5A 130101-"Matematika (yo'nalishlar bo'yicha)" mutaxasisligi bo'yicha magistri akademik darajasini olish uchun tayyorlangan Ismailova Nargizaning «Dirak operatori uchun izlar formulasi va teskari masala» mavzusidagi magistrlik dissertatsiyasiga

T A Q R I Z

Kvant mexanikasining fundamental tenglamalaridan biri bu Dirak tenglamasidir. U amaliy jihatdan muhim tadbiqlarga ega bo'lib, umuman olganda u yarim chegaralanmagan operatordir. Shuning uchun yarim chegaralangan operatorlar spektral xossalarini o'rganishda qo'llanilgan ko'plab usullarni bu yerda to'g'ridan to'g'ri qo'llashning imkoni bo'lmaydi.

Dirak operatori teskari spektral masalalar nazariyasida muhim natijalar M.G.Gasymov, B.M.Levitan, I.S.Frolov, V.E.Zaxarov, A.B.Shabat, M.Ablovits, X.Sigur, D.Kaup, A.Nyuel, T.V.Misyur, M.Z.Zamonov, A.B.Xasanov, P.Djakov, B.S.Mityagin, S.Albeverio, R.Xruniv, YA.Mukutyuk va boshqa olimlar ishlarida olingan.

V.E.Zaxarov, A.B.Shabat, S.V.Manakovlarning ishida ushbu

$$u_t = 2iu|u|^2 - iu_{xx}$$

nochiziqli Shredinger tenglamasi Dirak opertoriga qo'yilgan teskari masala usulidan foydalanib integrallanishi mumkinligi ko'rsatildi. Bu esa Dirak operatoriga qo'yilgan teskari masalalarni o'rganishga bo'lgan qiziqishni yanada kuchaytirdi. Hozirgi kunda bu sohaning kvant fizikasiga, chiziqli va nochiziqli xususiy hosilali tenglamalar nazariyasiga, kristallografiyaga, geologo-razvedka masalalariga muhim tadbiqlari topilganligi bois bu mavzuning dolzarbligi yanada ortmoqda.

Ismailova Nargizaning magistrlik dissertatsiyasida Dirak operatori uchun izlar formulasi yangicha elementar usulda keltirib chiqarilgan va teskari spektral masala atroflicha o‘rganilgan, ya’ni davriy potensalli Dirak operatorining spektral berilganlari yordamida uning koeffitsiyentlarini tiklash algoritmi ko‘rsatilgan.

Magistrlik dissertatsiya kirish, uchta bob, xulosa va adabiyotlar ro‘yhatidan iborat. Birinchi bobda Dirak tenglamasining kelib chiqishi va Dirak operatori haqida asosiy ma’lumotlar keltirilgan. Magistrlik dissertatsiyasining ikkinchi bobida har xil chegaraviy shartlarda Dirak operatori uchun izlar formulasi yangicha elementar usulda keltirib chiqarilgan. Uchinchi bobda esa davriy potensalli Dirak operatorining spektral berilganlari yordamida uning koeffitsiyentlarini tiklash algoritmi ko‘rsatilgan.

Dissertatsiya ishi yuqori saviyada ilmiy xatolarsiz tayyorlangan. Muallifning dissertatsiya bo‘yicha 2 ta ilmiy maqolasi “Ilm sarchashmalari” jurnalida chop etilgan.

Yuqorida keltirib o‘tilganlarni hisobga olib, dissertatsiya ishi Oliy va O‘rta Maxsus Ta’lim Vazirligi tomonidan magistrlik dissertatsiyasiga qo‘yilgan barcha talablarga to‘liq javob beradi deb hisoblayman.

Ismailova Nargizaning «Dirak operatori uchun izlar formulasi va teskari masala» mavzusidagi magistrlik dissertatsiyasini Davlat attestatsiya komissiyasi oldida himoya qilishga tavsiya qilaman va muvaffaqiyatli himoyadan keyin muallifni 5A 130101-“Matematika (yo‘nalishlar bo‘yicha)” ixtisosligi bo‘yicha matematika magistri akademik darajasiga loyiq deb bilaman.

Taqrizchi: **Qoraqolpoq memleketlik universiteti Matematika fakulteti dotsenti, f-m.f.n. Tanirbergenov M.B.**

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5A 130101-“Matematika (yo‘nalishlar bo‘yicha)” mutaxasisligi bo‘yicha magistri akademik darajasini olish uchun tayyorlangan Ismailova Nargizaning «Dirak operatori uchun izlar formulasi va teskari masala» mavzusidagi magistrlik dissertatsiyasiga

T A Q R I Z

Regulyar Shturm–Liuvill operatorining izi ilk bor 1953 yilda I.M.Gelfand va B.M.Levitan tomonidan hisoblandi. Bu ishdan keyin tez orada L.A.Dikiy tomonidan regulyar iz formulasi boshqacha usul yordamida keltirib chiqarildi va buning natijasida u Shturm-Liuvill operatorning barcha darajalari uchun regulyarlashgan izlar formulasini hisoblash uchun rekkurent formula olishga muvofiq bo‘ldi. Dirak operatori uchun regulyarlashgan izlar formulasi E.Abduqodirov tomonidan olindi. Davriy potensialli Dirak operatori uchun izlar formulasi V.A.Marchenko tomonidan keltirib chiqarilgan. Bu natija A.B.Yaxshimurotov ishida Laks usuli yordamida keltirib chiqarilgan. Izlar formulasi differensial operatorlarga qo‘yilgan teskari masalalarni o‘rganishda muhim o‘rin egallaydi.

Teskari spektral masalalarni o‘rganish XX asr boshlarida boshlangan bo‘lib, bu yo‘nalishning rivojiga muhim turtki bo‘lgan ilk natija 1929 yilda V.A.Ambarsumyan tomonidan olingan.

Dirak operatori teskari spektral masalalar nazariyasida muhim natijalar M.G.Gasymov, B.M.Levitan, I.S.Frolov, V.E.Zaxarov, A.B.Shabat, M.Ablovits, X.Sigur, D.Kaup, A.Nyuel, T.V.Misyur, M.Z.Zamonov, A.B.Xasanov, P.Djakov, B.S.Mityagin, S.Albeverio, R.Xruniv, YA.Mykytyuk va boshqa olimlar ishlarida olingan.

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Yuqorida keltirib o'tilganlarni hisobga olib, dissertatsiya ishi Oliy va O'rta Maxsus Ta'lim Vazirligi tomonidan magistrlik dissertatsiyasiga qo'yilgan barcha talablarga to'liq javob beradi deb hisoblayman.

Ismailova Nargizaning «Dirak operatori uchun izlar formulasi va teskari masala» mavzusidagi magistrlik dissertatsiyasini Davlat attestatsiya komissiyasi oldida himoya qilishga tavsiya qilaman va muvaffaqiyatli himoyadan keyin magistrantni 5A 130101-"Matematika (yo'nalishlar bo'yicha)" ixtisosligi bo'yicha matematika magistri akademik darajasiga loyiq deb bilaman.

Urganch Davlat Universiteti,
„Amaliy matematika va matematik- fizika"
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Taqrizchi: **f.-m.f.n. O.R. Allaberganov**

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O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

AL - XORAZMIY NOMIDAGI URGANCH DAVLAT UNIVERSITETI

Qo‘l yozma huquqida

UDK 517.946

ISMAILOVA NARGIZA BAXTIYAROVNA

“Dirak operatori uchun izlar formulasi va teskari masala ”

Mutaxassislik: 5A 30101 “Matematika” (yo‘nalishlar bo‘yicha)

Magistr akademik darajasini olish uchun yozilgan dissertatsiya

Ilmiy rahbar: **f-m.f.n. Babajanov B.A.**

Urganch - 2013

M U N D A R I J A

Kirish

Bizga n - o‘lchamli chiziqli fazoda A chiziqli operator biror bazisda

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

matritsa bilan berilgan bo‘lsin. Chiziqli algebra kursidan ma’lumki, agar $\lambda_1, \lambda_2, ..., \lambda_n$ sonlar A chiziqli operatorning xos qiymatlari bo‘lsa, u holda bu xos qiymatlar yig‘indisiga berilgan operatorning spektral izi deyiladi, A matritsaning diagonal elementlari yig‘indisiga esa, matritsaviy izi deyiladi va quyidagi tenglik o‘rinli bo‘ladi:

$$\sum_{k=1}^{n} \lambda_k = \sum_{k=1}^{n} a_{kk} .$$

Agar ushbu

$$\begin{cases} Ly \equiv -y'' + q(x)y - \lambda y = 0, \quad (0 \le x \le \pi) \\ y(0)\cos\alpha + y'(0)\sin\alpha = 0, \quad \alpha \in R^1 \\ y(\pi)\cos\beta + y'(\pi)\sin\beta = 0, \quad \beta \in R^1 \end{cases}$$

regulyar Shturm–Liuvill chegaraviy masalasining $\lambda_1, \lambda_2, ..., \lambda_n, \dots$ - xos qiymatlar ketma–ketligi yordamida quyidagi

$$\sum_{\mathrm{n}=1}^{\infty} \lambda_n$$

qatorni tuzsak shubhasiz u uzoqlashadi, chunki λ_n lar uchun ushbu

$$\lambda_n = n^2 + c_0 + o\left(\frac{1}{n^2}\right), \quad n \to \infty$$

asimptotika o‘rinli

Ta’rif. Ushbu

$$\sum_{n=1}^{\infty} (\lambda_n - \lambda_n^0) < \infty$$

sonli qatorning yig‘indisiga regulyar Shturm–Liuvill operatorining regulyarlashtirilgan izi deyiladi.

Bu yerda λ_n^0 ushbu

$$\begin{cases} -y''=\lambda y, \quad 0 \le x \le \pi \\ y(0)\cos\alpha + y'(0)\sin\alpha = 0, \quad \alpha \in R^1 \\ y(\pi)\cos\beta + y'(\pi)\sin\beta = 0, \quad \beta \in R^1 \end{cases}$$

masalaning xos qiymatlari.

Regulyar Shturm–Liuvill operatorining izi, ya’ni

$$\sum_{n=1}^{\infty} (\lambda_n - \lambda_n^0) < \infty$$

sonli qatorning yig‘indisi ilk bor 1953 yilda I.M.Gelfand va B.M.Levitan [16] tomonidan hisoblanildi va ular quyidagi ajoyib natijani qo‘lga kiritdilar

$$\sum_{n=0}^{\infty}\left[\lambda_n - n^2 - \frac{1}{\pi}\int_0^{\pi} q(x)dx\right] = \frac{q(0)+q(\pi)}{4} - \frac{1}{2\pi}\int_0^{\pi} q(x)dx$$

bunda λ_n ushbu

$$\begin{gathered} -y'' + q(x)y = \lambda y, \\ y'(0) = y'(\pi) = 0, \end{gathered}$$

Shturm-Liuvill masalasining xos qiymatidir.

Bu ishdan keyin tez orada L.A.Dikiy [21] tomonidan yuqoridagi formula boshqacha usul yordamida keltirib chiqariladi va buning natijasida u L operatorning barcha darajalari uchun regulyarlashgan izlar formulasini hisoblash uchun rekkurent formula olishga muvofiq bo‘ldi. Bu sohadagi keyingi muhim natija V.B.Lidskiy va V.A.Sadovnichiylar [39] tomonidan olindi. Dirak operatori uchun regulyarlashgan izlar formulasi E.Abduqodirov [1] tomonidan olindi. Davriy potentsialli Shturm – Liuvill operatori uchun izlar formulasi X.Xoxshtad

[58] tomonidan keltirib chiqarildi. Davriy potensialli Dirak operatori uchun izlar formulasi V.A.Marchenko [40] kitobida keltirib chiqarilgan. Bu natija A.B.Yaxshimurodov [61] ishida Laks usuli yordamida keltirib chiqarilgan. Izlar formulasi differensial operatorlarga qo‘yilgan teskari masalalarni o‘rganishda muhim o‘rin egallaydi.

Berilgan potensial bo‘yicha Shturm-Liuvill operatorining spektral xarakteristikalarini topish masalasiga spektral analizning to‘g‘ri masalasi deyiladi, aksincha, spektral xarakteristikalar bo‘yicha Shturm-Liuvill operatorini tiklash masalasiga spektral analizning teskari masalasi deyiladi. Turli xil chegaraviy shartlardagi spektrlar, spektral funksiya, sochilish nazariyasining berilganlari va shunga o‘xshash xarakteristikalar spektral xarakteristikalar bo‘la olishi mumkin.

Teskari spektral masalalarni o‘rganish XX asr boshlarida boshlangan bo‘lib, bu yo‘nalishning rivojiga muhim turtki bo‘lgan ilk natija 1929 yilda V.A.Ambarsumyan [5] tomonidan olingan.

Shturm-Liuvill operatorining teskari spektral masalalari nazariyasining keyingi rivojiga G.Borg [9], N.Levinson [35], A.N.Tixonov [55], L.A.Chudov [59], V.A.Marchenko [42], I.M.Gelfand, B.M.Levitan [16], B.A.Dubrovin [24], Yu.M.Berezansko [7], M.G.Gasymov [14], A.Sh.Blox [8], F.S.Rofe-Beketov[49], F.Gestezi, B.Saymon [19], V.A.Yurko [73], X.Xoxshtadt [58], N.I.Axiezer [6], I.V.Stankevich [54], V.A.Marchenko va I.V.Ostrovskiy [42], X.P.Mak-Kin va E.Trubovits [46], E.Trubovits [56] va boshqa olimlarning ishlarida sezilarli darajada hissa qo‘shilgan.

K.Gardner, Dj.Grin, M.Kruskal va R.Miuralar [11] tomonidan teskari spektral masalalarni nochiziqli tenglamalarni yechishga tadbiq qilinishi teskari spektral masalalarni o‘rganishga bo‘lgan qiziqishni yanada orttirdi va Shturm-Liuvill operatoridan boshqa operatorlar uchun ham teskari spektral masalalarni turli sinflarda (davriy potensialli, deyarli davriy potensialli, chekli zonali potensialli, cheksiz zonali potensialli cheksiz kamayuvchi potensialli va hokazo) chuqurroq o‘rganish zarurligini ko‘rsatdi.

Shunday operatorladan biri Dirak operatoridir. U amaliy jihatdan muhim tadbiqlarga ega bo‘lib, umuman olganda yarim chegaralanmagan operator va yarim chegaralangan operatorlar spektral xossalarini o‘rganishda qo‘llanilgan ko‘plab usullarni bu yerda to‘g‘ridan to‘g‘ri qo‘llashni imkoni bo‘lmaydi.

Dirak operatori teskari spektral masalalar nazariyasida muhim natijalar M.G.Gasymov, B.M.Levitan [15], I.S.Frolov [57], V.E.Zaxarov, A.B.Shabat [28], M.Ablovits, X.Sigur, D.Kaup, A.Nyuel [2], T.V.Misyur [43-45], M.Z.Zamonov, A.B.Xasanov [26], P.Djakov, B.S.Mityagin [22], S.Albeverio, R.Xruniv, YA.Mukutyuk [3] va boshqa olimlar ishlarida olingan.

V.E.Zaxarov, A.B.Shabat, S.V.Manakov ([27-28]) ishida nochiziqli Shredinger tenglamasi:

$$u_t = 2iu|u|^2 - iu_{xx}$$

Dirak opertoriga qo‘yilgan teskari masala usulidan foydalanib integrallanishi mumkinligi ko‘rsatildi. Bu esa Dirak operatoriga qo‘yilgan teskari masalalarni o‘rganishga bo‘lgan qiziqishni yanada kuchaytirdi. Hozirgi kunda, bu sohaning kvant fizikasiga, chiziqli va nochiziqli xususiy hosilali tenglamalar nazariyasiga, kristallografiyaga, geologo-razvedka masalalariga muhim tadbiqlari topilganligi bois bu mavzuning dolzarbligi yanada ortmoqda.

Mazkur dissertatsiya ishida davriy potensialli Dirak operatori uchun izlar formulasi yangicha elementar usulda keltirib chiqarilgan va teskari spektral masala atroflicha o‘rganilgan, ya’ni davriy potensalli Dirak operatorining spektral berilganlari yordamida uning koeffitsiyentlarini tiklash algoritmi ko‘rsatilgan.

Dissertatsiyada olingan asosiy natijalarning qisqacha bayoni

Quyidagi Dirak operatorini qaraymiz

$$Ly \equiv \mathrm{B}\frac{dy}{dx} + \Omega(x)y = \lambda y, \quad 0 < x < \pi, \tag{1}$$

$$\begin{cases} y_1(0)\cos\alpha + y_2(0)\sin\alpha = 0 \\ y_1(\pi)\cos\beta + y_2(\pi)\sin\beta = 0, \end{cases} \tag{2}$$

bu yerda

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \Omega(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}, \quad y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}.$$

$p(x), q(x) \in C^1[0,\pi]$, $\alpha, \beta \in R$. $y(x,\lambda)$ orqali (1) tenglamaning

$$y_1(0) = \sin\alpha, \quad y_2(0) = -\cos\alpha. \tag{3}$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimini belgilaymiz.

1. $\alpha = \beta = 0$ bo‘lsin, u holda (2) chegaraviy shart Dirixle chegaraviy shart bilan ustma-ust tushadi, ya’ni

$$\begin{cases} y_1(0) = 0, \\ y_1(\pi) = 0 \end{cases} \tag{4}$$

bo‘lib, (1)+(4) masalaga Dirixle chegaraviy masalasi deyiladi. $\{\xi_n\}$, $n \in Z$ orqali Dirixle chegaraviy masalaning xos qiymatlarini belgilaymiz.

2. $\alpha = \beta = \frac{\pi}{2}$ bo‘lsin. Bu holda (2) chegaraviy shart Neyman chegaraviy shart bilan ustma-ust tushadi, ya’ni

$$\begin{cases} y_2(0) = 0, \\ y_2(\pi) = 0, \end{cases} \tag{5}$$

bo‘lib, (1)+(4) masalaga Neyman chegaraviy masalasi deyiladi. $\{\eta_n\}$, $n \in Z$ orqali Neyman chegaraviy masalaning xos qiymatlarini belgilaymiz. $\delta_n = n$, $n \in Z$ orqali ushbu $w_0(\lambda) = \sin\lambda\pi$ funksiyaning nollarini belgilaymiz.

3. Ushbu chegaraviy shartlarga mos ravishda

$$y_1(0) = y_1(\pi), \quad y_2(0) = y_2(\pi) \tag{6}$$

$$y_1(0) = -y_1(\pi), \quad y_2(0) = -y_2(\pi) \tag{7}$$

davriy va antidavriy chegaraviy shartlar deyiladi. (1)+(6) masalaga Dirak operatori uchun qo‘yilgan Davriy chegaraviy masala, (1)+(7) masalaga esa antidavriy chegaraviy masala deyiladi

Lemma 1. (1) tenglamaga qo‘yilgan Dirixle va Neyman chegaraviy masalalari uchun mos ravishda quyidagi izlar formulasi o‘rinli:

$$\sum_{n=-\infty}^{\infty}(\xi_n - n) = -\frac{p(0)+p(\pi)}{2},$$

$$\sum_{n=-\infty}^{\infty}(\eta_n - n) = \frac{p(0)+p(\pi)}{2}.$$

Lemma 2. (1) sistemaga qo‘yilgan davriy va antidavriy chegaraviy masalalar uchun mos ravishda quyidagi izlar formulasi o‘rinli

$$\sum_{n=-\infty}^{\infty}(\lambda_{2n} + \lambda_{2n+1} - 4n) = 0,$$

$$\sum_{n=-\infty}^{\infty}[\mu_{2n} + \mu_{2n+1} - 2(2n+1)] = 0.$$

Bu yerda λ_n, μ_n −lar mos ravishda davriy va antidavriy masalaning xos qiymatlari.

Teorema 1. Quyidagi izlar formulalari o‘rinli

$$\sum_{n=-\infty}^{\infty}[\lambda_n + \mu_n - 2\xi_n] = p(0) + p(\pi),$$

$$\sum_{n=-\infty}^{\infty}[\lambda_n + \mu_n - 2\eta_n] = -p(0) - p(\pi).$$

Quyida butun o‘qda berilgan Dirak tenglamalar sistemasini ko‘rib chiqamiz:

$$Ly \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad x \in (-\infty, \infty), \tag{8}$$

bu yerda $p(x)$ va $q(x)$ haqiqiy uzluksiz π davrli funksiyalar, λ esa kompleks parametr. (8) tenglamaning ushbu

$$c(0,\lambda) = (1,0)^T \quad \text{va} \quad s(0,\lambda) = (0,1)^T$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini

$$c(x,\lambda)=\left(c_1(x,\lambda),c_2(x,\lambda)\right)^T \text{ va } s(x,\lambda)=\left(s_1(x,\lambda),s_2(x,\lambda)\right)^T$$

orqali belgilaymiz.

Ta’rif 1. $\Delta(\lambda)=c_1(\pi,\lambda)+s_2(\pi,\lambda)$ funksiyaga (8) Dirak operatori uchun Lyapunov funksiyasi yoki Xill diskriminanti deyiladi.

(8) operatorning spektri quyidagi to‘plamdan iborat:

$$E=\{\lambda\in R^1:\,-2\le\Delta(\lambda)\le 2\}=R^1\setminus\left\{\bigcup_{n=-\infty}^{\infty}(\lambda_{2n-1},\lambda_{2n})\right\}. \tag{9}$$

Ushbu $(\lambda_{2n-1},\lambda_{2n})$, $n\in Z$ intervallarga lakunalar deyiladi. Spektrning chekka nuqtalari davriy va antidavriy masalaning xos qiymatlari bilan ustma ust tushadi.

$s_1(\pi,\lambda)=0$ tenglamaning ildizlarini ξ_n, $n\in Z$ orqali belgilaymiz. ξ_n, $n\in Z$ sonlar (1) sistemaga qo‘yilgan Dirixle masalasining xos qiymatlari bilan ustma –ust tushadi va ushbu $\xi_n\in[\lambda_{2n-1},\lambda_{2n}]$, $n\in Z$ munosabat bajariladi.

Ta’rif 2. $\xi_n\in[\lambda_{2n-1},\lambda_{2n}]$, $n\in Z$ sonlar ketma-ketligi va ushbu $\sigma_n=sign\left\{s_2(\pi,\xi_n)-\dfrac{1}{s_2(\pi,\xi_n)}\right\}$, $n\in Z$ ishoralar ketma-ketligiga (8) masalaning spektral berilganlari deyiladi.

Ta’rif 3. Spektral parametrlar ξ_n,σ_n, $n\in Z$ va spektrning chekka nuqtalari λ_n, $n\in Z$ larga (8) masalaning spektral berilganlari deyiladi.

(8) masalaning spektral berilganlarini topish masalasiga to‘g‘ri masala va aksincha spektral berilganlar yordamida $p(x)$ va $q(x)$ koeffitsiyentlarni tiklash masalasiga teskari masala deyiladi.

Agar (8) masalada $p(x)$ va $q(x)$ o‘rniga $p(x+\tau)$ va $q(x+\tau)$ funksiyalarni olsak, u holda hosil bo‘lgan yangi masalaning spektri τ parametrga bog‘liq bo‘lmaydi, ya’ni $\lambda_n(\tau)\equiv\lambda_n$, $n\in Z$, ammo spektral parametrlar τ parametrga bog‘lik bo‘ladi: $\xi_n(\tau)$, $\sigma_n(\tau)$, $n\in Z$ va bu masala uchun izlar formulasi ushbu

$$p(\tau)=\sum_{k=-\infty}^{\infty}\left(\frac{\lambda_{2k-1}+\lambda_{2k}}{2}-\xi_k(\tau)\right), \tag{10}$$

$$q(\tau)=\sum_{n=-\infty}^{\infty}(-1)^{n-1}\sigma_n(\tau)\sqrt{(\xi_n(\tau)-\lambda_{2n-1})(\lambda_{2n}-\xi_n(\tau))}\cdot\sqrt{\prod_{\substack{k=-\infty\\k\neq n}}^{\infty}\frac{(\lambda_{2k-1}-\xi_n(\tau))(\lambda_{2k}-\xi_n(\tau))}{(\xi_k(\tau)-\xi_n(\tau))^2}} \tag{11}$$

ko‘rinishda boladi.

Teorema 2. Agar $p(x), q(x)\in C^1(R^1)$ haqiqiy π-davrli funksiyalar bo‘lsa, u holda (8) masalaning $\xi_n(t)$, $n\in Z$ spektral parametrlari quyidagi Dubrovin differensial tenglamalar sistemasini qanoatlantiradi:

$$\dot{\xi}_n(t)=(-1)^{n-1}\sigma_n(t)\sqrt{(\xi_n(t)-\lambda_{2n-1})(\lambda_{2n}-\xi_n(t))}\times$$
$$\times[2\xi_n(t)+\sum_{k=-\infty}^{\infty}(\lambda_{2k-1}+\lambda_{2k}-2\xi_k(t))]\times$$
$$\times\sqrt{\prod_{\substack{k=-\infty,\\k\neq n}}^{\infty}\frac{(\lambda_{2k-1}-\xi_n(t))(\lambda_{2k}-\xi_n(t))}{(\xi_k(t)-\xi_n(t))^2}},\ \ n\in Z. \tag{12}$$

(12) Dubrovin tenglamalar sistemasi quyidagi boshlang‘ich shartlar bilan qaraladi:

$$\xi_n(t)\big|_{t=0}=\xi_n(0),\qquad \sigma_n(t)\big|_{t=0}=\sigma_n(0),\qquad n\in Z.$$

$\xi_n(t)$ spektral parametr o‘z lakunasining chetiga har safar urilganda $\sigma_n(t)$ ishora qarama-qarshisiga o‘zgaradi.

(12) Dubrovin tenglamalar sistemasi hamda (10) va (11) izlar formulasi (8) masalaga qo‘yilgan teskari spektral masalani yechish algoritmini beradi.

Quyidagi teoremada (10) va (11) izlar formulasi yordamida potensialni silliqlik darajasi va lakunalar uzunliklarini kamayish tartibi orasidagi bog‘lanish ko‘rsatilgan.

Teorema 3. Agar $p(x)$, $q(x)$ π-davrli haqiqiy funksiyalar $C^2(-\infty,\infty)$ sinfdan bo‘lsa va $\lambda_{2n}-\lambda_{2n-1}$ lakunalar uzunligi eksponensial tarzda kamaysa, ya’ni agarda ixtiyoriy butun n larda $\lambda_{2n}-\lambda_{2n-1}<ae^{-b|n|}$ tengsizlik o‘rinli bo‘ladigan

shunday $a>0, b>0$ o'zgarmas sonlar mavjud bo'lsa, u holda $p(x)$ va $q(x)$ lar butun o'qda haqiqiy analitik funksiyalardan iborat bo'ladi.

Теорема 4. Agar $p(x)$ va $q(x)$ haqiqiy analitik π-davrli funksiyalar bo'lsa, u holda lakunalar uzunligi $\lambda_{2n}-\lambda_{2n-1}$ eksponensial kamayadi.

G.Borg tomonidan davriy potensialli Shturm-Liuvill operatori uchun $\frac{\pi}{2}$ son davr bo'lishining zarur va yetarlilik shartlari keltirilgan. Ushbu teoremada G.Borg teoremasining Dirak sistemasi uchun anologi keltiriladi.

Teorema 5. (8) tenglamalar sistemasining $p(x)$ va $q(x)$ koeffitsiyentlari $\frac{\pi}{2}$ davrli bo'lishi uchun, (1)+(7) antidavriy masalaning barcha xos qiymatlari ikki karrali bo'lishi zarur va yetarli.

X.P.Mak-Kin, E.Trubovits va P.Deyft ishlarida davriy va antidavriy chegaraviy shartli Shturm-Liuvill operatorining λ_{2k} xos qiymatlariga mos $\varphi_{2k}(x)$ ortonormallangan xos funksiyalarning kvadrati uchun

$$\sum_{k=0}^{\infty}\varepsilon_k\varphi_{2k}^2(x)=1$$

ayniyat olingan. Quyidagi teoremada bu ayniyatni Dirak sistemasi uchun analogi keltiriladi.Ushbu

$$\ldots, y_{-2}(x), y_{-1}(x), y_0(x), y_1(x), y_2(x), \ldots \qquad (13)$$

funksiyalar (1) sistema uchun $[0,\pi]$ kesmada qo'yilgan davriy va antidavriy masalaning

$$\ldots\le\lambda_{-2}<\lambda_{-1}\le\lambda_0<\lambda_1\le\lambda_2<\ldots$$

xos qiymatlarga mos keluvchi normallangan xos vektor-funksiyalar bo'lsin.

Teorema 6. (13) funksiyalar uchun quyidagi ayniyatlar o'rinli:

$$\sum_{k=-\infty}^{\infty}a_k y_{2k-1,1}^2(t)=-p(t)+\frac{c}{2}, \quad \sum_{k=-\infty}^{\infty}b_k y_{2k,1}^2(t)=p(t)+\frac{c}{2},$$

$$\sum_{k=-\infty}^{\infty}a_k y_{2k-1,2}^2(t)=p(t)+\frac{c}{2}, \quad \sum_{k=-\infty}^{\infty}b_k y_{2k,2}^2(t)=-p(t)+\frac{c}{2},$$

$$\sum_{k=-\infty}^{\infty} a_k y_{2k-1,1}(t) y_{2k-1,2}(t) = q(t), \quad \sum_{k=-\infty}^{\infty} b_k y_{2k,1}(t) y_{2k,2}(t) = -q(t),$$

bu yerda

$$a_k = \frac{-\Delta'(\lambda_{2k-1})}{f'(\lambda_{2k-1})}, \qquad b_k = \frac{\Delta'(\lambda_{2k})}{g'(\lambda_{2k})}, \qquad c = \sum_{k=-\infty}^{\infty} (\lambda_{2k} - \lambda_{2k-1}),$$

$$f(\lambda) = \pi(\lambda_{-1} - \lambda) \prod_{0 \neq k=-\infty}^{\infty} \frac{\lambda_{2k-1} - \lambda}{k}, \quad g(\lambda) = \pi(\lambda_0 - \lambda) \prod_{0 \neq k=-\infty}^{\infty} \frac{\lambda_{2k} - \lambda}{k}.$$

I BOB. DIRAK OPERATORI HAQIDA ASOSIY MA'LUMOTLAR

1. Dirak tenglamasini keltirib chiqarish

Kvant mexanikasining fundamental tenglamalaridan biri bu Dirak tenglamasidir. Odatda uni Shredinger tenglamasining relyativistik umumlashmasi deb qaraladi. Ma'lumki, Shredinger tenglamasi norelyativistik zarralarni vaqt bo'yicha holatini aniqlovchi to'lqin tenglamasi bo'lib, u 1926 yil Avstraliyalik fizik E.Shredinger tomonidan keltirib chiqarilgan

$$i\hbar\frac{\partial\psi(r,t)}{\partial t}=\frac{-\hbar^2}{2m}\nabla^2\psi(r,t)+V(r)\psi(r,t),\; r\in R^3$$

bu yerda m zarrani massasi, $2\pi\hbar$ Plank doimiysi. Shredinger bu tenglamani keltirib chiqarishda norelyativistik energiya tenglamasi $E=V+\frac{p^2}{2m}$, hamda energiya va impulsning operator $E\to i\hbar\partial_t, p\to -i\hbar\nabla$ ko'rinishidan foydalangan. Shredingerning bu natijasiga ko'ra kvant mexanikasida yangi tushuncha ya'ni to'lqin funksiyasi $\psi(r,t)$ tushunchasi paydo bo'ldi. Unga ko'ra, agar to'lqin funksiyasi $\psi(r,t)$ to'g'ri normallansa, $|\psi(r,t)|^2$ kattalik zarraning fazoni biror Ω sohasida bo'lish ehtimolini beradi.

Shundan keyin, relyativistik zarralar uchun ham to'lqin tenglamasini keltirib chiqarish mumkinmi? degan savolga javob topish o'sha davrning ko'plab olimlarini bu yo'nalishda ilmiy izlanishlar olib borishiga sababchi bo'ldi. Natijada, 1926 yilda bir-biridan bexabar ravishda Klein va Gardonlar tomonidan dastlabki relyativistik to'lqin tenglamasi olindi

$$\left(\nabla^2-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\psi(r,t)=\frac{m^2c_0^2}{\hbar^2}\psi(r,t)\;,$$

bu yerda c_0 yorug'lik tezligi, m zarrani massasi. Hozirgi kunda bu tenglama Klein-Gardon tenglamasi deb yuritiladi va uning yechimi $\psi(r,t)$ spinsiz skalyar Bozonlar holatini ifodalaydi. Bu tenglamadan keyin 1927 yilda Angliyalik fizik

Dirak tomonidan relyativistik kvant nazariyasining yangi vaqtga nisbatan chiziqli bo‘lgan “relyativistik to‘lqin tenglamasi” keltirib chiqarildi. Bu tenglama hozirgi kunda Dirak tenglamasi deb yuritiladi va spinli zarralarning holatini ifodalaydi. Uning ko‘rinishi quyidagicha:

$$\hat{H}_0 \psi(r,t) = i\hbar \frac{\partial \psi(r,t)}{\partial t},$$

bu yerda

$$\hat{H}_0 = \frac{\hbar c}{i}\left(\alpha_1 \frac{\partial}{\partial x_1} + \alpha_2 \frac{\partial}{\partial x_2} + \alpha_3 \frac{\partial}{\partial x_3}\right) + \beta mc^2 = c\alpha p + \beta mc^2$$

$$p \to -i\hbar\nabla, \nabla = i\frac{\partial}{\partial x_1} + j\frac{\partial}{\partial x_2} + k\frac{\partial}{\partial x_3} and E \to i\hbar\frac{\partial}{\partial t}$$

$$\psi(r,t) = (\psi_1(r,t), \psi_2(r,t), \psi_3(r,t), \psi_4(r,t))^T;$$

$$\alpha = \alpha_1 i + \alpha_2 j + \alpha_3 k$$

$$\alpha_1 = \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & o \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} o & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \beta = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}$$

I_2-2x2 o‘lchamli birlik matritsa.

Eynshteyn nisbiylik nazariyasiga ko‘ra relyativistik energiya tenglamasi ushbu

$$E = \sqrt{p^2c^2 + m^2c^4} \tag{1.1}$$

ko‘rinishda yoziladi. Bu yerda $p = p(p_1, p_2, p_3)$ bo‘lib u zarraning impulsini bildiradi. Energiya va impulsning operator quyidagicha

$$p \to -i\hbar\nabla, \nabla = i\frac{\partial}{\partial x_1} + j\frac{\partial}{\partial x_2} + k\frac{\partial}{\partial x_3} and E \to i\hbar\frac{\partial}{\partial t}$$

ko‘rinishini (1.1) tenglikka qo‘yib, Shredinger nazariyasidan foydalansak, relyativistik zarralar uchun to‘lqin tenglamasiga ega bo‘lamiz:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi = \frac{m^2c^2}{\hbar^2}\psi. \tag{1.2}$$

Bu tenglama vaqtga nisbatan chiziqli emas, bizni qiziqtirgan relyativistik to‘lqin tenglamasi vaqtga nisbatan chiziqli bo‘lishi zarur, shu maqsadda (1.2) tenglikning chap tomonini quyidagicha ko‘rinishda yozib olamiz:

$$\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} = \left(\alpha_1\partial_{x_1} + \alpha_2\partial_{x_2} + \alpha_3\partial_{x_3} + \frac{\mathrm{i}}{\mathrm{c}}\beta\partial_t\right)\left(\alpha_1\partial_{x_1} + \alpha_2\partial_{x_2} + \alpha_3\partial_{x_3} + \frac{i}{c}\beta\partial_t\right). \quad (1.3)$$

Bizning maqsad (1.3) tenglikni qanoatlantiruvchi α_i va β kattaliklarni mavjudligini ko‘rsatishdir. Dirakgacha bo‘lgan olimlar (1.3) tenglikni qanoatlantiruvchi kattaliklar haqiqiy va kompleks sonlar ichida, hattoki 2x2 o‘lchamli matritsalar ichida ham mavjud emasligini bilganlar. Dirakni xizmati shundaki, u bunday kattaliklarni 4x4-o‘lchamli matritsalar ko‘rinishida qidirgan va natijada ularning mavjudligini ko‘rsatishga muvaffaq bo‘lgan. (1.3) tenglikni bajarilishi uchun bu kattliklardan ushbu

$$\beta^2 = I, \alpha_1^2 = \alpha_2^2 = \alpha_3^2 = I, \beta\alpha_i = 0, \alpha_i\alpha_j + \alpha_j\alpha_i = 0, i = 1,2,3; j = 1,2,3; i \neq j$$

shartlarni bajarilishi talab qilinadi. Bu shartlarni qanoatlantiruvchi matritsalar ushbu

$$\alpha_1 = \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & o \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} o & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \beta = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}$$

ko‘rinishda bo‘lib, (1.3) tenglikning o‘rinli ekanligini oddiy hisoblashlar yordamida ko‘rsatish mumkin. Shundan keyin, (1.2) va (1.3) tengliklardan darhol quyidagi ayniyatni yoza olamiz

$$\left(\alpha_1\partial_{x_1} + \alpha_2\partial_{x_2} + \alpha_3\partial_{x_3} + \frac{\mathrm{i}}{\mathrm{c}}\beta\partial_t\right)\psi = \frac{\mathrm{mc}}{\hbar}\psi . \quad (1.4)$$

Agar (1.4) tenglikning har ikkala tomoniga qaytadan bu matritsa operatorni qo‘llasak, natijada (1.2) formulani hosil qilamiz, ya`ni

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi = \frac{m^2c^2}{\hbar^2}\psi$$

bo‘ladi. Shunday qilib, biz relyativistik zarralar holatini ifodalovchi, vaqtga nisbatan chiziqli bo‘lgan ushbu

$$i\hbar\frac{\partial\psi(r,t)}{\partial t}=\left[\frac{\hbar c}{i}\left(\alpha_1\frac{\partial}{\partial x_1}+\alpha_2\frac{\partial}{\partial x_2}+\alpha_3\frac{\partial}{\partial x_3}\right)+\beta mc^2\right]\psi(r,t) \qquad (1.5)$$

Dirak relyativistik to‘lqin tenglamasini keltirib chiqardik.

Dirak operatori uchun spektral masalalarni o‘rganishda ko‘p hollarda statsionar Dirak tenglamasi bilan ishlashga to‘g‘ri keladi. (1.5) tenglamada ushbu

$$\psi(r,t)=\psi(r)e^{-\frac{iEt}{\hbar}}$$

almashtirishni bajarsak, natijada quyidagi

$$\hat{H}_0\psi(r)=E\psi(r), \qquad (1.6)$$

statsionar Dirak tenglamasiga ega bo‘lamiz. Bu yerda E zarraning relyativistik energiyasi. Agar zarra biror skalyar $V(r)$ potensial maydon ta’sirida bo‘lsa, u holda (1.6) tenglama quyidagicha yoziladi

$$\hat{H}_0\psi(r,E)=E\psi(r,E),$$

bunda

$$\hat{H}=c\alpha\hat{p}+\beta mc^2+V(r)=\hat{H}_0+V(r).$$

Bu tenglik yordamida aniqlangan ifoda odatda Dirak operatori deb yuritiladi.

2. Davriy potensialli Dirak operatori uchun Lyapunov funksiyasi va uning xossalari

Quyida butun o‘qda berilgan Dirak tenglamalar sistemasini ko‘rib chiqamiz

$$Ly \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad x \in (-\infty, \infty), \quad (2.1)$$

bu yerda $p(x)$ va $q(x)$ haqiqiy uzluksiz π davrli funksiyalar, λ esa kompleks parametr. (2.1) tenglamaning ushbu

$$c(0,\lambda) = (1, 0)^T \quad \text{va} \quad s(0,\lambda) = (0,1)^T$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini

$$c(x,\lambda) = \left(c_1(x,\lambda), c_2(x,\lambda)\right)^T \text{ va } s(x,\lambda) = \left(s_1(x,\lambda), s_2(x,\lambda)\right)^T$$

orqali belgilaymiz.

$c(x,\lambda)$ va $s(x,\lambda)$ vektor-funksiyalar x ning har bir tayinlangan qiymatida λ parametrga nisbatan butun funksiya bo‘lib, (2.1) tenglamaning fundamental yechimlar sistemasini tashkil qiladi, hamda bu yechimlar uchun quyidagi ayniyat bajariladi:

$$c_1(x,\lambda)s_2(x,\lambda) - c_2(x,\lambda)s_1(x,\lambda) = 1.$$

Ta’rif 2.1. $\Delta(\lambda) = c_1(\pi,\lambda) + s_2(\pi,\lambda)$ funksiyaga (2.1) Dirak operatori uchun Lyapunov funksiyasi yoki Xill diskriminanti deyiladi.

Teorema 2.1. a) $[0,\pi]$ kesmada Dirak sistemasi uchun qo‘yilgan $y_1(0) = y_1(\pi)$, $y_2(0) = y_2(\pi)$ davriy chegaraviy shartli masalaning xos qiymatlari haqiqiy bo‘ladi va ular $\Delta(\lambda) - 2 = 0$ tenglamaning ildizlari bilan ustma-ust tushadi;

b) $[0,\pi]$ kesmada Dirak sistemasi uchun qo‘yilgan $y_1(0) = -y_1(\pi)$, $y_2(0) = -y_2(\pi)$ antidavriy chegaraviy shartli masalaning xos qiymatlari haqiqiy bo‘ladi va ular $\Delta(\lambda) + 2 = 0$ tenglamaning ildizlari bilan ustma-ust tushadi.

Natija 2.1. Ushbu $\Delta(\lambda) - 2 = 0$, $\Delta(\lambda) + 2 = 0$ tenlamalarning ildizlari haqiqiy bo‘ladi.

Misol 2.1. Agar $p(x)\equiv 0$, $q(x)\equiv 0$ bo‘lsa, u holda

$$c(x,\lambda)=\begin{pmatrix}\cos\lambda x\\ \sin\lambda x\end{pmatrix},\quad s(x,\lambda)=\begin{pmatrix}-\sin\lambda x\\ \cos\lambda x\end{pmatrix},\quad \Delta(\lambda)=2\cos\lambda\pi$$

bo‘ladi. $\Delta(\lambda)-2=0$ tenglamaning ildizlari $\lambda_{4n-1}=\lambda_{4n}=2n,\ \ n\in Z$ bo‘ladi; $\Delta(\lambda)+2=0$ tenglamaning ildizlari esa $\lambda_{4n+1}=\lambda_{4n+2}=2n+1,\ \ n\in Z$ bo‘ladi.

Ushbu

$$c(x,\lambda)=\begin{pmatrix}\cos\lambda x\\ \sin\lambda x\end{pmatrix}+\int_0^x\begin{pmatrix}\sin\lambda(x-t) & \cos\lambda(x-t)\\ -\cos\lambda(x-t) & \sin\lambda(x-t)\end{pmatrix}\begin{pmatrix}p(t) & q(t)\\ q(t) & -p(t)\end{pmatrix}c(t,\lambda)dt,$$

$$s(x,\lambda)=\begin{pmatrix}-\sin\lambda x\\ \cos\lambda x\end{pmatrix}+\int_0^x\begin{pmatrix}\sin\lambda(x-t) & \cos\lambda(x-t)\\ -\cos\lambda(x-t) & \sin\lambda(x-t)\end{pmatrix}\begin{pmatrix}p(t) & q(t)\\ q(t) & -p(t)\end{pmatrix}s(t,\lambda)dt$$

integral tenglamalar yordamida $c(x,\lambda)$ va $s(x,\lambda)$ yechimlarning katta $|\lambda|$ lardagi asimptotikasini o‘rganish mumkin. Bunda quyidagi asimptotik formulalar kelib chiqadi:

$$c(x,\lambda)=\begin{pmatrix}\cos\lambda x\\ \sin\lambda x\end{pmatrix}+\underline{\underline{O}}\left(\frac{e^{|\operatorname{Im}\lambda|x}}{\lambda}\right),\ \ |\lambda|\to\infty,\ \ x\in[0,\pi],$$

$$s(x,\lambda)=\begin{pmatrix}-\sin\lambda x\\ \cos\lambda x\end{pmatrix}+\underline{\underline{O}}\left(\frac{e^{|\operatorname{Im}\lambda|x}}{\lambda}\right),\ \ |\lambda|\to\infty,\ \ x\in[0,\pi].$$

Bulardan, xususan Lyapunov funksiyasining haqiqiy λ lardagi asimptotikasi kelib chiqadi:

$$\Delta(\lambda)=2\cos\lambda\pi+\underline{\underline{O}}\left(\frac{1}{\lambda}\right),\qquad \lambda\to\infty.$$

Teorema 2.2. Lyapunov funksiyasi uchun quyidagi tenglik o‘rinli:

$$\frac{d\Delta(\lambda)}{d\lambda}=-\int_0^\pi\{c_2(\pi,\lambda)s_1^2(t,\lambda)+[c_1(\pi,\lambda)-s_2(\pi,\lambda)]c_1(t,\lambda)s_1(t,\lambda)-s_1(\pi,\lambda)c_1^2(t,\lambda)+$$
$$+c_2(\pi,\lambda)s_2^2(t,\lambda)+[c_1(\pi,\lambda)-s_2(\pi,\lambda)]c_2(t,\lambda)s_2(t,\lambda)-s_1(\pi,\lambda)c_2^2(t,\lambda)\}dt.$$

Teorema 2.3. Agar haqiqiy λ son uchun ushbu $-2<\Delta(\lambda)<2$ tengsizlik bajarilsa, u holda quyidagi tengsizlik o‘rinli bo‘ladi:

$$\frac{d\Delta(\lambda)}{d\lambda}\neq 0.$$

Natija 2.2. Ushbu $-2 < \Delta(\lambda) < 2$ polosa ichida Lyapunov funksiyasing ekstremal qiymatlari yo'q.

Teorema 2.4. a) λ son $\Delta(\lambda) = 2$ tenglamaning ikki karrali ildizi bo'lishi uchun quyidagi tengliklar bajarilishi zarur va yetarlidir:

$$s_1(\pi,\lambda) = 0,\ s_2(\pi,\lambda) = 1,\ c_1(\pi,\lambda) = 1, c_2(\pi,\lambda) = 0;$$

b) λ son $\Delta(\lambda) = -2$ tenglamaning ikki karrali ildizi bo'lishi uchun quyidagi tengliklar bajarilishi zarur va yetarlidir:

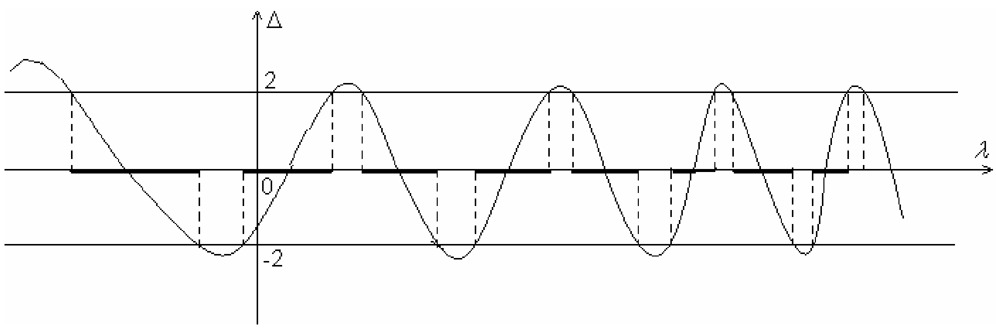
$$s_1(\pi,\lambda) = 0,\ s_2(\pi,\lambda) = -1,\ c_1(\pi,\lambda) = -1,\ c_2(\pi,\lambda) = 0.$$

Teorema 2.5. a) agar λ son $\Delta(\lambda) = 2$ tenglamaning ikki karrali ildizi bo'lsa, u holda $\ddot{\Delta}(\lambda) < 0$ bo'ladi. Xususan, bu nuqtada Lyapunov funksiyasi lokal maksimumga erishadi.

b) agar λ son $\Delta(\lambda) = -2$ tenglamaning ikki karrali ildizi bo'lsa, u holda $\ddot{\Delta}(\lambda) > 0$ bo'ladi. Hususan, bu nuqtada Lyapunov funksiyasi lokal minimumga erishadi.

Natija 2.3. Ushbu $\Delta(\lambda) \pm 2 = 0$ tenglamalar ildizlarining karrasi ikkidan oshmaydi.

Yuqorida keltirilgan teoremalar va $\Delta(\lambda)$ Lyapunov funksiyasining asimptotikasiga ko'ra, umuman olganda $\Delta(\lambda)$ funksiyaning grafigi quyidagi ko'rinishda bo'lishi kelib chiqadi:



(1-rasm)

E orqali $-2 \le \Delta(\lambda) \le 2$ tengsizlikning yechimlari to'plamini belgilaymiz, tushunarliki, E to'plam

$$E=\bigcup_{n=-\infty}^{\infty}[\lambda_{2n},\lambda_{2n+1}]=R^1\setminus\bigcup_{n=-\infty}^{\infty}(\lambda_{2n-1},\lambda_{2n})$$

kesmalar birlashmasidan iborat. Bu kesmalar orasidagi intervallar lakunalar deyiladi: $(\lambda_{2n-1},\lambda_{2n})$, $n\in Z$.

Teorema 2.6. a) har bir yopiq $[\lambda_{2n-1},\lambda_{2n}]$, $n\in Z$ lakunda $s_1(\pi,\lambda)=0$ tenglama bitta $\xi_n\in[\lambda_{2n-1},\lambda_{2n}]$, $n\in Z$ ildizga ega va komleks tekislikning qolgan qismlarida yechimga ega emas, xususan $\overset{0}{E}$ to‘plamda yechimga ega bo‘lmaydi.

б) har bir yopiq $[\lambda_{2n-1},\lambda_{2n}]$, $n\in Z$ lakunada $c_2(\pi,\lambda)=0$ tenglama bitta $\eta_n\in[\lambda_{2n-1},\lambda_{2n}]$, $n\in Z$ ildizga ega va kompleks tekislikning qolgan qismlarida yechimga ega emas, xususan $\overset{0}{E}$ to‘plamda yechimga ega bo‘lmaydi.

Teorema 2.7. Quyidagi yoyilmalar o‘rinli:

$$\Delta^2(\lambda)-4=-4\pi^2\prod_{k=-\infty}^{\infty}\frac{(\lambda_{2k-1}-\lambda)(\lambda_{2k}-\lambda)}{a_k^2},$$

$$c_2(\pi,\lambda)=-\pi\prod_{k=-\infty}^{\infty}\frac{\eta_k-\lambda}{a_k},\quad s_1(\pi,\lambda)=\pi\prod_{k=-\infty}^{\infty}\frac{\xi_k-\lambda}{a_k}.$$

Bu yerda $a_0=1$ va $k\neq 0$ da $a_k=k$.

Eslatma 2.1. Bu yerda va bundan keyin, $\Pi_{k=-\infty}^{\infty}$ ko‘paytmalar va $\Sigma_{k=-\infty}^{\infty}$ yig‘indilarda xususiy ko‘paytmalar va xususiy yig‘indilar simmetrik deb olinadi.

Lemma 2.1. (2.1) sistemaning $c(x,\lambda)$ va $s(x,\lambda)$ yechimlari uchun quyidagi ayniyatlar bajariladi:

$$c(x+\pi,\lambda)=c_1(\pi,\lambda)c(x,\lambda)+c_2(\pi,\lambda)s(x,\lambda),$$

$$s(x+\pi,\lambda)=s_1(\pi,\lambda)c(x,\lambda)+s_2(\pi,\lambda)s(x,\lambda).$$

Quyidagi Dirak tenglamalar sistemasini ko‘rib chiqamiz

$$L(\tau)y\equiv\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{pmatrix}y_1'\\y_2'\end{pmatrix}+\begin{pmatrix}p(x+\tau)&q(x+\tau)\\q(x+\tau)&-p(x+\tau)\end{pmatrix}\begin{pmatrix}y_1\\y_2\end{pmatrix}=\lambda\begin{pmatrix}y_1\\y_2\end{pmatrix},\ x\in(-\infty,\infty),\ (2.2)$$

bu yerda t haqiqiy parametr. $c(x,\lambda,\tau)$ va $s(x,\lambda,\tau)$ orqali (2.2) tenglamaning quyidagi

$$c(0,\lambda,t)=(1,0)^T\ \text{ va }\ s(0,\lambda,t)=(0,1)^T$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini belgilaymiz. Lyapunov funksiyasining ta’rifiga ko‘ra $\Delta(\lambda,\tau)=c_1(\pi,\lambda,\tau)+s_2(\pi,\lambda,\tau)$ bo‘ladi.

Lemma 2.2. Quyidagi ayniyatlar o‘rinli:

$$c(x,\lambda,\tau)=s_2(\tau,\lambda)c(x+\tau,\lambda)-c_2(\tau,\lambda)s(x+\tau,\lambda),$$
$$s(x,\lambda,\tau)=-s_1(\tau,\lambda)c(x+\tau,\lambda)+c_1(\tau,\lambda)s(x+\tau,\lambda).$$

Lemma 2.1 va 2.2 lardan foydalanib quyidagi muhim teorema oson isbotlanadi.

Teorema 2.8. Ushbu $\Delta(\lambda,\tau)\equiv\Delta(\lambda)$ ayniyat bajariladi, ya’ni Lyapunov funksiyasi τ parametrga bog‘liq emas.

3. Floke yechimlari va ularning xossalari

Teorema 3.1. (Floke). a) agar $\Delta^2(\lambda)-4\neq 0$ bo‘lsa, u holda (2.1) tenglamaning quyidagi ko‘rinishdagi ikkita chiziqli erkli yechimi mavjud:

$$\psi_-(x,\lambda)=\rho_-^{\frac{x}{\pi}}\cdot p_-(x,\lambda),\ \psi_+(x,\lambda)=\rho_+^{\frac{x}{\pi}}\cdot p_+(x,\lambda),$$

bunda $p_-(x,\lambda)$ va $p_+(x,\lambda)$ funksiyalar x bo‘yicha π davrli bo‘lib,

$$\rho_{\mp}=\frac{\Delta(\lambda)\pm\sqrt{\Delta^2(\lambda)-4}}{2};$$

b) agar $\Delta(\lambda)=2$ bo‘lsa, u holda (2.1) tenglamaning π davrli yechimi mavjud bo‘ladi;

c) agar $\Delta(\lambda)=-2$ bo‘lsa, u holda (2.1) tenglamaning π antidavrli yechimi mavjud bo‘ladi.

Agar $\psi_1^{\pm}(0,\lambda)=1$ deb olinsa, u holda

$$\psi^{\pm}(x,\lambda)=c(x,\lambda)+\frac{s_2(\pi,\lambda)-c_1(\pi,\lambda)\mp\sqrt{\Delta^2(\lambda)-4}}{2s_1(\pi,\lambda)}s(x,\lambda)$$

bo‘ladi.

Bu yerdagi ildiz analitik ildiz bo‘lib, uning quyidagi shartni qanoatlantiruvchi shohchasi olingan

$$\sqrt{\Delta^2(\lambda)-4}-2i\sin\lambda\pi=\overline{\overline{o}}\left(e^{|\mathrm{Im}\,\lambda|\pi}\right),\qquad \lambda\to i\infty.$$

shart o‘rinli bo‘ladigan shohchasi olingan.
Shuning bilan birga $C\setminus E$ sohada analitik bo‘ladigan funksiyaga ega bo‘lamiz, bu yerda E orqali ushbu $-2\le\Delta(\lambda)\le 2$ tengsizlik bajariladigan haqiqiy o‘qning qismi belgilangan. E to‘plamning har bir kesmasida bu funksiya uzilishga ega bo‘ladi. $\psi_-(x,\lambda)$ va $\psi_+(x,\lambda)$ yechimlarga Floke yechimlari deyiladi.

Teorema 3.2. a) agar $|\Delta(\lambda)|>2$ bo‘lsa, u holda $|\rho_-(\lambda)|>1, |\rho_+(\lambda)|<1$ bo‘ladi;

b) agar $|\Delta(\lambda)|\le 2$ bo‘lsa, u holda $|\rho_-(\lambda)|=|\rho_+(\lambda)|=1$ bo‘ladi.

Teorema 3.3. Floke yechimlari Veyl yechimlariga proporsional bo‘ladi va Veyl-Titchmarsh funksiyasi quyidagi formula bilan beriladi:

$$m^{\mp}(\lambda)=\frac{s_2(\pi,\lambda)-c_1(\pi,\lambda)\pm\sqrt{\Delta^2(\lambda)-4}}{2s_1(\pi,\lambda)}.$$

Eslatma 3.1. Lemma 2.1 va2.2 lardan quyidagilar kelib chiqadi:

$$c(\pi,\lambda,\tau)=-s_1(\pi,\lambda)c_2(\tau,\lambda)c(\tau,\lambda)+c_1(\pi,\lambda)s_2(\tau,\lambda)c(\tau,\lambda)-$$

$$-s_2(\pi,\lambda)c_2(\tau,\lambda)s(\tau,\lambda)+c_2(\pi,\lambda)s_2(\tau,\lambda)s(\tau,\lambda),$$

$$s(\pi,\lambda,\tau)=s_1(\pi,\lambda)c_1(\tau,\lambda)c(\tau,\lambda)+s_2(\pi,\lambda)c_1(\tau,\lambda)s(\tau,\lambda)-$$

$$-c_1(\pi,\lambda)s_1(\tau,\lambda)c(\tau,\lambda)-c_2(\pi,\lambda)s_1(\tau,\lambda)s(\tau,\lambda).$$

Bu ifodalardan foydalanib Floke yechimlari uchun quyidagi

$$s_1(\pi,\lambda)[\psi_1^+(\tau,\lambda)\psi_2^-(\tau,\lambda)+\psi_1^-(\tau,\lambda)\psi_2^+(\tau,\lambda)]=s_2(\pi,\lambda,\tau)-c_1(\pi,\lambda,\tau), \quad (3.1)$$

$$s_1(\pi,\lambda)[\psi_1^+(\tau,\lambda)\psi_1^-(\tau,\lambda)-\psi_2^-(\tau,\lambda)\psi_2^+(\tau,\lambda)]=s_1(\pi,\lambda,\tau)+c_2(\pi,\lambda,\tau) \quad (3.2)$$

tengliklarni hosil qilamiz.

Natija 3.1.. Davriy potensialli (2.1) Dirak operatori xos qiymatga ega emas. Uning spektri uzluksiz bo‘lib, u quyidagi to‘plamdan iborat:

$$E=\left\{\lambda\in R^1\,\middle|\,-2\le\Delta(\lambda)\le 2\right\}=R^1\setminus\bigcup_{n=-\infty}^{\infty}(\lambda_{2n-1},\lambda_{2n}). \quad (3.3)$$

Ushbu $(\lambda_{2n-1},\lambda_{2n})$, $n\in Z$ intervallarga lakunalar deyiladi.

Ushbu $s_1(\pi,\lambda)=0$ tenglamaning ildizlarini ξ_n, $n\in Z$ orqali belgilaymiz. (Bu yerda Veyl-Titchmarsh funksiyasining mahrajidan kelib chiqdik.) ξ_n, $n\in Z$ sonlar (2.1) Dirak sistemasi uchun qo‘yilgan quyidagi

$$y_1(0)=0,\ \ y_1(\pi)=0$$

Dirixle masalasining xos qiymatlari bilan ustma-ust tushadi.

Bundan tashqari $\xi_n\in[\lambda_{2n-1},\lambda_{2n}]$, $n\in Z$ bo‘ladi.

Ta’rif 3.1. Usbu $\xi_n\in[\lambda_{2n-1},\lambda_{2n}]$, $n\in Z$ sonlar va $\sigma_n=sign\{s_2(\pi,\xi_n)-c_1(\pi,\xi_n)\}$, $n\in Z$ ishoralarga (2.1) masalaning spektral parametrlari deyiladi.

Eslatma 3.2. Agar $\xi_n = \lambda_{2n-1}$ yoki $\xi_n = \lambda_{2n}$ bo‘lsa, u holda $s_2(\pi,\xi_n) - \dfrac{1}{s_2(\pi,\xi_n)} = 0$ bo‘ladi. Shu sababli bu holda aniqlik uchun $\sigma_n = 1$ deb olamiz.

Ta’rif 3.2. Lakunalarning chetlari λ_n, $n \in Z$ hamda ξ_n, $\sigma_n = \pm 1$, $n \in Z$ spektral parametrlarga (2.1) masalaning spektral berilganlari deyiladi.

Ta’rif 3.3. (2.1) sistemaning spektral berilganlarini topish masalasiga to‘g‘ri masala deyiladi, aksincha, spektral berilganlar orqali (2.1) sistemaning koeffitsientlarini topish masalasiga teskari spektral masala deyiladi.

I bob bo’yicha xulosa

Dissertatsiyaning ushbu bobida Dirak tenglamasining kelib chiqish tarixi, hamda Dirak operatori haqida asosiy ma’lumotlar keltirilgan. Davriy potensialli Dirak operatori uchun Lyapunov funksiyasi haqida tushunchalar berilib, uning xossalari o‘rganilgan, hamda bir nechta teorema, lemmalar keltirilgan hamda ularning tadbiqiga doir misollar keltirildi. Bundan tashqari potensial argumentining haqiqiy siljishida Lyapunov funksiyasining o‘zgarmaslik xossalari ham o‘rganilgan. Keltirilgan bu lemma va teoremalar Dirak operatori uchun qo‘yilgan teskari masalani yechishda muhim ahamiyatga ega. Floke yechimlari va bu yechimlarning xossalari o□rganilgan.

II BOB. DIRAK OPERATORI UCHUN IZLAR FORMULALARI

1. Dirixle va Neyman chegaraviy shartli Dirak operatori uchun izlar formulasi.

Quyidagi Dirak operatorini qaraylik

$$Ly \equiv \mathrm{B}\frac{dy}{dx} + \Omega(x)y = \lambda y, \quad 0 < x < \pi , \tag{1.1}$$

$$\begin{cases} y_1(0)\cos\alpha + y_2(0)\sin\alpha = 0 \\ y_1(\pi)\cos\beta + y_2(\pi)\sin\beta = 0, \end{cases} \tag{1.2}$$

bu yerda

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \Omega(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}, \quad y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix},$$

$p(x), q(x) \in C^1[0,\pi]$, $\alpha, \beta \in R$. $y(x,\lambda)$ orqali (1.1) tenglamaning

$$y_1(0) = \sin\alpha , \quad y_2(0) = -\cos\alpha . \tag{1.3}$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimini belgilaylik. Koshi funksiyasi yordamida (1.1)+(1.3) masalani

$$y_1(x,\lambda) = \sin(\lambda x + \alpha) + \int_0^x \{[p(t)\sin\lambda(x-t) + q(t)\cos\lambda(x-t)]y_1(t) +$$

$$+ [q(t)\sin\lambda(x-t) - p(t)\cos\lambda(x-t)]y_2(t)\}dt , \tag{1.4}$$

$$y_2(x,\lambda) = -\cos(\lambda x + \alpha) + \int_0^x \{[q(t)\sin\lambda(x-t) - p(t)\cos\lambda(x-t)]y_1(t) -$$

$$- [p(t)\sin\lambda(x-t) + q(t)\cos\lambda(x-t)]y_2(t)\}dt . \tag{1.5}$$

integral tenglamalar sistemasiga keltiramiz. Agar $p(x), q(x) \in C^1[0,\pi]$ bo‘lsa, u holda har bir fiksirlangan x larda (1.4)+(1.5) integral tenglamalar sistemasining $y(x,\lambda)$ yechimi uchun

$$y_1(x,\lambda)=\sin(\lambda x+\alpha)+\frac{1}{2\lambda}\{p(x)\sin(\lambda x+\alpha)+p(0)\sin(\lambda x-\alpha)-$$

$$-q(x)\cos(\lambda x+\alpha)+q(0)\cos(\lambda x-\alpha)-a(x)\cos(\lambda x+\alpha)\}+\underline{\underline{O}}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda^2}\right),$$

$$|\lambda|\to\infty$$

$$y_2(x,\lambda)=-cos(\lambda x+\alpha)+\frac{1}{2\lambda}\{p(x)\cos(\lambda x+\alpha)-p(0)\cos(\lambda x-\alpha)+$$

$$+q(x)\sin(\lambda x+\alpha)+q(0)\sin(\lambda x-\alpha)-a(x)\sin(\lambda x+\alpha)\}+\underline{\underline{O}}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda^2}\right),$$

$$|\lambda|\to\infty,$$

assimptotik formula o‘rinli bo‘ladi, bu yerda

$$a(x)=\int_0^x[p^2(s)+q^2(s)]ds.$$

$w(\lambda)$ orqali (1.1)+(1.2) masalaning xarakteristik tenglamasini belgilaymiz:

$$w(\lambda)=y_1(\pi)\cos\beta+y_2(\pi)\sin\beta. \qquad (1.6)$$

$y_1(\pi)$ va $y_2(\pi)$ lar o‘rniga ularning assimptotik ifodalarini qo‘yib, quyidagiga ega bo‘lamiz:

$$w(\lambda)=\sin(\lambda\pi+\alpha-\beta)+\frac{1}{2\lambda}\{p(\pi)\sin(\lambda\pi+\alpha+\beta)+$$
$$+p(0)\sin(\lambda\pi-\alpha-\beta)-q(\pi)\cos(\lambda\pi+\alpha+\beta)+$$
$$+q(0)\cos(\lambda\pi-\alpha-\beta)-a(\pi)\cos(\lambda\pi+\alpha-\beta)\}+\underline{\underline{O}}\left(\frac{e^{|\pi\operatorname{Im}\lambda|}}{\lambda^2}\right). \qquad (1.7)$$

(1.7) ifodani

$$w(\lambda)=\sin(\lambda\pi+\alpha-\beta)+\frac{1}{2\lambda}\{[p(\pi)\cos 2\beta+p(0)\cos 2\alpha+$$
$$+q(\pi)\sin 2\beta+q(0)\sin 2\alpha]\sin(\lambda\pi+\alpha-\beta)-$$
$$-[-p(\pi)\sin 2\beta+p(0)\sin 2\alpha+q(\pi)\cos 2\beta-$$
$$-q(0)\cos 2\alpha+a(\pi)]\cos(\lambda\pi+\alpha-\beta)\}+\underline{\underline{O}}\left(\frac{e^{|\pi\operatorname{Im}\lambda|}}{\lambda^2}\right)|\lambda|\to\infty.$$

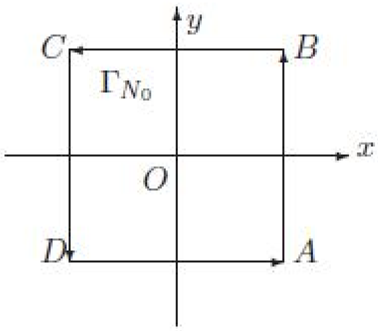
ko‘rinishda yozishimiz mumkin.

$$w_0(\lambda) = \sin(\lambda\pi + \alpha - \beta). \tag{1.8}$$

$w_0(\lambda)$ funksiya nollarini μ_n orqali belgilaylik. Ko'rish qiyinmaski,

$$\mu_n = n - \frac{\alpha - \beta}{\pi}, \ n \in Z.$$

Γ_{N_0} −kvadrat kontur bo'lsin.



1-rasm.

$$A = \left(N_0 + \frac{1}{2}\right)(1 - i), \ B = (N_0 + \frac{1}{2})(1 + i),$$

$$C = \left(N_0 + \frac{1}{2}\right)(-1 + i), \ D = \left(N_0 + \frac{1}{2}\right)(-1 - i),$$

bu yerda N_0 natural son.

Quyidagi munosabatni qaraylik:

$$\frac{w(\lambda)}{w_0(\lambda)} = 1 + \frac{1}{2\lambda}\{p(\pi)\cos 2\beta + p(0)\cos 2\alpha + q(\pi)\sin 2\beta + q(0)\sin 2\alpha\} -$$

$$-\frac{ctg(\lambda\pi + \alpha - \beta)}{2\lambda} \cdot \{p(\pi)\sin 2\beta + p(0)\sin 2\alpha + q(\pi)\cos 2\beta -$$

$$-q(0)\cos 2\alpha + a(\pi)\} + \underline{\underline{O}}\left(\frac{1}{\lambda^2}\right). \tag{1.9}$$

1. $\alpha = \beta = 0$ bo'lsin, u holda (1.2) chegaraviy shart Dirixle chegaraviy shart bilan ustma-ust tushadi, ya'ni

$$\begin{cases} y_1(0) = 0, \\ y_1(\pi) = 0. \end{cases} \tag{1.10}$$

$\{\xi_n\}$, $n \in Z$ orqali (1.1)+(1.10) masalaning xos qiymatlarini belgilaylik. $\alpha = \beta = 0$ da $\mu_n = n, n \in Z$ ga ega bo‘lamiz. Quyidagi tengliklarni o‘rinli ekanini isbotlash qiyin emas:

$$\xi_0 + \sum_{n=1}^{N_0} (\xi_n + \xi_{-n}) = \frac{1}{2\pi i} \oint_{\Gamma_{N_0}} \lambda \left[\frac{w'(\lambda)}{w(\lambda)} - \frac{w_0'(\lambda)}{w_0(\lambda)} \right] d\lambda =$$

$$= \frac{1}{2\pi i} \oint_{\Gamma_{N_0}} \lambda d \ln \frac{w(\lambda)}{w_0(\lambda)} = \frac{1}{2\pi i} \oint_{\Gamma_{N_0}} \ln \frac{w(\lambda)}{w_0(\lambda)} d\lambda. \qquad (1.11)$$

Qaralayotgan holda (1.9) tenglik ushbu ko‘rinishga keladi:

$$\frac{w(\lambda)}{w_0(\lambda)} = 1 + \frac{1}{2\lambda} \{ [p(\pi) + p(0)] - [q(\pi) - q(0) + a(\pi)] \cdot ctg\lambda\pi \} + \underline{\underline{O}}\left(\frac{1}{\lambda^2} \right).$$

Agar $\ln \dfrac{w(\lambda)}{w_0(\lambda)}$ funksiyani Makloren qatoriga yoysak, u holda

$$\ln \frac{w(\lambda)}{w_0(\lambda)} = \frac{1}{2\lambda} \{ [p(\pi) + p(0)] -$$

$$- [q(\pi) - q(0) + a(\pi)] \cdot ctg\lambda\pi \} + \underline{\underline{O}}\left(\frac{1}{\lambda^2} \right) \qquad (1.12)$$

ni olamiz. (1.11) formulaga (1.12) ni qo‘yib,

$$\xi_0 + \sum_{n=1}^{N_0} (\xi_n + \xi_{-n}) = -\frac{1}{2\pi i} \oint_{\Gamma_{N_0}} \left\{ \frac{1}{2\lambda} [p(\pi) + p(0) - \right.$$

$$\left. - (q(\pi) - q(0) + a(\pi)) ctg\lambda\pi] + \underline{\underline{O}}\left(\frac{1}{\lambda^2} \right) \right\} d\lambda. \qquad (1.13)$$

tenglikni keltirib chiqaramiz.

$$ctgz = \frac{1}{z} + 2z \sum_{n=1}^{\infty} \frac{1}{z^2 - n^2\pi^2} \qquad (1.14)$$

ayniyatdan foydalanib, quyidagiga ega bo‘lamiz:

$$\frac{1}{2\pi i} \oint_{\Gamma_{N_0}} \frac{ctg\lambda\pi}{\lambda} d\lambda = 0. \qquad (1.15)$$

Yetarlicha katta N_0 uchun ushbu baholash o‘rinli bo‘ladi:

$$\left| \oint\limits_{\Gamma_{N_0}} \underline{\underline{O}}\left(\frac{1}{\lambda^2}\right) d\lambda \right| = \underline{\underline{O}}\left(\frac{1}{N_0}\right). \tag{1.16}$$

(1.15) va (1.16) ga asosan, (1.13) dan

$$\xi_0 + \sum_{n=1}^{N_0} (\xi_n + \xi_{-n}) = -\frac{p(\pi) + p(0)}{2} + \underline{\underline{O}}\left(\frac{1}{N_0}\right) \tag{1.17}$$

ni hosil qilamiz. (1.17) tenglikda $N_0 \to \infty$ da limitga o‘tib

$$\xi_0 + \sum_{n=1}^{\infty} (\xi_n + \xi_{-n}) = -\frac{p(0) + p(\pi)}{2} \tag{1.18}$$

ga ega bo‘lamiz. (1.18) tenglikni quyidagi ko‘rinishda yozish mumkin:

$$\sum_{n=-\infty}^{\infty} (\xi_n - n) = -\frac{p(0) + p(\pi)}{2}.$$

2. $\alpha = \beta = \dfrac{\pi}{2}$ bo‘lsin. Bu holda (1.2) chegaraviy shart Neyman chegaraviy shart bilan ustma-ust tushadi, ya’ni

$$\begin{cases} y_2(0) = 0, \\ y_2(\pi) = 0. \end{cases} \tag{1.19}$$

$\{\eta_n\}$, $n \in Z$ orqali (1.1)+(1.19) masalaning xos qiymatlarini belgilaymiz. Bu holda $\mu_n = n, \quad n \in Z$

va

$$\frac{w(\lambda)}{w_0(\lambda)} = 1 + \frac{1}{2\lambda}\{[-p(\pi) - p(0)] + [q(\pi) - q(0) - a(\pi)]ctg\lambda\pi\} + \underline{\underline{O}}\left(\frac{1}{\lambda^2}\right).$$

Birinchi holdagidek, ko‘rish mumkinki,

$$\eta_0 + \sum_{n=1}^{\infty} (\eta_n + \eta_{-n}) = \frac{p(0) + p(\pi)}{2}$$

va

$$\sum_{n=-\infty}^{\infty} (\eta_n - n) = \frac{p(0) + p(\pi)}{2}.$$

2. Davriy va antidavriy chegaraviy shartli Dirak operatori uchun izlar formulasi

1. Davriy chegaraviy shart, ya’ni

$$y_1(0) = y_1(\pi), \ \ y_2(0) = y_2(\pi) \tag{2.1}$$

da regulyarlashtirilgan iz formulasi

$$\sum_{n=-\infty}^{\infty}\left(\lambda_{2n} + \lambda_{2n+1} - 4n\right) = 0,$$

ko‘rinishga ega bo‘ladi, bu yerda $\lambda_{2n}, \lambda_{2n+1}$ −(1.1)+(2.1) masalaning xos qiymatlari.Haqiqatan, bu holda

$$w(\lambda) = \Delta(\lambda) - 2, \tag{2.2}$$

bu yerda

$$\Delta(\lambda) = \theta_1(\pi, \lambda) + \varphi_2(\pi, \lambda). \tag{2.3}$$

$\theta(x,\lambda) = \left(\theta_1(x,\lambda), \theta_2(x,\lambda)\right)^T$ va $\varphi(x,\lambda) = \left(\varphi_1(x,\lambda), \varphi_2(x,\lambda)\right)^T$ orqali (1.1) tenglamaning

$$\theta_1(0,\lambda) = 0, \ \theta_2(0,\lambda) = 0, \ \varphi_1(0,\lambda) = 0, \ \varphi_2(0,\lambda) = 1$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini belgilaymiz. $\theta(x,\lambda)$ va $\varphi(x,\lambda)$ funksiyalarning assimptotik formulalaridan

$$\theta_1(\pi,\lambda) = \cos\lambda\pi + \frac{1}{2\lambda}\left[\left(p(\pi) - p(0)\right)\cos\lambda\pi + (q(\pi) + q(0))\sin\lambda\pi + \right.$$

$$\left. + a(\pi)\sin\lambda\pi\right] + \underline{\underline{O}}\left(\frac{e^{|\pi \operatorname{Im}\lambda|}}{\lambda^2}\right), |\lambda| \to \infty,$$

$$\theta_2(\pi,\lambda) = \sin\lambda\pi + \frac{1}{2\lambda}\left[-\left(p(\pi) + p(0)\right)\sin\lambda\pi + (q(\pi) - q(0))\cos\lambda\pi - \right.$$

$$\left. - a(\pi)\cos\lambda\pi\right] + \underline{\underline{O}}\left(\frac{e^{|\pi \operatorname{Im}\lambda|}}{\lambda^2}\right), |\lambda| \to \infty,$$

$$\varphi_1(\pi,\lambda) = -\sin\lambda\pi + \frac{1}{2\lambda}\left[-(p(\pi) + p(0))\sin\lambda\pi + (q(\pi) - q(0))\cos\lambda\pi + \right. \tag{2.4}$$

$$\left. + a(\pi)\cos\lambda\pi\right] + \underline{\underline{O}}\left(\frac{e^{|\pi \operatorname{Im}\lambda|}}{\lambda^2}\right), |\lambda| \to \infty,$$

$$\varphi_2(\pi,\lambda)=\cos\lambda\pi+\frac{1}{2\lambda}\big[(p(\pi)-p(0))\cos\lambda\pi-(q(\pi)+q(0))\sin\lambda\pi+$$

$$+a(\pi)\sin\lambda\pi\big]+\underline{\underline{O}}\left(\frac{e^{|\pi\,\mathrm{Im}\,\lambda|}}{\lambda^2}\right),|\lambda|\to\infty$$

ga ega bo‘lamiz, bu yerda

$$a(\pi)=\int_0^\pi\left[p^2(s)+q^2(s)\right]ds, \tag{2.5}$$

(2.4) assimptotikadan

$$w(\lambda)=2\cos\lambda\pi-2+\frac{1}{\lambda}a(\pi)\sin\lambda\pi+\underline{\underline{O}}\left(\frac{e^{|\pi\,\mathrm{Im}\,\lambda|}}{\lambda^2}\right)|\lambda|\to\infty \tag{2.6}$$

kelib chiqadi.

$$w_0(\lambda)=2\cos\lambda\pi-2, \tag{2.7}$$

bo‘lsin. U holda $w_0(\lambda)$ funksiyaning ildizlari ikki karrali bo‘ladi va ularni μ_n, $n\in Z$ orqali belgilasak,

$$\mu_n=\mu_{2n+1}=\mu_{2n}=2n,\ n\in Z.$$

(2.6) va (2.7) dan quyidagiga ega bo‘lamiz:

$$\frac{w(\lambda)}{w_0(\lambda)}=1+\frac{1}{2\lambda}a(\pi)ctg\frac{\lambda\pi}{2}+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right), \tag{2.8}$$

Ko‘rish mumkinki,

$$\sum_{n=-N_0}^{N_0}\left[\lambda_{2n}+\lambda_{2n+1}-4n\right]=\frac{1}{2\pi i}\oint_{\Gamma_{N_0}}\lambda\left[\frac{w'(\lambda)}{w(\lambda)}-\frac{w_0'(\lambda)}{w_0(\lambda)}\right]d\lambda=-\frac{1}{2\pi i}\oint_{\Gamma_{N_0}}\ln\frac{w(\lambda)}{w_0(\lambda)}d\lambda. \tag{2.9}$$

(2.8) tenglikdan

$$\ln\frac{w(\lambda)}{w_0(\lambda)}=\frac{1}{2\lambda}a(\pi)ctg\frac{\lambda\pi}{2}+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right)$$

ga ega bo‘lamiz. (2.9) tenglikning o‘ng qismi hisoblansa

$$-\frac{1}{2\pi i}\oint_{\Gamma_{N_0}}\left[\frac{a(\pi)}{2}\cdot\frac{ctg\frac{\lambda\pi}{2}}{\lambda}+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right)\right]d\lambda=\underline{\underline{O}}\left(\frac{1}{N_0}\right). \tag{2.10}$$

(2.10) ni (2.9) ga qo‘yib va $N_0 \to \infty,$ da limitga o‘tsak, u holda davriy masala uchun iz formulasini hosil qilamiz:

$$\sum_{n=\infty}^{\infty}\left[\lambda_{2n}+\lambda_{2n+1}-4n\right]=0$$

2. Antidavriy chegaraviy shart, ya’ni

$$y_1(0)=-y_1(\pi), \quad y_2(0)=-y_2(\pi) \qquad (2.11)$$

da regulyarlashtirilgan iz formulasi

$$\sum_{n=\infty}^{\infty}\left[\lambda_{2n}+\lambda_{2n+1}-2(2n+1)\right]=0 \ ,$$

ko‘rinishga ega bo‘ladi, bu yerda $\lambda_{2n}, \lambda_{2n+1} -$ (1.1)+(2.11) masalaning xos qiymatlari.

Yuqorida keltirilgan mulohazalardan quyidagi teoremaning o‘rinli ekani kelib chiqadi:

Teorema. Quyidagi iz formulalari o‘rinli

$$\sum_{n=-\infty}^{\infty}\left[\lambda_n+\mu_n-2\xi_n\right]=p(0)+p(\pi)\,,$$

$$\sum_{n=-\infty}^{\infty}\left[\lambda_n+\mu_n-2\eta_n\right]=-p(0)-p(\pi),$$

bu yerda $\lambda_n,\ \mu_n,\ n\in Z$ –davriy va antidavriy masalaning xos qiymatlari, $\xi_n,\eta_n\,,n\in Z$ – Dirak operatori uchun mos ravishda Dirixle va Neyman masalalarining xos qiymatlari.

II bob bo‘yicha xulosa

Dissertatsiyaning ikkinchi bobida Dirak operatori uchun izlar formulasini keltirib chiqarishning yangicha elementar usuli keltirilgan. Bunda birinchi bo’limda Dirak operatori uchun qo’yilgan Dirixle va Neyman chegaraviy shartlari hollarida izlar formulalari keltirib chiqarildi. Ikkinchi bo’limda esa Dirak operatori uchun qo□yilgan davriy va antidavriy chegaraviy shartlari hollarida izlar formulalari keltirib chiqarildi. Har ikkala bo□limda izlar formulalari yangicha elementar usulda keltirib chiqarildi.

III BOB. DIRAK OPERATORI UCHUN TESKARI MASALA

1. Butun o‘qda berilgan davriy potensialli Dirak operatori uchun Dubrovin tenglamalar sistemasi analogini keltirib chiqarish.

I bobdagi (2.2) masalaning spektri t ga bog‘liq emas, shuning uchun u I bobdagi (3.3) to‘plamdan iborat, lekin spektral parametrlar t ga bog‘liq. Ularni $\xi_n(t), \sigma_n(t)$, $n \in Z$ orqali belgilaymiz.

Teorema 1.1. Agar $p(x), q(x) \in C^1(R^1)$ haqiqiy π-davrli funksiyalar bo‘lsa, u holda (2.2) masalaning $\xi_n(t)$, $n \in Z$ spektral parametrlari quyidagi Dubrovin differensial tenglamalar sistemasini qanoatlantiradi:

$$\dot{\xi}_n(t) = (-1)^{n-1}\sigma_n(t)\sqrt{(\xi_n(t)-\lambda_{2n-1})(\lambda_{2n}-\xi_n(t))} \times$$

$$\times [2\xi_n(t) + \sum_{k=-\infty}^{\infty}(\lambda_{2k-1}+\lambda_{2k}-2\xi_k(t))] \times$$

$$\times \sqrt{\prod_{\substack{k=-\infty,\\ k\neq n}}^{\infty} \frac{(\lambda_{2k-1}-\xi_n(t))(\lambda_{2k}-\xi_n(t))}{(\xi_k(t)-\xi_n(t))^2}}, \quad n \in Z. \tag{1.1}$$

(1.1) Dubrovin tenglamalar sistemasi quyidagi boshlang‘ich shartlar bilan qaraladi:

$$\xi_n(t)\big|_{t=0} = \xi_n(0), \quad \sigma_n(t)\big|_{t=0} = \sigma_n(0), \quad n \in Z.$$

$\xi_n(t)$ spektral parametr o‘z lakunasining chetiga har safar urilganda $\sigma_n(t)$ ishora qarama-qarshisiga o‘zgaradi.

Isbot. I bobdagi (2.2) differensial ifoda va ushbu

$$y_1(0) = 0, \quad y_1(\pi) = 0 \tag{1.2}$$

chegaraviy shartlar yordamida tuzilgan $L(t)$, $t \in R^1$ operatorlar oilalarining $L_2^2(0,\pi)$ vektor-funksiyalar fazosini qaraymiz.

$L(t)$ operatorning $\xi_n(t)$, $n \in Z$ xos qiymatlariga mos ortonormallangan xos vektor-funksiyalar $y_n(x,t)$, $n \in Z$ bo‘lsa, u holda $L(t)y_n = \xi_n(t)y_n$,

$\xi_n(t)=\left(L(t)y_n,y_n\right)$ bo‘ladi. Oxirgi tenglikni t bo‘yicha differensiallasak quyidagiga ega bo‘lamiz:

$$\dot{\xi}_n(t)=\left((L(t)y_n\dot{)},y_n\right)+\left(L(t)y_n,\dot{y}_n\right),$$

$$\dot{\xi}_n(t)=\left(L(t)\dot{y}_n,y_n\right)+\left(\Omega'(x+t)y_n,y_n\right)+\xi_n(t)(y_n,\dot{y}_n).$$

Bu yerda

$$\dot{\vartheta}\equiv\frac{d\vartheta}{dt},\ \ \vartheta'\equiv\frac{d\vartheta}{dx}\ ,\ \ \Omega'(x+t)=\begin{pmatrix} p'(x+t) & q'(x+t)\\ q'(x+t) & -p'(x+t)\end{pmatrix}.$$

$L(t)$ operatorning simmetrikligidan va uning xos vektor-funksiyalari normallanganligidan foydalansak

$$\dot{\xi}_n(t)=\left(\Omega'(x+t)y_n,y_n\right)$$

tenglikni hosil qilamiz, ya’ni

$$\dot{\xi}_n(t)=\int_0^\pi\{p'(x+t)[y_{n1}^2(x,t)-y_{n2}^2(x,t)]+q'(x+t)\cdot 2y_{n1}(x,t)y_{n2}(x,t)\}dx. \quad (1.3)$$

(1.3) tenglikni bo‘laklab integrallasak va (1.2) chegaraviy shartlarni inobatga olsak,

$$\dot{\xi}_n(t)=p(t)[y_{n2}^2(0,t)-y_{n2}^2(\pi,t)]-$$

$$-\int_0^\pi\{2y_{n1}(x,t)y'_{n1}(x,t)-2y_{n2}(x,t)y'_{n2}(x,t)\}p(x+t)dx-$$

$$-\int_0^\pi\{2y_{n1}(x,t)y'_{n2}(x,t)+2y_{n2}(x,t)y'_{n1}(x,t)\}q(x+t)dx.$$

tenglikka ega bo‘lamiz. Bu ayniyatni quyidagi ko‘rinishda yozish mumkin:

$$\dot{\xi}_n(t)=p(t)[y_{n2}^2(0,t)-y_{n2}^2(\pi,t)]-$$

$$-2\int_0^\pi\{y'_{n1}(x,t)[y'_{n2}(x,t)+p(x+t)y_{n1}(x,t)+q(x+t)y_{n2}(x,t)]+$$

$$+y'_{n2}(x,t)[-y'_{n1}(x,t)+q(x+t)y_{n1}(x,t)-p(x+t)y_{n2}(x,t)]\}dx.$$

I bobdagi (2.2) tenglamalar sistemasi va (1.2) chegaraviy shartlardan foydalanib,

$$\dot{\xi}_n(t)=p(t)[y_{n2}^2(0,t)-y_{n2}^2(\pi,t)]-\xi_n(t)\int_0^\pi\{y_{n1}^2(x,t)+y_{n2}^2(x,t)\}'dx,$$

$$\dot{\xi}_n(t)=-[p(t)+\xi_n(t)]\cdot[y_{n2}^2(\pi,t)-y_{n2}^2(0,t)] \quad (1.4)$$

ayniyatni hosil qilamiz. Ushbu

$$y_n(x,t)=\frac{1}{c_n(t)}s(x,\xi_n(t),t)$$

tenglikni hisobga olib, (1.4) tenglikni

$$\dot{\xi}_n(t)=-\frac{[p(t)+\xi_n(t)]\cdot[s_2^2(\pi,\xi_n(t),t)-1]}{c_n^2(t)} \tag{1.5}$$

bilan almashtiramiz. Dirak sistemalarining yechimlari uchun yozilgan

$$\int_0^\pi [y_1^2(x,\lambda)+y_2^2(x,\lambda)]dx=\left(y_1(x,\lambda)\cdot\frac{\partial y_2(x,\lambda)}{\partial\lambda}-y_2(x,\lambda)\cdot\frac{\partial y_1(x,\lambda)}{\partial\lambda}\right)\Bigg|_0^\pi$$

tenglikdan, I bobdagi (2.2)+(1.2) masalaning normallangan o‘zgarmaslari uchun quyidagi formulani hosil qilamiz:

$$c_n^2(t)=\int_0^\pi [s_1^2(x,\xi_n(t),t)+s_2^2(x,\xi_n(t),t)]dx=-\frac{\partial s_1(\pi,\xi_n(t),t)}{\partial\lambda}\cdot s_2(\pi,\xi_n(t),t). \tag{1.6}$$

(1.6) ifodani (1.5) tenglikka qo‘yib

$$\dot{\xi}_n(t)=\frac{[p(t)+\xi_n(t)]\cdot\left(s_2(\pi,\xi_n(t),t)-\dfrac{1}{s_2(\pi,\xi_n(t),t)}\right)}{\dfrac{\partial s_1(\pi,\xi_n(t),t)}{\partial\lambda}}$$

tenglikka ega bo‘lamiz.

$x=\pi$ va $\lambda=\xi_n(t)$ qiymatlarni

$$c_1(x,\lambda,t)s_2(x,\lambda,t)-c_2(x,\lambda,t)s_1(x,\lambda,t)=1$$

ayniyatga qo‘ysak,

$$c_1(\pi,\xi_n(t),t)=\frac{1}{s_2(\pi,\xi_n(t),t)}$$

hosil bo‘ladi.

Bu tenglik va

$$[c_1(\pi,\lambda,t)-s_2(\pi,\lambda,t)]^2=(\Delta^2(\lambda)-4)-4c_2(\pi,\lambda,t)s_1(\pi,\lambda,t)$$

ayniyatni hisobga olgan holda, quyidagini hosil qilamiz:

$$s_2(\pi,\xi_n(t),t)-\frac{1}{s_2(\pi,\xi_n(t),t)}=\sigma_n(t)\sqrt{\Delta^2(\xi_n(t))-4},$$

bu yerda

$$\Delta(\lambda) = c_1(\pi,\lambda,t) + s_2(\pi,\lambda,t), \quad \sigma_n(t) = sign\left\{ s_2(\pi,\xi_n(t),t) - \frac{1}{s_2(\pi,\xi_n(t),t)} \right\}.$$

Demak,

$$\dot{\xi}_n(t) = \frac{[p(t)+\xi_n(t)]\cdot\sigma_n(t)\sqrt{\Delta^2(\xi_n(t))-4}}{\dfrac{\partial s_1(\pi,\xi_n(t),t)}{\partial\lambda}}. \quad (1.7)$$

bo‘lar ekan.

Ushbu

$$\Delta^2(\lambda) - 4 = -4\pi^2 \prod_{k=-\infty}^{\infty} \frac{(\lambda_{2k-1}-\lambda)(\lambda_{2k}-\lambda)}{a_k^2}, \qquad s_1(\pi,\lambda,t) = \pi\prod_{k=-\infty}^{\infty}\frac{\xi_k-\lambda}{a_k},$$

yoyilmalardan foydalanib, (1.7) tenglikni quyidagi ko‘rinishda yozishimiz mumkin:

$$\dot{\xi}_n(t) = 2(-1)^{n-1}\sigma_n(t)\sqrt{(\xi_n(t)-\lambda_{2n-1})(\lambda_{2n}-\xi_n(t))}\times[p(t)+\xi_n(t)]\times$$

$$\times\sqrt{\prod_{\substack{k=-\infty,\\ k\neq n}}^{\infty}\frac{(\lambda_{2k-1}-\xi_n(t))(\lambda_{2k}-\xi_n(t))}{(\xi_k(t)-\xi_n(t))^2}}, \quad n\in Z \quad (1.8)$$

bu yerda $a_0 = 1$ va $k \neq 0$ da $a_k = k$.

Bunda biz

$$sign\left\{ -\frac{\pi}{a_n}\prod_{\substack{k=-\infty\\ k\neq n}}^{\infty}\frac{\xi_k-\xi_n}{a_k} \right\} = (-1)^{n-1}$$

tenglikdan foydalandik. Quyidagi

$$p(t) = \sum_{k=-\infty}^{\infty}\left(\frac{\lambda_{2k-1}+\lambda_{2k}}{2} - \xi_k(t)\right)$$

birinchi iz formulasidan foydalanib, (1.1) tenglamani keltirib chiqaramiz.

(1.1) tenglamada barcha ildizlar arifmetik ma’noda tushuniladi. $\xi_n(t)$ nuqtalar bilan $[\lambda_{2n-1},\lambda_{2n}]$ lakunalar chegaralarining har bir to‘qnashuvida $\sigma_n(t)$ ishora teskarisiga o‘zgaradi. I bobdagi (2.2) tenglamaning Lyapunov funksiyasi t

ga bog‘liq emasligidan bu tenglamaning spektri t ga bog‘liq emasligi kelib chiqadi. **Teorema isbotlandi.**

Hozir Dubrovin tenglamalar sistemasi uchun qo‘yilgan Koshi masalasini o‘rganamiz. Shu maqsadda (1.1) Dubrovin tenglamalar sistemasida o‘zgaruvchilarni almashtirish bajaramiz:

$$\xi_n(t) = \lambda_{2n-1} + (\lambda_{2n} - \lambda_{2n-1})\sin^2 x_n(t), \qquad n \in Z. \tag{1.9}$$

U holda

$$\frac{dx_n}{dt} = \frac{1}{2}(-1)^{n-1}\sigma_n(0)\, h_n(\ldots,\xi_{-1},\xi_0,\xi_1,\ldots), \quad n \in Z \tag{1.10}$$

bo‘ladi, bu yerda

$$h_n = [2\xi_n + \sum_{k=-\infty}^{\infty}(\lambda_{2k-1} + \lambda_{2k} - 2\xi_k)] \times \sqrt{\prod_{\substack{k=-\infty,\\ k\neq n}}^{\infty} \frac{(\lambda_{2k-1} - \xi_n)(\lambda_{2k} - \xi_n)}{(\xi_n - \xi_k)^2}}.$$

Boshlang‘ich shartni

$$x_n(0) = \arcsin\sqrt{\frac{\xi_n(0) - \lambda_{2n-1}}{\lambda_{2n} - \lambda_{2n-1}}}, \quad n \in Z \tag{1.11}$$

ko‘rinishda olamiz. Nihoyat (1.10) tenglikni quyidagicha yozish mumkin:

$$\frac{dx_n}{dt} = H_n(\ldots,x_{-1},x_0,x_1,\ldots), \quad n \in Z, \tag{1.12}$$

bu yerda

$$H_n(x) = \frac{1}{2}(-1)^{n-1}\sigma_n(0)\, h_n(\xi) \tag{1.13}$$

va

$$\xi_n = \xi_n(t) = \lambda_{2n-1} + (\lambda_{2n} - \lambda_{2n-1})\sin^2 x_n(t), \qquad n \in Z. \tag{1.14}$$

(1.11), (1.12) Koshi masalasini o‘rganish maqsadida

$$K = \left\{ x = (\ldots,x_{-1},x_0,x_1,\ldots) : \|x\| = \sum_{n=-\infty}^{\infty}(\lambda_{2n} - \lambda_{2n-1})|x_n| < \infty \right\},$$

Banax fazosini kiritamiz va

$$H = (\ldots,H_{-1},H_0,H_1,\ldots)$$

deb belgilaymiz. Dubrovin sistemasini K banax fazosida bitta tenglama ko‘rinishida yozamiz:

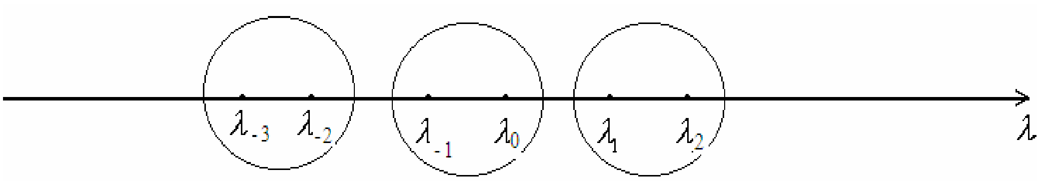
$$\frac{dx}{dt} = H(x). \tag{1.15}$$

Ma’lumki, $y' = F(y), \quad y(0) = y_0$ Koshi masalasi K banax fazosida yagona yechimga ega bo‘lishi uchun Lipshits shartini bajarishi yetarli edi, ya’ni barcha $x, y \in K$ larda

$$\|F(x) - F(y)\| \le const \cdot \|x - y\|$$

bo‘lishi yetarli.

$H(x)$ K fazoda Lipshits shartini qanoatlantirishini isbotlaymiz. $D = \prod\limits_{k=-\infty}^{\infty} D_k$ to‘plamni kiritamiz, bu yerda $D_k, k \in Z$ lar kesishmaydigan ochiq doiralar (1 rasmga qaralsin). D_k doiralar λ-tekislikda joylashadi, $[\lambda_{2k-1}, \lambda_{2k}]$ yopiq lakunalarni o‘z ichiga oladi, markazi $\frac{\lambda_{2k-1} + \lambda_{2k}}{2}$ nuqtalarda joylashadi va yetarlicha katta $|k|$ lar uchun r_k radiuslari $r_\kappa = \frac{\lambda_{2k} - \lambda_{2k-1}}{2} + \frac{1}{k^2}$ formula bo‘yicha aniqlanadi.



(1 rasm)

Lemma 1.1. $\xi \in D$ bo‘lsin. U holda

$$h_n = \underline{\underline{O}}(n) \ , \ n \to \infty \ , \quad \frac{\partial h_n}{\partial \xi_k} = \underline{\underline{O}}(1), \quad n \to \infty$$

baholashlar o‘rinli bo‘ladi.

Isbot. D to‘plamda quyidagi baholashlar o‘rinli:

$$|h_n|^2 = \left| 2\xi_n + \sum_{k=-\infty}^{\infty} (\lambda_{2k-1} + \lambda_{2k} - 2\xi_k) \right|^2 \cdot \prod_{k \ne n} \left| \frac{\xi_n - \lambda_{2k-1}}{\xi_n - \xi_k} \right| \cdot \left| \frac{\xi_n - \lambda_{2k}}{\xi_n - \xi_k} \right| \le$$

$$\le (2|n|+c)^2 \cdot \prod_{k\ne n}\left|\left(1+\frac{\xi_k-\lambda_{2k-1}}{\xi_n-\xi_k}\right)\left(1+\frac{\xi_k-\lambda_{2k}}{\xi_n-\xi_k}\right)\right| \le$$

$$\le (2|n|+c)^2 \cdot \prod_{k\ne n}\left|1+\frac{\lambda_{2k-1}+\lambda_{2k}-2\xi_k}{\xi_k-\xi_n}+\frac{(\xi_k-\lambda_{2k-1})(\xi_k-\lambda_{2k})}{(\xi_n-\xi_k)^2}\right| \le$$

$$\le (2|n|+c)^2 \cdot \exp\left\{\sum_{k\ne n}\left|\frac{\lambda_{2k-1}+\lambda_{2k}-2\xi_k}{\xi_k-\xi_n}\right| + \sum_{k\ne n}\left|\frac{(\xi_k-\lambda_{2k-1})(\xi_k-\lambda_{2k})}{(\xi_n-\xi_k)^2}\right|\right\}. \quad (1.16)$$

λ_{2k-1} va λ_{2k} lar uchun assimptotikalardan foydalansak, hamda D_k to‘plamlar tuzilishiga ko‘ra (1.16) da $h_n = \underline{\underline{O}}(n)$, $n\to\infty$ baholashga ega bo‘lamiz.

Koshi integral formulasidan quyidagiga ega bo‘lamiz:

$$\frac{\partial h_n}{\partial \xi_k} = \frac{1}{2\pi i}\int_{\partial D_k}\frac{h_n(\ldots,\xi_{k-1},s,\xi_{k+1},\ldots)}{(s-\xi_k)^2}ds,$$

bundan $\dfrac{\partial h_n}{\partial \xi_k} = \underline{\underline{O}}(1)$, $n\to\infty$ kelib chiqadi. **Lemma isbotlandi.**

Agar $x, y \in K$ bo‘lsa, u holda $\xi,\eta \in D$, bu yerda

$$\xi = (\ldots,\xi_{-1},\xi_0,\xi_1,\ldots), \qquad \eta = (\ldots,\eta_{-1},\eta_0,\eta_1,\ldots),$$

$$\xi_n = \lambda_{2k-1} + (\lambda_{2k}-\lambda_{2k-1})\sin^2 x_n, \quad \eta_n = \lambda_{2k-1} + (\lambda_{2k}-\lambda_{2k-1})\sin^2 y_n.$$

Shuning uchun

$$|h_n(\xi)-h_n(\eta)| \le \sum_{k=-\infty}^{\infty}\left|\frac{\partial h_n(\theta)}{\partial \xi_k}\right|\cdot|\xi_k-\eta_k| \le$$

$$\le \sum_{k=-\infty}^{\infty}\left[c\cdot|\lambda_{2k}-\lambda_{2k-1}|\cdot\left|\sin^2 x_k-\sin^2 y_k\right|\right] \le c\cdot\sum_{k=-\infty}^{\infty}(\lambda_{2k}-\lambda_{2k-1})\cdot|x_k-y_k| = c\cdot\|x-y\|$$

va

$$\|H(x)-H(y)\| = \sum_{n=-\infty}^{\infty}(\lambda_{2n}-\lambda_{2n-1})|H_n(x)-H_n(y)| =$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty}(\lambda_{2n}-\lambda_{2n-1})|h_n(\xi)-h_n(\eta)| \le \tilde{c}\cdot\|x-y\|,$$

ya’ni Lipshits sharti bajariladi.

Shunday qilib, (1.12) cheksizta tenglamalar sistemasi ixtiyoriy boshlang‘ich berilganlarda yagona yechimga ega va barcha $t\in R^1$ lar uchun yechim mavjud.

2. Koeffitsiyentlar analitikligi va lakuna uzunligining kamayishi orasidagi bog‘lanish

Bu paragrafda Dubrovin tenglamalar sistemasi yordamida lakunalar uzunligi bilan quyidagi

$$Ly \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \ x \in (-\infty, \infty) \qquad (2.1)$$

Dirak tenglamalar sistemasining $p(x)$ va $q(x)$ koeffitsiyentlari analitikligi orasidagi bog‘lanish o‘rganiladi, bu yerda $p(x)$ va $q(x)$ π davrli haqiqiy uzluksiz funksiyalar, λ esa kompleks parametr.

Teorema 2.1. Agar $p(x)$, $q(x)$ π-davrli haqiqiy funksiyalar $C^2(-\infty, \infty)$ sinfdan bo‘lsa va $\lambda_{2n} - \lambda_{2n-1}$ lakunalar uzunligi eksponensial tarzda kamaysa, ya’ni agarda ixtiyoriy butun n larda $\lambda_{2n} - \lambda_{2n-1} < ae^{-b|n|}$ tengsizlik o‘rinli bo‘ladigan shunday $a > 0, b > 0$ o‘zgarmas sonlar mavjud bo‘lsa, u holda $p(x)$ va $q(x)$ lar butun o‘qda haqiqiy analitik funksiyalardan iborat bo‘ladi.

Isbot. Teorema shartiga ko‘ra $\lambda_{2n} - \lambda_{2n-1} < ae^{-b|n|}$ $(a > 0,\ b > 0)$.

$$\xi_n = \lambda_{2n-1} + (\lambda_{2n} - \lambda_{2n-1})\sin^2 z_n \ \text{ bo‘lsin,}$$

bu yerda $z_n = x_n + iy_n$, $n \in Z$.

$|y_n| \le \frac{b}{4}|n|$ bo‘lganda ξ_n qiymatlar D_n doirada joylashishini ko‘rsatamiz. Haqiqatan ham,

$$\left|\frac{\lambda_{2n-1} + \lambda_{2n}}{2} - \xi_n\right| = \left|\frac{\lambda_{2n} - \lambda_{2n-1}}{2} - (\lambda_{2n} - \lambda_{2n-1})\sin^2 z_n\right| =$$

$$= (\lambda_{2n} - \lambda_{2n-1}) \times \frac{1}{2}|\cos 2z_n| \le ae^{-b|n|} \times \frac{1}{2}e^{2|y_n|} \le \frac{a}{2}e^{-\frac{b}{2}|n|} < r_n .$$

$x = (..., x_{-1}, x_0, x_1, ...)$ haqiqiy qismlari K to‘plamdan va mavhum qismlari $|y_n| \le \frac{b}{4}|n|$, $n \in Z$ bo‘lgan $z = (..., z_{-1}, z_0, z_1, ...)$ kompleks ketma-ketlikdan iborat F to‘plamni qaraylik.

Lemma 1.1. dan foydalanib, F to‘plamda $H(z)$ uchun Lipshits sharti bajarilishini ko‘rish mumkin.

Quyidagi vektor-funksiyalar ketma-ketligini o‘rganaylik

$$z_n^{(0)}(t) = x_n(0) = \arcsin\sqrt{\frac{\xi_n(0) - \lambda_{2n=1}}{\lambda_{2n} - \lambda_{2n-1}}}, \quad n \in Z,$$

$$z_n^{(m+1)}(t) = x_n(0) + \int_0^t H_n(\ldots, z_{-1}^{(m)}(s), z_0^{(m)}(s), z_1^{(m)}(s), \ldots)ds, \ m = 0, 1, 2, \ldots,$$

bu yerda t kompleks argument. $|t| \le \frac{b}{4c}$ da

$$\left|y_n^{(m+1)}(t)\right| \le \left|\int_0^t H_n(\ldots, z_{-1}^{(m)}(s), z_0^{(m)}(s), z_1^{(m)}(s), \ldots)ds\right| \le c \cdot |t| \cdot |n|,$$

tengsizlikdan

$$\left|y_n^{(m)}(t)\right| \le \frac{b}{4}|n|, \ \ m = 0, 1, 2, \ldots$$

baholashlarni olamiz.

Demak, $|t| \le \frac{b}{4c}$ uchun $z^{(m)}(t)$ vektor-funksiyalar ketma-ketligi F ga tegishli bo‘ladi. Lipshits shartidan foydalanib, $z^{(m)}(t)$ vektor-funksiyalar ketma-ketligi $|t| \le \frac{b}{4c}$ doirada tekis yaqinlashishini ko‘rish mumkin.

$z^{(m)}(t)$ ketma-ketlik hadlari $|t| \le \frac{b}{4c}$ doirada golomorf vektor-funksiyadan iborat bo‘lganligi uchun Veyershtrass teoremasiga ko‘ra, $z(t)$ limitik vektor-funksiya ham shu doirada golomorfligi kelib chiqadi. Bunda $|t| \le \frac{b}{4c}$ doirada

$$\xi_n(t) = \lambda_{2n-1} + (\lambda_{2n} - \lambda_{2n-1})\sin^2 z_n(t), \ n \in Z$$

funksiyalarning golomorfligiga ega bo‘lamiz.

Xususan $-\dfrac{b}{4c} \le t \le \dfrac{b}{4c}$ da, $z(t)$ vektor-funksiyalar haqiqiy, analitik bo‘ladi va (1.12) tenglamalar sistemasini hamda (1.11) boshlang‘ich shartlarni qanoatlantiradi. Quyidagi

$$\tilde{p}(t) = \sum_{k=-\infty}^{\infty} \left(\frac{\lambda_{2k-1} + \lambda_{2k}}{2} - \xi_k(t) \right), \tag{2.2}$$

$$\tilde{q}^2(t) + \tilde{q}'(t) = \sum_{k=-\infty}^{\infty} \left(\frac{\lambda_{2k-1}^2 + \lambda_{2k}^2}{2} - \xi_k^2(t) \right) \tag{2.3}$$

qatorlarning tekis yaqinlashuvchiligidan, bu qatorlarning $t = 0$ nuqta atrofidagi $\tilde{p}(t)$, $\tilde{q}^2(t) + \tilde{q}'(t)$ yig‘indilarining ham golomorfligi kelib chiqadi.

Agar (1.2) Dirixle chegaraviy shartlar o‘rniga Neyman chegaraviy shartlari ($y_2(0) = 0$, $y_2(\pi) = 0$) olinsa, u holda yuqoridagi usul yordamida quyidagi

$$\tilde{q}^2(t) - \tilde{q}'(t) = \sum_{k=-\infty}^{\infty} \left(\frac{\lambda_{2k-1}^2 + \lambda_{2k}^2}{2} - \eta_k^2(t) \right) \tag{2.4}$$

qator yig‘indisining ham golomorfligini keltirib chiqarish mumkin. (2.3) va (2.4) tengliklardan $t = 0$ nuqta atrofida $\tilde{q}\,(t)$ funksiyaning golomorfligi kelib chiqadi.

Haqiqiy t larda (2.2), (2.3) va (2.4) qator yig‘indilari izlar formulalariga ko‘ra mos ravishda $p(t)$, $q^2(t) + q'(t)$ va $q^2(t) - q'(t)$ funksiyalarga o‘tadi. Bundan $p(t)$ va $q(t)$ koeffitsiyentlar nol nuqta atrofida analitikligi kelib chiqadi.

(2.1) tenglamalar sistemasi o‘rniga

$$L(t_0)y \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} p(x+t_0) & q(x+t_0) \\ q(x+t_0) & -p(x+t_0) \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \; x \in R^1,$$

tenglamani qarasak, $t = 0$ nuqtada $p(t + t_0)$ va $q(t + t_0)$ funksiyalar analitikligi kelib chiqadi, bu yerda t_0 haqiqiy tayinlangan son. Demak, $p(t)$ va $q(t)$ funksiyalar t_0 nuqtada analitik. **Teorema isbotlandi.**

Natija 2.1. Isbotlangan teoremalardan $p(t)$ va $q(t)$ funksiyalar har bir haqiqiy t_0 nuqta atrofida analitik davom etishi kelib chiqadi. Bu atroflar $[0, \pi]$ kompaktning qoplamasi bo‘ladi, shuning uchun ulardan chekli qismqoplama ajratish mumkin. Bu chekli atroflar sistemasi $[0,\pi]$ kesmani o‘z ichiga olgan polosa uchun ham qoplama bo‘ladi. Bundan , π-davrli $p(t)$ va $q(t)$ funksiyalarning π-davrli ekanligidan foydalanib, ularning haqiqiy o‘qni o‘z ichiga oluvchi biror polosaga analitik davom qilishi kelib chiqadi.

Teorema 2.2. Agar $p(x)$ va $q(x)$ π-davrli haqiqiy analitik funksiyalar bo‘lsa, u holda $\lambda_{2n}-\lambda_{2n-1}$ lakun uzunliklari eksponensial ravishda kamayadi..

Isbot. Dastlab haqiqiy t larda quyidagi chegaraviy masalaning xos qiymatlari assimptotikasini o‘rganamiz:

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix}+\begin{pmatrix} p(x+it) & q(x+it) \\ q(x+it) & -p(x+it) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}=\lambda\begin{pmatrix} y_1 \\ y_2 \end{pmatrix},\ x\in(0,\pi), \qquad (2.5)$$

$$y_1(0)=0\,,\quad y_1(\pi)=0\,. \qquad (2.6)$$

(2.5)+(2.6) masalaning xos qiymatlari

$$s_1(\pi,\lambda,it)=0 \qquad (2.7)$$

tenglamadan aniqlanadi. $s_1(x,\lambda,it)$ uchun quyidagicha assimptotika olish mumkin:

$$s_1(x,\lambda,it)=-\sin\lambda x+\frac{1}{2\lambda}\{-[p(x+it)+p(it)]\sin\lambda x+$$

$$+[q(x+it)-q(it)]\cos\lambda x+a(x,t)\cos\lambda x\}+\underline{\underline{O}}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda^2}\right),\quad |\lambda|\to\infty, \qquad (2.8)$$

bu yerda

$$a(x,t)=\int_0^x[p^2(s+it)+q^2(s+it)]ds\,.$$

(2.7) va (2.8) formulalardan

$$-\sin\lambda\pi+\frac{1}{2\lambda}\{-[p(\pi+it)+p(it)]\sin\lambda\pi+$$

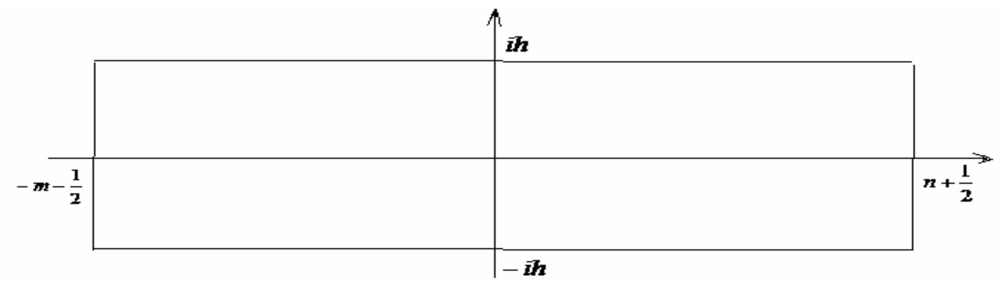
$$+[q(\pi+it)-q(it)]\cos\lambda\pi+a(\pi,t)\cos\lambda\pi\}+\underline{\underline{O}}\left(\frac{e^{|\pi\operatorname{Im}\lambda|}}{\lambda^2}\right)=0 \qquad (2.9)$$

tenglama kelib chiqadi.

Quyidagicha funksiyalarni kiritaylik:

$$f(\lambda) = -\sin\lambda\pi\,,\ g(\lambda) = s_1(\pi, \lambda, it) + \sin\lambda\pi\,.$$

$f(\lambda)$ va $g(\lambda)$ uchun $P_{m,n,h}$ (2 rasm) to‘g‘ri to‘rtburchakda Rushe teoremasini qo‘llaymiz



(2 rasm)

Yetarlicha katta $m, n \in N$ va $h \in R_+^1$ sonlar bilan boshlab (2.9) baholashga ko‘ra $P_{m,n,h}$ to‘g‘ri to‘rtburchak chegarasida $|g(\lambda)| < |f(\lambda)|$ tengsizlik bajariladi. Shunday to‘g‘ri to‘rtburchaklarning eng kichigini P orqali belgilaymiz. Rushe teoremasi bo‘yicha, $P_{m,n,h}$ to‘g‘ri to‘rtburchakda $f(\lambda) + g(\lambda) = s_1(\pi, it, \lambda)$ funksiya $m+n+1$ ta nollarga ega bo‘ladi. Bu nollarni

$$\xi_{-m}(it),\ \ldots,\ \xi_0(it),\ \ldots,\ \xi_n(it)$$

orqali belgilaymiz. $P_{m,n+1,h}$ to‘g‘ri to‘rtburchak esa bu nollardan boshqa yana bitta nolga ega bo‘ladi, uni $\xi_{n+1}(it)$ orqali belgilaymiz. Shunday qilib, yetarlicha katta $n = n_0$ qiymatlarda quyidagi tengsizlik bajariladi

$$n - \frac{1}{2} < \mathrm{Re}\,\xi_n(it) < n + \frac{1}{2}\,.$$

Markazi $n \in Z \setminus P$ nuqtada va radiusi $\frac{1}{|n|^\beta}$, $(0 < \beta < 1)$ bo‘lgan B_n doirada $f(\lambda)$ va $g(\lambda)$ funksiyalar uchun Rushe teoremasini qo‘llaymiz. Natijada B_n doirada $\xi_n(it)$ xos qiymat yotishi kelib chiqadi. Demak

$$\xi_n(it) = n + \underline{\underline{O}}\left(\frac{1}{|n|^\beta}\right), \quad |n| \to \infty \tag{2.10}$$

O‘rinli bo‘ladi. $\xi_n(it)$ funksiyalarning t bo‘yicha uzluksizligidan (2.10) baholash chekli oraliqlarda tekis ekanligi kelib chiqadi.

Lemma 2.1. Yetarlicha katta $|n|$ va musbat c o'zgarmas uchun D to'plamda

$$|\operatorname{Re} h_n| > c \cdot |n| \tag{2.11}$$

baholash o'rinli bo'ladi.

Isbot.

$$A_n = \prod_{k \neq n} \frac{(\xi_n - \lambda_{2k-1})(\xi_n - \lambda_{2k})}{(\xi_n - \xi_k)^2}, \quad \xi \in D$$

bo'lsin. U holda D to'plamning tuzilishiga ko'ra va $\lambda_{2k-1}, \lambda_{2k}$ lar uchun assimptotikalardan

$$A_n = \prod_{k \neq n} \left(1 + \frac{\lambda_{2k-1}\lambda_{2k} - \xi_k{}^2 - \xi_n \cdot (\lambda_{2k-1} + \lambda_{2k} - 2\xi_k)}{(\xi_n - \xi_k)^2} \right) =$$

$$= \exp\left\{ \sum_{k \neq n}^{\infty} \ln\left(1 + \frac{\lambda_{2k-1}\lambda_{2k-1} - \xi_k{}^2 - \xi_n \cdot (\lambda_{2k-1} + \lambda_{2k} - 2\xi_k)}{(\xi_n - \xi_k)^2} \right) \right\} =$$

$$= \exp\left\{ \sum_{k \neq n}^{\infty} \ln\left(1 - \frac{1}{\xi_n} \cdot (\lambda_{2k-1} + \lambda_{2k} - 2\xi_k) + \underline{\underline{O}}\left(\frac{1}{\xi_n^2} \right) \right) \right\} =$$

$$= \exp\left\{ -\frac{1}{\xi_n} \cdot \sum_{k \neq n}^{\infty} (\lambda_{2k-1} + \lambda_{2k} - 2\xi_k) + \underline{\underline{O}}\left(\frac{1}{\xi_n^2} \right) \right\} = 1 + \underline{\underline{O}}\left(\frac{1}{n} \right), \; |n| \to \infty$$

ni keltirib chiqarish mumkin. Shuning uchun yetarlicha katta $|n|$ larda $\operatorname{Re} A_n > \frac{1}{2}$ tengsizlik bajariladi. Bundan yetarlicha katta $|n|$ lar uchun

$$|\operatorname{Re} h_n|^2 \geq \operatorname{Re} h_n^2 = \operatorname{Re}\left(2\xi_n + \sum_{k=-\infty}^{\infty} (\lambda_{2k-1} + \lambda_{2k} - 2\xi_k) \right)^2 \times \operatorname{Re} A_n -$$

$$- \operatorname{Im}\left(2\xi_n + \sum_{k=-\infty}^{\infty} (\lambda_{2k-1} + \lambda_{2k} - 2\xi_k) \right)^2 \times \operatorname{Im} A_n \geq c^2 \cdot n^2$$

tengsizlikka ega bo'lamiz. **2.1 lemma isbotlandi.**

(2.1) tenglamadan foydalangan holda, $\xi_n(t)$ spektral parametrlarning analitikligini ko'rsatish qiyin emas. (1.2) va (1.4) tengliklardan $x_n(t)$ ning ham analitikligi kelib chiqadi. $x_n(t),\, n \in Z$ yechimni mavhum argumentga davom ettiramiz. Dubrovin tenglamasidan

$$x_n(t) = x_n(0) + \int_0^t H_n(\ldots, x_{-1}(\tau), x_0(\tau), x_1(\tau), \ldots)\, d\tau \qquad (2.12)$$

kelib chiqadi. (2.12) ayniyat yetarlicha kichik kompleks t larda o‘rinli bo‘ladi. Xususan $t = is,\ s \in R^1$ (yetarlicha kichik sonlar) da

$$x_n(is) = x_n(0) + \int_0^{is} H_n(\ldots, x_{-1}(\tau), x_0(\tau), x_1(\tau), \ldots)\, d\tau \qquad (2.13)$$

bo‘ladi.

(2.13) tenglikda $\tau = i\omega$ almashtirish bajarsak, u holda u

$$x_n(is) = x_n(0) + \int_0^{s} i \cdot H_n(\ldots, x_{-1}(i\omega),\, x_0(i\omega), x_1(i\omega), \ldots)\, d\omega \qquad (2.14)$$

ko‘rinishda bo‘ladi. (2.14) tenglikdan kichik s va yetarlicha katta $|n|$ lar uchun

$$|\mathrm{Im}\, x_n(is)| = \left| \int_0^{s} \mathrm{Re}\, H_n(\ldots, x_{-1}(i\omega), x_0(i\omega), x_1(i\omega), \ldots)\, d\omega \right| =$$

$$= \left| s \cdot \mathrm{Re}\, H_n(\ldots, x_{-1}(i\omega_0),\, x_0(i\omega), x_1(i\omega_0), \ldots) \right| \geq C_1 \cdot |n|,$$

ni keltirib chiqaramiz. $\xi_n(is)$ uchun assimptotikalardan va oxirgi tengsizlikdan quyidagiga ega bo‘lamiz:

$$\overset{=}{o}(1) = |\xi_n(is) - \lambda_{2n-1}| = (\lambda_{2n} - \lambda_{2n-1}) |\sin x_n(is)|^2 \geq$$

$$\geq (\lambda_{2n} - \lambda_{2n-1})\, \mathrm{sh}^2(\mathrm{Im}\, x_n(is)) \geq (\lambda_{2n} - \lambda_{2n-1})\, \mathrm{sh}^2(C_1 \cdot |n|).$$

Shunday qilib, $a > 0,\ b > 0$ lar uchun $|n|$ ning biror qiymatidan boshlab $\lambda_{2n} - \lambda_{2n-1} \leq a e^{-b|n|}$ tengsizlik bajariladi. **Teorema isbotlandi.**

3. Dirak operatori uchun Borg teskari teoremasi analogi

Navbatdagi paragrafda Dirak operatori uchun Borg teoremalarining teskari ikkita analogi isbotlanadi.

Dirak tenglamalar sistemasini qaraylik

$$Ly \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \; x \in (-\infty, \infty), \tag{3.1}$$

bu yerda $p(x)$ va $q(x)$ $C^1(R^1)$ sinfdan olingan π davrli haqiqiy funksiyalar.

$$y(0) - y(\pi) = 0 \tag{3.2}$$

$$y(0) + y(\pi) = 0 \tag{3.3}$$

chegaraviy shartlar mos ravishda davriy va antidavriy chegaraviy shart deyiladi.

(3.1) tenglamalar sistemasining $c_1(0,\lambda) = 1$, $c_2(0,\lambda) = 0$, $s_1(0,\lambda) = 0$, $s_2(0,\lambda) = 1$ boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini $c(x,\lambda)$ va $s(x,\lambda)$ bilan belgilaymiz.

Davriy va antidavriy masalalar uchun xarakteristik tenglamalar mos ravishda quyidagi ko‘rinishga ega :

$$\Delta(\lambda) - 2 = 0, \quad \Delta(\lambda) + 2 = 0,$$

bu yerda $\Delta(\lambda) = c_1(\pi,\lambda) + s_2(\pi,\lambda)$.

Teorema 3.1. (3.1) tenglamalar sistemasining $p(x)$ va $q(x)$ koeffitsiyentlari $\frac{\pi}{2}$ davrli bo‘lishi uchun, (3.1)+(3.3) antidavriy masalaning barcha xos qiymatlari ikki karrali bo‘lishi zarur va yetarli .

Isbot. Zaruriyligi. $p(x)$ va $q(x)$ funksiyalar $\frac{\pi}{2}$ davrga ega bo‘lsin. Agar biror $\rho = \rho(\lambda)$ son uchun (3.1) tenglamalar sistemasining

$$y\left(x + \frac{\pi}{2}\right) = \rho \cdot y(x) \tag{3.4}$$

tenglikni qanoatlantiruvchi nolmas yechimlarini izlasak, u holda ρ ni aniqlash uchun quyidagi tenglamaga ega bo‘lamiz:

$$\rho^2-\delta(\lambda)\rho+1=0, \tag{3.5}$$

bu yerda $\delta(\lambda)=c\left(\frac{\pi}{2},\lambda\right)+s'\left(\frac{\pi}{2},\lambda\right)$ - (3.1) tenglamaning $\frac{\pi}{2}$ davrga mos Lyapunov funksiyasi. (3.4) tenglikdan $y(x+\pi)=\rho^2 y(x)$ ayniyatga ega bo‘lamiz, bundan esa

(3.6)

kelib chiqadi. (3.5) va (3.6) tengliklardan $\delta(\lambda)=\rho+\frac{1}{\rho}$, $\Delta(\lambda)=\rho^2+\frac{1}{\rho^2}$ bo‘lishi kelib chiqadi, bunga ko‘ra $\Delta(\lambda)+2=\delta^2(\lambda)$ bog‘lanish o‘rinli bo‘lishi kelib chiqadi. Bu bog‘lanish (3.1) + (3.3) masalaning barcha xos qiymatlari ikki karrali ekanini ko‘rsatadi.

Yetarliligi. (3.1) + (3.3) antidavriy masalaning barcha xos qiymatlari ikki karrali bo‘lsin.

Dastlab katta kompleks λ larda (3.1) sistemaning

$$y_1(0,\lambda)=\sin\alpha, \quad y_2(0,\lambda)=-\cos\alpha \tag{3.7}$$

boshlang‘ich shartlarni qanoatlantiruvchi $y(x,\lambda)$ yechimining assimptotikasini o‘rganamiz. Yaxshi ma`lumki, (3.1)+(3.7) Koshi masalasi

$$y_1(x,\lambda)=\sin(\lambda x+\alpha)+\int_0^x\{[p(t)\sin\lambda(x-t)+q(t)\cos\lambda(x-t)]y_1(t,\lambda)+$$

$$+[q(t)\sin\lambda(x-t)-p(t)\cos\lambda(x-t)]y_2(t,\lambda)\}dt,$$

$$y_2(x,\lambda)=-\cos(\lambda x+\alpha)+\int_0^x\{[q(t)\sin\lambda(x-t)-p(t)\cos\lambda(x-t)]y_1(t,\lambda)-$$

$$-[p(t)\sin\lambda(x-t)+q(t)\cos\lambda(x-t)]y_2(t,\lambda)\}dt \tag{3.8}$$

integral tenglamalar sistemasiga ekvivalent.

Lemma 3.1. Har bir fiksirlangan x larda, (3.8) integral tenglamalar sistemasining $y(x,\lambda)$ yechimlari uchun ($|\lambda|\to\infty$) da quyidagi assimptotik formulalar o‘rinli:

$$y_1(x,\lambda)=\sin(\lambda x+\alpha)+\frac{1}{2\lambda}\{p(x)\sin(\lambda x+\alpha)+p(0)\sin(\lambda x-\alpha)-$$

$$-q(x)\cos(\lambda x+\alpha)+q(0)\cos(\lambda x-\alpha)-a(x)\cos(\lambda x+\alpha)\}+\overline{\overline{o}}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda}\right),$$

$$y_2(x,\lambda)=-\cos(\lambda x+\alpha)+\frac{1}{2\lambda}\{p(x)\cos(\lambda x+\alpha)-p(0)\cos(\lambda x-\alpha)+$$

$$+q(x)\sin(\lambda x+\alpha)+q(0)\sin(\lambda x-\alpha)-a(x)\sin(\lambda x+\alpha)\}+\overline{\overline{o}}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda}\right), \quad (3.9)$$

bu yerda

$$a(x)=\int_0^x [p^2(s)+q^2(s)]ds.$$

Lemmaning isboti. (3.8) integral tenglamalar sistemasida noma'lum funksiyalarni quyidagicha almashtiramiz:

$$y_1(x,\lambda)=\sin(\lambda x+\alpha)+\frac{1}{2\lambda}\{p(x)\sin(\lambda x+\alpha)+p(0)\sin(\lambda x-\alpha)-$$

$$-q(x)\cos(\lambda x+\alpha)+q(0)\cos(\lambda x-\alpha)-a(x)\cos(\lambda x+\alpha)\}+z_1(x,\lambda),$$

$$y_2(x,\lambda)=-\cos(\lambda x+\alpha)+\frac{1}{2\lambda}\{p(x)\cos(\lambda x+\alpha)-p(0)\cos(\lambda x-\alpha)+$$

$$+q(x)\sin(\lambda x+\alpha)+q(0)\sin(\lambda x-\alpha)-a(x)\sin(\lambda x+\alpha)\}+z_2(x,\lambda).$$

U holda (3.8) sistema

$$z_1(x,\lambda)=\frac{1}{2\lambda}\int_0^x\{[q'(t)-q(t)a(t)]\cos(\lambda x-2\lambda t-\alpha)+$$

$$+[p'(t)-p(t)a(t)]\sin(\lambda x-2\lambda t-\alpha)+$$

$$+[p(t)p(0)+q(t)q(0)]\cos(\lambda x-2\lambda t+\alpha)+$$

$$+[p(t)q(0)+q(t)p(0)]\sin(\lambda x-2\lambda t-\alpha)\}dt+$$

$$+\int_0^x\{[p(t)\sin\lambda(x-t)+q(t)\cos\lambda(x-t)]z_1(t,\lambda)+$$

$$+[q(t)\sin\lambda(x-t)-p(t)\cos\lambda(x-t)]z_2(t,\lambda)\}dt,$$

$$z_2(x,\lambda)=\frac{1}{2\lambda}\int_0^x\{[q'(t)-q(t)a(t)]\sin(\lambda x-2\lambda t-\alpha)-$$

$$-[p'(t)-p(t)a(t)]\cos(\lambda x-2\lambda t-\alpha)+$$

$$+[p(t)p(0)+q(t)q(0)]\sin(\lambda x-2\lambda t+\alpha)-$$

$$-[p(t)q(0)-q(t)p(0)]\cos(\lambda x-2\lambda t+\alpha)\}dt+$$

$$+\int_0^x\{[q(t)\sin\lambda(x-t)-p(t)\cos\lambda(x-t)]z_1(t,\lambda)-$$

$$-[p(t)\sin\lambda(x-t)+q(t)\cos\lambda(x-t)]z_2(t,\lambda)\}dt \qquad (3.10)$$

ko‘rinishda bo‘ladi. $|z_1(x,\lambda)|+|z_2(x,\lambda)|$ funksiyalar uchun Gronoull lemmasidan foydalansak, 3.1 lemma tasdig‘i o‘rinli ekanligini ko‘ramiz.

$\alpha=\frac{\pi}{2}$ va $\alpha=\pi$ da $c(x,\lambda)$ va $s(x,\lambda)$ yechimlar uchun quyidagi assimptotik formulalar olinadi:

$$c_1(x,\lambda)=\cos\lambda x+\frac{1}{2\lambda}\{[p(x)-p(0)]\cos\lambda x+$$

$$+[q(x)+q(0)]\sin\lambda x+a(x)\sin\lambda x\}+\overset{=}{o}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda}\right),$$

$$c_2(x,\lambda)=\sin\lambda x+\frac{1}{2\lambda}\{-[p(x)+p(0)]\sin\lambda x+$$

$$+[q(x)-q(0)]\cos\lambda x-a(x)\cos\lambda x\}+\overset{=}{o}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda}\right),$$

$$s_1(x,\lambda)=-\sin\lambda x+\frac{1}{2\lambda}\{-[p(x)+p(0)]\sin\lambda x+$$

$$+[q(x)-q(0)]\cos\lambda x+a(x)\cos\lambda x\}+\overset{=}{o}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda}\right),$$

$$s_2(x,\lambda)=\cos\lambda x+\frac{1}{2\lambda}\{-[p(x)-p(0)]\cos\lambda x-$$

$$-[q(x)+q(0)]\sin\lambda x+a(x)\sin\lambda x\}+\overset{=}{o}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda}\right),\quad (|\lambda|\to\infty) \qquad (3.11)$$

Bundan $\Delta(\lambda)$ Lyapunov funksiyasi uchun

$$\Delta(\lambda)=2\cos\lambda\pi+\frac{1}{\lambda}a(\pi)\sin\lambda\pi+\overset{=}{o}\left(\frac{e^{|\pi\operatorname{Im}\lambda|}}{\lambda}\right),\quad (|\lambda|\to\infty),$$

assimptotikaga ega bo‘lamiz. Bundan :

$$2+\Delta(\lambda)=\left(2\cos\frac{\lambda\pi}{2}+\frac{1}{2\lambda}a(\pi)\sin\frac{\lambda\pi}{2}\right)^2+\overset{=}{o}\left(\frac{e^{|\pi\,\mathrm{Im}\,\lambda|}}{\lambda}\right),\quad (|\lambda|\to\infty).$$

tenglik kelib chiqadi. $f(\lambda)=\sqrt{2+\Delta(\lambda)}$ funksiyani kiritamiz, bu yerda $f(\lambda)=2\cos\frac{\lambda\pi}{2}+\overset{=}{o}\left(e^{\frac{\pi}{2}|\mathrm{Im}\,\lambda|}\right)$, $(|\lambda|\to\infty)$ assimptotika o‘rinli bo‘ladigan radikalning shoxchasi olinadi. Ko‘rinib turibdiki, $f(\lambda)$ butun funksiya bo‘ladi.

funksiyalar uchun

$$f(\lambda)=2\cos\frac{\lambda\pi}{2}+\frac{a(\pi)}{2\lambda}\sin\frac{\lambda\pi}{2}+\overset{=}{o}\left(\frac{e^{\frac{\pi}{2}|\mathrm{Im}\,\lambda|}}{\lambda}\right),\quad |\lambda|\to\infty \tag{3.12}$$

assimptotikaga ega bo‘lamiz.

Quyidagi lemmani isbotlash qiyin emas.

Lemma 3.2. λ son (3.1)+(3.3) antidavriy masalaning ikki karrali xos qiymati bo‘lishi uchun

$$c_2(\pi,\lambda)=s_1(\pi,\lambda)=c_1(\pi,\lambda)+1=s_2(\pi,\lambda)+1=0$$

tenglikning bajarilishi zarur va yetarli.

Quyidagi matritsa-funksiyani qaraylik

$$F(x,\lambda)=\left(\frac{c(x,\lambda)+c(x-\pi,\lambda)}{f(\lambda)},\ \frac{s(x,\lambda)+s(x-\pi,\lambda)}{f(\lambda)}\right).$$

Lemma 3.3. Har bir tayinlangan x da $F(x,\lambda)$ matritsa-funksiya λ ning butun matritsa-funksiyasi bo‘ladi.

Isbot. Ushbu

$$c(x-\pi,\lambda)=s_2(\pi,\lambda)c(x,\lambda)-c_2(\pi,\lambda)s(x,\lambda),$$

$$s(x-\pi,\lambda)=c_1(\pi,\lambda)s(x,\lambda)-s_1(\pi,\lambda)c(x,\lambda)$$

ayniyatlardan foydalanib, $F(x,\lambda)$ matritsa-funksiyani

$$F(x,\lambda)=\left(\frac{s_2(\pi,\lambda)+1}{f(\lambda)}c(x,\lambda)-\frac{c_2(\pi,\lambda)}{f(\lambda)}s(x,\lambda),\ \frac{c_1(\pi,\lambda)+1}{f(\lambda)}s(x,\lambda)-\frac{s_1(\pi,\lambda)}{f(\lambda)}c(x,\lambda)\right)$$

ko‘rinishda ifodalash mumkin. $f(\lambda)$ funksiyaning ildizlari oddiyligini hisobga olgan holda, oxirgi ifoda, 3.2 lemma va $f(\lambda)$ funksiya butunligidan, 3.3 lemmani tasdig‘iga ega bo‘lamiz.

Lemma 3.4. Barcha $\lambda \in C$ larda

$$F\left(\frac{\pi}{2}, \lambda\right) \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.13}$$

ayniyat o‘rinli bo‘ladi.

Isbot. Assimptotik formulalardan

$$c_1\left(\frac{\pi}{2}, \lambda\right) + c_1\left(-\frac{\pi}{2}, \lambda\right) = 2\cos\frac{\lambda\pi}{2} + \frac{1}{2\lambda}\left\{2\left[p\left(\frac{\pi}{2}\right) - p(0)\right]\cos\frac{\lambda\pi}{2} + \right.$$

$$\left. + \left[a\left(\frac{\pi}{2}\right) - a\left(-\frac{\pi}{2}\right)\right]\sin\frac{\lambda\pi}{2}\right\} + \bar{\bar{o}}\left(\frac{e^{\frac{\pi}{2}|\operatorname{Im}\lambda|}}{\lambda}\right),$$

$$c_2\left(\frac{\pi}{2}, \lambda\right) + c_2\left(-\frac{\pi}{2}, \lambda\right) = \frac{1}{2\lambda}\left\{2\left[q\left(\frac{\pi}{2}\right) - q(0)\right]\cos\frac{\lambda\pi}{2} - \right.$$

$$\left. - \left[a\left(\frac{\pi}{2}\right) + a\left(-\frac{\pi}{2}\right)\right]\cos\frac{\lambda\pi}{2}\right\} + \bar{\bar{o}}\left(\frac{e^{\frac{\pi}{2}|\operatorname{Im}\lambda|}}{\lambda}\right),$$

$$s_1\left(\frac{\pi}{2}, \lambda\right) + s_1\left(-\frac{\pi}{2}, \lambda\right) = \frac{1}{2\lambda}\left\{2\left[q\left(\frac{\pi}{2}\right) - q(0)\right]\cos\frac{\lambda\pi}{2} + \right.$$

$$\left. + \left[a\left(\frac{\pi}{2}\right) + a\left(-\frac{\pi}{2}\right)\right]\cos\frac{\lambda\pi}{2}\right\} + \bar{\bar{o}}\left(\frac{e^{\frac{\pi}{2}|\operatorname{Im}\lambda|}}{\lambda}\right),$$

$$s_2\left(\frac{\pi}{2}, \lambda\right) + s_2\left(-\frac{\pi}{2}, \lambda\right) = 2\cos\frac{\lambda\pi}{2} + \frac{1}{2\lambda}\left\{-2\left[p\left(\frac{\pi}{2}\right) - p(0)\right]\cos\frac{\lambda\pi}{2} + \right.$$

$$\left. + \left[a\left(\frac{\pi}{2}\right) - a\left(-\frac{\pi}{2}\right)\right]\sin\frac{\lambda\pi}{2}\right\} + \bar{\bar{o}}\left(\frac{e^{\frac{\pi}{2}|\operatorname{Im}\lambda|}}{\lambda}\right),$$

bo‘lishi kelib chiqadi. Bundan

$$\lim_{|\lambda|\to\infty} F\left(\frac{\pi}{2},\lambda\right)=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

kelib chiqadi. Bundan, xususan $F\left(\frac{\pi}{2},\lambda\right)$ butun matritsa-funksiya chegaralanganligi kelib chiqadi. Liuvill teoremasidan $F\left(\frac{\pi}{2},\lambda\right)$ matritsa-funksiya o‘zgarmas ekanligi kelib chiqadi. Demak, (3.13) ayniyat o‘rinli. **3.4 lemma isbotlandi.**

3.4 lemmadan quyidagi

$$\left(c\left(\frac{\pi}{2},\lambda\right)+c\left(-\frac{\pi}{2},\lambda\right),\, s\left(\frac{\pi}{2},\lambda\right)+s\left(-\frac{\pi}{2},\lambda\right)\right)\equiv\begin{pmatrix}f(\lambda) & 0\\ 0 & f(\lambda)\end{pmatrix} \qquad (3.14)$$

ayniyat olinadi. (3.14) ayniyatda, assimptotik tengliklardan foydalanib, $\frac{1}{\lambda}$ oldidagi koeffitsiyentlardan bir-biriga tenglashtirib, quyidagi munosabatlarga ega bo‘lamiz:

$$p\left(\frac{\pi}{2}\right)-p(0)=0\,, \qquad 2\left[q\left(\frac{\pi}{2}\right)-q(0)\right]-a\left(\frac{\pi}{2}\right)-a\left(-\frac{\pi}{2}\right)=0\,,$$

$$2\left[q\left(\frac{\pi}{2}\right)-q(0)\right]+a\left(\frac{\pi}{2}\right)+a\left(-\frac{\pi}{2}\right)=0\,,$$

ya’ni

$$p\left(\frac{\pi}{2}\right)-p(0)=0\,, \quad q\left(\frac{\pi}{2}\right)-q(0)=0\,. \qquad (3.15)$$

Agar (3.1) tenglamalar sistemasi o‘rniga

$$L(t)y\equiv\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}y_1'\\ y_2'\end{pmatrix}+\begin{pmatrix}p(x+t) & q(x+t)\\ q(x+t) & -p(x+t)\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}=\lambda\begin{pmatrix}y_1\\ y_2\end{pmatrix},\ x\in(-\infty,\infty),$$

tenglamalar sistemasini qarasak, bu yerda t haqiqiy parametr, u holda Lyapunov funksiyasining t parametrga bog‘liq emasligini e’tiborga olsak,

$$p\left(\frac{\pi}{2}+t\right)-p(t)=0\,, \quad q\left(\frac{\pi}{2}+t\right)-q(t)=0\,,$$

tengliklarga ega bo‘lamiz. **3.1 teorema isbotlandi.**

Quyidagi teorema ham shunga o‘xshash isbotlanadi.

Teorema 3.2. Agar davriy masalaning barcha xos qiymatlari ikki karrali bo‘lsa, u holda $\frac{\pi}{2}$ son (2.41) tenglamalar sistemasining $p(x)$ va $q(x)$ koeffitsiyantlari uchun antidavr bo‘ladi, ya’ni quyidagi ayniyatlar o‘rinli bo‘ladi:

$$p\left(\frac{\pi}{2}+x\right)=-p(x), \quad q\left(\frac{\pi}{2}+x\right)=-q(x).$$

Endi quyidagi kanonik ko‘rinishdagi Dirak sistemasini qaraylik

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix}+\begin{pmatrix} p(x) & 0 \\ 0 & q(x) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}=\lambda\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \, x\in(-\infty,\infty), \tag{3.16}$$

bu yerda $p(x)$ va $q(x)$ funksiyalar $C^1[0,\pi]$ sinfdan olingan π davrli haqiqiy funksiyalar. Bu holda Borg teskari teoremasining analogi o‘rinli emas, masalan $q(x)\equiv p(x)$ da bu tenglamalar sistemasining umumiy yechimi quyidagi ko‘rinishda bo‘ladi:

$$y_1(x,\lambda)=C_1\cos\left(\lambda x-\int_0^x p(t)dt\right)-C_2\sin\left(\lambda x-\int_0^x p(t)dt\right),$$

$$y_2(x,\lambda)=C_1\sin\left(\lambda x-\int_0^x p(t)dt\right)+C_2\cos\left(\lambda x-\int_0^x p(t)dt\right).$$

Bundan

$$\Delta(\lambda)=2\cos\left(\lambda\pi-\int_0^\pi p(t)dt\right)$$

tenglikni olamiz. Demak, bu holda davriy va antidavriy masalalarning barcha xos qiymatlari ikki karrali bo‘ladi. Biroq, (3.16) tenglamalar sistemasi uchun quyidagi teorema o‘rinli.

Teorema 3.3. Agar antidavriy masalaning barcha xos qiymatlari ikki karrali bo‘lsa, u holda $\frac{\pi}{2}$ son $(p(x)-q(x))^2$ funksiya uchun davr bo‘ladi.

Isbot. Quyidagi belgilashlarni kiritaylik

$$A(x)=p(x)+q(x), \; B(x)=p(x)-q(x), \; \eta(x)=-\frac{1}{2}\int_0^x A(t)dt.$$

$|\lambda|\to\infty$ da ushbu assimptotik tengliklar o‘rinli bo‘ladi:

$$c_1(x,\lambda)=\cos(\lambda x+\eta(x))+\frac{1}{\lambda}\left\{\frac{1}{4}[B(x)-B(0)]\cos(\lambda x+\eta(x))+\right.$$

$$\left.+\frac{1}{8}\int_0^x B^2(t)dt\sin(\lambda x+\eta(x))\right\}+\underline{\underline{O}}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda^2}\right),$$

$$c_2(x,\lambda)=\sin(\lambda x+\eta(x))+\frac{1}{\lambda}\left\{-\frac{1}{4}[B(x)-B(0)]\sin(\lambda x+\eta(x))-\right.$$

$$\left.-\frac{1}{8}\int_0^x B^2(t)dt\cos(\lambda x+\eta(x))\right\}+\underline{\underline{O}}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda^2}\right),$$

$$s_1(x,\lambda)=-\sin(\lambda x+\eta(x))+\frac{1}{\lambda}\left\{-\frac{1}{4}[B(x)+B(0)]\sin(\lambda x+\eta(x))+\right.$$

$$\left.+\frac{1}{8}\int_0^x B^2(t)dt\cos(\lambda x+\eta(x))\right\}+\underline{\underline{O}}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda^2}\right),$$

$$s_2(x,\lambda)=\cos(\lambda x+\eta(x))+\frac{1}{\lambda}\left\{-\frac{1}{4}[B(x)-B(0)]\cos(\lambda x+\eta(x))+\right.$$

$$\left.+\frac{1}{8}\int_0^x B^2(t)dt\sin(\lambda x+\eta(x))\right\}+\underline{\underline{O}}\left(\frac{e^{|x\operatorname{Im}\lambda|}}{\lambda^2}\right).$$

Bu assimptotikalardan Lyapunov funksiyasi uchun

$$\Delta(\lambda)=2\cos(\lambda\pi+\eta(\pi))+\frac{1}{\lambda}\left\{\frac{1}{4}\int_0^\pi B^2(t)dt\sin(\lambda\pi+\eta(\pi))\right\}+\underline{\underline{O}}\left(\frac{e^{|\pi\operatorname{Im}\lambda|}}{\lambda^2}\right),\quad |\lambda|\to\infty$$

assimptotikaga ega bo‘lamiz.

Faraz qilaylik, $\eta(\pi)=0$ bo‘lsin. U holda

$$\Delta(\lambda)+2=\left(2\cos\frac{\lambda\pi}{2}+\frac{1}{8\lambda}\int_0^\pi B^2(t)dt\sin\frac{\lambda\pi}{2}\right)^2+\underline{\underline{O}}\left(\frac{e^{|\pi\operatorname{Im}\lambda|}}{\lambda^2}\right),\qquad .$$

$f(\lambda)=\sqrt{2+\Delta(\lambda)}$ belgilashni kiritamiz. $f(\lambda)$ funksiya uchun

$$f(\lambda)=2\cos\frac{\lambda\pi}{2}+\frac{1}{8\lambda}\int_0^\pi B^2(t)dt\sin\frac{\lambda\pi}{2}+\underline{\underline{O}}\left(\frac{e^{|\pi\operatorname{Im}\lambda|/2}}{\lambda^2}\right),\quad |\lambda|\to\infty$$

assimptotik formulaga ega bo‘lamiz. Teorema shartidan $f(\lambda)$ butun funksiya bo‘lishi kelib chiqadi.

$$F(x,\lambda)=\left(\frac{c(x,\lambda)+c(x-\pi,\lambda)}{f(\lambda)},\frac{s(x,\lambda)+s(x-\pi,\lambda)}{f(\lambda)}\right)$$

matritsa funksiyani qaraylik.

Har bir tayinlangan x larda $F(x,\lambda)$ matritsa-funksiyani butun funksiya bo‘lishini ko‘rsatish qiyin emas.

Lemma 3.5. Agar antidavriy masalaning barcha xos qiymatlari ikki karrali bo‘lib, $\eta(\pi)=0$ bo‘lsa, u holda quyidagi ayniyat o‘rinli bo‘ladi:

$$F\left(\frac{\pi}{2},\lambda\right)\equiv\begin{pmatrix}\cos\eta & -\sin\eta\\ \sin\eta & \cos\eta\end{pmatrix}, \tag{3.17}$$

bu yerda $\eta=\eta\left(\frac{\pi}{2}\right)$.

Lemmaning isboti. Assimptotik formulalardan

$$\lim_{\lambda\to\infty}F\left(\frac{\pi}{2},\lambda\right)=\begin{pmatrix}\cos\eta & -\sin\eta\\ \sin\eta & \cos\eta\end{pmatrix}$$

kelib chiqadi, shuning uchun u butun kompleks tekislikda chegaralangan bo‘ladi. Liuvill teoremasiga ko‘ra 3.5 lemma tasdig‘iga ega bo‘lamiz.

Lemma 3.6. Agar antidavriy masalaning barcha xos qiymatlari ikki karrali bo‘lib, $\eta(\pi)=0$ bo‘lsa, u holda ushbu tengliklar o‘rinli bo‘ladi:

$$4\left[B\left(\frac{\pi}{2}\right)-B(0)\right]\cos\eta+\left\{\int_0^{\frac{\pi}{2}}B^2(t)dt+\int_0^{-\frac{\pi}{2}}B^2(t)dt\right\}\sin\eta=0,$$

$$4\left[B\left(\frac{\pi}{2}\right)+B(0)\right]\sin\eta+\left\{\int_0^{\frac{\pi}{2}}B^2(t)dt+\int_0^{-\frac{\pi}{2}}B^2(t)dt\right\}\cos\eta=0,$$

$$4\left[B\left(\frac{\pi}{2}\right)+B(0)\right]\sin\eta-\left\{\int_0^{\frac{\pi}{2}}B^2(t)dt+\int_0^{-\frac{\pi}{2}}B^2(t)dt\right\}\cos\eta=0,$$

$$4\left[B\left(\frac{\pi}{2}\right)-B(0)\right]\cos\eta-\left\{\int_0^{\frac{\pi}{2}}B^2(t)dt+\int_0^{-\frac{\pi}{2}}B^2(t)dt\right\}\sin\eta=0,$$

bu yerda $\eta=\eta\left(\frac{\pi}{2}\right)$.

Lemmaning isboti. Yechimlarning assimptotikalariga ko‘ra, haqiqiy $\lambda\to\infty$ da quyidagi assimptotik ifodalarga ega bo‘lamiz:

$$c_1\left(\frac{\pi}{2},\lambda\right)+c_1\left(-\frac{\pi}{2},\lambda\right)=2\cos\eta\cos\frac{\lambda\pi}{2}+$$

$$+\frac{1}{8\lambda}\int_0^{\frac{\pi}{2}}B^2(t)dt\cos\eta\sin\frac{\lambda\pi}{2}+\frac{1}{2\lambda}\left[B\left(\frac{\pi}{2}\right)-B(0)\right]\cos\eta\cos\frac{\lambda\pi}{2}+$$

$$+\frac{1}{8\lambda}\left\{\int_0^{\frac{\pi}{2}}B^2(t)dt+\int_0^{-\frac{\pi}{2}}B^2(t)dt\right\}\sin\eta\cos\frac{\lambda\pi}{2}+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right),$$

$$c_2\left(\frac{\pi}{2},\lambda\right)+c_2\left(-\frac{\pi}{2},\lambda\right)=2\sin\eta\cos\frac{\lambda\pi}{2}+$$

$$+\frac{1}{8\lambda}\int_0^{\frac{\pi}{2}}B^2(t)dt\sin\eta\sin\frac{\lambda\pi}{2}-\frac{1}{2\lambda}\left[B\left(\frac{\pi}{2}\right)+B(0)\right]\sin\eta\cos\frac{\lambda\pi}{2}-$$

$$-\frac{1}{8\lambda}\left\{\int_0^{\frac{\pi}{2}}B^2(t)dt+\int_0^{-\frac{\pi}{2}}B^2(t)dt\right\}\sin\eta\cos\frac{\lambda\pi}{2}+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right),$$

$$s_1\left(\frac{\pi}{2},\lambda\right)+s_1\left(-\frac{\pi}{2},\lambda\right)=-2\sin\eta\cos\frac{\lambda\pi}{2}-$$

$$-\frac{1}{8\lambda}\int_0^{\pi}B^2(t)dt\sin\eta\sin\frac{\lambda\pi}{2}-\frac{1}{2\lambda}\left[B\left(\frac{\pi}{2}\right)+B(0)\right]\sin\eta\cos\frac{\lambda\pi}{2}+$$

$$+\frac{1}{8\lambda}\left\{\int_0^{\frac{\pi}{2}}B^2(t)dt+\int_0^{-\frac{\pi}{2}}B^2(t)dt\right\}\cos\eta\cos\frac{\lambda\pi}{2}+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right),$$

$$s_2\left(\frac{\pi}{2},\lambda\right)+s_2\left(-\frac{\pi}{2},\lambda\right)=-2\cos\eta\cos\frac{\lambda\pi}{2}+$$

$$+\frac{1}{8\lambda}\int_0^{\pi}B^2(t)dt\cos\eta\sin\frac{\lambda\pi}{2}-\frac{1}{2\lambda}\left[B\left(\frac{\pi}{2}\right)-B(0)\right]\cos\eta\cos\frac{\lambda\pi}{2}+$$

$$+\frac{1}{8\lambda}\left\{\int_0^{\frac{\pi}{2}} B^2(t)dt+\int_0^{-\frac{\pi}{2}} B^2(t)dt\right\}\sin\eta\cos\frac{\lambda\pi}{2}+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right).$$

(3.17) tenglikda $\frac{1}{\lambda}$ ning bir xil darajalari oldidagi koeffitsiyentlarni tenglasak, 3.6 lemmaning tasdig‘iga ega bo‘lamiz.

Demak, agar antidavriy masalaning barcha xos qiymatlari ikki karrali bo‘lib, $\eta(\pi)=0$ tenglik bajarilsa, u holda

$$\left[B(\frac{\pi}{2})-B(0)\right]\cos\eta=0,\quad \left[B(\frac{\pi}{2})+B(0)\right]\sin\eta=0, \tag{3.18}$$

$$\left\{\int_0^{\frac{\pi}{2}} B^2(t)dt+\int_0^{-\frac{\pi}{2}} B^2(t)dt\right\}\sin\eta=0,\quad \left\{\int_0^{\frac{\pi}{2}} B^2(t)dt+\int_0^{-\frac{\pi}{2}} B^2(t)dt\right\}\cos\eta=0, \tag{3.19}$$

bo‘ladi, bu yerda $\eta=\eta\left(\frac{\pi}{2}\right)$.

Antidavriy masalaning barcha xos qiymatlari ikki karrali bo‘lib, $\eta(\pi)=0$ tenglik bajarilishi shart bo‘lmasin. U holda $p(x)$ va $q(x)$ funksiyalar o‘rniga $\widetilde{p}(x)=p(x)+c$, $\widetilde{q}(x)=q(x)+c$ funksiyalarni qaraymiz, bu yerda

$$c=-\frac{1}{2\pi}\int_0^{\pi} A(t)dt.$$

Bu holda

$$\widetilde{A}(t)=A(t)+2c,\ \widetilde{B}(t)=B(t),\ \widetilde{\eta}(\pi)=0,$$

$$\widetilde{\eta}\left(\frac{\pi}{2}\right)=-\frac{1}{2}\int_0^{\frac{\pi}{2}}\widetilde{A}(t)dt=\frac{1}{4}\int_0^{\frac{\pi}{2}}\left[A\left(t+\frac{\pi}{2}\right)-A(t)\right]dt$$

bo‘lishini ko‘rish qiyin emas. Shuning uchun (3.18) tengliklar quyidagi ko‘rinishga ega bo‘ladi:

$$\left[B\left(\frac{\pi}{2}\right)-B(0)\right]\cos\left\{\frac{1}{4}\int_0^{\frac{\pi}{2}}\left[A\left(t+\frac{\pi}{2}\right)-A(t)\right]dt\right\}=0, \tag{3.20}$$

$$\left[B\left(\frac{\pi}{2}\right)+B(0)\right]\sin\left\{\frac{1}{4}\int_0^{\frac{\pi}{2}}\left[A\left(t+\frac{\pi}{2}\right)-A(t)\right]dt\right\}=0. \qquad (3.21)$$

Agar antidavriy masalaning barcha xos qiymatlari ikki karrali bo'lsa, u holda quyidagi

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\cdot\begin{pmatrix} y_1'(x) \\ y_2'(x) \end{pmatrix}+\begin{pmatrix} p(x+s) & 0 \\ 0 & q(x+s) \end{pmatrix}\cdot\begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}=\lambda\cdot\begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix},\ x\in R^1, \quad (3.22)$$

$$y(0)+y(\pi)=0, \qquad (3.23)$$

masalaning xos qiymatlari ham ikki karrali bo'ladi, bu yerda s haqiqiy parametr. Bu holda (3.20) va (3.21) tengliklar

$$\left[B\left(\frac{\pi}{2}+s\right)-B(s)\right]\cos\left\{\frac{1}{4}\int_0^{\frac{\pi}{2}}\left[A\left(t+\frac{\pi}{2}+s\right)-A(t+s)\right]dt\right\}=0, \qquad (3.24)$$

$$\left[B\left(\frac{\pi}{2}+s\right)+B(s)\right]\sin\left\{\frac{1}{4}\int_0^{\frac{\pi}{2}}\left[A\left(t+\frac{\pi}{2}+s\right)-A(t+s)\right]dt\right\}=0 \qquad (3.25)$$

ko'rinishda bo'ladi. (3.24) va (3.25) tengliklarni mos ravishda $B\left(\frac{\pi}{2}+s\right)+B(s)$ va $B\left(\frac{\pi}{2}+s\right)-B(s)$ ga ko'paytirib, hosil bo'lgan tengliklarni kvadratlarini qo'shsak, $B^2\left(\frac{\pi}{2}+s\right)-B^2(s)\equiv 0$ kelib chiqadi. Bu esa $B^2(x)=(p(x)-q(x))^2$ funksiyalar uchun ham davr $\frac{\pi}{2}$ ekanini bildiradi. **Teorema isbotlandi.**

4. Davriy va antidavriy chegaraviy shartli Dirak operatorining xos funksiyalarining kvadrati uchun ayniyatlar

X.P.Mak-Kin, E.Trubovits [46] va P.Deyft, E.Trubovits [20] ishlarida davriy va antidavriy chegaraviy shartli Shturm-Liuvill operatorining λ_{2k} xos qiymatlariga mos $\varphi_{2k}(x)$ ortonormallangan xos funksiyalarning kvadrati uchun

$$\sum_{k=0}^{\infty}\varepsilon_k\varphi_{2k}^2(x)=1$$

ayniyat olingan. Bu yerda ε_k faqat $\lambda_0,\lambda_1,\lambda_2,...$ larga bog'liq, agar $\lambda_{2k}-\lambda_{2k-1}>0$ bo'lsa, u holda musbat bo'ladi, agar $\lambda_{2k}-\lambda_{2k-1}=0$ bo'lsa, u holda nolga teng bo'ladi.

Ushbu paragrafda silliq davriy potensialli Dirak tenglamalar sistemasining xos vektor-funksiyalari komponentalari kvadratlari uchun ba'zi bir ayniyatlar olinadi.

$L_2^2\left(R^1\right)$ vektor-funksiyalar fazosida quyidagi Dirak operatorini qaraylik

$$Ly\equiv\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}y_1'\\ y_2'\end{pmatrix}+\begin{pmatrix}p(x) & q(x)\\ q(x) & -p(x)\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}=\lambda\begin{pmatrix}y_1\\ y_2\end{pmatrix},\ x\in(-\infty,\infty), \qquad (4.1)$$

bu yerda $p(x)$ va $q(x)$ - $C^1\left(R^1\right)$ sinfdan olingan π davrli haqiqiy funksiyalar.

(4.1) tenglamalar sistemasining, $c_1(0,\lambda)=1$, $c_2(0,\lambda)=0$, $s_1(0,\lambda)=0$, $s_2(0,\lambda)=1$ boshlang'ich shartlarni qanoatlantiruvchi yechimlarini $c(x,\lambda)$ va $s(x,\lambda)$ orqali belgilaymiz.

L operatorning spektri butun o'qdan sanoqlita ko'p bo'lmagan o'zaro kesishmaydigan, chegaralangan $(\lambda_{2n},\lambda_{2n-1}),n\in Z$ oraliqlar (lakunalar) ni chiqarib tashlashdan hosil qilinadi. Lakun oxirlari $[0,\pi]$ kesmada (4.1) sistemalar uchun yo davriy, yo antidavriy masalalarning xos qiymatlaridan iborat bo'ladi va $\Delta^2(\lambda)-4$ butun funksiya nollaridan tuzilgan bo'ladi, bu yerda $\Delta(\lambda)=c_1(\pi,\lambda)+s_2(\pi,\lambda)$. Bundan tashqari quyidagi assimptotika bajariladi:

$$\lambda_{2n-1}, \lambda_{2n} = n + \frac{c_1}{n} + \underline{\underline{O}}\left(\frac{1}{n^2}\right), \quad |n| \to \infty .$$

Ushbu

$$\ldots, y_{-2}(x), y_{-1}(x), y_0(x), y_1(x), y_2(x), \ldots \tag{4.2}$$

Funksiyalar (4.1) sistemaning $[0, \pi]$ kesmadagi

$$\ldots \le \lambda_{-2} < \lambda_{-1} \le \lambda_0 < \lambda_1 \le \lambda_2 < \ldots$$

xos qiymatlarga mos keluvchi normallangan xos vektor-funksiyalar bo‘lsin.

Teorema 4.1. (4.2) funksiyalar uchun quyidagi ayniyatlar o‘rinli:

$$\sum_{k=-\infty}^{\infty} a_k y_{2k-1,1}^2(t) = -p(t) + \frac{c}{2}, \quad \sum_{k=-\infty}^{\infty} b_k y_{2k,1}^2(t) = p(t) + \frac{c}{2},$$

$$\sum_{k=-\infty}^{\infty} a_k y_{2k-1,2}^2(t) = p(t) + \frac{c}{2}, \quad \sum_{k=-\infty}^{\infty} b_k y_{2k,2}^2(t) = -p(t) + \frac{c}{2},$$

$$\sum_{k=-\infty}^{\infty} a_k y_{2k-1,1}(t) y_{2k-1,2}(t) = q(t), \quad \sum_{k=-\infty}^{\infty} b_k y_{2k,1}(t) y_{2k,2}(t) = -q(t),$$

bu yerda

$$a_k = \frac{-\Delta'(\lambda_{2k-1})}{f'(\lambda_{2k-1})}, \qquad b_k = \frac{\Delta'(\lambda_{2k})}{g'(\lambda_{2k})}, \qquad c = \sum_{k=-\infty}^{\infty} (\lambda_{2k} - \lambda_{2k-1}),$$

$$f(\lambda) = \pi(\lambda_{-1} - \lambda) \prod_{0 \ne k=-\infty}^{\infty} \frac{\lambda_{2k-1} - \lambda}{k}, \quad g(\lambda) = \pi(\lambda_0 - \lambda) \prod_{0 \ne k=-\infty}^{\infty} \frac{\lambda_{2k} - \lambda}{k}. \tag{4.3}$$

Teoremaning isboti. Ushbu

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} p(x+t) & q(x+t) \\ q(x+t) & -p(x+t) \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \; x \in (-\infty, \infty),$$

tenglamalar sistemasining $c_1(0,\lambda,t) = 1$, $c_2(0,\lambda,t) = 0$, $s_1(0,\lambda,t) = 0$, $s_2(0,\lambda,t) = 1$ boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini $c(x,\lambda,t)$ va $s(x,\lambda,t)$ orqali belgilaymiz, bu yerda t - haqiqiy parametr.

$|\lambda| \to \infty$ da quyidagi

$$\frac{s_1(\pi,\lambda,t)}{f(\lambda)}, \quad \frac{-c_2(\pi,\lambda,t)}{f(\lambda)}, \quad \frac{c_1(\pi,\lambda,t) - s_2(\pi,\lambda,t)}{f(\lambda)},$$

$$\frac{s_1(\pi,\lambda,t)}{g(\lambda)}, \quad \frac{-c_2(\pi,\lambda,t)}{g(\lambda)}, \quad \frac{c_1(\pi,\lambda,t) - s_2(\pi,\lambda,t)}{g(\lambda)}.$$

funksiyalar assimptotikasini o‘rganamiz. Quyidagi

$$s_1(\pi,\lambda,t)=\pi(\xi_0(t)-\lambda)\prod_{\substack{k=-\infty,\\ k\neq 0}}^{\infty}\frac{\xi_k(t)-\lambda}{k},$$

ayniyatdan foydalanib

$$\frac{s_1(\pi,\lambda,t)}{f(\lambda)}=\sqrt{\prod_{k=-\infty}^{\infty}\frac{\lambda-\lambda_{2k}}{\lambda-\lambda_{2k-1}}}\cdot\sqrt{\prod_{k=-\infty}^{\infty}\frac{\lambda-\xi_k(t)}{\lambda-\lambda_{2k-1}}\cdot\frac{\lambda-\xi_k(t)}{\lambda-\lambda_{2k}}}=$$

$$=\sqrt{\exp\left\{\sum_{k=-\infty}^{\infty}\ln\left(1+\frac{\lambda_{2k-1}-\lambda_{2k}}{\lambda-\lambda_{2k-1}}\right)\right\}}\times$$

$$\times\sqrt{\exp\left\{\sum_{k=-\infty}^{\infty}\ln\left[1+\frac{\lambda_{2k-1}-\xi_k(t)}{\lambda-\lambda_{2k-1}}+\frac{\lambda_{2k}-\xi_k(t)}{\lambda-\lambda_{2k}}+\frac{(\lambda_{2k-1}-\xi_k(t))(\lambda_{2k}-\xi_k(t))}{(\lambda-\lambda_{2k-1})(\lambda-\lambda_{2k})}\right]\right\}}=$$

$$=\sqrt{\exp\left\{\frac{1}{\lambda}\sum_{k=-\infty}^{\infty}(\lambda_{2k-1}-\lambda_{2k})+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right)\right\}}\times$$

$$\times\sqrt{\exp\left\{\frac{1}{\lambda}\sum_{k=-\infty}^{\infty}(\lambda_{2k-1}+\lambda_{2k}-2\xi_k(t))+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right)\right\}},\quad |\lambda|\to\infty \tag{4.4}$$

ga ega bo‘lamiz. (4.3) belgilashlar va

$$\sum_{k=-\infty}^{\infty}\left(\frac{\lambda_{2k-1}+\lambda_{2k}}{2}-\xi_k(t)\right)=p(t),$$

izlar formulasini e’tiborga olib, (4.4) dan

$$\frac{s_1(\pi,\lambda,t)}{f(\lambda)}=1+\frac{1}{\lambda}\left(p(t)-\frac{c}{2}\right)+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right),\quad |\lambda|\to\infty \tag{4.5}$$

ni hosil qilamiz. Ushbu

$$c_1(\pi,\lambda,t)-s_2(\pi,\lambda,t)=s_1(\pi,\lambda,t)\sum_{k=-\infty}^{\infty}\frac{\sigma_k(t)\sqrt{\Delta^2(\xi_k(t))-4}}{\left(\left.\dfrac{\partial s_1(\pi,\lambda,t)}{\partial\lambda}\right|_{\lambda=\xi_k(t)}\right)\cdot(\lambda-\xi_k(t))}$$

tenglik va

$$\sum_{k=-\infty}^{\infty}\frac{\sigma_k(t)\sqrt{\Delta^2(\xi_k(t))-4}}{\left.\dfrac{\partial s_1(\pi,\lambda,t)}{\partial\lambda}\right|_{\lambda=\xi_k(t)}}=q(t)$$

iz formulalari asosida (4.5) assimptotikadan foydalangan holda quyidagi

$$\frac{c_1(\pi,\lambda,t)-s_2(\pi,\lambda,t)}{f(\lambda)}=\frac{1}{\lambda}q(t)+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right),\quad |\lambda|\to\infty \tag{4.6}$$

tenglikni hosil qilamiz, bu yerda $\sigma_k(t)=sign\{c_1(\pi,\xi_k(t),t)-s_2(\pi,\xi_k(t),t)\}$.

Shunga o‘xshash quyidagi assimptotikalar olinadi:

$$\frac{-c_2(\pi,\lambda,t)}{f(\lambda)}=1+\frac{1}{\lambda}\left(-p(t)-\frac{c}{2}\right)+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right),\quad |\lambda|\to\infty,$$

$$\frac{s_1(\pi,\lambda,t)}{g(\lambda)}=1+\frac{1}{\lambda}\left(p(t)+\frac{c}{2}\right)+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right),\quad |\lambda|\to\infty,$$

$$\frac{c_1(\pi,\lambda,t)-s_2(\pi,\lambda,t)}{g(\lambda)}=\frac{1}{\lambda}q(t)+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right),\quad |\lambda|\to\infty$$

$$\frac{-c_2(\pi,\lambda,t)}{g(\lambda)}=1+\frac{1}{\lambda}\left(-p(t)+\frac{c}{2}\right)+\underline{\underline{O}}\left(\frac{1}{\lambda^2}\right),\quad |\lambda|\to\infty. \tag{4.7}$$

Olingan assimptotikalar va Mittag-Leffler teoremasidan foydalanib, ushbu ayniyatni keltirib chiqaramiz:

$$\frac{s_1(\pi,\lambda,t)}{f(\lambda)}=1+\sum_{k=-\infty}^{\infty}\frac{s_1(\pi,\lambda_{2k-1},t)}{f'(\lambda_{2k-1})}\cdot\frac{1}{\lambda-\lambda_{2k-1}},$$

$$\frac{-c_2(\pi,\lambda,t)}{f(\lambda)}=1+\sum_{k=-\infty}^{\infty}\frac{-c_2(\pi,\lambda_{2k-1},t)}{f'(\lambda_{2k-1})}\cdot\frac{1}{\lambda-\lambda_{2k-1}},$$

$$\frac{c_1(\pi,\lambda,t)-s_2(\pi,\lambda,t)}{f(\lambda)}=\sum_{k=-\infty}^{\infty}\frac{c_1(\pi,\lambda_{2k-1},t)-s_2(\pi,\lambda_{2k-1},t)}{f'(\lambda_{2k-1})}\cdot\frac{1}{\lambda-\lambda_{2k-1}},$$

$$\frac{s_1(\pi,\lambda,t)}{g(\lambda)}=1+\sum_{k=-\infty}^{\infty}\frac{s_1(\pi,\lambda_{2k},t)}{g'(\lambda_{2k})}\cdot\frac{1}{\lambda-\lambda_{2k}},$$

$$\frac{-c_2(\pi,\lambda,t)}{g(\lambda)}=1+\sum_{k=-\infty}^{\infty}\frac{-c_2(\pi,\lambda_{2k},t)}{g'(\lambda_{2k})}\cdot\frac{1}{\lambda-\lambda_{2k}},$$

$$\frac{c_1(\pi,\lambda,t)-s_2(\pi,\lambda,t)}{g(\lambda)}=\sum_{k=-\infty}^{\infty}\frac{c_1(\pi,\lambda_{2k},t)-s_2(\pi,\lambda_{2k},t)}{g'(\lambda_{2k-1})}\cdot\frac{1}{\lambda-\lambda_{2k}}. \tag{4.8}$$

(4.5)-(4.8) tengliklarda $\frac{1}{\lambda}$ da koeffitsiyentlarni tenglashtirib quyidgi ayniyatlarni keltirib chiqaramiz:

$$\sum_{k=-\infty}^{\infty}\frac{s_1(\pi,\lambda_{2k-1},t)}{f'(\lambda_{2k-1})}=p(t)-\frac{c}{2},\qquad \sum_{k=-\infty}^{\infty}\frac{-c_2(\pi,\lambda_{2k-1},t)}{f'(\lambda_{2k-1})}=q(t),$$

$$\sum_{k=-\infty}^{\infty} \frac{c_1(\pi,\lambda_{2k-1},t) - s_2(\pi,\lambda_{2k-1},t)}{f'(\lambda_{2k-1})} = -p(t) - \frac{c}{2},$$

$$\sum_{k=-\infty}^{\infty} \frac{s_1(\pi,\lambda_{2k},t)}{g'(\lambda_{2k})} = p(t) + \frac{c}{2}, \qquad \sum_{k=-\infty}^{\infty} \frac{-c_2(\pi,\lambda_{2k},t)}{g'(\lambda_{2k})} = q(t),$$

$$\sum_{k=-\infty}^{\infty} \frac{c_1(\pi,\lambda_{2k},t) - s_2(\pi,\lambda_{2k},t)}{g'(\lambda_{2k-1})} = -p(t) + \frac{c}{2}. \qquad (4.9)$$

(4.9) tenglik va quyidagi lemmadan teorema isboti kelib chiqadi.

Lemma 4.1. Quyidagi tengliklar o‘rinli:

$$s_1(\pi,\lambda_n,t) = \Delta'(\lambda_n) y_{n,1}^2(t), \quad c_2(\pi,\lambda_n,t) = -\Delta'(\lambda_n) y_{n,2}^2(t),$$

$$c_1(\pi,\lambda_n,t) = \frac{1}{2}\Delta(\lambda_n) - \Delta'(\lambda_n) y_{n,1}(t) y_{n,2}(t),$$

$$s_2(\pi,\lambda_n,t) = \frac{1}{2}\Delta(\lambda_n) + \Delta'(\lambda_n) y_{n,1}(t) y_{n,2}(t). \qquad (4.10)$$

Lemmaning isboti. (4.10) tenglikni birinchisini isbotlaylik. λ_n ikki karrali bo‘lgan holda lemma tasdig‘i o‘rinli ekani ko‘rinib turibdi. λ_n ikki karrali bo‘lmasin deb olaylik. Quyidagi

$$c(t+\pi,\lambda) = c_1(\pi,\lambda)c(t,\lambda) + c_2(\pi,\lambda)s(t,\lambda),$$

$$s(t+\pi,\lambda) = s_1(\pi,\lambda)c(t,\lambda) + s_2(\pi,\lambda)s(t,\lambda),$$

$$s(x,\lambda,t) = c_1(t,\lambda)s(x+t,\lambda) - s_1(t,\lambda)c(x+t,\lambda)$$

tengliklardan

$$s_1(\pi,\lambda,t) = s_1(\pi,\lambda)c_1^2(t,\lambda) + [s_2(\pi,\lambda) - c_1(\pi,\lambda)]c_1(t,\lambda)s_1(t,\lambda) - c_2(\pi,\lambda)s_1^2(t,\lambda). \qquad (4.11)$$

ayniyatni keltirib chiqaramiz.

$\psi_{\pm}(x,\lambda)$ - (4.1) tenglamalar sistemasi uchun Floke yechimlari bo‘lsin. Ular uchun quyidagi yaxshi ma’lum bo‘lgan tasvirlar o‘rinli:

$$\psi_{\pm}(x,\lambda) = c(x,\lambda) + \frac{s_2(\pi,\lambda) - c_1(\pi,\lambda) \mp \sqrt{\Delta^2(\lambda) - 4}}{2s_1(\pi,\lambda)} \cdot s(x,\lambda).$$

Quyidagicha belgilash kiritaylik:

$$\psi(x,\lambda_n)=\psi_{\pm}(x,\lambda_n)=c(x,\lambda_n)+\frac{s_2(\pi,\lambda_n)-c_1(\pi,\lambda_n)}{2s_1(\pi,\lambda_n)}\cdot s(x,\lambda_n). \qquad (4.12)$$

Ushbu

$$\Delta'(\lambda)=s_1(\pi,\lambda_n)\int_0^\pi\left\{\left[c_1(t,\lambda)+\frac{s_2(\pi,\lambda)-c_1(\pi,\lambda)}{2s_1(\pi,\lambda)}\cdot s_1(t,\lambda)\right]^2+\right.$$

$$\left.+\left[c_2(t,\lambda)+\frac{s_2(\pi,\lambda)-c_1(\pi,\lambda)}{2s_1(\pi,\lambda)}\cdot s_2(t,\lambda)\right]^2\right\}dt-$$

$$-\frac{\Delta^2(\lambda)-4}{4s_1(\pi,\lambda)}\cdot\int_0^\pi[s_1^2(t,\lambda)+s_2^2(t,\lambda)]dt$$

formuladan

$$\Delta'(\lambda)=s_1(\pi,\lambda_n)\cdot\int_0^\pi[\psi_1^2(t,\lambda_n)+\psi_2^2(t,\lambda_n)]dt$$

kelib chiqadi. λ_n xos qiymatlar oddiy bo‘lganligi uchun

$$\psi(t,\lambda_n)=\pm\sqrt{\frac{\Delta'(\lambda)}{s_1(\pi,\lambda_n)}}\cdot y_n(t)$$

tenglikni keltirib chiqaramiz. Bundan

$$s_1(\pi,\lambda_n)\psi_1^2(t,\lambda_n)=\Delta'(\lambda)y_{n,1}^2(t) \qquad (4.13)$$

kelib chiqadi. (4.11), (4.12) va

$$[c_1(\pi,\lambda)-s_2(\pi,\lambda)]^2=\Delta^2(\lambda)-4-4s_1(\pi,\lambda)c_2(\pi,\lambda)$$

ayniyatdan foydalanib, (4.13) tenglikning chap qismini quyidagicha yozib olamiz:

$$s_1(\pi,\lambda_n)\psi_1^2(t,\lambda_n)=$$

$$=\frac{1}{4s_1(\pi,\lambda_n)}\{2s_1(\pi,\lambda_n)c_1(t,\lambda_n)+[s_2(\pi,\lambda_n)-c_1(\pi,\lambda_n)]s_1(t,\lambda_n)\}^2=$$

$$=\frac{1}{4s_1(\pi,\lambda_n)}\{4s_1^2(\pi,\lambda_n)c_1^2(t,\lambda_n)+$$

$$+4s_1(\pi,\lambda_n)[s_2(\pi,\lambda_n)-c_1(\pi,\lambda_n)]c_1(t,\lambda_n)s_1(t,\lambda_n)-4s_1(\pi,\lambda_n)c_2(\pi,\lambda_n)s_1^2(t,\lambda_n)\}=$$

$$=s_1(\pi,\lambda_n)c_1^2(t,\lambda_n)+[s_2(\pi,\lambda_n)-c_1(\pi,\lambda_n)]c_1(t,\lambda_n)s_1(t,\lambda_n)-c_2(\pi,\lambda_n)s_1^2(t,\lambda_n)=$$

$$=s_1(\pi,\lambda_n,t).$$

(4.13) tenglikdan foydalangan holda, lemmaning birinchi ayniyatini keltirib chiqaramiz; qolgan tengliklar ham shunga oʻxshash isbotlanadi. **Lemma isbotlandi.**

Ta'kidlash kerakki, $p(x) \equiv q(x) \equiv 0$ va $p(x) \equiv m, m > 0, q(x) \equiv 0$ holda olingan ayniyatlar sonli tenglik koʻrinishiga keladi.

III bob bo‘yicha xulosa

Ushbu bobda Dirak operatori uchun to’g’ri va teskari masalalar o’rganilgan.. Bunda butun o‘qda berilgan davriy potensialli Dirak operatori uchun Dubrovin tenglamalar sistemasi analogini keltirib chiqarilgan, koeffitsiyentlar analitikligi va lakuna uzunligining kamayishi orasidagi bog‘lanish keltirib chiqarilgan, Dirak operatori uchun Borg teskari teoremasi analogi olingan, hamda Davriy va antidavriy chegaraviy shartli Dirak operatori xos funksiyalarining kvadrati uchun ayniyatlar olingan.

XULOSA

Mazkur magistrlik dissertatsiyasi Dirak operatori uchun qo□yilgan har xil chegaraviy shartlarda izlar ormulalarni yangicha elenemtar usulda keltirib chiqarish va bu formulalarni teskari masalaga qo□llashga bag□ishlangan.

Magistrlik dissertatsiyasining birinchi bobida Dirak tenglamasining kelib chiqish tarixi, hamda Dirak operatori haqida asosiy ma'lumotlar keltirilgan. Davriy potensialli Dirak operatori uchun Lyapunov funksiyasi haqida tushunchalar berilib, uning xossalari o'rganilgan, hamda bir nechta teorema, lemmalar keltirilgan hamda ularning tadbiqiga doir misollar keltirildi. Bundan tashqari potensial argumentining haqiqiy siljishida Lyapunov funksiyasining o'zgarmaslik xossalari ham o'rganilgan. Keltirilgan bu lemma va teoremalar Dirak operatori uchun qo'yilgan teskari masalani yechishda muhim ahamiyatga ega. Floke yechimlari va bu yechimlarning xossalari o□rganilgan.

Magistrlik dissertatsiyasining ikkinchi bobida Dirak operatori uchun izlar formulasini keltirib chiqarishning yangicha elementar usuli keltirilgan. Bunda birinchi bo'limda Dirak operatori uchun qo'yilgan Dirixle va Neyman chegaraviy shartlari hollarida izlar formulalari keltirib chiqarildi. Ikkinchi bo'limda esa Dirak operatori uchun qo□yilgan davriy va antidavriy chegaraviy shartlari hollarida izlar formulalari keltirib chiqarildi. Har ikkala bo□limda izlar formulalari yangicha elementar usulda keltirib chiqarildi.

Magistrlik dissertatsiyasining uchinchi bobida Dirak operatori uchun to'g'ri va teskari masalalar o'rganilgan.. Bunda butun o'qda berilgan davriy potensialli Dirak operatori uchun Dubrovin tenglamalar sistemasi analogini keltirib chiqarilgan, koeffitsiyentlar analitikligi va lakuna uzunligining kamayishi orasidagi bog'lanish keltirib chiqarilgan, Dirak operatori uchun Borg teskari teoremasi analogi olingan, hamda Davriy va antidavriy chegaraviy shartli Dirak operatori xos funksiyalarining kvadrati uchun ayniyatlar olingan.

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