II QISM. QATORLAR NAZARIYASIDAN MASHQLAR

I BOB. SONLI QATORLAR

1-§. Sonli qatorlar

1-misol. Umumiy hadi $a_n = \dfrac{1}{(2n-1)(2n+1)}$ bo'lgan qator yig'indisini toping.

Yechish. n ga ketma-ket $1, 2, 3, \ldots,$ qiymatlar berib quyidagi

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots+\frac{1}{(2n-1)(2n+1)}+\ldots=\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}$$

qatorni hosil qilamiz. Qatorning birinchi n ta hadlarining yig'indisi

$$S_n=\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots+\frac{1}{(2n-1)(2n+1)}$$

ga teng. S_n xususiy yig'indini soddaroq ko'rinishga keltirish uchun qatorning umumiy $a_n = \dfrac{1}{(2n-1)(2n+1)}$ hadini aniqmas koeffitsientlar usuli bo'yicha sodda kasrlarga ajratamiz:

$$\frac{1}{(2n-1)(2n+1)}=\frac{A}{2n-1}+\frac{B}{2n+1}.$$

Umumiy mahraj berib quyidagi ifodani hosil qilamiz

$$1=2An+A+2Bn-B \;\Rightarrow\; 1=(2A+2B)n+A-B.$$

Bundan

$$\begin{cases} 2A+2B=0 \\ A-B=1 \end{cases}$$

sistemani yozamiz.

Bu sistemadan $A=\frac{1}{2}$, $B=-\frac{1}{2}$ kelib chiqadi.

$$\frac{1}{(2n-1)(2n+1)}=\frac{1}{2(2n-1)}-\frac{1}{2(2n+1)}=\frac{1}{2}\left(\frac{1}{(2n-1)}-\frac{1}{(2n+1)}\right).$$

Qatorning har bir hadini ikki qo' shiluvchi yig' indisi ko' rinishida ifodalasak, S_n xususiy yig' indi quyidagi ko' rinishga keladi:

$$S_n=\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+...+\frac{1}{(2n-1)(2n+1)}=\frac{1}{2}\left(1-\frac{1}{3}\right)+\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)+$$

$$+\frac{1}{2}\left(\frac{1}{5}-\frac{1}{7}\right)+...+\frac{1}{2}\left(\frac{1}{2n-3}-\frac{1}{2n-1}\right)+\frac{1}{2}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)=\frac{1}{2}\left(1-\frac{1}{2n+1}\right).$$

Qator yig' indisi S ni quyidagicha topamiz

$$S=\lim_{n\to\infty}S_n=\frac{1}{2}\lim_{n\to\infty}\left(1-\frac{1}{2n+1}\right)=\frac{1}{2}.$$

Demak, berilgan qator yaqinlashuvchi bo' lib, qatorning yig' indisi $\frac{1}{2}$ ga teng bo' ladi.

2-misol. $\frac{2}{3}+\frac{4}{5}+\frac{6}{7}+...$ qator yaqinlashishning zaruriy shart bajarilishini tekshiring.

Yechish. Berilgan qatorning umumiy hadi $\frac{2n}{2n+1}$ ko' rinishda bo' ladi. U holda

$$\frac{2}{3}+\frac{4}{5}+\frac{6}{7}+...+\frac{2n}{2n+1}+...,\quad a_n=\frac{2n}{2n+1}.$$

Ma’lumki, qator yaqinlashuvchi bo’lsa, uning umumiy hadi $n\to\infty$ da nolga intiladi. Buni e’tiborga olgan holda

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{2n}{2n+1} = \lim_{n\to\infty}\frac{2}{2+\frac{1}{n}} = 1$$

bo’ladi.

Qator yaqinlashishining zaruriy sharti bajarilmadi, shuning uchun berilgan qator uzoqlashuvchi.

Mustaqil yechish uchun mashqlar

Quyidagi qatorlarinng yig’indilarini toping.

1. $\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\ldots+\frac{1}{3\cdot 2^{n-1}}+\ldots$ Javob: $\frac{2}{3}$
2. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{n(n+1)}+\ldots$ Javob: 1
3. $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}+\ldots+$
$+\frac{1}{n(n+1)(n+2)}+\ldots$ Javob: $\frac{1}{4}$
4. $\frac{3}{1^2\cdot 2^2}+\frac{5}{2^2\cdot 3^2}+\ldots+\frac{2n+1}{n^2(n+1)^2}+\ldots$ Javob: 1
5. $\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\ldots$ Javob: $\frac{3}{4}$
6. $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots$ Javob: $\frac{1}{2}$

Quyidagi qatorlarning yaqinlashishini zaruriy sharti yordamida tekshiring.

1. $\frac{2}{1}+\frac{5}{8}+\frac{10}{27}+...+\frac{n^2+1}{n^2}+...$ Javob: uzoqlashuvchi

2. $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}+...+\frac{2n-1}{2n}+...$ Javob: uzoqlashuvchi

3. $\cos 1+\cos\frac{1}{2}+\cos\frac{1}{3}+...+\cos\frac{1}{n}+...$ Javob: uzoqlashuvchi

4. $\ln\frac{2}{1}+\ln\frac{3}{2}+\ln\frac{4}{3}+...$ Javob: uzoqlashuvchi

5. $\frac{2}{3}+\frac{3}{5}+\frac{4}{7}+...+\frac{n+1}{2n+1}+...$ Javob: uzoqlashuvchi

6. $(1+1)+\left(1+\frac{1}{2}\right)^2+\left(1+\frac{1}{3}\right)^3+...$ Javob: uzoqlashuvchi

2-§. Musbat hadli qatorlar

Quyidagi qatorlarning yaqinlashish yoki uzoqlashishini tekshiring.

1-misol. $\sum_{n=1}^{\infty}\frac{1}{n\cdot 3^{n-1}}$.

Yechish. Berilgan qatorning $a_n=\frac{1}{n\cdot 3^{n-1}}$ umumiy hadi yaqinlashuvchi bo’lgan quyidagi $\sum_{n=1}^{\infty}\frac{1}{3^{n-1}}$ geometrik $\left(q=\frac{1}{3}<1\right)$ qatorning umumiy hadidan kichik, ya’ni

$a_n = \frac{1}{n\cdot 3^{n-1}} \le \frac{1}{3^{n-1}} = b_n$ solishtirish alomatiga asosan, berilgan qator yaqinlashuvchi bo'ladi.

2-misol. $\sum_{n=1}^{\infty} \frac{1}{2n-1}$.

Yechish. Umumiy hadi $a_n = \frac{1}{2n-1}$ bo'lgan berilgan qatorni umumiy hadi $b_n = \frac{1}{n}$ bo'lgan uzoqlashuvchi $\sum_{n=1}^{\infty} \frac{1}{n}$ garmonik qator bilan solishtiramiz.

Ikkinchi solishtirish alomatiga asosan

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{2n-1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{2n-1} = \frac{1}{2} \neq 0$$

bo'lgani uchun berilgan qator ham uzoqlashuvchi bo'ladi.

3-misol. $\sum_{n=1}^{\infty} \frac{n}{3^n}$.

Yechish. Bu qatorning umumiy hadi $a_n = \frac{n}{3^n}$, keyingi hadi $a_{n+1} = \frac{n+1}{3^{n+1}}$. Dalamber alomatiga asosan

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}} = \lim_{n\to\infty} \frac{(n+1)\cdot 3^n}{n\cdot 3^{n+1}} = \frac{1}{3}\lim_{n\to\infty} \frac{(n+1)}{n} = \frac{1}{3} < 1$$

bo'lgani uchun berilgan qator yaqinlashuvchi bo'ladi.

4-misol. $\sum_{n=1}^{\infty} \frac{3^n \cdot n!}{n^n}$.

Yechish. Bu yerda $a_n = \dfrac{3^n \cdot n!}{n^n}$, $a_{n+1} = \dfrac{3^{n+1} \cdot (n+1)!}{(n+1)^{n+1}}$.

Dalamber alomatiga asosan

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{3^{n+1}(n+1)!}{(n+1)^{n+1}} : \frac{3^n \cdot n!}{n^n} = \lim_{n\to\infty} \frac{3^{n+1}(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{3^n \cdot n!} =$$

$$= \lim_{n\to\infty} \frac{3n^n}{(n+1)^n} = 3\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = 3\lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n} = \frac{3}{\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n} = \frac{3}{e} > 1$$

bo'lgani uchun berilgan qator uzoqlashuvchi.

5-misol. $\sum\limits_{n=1}^{\infty}\left(\dfrac{n}{n+1}\right)^{n^2}$.

Yechish. Koshining radikal alomatiga ko'ra

$$\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\left(\frac{n}{n+1}\right)^{n^2}} = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty}\frac{1}{\left(1+\frac{1}{n}\right)^n} = \frac{1}{e} < 1$$

bo'lgani uchun qator yaqinlashuvchi.

6-misol. $\sum\limits_{n=1}^{\infty}\dfrac{1}{n^\alpha}$ (Dirixle qatori) qatorni yaqinlashishga tekshiring.

Yechish. Qatorni yaqinlashishga tekshirish uchun Koshining integral alomatidan foydalanamiz.

$f(x) = \dfrac{1}{x^\alpha}$ deb olib, ushbu $\int\limits_1^{\infty}\dfrac{dx}{x^\alpha}$ xosmas integralni tekshiramiz.

$$\int_1^{\infty}\frac{dx}{x^{\alpha}}=\lim_{b\to\infty}\int_1^{b}x^{-\alpha}dx=\lim_{b\to\infty}\begin{cases}\frac{1}{1-\alpha}\left(b^{-\alpha}-1\right), & \alpha\neq 1\\ \ln b, & \alpha=1\end{cases}=$$

$$=\begin{cases}\frac{1}{\alpha-1}, & \text{agar } \alpha>1 \text{ bo'lsa,} & \text{yaqinlashuvchi}\\ \infty, & \text{agar } \alpha<1 \text{ bo'lsa,} & \text{uzoqlashuvchi}\\ \infty, & \text{agar } \alpha=1 \text{ bo'lsa,} & \text{uzoqlashuvchi}\end{cases}$$

Demak, xosmas integral $\alpha>1$ da yaqinlashuvchi, $\alpha\leq 1$ da uzoqlashuvchi . Koshining integral alomatiga ko’ra $\sum_{n=1}^{\infty}\frac{1}{n^{\alpha}}$ qator $\alpha>1$ da yaqinlashuvchi, $\alpha\leq 1$ da uzoqlashuvchi bo’ladi. $\alpha=1$ bo’lganda $\sum_{n=1}^{\infty}\frac{1}{n}$ garmonik qator hosil bo’ladi.

Mustaqil yechish uchun mashqlar

Solishtirish alomatlaridan foydalanib quyidagi qatorlarning yaqinlashishi yoki uzoqlashishini ko’rsating.

1. $\sum_{n=1}^{\infty}\frac{1}{n\cdot 2^{n}}$ Javob: yaqinlashuvchi
2. $\sum_{n=2}^{\infty}\frac{\ln n}{n}$ Javob: uzoqlashuvchi
3. $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2\left(1+2^{2}\right)}}+\frac{1}{\sqrt{3\left(1+3^{2}\right)}}+\ldots+$

$+\frac{1}{\sqrt{n\left(1+n^{2}\right)}}+\ldots$ Javob: yaqinlashuvchi

4. $\sin\frac{\pi}{3}+\sin\frac{\pi}{9}+...+\sin\frac{\pi}{3^n}+...$ Javob: yaqinlashuvchi

5. $\frac{1}{\sqrt{1\cdot 2}}+\frac{1}{\sqrt{2\cdot 3}}+\frac{1}{\sqrt{3\cdot 4}}+\ldots+\frac{1}{\sqrt{n(n+1)}}+\ldots$ Javob: uzoqlashuvchi

6. $1+\frac{1+2}{1+2^2}+\frac{1+3}{1+3^2}+\ldots+\frac{1+n}{1+n^2}+\ldots$ Javob: uzoqlashuvchi

7. $\sin 1+\sin\frac{1}{2}+...+\sin\frac{1}{n}+...$ Javob: yaqinlashuvchi

8. $\ln\frac{3}{2}+\ln\frac{2^2+2}{2^2+1}+\ln\frac{3^2+2}{3^2+1}+\ldots+\ln\frac{n^2+2}{n^2+1}+\ldots$ Javob: yaqinlashuvchi

9. $2\sin\frac{\pi}{3}+2^2\sin\frac{\pi}{3^2}+\ldots+2^n\sin\frac{\pi}{3^n}+\ldots$ Javob: uzoqlashuvchi

Dalamber va Koshi alomatlaridan foydalanib quyidagi qatorlarning yaqinlashish yoki uzoqlashishini tekshiring.

1. $\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+...+\frac{n}{2^n}+...$ Javob: yaqinlashuvchi

2. $\frac{5}{1!}+\frac{5^2}{2!}+\frac{5^3}{3!}+...+\frac{5^n}{n!}+...$ Javob: uzoqlashuvchi

3. $1+\frac{2^2}{2!}+\frac{3^3}{3!}+...+\frac{n^n}{n!}+...$ Javob: uzoqlashuvchi

4. $2+\left(\frac{2+1}{2\cdot 2-1}\right)^2+\left(\frac{3+1}{2\cdot 3-1}\right)^3+\ldots+\left(\frac{n+1}{2n-1}\right)^n+\ldots$ Javob: yaqinlashuvchi

5. $\frac{2}{3}+\frac{\left(\frac{3}{2}\right)^4}{9}+...+\frac{\left(\frac{n+1}{n}\right)^{n^2}}{3^n}+...$ Javob: yaqinlashuvchi

6. $2+\frac{2\cdot 5}{1\cdot 5}+\frac{2\cdot 5\cdot 8}{1\cdot 5\cdot 9}+\ ...\ +$

$+\frac{2\cdot 5\cdot 8\cdot\ ...\ \cdot(3n-1)}{1\cdot 5\cdot 9\cdot\ ...\ \cdot(4n-3)}+\ ...$ Javob: yaqinlashuvchi

7. $1+\frac{1\cdot 4}{3!!}+\frac{1\cdot 4\cdot 7}{5!!}+\ ...\ +$

$+\frac{1\cdot 4\cdot 7\cdot\ ...\ \cdot(3n-2)}{(2n-1)!!}+\ ...$ Javob: uzoqlashuvchi

8. $\sin\frac{\pi}{2}+\sin^2\frac{\pi}{4}+...+\sin^n\frac{\pi}{2n}+...$ Javob: yaqinlashuvchi

9. $\frac{3}{2^{-1}}+\frac{3^3}{2^2}+\frac{3^5}{2^5}+...+\frac{3^{2n+1}}{2^{3n-1}}+...$ Javob: uzoqlashuvchi

10. $\frac{1}{\sqrt{3}}+\frac{5}{\sqrt{2\cdot 3^2}}+\frac{9}{\sqrt{3\cdot 3^3}}+...+\frac{4n-3}{\sqrt{n\cdot 3^n}}+...$ Javob: yaqinlashuvchi

Koshining integral alomatidan foydalanib, quyidagi qatorlarning yaqinlashishini tekshiring.

1. $\frac{1}{2(\ln 2)^2}+\frac{1}{3(\ln 3)^2}+\ ...\ +\frac{1}{n(\ln n)^2}+\ ..$ Javob: yaqinlashuvchi

2. $1+\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{9}}+...+\frac{1}{\sqrt{4n+1}}+...$ Javob: uzoqlashuvchi

3. $\frac{2}{3+1^2}+\frac{2}{3+2^2}+\frac{2}{3+3^2}+...+\frac{2}{3+n^2}+...$ Javob: yaqinlashuvchi

4. $\left(\frac{1+1}{1+1^2}\right)^2+\left(\frac{1+2}{1+2^2}\right)^2+\ldots+\left(\frac{1+n}{1+n^2}\right)^2+\ldots$ Javob: yaqinlashuvchi

5. $\sum_{n=2}^{\infty}\frac{\ln n}{n^2}$ Javob: yaqinlashuvchi

6. $\frac{e^{-\sqrt{1}}}{\sqrt{1}}+\frac{e^{-\sqrt{2}}}{\sqrt{2}}+\frac{e^{-\sqrt{3}}}{\sqrt{3}}+\ldots+\frac{e^{-\sqrt{n}}}{\sqrt{n}}+\ldots$ Javob: yaqinlashuvchi

Quyidagi qatorlarni yaqinlashishga tekshiring.

1. $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}+\ldots$ Javob: uzoqlashuvchi

2. $1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}+\ldots$ Javob: yaqinlashuvchi

3. $\frac{1}{2^3}+\frac{2}{3^3}+\frac{3}{4^3}+\ldots+\frac{n}{(n+1)^3}+\ldots$ Javob: yaqinlashuvchi

4. $\sum_{n=2}^{\infty}\frac{1}{\sqrt{n(n-1)}}$ Javob: uzoqlashuvchi

5. $\sum_{n=1}^{\infty}\frac{n}{2n-1}$ Javob: uzoqlashuvchi

6. $\sum_{n=1}^{\infty}\frac{n!}{3^{n-1}}$ Javob: uzoqlashuvchi

7. $\sum_{n=1}^{\infty}\frac{2n-1}{\left(\sqrt{3}\right)^n}$ Javob: yaqinlashuvchi

8. $\sum_{n=1}^{\infty}\frac{n^2}{2n^2+1}$ Javob: uzoqlashuvchi

9. $\sum_{n=1}^{\infty}\frac{n^3}{e^n}$ Javob: yaqinlashuvchi

10. $\sum_{n=1}^{\infty}\left(\frac{1+n}{1+n^2}\right)^2$ Javob: yaqinlashuvchi

11. $\sum_{n=0}^{\infty}\frac{1}{\sqrt{4n+1}}$ Javob: uzoqlashuvchi

12. $\sum_{n=3}^{\infty}\frac{n}{n^4-9}$ Javob: yaqinlashuvchi

13. $\sum_{n=1}^{\infty}\frac{2}{2^n}\left(\frac{n+1}{n}\right)^{n^2}$ Javob: uzoqlashuvchi

3-§. Ishoralari o'zgaruvchi qatorlar

Quyidagi misollarda ishoralari o'zgaruvchi qatorlarning absolyut, shartli yaqinlashishi yoki uzoqlashishini tekshiring.

1-misol. $\sum_{n=1}^{\infty}\frac{\sin n\alpha}{3^n}$ $(\alpha$ – ихтиёрий сон$)$.

Yechish. Berilgan qator hadlarining absolyut qiymatlaridan yangi qator tuzamiz

$$\sum_{n=1}^{\infty}\frac{|\sin n\alpha|}{3^n}$$

va bu qatorni $\sum_{n=1}^{\infty}\frac{1}{3^n}$ qator bilan taqqoslaymiz. Ma'lumki $\frac{|\sin n\alpha|}{3^n}\le\frac{1}{3^n}$. Geometrik qator $\left(q=\frac{1}{3}<1\right)$ $\sum_{n=1}^{\infty}\frac{1}{3^n}$ yaqinlashuvchi bo'lgani uchun, musbat hadli qatorlarni solishtirish alomatiga ko'ra qator yaqinlashuvchi bo'ladi. Demak, berilgan qator absolyut yaqinlashuvchidir.

2-misol. $\sum_{n=1}^{\infty} \cos \frac{n\pi}{4}$.

Yechish. Berilgan ishoralari o'zgaruvchi qator uchun qator yaqinlashishining zaruriy sharti bajarilmaydi. Haqiqatan,

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \cos \frac{n\pi}{4}$$

limit mavjud emas. Demak, berilgan qator uzoqlashuvchi.

3-misol. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n \cdot 2^n}$.

Yechish. Ishoralari almashinuvchi berilgan qatorning hadlari absolyut qiymat bo'yicha monoton kamayadi, ya'ni

$$\frac{1}{2} > \frac{1}{2 \cdot 2^2} > \frac{1}{3 \cdot 2^3} > \ldots$$

va

$$\lim_{n\to\infty} \frac{1}{n \cdot 2^n} = 0.$$

Leybnis teoremasiga asosan berilgan qator yaqinlashuvchi. Qatorning absolyut yoki shartli yaqinlashishini tekshirish uchun berilgan qator hadlarining absolyut qiymatlaridan $\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^n}$ qator tuzamiz. Bu qatorning yaqinlashishini Dalamber alomatiga ko'ra tekshiramiz.

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\dfrac{1}{(n+1)\cdot 2^{n+1}}}{\dfrac{1}{n\cdot 2^n}}=\lim_{n\to\infty}\frac{n\cdot 2^n}{(n+1)\cdot 2^{n+1}}=\lim_{n\to\infty}\frac{n}{2n+2}=\frac{1}{2}<1\cdot$$

Demak, musbat hadli qator yaqinlashuvchi. Shuning uchun, berilgan qator absolyut yaqinlashuvchi bo’ladi.

4-misol. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}$.

Yechish. Berilgan qator Leybnis teoremasi shartlarini qanoatlantiradi, ya’ni

$$1>\frac{1}{3}>\frac{1}{5}>\ldots$$

va

$$\lim_{n\to\infty}\frac{1}{2n-1}=0\cdot$$

Demak, berilgan qator yaqinlashuvchi. Berilgan qatorning absolyut yoki shartli yaqinlashishini tekshiramiz. Buning uchun qator hadlarining absolyut qiymatlaridan tuzilgan $\sum_{n=1}^{\infty}\frac{1}{2n-1}$ qatorni tekshiramiz. Koshining integral alomatidan foydalanamiz.

$$\int_1^{\infty}\frac{dx}{2x-1}=\frac{1}{2}\lim_{n\to\infty}\int_1^{n}\frac{d(2x-1)}{2x-1}=\frac{1}{2}\lim_{n\to\infty}\ln(2x-1)\Big|_1^n=\frac{1}{2}\lim_{n\to\infty}\ln(2n-1)=+\infty$$

bo’lgani uchun xosmas integral uzoqlashuvchi. Demak, musbat hadli $\sum_{n=1}^{\infty}\frac{1}{2n-1}$ qator uzoqlashuvchi. U holda berilgan qator shartli yaqinlashuvchi.

Mustaqil yechish uchun mashqlar

1. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ Javob: shartli yaqinlashuvchi

2. $\frac{1}{\ln 2}-\frac{1}{\ln 3}+\frac{1}{\ln 4}-\ldots$ Javob: shartli yaqinlashuvchi

3. $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\ldots+(-1)^{n-1}\frac{1}{\sqrt{n}}+\ldots$ Javob: shartli yaqinlashuvchi

4. $\frac{\sin\alpha}{1}+\frac{\sin 2\alpha}{2^2}+\frac{\sin 3\alpha}{3^2}+\ldots$ Javob: absolyut yaqinlashuvchi

5. $-\frac{3}{4}+\left(\frac{5}{7}\right)^2-\left(\frac{7}{10}\right)^3+\ldots+$

$+(-1)^n\left(\frac{2n+1}{3n+1}\right)^n+\ldots$ Javob: absolyut yaqinlashuvchi

6. $\sum_{n=1}^{\infty}\frac{\cos n\alpha}{(\ln 10)^n}$ Javob: absolyut yaqinlashuvchi

7. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2n+1}{n(n+1)}$ Javob: absolyut yaqinlashuvchi

8. $\sum_{n=1}^{\infty}(-1)^{n-1}\cos\frac{\pi}{3n}$ Javob: uzoqlashuvchi

9. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(n+1)a^{2n}}$ Javob: $\begin{cases}|a|>1\text{da abs. yaqinlash.}\\ |a|<1\text{da uzoqlashuvchi}\end{cases}$

10. $1-\frac{1}{3}+\frac{1}{2}-\frac{1}{3^3}+\frac{1}{2^2}-\frac{1}{3^5}+\ldots$ Javob: absolyut yaqinlashuvchi

11. $\frac{1}{2}\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\right)=1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+\ldots$ tenglikni isbotlang.

II BOB. FUNKSIONAL VA DARAJALI QATORLAR

1-§. Funksional qatorlar

1-misol. Quyidagi funksional qatorning yaqinlashish sohasini toping:

$$\sum_{n=1}^{\infty} \frac{1}{n(x+3)^n}.$$

Yechish. Dalamber alomatidan foydalanamiz.

$$u_n(x) = \frac{1}{n(x+3)^n}\ ;\ u_{n+1}(x) = \frac{1}{(n+1)(x+3)^{n+1}},$$

$$l = \lim_{n\to\infty}\left|\frac{u_{n+1}(x)}{u_n(x)}\right| = \lim_{n\to\infty}\left|\frac{n(x+3)^n}{(n+1)(x+3)^{n+1}}\right| = \lim_{n\to\infty}\left|\frac{n}{(n+1)(x+3)}\right| =$$

$$= \frac{1}{|x+3|}\lim_{n\to\infty}\frac{n}{n+1} = \frac{1}{|x+3|}.$$

Dalamber alomatiga ko'ra qator yaqinlashishi uchun $l<1$ bo'lishi kerak. Bu holda

$$\frac{1}{|x+3|} < 1 \ \Rightarrow\ |x+3| > 1 \Rightarrow\ x+3 > 1$$

va

$$x+3 < -1 \Rightarrow x > -2 \ \text{ва}\ \ x < -4.$$

Demak, qatorning yaqinlashish sohasi $(-\infty;\,-4)\cup(-2;\,+\infty)$ dan iborat bo'ladi. Berilgan qatorni topilgan interval chegaralarida tekshiramiz.

$x=-4$ bo'lganda $\sum_{n=1}^{\infty}\frac{1}{n(-1)^n}$ ishoralari almashinuvchi qator hosil bo'lib, bu qator Leybnis alomatiga ko'ra yaqinlashuvchi bo'ladi. $x=-2$ bo'lganda $\sum_{n=1}^{\infty}\frac{1}{n}$ uzoqlashuvchi bo'lgan garmonik qator hosil bo'ladi.

Shunday qilib, berilgan qatorning yaqinlashish sohasi $(-\infty;\,-4]\cup(-2;\,+\infty)$ dan iborat.

2-misol. Quyidagi

$$\sum_{n=1}^{\infty}\left(8-x^2\right)^n$$

funksional qatorning yaqinlashish sohasini toping.

Yechish. Berilgan funksional qatorni mahraji $q=8-x^2$ bo'lgan geometrik qator deb qarash mumkin. $|q|=\left|8-x^2\right|<1$ bo'lganda ma'lumki qator yaqinlashuvchi bo'ladi. $\left|8-x^2\right|<1$ tengsizlikni echamiz. $-1<8-x^2<1$ yoki $7<x^2<9$, bundan $\sqrt{7}<|x|<3$. Qatorning yaqinlashish sohasi $\left(-3;\,-\sqrt{7}\right)\cup\left(\sqrt{7};\,3\right)$ dan iborat. Endi qatorning yaqinlashishini oraliq chegaralarida tekshirib ko'ramiz: $x=-3$, $x=-\sqrt{7}$, $x=\sqrt{7}$ va $x=3$ bo'lganda $\sum_{n=1}^{\infty}(-1)^n$ va $\sum_{n=1}^{\infty}1^n$ uzoqlashuvchi qatorlar hosil bo'ladi.

Demak, berilgan qatorning yaqinlashish sohasi $\left(-3;\,-\sqrt{7}\right)\cup\left(\sqrt{7};\,3\right)$ dan iborat.

3-misol. Funksional qatorning yaqinlashish sohasini toping:

$$\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{|x|}{n}\right)^{n}, \ x\neq 0.$$

Yechish. Funksional qatorning hadlari sonlar o‘qining $x=0$ dan boshqa barcha nuqtalarda aniqlangan. Agar $x<0$ bo‘lsa $\frac{|x|}{x}=\frac{-x}{x}=-1$ bo‘ladi. U holda ishoralari almashinuvchi $\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{|x|}{n}\right)^{n}=\sum_{n=1}^{\infty}\frac{(-1)^{n}}{n}$ qator Leybnis alomatiga ko‘ra yaqinlashuvchi bo‘ladi. Agar $x>0$ bo‘lsa $\frac{|x|}{x}=\frac{x}{x}=1$ bo‘ladi. U holda $\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{|x|}{n}\right)^{n}=\sum_{n=1}^{\infty}\frac{1}{n}$ uzoqlashuvchi garmonik qator hosil bo‘ladi. Demak, berilgan qatorning yaqinlashish sohasi $(-\infty;\,0)$ dan iborat bo‘ladi.

4-misol. Quyidagi funksional qatorning yaqinlashish sohasini toping:

$$\sum_{n=1}^{\infty}2^{n-1}\cdot x^{2n-2}.$$

Yechish. Funksional qatorda x ning barcha natural (x ning toq darajalari qatnashmaydi) darajalari qatnashmaganligi uchun qatorning yaqinlashish sohasini topishda Dalamber alomatidan foydalanamiz.

$$u_n(x)=2^{n-1}\cdot x^{2n-2}\ ;\ u_{n+1}(x)=2^{n}\cdot x^{2n},$$

$$l=\lim_{n\to\infty}\frac{u_{n+1}(x)}{u_n(x)}=\lim_{n\to\infty}\frac{2^n\cdot x^{2n}}{2^{n-1}\cdot x^{2n-2}}=\lim_{n\to\infty}2x^2=2x^2<1\cdot$$

Bundan $x^2<\frac{1}{2}$ yoki $|x|<\frac{1}{\sqrt{2}}$ bo'ladi. Qatorning yaqinlashish intervali $\left(-\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}}\right)$ bo'ladi.

Endi qator yaqinlashishini intervalning chegaralarida tekshiramiz. $x=\pm\frac{1}{\sqrt{2}}$ bo'lganda

$$1+1+1+\ldots+1+\ldots$$

uzoqlashuvchi qator hosil bo'ladi. Shunday qilib, berilgan darajali qator $\left(-\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}}\right)$ intervalda yaqinlashuvchi bo'ladi.

5-misol. Quyidagi funksional qatorning yaqinlashish sohasini toping:

$$\sum_{n=1}^{\infty}\frac{1}{n^4+x^2}.$$

Yechish. x ning barcha qiymatlari uchun

$$\frac{1}{n^4+x^2}\le\frac{1}{n^4}$$

tengsizlik o'rinli. Ma'lumki, $\sum_{n=1}^{\infty}\frac{1}{n^4}$ qator ($\alpha=4>1$ bo'lganligi uchun, umumlashgan garmonik qatorni eslang) yaqinlashuvchi. $\frac{1}{n^4+x^2}\le\frac{1}{n^4}$ tengsizlik o'rinli bo'lgani uchun Veyershtrass alomatiga ko'ra, berilgan qator x ning barcha qiymatlarida tekis yaqinlashuvchi bo'ladi.

6-misol. $\sum_{n=1}^{\infty}\frac{\sin 3^n \pi x}{3^n}$ funksional qatorning tekis yaqinlashishini ko'rsating.

Yechish. $|\sin kx| \le 1$ bo'lgani uchun

$$\left|\frac{\sin 3^n \pi x}{3^n}\right| \le \frac{1}{3^n} \qquad (*)$$

tengsizlik o'rinli. n - hadi $\frac{1}{3^n}$ bo'lgan $\sum_{n=1}^{\infty}\frac{1}{3^n}$ qator geometrik qator bo'lib, $\left(q=\frac{1}{3}<1\right)$ bo'lganligi uchun yaqinlashuvchi. Yuqoridagi (*) tengsizlik o'rinli bo'lgani uchun, Veyershtrass alomatiga ko'ra, berilgan qator x ning barcha qiymatlarida tekis yaqinlashuvchi bo'ladi.

7-misol. $\sum_{n=1}^{\infty}\frac{1}{2^n\sqrt{1+(2n+1)x}}$ $(0 \le x < \infty)$ funksional qatorning tekis yaqinlashishini ko'rsating. Qanday n lar uchun $|\varphi_n(x)| < 0{,}01$ tengsizlik bajariladi?

Yechish. Qaralayotgan oraliqda quyidagi

$$\frac{1}{2^n\sqrt{1+(2n+1)x}} \le \frac{1}{2^n} \qquad (**)$$

tengsizlik o'rinli. Mahraji $q=\frac{1}{2}<1$ bo'lgan $\sum_{n=1}^{\infty}\frac{1}{2^n}$ geometrik qator yaqinlashuvchi bo'lganligi uchun (**) Veyershrass teoremasiga ko'ra, berilgan qator qaralayotgan oraliqda tekis yaqinlashuvchi bo'ladi. Funksional qatorning qoldig'i $r_n(x)=S(x)-S_n(x)$ ni baholash uchun $\sum_{n=1}^{\infty}\frac{1}{2^n}$ sonli qatorning qoldig' i $r_n = S - S_n$ ni baholaymiz. Ma' lumki, $S_n = \frac{\frac{1}{2}-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}$,

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\frac{\frac{1}{2}-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} = 1$$

va

$$r_n = S - S_n = \frac{1}{2^n}.$$

Berilgan funksional qator $\varphi_n(x)$ qoldig' i $\sum_{n=1}^{\infty}\frac{1}{2^n}$ sonli qatorning r_n qoldig' idan katta bo' la olmaydi. Shuning uchun $\varphi_n(x) \le \frac{1}{2^n}$ bo' ladi. $\varphi_n(x) < 0{,}01$ tengsizlik o' rinli bo' lishi uchun $\frac{1}{2^n} < 0{,}01$ tengsizlikni echamiz. U holda $2^n > 100$ bo' lib, $n \ge 7$ bo' ladi.

Mustaqil yechish uchun mashqlar

Quyidagi funksional qatorlarning yaqinlashish sohasini toping.

1. $\sum_{n=1}^{\infty} e^{-nx}$ Javob: $(0;\ \infty)$

2. $\sum\limits_{n=1}^{\infty}\frac{1}{x^n}$ Javob: $(-\infty\,;\,-1)\cup(1\,;\,\infty)$

3. $\sum\limits_{n=1}^{\infty}\frac{n!}{x^n}$ Javob: butun sonlar o′ qida uzoqlashuvchi

4. $\sum\limits_{n=1}^{\infty}\left(2-x^2\right)^n$ Javob: $\left(-\sqrt{3}\,;\,-1\right)\cup\left(1\,;\,\sqrt{3}\right)$

5. $\sum\limits_{n=1}^{\infty}2^n\sin\frac{x}{2^n}$ Javob: $(-\infty\,;+\infty)$

6. $\sum\limits_{n=1}^{\infty}\ln^n x$ Javob: $\left(\frac{1}{e}\,;\,e\right)$

7. $\sum\limits_{n=1}^{\infty}n\sqrt[3]{\sin^n x}$ Javob: $x_k\neq\frac{\pi}{2}+k\pi\ \left(k=0,\pm1,\pm2,\ldots\right)$

8. $\sum\limits_{n=1}^{\infty}\frac{\cos nx}{n^2}$ Javob: $(-\infty\,;+\infty)$

9. $\sum\limits_{n=1}^{\infty}\frac{n(x+3)}{3^{n+1}}$ Javob: $(-6\,;\,0)$

10. $\sum\limits_{n=1}^{\infty}(3x)^{n^2}$ Javob: $\left(-\frac{1}{3}\,;\,\frac{1}{3}\right)$

11. $\sum\limits_{n=1}^{\infty}\frac{\cos nx}{n^n}$ Javob: $(-\infty\,;+\infty)$

12. $\sum\limits_{n=1}^{\infty}\frac{\cos 2^n x}{n^2}$ Javob: $(-\infty\,;+\infty)$

13. $\sum\limits_{n=1}^{\infty}\frac{1}{n^2+\cos^2 x}$ Javob: $(-\infty\,;+\infty)$

14. $\sum\limits_{n=1}^{\infty}\frac{(-1)^{n+1}}{x^{2n}+n}$ Javob: $(-\infty\,;\,-1]\cup[1\,;\,\infty)$

2-§. Darajali qatorlar

Quyidagi qatorlarning yaqinlashish radiuslari va yaqinlashish intervallarini toping. Yaqinlashish intervalining chegaralarida qatorning yaqinlashishini tekshiring.

1-misol. $\sum_{n=1}^{\infty} \frac{2^n x^n}{\sqrt{3^n}}$.

Yechish. Bu yerda $a_n = \frac{2^n x^n}{\sqrt{3^n}}$. Qatorning yaqinlashish radiusini quyidagi Koshi alomati yordamida topamiz.

$$R = \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{a_n}} = \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{\frac{2^n}{\sqrt{3^n}}}} = \frac{1}{\frac{2}{\sqrt{3}}} = \frac{\sqrt{3}}{2}, \quad R = \frac{\sqrt{3}}{2}.$$

Demak, yaqinlashish intervali $\left(-\frac{\sqrt{3}}{2}; \frac{\sqrt{3}}{2}\right)$ bo' ladi. Intervalning chegaralarida qatorning yaqinlashishini alohida tekshiramiz.

$x = -\frac{\sqrt{3}}{2}$ bo' lsa $\sum_{n=1}^{\infty} (-1)^n$ uzoqlashuvchi qator,

$x = \frac{\sqrt{3}}{2}$ bo' lsa $\sum_{n=1}^{\infty} 1^n = 1 + 1 + 1 + \ldots + 1 + \ldots$ uzoqlashuvchi qator hosil bo' ladi. Demak berilgan qatorning yaqinlashish intervali $\left(-\frac{\sqrt{3}}{2}; \frac{\sqrt{3}}{2}\right)$ dan iborat.

2-misol. $\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 2^n}$.

Yechish. n - hadining koeffitsienti $a_n = \dfrac{1}{n \cdot 2^n}$.

Qatorning yaqinlashish radiusini topish formulaga asosan topamiz:

$$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\frac{(n+1)\cdot 2^{n+1}}{n \cdot 2^n} = \lim_{n\to\infty}\frac{2n+2}{n} = 2,$$

$R = 2$ va yaqinlashish intervali $(-2\ ;\ 2)$ bo' ladi. Interval chegaralarida, ya' ni $x = -2$ va $x = 2$ nuqtalarda tekshiramiz.

$x = -2$ bo' lganda $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n}{n}$ qator hosil bo' ladi va u Leybnis teoremasi shartlarini qanoatlantiradi. Demak, qator $x = -2$ nuqtada yaqinlashadi.

$x = 2$ bo' lganda $\sum\limits_{n=1}^{\infty}\dfrac{1}{n}$ uzoqlashuvchi garmonik qator hosil bo' ladi. Shunday qilib, berilgan darajali qator $[-2\ ;\ 2)$ yarim oraliqda yaqinlashuvchi.

3-misol. $\sum\limits_{n=1}^{\infty}\dfrac{4^n}{n^3\left(x^2 - 4x + 7\right)^n}$.

Yechish. Qatorning n-hadi va $(n+1)$- hadini yozamiz:

$$u_n(x) = \frac{4^n}{n^3\left(x^2 - 4x + 7\right)^n}\ ;\ u_{n+1}(x) = \frac{4^{n+1}}{(n+1)^3\left(x^2 - 4x + 7\right)^{n+1}}.$$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{4^{n+1}}{(n+1)^3\left(x^2 - 4x + 7\right)^{n+1}} \cdot \frac{n^3\left(x^2 - 4x + 7\right)^n}{4^n}\right| =$$

$$= \lim_{n\to\infty}\left|\frac{4n^3}{(n+1)^3\left(x^2 - 4x + 7\right)}\right| = \frac{4}{x^2 - 4x + 7} < 1$$

Bu tengsizlikni yechib yaqinlashish oralig' ini topamiz.

$$x \in (-\infty,\, 1) \cup (3,\, \infty)$$

Endi qatorni chegaradagi nuqtalarda yaqinlashishini tekshiramiz. $x = 1$ da

$$\sum_{n=1}^{\infty} \frac{1}{n^3}$$

qator hosil bo' ladi. Bu garmonik qatorning umumlashgan $p = 3$ bo' lgani uchun yaqinlashuvchidir $x = 3$ da

$$\sum_{n=1}^{\infty} \frac{1}{n^3}$$

hosil bo' ladi. Bu qator ham yaqinlashuvchi. Demak yaqinlashish oralig' i $(-\infty,\, 1] \cup [3,\, \infty)$.

Mustaqil yechish uchun mashqlar

Quyidagi qatorlarning yaqinlashish radiuslari va yaqinlashish intervallarini toping. Yaqinlashish intervalining chegaralarida qatorning yaqinlashishini tekshiring.

1. $\sum\limits_{n=1}^{\infty} x^n$ Javob: $R = 1,\; (-1;\, 1)$
2. $\sum\limits_{n=1}^{\infty} \frac{(x-4)^n}{n}$ Javob: $R = 1,\; [3;\, 5]$
3. $\sum\limits_{n=1}^{\infty} n! \cdot x^n$ Javob: $R = 0,\; x = 0$
4. $\sum\limits_{n=1}^{\infty} (-2)^n x^{2n}$ Javob: $R = \frac{1}{\sqrt{2}},\; \left(-\frac{1}{\sqrt{2}};\, \frac{1}{\sqrt{2}}\right)$
5. $\sum\limits_{n=1}^{\infty} \frac{x^n}{n(n+1)}$ Javob: $R = 1,\; [-1;\, 1]$

6. $\sum_{n=1}^{\infty}\frac{x^n}{\sqrt{n}}$ Javob: $R=1,\ [-1;\,1]$

7. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-2)^{2n}}{n\cdot 4^n}$ Javob: $R=2,\ [0;\,4]$

8. $\sum_{n=1}^{\infty}\frac{n}{n+1}\left(\frac{x}{2}\right)^n$ Javob: $R=2,\ (-2;\,2)$

9. $\sum_{n=1}^{\infty}\frac{n^5}{(n+1)!}(x+5)^{2n+1}$ Javob: $R=\infty, (-\infty;\,\infty)$

10. $\sum_{n=1}^{\infty}\frac{10^n x^n}{\sqrt{n}}$ Javob: $R=\frac{1}{10},\ (-0{,}1;0{,}1)$

11. $\sum_{n=1}^{\infty}\frac{(2x+1)^n}{3n-2}$ Javob: $R=1, [-1;\,0)$

12. $\sum_{n=1}^{\infty}\frac{(-x)^n}{3^{n-1}\sqrt{n}}$ Javob: $R=3,\ (-3;\,3]$

13. $\sum_{n=1}^{\infty}\frac{2^n\cdot n!}{(2n)!}x^{2n}$ Javob: $R=\infty,\ (-\infty;\,\infty)$

14. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}x^n$ Javob: $R=\frac{1}{e},\ \left(-\frac{1}{e};\,\frac{1}{e}\right)$

3-§. Teylor va Makloren qatorlari

1-misol. $f(x)=\sin\frac{\pi x}{4}$ funksiyani $x=2$ nuqta atrofida Teylor qatoriga yoying.

Yechish. Berilgan funksiyaning hosilalarini va $x=2$ nuqtadagi qiymatlarini topamiz.

$$f(x)=\sin\frac{\pi x}{4},\quad f(2)=\sin\frac{2\pi}{4}=1,$$

$$f'(x)=\frac{\pi}{4}\cos\frac{\pi x}{4}=\frac{\pi}{4}\sin\left(\frac{\pi x}{4}+\frac{\pi}{2}\right),\quad f'(2)=\sin\left(\frac{2\pi}{4}+\frac{\pi}{2}\right)=0,$$

$$f''(x)=-\frac{\pi^2}{4^2}\sin\frac{\pi x}{4}=\frac{\pi^2}{4^2}\sin\left(\frac{\pi x}{4}+2\cdot\frac{\pi}{2}\right),$$

$$f''(2)=\frac{\pi^2}{4^2}\sin\left(\frac{2\pi}{4}+\pi\right)=-\frac{\pi^2}{4^2},$$

$$f'''(x)=-\frac{\pi^3}{4^3}\cos\frac{\pi x}{4}=\frac{\pi^3}{4^3}\sin\left(\frac{\pi x}{4}+3\cdot\frac{\pi}{2}\right),$$

$$f'''(2)=\frac{\pi^3}{4^3}\sin\left(\frac{2\pi}{4}+\frac{3\pi}{2}\right)=0,$$

$$f^{IV}(x)=\frac{\pi^4}{4^4}\sin\frac{\pi x}{4}=\frac{\pi^4}{4^4}\sin\left(\frac{\pi x}{4}+4\cdot\frac{\pi}{2}\right),$$

$$f^{IV}(2)=\frac{\pi^4}{4^4}\sin\left(\frac{2\pi}{4}+2\pi\right)=\frac{\pi^4}{4^4},$$

. . . .

$$f^{(2k)}(x)=\frac{\pi^{2k}}{4^{2k}}\sin\left(\frac{\pi x}{4}+2k\cdot\frac{\pi}{2}\right),$$

$$f^{(2k)}(2)=\frac{\pi^{2k}}{4^{2k}}\sin\left(\frac{2\pi}{4}+k\pi\right)=(-1)^k\,\frac{\pi^{(2k)}}{4^{(2k)}},$$

$$f^{(2k+1)}(x)=\frac{\pi^{2k+1}}{4^{2k+1}}\sin\left(\frac{\pi x}{4}+(2k+1)\cdot\frac{\pi}{2}\right),$$

$$f^{(2k+1)}(2)=\frac{\pi^{2k+1}}{4^{2k+1}}\sin\left(\frac{2\pi}{4}+\frac{(2k+1)\pi}{2}\right)=0.$$

Topilganlarni Teylor qatoriga qo’ysak, quyidagiga ega bo’lamiz

$$\sin\frac{\pi x}{4}=1-\frac{\pi^2}{4^2}\frac{(x-2)^2}{2!}+\frac{\pi^4}{4^4}\frac{(x-2)^4}{4!}-\ldots+$$

$$+\ldots+(-1)^k\,\frac{\pi^{2k}}{4^{2k}}\frac{(x-2)^{2k}}{(2k)!}+\ldots.$$

Bu darajali qatorning yaqinlashish intervalini Dalamber alomati yordamida topamiz.

$$\lim_{k\to\infty}\left|\frac{u_{2k+2}(x)}{u_{2k}(x)}\right|=\lim_{k\to\infty}\left|\frac{\pi^{2k+2}}{4^{2k+2}}\frac{(x-2)^{2k+2}}{(2k+2)!}\cdot\frac{4^{2k}}{\pi^{2k}}\frac{(2k)!}{(x-2)^{2k}}\right|=$$

$$=\frac{\pi^2}{4^2}(x-2)^2\lim_{k\to\infty}\frac{1}{(2k+1)(2k+2)}=0<1$$

bo’lgani uchun x ning barcha qiymatlarida yuqorida berilgan qator yaqinlashuvchi, ya’ni qatorning yaqinlashish intervali $(-\infty\,;\,\infty)$ dan iborat. Endi Teylor formulasidan

$$R_{k+1}=\frac{\pi^{2k+1}}{4^{2k+1}}\frac{(x-2)^{2k+1}}{(2k+1)!}\,\sin\left(\frac{\pi\xi}{4}+(2k+1)\frac{\pi}{2}\right)$$

qoldiq hadni tekshiramiz. Har qanday k va ξ uchun $\left|\sin\left(\frac{\pi\xi}{4}+(2k+1)\frac{\pi}{2}\right)\right|\le 1\quad \lim\limits_{k\to\infty}\left(\frac{\pi}{4}\right)^{2k+1}=0$ o’rinli. Har qanday chekli x uchun

$$\lim_{k\to\infty}\frac{(x-2)^{2k+1}}{(2k+1)!}=0 \cdot$$

U holda $\lim_{k\to\infty}R_{2k+1}(x)=0$. Demak, berilgan $f(x)=\sin\frac{\pi x}{4}$ funksiya ixtiyoriy tartibdagi hosilaga ega va $\lim_{k\to\infty}R_{2k+1}(x)=0$ bo' lgani uchun berilgan funksiya $(x-2)$ ning darajalari bo' yicha Teylor qatoriga yoyiladi.

$$\sin\frac{\pi x}{4}=1-\frac{\pi^2}{4^2}\frac{(x-2)^2}{2!}+\frac{\pi^4}{4^4}\frac{(x-2)^4}{4!}-\ldots+(-1)^k\frac{\pi^{2k}}{4^{2k}}\frac{(x-2)^{2k}}{(2k)!}+\ldots\cdot$$

2-misol. $f(x)=\frac{1}{x}$ funksiyani $x=-3$ nuqta atrofida Teylor qatoriga yoying.

Yechish. Berilgan $f(x)=\frac{1}{x}$ funksiya hosilalarini va ularning $x=-3$ nuqtadagi qiymatlarini topamiz.

$$f(x)=\frac{1}{x}=x^{-1}, \qquad f(-3)=-\frac{1}{3},$$

$$f'(x)=-1\cdot x^{-2}=-\frac{1}{x^2}, \qquad f'(-3)=-\frac{1}{3^2},$$

$$f''(x)=1\cdot 2\cdot x^{-3}=\frac{2!}{x^3}, \qquad f''(-3)=-\frac{2!}{3^3},$$

$$f'''(x)=-1\cdot 2\cdot 3\cdot x^{-4}=-\frac{3!}{x^4}, \qquad f'''(-3)=-\frac{3!}{3^4},$$

. . . .

$$f^{(n)}(x)=(-1)^n\frac{n!}{x^{n+1}}, \qquad f^{(n)}(-3)=-\frac{n!}{3^{n+1}},$$

. . . .

Topilganlarni Teylor qatoriga qo’ysak, $f(x)=\frac{1}{x}$ funksiya uchun Teylor qatori

$$-\frac{1}{3}-\frac{1!}{3^2}\frac{x+3}{1!}-\frac{2!}{3^4}\frac{(x+3)^2}{2!}-\ldots-\frac{n!}{3^{n+1}}\frac{(x+3)^n}{n!}-\ldots=$$

$$=-\frac{1}{3}\left(1+\frac{x+3}{3}+\frac{(x+3)^2}{3^2}+\ldots+\frac{(x+3)^n}{3^n}+\ldots\right).$$

ko’rinishda bo’ladi. Bu qatorning yaqinlashish intervalini Dalamber alomati bo’yicha topamiz.

$$u_n(x)=\frac{(x+3)^n}{3^n}\ ;\ u_{n+1}(x)=\frac{(x+3)^{n+1}}{3^{n+1}}.$$

$$l=\lim_{n\to\infty}\left|\frac{u_{n+1}(x)}{u_n(x)}\right|=\lim_{n\to\infty}\left|\frac{(x+3)^{n+1}}{3^{n+1}}\cdot\frac{3^n}{(x+3)^n}\right|==\lim_{n\to\infty}\frac{|x+3|}{3}=\frac{|x+3|}{3}<1.$$

Bundan $|x+3|<3$, $-3<x+3<3$, $-6<x<0$. Intervalning chegaralarida, ya’ni $x=-6$, $x=0$ da mos ravishda

$$1-1+1-1+\ldots\ ,\qquad 1+1+1+1+\ldots$$

uzoqlashuvchi qatorlar hosil bo’ladi. Demak, qatorning yaqinlashish intervali $(-6;\,0)$ bo’ladi. Hosil bo’lgan qatorda

$$\lim_{n\to\infty}R_n(x)=0$$

ekanligini oson isbotlash mumkin. Demak, quyidagi tenglik o’rinli:

$$\frac{1}{x}=-\frac{1}{3}\left(1+\frac{x+3}{3}+\frac{(x+3)^2}{3^2}+\ldots+\frac{(x+3)^n}{3^n}+\ldots\right).$$

3-misol. $f(x)=\dfrac{1}{x^2-3x+2}$ funksiyani x ning darajalari bo'yicha Makloren qatoriga yoying.

Yechish. Berilgan funksiyani sodda kasrlarga ajratamiz

$$\frac{1}{x^2-3x+2}=\frac{1}{x-2}-\frac{1}{x-1}.$$

Bu misolni yechish uchun quyidagi binomial qator yoyilmasidan foydalanamiz

$$(1+x)^m=1+mx+\frac{m(m-1)}{2!}x^2+\ldots+\frac{m(m-1)\ldots(m-n+1)}{n!}x^n+\ldots,$$

$(-1<x<1)$.

Demak,

$$\frac{1}{x-2}=\frac{-1}{2\left(1-\frac{x}{2}\right)}=-\frac{1}{2}\left(1+\frac{x}{2}+\frac{x^2}{2^2}+\ldots+\frac{x^n}{2^n}+\ldots\right)=$$

$$=-\frac{1}{2}-\frac{x}{2^2}-\frac{x^2}{2^3}-\ldots-\frac{x^n}{2^{n+1}}-\ldots,\quad (-2<x<2)$$

$$\frac{1}{x-1}=-\frac{1}{1-x}=-(1+x+x^2+\ldots+x^n+\ldots)=$$

$$=-1-x-x^2-\ldots-x^n-\ldots,\quad (-1<x<1).$$

Qatorlarni hadma-had ayirish natijasida quyidagiga ega bo'lamiz

$$\frac{1}{x^2-3x+2}=\frac{1}{x-2}-\frac{1}{x-1}=\frac{1}{2}+\frac{3}{4}x+\ldots+\frac{2^{n+1}-1}{2^{n+1}}x^n+\ldots$$

$(-1<x<1)$.

4-misol. $f(x)=e^{-x}\sin x$ funksiyani Makloren qatoriga yoying.

Echish. Ma’lumki

$$e^x = 1 + x + \frac{1}{2!}x^2 + \ldots + \frac{1}{n!}x^n + \ldots, \quad (-\infty < x < \infty),$$

$$\sin x = x - \frac{x^3}{3!} + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots, (-\infty < x < \infty).$$

U holda

$$e^{-x} = 1 - x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \ldots + (-1)^n \frac{1}{n!}x^n + \ldots .$$

Ko’paytmani aniqlaymiz

$$f(x) = e^{-x}\sin x = \left(1 - x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \ldots + (-1)^n \frac{1}{n!}x^n + \ldots\right) \times$$

$$\times \left(x - \frac{x^3}{3!} + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots\right) = x - x^2 + \frac{1}{3}x^3 - \frac{3}{40}x^5 + \ldots,$$

$(-\infty < x < \infty)$.

5-misol. $f(x) = \ln\sqrt{\dfrac{1+3x}{1-x}}$ funksiyani Makloren qatoriga yoying.

Yechish. Ma’lumki

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}, \qquad (-1 < x < 1).$$

U holda

$$\ln(1+3x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n \cdot x^n}{n}, \qquad \left(-\frac{1}{3} < x < \frac{1}{3}\right),$$

$$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}, \qquad (-1 < x < 1),$$

$$\ln\sqrt{\frac{1+3x}{1-x}} = \frac{1}{2}\left(\ln(1+3x) - \ln(1-x)\right) =$$

$$=\frac{1}{2}\sum_{n=1}^{\infty}\left((-1)^{n+1}3^n+1\right)\frac{x^n}{n}, \qquad \left(-\frac{1}{3}<x<\frac{1}{3}\right).$$

Mustaqil yechish uchun mashqlar

Quyidagi funksiyalarni $(x-x_0)$ ning darajalari bo' yicha Teylor qatoriga yoying.

1. $f(x)=\frac{1}{x};\ x_0=3$

Javob: $f(x)=\frac{1}{3}-\frac{x-3}{3^2}+\ldots+(-1)^{n+1}\frac{(x-3)^{n-1}}{3^n}+\ldots, 0<x<6.$

2. $f(x)=x^2-3x;\quad x_0=1$

Javob: $f(x)=-2+3(x-1)^2+(x-1)^3.$

3. $f(x)=x^4-3x^2;\quad x_0=-2$

Javob: $f(x)=-16(x+2)+20(x+2)^2-8(x+2)^3+(x+2)^4.$

4. $f(x)=\ln(x+2);\quad x_0=1$

Javob: $f(x)=\ln 3+\frac{1}{3}(x-1)-\frac{1}{3^2}\frac{(x-1)}{2}+\frac{1}{3^3}\frac{(x-1)}{3}+\ldots+$

$+\frac{(-1)^{n-1}}{3^n}\frac{(x-1)^{n-1}}{n}+\ldots\ ,\ -2\le x\le 4.$

5. $f(x)=\cos x;\quad x_0=\frac{\pi}{4}$

Javob: $f(x)=\frac{\sqrt{2}}{2}\left(1-\frac{x-\frac{\pi}{4}}{1!}-\frac{\left(x-\frac{\pi}{4}\right)^2}{2!}+\frac{\left(x-\frac{\pi}{4}\right)^3}{3!}+\ldots\right)-\infty\le x\le+\infty.$

6. $f(x)=e^x;\quad x_0=-2$

Javob: $f(x)=e^{-2}\left(1+\frac{x+2}{1!}-\frac{(x+2)^2}{2!}+\ldots+\frac{(x+2)^n}{n!}+\ldots\right)-\infty\leq x\leq+\infty$.

Quyidagi funksiyalarni x ning darajalari bo' yicha Makloren qatoriga yoying.

1. $f(x)=5^x$

Javob: $f(x)=5^x=1+\sum_{n=1}^{\infty}\frac{x^n\ln^n 5}{n!};\quad(-\infty<x<\infty)$.

2. $f(x)=\frac{e^x-1}{x}$

Javob: $f(x)=\frac{e^x-1}{x}=1+\frac{x}{2!}+\ldots+\frac{x^{n-1}}{n!}+\ldots,(-\infty<x<\infty)$.

3. $f(x)=\ln(10+x)$

Javob: $f(x)=\ln(10+x)=\ln 10+\frac{x}{10}-\frac{x^2}{2\cdot 10^2}+\frac{x^3}{3\cdot 10^3}-\ldots+$

$+(-1)^{n-1}\frac{x^n}{n\cdot 10^n}+\ldots,\ (-\infty<x<\infty)$.

4. $f(x)=\sin^2 x$

Javob: $f(x)=\sin^2 x=\frac{2x^2}{2!}-\frac{8x^4}{4!}+\ldots+(-1)^{n-1}\frac{2^{2n-1}x^{2n}}{(2n)!}+\ldots$

$(-\infty<x<\infty)$.

5. $f(x)=\sin^2 x\cos^2 x$

Javob: $f(x)=\sin^2 x\cos^2 x=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^{4n-3}x^{2n}}{(2n)!}$,

$\sin^2 x\cos^2 x=\frac{1}{4}\sin^2 2x,\ (-\infty<x<\infty)$.

6. $f(x)=\frac{3}{(1-x)(1+2x)}$

Javob: $f(x)=\frac{3}{(1-x)(1+2x)}=\sum_{n=0}^{\infty}\left(1+(-1)^n 2^{n+1}\right)x^n$,

$$\frac{3}{(1-x)(1+2x)}=\frac{1}{1-x}+\frac{2}{1+2x}\ ,\quad \frac{1}{1-x}=1+x+x^2+\ldots=\sum_{n=0}^{\infty}x^n\ ,$$

$$\frac{1}{1+2x}=1-2x+(2x)^2-\ldots+(-1)^n 2^n\cdot x^n+\ldots=\sum_{n=0}^{\infty}(-1)^n 2^n x^n\ ,$$

$$f(x)=\sum_{n=0}^{\infty}x^n+2\sum_{n=0}^{\infty}(-1)^n 2^n x^n=\sum_{n=0}^{\infty}\left(1+(-1)^n 2^{n+1}\right)x^n\ .$$

7. $f(x)=(1+x)e^{-x}$

Javob: $f(x)=(1+x)e^{-x}=1+\sum_{n=2}^{\infty}(-1)^{n-1}\frac{n-1}{n!}x^n, (-\infty<x<\infty)$.

8. $f(x)=\ln\frac{1+x}{1-x}$

Javob: $f(x)=\ln\frac{1+x}{1-x}=2x+\frac{2x^3}{3}+\ldots+\frac{2x^{2n+1}}{2n+1}+\ldots,(-1<x<1)$

9. $f(x)=\frac{x^2}{\sqrt{1-x^2}}$

Javob: $f(x)=\frac{x^2}{\sqrt{1-x^2}}=x^2+\frac{1}{2}x^2+\ldots+\frac{(2n-1)!!}{(2n)!!}x^{2n-2},$

$(-1<x<1))$

10. $f(x)=\frac{\ln(1+x)}{1+x}$

Javob: $f(x)=\frac{\ln(1+x)}{1+x}=x-\left(1+\frac{1}{2}\right)x^2+\left(1+\frac{1}{2}+\frac{1}{3}\right)x^3-$

$-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)x^4+\ldots,\ (-1<x<1)$

11. $f(x)=\ln(1+3x+2x^2)$

Javob: $f(x)=\ln(1+3x+2x^2)=(-1)^{n-1}(1+2^n)\frac{x^n}{n},\ \left(-\frac{1}{2}<x\le\frac{1}{2}\right)$

12. Binomial qator yordami bilan $|x|<1$ bo'lganda

$$\frac{1}{\sqrt{1-x^2}}=1+\frac{1}{2}x^2+\frac{1\cdot 3}{2\cdot 4}x^4+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^6+\ldots+$$

$$+\frac{1\cdot 3\cdot \ldots\cdot (2n-1)}{2\cdot 4\cdot \ldots \cdot (2n)}x^{2n}+\ldots$$

ekanini ko'rsating va qatorni hadma-had integrallab, $\arcsin x$ uchun qatorga yoying.

Javob: $\arcsin x = x+\frac{1}{2}\frac{x^3}{3}+\frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5}+\ldots+\frac{1\cdot 3\cdot \ldots\cdot (2n-1)}{2\cdot 4\cdot \ldots\cdot (2n)}\frac{x^{2n+1}}{2n+1}+\ldots$.

13. Binomial qator yordami bilan $|x|<1$ bo'lganda

$$\frac{1}{\sqrt{1+x^2}}=1-\frac{1}{2}x^2+\frac{1\cdot 3}{2^2\cdot 2!}x^4-\frac{1\cdot 3\cdot 5}{2^3\cdot 3!}x^6+\ldots$$

ekanligini ko'rsating va qatorni hadma-had integrallab, $\ln\left(x+\sqrt{1+x^2}\right)$ funksiya uchun qatorga yoying.

4-§. Qatorlarning taqribiy hisoblashlarga tatbiqi

1-misol. $\ln 1{,}1$ ни 10^{-4} gacha aniqlikda hisoblang.

Yechish. Ma'lumki

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots+(-1)^{n+1}\frac{x^n}{n}+\ldots$$

$(-1<x\le 1)$ da $x=0{,}1$ deb olsak

$$\ln(1{,}1)=0{,}1-\frac{(0{,}1)^2}{2}+\frac{(0{,}1)^3}{3}-\frac{(0{,}1)^4}{4}+\ldots.$$

Bu ishoralari almashinuvchi Leybnis qatori. Qator to'rtinchi hadining absolyut qiymati 10^{-4} dan kichik bo'lgani sababli, $\ln 1{,}1$ ни 10^{-4} gacha aniqlikda hisoblash uchun qatorning uchta $(n=3)$ hadini olish yetarlidir. Demak,

$$\ln(1{,}1)\approx 0{,}1-\frac{0{,}01}{2}+\frac{0{,}001}{3}\approx 0{,}0953.$$

$$\ln(1{,}1) \approx 0{,}0953.$$

2-misol. $\ln 2$ ни 10^{-5} gacha aniqlikda hisoblang.

Yechish. Logarifmlarni taqribiy hisoblashda

$$\ln(N+1) = \ln N + 2\left(\frac{1}{2N+1} + \frac{1}{3(2N+1)^3} + \right.$$

$$\left. + \frac{1}{5(2N+1)^5} + \ldots\right)$$

dan foydalanamiz. Bu yerda $N=1$ bo'ladi.

$$\ln 2 = 2\left(\frac{1}{3} + \frac{1}{3\cdot 3^3} + \frac{1}{5\cdot 3^5} + \ldots + \frac{1}{(2n+1)\cdot 3^{2n+1}} + \ldots\right).$$

R_{n+1} qoldiq hadni baholaymiz.

$$R_{n+1} = 2\left(\frac{1}{2n+3}\cdot\frac{1}{3^{2n+1}} + \frac{1}{2n+5}\cdot\frac{1}{3^{2n+5}} + \ldots\right) <$$

$$< \frac{2}{(2n+3)\cdot 3^{2n+3}}\left(1 + \frac{1}{3^2} + \frac{1}{3^n}\right) = \frac{2\cdot 9}{(2n+3)\cdot 3^{2n+3}\cdot 8} =$$

$$= \frac{1}{4(2n+3)\cdot 3^{2n+1}} < 10^{-5}.$$

Bundan $4(2n+3)\cdot 3^{2n+1} > 10^5$ va bu tengsizlik $n=4$ bo'lganda o'rinli bo'ladi. Demak,

$$\ln 2 = 2\left(\frac{1}{3} + \frac{1}{3\cdot 3^3} + \frac{1}{5\cdot 3^5} + \frac{1}{7\cdot 3^7} + \frac{1}{9\cdot 3^9}\right) \approx 0{,}69314 \text{ yoki } \ln 2 \approx 0{,}69314.$$

3-misol. $e^{0{,}1}$ ни 10^{-3} gacha aniqlikda hisoblang.

Yechish. e^x ning yoyilmasidan foydalanamiz.

$$e^x = 1 + x + \frac{1}{2!}x^2 + \ldots + \frac{1}{(n-1)!}x^{n-1} + R_n\ ,$$

$R_n = \frac{1}{n!} x^{\xi}$, $0 < \xi < x$. $x = 0{,}1$ desak, $e^{\xi} < e^{0,1} < e < 3$ bo'ladi. U holda $R_n < \frac{3}{10^n \cdot n!} < 0{,}001$ tengsizlik $n = 3$ bo'lganda o'rinli bo'ladi. Demak, $e^{0,1}$ ни 10^{-3} gacha aniqlikda hisoblash uchun qatorning uchta hadini hisoblash kifoya, ya'ni

$$e^{0,1} \approx 1 + \frac{1}{10} + \frac{1}{200} \approx 1{,}105$$

yoki $e^{0,1} \approx 1{,}105$.

4-misol. $\sqrt[3]{130}$ ни $0{,}0001$ gacha aniqlikda taqribiy hisoblang.

Yechish. Ma'lumki

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 +$$

$$+ \ldots + \frac{m(m-1)\ldots(m-n+1)}{n!}x^n + \ldots, \ -1 < x < 1$$

$\sqrt[3]{130}$ ni quyidagi ko'rinishda yozamiz

$\sqrt[3]{130} = \sqrt[3]{125+5} = \sqrt[3]{125\left(1+\frac{1}{25}\right)} = 5\left(1+\frac{1}{25}\right)^{\frac{1}{3}}$. $m = \frac{1}{3}$ bo'lsa

$$(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}x^2 + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}x^3 + \ldots =$$

$$= 1 + \frac{1}{3}x - \frac{1 \cdot 2}{3^2 \cdot 2!}x^2 + \frac{1 \cdot 2 \cdot 5}{3^3 \cdot 3!}x^3 - \frac{1 \cdot 2 \cdot 5 \cdot 8}{3^4 \cdot 4!}x^4 + \ldots.$$

Endi hosil bo'lgan qatorda x ning o'rniga $\frac{1}{25}$ ni qo'yamiz.

$$\left(1+\frac{1}{25}\right)^{\frac{1}{3}}=1+\frac{1}{3\cdot 5^2}-\frac{1\cdot 2}{3^2\cdot 2!\cdot 5^4}+\frac{1\cdot 2\cdot 5}{3^3\cdot 3!\cdot 5^6}-\frac{1\cdot 2\cdot 5\cdot 8}{3^4\cdot 4!\cdot 5^8}+\ldots.$$

Ishoralari almashinuvchi qator hosil bo'ldi. Ildizning qiymatini $0{,}0001$ gacha aniqlikda taqribiy hisoblash uchun qatorning 3 ta hadini olish kifoya, chunki to'rtinchi hadi

$$\frac{5\cdot 1\cdot 2\cdot 5}{3^3\cdot 3!\cdot 5^6}=\frac{1\cdot 2}{3^3\cdot 6\cdot 5^4}=\frac{1}{81\cdot 625}<0{,}0001$$

tengsizlik o'rinli. Demak,

$$\sqrt[3]{130}\approx 5\left(1+\frac{1}{3\cdot 5^2}-\frac{1\cdot 2}{3^2\cdot 2!\cdot 5^4}\right)\approx 5{,}0000+0{,}0667-0{,}0009\approx 5{,}0658$$

$\sqrt[3]{130}\approx 5{,}0658$.

5-misol. $\sin 5^0$ ni 10^{-6} gacha aniqlikda taqribiy hisoblang.

Yechish. $\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots$

Foydalanib, 5^0 radian hisobida $\frac{\pi}{36}$ bo'lganligi uchun $\left(\frac{2\pi}{360}\cdot 5=\frac{\pi}{36}\right)$.

$$\sin\frac{\pi}{36}=\frac{\pi}{36}-\frac{\pi^3}{36^3\cdot 3!}+\frac{\pi^5}{36^5\cdot 5!}-\frac{\pi^7}{36^7\cdot 7!}+\ldots$$

bo'ladi. Qatorning uchinchi hadini baholaymiz.

$$\frac{1}{5!}\left(\frac{\pi}{36}\right)^5<\frac{1}{5!}(0{,}1)^5=\frac{1}{120}\cdot 10^{-5}=\frac{5}{6}\cdot 10^{-7}.$$

Qatorning ikkita hadi bilan chegaralansak ham bo'lar ekan, chunki hisoblashda qilingan xatolik $\frac{5}{6}\cdot 10^{-7}$ dan kichik bo'ladi. Shuning uchun

$$\sin\frac{\pi}{36}\approx\frac{\pi}{36}-\frac{\pi^3}{36^3\cdot 3!}\approx 0{,}0872665-0{,}0001107=0{,}0871558$$

$$\sin\frac{\pi}{36}\approx 0{,}0871558.$$

6-misol. $\int_0^1 \cos\sqrt{x}\,dx$ integralni $0{,}0001$ gacha aniqlikda taqribiy hisoblang.

Yechish. Ma' lumki

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-...+(-1)^n\frac{x^{2n}}{(2n)!}+....$$

Agar x ni $\sqrt{x}$ bilan almashtirsak,

$$\cos\sqrt{x}=1-\frac{x}{2!}+\frac{x^2}{4!}-\frac{x^3}{6!}+...+(-1)^n\frac{x^n}{(2n)!}+...,\ (x\geq 0)$$

hosil bo' ladi. Bu tenglikning ikkala tomonini 0 dan 1 gacha chegaralarda integrallab quyidagini topamiz:

$$\int_0^1 \cos\sqrt{x}\,dx=\left(x-\frac{x^2}{2\cdot 2!}+\frac{x^3}{3\cdot 4!}-\frac{x^4}{4\cdot 6!}+...\right)\Bigg|_0^1=1-\frac{1}{2\cdot 2!}+\frac{1}{3\cdot 4!}-\frac{1}{4\cdot 6!}+....$$

hosil bo' lgan ishoralari almashinuvchi qator 5-hadining absolyut qiymati $0{,}0001$ dan kichik bo' lganligi sababli qatorning birinchi 4 ta hadini olish kifoya. Demak,

$$\int_0^1 \cos\sqrt{x}\,dx\approx 1-\frac{1}{4}+\frac{1}{72}-\frac{1}{2880}\approx 0{,}7635$$

$$\int_0^1 \cos\sqrt{x}\,dx\approx 0{,}7635.$$

Mustaqil yechish uchun mashqlar

Quyidagi ifodalarni berilgan ε aniqlikda taqribiy hisoblang.

1. $\ln 5;\ \varepsilon = 0{,}001$ Javob: $1{,}609$
2. $\lg 101;\ \varepsilon = 0{,}0001$ Javob: $2{,}0043$
3. $\ln 17;\ \varepsilon = 0{,}001$ Javob: $2{,}833$
4. $\sqrt[3]{30};\ \varepsilon = 0{,}001$ Javob: $3{,}1072$
5. $\sqrt[3]{10};\ \varepsilon = 0{,}001$ Javob: $2{,}154$
6. $\sqrt[5]{250};\ \varepsilon = 0{,}001$ Javob: $3{,}617$
7. $\cos 10^0;\ \varepsilon = 0{,}0001$ Javob: $0{,}9948$
8. $\sin 0{,}4;\ \varepsilon = 0{,}001$ Javob: $0{,}3894$
9. $\sin 18^0;\ \varepsilon = 0{,}001$ Javob: $0{,}309$
10. $\mathrm{arctg}\, 0{,}2;\ \varepsilon = 0{,}0001$ Javob: $0{,}1973$
11. $\sqrt{e};\ \varepsilon = 0{,}001$ Javob: $1{,}649$
12. $\frac{1}{\sqrt[4]{e}};\ \varepsilon = 0{,}001$ Javob: $0{,}779$

Quyidagi integrallarni $0{,}001$ aniqlikda taqribiy hisoblang.

1. $\int_0^1 \sqrt[3]{x} \cos x\, dx$ Javob: $0{,}608$
2. $\int_0^1 \sin x^2\, dx$ Javob: $0{,}310$
3. $\int_0^{\pi/4} \frac{\sin x}{x}\, dx$ Javob: $0{,}758$
4. $\int_0^1 e^{-x^2}\, dx$ Javob: $0{,}747$

5. $\int_0^{1/2} \frac{\operatorname{arctg} x}{x} dx$ Javob: 0,487

6. $\int_0^{1/2} \cos \frac{x^2}{4} dx$ Javob: 0,500

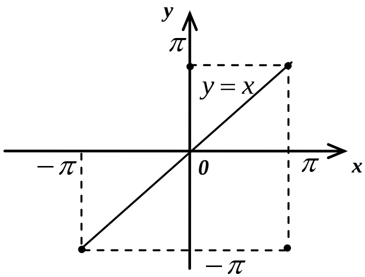
7. $\int_0^{1/4} \ln\left(1+\sqrt{x}\right) dx$ Javob: 0,072

8. $\int_0^{1/2} \sqrt{1+x^3}\, dx$ Javob: 0,508

III BOB. FURYE QATORI

1-§. Davri 2π bo'lgan funksiyalarni Furye qatoriga yoyish

1-misol. Davri $T = 2\pi$ bo'lgan $f(x) = x$ funksiyani $[-\pi;\pi]$ kesmada Furye qatoriga yoying (8-rasm).



8-rasm.

Yechish. $f(x) = x$ funksiya $[-\pi;\pi]$ da Dirixle shartlarini qanoatlantiradi. Shuning uchun Furye qatoriga yoyish mumkin va a_0, a_k, b_k koeffitsientlarini hammasini quyidagi formulalar orqali topamiz:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\,dx = \frac{1}{\pi}\frac{x^2}{2}\Bigg|_{-\pi}^{\pi} = \frac{1}{2\pi}\left(\pi^2 - \pi^2\right) = 0,$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos kx dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos kx dx = \begin{bmatrix} u = x, & dv = \cos kx dx \\ du = dx, & v = \frac{1}{k}\sin kx \end{bmatrix} =$$

$$= \frac{1}{\pi}\left[\frac{x}{k}\sin kx\Big|_{-\pi}^{\pi} - \frac{1}{k}\int_{-\pi}^{\pi}\sin kx dx\right] = \frac{1}{\pi}\left[0 + \frac{1}{k^2}\cos kx\Big|_{-\pi}^{\pi}\right] =$$

$$= \frac{1}{k^3\pi}(\cos k\pi - \cos k\pi) = 0,$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin kx dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin kx dx = \begin{bmatrix} u = x, & dv = \sin kx dx \\ du = dx, & v = -\frac{1}{k}\cos kx \end{bmatrix} =$$

$$= \frac{1}{\pi}\left[-\frac{x}{k}\cos kx\Big|_{-\pi}^{\pi} + \frac{1}{k}\int_{-\pi}^{\pi}\cos kx dx\right] = \frac{1}{\pi}\left[\left(-\frac{\pi}{k} - \frac{\pi}{k}\right)\cos k\pi + 0\right] =$$

$$= -\frac{2}{k}\cos k\pi = (-1)^{k+1}\frac{2}{k}.$$

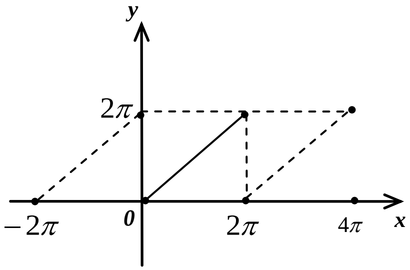
$$b_k = (-1)^{k+1}\frac{2}{k}.$$

Javob: $f(x) = x = 2\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sin kx}{k}$

yoki

$$x = 2\left(\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \ldots + (-1)^{k+1}\frac{\sin kx}{k} + \ldots\right).$$

2-misol. Davri $T = 2\pi$ bo’lgan $f(x) = x$ funksiyani $[0;\, 2\pi]$ kesmada Furye qatoriga yoying (9-rasm).



9-rasm.

Yechish. Davri $T = 2\pi$ bo'lgan $f(x) = x$ funksiya uchun quyidagi tenglik o'rinlidir:

$$\int_{-\pi}^{\pi} f(x)dx = \int_{\lambda}^{\lambda+2\pi} f(x)dx$$

va $\lambda = 0$ bo'lsa

$$\int_{-\pi}^{\pi} f(x)dx = \int_{0}^{2\pi} f(x)dx. \qquad (1)$$

Bu formulalardan foydalanib, a_0, a_k, b_k koeffitsientlarni topamiz.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_{0}^{2\pi} x\,dx = \frac{1}{\pi}\left.\frac{x^2}{2}\right|_0^{2\pi} = \frac{1}{2\pi}\left(4\pi^2 - 0\right) = 2\pi,$$

$$a_k = \frac{1}{\pi}\int_{0}^{2\pi} x\cos kx dx = \begin{bmatrix} u = x, & dv = \cos kx dx \\ du = dx, & v = \dfrac{1}{k}\sin kx \end{bmatrix} = \frac{1}{\pi}\left[\left.\frac{x}{k}\sin kx\right|_0^{2\pi} -\right.$$

$$\left. - \frac{1}{k}\int_{0}^{2\pi}\sin kx dx\right] = \frac{1}{\pi}\left[0 + \left.\frac{1}{k^2}\cos kx\right|_0^{2\pi}\right] = \frac{1}{k^2\pi}\left(\cos 2k\pi - \cos 0\right) = 0,$$

$$b_k = \frac{1}{\pi}\int_0^{2\pi} x\sin kx\,dx = \begin{bmatrix} u = x, & dv = \sin kx\,dx \\ du = dx, & v = -\frac{1}{k}\cos kx \end{bmatrix} = \frac{1}{\pi}\left[-\frac{x}{k}\cos kx\Big|_0^{2\pi} + \right.$$

$$\left. + \frac{1}{k}\int_0^{2\pi}\cos kx\,dx \right] = \frac{1}{\pi}\left[-\frac{2\pi}{k} + 0 - \frac{1}{k^2}\sin kx\Big|_0^{2\pi} \right] = \frac{1}{\pi}\left(-\frac{2\pi}{k} \right) = -\frac{2}{k}.$$

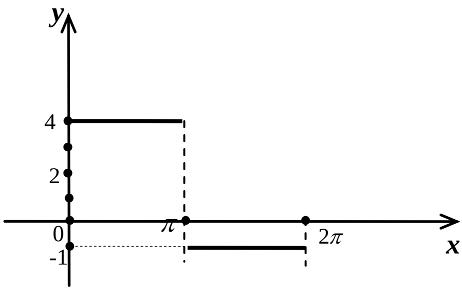
Javob: $x = \pi - 2\sum_{k=1}^{\infty}\frac{\sin kx}{k}$

yoki

$$x = \pi - 2\left(\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \ldots + \frac{\sin kx}{k} + \ldots \right).$$

3-misol. Davri $T = 2\pi$ bo' lgan quyidagi funksiyani $[0;\, 2\pi]$ kesmada Furye qatoriga yoying: (10-rasm).

$$f(x) = \begin{cases} 4, & \text{agar } 0 \le x < \pi \\ -1, & \text{agar } \pi \le x \le 2\pi \end{cases}$$



10-rasm.

Yechish. 2-misoldagi (1) tenglikdan foydalanib, a_0, a_k, b_k koeffitsientlarni topamiz.

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx = \frac{1}{\pi}\left(\int_0^{\pi} 4\,dx - \int_{\pi}^{2\pi} dx \right) = \frac{1}{\pi}\left(4x\Big|_0^{\pi} - x\Big|_{\pi}^{2\pi} \right) = 3,$$

$$a_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos kx dx = \frac{1}{\pi}\left[\int_0^{\pi} 4\cos kx dx - \int_{\pi}^{2\pi} \cos kx dx\right] =$$

$$= \frac{1}{\pi}\left[\frac{4}{k}\sin kx\Big|_0^{\pi} - \frac{1}{k}\sin kx\Big|_{\pi}^{2\pi}\right] = \frac{1}{\pi}(0-0) = 0,$$

$$b_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin kx dx = \frac{1}{\pi}\left[\int_0^{\pi} 4\sin kx dx - \int_{\pi}^{2\pi} \sin kx dx\right] =$$

$$= \frac{1}{\pi}\left[-\frac{4}{k}\cos kx\Big|_0^{\pi} + \frac{1}{k}\cos kx\Big|_{\pi}^{2\pi}\right] = \frac{1}{\pi}\left(-\frac{4}{k}(\cos k\pi - 1) + \frac{1}{k} - \frac{1}{k}\cos k\pi\right) =$$

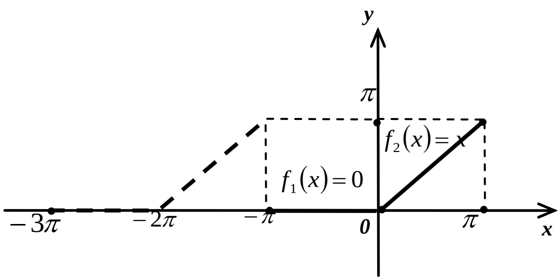
$$= (-1)^{k+1}\frac{5}{k\pi} + \frac{5}{k\pi} = \frac{5}{k\pi}\left[1-(-1)^k\right] = \begin{cases} 0 & \text{, agar } k = 2n \\ \dfrac{10}{k\pi} & \text{, agar } k = 2n+1 \end{cases}.$$

Javob: $f(x) = \dfrac{3}{2} + \dfrac{10}{\pi}\sum_{k=0}^{\infty}\dfrac{\sin(2k+1)x}{2k+1}.$

4-misol. Davri $T = 2\pi$ bo' lgan quyidagi funksiyani:

$$f(x) = \begin{cases} 0, & \text{agar } -\pi \le x < 0 \\ x, & \text{agar } 0 \le x \le \pi \end{cases}$$

$[-\pi; \pi]$ kesmada Furye qatoriga yoying. (11-rasm)



11-rasm.

Yechish. Berilgan funksiya $[-\pi;\pi]$ da Dirixle shartlarini qanoatlantiradi. a_0, a_k, b_k koeffitsientlarini topamiz.

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx=\frac{1}{\pi}\int_{-\pi}^{0}f_1(x)dx+\frac{1}{\pi}\int_{0}^{\pi}f_2(x)dx=$$

$$=\frac{1}{\pi}\left[\int_{-\pi}^{0}0\cdot dx+\int_{0}^{\pi}x\,dx\right]=\frac{1}{\pi}\cdot\frac{x^2}{2}\bigg|_0^{\pi}=\frac{\pi^2}{2\pi}=\frac{\pi}{2},$$

$$a_k=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos kx dx=\frac{1}{\pi}\int_{-\pi}^{0}f_1(x)\cos kx dx+\frac{1}{\pi}\int_{0}^{\pi}f_2(x)\cos kx dx=$$

$$=\begin{bmatrix}u=x, & dv=\cos kx dx\\ du=dx, & v=\frac{1}{k}\sin kx\end{bmatrix}=\frac{1}{\pi}\left[\frac{x}{k}\sin kx\bigg|_0^{\pi}-\frac{1}{k}\int_0^{\pi}\sin kx dx\right]=$$

$$=\frac{1}{\pi}\left[0+\frac{1}{k^2}\cos kx\bigg|_0^{\pi}\right]=\frac{1}{k^2\pi}(\cos k\pi-\cos 0)=\frac{1}{k^2\pi}\left[(-1)^k-1\right]=$$

$$=\begin{cases}-\dfrac{2}{k^2\pi}, & \text{agar } k=2n+1,\\ 0, & \text{agar } k=2n\end{cases}$$

$$b_k=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin kx dx=\frac{1}{\pi}\int_{-\pi}^{0}0\cdot\sin kx dx+\frac{1}{\pi}\int_{0}^{\pi}x\sin kx dx=$$

$$=\frac{1}{\pi}\int_0^{\pi}x\sin kx dx=\begin{bmatrix}u=x, & dv=\sin kx dx\\ du=dx, & v=-\frac{1}{k}\cos kx\end{bmatrix}=\frac{1}{\pi}\left[-\frac{x}{k}\cos kx\bigg|_0^{\pi}+\right.$$

$$\left.+\frac{1}{k}\int_0^{\pi}\cos kx dx\right]=-(-1)^k\frac{1}{\pi}\cdot\frac{\pi}{k}+0=(-1)^{k+1}\frac{1}{k}.$$

Javob: $f(x)=\dfrac{\pi}{4}-\dfrac{2}{\pi}\sum\limits_{k=1}^{\infty}\dfrac{\cos(2k+1)x}{(2k+1)^2}+\sum\limits_{k=1}^{\infty}\dfrac{(-1)^{k+1}}{k}\sin kx\cdot$

Mustaqil yechish uchun mashqlar

1. Davri $T=2\pi$ bo' lgan $f(x)=x^2$ funksiyani $[0;\,2\pi]$ kesmada Furye qatoriga yoying.

Javob: $f(x)=\dfrac{4\pi}{3}+4\sum_{k=1}^{\infty}\left(\dfrac{\cos kx}{k^2}-\pi\dfrac{\sin kx}{k}\right).$

2. Davri $T=2\pi$ bo' lgan $f(x)=e^x$ funksiyani $[-\pi;\,\pi]$ kesmada Furye qatoriga yoying.

Javob: $f(x)=\dfrac{2}{\pi}\operatorname{sh}\pi\left[\dfrac{1}{2}+4\sum_{k=1}^{\infty}\dfrac{(-1)^k}{k^2+1}(\cos kx-k\sin kx)\right].$

3. Davri $T=2\pi$ bo' lgan $f(x)=x-2$ funksiyani $[-\pi;\,\pi]$ kesmada Furye qatoriga yoying.

Javob:

$$f(x)=\frac{4+\pi^2}{2\pi}+\frac{2}{\pi}\sum_{k=1}^{\infty}\left(\frac{\left((-1)^k 2k-\sin 2k\right)\sin kx}{k^2}+\frac{\left((-1)^k-\cos 2k\right)\cos kx}{k^2}\right).$$

4. Davri $T=2\pi$ bo' lgan quyidagi:

$$f(x)=\begin{cases}0, & \text{agar } -\pi\le x<0\\ x^2, & \text{agar } 0\le x\le\pi\end{cases}$$

funksiyani $[-\pi;\,\pi]$ kesmada Furye qatoriga yoying.

Javob:

$$f(x)=\frac{\pi^2}{6}+2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\cos kx+\pi\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\sin kx-\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin kx}{k^3}.$$

5. Davri $T=2\pi$ bo' lgan quyidagi:

$$f(x)=\begin{cases}-2x, & \text{agar } -\pi\le x<0\\ 3x, & \text{agar } 0\le x\le\pi\end{cases}$$

funksiyani $[-\pi;\,\pi]$ kesmada Furye qatoriga yoying.

Javob: $f(x)=\frac{5\pi}{4}-\frac{10}{\pi}\sum_{k=0}^{\infty}\frac{\cos(2k+1)x}{(2k+1)^2}+\sum_{k=1}^{\infty}\frac{\sin kx}{k}$.

6. Davri $T=2\pi$ bo'lgan $f(x)=\pi-x$ funksiyani $[0;\,2\pi]$ kesmada Furye qatoriga yoying.

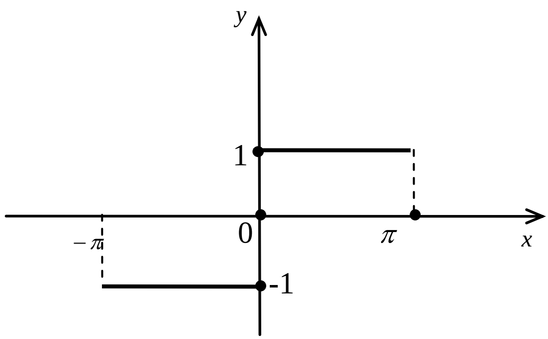
Javob: $f(x)=2\sum_{k=1}^{\infty}\frac{\sin kx}{k}$.

2-§. Juft va toq funksiyalarni Furye qatoriga yoyish

1-misol. Davri $T=2\pi$ bo'lgan quyidagi:

$$f(x)=\begin{cases}-1, & \text{agar } -\pi\le x<0\\ 1, & \text{agar } 0\le x\le\pi\end{cases}$$

funksiyani $[-\pi;\,\pi]$ kesmada Furye qatoriga yoying. (12-rasm)



12-rasm.

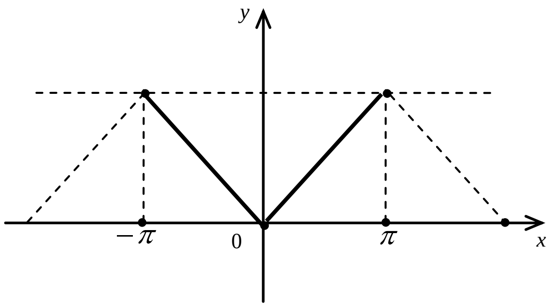
Yechish. Bu funksiya toq funksiya. U holda $a_0 = 0$, $a_k = 0$. b_k koeffitsientni topamiz.

$$b_k = \frac{2}{\pi}\int_0^{\pi} f(x)\sin kx\,dx = \frac{2}{\pi}\int_0^{\pi}\sin kx\,dx = -\frac{2}{k\pi}\cos kx\Big|_0^{\pi} =$$

$$= -\frac{2}{k\pi}(\cos k\pi - \cos 0) = -\frac{2}{k\pi}\left[(-1)^k - 1\right] = \begin{cases} 0, & \text{agar } k = 2n \\ \dfrac{4}{k\pi}, & \text{agar } k = 2n+1 \end{cases}.$$

Javob: $f(x) = \dfrac{4}{\pi}\sum\limits_{k=0}^{\infty}\dfrac{\sin(2k+1)x}{2k+1}$.

2-misol. Davri $T = 2\pi$ bo' lgan $f(x) = |x|$ funksiyani $[-\pi;\pi]$ kesmada Furye qatoriga yoying (13-rasm).



13-rasm.

Yechish. Bu funksiya juft funksiya. U holda $b_k = 0$. a_0, a_k koeffitsientlarni topamiz.

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx = \frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{2}{\pi}\frac{x^2}{2}\Big|_0^{\pi} = \pi,$$

$$a_k=\frac{2}{\pi}\int_0^{\pi}x\cos kx\,dx=\begin{bmatrix}u=x, & dv=\cos kx\,dx\\ du=dx, & v=\dfrac{1}{k}\sin kx\end{bmatrix}=\frac{2}{\pi}\left[\frac{x}{k}\sin kx\Big|_0^{\pi}-\right.$$

$$\left.-\frac{1}{k}\int_0^{\pi}\sin kx\,dx\right]=\frac{2}{k^2\pi}\cos kx\Big|_0^{\pi}=\frac{2}{k^2\pi}(\cos k\pi-\cos 0)=$$

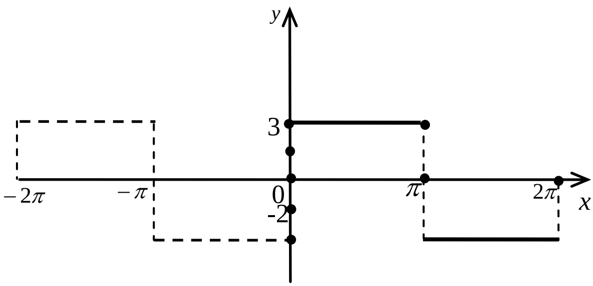
$$=\frac{2}{k^2\pi}\left[(-1)^k-1\right]=\begin{cases}0\ , & \text{agar }\ k=2n\\ -\dfrac{4}{k^2\pi}, & \text{agar }\ k=2n+1\end{cases}.$$

Javob: $f(x)=\dfrac{\pi}{2}-\dfrac{4}{\pi}\sum\limits_{k=0}^{\infty}\dfrac{\cos(2k+1)x}{(2k+1)^2}$.

3-misol. Davri $T=2\pi$ bo' lgan quyidagi

$$f(x)=\begin{cases}3, & \text{agar }\ 0\le x<\pi\\ -2, & \text{agar }\ \pi\le x\le 2\pi\end{cases}$$

funksiyani $[0;\,2\pi]$ kesmada Furye qatoriga yoying. (14-rasm)



14-rasm.

Yechish. Bu funksiya 2π davrli bo' lib, u toq ham, juft ham emas. Uning Furye koeffitsientlarini topish uchun ko' rilgan lemma shartidan foydalanamiz.

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx = \frac{1}{\pi}\left(\int_0^{\pi} 3dx - \int_{\pi}^{2\pi} 2dx\right) = \frac{1}{\pi}\left(3x\Big|_0^{\pi} - 2x\Big|_{\pi}^{2\pi}\right) = 1,$$

$$a_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos kx dx = \frac{1}{\pi}\left[\int_0^{\pi} 3\cos kx dx - \int_{\pi}^{2\pi} 2\cos kx dx\right] =$$

$$= \frac{1}{\pi}\left[\frac{3}{k}\sin kx\Big|_0^{\pi} - \frac{2}{k}\sin kx\Big|_{\pi}^{2\pi}\right] = \frac{1}{\pi}(0-0) = 0,$$

$$b_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin kx dx = \frac{1}{\pi}\left[\int_0^{\pi} 3\sin kx dx - \int_{\pi}^{2\pi} 2\sin kx dx\right] =$$

$$= \frac{1}{\pi}\left[-\frac{3}{k}\cos kx\Big|_0^{\pi} + \frac{2}{k}\cos kx\Big|_{\pi}^{2\pi}\right] = \frac{1}{\pi}\left(-\frac{3}{k}\left[(-1)^k - 1\right] + \frac{2}{k}\left[1-(-1)^k\right]\right) =$$

$$= \frac{5}{k\pi}\left[1-(-1)^k\right] = \begin{cases} 0 \quad , \text{ агар } \quad k = 2n \\ \dfrac{10}{k\pi}, \text{ агар } \quad k = 2n+1 \end{cases}.$$

Javob: $f(x) = \dfrac{1}{2} + \dfrac{10}{\pi}\sum\limits_{k=0}^{\infty}\dfrac{\sin(2k+1)x}{2k+1}.$

Mustaqil yechish uchun mashqlar

1. Davri $T = 2\pi$ bo' lgan $f(x) = x^3$ funksiyani $[-\pi;\pi]$ kesmada Furye qatoriga yoying.

Javob: $f(x) = \sum\limits_{k=1}^{\infty}(-1)^k\left(\dfrac{12}{k^3} - \dfrac{2\pi^2}{k}\right)\sin kx.$

2. Davri $T = 2\pi$ bo‘lgan quyidagi

$$f(x)=\begin{cases}-2, & \text{agar } -\pi \le x < 0\\ 2, & \text{agar } 0 \le x \le \pi\end{cases}$$

funksiyani $[-\pi;\, \pi]$ kesmada Furye qatoriga yoying.

Javob: $f(x)=\dfrac{8}{\pi}\sum\limits_{k=0}^{\infty}\dfrac{\sin(2k+1)x}{2k+1}$.

3. Davri $T = 2\pi$ bo‘lgan $f(x)=x^2$ funksiyani $[0;\, 2\pi]$ kesmada Furye qatoriga yoying.

Javob: $f(x)=\dfrac{4\pi}{3}+4\sum\limits_{k=1}^{\infty}\left(\dfrac{1}{k^2}\cos kx-\dfrac{\pi}{k}\sin kx\right)$.

4. Davri $T = 2\pi$ bo‘lgan $f(x)=x$ funksiyani $[-\pi;\, \pi]$ kesmada Furye qatoriga yoying.

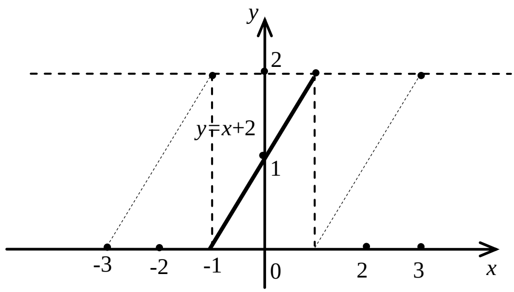
Javob: $f(x)=2\sum\limits_{k=1}^{\infty}(-1)^{k+1}\dfrac{\sin kx}{k}$.

5. Davri $T = 2\pi$ bo‘lgan $f(x)=\sin x$ funksiyani $[-\pi;\, \pi]$ kesmada Furye qatoriga yoying.

Javob: $f(x)=\dfrac{4}{\pi}\sum\limits_{k=1}^{\infty}\dfrac{\sin 2kx}{4k^2-1}$.

3-§. Davri $2l$ bo'lgan funksiyalarni Furye qatoriga yoyish

1-misol. Davri $T=2$ bo'lgan $f(x)=x+1$ funksiyani $[-1;1]$ kesmada Furye qatoriga yoying (15-rasm).



15-rasm.

Yechish. $f(x)=x+1$ funksiya toq ham, juft ham emas. Shuning uchun a_0, a_k, b_k koeffitsientlarni topamiz. Bunda $l=1$.

$$a_0=\frac{1}{l}\int_{-l}^{l} f(x)dx=\int_{-1}^{1}(x+1)dx=\left(\frac{x^2}{2}+x\right)\Bigg|_{-1}^{1}=2,$$

$$a_k=\frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{k\pi}{l}x\,dx=\int_{-1}^{1}(x+1)\cos k\pi x\,dx=$$

$$=\begin{bmatrix} u=x+1, & dv=\cos k\pi x\,dx \\ du=dx, & v=\dfrac{1}{k\pi}\sin k\pi x \end{bmatrix}=\frac{x+1}{k\pi}\sin k\pi x\Bigg|_{-1}^{1}-\frac{1}{k\pi}\int_{-1}^{1}\sin k\pi x\,dx=$$

$$= \frac{2}{k\pi}\sin k\pi - 0 + \frac{1}{k^2\pi^2}\cos k\pi x\Big|_{-1}^{1} = \frac{2}{k^2\pi^2}\left(\cos k\pi - \cos k\pi\right) = 0,$$

$$b_k = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{k\pi}{l}x\,dx = \int_{-1}^{1}(x+1)\sin k\pi x\,dx =$$

$$= \begin{bmatrix} u = x+1, & dv = \sin k\pi x\,dx \\ du = dx, & v = -\frac{1}{k\pi}\cos k\pi x \end{bmatrix} = -\frac{x+1}{k\pi}\cos k\pi x\Big|_{-1}^{1} +$$

$$+\frac{1}{k\pi}\int_{-1}^{1}\cos k\pi x\,dx = -\frac{2}{k\pi}\cos k\pi - 0 + \frac{1}{k^2\pi^2}\sin k\pi x\Big|_{-1}^{1} =$$

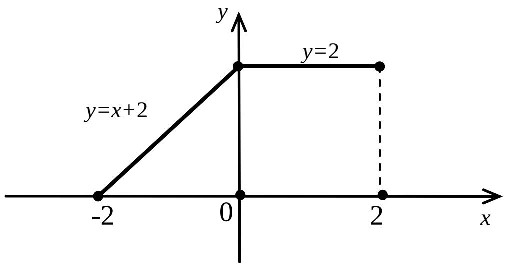
$$= -\frac{2}{k\pi}(-1)^k + \frac{1}{k^2\pi^2}\left(\sin k\pi - \sin k\pi\right) = -\frac{2}{k\pi}(-1)^k = (-1)^{k+1}\frac{2}{k\pi}.$$

Javob: $f(x) = 1 + \frac{2}{\pi}\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sin kx}{k}.$

2-misol. Davri $T = 4$ bo' lgan quyidagi:

$$f(x) = \begin{cases} 2+x, & \text{agar } -2 \le x < 0 \\ 2, & \text{agar } 0 \le x \le 2 \end{cases}$$

funksiyani $[-2;\,2]$ kesmada Furye qatoriga yoying. (16-rasm)



16-rasm.

Yechish. Bu funksiya juft ham, toq ham emas, $l=2$. Shuning uchun a_0, a_k va b_k koeffitsientlarning topamiz.

$$a_0=\frac{1}{l}\int_{-l}^{l}f(x)dx=\frac{1}{2}\int_{-2}^{0}(x+2)dx+\frac{1}{2}\int_{0}^{2}2\,dx=\frac{1}{2}\left(\frac{x^2}{2}+2x\right)\Bigg|_{-2}^{0}+\frac{1}{2}\cdot 2x\Big|_{0}^{2}=3\cdot$$

$$a_k=\frac{1}{2}\int_{-2}^{0}(x+2)\cos\frac{k\pi}{2}x\,dx+\frac{1}{2}\int_{0}^{2}2\cos\frac{k\pi}{2}x\,dx=$$

$$=\begin{bmatrix} u=x+2, & dv=\cos\dfrac{k\pi}{2}x\,dx \\ du=dx, & v=\dfrac{2}{k\pi}\sin\dfrac{k\pi}{2}x \end{bmatrix}=\frac{1}{2}\left[\frac{2(x+2)}{k\pi}\sin\frac{k\pi}{2}x\Big|_{-2}^{0}-\right.$$

$$\left.-\frac{2}{k\pi}\int_{-2}^{0}\sin\frac{k\pi}{2}x\,dx+\frac{2}{k\pi}\sin\frac{k\pi}{2}x\Big|_{0}^{2}\right]=\frac{2}{k^2\pi^2}\cos\frac{k\pi}{2}x\Big|_{-2}^{0}=$$

$$=\frac{2}{k^2\pi^2}\left(1-(-1)^k\right)=\begin{cases}\dfrac{4}{k^2\pi^2}, & k=2n+1\\ 0, & k=2n\end{cases},$$

$$b_k=\frac{1}{2}\int_{-2}^{0}(x+2)\sin\frac{k\pi}{2}x\,dx+\frac{1}{2}\int_{0}^{2}2\sin\frac{k\pi}{2}x\,dx=$$

$$=\begin{bmatrix} u=x+2, & dv=\sin\dfrac{k\pi}{2}x\,dx \\ du=dx, & v=-\dfrac{2}{k\pi}\cos\dfrac{k\pi}{2}x \end{bmatrix}=\frac{1}{2}\left[-\frac{2(x+2)}{k\pi}\cos\frac{k\pi}{2}x\Big|_{-2}^{0}+\right.$$

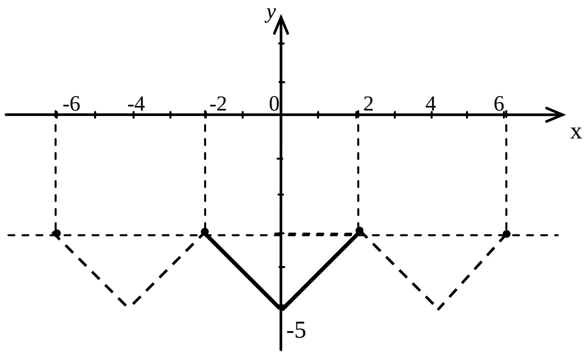
$$\left.+\frac{2}{k\pi}\int_{-2}^{0}\cos\frac{k\pi}{2}x\,dx-\frac{2}{k\pi}\cos\frac{k\pi}{2}x\Big|_{0}^{2}\right]=-\frac{2}{k\pi}+\frac{2}{k^2\pi^2}\sin\frac{k\pi}{2}x\Big|_{-2}^{0}-$$

$$-\frac{2}{k\pi}\left((-1)^k-1\right)=(-1)^{k+1}\frac{2}{k\pi}.$$

Javob:

$$f(x)=\frac{3}{2}+\frac{4}{\pi^2}\sum_{k=0}^{\infty}\frac{1}{2k+1}\cos\frac{(2k+1)\pi}{2}x+\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\sin\frac{k\pi}{2}x.$$

3-misol. Davri $T=4$ bo'lgan $f(x)=|x|-5$ funksiyani $[-2,\ 2]$ kesmada Furye qatoriga yoying (17-rasm).



17-rasm.

Yechish. Berilgan funksiya juft bo'lgani uchun faqat a_0 va a_n koeffitsientlarni topamiz, $b_k=0$ bo'ladi. $l=2$

$$a_0=\frac{2}{l}\int_0^l f(x)dx=\frac{2}{2}\int_0^2(x-5)dx=\frac{x^2}{2}\Big|_0^2=2-10=-8$$

$$a_k=\frac{2}{l}\int_0^l f(x)\cos\frac{k\pi}{l}xdx=\frac{2}{2}\int_0^2(x-5)\cos\frac{k\pi}{2}xdx=$$

$$=\begin{bmatrix} u=x-5\Rightarrow du=dx \\ dv=\cos\frac{k\pi}{2}xdx\Rightarrow v=\frac{2}{k\pi}\sin\frac{k\pi}{2}x \end{bmatrix}=$$

$$=\frac{2(x-5)}{k\pi}\sin\frac{k\pi}{2}x\Big|_0^2-\frac{2}{k\pi}\int_0^2\sin\frac{k\pi}{2}xdx=$$

$$=\frac{4}{k^2\pi^2}\cos\frac{k\pi}{2}x\Big|_0^2=\frac{4}{k^2\pi^2}(\cos k\pi-\cos 0)=$$

$$\frac{4}{k^2\pi^2}\left((-1)^k-1\right)=\begin{cases} 0, & \text{agar } k=2n \\ -\dfrac{8}{k^2\pi^2}, & \text{agar } k=2n+1 \end{cases}$$

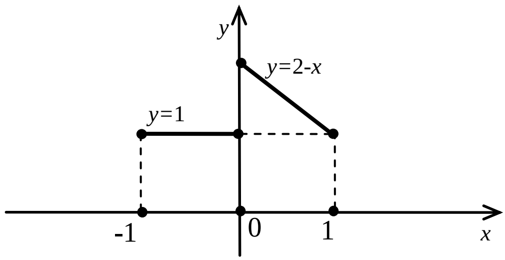
Javob:

$$|x|-5=-4-\frac{8}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{(2n+1)^2}\cos\frac{(2n+1)\pi}{2}x$$

4-misol. Davri $T=2$ bo’ lgan quyidagi

$$f(x)=\begin{cases} 1, & \text{agar } -1\le x<0 \\ 2-x, & \text{agar } \ 0\le x\le 1 \end{cases}$$

funksiyani $[-1;1]$ kesmada Furye qatoriga yoying. (18-rasm)



18-rasm.

Yechish. Berilgan funksiya juft ham, toq ham emas, $l=1$. Demak, a_0, a_k va b_k koeffitsientlarning hammasini topamiz.

$$a_0=\frac{1}{l}\int_{-l}^{l}f(x)dx=\int_{-1}^{0}dx+\int_{0}^{1}(2-x)dx=x\Big|_{-1}^{0}+\left(2x-\frac{x^2}{2}\right)\Bigg|_{0}^{1}=\frac{5}{2},$$

$$a_k=\int_{-1}^{0}\cos\,k\pi x\,dx+\int_{0}^{1}(2-x)\cos k\pi x\,dx=\begin{bmatrix}u=2-x,\ dv=\cos\,k\pi x\,dx\\ du=-dx,\ v=\dfrac{1}{k\pi}\sin\,k\pi x\end{bmatrix}=$$

$$=\frac{1}{k\pi}\sin k\pi x\Big|_{-1}^{0}+\frac{2-x}{k\pi}\sin k\pi x\Big|_{0}^{1}+\frac{1}{k\pi}\int_{0}^{1}\sin k\pi x\,dx=$$

$$=-\frac{1}{k^2\pi^2}\cos k\pi x\Big|_{0}^{1}=\frac{1}{k^2\pi^2}\left[1-(-1)^k\right]=\begin{cases}\dfrac{2}{k^2\pi^2}\ ,\ \text{agar}\ \ k=2n+1,\\ \ \ 0\quad ,\ \text{agar}\ \ k=2n\end{cases}$$

$$b_k=\int_{-1}^{0}\sin k\pi x\,dx+\int_{0}^{1}(2-x)\sin k\pi x\,dx=\begin{bmatrix}u=2-x,\ dv=\sin k\pi x\,dx\\ du=-dx,\ v=-\dfrac{1}{k\pi}\cos k\pi x\end{bmatrix}=$$

$$= -\frac{1}{k\pi}\cos k\pi x\Big|_{-1}^{0} - \frac{2-x}{k\pi}\cos k\pi x\Big|_{0}^{1} - \frac{1}{k\pi}\int_0^1 \cos k\pi x\,dx =$$

$$= \frac{1}{k\pi}\left[(-1)^k - 1\right] - \frac{1}{k\pi}\cos k\pi + \frac{2}{k\pi} - \frac{2}{k^2\pi^2}\sin k\pi x\Big|_0^1 = \frac{1}{k\pi}.$$

Javob: $f(x) = \frac{5}{4} + \frac{2}{\pi^2}\sum_{k=0}^{\infty}\frac{\cos(2k+1)\pi x}{2k+1} + \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{\sin k\pi x}{k}$.

Mustaqil yechish uchun mashqlar

1. Davri $T = 2$ bo‘lgan quyidagi:

$$f(x) = \begin{cases} -\frac{3x+1}{2}, & \text{agar } -1 \le x < 0 \\ 1, & \text{agar } \ 0 \le x \le 1 \end{cases}$$

funksiyani $[-1;1]$ kesmada Furye qatoriga yoying.

Javob: $f(x) = \frac{5}{8} + 3\sum_{k=1}^{\infty}\left(\frac{\cos k\pi x}{k^2\pi^2} + \frac{\sin k\pi x}{2k\pi}\right)$.

2. Davri $T = 2$ bo‘lgan $f(x) = x^2$ funksiyani $[-1;1]$ kesmada Furye qatoriga yoying.

Javob: $f(x) = \frac{1}{3} + \frac{4}{\pi^2}\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\cos k\pi x$.

3. Davri $T = 2$ bo‘lgan $f(x) = |x|$ funksiyani $[-1;1]$ kesmada Furye qatoriga yoying.

Javob: $f(x) = \frac{1}{2} - \frac{4}{\pi^2}\sum_{k=0}^{\infty}\frac{\cos(2k+1)\pi x}{(2k+1)^2}$.

4. Davri $T = 2$ bo‘lgan $f(x) = x - [x]$ funksiyani $[-1;1]$ kesmada Furye qatoriga yoying.

Javob: $f(x)=\frac{1}{2}-\frac{1}{\pi}\sum_{k=1}^{\infty}\frac{\sin k\pi x}{k}$.

5. Davri $T=4$ bo'lgan $f(x)=|x|$ funksiyani $[-2;\,2]$ kesmada Furye qatoriga yoying.

Javob: $f(x)=1-\frac{8}{\pi^2}\sum_{k=0}^{\infty}\frac{1}{(2k+1)^2}\cos\frac{(2k+1)\pi x}{2}$.

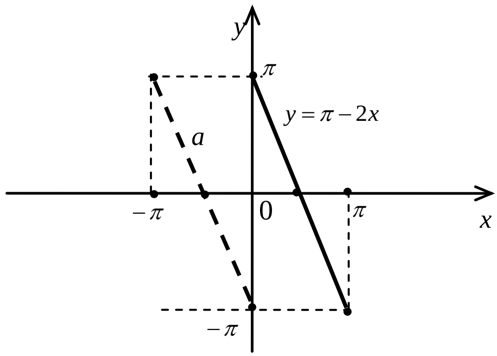
4-§. Davriy bo'lmagan funksiyalarni Furye qatoriga yoyish

Biror $[0;\,l]$ kesmada uzluksiz va bo'lakli monoton $f(x)$ funksiyani $[-l;\,0]$ kesmada davom ettirib, bu funksiyani $[-l;\,l]$ kesmada toq yoki juft holdagi funksiyaga to'ldirib uni Furye qatoriga yoyish mumkin.

Davom ettirilgan toq funksiya uchun Furye qatori faqat sinuslarni, juft funksiya uchun Furye qatori faqat kosinuslarni o'z ichiga oladi.

1-misol. $f(x)=\pi-2x$ funksiyani $[0;\,\pi]$ kesmada toq holda davom ettirib, $[-\pi;\,\pi]$ kesmada Furye qatoriga yoying. (19-rasm).

Yechish. Bu funksiya $T=2\pi$ davrli bo'lib, $[-\pi;\,\pi]$kesmada toq funksiya bo'ladi. U holda $a_0=0$, $a_k=0$, $b_k\neq 0$ bo'lib, $b_k$ni umumiy formuladan topish kifoya. Bu yerda $l=\pi$.



19-rasm.

$$b_k=\frac{2}{\pi}\int_0^{\pi}(\pi-2x)\sin kx\,dx=\begin{bmatrix} u=\pi-2x, & dv=\sin kx\,dx \\ du=-2dx, & v=-\frac{1}{k}\cos kx \end{bmatrix}=$$

$$=\frac{2}{\pi}\left[-\frac{\pi-2x}{k}\cos kx\Big|_0^{\pi}-\frac{2}{k}\int_0^{\pi}\cos kx\,dx\right]=\frac{2}{\pi}\left[\frac{\pi}{k}\cos k\pi+\frac{\pi}{k}\cos 0-0\right]=$$

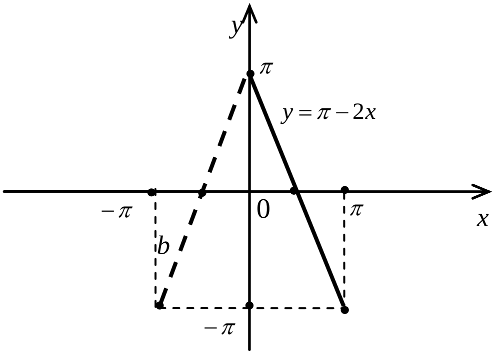
$$=\frac{2}{\pi}\cdot\frac{\pi}{k}\left[(-1)^k+1\right]=\frac{2}{k}\left[(-1)^k+1\right]=\begin{cases}\frac{4}{k}, & \text{agar } k=2n \\ 0, & \text{agar } k=2n+1\end{cases}.$$

Javob: $f(x)=4\sum_{k=1}^{\infty}\frac{\sin 2kx}{2k}$.

2-misol. $f(x)=\pi-2x$ funksiyani $[0;\pi]$ kesmada juft holda davom ettirib, $[-\pi;\pi]$ kesmada Furye qatoriga yoying. (20-rasm).

Yechish. Bu davriy bo’lmagan $f(x)=\pi-2x$ funksiyani juft holda davom ettirib, $[-\pi;\pi]$ kesmada $T=2\pi$ davrli juft funksiyaga to’ldirsak bo’ladi. U holda

$b_k = 0$ bo' lib, a_0, a_k larni umumiy formulalardan topish kifoya. Bu yerda $l = \pi$.



20-rasm.

$$a_0 = \frac{2}{\pi}\int_0^{\pi}(\pi - 2x)dx = \frac{2}{\pi}\left(\pi x - 2\cdot\frac{x^2}{2}\right)\Bigg|_0^{\pi} = \frac{2}{\pi}\left(\pi^2 - \pi^2\right) = 0,$$

$$a_k = \frac{2}{\pi}\int_0^{\pi}(\pi - 2x)\cos kx dx = \begin{bmatrix} u = \pi - 2x, & dv = \cos kx dx \\ du = -2dx, & v = \frac{1}{k}\sin kx \end{bmatrix} =$$

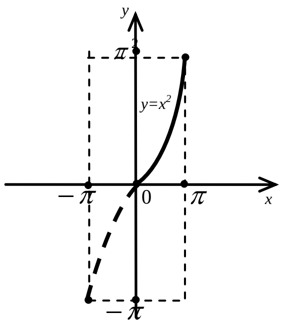
$$= \frac{2}{\pi}\left[\frac{\pi - 2x}{k}\sin kx\Bigg|_0^{\pi} + \frac{2}{k}\int_0^{\pi}\sin kx dx\right] = \frac{2}{\pi}\left[-\frac{2}{k}\cdot\frac{1}{k}\cos kx\Bigg|_0^{\pi}\right] =$$

$$= -\frac{4}{k^2\pi}(\cos k\pi - 1) = \frac{4}{k^2\pi}\left[1 - (-1)^k\right] = \begin{cases} \dfrac{8}{k^2\pi}, & \text{agar } k = 2n+1 \\ 0, & \text{agar } k = 2n \end{cases}.$$

Javob: $f(x) = \pi - 2x = \frac{8}{\pi}\sum_{k=0}^{\infty}\frac{\cos(2k+1)x}{2k+1}$.

3-misol. $f(x)=x^2$ funksiyani $[0;\pi]$ kesmada toq holda davom ettirib, $[-\pi;\pi]$ kesmada Furye qatoriga yoying. (21-rasm).

Yechish. Agar $f(x)=x^2$ funksiyani toq holda davom ettirsak u $[-\pi;\pi]$ kesmada $T=2\pi$ davrli toq funksiyaga aylanadi. Demak, uni $[-\pi;\pi]$ kesmada toq funksiya deb qarab, uning Furye koeffitsientlarini topamiz. Bunda $a_0=0$, $a_k=0$, $b_k\neq 0$, $l=\pi$.



21-rasm.

$$b_k=\frac{2}{\pi}\int_0^{\pi}x^2\sin kx\,dx=\begin{bmatrix}u=x^2, & dv=\sin kx\,dx\\ du=2x\,dx, & v=-\frac{1}{k}\cos kx\end{bmatrix}=$$

$$=\frac{2}{\pi}\left[-\frac{x^2}{k}\cos kx\Big|_0^{\pi}+\frac{2}{k}\int_0^{\pi}x\cos kx\,dx\right]=\begin{bmatrix}u=x, & dv=\cos kx\,dx\\ du=dx, & v=\frac{1}{k}\sin kx\end{bmatrix}=$$

$$=\frac{2}{\pi}\left[-\frac{\pi^2}{k}\cos k\pi+0\right]+\frac{4}{k\pi}\left[\frac{x}{k}\sin kx\Big|_0^{\pi}-\frac{1}{k}\int_0^{\pi}\sin kx\,dx\right]=-\frac{2\pi}{k}\cos k\pi+$$

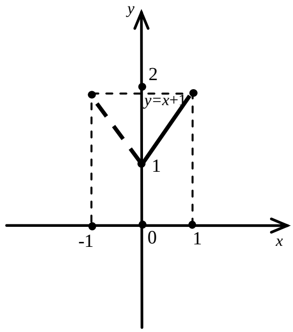
$$+\frac{4}{k\pi}\left[0+\frac{1}{k^2}\cos kx\Big|_0^{\pi}\right]=(-1)^{k+1}\frac{2\pi}{k}+\frac{4}{k^3\pi}[\cos k\pi-\cos 0]=$$

$$=(-1)^{k+1}\frac{2\pi}{k}+\frac{4}{k^3\pi}\left[(-1)^k-1\right]=(-1)^{k+1}\frac{2\pi}{k}-\begin{cases}\dfrac{8}{k^3\pi}, & \text{agar } k=2n+1\\ 0, & \text{agar } k=2n\end{cases}.$$

Javob: $f(x)=2\pi\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sin kx}{k}-\frac{8}{\pi}\sum_{k=0}^{\infty}\frac{\sin(2k+1)x}{(2k+1)^3}$.

4-misol. $f(x)=x+1$ funksiyani $[0;1]$ kesmada juft holda davom ettirib, Furye qatoriga yoying. (22-rasm).

Yechish. Agar $f(x)=x+1$ funksiyani juft holda davom ettirsak u $[-1;1]$ kesmada $T=2$ davrli juft funksiyaga aylanadi. Uning Furye koeffitsientlarini topamiz. Bunda $b_k=0$, $l=1$.



22-rasm.

$$a_0=\frac{1}{l}\int_{-l}^{l}f(x)dx=2\int_0^1(x+1)dx=2\left(\frac{x^2}{2}+x\right)\Bigg|_0^1=2\left(\frac{1}{2}+1\right)=3,$$

$$a_k = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{k\pi}{l}x\,dx = 2\int_{0}^{1}(x+1)\cos k\pi x\,dx =$$

$$= \begin{bmatrix} u = x+1, & dv = \cos\, k\pi x\,dx \\ du = dx, & v = \dfrac{1}{k\pi}\sin\, k\pi x \end{bmatrix} = 2\left[\frac{x+1}{k\pi}\sin k\pi x\Big|_0^1 - \frac{1}{k\pi}\int_0^1 \sin k\pi x\,dx\right]$$

$$= 0 + \frac{2}{k^2\pi^2}\cos k\pi x\Big|_0^1 = \frac{2}{k^2\pi^2}(\cos k\pi - \cos 0) = \frac{2}{k^2\pi^2}\left[(-1)^k - 1\right] =$$

$$= \begin{cases} -\dfrac{4}{k^2\pi^2}, & \text{агар} \quad k = 2n+1 \\ \quad 0 \quad, & \text{агар} \quad k = 2n \end{cases}$$

Javob: $f(x) = \dfrac{3}{2} - \dfrac{4}{\pi^2}\sum\limits_{k=0}^{\infty}\dfrac{\cos(2k+1)\pi x}{(2k+1)^2}$.

Mustaqil yechish uchun mashqlar.

1. Davriy bo‘ lmagan $f(x) = \cos 2x$ funksiyani $[0;\pi]$ kesmada toq holda davom ettirib (sinuslar buyicha), $[-\pi;\pi]$ da Furye qatoriga yoying.

Javob: $f(x) = -\dfrac{4}{\pi}\sum\limits_{k=0}^{\infty}\dfrac{(2k+1)\sin(2k+1)x}{4-(2k+1)^2}$.

2. Davriy bo‘ lmagan quyidagi:

$$f(x) = \begin{cases} x \quad, & \text{agar} \quad 0 \le x \le 1 \\ 2-x, & \text{agar} \quad 1 < x \le 2 \end{cases}$$

funksiyani $[0;2]$ kesmada juft holda davom ettirib, $[-2;2]$ da Furye qatoriga yoying.

Javob: $f(x) = \dfrac{1}{2} - \dfrac{4}{\pi^2}\sum\limits_{k=0}^{\infty}\dfrac{\cos(2k+1)\pi x}{(2k+1)^2}$.

3. Davriy bo' lmagan $f(x)=x-\frac{1}{2}x^2$ funksiyani $[0;\,2]$ kesmada juft holda davom ettirib, $[-2;\,2]$ kesmada Furye qatoriga yoying.

Javob: $f(x)=\frac{1}{3}-\frac{4}{\pi^2}\sum_{k=1}^{\infty}\frac{1+(-1)^k}{k^2}\cos\frac{k\pi}{2}x$.

4. Davriy bo' lmagan $f(x)=x-\frac{1}{2}x^2$ funksiyani $[0;\,2]$ kesmada toq holda davom ettirib, $[-2;\,2]$ kesmada Furye qatoriga yoying.

Javob: $f(x)=\frac{8}{\pi^3}\sum_{k=1}^{\infty}\frac{1-(-1)^k}{k^3}\sin\frac{k\pi}{2}x$.

5. Davriy bo' lmagan $f(x)=\frac{\pi}{4}-\frac{x}{2}$ funksiyani $[0;\,\pi]$ kesmada toq holda davom ettirib, $[-\pi;\,\pi]$ da Furye qatoriga yoying.

Javob: $f(x)=\frac{2}{\pi}\sum_{k=0}^{\infty}\frac{\cos(2k+1)x}{(2k+1)^2}$.

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