Urganch Davlat universiteti fizika-matematika fakulteti 301-amaliy matematika va informatika guruh talabasi Iskandarov Jo‘shqinning hisoblash usullari fanidan “Matritsaning xos son a xos vektorlarini hisoblash” mavzusidagi kurs ishiga

Taqriz

Ushbu ishda dastlab mavzuni yoritish uchun zarur bo’lgan matritsa, matritsaning determinanti, birlik matritsa, matritsanining xos son va xos vektorlari, matritsaning xarakteristik tenglamasi, matritsaning xos yoki xarakteristik ko‘phadi, matritsaning minimal ko‘phadi kabi tayanch tushunchalar yoritilgan.

Matritsanining xos son va xos vektorlarini hisoblash uchun Krilov, Danilevskiy, Lavere va iteratsiya usullari o‘rganilgan va dasturi tuzilib natija olingan.

Talaba Iskandarov Jo‘shqinning “Matritsaning xos son va xos vektorlarini hisoblash” mavzusidagi kurs ishi Oliy va O‘rta maxsus ta’lim Vazirligi tomonidan kurs ishini bajarishga qo‘yilgan talablarga to‘la javob beradi.

Taqrizchi: dots. Xaitmetov U.

Urganch Davlat universiteti fizika-matematika fakulteti 301-amaliy matematika va informatika guruh talabasi Iskandarov Jo‘shqinning hisoblash usullari fanidan “Matritsaning xos son a xos vektorlarini hisoblash” mavzusidagi kurs ishiga

Taqriz

Ushbu ishda dastlab mavzuni yoritish uchun zarur bo’lgan matritsa, matritsaning determinanti, birlik matritsa, matritsanining xos son va xos vektorlari, matritsaning xarakteristik tenglamasi, matritsaning xos yoki xarakteristik ko‘phadi, matritsaning minimal ko‘phadi kabi tayanch tushunchalar yoritilgan. Matritsaning xos qiymat va xos vektorlarini hisoblash Jordan normal formasini hosil qilishda juda qo’l keladi.

Matritsanining xos son va xos vektorlarini hisoblash uchun Krilov, Danilevskiy, Lavere va iteratsiya usullari o‘rganilgan va dasturi tuzilib natija olingan.

Talaba Iskandarov Jo‘shqinning “Matritsaning xos son va xos vektorlarini hisoblash” mavzusidagi kurs ishi Oliy va O‘rta maxsus ta’lim Vazirligi tomonidan kurs ishini bajarishga qo‘yilgan talablarga to‘la javob beradi.

Taqrizchi: “Matematika” kafedrasi

o’qituvchisi Abdullayev J.

IMZONI TASDIQLAYMAN

KADRLAR BO‘LIMI BOSHLIGI

O`ZBEKISTON RESPUBLIKASI OLIY VA O`RTA MAXSUS TA`LIM VAZIRLIGI

URGANCH DAVLAT UNIVERSITETI

FIZIKA-MATEMATIKA FAKULTETI

«Amaliy matematika va matematik fizika» kafedrasi

80

301 – amaliy matematika va informatika

guruxi talabasi Iskandarov Jo'shqinning

hisoblash usullari fanidan

Kurs ishi

Topshirdi: Iskandarov J.

Qabul qildi: Madraximov U.

2014-2015-o'quv yili

Mavzu: Matritsaning xos son va xos vektorlarini hisoblash

Reja:

I. Kirish.

II. Asosiy qism.

1. Krilov metodi.
2. Danilevskiy metodi.
3. Lavere metodi.
4. Moduli bo‘yicha eng katta xos sonni va unga mos xos vektorni topish.

III. Xulosa.

IV. Foydalanilgan adabiyotlar.

Matritsalar haqida umumiy tushunchalar. Sistemalarni modellashtirishda ***matritsalar algebrasi*** degan tushuncha muhim ahamiyatga ega. Rejalashtirish muammolari, yalpi mahsulot, jami mehnat sarfi, narxni aniqlash va boshqa masalalar hamda ularda Kompyuterlarni qo‘llash matritsalar algebrasini qarashga olib keladi. Ishlab chiqarishni rejalashtirish, moddiy ishlab chiqarish orasidagi mavjud bog’lanishlarni ifodalashda va boshqalarda, ma`lum darajada tartiblangan axborotlar sistemasiga asoslangan bo‘lishi lozim. Bu tartiblangan axborotlar sistemasi muayyan jadvallar ko‘rinishida ifodalangan bo‘ladi. Misol o‘rnida moddiy ishlab chiqarish tarmoqlari orasidagi o‘zaro bog’liqlik axborotlari sistemasini qaraylik. Ishlab chiqarish 5 ta (masalan, mashinasozlik, elektroenergiya, metal, ko‘mir, rezina ishlab chiqarish sanoatlari) tarmoqdan iborat bo‘lsin. Bunda ular orasidagi o‘zaro bog’liqlik 1-jadval bilan ifodalansin.

1-jadval

Tarmoq-lar	**1**	**2**	**3**	**4**	**5**
1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}
3	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}
4	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}
5	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}

Bu jadvalda $a_{ij}\ (i, j = 1,2,3,4,5)$ lar bilan, i-tarmoqning j- tarmoqqa yetkazib beradigan (ta`minlaydigan) mahsuloti miqdori belgilangan, chunonchi, a_{21}, a_{22}, ..., a_{25} lar 2-tarmoqning mos ravishda hamma tarmoqlarga; a_{31}, a_{32}, ..., a_{35} lar esa 3-tarmoqning mos ravishda hamma tarmoqlarga yetkazib beradigan mahsulotlari miqdorini bildiradi. a_{22}, a_{33} lar mos ravishda 2,3-tarmoqlarning o‘z ehtiyojlariga sarfini ifodalaydi.

Yuqoridagiga o‘xshash ishlab chiqarish mezoni (normasi) axborotlari sistemasiga sonli misol qaraylik. Korxona 3 turdagi xom ashyo ishlatib 4 xildagi mahsulot ishlab chiqaradigan bo‘lsin, bunda xom ashyo sarfi normasi sistemasi 2-jadval bilan berilgan bo‘lsin.

2-jadval.

Xom ashyolar	Mahsulotlar			
	1	2	3	4
1	2	3	2	0
2	4	0	3	5
3	3	5	2	4

2-jadvalda masalan, 1-turdagi xom ashyo sarfi normasi mos ravishda 1,2,3,4-xildagi mahsulotlar ishlab chiqarish uchun 2,3,2,0 bo‘ladi.

1 va 2 jadvallar, matematikada o‘rganiladigan matritsalar tushunchasining misollari bo‘la oladi. Matritsalar iqtisodiy izlanishlarda keng qo‘llanilmoqda, Hususan, ulardan foydalanish ishlab chiqarishni rejalashtirishni osonlashtirib, mehnat sarfini kamaytiradi, hamda rejaning har xil variantlarini tuzishni ixchamlashtiradi. Bundan tashqari har xil iqtisodiy ko‘rsatkichlar orasidagi bog’liklikni tekshirishni osonlashtiradi. Bu holatlar matritsalarni umumiy holda qarashga olib keladi.

1-ta`rif. m ta satrli va n ta ustunli to‘g’ri burchakli $m \cdot n$ ta elementdan tuzilgan jadval

$$A=\begin{pmatrix} a_{11} & a_{12}\cdots a_{1n} \\ a_{21} & a_{22}\cdots a_{2n} \\ ---- & ---- \\ a_{m1} & a_{m2}\cdots a_{mn} \end{pmatrix}$$

$m\times n$ ***o'lchamli matritsa*** deyiladi. A matritsani qisqacha (a_{ij}) $(i=1,...,m,\ \ j=1,...,n)$ bilan ham belgilash mumkin. Matritsalarda satrlar soni ustunlar soniga teng bo'lsa, bunday matritsalar ***kvadrat matritsa*** deb ataladi. Har bir n tartibli kvadrat matritsa uchun uning elementlaridan tuzilgan *determinantni* hisoblash mumkin, aytaylik, biror *a, b, c, d* sonlar berilgan bo'lsin. Ushbu

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

ifoda *2-tartibli determinant, ad-bc* ayirma esa uning qiymati deyiladi. det $A=0$ bo'lsa, A matritsaga ***maxsus matritsa***, det $A\neq 0$ bo'lsa, ***maxsusmas matritsa*** deyiladi. Kvadrat matritsaning $a_{11}, a_{22}, \ldots, a_{nn}$ elementlar joylashgan diagonali ***bosh diagonal***, elementlari joylashgan diagonali ***yordamchi diagonal*** deyiladi. Bosh diagonaldagi elementlar 0 dan farqli boshqa barcha elementlari 0 ga teng kvadrat matritsa ***diagonal matritsa*** deyiladi. Masalan,

$$A=\begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

matritsa diagonal matritsadir. Diagonaldagi barcha elementlari 1 ga teng diagonal matritsa ***birlik matritsa*** deyiladi va

$$E = \begin{pmatrix} 1\,0 \ldots 0 \\ 0\,1 \ldots 0 \\ \ldots\ldots\ldots \\ 0\,0 \cdots 1 \end{pmatrix}$$

bilan belgilanadi.

Faqat bitta satrdan iborat $(a_{11}\ a_{12}\ a_{13}\ a_{14}\ )$ matritsaga satr matritsa deyiladi. Faqat bitta ustunga ega

$$\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix}$$

matritsaga ustun matritsa deb ataladi.

Barcha elementlari 0 lardan iborat bo‘lgan matritsaga ***no‘l matritsa*** deyiladi va O bilan belgilanadi. Agar biror noldan farqli x vektor uchun

$$Ax = \lambda x$$

tenglik bajarilsa, u holda λ son A kvadrat ***matritsaning xos soni*** deyiladi. Bu tenglikni qanoatlantiradigan noldan farqli x vektor A ***matritsaning*** λ ***xos soniga mos keladigan xos vektori*** deyiladi.

$$D(\lambda) = \det(A - \lambda \mathrm{E}) = \begin{vmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n-1} & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n-1} & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn-1} & a_{nn} - \lambda \end{vmatrix} = 0. \qquad (1)$$

tenglama A ***matritsaning xarakteristik tenglamasi*** deyiladi.

$$\det(A - \lambda E) = (-1)^n (\lambda^n - p_1 \lambda^{n-1} - \ldots - p_n) \qquad (2)$$

ko‘phad A ***matritsaning xos yoki xarakteristik ko‘phadi*** deyiladi.

1. Krilov metodi

Ixtiyoriy noldan farqli $y^{(0)}$ vektor olamiz va $y^{(k)} = A^k y^{(0)}$, $k = \overline{0,n}$ vektorlarni xosil qilamiz.

Keli-Gamilton munosabatini yozamiz:

$$A^n y^{(0)} - p_1 A^{n-1} y^{(0)} - \ldots - p_n y^{(0)} = 0$$

yoki

$$p_1 y^{(n-1)} + p_2 y^{(n-2)} + \ldots + p_n y^{(0)} = y^{(n)}$$

vektor tenglama hosil qilinadi. Buni ochib yozaylik

$$\left.\begin{array}{l} p_1 y_1^{(n-1)} + p_2 y_1^{(n-2)} + \ldots + p_n y_1^{(0)} = y_1^{(n)} \\ p_1 y_2^{(n-1)} + p_2 y_2^{(n-2)} + \ldots + p_n y_2^{(0)} = y_2^{(n)} \\ - \quad - \quad - \quad - \quad - \quad - \\ p_1 y_n^{(n-1)} + p_2 y_n^{(n-2)} + \ldots + p_n y_n^{(0)} = y_n^{(n)} \end{array}\right\} . \qquad (3)$$

(3) tenglamalar sistemasini misol uchun Gauss metodi bilan yechamiz va $p_1, p_2, \ldots, p_n$ larni topamiz, natijada (2) xos ko‘phad qurilgan bo‘ladi, so‘ng

$$D(\lambda) = 0$$

tenglamani yechilib $\lambda_1, \lambda_2, \ldots, \lambda_n$ lar topiladi.

Endi xos vektorlarni topamiz. $y^{(k)}$, $k = \overline{0, n-1}$ larni $x^{(i)}$, $i = \overline{1, n}$ vektorlar orqali yoyib olamiz

$$y^{(k)} = \sum_{i=1}^{n} c_i \cdot A^k x^{(i)} = \sum_{i=1}^{n} c_i \cdot \lambda_i^k x^{(i)}, \quad k = \overline{0, n-1}$$

Quyidagi ko‘phadni tuzamiz

$$\varphi_i(\lambda) = \lambda^{n-1} + q_{1,i}\lambda^{n-2} + \ldots + q_{n-1,i}, \quad i = \overline{1,n}$$

$y^{(k)}$, $k = \overline{0, n-1}$ vektorlarning quyidagi kombinatsiyasini tuzamiz

$$y^{(n-1)} + q_{1i}y^{(n-2)} + \ldots + q_{n-1i}y^{(0)} = c_1\varphi_i(\lambda_1)x^{(1)} + c_2\varphi_i(\lambda_2)x^{(2)} + \ldots + c_n\varphi_i(\lambda_n)x^{(n)}. \quad (5)$$

Agar $\varphi_i(\lambda) = \dfrac{D(\lambda)}{\lambda - \lambda_i}$ $\quad i = \overline{1,n}$ desak, $\varphi_i(\lambda_j) = 0,\ \ i \neq j$ bo‘lganligi uchun

$$C_i\varphi_i(\lambda_i)x^{(i)} = y^{(n-1)} + q_{1,i}y^{(n-2)} + \ldots + q_{n-1,i}y^{(0)} \qquad i = \overline{1,n}$$

bo‘ladi. $q_{j,i}$ koeffitsientlar esa

$$\begin{aligned} q_{0i} &= 1, \\ q_{ji} &= \lambda_i q_{j-1,i} + p_j \end{aligned}$$

rekurrent formula yordamida topiladi.

Agar (3) tenglamalar sistemasini yechishda Gauss usulini to‘g‘ri yo‘lini m ta qadami bajarilsa, u holda $y^{(0)}, y^{(1)} \ldots y^{(m-1)}$ vektorlar chiziqli erklidir. Shuning uchun (3) tenglamalar o‘rniga quyidagi

$$\left.\begin{array}{l} p_1y_1^{(m-1)} + p_2y_1^{(m-2)} + \ldots + p_my_1^{(0)} = y_1^{(m)} \\ \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \\ p_1y_n^{(m-1)} + p_2y_n^{(m-2)} + \ldots + p_my_n^{(0)} = y_n^{(m)} \end{array}\right\}$$

tenglamalar sistemasini yechib $p_1, p_2, \ldots, p_m$ lar topiladi va

$$\lambda^m - p_1\lambda^{m-1} - \ldots - p_m = 0$$

tenglamadan $\lambda_1, \lambda_2, \ldots, \lambda_m$ larni topamiz.

$\lambda^m - p_1\lambda^{m-1} - \ldots - p_m$ ko‘phad ***matritsaning minimal ko‘phadi*** deyiladi.

Xos vektor esa quyidagicha topiladi

$$x^{(i)} = \beta_{i1}y^{(0)} + \ldots + \beta_{im}y^{(m-1)},$$

bu yerda

$$\begin{aligned}
\beta_{im} &= 1\\
\beta_{im-1} &= \lambda_i - p_1\\
\beta_{im-2} &= \lambda_i^2 - p_1\lambda_i - p_2\\
\ldots \quad & \ldots \quad \ldots \quad \ldots\\
\beta_{i1} &= \lambda_i^{m-1} - p_1\lambda_i^{m-2} - \ldots - p_{m-1}
\end{aligned}$$

Misol 1.

$$A = \begin{bmatrix} -1 & 2 & 2 & 0\\ 2 & 1 & 3 & 2\\ 2 & -1 & 2 & 3\\ 0 & 2 & 1 & -2 \end{bmatrix}$$

matritsaning xarakteristik ko‘phadi topilsin.

Yechish. $y^{(0)} = (1,0,0,0)^1$, deb olamiz. U holda

$$\begin{aligned}
y^{(1)} = Ay^{(0)} &= (\ -1,\ 2,\ -2,\ 0\ )^1\\
y^{(2)} = Ay^{(1)} &= (\ 9,\ 6,\ 0,\ 6\ )^1\\
y^{(3)} = Ay^{(2)} &= (\ 3,\ 36,\ 30,\ 0\ )^1\\
y^{(4)} = Ay^{(3)} &= (\ 129,\ 132,\ 30,\ 102\ )^1 .
\end{aligned}$$

Endi (3) tenglamalarni yozamiz

$$\left.\begin{aligned}
3P_1 + 9P_2 - P_3 + P_4 &= 129\\
36P_1 + 6P_2 + 2P_3 &= 132\\
30P_1 + 2P_3 &= 30\\
6P_2 &= 102
\end{aligned}\right\} \Rightarrow$$

$$P_1 = 0, P_2 = 17, \quad P_3 = 15, \quad P_4 = -9,$$

$$D(x) = \lambda^4 - 17\lambda^2 - 15\lambda + 9.$$

Misol 2.

$$A=\begin{bmatrix} 5 & 30 & -48 \\ 3 & 14 & -24 \\ 3 & 15 & -25 \end{bmatrix}$$

matritsaning xos son sonlari va xos vektorlari topilsin.

Yechish. $y^{(0)}=(1,0,0)^1$ deb

$$y^{(1)}=(5,3,3)^1, \qquad y^{(2)}=(-29,-15,-15)^1, \quad y^{(3)}=(125,63,63)^1$$

larni hosil qilamiz va (3) sistemani yozamiz

$$\begin{array}{llll} -29p_1 & +5p_2 & +p_3 & =125 \\ -15p_1 & +3p_2 & & =63 \\ -15p_1 & +3p_2 & & =63 \end{array}.$$

Bu sistemani yechishda Gauss usulining uchinchi qadami bajarilmaydi, chunki 2 va 3-tenglamalar bir xil, demak $y^{(0)}$ $y^{(1)}$ $y^{(2)}$ $y^{(3)}$ lar chiziqli bog‘liq.

$y^{(0)}$ $y^{(1)}$ $y^{(2)}$ larga bog‘liq

$$\left.\begin{array}{l} 5p_1+p_2=-29 \\ 3p_1 \qquad =-15 \end{array}\right\}$$

sistemani tuzamiz. Bundan $p_1=-5, \quad p_2=-4$ bo‘ladi. $\lambda^2+5\lambda+4=0$ deb $\lambda_1=-4, \quad \lambda_2=-1$ ni topamiz. λ_3 ni topish uchun, bizga ma’lum

$$a_{11}+a_{22}+a_{33}=\lambda_1+\lambda_2+\lambda_3$$

munosabatdan foydalanamiz

$$5+14-25=-4-1+\lambda_3$$

$$-6=-5+\lambda_3$$

$$\lambda_3=-1.$$

Endi xos vektorlarni topamiz:

$$x^{(i)} = \beta_{i1} y^{(0)} + \beta_{i2} y^{(1)} = (\lambda_i - p_1) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} \lambda_i - p_1 + 5 \\ 3 \\ 3 \end{pmatrix}$$

$$x^{(1)} = \begin{pmatrix} -4 - (-5) + 5 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix},$$

$$x^{(2)} = \begin{pmatrix} -1 - (-5) + 5 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \\ 3 \end{pmatrix}.$$

$x^{(3)}$ ni topish uchun $y^{(0)}$ vektorni boshqacha tanlash kerak.

2. Danilevskiy metodi

Berilgan matritsa o‘xshash almashtirish yordamida Frobenius

$$P = \begin{bmatrix} p_1 & p_2 & \cdots & p_{n-1} & p_n \\ 1 & 0 & \cdots & 0 & 0 \\ & \cdots & \cdots & \cdots & \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

normal ko‘rinishiga keltiriladi. Ma’lumki, P matritsaning xarakteristik ko‘phadi $P(\lambda) = \lambda^n - p_1\lambda^{n-1} - ... - p_n$ bo‘ladi [1].

$$A^{(1)} = M_{n-1}^{-1} A M_{n-1} = \begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & \cdots & a_{1n-1}^{(1)} & a_{1n}^{(1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n-11}^{(1)} & a_{n-12}^{(1)} & \cdots & a_{n-1n-1}^{(1)} & a_{n-1n}^{(1)} \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

hosil qilinadi, so‘ng $A^{(2)}, \ldots,$ $A^{(n-1)} = P$ hosil bo‘ladi.

Har qadamdagi o‘ngdan va chapdan ko‘paytiriladigan matritsalarni ko‘rinishini yozamiz

$$M_{n-1}=\begin{bmatrix} 1 & 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ -\frac{a_{n1}}{a_{nn-1}} & -\frac{a_{n2}}{a_{nn-1}} & \dots & -\frac{a_{nn-2}}{a_{nn-1}} & \frac{1}{a_{nn-1}} & -\frac{a_{nn}}{a_{n\,n-1}} \\ 0 & 0 & \dots & 0 & 0 & 1 \end{bmatrix}$$

,

$$M_{n-1}^{-1}=\begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \cdots & a_{nn-1} & a_{nn} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix},$$

$$M_{n-2}=\begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ -\frac{a_{n-11}^{(1)}}{a_{n-1n-2}^{(1)}} & -\frac{a_{n-1,2}^{(1)}}{a_{n-1n-2}^{(1)}} & \cdots & \frac{1}{a_{n-1n-2}^{(1)}} & -\frac{a_{n-1n-1}^{(1)}}{a_{n-1n-2}^{(1)}} & -\frac{a_{n-1n}^{(1)}}{a_{n\,n-1}} \\ 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix},$$

$$M_{n-2}^{-1}=\begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ a_{n-1,1}^{(1)} & a_{n-1,2}^{(1)} & \cdots & a_{n-1,n-1}^{(1)} & a_{n-1,n}^{(1)} \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

va hokazo. Natijada A matritsa Frobenius normal ko‘rinishiga keladi.

$$A^{(n-1)}=M_1^{-1}M_2^{-1}\cdots M_{n-1}^{-1}AM_{n-1}M_{n-2}\cdots M_1=S^{-1}AS=P,$$

$$S=M_{n-1}M_{n-2}\cdots M_2\cdot M_1\ .$$

Danilevskiy usulida xos vektor x quyidagicha topiladi:

$$x=M_{n-1}M_{n-2}\cdots M_2\cdot M_1 y$$

bu yerda

$$y = \begin{pmatrix} \lambda^{n-1} \\ \lambda^{n-2} \\ \vdots \\ \lambda \\ 1 \end{pmatrix}$$

bo‘lib, u P matritsaning xos vektoridir.

Danilevskiy metodidagi noregulyar hol. Danilevskiy metodining $(n-k)-$ qadami bajarilgan bo‘lsin va $A^{(n-k)}$) matritsa¬ning $a_{k,k-1}^{(n-k)}$ elementi nolga teng bo‘lsin. Navbatdagi $(n-k+1)-$qadamni odatdagidek bajarib bo‘lmaydi. Bunda agar $A^{(n-k)}$ matritsaning $a_{k,k-1}^{(n-k)}$ elementidan hamda, masalan, $i-$element $(i<k-1)$ $a_{ki}^{(n-k)} \neq 0$ bo‘lsa, $(k-1)-$ustunni $i-$ustun bilan almashtiramiz va xuddi shu nomerli satrlarni almashtirib yozamiz. Bunday almashtirishdan so‘ng odatdagidek Danilevskiy usulini davom ettiramiz. Faraz qilaylik,

$$a_{k1}^{(n-k)} = a_{k2}^{(n-k)} = \cdots = a_{k\,k-1}^{(n-k)} = 0$$

bo‘lsin. U holda $A^{(n-k)}$ quyidagicha ko‘rinishga ega bo‘ladi

$$A^{(n-k)} = \begin{bmatrix} B^{(n-k)} & C^{(n-k)} \\ 0 & P^{(n-k)} \end{bmatrix}$$

Bu yerda $P^{(n-k)}$ Frobenius normal formasiga ega bo‘lgan $(n-k+1)$ tartibli kvadrat matritsadir. $B^{(n-k)}$ esa $(k-1)-$tartibli kvadrat matritsa bo‘lib, uni odatdagidek Danilevskiy usuli bilan Frobenius normal ko‘rinishga keltirish mumkin.

Misol. Matritsaning xos son va xos vektorlari topilsin.

$$A = \begin{bmatrix} 23 & -9 & -2 & 0 \\ -4 & 23 & 0 & -2 \\ -8 & 0 & 23 & -9 \\ 0 & -8 & -4 & 23 \end{bmatrix}$$

Yechish.

$$M_3=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ -\frac{a_{41}}{a_{43}} & -\frac{a_{42}}{a_{43}} & \frac{1}{a_{43}} & -\frac{a_{44}}{a_{43}}\\ 0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & -2 & -\frac{1}{4} & \frac{23}{4}\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$M_3^{-1}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ a_{41} & a_{42} & a_{43} & a_{44}\\ 0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & -8 & -4 & 23\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$A^{(1)}=M_3^{-1}AM_3=\begin{bmatrix}-23 & -5 & \frac{1}{2} & -\frac{23}{2}\\ -4 & 23 & 0 & -2\\ 64 & 0 & 46 & -477\\ 0 & 0 & 1 & 0\end{bmatrix}$$

$A(1)$ matritsada $a_{32}^{(1)}=0$, shuning uchun $A(1)$ matritsaning 1- va 2-ustunlarini, so‘ng 1- va 2-satrlarni almashtirib yozamiz. Bu amallarni bajarish matritsasini keltiramiz:

$$U=\begin{bmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Natijada quyidagini hosil qilamiz:

$$A^{(1)}U=\begin{bmatrix}23 & -4 & 0 & -2\\ -5 & 23 & \frac{1}{2} & -\frac{23}{2}\\ 0 & 64 & 46 & -477\\ 0 & 0 & 1 & 0\end{bmatrix}.$$

Endi $A(2)$ ni topamiz, buning uchun M_2, M_2^{-1} larni yozamiz

$$M_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{64} & -\frac{46}{64} & \frac{477}{64} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad M_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 64 & 46 & -477 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A^{(2)} = M_2^{-1} U A^{(1)} U M_2 = \begin{bmatrix} 23 & -\frac{1}{16} & \frac{46}{16} & -\frac{509}{16} \\ -320 & 69 & -1503 & 10235 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Nihoyat, $A^{(3)} = M_1^{-1} A^{(2)} M_1$,

$$M_1^{-1} = \begin{bmatrix} -320 & 69 & -1503 & 10235 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad M_1 = \begin{bmatrix} -\dfrac{1}{320} & \dfrac{69}{320} & -\dfrac{1503}{320} & \dfrac{10235}{320} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{(3)} = \begin{bmatrix} 92 & -3070 & 43884 & -225225 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Demak,

$$P(\lambda) = \lambda^4 - 92\lambda^3 + 3070\lambda^2 - 43884\lambda + 225225 \,.$$

$P(\lambda) = 0$ ni yechib, $\lambda_1 = 13, \quad \lambda_2 = 21, \quad \lambda_3 = 25, \quad \lambda_4 = 33$ ekanligini topamiz.

$x^{(1)} = M_3 U M_2 M_1 y^{(1)}$ va

$$U = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

bo‘lib, A - (1) matritsaning 1- va 2-ustunlarini almashtirish matritsasidir.

$$x^{(1)} = M_3UM_2M_1y^{(1)}\begin{bmatrix} -\frac{1}{320} & \frac{69}{320} & \frac{1503}{320} & \frac{10235}{320} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\cdot\begin{pmatrix} 13^3 \\ 13^2 \\ 13^1 \\ 1 \end{pmatrix} = M_3U\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{64} & -\frac{46}{64} & \frac{477}{64} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\times$$

$$\times\begin{pmatrix} 1/2 \\ 13^2 \\ 13^1 \\ 1 \end{pmatrix} == M_3\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\cdot\begin{pmatrix} 1/2 \\ 3/4 \\ 13 \\ 1 \end{pmatrix} = M_3\begin{pmatrix} 3/4 \\ 1/2 \\ 13 \\ 1 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & \frac{1}{4} & \frac{23}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}\cdot\begin{pmatrix} 3/4 \\ 1/2 \\ 13 \\ 1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 3 \\ 2 \\ 6 \\ 4 \end{pmatrix}$$

Shunday qilib $x^{(1)} = (3,\ 2,\ 6,\ 4)^1$. Shu yo‘l bilan

$$x^{(2)} = (3,\ 2,\ -6,\ 4)^T$$
$$x^{(3)} = (3, -2,\ 6,\ -4)^T$$
$$x^{(4)} = (3,\ -2,\ -6,\ 4)^T$$

topiladi.

3. Lavere metodi.

Quyidagi

$$s_1 = \lambda_1 + \lambda_2 + ... + \lambda_n = SpA$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$s_k = \lambda_1^k + \lambda_2^k + ... + \lambda_n^k = SpA^k$$

$$A^k = A^{k-1}A, \qquad k = \overline{0,n}. \qquad\qquad k \le n,$$

Munosabatlardan foydalanib, $\det(\lambda E - A) = \lambda^n + p_1\lambda^{n-1} + ... + p_n$ ko‘phadning p_k koeffitsientlarini ketma-ket topiladi:

$$p_k = -\frac{1}{k}\left(s_k + p_1s_{k-1} + p_2s_{k-2} + ... + p_{k-1}s_1\right), \qquad k = \overline{1,n}.$$

Misol. Quyidagi matritsaning xarakteristik ko‘phadi topilsin.

$$A=\begin{bmatrix}5 & 30 & -48\\ 3 & 14 & -24\\ 3 & 15 & -25\end{bmatrix}$$

Yechish.

$$s_1 = SpA = 5+14-25=-6$$

$$A^2 = A\cdot A = \begin{bmatrix}-29 & -150 & 240\\ -15 & -74 & 120\\ -15 & -75 & 121\end{bmatrix}$$

$$s_2 = SpA^2 = -29-74+121=18$$

$$A^3 = A^2\cdot A = \begin{bmatrix}125 & 630 & -1008\\ 63 & 314 & -504\\ 63 & 315 & -505\end{bmatrix}$$

$$s_3 = SpA^3 = 125+314-505=66$$

$$p_1 = -\frac{1}{1}s_1 = 6,$$

$$p_k = -\frac{1}{2}(s_2+p_1s_1) = -\frac{1}{2}(18+6\cdot(-6)) = 9,$$

$$p_3 = -\frac{1}{3}(s_3+p_1s_2+p_2s_1) = -\frac{1}{3}(-66+6\cdot 18+9\cdot(-6)) = 4,$$

$\det(\lambda E - A) = \lambda^3+6\lambda^2+9\lambda+4=0,$ bundan $\lambda_1=-1,$ $\lambda_2=-4,$ $\lambda_1=-1,$

4. Moduli bo‘yicha eng katta xos sonni va unga mos xos vektorni topish

1-hol. $|\lambda_1|>|\lambda_2|\geq\ldots\geq|\lambda_n|$ bo‘lsin.

$$\lambda_1 \approx \frac{y_i^{(k+1)}}{y_i^{(k)}}, \qquad i=\overline{1,n}, \tag{6}$$

bu yerda $y^{(k)} = A^k y^{(0)},$ $y^{(0)} \neq (0,0,\cdots,0)^1,$ $y^{(k)} = (y_i^{(k)},\cdots,y_n^{(k)})^1$

$$y^{(k)} = A^k y^{(o)} = \sum_{j=1}^{n} b_j A^k x^{(j)} = \sum_{j=1}^{n} b_j \lambda_j^k x^{(j)};$$

Agar (6) taqribiy tenglik barcha $i = \overline{1,n}$ uchun berilgan aniqlikda bajarilsa, λ_1 ning qiymati topilgan bo‘ladi.

Xos vektorini $y^{(k)} \approx b_1 \lambda_1^k x^{(1)}$ deb olish mumkin, chunki $y^{(k)}$ $x^{(1)}$ dan sonli ko‘paytuvchi bilan farq qiladi.

2-hol. $\lambda_1 = \lambda_2 = \cdots = \lambda_s$ bo‘lsin

$$|\lambda_1| > |\lambda_{s+1}| \geq \cdots \geq |\lambda_n|,$$

$$\lambda_1 \cong \frac{y_i^{(k+1)}}{y_i^{(k)}}, \qquad i = \overline{1,n}$$

Xos vektor $x^{(1)}$ 1-holdagidek topiladi.

3-hol. $\lambda_1 = \cdots = \lambda_r = -\lambda_{r+1} = \cdots = -\lambda_{r+p}$

$$|\lambda_1| = \cdots |\lambda_{r+p}| > |\lambda_{r+p+1}| \geq \cdots \geq |\lambda_n|$$

$$\lambda_1^2 \cong \frac{y_i^{(2k+2)}}{y_i^{(2k)}}, \qquad i = 1,n$$

$$\lambda_1^2 \cong \frac{y_i^{(2k+1)}}{y_i^{(2k-1)}}, \qquad i = 1,n$$

taqribiy tengliklardan λ_1, $-\lambda_1$ topiladi. $\frac{y_i^{(k+1)}}{y_i^{(k)}}$ nisbat esa $k \to \infty$ da limitga ega bo‘lmaydi. λ_1 ga mos kelgan xos vektor sifatida $y^{(k+1)} + \lambda_1 y^{(k)}$ ni olamiz. Agar r va p yoki bularning birortasi birdan katta bo‘lsa, u holda boshqa dastlabki vektor tanlab jarayonni bajarish kerak.

Xulosa:

Men ushbu kurs ishida “Matritsaning xos son va xos vektorlarini hisoblash” mavzusini o‘rgandim, unda matritsa, matritsaning tartibi, matritsaning determinanti, nol matritsa, birlik matritsa, matritsanining xos son va xos vektorlari, matritsaning xarakteristik tenglamasi, matritsaning xos yoki xarakteristik ko‘phadi, matritsaning minimal ko‘phadi, matritsanining xos son va xos vektorlarini amaliy hisoblashda Krilov, Danilevskiy, Lavere metodlarini va moduli bo‘yicha eng katta xos sonni va unga mos xos vektorni topishni o‘rgandim. O‘rgangan metodlarni amaliy jihatdan Ms Excel va Paskalda dasturlarini tuzdim.

Foydalanilgan adabiyotlar:

1. Isroilov M.I. Hisoblash metodlari, I. Toshkent, O‘qituvchi, 2000.
2. Демидович Б.П., Марон И.А. Основы вычислительной математики. – М., Наука, 1970.
3. Копченова И.В., Марон И.А. Вычислительная математика в примерах и задачах. –М., Наука, 1972. – 368 с.
4. Крылов В.И., Бобков В.В., Монастырный П.И. Вычислительные методы, том I. –М., Наука, 1976.
5. *www.google.uz*