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« Chiziqli algebraik tenglamalar tizimini yechish. Iteratsion usullar. Oddiy iteratsiya usuli»

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Chiziqli algebraik tenglamalar tizimini yechish. Iteratsion usullar. Oddiy iteratsiya usuli

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Iteratsiya, statsonar, rekkurent, nostatsionar, xatolik, parametr, empirik, boshlangich yaqinlashish, diogonal elementlar, oshkor usul.

Kirish

Bugungi kunda turli tamoyil (printsip)larga asoslangan juda ko`plab iteratsion usullar mavjud. Umuman, bu usullarning, o`ziga xos tomonlaridan biri shundan iboratki, yo’l qo’yilgan xatoliklari har qadamda to`g’rilanib boradi. Aniq usullar bilan ishlayotganda, agar biror qadamda xatoga yo’l qo’yilsa, bu xato oxirgi natijaga ham ta`sir kiladi. Yaqinlashuvchi iteratsion jarayonning biror qadamida yo`l qo`yilgan xatolik esa faqat bir necha iteratsiya qadamini ortiqcha bajarishgagina olib keladi xolos. Biror qadamda yo`l qo`yilgan xatolik keyingi qadamlarda tuzatilib boriladi. Boz ustiga bu usullarning hisoblash tartibi sodda bo`lib, ularni EHM larda hisoblash qulaydir. Lekin har bir iteratsion usulning qo`llanish soxasi chegaralangandir. Chunki iteratsiya jarayoni berilgan tizim uchun uzoqlashi-shi yoki shuningdek, sekin yaqinlashishi mumkinki, buning oqibatida amalda yechimni qoniqarli aniqlikda topib bo`lmaydi.

Shuning uchun ham iteratsion usullarda faqat yaqinlashish masalasigina emas, balki yaqinlashish tezligi masalasi ham katta axamiyatga egadir. Yaqinlashish tezligi dastlabki yaqinlashish vektorining qulay tanlanishiga ham bog’liqdir.

Bu kurs ishida avval iteratsion usullarning umumiy xarakteristikasini ko’rib chiqamiz, so`ngra esa hisoblash amaliyotida keng qo`llaniladigan iteratsion usullarni keltiramiz.

ITERATSION USULLARNING UMUMIY XARAKTERISTIKASI

Yuqorida qayd etilganidek, iteratsion usullar tizimning izlangan *x* echimiga yaqinlashadigan y_0, y_1, y_2, … iteratsion ketma-ketliklarni kurishga asoslangan. Har bir shunday usul navbatdagi y_{k+1} yaqinlashishni avvalgilari yordamida hisoblashga imkon beradigan iteratsion formulalar bilan xarakterlanadi. Eng sodda xolda y_{k+1} ni hisoblashda faqat bitta avvalgi y_k iteratsiyadan foydalaniladi. Bunday usullar bir qadamli deyiladi. Bir qadamli usullar uchun iteratsion formulani quyidagi

$$B_{k+1}\frac{y_{k+1}-y_k}{\tau_{k+1}}+Ay_k=f \tag{1.1}$$

standart kanonik ko`rinishda yozish qabul kilingan; bunda $\tau_{k+1}$ - iteratsion parametrlar ($\tau_{k+1}>0$), $B_{k+1}$ – yordamchi maxsusmas matritsalar. Agar $\tau$ va B lar k+1 indeksga bog'liq bo`lmasa, ya`ni (1.1) formula ixtiyoriy k lar uchun bir xil ko`rinishga ega bo`lsa, u xolda bu iteratsion usul s t a t s i o n a r u s u l deyiladi. Statsionar usullar hisob-lash jarayonini tashkil etish nuqtai nazaridan soddadir. Ammo nostatsionar usullar boshqa ustunliklarga ega: ular $\{\tau_{k+1}\}$, $\{B_{k+1}\}$ ketma-ketliklarni tanlash bilan boglangan kushimcha «erkinlik darajasiga» ega. Bundan y_k iteratsiyalar tizimning *x* echimiga yaqinlashish tezligini oshirishda foydalanish mumkin.

(1.1) iteratsion formula yordamida navbatdagi y_{k+1} yaqinlashishni topish ushbu

$$B_{k+1}\, y_{k+1} = F_{k+1} \tag{1.2}$$

tenglamalar tizimini echishni talab etadi. Bunda

$$F_{k+1} = (B_{k+1} - \tau_{k+1} A)\, y_k + \tau_{k+1} f$$

Shunday hisoblashni kar bir qadamda bajarishga turri keladi. B_{k+1} matritsa sifatida birlik $B_{k+1} = E$ matritsa olsak, iteratsion ketma-ketlik xadlarini hisoblash uchun eng sodda tarxga ega bula-miz. Bu xolda (1.1) formula ketma-ketlikning navbatdagi y_{k+1} xadini uning avvalgi y_k xadi orqali oshkor ifodalash imkonini beradi:

$$y_{k+1} = y_k - \tau_{k+1} A\, y_{k+1} + \tau_{k+1} f \tag{1.3}$$

Ana shunday rekkurent formulaga asoslangan iteratsion usullar oshkor usullar deyiladi.

Oshkormas usullar ($B_{k+1} \neq E$ orasida B_{k+1} matritsani uchburchakli kilib tanlanadigan usullar eng ko`p tarqalgan. Bu kolda navbatdagi y_{k+1} iteratsiyani topish uchun y_{k+1} ning komponentlarini (1.2) uchburchakli tizimdan birin-ketin Gauss usulining teskari yurishiga kilinganidek topishga keltiriladi.

Qandaydir iteratsion usulning qo`llanishi $\{y_k\}$ ketma-ketlik tizimning x echimiga yaqinlashishni bildiradi:

$$\lim_{k\to\infty} y_k = x \tag{1.4}$$

(1.4) tenglik quyidagini anglatadi:

$$\sqrt{\left(y_1^{(k)} - x_1\right)^2 + \left(y_2^{(k)} + x_2 + \ldots + \left(y_n^{(k)} - x_n\right)\right.} \to 0 \tag{1.5}$$

(1.5) dan kurinadiki, u vektorlar ketma-ketligining x vektorga yaqinlashishining zaruriy va etarli sharti kar bir komponentning yaqinlashuvchiligidan iborat:

$$\lim_{k\to\infty} y_1^{(k)} = x_i, \qquad i = 1,2,\ldots,n$$

Ushbu ayirma $z_k = y_k - x$ xatolik deyiladi. y_k ni $y_k = x + z_k$ ko`rinishda yozib va (1.1) ga kuyib, xatolik uchun,

$$B_{k+1}\frac{z_{k+1} - z_k}{\tau_{k+1}} + Az_k = 0 \tag{1.6}$$

iteratsion formulami hosil kilamiz. (1.1) dan farqli ularok, u tizimning ung tomoni (f) ni o`z ichiga olmaydi, ya`ni bir jinslidir. (1.4) yaqinlashishni talab etish z_k ning nolga intilishi lozimligini anglatadi:

$$\lim_{k\to\infty} z_k = 0 \tag{1.7}$$

Har bir iteratsion usul yaqinlashuvchiligining etarlilik shartlari A, B_{k+1} matritsalar va τ_{k+1} iteratsion parametrlar kanoatlantirishi lozim bo`lgan ko`rinishda ifodalanadi. Ulardan ba`zilarini, ayniksa, iteratsion parametrlarni optimal tanlashga oid shartlarni tekshirish kiyin. Natijada hisoblashlarni bajarayotganda iteratsion parametrlarni ko`pincha tajriba yuli bilan (empirik) tanlashga turri keladi.

II. Iteratsion usullarning turlari.

2.1. ODDIY ITERATSION USUL

Faraz kilaylik,

$$Ax = b \qquad (2.1)$$

tizim biror usul bilan

$$x + Cx + f \qquad (2.2)$$

ko`rinishga keltirilgan bo`lsin, bu erda *S* — qandaydir matritsa, *f* - vektor ustun. Dastlabki yaqinlashish vektori $x^{(0)}$ biror usul bilan (masalan, $x^{(0)} = 0$) topilgan bo`lsin. Agar keyingi yaqinlashishlar

$$x^{(k+1)} = Cx^{(k)} + f \qquad (k=0,1,2,\ \ldots)$$

rekkurent formula yordamida topilsa, bunday usul oddiy iteratsiya usuli deyiladi.

Agarda *S* matritsa elementlari

$$\sum_{i=1}^{n} \left| C_{ij} \right| \le a \le 1 \qquad \left(i = 1,2,\ldots,n \right) \qquad (2.3)$$

va

$$\sum_{i=1}^{n} \left| C_{ij} \right| \le \beta \le 1 \qquad \left(j = 1,2,\ldots,n \right) \qquad (2.4)$$

shartlardan birortasini kanoatlantirsa, u xolda iteratsion jarayon berilgan tenglamaning *x* echimiga ixtiyoriy boshlangich $x^{(0)}$ vektorda yaqinlashishi isbotlangan, ya`ni

$$x = \lim_{k \to \infty} x^{(k)}$$

Shunday kilib, tizimning aniq echimi cheksiz qadamlar natijasida -hosil qilinadi va hosil kilingan ketma-ketlikning ixtiyoriy vektori taqribiy echimni beradi. Bu taqribiy echimning xatoligini quyidagi formulalardan biri orqali ifodalash mumkin:

$$\left(x_i - x_i^{(k)} \right| \le \frac{\alpha}{1-\alpha} \max_{j=1,2,\ldots n} \left| x_j^{(k)} - x_j^{(k-1)} \right| \qquad (2.5)$$

agarda (2.3) shart bajarilsa, yoki

$$\left| x_i - x_i^{(k)} \right| \le \frac{\beta}{1-\beta} \sum_{j=1}^{n} \left| x_j^{(k)} - x_j^{(k-1)} \right| \qquad (2.6)$$

agarda (2.4) shart bajarilsa. Bu baxolarni moc ravishda quyidagicha kuchaytirish mumkin:

$$m\left(x_i - x_i^{(k)}\right) \le \frac{\alpha}{1-\alpha} \max \left| x_i^{(k)} - x_i^{(k-1)} \right|$$

eki

$$\sum_{j=1}^{n} \left| x_i - x_i^{(k)} \right| \le \frac{\beta}{1-\beta} \sum_{j=1}^{n} \left| x_j^{(k)} - x_j^{(k-1)} \right|$$

Iteratsion jarayonlarni yuqoridagi baxolar oldindan berilgan aniqlikni kanoatlantirganda tugallaydilar.

Boshlangich $x^{(0)}$ vektor, umuman olganda, ixtiyoriy tanlanishi mumkin. Ba`zan $x^{(0)} = f$ deb olishadi. Ammo $x^{(0)}$ vektorning komponentlari sifatida noma`lumlarning ko`pol taxminlarda aniqla-ngan qiymatlari olinadi.

(2.1) tizimni (2.2) ko`rinishga keltirishni bir necha xil usullarda amalga oshirish mumkin. Faqat (2.3) yoki (2.4) shartlardan birortasining bajarilishi lozim. Shunday usullardan ikkitasiga tuxtalamiz.

"Birinchi usul. Agarda A matritsaning diagonal elementlari noldan farqli bo`lsa, ya`ni

$$a_{ii} \ne 0 \qquad (I=1,2,\dots,n)$$

u xolda berilgan tizimni

$$\begin{cases} x_1 = \dfrac{1}{a_{11}}\left(b_1 - a_{12}x_2 - \dots - a_{1n}x_n\right) \\ x_2 = \dfrac{1}{a_{22}}\left(b_2 - a_{22}x_1 - a_{23}x_3 - \dots - a_{2n}x_n\right) \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ x_n = \dfrac{1}{a_{nn}}\left(b_n - a_{n1}x_1 - \dots - a_{n,n-1}x_{n-1}\right) \end{cases} \tag{2.7}$$

ko`rinishda yozish mumkin. Bu xolda S matritsa elementlari quyida-gicha aniqlanadi:

$$C_{ij} = -\frac{a_{ij}}{a_{ii}} \ (i \ne j), \quad C_{ii} = 0$$

hamda (2.3)va (2.4)shartlar mos ravishda quyidagi ko`rinishni qabul kiladi:

$$\sum_{\substack{j=1 \\ j\ne i}}^{n} \left| \frac{a_{ij}}{a_{ii}} \right| \le a < 1 \qquad (i = 1,2,\dots,n) \tag{2.8}$$

$$\sum_{i=1}^{n} \left| \frac{a_{ij}}{a_{ii}} \right| \le \beta < 1 \qquad (j = 1,2,...,n) \qquad (2.9)$$

(2.8)va (2.9)tengsizliklar A matritsaning diagonal elementlari

$$|a_{ii}| > \sum_{j \ne i} |a_{ij}| \qquad (i = 1,2,...,n) \qquad (2.10)$$

shartlartlarni kanoatlantirganda urinli bo`ladi.

Ikkinchi usul. Bu usulni quyidagi misol orqali namoyish qilamiz.

Umuman olganda, har qanday keltirilmagan matritsali tizim uchun yaqinlashuvchi iteratsion usullar mavjud, ammo ularning barchasi kisoblash uchun qulay emas.

Agarda iteratsiya usuli yaqinlashuvchi bo`lsa, u xolda bu usul yuko-rida kurilgan usullardan quyidagi afzalliklarga ega bo`ladi:

1. Iteratsion jarayon tezrok yaqinlashsa, ya`ni tizimning echimini aniqlash uchun *p* dan kamrok iteratsiya talab kilinsa, u xolda vaktdan yutiladi, chunki arifmetik emallar soni p^2 ga mutanosib (proportsional) (Gauss usuli uchun esa bu son p^3 ga mutanosib).

2. Yaxlitlash xatoliklari iteratsiya usulida natijaga kamrok ta`-sir etadi. Bundan tashqari iteratsiya usuli o`z xatoligini to`g'rilab boruvchi usuldir.

3. Iteratsiya usuli tizimning muayyan koeffitsientlari nolga teng bo`lgan kolda juda ham qulaylashadi. Bunday tizimlar xususiy hosilali differentsial tenglamalarni echganda ko`prok uchraydi.

4. Iteratsiya jarayonida bir xil turdagi amallar bajariladi, bu esa eX.M uchun programmalashtirishni osonlashtiradi.

1- misol. Quyidagi tizim oddiy iteratsiya usuli bilan yechilsin:

$$\begin{cases} 10x_1 + x_2 - 3x_3 - 2x_4 + x_5 = 6 \\ -x_1 + 25x_2 - x_3 - 5x_4 - 2x_5 = 11 \\ 2x_1 + x_2 - 20x_3 + 2x_4 - 3x_5 = -19 \\ x_2 - x_3 + 10x_4 - 5x_5 = 10 \\ x_1 + 2x_2 - x_3 - 2x_4 - 20x_5 = -32 \end{cases}$$

E c h i s h . Birinchi usulda aytilganidek, bu tizimning tenglamalarini mos ravishda 10, 25, - 20, 10, 20 larga bo`lib, quyidagi ko`rinishda yozib olamiz:

$$\begin{cases} x_1 = 0,6 \ -0,1x_2 \ +0,3x_3 + 0,2x_4 \ - \ 0,1x_5 \\ x_2 = 0,44 + 0,04x_1 - 0,04x_3 + 0,2x_4 + 0,08x_5 \\ x_3 = 0,95 + 0,1x_1 + 0,05x_2 + 0,1x_4 \ - \ 0,15x_5 \\ x_4 = 1 - 0,1x_2 + 0,1x_3 + 0,5x_5 \\ x_5 = 1,6 + 0,05x_1 \ + \ 0,1x_2 + 0,05x_3 \ + \ 0,1x_4 \end{cases}$$

bu erda (2.8)shart bajariladi. Xaqiqatdan ham,

$$\sum_{j=1}^{5} \left| C_{1j} \right| = 0,3 < 1; \qquad \sum_{j=1}^{5} \left| C_{2j} \right| = 0,28 < 1;$$

$$\sum_{j=1}^{5} \left| C_{3j} \right| = 0,41 < 1; \qquad \sum_{j=1}^{5} \left| C_{4j} \right| = 0,5 < 1;$$

$$\sum_{j=1}^{5} \left| C_{5j} \right| = 0,3 < 1;$$

Dastlabki yaqinlashish $x^{(0)}$ sifatida ozod xadlar ustuni (0,6; 0,44; 0,95; 1; 1,6) ni olib keyingi yaqinlashishlarni topamiz:

$x_1^{(1)} = 0,6 - 0,1x_2^{(0)} + 0,3x_3^{(0)} + 0,2x_4^{(0)} - 0,1x_5^{(0)} =$

0,6 – 0,1 · 0,44 + 0,3 · 0,95 + 0,2 · 1 – 0,1 · 1,6 = 0.881

$x_2^{(1)}$= 0,44 + 0.04 · 0,6 – 0,04 · 0,95 + 0,2 · 1 + 0,08 · 1,6 = 0,754

Shunga o`xshash $x_3^{(1)}$= 0,892; $x_4^{(1)}$= 1,851; $x_5^{(1)}$= 1,72. Hisoblashlarning davomini 1- jadvalda keltiramiz:

1-jadval

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$x_4^{(k)}$	$x_5^{(k)}$
0	0,6	0,44	0,95	1	1,6
1	0.881	0,754	0.892	1,851	1,72
2	0.9884	0.9482	1,0029	1,9147	1,9859
3	0,9904	0,9814	0,9908	1,9939	1,9854
4	0,99944	0.99753	0,99789	1,99364	1.99897
5	0,99839	0,99865	0,99929	1,99954	1,99970
6	0.99986	0,99989	0,99977	1,99976	1.99960
7	0,999934	0,999920	1,000018	1,999788	1,999947
8	0.999974	0,999951	0,999976	2,000042	1,999978

Yuqoridagi 3.4- jadvaldan ko`ramizki, 8-iteratsiya x_1= 0,999974; x_2= 0,99951; x_3= 0,99998; x_4 = 2,00004; x_5= 1,99998 echimdan iborat. Bu topilgan taqribiy echim aniq echim

$$x_1^* = x_2^* = x_3^* = 1; \qquad x_4^* = x_5^* = 2$$

dan beshinchi xonaning birliklari buyichagina farqlanadi.

2- misol.

$$\begin{cases} 1{,}02x_1 - 0{,}05x_2 - 0{,}10x_3 = 0{,}795 \\ -0{,}11x_1 - 1{,}03x_2 - 0{,}05x_3 = 0{,}849 \\ -0{,}11x_1 - 0{,}12x_2 + 1{,}04x_3 = 1{,}398 \end{cases}$$

tizimni Z ta iteratsiya bajarib eching va xatoligini baxolang.

E c h i s h . Berilgan tizim-matritsaning diaganal elementlari birga yaqin, kolganlari esa birdan ancha kichik.

Shu sababli iteratsiya usulini qo`llash uchun berilgan tizimni quyidagicha yozib olamiz:

$$x_1 = 0{,}795 - 0{,}02x_1 + 0{,}05x_2 + 0{,}10x_3;$$
$$X_2 = 0{,}849 + 0{,}11x_1 - 0{,}03x_2 + 0{,}05x_3;$$
$$x_3 = 1{,}398 + 0{,}11x_1 + 0{,}12x_2 - 0{,}04x_3.$$

(2.8)yaqinlashish sharti bu tizim uchun bajariladi. Xakikatan ham,

$$\sum_{j=1}^{3} \left| C_{1j} \right| = 0{,}02 + 0{,}05 + 0{,}10 = 0{,}17 < 1$$
$$\sum_{j=1}^{3} \left| C_{2j} \right| = 0{,}11 + 0{,}03 + 0{,}05 = 0{,}19 < 1$$
$$\sum_{j=1}^{3} \left| C_{3j} \right| = 0{,}11 + 0{,}12 + 0{,}04 = 0{,}27 < 1$$

Boshlangich yaqinlashish $x^{(0)}$ sifatida ozod xadlar ustuni elementlarini ikki xona aniqlikda olamiz

$$x^{(0)} = \begin{pmatrix} 0{,}80 \\ 0{,}85 \\ 1{,}40 \end{pmatrix}$$

Endi ketma-ket quyidagilarni aniqlaymiz:

k = 1 da

$x_1^{(1)}$ = 0,795 – 0,016 + 0,0425 + 0,140 = 0,9615 ≈ 0,962

$x_2^{(1)}$ = 0,849 + 0,088 – 0,255 + 0,070 = 0,9815 ≈ 0,982

$x_3^{(1)} = 1,398 + 0,088 + 0,1020 – 0,056 =1,532$

k = 2 da

$x_1^{(3)} = 0,980,$ $x_2^{(3)} = 1,004,$ $x_3^{(3)} = 1,563$

k = 3 da

$x_1^{(3)} = 0,980,$ $x_2^{(3)} = 1,004,$ $x_3^{(3)} = 1,563$

Noma`lumlarning k=2 va k=3 dagi kiimatlari $3\cdot 10^{-3}$ dan kamrok farq kilayapti, shuning uchun noma`lumlarning taqribiy qiymatlari sifatida

$$x_1 \approx 0,980, \qquad x_2 \approx 1,004, \qquad x_3 \approx 1,563$$

larni olamiz.

2.2. ZEYDEL USULI

Zeydel usuli chiziqli bir qadamli birinchi tartibli iteratsion usuldir. Bu usul oddiy iteratsion usuldan shu bilan farq qiladiki, dastlabki yaqinlashish $x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}$ga ko`ra $x_1^{(1)}$ topiladi. So`ngra $x_1^{(1)}, x_2^{(0)}, \ldots, x_n^{(0)}$ ko`ra $x_2^{(1)}$topiladi va x.k. Barcha $x_1^{(1)}$ lar aniqlangandan so`ng $x_i^{(2)}, x_i^{(3)}, \ldots$lar topiladi. Aniqroq aytganda, hisoblashlar quyidagi tarx (sxema) buyicha olib boriladi:

$$x_1^{(k+1)} = \frac{b_1}{a_{11}} - \sum_{j=2}^{n} \frac{a_{1j}}{a_{11}} x_j^{(k)}$$

$$x_2^{(k+1)} = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}} x_1^{(k+1)} - \sum_{j=3}^{n} \frac{a_{2j}}{a_{22}} x_j^{(k)}$$

$$x_i^{(k+1)} = \frac{b_i}{a_{ii}} = \frac{b_i}{a_{ii}} - \sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^{(k+1)} - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{(k)}$$

$$x_n^{(k+1)} = \frac{b_n}{a_{nn}} - \sum_{j=1}^{n-1} x_j^{(k+1)}$$

Odiy iteratsiya usulidagi yaqinlashish shartlari Zeydel usuli uchun ham urinlidir. Ko`pincha Zeydel usuli oddiy iteratsiya usuliga nisbatan yaxshirok yaqinlashadi, ammo har doim ham bunday bulavermaydi. Bundan tash-kari Zeydel usuli programmalashtirish uchun qulaydir, chunki $x_i^{(k+1)}$ ning qiymati hisoblanayotganda $x_1^{(k)}, \ldots, x_{i-1}^{(k)}$ larning qiymatini saklab kolishning xojati yo`q.

Misol. Zeydel usuli bilan 2.1 dagi 1- misolning echimi 5 xona aniqlikda topilsin.

E c h i s h . Tizimni

$$x_1=0,6 - 0,1x_2 + 0,3x_3 + 0,2x_4 - 0,1x_5,$$
$$x_2 = 0,44 + 0,04x_1 - 0,04x_3 + 0,2x_4 + 0,08x_5,$$
$$x_3 = 0,95 + 0,1x_1 + 0,05x_2 + 0,1x_4 - 0,15x_5,$$
$$x_4 = 1 - 0,1x_2 + 0,1x_3 + 0,5x_5,$$
$$x_5 = 1,6 + 0,05x_1 + 0,1x_2 + 0,05x_3 + 0,1x_4$$

ko`rinishda yozib olamiz va dastlabki yaqinlashish x sifatida oddiy iteratsiya usulidagidek x =(0,6; 0,44; 0,95;1; 1,6) deb olamiz.

Iteratsiyaning birinchi qadamini bajaramiz:

$$x_1^{(1)} = 0,6 – 0,1\ x_2^{(0)} + 0,3x_3^{(0)} +0,2x_4^{(0)} – 0,1x_5^{(0)} =$$
$$=0,6 – 0,1 \cdot 0,44 + 0,3 \cdot 0,95 + 0,2 \cdot 1 – 0,1 \cdot 1,6 = 0,881$$
$$x_2^{(1)} = 0,44 + 0,04\ x_1^{(4)} - 0,04x_3^{(0)} +0,2x_4^{(0)} + 0,08x_5^{(0)} =$$
$$= 0,44 + 0,04 \cdot 0,881 - 0,04 \cdot 0,95 + 0,2 \cdot 1 – 0,08 \cdot 1,6 = 0,771$$
$$x_3^{(1)} = 0,95 + 0,1\ x_1^{(1)} + 0,05x_2^{(1)} +0,1x_4^{(0)} – 0,1x_5^{(0)} =$$
$$= 0,95 + 0,1 \cdot 0,881 + 0,05 \cdot 0,771 + 0,1 \cdot 1 – 0,15 \cdot 1,6 = 0,937$$
$$x_4^{(1)} = 1 – 0,1\ x_2^{(1)} + 0,1x_3^{(1)} +0,5x_5^{(0)} = 1,817$$
$$x_5^{(1)} = 1,6 + 0,05x_1^{(1)} + 0,1x_2^{(1)} + 0,05x_3^{(1)} +0,1x_4^{(1)} = 1,948$$

Keyingi yaqinlashishlarni 2- jadvalda keltiramiz:

2-jadval

K	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$x_4^{(k)}$	$x_5^{(k)}$
0	0,6	0,44	0,95	1	1,6
1	0.881	0,771	0,937	1,817	1,948
2	0,973	0,961	0,985	1.974	1,992
3	0,995	0,995	0,999	1,996	1,999
4	0,9995	0,9991	0,9997	1,9995	1,9998
5	0,99992	0,99989	0,99997	1.99991	1,99997
6	0,99999	0,99998	0,99999	1,99999	2.00000

Ko`rinib turibdiki, Zeydel usuli oddiy iteratsiya usuliga nisbatan tezrok yaqinlashmokda.

2.3.Usullarning ishchi algoritmlari.

3.1-masala. Oddiy iteratsiya usuli bilan quyidagi chiziqli tenglamalar sistemasini $\varepsilon=10^{-3}$ aniqlikda yeching.

$$\left.\begin{array}{l} 20.91x_1 + 1.2x_2 + 2.1x_3 + 0.9x_4 = 21.70 \\ 1.2x_1 + 21.2x_2 + 1.5x_3 + 2.5x_4 = 27.46 \\ 2.1x_1 + 1.5x_2 + 19.8x_3 + 1.3x_4 = 28.76 \\ 0.9x_1 + 2.5x_2 + 1.3x_3 + 32.1x_4 = 49.72 \end{array}\right\} \quad (3.1)$$

Yechish. Berilgan (3.1) sistemani ushbu ko‘rinishga keltiramiz:

$$x_1 = \frac{1}{20.9}(21.7 - 1.2x_2 - 2.1x_3 - 0.9x_4)$$

$$x_2 = \frac{1}{21.2}(27.46 - 1.2x_2 - 1.5x_3 - 2.5x_4)$$

$$x_3 = \frac{1}{19.8}(28.76 - 2.1x_1 - 1.5x_2 - 1.3x_4)$$

$$x_4 = \frac{1}{32.1}(49.72 - 0.9x_1 - 2.5x_2 - 1.3x_3)$$

hosil bo‘lgan sistemaning koeffitsientlari ushbu shartni qanoatlantirishini tekshiramiz.

$$\sum_{j\neq 1}\left|C_{1j}\right| \approx 0.20 < 1, \qquad \sum_{j\neq 2}\left|C_{2j}\right| \approx 0.24 < 1,$$

$$\sum_{j\neq 3}\left|C_{3j}\right| \approx 0.25 < 1, \qquad \sum_{j\neq 4}\left|C_{4j}\right| \approx 0.15 < 1,$$

iteratsiya jarayoni yaqinlashuvchi bo‘lib, oxirgilardan $\alpha = 0.25<1$ ekanligini olamiz. Bunda $\frac{\alpha}{1-\alpha} = \frac{1}{3}$ bo‘ladi. Sistemaning ozod hadlarini boshlang‘ich $\vec{x}^{(0)}$ vektorning mos elementlari uchun qabul qilamiz, ya’ni

$$\vec{x}^{(0)} = \begin{pmatrix} 21.7/20.9 \\ 27.46/21.2 \\ 28.76/19.8 \\ 49.72/32.1 \end{pmatrix} = \begin{pmatrix} 1{,}04 \\ 1{,}03 \\ 1{,}45 \\ 1{,}55 \end{pmatrix}$$

Hisoblash jarayonini

$$\max \left| x_i^{(k)} - x_i^{(k-1)} \right| \leq \frac{0.001}{1/3}, \qquad (i = 1,2,3,4)$$

shart bajarilguncha davom ettiramiz.

Hisoblashni ketma-ket bajara borib, quyidagilarni olamiz:

K=1 da

$$x_1^{(1)} = \frac{1}{20.9}(21.7 - 1.56 - 3.045 - 1.395) = 0.75$$
$$x_2^{(1)} = \frac{1}{21.2}(27.46 - 1.248 - 2.175 - 3.875) = 0.95$$
$$x_3^{(1)} = \frac{1}{19.8}(28.76 - 2.184 - 1.95 - 2.015) = 1.14$$
$$x_4^{(1)} = \frac{1}{32.1}(49.72 - 0.936 - 3.25 - 1.885) = 1.36$$
$$\max\left|x_i^{(1)} - x_i^{(0)}\right| = \max\{0,29; 0,08; 0,34; 0,19\} = 0,34 > \frac{0,001}{1/3}.$$

K=2 bo‘lganda

$$x_1^{(2)} = \frac{16.942}{20.9} = 0.8106, \qquad x_3^{(2)} = \frac{23.992}{19.8} = 1.2117$$
$$x_2^{(2)} = \frac{21.450}{21.2} = 10118, \qquad x_4^{(2)} = \frac{45.1888}{32.1} = 1.4077$$
$$\max\left|x_i^{(2)} - x_i^{(1)}\right| = \max\{0,0606; 0,0618; 0,0717; 0,0477\} = 0,0717 > \frac{0,001}{1/3} = 3 \cdot 0,001$$

K=3 bo‘lganda

$$x_1^{(3)} = \frac{16.67434}{20.9} = 0.7978, \qquad x_3^{(3)} = \frac{23.71003}{19.8} = 1.1975$$
$$x_2^{(3)} = \frac{21.1503}{21.2} = 0.9977, \qquad x_4^{(3)} = \frac{44.88575}{32.1} = 1.3983$$
$$\max\left|x_i^{(3)} - x_i^{(2)}\right| = \max\{0,0128; 0,0113; 0,0112; 0,0072\} = 0,0128 > \frac{0,001}{1/3} = 3 \cdot 0,001$$

K=4 bo‘lganda

$$x_1^{(4)} = \frac{16.7295}{20.9} = 0.8104, \qquad x_3^{(4)} = \frac{23.7703}{19.8} = 1.2005$$
$$x_2^{(4)} = \frac{21.2106}{21.2} = 1.0005, \qquad x_4^{(4)} = \frac{44.9510}{32.1} = 1.4003$$
$$\max\left|x_i^{(4)} - x_i^{(3)}\right| = \max\{0,0126; 0,0028; 0,0030; 0,0020\} = 0,0126 > 3 \cdot 0,001.$$

K=5 bo‘lganda

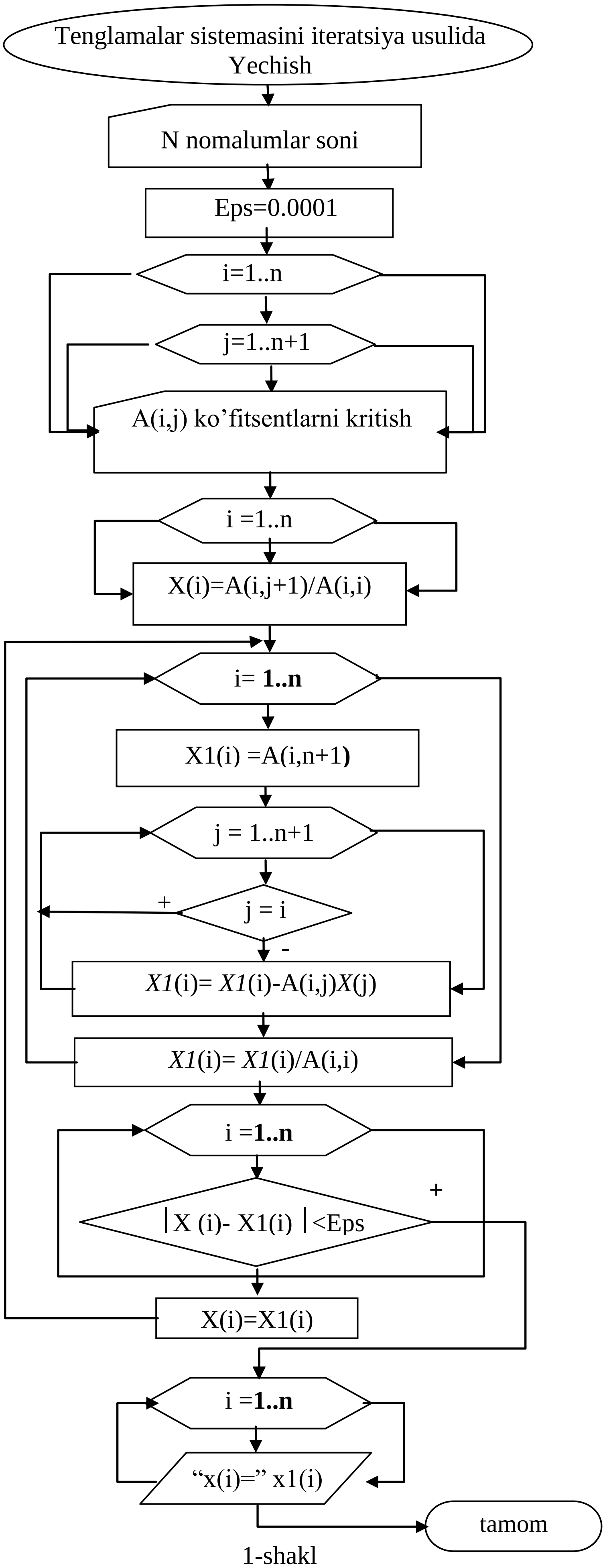
$$x_1^{(5)} = \frac{16.71809}{20.9} = 0.7999, \qquad x_3^{(5)} = \frac{23.7582}{19.8} = 1.1999$$
$$x_2^{(5)} = \frac{21.19802}{21.2} = 0.9999, \qquad x_4^{(5)} = \frac{44.93774}{32.1} = 1.3999$$
$$\max\left|x_i^{(5)} - x_i^{(4)}\right| = \max\{0,0105; 0,0006; 0,0006; 0,0004\} = 0,0105 > 3 \cdot 0,001.$$
$$\left|x_1^{(4)} - x_1^{(5)}\right| = 0.0105, \qquad \left|x_3^{(4)} - x_3^{(5)}\right| = 0.0006$$
$$\left|x_2^{(4)} - x_2^{(5)}\right| = 0.0006, \qquad \left|x_4^{(4)} - x_4^{(5)}\right| = 0.0004$$

K=6 uchun hisoblash kerak:

$$x_1^{(6)} = \frac{16,7204}{20,9} = 0,8000; \qquad x_3^{(6)} = \frac{23,7604}{19,8} = 1,2000;$$
$$x_2^{(6)} = \frac{21,2004}{21,2} = 1,0000; \qquad x_4^{(6)} = \frac{44,9404}{32,1} = 1,4000;$$
$$\max\left|x_i^{(6)} - x_i^{(5)}\right| = \max\{0.0001; 0.0001; 0.0001\} = 0.0001 < 3 \cdot 0.001$$

Demak, sistemaning yechimi

x_1=0.8000, x_2=1.0000, x_3=1.2000, x_4=1.4000.


Tenglamalar sistemasini iteratsiya usulida Yechish
N nomalumlar soni
Eps=0.0001
i=1..n
j=1..n+1
A(i,j) ko'fitsentlarni kritish
i =1..n
X(i)=A(i,j+1)/A(i,i)
i= 1..n
X1(i) =A(i,n+1)
j = 1..n+1
j = i
+
-
X1(i)= X1(i)-A(i,j)X(j)
X1(i)= X1(i)/A(i,i)
i =1..n
|X (i)- X1(i) |<Eps
+
–
X(i)=X1(i)
i =1..n
"x(i)=" x1(i)
tamom

1-shakl

Oddiy iteratsiya usuli asosida chiziqli tenglamalar sistemasining hisoblash dasturini tuzamiz:

```
{ * --3.11- Paskal tilida dastur - *}
 uses crt;
label 40,90,100;
var
   n,i,j:integer;
   c,c1:real;
   a:array[1..5,1..5] of real;
   b:array[1..5] of real;
   x:array[1..5] of real;
   x1:array[1..5] of real;
begin
    clrscr;
    writeln(' Oddiy iteratsiya usulida ');
    writeln(‘ chiziqli tenglamalar sistemasini yechish’);
    write('tenglamalar soni N=');
    readln(n);
    for i:=1 to n do
    begin
          for j:=1 to n do
                    begin
                    gotoxy(16*j,4*i);
                    write(‘a[‘,i,’:’,j,’]=’);
                    read(a[i,j]);
                    end;
       gotoxy(22*j,4*i);
       write(‘b[‘,i,’]=’);
       read(b[i]);
    end;
    for i:=1 to n do
    begin
        c:=a[i,i];
        for j:=1 to n do a[i,j]:=a[i,j]/c;
        b[i]:=b[i]/c;
        x[i]:=b[i];
   end;
40:   for i:=1 to n do  a[i,i]:=0;
      for i:=1 to n do
   begin
      c1:=0;
      for j:=1 to n do c1:=c1+a[i,j]*x[j];
      x1[i]:=b[i]-c1;
```

```
  end;
    for i:=1 to n do
    if abs(x[i]-x1[i])>0.01 then goto 90;
    goto 100;
90:   for i:=1 to n do x[i]:=x1[i];
    goto 40;
100:   clrscr;
    writeln('YECHIM:');
    for i:=1 to n do
    writeln('x[',i,']=',x[i]);
    readln;
      end.
```

Oddiy iteratsiya usulida chiziqli tenglamalar sistemasini yechish

tenglamalar soni N=4

a[1,1]=20.9	a[1,2] =1.2	a[1,3]:=2.1	a[1,4]=0.9	a[1,5]=21.7
a[2,1]=1.2	a[2,2]=21.2	a[2,3]=1.5	a[2,4]=2.5	a[2,5]=27.46
a[3,1]=2.1	a[3,2]=1.5	a[3,3]=19.8	a[3,4]=1.3	a[3,5]:=28.76
a[4,1]=0.9	a[4,2]=2.5	a[4,3]=1.3	a[4,4]=32.1	a[4,5]=49.72

YECHIM:

x[1]=0.7999
x[2]=0.9999
x[3]=1.1999
x[4]=1.3999

3.2 – misol. Quyidagi chiziqli tenglamalar sistemasini iteratsiya usuli bilan 10^{-3} aniqlikda yeching.

$$\left.\begin{array}{l} 1.02x_1 - 0.05x_2 - 0.10x_3 = -0.795 \\ -0.11x_1 + 0.97x_2 - 0.05x_3 = 0.849 \\ -0.11x_1 - 0.12x_2 + 1.04x_3 = 1.398 \end{array}\right\} \quad (*)$$

Yechish. Berilgan sistema matritsasining diagonal elementlari birga yaqin bo'lib, qolganlari modul jihatdan birdan ancha kichik. Iteratsiya usulini qo'llab yechish uchun (*) sistemani quyidagi ko'rinishga keltiramiz:

$$\left.\begin{array}{l} x_1 = 0.795 - 0.02x_1 + 0.05x_2 + 0.1x_3 \\ x_2 = 0.849 + 0.11x_1 + 0.03x_2 + 0.05x_3 \\ x_3 = 1.398 + 0.11x_1 + 0.12x_2 - 0.04x_3 \end{array}\right\}$$

Olingan bu sistema uchun yaqinlashish shartini tekshiramiz:

$$\sum_{j=1}^{3} |c_{1j}| = 0.02+0.05+0.10=0.17<1;$$

$$\sum_{j=1}^{3} |c_{2j}| = 0.11+0.05+0.03=0.19<1;$$

$$\sum_{j=1}^{3} |c_{3j}| = 0.11+0.12+0.04=0.27<1.$$

Bulardan, α=0.27<1 bo'lib,

$$\frac{\alpha}{1-\alpha}=\frac{0.27}{1-0.27}=0.369...<0.37$$

Demak, hosil bo‘lgan oxirgi sistemaga qo‘llaniladigan iteratsiya yaqinlashuvchi bo‘lar ekan.

Boshlang‘ich $\vec{x}^0$ vektorning elementlari sifatida ozod hadlarni verguldan so‘ng ikki xonagacha aniqlik bilan quyidagicha tanlaymiz:

$$\vec{x}^0=\begin{pmatrix}0.80\\0.85\\1.40\end{pmatrix}$$

Endi hosil bo‘lgan sistemaga iteratsiya usulini qo‘llash bilan yechimni ketma-ket quyidagicha topamiz:

K=1 bo‘lganda

$$x_1^{(1)}=0.795-0.013+0.0425+0.140=0.9613$$

$$x_2^{(1)}=0.849+0.088-0.0255+0.070=0.9813$$

$$x_3^{(1)}=1.398+0.088+0.1020-0.056=1.532$$

K=2 bo‘lganda

$$x_1^{(2)}\approx 0.978,\ x_2^{(2)}\approx 1.002,\ x_3^{(2)}\approx 1.560$$

K=3 bo‘lganda

$$x_1^{(3)}\approx 0.980,\ x_2^{(3)}\approx 1.004,\ x_3^{(3)}\approx 1.563$$

K=2 va K=3 bo‘lganda yechim qiymatlarining farqi modul jihatdan $0{,}37\cdot 10^{-3}$ dan katta emas, shuning uchun taqribiy yechimni quyidagicha olamiz:

$$x_1\approx 0.980,\ x_2\approx 1.004,\ x_3\approx 1.563$$

2.4. Gauss – Zeydelning iteratsiya usuli

Quyidagi chiziqli tenglamalar sistemasini Gayss-Zeydel usulida yechamiz.

$$\begin{cases}a_{11}x_1+a_{12}x_2+a_{13}x_3+a_{14}x_4=b_1\\a_{21}x_1+a_{22}x_2+a_{23}x_3+a_{24}x_4=b_2\\a_{31}x_1+a_{32}x_2+a_{33}x_3+a_{34}x_4=b_3\\a_{41}x_1+a_{42}x_2+a_{43}x_3+a_{44}x_4=b_4\end{cases}\qquad(4.1)$$

Aytaylik, $a_{ii}\neq 0$ i=1,2,3,4 bo‘lsin. Berilgan sistemani

$$\begin{cases}x_1=(b_1-a_{12}x_2-a_{13}x_3-a_{14}x_4)/a_{11}\\x_2=(b_2-a_{11}x_1-a_{23}x_3-a_{24}x_4)/a_{22}\\x_3=(b_3-a_{31}x_1-a_{32}x_2-a_{34}x_4)/a_{33}\\x_4=(b_4-a_{41}x_1-a_{42}x_2-a_{43}x_3)/a_{44}\end{cases}\qquad(4.2)$$

ko‘rinishga keltiramiz.

Bu sistemaning yechimini topish uchun birorta boshlang‘ich yaqinlashishni tanlab

$$x_1^{(0)}, x_2^{(0)}, x_3^{(0)}, x_4^{(0)}$$

larni olamiz. Bu boshlang‘ich yaqinlashish asosida (4.2) tenglamaning birinchi tenglamasidan

$$x_1^{(1)} = (b_2 - a_{12}x_2^{(0)} - a_{13}x_3^{(0)} - a_{14}x_4^{(0)}) / a_{11}$$

ikkinchi tenglamasidan

$$x_2^{(1)} = (b_2 - a_{21}x_1^{(1)} - a_{23}x_3^{(0)} - a_{24}x_4^{(0)}) / a_{22}$$

uchinchi tenglamasidan

$$x_3^{(1)} = (b_3 - a_{31}x_1^{(1)} - a_{32}x_2^{(1)} - a_{34}x_4^{(0)}) / a_{33}$$

to‘rtinchi tenglamasidan esa

$$x_4^{(1)} = (b4 - a_{41}x_1^{(1)} - a_{42}x_2^{(1)} - a_{43}x_3^{(1)}) / a_{44}$$

larni hisoblab topamiz.

Xuddi shu yo‘l bilan *k-1* yaqinlashish asosida *k*-chi yaqinlashishni quyidagicha topamiz:

$$x_1^{(k)} = (b_1 - a_{12}x_2^{(k-1)} - a_{13}x_3^{(k-1)} - a_{14}x_4^{(k-1)}) / a_{11}$$
$$x_2^{(k)} = (b_2 - a_{21}x_1^{(k)} - a_{23}x_3^{(k-1)} - a_{24}x_4^{(k-1)}) / a_{22}$$
$$x_3^{(k)} = (b_3 - a_{31}x_1^{(k)} - a_{32}x_2^{(k)} - a_{34}x_4^{(k-1)}) / a_{33}$$
$$x_4^{(k)} = (b_4 - a_{41}x_1^{(k)} - a_{42}x_2^{(k)} - a_{34}x_{43}^{(k)}) / a_{44}$$

Umuman, agar (3.2) tenglamalar sistemasi o‘rniga n noma’lum n chiziqli tenglamalar sistemasi berilgan bo‘lib, $a_{ii} \neq 0, i = \overline{1,n}$ bo‘lsa, *k*-yaqinlashish uchun

$$x_i^{(k)} = (b_i - a_{i1}x_1^{(k)} - \ldots - a_{i,i-1}x_{i-1}^{(k)} - a_{i,i+1}x_{i+1}^{(k-1)} - \ldots - a_{i,n_{-1}}x_{n-1}^{(k-1)}) / a_{ii}$$

formula hosil bo‘ladi.

Iteratsiya jarayoni

$$\max\left|x_i^{(k)} - x_i^{(k-1)}\right| < \varepsilon$$

shart bajarilguncha davom etadi (ε>0 berilgan aniqlik).

Bu iteratsiya jarayonining yaqinlashishi uchun

$$\sum_{j=1, i \neq j} |a_{ij}| < |a_{ii}|, \qquad i = \overline{1,n} \qquad (4.3)$$

tengsizliklarning bajarilishi etarlidir.

3.3-misol. Quyidagi chiziqli tenglamalar sistemasi $\varepsilon=10^{-3}$ aniqlikda Zeydel usuli bilan yeching.

$$\begin{cases} 20.9x_1 + 1.2x_2 + 2.1x_3 + 0.9x_4 = 21.7 \\ 1.2x_1 + 21.9x_2 + 1.5x_3 + 2.5x_4 = 27.46 \\ 2.1x_1 + 1.5x_2 + 19.8x_3 + 1.3x_4 = 28.76 \\ 0.9x_1 + 2.5x_2 + 1.3x_3 + 32.1x_4 = 49.72 \end{cases}$$

Yechish. Bu tenglamalar sistemasi uchun (4.3) shartning bajarilishini tekshirib ko‘rish qiyin emas. Uni (3.40) ko‘rinishga keltiramiz.

x_1=(21.70-1.2x_2-2.1x_3-0.9x_4)/20.9
x_2=(27.46-1.2x_1-1.5x_3-2.5x_4)/21.2
x_3=(28.76-2.1x_1-1.5x_2-1.3x_4)/19.8
x_4=(49.72-0.9x_1-2.5x_2-1.3x_3)/32.1

boshlang‘ich $\vec{x}^0$ vektorni

$$\vec{x}^0=\begin{pmatrix}1.04\\1.30\\1.45\\1.55\end{pmatrix} \quad eku \quad x_1^{(0)}=1.04,\ x_2^{(0)}=1.3,\ x_3^{(0)}=1.45,\ x_4^{(0)}=1.55$$

kabi tanlab, Zeydel usulini qo‘llaymiz.

K=1 deb, birinchi yaqinlashishni topamiz:

$$x_1^{(1)}=(21.7\text{-}1.1\,x_2^{(0)}\text{-}2.1\,x_3^{(0)}\text{-}0.9\,x_4^{(0)})/20.9=$$
$$=(21.7\text{-}1.56\text{-}3.045\text{-}1.395)/20.9=0.7512$$
$$x_2^{(1)}=(27.46\text{-}1.2\,x_1^{(1)}\text{-}1.5\,x_3^{(0)}\text{-}2.5\,x_4^{(0)})/21.2=$$
$$=(27.46\text{-}0.900\text{-}2.175\text{-}3.875)=0.9674$$
$$x_3^{(1)}=(28.76\text{-}2.1\,x_2^{(1)}\text{-}1.5\,x_2^{(1)}\text{-}1.3\,x_4^{(0)})/19.8=$$
$$=(28.76\text{-}1.575\text{-}1.455\text{-}2.013)=1.1977$$
$$x_4^{(1)}=(49.72\text{-}0.9\,x_1^{(1)}\text{-}2.5\,x_2^{(1)}\text{-}1.3\,x_3^{(1)})/32.1=$$
$$=(49.72\text{-}0.675\text{-}2.425\text{-}1.56)=1.4037$$

$$\max\left|x_i^{(1)}-x_i^{(0)}\right|=\max\{0{,}2888;\ 0{,}3004;\ 0{,}2504;\ 0{,}1500\}=0{,}3004>10^{-3}.$$

K=2 bo‘lganda: $x_1^{(2)}$=13.76062/20.9=0,6558

$x_2^{(2)}$=21.9902/21.2=0.9996

$x_3^{(2)}$=23.75180/19,8=1,19959

$x_4^{(2)}$=44.93971/32.1=1.4000

$$\max\left|x_i^{(2)}-x_i^{(1)}\right|=\max\{0{,}0954;\ 0{,}0322;\ 0{,}0019;\ 0{,}0037\}=0{,}0954>10^{-3}.$$

K=3 bo‘lganda: $x_1^{(3)}$=13.7213/20.9=0.6557

$x_2^{(3)}$=21.200528/21.2=1.00002

$x_3^{(3)}$=23.759844/19.8=1.19999

$x_4^{(3)}$=44.939909/32.1=1.4000

$$\max\left|x_i^{(3)}-x_i^{(4)}\right|=\max\{0{,}0001;\ 0{,}0004;\ 0{,}0004; 0{,}3=0{,}0004<\varepsilon=10^{-3}\}$$

Bu qadam uchun yaqinlashish sharti bajarildi, demak, berilgan aniqlikdagi yechim:

$$x_1\approx 0{,}656;\quad x_2\approx 1{,}000;\quad x_3\approx 1{,}200;\quad x_4=1{,}400.$$

CHiziqli tenglamalar sistemasini Zeydel usuli bilan hisoblash dasturini beramiz:

```
{ Delphi tilida dastur}
uses crt;
label 40,90,100;
var
```

```
   n,i,j:integer;
   c,c1:real;
   a:array[1..5,1..5] of real;
   b:array[1..5] of real;
   x:array[1..5] of real;
   x1:array[1..5] of real;
begin
    clrscr;
    writeln(' Zeydel usulida xisoblash ');
    write('Tenglamalar sonini kiriting‘);
    write('  n=');
    readln(n);
    for i:=1 to n do
    begin
          for j:=1 to n do
                    begin
                    gotoxy(16*j,4*i);
                    write(‘a[‘,i,’:’,j,’]=’);
                    read(a[i,j]);
                    end;
       gotoxy(22*j,4*i);
       write(‘b[‘,i,’]=’);
       read(b[i]);
    end;
    for i:=1 to n do
    begin
        c:=a[i,i];
        for j:=1 to n do a[i,j]:=a[i,j]/c;
        b[i]:=b[i]/c;
        x[i]:=b[i];
   end;
40:   for i:=1 to n do  a[i,i]:=0;
      for i:=1 to n do
   begin
      c1:=0;
      for j:=1 to n do c1:=c1+a[i,j]*x[j];
      x1[i]:=b[i]-c1;
   end;
       for i:=1 to n do
       if abs(x[i]-x1[i])>0.01 then goto 90;
       goto 100;
90:    for i:=1 to n do x[i]:=x1[i];
       goto 40;
100:   clrscr;
       writeln(‘Sistema yechimlari:’);
```

```
    for i:=1 to n do
    writeln('x[',i,']=',x[i]);
    readln;
readln;
end.
```

Chiziqli tenglamalar sistemasining yechimini Zydel uculida ehisoblash
Nomalumlar sonini kriting n=4

a[1,1]=20.9 a[1,2] =1.2 a[1,3]:=2.1 a[1,4]=0.9 a[1,5]=21.7
a[2,1]=1.2 a[2,2]=21.2 a[2,3]=1.5 a[2,4]=2.5 a[2,5]=27.46
a[3,1]=2.1 a[3,2]=1.5 a[3,3]=19.8 a[3,4]=1.3
a[3,5]:=28.76
a[4,1]=0.9 a[4,2]=2.5 a[4,3]=1.3 a[4,4]=32.1 a[4,5]=49.72

Sistema yechimi

x[1]=0.7999
x[2]=0.9999
x[3]=1.1999
x[4]=1.3999

XULOSA

Bugungi kunda chiziqli algebraik teglamalar sistemasini taqribiy yechish usullari keng qo'llanilmoqda. Masalaning yechimini topishning aniq usullarida hisoblash jarayonida yo'l qo'yilgan xatoliklar masala yechimiga jiddiy ta'sir ko'rsatadi. Shuning uchun ham taqribiy usullardan biri bo'lgan iteratsion usullar bugungi kunda ommabop usullardan hisoblanadi.

Ushbu kurs ishini bajarish davomida quyidagi ishlar amalga oshirildi:

- ❖ Iteratsion usullar haqida qisqacha ma'lumotlar berib o'tildi;
- ❖ Oddiy iteratsiya usulining tavsifi keltirildi;
- ❖ Zeydel usulining tavsifi keltirildi;
- ❖ Gauss- Zeydelning iteratsion usuli tavsifi keltirildi;
- ❖ Usullarning ishchi algoritmlari keltirildi.

Ushbu ishda keltirilgan ma'lumotlardan taqribiy yechishning iteratsion usullarini qo'llashni yo'lga qo'yishni , ya'ni kengroq o'rganishni istovchilarning faydalanishlari yaxshi natija beradi deb hisoblaymiz.

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