel bo’lgan to’g’ri chiziqning parametrik tenglamalarini tuzing.

b) Uchlari A(3, -1, 0), B(0, -7, 3), C(-2, 1, -1) va D(3, 2, 6) nuqtalarda yotgan tetraedrning qarama-qarshi qirralari orasidagi burchakni toping.

3. *a)* Koordinatalar o’qlarining parametrik tenglamalarini tuzing.

b) $\begin{cases} x = x_0 + lt \\ y = y_0 + mt \\ z = z_0 + nt \end{cases}$ to’g’ri chiziqning koordinatalar o’qi bilan hosil qilgan burchaklarining kosinuslarini toping.

4. *a)* $\dfrac{x+2}{-2} = \dfrac{y-1}{1} = \dfrac{z+5}{3}$ to’g’ri chiziqda abstsissasi ikkiga teng bo’lgan nuqtaning koordinatalarini toping.

b) Kubning diagonallari orasidagi burchakning kosinusini hisoblang.

5. *a)* $\begin{cases} x + y + 5 = 0 \\ 2x - y + z + 3 = 0 \end{cases}$ to’g’ri chiziqdagi abstsissasi 5 bo’lgan M_1 nuqta va $M_2(-1, 3, 5)$ nuqtadan o’tuvchi to’g’ri chiziqning tenglamalarini toping.

b) Quyidagi to’g’ri chiziqlarning parametrik va kanonik tenglamalarini toping.

$$u: \begin{cases} x = 0 \\ y + z = 0 \end{cases}$$

6. *a)* Berilgan $M_1(1, -2, -5)$ va $M_2(3, -1, 0)$ nuqtalardan o’tuvchi to’g’ri chiziq tenglamalarini toping.

b) Quyidagi to’g’ri chiziqlarning o’zaro vaziyatini aniqlang:

$$\frac{x-1}{2} = \frac{x+3}{3} = \frac{z-1}{-2} \quad va \quad \frac{x}{-2} = \frac{y+1}{-3} = \frac{z+3}{2}$$

7. *a)* Abstsissa va ordinata o’qlaridan bir birlikdagi kesma kesuvchi to’g’ri chiziq tenglamalarini toping.

b) $\dfrac{x+2}{-2} = \dfrac{y-1}{1} = \dfrac{z+5}{3}$ to’g’ri chiziqda applikatasi beshga teng bo’lgan nuqtaning koordinatalarini toping.

8. *a)* $2x - 3y + z + 5 = 0$ tekislikning koordinatalar tekisliklari bilan kesishish chiziqlarining tenglamalarini yozing.

b) To’g’ri chiziqning quyidagi tenglamalari kanonik shaklga keltirilsin:

$$\begin{cases} 5x - 6y + 2z + 21 = 0, \\ x - z + 3 = 0. \end{cases}$$

9. *a)* $2x - y + z + 1 = 0$ tekislik bilan $M_1(3,2,0)$, $M_2(1,-1,1)$ va $M_3(1,-3,2)$ nuqtalardan o'tuvchi tekislik kesishishidan hosil bo'lgan to'g'ri chiziq tenglamalarini toping.

b) Quyidagi to'g'ri chiziqlarni kesishishini isbot qiling va kesishgan nuqtasining koordinatalarini toping:

$$v_1: \begin{cases} x + z - 1 = 0 \\ 3x + y - z + 13 = 0 \end{cases} \qquad v_2: \begin{cases} x - 2y + 3 = 0 \\ y + 2z - 8 = 0 \end{cases}$$

10. *a)* Quyidagi to'g'ri chiziqlarning parametrik va kanonik tenglamalarini toping.

$$u: \begin{cases} x + 2y + z - 1 = 0, \\ x + \quad y + \quad\quad 1 = 0 \end{cases}$$

b) $\frac{x+2}{-2} = \frac{y-1}{1} = \frac{z+5}{3}$ to'g'ri chiziqda bu to'g'ri chiziqning tenglamalarini ikki tekislikning kesishish chizig'i sifatida ifodalang.

11. *a)* Quyidagi to'g'ri chiziqlarning koordinatalar sistemasiga nisbatan qanday joylashishini aniqlang:

$$u_1: \begin{cases} 2y - z + 1 = 0 \\ 3y + z + 4 = 0 \end{cases} \quad u_2: \begin{cases} 7x + 8y - 3z + 6 = 0 \\ 3x + y - 3z + 6 = 0 \end{cases}$$

b) Quyidagi to'g'ri chiziqlarni kesishishini isbot qiling va kesishgan nuqtasining koordinatalarini toping:

$$w_1: \begin{cases} 7x + 3y + z - 5 = 0 \\ 5y - 2z - 1 = 0 \end{cases} \qquad w_2: \begin{cases} x + y + z - 3 = 0 \\ 11x - 3z + 6 = 0 \end{cases}$$

12. *a)* $\begin{cases} 2x - 3y + 5z - 6 = 0 \\ x + 5y - 7z + 10 = 0 \end{cases}$ to'g'ri chiziqning *Oy* o'q bilan kesishishini isbot qiling.

b) Quyidagi to'g'ri chiziqning yo'naltiruvchi kosinuslari hisoblab topilsin:

$$\frac{x-1}{4} = \frac{y-2}{-6} = \frac{z+3}{9}.$$

13. *a)* Quyidagi to'g'ri chiziqlarning parametrik va kanonik tenglamalarini toping.

$$u: \begin{cases} x + 2y + z - 1 = 0, \\ x - y + 1 = 0 \end{cases}$$

b) Quyidagi to'g'ri chiziqlarning o'zaro vaziyatini aniqlang:

$$\begin{cases} 3x - y - 5z + 7 = 0 \\ 2x + 3y + 4 = 0 \end{cases} \quad va \quad \begin{cases} x - 2y + z = 0 \\ 2y - 2z - 5 = 0 \end{cases}$$

14. *a)* $M_0(-3,\ 1,\ -4)$ nuqtadan o'tib, $\begin{cases} 2x - y + z - 3 = 0 \\ x + 3y - z - 1 = 0 \end{cases}$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziqning parametrik tenglamalarini toping.

b) $A(4,\ -6,\ -9)$ nuqtadan $\begin{cases} 2x - y + 3z - 1 = 0, \\ 5x + 4y - z - 7 = 0 \end{cases}$ to'g'ri chiziqqa parallel to'g'ri chiziq o'tkazilsin.

15. *a)* Quyidagi to'g'ri chiziqlarning o'zaro vaziyatini aniqlang:

$$1)\ \begin{cases} x = 4t + 3 \\ y = -7t - 2 \\ z = -3 + 2t \end{cases} \quad va \quad \begin{cases} x = 27 - 4t \\ y = 15 + 7t \\ z = -2t \end{cases}$$

$$2)\ \begin{cases} x - 2y + 3z - 4 = 0 \\ x + 2y - 5z + 1 = 0 \end{cases} \quad va \quad \begin{cases} x + y + z + 1 = 0 \\ y - 3z = 0 \end{cases}$$

b) Quyidagi to'g'ri chiziqlar orasidagi burchakni hisoblang:

$$\frac{x-1}{2} = \frac{y+4}{5} = \frac{z+16}{-6} \quad va \quad \begin{cases} x = 4 + 3t \\ y = -10t \\ z = 5 + t \end{cases}$$

16. *a)* Quyidagi to'g'ri chiziqlar bir tekislikda yotishini isbot qiling va bu tekislikning tenglamasini tuzing:

$$u_1 : \begin{cases} x = 2 + 4t \\ y = -6t \\ z = -1 - 8t \end{cases} \quad va \quad u_2 : \begin{cases} x = 7 - 6t \\ y = 2 + 9t \\ z = 12t \end{cases}$$

b) $A(-1,\ 3,\ 7)$ nuqtadan $\dfrac{x-2}{5} = \dfrac{y+3}{-2} = \dfrac{z-1}{2}$ to'g'ri chiziqqa parallel to'g'ri chiziq o'tkazilsin.

17. *a)* Quyidagi to'g'ri chiziqlarning o'zaro kesishishini ko'rsating va ular orqali o'tuvchi tekislikning tenglamasini tuzing:

$$u_1 : \begin{cases} x + z + 2 = 0 \\ 2x - y + 1 = 0 \end{cases} \qquad u_2 : \begin{cases} 5x + 4z + 3 = 0 \\ 2x + y + 3z = 0 \end{cases}$$

b) Quyidagi to'g'ri chiziqning yo'naltiruvchi kosinuslari hisoblab topilsin:

$$\begin{cases} 2x - 3y - 3z - 9 = 0, \\ x - 2y + z + 3 = 0. \end{cases}$$

18. *a)* 1) (0, 0, 1) nuqtadan o'tuvchi va $\begin{cases} x - 2y + z - 1 = 0 \\ 2x - y + 2z - 3 = 0 \end{cases}$, $\begin{cases} x = 3t + 1 \\ y = 2t \\ z = -t - 1 \end{cases}$ to'g'ri chiziqlarning har biri bilan kesishuvchi to'g'ri chiziqning kanonik tenglamalarini toping.

2) koordinatalar boshidan o’tuvchi, $\begin{cases} x = t \\ y = 1 - t \\ z = 3 + t \end{cases}$ *va* $\begin{cases} x = 2 + 2t \\ y = 3 - t \\ z = 4 + 3t \end{cases}$ to’g’ri chiziqlarning har biri bilan kesishuvchi to’g’ri chiziqning parametrik tenglamalarini toping.

b) Quyidagi to’g’ri chiziqning yo’naltiruvchi kosinuslari hisoblab topilsin:

$$\frac{x-3}{2} = \frac{y+1}{4} = \frac{z-4}{-3}.$$

19. *a)* Quyidagi to’g’ri chiziqlarni kesishishini isbot qiling va kesishgan nuqtasining koordinatalarini toping:

$$u_1: \quad \frac{x-1}{2} = \frac{y-7}{1} = \frac{z-3}{4}; \quad u_2: \quad \frac{x-6}{3} = \frac{y+1}{-2} = z+2.$$

b) $A(2, -5, 3)$ nuqtadan z o’qiga parallel to’g’ri chiziq o’tkazilsin.

20. *a)* Quyidagi to’g’ri chiziqlarning koordinatalar sistemasiga nisbatan qanday joylashishini aniqlang:

$$u_1: \begin{cases} 2x - 3y = 0 \\ 5x + y = 0 \end{cases} \qquad u_2: \begin{cases} x - 2y + 3z = 0 \\ 2x + y - z = 0 \end{cases}$$

b) $N(-1, 0, -4)$ nuqtadan Ox o’qiga parallel to’g’ri chiziq o’tkazilsin.

21.*a)* $M_0(1, -2, 5)$ nuqtadan o’tib, $\begin{cases} 2x - y + z - 3 = 0 \\ x + 3y - z - 1 = 0 \end{cases}$ to’g’ri chiziqqa parallel bo’lgan to’g’ri chiziqning parametrik tenglamalarini toping.

b) $M(6, -3, -7)$ nuqtadan Oy o’qiga parallel to’g’ri chiziq o’tkazilsin.

2-topshiriq.

1. $\Pi: 3x + 2y - 5z - 1 = 0$ tekislik bilan $u: \begin{cases} x = 4t + 2 \\ y = -3t + 2 \\ z = 2t + 1 \end{cases}$ to’g’ri chiziqning kesishish nuqtasini toping.

2. $u: \frac{x+6}{-2} = \frac{y-1}{3} = z - 1$ to’g’ri chiziq bilan $\Pi: 2x - 5y + 6z - 1 = 0$ tekislikning o’zaro vaziyatini aniqlang.

3. $u: \begin{cases} x = 5t + 2 \\ y = -8t - 3 \\ z = 3t + 4 \end{cases}$ to’g’ri chiziq bilan $\Pi: 7x + y - 9z + 53 = 0$ tekislikning o’zaro vaziyatini aniqlang.

4. $u:\begin{cases} x=t+1 \\ y=-8t-3 \\ z=\alpha t+2 \end{cases}$ to'g'ri chiziq va $\Pi: 3x+4y+7z-2=0$ tekislik berilgan α ning qanday qiymatida to'g'ri chiziq tekislikka parallel bo'ladi?

5. Shunday to'g'ri chiziq va tekislik tenglamasini yozingki, ular 1) o'zaro parallel bo'lsin; 2) kesishsin.

6. M(1, -1, 3) nuqtadan va $\begin{cases} x=4t \\ y=6t+5 \\ z=t \end{cases}$ to'g'ri chiziqdan o'tuvchi tekislik tenglamasini toping.

7. $\frac{x-1}{2}=y+3=\frac{z}{4}$ to'g'ri chiziq orqali o'tib, $2x-y+z+1=0$ tekislikka perpendikulyar bo'lgan tekislik tenglamasini toping.

8. $M_0(3, -5, 1)$ nuqtadan o'tib, $\frac{x-3}{2}=\frac{y+5}{3}=\frac{z-1}{5}$ to'g'ri chiziqqa perpendikulyar bo'lgan tekislik tenglamasini toping.

9. $\frac{x-5}{6}=\frac{y-1}{-3}=\frac{z-1}{2}$ to'g'ri chiziq bilan $4x+y-8z+16=0$ tekislik orasidagi burchakni hisoblang.

10. $y=2x+1$, $2z=5x+6$ to'g'ri chiziq va $x+2y-z+3=0$ tekislik orasidagi burchak topilsin.

11. $\frac{x-4}{2}=\frac{y+2}{1}=\frac{z-3}{4}$ to'g'ri chiziqdan o'tib, $x-3y+5z=0$ tekislikka perpendikulyar tekislik tenglamasi tuzilsin.

12. $\frac{x-2}{2}=\frac{y-1}{1}=\frac{z-3}{2}$ va $\frac{x+1}{2}=\frac{y-1}{2}=\frac{z}{2}$ parallel to'g'ri chiziqlardan o'tuvchi tekislik tenglamasi tuzilsin.

13. $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z+1}{2}$ to'g'ri chiziq bilan $3x+5y-z+8=0$ tekislikning kesishgan nuqtasini toping.

14. $y=2x+5$, $z=3x-6$ to'g'ri chiziq bilan $5x-3y+z+7=0$ tekislikning kesishgan nuqtasi topilsin.

15. $M_0(3, -2, 4)$ nuqtadan $5x+3y-7z+1=0$ tekislikka perpendikulyar tushirilsin.

16. Koordinatalar boshidan $\frac{x+2}{4}=\frac{y-3}{5}=\frac{z-1}{-2}$ to'g'ri chiziqqa perpendikulyar tekislik o'tkazilsin.

17. Quyidagi ikki parallel to'g'ri chiziq orasidagi masofa topilsin:

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z}{2} \quad \text{va} \quad \frac{x-7}{3}=\frac{y-1}{4}=\frac{z-3}{2}.$$

18. $M(4,\,-3,\,1)$ nuqtadan o'tuvchi hamda

$$\frac{x}{6}=\frac{y}{2}=\frac{z}{-3} \quad \text{va} \quad \frac{x+1}{5}=\frac{y-3}{4}=\frac{z-4}{2}$$

to'g'ri chiziqlarga parallel tekislikning tenglamasi tuzilsin.

19. Mana bu $\frac{x+5}{3}=\frac{y-2}{1}=\frac{z}{4}$ to'g'ri chiziq orqali $x+y-z+15=0$ tekislikka parallel tekislik o'tkazilsin.

20. Quyidagi ikki parallel to'g'ri chiziq orasidagi masofa topilsin:

$$\frac{x+1}{-3}=\frac{y-3}{1}=\frac{z-2}{4} \quad \text{va} \quad \frac{x-2}{3}=\frac{y+1}{-1}=\frac{z+5}{-4}.$$

21. *M_0(-2, 5, -1)* nuqtadan $-2x+5y+7z-6=0$ tekislikka perpendikulyar tushirilsin.

Ishni bajarish tartibi.

1. Talaba laboratoriya ishi bilan tanishadi.
2. Laboratoriya ishi bo'yicha hisobot tayyorlaydi.
3. Nazorat savollariga javob beradi.

Nazorat savollari.

1. Fazoda to'g'ri chiziq qanday usullar bilan berilishi mumkin ?
2. Qanday ko'rinishdagi tenglamalarga to'g'ri chiziqning vektor va parametrik tenglamalari deyiladi ?
3. To'g'ri chiziqning kanonik tenglamasi qanday ko'rinishda bo'ladi ?
4. Ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini ko'rsating?
5. To'g'ri chiziqning umumiy tenglamasi deb qanday ko'rinishdagi tenglamaga aytiladi ?

6.Qanday shartlarda to'g'ri chiziqlar parallel va perpendikulyar bo'ladi?
7.To'g'ri chiziq bilan tekislik orasidagi burchak qanday formula bilan topiladi?
8. Qanday shartlarda to'g'ri chiziq va tekisliklar parallel va perpendikulyar bo'ladi?
9. Tekisliklar bog'lami deb nimaga aytiladi?
10. Tekisliklar dastasi deb nimaga aytiladi?

Foydalanilgan adabiyotlar.

1. Suberbiller O.N.«Analitik geometriyadan masalalar va mashqlar». Toshkent-1960 y.
2. Bakelman I.YA. Analitik geometriya va chiziqli algebra. Toshkent, O'qituvchi,1978 y.
3. X.X.Nazarov, X.O.Ochilova, E.G.Podgarnova. «Geometriyadan masalalar to'plami».I-kism. Toshkent-1983 y.
4. L.S.Atanasyan, V.A.Atanasyan. «Sbornik zadach po geometrii». CHast-I. Moskva-1973 g.
5. N.D.Dodajonov, M.Sh.Jo'rayeva. Geometriya.1-qism.Toshkent-1996 y.
6.X.R.Latipov, F.U.Nosirov, Sh.I. Tojiyev. Analitik geometriya va chiziqli algebradan masalalar yechish bo'yicha qo'llanma. Toshkent-1999y.

9 - laboratoriya ishi.
Mavzu: Ikkinchi tartibli chiziqlar: ellips, giperbola va parabola

Ishdan maqsad: Ikkinchi tartibli chiziqlarni yasashni, o'qlarini, fokuslarini, ekssentrisitetini, direktrisasini va urinma to'g'ri chiziqlarini topishni o'rganish.

Nazariy qism.

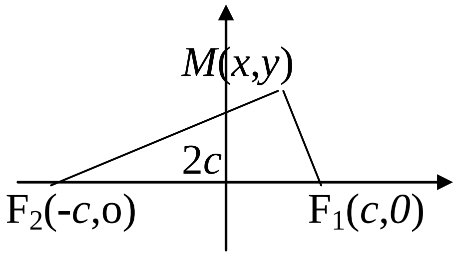
1. Ellips.

Ta'rif. Ellips deb, har bir nuqtasidan berilgan ikki nuqtagacha (fokuslargacha) masofalarning yig'indisi o'zgarmas songa teng bo'lgan tekislik nuqtalarining geometrik o'rniga aytiladi.

Bu ikki berilgan nuqta ellipsning fokuslari, nuqtalar orasidagi masofa esa fokal masofa deyiladi.

Ellips tenglamasini keltirib chiqaraylik.

Buning uchun koordinatalar sistemasini shunday joylashtiramiz:



ya'ni Ox abstsissa o'qi fokuslar orqali o't sin, ordinata o'qi esa fokuslar orasidagi masofani teng ikkiga bo'lib, Ox o'qqa perpendikulyar ravishda o'tsin.

Shartga ko'ra

$$F_1 M + F_2 M = 2a \qquad (1)$$

$$\sqrt{(x-c)^2+y^2}+\sqrt{(x+c)^2+y^2}=2a \qquad (2)$$

bu ellips tenglamasidir.

$$\sqrt{(x+c)^2+y^2}=2a-\sqrt{(x-c)^2+y^2}$$

$$(x+c)^2+y^2=4a^2-4a\sqrt{(x-c)^2+y^2}+(x-c)^2+y^2$$

$$x^2+2cx++c^2+y^2=4a^2-4a\sqrt{(x-c)^2+y^2}+x^2-2xc+c^2+y^2$$

$$\sqrt{(x-c)^2+y^2}=a-\frac{xc}{a}$$

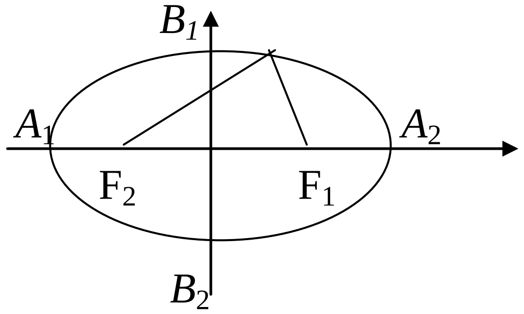
$$(x-c)^2+y^2=a^2-2cx+\frac{x^2c^2}{a^2}$$

$$x^2-2cx+c^2+y^2=a^2-2cx+\frac{x^2c^2}{a^2}$$

$$\left(1-\frac{c^2}{a^2}\right)x^2+y^2=a^2-c^2$$

$$\frac{a^2-c^2}{a^2}x^2+y^2=a^2-c^2$$

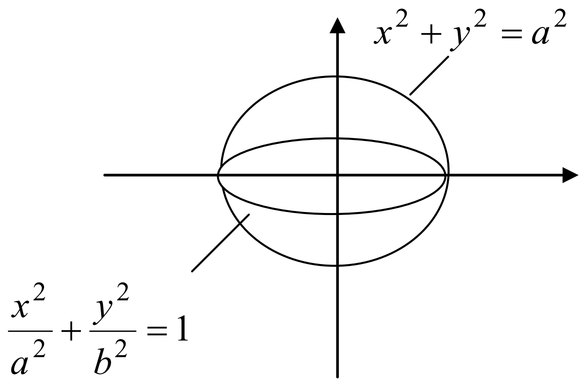
$a^2-c^2=b^2$ desak, ($a>c$) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ bu ellipsning *kanonik tenglamasi*,



$A_1(a,0)$, $A_2(-a,0)$, $B_1(0,b)$, $B_2(0,-b)$ nuqtalar *ellipsning uchlari*, $\overrightarrow{AA_1}=2a$ ga *ellipsning katta o'qi*, $\overrightarrow{BB_1}=2b$ *ellipsning kichik o'qi* deyiladi ($a>b$).

a va b sonlar *ellipsning yarim o'qlari* deyiladi.

$x^2+y^2=a^2$ aylana tenglamasidan ellips tenglamasini keltirib chiqarish uchun nuqta ordinatasini $\frac{b}{a}$ marta kamaytirish kerak, ya'ni



$$x^2+y^2=a^2$$

$$\frac{x^2}{a^2}+\frac{\left(\frac{y}{b/a}\right)^2}{a^2}=1$$

$$\frac{x^2}{a^2}+\left(\frac{ay}{b}\right)^2\cdot\frac{1}{a^2}=1$$

Ta'rif. Ellipsning *ekstsentrisiteti* (e) deb, fokuslari orasidagi ($2c$) masofaning katta o'qi ($2a$) ga bo'lgan nisbatiga aytiladi, ya'ni

$$e=\frac{c}{a} \quad ,$$

bunda

$$e<1.$$

Ta'rif. Ellipsdagi nuqtadan fokuslargacha bo'lgan masofalar uning *fokal radius – vektorlari* (r_1 va r_2) deyiladi. Ellipsning ixtiyoriy $M(x,y)$ nuqtasi uchun

$$r_1=a-ex, \qquad r_2=a+ex,$$

va ellipsning tarifiga asosan

$$r_1+r_2=2a\,,$$

ya'ni ellipsning har qanday nuqtasining fokal radius – vektorlarining yig'indisi uning katta o'qiga teng.

Ta'rif. Ellipsning kichik o'qiga parallel va undan $\frac{a}{e}$ masofadan o'tgan ikki to'g'ri chiziq ellipsning *direktrisalari* deyiladi. Direktrisalar tenglamalari quyidagichadir:

$$x = \frac{a}{e} \quad va \quad x = -\frac{a}{e}.$$

2. Giperbola

Ta'rif. Giperbola deb, har bir nuqtasidan berilgan ikki nuqtagacha (fokuslargacha) masofalarning ayirmasi o'zgarmas songa teng bo'lgan tekislik nuqtalarining o'rniga aytiladi.

Berilgan nuqtaning giperbolaning fokuslari, ular orasidagi masofa esa fokal masofa deb ataladi. Giperbola tenglamasini keltirib chiqaramiz. Buning uchun koordinatalar sistemasini shunday joylashtiramizki, abstsissa o'qi giperbolaning fokuslaridan o'tsin, $F_1 F_2$ fokal masofani, ya'ni $F_1 F_2 = 2c$ deymiz. Ordinatalar o'qini F_1 va F_2 ni o'rtasidan perepndikulyar qilib o'tkazamiz. $F_1 M$ va $F_2 M$ lar orasidagi ayirma modulini $2a$ bilan belgilaymiz. U holda giperbola quyidagi tenglamani qanoatlantiradi:

$$\left|\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2}\right| = 2a$$

yoki

$$\left|\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2}\right| = \pm 2a$$

Tenglamaning chap tomoni musbat bo'lsa (+), manfiy bo'lsa (-) olinadi.

$$x^2 + 2cx + c^2 + y^2 = 4a^2 \pm 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2$$

$$\frac{cx}{a} - a = \pm\sqrt{(x-c)^2 + y^2}$$

$$\frac{c^2 - d^2}{a^2} x^2 - y^2 = c^2 - a^2$$

ta'rifga ko'ra $a < c$, $c^2 - a^2 = b^2$ musbat bo'ladi.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad (3)$$

giperbolaning kanonik tenglamasi.

Misol. $M\left(8\sqrt{5};\, 12\right)$ nuqtadan o'tuvchi giperbolani kanonik tenglamasi tuzilsin, agar fokal oraliq 2*0* ga teng bo'lsa.

Shartga ko'ra $2c = 20,$ $c = 10$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{(8\sqrt{5})^2}{a^2}-\frac{12^2}{b^2}=1 \qquad \frac{320}{a^2}-\frac{144}{b^2}=1$$

$$\begin{cases}\dfrac{320}{a^2}-\dfrac{144}{b^2}=1\\ b^2=100-a^2\end{cases} \qquad a^2=64 \quad b^2=36 \qquad \frac{x^2}{64}-\frac{y^2}{36}=1$$

Misol. $21x^2-43y^2=903$ giperbola tenglamasi berilgan. Fokuslarning koordinatasi topilsin.

$$\frac{21x^2}{903}-\frac{43y^2}{903}=1 \qquad \frac{x^2}{43}-\frac{y^2}{21}=1$$

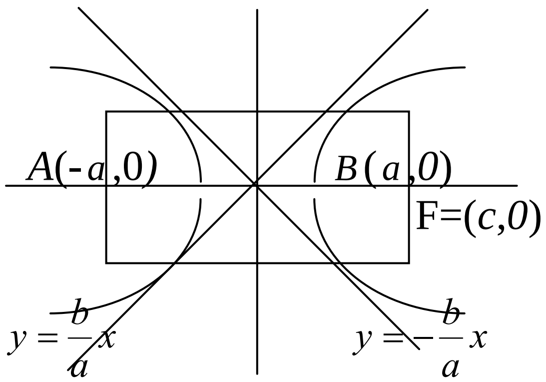
$$a^2=43 \qquad b^2=21 \qquad c^2=a^2+b^2 \qquad c^2=64 \qquad c=8$$

Demak, F_1 = (-8; *0*), F_2 = (8; *0*) ekan.

(3) tenglamadan ma'lumki, giperbola Ox, Oy o'qlariga nisbatan va koordinatalarning boshi *0* ga nisbatan simmetrikdir.

Giperbola abstsissalar o'qini $A(-a,\,0)$, $B(a,\,0)$ nuqtada kesib o'tadi. A va B nuqtalar giperbolaning uchlari deyiladi. AB kesma giperbolaning haqiqiy o'qi deyiladi. Uning uzunligi 2a ga teng.

$y=\dfrac{bx}{a}$ va $y=\dfrac{-bx}{a}$ to'g'ri chiziqlar giperbolaning asimptotalari deyiladi.



Giperbolaning ekssentrisiteti: $e=\dfrac{c}{a}$, bunda $e>1.$

(1) giperbola cheksizlikka cho'zilgan ikkita (o'ng va chap) tarmoqdan iborat: O'ng tarmoqning nuqtalari uchun fokal radius – vektorlar quyidagi formulalar bilan hisoblanadi:

$$\left.\begin{aligned} r_1&=ex-a,\\ r_2&=ex+a,\\ r_2-r_1&=2a.\end{aligned}\right\}$$

Chap tarmoqning nuqtalari uchun:

$$\left.\begin{aligned} r_1 &= ex - a, \\ r_2 &= ex + a, \\ r_2 - r_1 &= 2a \end{aligned}\right\}$$

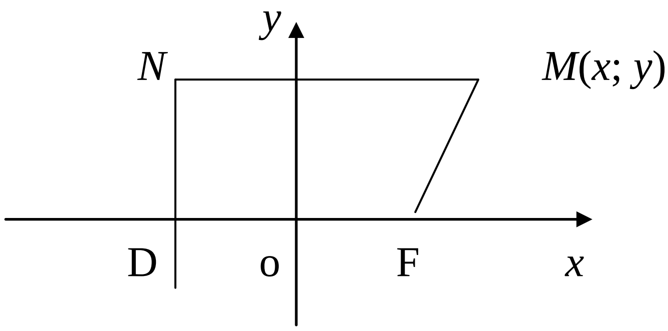
formulalar o'rinli.

Giperbolaning direktrisalari uchun: $x = \frac{a}{e}$ va $x = -\frac{a}{e}$ tengliklar o'rinli.

Asimptotaning tenglamalari: $\left.\begin{aligned} y &= \frac{b}{a}x \\ y &= -\frac{b}{a}x \end{aligned}\right\}$

3. Parabola

Ta'rif. Parabola deb, har bir nuqtasidan berilgan bir nuqtagacha (fokusgacha) va berilgan bir to'g'ri chiziqqacha (direktrisagacha) masofalari o'zaro teng bo'lgan tekislik nuqtalarining geometrik o'rniga aytiladi.



Fokusdan direktrisagacha bo'lgan masofa fokal parametri deyiladi va *R* bilan belgilanadi. Koordinatalar sistemasini shunday joylashtiramizki, Ox o'q *F* fokusdan o'tib direktrisaga perpendikular bo'lsin. Direktrisasi bo'lgan *O* o'q kesishish nuqtasini *D* bilan, koordinatalarning boshini *O* nuqta bilan belgilaymiz. *DF* ni o'rtasi *O* nuqta bo'ladi. U holda $F\left(\frac{p}{2};0\right)$ bo'ladi. Direktrissa esa $x = -\frac{p}{2}$, $x + \frac{p}{2} = 0$ tenglamaga ega bo'ladi. $M(x, y)$ parabolaning ixtiyoriy nuqtasi bo'lsin.

Parabolaning ta'rifiga ko'ra

$$\text{F}\,M = M\,N, \qquad FM = \sqrt{\left(x - \frac{p}{2}\right)^2 + y^2}\,.$$

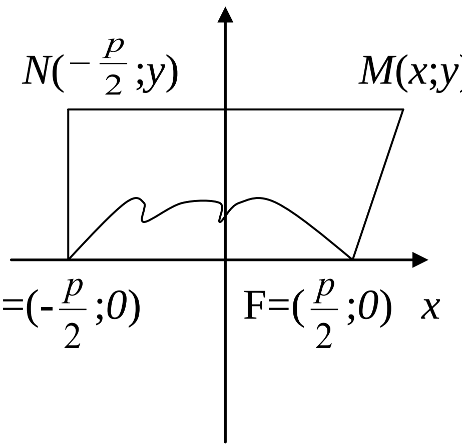
Ikki nuqta orasidagi masofani topish formulasiga binoan

y

$$MN=\sqrt{\left(x+\frac{p}{2}\right)^2+(y-y)^2}=\left|x+\frac{p}{2}\right|$$

$$\sqrt{\left(x-\frac{p}{2}\right)^2+y^2}=\left|x+\frac{p}{2}\right|$$

$$x^2-px+\frac{p^2}{4}+y^2=x^2+px+\frac{p^2}{4}$$



$$y^2=2px \qquad (4)$$

parabolaning kanonik tenglamasi deyiladi.

Misol. $y^2=3x$ parabola berilgan. Parabolaning shunday nuqtasini topaylikki, shu nuqtadan fokusgacha bo'lgan masofa 1 ga teng bo'lsin.

Yechish: Shartga ko'ra 2r = 3, $\frac{p}{2}=\frac{3}{4}$, $F\left(\frac{3}{4};0\right)$

$M(x,\ y)$ noma'lum

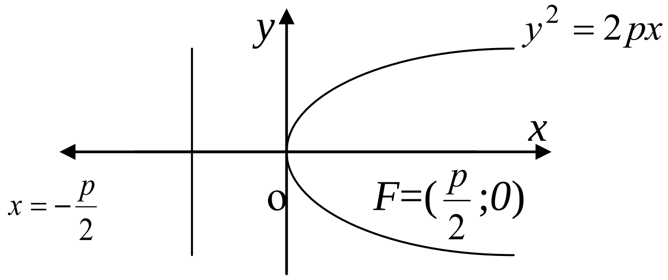
$$\begin{cases}\left(x-\frac{3}{4}\right)^2+y^2=1\\ y^2=3x\end{cases} \qquad x^2-\frac{3}{2}x+\frac{9}{16}+3x=1 \qquad \left(x+\frac{3}{4}\right)^2=1$$

$$x^2+\frac{3}{2}x+\frac{9}{16}=1 \qquad x+\frac{3}{4}=1 \qquad x=\frac{1}{4}$$

$$y^2=3x=3 \qquad \frac{1}{4} \quad =\frac{3}{4} \qquad y=\pm\frac{\sqrt{3}}{2}$$

tenglamadan ko'rinadiki, parabola $x\geq 0$ sohada yotadi va Ox o'qiga nisbatan simmetrik $M(x_0,\ y_0)$ bo'lsa, u holda $M_1(x_0,\ -y_0)$ bo'ladi.

Parabola koordinatalarning o'qini *O (0; 0)* nuqtada kesadi. Bu *O (0; 0)* nuqta *parabolaning uchi* deyiladi.



Parabolaning $x^2=2py\Rightarrow y=\frac{x^2}{2p}$ tenglamasi ham mavjud.

V A R I A N T L A R

1-topshiriq.

1. $\frac{x^2}{30}+\frac{y^2}{24}=1$ ellipsning $2x-y+17=0$ to'g'ri chiziqqa parellel bo'lgan urinmalari topilsin.

2. *A*(-6;+3) nuqtadan $\frac{x^2}{15}+\frac{y^2}{9}=1$ ellipsga o'tkazilgan urinmalarning tenglamasi tuzilsin.

3. $\frac{x^2}{36}+\frac{y^2}{25}=1$ ellipsga ichki chizilgan kvadrat tomonining uzunligi hisoblansin.

4. $A(2,\frac{3}{2}),\ B(2,\frac{2\sqrt{5}}{3}),\ C(-\frac{3}{2},-\frac{3}{2}),\ D(0,5),\ E(\frac{3\sqrt{5}}{2},1)$ nuqtalar $\frac{x^2}{9}+\frac{y^2}{4}=1$ ellipsga nisbatan qanday joylashganini aniqlang.

5. $\frac{x^2}{36}+\frac{y^2}{20}=1$ ellipsning *F(c, 0)* fokus nuqtasidan o'tib, katta o'qiga perpendikular bo'lgan vatarining uzunligini toping.

6. Quyidagi ellipslarning markazi va yarim o'qlarini toping.

   1) $\frac{(x+3)^2}{25}+\frac{(y-5)^2}{9}=1$
   2) $9x^2+16y^2+18x-96y+9=0$
   3) $4x^2+9y^2+16x+18y=11$

7. Har bir nuqtasidan A(1; 0) nuqtagacha bo'lgan masofa *x=0* to'g'ri chiziqqacha bo'lgan masofaga qaraganda uch marta yaqin bo'lgan figuraning tenglamasini tuzing.

8. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning kichik yarim o'qi uchidan o'tgan vatarlarining o'rta nuqtalaridan tashkil topgan figura tenglamasini tuzing.

9.Ushbu $\frac{x^2}{36}+\frac{y^2}{20}=1$ ellips direktrissalarining tenglamasini yozing.

10. x=$\pm$8 to'g'ri chiziqlar kichik o'qi 8 ga teng bo'lgan ellipsning direktrisalaridir. Shu ellipsning tenglamasi topilsin.

11. Yer sharining meridiani ellips shaklida bo'lib, o'qlarining nisbati $\frac{299}{300}$ ga teng. Yer sharining ekstsentrisiteti aniqlansin.

12. $\frac{x^2}{30}+\frac{y^2}{24}=1$ ellipsda uning kichik o'qidan 5 birlik masofadagi nuqta topilsin.

13. $\frac{x^2}{36}+\frac{y^2}{9}=1$ ellipsga ichki muntazam uchburchak ichki chizilgan. Uning uchlaridan biri ellips katta o'qining o'ng uchiga tushadi. Bu uchburchakning qolgan ikkita uchining koordinatalari topilsin.

14. $\frac{x^2}{49}+\frac{y^2}{16}=1$ ellipsda fokal radius – vektorlarining ko'paytmasi kichik yarim o'kning kvadratiga teng bo'lgan nuqta topilsin.

15. $\frac{x^2}{36}+\frac{y^2}{12}=1$ ellipsning $2x-y-9=0$ to'g'ri chiziq bilan kesishish nuqtalari topilsin.

16. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning F(c;0) fokusi orqali katta o'qiga perpendikular qilib vatar o'tkazilgan. Shu vatarning uzunligi topilsin.

17. $\frac{x^2}{16}+\frac{y^2}{9}=1$ ellips berilgan. Shu ellipsning koordinat burchagi bissektrissasi bo'ylab yo'nalgan diametrining uzunligi topilsin.

18. $\frac{x^2}{49}+\frac{y^2}{24}=1$ ellipsga to'g'ri to'rtburchak ichki chizilgan, uning ikkita qarama-qarshi tomoni fokuslardan o'tadi. Shu to'g'ri to'rtburchakning yuzi hisoblansin.

19. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning har qanday ichki $P(x_1,y_1)$ nuqtasi uchun $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}<1$ tengsizlik, har qanday tashqi $\frac{x_2^2}{a^2}+\frac{y_2^2}{b^2}>1$ tengsizlik o'rinli ekanligini isbotlang.

20. $\frac{x^2}{16}+\frac{y^2}{12}=1$ ellipsga (+2;-3) nuqtada urinuvchi to'g'ri chiziqning tenglamasi yozilsin.

21. Quyidagi ellipslarning markazi va yarim o'qlarini toping.

a) $9x^2+16y^2-54x+32y-47=0$

b) $x^2+2y^2+4x-4y=0$

2-topshiriq.

1. Quyidagi giperbolalarning markazini va yarim o'qlarini toping:

1) $9x^2-25y^2-18x-100y-316=0$

2) $5x^2-6y^2+10x-12y-31=0$

3) $x^2-4y^2+2x+16y-7=0$

4) $3x^2-y^2+12x-4y-4=0.$

2. $\frac{x^2}{25}-\frac{y^2}{144}=1$ giperbolaning:

1) yarim o'qlarini;
2) fokuslarini;
3) ekssentrisitetini;
4) asimptota tenglamalarini;
5) direktrisalar tenglamalarini toping.

3.Quyidagi har bir hol uchun giperbolaning kanonik tenglamasini tuzing.

1) *a=2, b=3;*

2) $a=\frac{1}{4},\ b=\frac{2}{5}$;

3) fokuslari orasidagi masofa $2c=12$ bo'lib, ekstsentrisiteti $e=\frac{6}{5}$

4) $A_1(4,3),\quad A_2(-5,\frac{3}{2}\sqrt{7})$ nuqtalardan o'tadi;

5) giperbola $M(\frac{9}{2},-1)$ nuqtadan o'tadi, uning asimptotalari tenglamasi $y=\pm\frac{2}{3}x$;

6) asimptotalari $y=\pm\frac{3}{4}x$; bo'lib, direktrisalari orasidagi masofa $12\frac{4}{5}$ ga teng;

7) giperbola $M_1(4,6)$ nuqtadan o'tib, uchlari orasidagi masofa 4 ga teng.

4. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning fokusidan o'tgan vatari haqiqiy o'qiga perpendikular bo'lsa, bu vatarning uzunligi $2r$ ni toping.

5. $x^2-4y^2=4$ giperbolaning asimptotalariga parallel bo'lib, (2, -5) nuqtadan o'tgan to'g'ri chiziqlar tenglamasini toping.

6. $9x^2-y^2=9$ giperbolaning har bir asimptotasiga perpendikular bo'lib, (2, 1) nuqtadan o'tgan to'g'ri chiziqlarning tenglamasini toping.

7. Fokuslari (0, 0) va (6, 0) nuqtalarda joylashgan, ekstsentrisiteti esa $e=\frac{3}{2}$ bo'lgan giperbolaning tenglamasini tuzing.

8.Bitta uchi $x^2-y^2=a^2$ giperbolada, ikkita tomoni esa bu giperbolaning asimptotalarida yotgan to'g'ri chiziq to'rtburchakning yuzini toping.

9. Giperbola direktrisasi fokusidan unga mos bo'lgan asimptotaga tushirilgan perpendikularning asosidan o'tishini isbot qiling. Bu perpendikularning uzunligini toping.

10. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning *A(a, 0)* uchidan vatarlar o'tkazilgan. Bu vatarlarning o'rta nuqtalari to'plamidan tashkil topgan figuraning tenglamasini toping.

11. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning o'ng fokusidan giperbolaning barcha nuqtalariga fokal radius vektorlar o'tkazilgan. Bu radius vektorlardan hosil bo'lgan kesmalarning o'rta nuqtalari to'plamining tenglamasini toping.

12. Quyidagi tenglamalar bilan aniqlangan giperbolalarni yasang, ularning yarim o'qlari va asimptotalarining tenglamalarini toping:

a) $\frac{x^2}{9}-\frac{y^2}{16}=1,\quad$ *b)* $\frac{(x-2)^2}{9}-\frac{(y+1)^2}{4}=1,\quad$ *c)* $\frac{(x+3)^2}{4}-y^2=1$

13. O'qlari koordinata o'qlari bilan ustma-ust tushgan va

a) uchlari orasidagi masofa 8 ga, fokuslari orasidagi masofa 10 ga teng bo'lgan;

b) haqiqiy o'qi 6 ga teng va (+9;-4) nuqtadan o'tgan giperbolaning tenglamasi tuzilsin.

14. $\frac{x^2}{49}-\frac{y^2}{25}=1$ giperbolaning fokuslari va asimptotalari yasalsin.

15. $\frac{x^2}{25}-\frac{y^2}{24}=1$ giperbolaning quyidagi to'g'ri chiziqlar bilan kesishish nuqtalari topilsin;

a) x-5y=0; *b)* 2x+y-18=0; *c)* x-y+5=0;

15. A(+3;-1) nuqtadan $\frac{x^2}{4}-y^2=1$ giperbolaning shunday vatari o'tkazilsinki, u *A* nuqtada teng ikkiga bo'linsin.

16. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning o'qlari, ular bilan teng ikkiga bo'lingan vatarlarga perpendikulyar bo'lgan birdan – bir diametrlari ekanligi tekshirilsin.

17. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaga ichki chizilgan kvadratning uchlari topilsin va qanday giperbolalarga ichki kvadrat chizish mumkinligi tekshirilsin.

18. $\frac{x^2}{5}-\frac{y^2}{4}=1$ giperbolaga (+2;0), (-4;+3),(+5;-1) nuqtalarning har biridan urinmalar o'tkazilsin.

19. Berilgan $\frac{x^2}{15}-\frac{y^2}{6}=1$ giperbolaga:

a) $x+y+7=0$ to'g'ri chiziqqa parallel;

b) $x-2y+3=0$ to'g'ri chiziqqa parallel;

s) $x-2y-5=0$ to'g'ri chiziqqa perpendikulyar bo'lgan urinmalar o'tkazilsin.

20. Giperbola asimptotalarining tenglamalari $y=\pm\frac{1}{2}x$ va urinmalaridan birining tenglamasi $5x-6y-8=0$ bo'lsa, giperbolaning tenglamasi tuzilsin.

21. Ushbu $\rho=\dfrac{9}{4-5\cos\varphi}$ chiziqning to'g'ri burchakli koordinatalar sistemasidagi tenglamasini toping.

3-topshiriq.

1. Quyidagilarga asoslanib, parabolaning kanonik tenglamasini tuzing:

1) fokusdan parabolaning uchigacha bo'lgan masofa 2 ga teng;

2) fokusdan direktrisagacha bo'lgan masofa 6 ga teng.

2. Quyidagi parabolalarning tenglamalarini soddaroq ko'rinishga keltiring va uning uchlarini toping. Parabolalarni yasang:

$a)\ y^2-2y-2x-5=0;$ $b)\ y=2x-x^2;$

$c)\ x^2-4x-2y+10=0;$ $d)\ x^2+3x=y.$

3. Quyidagi shartlarda parabolaning tenglamasi tuzilsin:

a) parabolaning uchidan fokusigacha bo'lgan masofa 4 ga teng;

b) fokusining koordinatalari (3, 0) bo'lib, ordinatalar o'qi direktrisa xizmatini qiladi.

4. $y^2=8x$ parabolada fokal radius – vektori 20 teng bo'lgan nuqta topilsin.

5. $y^2=4{,}5x$ parabolada direkrtisadan *d*=9,125 masofada turuvchi $M(x,y)$ nuqta olingan. Parabola uchidan bu nuqtagacha bo'lgan masofa hisoblansin.

6. $y^2=18x$ parabolaning quyidagi to'g'ri chiziqlar bilan kesishish nuqtalari topilsin:

$a)\ 6x+y-6=0;$ $b)\ 9x-2y+2=0;$ $c)\ 4x-y+5=0.$

7. Berilgan $y^2=12x$ parabolaga abstsissasi $x=3$ bo'lgan nuqtada bo'lgan urinma o'tkazilsin.

8. $y=kx+b$ to'g'ri chiziqning $y^2=2px$ parabolaga urinish sharti topilsin.

9. $y^2=64x$ paraboladan $4x+3y+46=0$ to'g'ri chiziqqacha bo'lgan eng qisqa masofa topilsin.

10. $y^2=2px$ parabolaning $x-2y+5=0$ to'g'ri chiziqqa urinishi ma'lum. Parabolaning parametri hisoblansin.

11. $\frac{x^2}{30}+\frac{y^2}{24}=1$ ellips bilan $y^2=\frac{20}{3}x$ parbolaning umumiy urinmalari topilsin.

12. $y^2=18x$ parabola bilan $(x+6)^2+y^2=100$ aylana umumiy vatarining tenglamasi tuzilsin.

13. Berilgan $y^2=12x$ parabolaga $2x-y+3=0$ to'g'ri chiziqqa parallel bo'lgan urinma o'tkazilsin.

14. Quyidagi shartlarda parabolaning tenglamasi tuzilsin:

a) fokusining koordinatalari (3, 0) bo'lib, ordinatalar o'qi direktrisa xizmatini qiladi;

b) parabola y o'qiga nisbatan simmetrik bo'lib, $M(6, -2)$ nuqtadan va koordinatalar boshidan o'tadi.

15. Quyidagilarga asoslanib, parabolaning kanonik tenglamasini tuzing:

1) fokusdan direktrisagacha bo'lgan masofa 6 ga teng;

2) parabolaning uchidan direktrisagacha bo'lgan masofa 1 ga teng.

16. Uchi $A(-4, -2)$ nuqtada, simmetriya o'qi Ox o'qqa parallel bo'lgan va $M(1, 3)$ nuqtadan o'tadigan parabolaning tenglamasini tuzing.

17. Berilgan $y^2 = 12x$ parabolaga $3x - y + 5 = 0$ to'g'ri chiziqqa perpendikulyar bo'lgan urinma o'tkazilsin.

18. Uchi $A(5, -5)$ nuqtada, simmetriya o'qi Oy o'qqa parallel bo'lgan va koordinatalar boshidan o'tadigan parabolaning tenglamasini tuzing.

19. Berilgan $y^2 = 12x$ parabolaga $6x - 3y + 5 = 0$ to'g'ri chiziq bilan $\frac{\pi}{4}$ burchak tashkil etgan urinma o'tkazilsin.

20. Quyidagi parabolalarning tenglamalarini soddaroq ko'rinishga keltiring va uning uchlarini toping. Parabolalarni yasang:

$a)$ $x^2 + 4x - y + 4 = 0;$ $b)$ $y^2 + 2x + 2y - 1 = 0;$ $c)$ $x^2 - 4x + 2y - 2 = 0;$ 21

21. $y^2 = 18x$ paraboladan $6x + 8y - 5 = 0$ to'g'ri chiziqqacha bo'lgan eng qisqa masofa topilsin.

Ishni bajarish tartibi.

1. Talaba laboratoriya ishi bilan tanishadi.
2. Laboratoriya ishi bo'yicha hisobot tayyorlaydi.
3. Nazorat savollariga javob beradi.

Nazorat savollari.

1. Ellips tenglamasini yozing?
2. Ellipsning taorifini ayting?
3. Ellipsning qanday qilib aylanaga keltirish mumkin?
4. Giperbola tenglamasini yozing?
5. Giperbolaning haqiqiy o'qi nimadan iborat?
6. Qaysi to'g'ri chiziqlar giperbolaning asimptotalari bo'ladi?
7. Parabola tenglamasini yozing?
8. Parabola deb qanday figuraga aytiladi?

9. Parabolaning qanday tenglamalari mavjud?

Foydalanilgan adabiyotlar.

1. Suberbiller O.N.«Analitik geometriyadan masalalar va mashqlar». Toshkent-1960 y.
2. Bakelman I.YA. Analitik geometriya va chiziqli algebra. Toshkent, O'qituvchi,1978 y.
3. X.X.Nazarov, X.O.Ochilova, E.G.Podgarnova. «Geometriyadan masalalar to'plami».I-kism. Toshkent-1983 y.
4. L.S.Atanasyan, V.A.Atanasyan. «Sbornik zadach po geometrii». CHast-I. Moskva-1973 g.
5. N.D.Dodajonov, M.Sh.Jo'rayeva. Geometriya.1-qism.Toshkent-1996 y.
6.X.R.Latipov, F.U.Nosirov, Sh.I. Tojiyev. Analitik geometriya va chiziqli algebradan masalalar yechish bo'yicha qo'llanma. Toshkent-1999y.

3-laboratoriya ishi.
Mavzu: Evklid algoritmi. Eng katta umumiy bo'luvchi

Ishdan maqsad: Evklid algoritmi yordamida ko'phadlarni eng katta umumiy bo'luvchisini topishni o'rganish.

Nazariy qism:

Butun sonlar uchun ma'lum bo'lgvan Evklid algoritmi va uning natijalarining ko'phadlarga ham tatbiq etilishini ko'rib o'taylik.

$f(x) \neq 0$ ko'phadning darajasi $\varphi(x) \neq 0$ ko'phadning darajasidan kichik emas deb faraz qilamiz va $f(x)$ ni $\varphi(x)$ ga bo'lamiz. Hosil bo'lgan bo'linma va qoldiqni, mos ravishda $q_1(x)$ va $r_1(x)$ bilan belgilaymiz. Ma'lumki, $r_1(x)$ ning darajasi $\varphi(x)$ ning darajasidan kichikdir. Endi $\varphi(x)$ ni $r_1(x)$ ga bo'lib, bo'linma va qoldiqni $q_2(x)$ va $r_2(x)$ orqali belgilaymiz. Yana $r_2(x)$ ning darajasi $r_1(x)$ nikidan kichikligini e'tiborga olib, $r_1(x)$ ni $r_2(x)$ ga bo'lamiz; hosil bo'lgan bo'linma va qoldiqni $q_3(x)$ va $r_3(x)$ bilan belgilaymiz va h. k. Umuman har bir qoldiqni undan keyingi qoldiqqa bo'lamiz. Natijada darajalari kamayib boruvchi:

$$r_1(x),\ r_2(x),\ r_3(x),\ r_4(x),\ \ldots$$

qoldiqlar vujudga keladi. Bu qoldiqlarning soni albatta cheklidir, chunki ularning darajalari kamayib boruvchi (lekin manfiy emas) butun sonlar ketma-ketligini tashkil etadiki, bunday qator cheksiz bo'la olmaydi. SHu sababli, yuqoridagi bo'lish protsessi chekli bo'lib, biz shunday $r_k(x)$ qoldiqqa kelamizki, unga oldingi $r_{k-1}(x)$ qoldiq raso bo'linadigan bo'ladi. Natijada ushbu:

$$\begin{aligned}
&f(x) = \varphi(x)q_1(x) + r_1(x),\\
&\varphi(x) = r_1(x)q_2(x) + r_2(x),\\
&r_1(x) = r_2(x)q_3(x) + r_3(x),\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad\qquad (1)\\
&r_{k-3}(x) = r_{k-2}(x)q_{k-1}(x) + r_{k-1}(x),\\
&r_{k-2}(x) = r_{k-1}(x)q_k(x) + r_k(x),\\
&r_{k-1}(x) = r_k(x)q_{k+1}(x)
\end{aligned}$$

tengliklarni hosil qilamiz Bu ketma-ket bo'lish protsessi, odatda, *Evklid algoritmi* deyiladi.

Endi, ko'phadlarning umumiy bo'luvchilari tushunchasiga o'taylik.

1-ta'rif. Agar $f(x)$ va $\varphi(x)$ ko'phadlarning ikkalasi ham $g(x)$ ko'phadga bo'linsa, $g(x)$ ko'phad $f(x)$ va $\varphi(x)$ ning *umumiy bo'luvchisi* deyiladi.

Ikki $f(x)$ va $\varphi(x)$ ko'phadning bir necha umumiy bo'luvchilari mavjud bo'lishi mumkin.

Masalan,

$$f(x) = x^4 + x^3 - 7x^2 - x + 6$$

va

$$\varphi(x) = x^4 - 5x^2 + 4$$

ko'phadlar uchun

$$g_1(x) = x - 1,$$
$$g_2(x) = x + 1,$$
$$g_3(x) = x - 2,$$
$$g_4(x) = x^2 - 1,$$
$$g_5(x) = x^2 - 3x + 2,$$
$$g_6(x) = x^2 - x - 2,$$
$$g_7(x) = x^3 - 2x^2 - x + 2$$

ko'phadlarning har qaysisi umumiy bo'luvchidir.

2-ta'rif. Agar $f(x)$ va $\varphi(x)$ ning $d(x)$ dan iborat umumiy bo'luvchisi bu ikki ko'phadning har bir umumiy bo'luvchisiga bo'linsa, $d(x)$ bo'luvchi $f(x)$ va $\varphi(x)$ ning *eng katta umumiy bo'luvchisi* deyiladi.

Masalan, yuqoridagi ikki $f(x)$ va $\varphi(x)$ ko'phadning *eng katta* umumiy bo'luvchisi $g_7(x) = x^3 - 2x^2 - x + 2$ dir.

3-ta'rif. Agar $f(x)$ va $\varphi(x)$ ko'phadlarning eng katta umumiy bo'luvchisi nolinchi darajali ko'phad bo'lsa, $f(x)$ va $\varphi(x)$- *o'zaro tub ko'phadlar* deyiladi.

Teorema. Agar $d(x)$ ko'phad $f(x)$ va $\varphi(x)$ ning eng katta umumiy bo'luvchisi bo'lsa, $ad(x)$ ham $f(x)$ va $\varphi(x)$ ning eng katta umumiy bo'luvchisini ifodalaydi, bunda $a-$istalgan nolinchi darajali ko'phad.

Teorema. P maydondagi $f(x)$ va $\varphi(x)$ ko'phadlarning eng katta umumiy bo'luvchisi $d(x)$ bo'lsa, bu maydonda ular uchun:

$$f(x)g(x) + \varphi(x)h(x) = d(x) \tag{2}$$

tenglikni qanoatlantiruvchi $g(x)$ va $h(x)$ ko'phadlar mavjud.

Misol. $f(x) = x^4 + x^3 - 7x^2 - x + 6$ va $\varphi(x) = x^4 - 5x^2 + 4$ ko'phadlarning eng katta umumiy bo'luvchisini aniqlang.

Yechilishi. Buning uchun $f(x)$ ni $\varphi(x)$ ga bo'lamiz:

$$\begin{array}{r|l} -\begin{array}{l} x^4 + x^3 - 7x^2 - x + 6 \\ x^4 \qquad\;\; - 5x^2 \qquad + 4 \end{array} & \begin{array}{c} x^4 - 5x^2 + 4 \\ \hline 1 \end{array} \\ \hline \end{array}$$

$$r_1(x) = x^3 - 2x^2 - x + 2$$

$\varphi(x)$ ni $r_1(x)$ ga bo'lamiz:

$$
\begin{array}{r|l}
-\begin{array}{l} x^4-5x^2+4 \\ \underline{x^4-2x^3-x^2+2x} \end{array} & \begin{array}{c} x^3-2x^2-x+2 \\ \hline x+2 \end{array} \\
\quad -\begin{array}{l} 2x^3-4x^2-2x+4 \\ \underline{2x^3-4x^2-2x+4} \end{array} & \\
0 &
\end{array}
$$

Demak, biz izlagan eng katta umumiy bo'luvchi $d(x)=x^3-2x^2-x+2$.

Keltiriladigan va keltirilmaydigan ko'phadlar.

4-ta'rif. Agar P maydonda darajasi nolga teng bo'lmagan $f(x)$ ko'phadni P maydonda va darajalari $f(x)$ ning darajasidan kichik ikkita $g(x)$ va $h(x)$ ko'phad ko'paytmasi sifatida ifodalash (ko'paytmaga keltirish) mumkin bo'lsa, $f(x)$ ni P maydonda *keltiriladigan ko'phad* deymiz.

P maydonda darajasi nolga teng emas $f(x)$ ko'phadni bunday ko'paytma sifatida ifodalash (bunday ko'paytmaga keltirish) mumkin bo'lmasa, u, P maydonda *keltirilmaydigan ko'phad* deb ataladi.

Masalan, ratsional sonlar maydonidagi $f(x)=x^5+2x^3+x^2+x+1$ ko'phad shu maydonda keltiriladi, chunki

$$x^5+2x^3+x^2+x+1=(x^3+x+1)(x^2+1).$$

Ratsional sonlar maydonidagi $f(x)=x^2-3$ ko'phad esa bu maydonda keltirilmaydi. Haqiqatan, bu ko'phadni ratsional sonlar maydonida keltiriladigan desak,

$$f(x)=g(x)h(x) \tag{3}$$

tenglik bajarilib, $g(x)$ va $h(x)$ ning darajalari 2 dan kichik va koeffitsientlari ratsional bo'lishi lozim. Demak, $g(x)$ va $h(x)$ birinchi darajali ko'phadlar bo'lgandagina (3) tenglik bajarilishi mumkin; shu sababli:

$$x^2-3=(ax+b)(cx+d)$$

tenglik o'rinli bo'lsa, a, b, c, d ratsional sonlar bo'lishi kerak.

So'nggi tenglikning o'ng tomoni va, demak, chap tomoni ham $x=-\frac{b}{a}$, qiymatda nolga aylanadi, ya'ni $\frac{b^2}{a^2}-3=0$, bundan $\sqrt{3}=\pm\frac{b}{a}$. Lekin bunday tenglik mavjud bo'lishi mumkin emas, chunki $\sqrt{3}$ irratsional son $\pm\frac{b}{a}$ ratsional songa teng bo'lolmaydi.

Har qanday P sonlar maydonidagi birinchi darajali istalgan ko'phad – shu maydonda keltirilmaydigan ko'phaddir. Haqiqatan, darajasi *1* dan kichik ko'phad faqat nolinchi darajali bo'lishi mumkin. Lekin, birinchi darajali ko'phadni ikkita nolinchi darajali ko'phadning ko'paytmasi qilib yozish hech mumkin emas.

Birinchidan yuqori darajali bo'lib, P maydonda keltirilmaydigan $f(x)$ ko'phad P ni o'z ichiga olgan kengroq maydonda keltiriladigan bo'lishi mumkin. Masalan, ratsional sonlar maydonida keltirilmaydigan x^2-3 ko'phad haqiqiy sonlar maydonida keltiriladi, chunki

$$x^2-3=(x-\sqrt{3})(x+\sqrt{3}).$$

Shuningdek, haqiqiy sonlar maydonida keltirilmaydigan x^2+1 ko'phad kompleks sonlar maydonida keltiriladi:

$$x^2+1=(x-i)(x+i).$$

Shu sababli $f(x)$ ko'phadning *keltiriladiganligi* yoki *keltirilmaydiganligi* haqida biror maydonni ko'zda tutibgina gapirish mumkin.

Keltirilmaydigan ko'phadlar quyidagi xossalarga ega.

1-xossa. Agar keltirilmaydigan $p(x)$ ko'phad keltirilmaydigan ikkinchi $q(x)$ ko'phadga bo'linsa, $p(x)$ va $q(x)$ bir-biridan o'zgarmas ko'paytuvchi bilangina farq qiladi.

2-xossa. Istalgan $f(x)$ ko'phad keltirilmaydigan ixtiyoriy $p(x)$ ko'phadga yo bo'linadi, yoki u, $p(x)$ bilan o'zaro tub bo'ladi.

3-xossa. Agar $f_1(x),\ f_2(x),\ \ldots,\ f_m(x)$ ko'phadlarning hech qaysisi keltirilmaydigan $p(x)$ ko'phadga bo'linmasa, ularning $f_1(x),\ f_2(x),\ \ldots,\ f_m(x)$ ko'paytmasi ham $p(x)$ ga bo'linmaydi.

4-xossa. Agar $f_1(x),\ f_2(x),\ \ldots,\ f_m(x)$ ko'paytma keltirilmaydigan $p(x)$ ko'phadga bo'linsa, $f_1(x),\ f_2(x),\ \ldots,\ f_m(x)$ ko'phadlarning aqalli bittasi $p(x)$ ga bo'linadi.

5-xossa. $p(x)$ - keltirilmaydigan ko'phad bo'lsa, $ap(x)$ ham keltirilmaydigan ko'phadni ifodalaydi, bunda $a-$ istalgan nolinchi darajali ko'phad.

Teorema. P maydonda berilgan va darajasi *1* dan kichik bo'lmagan har bir $f(x)$ ko'phad shu maydonda keltirilmaydigan ko'phadlar ko'paytmasiga yoyiladi:

$$f(x)=p_1(x)p_2(x)\ \ldots\ p_r(x)$$

va bu yoyilma o'zgarmas ko'paytuvchilargacha aniqlik bilan yagonadir.

V A R I A N T L A R

1-topshiriq. Quyidagi ko'phadlarning EKUB ini toping.

1. $f(x)=x^4-1,\qquad g(x)=2x^3+x^2-2x-1.$

2. $f(x)=x^4-2x^3-4x^2+4x-3,$ $\quad g(x)=2x^3-5x^2-4x+3.$

3. $f(x)=x^4+x^3+x^2+x+1,$ $\quad g(x)=3x^3+x^2+3x-1.$

4. $f(x)=x^4+2x^3-x^2-4x-2,$ $\quad g(x)=x^4+x^3-x^2-2x-2.$

5. $f(x)=x^5+3x^4+x^3+x^2+3x+1,$ $\quad g(x)=x^4+2x^3+x+2.$

6. $f(x)=3x^5+5x^4-16x^3-6x^2-5x-6,$ $\quad g(x)=3x^4-4x^3-x^2-x-2.$

7. $f(x)=4x^4-2x^3-16x^2+5x+9,$ $\quad g(x)=2x^3-x^2-5x+4.$

8. $f(x)=x^5-5x^4-2x^3+12x^2-2x+12,$ $\quad g(x)=x^3-5x^2-3x+17.$

9. $\begin{cases} f(x)=x^6-4x^5+11x^4-27x^3+37x^2-35x+35, \\ g(x)=x^5-3x^4+7x^3-20x^2+10x-25. \end{cases}$

10. $\begin{cases} f(x)=3x^7+6x^6-3x^5+4x^4+14x^3-6x^2-4x+4, \\ g(x)=3x^6-3x^4+7x^3-6x+2. \end{cases}$

11. $f(x)=x^5-7x^4-2x^3+2x^2-3x+4,$ $\quad g(x)=x^3-2x^2-3x+7.$

12. $f(x)=x^5+5x^4+9x^3+7x^2+5x+3,$ $\quad g(x)=x^4+2x^3+2x^2+x+1.$

13. $f(x)=x^4-x^3-4x^2+4x+1,$ $\quad g(x)=x^2-x-1.$

14. $f(x)=x^5+4x^4-10x^2-x+6,$ $\quad g(x)=5x^4+16x^3-20x^2-1.$

15. $f(x)=x^5+3x^4-x^3-7x^2+4,$ $\quad g(x)=5x^4+12x^3-3x^2-14x.$

16. $f(x)=x^5+x^4-x^3-2x-1,$ $\quad g(x)=3x^4+2x^3+x^2-2.$

17. $f(x)=x^6+2x^4-4x^3-3x^2+8x-5,$ $\quad g(x)=x^5+x^2-x+1.$

18. $f(x)=x^3+5x^2-x-5,$ $\quad g(x)=3x^2+10x-1.$

19. $f(x)=x^5-15x^3+16x^2+36x-48,$ $\quad g(x)=x^4+x^3-2x^2-3x-3.$

20. $f(x)=x^6+3x^5+x^4-8x^3-24x^2-x,$ $\quad g(x)=x^4-4x^3-11x^2-30x.$

21. $f(x)=4x^4-2x^3-16x^2+5x+9,$ $\quad g(x)=2x^3-x^2-5x+4.$

22. $f(x)=x^5+x^4-x^3-3x^2-3x-1,$ $\quad g(x)=x^4-2x^3-x^2-2x+1.$

23. $f(x)=x^4-2x^3-4x^2+6x+1,$ $\quad g(x)=x^3-5x-3.$

24. $f(x)=x^4+x^3-3x^2-4x-1,$ $\quad g(x)=x^3+x^2-x-1.$

25. $f(x)=x^6-7x^4+8x^3-7x+7,$ $\quad g(x)=3x^5-7x^3+3x^2-7.$

26. $f(x)=x^5-2x^4+x^3+7x^2-12x+10,$ $\quad g(x)=3x^4-6x^3+5x^2+2x-2.$

2-topshiriq. Evklid algoritmidan foydalanib, $f_1(x)M_2(x)+f_2(x)M_1(x)=\delta(x)$ tenglikdan $M_1(x),\ M_2(x)$ ko’phadlarni toping, bu yerda $\delta(x)-f_1(x)$ va $f_2(x)$ ko’phadlarning EKUB idir.

1. $f_1(x)=x^3+5x^2-x-5,$ $\quad f_2(x)=3x^2+10x-1.$

2. $f_1(x)=x^4+x^3-3x^2-4x-1,$ $\quad f_2(x)=x^3+x^2-x-1.$

3. $f_1(x)=x^6+3x^5+x^4-8x^3-24x^2-x,$ $\quad f_2(x)=x^4-4x^3-11x^2-30x.$

4. $f_1(x)=x^5+x^4-x^3-2x-1,$ $f_2(x)=3x^4+2x^3+x^2+2x-2.$

5. $f_1(x)=x^5-15x^3+16x^2+36x-48,$ $f_2(x)=x^4+x^3-2x^2-3x-3.$

6. $f_1(x)=x^6-7x^4+8x^3-7x+7,$ $f_2(x)=3x^5-7x^3+3x^2-7.$

7. $f_1(x)=4x^4-2x^3-16x^2+5x+9,$ $f_2(x)=2x^3-x^2-5x+4.$

8. $f_1(x)=x^5+3x^4-12x^3-52x^2-52x-12,$ $f_2(x)=x^4+3x^3-6x^2-22x-12.$

9. $f_1(x)=x^5+5x^4+9x^3+7x^2+5x+3,$ $f_2(x)=x^4+2x^3+2x^2+x+1.$

10. $f_1(x)=x^4-4x^3+1,$ $f_2(x)=x^3-3x^2+1.$

11. $f_1(x)=x^4-1,$ $f_2(x)=2x^3+x^2-2x-1.$

12. $f_1(x)=x^5-2x^4+x^3+7x^2-12x+10,$ $f_2(x)=3x^4-6x^3+5x^2+2x-2.$

13. $f_1(x)=x^4+x^3-7x^2-x+6,$ $f_2(x)=x^4-5x^2+4.$

14. $f_1(x)=x^5+x^4-x^3-3x^2-3x-1,$ $f_2(x)=x^4-2x^3-x^2-2x+1.$

15. $f_1(x)=x^4-2x^3-4x^2+4x-3,$ $f_2(x)=2x^3-5x^2-4x+3.$

16. $f_1(x)=x^4+7x^3+19x^2+23x+10,$ $f_2(x)=x^4+7x^3+18x^2+22x+12.$

17. $f_1(x)=x^4+x^3+x^2+x+1,$ $f_2(x)=3x^3+x^2+3x-1.$

18. $f_1(x)=x^4-2x^3-4x^2+6x+1,$ $f_2(x)=x^3-5x-3.$

19. $f_1(x)=x^4+2x^3-x^2-4x-2,$ $f_2(x)=x^4+x^3-x^2-2x-2.$

20. $f_1(x)=x^4+2x^3+x+1,$ $f_2(x)=x^4+x^3-2x^2+2x-1.$

21. $f_1(x)=x^5+3x^4+x^3+x^2+3x+1,$ $f_2(x)=x^4+2x^3+x+2.$

22. $f_1(x)=3x^5+5x^4-16x^3-6x^2-5x-6,$ $f_2(x)=3x^4-4x^3-x^2-x-2.$

23. $f_1(x)=4x^4-2x^3-16x^2+5x+9,$ $f_2(x)=2x^3-x^2-5x+4.$

24. $f_1(x)=x^5-5x^4-2x^3+12x^2-2x+12,$ $f_2(x)=x^3-5x^2-3x+17.$

25. $f_1(x)=x^4-x^3+x^2-3x-6,$ $f_2(x)=x^5-x^4-2x^3+2x^2+x-2.$

26. $f_1(x)=x^5-x^4-5x^3+7x^2-10,$ $f_2(x)=x^4+x^3-2x^2-5x-15.$

3-topshiriq. Quyida berilgan ko’phadlar qaysi sonlar maydonida keltiriladi va qaysi sonlar maydonida keltirilmaydi?

1. $f(x)=x^5+2x^3+x^2+x+1$

2. $f(x)=x^4+49x^3+14x^2$

3. $f(x)=x^5-10x^3-20x^2-15x-4$

4. $f(x)=x^3+3x^2-4$

5. $f(x)=x^4+2x^3+x+2$

6. $f(x)=x^4-2x^3+18x^2-27$

7. $f(x)=x^4-2x^3-x+2$

8. $f(x)=x^3+3x^2+3x-36$

9. $f(x)=27x^3-81x^2+297x+242$

10. $f(x)=x^3+9x^2+18x+28$
11. $f(x)=x^3+12x^2+45x+54$
12. $f(x)=x^4-4x^3-22x^2+100x-75$
13. $f(x)=x^4+8x^3+32x^2+80x+100$
14. $f(x)=x^4-8x^3+24x^2-9x-8$
15. $f(x)=16x^4-8x^2+64ix-65$
16. $f(x)=x^4+4x+3$
17. $f(x)=x^4+x^3+x^2+x+2$
18. $f(x)=x^3-3x^2-3x+11$
19. $f(x)=8x^3+24x^2-81$
20. $f(x)=x^3+12x+63$
21. $f(x)=x^4-x^3+x^2-3x-6$

Quyidagi ko'phadlarning ratsional sonlar maydonida keltirilmasligini isbotlang:

22. $f(x)=x^4-2x+3$
23. $f(x)=\frac{1}{3}x^4-\frac{2}{3}x^3+2x^2-\frac{4}{3}x+2$
24. $f(x)=x^5-12x^3+36x-12$
25. $f(x)=\frac{1}{2}x^4-\frac{1}{2}x^3+x+\frac{1}{2}$
26. $f(x)=x^4+16$

Ishni bajarish tartibi.

1. Talaba laboratoriya ishi bilan tanishadi.
2. Laboratoriya ishi bo'yicha hisobot tayyorlaydi.
3. Nazorat savollariga javob beradi.

Nazorat savollari.

1. Evklid algoritmi deb nimaga aytiladi?
2. Ko'phadlarning umumiy bo'luvchisi deb nimaga aytiladi?
3. Ko'phadlarning eng katta umumiy bo'luvchisi deb nimaga aytiladi?
4. O'zaro tub ko'phadlar deb qanday ko'phadlarga aytiladi?
5. Qanday ko'phadga keltiriladigan ko'phad deyiladi?
6. Qanday ko'phadga keltirilmaydigan ko'phad deyiladi?
7. Keltirilmaydigan ko'phadlarning xossalarini sanab bering?

Foydalanilgan adabiyotlar.

1. A.G.Kurosh. «Oliy algebra kursi». Toshkent, O'qituvchi, 1976.
2. R.Iskandarov. «Oliy algebra», I-qism. Toshkent, 1963.
3. D.K.Fadeev, I.S.Sominskiy. «Sbornik zadach po vыsshey algebre». Moskva, Nauka, 1977.
4. I.V.Proskuryakov. «Sbornik zadach po lineynoy algebre». Moskva, Nauka, 1984.

Laboratoriya ishi.
Mavzu: Uchinchi va to'rtinchi darajali tenglamalar

Ishdan maqsad: Uchinchi, to'rtinchi darajali tenglamalarni yechishning mos ravishda Kardano, Ferrari va Lobachevskiy usullarini o'rganish.

Nazariy qism:

Uchinchi darajali tenglamalar. Kompleks sonlar maydonidagi uchinchi darajali tenglamaning ikkala tomonini bosh koeffitsientga bo'lib, uni:

$$x^3+ax^2+bx+c=0 \tag{1}$$

ko'rinishga keltirish mumkin.

Bu tenglama quyidagi metod bilan echiladi.

(1) tenglamani yangi noma'lum y ga nisbatan ikkinchi darajali had ishtirok etmagan uchinchi darajali tenglamaga quyidagicha keltirish mumkin: α ni (1) tenglamada $x=y+\alpha$ almashtirishni bajargandan keyin yuqoridagi shartni qanoatlantiruvchi uchinchi darajali tenglama hosil bo'ladigan qilib tanlaymiz.

(1) da x o'rniga $y+\alpha$ ni qo'yib y^2 ning koeffitsientini nolga tenglashdan $3\alpha+a=0$ tenglama kelib chiqadi. Bu tenglamadan $\alpha=-\frac{a}{3}$ topiladi.

Aytilganlarga asosan (1) tenglamada

$$x=y-\frac{a}{3} \tag{2}$$

almashtirishni bajarsak,

$$y^3+py+q=0 \tag{3}$$

hosil bo'ladi, bunda:

$$p=b-\frac{a^2}{3}, \qquad q=\frac{2a^3}{27}-\frac{ab}{3}+c. \tag{4}$$

(3)-uchinchi darajali tenglamaning *normal* shakli deb ataladi.
(3) normal tenglamani yechish uchun

$$y = u + v \tag{5}$$

deymiz, bunda u va v-yangi noma'lumlar. Bu ifodani (3) tenglamaga qo'ysak, quyidagi kelib chiqadi:

$$(u+v)^3 + p(u+v) + q = 0,$$

bundan:

$$(u^3 + v^3 + q) + (3uv + p)(u+v) = 0. \tag{6}$$

Endi , u va v noma'lumlarni shunday aniqlaylikki ,

$$3uv + p = 0 \quad \text{yoki} \quad uv = -\frac{p}{3} \tag{7}$$

bajarilsin. Bu vaqtda (6) va (7) dan:

$$u^3 + v^3 = -q \ , \qquad u^3 v^3 = -\frac{p^3}{27}$$

hosil bo'ladi. Ko'ramizki, u^3 va v^3 ushbu:

$$z^2 + qz - \frac{p^3}{27} = 0$$

kvadrat tenglamaning ildizlaridan iborat. Bu tenglamani yechib, quyidagini topamiz:

$$z_{1,2} = -\frac{q}{2} \pm \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$$

yoki

$$u^3 = z_1 = -\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \quad \text{va} \quad v^3 = z_2 = -\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}},$$

bundan, (5) ga ko'ra:

$$y = u + v = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}. \tag{8}$$

(8) tenglik, odatda, *Kardano formulasi* deb ataladi. Bu tenglik-ikkita ildizning yig'indisidan iborat bo'lib, har bir ildiz uchta qiymatga ega; u ning har bir qiymatini v ning har bir qiymati bilan olsak, $y = u + v$ uchun hammasi bo'lib to'qqizta qiymatni hosil qilamiz. Ammo (3) tenglama faqat uchta ildizga ega; shu sababli, yuqoridagi to'qqizta qiymatdan uchtasini, ya'ni $y = u + v$ yig'indining (7) shartni qanoatlantiruvchi qiymatlarini olishimiz kerak. Shu maqsadda avval:

$$u = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

ildizning uchta qiymatini topamiz. Buning uchun, ma’lumki, u ning bitta, masalan, u_1 ildizini 1 ning uchinchi darajali

$$\sqrt[3]{1}=\cos 0+i\sin 0=1,$$

$$\sqrt[3]{1}=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}=-\frac{1}{2}+i\frac{\sqrt{3}}{2}=\varepsilon,$$

$$\sqrt[3]{1}=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=-\frac{1}{2}-i\frac{\sqrt{3}}{2}=\varepsilon^2$$

ildizlariga ko’paytirishimiz lozim. Natijada u ning uchinchi darajali ildizlari $u_1, u_2=\varepsilon u_1, u_3=\varepsilon^2 u_1$ bo’ladi.

Endi v ning tegishli qiymatlarini (7) shartdan topamiz:

$$v_1=-\frac{p}{3u_1};\quad v_2=-\frac{p}{3u_2}=-\frac{p}{3\varepsilon u_1}=\varepsilon^2\left(-\frac{p}{3u_1}\right)=\varepsilon^2 v_1;$$

$$v_3=-\frac{p}{3u_3}=-\frac{p}{3\varepsilon^2 u_1}=\varepsilon\left(-\frac{p}{3u_1}\right)=\varepsilon v_1,$$

bunda $\varepsilon^3=1$ dan foydalandik. Shunday qilib, u ning har bir qiymatini v ning mos qiymatiga qo’shsak, y uchun quyidagi uchta qiymat kelib chiqadi:

$$y_1=u_1+v_1,\quad y_2=\varepsilon u_1+\varepsilon^2 v_1,\quad y_3=\varepsilon^2 u_1+\varepsilon v_1.$$

Agar bu tenglamalarga ε va ε^2 ning qiymatlarini qo’ysak, (3) normal tenglamaning ildizlari quyidagilarga teng bo’ladi:

$$\begin{aligned} y_1&=u_1+v_1,\\ y_2&=-\frac{1}{2}(u_1+v_1)+i\frac{\sqrt{3}}{2}(u_1-v_1),\\ y_3&=-\frac{1}{2}(u_1+v_1)-i\frac{\sqrt{3}}{2}(u_1-v_1). \end{aligned}\tag{9}$$

Endi, (2) tenglikdan foydalanib, (1) tenglamaning ildizlarini topamiz:

$$\begin{aligned} x_1&=u_1+v_1-\frac{a}{3},\\ x_2&=-\frac{1}{2}(u_1+v_1)+i\frac{\sqrt{3}}{2}(u_1-v_1)-\frac{a}{3},\\ x_3&=-\frac{1}{2}(u_1+v_1)-i\frac{\sqrt{3}}{2}(u_1-v_1)-\frac{a}{3}. \end{aligned}\tag{10}$$

Misol. $x^3+3x^2+15x+13=0$ tenglamani yechaylik. Bunda $a=3,\ b=15,\ c=13$ bo’lgani uchun, (4) tengliklarga asosan, $p=15-\frac{9}{3}=12$ va $q=\frac{2\cdot 3^3}{27}-\frac{3\cdot 15}{3}+13=0$. Endi

$$u=\sqrt[3]{o+\sqrt{0+\frac{12^3}{27}}}=\sqrt[3]{\sqrt{64}}=\sqrt[3]{8}.$$

Agar $u_1=2$ desak, $v_1=-\frac{12}{3\cdot 2}=-2$ hosil bo'ladi.

Demak, (10) ga binoan, berilgan tenglamaning ildizlari quyidagilardan iborat:

$$x_1=2-2-1=-1,\quad x_2=i\frac{\sqrt{3}}{2}(2+2)-1=2\sqrt{3}i-1,\quad x_3=-i\frac{\sqrt{3}}{2}(2+2)-1=-2\sqrt{3}i-1.$$

To'rtinchi darajali tenglamalar. Kompleks sonlar maydonidagi to'rtinchi darajali tenglamaning ikkala tomonini bosh koeffitsientga bo'lib, uni:

$$x^4+ax^3+bx^2+cx+d=0 \tag{1}$$

ko'rinishga keltira olamiz.

To'rtinchi darajali tenglamani echishning usullari bor. Biz ularning ba'zilarini ko'rib o'tamiz.

1. Ferrari usuli. (1) tenglamaning keyingi uchta hadini o'ng tomonga o'tkazib, ikkala tomonga $\frac{a^2x^2}{4}$ ni qo'shamiz. Natijada:

$$\left(x^2+\frac{ax}{2}\right)^2=\left(\frac{a^2}{4}-b\right)x^2-cx-d$$

hosil bo'ladi. Endi, so'nggi tenglamaning ikkala tomoniga $\left(x^2+\frac{ax}{2}\right)^2y+\frac{y^2}{4}$ yig'indini qo'shib, ushbuga ega bo'lamiz:

$$\left(x^2+\frac{ax}{2}+\frac{y}{2}\right)^2=\left(\frac{a^2}{4}-b+y\right)x^2+\left(\frac{ay}{2}-c\right)x+\left(\frac{y^2}{4}-d\right). \tag{2}$$

Yangi y noma'lumni (2) tenglamaning o'ng tomoni to'liq kvadratdan iborat bo'lib qoladigan qilib tanlaymiz. Buning uchun:

$$\frac{a^2}{4}-b+y=A^2,\quad \frac{ay}{2}-c=2AB,\quad \frac{y^2}{4}-d=B^2 \tag{3}$$

deb olishimiz kerak. Ushbu:

$$4A^2B^2=(2AB)^2$$

ayniyatga asosan, quyidagi natijaga kelamiz:

$$4\left(\frac{a^2}{4}-b+y\right)\left(\frac{y^2}{4}-d\right)=\left(\frac{ay}{2}-c\right)^2. \tag{4}$$

Qavslarni ochib, y ning darajalariga nisbatan o'xshash hadlarni yig'sak,

$$y^3-by^2+(ac-4a)y-[d(a^2-4b)+c^2]=0 \tag{5}$$

shakldagi uchinchi darajali tenglamaga kelamiz. Ko'ramizki, y noma'lumning qiymatlari - shu (4) tenglamaning ildizlaridan iborat. Bu tenglamani *hal qiluvchi* tenglama yoki (1) tenglamaning *rezolventasi* deyiladi.

Agar (5) tenglamaning bironta ildizini y_0 bilan belgilasak, bu qiymatda (4) tenglik bajarilib, (3) tengliklarga asosan, (2) tenglama quyidagi:

$$\left(x^2+\frac{ax}{2}+\frac{y_0}{2}\right)^2=(Ax+B)^2$$

yoki

$$x^2+\frac{ax}{2}+\frac{y_0}{2}=\pm(Ax+B)$$

ko'rinishni oladim va, demak, berilgan to'rtinchi darajali tenglama ikkita:

$$x^2+\frac{ax}{2}+\frac{y_0}{2}=Ax+B,$$

$$x^2+\frac{ax}{2}+\frac{y_0}{2}=-Ax-B$$

kvadrat tenglamaning ko'paytmasiga yoyiladi. Bu tenglamalarni yechib, berilgan to'rtinchi darajali tenglamaning to'rtta ildizini topamiz.

Misol. $x^4+3x^3-5x-3=0$ tenglamani yechaylik. Bunda $a=3,\ \ b=0,\ \ c=-5,\ \ d=-3$.

Avval (5) tenglamani tuzamiz; a,b,c,d ning qiymatlarini (5) ga qo'yib, ushbuni topamiz:

$$y^3-3y+2=0.$$

Bu tenglamani yechamiz:

$$u=\sqrt[3]{-1+\sqrt{1-1}}=\sqrt[3]{-1},$$

bundan $u_1=-1;\quad v_1=-\dfrac{-3}{3(-1)}=-1;$ Demak, $y_0-1-1=-2.$

(3) tengliklardan A va B ni aniqlaymiz: $A^2=\frac{9}{4}-2=\frac{1}{4};\quad A=\pm\frac{1}{2};$ masalan, $A=\frac{1}{2}$ ni olsak, $2AB=\frac{3(-2)}{2}+5=2$ dan $B=2$ ni hosil qilamiz. Shunday qilib:

$$x^2+\frac{3x}{2}-1=\frac{1}{2}x+2,$$

$$x^2+\frac{3x}{2}-1=-\frac{1}{2}x-2.$$

Bu tenglamalarni yechib, berilgan tenglamaning ildizlarini topamiz:

$$x_1=\frac{-1+\sqrt{13}}{2},\qquad x_1=\frac{-1-\sqrt{13}}{2},\quad x_3=x_4=-1.$$

2.Lobachevskiy usuli.

$$x=y-\frac{a}{4} \tag{6}$$

almashtirish yordami bilan (1) tenglamani:

$$y^4+py^2+qy+r=0 \qquad (7)$$

shaklga keltiramiz. Buni to'rtinchi darajali tenglamaning *normal* shakli deyiladi.

Bu normal tenglamaning istalgan ildizini y bilan belgilab, ushbu:

$$z^3-yz^2+az+\beta=0 \qquad (8)$$

yordamchi tenglamani tuzamiz; shuni aytish kerakki, α ning qiymati bizga kerak bo'lmaydi, β ning qiymati esa keyinroq aniqlanadi. Agar (8) tenglamada z ni $-z$ bilan almashtirsak,

$$z^3+yz^2+az-\beta=0 \qquad (9)$$

tenglama hosil bo'ladi. So'nggi (8) va (9) tenglamalarni o'zaro ko'paytirib,

$$u^3+lu^2+mu-n=0 \qquad (10)$$

tenglamani hosil qilamiz, bunda $u=z^2$ va

$$l=2\alpha-y^2,\ m=\alpha^2+2\beta y,\ n=\beta^2.$$

Bu tengliklarning birinchi va uchinchisidan α va β ni aniqlab, ikkinchisiga qo'ysak,

$$y^4+2ly^2+8\sqrt{n}y+(l^2-4m)=0$$

tenglama kelib chiqadi. Bu tenglamaning xuddi yuqoridagi (7) tenglamadan iborat bo'lishini talab qilib,

$$2l=p,\ 8\sqrt{n}=q,\ l^2-4m=r$$

deymiz, bundan:

$$l=\frac{p}{2},\ n=\frac{q^2}{64},\ m=\frac{1}{4}\left(\frac{p^2}{4}-r\right) \qquad (11)$$

hosil bo'ladi; $\beta^2=n$ va $n=\frac{q^2}{64}$ tengliklarga asosan, $\beta=\frac{q}{8}$ deb hisoblashimiz mumkin.

(11) tengliklardan foydalanib, (10) tenglamani:

$$u^3+\frac{p}{2}u^2+\frac{1}{4}\left(\frac{p^2}{4}-r\right)u-\frac{q^2}{64}=0 \qquad (12)$$

shaklga keltiramiz. Bu-*hal qiluvchi* tenglama yoki (7) tenglamaning *rezolventasidir*.

Agar u_1,u_2,u_3 bilan (12) tenglamaning ildizlarini belgilasak, $z^2=u$ ga asosan $z_1=\sqrt{u_1},\ z_2=\sqrt{u_2},\ z_3=\sqrt{u_3}$ sonlar (8) yordamchi tenglamaning ildizlariniifoda laydi. Shu sababli:

$$z_1+z_2+z_3=y,\quad z_1z_2z_3=-\beta=-\frac{q}{8}$$

shartlar bajariladi.

Ko'ramizki, $z_1,\ z_2,\ z_3$ qiymatlar $z_1z_2z_3=-\frac{q}{8}$ shartni qanoatlantiradigan bo'lsa,

$$z_1, \quad -z_2, \quad -z_3;$$
$$-z_1, \quad z_2, \quad -z_3;$$
$$-z_1, \quad -z_2, \quad z_3$$

qiymatlar ham bu shartni qanoatlantiradi. Demak, (7) tenglamaning ildizlari quyidagilardan iborat bo’ladi:

$$y_1 = z_1 + z_2 + z_3,$$
$$y_2 = z_1 - z_2 - z_3,$$
$$y_3 = -z_1 + z_2 - z_3,$$
$$y_4 = -z_1 - z_2 + z_3.$$

Bu qiymatlarni (6) ga qo’yib, (1) tenglamaning ildizlarini topamiz.

Misol. $x^4 + 4x^3 + \frac{5}{2}x^2 - 2x - \frac{3}{16} = 0$ tenglamani yechaylik. Avval $x = y - 1$ almashtirishni bajarib, normal tenglamaga o’tamiz:

$$y^4 - \frac{7}{2}y^2 + y + \frac{21}{16} = 0.$$

Bunda $p = -\frac{7}{2}$, $q = 1$, $r = \frac{21}{16}$ ekanini e’tiborga olib, (12) rezolventani tuzamiz:

$$u^3 - \frac{7}{4}u^2 + \frac{7}{16}u - \frac{1}{64} = 0.$$

Bu tenglamani ko’paytuvchilarga ajratib yechish engildir.

$$\left(u^3 - \frac{1}{64}\right) - \frac{7}{4}u\left(u - \frac{1}{4}\right) = \left(u - \frac{1}{4}\right)\left(u^2 + \frac{1}{4}u + \frac{1}{16}\right) - \frac{7}{4}u\left(u - \frac{1}{4}\right) =$$
$$= \left(u - \frac{1}{4}\right)\left(u^2 - \frac{3}{2}u + \frac{1}{16}\right) = 0.$$

Demak, $u_1 = \frac{1}{4}$, $\quad u_{2,3} = \frac{3 \pm 2\sqrt{2}}{4}$.

Bunda $q = 1 > 0$ bo’lgani uchun, $z_1 z_2 z_3 = -\frac{q}{8} = -\frac{1}{8}$ shartni qanoatlantirish maqsadida: $z_1 = -\sqrt{\frac{1}{4}} = -\frac{1}{2}$, $\quad z_{2,3} = +\sqrt{\frac{3 \pm 2\sqrt{2}}{4}} = \frac{\sqrt{2} \pm 1}{2}$ deb olamiz. Natijada:

$$x_1 = -\frac{1}{2} + \frac{\sqrt{2}+1}{2} + \frac{\sqrt{2}-1}{2} - 1 = \frac{-3 + 2\sqrt{2}}{2},$$
$$x_2 = -\frac{1}{2} - \frac{\sqrt{2}+1}{2} - \frac{\sqrt{2}-1}{2} - 1 = \frac{-3 - 2\sqrt{2}}{2},$$
$$x_3 = \frac{1}{2} + \frac{\sqrt{2}+1}{2} - \frac{\sqrt{2}-1}{2} - 1 = \frac{1}{2},$$
$$x_4 = \frac{1}{2} - \frac{\sqrt{2}+1}{2} + \frac{\sqrt{2}-1}{2} - 1 = -\frac{3}{2}.$$

V A R I A N T L A R

1-topshiriq. Kardano formulasidan foydalanib, quyidagi tenglamalarning yechimlarini toping:

1. $x^3+3x^2+15x+13=0$
2. $x^3-2x-4=0$
3. $x^3-3x+2=0$
4. $x^3-7x+6=0$
5. $x^3-6x+9=0$
6. $x^3+12x+63=0$
7. $x^3+3x^2-4=0$
8. $8x^3+24x^2-81=0$
9. $x^3+3x^2+3x-36=0$
10. $x^3+9x^2+18x+28=0$
11. $x^3+12x^2+45x+54=0$
12. $x^3-3x^2-3x+11=0$
13. $27x^3+108x^2+144x+118=0$
14. $27x^3-81x^2+297x+242=0$
15. $x^3+4x^2+x-6=0$
16. $x^3+8x^2+11x-20=0$
17. $x^3-2x^2+9x-18=0$
18. $x^3-x^2+4x-4=0$
19. $x^3+3x^2+3x-36=0$
20. $x^3-5x^2+x-5=0$
21. $x^3+2x^2+89x+18=0$
22. $x^3+3x^2-10x-24=0$
23. $x^3+5x^2-x-5=0$
24. $x^3-5x^2+2x+8=0$
25. $x^3-2x^2-3x+10=0$
26. $x^3+4x^2+9x+36=0$
27. $x^3-7x-6=0$
28. $x^3-x^2+4x-4=0$

2-topshiriq. Quyida berilgan to’rtinchi darajali tenglamalarni yeching:

Ferrari usuli bilan

1. $x^4+4x+3=0$
2. $x^4+2x^3+x+2=0$
3. $x^4-4x+3=0$
4. $x^4-2x^3-x+2=0$
5. $x^4-8x^3+18x^2-27=0$
6. $x^4+3x^3-5x-3=0$
7. $x^4-2x^3-8x^2+13x-24=0$
8. $x^4+2x^3-13x^2-38x-24=0$
9. $4x^4-7x^2-5x-1=0$
10. $x^4+4x^3-2x^2-12x+9=0$
11. $x^4-3x^3+2x^2+3x-9=0$
12. $x^4+x^3-10x^2-2x+4=0$
13. $x^4-2x^3-7x^2+8x+12=0$

Lobachevskiy usuli bilan

14. $x^4-4x^3-22x^2+100x-75=0$
15. $x^4+8x^3 32x^2+80x+100=0$
16. $x^4-8x^3+24x^2x-8=0$
17. $x^4+x^3-3x^2-4x-1=0$
18. $x^4-3x^3-6x^2+22x-12=0$
19. $x^4-4x^3+1=0$
20. $3x^4-4x^3-x^2-x-2=0$
21. $4x^4-2x^3-16x^2+5x+9=0$
22. $x^4+5x^3+2x^2-20x-24=0$
23. $x^4+2x^3-8x^2-18x-9=0$
24. $x^4+4x^3+x^2-16x-20=0$
25. $x^4+3x^3-11x^2-3x+10=0$
26. $x^4+x^3-6x^2-4x+8=0$
27. $x^4-2x^3+2x^2+4x-8=0$
28. $x^4-4x^3+5x^2-2x-6=0$

Ishni bajarish tartibi.

1. Talaba laboratoriya ishi bilan tanishadi.
2. Laboratoriya ishi bo’yicha hisobot tayyorlaydi.
3. Nazorat savollariga javob beradi.

Nazorat savollari.

1. Uchinchi darajali tenglama deb nimaga aytiladi?
2. Uchinchi darajali tenglamaning normal shakli qanday?
3. Kardano formulasi deb qanday formulaga aytiladi?
4. To'rtinchi darajali tenglamalarni yechishning qanday usullarini bilasiz?
5. To'rtinchi darajali tenglamaning normal shakli qanday?

Foydalanilgan adabiyotlar.

1. A.G.Kurosh. «Oliy algebra kursi». Toshkent, O'qituvchi, 1976.
2. R.Iskandarov. «Oliy algebra», I-qism. Toshkent, 1963.
3. D.K.Fadeev, I.S.Sominskiy. «Sbornik zadach po vыsshey algebre». Moskva, Nauka, 1977.
4. I.V.Proskuryakov. «Sbornik zadach po lineynoy algebre». Moskva, Nauka, 1984.

M U N D A R A J A