O'ZBEKISTON RESPUBLIKASI
OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

MIRZO ULUG’BEK NOMIDAGI SAMARQAND DAVLAT ARXITEKTURA-QURILISH INSTITUTI

«MUHANDISLIK KOMMUNIKATSIYALARI QURILISHI» FAKULTETI

«Oliy matematika va fizika» kafedrasi

Oliy matematikadan mustaqil ish topshiriqlari va ularni bajarish bo’yicha

USLUBIY KO’RSATMA

1-QISM

Samarqand – 2015

O'ZBEKISTON RESPUBLIKASI
OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

MIRZO ULUG'BEK NOMIDAGI SAMARQAND DAVLAT ARXITEKTURA-QURILISH INSTITUTI

«MUHANDISLIK KOMMUNIKATSIYALARI QURILISHI» FAKULTETI

«OLIY MATEMATIKA VA FIZIKA » KAFEDRASI

Institutning Ilmiy-uslubiy
kengashida ko'rib chiqildi va chop
etishga ruxsat berildi
Ro'yxatga olindi: ________________
Bayonnoma № __________________
«_____»______________ 2015 y.

«TASDIQLAYMAN»
Institutning Ilmiy-uslubiy kengashi
raisi t.f.n., dotsent A.T.Quldashev
____
«____» ____________ 2015 y.

OLIY MATEMATIKADAN MUSTAQIL ISH TOPSHIRIQLARI VA ULARNI BAJARISH BO'YICHA

USLUBIY KO'RSATMA

1-QISM

Bakalavriatning

5111007 – «Kasb ta'limi: bino va inshootlar qurilishi»
5111028 – «Kasb ta'limi: muhandislik kommunikatsiyalari qurilishi va montaji»
5340200 – «Bino va inshootlar qurilishi»
5340300 – «Shahar qurilishi va xo`jaligi»
5340400 – «Muhandislik kommunikatsiyalari qurilishi va montaji»
5340500 – «Qurilish materiallari, buyumlari va konstruksiyalarini ishlab chiqarish»
5340900 – «Ko`chmas mulk ekspertizasi va uni boshqarish»
5610100 – «Xizmatlar sohasi (uy-joy, communal va maishiy xizmatlar)»

ta'lim yo'nalashlarida o'qiyotgan 1-bosqich talabalari uchun mo'ljallangan

Samarqand – 2015

Mazkur **uslubiy ko'rsatma** O'zbekiston Respublikasida amaldagi davlat ta'lim standartlari asosida bakalavriat «Oliy matematika» fani o'quv dasturiga mos tayyorlangan. U birinchi bosqichning birinchi semestrlari bo'yicha bajariladigan mustaqil ishlarga doir yetarlicha nazariy ma'lumotlarni, namunaviy yechimlar va mustaqil bajarish uchun nazariy savollarni hamda amaliy topshiriqlarni o'z ichiga olgan.

Uslubiy ko'rsatma bakalavriatning qurilish ta'lim yo'nolishlarida o'qiydigan birinchi bosqich talabalar uchun mo'ljallangan.

Rasmlar – 22 ta, bibliografiya – 10 ta.

Tuzuvchilar:

Qozoqboy Mamasoliyev - fizika-matematika fanlari nomzodli, SamDAQI dotsenti
Mamasoli Jabborov - fizika-matematika fanlari nomzodli, SamDAQI dotsenti
Bahodir Mardonov - katta o'qituvchi
Kamo Filiposyan - o'qituvchi

Muharrir: SamDAQI «Oliy matematika va fizika» kafedrasi professori, fizika-matematika fanlari doktori **Jahon Akilov**

Taqrizchilar: SamDAQI «Oliy matematika va fizika» kafedrasi dotsenti, fizika-matematika fanlari nomzodi **Yusuf Nishonov,**
SamDU «Matematik analiz» kafedrasi dotsenti, fizika-matematika fanlari nomzodi **Ulug'bek Mamirov**

«Oliy matematika va fizika» kafedrasi majlisida («___» _______ 2015 yil. Bayonnoma №___) va «Muhandislik kommunikatsiyalari qurilishi» fakultetining ilmiy-uslubiy kengash yig'ilishida («___» _______ 2015 yil. Bayonnoma №___) ko'rib chiqilgan va ma'qullangan.

Chiqish belgilarii: © SamDAQI. Shakli A5. Buyurtma №___. Adadi__. Hajmi__.

MUNDARIJA

K I R I S H

Mazkur «Mustaqil ishlar topshiriqlari va ularni bajarish bo'yica uslubiy ko'rsatmalar(1-qism.)» birinchi bosqichning birinchi semestrida mo'ljallangan b'lib, u "Oliy matematika" fanining «Chiziqli algebra elementlari», «Analitik geometriya elementlari», «Matematik analizga kirish», «Bir o'zgaruvchili funksiyaning differensial hisobi», «Differensial hisobning funksiyani tekshirish va grafigini yasashga tadbiqi» bo'limlari bo'yicha bajariladigan mustaqil ishlar topshiriqlariga doir nazatiy ma'lumotlar, namunaviy yechimlar va topshiriqlar - misol va masalalarni o'z ichiga oladi.

Uslubiy ko'rsatmada barcha topshiriqlarga doir qisqacha nazariy ma'lumotlar berilgan, keyin namunaviy misol-masalalar yetarlicha izohlar bilan bajarib ko'rsatilgan. Ilova qismida **Maple** kompyuter matematik sistemasi(**KMS**) haqida qisqacha ma'lumot berilib, **Maple KMS**da namunaviy misol-masalalar yechib ko'rsatilgan. Talablarga mustaqil ishlarni **Maple KMS**da ham bajarib ko'rish tavsiya qilinadi.

Har bir topshiriqqa doir 30 ta variantdan iborat misol yoki masalalar berilgan. Topshiriqlardagi misol va masalalar tartib raqamlari orqali berilgan. Bunda, agar talabaning guruh jurnalidagi tartib raqami 15 bo'lsa, talaba har bir topshiriqning 15-misol yoki masalalarni bajaradi.

Foydalanilgan shartli belgilar:

Δ – misol yoki masalani yechishning boshlanishi;
▲ – misol yoki masalani yechishning tugashi.
J: - misol yoki masalaning javobi.

1-TOPSHIRIQGA DOIR KO'RSATMALAR

Tenglamalar sistemasini Kramer va matritsa usullari bilan yechish

Qisqacha nazariy ma'lumotlar

Ikkinchi va uchinchi tartibli determinatlar

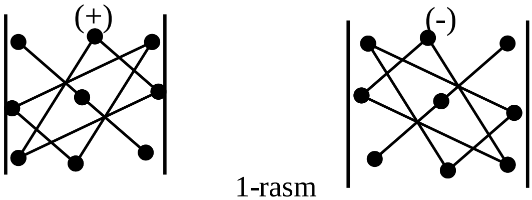
Ikkinchi tartibli determinant deb, $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ simvol bilan belgilanuvchi $a_{11}a_{22} - a_{12}a_{21}$ songa aytiadi, ya'ni

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}. \qquad (1)$$

Uchinchi tartibli determinant deb, $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ simvol bilan belgilanuvchi $a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$ songa aytiladi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}. \quad (2)$$

$a_{11}, a_{12}, ..., a_{33}$ sonlar determinantning elementlari deyiladi. Uchinchi tartibli determinant uchta satr va uchta ustunga ega. (2) ifodaga kiruvchi musbat va manfiy ishorali qo'shiluvchilarni quyidagi sxemada ko'rsatilgani kabi uchtadan elementlarni ko'paytirib hosil qilish mumkin (1-rasm).



1-rasm

Uchinchi tartibli determinantni ikkinchi tartibli determinant yrdamida quyidagicha hisoblash mumkin:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}. \quad (3)$$

Chiziqli tenglamalar sistemalarini Kramer usuli bilan yechish

Quyidagi

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases} \qquad (1)$$

uch noma'lumli uchta chiziqli tenglamalar sistemasi berilgan bo'lsin. Agar uning determinant (sistema determinanti) noldan farqli, ya'ni

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0 \qquad (4)$$

bo'lsa, bu sistema yagona yechimga ega va bu yechim

$$x = \frac{\Delta_x}{\Delta}, \quad y = \frac{\Delta_y}{\Delta}, \quad z = \frac{\Delta_z}{\Delta} \qquad (5)$$

formulalar bilan topiladi, bu yerda

$$\Delta_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \ \Delta_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \ \Delta_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} \qquad (6)$$

Bu usul chiziqli tenglamalar sistemasini yechishning Kramer usuli deyiladi.

Misollarni yechishga doir namunalar

1-*misol.* Tenglamalar sistemasini yeching.

$$\begin{cases} x + 2y - z = 2 \\ 2x - 3y + 2z = 2 \\ 3x + y + z = 8 \end{cases}$$

Δ Sistema determinantini topamiz:

$$\Delta=\begin{vmatrix}1 & 2 & -1\\ 2 & -3 & 2\\ 3 & 1 & 1\end{vmatrix}=1\cdot(-3)\cdot 1+2\cdot 2\cdot 3+2\cdot(-1)\cdot 1-(-1)\cdot(-3)\cdot 3-1\cdot 1\cdot 2-2\cdot 2\cdot 1=-8,$$

$\Delta\neq 0$ bo'lganligi uchun berilgan sistema yagona yechimga ega. Uni (5), (6) formulalar bilan topamiz:

$$\Delta_x=\begin{vmatrix}2 & 2 & -1\\ 2 & -3 & 2\\ 8 & 1 & 1\end{vmatrix}=-8,\ \Delta_y=\begin{vmatrix}1 & 2 & -1\\ 2 & 2 & 2\\ 3 & 8 & 1\end{vmatrix}=-16,\ \Delta_z=\begin{vmatrix}1 & 2 & 2\\ 2 & -3 & 2\\ 3 & 1 & 8\end{vmatrix}=-24$$

$$x=\frac{\Delta_x}{\Delta}=\frac{-8}{8}=1,\quad y=\frac{\Delta_y}{\Delta}=\frac{-16}{-8}=2,\qquad z=\frac{\Delta_z}{\Delta}=\frac{-24}{-8}=3.$$

J: (1, 2, 3) ▲

Chiziqli tenglamalar sistemasini matritsa usulida yechish

Determinanti noldan farqli kvadrat matritsa xosmas(maxsusms) matritsa deyiladi. Xosmas matritsa uchun teskari matritsa tushunchasi o'rinli. Agar A kvadrat matritsa xosmas matritsa, ya'ni $|A|\neq 0$ bo'lsa, u holda $A\cdot A^{-1}=A^{-1}\cdot A=E$ tenglikni qanoatlantiruvchi yagona A^{-1} matritsa mavjud va u A matritsaga teskari matritsa deyiladi, bu yerda E - tartibi A matritsa bilan bir xil birlik matritsa.

A matritsaga teskari bo'lgan A^{-1} matritsani quyidagi formula bilan topish mumkin:

$$A^{-1}=\frac{1}{\det A}\begin{pmatrix}A_{11} & A_{21} & \cdots & A_{n1}\\ A_{12} & A_{22} & \cdots & A_{n2}\\ \vdots & \vdots & \vdots & \vdots\\ A_{1n} & A_{2n} & \cdots & A_{nn}\end{pmatrix}.$$

Bu yerda A_{ik} lar $|A|=\det A$ determinant a_{ik} elementining algebraik to'ldiruvchilar, $i, j = 1, \dots, n$.

n noma'lumli n ta chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases}a_{11}x_1+a_{12}x_2+....+a_{1n}x_n=b_1,\\ a_{21}x_1+a_{22}x_2+...+a_{2n}x_n=b_2,\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ a_{n1}x_1+a_{n2}x_2+...+a_{nn}x_n=b_n\end{cases}\qquad (1)$$

Bu sistema koeffisiyentlaridan tuzilgan matritsa – sistema matritsasini tuzamiz:

$$A=\begin{pmatrix} a_{11} & a_{12} \dots a_{1n} \\ a_{21} & a_{22} \dots \ a_{2n} \\ \dots\dots\dots\dots\dots\dots \\ a_{n1} & a_{2n} \dots \ a_{nn} \end{pmatrix}$$

A maxsusmas matritsa bo’lgan holnigina qaraymiz. (1) sistemaning chap tomoni sistema matritsasi A ni $X=\begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{pmatrix}$ matritsaga ko’paytirishdan kelib chiqadigan n ta satrli, bir ustunli matritsaning elementlari, sistemaning o’ng tomoni esa $B=\begin{pmatrix} b_1 \\ b_2 \\ . \\ . \\ b_n \end{pmatrix}$ matritsaning elementlaridan iborat, deb qarash mumkin. Shu sababli ikki matritsaning tenglik ta’rifiga asosan, (1) ni

$$\begin{pmatrix} a_{11} & a_{12} \dots a_{1n} \\ a_{21} & a_{22} \dots a_{2n} \\ \dots\dots\dots\dots\dots\dots \\ a_{n1} & a_{n2} \dots a_{nn} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}=\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

yoki

$$A\cdot X=B \qquad (2)$$

ko’rinishda yozish mumkin. (2) - *marisaviy tenglama* yoki (1) sistemaning matritsaviy yozuvi (ko’rinishi) deyiladi.

A maxsusmas matritsa bo’lgani sababli, unga teskari bulgan A^{-1} matritsa mavjud, shu sababli (2) ni chap tomondan A^{-1} ga ko’paytiramiz:

$$A^{-1}\cdot(A\cdot X)=A^{-1}\cdot B,$$

lekin

$$A^{-1}\cdot(A\cdot X)=(A^{-1}\cdot A)\times X=EX=X\ ,$$

demak,

$$X=A^{-1}\cdot B\,. \qquad (3)$$

Bu - (2) matritsaviy tenglamaning yechimi, ya’ni berilgan sistemaning yechimi.

Chiziqli tenglamalar sistemasini yechishning bu usuli – *matritsaviy usul* deyiladi.

2-*misol*.
$$A=\begin{pmatrix}1 & 2 & 0\\ 3 & 2 & 1\\ 0 & 1 & 2\end{pmatrix}$$
matritsaga teskari matritsani toping.

Δ Bu matritsaning determinanti:
$$|A|=\begin{vmatrix}1 & 2 & 0\\ 3 & 2 & 1\\ 0 & 1 & 2\end{vmatrix}=1\cdot2\cdot2+0\cdot2\cdot1+3\cdot0\cdot1-0\cdot2\cdot0-2\cdot2\cdot3-1\cdot1\cdot1=-9$$

$|A|\neq 0$ bo’lgani uchun A matritsa maxsusmas matritsadir va demak, unga teskari matritsa mavjud.

Algebraik to’ldiruvchilarini hisoblaymiz:
$$A_{11}=(-1)^{1+1}\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix}=4-1=3;\quad A_{12}=(-1)^{1+2}\begin{vmatrix}3 & 1\\ 0 & 2\end{vmatrix}=-6;\quad A_{13}=(-1)^{1+3}\begin{vmatrix}3 & 2\\ 0 & 1\end{vmatrix}=3;$$
$$A_{21}=(-1)^{2+1}\begin{vmatrix}2 & 0\\ 1 & 2\end{vmatrix}=-4,\quad A_{22}=(-1)^{2+2}\begin{vmatrix}1 & 0\\ 0 & 2\end{vmatrix}=2,\quad A_{23}=(-1)^{2+3}\begin{vmatrix}1 & 2\\ 0 & 1\end{vmatrix}=-1,$$
$$A_{31}=(-1)^{3+1}\begin{vmatrix}2 & 0\\ 2 & 1\end{vmatrix}=2,\quad A_{32}=(-1)^{3+2}\begin{vmatrix}1 & 0\\ 3 & 1\end{vmatrix}=-1,\quad A_{33}=(-1)^{3+3}\begin{vmatrix}1 & 2\\ 3 & 2\end{vmatrix}=-4.$$

Unda
$$A^{-1}=\frac{1}{|A|}\begin{pmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{pmatrix}=\frac{1}{-9}\begin{pmatrix}3 & -4 & 2\\ -6 & 2 & -1\\ 3 & -1 & -4\end{pmatrix}=\begin{pmatrix}-\frac{1}{3} & \frac{4}{9} & -\frac{2}{9}\\ \frac{2}{3} & -\frac{2}{9} & \frac{1}{9}\\ -\frac{1}{3} & \frac{1}{9} & \frac{4}{9}\end{pmatrix}\ \blacktriangle.$$

3-*misol*. Tenglamalar sistemasini matritsaviy usulda yeching.
$$\begin{cases}2x_1+x_2-x_3=5,\\ 5x_1+2x_2+4x_3=1,\\ 7x_1+3x_2+2x_3=4.\end{cases}$$

Δ Berilgan sistemaning matritsasini, noma’lumlar matritsasini va ozod hadlar matritsasini yozamiz:

$$A=\begin{pmatrix}2&1&-1\\5&2&4\\7&3&2\end{pmatrix};\qquad X=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix};\ B=\begin{pmatrix}5\\1\\4\end{pmatrix}$$

U holda berilgan sistemaning matritsaviy ko'rinishi

$$A\cdot X=B \qquad (4)$$

bo'ladi. A ga teskari A^{-1} matritsa matritsani topamiz (1-misoldagi kabi):

$$A^{-1}=\begin{pmatrix}-8&-5&6\\18&11&-13\\1&1&-1\end{pmatrix}$$

(4) ni chap tomondan A^{-1}ga ko'paytiramiz:

$$A^{-1}\cdot A\cdot X=A^{-1}\cdot B;$$

$$X=A^{-1}\cdot B.$$

$$X=A^{-1}\cdot B=\begin{pmatrix}-8&-5&6\\18&11&-13\\1&1&-1\end{pmatrix}\cdot\begin{pmatrix}5\\1\\4\end{pmatrix}=\begin{pmatrix}(-8)\cdot5+(-5)\cdot1+6\cdot4\\18\cdot5+11\cdot1+(-13)\cdot4\\1\cdot5+1\cdot1+(-1)\cdot4\end{pmatrix}=\begin{pmatrix}-21\\49\\2\end{pmatrix};$$

$$X=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}-21\\49\\2\end{pmatrix}.$$

Demak, tenglamalar sistemasining yechimi: $x_1=-21;\ \ x_2=49;\ \ x_3=2.$ ▲

2-TOPSHIRIQGA DOIR KO'RSATMALAR

Vektorlar algebrasi usullaridan foydalanib misollar yechish

Qisqacha nazariy ma'lumotlar

I. *Fazoda* $A(x_1,\, y_1,\, z_1)$ *va* $B(x_2,\, y_2,\, z_2)$ *nuqtalar berilgan bo'lsin:*

1. Boshi $A(x_1,\, y_1,\, z_1)$ oxiri $B(x_2,\, y_2,\, z_2)$ nuqtada bo'lgan $\overrightarrow{AB}$ vektor koordinatalari yordamida quyidagicha yozadi:

$$\overrightarrow{AB}=(x_2-x_1)\vec{i}+(y_2-y_1)\vec{j}+(z_2-z_1)\vec{k}, \qquad (1)$$

yoki

$$\overrightarrow{AB}=\vec{a}=\{x_2-x_1;\ y_2-y_1;\ z_2-z_1\} \qquad (1')$$

2. $\overrightarrow{AB}$ vektorning uzunligi (moduli, absolyut qiymati)

$$|\overrightarrow{AB}|=AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \qquad (2)$$

formula bilan topiladi.

3. Ikkita $A(x_1,\ y_1,\ z_1)$ va $B(x_2,\ y_2,\ z_2)$ nuqtalardan o'tuvchi AB to'g'ri chiziqning tenglamasi:

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}. \qquad (3)$$

II. $\overrightarrow{AB}$, $\overrightarrow{AC}$ *va* $\overrightarrow{AD}$ *vektorlar koordinatalari bilan berilgan bo'lsin*:

$$\overrightarrow{AB}=x_1\vec{i}+y_1\vec{j}+z_1\vec{k},\quad \overrightarrow{AC}=x_2\vec{i}+y_2\vec{j}+z_2\vec{k},\quad \overrightarrow{AD}=x_3\vec{i}+y_3\vec{j}+z_3\vec{k}$$

1. $\overrightarrow{AB}$ va $\overrightarrow{AC}$ vektorlarning skalyar ko'paytmasi

$$\overrightarrow{AB}\cdot\overrightarrow{AC}=x_1\cdot x_2+y_1\cdot y_2+z_1\cdot z_2 \qquad (4)$$

formula bilan topiladi;

2. $\overrightarrow{AB}$ va $\overrightarrow{AC}$ vektorlar orasidagi burchakni topish formulasi:

$$\cos\left(\overrightarrow{AB}\,\hat{,}\,\overrightarrow{AC}\right)=\frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}|\cdot|\overrightarrow{AC}|}; \qquad (5)$$

3. $\overrightarrow{AB}$ vektorning $\overrightarrow{AC}$ vektorga poeksiyasi

$$pr_{\overline{AC}}\overrightarrow{AB}=\frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AC}|} \qquad (6)$$

formula bilan topiladi;

4. $\overrightarrow{AB}$ va $\overrightarrow{AC}$ vektorlarning vektor ko'paytmasi

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\end{vmatrix} \qquad (7)$$

formula bilan topiladi;

5. Uchlari $A,\ B,\ C$ nuqtalarda bo'lgan uchburchakning yuzi vektor ko'paytma yotdamida

$$S_{\Delta ABC}=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| \qquad (8)$$

formula bilan topiladi;

6. $\overrightarrow{AB}$, $\overrightarrow{AC}$ va $\overrightarrow{AD}$ vektorlarga qurilgan $ABCD$ piramidaning hajmi

$$V=\pm\frac{1}{6}\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD} \quad \text{yoki} \quad V=\pm\frac{1}{6}\cdot\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \qquad (9)$$

formula bilan topiladi, bunda ishora vektorlar o'ng bog'lamni tashkil etsa musbat, chap bog'lamni tashkil etsa, manfiy olinadi.

7. $\overrightarrow{AB}$, $\overrightarrow{AC}$ va $\overrightarrow{AD}$ vektorlarga qurilgan $ABCD$ piramidaning ABC yog'iga tushirilgan balandlik

$$H=\frac{6V}{S_{\Delta ABC}} \qquad (10)$$

formula bilan topiladi.

Misollarni yechishga doir namunalar

1-*misol*: Piramida $A,\ B,\ C,\ D$ uchlarining koordinatalari berilgan:

$$A(3;1;-2),\ B(1;-2;0),\ C(-2;1;0),\ D(2;2;5).$$

Vektorlar algebrasi usullaridan foydalanib, quyidagilarni toping:

1) $\overrightarrow{AB}$ vektorning uzunligini va AB to'g'ri chiziq tenglamasini;

2) $\overrightarrow{AB}$ va $\overrightarrow{AC}$ qirralar orasidagi burchakni;

3) $\overrightarrow{AC}$ vektorning $\overrightarrow{AD}$ vektorga proeksiyasini;

4) $\overrightarrow{AB}$ va $\overrightarrow{AC}$ vektorlarning vektor ko'paytmasini;

5) ABC yoqning yuzini;

6) $ABCD$ piramidaning hajmini, ABC yog'iga tushirilgan balandlikni toping va piramidani yasang.

Δ 1) Boshi $A(3;1;-2)$ va oxiri $B(1;-2;1)$ nuqtada bo'lgan $\overrightarrow{AB}$ vektorni (1) formulaga ko'ra koordnatalari bo'yicha yozamiz:

$$\overrightarrow{AB}=(x_2-x_1)\vec{i}+(y_2-y_1)\vec{j}+(z_2-z_1)\vec{k}=2\vec{i}-3\vec{j}+3\vec{k}\ .$$

Unda (2) va (3) formulalardan foydalanib $\overrightarrow{AB}$ vektorning uzunligini va AB to'g'ri chiziq tenglamasini topish mumkin.

$$\left|\overrightarrow{AB}\right|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}=\sqrt{(-2)^2+(-3)^2+3^2}=\sqrt{4+9+9}=\sqrt{22}$$

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1};\quad \frac{x-3}{1-3}=\frac{y-1}{-2-1}=\frac{z-(-2)}{1-(-2)};\quad \frac{x-3}{-2}=\frac{y-1}{-3}=\frac{z+2}{3}\ .$$

2) AB va AC qirralar orasidagi burchakni $\overrightarrow{AB}$ va $\overrightarrow{AC}$ vektorlar orasidagi burchak sifatida qaraymiz, shuning uchun uni (5) formuladan foydalanib toppish mumkin.

$$\overrightarrow{AB}=-2\vec{i}-3\vec{j}+3\vec{k},\quad \left|\overrightarrow{AB}\right|=\sqrt{22}\ ,$$

$$\overrightarrow{AC}=(-2-3)\vec{i}+(1-1)\,\vec{\mathrm{j}}+(0+2)\vec{\mathrm{k}}=-5\vec{i}+2\vec{k};\overrightarrow{AC}\{-5;\,0;\,2\},\ \left|\overrightarrow{AC}\right|=\sqrt{25+4}=\sqrt{29}.$$

$$\cos\left(\overrightarrow{AB}\,\hat{,}\,\overrightarrow{AC}\right)=\frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|}=\frac{-2(-5)+0(-3)+3\cdot 2}{\sqrt{22}\sqrt{29}}=\frac{16}{\sqrt{638}}=\frac{8\sqrt{638}}{319},$$

bundan $\left(\overrightarrow{AB}\,\hat{,}\,\overrightarrow{AC}\right)=\arccos\dfrac{8\sqrt{638}}{319}.$

3) $\overrightarrow{AC}$ vektorning $\overrightarrow{AD}$ vektorga poeksiyasini (6) formula bilan topamiz:

$$pr_{\overrightarrow{AD}}\overrightarrow{AC}=\frac{\overrightarrow{AC}\cdot\overrightarrow{AD}}{\left|\overrightarrow{AD}\right|}\ .$$

$$\overrightarrow{AC}=-5\vec{i}+2\vec{k};\quad \overrightarrow{AD}=(2-3)\vec{i}+(2-1)\vec{j}+(5+2)\vec{k}=-\vec{i}+\vec{j}+7\vec{k}\ ;$$

$$\left|\overrightarrow{AD}\right|=\sqrt{(-1)^2+1^2+7^2}=\sqrt{51}\ ;\quad \overrightarrow{AC}\cdot\overrightarrow{AD}=(5\vec{i}+2\vec{k})(-\vec{i}+\vec{j}+7\vec{k})=$$

$=(-5)(-1)+0\cdot 1+2\cdot 7=19$ bo'lgani uchun:

$$pr_{\overrightarrow{AD}}\overrightarrow{AC}=\frac{19}{\sqrt{51}}=\frac{19\sqrt{51}}{51}.$$

4) $\overrightarrow{AB}=-2\vec{i}-3\vec{j}+3\vec{k}$ va $\overrightarrow{AC}=-5\vec{i}+2\vec{k}$ yoki $\overrightarrow{AB}=\{-2;-3;3\}$, $\overrightarrow{AC}=\{-5;0;2\}$ vektorlarning vektor ko'paytmasini (7) formulaga ko'ra topamiz:

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -2 & -3 & 3\\ -5 & 0 & 2\end{vmatrix}=-6\vec{i}-15\vec{j}-15\vec{k}+4\vec{j}=-6\vec{i}-11\vec{j}-15\vec{k}$$

5) ABC yoqning yuzini (8) formula bilan topamiz:

$$S_{\Delta ABC}=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\frac{1}{2}\left|-6i-11j-15k\right|=\frac{1}{2}\sqrt{(-6)^2+(-11)^2+(-15)^2}=$$

$$=\frac{1}{2}\sqrt{36+121+225}=\frac{1}{2}\sqrt{382}\text{ kv.b.}$$

6) $ABCD$ piramidaning hajmini topamiz.

$$\overrightarrow{AB}=\{-2;-3;3\},\ \overrightarrow{AC}=\{-5;0;2\},\ \overrightarrow{AD}=\{-1;1;7\}$$

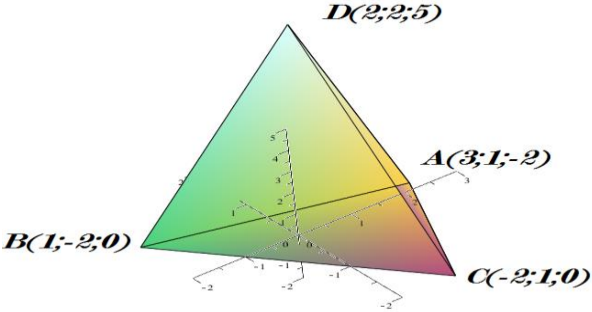
bo'lgani uchun (9) formulaga ko'ra

$$V=\pm\frac{1}{6}\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD}=\pm\frac{1}{6}\cdot\begin{vmatrix}-2 & -3 & 3\\ -5 & 0 & 2\\ -1 & 1 & 7\end{vmatrix}=\pm\frac{1}{6}\cdot(-110)=\frac{55}{3}=18\frac{1}{3}\text{ kub b.r.}$$

$ABCD$ piramida ABC yog'iga tushirilgan balandlikni (10) formula bilan topamiz:

$$H=\frac{6V}{S_{\Delta ABC}}=\frac{6\cdot 55/3}{\sqrt{382}/2}=\frac{220\sqrt{382}}{382}=\frac{110}{191}\sqrt{382}.$$

A, B, C, D uchlarining koordinatalari bo'ycha $ABCD$ piramidani yasaymiz (2-rasm). ▲



2-rasm

3-TOPSHIRIQGA DOIR KO'RSATMALAR

Tekislikda nuqta va to'g'ri chiqqa doir misollar yechish

Qisqacha nazariy ma'lumotlar

Oxy tekislikda $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalar orasidagi masofa quyidagi formula bilan topiladi:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\ . \qquad (1)$$

$A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1} \qquad (2)$$

Berilgan $M(x_0;y_0)$ nuqtadan $Ax+By+C=0$ to'g'ri chiziqqacha bo'lgan masofa

$$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}} \qquad (3)$$

formula bilan topiladi.

Uchlari $A(x_1;\ y_1)$, $B(x_2;\ y_2)$, $C(x_3;\ y_3)$ nuqtalarda bo'lgan uchburchakning yuzini hisoblash formulasi:

$$S=\pm\frac{1}{2}\left[\begin{vmatrix}x_1 & y_1\\ x_2 & y_2\end{vmatrix}+\begin{vmatrix}x_2 & y_2\\ x_3 & y_3\end{vmatrix}+\begin{vmatrix}x_3 & y_3\\ x_1 & y_1\end{vmatrix}\right] \qquad (4)$$

Uchlari $A(x_1;\ y_1)$, $B(x_2;\ y_2)$, $C(x_3;\ y_3), \ldots, D(x_n;\ y_n)$ nuqtalarda bo'lgan ko'pburchakning yuzini hisoblash formulasi:

$$S=\pm\frac{1}{2}\left[\begin{vmatrix}x_1 & y_1\\ x_2 & y_2\end{vmatrix}+\begin{vmatrix}x_2 & y_2\\ x_3 & y_3\end{vmatrix}+\ldots+\begin{vmatrix}x_n & y_n\\ x_1 & y_1\end{vmatrix}\right].$$

Berilgan $M(x_0;y_0)$ nuqtadan o'tib, berilgan $Ax+By+C=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziqning tenglamasi:

$$y-y_0=\frac{B}{A}(x-x_0) \qquad (5)$$

Misollarni yechishga doir namunalar

1-*misol*. ABC uchburchak uchlarining koordinatlari berilgan: $A(5;3)$, $B(2;-1)$, $C(-1;4)$. Quyidagilarni toping:

a) AC tomonning uzunligini va AC to'g'ri chiziqning tenglamasini;
b) B uchdan tushirilgan balandlikning uzunligini va tenglamasini;
c) ABC uchburchakning yuzini. Uchburchakni yasang

Δ a) Ikki nuqta orasidagi masofani topish formulasi (1) ga ko'ra AC tomonning uzunligini topamiz:

$$AC=\sqrt{(-1-5)^2+(4-3)^2}=\sqrt{36+1}=\sqrt{37}.$$

AC to'g'ri chiziqning tenglamasini ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini (2) dan foydalanib topamiz:

$$AC:\quad \frac{x-5}{-1-5}=\frac{y-3}{4-3};\quad x+6y-23=0.$$

b) Berilgan nuqtadan berilgan to'g'ri chiziqqacha bo'lgan masofani hisoblash formulasi (3) ga ko'ra, uchburchakning B uchidan AC tomoniga tushirilgan balandlikning uzunligi d_B ni topamiz: $d_B=\dfrac{|2+6\cdot(-1)-23|}{\sqrt{1^2+6^2}}=\dfrac{27}{\sqrt{37}}.$

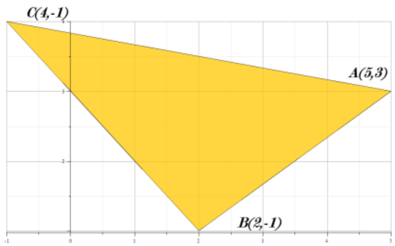
B uchidan tushirilgan balandlikning tenglamasi berilgan nuqtadan o'tib, berilgan to'g'ri chiqqa perpendikulyar bolgan to'g'ri chiziq tenglamasi (5) ga ko'ra topiladi: $y+1=\dfrac{6}{1}(x-2);\qquad 6x-y-13=0.$

c) $\triangle ABC$ uchburchakning yuzini, uchlarining koordinatalari berilgan bo'lganligi uchun, (4) formula bilan hisoblash mumkin:

$$S=\pm\frac{1}{2}\left[\begin{vmatrix}5 & 3\\ 2 & -1\end{vmatrix}+\begin{vmatrix}2 & -1\\ -1 & 4\end{vmatrix}+\begin{vmatrix}-1 & 4\\ 5 & 3\end{vmatrix}\right]=$$

$$=\pm\frac{1}{2}\Big[\big(5\cdot(-1)-2\cdot 3\big)+\big(2\cdot 4-(-1)\cdot(-1)\big)+\big((-1)\cdot 3-5\cdot 4\big)\Big]=13\frac{1}{2}\ \text{kv.b.}$$

A, B, C uchlarining koordinatalari bo'ycha ABC uchburchakni yasaymiz (3-rasm). ▲



3-rasm

4-TOPSHIRIQGA DOIR KO'RSATMALAR

Ikkinchi tartibli chiziq tenglamalariga doir misollar yechish

Qisqacha nazariy ma'lumotlar

Ikkinchi tartibli chiziq deb tenglamasi x va y o'zgaruvchilarga nisbatan ikkinchi darajali algebraik tenglama bo'lgan chiziqqa aytiladi. Ikkinchi tartibli chiziq tenglamasining umumiy ko'rinishi:

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0, \quad (1)$$

bu yerda A, B, C lardan kamida biri noldan farqli. (1) tenglama xususiy holda aylana, ellips, giperbola, parabola tenglamalari bolishi mumkin.

1. Aylana

Ta'rif. Aylana deb berilgan nuqtadan bir xil masofada yotuvchi nuqtalar geometrik o'rnidan iborat chiziqqa aytiladi. Berigan nuqta – markaz, ta'rifda aytilgan masofa – radius deyiladi.

Agar aylana markazi C(a, b) nuqtada, radiusi R ga teng desak, aylananing kanonik (sodda) tenglamasi quyidagicha yoziladi:

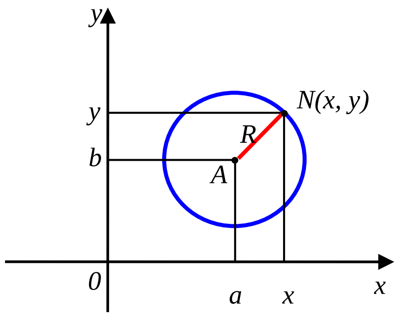
$$(x-a)^2 + (y-b)^2 = R^2 \quad (1)$$

(1) tenglamadagi qavslarni ochib, uni

$$x^2 + y^2 + mx + ny + p = 0 \quad (2)$$

ko'rinishga keltirish mumkin. Ko'rinadiki, aylana ikkinchi tartibli egri chiziq ekan. (2) – aylananing umumiy tenglamasi deyiladi. Aylana umumiy tenglama bilan berilganida (1) kanonik tenglamaga keltirish uchun (2) ning chap tomonidan to'la kvadratlar ajratish kerak:

$$\left(x + \frac{m}{2}\right)^2 + \left(y + \frac{n}{2}\right)^2 = \frac{m^2}{4} + \frac{n^2}{4} - p\,. \quad (3)$$



4-rasm

Misollarni yechishga doir namunalar

1-*misol*. $x^2+y^2-6x+4y-23=0$ aylananing markazi va radiusi topilsin.

Δ Berilgan tenglama aylananing umumiy tenglamasi bo'lganligi uchun uning chap tomonini ni to'la kvadratdan iborat ifodalarga ajratamiz:

$$(x^2-6x+9)+(y^2+4x+4)-13-23=0;$$
$$(x-3)^2+(y+2)^2-36=0;$$
$$(x-3)^2+(y+2)^2=6^2.$$

Bu tenglamani (1) tenglama bilan solishtib, aylananing markazi C(3; -2) nuqta, radiusi $R=6$ ekanini topamiz. ▲

2. Ellips

Ta'rif. Ellips deb istalgan nuqtasidan berilgan ikkita nuqtagacha (fokuslargacha) masofalarining yig'indisi o'zgarmas songa teng bo'lgan nuqtalar geometric o'rnidan iborat chiziqqa aytiladi.

Fokuslarni F_1 va F_2, ta'rifda aytilgan masofani 2a desak, ellipsning kanonik tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (1)$$

ko'rinishda bo'ladi, bu yerda a va b – *ellipsning katta va kichik yarim o'qlari* deyiladi. Ellips koordinata o'qlariga nisbatan simmetrikdir (5-rasm).

Agar $a>b$ bo'lsa, fokuslar Ox o'qida bo'lib, koordinatlar boshidan $c=\sqrt{a^2-b^2}$ masofada bo'ladi(c – fokus masofasi ham deyiladi), ya'ni $F_1(c;0)$ *va* $F_2(-c;0)$.

$$\varepsilon = \frac{c}{a} = \sqrt{1 - \frac{b^2}{a^2}}$$

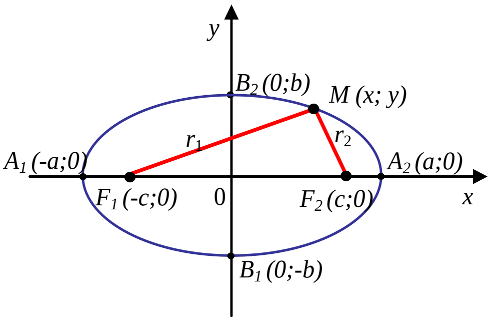
nisbat ellipsning *ekssentrisiteti* deyilib, u ellipsning shaklini xarakterlaydi ($0 \le \varepsilon < 1$). $\varepsilon = 0$ da ellips aylana shaklini oladi, ε oshishi bilan ellips katta o'qi bo'ylab cho'zilgan shaklni oladi.

Ellipsning $M(x, y)$ nuqtasidan fokuslargacha bo'lgan $r_1 = F_1M$, $r_2 = F_2M$ masofalar $M(x, y)$ nuqtaning *fokal radiuslari* deyiladi. Ular

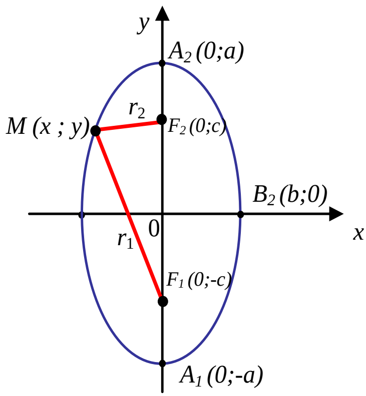
$$r_1 = |a - \varepsilon x|, \quad r_2 = |a + \varepsilon x| \tag{2}$$

formulalar bilan topiladi.

Agar $a < b$ bo'lsa, fokuslar Oy o'qda bo'lib, $c = \sqrt{b^2 - a^2}$, $\varepsilon = \frac{c}{b}$, $r_1 = |b - \varepsilon y|$, $r_2 = |b + \varepsilon y|$ bo'ladi (6-rasm).



5-rasm



6-rasm

3. Giperbola

Ta'rif. Giperbola deb istalgan nuqtasidan berilgan ikkita nuqtagacha (fokuslargacha) masofalari ayrmasining absolyut qiymati o'zgarmas songa teng bo'lgan nuqtalar geometric o'rniga aytiladi.

Fokuslarni F_1 va F_2, ta'rifda aytilgan masofani $2a$ desak ($0 < 2a < F_1F_2$), giperbolaning kanonik tenglama

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \tag{1}$$

ko'rinishda bo'ladi.

Giperbola koordinata o'qlariga nisbatan simmetrikdir (7-rasm).

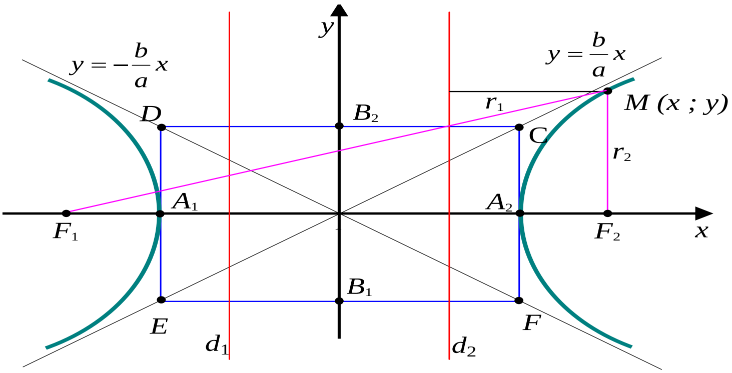
Giperbola Ox o'qini *uchlar* deb ataluvchi $A_1(a;0)$, $A_2(-a;0)$ nuqtalarda kesib o'tadi, Oy o'qi bilan esa kesishmaydi. a - giperbolaning *haqiqiy yarim o'qi*, b esa *mavqum yarim o'qi* deyiladi. $c=\sqrt{a^2+b^2}$ parametr koordinatlar boshidan fokuslargacha bo'lgan masofani bildiradi. $\frac{c}{a}=\varepsilon>1$ nisbat giperbolaning *ekssentrisiteti* deyiladi.

$y=\pm\frac{b}{a}x$ to'g'ri chiziqlar giperbolaning *asimptotalari* deyiladi. $M(x;y)$ nuqtalardan fokuslargacha bo'lgan masofalar bu nuqtaning *fokal radiuslari* deyilib

$$r_1=|\varepsilon\, x-a|,\quad r_2=|\varepsilon\, x+a| \qquad (2)$$

formulalar orqali aniqlanadi.

Agar $a=b$ bo'lsa, giperbola *teng tomonli giperbola* deb ataladi. Uning tenglamasi $x^2-y^2=a^2$, asimptotalarining tenglamalari esa $y=\pm x$ bo'ladi. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ va $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ giperbolalar *o'zaro qo'shma* giperbolalar deyiladi.



7-rasm

4. Parabola

Ta'rif. Berilgan nuqtadan(fokusdan) va berilgan to'g'ri chiziqdan (direktrisadan) bir xil uzoqliqda bo'lgan nuqtalarning geometrik o'rni parabola deyiladi.

Parabolaning kanonik tenglamasi quyidagi ikki ko’rinishga ega:

1) $y^2 = 2px$ - Ox o’qqa nisbatan simmetrik parabola tenglamasi;

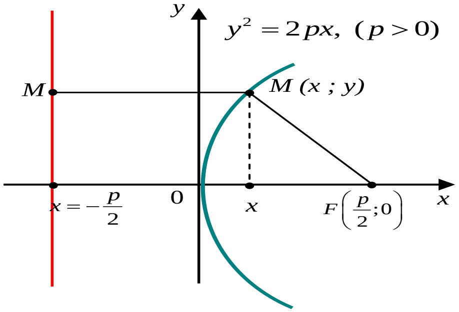
2) $x^2 = 2py$ Oy o’qqa nisbatan simmetrik parabola tenglamasi,

bu yerda p – parabolaning *parametri* deyiladi.

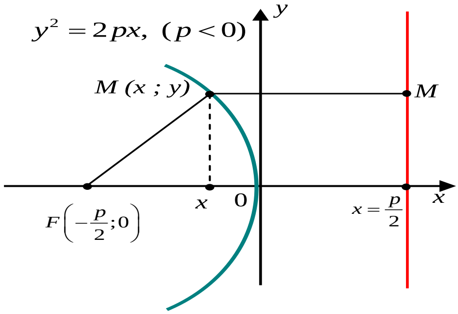
Har ikki holda qam parabolaning *uchi* - *simmetriya o’qi*da yotuvchi nuqtasi, koordinatlar boshida bo’ladi.

$y^2 = 2px$ parabolaning(8-9-rasmlar) *fokusi* $F(\frac{p}{2};\ 0)$ nuqtada, *direktrisasi tenglamasi* $x = -\frac{p}{2}$, uning $M(x;\ y)$ nuqtasining *fokal radiusi* $r = x + \frac{p}{2}$ bo’ladi.

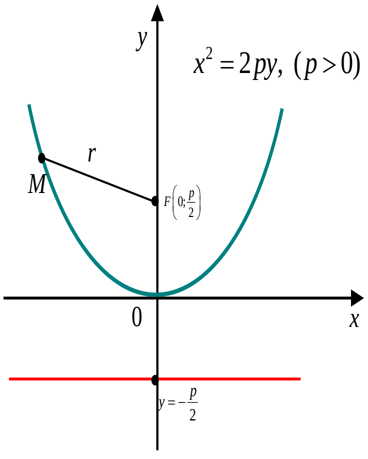
$x^2 = 2py$ parabolaning(10-11-rasmlar) *fokusi* $F(0;\ \frac{p}{2})$ nuqtada, *direktrisasi tenglamasi* $y = -\frac{p}{2}$, $M(x;\ y)$ nuqtasining *fokal radiusi* $r = y + \frac{p}{2}$ bo’ladi.



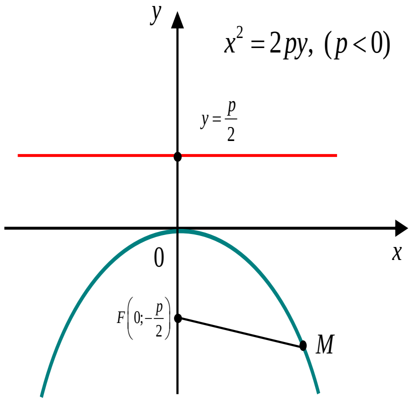
8-rasm



9-rasm



10-rasm

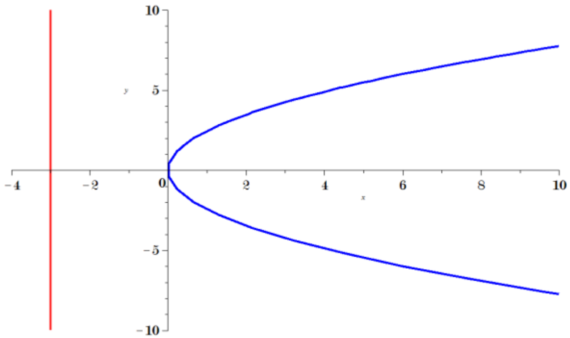


11-rasm

Misollarni yechishga doir namunalar

2-*misol*. $y^2 = 6x$ parabola berilgan. Uning direktrisasi tenglamasini tuzing va fokusini toping.

Δ Berilgan tenglamani parabolaning kanonik tenglamasi $y^2 = 2px$ bilan taqqoslab ko'ramizki, $2p = 6,\ p = 3$. Parabola direktrisasining tenglamasi $x = -\dfrac{p}{2}$ va fokusi $\dfrac{p}{2}$ va 0 koordinatlarga ega bo'lganidan, ko'rilayotgan qol uchun direktrisa tenglamasi $x = -\dfrac{3}{2}$ va fokus $F(\dfrac{3}{2};\,0)$ bo'ladi (12-rasm). ▲



12-rasm

5-TOPSHIRIQQA DOIR KO'RSATMALAR

Parametrik tenglamalar bilan berilgan chiziq tenglamalariga doir misollar yechish

Qisqacha nazariy ma'lumotlar

Mexanika va texnikada moddiy nuqtaning harakat trayektoriyasi qaraladi. Bu trayektoriya t vaqtga bog'liq ravishda o'zgaradi. Shuning uchun trayektoriya ixtiyoriy nuqtasining koordinatalari vaqt t ning o'zgarishi bilan o'zgaradi - t ning funksiyalari bo'ladi. Shuning uchun tekislikda

$$x=\varphi(t),\quad y=\psi(t), \qquad (1)$$

fazoda

$$x=\varphi(t),\quad y=\psi(t),\quad z=\theta(t) \qquad (2)$$

deb yozish mumkin. Bu tenglamalar moddiy nuqtaning *harakat tenglamalari* deyiladi. Geometriya tilida (1) va (2) tenglamalarni egri chiziqning *parametrik tenglamalari* deb ataladi. Bunda t o'zgaruvchi *parametr* deyiladi. t har doim vaqtni bildirishi shart emas, t parametr koordinatalar qanday o'zgarishini aniq tasvirlash uchun qulaylik beradigan yordamchi o'zgaruvchidir.

(1) tenglamadan t parametr chiqarilsa(yo'qotilsa), egri chiziqning Dekart sistemasidagi

$$F(x,y)=0 \quad \text{yoki} \quad y=f(x) \qquad (3)$$

tenglamasi hosil bo'ladi. Bu - parametrni yo'qotish(chiqarish) ham deyiladi. Ko'rinadiki, egri chiziq parametric shaklda tekislikda ikkita tanglama bilan, fazoda esa uchta tenglama bilan beriladi.

Misollarni yechishga doir namunalar

1-*misol*. Markazi koordinatalar boshida bo'lib, radiusi r gat eng bo'gan aylananing parametrik tenglamalarini tuzing.

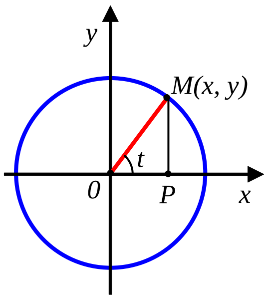
Δ $M(x,y)$ aylananing ixtiyoriy nuqtasi hamda

$$x=OP,\quad y=PM$$

bo'lsin(13-rasm). MOP burchakni t bilan belgilaymiz, bu holda MOP to'g'ri burchakli uchburchakdan

$$\left.\begin{aligned} OP&=OM\cos t\\ PM&=OM\sin t\end{aligned}\right\}$$

yoki $OM=r$ bo'lgani uchun



13-rasm

$$\left.\begin{aligned} x &= r\cos t \\ y &= r\sin t \end{aligned}\right\} \qquad (2)$$

tenglamalar hosil bo’ladi. Bu - aylananing parametrik tenglamasi.

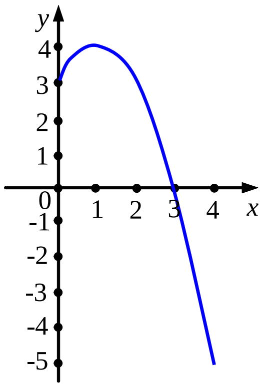
(2) tenglamalardan t parametrni yo'qotib, aylananing Dekart sistemasilagi tenglamasini hosil qilamiz. Buning uchun (2) tenglamalarni kvadratga ko'tarib, natijani hadlab qo'shamiz:

$$x^2 + y^2 = r^2(\cos^2 t + \sin^2 t)$$

yoki

$$x^2 + y^2 = r^2$$

Bu tenglama markazi koordinatalar boshida bo’lib, radiusi r ga teng bo’lgan aylananing Dekart sistemasidagi tenglamasidir. ▲



14-rasm

3-*misol*. Paramatrik tenglamalari bilan berilgan chiziqni:

1) chiziing;

2) tenglamasini to’g’ri burchakli dekart koordinatalarida yozing(parametrni yo’qoting).

$$x = \sqrt{t},\ y = 3 + 2\sqrt{t} - t, \quad t \geq 0.$$

Δ a) t parametning bir necha nomanfiy $(t \geq 0)$ qiymatlariga mos $(x;\ y)$ nuqtalarni topamiz. Buning uchun quyidagi jadvalni tuzamiz:
Oxirgi satrda koordinatalari ko’rsatilgan nuqtalarni to’g’ri burchakli dekart koordinata sistemasida belgilab, ularni silliq chiziq bilan tutashtirib, izlangan chiziqni hosil qilamiz(14-rasm).

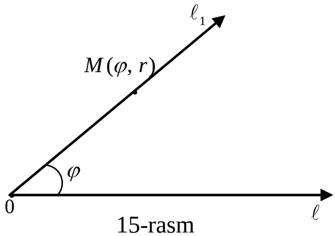
t	0	1	2	4	9	12	16
$x = \sqrt{t}$	0	1	$\sqrt{2} \approx 1,4$	2	3	$\sqrt{12} \approx 3,5$	4
$y = 3 + 2\sqrt{t} - t$	3	4	4,8	3	0	-2,3	-5
$(x;\ y)$	$(0;\ 3)$	$(1;\ 4)$	$(1,4;\ 4,8)$	$(2;\ 3)$	$(3;\ 0)$	$(3,5;\ -2,3)$	$(4;-5)$

b) chiziqning berilgan tenglamasini to’g’ri burchakli dekart koordinatalarida yozish, ya’ni parametrni yo’qotish uchun berilgan tenglamalarning ikkinchisiga $\sqrt{t} = x,\ t = x^2$ larni qo’yib $y = 3 + 2x - x^2, \quad x \geq 0$ tenglamani hosil qilamiz. Bu - $y = 3 + 2x - x^2$ parabolaning ordinata o’qidan o’ng tomondagi qismit. ▲

6-TOPSHIRIQGA DOIR KO'RSATMALAR

Tenglamasi qutb koordinatalar sistemasida berilgan chiziq tenglamalariga doir misollar yechish

Qisqacha nazariy ma'lumotlar

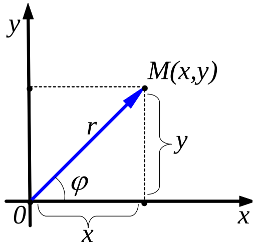


15-rasm

Tekislikda biror ℓ son o'qi - sanoq boshi O ga, musbar yo'nalish va masshtab birligiga ega bo'lgan to'g'ri chiziqni qaraymiz(15-rasm). Bu o'qni *qutb o'qi*, uning O sanoq boshini esa *qutb* deb ataymiz.

M - tekislikdagi qutbdan boshqa biror nuqta bo'lsin. Bu nuqta va qutb orqali ℓ_1 o'qni o'tkazamiz. ℓ va ℓ_1 o'qlar orasidagi φ burchak - *qutb burchagi*, M nuqtadan qutbgacha bo'lgan masofa $OM = r$ esa nuqtaning *qutb radiusi* deyiladi. φ va r lar M nuqtaning *qutb koordinatalari* deyiladi va $M(\varphi, r)$ shaklda yoziladi. Bu kiritilgan koordinata sistemasi *qutb koordinatalar sistemasi* deyiladi.

Qutb koordinatalar sistemasida nuqta o'rnini topishda φ va r istalgan nusbat va manfiy qiymatlarni qabul qilishi mumkun, deb qaraladi. Bunda manfiy burchaklar soat mili yo'nalishi bo'ylab hisoblanadi, manfiy qutb radiusi esa qaralayotgan nur yo'nalishi bo'ylab emas, balki qutbning davomida olinadi.



16-rasm

Agar qutb O ni Dekart sistemasining boshi, ℓ qutb o'qini esa Ox abssissa o'qi deb qabul qilsak, unda M nuqtaning (x, y) Dekart koordinatalari bilan (φ, r) qutb koordinatalari orasidagi bog'lanishni topish mumkin(16-rasm):

$$x = r\cos\varphi, \quad y = r\sin\varphi; \qquad (1)$$

$$r = \sqrt{x^2 + y^2}, \quad \varphi = arctg\,\frac{y}{x}. \qquad (2)$$

Misollarni yechishga doir namunalar

1-misol. Qutb koordinatlar sistemasida berilgan $A(\frac{\pi}{2}, 3)$ va $B(-\frac{\pi}{4}, 4)$ nuqtalarning Dekart koordinatlari sistemasidagi koordinatlarini toping.

Δ a) $A(\frac{\pi}{2}, 3)$; $\varphi = \frac{\pi}{2}$; $r = 3$. (1) formulalarga ko'ra:

$$x = r\cos\varphi = 3\cos\frac{\pi}{2} = 3\cdot 0 = 0; \quad y = r\cos\varphi = 3\sin\frac{\pi}{2} = 3\cdot 1 = 3\,.$$

Demak, Dekart koordinatalar sistemasida: $A(0, 3)$.

b) Xuddi yuqoridagiga o'xshash topamiz: $B(-\frac{\pi}{4}, 4)$; $\varphi = -\frac{\pi}{4}$; $r = 4$.

$x = r\cos\varphi = 4\cos(-\frac{\pi}{4}) = 4\cdot\frac{\sqrt{2}}{2} = 2\sqrt{2}$; $y = r\sin\varphi = 4\sin(-\frac{\pi}{4}) = -4\cdot\frac{\sqrt{2}}{2} = -2\sqrt{2}$.

Demak, Dekart koordinatlar sistemasida: $B(2\sqrt{2}; -2\sqrt{2})$. ▲

2-*misol*. Qutb koordinatalar sistemasida $r = \frac{2}{1-\cos\varphi}$ tenglama bilan berilgan chiziqni chizing va tenglamani to'g'ri burchakli dekart koordinatalarida yozing.

Δ Qutb burchagi φ ning 0 dan 2π gacha qiymatlariga mos qutb radiusining qiymatlarinini hisoblaymiz:

$$\varphi = 0,\ r = \frac{2}{1-\cos 0} = \frac{2}{1-1} = +\infty; \quad M_4(0; +\infty).$$

$$\varphi = \frac{\pi}{4},\ r = \frac{2}{1-\cos\frac{\pi}{4}} = \frac{2}{1-\frac{\sqrt{2}}{2}} = 6{,}828; \quad M_1(\frac{\pi}{4}; 6{,}828);$$

$$\varphi = \frac{\pi}{2},\ r = \frac{2}{1-\cos\frac{\pi}{2}} = \frac{2}{1-0} = 2; \quad M_2(\frac{\pi}{2}; 2);$$

$$\varphi = \frac{3\pi}{4},\ r = \frac{2}{1-\cos\frac{3\pi}{4}} = \frac{2}{1+\frac{\sqrt{2}}{2}} = 1{,}172; \quad M_3(\frac{3\pi}{4}; 1{,}172);$$

$$\varphi = \pi,\ r = \frac{2}{1-\cos\pi} = \frac{2}{1+1} = 1; \quad M_4(\pi; 1);$$

$$\varphi = \frac{5\pi}{4},\ r = \frac{2}{1-\cos\frac{5\pi}{4}} = \frac{2}{1+\frac{\sqrt{2}}{2}} = 1{,}172; \quad M_5(\frac{5\pi}{4}; 1{,}172);$$

$$\varphi = \frac{3\pi}{2},\ r = \frac{2}{1-\cos\frac{3\pi}{2}} = \frac{2}{1-0} = 2; \quad M_6(\frac{3\pi}{2}; 2);$$

$$\varphi = \frac{7\pi}{4},\ r = \frac{2}{1-\cos\frac{7\pi}{4}} = \frac{2}{1-\frac{\sqrt{2}}{2}} = 6{,}828; \quad M_7(\frac{7\pi}{4}; 6{,}828);$$

$$\varphi = 2\pi,\ r = \frac{2}{1-\cos 2\pi} = \frac{2}{1-1} = +\infty; \quad M_8(2\pi; +\infty).$$

Topilgan qiymatlar va ularga mos nuqtalarni quyidagi jadvalda ko'rsatish mumkin:

φ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$r=\dfrac{2}{1-\cos\varphi}$	$+\infty$	6,828	2	1,172	1	1,172	1	6,828	$+\infty$
$M_i=M_i(\varphi;\, r)$	M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8

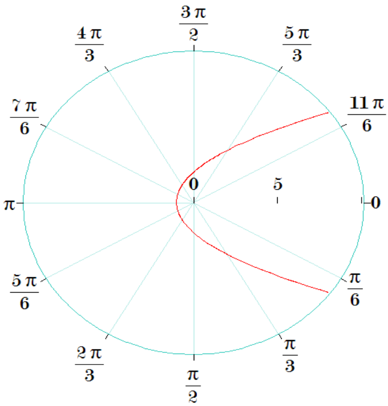
Qutb koordinata sistemasida chiziqqa tegishli bu nuqtalarni topib, ularni tutash chiziq bilan tutshtirib, chiziqning grafigini hosil qilamiz(17-rasm). Ko'rinib turibdiki, u paraboladan iborat.

Qutb koordinatalarida berilgan chiziqning tenglamasini to'g'ri burchakli dekart koordinatalarida yozish uchun $r=\sqrt{x^2+y^2}$, $x=r\cos\varphi$, $\cos\varphi=\dfrac{x}{r}=\dfrac{x}{\sqrt{x^2+y^2}}$, formulalardan foydalanamiz:

$$r=\frac{2}{1-\cos\varphi};\qquad \sqrt{x^2+y^2}=\frac{2}{2-\dfrac{x}{\sqrt{x^2+y^2}}};\qquad \left(x-\sqrt{x^2+y^2}\right)\sqrt{x^2+y^2}=2\sqrt{x^2+y^2};$$

$$\sqrt{x^2+y^2}-x=2;\quad \sqrt{x^2+y^2}=2+x;\quad x^2+y^2=x^2+4x+4;\quad y^2=4(x+1).$$

Bu – uchi $(-1;\,0)$ nuqtada bo'lib, abscissa o'qiga nisbatan simmetrik parabolaning tenglamasi. ▲



17-rasm

7-TOPSHIRIQGA DOIR KO'RSATMALAR

Limitni Lopital qoidasidan foydalanmasdan hisoblash

Qisqacha nazariy ma'lumotlar

$y=f(x)$ funksiya $x=x_0$ nuqtaning biror δ–atrofida, ya'ni $x_0-\delta<x<x_0+\delta$ yoki $(x_0-\delta;x_0+\delta)$ oraliqda aniqlangan bo'lsin. x_0 nuqtaning o'zida aniqlangan bo'lishi shart emas.

Ta'riflar:

1. Agar har qanday $\varepsilon>0$ bo'yicha unga bog'liq bo'lgan shunday $\delta>0$ sonni ko'rsatish mumkin bo'lsaki, $0<|x-x_0|<\delta$ shartni qanoatlantiradigan barcha x lar uchun $|f(x)-A|<\varepsilon$ tengsizlik o'rinli bo'lsa, A soni $f(x)$ *funksiyaning* $x\to x_0$ *dagi limiti* deyiladi. Bu

$$\lim_{x\to x_0} f(x)=A$$

deb yoziladi. Agar $A=0$ bo'lsa, $f(x)$ funksiya $x\to x_0$ da *cheksiz kichik* deyiladi.

2. Agar har qanday musbat M soni uchun, u qanday katta bo'lsa ham, shunday $\delta>0$ topilsaki, $0<|x-x_0|<\delta$ tengsizlikni qanoatlantiruvchi barcha x lar uchun $|f(x)|>M$ tengsizlik bajarilsa, $y=f(x)$ funksiya $x\to x_0$ da *cheksiz katta* deyiladi va bu $\lim\limits_{x\to x_0} f(x)=\infty$ kabi yoziladi.

Cheksiz kichik va cheksiz kattalar orasida quyidagi munosabatlar mavjud:

a) Agar $f(x)$ cheksiz kichik bo'lsa, u holda $\dfrac{1}{f(x)}$ cheksiz katta bo'ladi.

b) Agar $f(x)$ cheksiz katta bo'lsa, u holda $\dfrac{1}{f(x)}$ cheksiz kichikdir.

3. $f(x)$ funksiya $N>0$ son istalgancha katta bo'lsa ham $|x|>N$ tengsizlikni qanoatlantiruvchi barcha x larda aniqlangan bo'lsin. A son $y=f(x)$ *funksiyaning* $x\to\infty$ dagi *limiti* deyiladi, agar har qanday $\varepsilon>0$ uchunshunday $N>0$ sonni ko'rsatish mumkin bo'lib, $|x|>N$ ni qanoatlantiradigan barcha x lar uchun $|f(x)-A|<\varepsilon$ tengsizlik bajarilsa. Bu $A=\lim\limits_{x\to\infty} f(x)$ kabi yoziladi.

Limitlarni hisoblashda quyidagi qoidalardan foydalaniladi

$\lim\limits_{x\to x_0} f(x)=A$, $\lim\limits_{x\to x_0} g(x)=B$ chekli limitlar mavjud bo'lsin. U holda quyidagilar o'rinli:

1. $\lim\limits_{x\to x_0}\left(f(x)\pm g(x)\right)=A\pm B$

2. $\lim\limits_{x\to x_0}\left(f(x)\cdot g(x)\right)=A\cdot B$

3. $\lim\limits_{x\to x_0}\dfrac{f(x)}{g(x)}=\dfrac{A}{B},\ \left(B\neq 0\right)$

Agar $\lim\limits_{x\to x_0} f(x)=\lim\limits_{x\to x_0} g(x)=0$ (yoki $\lim\limits_{x\to x_0} f(x)=\lim\limits_{x\to x_0} g(x)=\infty$) bo'lsa, u holda $\dfrac{f(x)}{g(x)}$ nisbat $x\to x_0$ da $\dfrac{0}{0}\left(\text{yoki } \dfrac{\infty}{\infty}\right)$ ko'rinishidagi *aniqmaslikni* beradi. Bu hollarda limitni topish - berilgan ifodalar ustida algebraik almashtirishlar bajarib aniqmaslikdan qutilishdan iborat (aniqmaslikni ochish ham deyiladi).

Limitlarni hisoblashda *ajoyib limitlar* deb ataladigan limitlar alohida o'ringa ega. Ular quyidagilardir:

a) $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$ – *birinchi ajoyib limit* ($\dfrac{0}{0}$ ko'rinishidagi aniqmaslik);

b) $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x=e$ - *ikkinchi ajoyib limit* (1^∞ ko'rinishidagi aniqmaslik),

bunda $e\approx 2{,}7182818...$ - irratsional son. $\alpha=\dfrac{1}{x}$ deb ikkinchi ajoyib limitni ushbu

$$\lim_{\alpha\to 0}\left(1+\alpha\right)^{1/\alpha}=e$$

ko'rinishda ham yozish mumkin.

Birinchi ajoyib limit formulasida x ning o'rnida biror cheksiz kichik funksiya bo'lishi ham mumkin. Masalan:

$$\lim_{x\to 0}\frac{\sin(kx^2)}{kx^2}=1,\qquad \lim_{x\to\infty}\frac{\sin\frac{1}{2x}}{\frac{1}{2x}}=1,\qquad \text{va hokazo.}$$

Ikkinchi ajoyib limit formulalarida ham x ning o'rnida cheksiz katta funksiya, α ning o'rnida cheksiz kichik funksiya bo'lishi mumkin. Masalan:

$$\lim_{x\to\infty}\left(1+\frac{5}{2x}\right)^{\frac{2x}{5}}=e\quad \lim_{x\to -1}\left[1+\left(1+x\right)\right]^{\frac{1}{1+x}}=e,\quad \lim_{x\to 0}\left[1+tgx\right]^{\frac{1}{tgx}}=e$$

va hokazo.

Misollarni yechishga doir namunalar

1- misol. Limitlarni hisoblang.

a) $\lim\limits_{x\to\infty}\dfrac{8x^9-9x^3+3}{4x^9+x^6-x^2}$ b) $\lim\limits_{x\to1}\dfrac{\sqrt{x+3}-\sqrt{5-x}}{x^2+5x-6}$

d) $\lim\limits_{x\to0}\dfrac{tg2x-\sin 2x}{x^3}$ e) $\lim\limits_{x\to\infty}(4x-1)\left[\ln(2x-3)-\ln 2x\right]$.

Δ a) Bu misolda, agar x ning o'rniga ∞ ni qo'ysak, $\dfrac{\infty}{\infty}$ ko'rinishidagi aniqmaslikka ega bo'lamiz. Bunday ko'rinishdagi misollarni yechishda limit ostidagi kasrning surat va maxrajida x ning eng katta darajasini qavsdan tashqariga ko'paytuvchi qilib chiqarish, so'ngra x ning mos darajasiga qisqartirish kerak bo'ladi.

$$\lim_{x\to\infty}\frac{8x^9-9x^8+3}{4x^9+x^6-x^2}=\lim_{x\to\infty}\frac{x^9\left(8-\dfrac{9x^8}{x^9}+\dfrac{3}{x^9}\right)}{x^9\left(4+\dfrac{x^6}{x^9}-\dfrac{x^2}{x^9}\right)}=\lim_{x\to+\infty}\frac{8-\dfrac{9}{x}+\dfrac{3}{x^9}}{4+\dfrac{1}{x^3}-\dfrac{1}{x^7}}=\frac{8}{4}=2.$$

Bu yerda $x\to+\infty$ da $\dfrac{9}{x},\ \dfrac{3}{x^2},\ \dfrac{1}{x^3},\ \dfrac{1}{x^7}\to0.$

b) Bu misolda $x\to1$ da limit ostidagi kasrning surati va maxraji 0 ga intilishini ko'rish qiyin emas. Demak biz $\dfrac{0}{0}$ ko'rinishdagi aniqmaslikka egamiz. Bunday hollarda (agar surat va maxrajda yoki faqat suratda (maxrajda) irratsionallik bo'lsa, undan qutulish uchun) kasrning surati va maxrajini misoldagi irrasionallikning qo'shmasiga ko'paytirib, bo'lish kerak.

$$\lim_{x\to1}\frac{\sqrt{x+3}-\sqrt{5-x}}{x^2+5x-6}=\lim_{x\to1}\frac{\left(\sqrt{x+3}-\sqrt{5-x}\right)}{x^2+5x-6}\cdot\frac{\left(\sqrt{x+3}+\sqrt{5-x}\right)}{\sqrt{x+3}+\sqrt{5-x}}=$$

$$=\lim_{x\to1}\frac{(\sqrt{x+3})^2-(\sqrt{5-x})^2}{(x-1)(x+6)(\sqrt{x+3}+\sqrt{5-x})}=\lim_{x\to1}\frac{x+3-(5-x)}{(x-1)(x+6)(\sqrt{x+3}+\sqrt{5-x})}$$

$$=\lim_{x\to1}\frac{2(x-1)}{(x-1)(x+6)(\sqrt{x+3}+\sqrt{5-x})}=\lim_{x\to1}\frac{2}{(x+6)(\sqrt{x+3}+\sqrt{5-x})}=\frac{2}{7\cdot4}=\frac{1}{14}.$$

d) Bu misolda ham birdaniga limitga o'tsak $\dfrac{0}{0}$ ko'rinishidagi aniqmaslikka ega bo'lamiz. Limit ostidagi ifodada shakl o'zgartirishlar bajarib birinchi ajoyib limitni qo'llash mumkin bo'lgan ko'rinishga keltiramiz:

$$\lim_{x\to -0}\frac{tg2x-\sin 2x}{x^3}=\lim_{x\to 0}\frac{\dfrac{\sin 2x}{\cos 2x}-\sin 2x}{x^3}=\lim_{x\to 0}\frac{\sin 2x\left(\dfrac{1}{\cos 2x}-1\right)}{x^3}=$$

$$=\lim_{x\to 0}\frac{\sin 2x(1-\cos 2x)}{x^3\cdot\cos 2x}=\lim_{x\to 0}2\cdot\frac{\sin 2x}{2x}\cdot\lim_{x\to 0}\frac{1-\cos 2x}{x^2}\cdot\lim_{x\to 0}\frac{1}{\cos 2x}=$$

$$=2\cdot 1\cdot 1\cdot\lim_{x\to 0}\frac{1-\cos 2x}{x^2}\left(\frac{0}{0}\right)=2\lim_{x\to 0}\frac{2\sin^2 x}{x^2}=2\cdot 2\cdot\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\sin x}{x}=4.$$

e) Bu misolda bevosita limitga o'tish $(\infty-\infty)$ ko'rinishidagi aniqmaslikka olib keladi. Logarifmning xossalaridan foydalanib, limit ostidagi ifodaning shaklini o'zgartirib:

$$(4x-1)[\ln(2x-3)-\ln 2x]=(4x-1)\ln\frac{2x-3}{2x}=\ln\left(\frac{2x-3}{2x}\right)^{4x-1},$$

ikkinchi ajoyib limit formulasidan foydalanish mumkin:

$$\lim_{x\to+\infty}(4x-1)[\ln(x-3)-\ln 2x]=\lim_{x\to+\infty}\ln\left[\frac{2x-3}{2x}\right]^{4x-1}=\ln\left[\lim_{x\to+\infty}\left(1+\frac{-3}{2x}\right)^{4x-1}\right]=$$

$$=\ln\left[\lim_{x\to+\infty}\left(1+\frac{1}{\dfrac{2x}{-3}}\right)^{-\frac{3}{2x}\left(-\frac{2x}{3}\right)\cdot(4x-1)}\right]=\ln\lim_{x\to+\infty}\left[\left(1+\frac{1}{\dfrac{2x}{-3}}\right)^{-\frac{2x}{3}}\right]^{\lim\limits_{x\to\infty}\frac{-3(4x-1)}{2x}}=$$

$$=\ln e^{\lim\limits_{x\to+\infty}\frac{-12+\frac{3}{x}}{2}}=\ln e^{-6}=-6.\ \blacktriangle$$

Izoh. Funksiyaning uzluksizligi mavzusida funksiya belgisi ostida limitga o'tish mumkinligi ko'rsatiladi: $x\to a$ da $f(x)$ uzluksiz bo'lsa, u holda

$$\lim_{x\to a}f(x)=f\left(\lim_{x\to a}x\right)=f(a).$$

8-TOPSHIRIQGA DOIR KO'RSATMALAR

Funksiyaning uzilish nuqtalarini topish va grafigini yasash

Qisqacha nazariy ma'lumotlar

$y=f(x)$ funksiya $x=x_0$ nuqtaning biror tomonida masalan $x<x_0$ da yoki $x>x_0$ da aniqlangan bo'lsin. Bu holda $f(x)$ funksiyaning $x\to x_0$ dagi bir tomonli limitlari haqida gapirish mumkin bo'ladi. $f(x)$ funksiyaning $x=x_0$ nuqtadagi *chap limiti* deb $f(x)$ ning x, x_0 ga chapdan(ya'ni $x<x_0$ bo'lib) intilgandagi limitga aytiladi va u $f(x_0-0)$ kabi belgilanadi:

$$\lim_{x\to x_0-0} f(x)=\lim_{\substack{x\to x_0\\ x<x_0}} f(x)=f(x_0-0).$$

Shunga o'xshash $f(x)$ ning $x=x_0$ nuqtadagi *o'ng limiti* $f(x_0+0)$ aniqlanadi.

$$\lim_{x\to x_0+0} f(x)=\lim_{\substack{x\to x_0\\ x>x_0}} f(x)=f(x_0+0).$$

Ta'rif. Agar $f(x)$ funksiya $x=x_0$ nuqta va uning biror atrofida aniqlangan bolib,

$$\lim_{x\to x_0} f(x)=f(x_0)$$

bo'lsa, *funksiya* $x=x_0$ *nuqtada uzluksiz* deyiladi.

Bu ta'rif quyidagiga teng kuchli: $y=f(x)$ funksiya

1) $x=x_0$ nuqtada aniqlangan, ya'ni $f(x_0)$ mavjud;
2) $x=x_0$ nuqtaning biror atrofida aniqlangan;
3) $x=x_0$ nuqtada chap va o'ng limitlarga ega va ular chekli;
4) chap va o'ng limitlar $f(x_0)$ ga teng, ya'ni

$$f(x_0-0)=f(x_0+0)=f(x_0)\,.$$

Agar bu shartlardan birortasi bajarilmasa u holda $f(x)$ funksiya berilgan *nuqtada uzilishga ega* deyiladi. Funksiya uzluksiz bo'lmagan nuqta – *funksiyaning uzilish nuqtasi* deyiladi. Uzilish nuqtalari ikki xil bo'ladi: *birinchi va ikkinchi tur uzilish* nuqtalari.

Agar funksiyaning $x=x_0$ nuqtadagi chap va o'ng limitlari chekli sonlar, lekin o'zaro teng bo'lmasa: $f(x_0-0)\neq f(x_0+0)$, x_0 $f(x)$ funksiyaning *birinchi tur uzilish nuqtasi* deyiladi. $D=\left|f(a-0)-f(a+0)\right|$ son *funksiyaning* x_0 *nuqtadagi sakrashi* deyiladi. Bu holda

$f(x_0-0)=f(x_0)$ yoki $f(x_0+0)=f(x_0)$ yoki $f(x_0)\neq f(x_0-0)\neq f(x_0+0)$ bo'lishi mumkin.

Agar $f(x_0-0)=f(x_0+0)\neq f(x_0)$ bo'lsa - x_0 nuqta *yuqotilishi mumkin bo'lgan uzilish nuqtasi* deyiladi.

Agar $f(x_0-0)$ va $f(x_0+0)$ limitlardan kamida biri mavjud bo'lmasa yoki cheksiz bo'lsa - x_0 nuqta $f(x)$ funksiyaning *ikkinchi tur uzilish nuqtasi* deyiladi.

$y=f(u)$ va $u=\varphi(x)$ bo'lsin. U holda $y=f(\varphi(x))$ *murakkab funksiya* (funksiyaning funksiyasi) deyiladi. Quyidagi:

$y=C$ ($C=const$) - o'zgarmas funksiya;

$y=x^n$ - darajali funksiya;

$y=a^x$ $(0<a\neq 1)$ - ko'rsatkichli funksiya;

$y=\log_a x$ $x>0,\ (0<a\neq 1)$ - logarifmik funksiya;

$y=\sin x$, $y=\cos x$ $y=tgx$, $y=ctgx$ - trigonometrik funksiyalar

asosiy elementar funksiyalar deyiladi. Asosiy elementar funksiyalar ustida arifmetik amallar bajarish yoki murakkab funksiya tuzish yo'li bilan yangi murakkab funksiyalar hosil qilish mumkin. Ular *elementlar funksiyalar* deyiladi. Masalan,

$$y=e^{\sin x},\quad y=\sin(e^x+\sqrt{1-x^2})$$

elementar funksiyalardir.

Agar $y=f(x)$ funksiya biror to'plamning hamma nuqtalarida uzluksiz bo'lsa, u holda funksiya shu to'plamda uzluksiz deyiladi. Elementar funksiyalarning hammasi o'z aniqlanish sohasida uzluksizdir.

$y=f(x)$ funksiya $[a,b]$ kesmada uzluksiz bo'lishi uchun u $(a,b)\subset[a,b]$ oraliqda uzluksiz bo'lishi, a nuqtaga o'ngdan, b nuqtada esa chapdan uzluksuz bo'lishi kerak.

Misollarni yechishga doir namunalar

1-*misol*. $y=2^{\frac{1}{x-4}}$ funksiyani uzilish nuqtalarini toping va ularning xarakterini tekshiring.

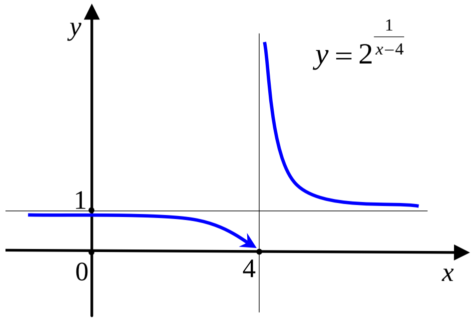
Δ Berilgan funksiya elementar funksiya bo'lgani uchun o'zining aniqlanish sohasi: $x-4\neq 0$, ya'ni $(-\infty,4)\cup(4,+\infty)$ to'plamda uzluksiz. $x=4$ nuqtada esa uzilishga, chunki $x=4$ nuqtada aniqlanmagan. $x=4$ nuqta qaysi tur uzilish nuqtasi ekanini bilish uchun chap va o'ng limitlar $f\left(4-0\right)$, $f\left(4+0\right)$ni hisoblaymiz:

$$f(4-0)=\lim_{x\to 4-0}2^{\frac{1}{x-4}}=\begin{bmatrix} x-4=\alpha \\ x\to 4-0 \\ \alpha\to -0 \end{bmatrix}=\lim_{\alpha\to -0}2^{\frac{1}{\alpha}}=2^{\frac{1}{-0}}=2^{-\infty}=\frac{1}{2^{\infty}}=0;$$

$$f(4+0)=\lim_{x\to 4+0}2^{\frac{1}{x-4}}=\begin{bmatrix} x-4=\alpha \\ x\to 4+0 \\ \alpha\to +0 \end{bmatrix}=\lim_{\alpha\to +0}2^{\frac{1}{\alpha}}=2^{\frac{1}{+0}}=2^{+\infty}=+\infty.$$

Bir tomomli limitlardan biri - $f(4+0)$ o'ng limit cheksiz ekan, demak $x=4$ ikkinchi tur uzilish nuqtasi. Ravshanki, $\lim\limits_{x\to\pm\infty}2^{\frac{1}{x-4}}=2^{\frac{1}{\pm\infty}}=2^{0}=1.$ Bularga asoslanib, funksiyaning grafigini yasash mumkin(18-rasm).

▲



18-rasm

2-*misol*. $f(x)=\begin{cases} 2x^2+3, & -\infty<x\le 1 \quad da \\ 6-5x, & 1<x<3 \quad da \\ x-12, & 3\le x<+\infty \quad da \end{cases}$

funksiyani uzluksizlikka tekshiring.

Δ Funksiya son o'qida aniqlangan, har xil oraliqlarda turli xil formulalar bilan berilgan. Bu oraliqlar: $(-\infty,1]$, (1, 3), [3, +∞) ning har birida funksiya uzluksiz, chunki ularning har birida elementardir. Shuning uchun, oraliqlarning chegaralari $x=1$, $x=3$ nuqtalarni tekshirish kifoya, chunki bu nuqtalardan o'tganda funksiya o'zining ko'rinishini o'zgartiradi.

$x=1$ nuqtani tekshiramiz. $f(1-0)$, $f(1+0)$ va $f(1)$ larni topamiz:

$$f(1+0)=\lim_{x\to 1+0} f(x)=\lim_{\substack{x\to 1\\ x>1}}(6-5x)=6-5=1,$$

$$f(1-0)=\lim_{x\to 1-0} f(x)=\lim_{\substack{x\to 1\\ x<1}}(2x^2+3)=2\cdot 1+3=5,$$

$$f(1)=f(x)\Big|_{x=1}=\left[2x^2+3\right]_{x=1}=2+3=5.$$

Demak, $x=1$ nuqtaga funksiya 1-tur uzilishga ega, chunki

$$f(1-0)\neq f(1+0)=f(1).$$

Endi $x=3$ nuqtani tekshiramiz.

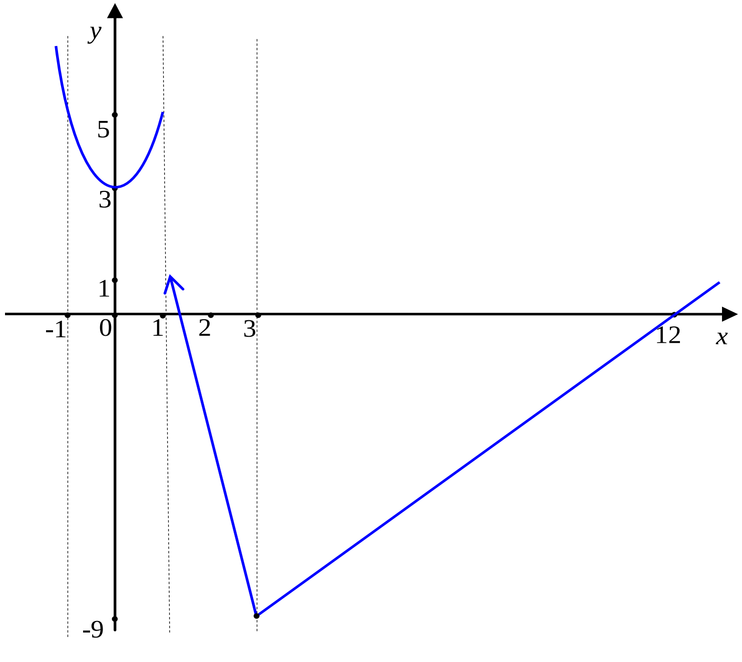
$$f(3-0)=\lim_{x\to 3-0}(6-5x)=6-5\cdot 3=-9,$$

$$f(3+0)=\lim_{x\to 3+0}(6-5x)=6-5\cdot 3=-9,$$

$$f(3)=f(x)\big|_{x=3}=(x-12)=3-12=-9.$$

Demak, funksiya $x=3$ nuqtada uzluksiz. Funksiyaning $x=1$ uzilish nuqtasidagi sakrashini topamiz: $D=|f(1-0)-f(1+0)|=|5-1|=4.$

Funksiya grafigini yasaymiz(19-rasm). ▲



19-rasm

a) $f(x)=2x^2+3$ - parabola, bunda $x\leq 1$.

b) $f(x)=6-5x$ - to'g'ri chiziq, bunda $1<x<3$.
v) $f(x)=x-12$ - to'g'ri chiziq, bunda $3\le x<+\infty$.

9-TOPSHIRIQGA DOIR KO'RSATMALAR

Funksiyaning hosilasini topish

Qisqacha nazariy ma'lumotlar

$y=f(x)$ funksiyaning x nuqtadagi argument orttirmasi Δx, funksiya orttirmasi $\Delta y=f(x+\Delta x)-f(x)$ bo'lsin.

Ta'rif: Funksiyaning berilgan *nuqtadagi hosilasi* deb, bu nuqtadagi funksiya orttirmasiing argument orttirmasiga nisbatining argument orttirmasi nolga intilgandagi limitiga aytiladi.

$y=f(x)$ ning hosilasi $f'(x),\ y',\ \dfrac{dy}{dx},\ \dfrac{df(x)}{dx}$ larning birortasi bilan belgilanadi. Demak, ta'rifga asosan:

$$f'(x)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} \qquad (1)$$

(1) limitning qiymati chekli bo'lsa, $y=f(x)$ funksiya x nuqtada *differensiallanuvchi* deyiladi. Hosilani topish amali *differensiallash* deyiladi.

Differensiallashning asosiy qoidalari

C – o'zgarmas son, $u=u(x),\ v=v(x)$ differensiallanuvchi funksiyalar bo'lsa, quyidagilar o'rini.

1. $C'=0$ - o'zgarmas miqdorning hosilasi nolga teng.
2. $\left[u(x)+v(x)\right]'=u'(x)+v'(x).$
3. $\left[Cu(x)\right]'=Cu'(x).$
4. $\left[u(x)v(x)\right]'=u'(x)v(x)+u(x)v'(x).$
5. $\left(\dfrac{u}{v}\right)'=\dfrac{u'\cdot v-u\cdot v'}{v^2};\ (v\neq 0\text{ da}).$

Oddiy elementar funksiyalarning hosilalari jadvali

1. $y=x^{\alpha},\ y'=\alpha x^{\alpha-1},\ (\alpha-$ixtiyoriy haqiqiy son).
2. $y=\sin x,\ y'=\cos x$
3. $y=\cos x,\ \ y'=-\sin x$

4. $y = tgx,\quad y' = \dfrac{1}{\cos^2 x}$

5. $y = ctgx,\quad y' = -\dfrac{1}{\sin^2 x}$

6. $y = \arcsin x,\quad y' = \dfrac{1}{\sqrt{1-x^2}}$

7. $y = \arccos x,\ y' = -\dfrac{1}{\sqrt{1-x^2}}$

8. $y = arctgx,\quad y' = \dfrac{1}{1+x^2}$

9. $y = arcctgx,\quad y' = -\dfrac{1}{1+x^2}$

10. $y = a^x,\ 0 < a \neq 1,\quad y' = a^x \ln a$

11. $y = \log_a x,\quad y' = \dfrac{1}{x}\log_a e$

12. $y = e^x,\quad y' = e^x$

13. $y = \ln|x|,\quad y' = \dfrac{1}{x}.$

Murakkab funksiyani differensiallash qoidasi

Agar $u = \varphi(x)$ funksiya x nuqtada $u' = \varphi'(x)$ hosilaga, $y = f(u)$ funksiya $u = \varphi(x)$ nuqtada $y'_u = f'(u)$ hosilaga ega bo'lsa, u holda $y = f\left(\varphi(x)\right)$ murakkab funksiya x nuqtada hosilaga ega bo'ladi va quyidagi o'rinli:

$$\left[f\left(\varphi(x)\right)\right]' = f'_u\ \left(\varphi(x)\right)\cdot\varphi'(x)$$

yoki

$$y'_x = y'_u \cdot u'_x \tag{2}$$

Shunday qilib, $y = f(u)$, $u = \varphi(x)$ bo'lsa, $y = f\left(\varphi(x)\right)$ murakkab funksiyani differensiallash uchun tashqi $f(u)$ funksiyaning u bo'yicha hosilasini ichki $u = \varphi(x)$ funksiyaning x bo'yicha hosilasiga ko'paytirish kerak.

Logarifmik differensiallash. $[u(x)]^{v(x)}$ ko'rinishidagi darajali-ko'rsatkichli funksiyani differensialash uchun quyidagi *logarifmik differensiallash* qoidasidan qoidasidan foydalaniladi:

1) Berilgan tenglik logarifmlanadi: $\ln y = \ln u^v$; $\ln y = v\ln u$;
2) $\ln y$ ni x ning murakkab funksiya deb qarab keyingi tenglikning har ikkala tomoni differensiallanadi:

$$(\ln y)' = (v\ln u)';\ \frac{y'}{y} = v'\ln u + v\frac{u'}{u};\ \mathrm{y}' = y\left(v'\ln u + v\frac{u'}{u}\right).$$

Bundan: $\mathrm{y}' = u^v\left(v'\ln u + v\dfrac{u'}{u}\right).$

Misollarni yechishga doir namunalar

1-*misol*. Quyida berilgan funksiyalarning y' hosilasi topilsin.

a) $y=\ln(x-2\sin x)$; b) $y=(7x-3)\sqrt{x^3-\cos x}$

v) $y=9^{\arcsin 4x-1}$; g) $y=\sin^9 x^2\cdot\cos^{10}2x$; d) $y=(\sin 8x)^{\frac{1}{2x}}$.

Δ 1) $y=\ln(x-2\sin x)$ murakkab funksiya. Shu sababli $u=x-2\sin x$ belgilash kiritamiz. Natijada $y=\ln u$ bo'ladi. Murakkab funksiyani hosilasini topish qoidasidan foydalanamiz: $y'=(\ln u)'=\frac{1}{u}\cdot u'$, biroq

$$u'=(x-2\sin x)'=1-2\cos x$$

Demak, $y'=\dfrac{1}{x-2\sin x}\cdot(1-2\cos x)=\dfrac{1-2\cos x}{x-2\sin x}$.

b) $y=\left(7x-3\right)\cdot\sqrt{x^3-\cos x}$ funksiya ikkita $u=7x-3$ va $v=\sqrt{x^3-\cos x}$ funksiyalar ko'paytmasidan iborat. Shuning uchun ko'paytmaning hosilasini topish qoidasidan foydalanamiz:

$$y'=\left[\left(7x-3\right)\cdot\sqrt{x^3-\cos x}\right]'=\left(7x-3\right)'\cdot\sqrt{x^3-\cos x}+\left(7x-3\right)\cdot\left(\sqrt{x^3-\sin x}\right)'.$$

Lekin

$$\left(7x-3\right)'=\left(7x\right)'-(3)'=7,\quad \left(\sqrt{x^3-\cos x}\right)'=\frac{\left(x^3-\cos x\right)'}{2\sqrt{x^3-\cos x}}=\frac{3x^2+\sin x}{2\sqrt{x^3-\cos x}},$$

bu yerda $(\sqrt{u})'=\dfrac{u'}{2\sqrt{u}}$ formuladan foydalanildi.

Shunday qilib, $y'=7\cdot\sqrt{x^3-\cos x}+\dfrac{\left(7x-3\right)\left(3x^2+\sin x\right)}{2\cdot\sqrt{x^3-\cos x}}$.

v) $y=9^{\arcsin(4x-1)}$ murakkab funksiya. Tashqi funksiya $9^{u(x)}$ ko'rsatkichli funksiya. $u=\arcsin(4x-1)$ teskari trigonometrik funksiya esa ichki funksiyadir. Shu sababli $y'=\left[9^{u(x)}\right]'=9^{u(x)}\cdot\ln 9$. O'z navbatida $u=\arcsin(4x-1)$ ham murakkab funksiya va shu qoidaga asosan:

$$u'=\left[\arcsin(4x-1)\right]'=\frac{1}{\sqrt{1-(4x-1)^2}}\cdot(4x-1)'=\frac{4}{\sqrt{1-(4x-1)^2}}=\frac{4}{\sqrt{8x-16x^2}}=$$
$$=\frac{2}{\sqrt{2x-4x^2}}.$$

Demak,

$$y'=\left[9^{\arcsin(4x-1)}\right]'=9^{\arcsin(4x-1)}\cdot\left[\arcsin(4x-1)\right]'\cdot\ln 9=\frac{2\ln 9}{\sqrt{2x-4x^2}}\cdot 9^{\arcsin(4x-1)}.$$

g) $y=\sin^9 x\cdot\cos^{10}2x$ funksiya hosilasini ko'paytmani differensiallsh qoidasiga asosan topamiz:

$$y'=(\sin^9 x\cdot\cos^{10}2x)'=(\sin^9 x)'\cdot\cos^{10}2x+\sin 9x\cdot(\cos^{10}2x)'=$$
$$=9\sin^8 x\cdot(\sin x)'\cdot\cos^{10}2x+10\sin^9 x\cdot\cos^9 x\cdot(\cos 2x)'=9\sin^8 x\cdot\cos x\cdot\cos^{10}2x+$$
$$+10\sin^9 x\cdot\cos^9 2x\cdot(-\sin 2x\cdot(2x)')=9\sin^8 x\cdot\cos x\cdot\cos^{10}2x-20\sin^9 x\times$$
$$\times\cos^9 2x\cdot\sin 2x=\left(9\cos x\cdot\cos 2x-20\sin x\cdot\sin 2x\right)\cdot\sin^8 x\cdot\cos^9 2x.$$

Izoh: $\left(\sin^9 x\right)'$, $\left(\cos^{10}2x\right)'$ hosilalarni hisoblashda darajali murakkab funksiya hosilasi: $\left(u^n\right)'=nu^{n-1}\cdot u'$ va $\left(\cos u\right)'=-\sin u\cdot u'$, $\left(\sin u\right)'=\cos u\cdot u'$ formulalardan foydalanildi.

d) $y=(\sin 8x)^{\frac{1}{2x}}$ funksiya $[u(x)]^{v(x)}$ ko'rinishdagi darajali-ko'rsatkichli funksiyadir. Bunday funksiya hosilasini topishda *logarifmik differensiallash* qoidasidan foydalanamiz.

$$y=(\sin 8x)^{\frac{1}{2x}}$$

ni logarifmlaymiz: $\ln y=\ln(\sin 8x)^{\frac{1}{2x}}=\frac{1}{2x}\ln(\sin 8x)$ va differensiallaymiz:

$$\frac{y'}{y}=\left(\frac{1}{2x}\right)'\cdot\ln\sin 8x+\frac{1}{2x}\cdot(\ln\sin 8x)'=-\frac{1}{2x^2}\cdot\ln\sin 8x+\frac{1}{2x}\cdot\frac{(\sin 8x)'}{\sin 8x}=$$
$$=-\frac{1}{2x^2}\cdot\ln\sin 8x+\frac{4\cos 8x}{x\sin 8x}.$$

Bundan: $\quad y'=y\cdot\left(-\frac{\ln\sin 8x}{2x^2}+\frac{4\cos 8x}{x\sin 8x}\right)$

yoki $\quad y'=(\sin 8x)^{\frac{1}{2x}}\left(-\frac{\ln\sin 8x}{2x^2}+\frac{4}{x}ctg 8x\right).$ ▲

10 - TOPSHIRIQGA DOIR KO'RSATMALAR

Parametrik tenglamalar bilan berilgan hamda oshkormas ko'rinishda berilgan funksiyalarning hosilalarini topish

Qisqacha nazariy ma'lumotlar

I. *Parametrik tenglamalar bilan berilgan funksiyalarning hosilasini topish.*

$y=f(x)$ funksiya $x=\varphi(t),\ y=\psi(t)$ ko'rinishdagi tenglamalar bilan berilgan bo'lsin, bunda t - parametr.

Ma'lumki, funksiyaning ikkinchi tartibli hosilasi deb uning birinchi tartibli hosilasidan olingan hosilaga aytiladi.

$x=\varphi(t),\ y=\psi(t)$ parametrik tenglamalar bilan berilgan funksiyaning birinchi tartibli $\frac{dy}{dx}=y_x'$ hosilasi qiyidagicha topish mumkin: $x=\varphi(t)$ funksiya uchun $t=\varphi^{-1}(x)$ teskari funksiya mavjud bo'lsin deb faraz qilaylik. Bu holda $x=\varphi(t),\ y=\psi(t)$ funksiyadan $y=\psi(\varphi^{-1}(x))$ murakkab funksiyaga ega bo'lamiz va murakkab funksiya hosilasini topish qoidasiga asosan topamiz:

$$y'_x=y'_t\cdot t'_x,\quad t'_x=\frac{dt}{dx}=\frac{1}{x'_t}$$

bo'lgani uchun(teskari funksiya hosilasi) quydagini hosil qilamiz:

$$y'_x=\frac{y'_t}{x'_t}\quad \text{yoki}\quad \frac{dy}{dx}=\frac{\psi'(t)}{\varphi'(t)}. \tag{1}$$

Endi ikkinchi tartibli hosilani topish formulasini keltiramiz:

$$y''_{xx}=(y'_x)'_x=\left(\frac{y'_t}{x'_t}\right)'_x=\frac{d}{dx}\left(\frac{y'_t}{x'_t}\right)=\frac{d}{dt}\left(\frac{y'_t}{x'_t}\right)\cdot\frac{dt}{dx}=\frac{y''_{tt}x'_t-x''_{tt}y'_t}{(x'_t)^2}\cdot\frac{1}{x'_t}.$$

Demak,

$$y''_{xx}=(y'_x)'_x=\frac{(y'_x)'_t}{x'_t}\quad \text{yoki}\quad y''_{xx}=\frac{y''_{tt}x'_t-x''_{tt}y'_t}{(x'_t)^3}. \tag{2}$$

Qulaylik uchun, shtrix bilan koordinana bo'yicha hosila, nuqta bilan esa parametr bo'yicha hosila belgilanib, (1) va (2) quyidagicha yoziladi:

$$y'=\frac{\dot{y}}{\dot{x}};\qquad y''=\frac{\ddot{y}\,\dot{x}-\ddot{x}\,\dot{y}}{\dot{x}^3}, \tag{3}$$

bu yerda

$$y' = \frac{dy}{dx},\ y'' = \frac{d^2y}{dx^2},\ \dot{y} = y_t' = \psi'(t),\ \dot{x} = x_t' = \varphi'(t),$$

$$\ddot{y} = y_t'' = \psi''(t),\ \ddot{x} = x_t'' = \varphi''(t).$$

Misollarni yechishga doir namunalar

1-*misol*. Parametrik ko'rinishda berilgan $x = a\cos t,\ y = b\sin t$ funksiyaning $\frac{dy}{dx},\ \frac{d^2y}{dx^2}$ hosilalarini toping.

Δ Oldin birinchi tartibli hosilasi $y' = \frac{dy}{dx}$ ni topamiz:

$$y' = \frac{dy}{dx} = \frac{y_t'}{x_t'} = \frac{(b\sin t)'}{(a\sin t)'} = \frac{b\cos t}{-a\sin t} = -\frac{b}{a}ctgt\,.$$

Ikkinchi tartibli hosilani $\frac{d^2y}{dx^2} = y'' = \frac{(y_x')_t'}{x_t'}$ formula bo'yicha topamiz:

$$\frac{d^2y}{dx^2} = y'' = \frac{(-\frac{b}{a}ctgt)_t'}{(a\cos t)_t'} = \frac{-\frac{b}{a}(-\frac{1}{\sin^2 t})}{-a\sin t} = -\frac{b}{a^2}\cdot\frac{1}{\sin^3 t}. \qquad \blacktriangle$$

II. *Oshkormas ko'rinishda berilgan funksiyaning hosilalari*

$$F(x,\ y) = 0 \qquad (1)$$

ko'rinishdagi tenglama birorta $y=f(x)$ funksiyani aniqlasa, u holda (1) tenglama bilan $y=f(x)$ oshkormas funksiya berilgan deyiladi. Bu holda $F(x,\ f(x)) \equiv 0$ bo'ladi. $F(x,\ f(x))$ ni x ning murakkab funksiyasi deyish mumkin. $F(x,\ f(x)) \equiv 0$ ayniyat bo'lgani uchun $F(x,\ f(x))$ funksiyaning hosilasi ham aynan nolga teng bo'ladi. Bu funksiyani differensiallab, quyidagini topamiz:

$$F_x'(x,y) + F_y'(x,y)\cdot y' = 0\,.$$

Bundan

$$y' = \frac{dy}{dx} = -\frac{F_x'(x,y)}{F_y'(x,y)}\,. \qquad (2)$$

Boshqacha qilib aytganda x o'zgarruvchining $y=f(x)$ funksiyasi oshkormas shaklda $F(x,\,y)=0$ tenglama bilan berilgan bo'lsa, u holda y' hosilani topish uchun $F(x,\,y)=0$ tenglikning ikkala qismini x bo'yicha differensiallab, so'ngra hosil bo'lgan tenglamani y' hosilaga nisbatan yechish kerak.

y'' ikkinchi hosila esa y' hosilaning hosilasi bo'lganligi uchun:

$$y''=\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=-\frac{d}{dx}\left(\frac{F'_x(x,y)}{F'_y(x,y)}\right)=-\left(\frac{F'_x(x,y)}{F'_y(x,y)}\right)'_x. \quad (3)$$

Misollarni yechishga doir namunalar

1-*misol*. Agar $x^2+y^2=R^2$ tenglama bilan berilgan oshkormas funksiyaning $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ hosilalarini toping.

Δ Murakkab funksiya hosilasini topish qoidasiga binoan topamiz:

$$(x^2)'_x+(y^2)'_x=(R^2)'; \quad 2x+2y\cdot y'=0; \quad y'=\frac{dy}{dx}=-\frac{x}{y}.$$

Ikkinchi tartibli hosilasini (3) formula bilan topamiz:

$$y''=\frac{d^2y}{dx^2}=\left(-\frac{x}{y}\right)'_x=-\frac{x'y-xy'}{y^2}=-\frac{y-x\cdot(-\frac{x}{y})}{y^2}=-\frac{y^2+x^2}{y^3}=\frac{R^2}{y^3}. \quad ▲$$

2-*misol*. Agar $\ln\sqrt{x^2+y^2}-arctg\,\frac{x}{y}=0$ bo'lsa, y', y'' hosilalarini toping. y''_{xx} topilsin.

Δ Oldingi misoldagi kabi topamiz:

$$(\ln\sqrt{x^2+y^2})'_x-(arctg\,\frac{x}{y})'_x=0, \quad \left[\frac{1}{2}\ln\left(x^2+y^2\right)\right]'-\frac{(\frac{x}{y})'_x}{1+(\frac{x}{y})^2}=0.$$

Bundan

$$\frac{(x^2+y^2)'}{2\left(x^2+y^2\right)}-\frac{1\cdot y-x\cdot y'}{y^2\cdot\frac{(y^2+x^2)}{y^2}}=0,\ \Rightarrow\ \frac{2x+2y\cdot y'}{2(x^2+y^2)}-\frac{y-xy'}{y^2+x^2}=0,$$

$$x+\mathrm{y}\cdot y'-y+xy'=0\Rightarrow(x-y)+(x+y)\cdot y'=0,\ y'=\frac{y-x}{y+x}.$$

Ikkinchi hosilasini topamiz :

$$y''=(\mathrm{y}')'=\left(\frac{y-x}{x+y}\right)'=\frac{(y-x)'\cdot(x+y)-(y-x)(x+y)'}{(x+y)^2}=$$

$$=\frac{(y'-1)(x+y)-(y-x)(1+y')}{(x+y)^2}=\frac{2x\cdot y'-2y}{(x+y)^2}=2\cdot\frac{x\cdot\frac{y-x}{x+y}-y}{(x+y)^2}=$$

$$=2\cdot\frac{x(y-x)-y(x+y)}{(x+y)^3}=-\frac{2(x^2+y^2)}{(x+y)^3}.$$

Shunday qilib, $\quad y''=-\dfrac{2(x^2+y^2)}{(x+y)^3}.\quad$ ▲

11-TOPSHIRIQGA DOIR KO’RSATMALAR

Berilgan chiziqqa o’tkazilgan urinma va normal tenglamalari. Funksiyaning differensiali

Qisqacha nazariy ma’lumotlar

$y=f(x)$ funksiya grafigiga absissasi $x=x_0$ nuqtada o'tkazilgan urinmaning tenglamasi:

$$y=y(x_0)+y'(x_0)(x-x_0)\qquad(1)$$

$y=f(x)$ funksiya grafigiga absissasi $x=x_0$ nuqtada o'tkazilgan normal tenglamasi:

$$y=y(x_0)-\frac{1}{y'(x_0)}(x-x_0)\qquad(2)$$

Agar $f(x)$ funksiya $(a,\ b)$ intervalning har bir $x\in(a,\ b)$ nuqtasida $f'(x)$ hosilaga ega bo’lib, bu $f'(x)$ funksiya $x_0\in(a,b)$ nuqtada hosilaga ega bo'lsa , u $f(x)$ funksiyaning x_0 nuqtadagi ikkinchi tartibli hosilasi deb ataladi. Funksiyaning ikkinchi tartibli hosilasi

$$y''_{x=x_0}, \quad f''(x_0), \quad \left(\frac{d^2y}{dx^2}\right)_{x=x_0}$$

belgilar bilan belgilanadi. $f(x)$ funksiyaning uchinchi , to'rtinchi hokazo tartibdagi hosilalari xuddi shunga o'xshash ta'riflanadi. Umuman, $f(x)$ funksiyaning *n* tartibli hosilasi $y^{(n)} = f^{(n)}(x)$, yoki $\dfrac{d^ny}{dx^n}$ bilan belgilanadi.

Misollarni yechishga doir namunalar

1-*misol.* $y = \ln(e^x + \sqrt{1+e^{2x}})$ funksiya grafigiga $x_0 = 0$ nuqtasiga o'tkazilgan urinma va normal tenglamalarini yozing.

Δ Berilgan funksiyaning hosilasini topamiz:

$$y' = \frac{1}{e^x + \sqrt{1+e^{2x}}} \cdot \left(e^x + \frac{1}{2\sqrt{1+e^{2x}}} \cdot e^{2x} \cdot 2 \right) = \frac{e^x}{\sqrt{1+e^{2x}}}$$

funksiya va uning hosilasi ó' ning x_0 nuqtadagi qiymatini topamiz:

$y_0 = y(0) = \ln(1+\sqrt{2})$, $\quad y'(0) = \dfrac{1}{\sqrt{2}}$, $\quad y = y(x)$ egri chiziqning (x_0, y_0) nuqtadagi urinma tenglamasi: $y = y_0 + y'(x_0)(x - x_0)$ va normal tenglamasi $y = y_0 - \dfrac{1}{y'(x_0)}(x - x_0)$ formulalardan berilgan misol uchun urinma tenglamasi

$$y = \ln(1+\sqrt{2}) + \frac{1}{\sqrt{2}}x$$

va normal tenglamasi

$$y = \ln(1+\sqrt{2}) - \sqrt{2}x$$

bo'ladi. ▲

2-misol. $y = arctg(x + \sqrt{1+x^2})$ funksiyaning ikkinchi tartibli hosilasini toping.

$$\Delta \; y' = \frac{1}{1+(x+\sqrt{1+x^2})} \cdot \left(1 + \frac{x}{\sqrt{1+x^2}}\right) = \frac{1}{2(1+x^2)},$$

$$y''=(y')'=\left(\frac{1}{2(1+x^2)}\right)'=-\frac{1}{2}\cdot\frac{2x}{(1+x^2)^2}=-\frac{x}{(1+x^2)^2}\ .\blacktriangle$$

12-TOPSHIRIQGA DOIR KO'RSATMALAR

Birinchi tartibli differensial yordamida funksiyaning taqribiy qiymatini topish

Qisqacha nazariy ma'lumotlar

Agar $y=f(x)$ funksiya x nuqtada differensiallanuvchi bo'lsa, ya'ni chekli y' hosilaga ega bo'lsa, u holda

$$y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x};\quad \frac{\Delta y}{\Delta x}=y'+\alpha\ \ (\Delta x\to 0\text{ da }\ \alpha\to 0).\ (1)$$

Bundan

$$\Delta y=y'\Delta x+\alpha\Delta x.\qquad (2)$$

Funksiya ortirmasi Δy ning argument orttirmasi Δx ga nisbatan chiziqli bo'lgan bosh qismi $y'\Delta x$ funksiyaning differensiali deyiladi va dy bilan belgilanadi:

$$dy=y'\Delta x\qquad (3)$$

(3) formulada $y=x$ deb $dx=x'\Delta x=1\cdot\Delta x=\Delta x$ ga ega bo'lamiz, shuning uchun

$$dy=y'dx\ \text{ yoki }\ dy=f'(x)dx\qquad (4)$$

Funksiyaning differensiali uning hosilasi bilan argument differensialining ko'paytmasiga teng.

Ixtiyoriy x nuqtada hosilaga ega funksiyaning Δy ortirmasini

$$\Delta y=f(x+\Delta x)-f(x)=F'(x)\Delta x+\alpha\cdot\Delta x=dy+\alpha\cdot\Delta x$$

(bunda $\Delta x\to 0$ da $\alpha\to 0$), ko'rinishda ifodalash mumkun. Kichik Δx larda $\alpha\cdot\Delta x$ cheksiz kichik bo'lganidan

$$\Delta y\approx dy\qquad (1)$$

taqribiy tenglik hosil bo'ladi va uning yordamida funksiyaning $f(x)$ qiymati ma'lum bo'lganda x ga yaqin bo'lgan nuqta(son)dagi taqribiy qiymatlari hisoblanadi. (1) formuladan quyidagi ko'rinishda foydalaniladi:

$$f(x+\Delta x)\approx f(x)+f'(x)\Delta x\ .$$

Misollarni yechishga doir namunalar

1-misol. $y=\frac{1}{6}\cdot\frac{\sin^2 6x}{\cos 12x}$ funksiyaning differensialini toping.

Δ Berilgan funksiyaning hosilasini topamiz:

$$y'=\frac{1}{6}\cdot\frac{2\sin 6x\cos 6x\cdot 6(\cos 12x)-\sin^2 6x(-\sin 12x)\cdot 12}{\cos^2 12x}=\frac{\sin 12x}{\cos^2 12x}.$$

$$y'=\frac{dy}{dx},\ dy=y'dx,\quad dy=\frac{\sin 12x}{\cos^2 12x}dx.\blacktriangle$$

2-*misol*. Birinchi tartibli differensial yordamida $f(x)=\frac{x^2-2x}{x+2}$ funksiyaning $x_0=3{,}01$ bo'lgandagi taqribiy qiymati topilsin.

$$\Delta\ f'(x)=\left(\frac{x^2-2x}{x+2}\right)'=\frac{(x^2-2x)'(x+2)-(x+2)'(x^2-2x)}{(x+2)^2}=\frac{x^2+4x-4}{(x+2)^2},$$

funksiyani $f(x+\Delta x)=\frac{(x+\Delta x)^2-2(x+\Delta x)}{(x+\Delta x)+2}$ ko'rinishda yozib, x_0 ni

$x+\Delta x=3{,}01$ ko'rinishda ifodalab, x=3, Δx =0,01 ni aniqlab, quyidagilarni topamiz:

$$f'(x)\Big|_{x=3}=\frac{x^2+4x-4}{(x+2)^2}\Big|_{x=3}=\frac{17}{25}\approx 0{,}68$$

$$f(x+\Delta x)\Big|_{\substack{x=3\\ \Delta x=0{,}01}}=\frac{(3.01)^2-2\cdot 3.01}{3.01+2}\approx 0{,}6068.$$

Taqribiy hisoblash formulasi: $f(x)=f(x+\Delta x)-f(x)\cong f'(x)\Delta x$ bunda

$$f(x+\Delta x)\cong f(x)+f'(x)\Delta x$$

demak, $f(x)=\frac{x^2-2x}{x+2}\approx 0{,}6068-0{,}68\cdot 0{,}01=0{,}6.$ $\blacktriangle$

3-*misol*. $\sqrt[3]{1+\alpha^3}$ miqdorning yetarli kichik α uchun taqribiy ifodasi topilsin va uning α=0,01 bo'lgandagi taqribiy qiymati hisoblansin.

Δ Differensialning taqribiy hisoblashlarga tadbiqi formulasidan foydalanamiz.

$$f(x+\Delta x)\cong f(x)+f'(x)\Delta x$$

Bunda $f(x)=\sqrt[3]{x}$ desak, $f(x+\Delta x)=\sqrt[3]{x+\Delta x}$, $f'(x)=\frac{1}{3\sqrt[3]{x^2}}$ bo'ladi va

$x+\Delta x=1+\alpha^3$ bo'lganda (x=1, $\Delta x=\alpha^3$):

$$f(\mathrm{x}+\Delta\mathrm{x})=\sqrt[3]{\mathrm{x}+\Delta\mathrm{x}}=\sqrt[3]{1+\alpha^3},\quad f(x)\big|_{x=1}=\sqrt[3]{1}=1,\quad f'(x)\big|_{x=1}=\frac{1}{3}.$$

Shunday qilib: $\sqrt[3]{1+\alpha^3}\approx 1+\frac{1}{3}\alpha^3$. Bu ifodadan α=0,01 da topamiz:

$$\sqrt[3]{1+(0,01)^3}\cong 1+\frac{1}{3}(0,01)^3$$

yoki $\sqrt[3]{1,000001}\approx 1+0,0000003=1,0000003$ ▲

13-TOPSHIRIQGA DOIR KO'RSATMALAR
Maksimum va minimum nazaryasining masalalar yechishda qo'llanilishi

Qisqacha nazariy ma'lumotlar

Funksiyaning maksimum va minimum nazaryasini amaliy masalalar yechishga qo'llash funksiyaning eng katta va eng kichik qiymatini topish bilan bog'liqdir.

[*a; b*] kesmada $y=f(x)$ funksiyani qaraylik. Bu funksiya eng katta va eng kichik qiymatiga kesmaning chegarasida yoki uning ichida erishadi. Bu qiymatlarni quyidagi tartibda topish tavsiya qilinadi:

1. Funksiyaning [*a; b*] kesmadagi barcha kritik nuqtalarni topib, ulardagi funksiya qiymatlarini hisoblash;
2. Funksiyaning kesma chegaralarida: $x=a$ va $x=b$ nuqtalardagi qiymatlarini hisoblash;
3. Bu qiymatlarning ichidan eng katta va eng kichigini aniqlash.

Izoh: Kesmada uzluksiz bo'lgan funksiya, kesma ichida bitta ekstremum nuqtasiga ega bo'lib, bu nuqtada maksimum(minimum)ga ega bo'lsa, bu eng katta (eng kichik) qiymat bo'ladi.

Misollarni yechishga doir namunalar

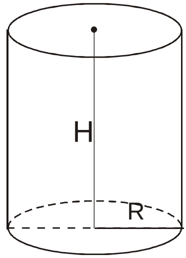
1-*masala*. *V* hajmli yopiq silindrik idish(bak) yasash zarur. Uning o'lchamlari qanday bo'lsa, idishni yasashga eng kam material sarflanadi?

Δ Masalada *V* hajmi berilgan silindrni yasashda kam material sarflanishi uchun, ya'ni silindrning to'la sirti eng kichik bo'lishi uchun, silindr asosi radiusi bilan balandligi qanday munosabatda bo'lishini aniqlash talab qilinadi(20-rasm).

Silindrning to'la sirti: $S=2\pi R^2+2\pi RH$

Masalada bu funksiyaning eng kichik qiymatini topish talab qilinadi.

Hajm berilgan: $V = \pi R^2 H$.



20-rasm

Bundan *H* ni *V* orqali ifodalaymiz: $H = \dfrac{V}{\pi R^2}$.

H ning bu qiymati bilan silindrning to'la sirti $S = 2\pi R^2 + \dfrac{2V}{R} = S(R)$ - *R* argumentning funksiya bo'ladi.

$$S'(R) = (\frac{2V}{R} + 2\pi R^2)' = -\frac{2V}{R^2} + 4\pi R = \frac{4\pi R^3 - 2V}{R^2}$$

bundan *R* o'zgaruvchi bo'yicha hosila olamiz: $S''(R) = \dfrac{4V}{R^3} + 4\pi$

har qanday *R* ga $S''(R) > 0$, $S'(R) = 0$ dan:

$$\frac{4\pi R^3 - 2V}{R^2} = 0 \;\Rightarrow\; 4\pi R^3 - 2V = 0, \quad R = \sqrt[3]{\frac{V}{2\pi}} \ .$$

Ravshanki, $R = \sqrt[3]{\dfrac{V}{2\pi}}$ qiymatda $S''(R) > 0$, ya'ni $S(R)$ funksiya minimumga ega bo'ladi va shu bilan birga u eng kichik qiymat bo'ladi. Ikkinchi tomondan

$$H = \frac{V}{\pi R^2} = \frac{V}{\pi\left(\sqrt[3]{\dfrac{V}{2\pi}}\right)^2} = 2 \cdot \sqrt[3]{\frac{V}{2\pi}} = 2R$$

Shunday qilib, silindr balandligi asos diametriga teng qilib olinsa, uni yasash uchun eng kam material sarflanar ekan. J: $H = 2R$ ▲

2-masala. To'g'ri konus berilgan. Qanday holda unga ichki chizilgan silindr eng katta hajmli bo'ladi?

Δ Konusning balandligi *h*, asosining radiusi *r* bo'lsin(21-rasm), *OB=h, OA=r*. Ichki chizilgan silindrning balandligini *y* bilan, asosining radiusini *x* bilan belgilaylik: *OD=y, OE=x*.

Ravshanki, silindr hajmi $V = \pi x^2 y$ (asos yuzi bilan balandligini ko'paytmasiga) teng va u ikkita *x* va *y* o'zgaruvchilarga bog'liq bo'lib qolmoqda. Bu o'zgaruvchilarni birini (masalan *y* ni), ikkinchisi orqali (*x* orqali) ifodalaymiz. Bu o'zgaruvchilarni bog'laydigan tenglamani tuzamiz: Shakldan: ΔAOB va ΔCDB lar o'xshashligidan $\dfrac{x}{h-y} = \dfrac{r}{h}$ ni topamiz. Bundan: $h - y = \dfrac{x}{r}h, \quad y = h - \dfrac{x}{r}h$.

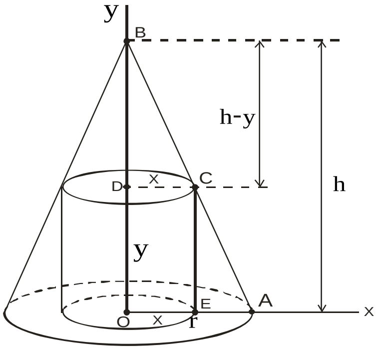
Bunga asosan silindr hajmi formulasi: $V = \pi x^2 (h - \frac{x}{r} h)$ bo'ladi. *V* hajmni *x* ning funksiyasi deb qarash mumkin. Ravshanki, *x* ning o'zgarish sohasi [*0; r*] oraliq bo'ladi. *V* funksiyaning (0; *r*) oraliqdagi eng katta qiymatni topamiz. Buning uchun *V* dan *x* bo'yicha hosila olamiz:

$$V' = \left[\pi x^2 (h - \frac{xh}{r} x) \right]' = 2\pi xh - 3\pi \frac{h}{r} x^2 = \pi h (2x - \frac{3}{r} x^2)$$

Hosilani *0* ga tenglab (0; *r*) oraliqdagi kritik nuqtalarni topamiz.

$$V' = 0; \quad \pi h (2x - \frac{3}{r} x^2) = 0; \quad 2x - \frac{3}{r} x^2 = 0; \quad x(2 - \frac{3}{r} x) = 0; \quad x_1 = 0; \quad 2 - \frac{3}{r} x = 0$$

$\Rightarrow \; x_1 = 0 \quad x_2 = \frac{2r}{3}$; bu yerda $x_1 = 0$ nuqta (0; *r*) ga tegishli emas. $x_2 = \frac{2r}{3}$ kritik nuqta.



21-rasm.

Ikkinchi hosila $V'' = \pi h (2 - \frac{6}{r} x)$ kritik nuqtada

$$V''(\frac{2r}{3}) = \pi h (2 - \frac{6}{r} \cdot \frac{2r}{3}) = \pi h (2 - 4) = -2\pi h < 0 .$$

Demak, *V* funksiya(hajm) $x = \frac{2r}{3}$ qiymatda (nuqtada) maksimumga erishadi va bu maksimum qiymat eng katta qiymat bo'ladi. Shunday qilib, to'g'ri konusga ichki chizilgan silindr asosi radiusi $x = \frac{2r}{3}$ (balandligi esa $y = h - \frac{h}{r} x = \frac{h}{3}$) bo'lgan holda eng katta hajmli bo'ladi. J: $x = \frac{2r}{3}$, $y = \frac{h}{3}$. ▲

14-TOPSHIRIQQA DOIR KO'RSATMALAR
Funksiyani to'la tekshirish va uning grafigini yasash

Qisqacha nazariy ma'lumotlar

Funksiyani to'la tekshirib, uning grafigini yasashni quyidagi tartibda bajarishni tavsiya qilamiz:

1. Funksiyaning aniqlanish sohasi topilsin.

2. Funksiya grafigining koordinata o'qlari bilan kesishish nuqtalari va funksiya o'z ishorasini saqlaydigan intervallar topilsin. Buning uchun tenglamalarning ikkita

$$\begin{cases} y = f(x) \\ y = 0 \end{cases} \text{ va } \begin{cases} y = f(x) \\ x = 0 \end{cases}$$

tenglamalar sistemasini yechish kerek. Birinchi sistema *Ox* o'q, ikkinchisi esa *Oy* o'q bilan kesishish nuqtalarini beradi.

3. Funksiya davriylik, juftlik va toqlikka tekshirilsin;

4. Funksiya uzluksizlikka tekshirilsin. Uzulish nuqtalari topilsin va uzulishlar turlari, uzluksizlik intervallari topilsin.

5. Funksiyaning kesmaning chetki nuqtalaridagi qiymatlari topilsin.

6. Funksiyaning qiymatlari sohasi topilsin.

7. Funksiyaning ekstrimum nuqtalari, bu nuqtalardagi qiymatlari, monotonlik intervallari topilsin.

8. Funksiya grafigining burilish nuqtalari qavariqlik va botiqlik intervallari topilsin.

9. Funksiya grafigi asimptotalari topilsin.

10. Funksiya grafigi yasalsin. Grafik aniqligini yanada oshirish uchun grafikning alohida nuqtalari topilishi kerak.

Grafikni funksiyaning tekshirish jarayonida yasab borish ma'qul bo'ladi. Ba'zan eng avvalo koordinatalar tekisligida, asimptotalarni, ekstremum nuqtalarni, grafikning bukilish nuqtalarini yasab olish qulay bo'ladi.

Misollar yechishga doir namunalar

1-misol. $y = \sqrt[3]{6x^2 - x^3}$ funksiya to'la tekshirilib, grafigi yasalsin.

Δ 1. Funksiyani aniqlovchi tenglik argument x ning har qanday (barcha) qiymatida ma'noga ega. Demak, funksiya $-\infty < x < +\infty$ da, ya'ni $D(f) = (-\infty; \infty)$ oraliqda aniqlangan.

2. Tenglamalarning $\begin{cases} y = f(x) \\ y = 0 \end{cases}$ va $\begin{cases} y = f(x) \\ x = 0 \end{cases}$ sistemalarini,

ya’ni $\begin{cases} y=\sqrt[3]{6x^2-x^3} \\ y=0 \end{cases}$ va $\begin{cases} y=\sqrt[3]{6x^2-x^3} \\ x=0 \end{cases}$ sistemalarini yechamiz. Grafikni *Ox* (y=0) o’q bilan kesishish nuqtasida $\sqrt[3]{6x^2-x^3}=0$ tenglamaga ega bo’lamiz. Bundan $x^2(6-x)=0$ yoki $x_1=x_2=0,\ x_3=6$. Demak, grafik *Ox* o’qni *O*(0; 0) va *A*(6; 0) nuqtalarda kesib o’tadi. Grafikning *Oy* (x=0) o’q bilan kesishish nuqtalarini topish uchun ikkinchi sistemani yechamiz. Undan x=0 ga y=0 ni topamiz. Demak, grafik *Oy* o’qni bitta *O*(0; 0) nuqtada kesib o’tadi. Funksiya bir xil ishoraga ega bo’lgan oraliqlarni topamiz:

a) har qanday $x\in(-\infty;\,6)$ nuqtalarda $y>0$;

b) har qanday $x\in(6;+\infty)$ nuqtalarda $y<0$.

3. Funksiya davriy, juft, toq funksiyalar sinfiga kirmaydi, ya’ni davriy ham, juft ham, toq ham emas.

4. Funksiya elementar funksiyalar sinfiga kiradi. Shu sababli u o’zining aniqlanish sohasi $D(y)=(-\infty;+\infty)$ da uzluksiz, ya’ni uzulish nuqtalariga ega emas.

5. Funksiyani uning aniqlanish sohasining chetlarida tekshiramiz:

$$\lim_{x\to-\infty} y=\lim_{x\to-\infty}\sqrt[3]{6x^2-x^3}=+\infty,\quad \lim_{x\to+\infty} y=\lim_{x\to+\infty}\sqrt[3]{6x^2-x^3}=\lim_{x\to\infty}\sqrt[3]{x^2(6-x)}=-\infty$$

6. Funksiyaning qiymatlari to’plami $E(f)=(-\infty;+\infty)$

7. Funksiyani ekstremumga tekshiramiz:

$$y'=\left[\sqrt[3]{6x^2-x^3}\right]'=\frac{(6x^2-x^3)'}{3\sqrt[3]{(6x^2-x^3)^2}}=\frac{x(4-x)}{\sqrt[3]{(6x^2-x^3)^2}}$$

Kritik nuqtalarni y'=0, ya’ni $\dfrac{x(4-x)}{\sqrt[3]{(6x^2-x^3)^2}}=0$ tenglamadan topamiz:

$x(4-x)=0;\ x_1=0,\ x_2=4.$

Ikkinchi tomondan, hosila x=6 nuqtada mavjud emas, ammo funksiya bu nuqtada aniqlangan.
Demak, x_3= 6 nuqta ham kritik nuqta bo’ladi. Shunday qilib, kritik nuqtalar uchta: $x_1=0;\ x_2=4;\ x_3=6.$

Bu nuqtalar funksiyaning aniqlanish sohasi $(-\infty;+\infty)$ ni to’rtta intervalga ajratadi: $(-\infty;0),\ (0;4),\ (4;6),\ (6;+\infty).$

Endi quyidagi jadvalni tuzamiz:

x	$(-\infty;\,0)$	0	(0; 4)	4	(4; 6)	6	$(6;\,+\infty)$
$y'=f'(x)$	—	0	+	0	—	aniqlanmagan	—

ishorasi								
$y=f(x)$ xarakteri	$\searrow$	min 0	$\nearrow$	max $\left(\sqrt[3]{2}\right)^5$	$\searrow$	0	$\searrow$

Bu yerda $\searrow$ - kamayish, $\nearrow$ - o'sish belgilari.

8. Bukilish nuqtalarini topamiz. Buning uchun ikkinchi y'' hosilani topib,uni nolga tenglashtiramiz:

$$y''=\left(\frac{4x-x^2}{\sqrt[3]{(6x^2-x^3)^2}}\right)'=\frac{-8}{(6-x)^3\cdot\sqrt[3]{(6x^2-x^3)^2}};$$

Bunda x=0 va x=6 nuqtalarda y'' hosila mavjud emasligini ko'rish qiyin emas.

Ma'lumki, y'' hosila 0 ga teng yoki mavjud bo'lmagan nuqtalarda funksiya «botiqlik» yo'nalishini o'zgartirishi mumkin, ya'ni bunday nuqtalar bukilish nuqtalari bo'lishi mumkin. Buning uchun ikkinchi hosila bu nuqtalardan o'tganda o'zining ishorasini almashtirishi kerak. Shuni aniqlash maqsadida quyidagi jadvalni tuzamiz:

x	(-∞;0)	0	(0;6)	6	(6;+∞)
y''	—	bukilish yo'q	—	bukilish nuqtasi	+
y	qavariq	0	qavariq	0	botiq

Shunday qilib, (-∞; 0) oraliqda $y''<0$, (0; 6) oraliqda ham $y''<0$ bo'lgani uchun x=0 nuqta bukilish nuqtasi emas va funksiya grafigi bu oraliqlarda qavariqdir. x=6 nuqta esa bukilish nuqtasi ekan va (6;+∞) oraliqda grafik botiqdir ($y''>0$).

9. Funksiya grafigining $y=kx+b$ ko'rinishdagi asimptotalarini aniqlaymiz, bunda

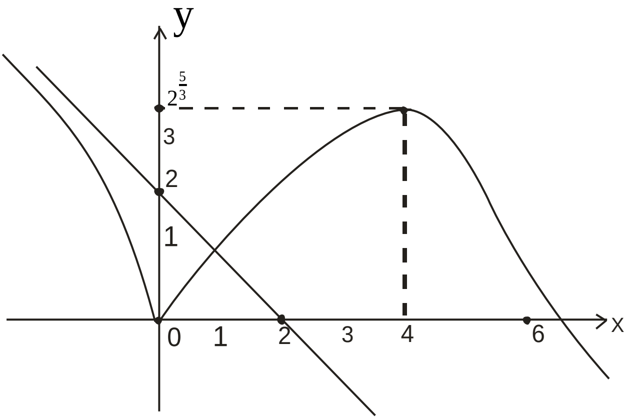
$$k=\lim_{x\to\infty}\frac{y}{x}=\lim_{x\to\infty}\frac{\sqrt[3]{6x^2-x^3}}{x}=\lim_{x\to\infty}\sqrt{\frac{6}{x}-1}=-1\,.$$

$$b=\lim_{x\to\infty}(y-kx)=\lim_{x\to\infty}(\sqrt[3]{6x^2-x^3}+x)=\lim_{x\to\infty}(x\sqrt[3]{\frac{6}{x}-1}+x)=\lim_{x\to\infty}x(\sqrt[3]{\frac{6}{x}-1}+1)=(\infty\cdot 0)=$$

$$=\lim_{x\to\infty}\frac{\sqrt[3]{\frac{6}{x}-1}+1}{\frac{1}{x}}=\lim_{x\to\infty}\frac{\left(\sqrt[3]{\frac{6}{x}-1}+1\right)'}{\left(\frac{1}{x}\right)'}=\lim_{x\to\infty}\frac{\frac{1}{3}(\frac{6}{x}-1)^{-\frac{2}{3}}(-\frac{6}{x^2})}{-\frac{1}{x^2}}=\lim_{x\to\infty}\frac{2}{\sqrt[3]{\left(\frac{6}{x}-1\right)^2}}=2;$$

$b=2.$

Demak, $y=-x+2$ yoki $x+y=2$ og'ma asimptotaning tenglamasi bo'ladi. Boshqa asimptotalar yo'q. Tekshirilgan funksiya grafigi shaklda tasvirlangan(22-rasm). ▲



22- rasm

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

I. NAZARIY SAVOLLAR

1 – topshiriqqa doir savollar

1. 2 - tartibli determinantlar(aniqlovchilar). Determinantning xossalari.
2. 3 - tartibli determinant, uning xossalari, hisoblash usullari.
3. Chiziqli tenglamalar sistemasini Kramer usulida yechish.
4. Matritsa tushunchasi. Matritsaning xususiy hollari (satr-matritsa, ustun-matritsa, birlik matritsa, diagonal-matritsa, nol-matritsa, kvadrat matritsa, simmetrik matritsa).
5. Matritsalar ustida chiziqli amallar (qo'shish, ayirish, songa ko'paytirish).
6. Matritsalarni ko'paytirish.
7. Teskari matritsa va uni topish.
8. Chiziqli tenglamalar sistemasini matritsa usuli bilan yechish.
9. Chiziqli algebraik tenglamalar sistemasini Gauss usuli bilan yechish.
10. Matritsaning rangi. Elementar almashtirishlar. Kengaytirilgan matritsa.
11. Istalgan chiziqli algebraik tenglamalar sistemasini yechish. Kroneker-Kapelli teoremasi.

2 – topshiriqqa doir savollar

Skalyar va vektor miqdorlar. Vektor tushunchasi.

1. Vektorlar ustida chiziqli amallar (qo'shish, ayirish va skalyarga ko'paytirish).
2. Vektorning tashkil etuvchilari va koordinatalari. Bazis tushunchasi.
3. Nuqtaning va vektorning fazoda to'gri burchakli dekart koordinatalari.
4. Ikki vector skalyar ko'paytmasi.
5. Ikki vector orasidagi burchak. Ikki vektorning perpendikulyarlik(ortogonallik) va kolleniarlik shartlari.
6. Ikki vector vektor ko'paytmasining ta'rifi, xossalari.
7. Ikki vector vektor ko'paytmasining geometrik va mexanik ma'nosi.
8. Uch vektorning aralash ko'paytmasi, uning xossalari.
9. Aralash ko'paytmaning geometrik ma'nosi. Uch vektorning komplanarlik sharti.

3 – topshiriqqa doir savollar

1. Son o'qida va tekislikda nuqtaning koordinatalari. Son o'qida va tekislikda ikki nuqta orasidagi masofa.
2. Uchlari berilban ucburchakning va ko'pburchakning yuzi.
3. Kesmani berilgan nisbatda bo'lish. Kesma o'rtasining koordinatalari.

4. Tekislikda analitik geometriya va uning ikki asosiy masalasi. Tekislikda chiziq tenglamasi.
5. Tekislikda to’g’ri chiziiq tenglamasi. To’g’ri chiziiqning umumiy tenglamasi va uni tekshirish.
6. To’g’ri chiziqning burchak koeffitsiyentli va kesmalar bo’yicha tenglamalari.
7. Berlga nuqtadan o’tib berilgan yo’nalishga ega to’g’ri chiziiq tenglamasi. Ikki nuqtadan o’tuvci to’g’ri chiziq tenglamasi.
8. To’g’ri chiziqning normal tenglamasi. Nuqtadan to’g’ri chiziqqacha masofa.
9. Tekislikda ikki to’g’ri chiziq orasidagi burchak.
10. Ikki to’g’ri chiziqning parallellik va perpendikulyarlik sharti.

4 – topshiriqqa doir savollar

1. Ikkinchi tartibli egri chiziq va unung tenglamasining kanonik shakli.
2. Aylana: tairifi, kanonik va umumiy tenglamasi.
3. Aylana va to’g’ri chiziiqning kesishishi.
4. Ellips: ta’rifi, kanonik tenglamasi, shakli.
5. Ellipsning fokusi, ekssenrisiteti, fokal radiuslari, direktrisasi.
6. Giperbola: ta’rifi, kanonik tenglamasi, shakli.
7. Giperbolaning fokusi, ekssenrisiteti, asimptotalari, fokal radiuslari, direktrisasi.
8. Parabola(abssissa o’qiga simmetrik): ta’rifi, kanonik tenglamasi, fokusi, fokal radiusi, direktrisasi tenglamasi, shakli.
9. Parabola(ordinata o’qiga simmetrik): ta’rifi, kanonik tenglamasi, fokusi, fokal radiusi, direktrisasi tenglamasi, shakli.
10. Qutb koordinatalar sistemasida aylana, ellips, giperbola, parabolaning kanonik tenglamalari.

5 – topshiriqqa doir savollar

1. Funksiyaning parametrik ko’rinishda berilishi va uning grafigi.
2. Chiziqning parametrik tenglamalari. Ularni dekart koordinatalarida ifodalash. Parametrni yo’qotish.
3. Parametrik tenglamalari bilan berilgan chiziqni yasash.
4. Aylananing kanonik tenglamasi va uni parametrik tenglamalar ko’rinishida yozish.
5. Ellipsning kanonik tenglamasi va uni parametrik tenglamalar ko’rinishida yozish.
6. Astroidaning parametric tenglamalarini va dekart koordinatalaridagi tenglamasini yozing.

7. $x = x_0 + a\cos t,\ y = y_0 + a\sin t$ tenglamalarni dekart koordinatalarida yozing.
8. $x = 5(t - \sin t),\ y = 5(1 - \cos t)$ tenglamalarni dekart koordinatalarida yozing.
9. $x = 3\cos t,\ y = 2\sin t$ tenglamalarni dekart koordinatalarida yozing.
10. Sikloidaning parametric tenglamalarini va dekart koordinatalaridagi tenglamasini yozing.

6 – topshiriqqa doir savollar

1. Qutb koordinatalar sistemasi va unda nuqtaning o'rnini topish. Misollar keltiring.
2. Nuqtaning dekart va qutb koordinatalari orasidagi bog'lanish formulalari.
3. To'g'ri chiziq tenglamalarini qutb koordinatalarida yozing. Misol keltiring.
4. Aylana va ellipsning tenglamalarini qutb koordinatalarida yozing. Misol keltiring.
5. Giperbola va parabola tenglamalarini qutb koordinatalarida yozing. Misol keltiring.
6. $x = 2,\ \ y = 2,\ y = x + 1 \triangleleft$ tenglamalarni qutb koordinatalarida yozing.
7. Qutb koordinatalar sistemasida $A(0;\ 3),\ B(0;\ -3),\ C(\pi;\ 3),\ D(-\pi;\ 3),\ E(\pi; -2),\ F(-\pi; -2)$ nuqtalarni yasang.
8. Qutb koordinatalar sistemasida nuqtalarni yasang:
$$A(\frac{\pi}{2};\ 4),\ B(\frac{2\pi}{3}; -3),\ C(-\frac{\pi}{3}; -3),\ D(-\frac{3\pi}{2};\ 3),\ E(-\frac{7\pi}{2};\ 5),\ F(-2\pi; -5)$$
9. Tenglamasi qutb koordinatalar sistemasida berilganchiziqni yasang:
$$r = 2\varphi;\quad r = a,\ (a > 0);\ \ r\sin\varphi = 2;\quad r\cos\varphi = -2$$
10. Kordioida, Bernulli lemniskatasi, uch yaproqli gul, Arximed spirali deb ataluvchi chiziqlarning tenglamasini yozing va shaklini chizing.

.

7 – topshiriqqa doir savollar

1. Funksiyaning ta'rifini ayting. Funksiyaning aniqlanish sohasi, qiymatlar to'plami, grafigi deb nimaga aytiladi? Asosiy elementar va elementar funksiyalar deb fanday funksiyalarga aytiladi?
2. Sonli ketma-ketlik deb nimaga aytiladi? O'suvchi, kamayuvchi, o'smaydigan, kamaymaydigan ketma-ketliklar deb qanday ketma-ketliklarga aytiladi.
3. Ketma-ketlik limitining ta'rifini ayting. Misol keltiring. Qanday ketma-ketlik limitga ega bo'ladi?

4. Funksiyaning nuqtadagi va cheksizlikdagi limitlari ta’riflarini ayting. Aniqmasliklar turlarini ayting, izohlang va misollar keltiring.
5. Cheksiz kichik va cheksiz katta funksiyalarning ta’riflarini va xossalarini ayting.
6. Limitning asosiy xossalarini ayting, ba’zilarini isbotlang.
7. Limitga ega funksiyaning chegaralanganligi. Oraliq funksiyaning limiti.
8. Bir tomonlama limitlarning ta’riflarini ayting, misollar keltiring.
9. Birinchi ajoyib limit formulasini ayting, isbotlang, misollar keltiring.
10. Ikkinchi ajoyib limit formulasini ayting, isbotlang, misollar keltiring. e soni va natural logarifm.

8 – topshiriqqa doir savollar

1. Funsiyaning nuqtadagi uzluksizligi ta’riflarini ayting.
2. Nuqtada uzluksiz funksiyaning xossalarini ayting.
3. Nuqtada uzluksiz funksiya uchun $f(x_0)=f(x_0-0)=f(x_0+0)$ shartni tushuntiring. Misol keltiring.
4. Funksiyaning uzilish nuqtalari nima va ular qanday turlarga bo’linadi? Misol keltiring. Funksiyaning berilgan nuqtadagi sakrashi deb nimaga aytiladi?
5. Funksiyaning chapdan va o’ngdan uzluksizligi.
6. Birinchi tur uzilish nuqtasi ta’rifini ayting, misol keltiring. Bo’lakli uzluksiz funksiya nima?
7. Ikkinchi tur uzilish nuqtasi ta’rifini ayting, misol keltiring.
8. Kesmada uzluksiz funksiyaning ta’rifini ayting.
9. Kesmada uzluksiz funksiyaning xossalarini ayting.
10. $y=\dfrac{2}{x-2}$, $y=2^{\frac{1}{x-1}}$, $f(x)=\dfrac{x^2-1}{x-1}$, $y=\begin{cases} x, & agar\ \ x<1; \\ 2x-1, & agar\ \ 1\le x\le 2; \\ x^2, & agar\ \ x>2 \end{cases}$

funksiyalarning uzilish nuqtalarini aniqlang, bu nuqtalarda chap va o’ng limitlarini toping.

9 – topshiriqqa doir savollar

1. Hosilaning ta’rifini ayting. Hosila uchun qanday belgilashlar ishlatiladi?
2. $y=x$, $y=x^2$ ning hosilasini ta’rif yordamida hisoblang.
3. $y=x^3$, $y=\dfrac{1}{x}$ funksiyalar hosilasini ta’rif yordamida hisoblang.

4. Qanday funksiya differensiallanuvchi deyiladi? Differensiallanuvchi funksiya uzluksiz ham bo'lishini isbotlang. Aksinchasi o'rinli bo'lmasligiga misol keltiring.
5. Differensiallash qoidalarini ayting, isbotlang, misollar keltiring.
6. $y = \sin x,\ y = \cos x,\ y = tgx,\ y = ctgx$ funksiyalarning hosilalarini ayting, birortasini isbotlang.
7. $y = a^x,\ y = e^x,\ y = \ln x$ funksiyalarning hosilalarini ayting, birortasini isbotlang.
8. $y = arc\sin x,\ y = \arccos x,\ y = arctgx$ funksiyalarning hosilalarini ayting, birortasini isbotlang.
9. Murakkab va teskari funksiyalar hosilalari qanday topiladi? Misollar keltiring.
10. Hosilalar jadvalini yozing. $y = 5^x + 3\arcsin x - 4$ ning hosilasini toping.

10 – topshiriqqa doir savollar

1. Funksiya differensialining ta'rifini va geometrik ma'nosini ayting.
2. Funksiya differensialining xossalarini ayting.
3. Differensialning invariantlik xossasi nimadan iborat.
4. Yuqori tartibli hosilalar va differensiallar. Leybnits formulasi.
5. Oshkormas funksiya nima va uning hosilasi qanday topiladi? Misol keltiring.
6. Oshkormas funksiyaning ikkinchi tartibli hosilasi qanday topiladi? Misol keltiring.
7. Parametrik ko'rinishda berilgan funksiya nima va uning hosilasi qanday topiladi? Misol keltiring.
8. Parametrik ko'rinishda berilgan funksiyaning ikkinchi tartibli hosilasi qanday topiladi? Misol keltiring.
9. $x(t) = t^3 - 3t + 2,\ y(t) = \cos 2t$ funksiyaning ikkinchi tartibli hosilasi $\frac{d^2y}{dx^2}$ ni toping.
10. $x^3e^{3y} + y^3x = 3x$ funksiyaning ikkinchi tartibli hosilasi $\frac{d^2y}{dx^2}$ ni toping.

11, 12- topshiriqlarqa doir savollar

1. Hosilaning geometrik ma'nosi. Ciziqqa berilgan nuqtada o'tkazilgan urinmaning burchak koeffitsiyenti.
2. Chiziqning berilgan nuqtasida o'tkazilgan urinma va normal tenglamalari.

3. $y=\sqrt{x}$ funksiya grafigiga $x_0=4$ absissali nuqtada o'tkazilgan urinma tenglamasini yozing. Ciziqni va urinmani yasang.
4. $y=\dfrac{4}{x}$ funksiya grafigiga $x_0=2$ absissali nuqtada o'tkazilgan normal tenglamasini yozing. Ciziqni va normalni yasang.
5. Hosilaning mexanik ma'nosini.
6. Moddiy nuqtaning harakat tenglamasi berilganida uning tezligi va tezlanishini topish. Misol keltiring.
7. Harakat qonuni $s(t)=0,25t^4-0,5t^2+4\sqrt{t}$ bo'lgan moddiy nuqtaning $t=4$ vaqt momentidagi tezligini toping.
8. Harakat qonuni $s(t)=\dfrac{4}{t+1}$ bo'lgan moddiy nuqtaning $t=1$ vaqt momentidagi tezlanishini toping.
9. Funksiya differensialining taqribiy hisobda qo'llanilishi formulasini ayting.
10. Funksiya differensialidan foydalanib $\sqrt{98}$, $\sqrt[5]{33}$, $\sqrt{102}$, $\cos 3$, $\sin 0,05$ larni hisoblang.

13 – topshiriqqa doir savollar

1. Funksiya maksimum va minimumining ta'rifini ayting. Ekstremum nima?
2. Ekstremumning zaruriy shartini ayting va isbotlang. Kritik nuqtalar nima?
3. Kritik nuqtada ekstremumga ega bo'lmaydigan funksiyaga misol keltiring.
4. Hosila cheksizga aylanadigan nuqtada ekstremumga ega funksiyaga misol keltiring.
5. Hosila mavjud bo'lmaydigan nuqtada ekstremumga ega funksiyaga misol keltiring.
6. Funksiya ekstremumining yetarli shartini ayting. Misol keltiring.
7. Funksiyani birinchi hosila yordamida ekstremumga tekshirish tartibi (sxemasi)ni ayting. Misol keltiring.
8. Funksiya ikkinchi tartibli hosila yordamida ekstremumga qanday tekshiriladi. Misol keltiring.

14 – topshiriqqa doir savollar

1. Funksiyaning kesmadagi eng katta va eng kichik qiymatlari qanday topiladi? Misol keltiring.
2. Funksiya grafigining asimptotalari nima? Og'ma va veritkal asimptotalar qanday topiladi? Misol keltiring.
3. Funksiya grafigining qavariqligi, botiqligi va burilish (egilish) nuqtalari deb nimaga aytiladi, ular qanday topiladi? Misol keltiring.

4. Funksiyani umumiy tekshirish va grafigini yasash qanday tartibda bajariladi.

II. AMALIY TOPSHIRIQLAR

1 – T O P S H I R I Q

Tenglamalar sistemasini Kramer va matritsa usullari bilan yeching

1. $\begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 11 \end{cases}$

2. $\begin{cases} x_1 - 2x_2 + 3x_3 = 6 \\ 2x_1 - 3x_2 - 4x_3 = 20 \\ 3x_1 - 2x_2 - 5x_3 = 6 \end{cases}$

3. $\begin{cases} 4x_1 - 3x_2 + 2x_3 = 9 \\ 2x_1 + 5x_2 - 3x_3 = 4 \\ 5x_1 + 6x_2 - 2x_3 = 18 \end{cases}$

4. $\begin{cases} x_1 + x_2 + 2x_3 = -1 \\ 2x_1 - x_2 + 2x_3 = -4 \\ 4x_1 + x_2 + 4x_3 = -2 \end{cases}$

5. $\begin{cases} 2x_1 - x_2 - x_3 = 4 \\ 3x_1 + 4x_2 - 2x_3 = 11 \\ 3x_1 - 2x_2 + 4x_3 = 11 \end{cases}$

6. $\begin{cases} 3x_1 + 4x_2 + 2x_3 = 8 \\ 2x_1 - x_2 - 3x_3 = -4 \\ x_1 + 5x_2 + x_3 = 0 \end{cases}$

7. $\begin{cases} x_1 + x_2 + x_3 = 1 \\ 8x_1 + 3x_2 - 6x_3 = 2 \\ 4x_1 + x_2 - 3x_3 = 3 \end{cases}$

8. $\begin{cases} x_1 - 4x_2 - 2x_3 = -3 \\ 3x_1 + x_2 + x_3 = 5 \\ 3x_1 - 5x_2 - 6x_3 = -9 \end{cases}$

9. $\begin{cases} 7x_1 - 5x_2 = 31 \\ 4x_1 + 11x_3 = -43 \\ 2x_1 + 3x_2 + 4x_3 = -20 \end{cases}$

10. $\begin{cases} x_1 + 2x_2 + 4x_3 = 31 \\ 5x_1 + x_2 + 2x_3 = 20 \\ 3x_1 - x_2 + x_3 = 9 \end{cases}$

11. $\begin{cases} x_1 - x_2 + x_3 = 6 \\ 2x_1 + x_2 + x_3 = 3 \\ x_1 + x_2 + x_3 = 5 \end{cases}$

12. $\begin{cases} x_1 + x_2 - x_3 = 2 \\ -2x_1 + x_2 + x_3 = 0 \\ x + x_2 + x_3 = 6 \end{cases}$

13. $\begin{cases} 3x_1 - x_2 = 5 \\ -2x_1 + x_2 + x_3 = 0 \\ 2x_1 - x_2 + 4x_3 = 15 \end{cases}$

14. $\begin{cases} 5x_1 + 4x_3 = 1 \\ x_1 - x_2 + 2x_3 = 0 \\ 4x_1 + x_2 + 2x_3 = 1 \end{cases}$

15. $\begin{cases} 2x_1 + 2x_2 - x_3 = 4 \\ 3x_1 + x_2 - 3x_3 = 7 \\ x_1 + x_2 + 2x_3 = 3 \end{cases}$

16. $\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 + 2x_2 + 2x_3 = 2 \\ 2x_1 + 3x_2 + 3x_3 = 3 \end{cases}$

17. $\begin{cases} 2x_1 + 3x_2 + 5x_3 = 10 \\ 3x_1 + 7x_2 + 4x_3 = 3 \\ x_1 + 2x_2 + 2x_3 = 3 \end{cases}$

18. $\begin{cases} 5x_1 - 6x_2 + 4x_3 = 3 \\ 3x_1 - 3x_2 + 2x_3 = 2 \\ 4x_1 - 5x_2 + 2x_3 = 1 \end{cases}$

19. $\begin{cases} 4x_1 - 3x_2 + 2x_3 = -4 \\ 6x_1 - 2x_2 + 3x_3 = -1 \\ 5x_1 - 3x_2 + 2x_3 = -3 \end{cases}$

20. $\begin{cases} 5x_1 + 2x_2 + 3x_3 = -2 \\ 2x_1 - 2x_2 + 5x_3 = 0 \\ 3x_1 + 4x_2 + 2x_3 = -10 \end{cases}$

21. $\begin{cases} 2x_1 + x_2 = 3 \\ 3x_1 + 4x_3 = -1 \\ x_1 - x_2 + 2x_3 = -2 \end{cases}$

22. $\begin{cases} 2x_1 + 3x_3 = 4 \\ 4x_1 + x_2 = 6 \\ 2x_1 - x_2 - 2x_3 = 0 \end{cases}$

23. $\begin{cases} 2x_1 + x_2 + 2x_3 = 6 \\ 3x_1 + 2x_3 = 8 \\ x_1 + x_2 - x_3 = 1 \end{cases}$

24. $\begin{cases} 3x_2 + 2x_3 = 5 \\ 2x_1 + x_2 - x_3 = 4 \\ -x_2 + 2x_3 = 1 \end{cases}$

25. $\begin{cases} x_1 + 2x_3 = 5 \\ 3x_1 - x_2 = 9 \\ x_1 + x_2 - 2x_3 = 1 \end{cases}$

26. $\begin{cases} x_1 + x_2 + 2x_3 = 4 \\ 2x_2 + x_3 = 3 \\ x_1 - x_2 = 2 \end{cases}$

27. $\begin{cases} 2x_1 + x_2 - 3x_3 = 4 \\ 4x_1 - x_2 = 2 \\ 2x_1 + 3x_2 - x_3 = 8 \end{cases}$

28. $\begin{cases} 7x_1 - 5x_2 + x_3 = 3 \\ 5x_1 - 2x_2 - x_3 = 2 \\ 2x_1 + x_2 = 3 \end{cases}$

29. $\begin{cases} 6x_1 - 2x_2 + x_3 = 2 \\ 2x_1 + x_2 - 3x_3 = 4 \\ x_1 - x_2 + x_3 = -1 \end{cases}$

30. $\begin{cases} 5x_1 + x_2 - x_3 = 6 \\ 4x_1 - 2x_2 = -3 \\ x_1 + 2x_2 + x_3 = 12 \end{cases}$

2 – T O P S H I R I Q

1 - 15 - *masalalarda* A_1, A_2, A_3, A_4 *nuqtalar berilgan. Vektorlar algebrasi usullaridan foydalanib, quyidagilarni toping:*

1) $\overrightarrow{A_1A_2}$ *vektorning uzunligini va* A_1A_2 *to'g'ri chiziq tenglamasini;*
2) $\overrightarrow{A_1A_2}$ *va* $\overrightarrow{A_1A_3}$ *vektorlar orasidagi burchakni;*
3) $\overrightarrow{A_1A_2}$ *vektorning* $\overrightarrow{A_1A_3}$ *vektorga preksiyasini;*
4) $\overrightarrow{A_1A_2}$ *va* $\overrightarrow{A_1A_3}$ *vektorlarning vektor ko'paytmasini* $A_1A_2A_3$ *yoqning yuzini;*
5) $A_1A_2A_3A_4$ *piramidani hajmini toping;*
6) $A_1A_2A_3$ *yoqqa tushirilgan balandlikni toping va yasang.*

1. $A_1(1;-2;1),\ A_2(-2;3;0),\ A_3(1;0;5),\ A_4(0;2;-3)$.
2. $A_1(-3;0;3),\ A_2(2;3;-1),\ A_3(1;-3;2),\ A_4(0;5;2)$.
3. $A_1(0;-3;3),\ A_2(2;0;3),\ A_3(2;-4;0),\ A_4(1;2;5)$.
4. $A_1(6;1;5),\ A_2(4;5;-2),\ A_3(-1;3;0),\ A_4(1;-1;6)$.
5. $A_1(1;-2;1),\ A_2(-2;3;0),\ A_3(1;0;5),\ A_4(0;2;-3)$.
6. $A_1(-2;1;0),\ A_2(2;-1;0),\ A_3(3;1-2),\ A_4(-1;1;6)$.
7. $A_1(2;2;5),\ A_2(-1;3;0),\ A_3(0;2;-3),\ A_4(3;1;2)$.
8. $A_1(4;0;0),\ A_2(-2;1;2),\ A_3(2;2;3),\ A_4(0;-2;5)$.
9. $A_1(3;-2;6),\ A_2(2;0;5),\ A_3(1;3;2),\ A_4(0;1;-3)$.
10. $A_1(5;6;-3),\ A_2(3;4;-2),\ A_3(0;-3;-2),\ A_4(2;3;4)$.
11. $A_1(-1;-2;-2),\ A_2(1;2;0),\ A_3(3;0\ 4),\ A_4(0;-2;5)$.
12. $A_1(3;0;1),\ A_2(1;-2;-1),\ A_3(2;2;2),\ A_4(1;0;4)$.
13. $A_1(2;-1;2),\ A_2(4;0;0),\ A_3(0;4;-1),\ A_4(1;1;5)$.
14. $A_1(2;-3;1),\ A_2(1;-3;-1),\ A_3(0;3;0),\ A_4(-1;1;4)$.
15. $A_1(3;-3;1),\ A_2(-1;3;-1),\ A_3(4;-3;2),\ A_4(4;2;0)$.

16 - 30 - masalalarda $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlar koordinatalari bilan berilgan. Vektorlar algebrasi usullaridan foydalanib, quyidagilarni toping:

1) $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchakni;
2) $\vec{a}$ va $\vec{b}$ vektorlarning vektor ko'paytmasini hamda $\vec{a}$ va $\vec{b}$ ga yasalgan uchburchakning yuzini;

3) $\vec{a}$ vektorning $\vec{c}$ vektorga proyeksiyasini;

4) $\vec{a}, \vec{b}, \vec{c}$ vektorlarning aralash ko'paytmasini va ularga yasalgan piramidaning hajmini;

5) piramidaning $\vec{a}$ va $\vec{b}$ vektorlardan hosil qilingan yog'iga tushirilgan balandligini.

16. $\vec{a} = 3\vec{i} - 4\vec{j} + \vec{k},\ \vec{b} = -\vec{i} + 3\vec{j} + 2\vec{k},\ \ \vec{c} = 2\vec{j} + 6\vec{k}$.

17. $\vec{a} = -\vec{i} - 3\vec{j} + \vec{k},\ \vec{b} = 2\vec{i} + 4\vec{j},\ \ \vec{c} = -4\vec{i} + 2\vec{j} + 3\vec{k}$.

18. $\vec{a} = 2\vec{i} - 3\vec{j} + 2\vec{k},\ \vec{b} = 4\vec{j} + 2\vec{k},\ \ \vec{c} = 5\vec{k}$.

19. $\vec{a} = 4\vec{j} - 3\vec{k},\ \vec{b} = \vec{i} + 3\vec{j} - \vec{k},\ \ \vec{c} = -\vec{i} - 2\vec{j} + 4\vec{k}$.

20. $\vec{a} = 3\vec{i} - 5\vec{j},\ \vec{b} = 2\vec{i} + \vec{j} + 2\vec{k},\ \ \vec{c} = -\vec{i} + 2\vec{j} - 4\vec{k}$.

21. $\vec{a} = -4\vec{i} + \vec{j} + \vec{k},\ \vec{b} = \vec{i} - 3\vec{j} - \vec{k},\ \ \vec{c} = 3\vec{j} + 2\vec{k}$.

22. $\vec{a} = -3\vec{i} + 3\vec{j} + \vec{k},\ \vec{b} = \vec{i} + \vec{j} - \vec{k},\ \ \vec{c} = -\vec{i} + 3\vec{j} - \vec{k}$.

23. $\vec{a} = 3\vec{i} - 5\vec{j} + 2\vec{k},\ \vec{b} = 2\vec{i} + \vec{j} + 2\vec{k},\ \ \vec{c} = -\vec{i} + 2\vec{j} - 4\vec{k}$.

24. $\vec{a} = 3\vec{i} - 5\vec{j} + 2\vec{k},\ \vec{b} = 2\vec{i} + \vec{j} + 2\vec{k},\ \ \vec{c} = -\vec{i} + 2\vec{j} - 4\vec{k}$.

25. $\vec{a} = 3\vec{i} - 5\vec{j} + 2\vec{k},\ \vec{b} = 2\vec{i} + \vec{j} + 2\vec{k},\ \ \vec{c} = -\vec{i} + 2\vec{j} - 4\vec{k}$.

26. $\vec{a} = 3\vec{i} - 5\vec{j} + 2\vec{k},\ \vec{b} = 2\vec{i} + \vec{j} + 2\vec{k},\ \ \vec{c} = -\vec{i} + 2\vec{j} - 4\vec{k}$.

27. $\vec{a} = 3\vec{i} - 5\vec{j} + 2\vec{k},\ \vec{b} = 2\vec{i} + \vec{j} + 2\vec{k},\ \ \vec{c} = -\vec{i} + 2\vec{j} - 4\vec{k}$.

28. $\vec{a} = 3\vec{i} - 5\vec{j} + 2\vec{k},\ \vec{b} = 2\vec{i} + \vec{j} + 2\vec{k},\ \ \vec{c} = -\vec{i} + 2\vec{j} - 4\vec{k}$.

29. $\vec{a} = 3\vec{i} - 5\vec{j} + 2\vec{k},\ \vec{b} = 2\vec{i} + \vec{j} + 2\vec{k},\ \ \vec{c} = -\vec{i} + 2\vec{j} - 4\vec{k}$.

30. $\vec{a} = 3\vec{i} - 5\vec{j} + 2\vec{k},\ \vec{b} = 2\vec{i} + \vec{j} + 2\vec{k},\ \ \vec{c} = -\vec{i} + 2\vec{j} - 4\vec{k}$.

3 – T O P S H I R I Q

Uchlari A, B, C nuqtalarda bo'lgan $\triangle ABC$ uchburchakning:

a) AC tomonining uzunligini va AC tomonining tenglamasini;

b) B uchidan tushirilgan balandlikning uzunligini va tenglamasini;

c) $\triangle ABC$ uchburchakning yuzini toping.

1. $A(-8; -2)$, $B(8; 0)$, $C(-4; -5)$
2. $A(-9; -1)$, $B(7; 1)$, $C(-5; -4)$
3. $A(8; 0)$, $B(-4; -5)$, $C(5; 4)$
4. $A(10; 11)$, $B(-2; -6)$, $C(6; 3)$
5. $A(6; 1)$, $B(-5; -4)$, $C(9; 5)$
6. $A(1; 1)$, $B(-4; 0)$, $C(-3; -3)$
7. $A(0; 4)$, $B(-4; 0)$, $C(3; 0)$
8. $A(-6; 0)$, $B(2; 3)$, $C(2; -6)$

9. $A(3; 0)$, $B(-4; 2)$, $C(7; -3)$
10. $A(6; 0)$, $B(2; 2)$, $C(3; 4)$
11. $A(4; 5)$, $B(1; 1)$, $C(8; 2)$
12. $A(8; 0)$, $B(-1; -1)$, $C(0; 6)$
13. $A(12; 2)$, $B(1; 1)$, $C(9; 6)$
14. $A(-7; 2)$, $B(-1; 8)$, $C(-10; 6)$
15. $A(-3; 0)$, $B(2; 3)$, $C(4; 1)$
16. $A(12; 9)$, $B(0; 6)$, $C(11; 2)$
17. $A(-3; -4)$, $B(-1; 1)$, $C(4; -3)$
18. $A(0; 7)$, $B(-4; 0)$, $C(1; 0)$
19. $A(0; -6)$, $B(-1; 2)$, $C(8; 0)$
20. $A(-3; -2)$, $B(-2; 4)$, $C(1; 1)$
21. $A(10; 4)$, $B(1; 6)$, $C(6; 1)$
22. $A(6; 2)$, $B(0; 4)$, $C(-2; -4)$
23. $A(-6; -2)$, $B(-4; 6)$, $C(2; 4)$
24. $A(-10; -4)$, $B(-8; 2)$, $C(-6; -1)$
25. $A(8; 0)$, $B(1; -4)$, $C(5; 4)$
26. $A(6; 2)$, $B(2; 4)$, $C(-6; -3)$
27. $A(6; 3)$, $B(1; 4)$, $C(-6; -2)$
28. $A(12; 0)$, $B(-2; 1)$, $C(0; 5)$
29. $A(7; 8)$, $B(0; 4)$, $C(2; -4)$
30. $A(4; 0)$, $B(1; 2)$, $C(0; -3)$.

4 – T O P S H I R I Q

Berilgan ikkinchi tartibli chiziqning turini aniqlang, fokuslarini va asimptotalarini toping, grafigini yasang.

1. $16x^2+4y-16=0$
2. $9x^2-16y^2-144=0$
3. $4x^2+16y^2-16=0$
4. $x^2-2x+y^2=0$
5. $x^2-4x+y^2=0$
6. $2x-y^2+4=0$
7. $9x^2+25y^2-225=0$
8. $\frac{x^2}{16}+\frac{y^2}{9}=1$
9. $\frac{x^2}{64}+\frac{y^2}{36}=1$
10. $\frac{x^2}{25}-\frac{y^2}{16}=1$
11. $x^2-4x+y^2+3=0$
12. $x^2+y^2-2y=0$
13. $x^2-2x+1+y^2-2y=0$
14. $y^2+4y=x$
15. $3x+y^2=9$
16. $x^2+3y=9$
17. $x^2=y+2$
18. $x^2-4y^2+8x-24y=24$
19. $x^2-4y^2=16$
20. $9x^2-25y^2=225$
21. $x^2+y^2-6y-9=0$
22. $x^2+y^2+x+y=0$
23. $x^2+4y^2-6x+8y=3$
24. $\frac{x^2}{25}+\frac{y^2}{16}=1$
25. $\frac{x^2}{100}+\frac{y^2}{36}=1$
26. $9x^2+16y^2=144$
27. $9x^2-36y^2=324$
28. $4y=4x-x^2$
29. $y^2=6x-6$
30. $y^2=x+1$

5 – T O P S H I R I Q

Parametrik tenglamalar bilan berilgan chiziqni:

a) *chizing;*

b) *tenglamadan* t *parametrni yo'qoting.*

1. $\begin{cases} x = \cos 2t, \\ y = \sin^2 t. \end{cases}$

2. $\begin{cases} x = \dfrac{t^1}{1+t^2}, \\ y = \dfrac{t^1}{1+t^2}. \end{cases}$

3. $\begin{cases} x = t^3, \\ y = t^2. \end{cases}$

4. $\begin{cases} x = \ln\left(1+t^2\right), \\ y = t^2. \end{cases}$

5. $\begin{cases} x = \arcsin t, \\ y = \sqrt{1-t^2}. \end{cases}$

6. $\begin{cases} x = 7t^2 - 3, \\ y = 5t. \end{cases}$

7. $\begin{cases} x = 2 - 3t - 6t^2, \\ y = 3 - \dfrac{3}{2}t - 3t^2. \end{cases}$

8. $\begin{cases} x = arctgt, \\ y = \sqrt{1+t^2}. \end{cases}$

9. $\begin{cases} x = \dfrac{1}{\cos t}, \\ y = tgt. \end{cases}$

10. $\begin{cases} x = 8\cos^2 \dfrac{\pi}{4}t - 7, \\ y = -8\sin^2 \dfrac{\pi}{4}t + 7. \end{cases}$

11. $\begin{cases} x = -4\cos \dfrac{\pi}{3}t, \\ y = -2\sin^2 \dfrac{\pi}{3}t - 3. \end{cases}$

12. $\begin{cases} x = \ln t, \\ y = t^3. \end{cases}$

13. $\begin{cases} x = e^{-t}, \\ y = t^3. \end{cases}$

14. $\begin{cases} x = 3\cos t, \\ y = 2\sin t. \end{cases}$

15. $\begin{cases} x = 2\cos 2t, \\ y = 3\sin 2t. \end{cases}$

16. $\begin{cases} x = t^2, \\ y = 2t. \end{cases}$

17. $\begin{cases} x = \dfrac{3t^2}{1+t^3}, \\ y = \dfrac{3t^2}{1+t^3}. \end{cases}$

18. $\begin{cases} x = 2t\cos t, \\ y = 2t\sin t. \end{cases}$

19. $\begin{cases} x = 2t^2, \\ y = t^3. \end{cases}$

20. $\begin{cases} x = \ln t, \\ y = t^2 - 1. \end{cases}$

21. $\begin{cases} x = 2\cos^3 t, \\ y = 2\sin^3 t. \end{cases}$

22. $\begin{cases} x = 3t^2 + 2, \\ y = -2t. \end{cases}$

23. $\begin{cases} x = \ln t, \\ y = \dfrac{1}{3+2t}. \end{cases}$

24. $\begin{cases} x = 2\left(t - \sin t\right), \\ y = 2\left(1 - \cos t\right). \end{cases}$

25. $\begin{cases} x = t^2, \\ y = t^2 + t. \end{cases}$

26. $\begin{cases} x = 4\cos t, \\ y = 3\sin t. \end{cases}$

27. $\begin{cases} x = 3t^2, \\ y = 3t - t^2. \end{cases}$

28. $\begin{cases} x = 3\cos^2 t, \\ y = 4\sin^2 t. \end{cases}$

29. $\begin{cases} x = -2t, \\ y = t^3. \end{cases}$

30. $\begin{cases} x = \ln 3t, \\ y = \dfrac{2}{t}. \end{cases}$

6 – T O P S H I R I Q

Tenglamasi qutb koordinatalar sistemasida berilgan chiziqni:
a) *chizing;*
b) *chiziq tenglamasini dekart koordinatalarida yozing.*

1. $r = 3 + \sin\varphi.$ 2. $r = \dfrac{9}{4 - 5\cos\varphi}.$ 3. $r = \dfrac{1}{\sin\varphi}.$

4. $r = 4\cos\varphi.$ 5. $r = 2\left(1 + \cos\varphi\right).$ 6. $r\cos\varphi = 4;$

7. $r = 2(1 - \cos\varphi).$ 8. $r^2 = 4\cos 2\varphi.$ 9. $r = 2\sin^2\varphi.$

10. $r = 6\cos^2\varphi.$ 11. $r = \dfrac{2}{2 + \cos\varphi}.$ 12. $r = \dfrac{1}{\cos^2\varphi}.$

13. $r = 3\cos 2\varphi.$ 14. $r = 2 - \sin\varphi.$ 15. $r = \dfrac{3}{2 + \sin\varphi}.$

16. $r = \dfrac{4}{1 - \cos\varphi}.$ 17. $r = 2 - \cos\varphi.$ 18. $r = \dfrac{2}{\cos\varphi}.$

19. $r = 4\sin\varphi.$ 20. $r = \dfrac{4}{2 - \cos\varphi}.$ 21. $\varphi = \dfrac{\pi}{3}.$

22. $r = 2\varphi.$ 23. $r = \dfrac{\varphi}{\pi}.$ 24. $r = 2\sin\varphi.$

25. $r = 3\sin 2\varphi.$ 26. $r = 2 - \cos 2\varphi.$ 27. $r = \dfrac{3}{2 + \cos\varphi}.$

28. $r = \dfrac{4}{2 + 2\cos\varphi}.$ 29. $r = 3 + 4\cos\varphi.$ 30. $r = \dfrac{1}{\cos^2 2\varphi}.$

7-TOPSHIRIQ

Limitni Lopital qoidasidan foydalanmasdan hisoblang

1. a) $\lim\limits_{x\to\infty}\dfrac{4x^5 - 3x^2 + 7}{6x^6 + 3x^3 + 1}$ b) $\lim\limits_{x\to 1}\dfrac{3x^2 - 4x + 1}{x^2 - 1}$

c) $\lim\limits_{x\to 0}\dfrac{\cos 4x - 1}{x tg 3x}$ d) $\lim\limits_{x\to\infty}\left(\dfrac{x+5}{x+8}\right)^{2x+3}.$

2. a) $\lim\limits_{x\to\infty}\dfrac{4+5x^2-3x^4}{8-3x-x^4}$ b) $\lim\limits_{x\to5}\dfrac{2-\sqrt{2x-6}}{1-\sqrt{x-4}}$;

c) $\lim\limits_{x\to0}\dfrac{\cos3x-\cos5x}{x^2}$ d) $\lim\limits_{x\to+\infty}(2x-3)[\ln(x-2)-\ln(x+1)]$.

3. a) $\lim\limits_{x\to\infty}\dfrac{5x^4-1}{x^4+3x^2+4}$ b) $\lim\limits_{x\to0}\dfrac{\sqrt{1-2x-x^2}-(1+x)}{x}$

c) $\lim\limits_{x\to0}\dfrac{\sin^2\frac{2x}{7}}{1-\sqrt{1-x^2}}$ d) $\lim\limits_{x\to0}(1-5x)^{\frac{1}{3x}}$.

4. a) $\lim\limits_{x\to\infty}\dfrac{2x^4-3x^2+1}{5x^4-3x+1}$ b) $\lim\limits_{x\to4}\dfrac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$

c) $\lim\limits_{x\to0}\dfrac{\cos x-\cos^3x}{x^2}$ d) $\lim\limits_{x\to\infty}\left(\dfrac{x^2+1}{x^2-1}\right)^{x^2}$.

5. a) $\lim\limits_{x\to\infty}\dfrac{x^8-x^7+1}{5x^8+7x^7+x^6}$ b) $\lim\limits_{x\to-3}\dfrac{9-x^2}{1-\sqrt{4+x}}$

c) $\lim\limits_{x\to0}\dfrac{8x^4}{\sin^4 5x}$ d) $\lim\limits_{x\to\infty}2x\ln(1+\dfrac{1}{x})$.

6. a) $\lim\limits_{x\to\infty}\dfrac{3x^6-2x^4-7}{9x^6+3x^3+5}$ b) $\lim\limits_{x\to-1}\dfrac{\sqrt{2x+3}-1}{\sqrt{5+x}-2}$

c) $\lim\limits_{x\to0}\dfrac{\cos x-\cos^4x}{4x^2}$ d) $\lim\limits_{x\to+\infty}(x+2)[\ln(2x+1)-\ln(2x-1)]$.

7. a) $\lim\limits_{x\to\infty}\dfrac{(x-1)^3}{1-5x^3}$ b) $\lim\limits_{x\to3}\dfrac{\sqrt{x+1}-2}{\sqrt{x-2}-1}$

c) $\lim\limits_{x\to0}\dfrac{1-\cos2x}{x^2}$ d) $\lim\limits_{x\to\infty}(\dfrac{x+1}{x-2})^{x^2}$.

8. a) $\lim\limits_{x\to\infty}\dfrac{6x^5-11x^2+3}{5x-x^5}$ b) $\lim\limits_{x\to+\infty}(\sqrt{x+2}-\sqrt{x})$

c) $\lim\limits_{x\to0}\dfrac{\arcsin2x}{3x}$ d) $\lim\limits_{x\to\infty}x[\ln(x+1)-\ln x]$.

9. a) $\lim\limits_{x\to\infty}\dfrac{8x^5-x^4+5}{x-3x^5}$ b) $\lim\limits_{x\to\infty}x(\sqrt{x^2+1}-x)$

c) $\lim\limits_{x\to 0}\dfrac{\cos 6x-1}{xtg2x}$ d) $\lim\limits_{x\to\infty}(\dfrac{x+5}{x+8})^{2x+3}$.

10. a) $\lim\limits_{x\to\infty}\dfrac{3x^2-5x+1}{x^3+1}$ b) $\lim\limits_{x\to -2}\dfrac{2-\sqrt{6+x}}{2+x}$

c) $\lim\limits_{x\to 0}\dfrac{arctg2x}{x}$ d) $\lim\limits_{x\to 0}(1+3x)^{\frac{2x+1}{3x}}$.

11. a) $\lim\limits_{x\to\infty}\dfrac{x-2x^2+7x^3}{2+3x^2+x^3}$ b) $\lim\limits_{x\to 1}\dfrac{\sqrt{3x+1}-2}{\sqrt{8x+1}-3}$

c) $\lim\limits_{x\to 0}\dfrac{x^2ctg2x}{\sin 3x}$ d) $\lim\limits_{x\to +\infty}(2x+3)[\ln(x+1)-\ln x]$.

12. a) $\lim\limits_{x\to\infty}\dfrac{2+x^2-3x^3}{1-3x+6x^3}$ b) $\lim\limits_{x\to 2}\dfrac{x^2-4}{\sqrt{2x}-2}$

b) $\lim\limits_{x\to 0}\dfrac{\arcsin 5x}{\sqrt{3+x}-\sqrt{3}}$ d) $\lim\limits_{x\to 0}(1-x^2)^{\frac{3}{x^2}}$.

13. a) $\lim\limits_{x\to\infty}\dfrac{(1+x^2)(2+3x)}{(1+2x^2)(3+2x)}$ b) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+4}-2}{x\sqrt{x+4}}$

c) $\lim\limits_{x\to 0}\dfrac{\arcsin 3x}{\sqrt{4+x}-2}$ d) $\lim\limits_{x\to 0}(\cos 2x)^{1/5x^2}$.

14. a) $\lim\limits_{x\to\infty}\dfrac{x^2+x-2}{x^2-1}$ b) $\lim\limits_{x\to\infty}(\sqrt{x^2-3x}-x)$

c) $\lim\limits_{x\to 0}\dfrac{\sqrt{9+x}-3}{arctg2x}$ d) $\lim\limits_{x\to\infty}2x[\ln(3+x)-\ln x]$.

15. a) $\lim\limits_{x\to\infty}\dfrac{3x^3-7x+5}{4x^3+8x-3}$ b) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x^2}-1}{x^2+x^3}$

c) $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{\cos 3x-\cos x}$ d) $\lim\limits_{x\to\infty}3x[\ln(5+2x)-\ln x]$.

16. a) $\lim\limits_{x\to 0}\dfrac{\arcsin 2x}{7x}$ b) $\lim\limits_{x\to 4}\dfrac{3x-12}{\sqrt{3x-8}-2}$

c) $\lim\limits_{x\to 0}\dfrac{1-\cos 4x}{\cos x-\cos 3x}$ d) $\lim\limits_{x\to\infty}(1-\dfrac{2}{x})^{5x}$.

17. a) $\lim\limits_{x\to\infty}\dfrac{(3x-2)(1-x)(2+x)}{2x^3+x^2-3}$ b) $\lim\limits_{x\to 0}\dfrac{\sqrt{16+x}-\sqrt{16-x}}{x^2+6x}$

c) $\lim\limits_{x\to 0}\dfrac{\arcsin 2x}{7x}$ d) $\lim\limits_{x\to 0}\dfrac{\ln(2-3x)}{5x}$.

18. a) $\lim\limits_{x\to\infty}\dfrac{(5-x)(2x-1)}{5x^2+4}$ b) $\lim\limits_{x\to -5}\dfrac{\sqrt{x^2-9}-4}{x^2+5x}$

c) $\lim\limits_{x\to 0}\dfrac{3xtg2x}{\sin^2 x}$; d) $\lim\limits_{x\to 0}\dfrac{\ln(5x-3)}{2x}$.

19. a) $\lim\limits_{x\to\infty}\dfrac{(1-2x)x^2}{(3x-2)(x^2-1)}$ b) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{3x}$

c) $\lim\limits_{x\to 0}\dfrac{1-\cos 6x}{3x\cdot tg3x}$ d) $\lim\limits_{x\to 0}(1+tg^2 3x)^{1/x^2}$.

20. a) $\lim\limits_{x\to\infty}\dfrac{x^3+4x-5}{2x^3+1}$ b) $\lim\limits_{x\to 6}\dfrac{\sqrt{3+x}-3}{x-6}$

c) $\lim\limits_{x\to 0}\dfrac{2x-\sin 2x}{1-\sqrt{\cos 2x}}$ d) $\lim\limits_{x\to 0}(1+\sin 4x)^{\frac{1}{\sin 2x}}$.

21. a) $\lim\limits_{x\to\infty}\dfrac{4x^5+x^2-5}{x^5+x-2}$ b) $\lim\limits_{x\to 1}\dfrac{x-\sqrt{x}}{x^2-x}$

c) $\lim\limits_{x\to 0}\dfrac{\sin\pi(x+3)}{\sqrt{4+x}-2}$ d) $\lim\limits_{x\to\infty}\left(\dfrac{x-3}{x+3}\right)^{2x}$.

22. a) $\lim\limits_{x\to\infty}\dfrac{5x^4+x^3-6}{2x^4-x+2}$ b) $\lim\limits_{x\to 0}\dfrac{3x}{\sqrt{1-5x}-1}$

c) $\lim\limits_{x\to 0}\dfrac{\sin\sqrt{3x}}{xe^x}$ d) $\lim\limits_{x\to 0}\left(1+\dfrac{2}{x}\right)^{\frac{3}{x}}$.

23. a) $\lim\limits_{x\to\infty}\dfrac{4x^2+6x-3}{5x^2-x+1}$ b) $\lim\limits_{x\to 0}\dfrac{\sqrt{1-x^2}-1}{x^2}$

c) $\lim\limits_{x\to 0}\dfrac{tg6x}{2xe^{2x}}$ d) $\lim\limits_{x\to 0}\left(\dfrac{1}{2x}\ln\sqrt{\dfrac{x+2}{x-2}}\right)$.

24. a) $\lim\limits_{x\to\infty}\dfrac{5-x^3+7x^4}{x^4-12x+1}$ b) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+5x}-\sqrt{1-3x}}{x+x^2}$

c) $\lim\limits_{x\to 0}\dfrac{x^2ctg5x}{\sin 7x}$ d) $\lim\limits_{x\to\infty}(x-5)[\ln(x-3)-\ln x]$.

25. a) $\lim\limits_{x\to\infty}\dfrac{x-3x^2+5x^4}{2+5x^2+x^4}$ b) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x^2}-1}{x^2+x^3}$

c) $\lim\limits_{x\to 0}\dfrac{1-\cos 6x}{1-\cos 2x}$ d) $\lim\limits_{x\to\infty}(2x+1)[\ln(x+3)-\ln x]$.

26. a) $\lim\limits_{x\to\infty}\dfrac{3x^2-x+1}{5x^2-x+5}$ b) $\lim\limits_{x\to 3}\dfrac{\sqrt{3x-1}-\sqrt{8}}{x-3}$

c) $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{1-\cos 4x}$ d) $\lim\limits_{x\to 0}(1+3x)^{\frac{1}{5x}}$.

27. a) $\lim\limits_{x\to\infty}\dfrac{9x^4-4x^3+2}{3x^4+5}$ b) $\lim\limits_{x\to 5}\dfrac{\sqrt{1+3x}-\sqrt{2x+6}}{x^2-5x}$

c) $\lim\limits_{x\to 2}\dfrac{tg(2-x)}{x-2}$ d) $\lim\limits_{x\to\infty}(\dfrac{3x-2}{3x+1})^{2x}$.

28. a) $\lim\limits_{x\to\infty}\dfrac{10x^5-3x^2+9}{2x^5+3x^2+4}$ b) $\lim\limits_{x\to 3}\dfrac{x-3}{\sqrt{3x}-3}$

c) $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin(x-\frac{\pi}{3})}{1-2\cos x}$ d) $\lim\limits_{x\to 3}(3x-8)^{\frac{2}{x-3}}$.

29. a) $\lim\limits_{x\to\infty}\dfrac{(2x-3)(3x+2)^2}{(2x+1)^4}$ b) $\lim\limits_{x\to\infty}3x(\sqrt{x^2+1}-x)$

c) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin(x-\frac{\pi}{4})}{1-\sqrt{2}\cos x}$ d) $\lim\limits_{x\to\infty}(\dfrac{4x+1}{4x-1})^{3x}$.

30. a) $\lim\limits_{x\to\infty}\dfrac{(1-x)(1+2x)^2}{x^3-2x+1}$ b) $\lim\limits_{x\to\infty}(\sqrt{x^2+3}-\sqrt{x^2-3})$

c) $\lim\limits_{x\to 0}\dfrac{\sin 2\pi x}{\sin 5\pi x}$ d) $\lim\limits_{x\to 3}(10-3x)^{\frac{3}{x-3}}$.

8-TOPSHIRIQ

Erkli o’zgaruvchi qiymatlar sohasining turli qismlarida har xil analitik ifodalar bilan berilgan funksiyaning uzilish nuqtasini toping(agar bo’sa) va grafigini yasang

1, 16. $y=\begin{cases} -x, & agar\ \ x\le 0; \\ x^2, & agar\ \ 0<x\le 2; \\ x+1, & agar\ \ x>2. \end{cases}$

2, 17. $y=\begin{cases} x^2+1, & agar\ \ x\le 1; \\ 2x^2, & agar\ \ 1<x\le 3; \\ x+2, & agar\ \ x>3. \end{cases}$

3, 18. $y=\begin{cases} x-3, & agar\ \ x\le 0; \\ x+1, & agar\ \ 0<x\le 4; \\ 3+\sqrt{x}, & agar\ \ x>4. \end{cases}$

4, 19. $y=\begin{cases} \sqrt{1-x}, & agar\ \ x\le 0; \\ 0, & agar\ \ 0<x\le 2; \\ x-2, & agar\ \ x>2. \end{cases}$

5, 20. $y=\begin{cases} 2x^2, & agar\ \ x\le 0; \\ x, & agar\ \ 0<x\le 1; \\ 2, & agar\ \ x>1. \end{cases}$

6, 21. $y=\begin{cases} \sin x, & agar\ \ x<0; \\ x, & agar\ \ 0\le x\le 2; \\ 1, & agar\ \ x>2. \end{cases}$

7, 22. $y=\begin{cases} \cos x, & agar\ \ x\le \dfrac{\pi}{2}; \\ 0, & agar\ \ \dfrac{\pi}{2}<x<\pi; \\ x, & agar\ \ x\ge \pi. \end{cases}$

8, 23. $y=\begin{cases} x-1, & agar\ \ x\le 0; \\ x^2, & agar\ \ 0<x<2; \\ 2x, & agar\ \ x\ge 2. \end{cases}$

9, 24. $y=\begin{cases} 3x+1, & agar\ \ x<0; \\ x^2+1, & agar\ \ 0\le x<2; \\ 0, & agar\ \ x\ge 2. \end{cases}$

10, 25. $y=\begin{cases} x+2, & agar\ \ x\le 0; \\ x^2+2, & agar\ \ 0<x<2; \\ 2x, & agar\ \ x\ge 2. \end{cases}$

11, 26. $y=\begin{cases} x+3, & agar\ \ x\le 1; \\ x^2, & agar\ \ 1<x<2; \\ 2x, & agar\ \ x\ge 2. \end{cases}$

12, 27. $y=\begin{cases} \sqrt{1-x}, & agar\ \ x\le 1; \\ x, & agar\ \ 1<x\le 2; \\ x+1, & agar\ \ x>2. \end{cases}$

13, 28. $y=\begin{cases} 2x-1, & agar\ \ x\le 0; \\ x+2, & agar\ \ 0<x\le 1; \\ 2+\sqrt{x}, & agar\ \ x>1. \end{cases}$

14, 29. $y=\begin{cases} x^2-1, & agar\ \ x\le 1; \\ x, & agar\ \ 1<x\le 3; \\ 6-x, & agar\ \ x>3. \end{cases}$

15, 30. $y=\begin{cases} 2^x, & agar \ \ x<0; \\ x+1, & agar \ \ 0\le x\le 3; \\ 4-x, & agar \ \ x>3. \end{cases}$

9 – TOPSHIRIQ

Berilgan funksiyalarning hosilalarini toping

1.

a) $y=\frac{1}{2}arctg\frac{e^x-3}{2}$

b) $y=(2x-1)\sqrt{x^2-1}$

c) $y=3^{\arcsin 4x}$

d) $y=\sin^3 5x\cos^3(2x+1)$

e) $y=(tgx)^{\frac{1}{x}}$.

2.

a) $y=\frac{\sin x}{\cos^2 x}$

b) $y=\ln(arctg\frac{x}{2}+\cos 4x)$

c) $y=x+\frac{1}{1+e^{x/4}}$

d) $y=tg(ctgx)$

e) $y=(\sin x)^x$.

3.

a) $y=(\sqrt{2x+5}+1)^3$

b) $y=\arcsin\frac{2x+1}{3}$

c) $y=e^{-\cos 5x}$

d) $y=ctg\sqrt{x+5}$

e) $y=(1+tg^2 3x)\cdot 2^{-x}$.

4.

a) $y=\sqrt[3]{\frac{x^2+x^3}{x^2-x^3}}$

b) $y=\ln(x^2+\sin 2x)$

c) $y=2^{ctg(5x+1)}$

d) $y=e^{2x}\cdot\arcsin(1-x^2)$

e) $y=tg(2x+3)\cdot\cos^2(2x+3)$.

5.

a) $y=\frac{1-x^3}{x^3-2}$

b) $y=e^x\arcsin e^x$

c) $y=\sin^2 x\cdot\cos^3 x$

d) $y=2^{ctg^2 x}$

e) $y=(tg2x)^{\sqrt[3]{x}}$.

6.

a) $y=(x^3-4x+1)\ln 4x$

b) $y=\frac{7^{1-x^2}}{\ln 3x}$

c) $y=\arcsin(\sqrt{\cos x})$

d) $y=\frac{\sin 4x}{4x}$

e) $y=(tg2x)^{ctg2x}$.

7.

a) $y=(x^2+a^x)\cdot\cos 2x$

b) $y=\ln\cos^3(2x-3)$

c) $y=\arcsin(3tgx)$

d) $y=x^4\cdot 4^x$

e) $y=x^{29x}\cdot 29^x$.

8.

a) $y=3x^3\cdot\sqrt[4]{4x-x^4}$

b) $y=\ln(\sin^2 x-\cos^2 x)$

c) $y=\dfrac{tg(x+2)}{x+2}$

d) $y=arctg\sqrt{e^x-1}$

e) $y=(\arcsin 3x)^{x^2}$.

9.

a) $y=\dfrac{7x^3}{\cos(5x+1)}$

b) $y=arctg\sqrt{3x}+arcctg\sqrt{3x}$

c) $y=\sqrt[x]{3}+3^{-2x}$

d) $y=\ln(\ln x^2)$

e) $y=(\sin 3x)^{\sqrt{x}}$.

10.

a) $y=3\sqrt[3]{x}\cdot\sin\sqrt{x}$

b) $y=arcctg(1+\sin 2x)$

c) $y=\ln^2(arctgx)$

d) $y=7^{\cos 7x}$

e) $y=(\arcsin x)^{\log_2 x}$.

11.

a) $y=\cos^2(2x^2+x-1)$

b) $y=arctg\dfrac{1-x^2}{1+x^2}$

c) $y=2^x\ln x$

d) $y=\log_4(1+\sin^2 x)$

e) $y=(ctg 2x)^{\sqrt{x}}$.

12.

a) $y=\dfrac{2}{\sin(x^3-x)}$

b) $y=\sqrt{arctgx^2}$

c) $y=\lg(tg^3x+3)$

d) $y=2^x+3^{2x}+4^{3x}$

e) $y=(\log_2 x)^{x^2}$.

13.

a) $y=\sin 2x\cdot\cos 4x$

b) $y=arcctg\dfrac{1-x^2}{4x}$

c) $y=4^{x^2+x-3}$

d) $y=\ln(2x+\sqrt{4x^2-1})$

e) $y=(\ln x)^{\sin x}$.

14.

a) $y=tg^2x+\dfrac{1}{5}tg5x$

b) $y=\arccos\sqrt{1-2x}$

c) $y=\lg\cos 2x$

d) $y=2^{\ln(1-x^3)}$

e) $y=(\ln 2x)^{\cos 2x}$.

15.

a) $y=\dfrac{1}{3}tg3x-\sin^2 x$

b) $y=\arccos\sqrt{5x}$

c) $y=e^{x^4-1}$

16.

a) $y=ctg(x^2+5)+\dfrac{1}{7}x^7$

b) $y=(\arcsin 2x)^3$

c) $y=\log_2\dfrac{2+x}{2-x}$

d) $y=\dfrac{(x^3+\ln^2 x)}{6^x}$

e) $y=(\arcsin 7x)^{3x^2}$.

17.

a) $y=2tg^3\dfrac{2}{x}$

b) $y=\arccos\sqrt{1-x^2}$

c) $y=2^{-\sin^2 x}$

d) $y=\ln\left(x^2-\dfrac{1}{x^3}\right)$

d) $y=(x-4)^{\ln 2x}$.

19.

a) $y=\sqrt{\sin(x^2-x)}$

b) $y=\arccos\dfrac{3}{3x-2}$

c) $y=3^{\frac{1}{4x}}$

d) $y=\lg(arctgx)$

e) $y=(x^2+2)^{\frac{2}{x}}$.

21.

a) $y=x^5-4\sqrt{x^3}+2\ln 4x-2$

b) $y=5\sin^2 2x+3\cos x$

c) $y=x^4\cdot e^{2x}$

d) $y=\dfrac{x^2}{\ln x}$

e) $y=x^{\sin 2x}$.

23.

a) $y=\sqrt[3]{\dfrac{2x+1}{x^2-5x+5}}$

b) $y=(1+x^2)arctgx-3x$

c) $y=(x^2-2x+2)e^{4x}$

d) $y=a^{5x}-2\sin x^5$

e) $y=x^{\sqrt{x}}$.

d) $y=e^{-x^2}\cdot\ln(e^{2x}+3)$

e) $y=(x^2-5)^{\ln x}$.

18.

a) $y=\dfrac{1}{3}ctg^3\dfrac{5}{x}$

b) $y=\arcsin(2x^3+5x-1)$

c) $y=\ln\dfrac{x^5}{1-x^2}$

d) $y=10^{\ln^2 x-4}$

e) $y=(x^2+5)^{x-2}$.

20.

a) $y=\sin^3 2x\cdot(\cos^3 2x+\dfrac{4}{5})$

b) $y=arctg\sqrt{x^2-3x+1}$

c) $y=x^{2\cos x}$

d) $y=\ln\dfrac{(x-2)^3}{x+2}$

e) $y=arctg(\sqrt[3]{x^2})$.

22.

a) $y=ax^m+bx^{m+n},\ a,\,b\in R$

b) $y=3tgx-ctg^2x$

c) $y=(x-1)^2e^x$

d) $y=\ln x\cdot\lg x-\ln a\cdot\log_a x$

e) $y=(3+2x^2)^{4x}$.

24.

a) $y=\dfrac{1+\sqrt[3]{x}}{1-\sqrt[3]{x}}$

b) $y=x\arcsin(1-x^3)$

c) $y=e^{2x}\arccos(2x+1)$

d) $y=2\sqrt{ctgx}-\sqrt{tg(2x-1)}$

e) $y=(\cos x)^{\sin 2x}$.

25.

a) $y=\dfrac{3}{\sqrt[3]{x^2}}-\dfrac{5}{x\cdot\sqrt[3]{x}}$

b) $y=2x\sin x-(x^2-2)\cos x$

c) $y=x^3\ln x-\dfrac{x^3}{3}$

d) $y=\sqrt[3]{a+bx^3}$

e) $y=x^{7x}$.

26.

a) $y=\dfrac{ax^4+b}{\sqrt{x^2-a^2}}$

b) $y=(x^2-2)ctg(1-x)$

c) $y=\dfrac{\ln(1+10x)}{\ln(1-10x)}$

d) $y=x\arcsin(x^3-x)$

e) $y=(2x)^x$.

27.

a) $y=\dfrac{a+b(1-x)}{c+d(1+x)}$

b) $y=x\arcsin(x^3-x)$

c) $y=\sqrt{xe^x-x^2}$

d) $y=tgx-\dfrac{1}{3}tg^3x+\dfrac{1}{5}tg^5x$

e) $y=(\cos 2x)^{\sin 2x}$.

28.

a) $y=\dfrac{2}{2x-1}$

b) $y=2x\sin x-(x^2-5)\cos 2x$

c) $y=x^3\ln x-\dfrac{x^3}{3}$

d) $y=\sqrt[3]{a+bx^{-3}}$

e) $y=(\sin 2x)^{\cos 2x}$.

29.

a) $y=x^2\cdot\sqrt[3]{x^2}+7$

b) $y=arctg(\log_3 x)$

c) $y=\dfrac{1}{x}+2\ln x-\dfrac{\lg x}{x}$

d) $y=(1+\sqrt{\sin x})^3$

e) $y=(1+tgx)^{ctgx}$.

30.

a) $y=3x^{\frac{2}{3}}-2x^{\frac{5}{2}}-x^{-3}$

b) $y=\dfrac{x^5}{e^{-x}}$

c) $y=\dfrac{3tgx}{\ln x}$

d) $y=\log_3(tg3x)$

e) $y=(arcctgx)^{arcctgx}$.

10-TOPSHIRIQ

Quyidagi parametrik tenglamalar bilan berilgan hamda oshkormas ko'rinishda berilgan funksiyalarning $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ *hosilalari topilsin*

1. a) $x=\ln 2t,\quad y=\dfrac{1}{3}t^3.$ b) $y^2=2px.$

2. a) $x=a\cos t,\quad y=a\sin t.$ b) $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1.$

3. a) $x = arctgt, \quad y = \ln(1+t^2);$ b) $y = x^3 + arctgy.$

4. a) $x = \arcsin t, \ y = \sqrt{1-t^2};$ b) $y = x^2 + \ln y.$

5. a) $x = \frac{1}{3}a\cos^3 t, \quad y = \frac{1}{3}a\sin^3 t;$ b) $x^2 + 5xy + y^2 - 2x + y - 6 = 0.$

6. a) $x = a(t - \sin t), \ y = a(1-\cos t);$ b) $x^4 - xy + y^4 = 1.$

7. a) $x = \cos 2t, \ y = \sin^2 t;$ b) $x^2 + 2x + y^2 - 4x - 2y - 2 = 0.$

8. a) $x = e^{-at}, \ y = e^{at};$ b) $(x^2 + y^2)^2 = a^2(x^2 - y^2).$

9. a) $x = \ln(1+t^2), \ y = t^2;$ b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$

10 a) $x = \sin t, \ y = ae^{\sqrt{2t}} + be^{-\sqrt{2t}};$ b) $x^3 + y^3 - 3axy = 0.$

11. a) $x = \sec t, \ y = tgt;$ b) $\ln\sqrt{x^2 + y^2} = arctg\frac{y}{x}.$

12. a) $x = 2e^{-t}, \ y = \frac{1}{3}t^3;$ b) $1 + xy - \ln(e^{xy} + e^{-xy}) = 0.$

13. a) $x = \ln t, \ y = t^3;$ b) $e^{x-y} + e^{\sqrt{xy}} = 0.$

14. a) $y = \arcsin(1-t^2), \quad y = \arccos(1-2t^2);$

b) $\sin(x^2 - y^2) + \cos(x^2 + y^2) = 0.$

15. a) $x = t^2 e^{t^3}, \ y = (1-t^2)e^{-t};$ b) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}.$

16. a) $x = \cos(3t+1), \quad y = \sin(3t-1);$ b) $x^3 - y + 3xy = 0.$

17. a) $x = \arcsin 2t, \quad y = \arccos 2t;$ b) $\sqrt{\frac{x}{3}} + \sqrt{\frac{y}{3}} = 1.$

18. a) $x = \sin^3 2t, \quad y = 2\cos^3 2t;$ b) $x^{-\frac{2}{3}} + y^{-\frac{2}{3}} = 1.$

19. a) $x = arctg(t-1), \quad y = arctg(1-t);$

b) $(x+y)^2 + (x-3y)^2 = 0.$

20. a) $x = \cos^2 t, \quad y = \arccos 2t;$ b) $arctg(3x) + arcctg(4y) = 0.$

21. a) $x = ctg^3 t, \quad y = arcctg(t + t^3);$ b) $xarctgy + yarctgx = 0..$

22. a) $y = \arccos 2t, \quad x = \arcsin(t^2 - 1);$ b) $(x+1)^3 + (y+1)^3 - 3y = 0.$

23. a) $x = \frac{1}{3}t^3 + \frac{1}{2}t^2 + t, \ y = \frac{t^2}{2} + \frac{1}{t};$ b) $x\ln(1+y^2) + y\ln(1+x^2) = 0..$

24. a) $y = t^3 + 8t, y = t^5 + 2t;$ b) $x^2 y - y^2 x + (x-y)^3 = 0.$

25. a) $x = \arccos t^2, \quad y = \cos(1-t^2);$ b) $x^2 y + y = 2x.$

26. a) $x = 2t-1,\ y = arctg(1-2t);$ b) $xy - 2\sqrt{y} = x^2.$

27. a) $x = t + \frac{1}{2}\sin 2t, \quad y = \cos^3 t;$ b) $x^2 + y^2 = R^2.$

28. a) $x = 1-10t,\ y = t^2 + t + 1;$ b) $(y^2 - x^2)^3 + x^2 y - y - x = 0.$

29. a) $x = \ln(t^2 - 1), \quad y = \ln(t^2 + 1);$ b) $x - xy - \sqrt{y} = 0.$

30. a) $x = \cos(t^2 - 1), \quad y = \sin(t^2 + 1);$ b) $2^{x+y} = e^{x-y}.$

11 – TOPSHIRIQ

a) Berilgan chiziqqa x_0 abssissali nuqtada o'tkazilgan urinma va normal tenglamalarini yozing;

b) Funksiyaning differensialini toping;

c) Funksiyaning ikkinchi tartibli hosilasini toping.

1.

a) $y = \dfrac{x}{cosx};\ x_0 = 0;$

b) $y = \sqrt{1-e^x};$

c) $y = 5^{-x^2}.$

2.

a) $y = \sqrt{\dfrac{x+2}{x-1}};\ x_0 = 2;$

b) $y = e^{2x} + \sqrt{x+1};$

c) $y = \sin x - \dfrac{\cos 6x}{\sin 30^0}.$

3.

a) $y = \dfrac{\sqrt{x+1}}{x-1};\ x_0 = 2;$

b) $y = tg2x;$

c) $y = (1-x^2)\sqrt{x}.$

4.

a) $y = \dfrac{x}{\sqrt{4+x}};\ x_0 = 0;$

b) $y = \cos x + \ln x;$

c) $y = x \cdot \log_3 x.$

5.

a) $y = \sqrt{x^2 + x + 1};\ x_0 = 0;$

b) $y = \dfrac{2}{3} \cdot \sqrt{e^x};$

c) $y = \ln \dfrac{1}{x}.$

6.

a) $y = \sqrt{x^3 + 4};\ x_0 = 0;$

b) $y = \dfrac{\sqrt{1+x^2}}{x^2};$

c) $y = \sqrt{1-x} + 5^x.$

7.

a) $y = ctgx - x$; $x_0 = \frac{3\pi}{4}$;

b) $y = \sqrt{\sin x}$;

c) $\text{ó} = \frac{x}{\sqrt{1-4x}}$.

8.

a) $y = \sqrt{x^3 + 7x - 7}$; $x_0 = 1$;

b) $y = \sqrt{1 - x^2}$;

c) $y = \ln\sqrt{1 + x^2}$.

9.

a) $y = \sqrt{1 + e^{2x}}$; $x_0 = 0$;

b) $y = \frac{x^2 - 4}{\sqrt{x^2 + 16}}$;

c) $y = \sqrt{x^3 + 7}$.

10.

a) $y = \frac{x+1}{x+2}$; $x_0 = 1$;

b) $y = \frac{2}{3}\sqrt{e^x}$;

c) $y = \frac{1}{\sqrt{2 + 4x}}$.

11.

a) $y = \frac{2}{\cos 14x}$; $x = \frac{\pi}{14}$;

b) $y = \sqrt{e^{2x} + 1}$;

c) $y = x^2 \cdot \ln x$.

12.

a) $y = \frac{2}{3}\sqrt{e^x}$; $x_0 = 0$;

b) $y = \sqrt{1 + \cos^2 x}$;

c) $y = x\sqrt{x^2 - 8}$.

13.

a) $y = \frac{1}{2}\ln(e^x + 1) - 2e^x$; $x_0 = 0$;

b) $y = \ln\frac{1}{\sqrt{x}}$;

c) $y = \frac{x}{\sqrt{1 - x}}$.

14.

a) $y = e^{2x}\cos 2x$; $x_0 = 0$;

b) $y = \sqrt{1 - e^{2x}}$;

c) $y = \frac{x}{\sqrt{1 - x^3}}$.

15.

a) $y = \sqrt{x^3 + 2x + 1}$; $x_0 = 0$;

b) $y = \log_2\frac{1}{x}$;

16.

a) $y = \frac{x-1}{x^2 + 5}$; $x_0 = 0$;

b) $y = arctg\,\frac{e^x}{2}$;

c) $y=\sqrt{\dfrac{x}{x+1}}$.

c) $y=\dfrac{\ln x-1}{2}$.

17.

a) $y=\dfrac{x^2}{2\sqrt{1-3x^4}}$; $x_0=0$;

a) $y=\dfrac{4+3x^2}{\sin x}$;

b) $y=3^{\sqrt{x+1}}$;

18.

a) $y=xe^{-x}+5^{2x}$; $x_0=0$;

b) $y=tg(e^x-e^{-x})$;

c) $y=x\sqrt{4-x^2}$;

19.

a) $y=\ln\dfrac{1+x}{1-x}$; $x_0=0$;

b) $y=\cos\sqrt{1+x}$;

c) $y=\ln 3x$;

20.

a) $y=\dfrac{2}{3}e^x$; $x_0=0$;

b) $y=x^2\cdot\sin 2x$;

c) $y=3^x\cdot\cos 2x$;

21.

a) $y=\sqrt{4+x^2}$; $x_0=0$;

b) $y=\ln\sin 3x$;

c) $y=\dfrac{2(3x^3+4x^2-x-2)}{15}$;

22.

a) $y=\dfrac{4x+3}{\sqrt{17}}$; $x_0=0$;

b) $y=x^2\cdot\cos\dfrac{1}{x}$;

c) $y=\dfrac{\cos^2 8x}{\sin 16x}$;

23.

a) $y=2\cdot\dfrac{1-\sqrt{x}}{1+\sqrt{x}}$; $x_0=0$;

b) $y=x^3+\cos 3x$;

c) $y=\dfrac{2}{3}\sqrt{\dfrac{x^2-1}{3}}$;

24.

a) $y=\dfrac{\sqrt{x^2-1}}{2x^3}$; $x_0=1$

b) $y=\ln\left(\sqrt{x}\right)$;

c) $y=\dfrac{1}{4}\dfrac{\sin 4x}{\cos 8x}$;

25.

a) $y=\dfrac{1}{x^3}+\dfrac{1}{\sqrt{x}}$; $x_0=1$

26.

a) $y=\dfrac{\sqrt{2x+3}}{x}$; $x_0=3$;

b) $y=\frac{1}{4}\frac{\sqrt{1+x}+x}{x}$;

c) $y=\frac{1}{18}tg\frac{x}{2}$;

b) $y=tg(\frac{x}{2}+\frac{\pi}{4})$;

c) $y=\frac{x^2-4}{\sqrt{x}}$;

27.

a) $y=\frac{2x}{\sqrt{x+4}}$; $x_0=0$

b) $y=tg\frac{x}{2}+\cos\sqrt{x}$;

c) $y=\ln\sin x$;

28.

a) $y=\frac{3x}{\sqrt{x+4}}$; $x_0=0$

b) $y=\sqrt{1+e^{2x}}$;

c) $y=\log_a\frac{2x+3}{2x-1}$;

29.

a) $y=\frac{1}{\sin x}$; $x_0=\frac{\pi}{2}$;

b) $y=x-e^{-x}$;

c) $y=arcctgx^2$;

30.

a) $y=\frac{x}{(x-1)^4}$; $x_0=2$;

b) $y=\sin\sqrt{x}$;

c) $y=\sin\frac{1}{x}$;

12 - TOPSHIRIQ

Birinchi tartibli differensial yordamida quyida berilgan funksiyalarning taqribiy qiymatini toping

1; 16.	$f(x)=\frac{x^3-x}{x-2}$	ning	x=2,98	bo'lganda.
2; 17.	$f(x)=\sqrt[3]{9x^2+x}$	ning	x= -0,88	bo'lganda.
3; 18.	$f(x)=\sqrt{\frac{x^2-4}{x+7}}$	ning	x=8,225	bo'lganda.
4; 19.	$f(x)=\ln(1+x)$	ning	x=0,056	bo'lganda.
5; 20.	$f(x)=(x^3-1)(x^2+1)$	ning	x=2,05	bo'lganda.
6; 21.	$f(x)=x^4-3x^2$	ning	x=2,99	bo'lganda.
7; 22.	$f(x)=\sqrt[10]{1-x}$	ning	x=0,06	bo'lganda.
8; 23.	$f(x)=\frac{1}{\sqrt[3]{1+x}}$	ning	x=0,0195	bo'lganda.

9; 24. $f(x)=(1+x)^6-1$ ning x=0,04 bo'lganda.

10; 25. $f(x)=(1+x)^5$ ning x=0,2 bo'lganda.

11; 26. $f(x)=\sqrt{1+x}$ ning x=0,02 bo'lganda.

12; 27. $f(x)=\sqrt[3]{x-1}$ ning x=9,025 bo'lganda.

13; 28. $f(x)=\dfrac{4x}{x+3}$ ning x=3,06 bo'lganda.

14; 29. $f(x)=x^4-2x^3+1$ ning x=2,03 bo'lganda.

15; 30. $f(x)=\sqrt[3]{1+x^2}$ ning $x=0,03$ bo'lganda.

13 - TOPSHIRIQ
O'zgaruvchi miqdorlarning eng katta va eng kichik qiymatlarini topishga doir masalani yeching

1. Tubi kvadrat shaklida bo'lgan , hajmi 32 m^3 bo'lgan ochiq hovuzning o'lchamlari shunday tanlansinki uning tubi va devorlarini qoplash uchun eng kam material sarf qilinadigan bo'lsin.

2. Trapetsiyaning yon tomonlari va kichik asosi a ga teng. Trapetsiyaning katta asosi b ni shunday tanlash kerakki , natijada uning yuzi eng katta bo'lsin.

3. Yig'indisi o'zgarmas a ga teng bo'lgan ikkita musbat sonlar ko'paytmasinining eng katta qiymatini toping.

4. 8 soni kvadratlarining yig'indisi eng kichik bo'ladigan ikkita musbat qo'shiluvchilarga ajratilsin.

5. Ko'paytmasi o'zgarmas a ga teng bo'lgan ikkita musbat sonlar yig'indisining eng kichik qiymatini toping.

6. Yuzi o'zgarmas S bo'lgan barcha to'g'ri to'rtburchaklardan perimetri eng kichigini toping.

7. Tomoni a ga teng bo'lgan kvadrat shaklidagi kartonning burchaklaridan bir xil kvadratlar qirqib olinadi , qolgan qismidan to'g'ri burchakli quticha yasaladi. Qutichaning hajmi eng katta bo'lishi uchun qirqib olingan kvadratning tomoni qanday bo'lishi kerak.

1.8. 9 soni kvadratlarining yig'indisi eng kichik bo'lgan ikkita musbat ko'paytuvchilarga ajratilsin.

1.9. Trapetsiyaning yon tomonlari va kichik asosi 10 sm. Uning katta asosini shunday tanlash kerakki , natijada trapetsiyaning yuzi eng katta bo'lsin.

10 Balandligi H va asosini radiusi R bo'lgan konusga eng katta hajmli silindr ichki yasalgan. Silindrning o'lchamlarini toping.

11. O'zgarmas V hajmli silindrik idish tayyorlashga eng kam material ketishi uchun o'lchamlari qanday bo'lishi kerak?

12. 9 sonini shunday ikkita ko'paytuvchiga ajratish kerakki ular kvadratlarining yig'indisi eng katta bo'lsin.

13. R radiusli yarim doiraga eng katta yuzali to'g'ri to'rtburchak ichki chizilgan. Uning o'lchamlarini toping.

14. Uzunligi 120 m bo'lgan sim setka bilan o'rab olingan eng katta yuzali to'gri to'rtburchakli maydonning o'lchamlari aniqlansin.

15. Gipotenuzasi m ga teng bo'lgan to'g'ri burchakli uchburchakning katetlari qanday bo'lganda uning yuzasi eng katta bo'ladi?

16. R radiusli yarim doiraga eng katta yuzali to'g'ri burchakli to'rtburchak ichki chizilgan. Uning o'lchamlarini toping.

17. R radiusli sharga eng katta hajmga ega bo'lgan silindr ichki chizilgan. Uning hajmini toping.

18. Hajmi 16 m^3 bo'gan silindr idishning o'lchamlari qanday bo'lganda uni tayyorlashga eng kam material ishlatiladi.

19. Sirti eng kichik bo'lgan V hajmli silindr radiusining balandligiga nisbatini toping.

20. Sirti eng kichik bo'lgan V hajmli konus radiusining balandligiga nisbatini toping.

21. Sirti S ga teng bo'lgan eng katta hajmli tsilindrning o'lchamlari topilsin.

22. Kanalning kesimi teng yonli trapetsiyadan iborat bo'lib, uning yon tomoni kichik asosiga teng. Yon tomonining og'ish burchagi qanday bo'lganda kanal kesimi yuzi eng katta bo'ladi?

23. Perimetri $2p$ bo'lgan uchburchakning bir tomoni a ga teng. Uning boshqa ikki tomonini shunday tanlangki natijada uchburchakning yuzi eng katta bo'lsin.

24. Hajmi V bo'lgan yopiq silindrik idish tayyorlash kerak. Uning o'lchamlari qanday bo'lganda tayyorlash uchun eng kam material ketadi.

25. R radiusi 4 dm, balandligi 6dm bo'lgan konusga eng katta hajmli silindr ichki chizilgan. Silindrning hajmini toping.

26. Doiraviy sektor ko'rinishidagi maydonni chegaralab olish uchun 20 m uzunlikdagi sim bor. Maydonnig yuzi eng katta bo'lishi uchun doira radiusi qanday bolishi karak?

27. Er osti yo'lining kesimi trapetsiya shaklida. Uning kichik asosi 6 m, yon tomonlari 10 m. dan. Trapetsiyaning katta asosi qanday bo'lganda uning yuzi eng katta bo'ladi?

28. Yuzasi S ga teng bo’lgan barcha to’g’ri to’rtburchaklar orasidan perimetri eng katta bo’lganini aniqlang.

29. 60 soni shunday ikki qismga ajratilsinki birinchi qismning ikkilangani bilan, ikkinchi qism kvadratining yig’indisi eng kichik bo’lsin.

30. Eni 11 sm bo’lgan tunikadan ko’ndalang kesimi teng yonli trapetsiya shakliga ega bo’lgan ochiq tarnov tayyorlash talab qilinadi. Tarnovning yuqori qismining eni qanday bolganda unga eng ko’p miqdordagi suv sig’adi?

14 – TOPSHIRIQ
Fuksiyani to’la tekshiring va grafigini yasang

1. $y=\frac{x}{e^x}$
2. $y=\frac{e^x}{x}$
3. $y=x\ln x$
4. $y=\frac{\ln x}{x}$
5. $y=\frac{x}{\ln x}$
6. $y=4xe^{-x}$
7. $y=x^3e^{-x}$
8. $y=x^3e^{-3x}$
9. $y=x^2e^{-x}$
10. $y=x^2e^{-2x}$
11. $y=x^2e^{-x^2}$
12. $y=\ln(x^2+1)$
13. $y=\ln(x^2-1)$
14. $y=\ln(4-x^2)$
15. $y=\frac{x}{x^2-1}$
16. $y=\frac{x}{1-x^2}$
17. $y=\frac{x^2}{4-x^2}$
18. $y=\frac{x}{1+x^2}$
19. $y=xe^{-\frac{x^2}{2}}$
20. $y=\frac{x^2}{x^2-1}$
21. $y=\frac{1}{x^2-4}$
22. $y=x+\frac{1}{x^2}$
23. $y=x-\frac{1}{x^2}$
24. $y=\frac{x^2}{x-1}$
25. $y=\frac{x^2}{2-x}$
26. $y=\ln(1-x^2)$
27. $y=\ln(x^2-4)$
28. $y=\ln(9-x^2)$
29. $y=\frac{x^2+1}{x^2-1}$
30. $y=\frac{x^2+1}{1-x^2}$

Maple kompyuter matematika sistemasi va unda ishlash haqida

Ma'lumki, boshqa fanlar qatori matematika fanini o'qitishda ham ko'rgazmalilik, texnika vositalari muhim ahamiyatga ega. Kompyuterlar va zamonaviy axborot texnologiyalarining kirib kelishi bilan bu sohada katta imkoniyatlar yuzaga keldi. Hozirgi zamon kompyuterlarning intllektual ko'rsatkichi ularning "Simvolli matematika" yoki "Kompyuter algebrasi" deb ataluvchi dasturlar sistemasi bilan ham belgilanadi. Matematik ifodalar ustida simvolli amallar bajarish uchun yaratilgan bunday sistemalar "Kompyuter matematika sistemalari(bundan keyin KMS deb ataymiz)" yoki "Matematik paketlar" deyiladi. Hozir dunyoda biror jiddiy ilmiy loyiha yo'qki, ulardan foydalanilmasin.

Matematika fanida grafik tasvirlarlar muhim rol o'ynaydi. Ko'p yillik tajribalar ko'rsatadiki,"Oliy matematika" fanining chizili algebra va analitik geometriyaga, matematik analizga kirish, differensial va integral hisob kabi fundamental bo'limlarini o'rganishda ko'rgazmali usullardan biri sifatida KMSdan foydalanish foydalidir.

Kompyuter va videoproyektor bilan jihozlangan o'quv xonalarida chiziqli algebraga oid tushunchalar, chiziqlar va sirtlar, funksiya tushunchasi, sonli ketma-ketlik va uning limiti, funksiyaning limiti, differensial va integral hisobga oid mavzularini talabalarga o'rgatishda, an'anaviy usullar (xususan, doskada bajarib ko'rsatish, tushuntirish) bilan bir qatorda **KMS** yordamida ko'rgazmali ravishda misollar bajarib ko'rsatish, katta uslubiy foyda beradi. Amaliy mustaqil ishlarni bajarganda, avval misol-masalalarni an'anviy usulda bajarilib, so'ng **KMS**da ham bajarilib ko'rish tavsiya qilinadi.

Hozirda kompyuter matematika sistemalarining turlri variantlari yaratilgan: **Maple, Mathematika, Matlab, Mathcad, Derive**. Quyida ularidan biri **- Maple KMS** haqida qisqacha tushuncha berilib, unda arifmetik amallarni bajarish, tenglama va tengsizliklarni yechish, grafiklar yasash, limitlar va hosilalarni topishga doir bir necha misollar keltirilgan.

Ulardan eng ko'p tarqalganlari birinchi uchtasi. Bu sistema 1980 yilda Kanadaning Waterloo universitetida Keyt Geddes va Gaston Gonnetlar tashkil qilgan simvolli hisoblashlar guruhi tomonidan "**Windows uchun Malpe"** nomi bilan yaratilgan. Uning bir necha versiyalari yaratilgan bo'lb, hozir eng ko'p ishlatiladiganlari **Maple 9.5** va **Maple 15**. Bundan keyin biz **Maple** deganda uning foydalanuvchi kompyuteriga o'rnatilgan versiyasini tushunamiz.

Maple sitemasiga kirish: buning uchun uning kompyuter ish stolidagi yorlig'idan foydalanish mumkin. Natijada **Maple** ish oynasi ochiladi. Biz bu yerda “dastur” deganda **Maple** da bajariladigan har qanday amallar (masalan, arifmetik yoki simvolli hisoblashlar, grafik tasvir yasash, tenglama yoki tengsizliklar yechish va h.k uchun ifodalar, munosabatlar) ni tushunamiz. O’rganishning oson yo’llaridan birii “men bajargan kabi bajarib ko’r” usuli bo’lgani uchun, **Maple** misollar yechib ko’rish vositasida o’rganamiz. Avval,

> *restart* :

komandasi bilan ekranni ilgari kiritilgan komandalardan tozalab olamiz. Ekranda **Maple** ning komandalar kiritish uchun taklifi:

>

belgisi paydo bo’ladi. Endi zarur komandalarni kiritib, ishlayverish mumkin.

1-misol. **Arifmetik amallarni bajarish. Maple**da o’zlashtirish(qiymat berish) operatori “ : = “ belgi yordamida bajariladi. Kiritilgan dastur(komanda) yoki uni bajarishning natijasi **Maple** oynasida ko’k rangda chiqadi. Misol sifatida, arifmetik amallarga doir quyidagilarni keltirish mumkin:

> $a := 1.5;$ $a := 1.5$

> $a := a + 2.5;$ $a := 4.0$

> $b := 2.5;$ $b := 2.5$

> $c := a + b;$ $c := 6.5$

> $d := a - b;$ $d := 1.5$

> $e := a \cdot b;$ $e := 10.0$

> $f := \frac{a}{b};$ $f := 1.600000000$

2-misol. **Tenglamani yechish. Maple**da tenglama ***solve(eq, x);*** dasturi yordamida yechish mumkin, bu yerda ***eq*** – tenglama, *x* – noma’lum. Masalan,

> *restart* :

> $eq := 2 \cdot x^2 - 3 \cdot x + 1 = 0;$ $eq := 2x^2 - 3x + 1 = 0$

> $solve(eq, x);$ $1, \frac{1}{2}$

Demak, bu tenglamaning ildizlari 1 va $\frac{1}{2}$ ekan.

Mapleda tenglamani formula shaklida(simvolli) yechish ham mumkin. Masalan,

> $eq := a \cdot x^2 + b \cdot x + c = 0;$ $eq := a x^2 + b x + c = 0$

tenglamaning yechilishi:

> $solve(eq, x);$ $\frac{1}{2} \frac{-b + \sqrt{b^2 - 4ac}}{a}, -\frac{1}{2} \frac{b + \sqrt{b^2 - 4ac}}{a}$

3-misol. **Tengsizlikni yechish** ham ***solve(eq, x);*** dasturi yordamida bajariladi. Masalan,

$$|x+2|+|x-2|\le 10$$

tengsizlik quyidagicha yechiladi:

> *solve((abs(x+2)+abs(x-2))<=10, x);*

Javobi biz odatlngan [-5, 5] o'rniga

RealRange (-5, 5)

ko'rinishda chiqadi, chunki **Maple** burchakli qavs [ ] ni boshqa maqsadda ishlatadi.

4-misol. **Funksiyaning aniqlanish sohasini topish** ham tengsizlikni yechishga keltiriladi. Masalan, $y = \lg(25 - x^2) + \dfrac{1}{x}$ funksiyaning aniqlanish sohasini quyidagcha toppish mumkin:

Logarifm ostidagi ifoda $25 - x^2 > 0$ da, ikkinchi qo'shiluvchi esa $x \neq 0$ da mavjud. Unda berilgan funksiyaning aniqlanish sohasi quyidagi sistemaning yechimidan iborat: $\begin{cases} 25 - x^2 > 0, \\ x \neq 0; \end{cases}$ $\begin{cases} x^2 < 25, \\ x \neq 0; \end{cases}$ $\begin{cases} |x| < 5, \\ x \neq 0; \end{cases}$ $\begin{cases} -5 < x < 5, \\ x \neq 0; \end{cases}$

Bu tengsizliklar sistemasining **Maple** da yechlilshi quyidagicha:

> *solve({25-x^2>0, x<>0}, x);*

Izoh: Mapleda sistema tengsizliklar to'plami deb qaraladi va qaysi o'zagruvchiga nisbatan yechilishi albatta ko'rsftiladi. Yechim quyidagi ko'rinishda yoziladi:

$$\{-5 < x, x < 0\}, \{x < 5, 0 < x\}$$

5-misol. **Grafiklar yasash.** Ikki o'lchovli va uch o'lchovli grafiklarni ysash uchun *plots* dasturlar to'plamining *plot* va *plot3d* dasturlaridan, oshkormas va bo'lakli uzluksiz funksiyalar grafiklarni yasash uchun esa *implicitplot* va *piesewise* dasturlaridan foydalaniladi.

Aniqlanish sohasi *[a; b]* bo'lgan *y= f(x)* funksiyaning grafigi

```
plot(f(x), x=a..b);
```

dasturdan foydalanib chiziladi. Berilgan oraliqlarda bir nechta funksiyalar grafiklarini bitta koordinata sistemasida yash ham mumkin. Masalan, ikkita *f(x)* va *g(x)* funksiylarning grafigini

```
plot([f(x), g(x)], x=a..b);
```

dasturdan foydalanib chiziladi. Masalan, $y=x^2-2x$ funksiyaning *[-2; 4]* oraliqdagi grafigini yasash quyidagicha bajariladi (I1-rasm):

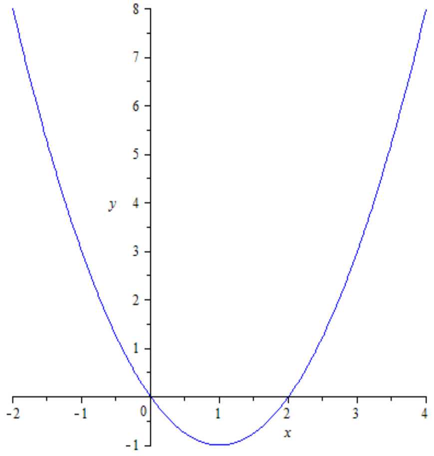
> $with(plots)$:

> $plot(x^2-2\cdot x,\ x=-2..4)$;

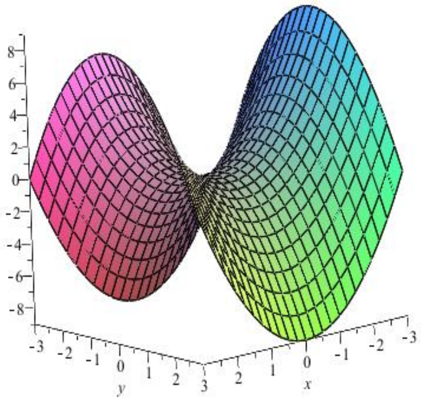
Quyidagi dastur yordamida $z=x^2-y^2$ tenglama bilan brilgan giperbolik paraboloid (egarsimon sirt) shaklini yasash mumkin(i2-rasm):

> $with(plots)$:

> $plot3d(x^2-y^2,\ x=-3..3,\ y=-3..3)$;



1I – rasm



I2-rasm

5-misol. Murakkab funksiyani tuzish, uning qiymatini hisoblash va grafigini yasash. Buni ushbu misolda qaraymiz: *f(x)=x²*, *g(x)=lgx* funksiyalar berilgan bo'lsa, quyidagilarni toping:

f[g (x)], f(g(10), g[f(x)], g[f(10)], g[f(-2)].

Yechish. y=f(x)=x^2, u=g(x)=lgx bo'lgani uchun: f[g(x)]=f(u)=u^2=lg^2x;
f[g(10)]=$lg^2 10=1^2$ =1; g[f(x)]= g(y)=lgy=lg(x^2);
g[f(10)]=lg10^2 = 2; g[f(-2)]=lg$(-2)^2$ = lg4=0,60206.

Masalaning **Maple** da yechlilshi quyidagicha:

```
> restart:
> f:=x->x^2; g:=x->log10(x);
```

ya'ni f(x) va g(x) funksiyalar **Maple** da shu tarzda aniqlanadi. Endi funksiyalarni hisoblash mumkin:

```
f(g(x));  f(g(10.));  g(f(x));
g(f(10.)); g(f(-2.));
```

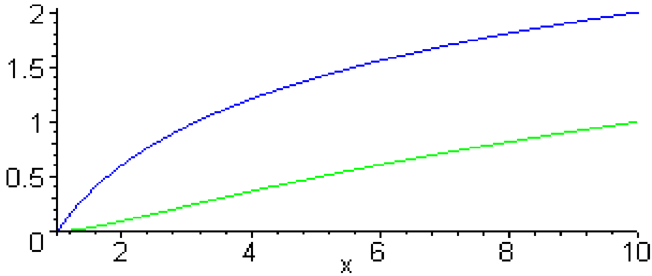
Natija kompyuter ekranida quyidagi shaklda chiqadi:

$$\frac{\ln(x)^2}{\ln(10)^2} \qquad 1. \qquad \frac{\ln(x^2)}{\ln(10)} \qquad 2.000000000 \qquad 0.6020599913$$

Berilgan funksiyalar haqida to'laroq tasavvurga ega bo'lish uchun [1, 10] oraliqdagi grafigini chizamiz:

```
> plot([f(g(x)), g(f(x))], x=1..10, color = [green, blue]);
```

f(g(x)) – yashil(green), g(f(x)) – ko'k(blue) rangda chiziladi (I3-rasm).



I3-rasm.

Bu yerda *plot* - funksiyalarning berilgan oraliqda grafigini chizish funksiyasi; color - chiziq rangi; green - yashil; blue - ko'k.

Maple sistemasida $\{x_n\}$ ketma-ketlikning dastlabki **k** ta hadini yozish uchun

seq(x(n), n=1..k);

funksiyasi ishlatiladi.

6-misol. Umumiy hadi $x_n = \frac{1}{n} \cdot \sin\frac{\pi n}{2}$ bilan berilgan ketma-ketlikning dastlabki beshta hadini quyidagicha topish mumkin :

> restart;

> x: = n -> sin(Pi*n/2)/n;   seq(x(n), n=1..5);

$$x := n \to \frac{\sin\left(\frac{1}{2}\pi n\right)}{n}$$

Javobi : $1, 0, \frac{-1}{3}, 0, \frac{1}{5}$

Maple sistemasida umumiy hadi x_n bo'lgan ketma-ketlikning limitini hisoblash uchun

limit(x(n), n=infinity);

funksiyalaridan foydalaniladi, bu yerda ***infinity*** cheksizlik belgisi - "∞" ni bildiradi. Bulardan birinchisi limitning ifodasini yozadi, ikkinchisi qiymatini hisoblaydi. Masalan, $\{x_n\}=\{(2n+1)/n\}$ ketma-ketlikning limiti quyidagicha hisoblanadi:

> $restart:$

> $x := n \rightarrow \frac{(2 \cdot n+1)}{n};$ $\quad x := n \rightarrow \frac{2n+1}{n}$

> $Limit(x(n), n = infinity) = limit(x(n), n = infinity);$ $\quad \lim_{n \rightarrow \infty} \frac{2n+1}{n} = 2$

Izoh: Maple oynasida, Limit so'zi bosh harf bilan boshlansa - limit ifodasining yozilishi, kichik harfda boshlansa - hisoblangan qiymati yoziladi.

Maple sistemasida *f(x)* funksiyaning *x=a* nuqtadagi limiti quyidagich topiladi:

$$limit(f(x), x = a);$$

7-misol. $\lim\limits_{x \to 3} \frac{2x^2 - 18}{x - 3}$ limitni toping.

Yechish. Bu limitning **Maple** da hisoblanishi quyidagicha:

> restart:

> $Limit\left(\frac{(2 \cdot x^2 - 18)}{x - 3}, x = 3\right) = limit\left(\frac{(2 \cdot x^2 - 18)}{x - 3}, x = 3\right);$

$$\lim_{x \to 3} \frac{2x^2 - 18}{x - 3} = 12$$

Maple sistemasida *f(x)* funksiyaning *x* bo'yicha hosilasi ***diff(f(x), x)*** dasturi bilan topiladi. Masalan,

$$f(x) = \frac{2 \cdot x^3}{cos3x}$$

funksiyaning hosilasini toppish quyidagicha b riladi:

> restart:

> $diff\left(\frac{2x^3}{\cos(3x)}, x\right);$ $\quad \frac{6x^2}{\cos(3x)} + \frac{6x^3 \sin(3x)}{\cos(3x)^2}$

Topshiriqlarni Maple kompyuter sistemasida bajarish namunalari

Quyida keltirilgan namunalardan foydalanib, o'z topshiriqlaringizni bajarib ko'ring.

1. **Tenglamalar sistemasini Kramer usuli bilan yeching.**

$$\begin{cases} x+2y-z=2, \\ 2x-3y+2z=2, \\ 3x+y+z=8. \end{cases}$$

Bajarilishi (ko'k rangda bajarish natijalari ko'rsatilgan):

```
> with(LinearAlgebra):
> A:=Matrix([[1,2,-1], [2,-3,2], [3,1,1]]);
> Delta:=Determinant(A);
```

$$A := \begin{bmatrix} 1 & 2 & -1 \\ 2 & -3 & 2 \\ 3 & 1 & 1 \end{bmatrix} \qquad \Delta := -8$$

```
> A1:=Matrix([[2,2,-1],[2,-3,2],[8,1,1]]);
> Delta1:=Determinant(A1);
```

$$\Delta 1 := -8 : \begin{bmatrix} 2 & 2 & -1 \\ 2 & -3 & 2 \\ 8 & 1 & 1 \end{bmatrix}$$

```
> A2:=Matrix([[1,2,-1],[2,2,2],[3,8,1]]);
> Delta2:=Determinant(A2);
```

$$A2 := \begin{bmatrix} 1 & 2 & -1 \\ 2 & 2 & 2 \\ 3 & 8 & 1 \end{bmatrix} \qquad \Delta 2 := -16$$

> A3:=Matrix([[1,2,2],[2,-3,2],[3,1,8]]);

> Delta3:=Determinant(A3);

$$A3 := \begin{bmatrix} 1 & 2 & 2 \\ 2 & -3 & 2 \\ 3 & 1 & 8 \end{bmatrix} \qquad \Delta 3 := -24$$

> x:=Delta1/Delta; y:=Delta2/Delta; z=Delta3/Delta;

$$x := 1 \qquad y := 2 \quad z = 3$$

2. Berilgan ikkinchi tartibli chiziqni yasang

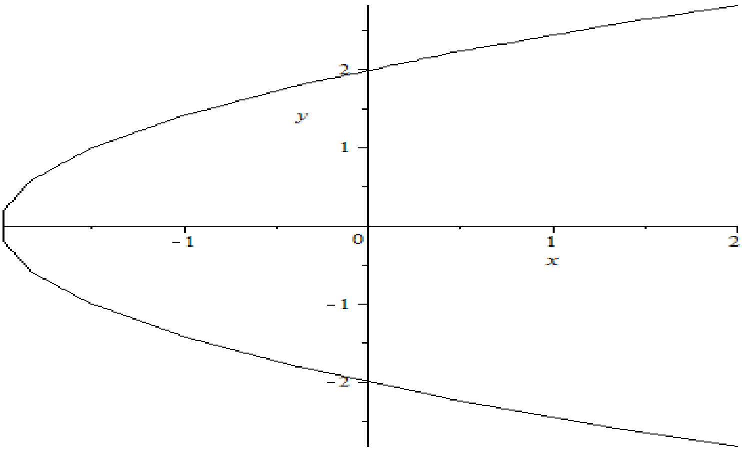
$$2x - y^2 + 4 = 0\,.$$

Bajarilishi: berilgan chiziqni *x* ning *[-2; 2]*, *y* ning *[-5; 5]* oraliqqa tegishli qiymatlari uchun yasaymiz:

> *restart* :

> with(plots):

> $implicitplot\left(2\cdot x - y^2 + 4 = 0,\ x=-2..2, y=-5..5\right);$



I4-rasm

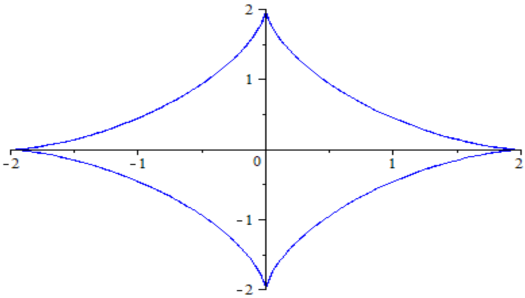
3. **Parametrik tenglmalari bilan berilgan chiziqni yasash**

$$x = 2\cos^3 t,\ y = 2\sin^3 t$$

chiziqni (astroidani) ko'k rangda yasang.
Bajarilishi (I5-rasm):

> $with(plots)$:

> $plot\left(\left[2\cdot(\cos(t))^3,\ 2\cdot(\sin(t))^3,\ t=-2\cdot\text{Pi}..2\cdot\text{Pi}\right], color = \text{"Blue"}\right)$



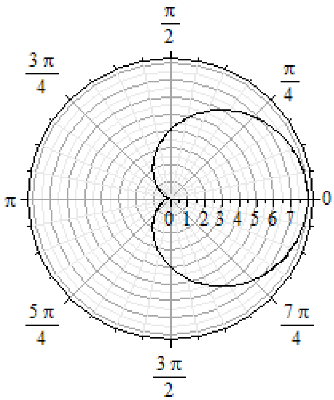
I5-rasm

4. **Tenglamasi qutb koordinatalarida berilgan chiziqni yasash**

Qutb koordinata sistemasida $r = 4(1 + cost)$ tenglama bilan berilgan chiziqni (kardioidani) qora rangda yasang.
Bajariishi:

> $with(plots)$:

> $polarplot(4\cdot(1 + \cos(t)), t=-\pi..\pi, color = \text{"Black"})$



I6-rasm

5. Limitni topish

Limitni toping.

1) $\lim\limits_{x\to 4}\dfrac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$; 2) $\lim\limits_{x\to 0}(1-5x)^{\frac{1}{x}}$; 3) $\lim\limits_{x\to\infty}(2x+1)\left(\ln(x+3)-\ln x\right)$.

Bajrilishi:

1)

> *restart* :

>

$$Limit\left(\frac{(\mathrm{sqrt}(2\cdot x+1)-3)}{\mathrm{sqrt}(x-2)-\mathrm{sqrt}(2)}, x=4\right)=limit\left(\frac{(\mathrm{sqrt}(2\cdot x+1)-3)}{\mathrm{sqrt}(x-2)-\mathrm{sqrt}(2)},\right.$$
$$\left. x=4\right);$$

$$\lim_{x\to 4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}=\frac{2}{3}\sqrt{2}$$

2) > Limit((1-5*x)^(1/x), x=0)=limit((1-5*x)^(1/x), x=0);

$$\lim_{x\to 0}(1-5x)^{\frac{1}{x}}=\mathrm{e}^{-5}$$

3) > $Limit((2\cdot x+1)\cdot(\log(x+3)-\log(x)),\ x=infinity)=limit((2\cdot x+1)$
$\cdot(\log(x+3)-\log(x)),\ x=infinity);$

6. Hosilani topish

Funksiyaning hosilasini toping.

1) $y=(4x+3)\sqrt{x^2-1}$; 2) $y=3^{\arcsin 4x}$; 3) $y=(\ln(\mathrm{x}^2+1))^{2x+1}$.

Bajarilishi.

1) > *restart* :

> $y1 := x\to(4\cdot x+1)\cdot\mathrm{sqrt}\left(x^2-1\right);$ $:= x\to(4x+1)\sqrt{x^2-1}$

> $diff(y1(x), x);$

$$4\sqrt{x^2-1}+\frac{(4x+1)\,x}{\sqrt{x^2-1}}$$

2) > $y2 := x \rightarrow 3^{\arcsin(4\cdot x)};$ $\quad y2 := x \rightarrow 3^{\arcsin(4\,x)}$

> $diff(y2(x),x);$

$$\frac{4\,3^{\arcsin(4\,x)}\ln(3)}{\sqrt{1-16\,x^2}}$$

3) > $y3 := x \rightarrow \left(\log\left(x^2+1\right)\right)^{(2\cdot x+1)};$ $\quad y3 := x \rightarrow \log\left(x^2+1\right)^{2\,x+1}$

> $diff(y3(x),x);$

$$\ln\left(x^2+1\right)^{2\,x+1}\left(2\ln\left(\ln\left(x^2+1\right)\right)+\frac{2\,(2\,x+1)\,x}{\left(x^2+1\right)\ln\left(x^2+1\right)}\right)$$

FOYDALANILGAN ADABIYOTLAR

1. Пискунов Н.С. Дифференциал ва интеграл ҳисоб. 1-қисм. –Тошкент: Ўқитувчи, 1985.
2. Соатов Ё.У. Олий математика. 1-жилд. – Т.: Ўқитувчи, 1994.
3. Соатов Ё.У. Олий математика. 3 -жилд. – Т.: Ўқитувчи, 1996.
4. Danko P.E., Popov A.G., Kojevnikova T.E. Oliy matematika mashqlar va masasalarda. 1-qism.– Toshkent, “O’qituvchi”, 2009 y.
5. Шнейдер В.Е., Слуцкий А.И., Шумов А.С. Олий математика киска курси. 1-кисм.
6. Rajabov F. va boshqalar. Oliy matematika. O’quv qo’llanma. –T.: Turon-iqbol, 2007. – 326 b.
7. Жўраев Т.Ж. ва бошқалар. Олий математика асослари. Т.: Ўқитувчи, 1997 й.
8. Oliy matematika. O’quv-uslubiy majmua. – Samarqand, SamDAQI. 2012 y.
9. Сборник задач по математике для втузов. Специальные разделы. Авт.коллектив под ред. А.В.Ефимова и А.П.Демидовича. – М.: Наука, 1994. 1-часть.
10. В.П.Дьяконов. Maple 9.5/10 в математики и физике иобразовании. – М.:СОЛОН-Пресс, 2006. – 720 с.