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REFERAT

Mavzu: Funksiyalar sistemasining to'liqligi. Funktsional
yopiq sinflar va post teoremasi

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SAMARQAND-2016

Reja:

- 1.Funksiyalar sistemasining to'liqligi.
- 2.Funksional yopiq sinflar.
- 3.Post teoremasi.

Funksional yopiq sinflar. Mantiq algebrasining $\Phi = \{\varphi_1, \dots, \varphi_n\}$ funksiyalar sistemasi berilgan bo'lsin.

1- ta'rif. Agar mantiq algebrasining istalgan funksiyasini $\Phi = \{\varphi_1, \dots, \varphi_n\}$ sistemadagi funksiyalar superpozitsiyasi orqali ifodalash mumkin bo'lsa, u holda Φ sistema **to'liq funksiyalar sistemasi** deb ataladi.

2- ta'rif. Mantiq algebrasining superpozitsiyaga nisbatan yopiq bo'lgan har qanday funksiyalar sistemasi **funksional yopiq sinf** deb ataladi.

3- ta'rif. O'z-o'zidan va mantiq algebrasining hamma funksiyalari sinfidan (P_2 dan) farq qiluvchi funksional yopiq sinflarga kirmaydigan xususiy funksional yopiq sinf **maksimal funksional yopiq sinf** deb ataladi.

Mantiq algebrasida hammasi bo'lib beshta maksimal funksional yopiq sinf mavjud. Bular quyidagilardir: P_0, P_1, M, S, L .

Post teoremasi. E. L. Post tomonidan funksiyalar sistemasi to'liqligining yetarli va zarur shartlari topilgan.

Post teoremasi. $\Phi = \{\varphi_1, \dots, \varphi_n\}$ funksiyalar sistemasi to'liq bo'lishi uchun bu sistemada P_0, P_1, M, S, L maksimal funksional yopiq sinflarning har biriga kirmaydigan kamida bitta funksiya mavjud bo'lishi

Ikki taraflama funksiya.

1- ta'rif. Quyidagicha aniqlangan

$$f^*(x_1, x_2, \dots, x_n) = \bar{f}(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)$$

1- jadval

Berilgan funksiya	Ikki taraflama funksiya
$f_1(x) = x$	$f_1^*(x) = x$
$f_2(x) = \bar{x}$	$f_2^*(x) = \bar{x}$
$f_3(x, y) = xy$	$f_3^* = x \vee y$
$f_4(x, y) = x \vee y$	$f_4^* = x y$
$f_5(x, y) = x \rightarrow y$	$f_5^* = \overline{y \rightarrow x}$

$f_6(x, y) = x \leftrightarrow y$	$f_6^* = \overline{x \leftrightarrow y}$
$f_7 = 1$	$f_7^* = 0$
$f_8 = 0$	$f_8^* = 1$

funksiyaga $f(x_1, x_2, \dots, x_n)$ funksiyaning **ikki taraflama funksiyasi** deb aytiladi.

2- ta'rif. Agar

$$f(x_1, x_2, \dots, x_n) = f^*(x_1, x_2, \dots, x_n) = \overline{f(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)}$$

munosabat bajarilsa, u holda $f(x_1, x_2, \dots, x_n)$ **o'z-o'ziga ikki taraflama funksiya** deb ataladi.

Jegalkin ko'phadi.

1- ta'rif. $\sum x_{i_1} x_{i_2} \dots x_{i_k} + a$ ko'rinishdagi ko'phad **Jegalkin ko'phadi** deb ataladi, bu yerda hamma x_{i_j} o'zgaruvchilar birinchi darajada qatnashadi, $(i_1, \dots, i_k)$ qiymatlar satrida hamma i_j lar har xil bo'ladi, $a \in E_2 = \{0, 1\}$.

2- ta'rif. $x_{i_1} + x_{i_2} + \dots + x_{i_k} + a$ ko'rinishdagi funksiya **chiziqli funksiya** deb ataladi.

Mantiq algebrasidagi monoton funksiyalar.

Tartiblash. $0 < 1$ munosabati orqali $\{0, 1\}$ to'plamini tartiblashtiramiz. $\alpha = (\alpha_1, \dots, \alpha_n)$ va $\beta = (\beta_1, \dots, \beta_n)$ qiymatlar satrlari bo'lsin.

1- ta'rif. Agar $\alpha_i \leq \beta_i$ tengsizlik hech bo'lmaganda bitta i uchun bajarilsa yoki α va β qiymatlar satrlari ustma-ust tushsa, u holda α **qiymatlar satri** β **qiymatlar satridan oldin keladi** deb aytamiz va $\alpha \prec \beta$ shaklda yozamiz.

2- ta'rif. Agar $\alpha \prec \beta$ munosabatdan $f(\alpha_1, \dots, \alpha_n) \leq f(\beta_1, \dots, \beta_n)$ tengsizlikning bajarilishi kelib chiqsa, u holda $f(x_1, \dots, x_n)$ funksiya **monoton funksiya** deb ataladi.

3- ta'rif Agar $\alpha \prec \beta$ munosabatdan $f(\alpha_1, \dots, \alpha_n) > f(\beta_1, \dots, \beta_n)$ tengsizlikning bajarilishi kelib chiqsa, u holda $f(x_1, \dots, x_n)$ **nomonoton funksiya** deb ataladi.

1- teorema. Monoton funksiyalarning superpozitsiyasidan hosil qilingan funksiya ham monoton funksiya bo'ladi.

2- teorema. Agar $f(x_1, \dots, x_n) \in M$ bo'lsa, u holda undan argumentlari o'rniga 0, 1 va x funksiyani qo'yish usuli bilan $\bar{x}$ funksiyani hosil qilish mumkin.

4-ta'rif. Agar $f(x_1, x_2, \dots, x_n)$ funksiya uchun $f(0, 0, \dots, 0) \equiv 0$ bo'lsa, u holda u **0 saqlovchi funksiya**, $f(1, 1, \dots, 1) \equiv 1$ bo'lganda esa **1 saqlovchi funksiya** deb ataladi.

Post jadvali					
	P_0	P_1	S	L	M
φ_1					
φ_2					
...	...	...	...	...	...
φ_n					

Amalda berilgan $\Phi = \{\varphi_1, \dots, \varphi_n\}$ funksiyalar sistemasining to'liq yoki to'liq emasligini aniqlash uchun **Post jadvali** deb ataluvchi jadvaldan foydalaniladi. Post jadvali quyida keltirilgan.

Jadvalning xonalariga o'sha satrdagi funksiya funksional yopiq sinflarning elementi bo'lsa "+" ishora, bo'lmasa "-" ishorasi qo'yiladi. $\Phi = \{\varphi_1, \dots, \varphi_n\}$ sistema to'liq funksiyalar sistemasi bo'lishi uchun, Post teoremasiga asosan, jadvalning har bir ustunida kamida bitta "-" ishorasi bo'lishi yetarli va zarur.

Demak, Post teoremasi shartidan P_0, P_1, M, S, L maksimal funksional yopiq sinflarning birortasini ham olib tashlash mumkin emas. Bu xulosadan, o'z navbatida, P_0, P_1, M, S, L maksimal funksional yopiq sinflarning birortasi ham boshqasining qism to'plami bo'la olmasligi kelib chiqadi. ■

XULOSA

x_1	x_2	x_3	$\overline{x_2}$	$\overline{x_3}$	$x_1 \rightarrow \overline{x_2}$	$x_2 \rightarrow \overline{x_3}$	$x_2 \rightarrow x_3$	$a \oplus b$	$(a \oplus b) \leftrightarrow c$
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0	0	0	1	1	1	1	1	0	0
0	0	1	1	0	1	1	1	0	0
0	1	0	0	1	1	1	0	0	1
0	1	1	0	0	1	0	1	1	1
1	0	0	1	1	1	1	1	0	0
1	0	1	1	0	1	1	1	0	0
1	1	0	0	1	0	1	0	1	0
1	1	1	0	0	0	0	1	0	0

Xulosa: Ushbu $((x_1 \rightarrow \bar{x}_2) \oplus (x_2 \rightarrow \bar{x}_3)) \leftrightarrow (x_2 \rightarrow x_3)$ formulaning chinlik jadvali {00110000}.

2-ish . Endi quyidagi formulani chinlik jadvalini yuqoridagi ta’riflardan foydalanib tuzamiz:

$$(x_2 \rightarrow x_1) \cdot (x_2 \downarrow x_2);$$

2.1-ish. $(x_2 \rightarrow x_1) \cdot (x_2 \downarrow x_2);$ ning qiymatini topamiz: (2-jadval)

2-jadval

x_1	x_2	$x_2 \rightarrow x_1$	$x_2 \downarrow x_2$	$(x_2 \rightarrow x_1) \cdot (x_2 \downarrow x_2)$
0	0	1	1	1
0	1	0	0	0
1	0	1	1	1
1	1	1	0	0

Xulosa: Ushbu $(x_2 \rightarrow x_1) \cdot (x_2 \downarrow x_2);$ formulaning chinlik jadvali $f=\{1010\}$.

3-ish . Endi quyidagi formulani chinlik jadvalini yuqoridagi ta’riflardan foydalanib tuzamiz:

$$((x_1 \vee x_2 \cdot \bar{x}_3) \rightarrow (x_2 \rightarrow x_1 \cdot x_3)) \leftrightarrow (x_1 \vee x_3);$$

3.1-ish. $((x_1 \vee x_2 \cdot \bar{x}_3) \rightarrow (x_2 \rightarrow x_1 \cdot x_3)) \leftrightarrow (x_1 \vee x_3);$ ning qiymatini topamiz.(3-jadval)

3-jadval

$a = x_1 \vee x_2 \cdot \overline{x_3}$; $b = x_2 \rightarrow x_1 \cdot x_3$; $c = x_1 \vee x_3$; deb belgilash kiritib oldim.

Xulosa: Ushbu $((x_1 \vee x_2 \cdot \overline{x_3}) \rightarrow (x_2 \rightarrow x_1 \cdot x_3)) \leftrightarrow (x_1 \vee x_3)$; formulaning chinlik jadvali $f = \{01111101\}$.

Kiyingi qiladigan ishim 3 ta funksiyani ham **Post** jadvaliga tekshiramiz.

1-ish. Formulalarni P_0 yopiq sinfga tegishli yoki tegishli emasligi tekshiramiz.

$$1) f_1(x, y, z) = ((x_1 \rightarrow \overline{x_2}) \oplus (x_2 \rightarrow \overline{x_3})) \leftrightarrow (x_2 \rightarrow x_3)$$

$f_1(0,0,0) = ((0 \rightarrow 1) \oplus (0 \rightarrow 1)) \leftrightarrow (0 \rightarrow 0) = 0$ demak f_1 formula P_0 yopiq sinfga tegishli ekan.

$$2) f_1(x, y, z) = (x_2 \rightarrow x_1) \cdot (x_2 \downarrow x_2);$$

$f_2(0,0,0) = (0 \rightarrow 0)(0 \downarrow 0) = 1$ demak f_2 formula P_0 yopiq sinfga tegishli emas ekan.

$$3) f_3(x, y, z) = ((x_1 \vee x_2 \cdot \overline{x_3}) \rightarrow (x_2 \rightarrow x_1 \cdot x_3)) \leftrightarrow (x_1 \vee x_3);$$

$f_3(0,0,0) = ((0 \vee 0 \cdot 1) \rightarrow (0 \rightarrow 0 \cdot 0)) \leftrightarrow (0 \vee 0) = 0$ demak f_3 formula P_0 yopiq sinfga tegishli ekan.

2-ish. Formulalarni P_1 yopiq sinfga tegishli yoki tegishli emasligi tekshiramiz.

$$1) f_1(x, y, z) = ((x_1 \rightarrow \overline{x_2}) \oplus (x_2 \rightarrow \overline{x_3})) \leftrightarrow (x_2 \rightarrow x_3)$$

x_1	x_2	x_3	$\overline{x_3}$	$x_2 \overline{x_3}$	a	$x_1 x_3$	b	$a \rightarrow b$	c	$(a \rightarrow b) \leftrightarrow c$
0	0	0	1	0	0	0	1	1	0	0
0	0	1	0	0	0	0	1	1	1	1
0	1	0	1	1	1	0	0	0	0	1
0	1	1	0	0	0	1	1	1	1	1
1	0	0	1	0	1	0	1	1	1	1
1	0	1	0	0	1	0	1	1	1	1
1	1	0	1	1	1	0	0	0	1	0
1	1	1	0	0	1	1	1	1	1	1

$f_1(1,1,1) = ((1 \rightarrow 0) \oplus (1 \rightarrow 0)) \leftrightarrow (1 \rightarrow 1) = 1$ demak f_1

formula P_1 yopiq sinfga tegishli emas ekan.

2)

$$f_1(x, y, z) = (x_2 \rightarrow x_1) \cdot (x_2 \downarrow x_2);$$

$$f_2(1,1,1) = (1 \rightarrow 1)(1 \downarrow 1) = 0$$

demak f_2 formula P_1 yopiq sinfga tegishli emas ekan.

$$3) f_3(x, y, z) = ((x_1 \vee x_2 \cdot \overline{x_3}) \rightarrow (x_2 \rightarrow x_1 \cdot x_3)) \leftrightarrow (x_1 \vee x_3);$$

$f_3(1,1,1) = ((1 \vee 1 \cdot 0) \rightarrow (1 \rightarrow 1 \cdot 1)) \leftrightarrow (1 \vee 1) = 1$ demak f_3 formula P_1 yopiq sinfga tegishli ekan.

3-ish. o'z-o'ziga ikki taraflama funksiyalar sinfi;

$$1) F = \overline{((\overline{x_1} \rightarrow x_2) \oplus (\overline{x_2} \rightarrow x_3)) \leftrightarrow (\overline{x_2} \rightarrow x_3)};$$

$a = \overline{x_1} \rightarrow x_2$; $b = \overline{x_2} \rightarrow x_3$; $c = \overline{x_2} \rightarrow \overline{x_3}$; deb belgilash kiritib olamiz.

x_1	x_2	x_3	$\overline{x_1}$	$\overline{x_2}$	$\overline{x_3}$	$\overline{x_1} \rightarrow x_2$	$\overline{x_2} \rightarrow x_3$	$a \oplus b$	$\overline{x_2} \rightarrow \overline{x_3}$	$(a \oplus b) \leftrightarrow c$	F^*
0	0	0	1	1	1	0	0	0	1	0	1
0	0	1	1	1	0	0	1	1	0	0	1
0	1	0	1	0	1	1	1	0	1	0	1
0	1	1	1	0	0	1	1	0	1	0	1
1	0	0	0	1	1	1	0	1	1	1	0
1	0	1	0	1	0	1	1	0	0	1	0
1	1	0	0	0	1	1	1	0	1	0	1
1	1	1	0	0	0	1	1	0	1	0	1

Demak: $F^* \neq F$ funksiya o'z-o'ziga ikki taraflama emas ekan.

$$2) F^* = \overline{(\overline{x_2} \rightarrow \overline{x_1})(\overline{x_2} \downarrow \overline{x_2})};$$

$\overline{x_1}$	$\overline{x_2}$	$\overline{x_2} \rightarrow \overline{x_1}$	$\overline{x_2} \downarrow \overline{x_2}$	F^*
1	1	1	0	1
1	0	1	1	0
0	1	0	0	1
0	0	1	1	0

Demak: $F^* = F$ funksiya o'z-o'ziga ikki taraflama ekan.

$$3) F^* = \overline{((\overline{x_1} \vee \overline{x_2} \cdot \overline{x_3}) \rightarrow (\overline{x_2} \rightarrow \overline{x_1} \cdot \overline{x_3})) \leftrightarrow (\overline{x_1} \vee \overline{x_3})};$$

$a = \overline{x_1} \vee \overline{x_2} \cdot \overline{x_3}$; $b = \overline{x_2} \rightarrow \overline{x_1} \cdot \overline{x_3}$; $c = \overline{x_1} \vee \overline{x_3}$; deb belgilash kiritib oldim.

Demak: $F^* \neq F$ funksiya o'z-o'ziga ikki taraflama emas ekan.

4-ish. Formulalarni chiziqli yoki chiziqli emasligiga tekshiramiz. Buning uchun

x_1	x_2	x_3	$\overline{x_1}$	$\overline{x_2}$	$\overline{x_3}$	$\overline{x_2 x_3}$	a	$\overline{x_1 x_3}$	b	c	$a \rightarrow b$	$(a \rightarrow b) \leftrightarrow c$	F^*
0	0	0	1	1	1	0	1	1	1	1	1	1	0
0	0	1	1	1	0	1	1	0	0	1	0	0	1
0	1	0	1	0	1	0	1	1	1	1	1	1	0
0	1	1	1	0	0	0	1	0	1	1	1	1	0
1	0	0	0	1	1	0	0	0	0	1	1	0	1
1	0	1	0	1	0	1	1	0	0	0	0	0	1
1	1	0	0	0	1	0	0	0	1	1	1	0	1
1	1	1	0	0	0	0	0	0	1	0	1	0	1

chinlik jadvalidagi oxirgi natijalardan foydalanamiz.

$$1) f_1(x, y, z) = ((x_1 \rightarrow \overline{x_2}) \oplus (x_2 \rightarrow \overline{x_3})) \leftrightarrow (x_2 \rightarrow x_3)$$

$$L_1 = a_0xyz + a_1xy + a_2xz + a_3yz + a_4x + a_5y + a_6z + b;$$

$$f(0,0,0) = 0 = a_0000 + a_100 + a_200 + a_30 + a_40 + a_50 + a_60 + b, \text{ demak } b = 0$$

$$f(0,0,1) = 0 = a_61 + 0 \text{ demak } a_6 = 0$$

$$f(0,1,0) = 1 = a_51 + 0 \text{ demak } a_5 = 1$$

$$f(0,1,1) = 1 = a_311 + a_51 + a_61 + 0 \text{ demak } a_3 = 0$$

$$f(1,0,0) = 0 = a_41 + 0 \text{ demak } a_4 = 0$$

$$f(1,0,1) = 0 = a_211 + a_4 + a_6 + 0 \text{ demak } a_2 = 0$$

$$f(1,1,0) = 0 = a_111 + a_4 + a_5 + 0 \text{ demak } a_1 = 1$$

$$f(1,1,1) = 0 = a_0111 + 1 + 0 + 0 + 0 + 1 + 0 + 0 \text{ demak } a_0 = 0 \text{ bundan kelib chiqadiki}$$

$$L = xy + y \text{ chiziqli emas ekan.}$$

$$2) f_1(x, y, z) = (x_2 \rightarrow x_1) \cdot (x_2 \downarrow x_2);$$

$$L_1 = a_0xy + a_1x + a_2y + b;$$

$$f(0,0) = 1 = a_000 + a_10 + a_20 + b, \text{ demak } b = 1$$

$$f(0,1) = 0 = a_001 + a_10 + a_21 + 1 \text{ demak } a_2 = 1$$

$$f(1,0)=1=a_010+a_11+a_20+1 \text{ demak } a_1=1$$

$$f(1,1)=0=a_011+a_11+a_21+1 \text{ demak } a_0=1 \text{ bundan kelib chiqadiki}$$

$$L=xy+x+y+1 \text{ chiziqli emas ekan.}$$

$$3) f_1(x,y,z) = ((x_1 \vee x_2 \cdot \overline{x_3}) \rightarrow (x_2 \rightarrow x_1 \cdot x_3)) \leftrightarrow (x_1 \vee x_3);$$

$$L_1 = a_0xyz + a_1xy + a_2xz + a_3yz + a_4x + a_5y + a_6z + b;$$

$$f(0,0,0)=0=a_0000+a_100+a_200+a_30+a_40+a_50+a_60+b, \text{ demak } b=0$$

$$f(0,0,1)=1=a_61+0 \text{ demak } a_6=1$$

$$f(0,1,0)=1=a_51+0 \text{ demak } a_5=1$$

$$f(0,1,1)=1=a_311+a_51+a_61+0 \text{ demak } a_3=1$$

$$f(1,0,0)=1=a_41+0 \text{ demak } a_4=1$$

$$f(1,0,1)=1=a_211+a_4+a_6+0 \text{ demak } a_2=1$$

$$f(1,1,0)=0=a_111+a_4+a_5+0 \text{ demak } a_1=0$$

$$f(1,1,1)=1=a_0111+0+1+1+1+1+1+0 \text{ demak } a_0=0 \text{ bundan kelib chiqadiki}$$

$$L=xz+yz+x+y+z \text{ chiziqli emas ekan.}$$

5-ish formulalarni monotonlikka tekshiramiz.

$$1) f_1(x,y,z) = ((x_1 \rightarrow \overline{x_2}) \oplus (x_2 \rightarrow \overline{x_3})) \leftrightarrow (x_2 \rightarrow x_3)$$

$$(0,1,1) \prec (1,0,0) \text{ va } f(0,1,1) > f(1,0,0) \text{ demak } f_1 \text{ formula monoton emas.}$$

$$2) f_1(x,y,z) = (x_2 \rightarrow x_1) \cdot (x_2 \downarrow x_2);$$

$$(0,0) \prec (0,1) \text{ va } f(0,0) > f(0,1) \text{ demak } f_2 \text{ formula monoton emas}$$

$$3) f_1(x,y,z) = ((x_1 \vee x_2 \cdot \overline{x_3}) \rightarrow (x_2 \rightarrow x_1 \cdot x_3)) \leftrightarrow (x_1 \vee x_3);$$

$$(1,0,1) \prec (1,1,0) \text{ va } f(1,0,1) > f(1,1,0) \text{ demak } f_3 \text{ formula monoton emas}$$

Endi Post jadvalini tuzamiz:

	P_0	P_1	S	L	M
f_1	+	-	-	-	-
f_2	-	-	+	-	-
f_3	+	+	-	-	-

