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5-o`lchamli filiformli komleksli Leybnits algebralari

mavzusidagi

BITIRUV MALAKAVIY ISHI

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M a z m u n i

K i r i sh

XX asrning 90 – yillari boshlarida frantsuz matematigi J.-L.Lodening ishlarida Leybnits algebralarini birinchi bo'lib kiritilgan. Lekin J.-L.Lode bu algebralarni kogomoliyalik nuqtayi nazardan o'rgangan. 1998 yildan boshlab Leybnits algebrasining strukturaviy nazariyasini Sh.A. Ayupov va B.A. Omirovlar o'rgana boshladi. Hozirgi vaqtda bu strukturaviy nazariya yaxshi o'rganilgan bo'lib va tadqiqot ishlari yanada rivojlanayapdi. Leybnits algebrasi Li algebrasining umumlashmalaridan biri hisoblanadi. Algebralarni tavsiflash masalasi asosan chekli o'lchovli algebralarda o'rganiladi. Chunki tavsiflash masalasi strukturaviy o'zgarmaslar uchun tenglamalar sistemasini yechishga olib kelinadi. Shu sababli ko'pgina masalalar asosan kompleks sonlar maydonida o'rganiladi. Li algebrasi nazariyasida chekli o'lchovli Li algebrasini o'rganish yechimli Li algebralarini o'rganishga olib kelinishi ma'lum. O'z navbatida yechimli Li algebralarini o'rganish nilpotent algebralarni o'rganishga olib kelinadi. Leybnits algebralarida hozircha bunday tasdiqlar yo'q. Shuning uchun yechimli algebralarni o'rganish kichik o'lchovdagi algebralarni tavsiflashdan boshlanadi. Hozirgacha yechimli Leybnits algebralari faqat to'rt o'lchovgacha o'rganilgan, to'rtdan boshlab tavsif qilish juda murakkab. Shuning uchun Li algebralar nazariyasida nilpotent algebralar o'rganiladi, chunki yuqarida ta'kidlaganimizday yechimli Li algebralari nilpotent Li algebralarini o'rganishga olib kelinadi. Nilpotent Li algebralari sakkiz o'lchovgacha tavsif qilingan. Leybnits algebralarida bo'lsa besh o'lchovgacha tavsiflangan. Bundan xulosa qilsak, algebraning o'lchovi qancha kattalashgan sari, uni tavsiflash shuncha murakkab bo'ladi. Nilpotent Leybnits algebralari bilan Ayupov Sh.A., Omirov B.A., Raximov I.S., Rixsiboev I.M., Xudoyberdiev A.X. va boshqalar shug'ullangan. Katta o'lchovdagi nilpotent Li algebralarini ham o'rganish murakkab bo'lgani uchun, nilpotent algebralar bir necha sinflarga bo'linadi. Masalan, nul filiform, filiform, kvazi filiform va boshqa sinflar.

Bitiruv malakaviy ishi kirish qismi, to`rtta paragraf, xulosa va adabiyotlar ruyxatidan iborat.

Ishning birinchi paragrafida Li va Leybnits algebrasiga oid dastlabki tushunchalar va belgilashlar keltirilgan.

Ikkinchi paragrafda filiformli Leybnits algebralari urganilgan.

Bitiruv malakaviy ishning uchinchi paragrafida filiformli kompleksli Leybnits algebralarining almashtirishlari qaraladi.

To`rtinchi paragrafida besh o`lchamli filiformli kompleksli Leybnits algebralari o`rganiladi.

Umuman bu nazariya bilan tanishish va terang o`rganish uchun [1-15] a'dabiyotlardan foydalanish mumkin.

§1. Dastlabki tushunchalar va belgilashlar

F maydonida berilgan *U* vektor fazo *algebra* deb ataladi, agar unda ko'paytmaning bichiziqli qonuni aniqlangan bo'lsa. U holda biz har bitta (x, y) juftligiga (x, y $\in U$) quyidagi bichiziqlilik shartlarini qanoatlandiradigan $x\cdot y\in U$ ko'paytmasiga mos qo'yamiz:

$$(x_1+x_2)y=x_1y+x_2y,\ x(y_1+y_2)=xy_1+xy_2, \tag{1.1}$$

$$\alpha(xy)=(\alpha x)y=x(\alpha y),\ \alpha\in F. \tag{1.2}$$

Biz aslida chekli o'lchovga ega bo'lgan algebralarni qaraymiz. Bunday algebralarda (e_1, e_2, … , e_n) ko'rinishidagi bazislar mavjud bo'lib va quyidagini yoza olamiz:

$$e_ie_j=\sum_{k=1}^{n}\gamma_{ijk}e_k\ ,$$

Bunda $\gamma_{ijk}\in F$, i, j, k = 1, 2, … ,n. Bu n^3 sondagi γ_{ijk} elementlari algebraning *strukturaviy o'zgarmaslar* deyiladi (olingan bazisga nisbatan). Bu o'zgarmaslar *U* dagi har bitta ko'paytmani aniqlaydi. Haqiqatdan ham, mayli *U* dagi ixtiyoriy ikki x va y elementlarini $x=\sum\xi_ie_i$, $y=\sum\eta_je_j$, ξ_i, $\eta_j\in F$ ko'rinishlarida yozamiz. *U* holda (1.1) va (1.2) shartlaridan,

$$xy=\left(\sum_i\xi_ie_i\right)\left(\sum_j\eta_je_j\right)=\sum_{i,j}(\xi_ie_i)(\eta_je_j)=$$

$$=\sum_{i,j}\xi_i(e_i(\eta_je_j))=\sum_{i,j}\xi_i\eta_j(e_ie_j),$$

ko'paytmasi e_ie_j ko'paytmalari yordamida aniqlanadi.

Bunaqa fikirlash chekli o'lchovdagi algebralarni tuzishning umumiy usulini beradi. Mayli *U* ixtiyoriy vektor fazo bo'lib va (e_i) undagi bazis bo'lsa, u holda har bitta (i, j) indekslar juftligi uchun ixtiyoriy ravishda e_ie_j ko'paytmasini *U* ning elementi deb aniqlaymiz. U holda $x=\sum_1^n\xi_ie_i$, $y=\sum_1^n\eta_je_j$ larda $x\cdot y$ ko'paytmasi

$$xy=\sum_{i,j=1}^{n}\xi_i\eta_j(e_ie_j) \tag{1.3}$$

formulasi yordamida aniqlanadi. Bu ko'paytma uchunda (1.1), (1.2) shartlari o'rinli.

Ta'rif 1.1. *U* algebrasi *assotsiativ* algebra deyiladi, agar uning ko'paytma amali assotsiativlik shartini qanoatlantirsa

$$(x\cdot y)\cdot z = x\cdot (y\cdot z). \tag{1.4}$$

Ta'rif 1.2. *U* algebrasi *Li* algebra deyiladi, agarda uning ko'paytma amali quyidagi shartlarni qanoatlantirsa:

$$\begin{gathered} x^2 = 0, \\ (x\cdot y)\cdot z + (y\cdot z)\cdot x + (z\cdot x)\cdot y = 0. \end{gathered} \tag{1.5}$$

(1.5) ayniyatlardagi birinchi shart antikommutativlik va ikkinchi shart Yakobi ayniyatlari deb ataladi.

Agar *U* algebrasi Li algebra va x, y$\in$*U* bo'lsa, u holda

$$0 = (x + y)^2 = x^2 + xy + yx + y^2 = xy + yx$$

bo'ladi. Bundan

$$xy = -yx \tag{1.6}$$

kelib chiqadi va bu shart ixtiyoriy Li algebrasida o'rinli bo'ladi.

Li algabrasiga quyidagicha ta'rif bersak ham bo'ladi.

Ta'rif 1.2′. F sonlar maydoni ustidagi G algebrasi Li algebrasi deyiladi, agar ixtiyoriy x, y, z$\in$G lar uchun quyidagi ayniyatlar bajarilsa:

$$[x, x]=0 \; - \; \text{antikommutativlik},$$

$$[[x, y], z]+[[y, z], x]+[[z, x], y]=0 \; - \; \text{Yakobi},$$

bu yerda [,] - G algebrasidagi ko'paytma.

Ta'rif 1.3. F sonlar maydoni ustidagi L algebrasi Leybnits algebrasi deyiladi, agar ixtiyoriy x, y, z $\in$ L lar uchun Leybnits ayniyati bajarilsa:

$$[x, [y, z]]=[[x, y], z] - [[x, z], y]$$

bu yerda [,] - L algebrasidagi ko'paytma.

Antikommutativlik [x,x] = 0 ayniyatidan

$$[x + y,x + y] = [x,x] + [x,y] + [y,x] + [y,y] = [x,y] + [y,x] = 0.$$

Bundan [x,y] = -[y,x] munosabatiga ega bo'lamiz.

Agar Leybnits ayniyatida antikommutativlik $[x,y] = -[y,x]$ ayniyati bajarilsa

$$-[[y, z],x] = [[x, y], z] - [-[z, x], y] = [[x, y], z] + [[z, x], y]$$

bo'ladi.

Bu oxirgi tenglikdan Yakobi ayniyati kelib chiqadi

$$[[x, y], z] + [[z, x], y] + [[y, z],x] = 0.$$

Shu sababli Leybnits algebrasi Li algebrasining umumlashmasi bo'lib topiladi.

Ixtiyoriy Leybnits L algebrasi uchun quyidagi qatorlarni aniqlaymiz:

a) $L^{<1>}=L,\ L^{<n+1>}=[L^{<n>},L]$;

b) $L^{[1]}=L,\ L^{[n+1]}=[L^{[n]},L^{[n]}]$;

v) $L^1=L,\ L^{n+1}=[L^1,L^n]+[L^2,L^{n-1}]+...+[L^{n-1},L^2]+[L^n,L^1]$.

L algebrasi *o'ng nilpotent* deyiladi, agar shunday $s\in N$ soni topilib $L^{<s>} = 0$ bo'lsa.

L algebrasi y*echimli* deyiladi, agar shunday $m\in N$ soni topilib $L^{[m]} = 0$ bo'lsa.

L algebrasi *nilpotent* deyiladi, agar $n\in N$ soni topilib $L^n = 0$ bo'lsa.

Leybnits algebralarida o'ng nilpotentlik va nilpotentlik ustma-ust tushadi.

$R(L)=\{x\in L\ :\ [y,x]=0$ ixtiyoriy $y\in L$ uchun$\}$ to'plami L Leybnits algebrasinig *o'ng nolga aylantiruvchi* deb ataladi.

Ta'rif 1.4. *G* Li algebrasi *filiformli* deyiladi, agarda $\dim G^i=n-i$, bunda $n=\dim G$ va $2\leq i\leq n$.

G algebraning tabiyiy graduirovkasini aniqlaymiz.

$G_i:=G^i/G^{i+1}$ dep alamiz. O` holda $\dim G_i=1$ va $\dim G_1=2$, $2\leq i\leq n-1$ bo'ladi. $grG := G_1\oplus G_2\oplus\ ...\ \oplus G_{n-1}$ deb belgilab olamiz. $[G^i, G^j]\subseteq G^{i+j}$ ichiga akslantirishni bajarsak biz $[G_i, G_j]\subseteq G_{i+j}$ munosabatiga ega bo'lamiz. Demak biz graduirovkani oldik.

Ta'rif 1.5. G algebra uchun yuqorida olingan graduirlash *tabiyiy graduirlash* deyiladi.

Ta'rif 1.6. L algebrasi *tabiyiy graduirlangan* deyiladi, agarda $L\cong grG$.

Tasdiq 1.1. Agar n toq bo`lsa, o` holda faqat izomorf bo`lmagan tabiyiy graduirlangan sheksiz sondagi elementlarni o`z ichiga oluvchi maydon ustinda aniqlangan $(n+1)$-o`lchamli filiformli μ_1^n va μ_2^n Li algebralari bor bo'ladi, bunda

μ_1^n: $\mu_1^n(e_0,e_i)=e_{i+1}$ $(1\le i\le n-1)$

$\mu_1^n(e_0,e_n)=0$

$\mu_1^n(e_i,e_j)=0$ $(1\le i,j\le n)$

μ_2^n: $\mu_2^n(e_0,e_i)=e_{i+1}$ $(1\le i\le n-1)$

$\mu_2^n(e_0,e_n)=0$

$\mu_2^n(e_i,e_j)=0$ $(1\le i,j\le n)$

$\mu_2^n(e_i,e_{n-i})=(-1)^i e_n$ $(1\le i\le n-1)$

bu erda $\mu_i^n(\cdot,\cdot)$ – μ_i^n algebradagi ko`paytpa, $i\in\{1, 2\}$.

Ta'rif 1.7. Mayli L algebrasi *n* o`lchamli Leybnits algebrasi bo`lsin, o` holda *nol-filiformli* deb ataladi, agar dim $L^i=(n+1)$ - i, $1\le i\le n+1$ bo`lsa.

Nol-filiformli *L* algebrasining aniqlanishi *L* algebrasining nilpotentli maksimal indeksga ega bo`lishi bilan teng ko`chli.

Teorema 1.1. Ixtiyoriy n-*o`lchamli nol-filiformli Leybnits* L *algebrasi uchin qo`yidagi ko`paytmaga ega bo`lgan* $\{e_1, e_2, \ldots, e_n\}$ *bazislari bar bo'ladi:*

$$[e_i, e_1]=e_{i+1} \quad 1\le i\le n-1, \tag{2.1}$$

(boshqa ko`paytmalar nolga teng).

Ta'rif 1.8. Leybnits L algebrasi *filiformli* deyiladi, agar dimL^i =n-i bo'lsa, bunda n=dim L va $2\le i\le n$.

§2. Filiformli kompleksli Leybnits algebralari

Bu paragrafta filiformli Leybnits algebralari o'rganiladi.

Sheksiz elementga ega bo'lgan maydon ustida aniqlangan *L* fazo uchun

$$[e_i,e_1]=e_{i+1}\ (2\leq i\leq n\text{-}1)\ \text{va}\ [e_n,e_1]=0$$

bo'ladigan $e_1, e_2\in L_1$, $e_i\in L_i$ $(3\leq i\leq n\text{-}1)$ bazislik elementlarini olamiz.

$[e_1, e_1]$ ko`paytma uchun mumkin bo'lgan hollarni qarashtiramiz.

Hol 1. Mayli $[e_1,e_1]=\alpha e_3$ $(\alpha\neq 0)$ bo'lsin, u holda $e_3\in R(L)\Rightarrow e_4, \ldots, e_n\in R(L)$ bo'ladi. $e_1^{'}=e_1, e_2=\alpha e_2, e_3^{'}=\alpha e_3, \ldots, e_n^{'}=\alpha e_n$ bazislik almastiricsh bajarib, $\alpha=1$ deb olish mumkin.

Shunday qilib

$$[e_1,e_1]=e_3,\ [e_i,e_1]=e_{i+1}\ (2\leq i\leq n\text{-}1)\ \text{va}\ [e_n,e_1]=0$$

bo'ladi.

Biz

$$[e_1,e_2]=\beta e_3\ \text{va}\ [e_2,e_2]=\gamma e_3$$

deb olamiz.

$$[e_1,[e_2,e_1]]=[[e_1,e_2],e_1]-[[e_1,e_1],e_2]$$

tenglikikni qaraymiz.

Agar $[e_2,e_1]\in R(L)$ bo'lsa, u holda

$$[[e_1,e_2],e_1]=[[e_1,e_1],e_2]$$

bo'ladi, yoki $\beta e_4=[e_3,e_2]$.

$$[e_2,[e_1,e_2]]=[[e_2,e_1],e_2]-[[e_2,e_2],e_1]$$

tengligidan uqshash holda $\gamma e_4=[e_3,e_2]$ munosabatiga ega bo'lamiz, bundan $\beta=\gamma$ kelib chiqadi.

Bazis elementlarining soni bo'yisha induktsiyani quyshdagi tenglikka qo'llab,

$$[e_{i0},[e_1,e_2]]=[[e_i,e_1],e_2]-[[e_i,e_2],e_1],$$

biz $[e_i,e_2]=\beta e_{i+1}$ ega bo'lamiz, yoki Hol 1 dagi kabi quydagi algebrani olamiz:

$$[e_1,e_1]=e_3,\ [e_i,e_1]=e_{i+1},\ [e_i,e_2]=\beta e_{i+1},\ [e_1,e_2]=\beta e_3\ (2\leq i\leq n\text{-}1).$$

Hol 2. Mayli $[e_1, e_1]=0$ va $[e_2, e_2]=\alpha e_3$ $(\alpha\neq 0)$ bo'lsin, u holda $e_3\in R(L)$ bo'ladi, bundan $e_3, e_4, ..., e_n\in R(L)$ hosil bo'ladi. O'rin almastirish quydagisha olsak:

$$e_1^{'}=\alpha e_1,\ e_1^{'}=e_2,\ e_3^{'}=\alpha e_3,\ \ e_4^{'}=\alpha^2 e_4,\ ...,\ e_n^{'}=\alpha^{n-2}e_n,$$

$\alpha=1$ deb olishga bo'ladi, yoki

$$[e_2,e_2]=e_3,\ [e_i,e_1]=e_{i+1}.$$

Biz $[e_1,e_2]=\beta e_3$ belgilash kiritamiz, u holda

$$[e_1,[e_2,e_1]] = [[e_1,e_2],e_1] - [[e_1,e_1],e_2]$$

tengligidan $[[e_1,e_2],e_1]=0$ ega bo'lamiz, yoki $\beta[e_3,e_1]=\beta e_4=0$ va bundan $\beta=0$.

Bazis elementlarining soni bo'yisha induksiyani quydagisha qo'llab

$$[e_i,[e_1,e_2]]=[e_{i+1},e_2] - [[e_i,e_2], e_1],$$

biz $[e_i,e_2]=e_{i+1}$ tengligiga ega bo'lamiz, yoki Hol 2 kabi quydagicha algebrani olamiz:

$$[e_i,e_1]=e_{i+1},\ [e_i,e_2]=e_{i+1}\quad (2 \leq i \leq n\text{-}1).$$

Bazislarni $e_1^{'}:=e_2-e_1$, $e_i^{'}:=e_i$ ko`rnishida olsak,

$$[e_1^{'},\ e_2^{'}]=e_3^{'},\ [e_i^{'},\ e_2^{'}]=e_{i+1}^{'}\ (2 \leq i \leq n\text{-}1)$$

ega bo'lamiz. Olingan bu algebra Hol 1 kabi algebraga $\beta=1$ bo'lganda izomorf bo'ladi ($e_1^{'}:=e_1-e_2,\ e_i^{'}:=e_i$).

Hol 3. Mayli $[e_1,e_1]=0$ va $[e_2,e_2]=0$ bo'lsin. $[e_1,e_2]=\alpha e_3$ deb belgilab olamiz.

Hol 3.1. Mayli $[e_1,e_2]=\alpha e_3$ (bunda $\alpha\neq$-1) bo'lsin, u holda $e_3 \in R(L) \Rightarrow e_4, ..., e_n\in R(L)$ bo'ladi. Bizda $\alpha\neq -1$ bo'lsa,u holda $e_2^{'}=e_2+e_1$ bo'ladi. Biz: $[e_2^{'},e_2^{'}]=(\alpha+1)e_3$, $[e_2^{'},e_1^{'}]=e_3$ ege bo'lamiz, yoki Hol 2 ga ustma- ust tushadi.

Hol 3.2. Mayli $[e_1,e_2]= -e_3$ bo'lsin, u holda quydagi lemmani kiritamiz.

Lemma 2.1. *Mayli* L - $\{e_1, e_2, ..., e_n\}$ *bazisli, quydagi tenglikni qanoatlantiruvchi n-olshamli tabiyiy graduirlangan filiformli Leybnits algebrasi bo'lsin*:

$$[e_1,e_1]=[e_2,e_2]=0,\ [e_1,e_2]= -e_3,\ [e_i,e_1]=e_{i+1}.$$

u holda L *algebrasi* Li *algebrasi bo'ladi*.

***Isboti*:** Bazis elementlarining soni bo'yicha induksiyani quydagi

$$[e_1,[e_i,e_1]]=[[e_1,e_i],e_1]-[[e_1,e_1],e_i],$$

tenglikga qo'llab

$$[e_1,e_i]=-[e_i,e_1]\ (1\le i\le n-1)$$

ega bo'lamiz.

$$[e_2,[e_2,e_1]]=[[e_2,e_2],e_1]-[[e_2,e_1],e_2]$$

tenglikdan $[e_2,e_3]=-[e_3,e_2]$ tengligi kelib chiqadi. Quydagi tengliklarning zanjirin qo'llab

$$[e_2,e_{i+1}]=[e_2,[e_i,e_1]]=[[e_2,e_i],e_1]-[[e_2,e_1],e_i]=-[e_1,[e_2,e_i]]+[[e_1,e_2],e_i]=$$

$$=-[[e_1,e_2],e_i]+[[e_1,e_i],e_2]+[[e_1,e_2],e_i]=[[e_1,e_i],e_2]]=-[e_{i+1},e_2]$$

va induksiyaga asoslanib $1\le i\le n$ uchun $[e_2,e_i]=-[e_i,e_2]$ ega bo'lamiz.

Shunday qilib biz $[e_1,e_i]=-[e_i,e_1]$ va $[e_2,e_i]=-[e_i,e_2]$ $(1\le i\le n)$ ega bo'ldik. Barcha i va j lar uchun

$$[e_i,e_j]=-[e_j,e_i]$$

tenglikni isbotlaymiz. Isbotlash Barcha j dagi i bo'yicha olib boramiz. i ni 2 dan katta deyish mumkin.

Quydagi tengliklarning zanjiri lemmaning isbotini yakunlaydi:

$$[e_j,e_{i+1}]=[e_j,[e_i,e_1]]=[[e_j,e_i],e_1]-[[e_j,e_1],e_i]=-[e_1,[e_j,e_i]]+[[e_1,e_j],e_i]=$$

$$=-[[e_1,e_j],e_i]+[[e_1,e_i],e_j]+[[e_1,e_j],e_i]=[[e_1,e_i],e_j]]=-[e_{i+1},e_j].$$

Shunday qilib Li bo'lmagan tabiyiy graduirlangan filiformli Leybnits algebralari quydagilar bo'ladi (ixtiyoriy β parametri uchun):

$$[e_1,e_1]=e_3,\ [e_i,e_1]=e_{i+1},\ [e_i,e_2]=\beta e_{i+1},\ [e_1,e_2]=\beta e_3.$$

Mayli $\beta\neq1$ bo'lsin, u holda quydagicha almashtirish kiritib:

$$e_1'=(1-\beta)e_1,\ e_2'=-\beta e_1+e_2,\ e_3'=(1-\beta)^2e_3,\dots,\ e_n'=(1-\beta)^n e_n$$

$\beta=0$ deb olish mumkin.

Endi $\beta=1$ bo'lgan holni qaraymiz, yoki

$$[e_1,e_1]=e_3,\ [e_i,e_1]=e_{i+1},\ [e_i,e_2]=e_{i+1},\ [e_1,e_2]=e_3\ (2\le i\le n).$$

Bazisni $e_2'=e_2-e_1$ deb almashtirsak

$$[e_1,e_1]=e_3,\ [e_i,e_1]=e_{i+1}\ (2\le i\le n)$$

ega bo‘lamiz.

Xosil bo’lgan

$$F_n^1: \ [e_1,e_1]=e_3,\ [e_i,e_1]=e_{i+1}\ (2\le i\le n-1),$$

$$F_n^2: \ [e_1,e_1]=e_3,\ [e_i,e_1]=e_{i+1}\ (3\le i\le n-1),$$

algebralarining izomorf emasligini ko’rsatamiz.

Aksinsha pikir yuritamiz. Mayli φ birinchi algebrani ikkinchi algebraga otkizuvchi izomorfizm bo’lsin, yoki $\varphi: F_n^1 \to F_n^2$. $\varphi(L(F_n^1))=L(F_n^2)$ va $\varphi(R(F_n^1))=R(F_n^2)$ ekanligin ko`rsatich qiyinmas, yana $\varphi(L(F_n^1)\cap R(F_n^1))=L(F_n^2)\cap R(F_n^2)$ ekanliginiyam ko`rish qiyinmas. Algebralarning mos ko`paytmalaridan:

$$L(F_n^1)=\{e_1-e_2\},\ R(F_n^1)=\{e_2, e_3, \ldots, e_n\},\ L(F_n^2)=\{e_2\},\ R(F_n^2)=\{e_2, e_2, e_3, \ldots, e_n\}$$

bo’ladi. Shunday qilib $L(F_n^1)\cap R(F_n^1)=\{0\}$ bo’ladi, lekin $L(F_n^2)\cap R(F_n^2)=\{e_2\}$, yoki $\varphi(0)=e_1$ bo’ladi va biz F_n^1 algebradan F_n^2 algebraga bo’lgan izomorfizmga zid kelamiz.

Bu ziddiyatdan quydagi teorema kelib chiqadi:

Teorema 2.1. *Barcha n-o`lchamli tabiyiy graduirlangan* Li *bo’lmagan kompleksli Leybnits algebrasi quydagi o’z-aro izomorf bo’lmagan algebralarning biriga izomorf bo’ladi*:

NF_n^1: $[e_1,e_1]=e_3$, $[e_i,e_1]=e_{i+1}$, *bunda* $2\le i\le n-1$.

NF_n^2: $[e_1,e_1]=e_3$, $[e_i,e_1]=e_{i+1}$, *bunda* $3\le i\le n-1$.

(*qolgan kopaytmalar nolga teng*).

Eslatma 2.1. Teoremani 2.1 dan barcha n-o`lchamli kompleksli Li bo’lmagan filiformli Leybnits algebrasida ungdan nolga aylantiruvchi n – 2 o`lchamga ega bo’ladi.

Shunday qilib, Tastdiq 1.1 va teorema 2.1 dan biz ta`biyiy graduirlangan Leybnits algebralarinin` klassifikatsiasini olamiz.

Teorema 2.2. *Barcha n-o`lshamli kompleksli* Li *bo’lmagan filiformli Leybnits algebrasi ushun shunday* $\{e_1, e_2, \ldots, e_n\}$ *bazisi topilib, uning ko`paytmasi quydagicha ko`rinishlarning biri bo’ladi*:

$F_n^1:$ $[e_1,e_1]=e_3$, $[e_i,e_1]=e_{i+1}$, $2 \le i \le n-1$,

$[e_1,e_2]=\alpha_4 e_4 + \alpha_5 e_5 + \ldots + \alpha_{n-1}e_{n-1} + \theta e_n$,

$[e_j,e_2]=\alpha_4 e_{j+2} + \alpha_5 e_{j+3} + \ldots + \alpha_{n+2-j}e_n$, $2 \le j \le n-2$,

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$F_n^2:$ $[e_1,e_1]=e_3$, $[e_i,e_1]=e_{i+1}$, $3 \le i \le n-1$,

$[e_1,e_2]=\beta_4 e_4 + \beta_5 e_5 + \ldots + \beta_n e_n$, $[e_2,e_2]=\gamma e_n$,

$[e_j,e_2]=\beta_4 e_{j+2} + \beta_5 e_{j+3} + \ldots + \beta_{n+2-j}e_n$, $3 \le j \le n-2$,

bunda $\alpha_i, \theta, \beta_i, \gamma \in \mathbb{C}$ va *qolgan kopaytmalar nolge ten`.*

***Isboti*:** Bu ko`paytmalarni Leybnits ayniyatiga qoyib tekshirsak, u holda Leybnits algebrasi ekenligi kelib shiqadi. Teorema 2.1 dan barcha n-o`lshamli kompleksli Li bo'lmagan filiformli Leybnits L algebrasi quydagicha izomorf bolmagan ikki algebralarning biriday bo'ladi:

$$NF_n^1+\beta,$$

bunda

$$\beta(e_i,e_1)=0,$$

bunda $2 \le i \le n-1$ va $i\neq 1$ da $\beta(e_i,e_j) \in lin(e_{i+j},\ldots,e_n)$ va $2 \le j \le n-2$ da $\beta(e_1,e_j)\in lin(e_{j+2},\ldots,e_n)$ bo'ladi.

$$NF_n^2+\beta,$$

bunda $\beta(e_1,e_1)=0$,

$$\beta(e_i,e_1)=0,$$

bunda $3 \le i \le n-1$ va $i,j\neq 1$ da $\beta(e_i,e_j) \in lin(e_{i+j},\ldots,e_n)$ va $2 \le j \le n-2$ da $\beta(e_1,e_j)\in lin(e_{j+2},\ldots,e_n)$ bo'ladi.

Hol 1. Mayli $L \cong NF_n^1+\beta$ bo'lsin, u holda L algebrasida $2 \le i \le n-1$ da quydagiga ega bo'lamiz:

$$[e_1,e_1]=e_3 \text{ va } [e_i,e_1]=e_{i+1}.$$

Bundan $e_3, e_4, \ldots, e_n \in R(L)$ bo'ladi, yoki $3 \le j \le n$, $1 \le i \le n$ larda

$$[e_i,e_j]=0.$$

Biz $[e_2,e_2]=\alpha_4 e_4 + \alpha_5 e_5 + \ldots + \alpha_n e_n$ deb olamiz.

Leybnits ayniyati bo'yicha

$$[e_i,[e_1,e_2]]=[[e_i,e_1],e_2] - [[e_i,e_2],e_1]$$

ega bo'lamiz.

Agar $[e_1,e_2] \in R(L)$ bo'lsa, u holda $[e_i,[e_1,e_2]]=0$ bo'ladi va barcha $i \geq 2$ lar uchun

$$[[e_i,e_1],e_2]=[[e_i,e_2],e_1]$$

bo'ladi. Shunday qilib $2 \leq i \leq n-2$ de

$$[e_i,e_2]=\alpha_4 e_{i+2} + \alpha_5 e_{i+3} + \ldots + \alpha_{n+2-i} e_n$$

bo'ladi.

$[e_1,e_2]=\beta_4 e_4 + \beta_5 e_5 + \ldots + \beta_n e_n$ deb olamiz.

Quydagi tenglikni qarasak

$$[e_1,[e_2,e_1]]=[[e_1,e_2],e_1] - [[e_1,e_1],e_2].$$

Oxirgi tenglikga uqshash tarzda quydagini olamiz

$$[[e_1,e_2],e_1]=[[e_1,e_1],e_2].$$

Bizda $[e_1,e_1]=e_3$ va $[e_i,e_1]=e_{i+1}$ bo'lganligidan

$$\beta_4 e_5 + \beta_5 e_6 + \ldots + \beta_{n-1} e_n=\alpha_4 e_5 + \alpha_5 e_6 +\ldots+\alpha_{n-1} e_n$$

bo'ladi. Bundan

$$[e_1,e_2]=\alpha_4 e_4 + \alpha_5 e_5 + \ldots + \alpha_{n-1} e_{n-1} + \beta_n e_n$$

kelib shiqadi.

Shunday qilib Hol 1 dan quydagi sinfni oldik:

$$[e_1,e_1]=e_3,\ [e_i,e_1]=e_{i+1}, \qquad 2 \leq i \leq n-1$$

$$[e_1,e_2]=\alpha_4 e_4 + \alpha_5 e_5 + \ldots + \alpha_{n-1} e_{n-1} + \beta_n e_n,$$

$$[e_j,e_2]=\alpha_4 e_{j+2} + \alpha_5 e_{j+3} + \ldots + \alpha_{n+2-j} e_n \qquad 2 \leq j \leq n-2$$

Hol 2. Mayli $L \cong NF_n^2 + \beta$ bo'lsin, u holda $3 \leq i \leq n-1$ da

$$[e_1,e_1]=e_3 \text{ va } [e_i,e_1]=e_{i+1}.$$

Bundan $\{e_3, e_4, \ldots, e_n\} \subseteq R(L)$ bo'ladi, yoki $3 \leq j \leq n$, $1 \leq i \leq n$ larda

$$[e_i,e_j]=0$$

bo'ladi.

Mayli $\beta(e_2,e_1)=\alpha_4e_4+\alpha_5e_5+\ldots+\alpha_ne_n$ bo'lsin, u holda bazisni $e_2^{'}:=e_2-\alpha_4e_3-\alpha_5e_4-\ldots-\alpha_ne_{n-1}$ deb olmashtirsak quydagiga ega bo'lamiz

$$[e_2^{'},e_1]=[-\alpha_4e_3-\alpha_5e_4-\ldots-\alpha_ne_{n-1},e_1]+\beta(e_2,e_1)=0.$$

Shunday qilib, $[e_2,e_1]=0$ deb olish mumkin.

Mayli $[e_1,e_2]=\beta_4e_4+\beta_5e_5+\ldots+\beta_ne_n$ bo'lsin.

$[e_2,e_1]\in R(L)$ bolganlikdan

$$[e_1,[e_2,e_1]]=[[e_1,e_2],e_1]-[[e_1,e_1],e_2]$$

tengligidan

$$[[e_1,e_2],e_1]=[[e_1,e_1],e_2]\Rightarrow[[e_1,e_2],e_1]=[e_3,e_2]$$

bo'ladi, yoki $[e_3,e_2]=\beta_4e_5+\beta_5e_6++\ldots+\beta_{n-1}e_n$.

Yuqoridagi kabi fikir yuritsak

$$[e_2,[e_1,e_2]]=[[e_2,e_1],e_2]-[[e_2,e_2],e_1]$$

tengligidan va $[e_1,e_2]\in R(L)$, $[e_2,e_1]=0$ dan $[[e_2,e_2],e_1]=0$ ega bo'lamiz. Bizda e_1 bazisi e_n da shap tomondan nolga aylantirganidan $[e_2,e_2]=\gamma e_n$ bo'ladi.

$3\le i\le n-1$ da

$$[e_i,[e_1,e_2]]=[[e_i,e_1],e_2]-[[e_i,e_2],e_1]$$

bolganligidan va$[e_1,e_2]\in R(L)$ dan

$$[[e_i,e_1],e_2]=[[e_i,e_2],e_1]$$

ega bo'lamiz. Bundan

$$[[e_{i+1},e_1],e_2]==[[e_i,e_2],e_1]$$

bo'ladi, yoki $3\le i\le n-2$ da

$$[e_i,e_2]=\beta_4e_{i+2}+\beta_5e_{i+3}+\ldots+\beta_{n+2-i}e_n$$

bo'ladi.

Shunday qilib biz Hol 2 da Leybnits algebrasining sinfini oldik:

$[e_1,e_1]=e_3$, $[e_i,e_1]=e_{i+1}$, $\qquad 3\le i\le n-1$,

$[e_1,e_2]=\beta_4e_4+\beta_5e_5+\ldots+\beta_ne_n$, $[e_2,e_2]=\gamma e_n$,

$[e_j,e_2]=\beta_4e_{j+2}+\beta_5e_{j+3}+\ldots+\beta_{n+2-j}e_n$, $\qquad 3\le j\le n-2$.

Teorema isbotlandi.

Eslatma 2.2. F_n^1 semeystvadagi algebralar F_n^2 semeystvadagi algebralarga izomorf bolmaydi, shundan Li bolmagan filiformli Leybnits algebralarining izomorflilikka o'rganish har bir sinfning ichidan izomorfizmga tekshirish yetarli.

Mayli L – *n*-o'lchamli algebra va $\{e_1, e_2, \dots, e_n\}$ L dagi bazis bo'lsin, u holda har bir $i,j \neq 0$ uchun $[e_i,e_j]\in\langle e_{i+j+1},\dots,e_n\rangle$.

Bundan $[e_i,e_1]=e_{i+1}+(*)e_{i+2}+\dots+(*)e_n$, $2\leq i \leq n-1$ bo'ladi.

Bazislarni $e_1'=e_1$, $e_2'=e_2$, $e_{i+1}'=[e_i',e_1']$ deb o`zgartirib, biz

$$[e_i,e_1]=e_{i+1},\ 2\leq i\leq n-1$$

tenglikka ega bo'lamiz.

Endi

$$[e_1,e_i]=-e_{i+1}+\alpha_{1,i}^{i+2}e_{i+2}+\alpha_{1,i}^{i+3}e_{i+3}+\dots+\alpha_{1,i}^{n}e_n,\ 2\leq i \leq n-1$$

ko`paytmani qarashtiramiz, u holda quydagiga ega bo'lamiz:

$$[e_i,e_1]+[e_1,e_i]=\alpha_{1,i}^{i+2}e_{i+2}+\alpha_{1,i}^{i+3}e_{i+3}+\dots+\alpha_{1,i}^{n}e_n,\ 2\leq i \leq n-1. \qquad (2.1)$$

Leybnits ayniyatidan har bir $x, y\in L$ lar uchun

$$[x, y] + [y, x]\in R(L),$$

bunda R(L) to'plami L dagi ung tomondan nolga aylantiradigan to'plam. Agar biz (2.1) ning ikkala tomoninda (n-i-3) marta e_1 ga ko'paytirsak $\alpha_{1,i}^{i+2}=0$ ega bo'lamiz. Bu protsessni davom ettirsak biz quydagiga ega bo'lamiz:

$$\alpha_{1,i}^{i+k}=0,\ 2\leq k\leq n-2-i.$$

Yuqoridagi $0\leq i\leq [\frac{n}{2}]$ da $[e_i, e_i]$ ga qo'llasak, biz

$$[e_i, e_i]=\alpha_{i,i}^{n}e_n$$

ko`paytmasiga ega bo'lamiz. Quyidagi

$$[e_1, e_i]=[e_1, [e_{i-1}, e_1]]=[[e_1, e_{i-1}], e_1]-[[e_1, e_1], e_{i-1}]=$$
$$=[-e_i+\alpha_{1,i-1}^{n}e_n, e_1]=-[e_i, e_1]=-e_{i+1}$$

tenglikni $3\leq i \leq n-1$ lar uchun

$$[e_i, e_1]=[e_1, e_i]=-e_{i+1}$$

tengligiga olib kelamiz, yoki barcha $x\in L^2$ uchun

$$[e_1, x]=-[x, e_1]$$

ega bo'lamiz.

Biz quydagicha bo'lsin deb olamiz

$$[e_i, e_j]= -[e_j, e_i], \quad 1\leq i < j \leq n.$$

Barcha j uchun induksiyani qo'llab quydagi ayniyatiga ega bo'lamiz

$$[e_i, e_{j+1}]=[e_i, [e_j, e_1]] = [[e_i, e_j], e_1] - [[e_i, e_1], e_j]= ([e_1,e_j]\in L^2)=[e_1, [e_i, e_j]]+$$

$$+[[e_1, e_i] - \alpha_{1,i}^n e_n, e_j] = - [e_1, [e_i, e_j]]+ [[e_1, e_i], e_j]= - [[e_1, e_i], e_j]+ [[e_1, e_j], e_i] +$$

$$+[[e_1, e_i],e_j] = - [e_{j+1}, e_i], \qquad 2\leq j\leq n-1.$$

Shuhday qilib, barcha bazisli ko`paytmalarni teorema 2.2 ga qo'yib, quydagi algebralar sinfiga ega bo'lamiz

$$F_n^3: \quad [e_i,e_1]= e_{i+1}, \qquad 2 \leq i \leq n-1,$$

$$[e_1,e_i]=- e_{i+1}, \qquad 3 \leq i \leq n-1,$$

$$[e_1,e_1]=\theta_1 e_n,\ [e_1,e_2]=-e_3 +\theta_2 e_n,\ [e_2,e_2]= \theta_3 e_n,$$

$$[e_2,e_j]=- [e_j,e_2] \in\{e_{j+2}, e_{j+3}, \dots, e_n \}, \qquad 3 \leq j \leq n-2,$$

$$[e_i,e_j]=- [e_j,e_i] \in\{e_{i+j}, e_{i+j+1}, \dots, e_n \}, \qquad 3 \leq i \leq [\frac{n}{2}],\ i \leq j \leq n-i,$$

Shunday qilib biz F_n^3 sinfini va teorema 2.2 ni birlashtirib quyidagi teoremani olamiz:

Teorema 2.3.[2] *Itiyoriy n-o`lchamli filiformli kompleksli Leybnits algebralarida* $\{e_1, e_2, \dots, e_n\}$ *bazisi mavjud bo'lib, u quydagi ko'rnishdagi ko`paytmalaridan biri bo'ladi*:

$$F_n^1: \quad [e_1,e_1]=e_3,\ [e_i,e_1]=e_{i+1}, \qquad 2 \leq i \leq n-1,$$

$$[e_1,e_2]=\alpha_4 e_4 + \alpha_5 e_5 + \dots + \alpha_{n-1}e_{n-1} + \theta e_n,$$

$$[e_j,e_2]=\alpha_4 e_{j+2} + \alpha_5 e_{j+3} + \dots + \alpha_{n+2-j}e_n, \qquad 2 \leq j \leq n-2,$$

bunda $\alpha_i, \theta\in \mathbb{C}$ *va qolgan ko`paytmalar barchasi nolga ten`*,

$$F_n^2: \quad [e_1,e_1]=e_3,\ [e_i,e_1]=e_{i+1}, \qquad 3 \leq i \leq n-1,$$

$$[e_1,e_2]=\beta_4 e_4 + \beta_5 e_5 + \dots + \beta_n e_n,\ [e_2,e_2]=\gamma e_n,$$

$$[e_j,e_2]=\beta_4 e_{j+2} + \beta_5 e_{j+3} + \dots + \beta_{n+2-j}e_n, \qquad 3 \leq j \leq n-2,$$

bunda $\beta_i, \gamma \in \mathbb{C}$ *va qolgan ko`paytmalarning barchasi nolga ten`*,

$$F_n^3: \quad [e_i,e_1]= e_{i+1}, \qquad 2 \le i \le n-1,$$

$$[e_1,e_i]=- e_{i+1}, \qquad 3 \le i \le n-1,$$

$$[e_1,e_1]=\theta_1 e_n,\ [e_1,e_2]=-e_3 +\theta_2 e_n,\ [e_2,e_2]= \theta_3 e_n,$$

$$[e_2,e_j]=- [e_j,e_2] \in \{e_{j+2}, e_{j+3}, \ldots, e_n \}, \qquad 3 \le j \le n-2,$$

$$[e_i,e_j]=- [e_j,e_i] \in \{e_{i+j}, e_{i+j+1}, \ldots, e_n \}, \qquad 3 \le i \le [\frac{n}{2}],\ i \le j \le n-i,$$

bunda $\theta_i \in \mathbb{C}$ *va ko`rsetilgen ko`paytmalar Leybnits ayniyatin qanoatlantiradi*.

§3. Filiformli kompleksli Leybnits algebralari almashtirishlari

Ixtiyoriy Li bo`lmagan filiformli Leybnits agebralari izomorflik aniqlikda teorema 2.2 dagi o`z-aro kesishmaydigan ikki sinfnting biriga tegishli bo’ladi. Biz bu sinflarning ichida izomorflikni tekshirish uchun bazislarning aynimagan almastirishi zarur. Bu paragraf shu masalani urganadi.

Dastlab Li bo`lmagan filiformli Leybnits algebralari uchun adaptirlangan bazis tushunchasini kiritamiz.

**Ta`rif 3.1.** Mayli L *n*-o`lchamli Li bo’lmagan filiformli kompleksli Leybnits algebrasi bo’lsin. L algebrasining $\{e_1, e_2, \ldots, e_n\}$ bazisini *adaptirlangan* deb ataymiz, agarda bu bazisda ko`paytma $F_n^1$ yoki $F_n^2$ ko`rinishda bo’lsa.

Mayli L algebrasi V vektor fazosida aniqlangan Li bo’lmagan filiformli kompleksli Leybnits algebrasi bo’lsin.

Ta’rif 3.2. $f \in GL(V)$ bazisining almashtirishi L algebrasining *ko`paytmasiga adaptirlangan* deyiladi, agarda f adaptirlangan bazisni adaptirlangan bazisga o`tqizsa.

GL(V) gruppasining adaptirlangan almashtirishidan tuzilgan yopiq qism gruppasini $GL_{ad}(V)$ orqali belgilaymiz.

Keyinchalik bizga quyidagi lemma kerak bo’ladi.

Lemma 3.1. *Ixtiyoriy* $0 \le p \le n\text{-}k$, $4 \le k \le n$ *uchun quyidagi tenglik urinli:*

$$\sum_{i=k}^{n} a(i) \sum_{j=i+p}^{n} b(i,j) e_j = \sum_{j=k+p}^{n} \sum_{i=k}^{j-p} a(i) b(i,j) e_j . \qquad (3.1)$$

Isboti. Bu tenglik quyidagi tengliklar zanjiridan kelib chiqadi:

$$\sum_{i=k}^{n} a(i) \sum_{j=i+p}^{n} b(i,j) e_j = \sum_{i=k}^{n} a(i)\Big(b(i,i+p)e_{i+p} + b(i,i+p+1)e_{i+p+1} + \ldots + b(i,n)e_n\Big) =$$

$$= a(k)[b(k,k+p)e_{k+p} + b(k,k+p+1)e_{k+p+1} + \ldots + b(k,n)e_n] + \ldots +$$

$$+a(t)[b(t,t+p)e_{t+p}+$$

$$+b(t,t+p+1)e_{t+p+1}+\ldots+b(t,n)e_n]+\ldots+a(n-p)b(n-p,n)e_n$$

$$=a(k)b(k,k+p)e_{k+p}+$$

$$+[a(k)b(k,k+p+1)+a(k+1)b(k+1,k+p+1)]e_{k+p+1}+\ldots+$$

$$[a(k)b(k,t+p)e_{t+p}+$$

$$+a(k+1)b(k+1,t+p)+\ldots+a(t)b(t,t+p)]e_{t+p}+\ldots+a(n-p)b(n-p,n)e_n=$$

$$=\sum_{t=k}^{n-p}\sum_{i=k}^{t}a(i)b(i,t+p)e_{t+p}=\sum_{j=k+p}^{n}\sum_{i=k}^{j-p}a(i)b(i,j)e_j\,.$$

Lemma isbotlandi.

Biz keyinchalik Li bo'lmagan filiformli Leybnits algebralarini qaraymiz.

Tasdiq 3.1. Mayli $f\in GL_{ad}(V)$.

a) Mayli L algebrasi F^1 sinfiga tegishli bo'lsin. U xolda f quyidagi ko`rinishga ega bo'ladi:

$$\begin{cases} f(e_1)=a_1e_1+a_2e_2+a_3e_3+\ldots+a_ne_n \\ f(e_2)=(a_1+a_2)e_2+a_3e_3+\ldots+a_{n-2}e_{n-2}+\left(a_{n-1}+a_2(\theta-\alpha_n)\right)e_{n-1}+b_ne_n \\ f(e_{i+1})=[f(e_i),f(e_1)] \quad 2\le i\le n\text{-}1 \\ f(e_3)=[f(e_1),f(e_1)]. \end{cases}$$

b) Mayli L algebrasi F^2 sinfiga tegishli bo'lsin. U xolda f quyidagi ko`rinishga ega bo'ladi:

$$\begin{cases} f(e_1)=a_1e_1+a_2e_2+a_3e_3+\ldots+a_ne_n \\ f(e_2)=b_2e_2-\dfrac{a_2b_2\gamma}{a_1}e_{n-1}+b_ne_n \\ f(e_{i+1})=[f(e_i),f(e_1)] \quad 3\le i\le n\text{-}1 \\ f(e_3)=[f(e_1),f(e_1)]. \end{cases}$$

Isboti. Mayli $f\in GL_{ad}(V)$.

$$f(e_1)=a_1e_1+a_2e_2+...+a_ne_n,\ f(e_2)=b_1e_1+b_2e_2+...+b_ne_n$$

deb olamiz.

a) xolni isbotlaymiz.

$f(e_3)=[f(e_1), f(e_1)]$ ko`paytmani qaraymiz.

(3.1) tenglikni qo'llab biz quyidagi tenglikning tug'riliga ega bo'lamiz:

$$[f(e_1),f(e_1)]=[a_1e_1+a_2e_2+...+a_ne_n,\ a_1e_1+a_2e_2+...+a_ne_n]=a_1(a_1+a_2)e_3+a_1\sum_{i=4}^{n}a_{i-1}e_i+$$

$$+a_1\sum_{i=4}^{n}a_{i-1}e_i+$$

$$+a_2\left(a_1+a_2\right)\sum_{i=4}^{n-1}\alpha_ie_i+a_2\left(a_1\theta+a_2\alpha_n\right)e_n+a_2\sum_{i=5}^{n}a_{i-2}\sum_{k=i}^{n}\alpha_{k+4-i}e_k=a_1(a_1+a_2)e_3+$$

$$+a_1\sum_{i=4}^{n}a_{i-1}e_i+a_2\left(a_1+a_2\right)\sum_{i=4}^{n-1}\alpha_ie_i+a_2(a_1\theta+a_2\alpha_n)e_n+a_2\sum_{k=5}^{n}\ \sum_{i=5}^{k}a_{i-2}\alpha_{k+4-i}e_k=$$

$$=a_1(a_1+a_2)e_3+\left(a_1a_3+a_2\left(a_1+a_2\right)\alpha_4\right)e_4+$$

$$\sum_{t=5}^{n-1}\left(a_1a_{t-1}+a_2\left(a_1+a_2\right)\alpha_t+a_2\sum_{i=5}^{t}a_{i-2}\alpha_{t+4-i}\right)e_t+$$

$$+\left(a_1a_{n-1}+a_2(a_1\theta+a_2\alpha_n)+a_2\sum_{i=5}^{n}a_{i-2}\alpha_{n+4-i}\right)e_n.$$

$[f(e_1),f(e_1)]\in L^2$ bo'lsa, u xolda $a_1(a_1+a_2)\neq 0$ bo'ladi, yani $a_1\neq 0$ va $(a_1+a_2)\neq 0$.

Quyidagi tenglikni qaraymiz:

$$[f(e_1),f(e_2)]=b_1\left(a_1+a_2\right)e_3+\sum_{i=4}^{n}c_ie_i.$$

$[f(e_1),f(e_2)]\notin L^2$ va $a_1+a_2\neq 0$ bolganligidan $b_1=0$ bo'ladi.

Adaptirlangan almashtirishning xossalaridan $f(e_3)=[f(e_2),f(e_1)]$ ega bo'lamiz.

$[f(e_2),f(e_1)]$ ko`rinishning $\{e_1, e_2, \dots, e_n\}$ bazisida yozilishi quyidagicha bo'ladi:

$$[f(e_2),f(e_1)]=a_1b_2e_3+\left(a_1b_3+a_2b_2\alpha_4\right)e_4+\sum_{t=5}^{n-1}\left(a_1b_{t-1}+a_2b_2\alpha_t+\right.$$
$$\left.+a_2\sum_{i=5}^{t}b_{i-2}\alpha_{t+4-i}\right)e_t+\left(a_1b_{n-1}+a_2b_2\alpha_n+a_2\sum_{i=5}^{n}b_{i-2}\alpha_{n+4-i}\right)e_n.$$

Bazis elementlarning oldidagi koeffitsientlarni taqqoslab *f* almashtirishning koeffitsientlariga chegaralashlarni olamiz:

$$\begin{cases} a_1+a_2=b_2, \\ a_3=b_3, \\ a_1a_{t-1}+a_2\sum\limits_{i=5}^{t}a_{i-2}\alpha_{t+4-i}=a_1b_{t-1}+a_2\sum\limits_{i=5}^{t}b_{i-2}\alpha_{t+4-i}, \\ a_1a_{n-1}+a_2\left(a_1\theta+a_2\alpha_n\right)+a_2\sum\limits_{i=5}^{n}a_{i-2}\alpha_{n+4-i}=a_1b_{n-1}+a_2b_2\alpha_n+a_2\sum\limits_{i=5}^{n}b_{i-2}\alpha_{n+4-i}. \end{cases}$$

Bundan

$$\begin{cases} b_2=a_1+a_2, \\ b_i=a_i \;\;,\; 3\le i\le n-2 \\ b_{n-1}=a_{n-1}+a_2\left(\theta-\alpha_n\right). \end{cases}$$

b) xol a) xolga uxshash isbotlanadi.

Tasdiq isbotlandi

Natija 3.1. Dim $GL_{ad}(V)$=n+2.

Ta'rif 3.3. Bazis almashtirishining quyidagi ko'rinishin elementar deb ataymiz:

birinchi tur - $\tau(a,b,k)=\begin{cases} f(e_1)=e_1+ae_k, \\ f(e_2)=e_2+be_k, \\ f(e_{i+1})=[f(e_i),f(e_1)], \\ f(e_3)=[f(e_1),f(e_1)], \end{cases}$ $2\le i\le n\text{-}1,\ 3\le k\le n,$

ikkinchi tur - $\vartheta(a,b)=\begin{cases} f(e_1)=ae_1+be_2, \\ f(e_2)=(a+b)e_2+b(\theta-\alpha_n)e_{n-1}, \\ f(e_{i+1})=[f(e_i),f(e_1)], \\ f(e_3)=[f(e_1),f(e_1)], \end{cases}$ $a(a+b)\neq 0,\ 2\leq i\leq n-1,$

uchinchi tur - $\sigma(b,n)=\begin{cases} f(e_1)=e_1, \\ f(e_2)=e_2+be_n, \\ f(e_{i+1})=[f(e_i),f(e_1)], \\ f(e_3)=[f(e_1),f(e_1)], \end{cases}$ $3\leq i\leq n-1,$

to`rtinchi tur - $\eta(a,k)=\begin{cases} f(e_1)=e_1+ae_k, \\ f(e_2)=e_2, \\ f(e_{i+1})=[f(e_i),f(e_1)], \\ f(e_3)=[f(e_1),f(e_1)], \end{cases}$ $3\leq i\leq n-1,\ 3\leq k\leq n,$

beshinchi tur - $\delta(a,b,d)=\begin{cases} f(e_1)=ae_1+be_2, \\ f(e_2)=de_2-\dfrac{bd\gamma}{a}e_{n-1}, \\ f(e_{i+1})=[f(e_i),f(e_1)], \\ f(e_3)=[f(e_1),f(e_1)], \end{cases}$ $ad\neq 0,\ 3\leq i\leq n-1,$

bunda a, b, d$\in$**C**.

Mayli *f* element $GL_{ad}(V)$ gruppaning ixtiyoriy elementi bo'lsin, u xolda f elementar almashtirishlar orqali anglatiladi, ya'niy quyidagi o'rinli.

Tasdiq 3.3.

i) Mayli f tasdiq 3.1 dagi a) ko`rinishda bo'lsin. U xolda

$$f=\tau(a_n,b_n,n)\circ\tau(a_{n-1},a_{n-1},n-1)\circ\ldots\circ\tau(a_3,a_3,3)\circ\vartheta(a_1,a_2).$$

ii) Mayli f tasdiq 3.1 dagi b) ko`rinishida bo'lsin. U xolda

$$f = y(b,n)\circ\eta(a_n,n)\circ\eta(a_{n-1},n-2)\circ\ldots\circ\eta(a_3,3)\circ\delta(a_1,a_2,b_2).$$

Isboti: Isbotlash tengliklarni tekshirishdan kelib chiqadi.

Tasdiq isbotlandi.

Quyidagi teorema Li bo’lmagan filiformli kompleksli Leybnits algebralarining izomorflik aniqlikdagi tavsiflash masalasini maxsus turdagi almashtirishlarni urganishga keltiriladi.

Teorema 3.1.[7] *Filiformli algebralarni izomorflik aniqlikda tavsiflash uchun:*

a) F_n^1 - *sinfdagi algebralarda quyidagi ko`rinishdagi bazis almashtirishlarni qarash etarli:*

$$\vartheta(a,b)=\begin{cases} f(e_1)=ae_1+be_2, \\ f(e_2)=(a+b)e_2+b(\theta-\alpha_n)e_{n-1}, \\ f(e_{i+1})=[f(e_i),f(e_1)], \\ f(e_3)=[f(e_1),f(e_1)], \end{cases} \quad 2\le i\le n\text{-}1,$$

bunda a, b$\in$C *va* a(a+b)$\neq$0,

b) F_n^2 - *sinfdagi algebralarda quyidagi ko`rinishdagi bazis almashtirishlarni qarash etarli:*

$$\delta(a,b,d)=\begin{cases} f(e_1)=ae_1+be_2, \\ f(e_2)=de_2-\dfrac{bd\gamma}{a}e_{n-1}, \\ f(e_{i+1})=[f(e_i),f(e_1)], \\ f(e_3)=[f(e_1),f(e_1)], \end{cases} \quad 3\le i\le n\text{-}1,$$

bunda a, b, d$\in$C *va* ad$\neq$0.

Isboti: a) tasdiqni isbotlash uchun

$g=\tau(a_n,b_n,n)\circ\tau(a_{n-1},a_{n-1},n-1)\circ\tau(a_{n-2},a_{n-2},n-2)\circ...\circ\tau(a_3,a_3,3)$ teorema 2.3 dagi birinchi sinfdagi strukturali o`zgarmaslarni o`zgarmasligin ko`rsatish etarli.

$\tau(a,b,k)$ almashtirishini qaraymiz:

$$\tau(a,b,k)=\begin{cases} f(e_1)=e_1+ae_k, \\ f(e_2)=e_2+be_k, \\ f(e_{i+1})=[f(e_i),f(e_1)], \\ f(e_3)=[f(e_1),f(e_1)]. \end{cases}$$

$3 \le k \le n$ da a=b deb olamiz, u holda $\tau(a,b,k)$ quyidagi ko`rinishge ega bo`ladi:

$$\tau(a,a,k)=\begin{cases} f(e_1)=e_1+ae_k, \\ f(e_2)=e_2+ae_k, \\ f(e_{i+1})=[f(e_i),f(e_1)], \\ f(e_3)=[f(e_1),f(e_1)]. \end{cases}$$

Berilgan almashtirish adaptirlangan ekanligi bizga ma`lum. Haqiqattan həm

$f(e_3)=[f(e_1),f(e_1)]=[e_1+ae_k,e_1+ae_k]=e_3+ae_{k+1}$,

$f(e_3)=[\ f(e_2),\ f(e_1)]=[e_2+ae_k,e_1+ae_k]=e_3+ae_{k+1}$,

$f(e_i)=[f(e_{i-1}),f(e_1)]=[e_{i-1}+ae_{k+i-2},e_1+ae_k]=e_i+ae_{k+i-1}$, $k+i-1 \le n$,

ko`paytmalarni qarashda $k + i - 1 > n$ da *a* koeffitsient nolga teng. Bundan $\tau(a,a,k)\in GL_{ad}(V)$ kelib chiqadi.

Parametrlar qatnashgan ko`paytmalarni qaraymiz:

$$[f(e_1),f(e_2)]=[e_1+ae_k,e_2+ae_k]=\sum_{i=4}^{n-1}\alpha_i e_i+\theta e_n+a\sum_{i=4}^{n-k+1}\alpha_i e_{k+i-1}=$$

$$=\sum_{i=4}^{n-k+1}\alpha_i\left(e_i+ae_{k+i-1}\right)+\sum_{i=n-k+2}^{n-1}\alpha_i e_i+\theta e_n=\sum_{i=4}^{n-1}\alpha_i f\left(e_i\right)+\theta f\left(e_n\right),$$

$$[f(e_2),f(e_2)]=[e_2+ae_k,e_2+ae_k]=\sum_{i=4}^{n}\alpha_i e_i+a\sum_{i=4}^{n-k+1}\alpha_i e_{k+i-1}=$$

$$=\sum_{i=4}^{n-k+1}\alpha_i\left(e_i+ae_{k+i-1}\right)+\sum_{i=n-k+2}^{n}\alpha_i e_i=\sum_{i=4}^{n}\alpha_i f\left(e_i\right).$$

Shunday qilib, birinshi tu`rdagi almashtirish $3 \le k \le n$ da ixtiyoriy $a\in C$ uchun $\alpha_i$, $\theta$ parametrlarini o`zgartirmaydi.

Uqshash turda $\tau\left(a,b,n\right)\in GL_{ad}(V)$ va bu almashtirish ixtiyoriy a, b $\in$ C uchun α_i, θ parametrlarini o`zgartirmaydi.

Adaptirlangan almashtirishlarning superpozitsiyasi yana adaptirlangan bo'lishidan

$$g=\tau\left(a_n,b_n,n\right)\circ\tau\left(a_{n-1},a_{n-1},n-1\right)\circ...\circ\tau\left(a_3,a_3,3\right)$$

almashtirishining teorema 2.3 dagi birinchi sinfning strukturali o`zgarmaslarini o`zgartmasligi kelib chiqadi.

Teoremaning ikkinchi tasdiqining isboti birinchi xolga uqshash $\phi=\sigma(b,n)\circ\eta(a_n,n)\circ\eta(a_{n-1},n-1)\circ...\circ\eta(a_3,3)$ almashtirishining teorema 2.3 dagi ikkinchi sinfining strukturali o`zgarmaslarini o`zgarmasligiga keltiriledi.

Teorema isbotlandi.

§ 4. Besh o`lchamli filiformli kompleksli Leybnits algebralari

Bu paragrafda besh o`lchamli filiformli kompleksli Leybnits algebralari urganiladi.

Ikkinchi paragrafdagi teorema 2.3 bo`yicha $\{e_1, e_2, e_3, e_4, e_5\}$ bazisli besh o`lchamli filiformli kompleksli Leybnits algebralari birinchi, ikkinchi va uchinchi sinflarda mos ravishda qo`yidagi ko`rinishda bo`ladi:

F_5^1: $[e_1,e_1] = e_3$, $[e_i,e_1] = e_{i+1}$, $2 \le i \le 4$, $[e_1,e_2] = \alpha_4 e_4 + \theta e_5$,

$[e_2,e_2] = \alpha_4 e_4 + \alpha_5 e_5$, $[e_3,e_2] = \alpha_4 e_5$,

F_5^2: $[e_1,e_1] = e_3$, $[e_i,e_1] = e_{i+1}$, $3 \le i \le 4$, $[e_1,e_2] = \beta_4 e_4 + \beta_5 e_5$, $[e_2,e_2] = \gamma e_5$,

$[e_3,e_2] = \beta_4 e_5$,

F_5^3: $[e_i,e_1] = e_{i+1}$, $2 \le i \le 4$, $[e_1,e_i] = -e_{i+1}$, $3 \le i \le 4$, $[e_1,e_1] = \theta_1 e_5$,

$[e_1,e_2] = -e_3 + \theta_2 e_5$, $[e_2,e_2] = \theta_3 e_5$, $[e_2,e_3] = -[e_3,e_2] = \theta_4 e_5$

bunda $\alpha_i,\theta_i,\beta_i,\gamma \in \mathbb{Q}_p$, qolgan barcha ko`paytmalar nolga teng.

Qo`yidagi teorema besh o`lchamli filiformli kompleksli Leybnits algebralarining tavsifini beradi:

Teorema 4.1. *Ixtiyoriy besh o`lchamli filiformli kompleksli Leybnits algebralari qo`yidagi o`z-aro izomorf bo`lmagan algebralarning biriga izomorf bo`ladi*:

L_1: $[e_1,e_1] = e_3$, $[e_i,e_1] = e_{i+1}$, $2 \le i \le 4$;

L_2: $[e_1,e_1] = e_3$, $[e_i,e_1] = e_{i+1}$, $2 \le i \le 4$, $[e_1,e_2] = e_5$;

L_3: $[e_1,e_1] = e_3$, $[e_i,e_1] = e_{i+1}$, $2 \le i \le 4$, $[e_2,e_2] = e_5$;

L_4: $[e_1,e_1] = e_3$, $[e_i,e_1] = e_{i+1}$, $2 \le i \le 4$, $[e_1,e_2] = 2e_5$, $[e_2,e_2] = e_5$;

$L_5(\theta)$: $[e_1,e_1] = e_3$, $[e_i,e_1] = e_{i+1}$, $2 \le i \le 4$, $[e_1,e_2] = \theta e_5$, $[e_2,e_2] = e_4$, $[e_3,e_2] = e_5$;

L_6: $[e_1,e_1] = e_3$, $[e_i,e_1] = e_{i+1}$, $2 \le i \le 4$, $[e_1,e_2] = e_4 - 2e_5$,

$[e_2,e_2] = e_4 - 2e_5$, $[e_3,e_2] = e_5$;

L_7: $[e_1,e_1] = e_3, [e_i,e_1] = e_{i+1},\ 2 \le i \le 4, [e_1,e_2] = e_4,$

$[e_2,e_2] = e_4 - 2e_5, [e_3,e_2] = e_5;$

G_1: $[e_1,e_1] = e_3, [e_i,e_1] = e_{i+1}, 3 \le i \le 4;$

G_2: $[e_1,e_1] = e_3, [e_i,e_1] = e_{i+1}, 3 \le i \le 4, [e_1,e_2] = e_5;$

G_3: $[e_1,e_1] = e_3, [e_i,e_1] = e_{i+1}, 3 \le i \le 4, [e_2,e_2] = e_5;$

G_4: $[e_1,e_1] = e_3, [e_i,e_1] = e_{i+1}, 3 \le i \le 4, [e_1,e_2] = e_4, [e_2,e_2] = 2e_5;$

G_5: $[e_1,e_1] = e_3, [e_i,e_1] = e_{i+1}, 3 \le i \le 4, [e_1,e_2] = e_4 + e_5, [e_2,e_2] = 2e_5, [e_3,e_2] = e_5;$

$G_6(\gamma)$: $[e_1,e_1] = e_3, [e_i,e_1] = e_{i+1}, 3 \le i \le 4, [e_1,e_2] = e_4 + e_5, [e_2,e_2] = \gamma e_5, [e_3,e_2] = e_5;$

T_1: $[e_i,e_1] = e_{i+1}, 2 \le i \le 4, [e_1,e_i] = -e_{i+1},\ 3 \le i \le 4;$

T_2: $[e_i,e_1] = e_{i+1}, 2 \le i \le 4, [e_1,e_i] = -e_{i+1},\ 3 \le i \le 4, [e_2,e_3] = -[e_3,e_2] = e_5;$

T_3: $[e_i,e_1] = e_{i+1}, 2 \le i \le 4, [e_1,e_i] = -e_{i+1},\ 3 \le i \le 4, [e_1,e_1] = e_5;$

T_4: $[e_i,e_1] = e_{i+1}, 2 \le i \le 4, [e_1,e_i] = -e_{i+1},\ 3 \le i \le 4, [e_1,e_1] = e_5,$

$[e_1,e_2] = -e_3 + e_5;$

$T_5(\alpha)$: $[e_i,e_1] = e_{i+1}, 2 \le i \le 4, [e_1,e_i] = -e_{i+1},\ 3 \le i \le 4, [e_1,e_1] = e_5,$

$[e_1,e_2] = -e_3 + \alpha e_5, [e_2,e_3] = -[e_3,e_2] = e_5;$

T_6: $[e_i,e_1] = e_{i+1}, 2 \le i \le 4, [e_1,e_i] = -e_{i+1},\ 3 \le i \le 4, [e_1,e_1] = e_5,$

$[e_1,e_2] = -e_3 + 2e_5, [e_2,e_2] = e_5;$

$T_7(\varphi)$: $[e_i,e_1] = e_{i+1}, 2 \le i \le 4, [e_1,e_i] = -e_{i+1},\ 3 \le i \le 4, [e_1,e_1] = e_5,$

$[e_1,e_2] = -e_3 + \sqrt{4 + \nu^4\varphi}\, e_n, [e_2,e_2] = e_5;$

bu erda $\theta, \gamma \in \mathbb{Q}_p$.

Isboti. F_5^1 sinfininng ichida izomorflikga tekshiramiz. Uchinchi paragrafdagi teorema 3.1 dagi F_5^1 sinf uchun ϑ almashtirishini olib, qo`yidagi bazis almashtirishini bajaramiz

$$e_1' = Ae_1 + Be_2,\ e_2' = (A + B)e_2 + B(\theta - \alpha_5)e_4.$$

U xolda qo`yidagiga ega bo`lamiz:

$e_3' = [e_1', e_1'] = A(A + B)e_3 + B(A + B)\alpha_4 e_4 + (AB\theta + B^2\alpha_5)e_5,$

$e_3' = [e_2', e_1'] = A(A + B)e_3 + B(A + B)\alpha_4 e_4 + (AB\theta + B^2\alpha_5)e_5,$

$e_4' = [e_3', e_1'] = A^2(A + B)e_4 + 2AB(A + B)\alpha_4 e_5,$

$e_5' = [e_4', e_1'] = A^3(A + B)e_5$, gde $A(A + B) \neq 0$.

F_5^1 algebradagi ko`paytmalarni yangi bazisda yozib va $\{e_1, e_2, e_3, e_4, e_5\}$ bazis elementlari oldidagi koeffitsientlarni taqqoslab, qo`yidagi munosabatlarga ega bo`lamiz:

$$\alpha_4' = \frac{A + B}{A^2}\alpha_4,$$

$$\theta' = \frac{A(A\theta + B\alpha_5) - 2(A + B)B\alpha_4^2}{A^4},$$

$$\alpha_5' = \frac{(A\alpha_5 - 2B\alpha_4^2)(A + B)}{A^3}.$$

Mumkin bo`lgan barcha xollarini qaraymiz.

Xol 1. Mayli $\alpha_4 = 0$ bo`lsin. U holda munosabatlar

$$\alpha_4' = 0,$$

$$\alpha_5' = \frac{\alpha_5(A + B)}{A^2},$$

$$\theta' = \frac{A\theta + B\alpha_5}{A^3}$$

ko`rinishda bo`ladi.

Xol 1.1. Mayli $\alpha_5 = 0$ bo`lsin. U holda

$$\alpha'_5 = 0 \text{ va } \theta' = \frac{\theta}{A^2}$$

bo`ladi.

Agar $\theta = 0$ bo`lsa, o` holda $\theta' = 0$ va L_1 algebrasiga ega bo`lamiz. Agar $\theta \neq 0$ bo`lsa, o` holda $A = \sqrt{\theta}$ deb olib, $\theta' = 1$ va $L_2$ algebrasiga ega bo`lamiz.

Xol 1.2. Aytaylik $\alpha_5 \neq 0$ bo`lsin. U holda $B = \dfrac{A^2 - A\alpha_5}{\alpha_5}$ deb olib

$$\alpha'_5 = 1,$$

$$\theta' = \frac{\theta - \alpha_5}{A^2} + 1$$

munosabatiga ega bo`lamiz.

Agar $\theta = \alpha_5$ bo`lsa, o` holda $\theta' = 1$ va L_3 algebrasiga ega bo`lamiz. Agar $\theta \neq \alpha_5$ bo`lsa, o` holda $A = \sqrt{\theta - \alpha_5}$ deb olib, $\theta' = 2$ va $L_4$ algebrasini olamiz. **Xol 2.** Mayli $\alpha_4 \neq 0$ bo`lsin. O` holda $B = \dfrac{A^2 - A\alpha_4}{\alpha_4}$ deb olib qo`yidagi munosabatlarga ega bo`lamiz:

$$\alpha'_4 = 1,$$

$$\alpha'_5 = \frac{\alpha_5 - 2A\alpha_4 + 2\alpha_4^2}{\alpha_4},$$

$$\theta' = \frac{\alpha_4(\theta - \alpha_5) + A\alpha_5 - 2A^2\alpha_4 + 2A\alpha_4^2}{A^2\alpha_4}.$$

Qo`yidagi munosabat o`rinli:

$$\alpha_5' + 2\alpha_4'^2 = \frac{A+B}{A^3}(\alpha_5 + 2\alpha_4^2),$$

o` holda $\alpha_5 + 2\alpha_4^2$ ifoda uchun izomorf bo`lmagan algebralar hosil bo`ladigan qo`yidagi ikki xolni qaraymiz.

Xol 2.1. Mayli $\alpha_5 + 2\alpha_4^2 \neq 0$. O` holda $A = \frac{\alpha_5 + 2\alpha_4^2}{2\alpha_4}$ deb olib

$$\alpha_5' = 0 \text{ va } \theta' = \frac{4\alpha_4^2(\theta - \alpha_5)}{(\alpha_5 + 2\alpha_4^2)^2}$$

munosabatlariga ega bo`lamiz, ya`ni $L_5(\theta)$ parametrli algebrasiga ega bo`lamiz.

Xol 2.2. Mayli $\alpha_5 + 2\alpha_4^2 = 0$. O` holda

$$\alpha_5' = -2,$$

$$\theta' = \frac{\theta + 2\alpha_4^2 - 2A^2}{A^2}.$$

Agar $\theta + 2\alpha_4^2 = 0$ bo`lsa, o` holda $\theta' = -2$ va L_6 algebrasi kelib chiqadi. Agar $\theta + 2\alpha_4^2 \neq 0$ bo`lsa, o` holda $A = \sqrt{\frac{\theta + 2\alpha_4^2}{2}}$ deb olib, $\theta' = 0$ va L_7 algebrasi hosil qilamiz.

Endi F_5^2 ikkinchi sinfdagi Leybnits algebrasini izomorflikga tekshiramiz. Uchinchi paragrafdagi teorema 3.1 dagi F_5^2 sinf uchun δ almashtirishini olib, qo`yidagi bazis almashtirishini bajaramiz

$$e_1' = Ae_1 + Be_2,$$

$$e_2' = De_2 - \frac{BD\gamma}{A}e_4,$$

bunda $AD \neq 0$.

o` holda qo`yidagiga ega bo`lamiz:

$$e_3' = [e_1', e_1'] = A^2 e_3 + AB\beta_4 e_4 + (AB\beta_5 + B^2\gamma)e_5,$$

$$e_4' = [e_3', e_1'] = A^3 e_4 + 2A^2 B\beta_4 e_5,$$

$$e_5' = [e_4', e_1'] = A^4 e_5.$$

F_5^2 algebradagi ko`paytmalarni yangi bazisda yozib va $\{e_1, e_2, e_3, e_4, e_5\}$ bazis elementlari oldidagi koeffitsientlarni taqqoslab, qo`yidagi munosabatlarga ega bo`lamiz:

$$\beta_4' = \frac{D\beta_4}{A^2},$$

$$\gamma' = \frac{D^2\gamma}{A^4},$$

$$\beta_5' = \frac{D(A\beta_5 + B\gamma + 2\beta_4^2 B)}{A^4}.$$

Xol 1. Aytaylik $\beta_4 = 0$ bo`lsin. O` holda

$$\beta_4' = 0,$$

$$\beta_5' = \frac{D(A\beta_5 + B\gamma)}{A^4},$$

$$\gamma' = \frac{D^2\gamma}{A^4}.$$

Xol 1.1. Mayli $\gamma = 0$ bo`lsin. O` holda

$$\gamma' = 0,$$

$$\beta_5' = \frac{D\beta_5}{A^3}.$$

Agar $\beta_5 = 0$ bo`lsa, o` holda $\beta'_5 = 0$ va G_1 algebrasini olamiz.

Agar $\beta_5 \neq 0$ bo`lsa, o` holda $D = \dfrac{A^3}{\beta_5}$ deb olib, $\beta'_5 = 1$ va G_2 algebrasiga ega bo`lamiz.

Xol 1.2. Mayli $\gamma \neq 0$ bo`lsin. O` holda $A = \sqrt[4]{\gamma D^2}$ va B= $-\dfrac{A\beta_5}{\gamma}$ deb olib, $\gamma' = 1$, $\beta'_5 = 0$ va G_3 algebrasi hosil buladi.

Xol 2. Mayli $\beta_4 \neq 0$ bo`lsin. O` holda $D = \dfrac{A^2}{\beta_4}$ deb olib qo`yidagiga ega bo`lamiz:

$$\beta'_4 = 1,$$

$$\beta'_5 = \frac{A\beta_5 + B\gamma - 2\beta_4^2 B}{A^4\beta_4},$$

$$\gamma' = \frac{\gamma}{\beta_4^2}.$$

Bizda

$$\gamma' - 2\beta_4'^2 = \frac{D^2}{A^4}(\gamma - 2\beta_4^2)$$

tengligi o`rinli bo`lganligidan izomorf bo`lmaydigan algebralar kelib chiqadigan qo`yidagi xolllarni qaraymiz.

Xol 2.1. Mayli $\gamma - 2\beta_4^2 = 0$ bulsin. O` holda

$$\beta'_5 = \frac{\beta_5}{A\beta_4},$$

$$\gamma' = 2$$

bo`ladi.

Xol 2.1.1. Mayli $\beta_5 = 0$. O` holda $\beta'_5 = 0$ va G_4 algebrasini olamiz.

Xol 2.1.2. Mayli $\beta_5 \neq 0$ bo`lsin. O` holda $A = \dfrac{\beta_5}{\beta_4}$ deb olib, $\beta'_5 = 1$ va G_5 algebrasiga ega bo`lamiz.

Xol 2.2. Aytaylik $\gamma - 2\beta_4^2 \neq 0$ bo`lsin. O` holda $B = \dfrac{A^2\beta_4 - A\beta_5}{\gamma - 2\beta_4^2}$ deb olib, $\beta'_5 = 1$ va $G_6(\gamma)$ parametrik algebrasini olamiz.

Keyingi qadam F_5^3 sinfning ichida izomorflikga tekshiramiz:

$$[e_i,e_1] = e_{i+1},\ 2 \leq i \leq 4,\ [e_1,e_1] = \theta_1 e_5,\ [e_1,e_2] = -e_3 + \theta_2 e_5,\ [e_2,e_2] = \theta_3 e_5,$$
$$[e_2,e_3] = -[e_3,e_2] = \theta_4 e_5,\ [e_1,e_i] = -e_{i+1},\ 3 \leq i \leq 4.$$

Mayli $(\theta_1, \theta_2, \theta_3) = (0, 0, 0)$ bo`lsin. O` holda algebramiz Li algebrasi o`lib topiladi va qo`yidagi ko`rinishga ega bo`ladi:

$$[e_i,e_1] = -[e_1,e_i] = e_{i+1},\ 2 \leq i \leq 4,\ [e_2,e_3] = -[e_3,e_2] = \theta_4 e_5.$$

Agar $\theta_4 = 0$ bo`lsa, o` holda T_1 algebrasiga ega bo`lamiz:

$$[e_i,e_1] = -[e_1,e_i] = e_{i+1},\ 2 \leq i \leq 4.$$

Agar $\theta_4 \neq 0$ bo`lsa, o` holda

$$e'_1 = \theta_4 e_1,\ e'_2 = \theta_4^2 e_2,\ e'_3 = \theta_4^3 e_3,\ e'_4 = \theta_4^4 e_4,\ e'_5 = \theta_4^5 e_5$$

bazis almashtirishida $\theta_4 = 1$ bo`ladi va T_2 algebrasi kelib chiqadi:

$$[e_i,e_1] = -[e_1,e_i] = e_{i+1},\ 2 \leq i \leq 4,\ [e_2,e_3] = -[e_3,e_2] = e_5.$$

Mayli $(\theta_1, \theta_2, \theta_3) \neq (0, 0, 0)$ bo`lsin. Agar $\theta_1 = 0$ bo`lsa, o` holda $(\theta_2,\theta_3) \neq (0, 0)$, va $e'_1 = e_1 + Ae_2$ bazislik almashtirish bajarib, $[e'_1,e'_1] = A^2\theta_3 e_5 \neq 0$ ega bo`lamiz, ya`ni ixtiyoriy $\theta_3$ uchun $A^2\theta_3 \neq 0$ bo`ladigan $A \neq 0$ mavjud bo`ladi. O` holda $\theta_1 \neq 0$ deb olishimiz mumkin va

$$e'_1 = \theta_1 e_1,\ e'_2 = \frac{1}{\theta_1} e_2,\ e'_3 = e_3,\ e'_4 = \theta_1 e_4,\ e'_5 = \theta_1^2 e_5$$

bazislik almashtirishida $\theta_1 = 1$ kelib chiqadi. Shunday qilib ko`paytma qo`yidagi ko`rinishga ega:

$$[e_i,e_1] = e_{i+1},\ 2 \le i \le 4,\ [e_1,e_1] = e_5,\ [e_1,e_2] = -e_3 + \alpha e_5,\ [e_2,e_2] = \beta e_5,$$

$$[e_2,e_3] = -[e_3,e_2] = \gamma e_5,\ [e_1,e_i] = -e_{i+1},\ 3 \le i \le 4.$$

Endi bu ksinfning ichida izomorflikni tekshiramiz:

$$e_1' = A_1e_1 + A_2e_2 + A_3e_3 + A_4e_4,\ e_2' = B_1e_1 + B_2e_2 + B_3e_3 + B_4e_4,$$

bunda $A_1B_2 - A_2B_1 \ne 0$.

O` holda

$$e_3' = [e_2', e_1'] = (A_1B_2 - A_2B_1)e_3 + (A_1B_3 - A_3B_1)e_4 +$$
$$+ (A_1B_1 + A_1B_4 + A_2B_1\alpha + A_2B_2\beta - A_2B_3\gamma + A_3B_2\gamma - A_4B_1)e_5,$$
$$e_4' = [e_3', e_1'] = A_1(A_1B_2 - A_2B_1)e_4 + (A_1^2B_3 - A_1A_3B_1 - A_1A_2B_2\gamma + A_2^2B_1\gamma)e_5,$$
$$e_5' = [e_4', e_1'] = A_1^2(A_1B_2 - A_2B_1)e_5,\ \text{gde } A_1(A_1B_2 - A_2B_1) \ne 0.$$

Qo`yidagi ko`paytmalarni qaraymiz

$$[e_1', e_1'] = (A_1^2 + A_1A_2\alpha + A_2^2\beta)e_5,$$
$$[e_1', e_2'] = (A_2B_1 - A_1B_2)e_3 + (A_3B_1 - A_1B_3)e_4 + (A_1B_1 - A_1B_4 + A_1B_2\alpha + A_2B_2\beta +$$
$$+ A_2B_3\gamma - A_3B_2\gamma - A_4B_1)e_5,$$
$$[e_2', e_2'] = (B_1^2 + B_1B_2\alpha + B_2^2\beta)e_5,$$
$$[e_2', e_3'] = B_1(A_2B_1 - A_1B_2)e_4 + (\gamma B_2(A_1B_2 - A_2B_1) + B_1(A_1B_3 - A_3B_1))e_5.$$

Ikkinchi tomondan

$$[e_1', e_1'] = e_5' = A_1^2(A_1B_2 - A_2B_1)e_5,$$
$$[e_1', e_2'] = -e_3' + \alpha' e_5' = (A_2B_1 - A_1B_2)e_3 + (A_3B_1 - A_1B_3)e_4 + (-A_1B_1 - A_1B_4 -$$
$$-A_2B_1\alpha - A_2B_2\beta + A_2B_3\gamma - A_3B_2\gamma + A_4B_1 + \alpha' A_1^2(A_1B_2 - A_2B_1))e_5,$$

$[e_2', e_2'] = \beta' e_5' = \beta' A_1^2 (A_1B_2 - A_2B_1)e_5,$

$[e_2', e_3'] = \gamma' e_5' = \gamma' A_1^2 (A_1B_2 - A_2B_1)e_5$

ega bo`lamiz.

Bazislik elementlar oldidagi koeffitsientlarni taqqoslab, qo`yidagi munosabatlarga ega bo`lamiz:

$$A_1^3 B_2 = A_1^2 + A_1A_2\alpha + A_2^2\beta,$$

$$\alpha' = \frac{\alpha A_1 + 2\beta A_2}{A_1^3},$$

$$\beta' = \frac{B_2\beta}{A_1^3},$$

$$\gamma' = \frac{\gamma}{A_1^2}.$$

Qo`yidagi tenglik o`rinli:

$$\alpha'^2 - 4\beta' = \frac{1}{A_1^4}(\alpha^2 - 4\beta).$$

Xol 1. Mayli $\beta=0$ bo`lsin. O` holda

$$\beta' = 0,$$

$$A_1^2 B_2 = A_1^2 + A_1A_2\alpha,$$

$$\alpha' = \frac{\alpha}{A_1^2},$$

$$\gamma' = \frac{\gamma}{A_1^2}.$$

Xol 1.1. Mayli $\gamma = 0$ bo`lsin. O` holda $\gamma' = 0$ va $\alpha' = \frac{\alpha}{A_1^2}$.

Agar $\alpha = 0$ bo`lsa, o` holda $\alpha' = 0$ bo`ladi va $T_3$ algebrasi kelib chiqadi. Agar $\alpha \neq 0$ bo`lsa, o` holda $A_1 = \sqrt{\alpha}$ deb olib, $\alpha' = 1$ ni va $T_4$ algebrasiga ega bo`lamiz.

Xol 1.2. Mayli $\gamma \neq 0$ bo`lsin. O` holda $A_1 = \sqrt{\gamma}$ deb olsak, $\gamma' = 1$ va $\alpha' = \frac{\alpha}{\gamma}$ bo`ladi. Demak biz $T_5(\alpha)$ algebrasiga ega bo`ldik, bunda $\alpha \neq 0$.

Xol 2. Aytaylik $\beta \neq 0$ bo`lsin. O` holda $B_2 = \frac{A_1^3}{\beta}$ deb olsak,

$$\beta' = 1,$$

$$A_1^6 = \beta(A_1^2 + A_1A_2\alpha + A_2^2\beta),$$

$$\alpha' = \frac{\alpha A_1 + 2\beta A_2}{A_1^3},$$

$$\gamma' = \frac{\gamma}{A_1^2}$$

munosabatlariga ega bo`lamiz.

Xol 2.1. Mayli $\gamma = 0$ bo`lsin. O` holda

$$\gamma' = 0,$$

$$A_1^6 = \beta(A_1^2 + A_1A_2\alpha + A_2^2\beta),$$

$$\alpha' = \frac{\alpha A_1 + 2\beta A_2}{A_1^3}.$$

Xol 2.1.1. Mayli $\alpha^2 - 4\beta = 0$ bo`lsin. O` holda $A_2 = \frac{1}{2\beta}(2A_1^3 - \alpha A_1)$ va $A_1 \neq 0$ deb olib, $\alpha' = 2$ ga va T_6 algebrasini olamiz.

Xol 2.1.2. Agar $\alpha^2 - 4\beta \neq 0$ bo\`lsa, o\` holda $A_1^4 = \dfrac{\alpha^2 - 4\beta}{4}$ deb olib, $\alpha' = 0$ va T_7 algebrasiga ega bo\`lamiz.

Teorema isbotlandi.

Xulosa

Leybnits algebrasining strukturaviy nazariyasini Sh.A. Ayupov va B.A. Omirovlar o'rgana boshladi. Hozirgi vaqtda bu strukturaviy nazariya yaxshi o'rganilgan bo'lib va tadqiqot ishlari yanada rivojlanayapdi. Leybnits algebrasi Li algebrasining umumlashmalaridan biri hisoblanadi. Algebralarni tavsiflash masalasi asosan chekli o'lchovli algebralarda o'rganiladi.

Ishning birinchi paragrafida Li va Leybnits algebrasiga oid dastlabki tushunchalar va belgilashlar keltirilgan.

Ikkinchi paragrafda filiformli Leybnits algebralari urganilgan.

Bitiruv malakaviy ishning uchinchi paragrafida filiformli kompleksli Leybnits algebralarining almashtirishlari qaraladi.

To`rtinchi paragrafida besh o`lchamli filiformli kompleksli Leybnits algebralari o`rganiladi.

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