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QOVUSHOQ SIQILUVCHAN SUYUQLIK TO'LDIRILGAN DOIRAVIY SILINDRIK QATLAM VA QOBIQLARNING NOSTATSIONAR TEBRANISHLARI

5A440202-Deformatsiyalanuvchi qattiq jism mexanikasi mutaxassisligi bo'yicha magistr akademik darajasini olish uchun

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Mavzuning dolzarbligi. Elastik jismlarda nostatsinar to'lqinlarning tarqalish jarayonlarini va qonuniyatlarini o'rganish mexanikaning dolzarb masalalaridan biridir. Shu nuqtai nazardan qaraganda tebranish jarayonlarining paydo bo'lish qonuniyatlarini o'rganish ham muhim ahamiyatga ega. Doiraviy silindrik qatlam va qobiqlarda va boshqa muhandislik inshootlari elementlarida (sterjenlar, plasnbnalar va boshqalar) to'lqinlar tarqalish jarayonlari va qonuniyatlarini tadqiq etish yuqorida ko'rsatilgan ana shunday dolzarb masalalarning eng muhimlaridan hisoblanadi.

Doiraviy silindrik elastik qatlam va qobiqlarining, bo'ylama-radial, ko'ndalang tebranishlari jarayonida, qovushoq suyuqliklar bilan o'zaro nostatsionar ta'sirlashuvini o'zgarish haqidagi masalalar ham ko'rsatib o'tilgan dolzarb masalalar qatoriga kiradi. Shu nuqtai-nazardan qaraganda dissertatsiya mavzusi doirasida tadqiq etilishi lozim bo'lgan suyuqlikni o'z ichida saqlovchi silindrik qatlam va qobiqlarningning nostatsionar bo'ylama-radial tebranishlari tenglamalarini keltirib chiqarish hamda kuchlanganlik-deformatsiyalanganlik holatini hisoblash algoritmini yaratish muammosi ham deformatsiyalanuvchi qattiq jism mexanikasining o'ta aktual masalasidir.

Yuqorida aytilganlar bilan bir qatorda shuni ham ta'kidlash lozimki, silindrik qobiqlarning suyuqliklar bilan o'zaro nostatsionar ta'sirlalashuvi haqidagi masalalar, juda ko'p hollarda faqat ideal suyuqlik uchungina yechilgan. Ayni paytda, suyuqlik qovushoq bo'lsa, bunday ishlarning soni juda kam. Silindrik qobiqning ichidagi suyuqlik qovushoq bo'lgan hol R.P. Darmond va W.T. Rouleau [1972], A.C.Volmir [1979], V.G.Gubenko va V.N.Kuznetsov [1980], V.D.Kubenko [1980], A.E. Babaev [1984] va boshqalar tomonidan tahlil etilgan.

Qovushoq suyuqlik va silindrik qobiqlarning, umuman qattiq jismlarning, o'zaro nostatsionar ta'siri masalasini tadqiq qilish ko'lamining nisbatan torligi, asosan, qovishoq suyuqlik uchun Nave-Stoks tenglamasini yechishga duch kelinadigan va shu vaqtdagacha hal bo'lmagan, matematik qiyinchiliklar bilan bog’liq. Xususan, shu vaqtgacha Nave-Stoks tenglamasining umumiy integrali topilmagan.

O’tgan asrning saksoninchi yillariga kelib Ukraina fanlar akademiyasining akademigi A.N.Guz tomonidan qovushoq suyuqlik uchun Nave-Stoks tenglamalari, to’rtta, bir-biriga bog’lik bo’lmagan, potensial funksiyalarga nisbatan tartibi ikkidan yuqori bo’lmagan differensial tenglamalarga keltirildi. Olingan tenglamalar qattiq jismlardagi to’lqin tarqalish tenglamalariga juda o’xshash bo’lib ularni yechish imkoniyati tug’ildi.

Xuddi ana shu munosabat bilan, endi qattiq jismlarning qovushoq suyuqliklar bilan o’zaro nostatsionar ta’sirlashuvi haqidagi masalalarni yangicha yo’nalishda tadqiq qilish imkoniyati paydo bo’ldi. Ikkinchi tomondan deformatsiyalanuvchi qattiq jismlarning, xususan silindrik qatlam va qobiqlarning, tutash muhitlar bilan o’zaro dinamik ta’sirlashuvi haqidagi masalalarni yechishda, qobiq ushun asosiy tenglamalar sifatida, taqribiy tebranish tenglamalardan foydalanadilar. Ana shunday tebranish tenglamalarni chiqarish ham bugungi kunda eng dolzarb bo’lib turibdi.

Dissertatsiyasining asosiy maqsadi, yuqorida sanab o’tilgan dolzarb masalalar mavzularidan kelib chiqqan holda, quyidagilardan iborat:

tashqi nostatsionar ta’sirlar ostidagi qovushoq siqiluvchi suyuqlik bilan to’ldirilgan doiraviy silindrik elastik qatlamning (xususiy holda qobiqning) bo’ylama-radial tebranishlari umumiy tenglamalarini keltirib chiqarish, bu tenglamalarning xususiy va limitik hollarini tekshirish; olingan umumiy tenglamalardan amaliyotda qo’llash mumkin bo’lgan taqribiy tenglamalarni chiqarish hamda amaliy masalalar yechishda lozim bo’ladigan boshlang’ich va chegaraviy shartlarni topishdan iborat.

Olingan asosiy natijalarning ilmiy yangiligi quyidagilardan iborat:

-ichi qovishoq siqiluvchi suyuqlik to’ldirilgan doiraviy silindrik elastik qatlamning bo’ylama-radial tebranishlari umumiy tenglamalari keltirib chiqarildi;

-qatlam uchun olingan umumiy tenglamalarning xususiy va limitik hollarini tadqiq qilish, jumladan, ichiga qovishoq siqiluvchi suyuqlik to’ldirilgan doiraviy silindrik elastik qobiqrning bo’ylama-radial tebranishlari umumiy tenglamalari olindi;

-tebranish tenglamalar bilan bir qatorda qatlam yoki qobiqning istalgan kesimi nuqtalaridagi kuchlanganlik-deformatsiyalanganlik holatini talab etilgan aniqlikda hisoblashga imkon beruvchi algoritm ishlab chiqildi;

-muhandislik amaliyotida qo’llash mumkin bo’lgan taqribiy tenglamalar va amaliy masalalarning boshlang’ich va chegaraviy shartlari topildi.

Dissertatsiya himoyasiga quyidagi natijalar olib chiqilmoqda:

-ichi qovishoq siqiluvchi suyuqlik to’ldirilgan doiraviy silindrik elastik qatlamning bo’ylama-radial tebranishlari umumiy tenglamalari;

-qatlam uchun olingan umumiy tenglamalarning xususiy va limitik hollari tadqiq qilish natijalari va ichiga qovishoq siqiluvchi suyuqlik to’ldirilgan doiraviy silindrik elastik qobiqlarning bo’ylama-radial tebranishlari umumiy tenglamalari;

-qatlam yoki qobiqning istalgan kesimi nuqtalaridagi kuchlanganlik-deformatsiyalanganlik holatini talab etilgan aniqlikda hisoblashga imkon beruvchi algoritm;

-amaliy masalalarning boshlang’ich va chegaraviy shartlari va muhandislik amaliyotida qo’llash mumkin bo’lgan, tartibi to’rtdan yuqori bo’lmagan taqribiydifferensial tenglamalar.

Tadqiqotlarning ilmiy ahamiyati ichida qovushoq suyuqlik to’ldirilgan doiraviy silindrik elastik qatlam va qobiqlarning bo’ylama-radial tebranishlari umumiy tenglamalarining yuqoridagi sistemalar tebranishlari haqidagi masalalarni yechishga qollash mumkinligi bo’lib, olingan natijalar tebranish jarayonidagi kuchlanganlik-deformatsiyalanganlik holatlarini aniqlashda ishlatilishi mumkinligidir.

Tadqiqotlarning amaliy ahamiyati olingan taqribiy tenglamalarning boshqa masalalarni, xususan yuqorida qaralgan gidroelastik sistemasinig majburiy tebranishlari haqidagi masalalarni ham yechishga qo’llash mumkin ekanligidir.

Dissertatsiya ishi 2 bobdan iborat bo’lib, 1-chi bobda doiraviy silindrik qatlam va qobiqlarning suyuqliklar bilan o’zaro nostatsionar ta’siriga bag’ishlangan ilmiy adabiyotlar sharhi, deformatsiyalanuvchi qattiq jism uchun potensial funksiyalarda ifodalangan to’lqin tenglamalari keltirilgan. Shuningdek siqiluvchi suyuqlik zarrachasi harakatning Nave-Stoks ko’rinishidagi tenglamalari va ularning potensial funksialar orqali to’lqin tenglamalariga o’xshash ko’rinishdagi ifodalanishi keltirilgan.

Ikkinchi bob qovushoq siqiluvchi suyuqlik bilan to’ldirilgan doiraviy silindrik elastik qatlam va qobiqlarning bo’ylama-radial tebranish tenglamalarini tadqiq qilishga bag’ishlangan. Bu bob ham, xuddi birinchi bob singari 4 ta paragrafdan iborat bo’lib, birinchi paragrafda ichida qovushoq siqiluvchi suyuqlik saqlovchi silindirik qatlamning buralma tebranishlari bayon qilingan. Ikkinchi paragrafi sistemaning kuchlangan – deformatsiyalangan holatini aniqlash, uchinchi paragrafi qovushoq siqluvchi suyuqlik to’ldirilgan doiraviy silindirik qatlam qobiqlarning bo’ylama radial tebranishlari tenglamalari tadqiq etilgan. To’rtinchi paragrafda esa ko’chishlar, kuchlanishlar va tezliklar maydonlarini aniqlash masalasi ko’rilgan.

Dissertatsiya ishining oxirgi bo’limi asosiy xulosalar deb ataladi va bu bo’limda dissertatsiya ichida olingan eng muhim natijalar asosida chiqarilgan xulosalar keltirilgan. Dissertatsiya ishi foydalanilgan adabiyotlar ro’yxati bilan yakunlanadi.

I-BOB. SUYUQLIK TO'LDIRILGAN DOIRAVIY SILINDRIK QATLAM VA QOBIQLARNING NOSTATSINAR TEBRANISHLARI: ADABIYOTLAR SHARHI; ASOSIY TENGLAMALAR VA MUNOSABATLAR

§1.1. Doiraviy silindrik qatlam va qobiqlarning suyuqliklar bilan o'zaro nostatsionar ta'sirlashuvi muammosini hal etishning hozirgi zamon holati

Biz ushbu paragraf doirasida silindrik qobiqlarning ideal va qovushoq suyuqliklar bilan o'zaro nostatsionar ta'sirlashuviga bag'ishlangan ilmiy tadqiqotlar va ularning natijalari bo'yicha ilmiy matbuotda e'lon qilingan maqolalarning ba'zilari haqida to'xtalib o'tamiz. Bunda ushbu tadqiqotlar yoki ular to'g'risidagi maqolalarni ikki yo'nalishga ajratamiz:

1) muxandislik konstruksiyalari elementlarining nostatsionar tebranishlar tenglamalarini chiqarish;
2) muxandislik konstruksiyalari elementlari va suyuq muhitlarning o'zaro ta'sirlashuv masalalari.

Dinamik masalalarni yechishning turli usullari ichida elastik va qovushoq-elastik sterjen, plastina va qobiqlarning dinamikasini taqribiy va aniqlashtirilgan tenglamalar asosida o'rganish alohida o'rin tutadi. Bu tadqiqotchilarning, xuddi statik masalalardagi kabi, dinamik masalalarni yechishda ham masalaning soddalashtirilgan matematik modeliga o'tish uchun intilishlari bilan bog'iq.

Ma'lum bo'lgan klassik tenglamalar juda past chastotali jarayonlarnigina yaxshi tavsiflaydilar. Jarayonlar yuqori chastotali bo'lishlari bilan bu tenglamalar yaroqsiz bo'lib qoladilar [14]. Shuning uchun ham tenglamalarni aniqlashtirishga urinishlar ko'p bo'lgan [17,18,23].

Bunday urinishlarda birinchilik L.Poxgammer, C. Kri, D.V.Reley va S.P.Timoshenkolarga tegishli [5]. Sterjenlarning klassik tebranish tenglamasini birinchi marta aniqlashtirish D.V.Releyga tegishli bo'lib, bunda u birinchi marta aylanish inersiyasini hisobga olgan. Keyinchalik D.V.Reley tenglamasi S.P.Timoshenko tomonidan ko'ndalang siljish deformatsiyasini hisobga olgan holda yanada aniqlashtirilgan. Klassik tenglamalarni aniqlashtirish masalasi bilan xuddi sterjenlar uchun bo’lgani kabi, plastina va qobiqlar uchun hozirgi kungacha davom etmoqda. Xususan bunday ish bilan L.G.Donnel, I.K.Nigul, V.I.Utesheva, E.B.Omenitskaya, K.Z.Galimov, I.Mirsky, H.Herrmann va boshqalar shig’ullanishgan [5].

Mexanik sistemalarning tebranish tenglamalarini aniqlashtirish hozirgi kunda ham davom etmoqda. Bu chet el va vatanimiz olimlarining eng yangi ilmiy maqolalaridan ham ko’rinib turibdi. Masalan I.G.Filippov [17], X.Xudoynazarov [18], Sh.Markus [19] va boshqalarning [22] ishlari yuqoridagi fikrimizni tasdiqlaydi. Ko’rsatilgan ishlarning ko’pchiligida elastik yoki qovushoq-elastik mexanik sistemalarning taqribiy tebranish tenglamalari u yoki bu fizik yoki geometrik xarakterdagi gipotezalar asosida keltirib chiqarilgan. Ammo, ushbu gipotezalar tebranish tenglamalarni soddalashtirish bilan bir qatorda ba’zi xato va kamchiliklarga ham olib keladi.

Bunday xato va kamchiliklarni birinchilardan bo’lib V.V.Novojilov va R.M.Finkelshteynlar 1943 yilda ko’rsatib o’tishgan va klassik nazariya yo’l qo’yadigan xato va kamchilklarni baholaganlar. Keyinchalik 1947 yilda X.M.Mushtari tomonidan qobiqlarning klassik Kirxgoff-Lyav nazariyasiga tegishli tebranish taqribiy tenglamalarning qo’llanish sohasi aniqlandi va uning nisbatan tor ekanligi ko’rsatildi. 1961 yilga kelib professor V.M.Darevskiy Kirxgoff-Lyav nazariyasining turli variantlari turli xil xatolarga olib kelishini isbotladi. Shuning uchun han umumiy holda, ya’ni ixtiyoriy qobiqlar va ixtiyoriy yuklamalar uchun elastiklik nazariyasining umumiy munosabatlarini sodda tenglik yoki munosabatlar bilan almashtirish masalasi doim ochiq holda qoladi.

Yuqorida nomlari zikr etilgan olimlarning ishlari va bu erda keltirilmagan ishlarning tahlili shuni ko’rsatadiki doiraviy silindrik elastik qobbiqlarning tebranish nazariyasi hozirgi kungacha to’liq asoslanmagan. Shuning uchun ushbu nazariyani asoslashga bo’lgan urinishlar bugungi kungacha davom etib kelmoqda. Klassik tebranish nazariyasini aniqlashtirish ham nazariyani asoslashning xuddi o’zidir. Klassik nazariyalarni aniqlashtirish yoki tebranish nazariyasini asoslashda tebranish tenglamalarini keltiriib chiqarish asosiy rol o’ynaydi.

Tebranish tenglamalarini keltirib chiqarish usullari uchta asosiy yo’nalishlarga bo’linadi. Bu yo’nalishlardan birinchisiga dinamikada variatsion prinsiplarni ishlatishga asoslangan usullarni keltirish mumkin [19].

Ikkinchi yo’nalishga elastik ko’chishlar maydoni tuzuvchilarini qatorga, shu jumladan darajali qatorga, yoyishga asoslangan usullar kiradi. Ushbu metodning statika uchun variantini N.A.Kil’chevskiy taklif etgan. Ushbu qator yoki darajali qatorga yoyish metodini dinamikaning keng sinflariga tadbiq etish birinchi bor P.S.Epstein tomonidan amalga oshirilgan. Bu avtorlar tadqiqotlaridan keyingi yillarda olib borilgan tadqiqotlar natijasidan ushbu metodlar sezilarli darajada rivojlantirildi.

Xuddi shu usul asosida V.Z.Vlasov o’zining boshlang’ich funksiyalar usulini yaratdi va uni qobiqli sistemalar uchun muvaffaqiyat bilan qo’lladi. Elastik ko’chishlarni darajali qatorga yoyish metodini G.I.Petrashen [14], tekis deformatsiyalanuvchi tekislik (qatlam) haqidagi dinamik masala misolida, matematik jihatdan qat’iy asosladi.

Tebranish tenglamalarini keltirib chiqarish usullarining uchinchi yo’nalishiga prof. I.G.Filippov va prof. X.X.Xudoynazarovlar [17,18] tomonidan sezilarli darajada rivojlantirilgan elastiklik dinamik nazariyasining uch o’lchovli masalalarining aniq yechimlaridan foydalanish usuli kiradi. Professorlar I.G.Filippov va X.X.Xudoynazarov hamda ularning ko’p sonli o’quvchilari tomonidan jismlar va moddalarning har xil xususiyatlarini hisobga olgan holda sterjenlar, plastinalar va qobiqlarning tebranish tenglamalari keltirib chiqarilgan. Moddalarning har xil xususiyatlariga reologik, anizotropik, bir jinslimaslik, haroratga qarab o’zgarishlik va boshqalar kiradi. Bundan tashqari jismlar, ya’ni tadqiq etilayotgan mexanik sistemalarning xususiyatlariga ularning ko’ndalang kesimlari geometriyasining o’zgarishlari, bikrligining o’zgaruvchanligi va shunga o’xshash xususiyatlarini hisobga olish kiradi.

Xuddi ana shu usulni qo’llab deformatsiyalanuvchi muhit bilan o’zaro ta’sirlashuvchi silindrik elastik va qovushoq-elastik qobiq va sterjenlarning tebranish nazariyasi prof. X.X.Xudoynazarov tomonidan yaratilgan [18]. Silindrik elastik va qovushoq-elastik qobiq va sterjenlarning ideal va qovushoq suyuqliklar bilan ta’sirlashuvi haqidagi masalalar tadqiqotiga juda ko’p avtorlarning ishlari qaratilgan.

Bunda ayniqsa mexanik sistemalarning ideal suyuqlik bilan ta’sirlashuvi masalalari juda keng va atroflicha tekshirilgan. Shu bilan bir qatorda, bunday sistemalarning qovushoq suyuqlik bilan ta’sirlashuvi haqidagi masalalar juda kam sonli tadqiqotchilar tomonidan tekshirilgan. Tadqiqotlar bu sohada kam o’tkazilishini sababi, qovushqoq suyuqlik harakat tenglamalarining yoki boshqacha aytganda Nave-Stoks tenglamasining echimi olinmaganligi, ya’ni bu tenglamalarni echish matematik jihatdan katta muammo ekanligidir.

O’tgan asrning 70-yillarida R.P.Darmond va W.T.Rouleau [26] lar tomonidan, qovushqoq siqilmaydigan suyuqliklar uchun Nave-Stoks tenglamasini to’rtta potensial funksiyalarga nisbatan tenglamalarga keltirirsh amalga oshiriladi. Nave-Stoks vektor tenglamasini eng umumiy holda ya’ni suyuqlik siqiluvchi va qovushoq bo’lgan holda potensial funksiyalarga nisbatan tenglamalar sistemasiga keltirish akademik A.N.Guz [8] tomonidan amalga oshirildi. Akademik A.N.Guzning ushbu ishlari asosida doiraviy silindrik elastik qobiqlarning qovushoq siqiluvchi suyuqliklar bilan o’zaro ta’sirlashuviga bag’ishlangan ishlar paydo bo’ldi. Xususan bunday ishlarga misol qilib tadqiqotchilardan B.Yalg’ashev ning [23], [24] ishlarini ko’rsatish mumkin.

Mazkur dissertatsiya ishida prof. X.Xudoynazarov va tadqiqotchi B.Yalg’ashevlar [23] tomonidan taklif etilgan nazariya asosida siqiluvchi qovushoq suyuqlik bilan to’ldirilgan doiraviy silindrik elastik qobiqning xususiy buralma tebranishlari haqidagi masala tadqiq qilingan. Olingan sonli natijalar asosida ba’zi ilmiy xulosalar chiqarilgan.

§1.2. Elastik silindrik qatlam va qovushoq suyuqliklarning harakat tenglamalari va ularni yechish

Biz ushbu paragraf doirasida chiziqli elastik muhit nuqtalarining harakat tenglamalarini skalyar va vektor potensiallar orqali ifodalaymiz. Buning uchun avvalo chiziqli elastik muhit deformatsiyasiga oid munosabatlarni qarab chiqamiz. Bunda biz tutash muhitlar kichik deformatsiyalari nazariyasiga tayanamiz. Ma’lumki, bu nazariyaga ko’ra ko’chishlar gradienti birdan ancha kichik bo’lishi talab etiladi. Ana shu holatda deformatsiyalarning kichikligi ta’minlanadi. Shunga ko’ra chiziqlimas deformatsiya tenzori komponentalarini chiziqlilashtirish mumkin bo’ladi. U holda cheksiz kichik deformatsiyalar simmetrik tenzorining komponentalari uchun quyidagi munosabatlarga ega bo’lamiz [28]:

$$\varepsilon_{xx}=\frac{du}{dx},\quad \varepsilon_{yy}=\frac{dv}{dy},\quad \varepsilon_{zz}=\frac{dw}{dz},\ \vec{S}=u\,\vec{i}+v\,\vec{j}+w\,\vec{k},$$

$$\varepsilon_{xy}=\frac{1}{2}\left(\frac{du}{dy}+\frac{dv}{dx}\right),\quad \varepsilon_{yz}=\frac{1}{2}\left(\frac{dv}{dz}+\frac{dw}{dy}\right),\ \varepsilon_{xz}=\frac{1}{2}\left(\frac{du}{dz}+\frac{dw}{dx}\right),\qquad (1.1)$$

bu erda $u,v,w-\vec{S}$ ko’chish vektorining x,y,z koordinat o’qlariga proeksiyalari; $\varepsilon_{ij}-$deformatsiya tenzori komponentalari. Bunda $\varepsilon_{ij}-$cho’zilish deformatsiyasi, agar $i=j$ bo’lsa, va siljish deformatsiyasi agar $i\neq j$ bo’lsa. Aniqlik uchun x,y,z larni Lagranj koordinatalari sistemasi deb qabul qilamiz. Ma’lumki klassik elastiklik chiziqli nazariyasida Eyler va Lagranj koordinatalari sistemalari bir-biridan farqlanmaydi va cheksiz kichik deformatsiyalarning Eyler va Lagranj komponentalari bir-biriga teng deb hisoblanadilar.

Deformatsiya natijasida muhit zarrachalarining, xuddi qattiq jism kabi, uning boshqa bir nuqtasi atrofida biror burchakka burilishi sodir bo’ladi va bu burilish ham cheksiz kichik bo’ladi. Bu burilish $\vec{\Omega}$ burchagi vektori bilan aniqlanadi. Burilish burchagi vektori

$$\vec{\Omega}=rot\,\vec{S}$$

kabi chiziqlanadi. Yuqoridagi (1.1) munosabatlar $oxyz-$Dekart koordinatalari sistemasi uchun keltirildi. Aslini olganda biz dissertatsiya doirasida doiraviy silindrik qobiqning xususiy tebranishlarini tadqiq etamiz. Shuning uchun (1.1) munosabatlar va bundan keyin zarur bo'ladigan asosiy munosabatlarni r,Θ,z – qutb koordinatalari sistemasida keltirish zarur bo'ladi.

Ushbu (1.1) Koshi munosabatlari qutb-(r,θ,z) koordinatalari sistemasida quyidagi ko'rinishni oladi [28]

$$\varepsilon_{rr}=\frac{du_r}{dr},\quad \varepsilon_{\theta\theta}=\frac{1}{r}\frac{du_\theta}{d\theta}+\frac{u_r}{r},\quad r_{zz}=\frac{du_z}{dz},\quad \varepsilon_{\theta z}=\frac{1}{2}\left(\frac{du_\theta}{dz}+\frac{du_z}{d\theta}\right),$$

$$\varepsilon_{r\theta}=\frac{1}{2}\left(\frac{1}{r}\frac{du_r}{d\theta}-\frac{u_\theta}{r}+\frac{du_\theta}{dr}\right),\quad \varepsilon_{2z}=\frac{1}{2}\left(\frac{du_r}{dz}+\frac{du_z}{dr}\right),\tag{1.2}$$

bu yerda u_r,u_θ,u_z – ko'chish $\vec{U}$-vektorining r,θ,z-koordinat o'qlaridagi tuzuvchilari.

Qutb koorditalari sistemasida elastik jismning harakat tenglamalari quyidagicha bo'ladi [28]

$$\frac{d\sigma_{rr}}{dr}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}+\frac{d\sigma_{rr}}{dz}+\frac{1}{r}\frac{d\sigma_{\theta r}}{d\theta}=\rho\frac{d^2u_r}{dt^2};$$

$$\frac{d\sigma_{rr}}{dz}+\frac{d\sigma_{rz}}{dr}+\frac{\sigma_{rz}}{z}+\frac{d\sigma_{\theta z}}{d\theta}=\rho\frac{d^2u_z}{dt^2};\qquad ()$$

$$\frac{d\sigma_{r\theta}}{dr}+\frac{d\sigma_{z\theta}}{dz}+\frac{1}{r}\frac{d\sigma_{\theta\theta}}{d\theta}+\frac{1}{r}\sigma_{r\theta}+\frac{\sigma_{\theta z}}{r}=\rho\frac{d^2u_\theta}{dt^2}.$$

Bu erda σ_{ij} – kuchlanish tenzori komponentalari bo'lib, $i=j$ bo'lganda ular kuchlanish tenzorining normal komponentalari, $i\neq j$ bo'lganda esa urinma kuchlanishlarni ifodalaydilar; ρ – elastik jism zichligi.

Elastiklik chiziqli nazriyasida kuchlanishlar bilan deformatsiyalar orasidagi bog'lanishlar [12]

$$\begin{aligned}\sigma_{rr}&=\lambda\varepsilon+2\mu\,\varepsilon_{rr}, & \sigma_{r\theta}&=2\mu\,\varepsilon_{r\theta},\\ \sigma_{\theta\theta}&=\lambda\varepsilon+2\mu\,\varepsilon_{\theta\theta}, & \sigma_{\theta z}&=2\mu\,\varepsilon_{\theta z},\\ \sigma_{zz}&=\lambda\varepsilon+2\mu\,\varepsilon_{zz}, & \sigma_{rz}&=2\mu\,\varepsilon_{rz},\end{aligned}\tag{1.4}$$

bu yerda

$\lambda=\dfrac{E\nu}{(1+\nu)(1-2\nu)},\quad \mu=\dfrac{E}{2(1+\nu)}$ - Lame koeffitsiyentlari;

$\varepsilon=\varepsilon_{rr}+\varepsilon_{\theta\theta}+\varepsilon_{zz}$ – hajmiy kengayish ; E – Yung moduli;

ν – Puasson koeffitsiyenti.

Keltirilgan (1.4) tenglamalar izotrop jismning uch o'lchovli kuchlanganlik holati uchun Guk qonuni deyiladi.

Doiraviy silindrik qobiqni uch o'lchovli silindrik qatlam deb hisoblaymiz. U holda bunday qobiq uchun harakat tenglamalari sifatida (1.3) tenglamalarni ishlatish mumkin bo'ladi. Ushbu tenglamalarni bundan keyin

$$(\lambda + 2\mu)\Delta\Phi = \rho\ddot{\Phi}, \quad \Delta\vec{\Psi} = \rho\ddot{\vec{\psi}}, \tag{1.5}$$

ko'rinishda [2] ishlatiladi. Bu erda $\Delta -$ Laplasning uch o'lchovli operatori:

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$$

Bo'ylama to'lqinlar potensiali – Φ va ko'ndalang to'lqinlar potensiali - $\vec{\Psi}$ lar

$$\vec{U} = grad\Phi + rot\left[\vec{e}_z\Psi_1 + rot(\vec{e}_z\Psi_2)\right] \tag{1.6}$$

formula [11] bilan kiritilgan.

Vektor maydoni-$\vec{\Psi}$ning solenoidallik sharti- $div\vec{\Psi} = 0$ avtomatik tarzda bajariladi, agar u quyidagicha tasvirlangan bo'lsa

$$\vec{\Psi} = \vec{e}_z\Psi_1 + rot(\vec{e}_z\Psi_2),$$

bunda $\vec{e}_z - oz$ o'qining birlik vektori.

Ko'chish vektori (1.6) ko'rinishda tasvirlanganda uning komponentalari

$$\begin{aligned}
u_r &= \frac{\partial}{\partial r}\left[\Phi + \frac{\partial}{\partial z}\Psi_2\right] + \frac{1}{r}\frac{\partial}{\partial\theta}\Psi_1, \\
u_\theta &= \frac{1}{r}\frac{\partial}{\partial\theta}\Phi - \frac{\partial}{\partial r}\Psi_1 + \frac{1}{r}\frac{\partial^2}{\partial z\partial\theta}\Psi_2, \\
u_z &= \frac{\partial}{\partial z}\Phi - \left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\Psi_2,
\end{aligned} \tag{1.7}$$

formulalar bilan aniqlanadi. Deformatsiyalar esa

$$\begin{aligned}
\varepsilon_{rr} &= \frac{\partial^2}{\partial r^2}\Phi - \frac{1}{r}\left(\frac{1}{r} - \frac{\partial}{\partial r}\right)\frac{\partial}{\partial\theta}\Psi_1 + \frac{\partial^3}{\partial r^2\partial z}\Psi_2, \\
\varepsilon_{\theta\theta} &= \frac{1}{r}\left[\left(\frac{1}{r}\frac{\partial^2}{\partial\theta^2} + \frac{\partial}{\partial r}\right)\Phi + \left(\frac{1}{r}\frac{\partial}{\partial\theta} - \frac{\partial^2}{\partial r\partial\theta}\right)\Psi_1 + \frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial^2}{\partial\theta^2} + \frac{\partial}{\partial r}\right)\Psi_2,\right. \\
\varepsilon_{zz} &= \frac{\partial^2}{\partial z^2}\Phi - \frac{\partial}{\partial z}\left(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\Psi_2,
\end{aligned}$$

$$\varepsilon_{rz} = 2\frac{\partial^2}{\partial r\partial z}\Phi + \frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}\Psi_1 + \frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial z^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) - \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\Psi_2, \qquad (1.8)$$

$$\varepsilon_{r\theta} = \frac{2}{r}\frac{\partial}{\partial\theta}\left(\frac{\partial}{\partial r} - \frac{1}{r}\right)\Phi + \left(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)\right)\Psi_1 + \frac{2}{r}\frac{\partial^2}{\partial z\partial z}\left(\frac{\partial}{\partial r} - \frac{1}{r}\right)\Psi_2,$$

$$\varepsilon_{z\theta} = \frac{\partial}{\partial z}\left(\frac{2}{r}\frac{\partial}{\partial\theta}\Phi - \frac{\partial}{\partial r}\Psi_1\right) - \frac{\partial}{\partial\theta}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^3}\frac{\partial^2}{\partial\theta^2} - \frac{1}{r}\frac{\partial^2}{\partial z^2}\right)\Psi_2,$$

$$\varepsilon = \varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz} = \Delta\Phi$$

ga teng.

Agar silindrik qobiqning tebranishlari uning o’qiga nisbatan simmetrik bo’lsalar, yuqoridagi kattaliklar Θ-burchakka bog’liq bo’lmaydi va ko’chshlar uchun formulalar quyidagi ko’rinishni oladilar

$$u_r = \frac{\partial}{\partial r}\left(\Phi + \frac{\partial}{\partial z}\Psi_2\right), \quad u_z = \frac{\partial}{\partial z}\Phi - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\Psi_2, \quad u_\theta = -\frac{\partial}{\partial r}\Psi_1, \qquad (1.9)$$

Deformatsiyalar uchun esa

$$\varepsilon_{rr} = \frac{\partial^2}{\partial r^2}\left(\Phi + \frac{\partial}{\partial z}\Psi_2\right), \quad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial}{\partial r}\left(\Phi + \frac{\partial}{\partial z}\Psi_2\right),$$

$$\varepsilon_{zz} = \frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}\Phi - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)\Psi_2,$$

(1.10)

$$\varepsilon_{rz} = 2\frac{\partial^2}{\partial r\partial z}\Phi + \frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial z^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)\Psi_2,$$

$$\varepsilon_{r\theta} = -r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)\Psi_1, \quad \varepsilon_{z\theta} = -\frac{\partial^2}{\partial r\partial z}\Psi_1.$$

ifodalarga ega bo’lamiz.

Izotrop qobiq uchun kuchlanish tenzorining σ_{ij}-komponentalari, masala o’qqa nisbatan simmetrik bo’lganligi uchun, ancha sodda ko’rinishlarga ega bo’ladilar

$$\sigma_{rr} = (\lambda + 2\mu)\left(\frac{\partial^2\Phi}{\partial r^2} + \frac{\partial^3\Psi_2}{\partial r^2\partial z}\right) + \lambda\left[\frac{\partial^2\Phi}{\partial z^2} - \frac{1}{r}\frac{\partial^2\Psi_2}{\partial r\partial z} - \frac{\partial^3\Psi_2}{\partial r^2\partial z} + \frac{1}{r}\left(\frac{\partial\Phi}{\partial r} + \frac{\partial^2\Psi_2}{\partial r\partial z}\right)\right]$$

$$\sigma_{\theta\theta} = \frac{(\lambda + 2\mu)}{r}\left(\frac{\partial\Phi}{\partial r} + \frac{\partial^2\Psi_2}{\partial r\partial z}\right) + \lambda\left[\frac{\partial^2\Phi}{\partial z^2} + \frac{\partial^2\Phi}{\partial z^2} - \frac{1}{r}\frac{\partial^2\Psi_2}{\partial r\partial z}\right]$$

$$\sigma_{zz} = (\lambda + 2\mu)\left(\frac{\partial^2\Phi}{\partial z^2} - \frac{1}{r}\frac{\partial^2\Psi_2}{\partial r\partial z} - \frac{\partial^3\Psi_2}{\partial r^2\partial z}\right) + \lambda\left[\frac{\partial^2\Phi}{\partial r^2} + \frac{\partial^3\Psi_2}{\partial r^2\partial z} + \frac{1}{r}\left(\frac{\partial\Phi}{\partial r} + \frac{\partial^2\Psi_2}{\partial r\partial z}\right)\right] \quad (1.11)$$

$$\sigma_{rz} = \mu\left\{2\frac{\partial^2\Phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left[\frac{\partial^2}{\partial z^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\Psi_2\right\}$$

$$\sigma_{\theta z} = -\mu\frac{\partial^2\Psi_1}{\partial r\partial z}\ ; \quad \sigma_{r\theta} = \mu\left[\frac{1}{r}\frac{\partial}{\partial r} - \frac{\partial^2}{\partial r^2}\right]\Psi_1 .$$

Doiraviy silindrik elastik qobiqning ichidagi suyuqlikni qovushoq va siqiluvchi deb hisoblaymiz. Bundan tashqari suyuqlik zarrachalarining tebranishlarini ham kichik deb qabul qilamiz. U holda bunday suyuqlikni tavsiflash uchun Lagranj koordinatalarini qo’llash mumkin bo’ladi, va demak mos ravishda, quyidagi munosabatlarga ega bo’lamiz [8]:

Nave-Stoksning vektor tenglamasi

$$\frac{\partial\vec{V}}{\partial t} - \nu'\Delta\vec{V} + \frac{1}{\rho_0'}gradP + \frac{1}{3}\nu' graddiv\vec{V} = 0, \qquad (r,\theta,z)\in V_2; \quad (1.12)$$

uzviylik tenglamasi

$$\frac{1}{\rho_0'}\frac{\partial\rho'}{\partial t} + div\vec{V} = 0, \qquad (r,\theta,z)\in V_2; \quad (1.13)$$

suyuqlikdagi kuchlanish tenzorining komponentalarini aniqlash formulalari

$$P_{ij} = -P\delta_{ij} + \lambda'\delta_{ij}div\vec{V} + \mu' e_{ij}, \qquad (r,\theta,z)\in V_2;$$
$$e_{ij} = \frac{1}{2}\left(V_{i,j} + V_{j,i}\right), \qquad (i,j = r,\theta,z); \quad (1.14)$$

va nihoyat holat tenglamasi

$$\frac{\partial\rho}{\partial\rho'} = a_0^2 \ , \qquad a_0 = const \quad (1.15)$$

Yuqorida qovushoq suyuqlik uchun keltirilgan asosiy munosabatlarda quyidagi belgilashlar qabul qilingan:

$\rho_{ij}(i,j = r,o,z)$ – suyuqlikda kuchlanish tenzori komponentalari;

ρ_o' – tinch holatdagi suyuqlik zichligi;

a_o – tinch holatdagi suyuqlikda tovush tarqalish tezligi;

$\vec{V}-$suyuqlik zarrashasining tezlik vektori;

ρ' $-$suyuqlik zichligining qo'zg'alishi (harakatga kelgan, qo'zg'algan suyuqlik zichligi);

$\rho-$suyuqlik bosimining qo'zg'alishi;

V_2 $-$fazoning suyuqlik egallagan qismining hajmi.

$V_i(i=r,\theta,z)-$ suyuqlik zarrashasi tezlik vektorining koordinat o'qlaridagi komponentalari;

$\lambda'-$qovushoqlik ikkinchi koeffitsiyenti, $\lambda'=-\frac{2}{3}\mu'$;

$\nu'-$qovushoqlik kinematik koeffitsiyenti;

$\mu'-$ qovushoqlik koeffitsiyenti, $\mu'=\rho_0'\nu'$.

Keltirilgan (1.5)-(1.8) hamda (1.12)-(1.15) munosabatlar qobiq va suyuqliklar uchun alohida-alohida beriluvchi chegaraviy va boshlang'ich shartlar hamda ikki muhitni (qattiq va suyuq) ajratib turuvchi tebranuvchi sirtda beriladigan kinematik va dinamik shartlar bilan birgalikda doiraviy silindrik qobiq va qovushoq siqiluvchi suyuqlik uchun gidroelastik munosabatlarning yopiq (to'liq) sistemasini tashkil etadilar.

Endi (r,θ,z) ortogonal koordinatalarning silindrik sistemasida keltirib chiqarilgan (1.12), (1.14) va (1.15) tenglamalarning umumiy yechimlarini topish bilan shug'ullanamiz. Suyuqlik uchun *G*-skalyar va $\vec{\chi}$-vektor potensiallarni quyidagicha kiritamiz [7,8]:

$$\vec{V}=\frac{\partial}{\partial t}\left(gradG+rot\vec{\chi}\right); \qquad (r,\theta,z)\in V_2\,, \tag{1.16}$$

Bu yerda vektor $\vec{\chi}$ quyidagi tenglamani qanoatlantiradi.

$$\left(\frac{\partial}{\partial t}-\nu'\Delta\right)\vec{\chi}=0\,, \qquad (r,\theta,z)\in V_2\,. \tag{1.17}$$

Oxirgi tenglama $\vec{\chi}$ vektorning χ_1, χ_2 va χ_3 komponentalariga nisbatan uchta skalyar tenglamalar sistemasiga ekvivalentdir.

Suyuqlikning harakat tenglamasi, uzviylik tenglamasi va holat tenglamasi hammasi bo'lib uchta tenglamani tashkil etadilar. Lekin izlanuvchi noma'lumlar soni to'rtta, ya'ni –G,χ_1,χ_2,χ_3 lar. Shuning uchun (1.16) vektor tenglamada faqatgina rotorlari noldan farqli bo'lgan qo'shiluvchilargina qoldiramiz. Boshqacha aytganda $\vec{\chi}$ vektorini quyidagicha kiritamiz [9,10]

$$\vec{\chi} = \vec{e}_z \chi_1 + rot(\vec{e}_z \chi_2) \tag{1.18}$$

ya'uni (1.15) ga qo'yib

$$\vec{V} = \frac{\partial}{\partial t}\left(gradG + rot\left[\vec{e}_z \chi_1 + rot(\vec{e}_z \chi_2)\right]\right); \quad (r,\theta,z) \in V_2, \tag{1.19}$$

ga ega bo'lamizki, bunda χ_1 va χ_2 lar, (1.17) ga asosan

$$\left(\frac{\partial}{\partial t} - \nu'\Delta\right)\chi_1 = 0 \quad \text{va} \quad \left(\frac{\partial}{\partial t} - \nu'\Delta\right)\chi_2 = 0; \quad (r,\theta,z) \in V_2. \tag{1.20}$$

tenglamalarni qanoatlantiradilar.

Kiritilgan (1.19) taqdimotdan suyuqlik zarrachalari tezliklarining V_r, V_θ, V_z komponentalari uchun ularni G,χ_1 va χ_2 skalyar potensiallar orqali ifodalovchi formulalarni keltirib chiqarish qiyin emas

$$\begin{aligned}
V_r &= \frac{\partial}{\partial t}\left[\frac{\partial G}{\partial r} + \frac{1}{r}\frac{\partial \chi_1}{\partial \theta} + \frac{\partial^2 \chi_2}{\partial r \partial z}\right];\\
V_\theta &= \frac{\partial}{\partial t}\left[\frac{1}{r}\frac{\partial G}{\partial \theta} - \frac{\partial \chi_1}{\partial r} + \frac{1}{r}\frac{\partial^2 \chi_2}{\partial \theta \partial z}\right]; \qquad (r,\theta,z) \in V_2,\\
V_z &= \frac{\partial}{\partial t}\left[\frac{\partial G}{\partial z} - \frac{\partial^2 \chi_2}{\partial r^2} - \frac{1}{r}\frac{\partial \chi_2}{\partial r} - \frac{1}{r^2}\frac{\partial^2 \chi_2}{\partial \theta^2}\right];
\end{aligned} \tag{1.21}$$

Silindrik koordinatlari sistemasida suyuqlik zarrachalari tezliklari divergensiyasi quyidagi ko'rinishga ega

$$div\vec{V} = \frac{1}{r}\left[V_r + r\frac{\partial V_r}{\partial r} + \frac{\partial V_\theta}{\partial \theta} + r\frac{\partial V_z}{\partial z}\right]. \tag{1.22}$$

(1.21) ifodani (1.22) formulaga qo'ysak

$$div\vec{V} = \Delta\frac{\partial G}{\partial t}, \qquad (r,\theta,z) \in V_2 \tag{1.23}$$

ga ega bo'lamiz.

Suyuqlik uchun deformatsiya tezliklari tenzorining e_{ij}-komponentalari quyidagi ko'rinishlarga ega bo'ladilar

$$e_{rr}=\frac{\partial^3 G}{\partial r^2\partial t}+\frac{\partial^4\chi_2}{\partial r^2\partial z\partial t}\quad;\quad e_{\theta\theta}=\frac{1}{r}\frac{\partial}{\partial t}\left[\frac{\partial G}{\partial r}+\frac{\partial^2\chi_2}{\partial r\partial z}\right];$$

$$e_{zz}=\frac{\partial^3 G}{\partial z^2\partial t}-\frac{1}{r}\frac{\partial^3\chi_2}{\partial r\partial z\partial t}-\frac{\partial^4\chi_2}{\partial r^2\partial z\partial t};\quad e_{\theta z}=-\frac{\partial^3\chi_1}{\partial r\partial z\partial t};\tag{1.24}$$

$$e_{rz}=2\frac{\partial^3 G}{\partial r\partial z\partial t}+\frac{\partial}{\partial r\partial t}\left[\frac{\partial^2}{\partial z^2}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\chi_2\quad;\quad e_{r\theta}=\frac{\partial}{\partial t}\left[\frac{1}{r}\frac{\partial\chi_1}{\partial r}-\frac{\partial^2\chi_1}{\partial r^2}\right]$$

Xuddi shunday suyuqlik uchun kuchlanish tenzorining P_{ij}-komponentalari quyidagicha yoziladi

$$P_{\theta\theta}=-\rho_0'\left(\frac{4}{3}\nu'\Delta_0-\frac{\partial}{\partial t}\right)\frac{\partial G}{\partial t}+\lambda'\Delta_0\frac{\partial G}{\partial t}+\mu'\frac{\partial}{\partial t}\left[\frac{1}{r}\frac{\partial G}{\partial r}+\frac{1}{r}\frac{\partial^2\chi_2}{\partial r\partial z}\right];$$

$$P_{zz}=-\rho_0'\left(\frac{4}{3}\nu'\Delta_0-\frac{\partial}{\partial t}\right)\frac{\partial G}{\partial t}+\lambda'\Delta_0\frac{\partial G}{\partial t}+\mu'\frac{\partial}{\partial t}\left[\frac{\partial^2 G}{\partial z^2}-\frac{1}{r}\frac{\partial^2\chi_2}{\partial r\partial z}-\frac{\partial^3\chi_2}{\partial r^2\partial z}\right];$$

$$P_{rr}=-\rho_0'\left(\frac{4}{3}\nu'\Delta_0-\frac{\partial}{\partial t}\right)\frac{\partial G}{\partial t}+\lambda'\Delta_0\frac{\partial G}{\partial t}+\mu'\frac{\partial}{\partial t}\left[\frac{\partial^2 G}{\partial r^2}+\frac{\partial^3\chi_2}{\partial r^2\partial z}\right];\tag{1.25}$$

$$P_{rz}=\mu'\frac{\partial}{\partial t}\left[2\frac{\partial^2 G}{\partial r\partial z}+\frac{\partial}{\partial r}\left[\frac{\partial^2}{\partial z^2}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\chi_2\right];$$

$$P_{r\theta}=\mu'\frac{\partial}{\partial t}\left(\frac{1}{r}-\frac{\partial}{\partial r}\right)\frac{\partial\chi_1}{\partial r}\quad;\quad P_{z\theta}=-\mu'\frac{\partial^3\chi_1}{\partial r\partial z\partial t}$$

$$\Delta_0=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$$

Endi (1.23) ni (1.12), (1.13) va (1.14) munosabatlarga qo’yib

$$\frac{\partial\vec{V}}{\partial t}-\nu'\Delta\vec{V}+\frac{1}{\rho_0'}gradP-\frac{1}{3}\nu' grad\Delta\frac{\partial G}{\partial t}=0\,;\tag{1.26}$$

$$\frac{1}{\rho_0'}\frac{\partial\rho'}{\partial t}+\Delta\frac{\partial G}{\partial t}=0\,;(r,\theta,z)\in V_2\tag{1.27}$$

$$P_{ij}=P\delta_{ij}+\lambda\delta_{ij}\Delta\frac{\partial G}{\partial t}+\mu' e_{ij}\,,\tag{1.28}$$

tenglamalarda ega bo’lamiz.

Ushbu olingan (1.26) tenglamadan quyidagini topamiz

$$\frac{1}{\rho_0'}gradP-\frac{1}{3}\nu' grad\Delta\frac{\partial G}{\partial t}=-\left(\frac{\partial}{\partial t}-\nu'\Delta\right)\vec{V}$$

va bu yerdagi $\vec{V}$ tezlik vektorining o'rniga uning (1.16) ifodasini qo'yib, differensial (chiziqli) operatorlarning o'rinlarini almashtiramiz hamda (1.17) ga asosan quyidagi ifodaga ega bo'lamiz

$$grad\left(\frac{\rho}{\rho_0'}-\frac{\nu'}{3}\Delta\frac{\partial G}{\partial t}\right)=-grad\left(\frac{\partial}{\partial t}-\nu'\Delta\right)\frac{\partial G}{\partial t}$$

bu yerdan

$$\frac{\rho}{\rho_0'}-\frac{\nu'}{3}\Delta\frac{\partial G}{\partial t}=-\frac{\partial^2 G}{\partial t^2}+\nu'\Delta\frac{\partial G}{\partial t}$$

tenglamaga ega bo'lamiz.

Olingan tenglama suyuqlik bosimi qo'zg'olishini kiritilgan G – skalyar funksiya orqali ifodalash imkonini beradi:

$$P=\rho_0'\left[\frac{4}{3}\nu'\Delta-\frac{\partial}{\partial t}\right]\frac{\partial G}{\partial t}\ ,\qquad (r,\theta,z)\in V_2\,. \tag{1.29}$$

Endi (1.15)-holat tenglamasini quyidagicha almashtiramiz

$$\frac{\partial\rho}{\partial\rho'}=a_0^2\ \Rightarrow\ \frac{\partial\rho}{\partial\rho'}\frac{\partial t}{\partial t}=a_0^2\ \Rightarrow\ \frac{\partial\rho}{\partial t}\frac{\partial t}{\partial\rho'}=a_0^2\ \Rightarrow\ \frac{\partial\rho}{\partial t}\frac{1}{\frac{\partial\rho'}{\partial t}}=a_0^2$$

bu yerda [3]

$$\frac{\partial\rho}{\partial t}=a_0^2\,\frac{\partial\rho'}{\partial t}$$

yoki

$$\frac{\partial\rho'}{\partial t}=\frac{1}{a_0^2}\frac{\partial\rho}{\partial t}\,.$$

Ushbu tenglamada P ning o'rniga uning (1.29) ifodasini qo'yamiz va natijada olamiz

$$\frac{\partial\rho'}{\partial t}=\frac{\rho_0'}{a_0^2}\left(\frac{4}{3}\nu'\Delta-\frac{\partial}{\partial t}\right)\frac{\partial^2 G}{\partial t^2},\quad (r,\theta,z)\in V_2\,. \tag{1.30}$$

Nihoyat (1.30) ifodani (1.27)-uzviylik tenglamasiga qo'yib quyidagini olamiz

$$\frac{1}{\rho_0'}\frac{\rho_0'}{a_0^2}\left(\frac{4}{3}\nu'\Delta-\frac{\partial}{\partial t}\right)\frac{\partial^2 G}{\partial t^2}+\Delta\frac{\partial G}{\partial t}=0$$

yoki

$$\frac{\partial}{\partial t}\left[\left(1+\frac{4\nu'}{3a_0^2}\right)\Delta-\frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right]G=0 \tag{1.31}$$

shunday qilib qovushoq siqiluvchi suyuqlikning (1.12), (1.13) va (1.15) harakat tenglamalari G, χ_1 va χ_2 potensiallarga nisbatan (1.20) va (1.31) to'lqin tenglamalariga keltiriladi va oxirgi tenglamalar yechimlari izlanadi. Bunda suyuqlik zarrachalari tezliklari vektorining komponentalari (1.21) formulalar yordamida, kuchlanish tenzori komponentalari (1.28) formulalar yordamida, suyuqlik bosimi qo'zg'olishi (1.29) formula yordamida va nihoyat suyuqlik zichligining qo'zg'olishi–(1.30) formula yordamida ushbu G, χ_1 va χ_2 skalyar potensiallar orqali ifodalanadi.

Qovushoq siqiluvchi suyuqlik uchun ushbu paragraf doirasida olingan natijalardan qovushoq siqilmaydigan suyuqlik uchun tenglamalar xususiy holda kelib chiqadi. Buning uchun [4] $a_0=\sqrt{\frac{dP}{d\rho}}$ ekanligini esga olish va suyuqlik siqilmaydigan bo'lgan holda, $\rho=const$ bo'lganligidan $d\rho\to 0$, va demak $a_0\to\infty$ ekanligini bilish kifoya. Agar $a_0\to\infty$ munosabatni (1.31) tenglamaga qo'ysak undan

$$\Delta\frac{\partial G}{\partial t}=0 \tag{1.32}$$

kelib chiqadi. Bu tenglama (1.20) tenglamalar bilan birgalikda siqilmaydigan qovushoq suyuqlik uchun asosiy tenglamalar sistemasini tashkil etadilar.

§1.3. Silindirik qatlam va qovushoq suyuqlikning o'qqa nisbatan simmetrik nostatsionar ta'sirlashuvi haqidagi masalalar.

Bundan oldingi paragraf doirasida silindrik qatlam va qovushoq suyuqlik uchun harakat tenglamalari keltirildi. Doiraviy silindrik qatlam uchun bu tenglamalar (1.5) ko'rinishiga ega va yoyilgan ko'rinishda quyidagi yoziladi

$$\Delta\phi=\frac{1}{a^2}\frac{\partial^2\phi}{\partial t^2};\quad \Delta\psi_1=\frac{1}{b^2}\frac{\partial^2\psi_1}{\partial t^2};\quad \Delta\psi_2=\frac{1}{b^2}\frac{\partial^2\psi_2}{\partial t^2} \tag{1.33}$$

bu yerda *a,b*- qatlam materialida bo’ylama va ko’ndalang to’lqinlar tarqalish tezliklari.

Qovushoq suyuqlik uchun esa harakat tenglamalari (1.20) va (1.31) ko’rinishlarda bo’ladi, ya’ni

$$\begin{gathered}(1+\frac{4\nu^{'}}{3a_0^2})\Delta\frac{\partial G}{\partial t}-\frac{1}{a_0^2}\frac{\partial^3 G}{\partial t^3}=0;\\ \left(\frac{\partial}{\partial t}-\nu^{'}\Delta\right)x_1=0;\quad \left(\frac{\partial}{\partial t}-\nu^{'}\Delta\right)x_2=0.\end{gathered} \tag{1.34}$$

Ya’na o’sha, bundan oldingi paragraf natijalariga ko’ra qovushoq suyuqlik to’ldirilgan doiraviy silindirik qatlam va qobiqlarning o’zaro nostatsinar ta’sirlashuvini o’zgarish masalasi, mos chegaraviy va boshlang’ich shartlar bilan (1.33), (1.34) tenglamalarini yechishga keltiriladi.

Bu yerda chegaraviy shartlardan bog’liq ravishda, uch xil tebranishlar bo’lishi mumkin. Ular ichida qovushoq suyuqlik bo’lgan silindirik qatlam ya’ni qobiqning buralma, bo’ylama-radial va ko’ndalang tebranishlaridir.

Yuqorida takidlanganidek, (1.33), (1.34) tenglamalar sistemasini integrallashda aniq bir yechimga ega bo’lish uchun, chegaraviy, kontakt va boshlang’ich shartlardan foydalanish zarur bo’ladi. Ma’lumki, ideal suyuqlikning silindrik qatlam ya’ni qobiq bilan nostatsinar ta’sirlashvida qatlamning suyuqlikka tegib turgan qattiq sirtidagi asosiy chegaraviy shart sirtining o’tkazmaslik shartidan iborat. Bu bilan bog’liq ravishda qaralayotgan sirt nuqtasi va ta’sirlashuvchi suyuqlik zarrachasining tezliklarining sirtga normal tezliklarining tengligi sharti kelib chiqadi. Qovushoq suyuqlik holida esa ushbu shart suyuqlik zarrachalarining qattiq devorga “yopishishi” sharti bilan almashtiriladi.

Bu shart qattiq sirtga nisbatan suyuqlik zarrachasining sirt normali va urunmasi yo’nalishlaridagi tezlik mavjud emasligini anglatadi. Boshqacha aytganda suyuqlik zarrachasi qattiq sirt nuqtasiga nisbatan normal yo’nalishda ham, urunma yo’nalishda ham harakatlanmaydi, ya’ni suyuqlikning sirt bo’ylab sirpanish tezligi mavjud emas.

Ushbu hodisa suyuqlikning “ho’llash” xususiyatidan tubdan farq qiladi. Masalan qo’rg’oshin o’zi oqayotgan shisha idishning ichi sirtini ho’llamaydi lekin “yopishadi” deb hisoblanadi. Ma’lumki ho’llash hodisasi “chegaraviy effekt” deb nomlangan hodisani xarakterlaydi va “menisk” hodisasining vujudga kelishiga sabab bo’ladi.

Shunday qilib, ideal suyuqlikdan farqli ravishda, qattiq sirtning qovushoq suyuqlik bilan yuvulishida qovushoq suyuqlik tezligining qo’zg’almas qattiq sirtda nolga tenglik chegaraviy sharti, ya’ni agar qattiq sirt harakatlanayotgan bo’lsa suyuqlik zarralari tezliklarining harakatlanuvchi qattiq sirt nuqtalari tezliklariga teng bo’lish chegaraviy sharti bajarilishi talab etiladi. Ushbu chegaraviy shart juda ko’p tatqiqotchilar tomonidan rad etiladi. Ammo, bugunga kelib bu shartni qabul qilish to’liq o’rnini oqladi.

Nostatsinar harakat to’g’risidagi masalalarda boshlang’ich shartlar mavjud bo’ladi. Bu shartlar oqim sohasidagi tezliklar taqsimotining vaqtning biror boshlang’ich paytida berilish, fazoning berilgan nuqtasidagi bosim ya’ni boshqa parametrlarning vaqtdan bog’liqligi va hakozolardan iboratdir. Doiraviy silindrik qatlamning (ichida qovushoq suyuqlik bo’lsa ham) o’qqa nisbatan simmetrik tebranishlari uning buralma va bo’ylama-radial tebranishlaridan iboratdir:

1) Ichiga qovushoq suyuqlik to’ldirilgan doiraviy elastik qatlamning buralma tebranishlarida kuchlanishlardan faqat $\tau_{r\theta}$, $\sigma_{r\theta}$ hamda $P_{r\theta}$, $P_{z\theta}$ lar, ko’chishlar esa faqat U_θ , suyuqlik zarrachasi tezlik vektori kompanentalaridan faqat ϑ_θ largina noldan farqli bo’ladilar. Bu parametrlarga mos ravishda kiritilgan potensial funksiyalardan esa faqatgina ψ_1 va χ_1 lar noldan farqli bo’ladilar. U holda qatlam uchun harakat tenglamasi

$$\frac{1}{b^2}\frac{\partial^2\psi_1}{\partial t^2}=\Delta_0\psi_1,\quad \Delta_0=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{\partial^2}{\partial z^2},\quad r_1\le r\le r_2.$$

(1.35)

qovushoq suyuqlik uchun harakat tenglamasi esa

$$\left(\frac{1}{\nu'}\frac{\partial}{\partial r}-\Delta_0\right)\chi_1=0,\quad r<r_1 \qquad (1.36)$$

bo'ladilar. Bu yerda r, r_1- silindrik qatlamning ichki va tashqi radiuslari, $r_1 < r_2$; r-radial, z–bo'ylama koordinatalari; b-qatlam materialida ko'ndalang to'lqin tarqalish tezligi.

Qaralayotgan gidroelastik sistemaning buralma tebranishlarini qatlamning $r = r_2$ sirtida ta'sir etuvchi kuchlar qo'zg'atadilar deb hisoblanadi, ya'ni masalaning chegaraviy sharti quyigagicha ko'rinishga ega bo'ladi

$$\sigma_{r\theta}(r,z,t)\big|_{r=r_2} = f_{r\theta}(z,t), \tag{1.37}$$

Bundan tashqari, silindrik qatlamning ichki suyuqlik bilan kontaktda bo'lgan sirtida, urunma kuchlanishlar tengligini ifodalovchi dinamik shart ham bajarilishi kerak, ya'ni

$$\sigma_{r\theta}(r,z,t)\big|_{r=r_1} = -P_{r\theta}(r,z,t)\big|_{r=r_1} \tag{1.38}$$

Yuqorida, kontakt sirtida qo'yilishi kerak bo'lgan shartlar haqidagi mulohazalar asosida, tebranayotgan kontakt sirtida yotuvchi nuqtalar bilan ularga tegib turgan suyuqlik zarrachalari orasida nisbiy tezlik mavjud emas, ya'ni nisbiy tezlikning normal va urunma tuzuvchilari aynan nolga teng deb faraz qilamiz. U holda ikki qattiq va suyuq muhitlarni ajratib turuvchi sirt ketida suyuqlik zarrachalari tezliklarni qatlamning ichki tebranuvchi sirtini nuqtalarning tezliklariga teng bo'lishlari kerak. Boshqacha aytganda kontakt sirtdagi kinematik shartlar ko'rinishi

$$\frac{\partial}{\partial t} U_\theta(r,z,t)\Big|_{r=r_1} = V_\theta(r,z,t)\big|_{r=r_1} \tag{1.39}$$

kabi bo'ladi.

Boshlang'ich shartlarni nolga teng deb hisoblaymiz.

Shunday qilib, siqiluvchi qovushoq suyuqlik to'ldirilgan doiraviy silindirik elastik qatlamning buralma tebranishlari haqidagi masala (1.35) va (1.36) differensial tenglamalarni (1.37)-chegaraviy, (1.38)-dinamik va (1.39)-kinematik hamda nolga teng boshlang'ich shartlarda integrallashga keltirildi.

2) Endi bo'ylama-radial tebranishlar bo'lganda qaralayotgan gidroelastik sistemaning harakat tenglamalari va chegaraviy hamda boshlang'ich shartlari

qanday bo'lishini qarab chiqamiz. Bu holda ham masala o'qqa nisbatan simmetrik. Shuning uchun kuchlanish tenzorlari kompanentalaridan faqat σ_{rr}, σ_{zz}, σ_{rz} $\sigma_{\theta\theta}$ P_{rr} P_{zz} P_{rz} $P_{\theta\theta}$ lar, ko'chish va tezlik vektorlari kompanentalaridan faqat U_r, U_z hamda V_r, V_z largina noldan farqli bo'ladilar. Ushbu holatda mos ravishda potensiallardan faqat Φ, Ψ_2 va G, χ_2lar noldan farqli bo'lishligini ko'rish qiyin emas. U holda silindirik qatlam uchun harakat tenglamalari

$$
\begin{aligned}
&\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{a^2}\frac{\partial^2 \phi}{\partial t^2}; \quad r_1 \le r \le r_2. \\
&\frac{\partial^2 \phi_2}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_2}{\partial r} + \frac{\partial^2 \psi_2}{\partial z^2} = \frac{1}{a^2}\frac{\partial^2 \psi_2}{\partial t^2};
\end{aligned}
\tag{1.40}
$$

qovushoq suyuqlik harakat tenglamalari esa

$$
\begin{aligned}
&\left(1 + \frac{4z'\phi}{3a_0^2}\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)\frac{\partial G}{\partial t} - \frac{1}{a_0^2}\frac{\partial^3 G}{\partial t^3} = 0; \\
&\frac{\partial^2 \chi_2}{\partial r^2} + \frac{1}{r}\frac{\partial\ \chi_2}{\partial r} + \frac{\partial^2 \chi_2}{\partial z^2} = \frac{1}{\nu'}\frac{\partial^2 \chi_2}{\partial t}; \qquad r < r_1,
\end{aligned}
\tag{1.41}
$$

ko'rinishlarga ega bo'ladilar. Bu yerda *a*-qatlam materialida bo'ylama to'lqinlar tezligi; a_0-tinch holatdagi suyuqlikda tovush tarqalish tezligi;

Qaralayotgan gidroelastik sistemaning bo'ylama-radial tebranishlarini qatlamning $r=r_2$ sirtida ta'sir qiluvchi tashqi kuchlar qo'zg'atadilar deb hisoblanadi, ya'ni masalaning chegaraviy shartlari quyidagi ko'rinishlarga ega bo'ladilar

$$
\begin{aligned}
\sigma_{rr}(r,z,t)\big|_{r=r_2} &= f_r(z,t), \\
\sigma_{rz}(r,z,t)\big|_{r=r_2} &= f_{rz}(z,t).
\end{aligned}
\tag{1.42}
$$

Kontakt shartlarni qanday bo'lishi kerakligi to'g'risida yuqorida keltirilgan mulohazalarga asosan, hamda suyuqlikning silindirik qatlamga nisbatan tinch holatda bo'lganligidan, qatlamning ichki sirtida quyidagi dinamik

$$
\begin{aligned}
\sigma_{r\theta}(r,z,t)\big|_{r=r_1} &= -P_{rr}(r,z,t)\big|_{r=r_1} \\
\sigma_{rz}(r,z,t)\big|_{r=r_1} &= -P_{rz}(r,z,t)\big|_{r=r_1}
\end{aligned}
\tag{1.43}
$$

va kinematik

$$
\left. v_r(r,z,t)\right|_{r=r_1} = \frac{\partial}{\partial t} U_r(r,z,t)\Big|_{r=r_1}, \qquad \left. v_z(r,z,t)\right|_{r=r_1} = \frac{\partial}{\partial t} U_z(r,z,t)\Big|_{r=r_1}. \tag{1.44}
$$

shartlar o'rinli bo'ladi. Boshlangh'ich shartlar bu yerda ham nolga teng deb hisoblanadi.

Shunday qilib qovushoq siqiluvchi suyuqlik to'ldirilgan doiraviy silidirik elastik qatlam va qobiqlarning buralma va bo'ylama-radial tebranishlari haqidagi o'qqa nisbatan simmetrik masalalar qo'yiladi. Endi bu masalalarni qanday yechish kerakligi bilan tanishamiz.

1.4 § Asosiy masalalarning umumiy yechimlari. Kuchlanish va deformatsiya tenzorlari komponentalari umumiy yechimlar orqali ifodalash.

Qaralayotgan silindirik qatlam va qovushoq siqiluvchi suyuqlik tenglamalri tebranish turlari uchun har xil bo'lishini bundan oldingi paragrafda ko'rib chiqdik. Bunda biz tekshirayotgan gidroelastik sistemaning buralma tebranishlari haqidagi nostatsionar masala (1.35) - (1.36) tenglamalar, (1.37) – (1.39) chegarviy shartlar va nolga teng boshlang'ich shartlar bilan to'liq tasiflanadi. Xuddi shunday bu sistemaning bo'ylama-radail tebranishlari haqidagi nostatsionar masala (1.40) – (1.41) tenglamalar sistemalari, (1.42) – (1.44) chegaraviy shartlar va nolga teng boshlang'ich shartlar bilan to'liq tavsiflanadi.

Endigi vazifa xuddi shu (1.35) – (1.36) yoki (1.40) – (1.41) tenglamalar singari σ_{ij}, P_{ij}- kuchlanishlar, u_θ, u_r, u_z - ko'chishlar, v_θ, v_r, v_z – suyuqlik zarrachalari tezliklari, $r_{ij}-$ deformatsiyalar hamda e_{ij} - deformatsiya tezliklari komponentalari ham Φ, Ψ_i, G, χ_i $(i = 1,2)$ potensiallar orqali ifodalashdan iboratdir. Shu munosabat bilan ixtiyoriy $\vec{A}-$ vektorining rotorini hamda ixtiyoriy $\Phi -$ skalyar funksiyaning gradiyentini hisoblash formulalarini keltiramiz.

Silindrik koordinatalari sistemasida $\vec{A}$ vektorining rotori quyidagi formula bilan hisoblanadi:

$$rot\vec{A} = \left(\frac{1}{r}\frac{\partial A_3}{\partial\theta} - \frac{\partial A_2}{\partial z}\right)\vec{e}_r + \left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial r}\right)\vec{e}_\theta + \left[\frac{1}{r}\frac{\partial}{\partial r}(rA_2) - \frac{1}{r}\frac{\partial A_1}{\partial\theta}\right]\vec{e}_z, \qquad (1.45)$$

bu yerda

$\vec{A} = A_1\vec{e}_r + A_2\vec{e}_\theta + A_3\vec{e}_z;$

$\vec{e}_r, \vec{e}_\theta, \vec{e}_z$ –mos koordinata o`qlari bo`ylab yo`nalgan birlik vektorlari.

$$rot(\vec{e}_z\Psi_1) = \frac{1}{r}\frac{\partial\Psi_1}{\partial\theta}\vec{e}_r - \frac{\partial\Psi_1}{\partial r}\vec{e}_\theta;$$
$$rotrot(\vec{e}_z\Psi_2) = \frac{\partial^2\Psi_2}{\partial r\partial z}\vec{e}_z + \frac{1}{r}\frac{\partial^2\Psi_2}{\partial\theta\partial z}\vec{e}_\theta + \left[\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Psi_2}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Psi_2}{\partial\theta}\right]\vec{e}_z \qquad (1.46)$$
$$grad\Phi = \frac{\partial\Phi}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\vec{e}_\theta + \frac{\partial\Phi}{\partial z}\vec{e}_z.$$

Yuqorida ta’kidlanganidek dissertatsiyada ikki masala, ya’ni ichiga qovushoq suyuqlik to`ldirilgan doiraviy elastik qatlam va qobiqlarning buralma va bo`ylama-radial tenglamalari asosidagi masalalar qaraladi. Bular esa qatlamning simmetriya o`qiga nisbatan simmetrik masalalardir. Demak, bu holda vektor va tenzor maydonlarning parametrlari $\theta$-burchak koordinatasidan bog`liq bo`lmaydi. Shuningdek potensiallarning $\theta$-burchak bo`yicha hosilalari ham nolga teng bo`ladilar. Aytilganlarni hisobga olsak (1.46) ifodalar quyidagi ko`rinishni oladilar:

$$rot(\vec{e}_z\Psi_1) = \frac{1}{r}\frac{\partial\Psi_1}{\partial\theta}\vec{e}_\theta;$$
$$rotrot(\vec{e}_z\Psi_2) = \frac{\partial^2\Psi_2}{\partial r\partial z}\vec{e}_z - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Psi_2}{\partial r}\right)\vec{e}_z; \qquad (1.47)$$
$$grad\Phi = \frac{\partial\Phi}{\partial r}\vec{e}_r + \frac{\partial\Phi}{\partial z}\vec{e}_z.$$

Olingan ifodalarni silindrik qatlam nuqtalari ko`chish vektori komponentalari uchun chiqarilgan formulalarga qo`yamiz va bu komponentalar uchun.

$$U_r = \frac{\partial\Phi}{\partial r} + \frac{\partial^2\Psi_2}{\partial r\partial z};$$
$$U_z = \frac{\partial\Phi}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Psi_2}{\partial r}\right); \qquad (1.48)$$
$$U_\theta = -\frac{\partial\Psi_1}{\partial r}.$$

ifodalarga ega bo`lamiz.

Suyuqlik zarrachalari suyuqlik vektori komponentalari uchun ham, xuddi yuqoridagidek, o`xshash almashtirish va $rot(\vec{e}_z, \chi_1)$, $rotrot(\vec{e}_z, \chi_2)$ va $gradG$ ifodalar uchun (1.47) ga o`xshash ifodalardan foydalanib, quyidagi formulalarni olish muammo emas:

$$V_z = \frac{\partial^2 G}{\partial t \partial z} - \frac{\partial^3 \chi_2}{\partial t \partial r^2} - \frac{1}{r}\frac{\partial^2 \chi_2}{\partial t \partial r}; \quad V_r = \frac{\partial^2 G}{\partial t \partial z} + \frac{\partial^3 \chi_2}{\partial t \partial r \partial z}; \quad V_\theta = -\frac{\partial^2 \chi_1}{\partial r \partial t}. \qquad (1.49)$$

Agar ko`chish komponentalari uchun (1.48) ifodalarni silindrik koordinatalar sistemasida yozilgan ε_{ij} deformatsiyalar va U_r, U_z, U_θ ko'chishlar orasidagi Koshi munosabatlariga qo'ysak

$$\varepsilon_{rr} = \frac{\partial^2 \Phi}{\partial r^2} + \frac{\partial^3 \Psi_2}{\partial r^2 \partial z}; \quad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial \Phi}{\partial r} + \frac{\partial \Psi_2}{\partial r \partial z}\right);$$

$$\varepsilon_{zz} = \frac{\partial^2 \Phi}{\partial z^2} + \frac{1}{r}\frac{\partial^2 \Psi_2}{\partial r \partial z} - \frac{\partial^3 \Psi_2}{\partial r^2 \partial z}; \quad \varepsilon_{\theta z} = -\frac{\partial^2 \Psi_1}{\partial r \partial z} \qquad (1.50)$$

$$\varepsilon_{rz} = \frac{\partial U_z}{\partial r} + \frac{\partial U_r}{\partial z} = 2\frac{\partial^2 \Phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left[\frac{\partial^2}{\partial z^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\Psi_2;$$

$$\varepsilon_{r\theta} = \frac{1}{r}\frac{\partial \Psi_1}{\partial r} - \frac{\partial^2 \Psi_1}{\partial r^2}$$

Xuddi shunday (1.49) ifodalarni e_{ij} – deformatsiya tezliklari komponentalari va V_r, V_θ, V_z tezliklar o`rtasidagi munosabatlarga qo`yib quyidagilarga ega bo`lamiz:

$$e_{rr} = \frac{\partial^3 G}{\partial r^2 \partial t} + \frac{\partial^4 \chi_2}{\partial r^2 \partial z \partial t}; \quad e_{\theta\theta} = \frac{1}{r}\frac{\partial}{\partial t}\left[\frac{\partial G}{\partial r} + \frac{\partial^2 \chi_2}{\partial r \partial z}\right]; \quad e_{r\theta} = \frac{\partial}{\partial t}\left[\frac{1}{r}\frac{\partial \chi_1}{\partial r} - \frac{\partial^2 \chi_1}{\partial r^2}\right];$$

$$e_{\theta z} = -\frac{\partial^3 \chi_1}{\partial r \partial z \partial t}; \quad e_{zz} = \frac{\partial^3 G}{\partial z^2 \partial t} - \frac{1}{r}\frac{\partial^3 \chi_2}{\partial r \partial z \partial t} - \frac{\partial^4 \chi_2}{\partial r^2 \partial z \partial t}; \qquad (1.51)$$

$$e_{rz} = 2\frac{\partial^3 G}{\partial r \partial z \partial t} + \frac{\partial}{\partial r \partial t}\left[\frac{\partial^2}{\partial z^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\chi_2.$$

Masala o`qqa nisbatan simmetrik bo`lganligi uchun σ_{ij} kuchlanish komponentalari ifodalari ham soddalashadi va quyidagi ko`rinishga ega bo`ladi:

$$\sigma_{rr} = (\lambda + 2\mu)\left(\frac{\partial^2 \Phi}{\partial r^2} + \frac{\partial^3 \Psi_2}{\partial r^2 \partial z}\right) + \lambda\left[\frac{\partial^2 \Phi}{\partial z^2} - \frac{1}{r}\frac{\partial^2 \Psi_2}{\partial r \partial z} - \frac{\partial^3 \Psi_2}{\partial r^2 \partial z} + \frac{1}{r}\left(\frac{\partial \Phi}{\partial r} + \frac{\partial^2 \Psi_2}{\partial r \partial z}\right)\right];$$

$$\sigma_{\theta\theta} = \frac{(\lambda + 2\mu)}{r}\left(\frac{\partial \Phi}{\partial r} + \frac{\partial^2 \Psi_2}{\partial r \partial z}\right) + \lambda\left[\frac{\partial^2 \Phi}{\partial z^2} + \frac{\partial^2 \Phi}{\partial z^2} - \frac{1}{r}\frac{\partial^2 \Psi_2}{\partial r \partial z}\right]; \quad \sigma_{\theta z} = -\mu\frac{\partial^2 \Psi_1}{\partial r \partial z};$$

$$\sigma_{zz}=(\lambda+2\mu)\left(\frac{\partial^2\Phi}{\partial z^2}-\frac{1}{r}\frac{\partial^2\Psi_2}{\partial r\partial z}-\frac{\partial^3\Psi_2}{\partial r^2\partial z}\right)+\lambda\left[\frac{\partial^2\Phi}{\partial r^2}+\frac{\partial^3\Psi_2}{\partial r^2\partial z}+\frac{1}{r}\left(\frac{\partial\Phi}{\partial r}+\frac{\partial^2\Psi_2}{\partial r\partial z}\right)\right]; \qquad (1.52)$$

$$\sigma_{rz}=\mu\left\{2\frac{\partial^2\Phi}{\partial r\partial z}+\frac{\partial}{\partial r}\left[\frac{\partial^2}{\partial z^2}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\Psi_2\right\}; \qquad \sigma_{r\theta}=\mu\left[\frac{1}{r}\frac{\partial}{\partial r}-\frac{\partial^2}{\partial r^2}\right]\Psi_1 .$$

Qovushoq suyuqlikning P_{ij} – kuchlanishlar komponentalari uchun ham, xuddi yuqoridagidek, ushbu ifodalarga ega bo`lamiz:

$$P_{\theta\theta}=-\rho_0'\left(\frac{4}{3}\nu'\Delta_0-\frac{\partial}{\partial t}\right)\frac{\partial G}{\partial t}+\lambda'\Delta_0\frac{\partial G}{\partial t}+\mu'\frac{\partial}{\partial t}\left[\frac{1}{r}\frac{\partial G}{\partial r}+\frac{1}{r}\frac{\partial^2\chi_2}{\partial r\partial z}\right]; \quad P_{r\theta}=\mu'\frac{\partial}{\partial t}\left(\frac{1}{r}-\frac{\partial}{\partial r}\right)\frac{\partial\chi_1}{\partial r};$$

$$P_{zz}=-\rho_0'\left(\frac{4}{3}\nu'\Delta_0-\frac{\partial}{\partial t}\right)\frac{\partial G}{\partial t}+\lambda'\Delta_0\frac{\partial G}{\partial t}+\mu'\frac{\partial}{\partial t}\left[\frac{\partial^2 G}{\partial z^2}-\frac{1}{r}\frac{\partial^2\chi_2}{\partial r\partial z}-\frac{\partial^3\chi_2}{\partial r^2\partial z}\right] \qquad (1.53)$$

$$P_{rr}=-\rho_0'\left(\frac{4}{3}\nu'\Delta_0-\frac{\partial}{\partial t}\right)\frac{\partial G}{\partial t}+\lambda'\Delta_0\frac{\partial G}{\partial t}+\mu'\frac{\partial}{\partial t}\left[\frac{\partial^2 G}{\partial r^2}+\frac{\partial^3\chi_2}{\partial r^2\partial z}\right]; \quad P_{z\theta}=-\mu'\frac{\partial^3\chi_1}{\partial r\partial z\partial t};$$

$$P_{rz}=\mu'\frac{\partial}{\partial t}\left[2\frac{\partial^2 G}{\partial r\partial z}+\frac{\partial}{\partial r}\left[\frac{\partial^2}{\partial z^2}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\chi_2\right]; \quad \Delta_0=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}.$$

Shunday qilib, qatlam uchun quyidagi to`lqin tenglamalari:

$$\Delta_0\Phi=\frac{1}{a^2}\frac{\partial^2\Phi}{\partial t^2}; \quad \Delta_0\psi_i=\frac{1}{b^2}\frac{\partial^2\psi_i}{\partial t^2}; \quad (i=1,2), \qquad (1.54)$$

va suyuqlik uchun:

$$\left[\left(1+\frac{4\nu'}{3a_0^2}\right)\Delta_0-\frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right]\frac{\partial G}{\partial t}=0; \quad \left(\frac{1}{\nu'}\frac{\partial}{\partial t}-\Delta_0\right)\chi_i=0; \quad (i=1,2) \qquad (1.55)$$

harakat tenglamalarining yechimlari topilsa (1.48) - (1.53) formulalar yordamida “Silindrik elastik qatlam (qobiq) - qovushoq siqiluvchi suyuqlik” gidroelastik sistemasining istalgan kesimidagi kuchlangan–deformatsiyalangan holatni vaqtning istalgan payti uchun bir qiymatli aniqlash mumkin.

Yuqoridagi (1.54) – to`lqin va (1.55) – harakat tenglamalarini yechish uchun (1.37) va (1.42) chegaraviy shartlardagi tashqi ta’sir kuchlari funksiyalari – $f_{r\theta}(z,t)$, $f_r(z,t)$ va $f_{rz}(z,t)$ larni, [2] ishdagi kabi, quyidagi ko`rinishda tasvirlanuvchi funksiyalar sinfida qaraymiz :

$$f_{rz}(z,t)=\int_0^\infty \left.\begin{matrix}\cos kz\\ \sin kz\end{matrix}\right\}dk\int_{(l)} f_{rz}^{(0)}(k,p)e^{pt}dp, \qquad (1.56)$$

$$[f_r(z,t), f_{r\theta}(z,t)] = \int_0^{\infty} \begin{Bmatrix} \sin kz \\ -\cos kz \end{Bmatrix} dk \int_{(l)} \left[f_r^{(0)}(k,p), f_{r\theta}^{(0)}(k,p)\right] e^{pt} dp,$$

bu yerda (l) – mavhum o\`qning $(-i\omega_0, i\omega_0)$ qismiga o\`ng tomondan yondashuvchi p tekisligidagi [2] ochiq kontur. Bundan tashqari $f_r(z,t)$, $f_{rz}(z,t)$ va $f_{r\theta}(z,t)$ - lar shunday funksiyalarki, ularning tasvirlari bo\`lmish $f_r^{(0)}(k,l), f_{rz}^{(0)}(k,l)$ va $f_{r\theta}^{(0)}(k,l)$ funksiyalari $\{0 < k \le k_0;\ |\mathrm{Im}\, P| \le \omega_0\}$ sohadan tashqarida hisobga olmaslik mumkin bo\`lgan darajada kichik bo\`ladi.

Xuddi shu tariqa qatlam uchun Φ, Ψ_1, Ψ_2 potensiallarni hamda suyuqlik uchun G, χ_1, χ_2 funksiyalarni, (1.56) ga mos ravishda, ushbu ko\`rinishda izlaymiz

$$[\Phi, \Psi_1] = \int_0^{\infty} \begin{Bmatrix} \sin kz \\ -\cos kz \end{Bmatrix} dk \int_{(l)} \left[\Phi^{(0)}, \Psi_1^{(0)}\right] e^{pt} dp;$$

$$[\Psi_2, \chi_2] = \int_0^{\infty} \begin{Bmatrix} \cos kz \\ \sin kz \end{Bmatrix} dk \int_{(l)} \left[\Psi_2^{(0)}, \chi_2^{(0)}\right] e^{pt} dp; \qquad (1.57)$$

$$[G, \chi_1] = \int_0^{\infty} \begin{Bmatrix} \sin kz \\ -\cos kz \end{Bmatrix} dk \int_{(l)} \left[G^{(0)}, \chi_1^{(0)}\right] e^{pt} dp,$$

keltirilgan (1.56) va (1.57) almashtirish shakllari, yuqoridagi farazlar doirasida, $\Phi, \Psi_i, G, \chi_i \ (i = 1,2)$ funksiyalarni *r,z* koordinatalar va *t*-vaqt bo\`yicha integral belgisi ostida qat'iy ravishda differensiallashga, hamda integrallash va yig'indi olish tartiblarini almashtirishga imkon beradi.

Endi ushbu (1.57) almashtirishlarni (1.54) va (1.55) tenglamalarga qo\`yib almashgan $\Phi^0, \Psi_i^{(0)}, G^{(0)}, \chi_i^{(0)} \ (i = 1,2)$ potensiallar uchun oddiy Bessel differensial tenglamalarini olamiz:

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \beta^2\right)\Psi_i^{(0)} = 0; \ (i = 1,2)$$

Suyuqlik uchun

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \delta^2\right)G^{(0)} = 0, \ \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \gamma^2\right)\chi_i^{(0)} = 0; \ (i = 1,2) \qquad (1.59)$$

bu yerda

$$\delta^2 = k^2 + \frac{3p^2}{3a_0^2 + 4\nu' p}; \ \alpha^2 = k^2 + \frac{\rho_c}{\lambda + 2\mu} p^2; \quad \beta^2 = k^2 + \frac{\rho_c}{\mu} p^2; \quad \gamma^2 = k^2 + \frac{p}{\nu'}; \quad (1.60)$$

ν'- qovushoqlik kinamatik koeffitsiyenti:

k,p- almashtirish parametrlari.

Olingan (1.58) va (1.59) tenglamalarining *r=0* va $r \to \infty$ bo`lganda chegaralangan umumiy yechimlari Besselning modifitsirlangan funksiyalari orqali ifodalanadilar va qatlam uchun

$$\Phi^0(r) = A_1 I_0(\alpha r) + A_2 K_0(\alpha r), \ \Psi_i^{(0)}(r) = B_{i1} I_0(\beta r) + B_{i2} K_0(\beta r); \ (i = 1{,}2) \quad (1.61)$$

va suyuqlik uchun

$$G^{(0)}(r) = C I_0(\delta r), \ \ \chi_i^{(0)}(r) = D_i I_0(\gamma r) \ ; \ (i = 1{,}2) \qquad (1.62)$$

ifodalarga teng, bu yerda $A_i(k,p)$, $B_{ij}(k,p)$ $(i,j = 1{,}2; \ B_{ij} \neq B_{ji})$, $C(k,p)$ va $D_i(k,p)$ - *r* o`zgaruvchiga nisbatan integrallash ixtiyoriy o`zgarmaslari.

Qaralayotgan silindrik qatlam uchun (σ_{ij}) kuchlanish tenzori va $\vec{u}(u_r, u_\theta, u_z)$ ko`chish vektori komponentalarini almashtiramiz. Buning uchun $\sigma_{ij}$ kuchlanishlar va $u_r, u_\theta, u_z$ ko`chishlarni ham

$$[\sigma_{rr}, \sigma_{r\theta}] = \int_0^\infty \left.\begin{matrix} \sin kz \\ -\cos kz \end{matrix}\right\} dk \int_{(l)} \left[\sigma_{rr}^{(0)}, \sigma_{r\theta}^{(0)}\right] e^{pt} dp \ ; \quad \sigma_{rz} = \int_0^\infty \left.\begin{matrix} \cos kz \\ \sin kz \end{matrix}\right\} dk \int_{(l)} \sigma_{rz}^{(0)} e^{pt} dp \ ; \quad (1.63)$$

va

$$[U_r, U_\theta] = \int_0^\infty \left.\begin{matrix} \sin kz \\ -\cos kz \end{matrix}\right\} dk \int_{(l)} \left[U_r^{(0)}, U_\theta^{(0)}\right] e^{pt} dp \ ; \quad U_z = \int_0^\infty \left.\begin{matrix} \cos kz \\ \sin kz \end{matrix}\right\} dk \int_{(l)} U_z^{(0)} e^{pt} dp \ . \quad (1.64)$$

Ko`rinishlarda tasvirlaymiz. Ushbu (1.63) va (1.64) almashtirishlarni qo’llab almashtirilgan $\sigma_{ij}^{(0)}$ kuchlanishlar uchun (1.61) yechimlar orqali ifodalangan formulalarni olamiz:

$$\sigma_{zz}^{(0)} = \mu \Big\{ \left[\left(\beta^2 + k^2\right) I_0(\alpha r) - \frac{2\alpha}{r} I_1(\alpha r) \right] A_1 + \left[\left(\beta^2 + k^2\right) K_0(\alpha r) + \frac{2\alpha}{r} K_1(\alpha r) \right] A_2 -$$

$$- 2k \left[\beta^2 I_0(\beta r) - \frac{\beta}{r} I_1(\beta r) \right] B_{21} - 2k \left[\beta^2 K_0(\beta r) + \frac{\beta}{r} K_1(\beta r) \right] B_{22} \ \Big\};$$

$$\sigma_{rz}^{(0)} = \mu \left\{ 2\alpha k \left[I_1(\alpha r) A_1 - K_1(\alpha r) A_2 \right] - \beta\left(\beta^2 + k^2\right)\left[I_1(\beta r) B_{21} - K_1(\beta r) B_{22} \right] \right\}$$

$$\sigma_{r\theta}^{(0)}=\mu\left\{\left[\frac{2\beta}{r}I_1(\beta r)-\beta^2 I_0(\beta r)\right]B_{11}-\left[\frac{2\beta}{r}K_1(\beta r)+\beta^2 K_0(\beta r)\right]B_{12}\right\};$$

$$\begin{aligned}\sigma_{z\theta}^{(0)}&=\mu\left\{\ \frac{2k}{r}[I_1(\alpha r)A_1+K_1(\alpha r)A_2]-k\left[\beta I_0(\beta r)-\frac{1}{r}I_1(\beta r)\right]B_{11}+\right.\\&\left.+k\left[\beta K_0(\beta r)+\frac{1}{r}K_1(\beta r)\right]B_{22}-\frac{(\beta^2+k^2)}{r}[I_1(\beta r)B_{21}+K_1(\beta r)B_{22}]\ \right\};\end{aligned} \qquad (1.65)$$

$$\sigma_{zz}^{(0)}=\mu\{(\beta^2-2\alpha^2-k^2)[I_1(\alpha r)A_1+K_1(\alpha r)A_2]+2k\beta^2[I_1(\beta r)B_{21}+K_1(\beta r)B_{22}]\};$$

$$\begin{aligned}\sigma_{\theta\theta}^{(0)}&=\mu\left\{\ \left(\beta^2-2\alpha^2+k^2-\frac{4}{r^2}\right)[I_1(\alpha r)A_1+K_1(\alpha r)A_2]-\frac{2\alpha}{r}[I_0(\alpha r)A_1+K_0(\alpha r)A_2]+\right.\\&+\frac{2}{r}\left[\beta I_0(\beta r)-\frac{2}{r}I_1(\beta r)\right]B_{11}-\frac{2}{r}\left[\beta K_0(\beta r)+\frac{2}{r}K_1(\beta r)\right]B_{12}-\\&\left.-\frac{2k}{r}\left[\beta I_0(\beta r)-\frac{2}{r}I_1(\beta r)\right]B_{21}+\frac{2k}{r}\left[\beta K_0(\beta r)+\frac{2}{r}K_1(\beta r)\right]B_{22}\ \right\}.\end{aligned}$$

Xuddi shunday, ko`chishlarning almashtirilgan $U_r^{(0)}, U_z^{(0)}$ va $U_\theta^{(0)}$ kattaliklari uchun quyidagilarga ega bo`lamiz:

$$\begin{aligned}U_r^{(0)}&=\alpha\,[I_1(\alpha r)A_1-K_1(\alpha r)A_2]-k\beta[I_1(\beta r)B_{21}-K_1(\beta r)B_{22}]\ ,\\U_z^{(0)}&=k\,[I_0(\alpha r)A_1+K_0(\alpha r)A_2]-\beta^2[I_0(\beta r)B_{21}+K_0(\beta r)B_{22}]\ ,\\U_\theta^{(0)}&=-\beta\,[B_{11}I_1(\beta r)-B_{12}K_1(\beta r)].\end{aligned} \qquad (1.66)$$

Endi suyuqlik uchun P_{ij} kuchlanishlarni yuqoridagidek almashtiramiz:

$$\begin{aligned}P_{rr}&=\int_0^\infty \left.\begin{matrix}\sin kz\\-\cos kz\end{matrix}\right\}dk\int_{(l)}P_{rr}^{(0)}e^{pt}dp,\\P_{rz}&=\int_0^\infty \left.\begin{matrix}\cos kz\\\sin kz\end{matrix}\right\}dk\int_{(l)}P_{rz}^{(0)}e^{pt}dp.\end{aligned} \qquad (1{,}67)$$

Bunda $P_{zz}, P_{\theta\theta}, P_{r\theta}$ kuchlanishlar xuddi P_{rr} kabi $P_{z\theta}$ esa xuddi P_{rz} – kabi almashtiriladi.

Endi (1.57) va (1.67) almashtirishlarni qo`llab, $P_{ij}^{(0)}$ kuchlanishlar uchun almashtirilgan $G^{(0)}$ va $\chi_{ij}^{(0)}$ potensiallar orqali munosabatlarga ega bo`lamiz:

$$P_{rr}^{(0)}=\rho_0'p^2G^{(0)}+\mu' p\frac{d^2}{dr^2}\left[G^{(0)}-k\chi_2^{(0)}\right];$$

$$P_{\theta\theta}^{(0)}=\rho_0'p^2G^{(0)}+\frac{\mu' p}{r}\frac{d}{dr}\left[G^{(0)}-k\chi_2^{(0)}\right];$$

$$P_{zz}^{(0)} = \rho_0' p^2 G^{(0)} + \mu' kp\left[\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)\chi_2^{(0)} - kG^{(0)}\right]; \qquad (1.69)$$
$$P_{rz}^{(0)} = \mu' p \frac{d}{dr}\left\{2kG^{(0)} - \left[k^2 + \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)\right]\chi_2^{(0)}\right\};$$
$$P_{r\theta}^{(0)} = \mu' p\left(\frac{1}{r} - \frac{d}{dr}\right)\frac{d\chi_1^{(0)}}{dr}; \quad P_{z\theta}^{(0)} = \mu' kp\frac{d\chi_1^{(0)}}{dr}.$$

Xuddi shunday, suyuqlik zarrachalarining almashtirilgan komponentalari uchun ushbu munosabatlarga ega bo`lamiz

$$V_z = p\left[kG^{(0)} - \left(\frac{d}{dr} + \frac{1}{r}\right)\frac{d\chi_2^{(0)}}{dr}\right]; \; V_r^{(0)} = p\frac{d}{dr}\left[G^{(0)} - k\chi_2^{(0)}\right]; \; V_\theta^{(0)} = -p\frac{d}{dr}\chi_1^{(0)}, \quad (1.69)$$

bu yerda V_r, V_θ, V_z lar ham quyidagicha almashtirilgan

$$\left[V_r, V_\theta\right] = \int_0^\infty \left.\begin{matrix}\sin kz \\ -\cos kz\end{matrix}\right\} dk \int_{(l)} \left[V_r^{(0)}, V_\theta^{(0)}\right] e^{pt} dp, \quad V_z = \int_0^\infty \left.\begin{matrix}\cos kz \\ \sin kz\end{matrix}\right\} dk \int_{(l)} V_z^{(0)} e^{pt} dp. \quad (1.70)$$

kuchlanishlarning (1.68) ifodalarini (1.62) yechimlardan foydalanib

$$P_{rr}^{(0)} = \rho_0' p^2 CI_0(\delta r) + \mu' p\left\{C\left[\delta^2 I_0(\delta r) - \frac{\delta}{r}I_1(\delta r)\right] - D_2 k\left[\gamma^2 I_0(\gamma r) - \frac{\gamma}{r}I_1(\gamma r)\right]\right\},$$
$$P_{\theta\theta}^{(0)} = \rho_0' p^2 CI_0(\delta r) + \frac{\mu' p}{r}\left[C\delta I_1(\delta r) - D_2 k\gamma I_1(\gamma r)\right],$$
$$P_{z\theta}^{(0)} = \mu' kp\gamma D_1 I_1(\gamma r), \qquad P_{r\theta}^{(0)} = \mu' p\gamma D_1\left\{\frac{2}{r}I_1(\gamma r) - \gamma I_0(\gamma r)\right\} \qquad (1.71)$$
$$P_{zz}^{(0)} = \rho_0' p^2 CI_0(\delta r) + \mu' kp\left[\gamma^2 D_2 I_0(\gamma r) - kCI_0(\delta r)\right],$$
$$P_{rz}^{(0)} = \mu' p\left\{2k\delta CI_1(\delta r) - D_2\gamma\left(k^2 + \gamma^2\right)I_0(\gamma r)\right\}, \quad .$$

ko`rinishlarga keltiramiz. Xuddi shunday suyuqlik zarrachalarining almashtirilgan tezlik vektori komponentalari uchun quyidagi ifodalarni olamiz

$$V_r^{(0)} = p\left[C\delta I_1(\delta r) - D_2 k\gamma I_1(\gamma r)\right],$$
$$V_z^{(0)} = p\left[kCI_0(\delta r) - D_2\gamma^2 I_0(\gamma r)\right], \qquad (1.72)$$
$$V_\theta^{(0)} = -\gamma p D_1 I_1(\gamma r).$$

Keltirilgan (1.65) (1.66) va (1.71) (1.72) formulalardan ko`rindiki, silindrik elastik qatlam va qovushoq siqiluvchi suyuqlikning buralma tebranishlariga o`zaro nostatsionar ta’sirlarini o`rganish haqidagi masala, bu gidroelastik sistemaning bo`ylama – radial tebranishlari holidagi masaladan bog`liqsiz ravishda o`rganilishi mumkin. Shuning uchun siqiluvchi suyuqlik saqlovchi doiraviy silindrik qatlamning buralma tebranishlarini haqidagi masalani uning bo`ylama – radial tebranishlari haqidagi masaladan alohida o`rganamiz.

II-BOB

QOVUSHOQ SIQILUVCHI SUYUQLIK BILAN TO'LDIRILGAN DOIRAVIY SILINDIRIK QATLAMNING NASTATSIONAR TEBRANISHLARI.

Ushbu bob doirasida ichiga qovushoq siqiluvchi suyuqlik to'ldirilgan doiraviy elastik qatlam va qobiqlarning bo'ylama simmetriya o'qiga nisbatan simmetrik nostatsionar tebranishlari tenglamalarini keltirib chiqaramiz. Olingan umumiy tenglamalar qatlam va qobiqlarning ikki xil – buralma va bo'ylama radial tebranishlarini tavsiflaydi. Bunda, tenglamalar bilan bir qatorda, "silindirik qatlam (qobiq) – qovushoq siqiluvchi suyuqlik" gidroelastik sistemasining ixtiyoriy kesmalaridagi nuqtalar kuchlangan-deformatsiyalangan holatini aniqlashga imkon beruvchi algaritim ishlab chiqilgan.

Shuningdek, olingan tebranish umumiy tenglamalaridan, xususiy hollarda, taqribiy, muhandislik hisoblarida ishlatish mumkin bo'lgan tenglamalar va formulalar keltirib chiqariladi. Bunda eng avvalo qaralayotgan gidroelastik sistemaning nostatsionar buralma tebranishlarini qaraymiz. Bunday masalani qatlam (qobiq)ning bo'ylama-radial tebranishlari haqidagi masaladan alohida qarash mumkin ekanligini birinchi bobning, oxirgi to'rtinchi paragrafi doirasida isbot qilingan edi.

§ 2.1. Ichida qovushoq siqiluvchi suyuqlik saqlovchi silindirik qatlamning buralma tebranishlari.

Ushbu paragraf doirasida ichiga qovushoq siqiluvchi suyuqlik to'ldirilgan silindirik qatlamning buralma tebranishlari umumiy va taqribiy tenglamalarini keltirib chiqarish masalasi bilan shug'ullanamiz, bunda silindirik qatlamning ko'ndalang kesimi doiraviy, uning ichki va tashqi radiuslari r mos ravishda r_1 va r_2 lar bo'lsin deb faraz qilamiz.

Bundan oldingi 1.3 paragrafda takidlanganidek siqiluvchi qovushoq suyuqlik to'ldirilgan doiraviy silindrik elastik qatlamning buralma tebranishlari haqidagi masala (1.35) va (1.36) differensial tenglamalari (1.37) – chegaraviy, (1.38) – dinamik va (1.39) – kinematik, hamda nolga teng boshlang'ich shartlarda integrallashga keltiriladi. Ushbu (1.35) va (1.36) tenglamalardan ko'rinadiki, qaralayotgan gidroelastik sistemaning kuchlangan – deformatsiyalangan holatini aniqlovchi hamma parametrlar faqat va faqat Ψ_1 va χ_1 potensiallardangina bog'liq bo'lishlari zarurligi kelib chiqadi. Yuqoridagi (1.48) – (1.53) munosabatlardan ko'rinadiki faqat Ψ_1 va χ_1 potensial funksiyalardan bog'liq bo'lgan parametrlar quyidagilar: $U_\theta, V_\theta, \varepsilon_{z\theta}, \varepsilon_{r\theta}, e_{z\theta}, e_{r\theta}, \sigma_{z\theta}, \sigma_{r\theta}, P_{z\theta}$ va $P_{r\theta}$.

Qo'yilgan masalani yechish uchun Ψ_1 va χ_1 potensial funksiyalarni (1.57) kabi tasvirlaymiz va bu tasvirlarni (1.35) va (1.36) tenglamarga qo'yamiz. U holda xuddi (1.58) va (1.59) tenglamalar kabi oddiy differensial (Bessel) tenglamalariga ega bo'lamiz:

$$\frac{d^2\Psi_1^{(0)}}{dr^2} + \frac{1}{r}\frac{d\Psi_1^{(0)}}{dr} - \beta^2\Psi_1^{(0)} = 0, \quad (2.1)$$

$$\frac{d^2\chi_1^{(0)}}{dr^2} + \frac{1}{r}\frac{d\chi_1^{(0)}}{dr} - \gamma^2\chi_1^{(0)} = 0, \quad (2.2)$$

bu yerda

$$\beta^2 = k^2 + \frac{\rho_k}{\mu}P^2; \quad \gamma^2 = k^2 + \frac{1}{\nu'}P;$$

ν' - qovushoqlik kinematik koeffitsiyenti; k, P – almashtirish parametrlari;
μ - Lamme koeffitsiyenti; ρ_k- qatlam materiali zichligi.

Ushbu (2.1) tenglamaning $r=0$ va $r \to \infty$ bo'lganda chegaralanganlik shartini qanoatlantiruvchi, hamda (2.2) tenglamaning $r=0$ bo'lganda chgaralangan yechimlari Besselning modifitsirlangan funksiyalari orqali ifodalanadi

$$\Psi_1^{(0)}(r) = B_{11}I_0(\beta r) + B_{12}K_0(\beta r) \quad (2.3)$$

va

$$\chi_1^{(0)}(r) = D_1 I_0(\gamma r) \quad (2.4)$$

bu yerda $I_0(\beta r)$, $I_0(\gamma r)$ va $K_0(\beta r)$ - Besselning nolinchi tartibli modifitsirlangan funksiyasi.

Qaralayotgan gidroelastik sistemaning buralma tebranish tenglamalarini keltirib chiqarish uchun U_θ- buralma ko'chishning bosh qismini ajratish zarur bo'ladi. Buning uchun (2.3) yechimni (1.48) ifodalardagi U_θ ko'chishning Ψ_1 orqali ifodasiga (1.57) va (1.64) almashtirishlarni qo'llab

$$U_\theta^{(0)} = -\frac{d}{dr}\Psi_1^{(0)}$$

ifodalarga ega bo'lamiz. Oxirgi formulaga (2.3) ni qo'yib

$$U_\theta^{(0)} = -\beta[B_{11}I_1(\beta r) - B_{12}K_1(\beta r)] \quad (2.5)$$

Bu formuladagi Bessel funksiyalarini darajali qatorga yoyamiz va ikkita bucheksiz qatorlarni bitta summaga birlashtirib ushu

$$U_\theta^{(0)} = \frac{B_{12}}{r} - \beta\sum_{n=0}^{\infty}[B_{11} - N(\beta r)B_{12}]\frac{\left({}^{\beta r}/_{r}\right)^{2n+1}}{n!(n+1)!} \quad (2.6)$$

formulani olamiz. Bu yerdagi $N(\beta r)$ funksiya

$$N(\beta r) = \ln\frac{\beta r}{r} - \frac{1}{r}\frac{1}{n+1} - \Psi(n+1) \quad (2.7)$$

ifoda bilan aniqlanadi. Bu yerda $\Psi(n)$ – digamma (Gamma funksiyaning logorigmik hosilasi) funksiya.

Endi qatlam ichki nuqtalarining buralma ko'chishini nuqtalarini sifatida qabul qilamiz. Buning uchun (2.6) formula $r=r_1$ deb olamiz va uning umumiy ko'rinishidan kelib chiqqan holda ikkita yangi funksiyani quyidagi ifodalar yordamida kiritamiz

$$U_{\theta,0}^{(0)} = \frac{B_{12}}{r_1}; \qquad U_{\theta,0}^{(0)} = -\frac{1}{r}\beta[B_{11} - B_{12}N(\beta r_1)] \quad (2.8)$$

Agar (2.6) ifodada $n=0$ va $r=r_1$ deb olinsa

$$U_\theta^{(0)}(r_1) = U_{\theta,0}^{(0)} + r_1 U_{\theta,1}^{(0)}$$

formulaga ega bo'lamiz.

Bu yerdan yangidan kiritilgan $U_{\theta,0}^{(0)}$ va $U_{\theta,1}^{(0)}$ funksiyalar buralma $U_\theta^{(0)}(r)$ ko'chishning bosh qismlari ekanligi ko'rinadi. Bunda $U_{\theta,0}^{(0)}$ ning o'lchov birligi xuddi ko'chishnikidek, $U_{\theta,1}^{(0)}$ ning birligi esa xuddi deformatsiyanikidek.

Endi chegaraviy, dinamik va kontakt (kinematik) shartlarni hamda kuchlanishlar, ko'chish va suyuqlik zarrachalari tezliklari vektorining komponentalariuchun chiqarilgan (1.65), (1.66), (1.69) va (1.71) formulalarni almashtirishga o'tamiz. Buning uchun (1.37), (1.38) va (1.39) shartlarga (1.56), (1.63) va (1.64) formulalarni qo'llab almashtirishni bajaramiz. Kuchlanishlar, ko'chish va suyuqlik zarrachalari tezliklari vektori komponentalari ifodalarini almashtirilgan chegaraviy va kontakt shartlariga qo'yib quyidagilarga ega bo'lsamiz:

$$\left[\frac{2\beta}{r_2}I_1(\beta r_2)-\beta^2 I_0(\beta r_2)\right]B_{11}-\left[\frac{2\beta}{r_2}K_1(\beta r_2)+\beta^2 K_0(\beta r_2)\right]B_{12}=\frac{1}{\mu}f_{r\theta}^{(0)};$$

$$\left[\frac{2\beta}{r_1}I_1(\beta r_1)-\beta^2 I_0(\beta r_1)\right]B_{11}-\left[\frac{2\beta}{r_1}K_1(\beta r_1)+\beta^2 K_0(\beta r_1)\right]B_{12}=$$

$$=-\frac{\mu'}{\mu}P\gamma E\left[\frac{r}{r_1}I_1(\gamma r_1)-\gamma I_0(\gamma r_1)\right]; \qquad (2.9)$$

$$\gamma E I_1(\gamma r_1)=\beta[B_{11}I_1(\beta r_1)-B_{12}K_1(\beta r_1)])$$

Oxirgi tenglikdan o'zgarmas E ni aniqlaymiz

$$E=\frac{\beta[B_{11}I_1(\beta r_1)-B_{12}K_1(\beta r_1)]}{\gamma I_1(\gamma r_1)} \qquad (2.9.1)$$

va bu ifodani (2.9) tenglamalar sistemasining ikkinchi tenglamasiga qo'yamiz

$$\frac{\mu'}{\mu}P\frac{\frac{r}{r_1}I_1(\gamma r_1)-\gamma I_0(\gamma r_1)}{I_1(\gamma r_1)}[B_{11}I_1(\beta r_1)-B_{12}K_1(\beta r_1)]=B_{12}\left[\frac{r}{r_1}K_1(\gamma r_1)+\beta K_0(\beta r_1)\right]-$$
$$-B_{11}\left[\frac{r}{r_1}I_1(\beta r_1)-\beta I_0(\beta r_1)\right].$$

Bu yerda quyidagicha belgilash kiritamiz.

$$R_k^{(0)}=\frac{\mu'}{\mu}P\frac{\frac{r}{r_1}I_1(\gamma r_1)-\gamma I_0(\gamma r_1)}{I_1(\gamma r_1)} \qquad (2.10)$$

va uni o'rniga qo'yib, hamda (2.9) sistemaning birinchi tenglamasini e'tiborga olgan holda, ikkita B_{11} va B_{12} no'malum o'zgarmaslarga nisbatan quyidagi ikkita chegaraviy shartlarga ega bo'lamiz:

$$\left[\frac{2\beta}{r_2}I_1(\beta r_2)-\beta^2 I_0(\beta r_2)\right]B_{11}-\left[\frac{2\beta}{r_2}K_1(\beta r_2)+\beta^2 K_0(\beta r_2)\right]B_{12}=\frac{1}{\mu}f_{r\theta}^{(0)}; \qquad (2.11)$$

$$\left[(\frac{r}{r_2}+R_k^{(0)})I_1(\beta r_2)-\beta^2 I_0(\beta r_1)\right]B_{11}-\left[(\frac{r}{r_1}+R_k^{(0)}K_1(\beta r_1)+\beta^2 K_0(\beta r_1)\right]B_{12}=0.$$

Yuqoridagi (2.8) formulalardan B_{11} va B_{12} o'zgarmaslarni topamiz:

$$B_{11}=r_1 N(\beta r_1)U_{\theta,0}^{(0)}-\frac{2}{\beta^2}U_{\theta,1}^{(0)};\ \ B_{12}=r_1 U_{\theta,0}^{(0)}. \qquad (2.12)$$

Topilgan ifodalarni (2.11) chegaraviy shartlarda B_{11} va B_{12} o'zgarmaslar o'rniga qo'yamiz va I_0, I_1, K_0, K_1 – Bessel funksiyalarini mos ravishda r_1 va r_2 radiuslar bo'yicha darajali qatorlarga yoyib

$$2\sum_{n=0}^{\infty}\left[U_{\theta,0}^{(0)}+\frac{r_2}{2}N_3^{(n)}(r_2)\beta^2 U_{\theta,1}^{(0)}\right]\beta^{2n+2}\frac{\left(r_2/2\right)^{2n+2}}{n!\,(n+2)!}+\frac{r_2}{2}\left(\beta^2-\frac{4}{r_2^2}\right)U_{\theta,1}^{(0)}=\frac{1}{\mu}f_{r\theta}^{(0)};$$

$$\sum_{n=0}^{\infty}\beta^{2n}\left\{\left(\frac{2\beta^2}{n+2}+R_k^{(0)}\right)U_{\theta,0}^{(0)}+\frac{r_1}{2}\left[\frac{2\beta^2}{n+2}N_3^{(0)}(r_1)+\frac{1}{r_1}R_k^{(0)}N_2^{(0)}(r_1)\right]\beta^2 U_{\theta,1}^{(0)}\right\}\frac{(r_1/2)^{2n+2}}{n!\,(n+1)!}$$
$$+\frac{r_1}{2}\left(\beta^2-\frac{4}{r_1^2}\right)U_{\theta,1}^{(0)}=0;$$

(2.13)

tenglmalarga ega bo'lamiz. Bu yerda

$$N_2^{(n)}(r)=N(\beta r)-N(\beta r_1)_{n=0;}$$

$$N_3^{(n)}(r)=ln\frac{r}{r_1}+\frac{n^2+5n+5}{2(n+1)(n+2)}-\sum_{k=1}^{n+2}\frac{1}{k}.$$

Bundan keyin $U_{\theta,0}$ va $U_{\theta,1}$ funksiyalar va λ^n $(n=1,2,3,\dots)$ operatorlarni quyidagi formulalar yordamida kiritamiz

$$(U_{\theta,0},U_{\theta,1})=\int_0^{\infty}\left.\begin{matrix}sinkz\\-coskz\end{matrix}\right\}dk\int_{(l)}\left[U_{\theta,0}^{(0)},U_{\theta,1}^{(0)}\right]e^{Pt}dP,$$

(2.14)

$$\lambda^n=\int_0^{\infty}\left.\begin{matrix}sinkz\\-coskz\end{matrix}\right\}dk\int_{(l)}(\beta^{2n}\varphi)e^{Pt}dP.$$

Endi (2.14) operatorlarni (2.13) tenglamarga qo'llab quyidagi tenglamarga ega bo'lamiz:

$$2\sum_{n=0}^{\infty}\lambda^n\left[U_{\theta,0}+\frac{r_2}{2}N_3^{(n)}(r_2)\lambda U_{\theta,1}\right]\frac{\left(r_2/2\right)^{2n+2}}{n!\,(n+2)!}+\frac{r_2}{2}\left(\lambda-\frac{4}{r_2^2}\right)U_{\theta,1}=\frac{1}{\mu}f_{r\theta};$$

$$\sum_{n=0}^{\infty}\lambda^n\left\{\left(\frac{2\lambda}{n+2}+R_k\right)U_{\theta,0}+\frac{r_1}{2}\left[\frac{2\lambda}{n+2}N_3^{(n)}(r_1)+\frac{1}{r_1}R_k\,N_2^{(n)}(r_1)\right]\lambda U_{\theta,1}\right\}\frac{(r_1/2)^{2n+2}}{n!\,(n+1)!}++\frac{r_1}{2}\Big(\lambda-$$
$$-\frac{4}{r_1^2}\Big)U_{\theta,1}=0,$$

(2.15)

bu yerda R_k - operatori $R_k^{(0)}$ - operatorining orginali va qovushoq siqiluvchi suyuqlikning doiraviy silindirik qatlamning buralma tebranishlariga reaksiyasini ifodaylaydi.

Yuqorida ko'rdikki $\beta^2 = k^2 + \frac{1}{r^2}p^2$. Bu yerda *k* va *P* lar (1.63) almashtirishlarda *z* koordinata va *t*-vaqt bo'yicha differensiallashga mos keladi. Shunga mos ravishda λ^n - operatorlarini *(z,t)* – o'zgaruvchilarda quyidagicha yozishmumkin

$$\lambda^n = \left[\frac{1}{b^2}\frac{\partial}{\partial t^2} - \frac{\partial^2}{\partial t^2}\right]^n, \ n = 1,2,\dots \qquad (2.16)$$

Keltirilgan λ^n $(n = 1,2,\dots)$ operatorlarining (2.16) ko'rinishlariga ko'ra (2.15) differensial tenglamalar sistemasining tartibi cheksiz katta va bu tenglamalar buralma U_θ ko'chishning $U_{\theta,0}$ va $U_{\theta,1}$ bosh qismlariga nisbatan chiqarildi. Bundan tashqari ushbu $U_{\theta,0}$ va $U_{\theta,1}$ funksiyalar silindirik qatlamning suyuqlik bilan kontaktda bo'lgan ichki sirti nuqtalari ko'chishlarini aniqlaydilar.

Yuqorida bayon qilingan mulohazalardaga ko'ra olingan (2.15) tenglamar sistemasi ichiga qovushoq siqiluvchi suyuqlik to'ldirilgan doiraviy silindirik elastik qatlamning buralma tebranishlari tenglamalaridan iboratdir. Ushbu tenglamalar silindirik qatlamning qovushoq siqiluvchi suyuqlik bilan kontaktda bo'lgan ichi sirti nuqtalari ko'chishlarining bosh qismlariga nisbatan chiqarildi. Ular *z* va *t* koordinatalar bo'yicha funksiyalarning ixtiyoriy hosilalariga ega hadlarni o'z ichiga oladi. Shuningdek bu tenglamalar qaralayotgan gidroelastik sistema sirtiga qo'yilgan tashqi yuklarni to'g'ridan-to'g'ri hisobga oladi hamda qatlam ichidagi qovushoq suyuqlikning qatlam tebranishlariga bo'lgan reaksiyasini o'z tarkibida saqlaydi.

§ 2.2. Sistemaning kuchlangan – deformatsiyalangan holatini aniqlash.

Ushbu paragrafning doirasida o'tkazilgan tatqiqotlarning asosiy maqsadi, defarmatsiyalanuvchi qattiq jismlar mexanikasining boshqa masalalari kabi, qaralayotgan gidroelastik sistemaning u yoki bu nuqtasidagi kuchlanganlik – deformatsiyalanganlik holatini aniqlashdan iborat. Shuning uchun bundan keyin qilinadigan asosiy ishlar ko'chish hamda kuchlanishlar tenzori kompanentalari uchun, (2.15) tenglamalar sistemasi tenglamalari tarkibiga kiruvchi asosiy izlanuvchi $U_{\theta,0}$ va $U_{\theta,1}$ funksiyalariga nisbatan, formulalar chiqarishdan iborat.

Yuqorida eslatilganidek, "silindirik qatlam qovushoq siqiluvchi suyuqlik" gidroelastik sistemasining buralma tebranishlarida qatlamning U_θ - ko'chishi va uning kesimlaridagi $\sigma_{r\theta}$ va $\sigma_{z\theta}$ kuchlanishlari, suyuqlikning zarrachalari tezliklari vektorining ϑ tuzuvchisi hamda suyuqlikdagi $P_{r\theta}$ va $P_{z\theta}$ kuchlanishlari noldan farqli bo'ladi. Shuning uchun kuchlanganlik – deformatsiyalanganlik holatlarini aniqlash uchun ushbu kompanentalari $U_{\theta,0}$ va $U_{\theta,1}$ funksiyalar orqali ifodalash yetarli bo'ladi.

Avvalo qatlam nuqtalarining buralma ko'chishi uchun formulani chiqaramiz. Buning uchun almashtirilgan ko'chishning (2.6) formulasini k va P parametrlar bo'yicha teskari almashtirish zarur. Buning uchun (2.6) ga B_{11} va B_{12} larning (2.12) qiymatlarini qo'yamiz va ushbu

$$U_\theta^{(0)} = \frac{r_1}{r} U_{\theta,0}^{(0)} + \sum_{n=0}^{\infty} \left[r_1 N_2^{(n)}(r) \beta^{2n+2} U_{\theta,0}^{(0)} + 2\beta^{2n} U_{\theta,1}^{(0)} \right] \frac{(r/r)^{2n+1}}{n!\,(n+1)!}$$

formulaga ega bo'lamiz. Bu ifodani teskari almashtirib

$$U_{\theta(r,z,t)} = \frac{r_1}{r} U_{\theta,0}(z,t) + \sum_{n=2}^{\infty} \left\{ r_1 N_2^{(n)}(r) \lambda^{n+1} U_{\theta,0} + 2\lambda^n U_{\theta,1} \right\} \frac{(r/2)^{2n+1}}{n!(n+1)!} \qquad (2.17)$$

ifodaga ega bo'alamiz. Keltirib chiqarilgan ushbu formula silindirik qobiq ixtiyoriy nuqtasining buralma U_θ ko'chishini $U_{\theta,0}$ va $U_{\theta,1}$ yordamchi funksiyalar qiymatlari orqali z va t koordinatalar bo'yicha talab qilingan aniqlikda hisoblashga imkon beradi.

Endi suyuqlik zarrachalarining tezliklarini aniqlaymiz. Ma'lumki buralma tebranish holida tezlikning faqat tangensial ϑ_θ kompanentalari noldan farqli bo'ladi. Bu ishni bajarmoq uchun (1.72) formulalardan foydalanamiz. Bu formulalardagi $v_\theta^{(0)}$ ning ifodasiga D_1 o'zgarmas o'rniga uning (2.9.1) qiymatini qo'yib

$$V_\theta^{(0)} = -P\beta \frac{I_1(\gamma r)}{I_1(\gamma r_1)}[B_{11} I_1(\beta r_1) - B_{12} K_1(\beta r_1)]$$

ifodaga ega bo'lamiz. Ammo, (1.66) ning oxirgi formulasini hisobga olsak oxirgi ifodani

$$V_\theta^{(0)} = -P\beta \frac{I_1(\gamma r)}{I_1(\gamma r_1)}[PU_\theta^{(0)}(r_1)] \tag{2.18}$$

ko'rinishda yozish mumkin.

Ushbu formuladagi Bessel funksiyalarining nisbati chekli qiymatga ega. Chunki fazoning suyuqlik bilan band qismi nuqtalari uchun $r \le r_1$ tengsizlik bajariladi. Bu yerda agar $I_n(z)$ funksiayaning o'suvchi ekanligi e'tiborga olinsa hamma $r \le r_1$ lar uchun $\frac{I_1(\gamma r)}{I_1(\gamma r_1)} \le 1$ deb hisoblash mumkin. Ushbu holatni hisobga olgan holda $I_1(\gamma r)$ lar yoyilmalarida birinchi uchta hadlar bilan chegaralanamiz. Bo'lishni bajarib $V_\theta^{(0)}$ uchun

$$V_\theta^{(0)} = \frac{r}{r_1}[1 + m_1(r_1 r_1)\gamma^2 + m_2(r_1 r_1)\gamma^4 + \dots]\left[PU_\theta^{(0)}(r_1)\right].$$

Ifodaga ega bo'lamiz va uni (1.64) hamda (1.70) larni hisobga olgan holda teskari almsashtirib suyuqlik zarrachalari urunma tezligi uchun

$$V_\theta = \frac{r}{r_1}[1 + m_1(r, r_1)\lambda_0 + m_2(r, r_1)\lambda_0^2 + \dots]\frac{\partial U_\theta(r_1)}{\partial t}.$$

(2.19)

formulaga ega bo’lamiz. Bu yerda

$$m_1(r, r_1) = \frac{r - r_1^2}{8}; \qquad m_2(r, r_1) = \frac{(r^2 - r_1^2)(2r^2 - r_1^2)}{192};$$

λ_0 – quyidagicha kiritilgan differensial aperator

$$\lambda_0^n(\varphi) = \int_0^\infty \left.\begin{matrix} sinkz \\ -coskz \end{matrix}\right\} dk \int_{(l)} (\gamma^{2n}\varphi) e^{Pt} dP. \qquad (2.20)$$

o’tgan bobdagi (1.60) formulaga ko’ra

$$\gamma^2 = \frac{P}{\nu'} + k^2.$$

Bu yerdan kelib chiqqan holda, xuddi yuqorida λ - operatordagi kabi, λ_0^n - operatorning *(z,t)* o’zgaruchilardagi ko’rinishi

$$\lambda_0^n = \left(\frac{1}{\nu'}\frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right)^n, \quad n = 0,1,2,\ldots \qquad (2.21)$$

Ekanligi to’g’risida xulosa chiqarish qiyin emas.

Shunday qilib (2.19) formula hamda λ_0^n operatorlarning (2.21) ko’rinishidan kelib chiqadiki, agar $U_{\theta,0}$ va $U_{\theta,1}$ yordamchi funksiyalar, va demak (2.17) formula bilan U_θ ko’chish ham, aniqlangan bo’lsa ϑ_θ tezlikni hisoblash qiyin emas. Bunda faqat yetarli darajada kichik tartibli hosilalarni hisoblash talab etiladi (*z* va *t* lar bo’yicha to’rtinchi tartibli va beshinchi tartibli hosilalar).

Qatlam va suyuqlik nuqtalaridagi kuchlanishlar uchun formulalar chiqarishga o’tamiz. Bunday formulalarni keltirib chiqarish U_θ ko’chish va ϑ_θ tezlik (2.17) va (2.19) formulalar yordamida aniqlanganliklari sababli qiyinchiliklar tug’dirmaydi. Bundan oldingi bobdagi (1.50) formulalardan $\varepsilon_{r\theta}$va $\varepsilon_{z\theta}$ deformatsiyalar uchun, mos ravishda, ushbu formulalarni olamiz

$$\varepsilon_{r\theta} = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)\psi_1; \qquad \varepsilon_{z\theta} = -\frac{\partial^2}{\partial z \partial r}\psi_1.$$

(2.22)

Takidlash lozimki ushbu ifodalar faqat buralma tebranishlar uchun o’rinli bo’ladi. Ikkinchi tomondan (1.48) dan

$$U_\theta = -\frac{\partial \Psi_1}{\partial z}.$$

Oxirgi ifodani (2.22) formulalarga qo’yamiz va olingan ifodalarniGuk qonuniga qo’yib

$$\sigma_{r\theta} = \mu\left(\frac{1}{r} - \frac{\partial}{\partial r}\right)U_\theta, \quad \sigma_{z\theta} = \mu\frac{\partial U_\theta}{\partial z}. \qquad (2.23)$$

Formulalarga ega bo’lamiz. Bu formulalarda U_θ o’rniga uning (2.17) ifodasini qo’yib kuchlanishlarni hisoblash oson.

Xuddi shunday (1.49) formulalardan buralma tebranishlar uchun

$$V_\theta = -\frac{\partial^2 \chi_1}{\partial t \partial r}$$

va (1.53) formulalardan

$$P_{r\theta} = \mu'\frac{\partial}{\partial t}\left(\frac{1}{r} - \frac{\partial}{\partial r}\right)\frac{\partial \chi_1}{\partial r}; \quad P_{z\theta} = -\mu'\frac{\partial^3 \chi_1}{\partial r \partial z \partial t}.$$

Ushbu formuladan ko’rinadiki

$$P_{r\theta} = \mu'\left(\frac{1}{r} - \frac{\partial}{\partial r}\right)V_\theta \quad \text{va} \quad P_{z\theta} = \mu'\frac{\partial V_\theta}{\partial z}.$$

tengliklar o’rinli. Bu tengliklarda V_θ ning o’niga uning (2.19) ifodasini qo’yib $P_{r\theta}$ va $P_{z\theta}$ kuchlanishlarni hisoblash qiyin ish emas.

Olingan oxirgi (2.17), (2.19), (2.23) va (2.24) formulalar qaralayotgan gidroelastik sistemaning istal kesimidagi kuchlangan – deformatsiyalangan holatini to’liq aniqlashga imkon beradilar. Bundan tashqari bu formulalar gidroelastik sistemaning buralma nostatsionar tebranishlari haqidagi amaliy masalalarni yechishda chegaraviy, kontakt va boshlang’ich shartlarni shakillantirish uchun ham xizmat qiladilar.

§ 2.3. Qovushoq siqiluvchi suyuqlik to’ldirilgan doiraviy silindirik qatlam va qobiqlarning bo’ylama radial tebranishlari tenglamalari

Ushbu paragraf doirasida silindirik elastik qatlamning ichi qovushoq suyuqlik bilan to’ldirilgan bo’lsin deb faraz qilamiz. Hosil qilingan gidroelastik sistemaning bo’ylama–radial nostatsionar tebranishlari tenglamalarini B.Yalg’ashov va Sh. Burqutboyevlarning ilmiy ishlaridan foydalangan holda keltirib chiqarishni o'rganamiz. Tenglamalarni keltirib chiqarish jarayonini qisqacha bayonini keltiramiz.

Yuqorida 1.3 paragrafda takidlanganidek, bu holda silindirik qatlamning Φ va Ψ_2 potensiallarga nisbatan harakat tenglamalari (1.40) ko'rinishga ega. Qovushoq suyuqlikning harakat tenglamalari esa (1.41) ko'rinishida. Masalaning chegaraviy, dinamik va kinematik shartlari (1.42), (1.43) va (1.44) lar kabi. Masalaning boshlang'ich shartlari nolga teng deb hisoblanadi. Masalani yechish uchun (1.57) ni (1.40) va (1.41) larga qo'yib

$$\widetilde{\Delta}_0\Phi_0 - \alpha^2\Phi_0 = 0, \quad \widetilde{\Delta}_0\Phi_{20} - \beta^2\Phi_{20} = 0, \quad r_1 \le r \le r_2 \text{ bo'lganda} \qquad (2.25)$$

$$\widetilde{\Delta}_0 \mathrm{G}_0 - \beta^2\mathrm{G}_0 = 0, \quad \widetilde{\Delta}_0\chi_{20} - \gamma^2\chi_{20} = 0, \quad r < r_1 \text{ bo'lganda} \qquad (2.26)$$

tenglamarga ega bo'lamiz. Bu yerda

$$\alpha^2 = k^2 + \frac{1}{a^2}p^2; \quad \beta^2 = k^2 + \frac{1}{b^2}p^2; \quad \delta^2 = k^2 + \frac{3}{3a_0^2 + 4\nu'}p^2;$$

$$\gamma^2 = \frac{p}{\nu'} + k^2; \quad \widetilde{\Delta}_0 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}. \qquad (2.27)$$

Ma'lumki (2.25) tenglamalrning umumiy yechimlari

$$\Phi_0(\mathrm{r}) = \mathrm{A}_1\mathrm{I}_0(\alpha\mathrm{r}) + \mathrm{A}_2\mathrm{K}_0(\alpha\mathrm{r}), \quad \Psi_{20}(\mathrm{r}) = \mathrm{B}_1\mathrm{I}_0(\beta\mathrm{r}) + \mathrm{B}_2\mathrm{K}_0(\beta\mathrm{r}), \quad r_1 \le r \le r_2 \qquad (2.28)$$

kabi, (2.26) tenglamarning umumiy yechimlari esa

$$\mathrm{G}_0(\mathrm{r}) = \mathrm{C}\mathrm{I}_0(\delta\mathrm{r}), \quad \chi_{20}(\mathrm{r}) = \mathrm{D}\mathrm{I}_0(\gamma\mathrm{r}), \quad r < r_1 \,. \qquad (2.29)$$

kabi ko'rinishga ega.

Silindirik qatlam nuqtalarining ko'chish vektori hamda suyuqlik zarrachalarining tezlik vektori kompanentalari, shuningdek qaralayotgan elastik jism (qatlam) va qovushoq suyuqlik nuqtalaridagi kuchlanish tenzorlarining no'lga bo'lmagan kompanentalarining kiritilgan Φ, Ψ_2, G va χ_2 potensial funksiayalar orqali ifodalari (1.48) – (1.49) va (1.52) – (1.53) formulalar orqali topilganligini ko'rsatib o'tamiz.

Chegaraviy va kontakt shartalarni ham almashtirilgan Φ_0, Ψ_{20}, G_0 va χ_0 potensaillar orqali ifodalsh uchun σ_{rr}, σ_{rz}, P_{rr}, P_{rz} kuchlanishlarni (1.63) va (1.67) ko'rinishlarda tasvirlaymiz va (1.52) hamda (1.53) formulalarga qo'yamiz. U holda

$$\begin{gathered} P_{rr}^{(0)} = p\left\{\lambda'\left(\widetilde{\Delta}_0 - k^2\right) + \rho_0'\left[p - \frac{4}{3}\nu'\left(\widetilde{\Delta}_0 - k^2\right)\right] + \mu'\frac{d^2}{dr^2}\right\}G_0 - \mu' pk\frac{d\chi_{20}}{dr^2}, \\ P_{rz}^{(0)} = \mu' p\left\{2k\frac{dG}{dr} - \frac{d}{dr}\left(k^2 + \widetilde{\Delta}_0\right)\chi_{20}\right\}, \qquad \widetilde{\Delta}_0 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}. \end{gathered} \tag{2.30}$$

$$\sigma_{rr}^{(0)} = \lambda\left(\widetilde{\Delta}_0 - k^2\right)\Phi_0 + 2\mu\frac{d^2}{dr^2}\left(\Phi_0 - k\Psi_{20}\right), \; \sigma_{rz}^{(0)} = \mu\frac{d}{dr}\left\{2k\Phi_0 + \left(k^2 - -\widetilde{\Delta}_0\right)\Psi_0\right\}. \tag{2.31}$$

Yuqoridagi (1.42) chegaraviy shartlardagi tashqi ta'sir funksiyalarini (1.56) ko'rinishida tasvirlab, (2.31) ifodalardan foydalangan holda

$$\left.\begin{gathered} \lambda'\left(\widetilde{\Delta}_0 - k^2\right)\Phi_0 + 2\mu\frac{d^2}{dr^2}\left(\Phi_0 - k\Psi_{20}\right) = f_r^{(0)}(k,p), \\ \mu\frac{d}{dr}\left[2k\Phi_0 - \left(k^2 + \widetilde{\Delta}_0\right)\Psi_{20}\right] = f_r^{(0)}(k,p). \end{gathered}\right. \qquad r = r_2 \text{ bo'lganda} \tag{2.32}$$

ko'rinishidagi chegaraviy shartlarga ega bo'lamiz.

Xuddi shunday (1.43) dinamik shartlardan $r = r_2$ bo'lganda (2.30) va (2.31) ifodalardan foydalanilgan holda

$$\begin{gathered} \lambda'\left(\widetilde{\Delta}_0 - k^2\right)\Phi_0 + 2\mu\frac{d^2}{dr^2}\left(\Phi_0 - k\Psi_{20}\right) = \mu' kp\frac{d^2\chi_{20}}{dr^2} - \left\{\left[\left(\widetilde{\Delta}_0 - k^2\right)\left(\lambda' - \right.\right.\right. \\ \left.\left.\left. - \frac{4}{3}\nu'\rho_0'\right) + p\rho_0'\right] + +\mu\frac{d^2}{dr^2}\right\}pG_0, \\ \mu\frac{d}{dr}\left[2k\Phi_0 + \left(k^2 + \widetilde{\Delta}_0\right)\Psi_{20}\right] = \mu' p\left[\left(k^2 + \widetilde{\Delta}_0\right)\frac{d\chi_{20}}{dr} - 2k\frac{dG_0}{dr}\right]. \end{gathered} \tag{2.33}$$

tenglamalarga ega bo'lamiz.

Endi (1.48) va (1.49) formulalarga (1.57), (1.64), (1.70) almashtirishlarni qo'llab

$$U_r^{(0)} = \frac{d\Phi_0}{dr} - k\frac{d\Psi_{20}}{dr}; \qquad U_z^{(0)} = k\Phi_0 - \widetilde{\Delta}_0\Psi_{20}. \tag{2.34}$$

$$V_r^{(0)} = p\frac{d}{dr}\left(G_0 - k\chi_{20}\right); \qquad V_z^{(0)} = p\left(kG_0 - \widetilde{\Delta}_0\chi_{20}\right). \tag{2.35}$$

ifodalarni olamiz

Nihoyat (1.44) kinematk shartlarga (1.57) va (1.70) almashirishlarni qo'llab

$$\begin{gathered} V_r^{(0)}(r_1,k,p) = pU_r^{(0)}(r_1,k,p) \\ V_z^{(0)}(r_1,k,p) = pU_z^{(0)}(r_1,k,p) \end{gathered} \tag{2.36}$$

Tenglamalarga ega bo'lamiz. Endi ana shu (2.36) tenglamalarga (2.34) va (2.35) ifodalarni qo'yib

$$p\left(\frac{dG_0}{dr}-k\frac{d\chi_{20}}{dr}\right)=p\left(\frac{d\Phi_0}{dr}-k\frac{d\Psi_{20}}{dr}\right);$$

$$p\left(kG_0-\tilde{\Delta}_0\chi_{20}\right)=p\left(k\Phi_0-\tilde{\Delta}_0\Psi_{20}\right);\quad (r=r_2). \qquad (2.37)$$

ko'rinishiga kinematik shartlarga ega bo'lamiz. Olingan (2.37) formulalarni (2.28) va (2.29) yechimlardan foydalanib

$$\delta I_1(\delta r_1)C-k\gamma I_1(\gamma r_1)D=E_1,\ \ kI_0(\delta r_1)C-\gamma^2 I_0(\gamma r_1)D=E_2, \qquad (2.38)$$

Ko'rinishida yozib olamiz. Bu yerda

$$E_1=\alpha\left[I_1(\alpha r_1)A_1-K_1(\alpha r_1)A_2\right]-k\beta\left[I_1(\beta r_1)B_1-K_1(\beta r_1)B_2\right],$$
$$E_2=k\left[I_0(\alpha r_1)A_1+K_0(\alpha r_1)A_2\right]-\beta^2\left[I_0(\beta r_1)B_1-K_0(\beta r_1)B_2\right].$$

Oxirgi (2.38) tenglamalar sistemasini C va D o'zgarmaslarga nisbatan yechamiz

$$D=\frac{\delta I_1(\delta r_1)E_2-kI_0(\delta r_1)E_1}{W},\quad C=\frac{k\gamma\ I_1(\gamma r_1)E_2-\gamma^2 I_0(\gamma r_1)E_1}{W} \qquad (2.39)$$

bu yerda

$$W=\gamma\left[k^2 I_0(\delta r_1)I_1(\gamma r_1)-\delta\gamma I_0(\gamma r_1)I_1(\delta r_1)\right]$$

Endi (2.28) yechimlarni (2.32) chegaraviy shartlarga qo'yamiz. Hosil qilingan ifodalarda Besselning modifitsirlangan $I_\nu(z),\ K_\nu(z)$ funksiyalari o'rniga ularning radial koordinata darajalari bo'yicha qatorlarga yoyilmalarni qo'yamiz. Murakkab bo'lmagan matematik almashtirishlarni bajarib, quyidagi tenglamalar sistemasiga ega bo'lamiz

$$\left(\beta^2+k^2\right)\sum_{m=0}^{\infty}\alpha^{2m}\left\{A_{10}-N_0^{(m)}(r)A_2\right\}\frac{(r_2/2)^{2m}}{(m!)^2}-\frac{2}{r}\sum_{m=0}^{\infty}\alpha^{2m+2}\left\{A_{10}-N_1^{(m)}(r)A_2\right\}\frac{(r/2)^{2m+1}}{m!(m+1)!}-2k\sum_{m=0}^{\infty}\beta^{2m+2}\times$$
$$\times\left\{B_{10}-N_0^{(m)}(r)B_2\right\}\frac{(r_2/2)^{2m}}{(m!)^2}+\frac{2k}{r}\sum_{m=0}^{\infty}\beta^{2m+2}\left\{B_{10}-N_0^{(m)}(r)B_2\right\}\frac{(r_2/2)^{2m+1}}{m!(m+1)!}+\frac{2}{r_2}\left(A_2-kB_2\right)=\frac{1}{\mu}f_r^{(0)}(k,p) \qquad (2.40)$$

$$2k\sum_{m=0}^{\infty}\alpha^{2m+2}\left\{A_{10}-N_1^{(m)}(r)A_2\right\}\frac{(r_2/2)^{2m+1}}{m!(m+1)!}-\left(\beta^2+k^2\right)\sum_{m=0}^{\infty}\beta^{2m+2}\left\{B_{10}-N_0^{(m)}(r)B_2\right\}\frac{(r_2/2)^{2m+1}}{m!(m+1)!}-$$
$$-\frac{2}{r_2}\left[kA_2-\left(\beta^2+k^2\right)B_2\right]=\frac{1}{\mu}f_{rz}^{(0)}(k,p),$$

bu yerda $\Psi(m)$ – Gamma funksiyasining logarifmik hosilasi;

$$A_{10}=A_1-\left(\ln\frac{\alpha r_1}{2}-\Psi(1)-\frac{1}{2}\right)A_2;\quad B_{10}=B_1-\left(\ln\frac{\alpha r_1}{2}-\Psi(1)-\frac{1}{2}\right)B_2 \tag{2.41}$$

$$N_0^{(m)}(r)=\ln\frac{r}{r_1}-\sum_{k=1}^{m}\frac{1}{k};\qquad N_1^{(m)}(r)=\ln\frac{r}{r_1}+\frac{1}{2}-\sum_{k=1}^{m}\frac{1}{k}\quad \text{при}\quad m=0\quad N_0^{(0)}(r)=\ln\frac{r}{r_1}$$

Endi A_2, B_2 o'zgarmaslar, hamda yangidan kiritilgan A_{10}, B_{10} o'zgarmaslarning mexanik ma'nosini aniqlaymiz. Buning uchun (2.28) yechimlarni (2.10) ifodalarga qo'yib almashtirilgan ko'chishlar uchun

$$\begin{aligned} U_r^{(0)}&=-k\beta\left[I_1(\beta r)B_1-K_1(\beta r)B_2\right]+\alpha\left[I_1(\alpha r)A_1-K_1(\alpha r)A_2\right],\\ U_z^{(0)}&=-\beta^2\left[I_0(\beta r)B_1-K_0(\beta r)B_2\right]+k\left[I_0(\alpha r)A_1-K_0(\alpha r)A_2\right]\end{aligned} \tag{2.42}$$

ifodalarni olamiz. Bu ifodalarningh o'ng tomonida Besselning modifitsirlangan funksiyalari o'rnida ularning standart yoyilmalaridan foydalanamiz. U holda

$$\begin{aligned} U_r^{(0)}&=\frac{kB_2-A_2}{r}-k\sum_{m=0}^{\infty}\beta^{2m+2}\left\{B_{10}-N_0^{(m)}(r)B_2\right\}\frac{\left(r/2\right)^{2m+1}}{m!(m+1)!}+\\ &+\sum_{m=0}^{\infty}\alpha^{2m+2}\left\{A_{10}-N_0^{(m)}(r)A_2\right\}\frac{\left(r/2\right)^{2m+1}}{m!(m+1)!},\end{aligned} \tag{2.43}$$

$$U_z^{(0)}=\sum_{m=0}^{\infty}k\alpha^{2m}\left\{A_{10}-N_1^{(m)}(r)A_2\right\}\frac{\left(r/2\right)^{2m+1}}{(m!)^2}-\sum_{m=0}^{\infty}\beta^{2m+2}\left\{B_{10}-N_1^{(m)}(r)B_2\right\}\frac{\left(r/2\right)^{2m}}{(m!)^2}.$$

Silindirik qobiqlar tebranishlarini tatqiq etishda asosiy izlanuvchi kattaliklar sifatida uning o'rta sirti nuqtalarining ko'chishlari qabul qilinadi. Ammo bunday qilish kontakt masalalarni yechishda qulay emas va ba'zi noaniqliklarni keltirib chiqaradi. Shu munosabat bilan biz, qaralayotgan masalada izlanuvchi kattaliklar sifatida, silindirik qatlamning ichki, suyuqlik bilan kontaktda bo'lgan sirti nuqtalarining ko'chishlarini qabul qilamiz. Buning uchun avvalo (2.43) ifodalarda $r=r_1$ deb olamiz va nolinchi *(m=0)* yaqinlashish bilan chegaralanamiz. U holda

$$\begin{aligned} U_r^{(0)}&=\frac{r}{2}\left\{\alpha^2A_{10}-k\beta^2B_{10}\right\}-\frac{A_2-kB_2}{r_1}+N_0^{(0)}(r_1)\left[k\beta^2B_2-\alpha^2A_2\right]\frac{r_1}{2};\\ U_z^{(0)}&=kA_{10}-\beta^2B_{10}-N_1^{(0)}(r_1)\left(kA_2-\beta^2B_2\right),\end{aligned} \tag{2.44}$$

bu yerda A_{10} va B_{10} o'zgarmaslar (2.41) formulalar bilan aniqlanadilar.

Almashtirilgan $U_r^{(0)}$ va $U_z^{(0)}$ ko'chishlarning (2.44) ko'rinishlaridan kelib chiqib, (2.41) ifodalarni hisobga olgan holda yangi izlanuvchi funksiyalarni quyidagicha kiritamiz:

$$U_{r,0}^{(0)}=\alpha^2 A_{10}-k\beta^2 B_{10}; \qquad U_{z,0}^{(0)}=kA_{10}-\beta^2 B_{10};$$
$$U_{r,1}^{(0)}=\frac{A_2-kB_2}{r_1}; \qquad U_{z,1}^{(0)}=\frac{kA_2-\beta^2 B_2}{r_1}. \tag{2.45}$$

olingan tenglamardan A_{10}, B_{10} , A_2, va B_2 o'zgarmaslarni topamiz va topilgan ifodalarni almashtirilgan ko'chishlarning (2.43) ifodalriga qo'yamiz va quyidagilarga ega bo'lamiz

$$U_r^{(0)}=\sum_{m=0}^{\infty}\left\{\beta^{2m}U_{r,0}^{(0)}+\alpha^2 q_1 P_m^{(0)}\left[U_{r,0}^{(0)}-kU_{z,0}^{(0)}\right]\right\}\frac{\left(r/2\right)^{2m+1}}{m!(m+1)!}-$$
$$-r_1\sum_{m=0}^{\infty}\left\{\beta^{2m+2}U_{r,1}^{(0)}+q_2 P_m^{(0)}\left[\beta^2 U_{r,1}^{(0)}-kU_{z,1}^{(0)}\right]\right\}N_0^{(m)}(r)\frac{\left(r/2\right)^{2m+1}}{m!(m+1)!}-\frac{r_1}{r}U_{r,1}^{(0)}; \tag{2.46}$$
$$U_z^{(0)}=\sum_{m=0}^{\infty}\left\{\beta^{2m}U_{z,0}^{(0)}+k\,q_1 P_m^{(0)}\left[U_{r,0}^{(0)}-kU_{z,0}^{(0)}\right]\right\}\frac{\left(r/2\right)^{2m}}{(m!)^2}-$$
$$-r_1\sum_{m=0}^{\infty}\left\{\alpha^{2m}U_{z,1}^{(0)}+\beta^2 q_2 P_m^{(0)}\left[kU_{r,1}^{(0)}-U_{z,1}^{(0)}\right]\right\}N_1^{(m)}(r)\frac{\left(r/2\right)^{2m}}{(m!)^2},$$

bu yerda

$$q_1=\frac{\alpha^2-\beta^2}{\alpha^2-k^2};\; q_2=\frac{\alpha^2-\beta^2}{\beta^2-k^2};\; P_m^{(0)}=\frac{\alpha^{2m}-\beta^{2m}}{\alpha^2-\beta^2};\;\; P_0^{(0)}\equiv 0;\; P_1^{(0)}\equiv 1;\;\; P_2^{(0)}=\alpha^2+\beta^2 \tag{2.47}$$

Yuqoridagi (2.27) formulalardan foydalanib (2.47) ni

$$q_1=-\frac{\lambda+\mu}{\mu};\quad q_2=-\frac{\lambda+\mu}{\lambda+2\mu};\quad P_m^{(0)}=\sum_{k=0}^{m-1}\alpha^{2(m-k-1)}\beta^{2k}. \tag{2.48}$$

ko'rinishda yozish mumkin. Agar (2.46) ifodalrda $m=0$ desak

$$U_r^{(0)}=\frac{r}{2}U_{r,0}^{(0)}+\frac{r_1 r}{2}\ln\left(\frac{r}{r_1}\right)\cdot\left[(1+q_2)\beta^2 U_{r,1}^{(0)}-kq_2 U_{z,1}^{(0)}\right],\; U_z^{(0)}=U_{z,0}^{(0)}-r_1\left(\ln\frac{r}{r_1}+\frac{1}{2}\right)U_{z,1}^{(0)}. \tag{2.49}$$

Bu yerda $U_{r,1}^{(0)}$ va $U_{z,0}^{(0)}$ funksiyalar ko'chishlar bilan bir xil o'lchovga, $U_{z,0}^{(0)}$ va $U_{z,1}^{(0)}$ funksiyalar esa deformatsiyalar bilan bir xil o'lchovga ekanligi ko'rinadi.

Endi yana chegarviy shartlarga qaytamiz. Buning uchun bundan keyin $p \gg ck$ tengsizlik qanoatlantiruvchi to'lqin jarayonlarini qaraymiz. Bu yerda $c=max\{a,a_0\}$; a_0 –sokin suyuqlikda tovush tezligi. U holda $r=r_1$ bo'lganda $E_1=U_r^{(0)}(r_1)$, $E_2=U_z^{(0)}(r_1)$ ekanligini hisobga olib, (2.39) dan D va C o'zgarmaslar uchun

$$D=\frac{\delta I_1(\delta r_1)}{W}U_z^{(0)}(r_1)-\frac{kI_0(\delta r_1)}{W}U_r^{(0)}(r_1),$$
$$C=\frac{k\gamma I_1(\gamma r_1)}{W}U_z^{(0)}(r_1)-\frac{\gamma^2 I_0(\gamma r_1)}{W}U_r^{(0)}(r_1) \tag{2.50}$$

ifodalarni olamiz.

Endi (2.29) yechimlarni yuqorida keltirilgan (2.33) dinamik shartlarning o'ng tomonlariga C va D o'zgarmaslarning (2.50) qiymatlarini hisobga olgan holda qo'yamiz, chap tamonlariga esa (2.29) yechimlarini qo'yamiz. Natijada dinamik chegaraviy shatrlarni (2.28) va (2.29) yechimlar orqali ifodalaymiz. Olingan tenglamarda ham (2.32) chegaraviy shartlarni almshtirganda qo'llangan almashtirishlarni qo'llab,quyidagi tenglamarni olamiz

$$\left(\beta^2+k^2\right)\sum_{m=0}^{\infty}\alpha^{2m}\left\{A_{10}-N_0^{(m)}(r_1)A_2\right\}\frac{\left(r_1/2\right)^{2m}}{(m!)^2}-\frac{2}{r_1}\sum_{m=0}^{\infty}\alpha^{2m+2}\left\{A_{10}-N_1^{(m)}(r_1)A_2\right\}\frac{\left(r_1/2\right)^{2m+1}}{m!(m+1)!}-2k\sum_{m=0}^{\infty}\beta^{2m+2}\times$$
$$\times\left\{B_{10}-N_0^{(m)}(r_1)B_2\right\}\frac{\left(r_1/2\right)^{2m}}{(m!)^2}+\frac{2k}{r_1}\sum_{m=0}^{\infty}\beta^{2m+2}\left\{B_{10}-N_0^{(m)}(r_1)B_2\right\}\frac{\left(r_1/2\right)^{2m+1}}{m!(m+1)!}+\frac{2}{r_1}\left(A_2-kB_2\right)=$$
$$=R_{z1}^{(0)}\left[\sum_{m=0}^{\infty}k\alpha^{2m}\left[A_{10}-N_1(r_1)A_2\right]\frac{\left(r/2\right)^{2m+1}}{m!(m+1)!}-\sum_{m=0}^{\infty}\beta^{2m+2}\left[B_{10}-N_1(r_1)B_2\right]\frac{\left(r_1/2\right)^{2m}}{(m!)^2}\right]-$$
$$-R_{r1}^{(0)}\left[\frac{kB_2-A_2}{r}+\sum_{m=0}^{\infty}\left[\alpha^{2m+2}\left(A_{10}-N_0(r_1)A_2\right)-k\beta^{2m+2}\left(B_{10}-N_0(r_1)B_2\right)\right]\frac{\left(r_1/2\right)^{2m+1}}{m!(m+1)!}\right],$$

$$2k\sum_{m=0}^{\infty}\alpha^{2m+2}\left\{A_{10}-N_1^{(m)}(r_1)A_2\right\}\frac{\left(r_1/2\right)^{2m+1}}{m!(m+1)!}-\left(\beta^2+k^2\right)\sum_{m=0}^{\infty}\beta^{2m+2}\left\{B_{10}-N_0^{(m)}(r_1)B_2\right\}\frac{\left(r_1/2\right)^{2m+1}}{m!(m+1)!}-$$
$$-\frac{2}{r_1}\left[kA_2-\left(\beta^2+k^2\right)B_2\right]=$$

$$= R_{z2}^{(0)}\left[\sum_{m=0}^{\infty} k\alpha^{2m}\left[A_{10}-N_1(r_1)A_2\right]\frac{\left(r_1/2\right)^{2m+1}}{m!(m+1)!}-\sum_{m=0}^{\infty}\beta^{2m+2}\left[B_{10}-N_1(r_1)B_2\right]\frac{\left(r_1/2\right)^{2m}}{(m!)^2}\right]-$$
$$- R_{r2}^{(0)}\left[\frac{kB_2-A_2}{r}+\sum_{m=0}^{\infty}\left[\alpha^{2m+2}\left(A_{10}-N_0(r_1)A_2\right)-k\beta^{2m+2}\left(B_{10}-N_0(r_1)B_2\right)\right]\frac{\left(r_1/2\right)^{2m+1}}{m!(m+1)!}\right], \qquad (2.51)$$

bu yerda

$$R_{z1}^{(0)}=\frac{\mu' pk}{\mu}\left[\frac{k\left(\delta^2-\gamma^2\right)I_1(\gamma r_1)I_0(\delta r_1)}{k^2 I_0(\delta r_1)I_1(\gamma r_1)-\delta\gamma I_0(\gamma r_1)I_1(\delta r_1)}-1\right],$$

$$R_{r1}^{(0)}=\frac{\mu' p}{\mu}\left[\frac{1}{r_1}+\frac{\gamma\left(\gamma^2-\delta^2\right)I_0(\gamma r_1)I_1(\delta r_1)}{k^2 I_0(\delta r_1)I_1(\gamma r_1)-\delta\gamma I_0(\gamma r_1)I_1(\delta r_1)}\right],$$

$$R_{z1}^{(0)}=\frac{\mu' p}{\mu}\frac{\delta\left(\gamma^2-k^2\right)I_1(\gamma r_1)I_1(\delta r_1)}{k^2 I_0(\delta r_1)I_1(\gamma r_1)-\delta\gamma I_0(\gamma r_1)I_1(\delta r_1)},$$

$$R_{r2}^{(0)}=-\frac{\mu' pk}{\mu}\left[\frac{\left(\gamma^2-k^2\right)I_1(\gamma r_1)I_0(\delta r_1)}{k^2 I_0(\delta r_1)I_1(\gamma r_1)-\delta\gamma I_0(\gamma r_1)I_1(\delta r_1)}+2\right].$$

Shunday qilib to'rtta A_{10}, B_{10} , A_2, va B_2 o'zgarmaslarni aniqlash uchun (2.40) va (2.51) lardan iborat to'rtta algebraic tenglamalar sistemasiga egamiz. Ana shu tenglamarda A_{10}, B_{10} , A_2, va B_2 integralash o'zgarmaslari o'rniga ularning (2.45) formulalar bilan aniqlanuvchi qiymatlarini qo'yamiz. Bundan tashqari yuqorida keltirilgan $p \gg ck,\ c = max\{a, a_0\}$ - fazoni hisobga olgan holda bir necha matematik almashtirishlardan keyin, quyidagi tenglamalar sistemasini olamiz

$$\sum_{m=0}^{\infty}\left\{\left(m(1-q_1)-q_1\right)\beta^{2m}-\left(\alpha^2-(m+1)\left(\beta^2+k^2\right)\right)q_1 P_m^{(0)}\right\}\frac{(r_1/2)^{2m}}{m!(m+1)!}U_{r,0}^{(0)}+$$
$$\sum_{m=0}^{\infty}\left\{m\left(\beta^2+k^2\right)q_1 P_m^{(0)}-\alpha^2 P_m^{(0)}q_1-(1+q_1)(m+1)\beta^{2m}\right\}\frac{(r_1/2)^{2m}}{m!(m+1)!}kU_{z,0}^{(0)}-$$
$$-r_1\sum_{m=0}^{\infty}\beta^2\left\{\left(\left(\beta^2+k^2\right)q_2 P_m^{(0)}+\beta^{2m}\right)N_1(r_1)-\left(q_2 P_{m+1}^{(0)}+\beta^{2m}\right)\frac{N_0(r_1)}{m+1}\right\}\frac{(r_1/2)^{2m}}{(m!)^2}U_{r,1}^{(0)}+$$
$$+\frac{2r_1}{r_2^2}U_{r,1}^{(0)}+r_1\sum_{m=0}^{\infty}\left\{\left(\beta^{2m}-\left(\beta^2+k^2\right)q_2 P_m^{(0)}\right)N_1(r_1)-q_2 P_{m+1}^{(0)}\frac{N_0(r_1)}{m+1}\right\}\frac{(r_1/2)^{2m}}{(m!)^2}kU_{z,1}^{(0)}=$$

$$= R_{z1}^{(0)}\left[\sum_{m=0}^{\infty}\left\{q_1 P_m^{(0)} k U_{r,0}^{(0)} - \left[k^2 P_m^{(0)} q_1 - \beta^{2m}\right] U_{z,0}^{(0)}\right\}\frac{(r_1/2)^{2m}}{(m!)^2} - r_1 \sum_{m=0}^{\infty}\left\{q_2 P_m^{(0)} \beta^2 k U_{r,1}^{(0)} - \right.\right.$$

$$\left. - \left[\beta^2 P_m^{(0)} q_2 - \alpha^{2m}\right] U_{z,1}^{(0)}\right]\Big\} N_1(r_1)\frac{(r_1/2)^{2m}}{(m!)^2} - R_{r1}^{(0)}\left[\sum_{m=0}^{\infty}\left\{\beta^{2m} U_{r,0}^{(0)} + \alpha^2 q_1 P_m^{(0)}\left[U_{r,0}^{(0)} + k U_{z,0}^{(0)}\right]\right]\right\}\times \quad (2.52)$$

$$\left.\times\frac{(r_1/2)^{2m+1}}{m!(m+1)!} - r_1 \sum_{m=0}^{\infty}\left\{\alpha^{2m+2} U_{r,1}^{(0)} + q_2 P_{m+1}^{(0)}\left[\beta U_{r,1}^{(0)} + k U_{z,1}^{(0)}\right]\right]\right\} N_0(r_1)\frac{(r_1/2)^{2m+1}}{m!(m+1)!} - \frac{r_1}{r} U_{r,1}^{(0)}\right];$$

$$\sum_{m=0}^{\infty}\left\{\left[\beta^{2m} + q_1\left(\alpha^2 P_m^{(0)} - P_{m+1}^{(0)}\right)\right] k U_{r,0}^{(0)} + \alpha^2\left[2k^2 q_1 P_m^{(0)} - \beta^{2m} q_1 + \beta^{2m}\right] U_{z,0}^{(0)} - \right.$$

$$\left. - r_1\left[2\beta^2 P_{m+1}^{(0)} q_2 + \beta^{2m}\right] k U_{r,1}^{(0)} + r_1\left[2k^2 q_2 P_{m+1}^{(0)} - \beta^{2m+2}\right] U_{z,1}^{(0)}\right\} N_0(r_2)\frac{(r_1/2)^{2m+1}}{m!(m+1)!} - \frac{r_2}{r_1}\left(k U_{r,1}^{(0)} + U_{z,1}^{(0)}\right) =$$

$$= \mu' p\left\{R_{z2}^{(0)}\left[\sum_{m=0}^{\infty}\left\{q_1 P_m^{(0)} k U_{r,0}^{(0)} - \left[k^2 P_m^{(0)} q_1 - \beta^{2m}\right] U_{z,0}^{(0)}\right\}\frac{(r_1/2)^{2m}}{(m!)^2} - r_1 \sum_{m=0}^{\infty}\left\{q_2 P_m^{(0)} \beta^2 k U_{r,1}^{(0)} - \right.\right.\right.$$

$$- \left[\beta^2 P_m^{(0)} q_2 - \alpha^{2m}\right] U_{z,1}^{(0)}\Big\} N_1(r_1)\frac{(r_1/2)^{2m}}{(m!)^2} - R_{r2}^{(0)}\left[\sum_{m=0}^{\infty}\left\{\beta^{2m} U_{r,0}^{(0)} + \alpha^2 q_1 P_m^{(0)}\left[U_{r,0}^{(0)} + k U_{z,0}^{(0)}\right]\right]\right\}\frac{(r_1/2)^{2m+1}}{m!(m+1)!} -$$

$$\left.\left.\left. - r_1 \sum_{m=0}^{\infty}\left\{\alpha^{2m+2} U_{r,1}^{(0)} + q_2 P_{m+1}^{(0)}\left[\beta U_{r,1}^{(0)} + k U_{z,1}^{(0)}\right]\right]\right\} N_0(0)\frac{(r_1/2)^{2m+1}}{m!(m+1)!} - \frac{r_1}{r} U_{r,1}^{(0)}\right]\right]\right\}; \quad (2.53)$$

$$\sum_{m=0}^{\infty}\left\{(m(1-q_1) - q_1)\beta^{2m} - \left(\alpha^2 - (m+1)\left(\beta^2 + k^2\right)\right) q_1 P_m^{(0)}\right\}\frac{(r_2/2)^{2m}}{m!(m+1)!} U_{r,0}^{(0)} +$$
$$\sum_{m=0}^{\infty}\left\{m\left(\beta^2 + k^2\right) q_1 P_m^{(0)} - \alpha^2 P_m^{(0)} q_1 - (1+q_1)(m+1)\beta^{2m}\right\}\frac{(r_2/2)^{2m}}{m!(m+1)!} k U_{z,0}^{(0)} -$$

$$- r_1 \sum_{m=0}^{\infty}\beta^2\left\{\left(\left(\beta^2 + k^2\right) q_2 P_m^{(0)} + \beta^{2m}\right) N_1(r_2) - \left(q_2 P_{m+1}^{(0)} + \beta^{2m}\right)\frac{N_0(r_2)}{m+1}\right\}\frac{(r_2/2)^{2m}}{(m!)^2} U_{r,1}^{(0)} + \frac{2r_2}{r_1^2} U_{r,1}^{(0)} +$$

$$+ r_1 \sum_{m=0}^{\infty}\left\{\left(\beta^{2m} - \left(\beta^2 + k^2\right) q_2 P_m^{(0)}\right) N_1(r_2) - q_2 P_{m+1}^{(0)}\frac{N_0(r_2)}{m+1}\right\}\frac{(r_2/2)^{2m}}{(m!)^2} k U_{r,1}^{(0)} = \frac{1}{\mu} f_r^{(0)}(k,p) \quad (2.54)$$

.

$$\sum_{m=0}^{\infty}\left\{\left[\beta^{2m} + q_1\left(\alpha^2 P_m^{(0)} - P_{m+1}^{(0)}\right)\right] k U_{r,0}^{(0)} + \alpha^2\left[2k^2 q_1 P_m^{(0)} - \beta^{2m} q_1 + \beta^{2m}\right] U_{z,0}^{(0)} - r_1\left[2\beta^2 P_{m+1}^{(0)} q_2 + \beta^{2m}\right] k U_{r,1}^{(0)} + \right.$$

$$\left. + r_1\left[2k^2 q_2 P_{m+1}^{(0)} - \beta^{2m+2}\right] U_{z,1}^{(0)}\right\} N_0(r_2)\frac{(r_2/2)^{2m+1}}{m!(m+1)!} - \frac{r_2}{r_1}\left(k U_{r,1}^{(0)} + U_{z,1}^{(0)}\right) = \frac{1}{\mu} f_{rz}^{(0)}(k,p), \quad (2.55)$$

bunda

$$q_1 = 1-\frac{a^2}{b^2};\quad q_2 = \frac{b^2}{a^2}-1;\quad \mathrm{P}_m^{(0)} = \sum_{k=0}^{m-1}\alpha^{2(n-k-1)}\beta^{2k};\quad \mathrm{P}_0^{(0)} \equiv 0;\quad \mathrm{P}_1^{(0)} \equiv 1;\qquad \mathrm{P}_2^{(0)} = \alpha^2+\beta^2;\ \ldots.$$

Shunday qilib kiritilgan $U_{r,1}^{(0)}, U_{z,0}^{(0)}$, $U_{z,0}^{(0)}$ va $U_{z,1}^{(0)}$ funksiyalarga nisbatan to'rtta algebraic tenglamaga ega bo'ldik. Quyidagi formulalar yordamida λ_1^n, λ_2^n $(n = 0,1,2\ldots)$ operatorlar hamda $U_{r,0}, U_{r,1}, U_{z,0}$ va $U_{z,1}$ funksiyalarni kiritamiz,

$$\begin{gathered}
\left[U_{r,0}, U_{r,1}\right] = \int_0^\infty \left.\begin{matrix}\sin kz \\ -\cos kz\end{matrix}\right\} \int_{(l)} \left[U_{r,0}^{(0)}, U_{r,1}^{(0)}\right] e^{pt}dp\,; \\
\left[\lambda_1^n, \lambda_2^n\right](\zeta) = \int_0^\infty \left.\begin{matrix}\sin kz \\ -\cos kz\end{matrix}\right\} dk \int_{(l)} \left[\alpha^{2n}, \beta^{2n}\right]\zeta e^{pt}dp \\
\left[U_{z,0}, U_{z,1}\right] = \int_0^\infty \left.\begin{matrix}\cos kz \\ \sin kz\end{matrix}\right\} \int_{(l)} \left[U_{z,0}^{(0)}, U_{z,1}^{(0)}\right] e^{pt}dp\,;
\end{gathered} \tag{2.56}$$

Ana shu (2.56) almashtirishlarni (2.52) – (2.55) tenglamalar sistemasiga qo'llab, quyidagi differensial tenglamalar sistemasini olamiz

$$\begin{gathered}
\left(c_{11} - R_{r1}a_{r11} - R_{z1}a_{z21}\right)U_{r,0} + \left(c_{12} - R_{r1}a_{r21} - R_{z1}a_{z11}\right)U_{z,0} - \\
-\left(c_{13} - R_{r1}r_1a_{r31} - R_{z1}r_1a_{z41}\right)U_{r,1} + \left(c_{14} - R_{r1}r_1a_{r41} + R_{z1}r_1a_{z31}\right)U_{z,1} = 0; \\
\left(d_{11} - R_{r2}a_{r11} - R_{z2}a_{z21}\right)U_{r,0} + \left(d_{12} - R_{r2}a_{r21} + R_{z2}a_{z11}\right)U_{z,0} - \\
-\left(d_{13} - R_{r2}r_1a_{r31} - R_{z2}r_1a_{z41}\right)U_{r,1} - \left(d_{14} - R_{r2}r_1a_{r41} + R_{z2}r_1a_{z31}\right)U_{z,1} = 0; \\
c_{21}U_{r,0} + c_{22}U_{z,0} - c_{23}U_{r,1} + c_{24}U_{z,1} = \frac{1}{\mu}f_r(r,t); \\
d_{21}U_{r,0} + d_{22}U_{z,0} - d_{23}U_{r,1} - d_{24}U_{z,1} = \frac{1}{\mu}f_{rz}(z,t),
\end{gathered} \tag{2.57}$$

bu yerda

$$c_{i1} = \sum_{m=0}^\infty \left[\left(m(1-q_1) - q_1\right)\lambda_2^m - \left(\lambda_1 - (m+1)\left(\lambda_2 - \frac{\partial^2}{\partial z^2}\right)\right)q_1P_m\right]\frac{(r_i/2)^{2m}}{m!(m+1)!};$$

$$c_{i2} = \sum_{m=0}^\infty \frac{\partial}{\partial z}\left[(m+1)\left(\lambda_2 - \frac{\partial^2}{\partial z^2}\right)q_1P_m - \lambda_1P_mq_1 - (1+q_1)(m+1)\lambda_2^m\right]\frac{(r_i/2)^{2m}}{m!(m+1)!};$$

$$c_{i3} = r_1\sum_{m=0}^\infty \lambda_2\left[(m+1)\left(\left(\lambda_2 - \frac{\partial^2}{\partial z^2}\right)q_2P_m + \lambda_2^m\right)\eta_{3,m} - \left(q_2P_{m+1} + \lambda_2^m\right)N_0(r_i)\right]\frac{(r_i/2)^{2m}}{m!(m+1)!} - \frac{2r_1}{r_i^2};$$

$$c_{i4}=r_1\sum_{m=0}^{\infty}\frac{\partial}{\partial z}\left[(m+1)\left(\lambda_2^m-\left(\lambda_2-\frac{\partial^2}{\partial z^2}\right)q_2P_m\right)N_1(r_2)-P_{m+1}q_2N_0(r_2)\right]\frac{(r_i/2)^{2m}}{m!(m+1)!};\qquad(2.58)$$

$$d_{i2}=\sum_{m=0}^{\infty}\frac{\partial}{\partial z}\left[\lambda_1q_1P_m-P_{m+1}q_1+\lambda_2^m\right]\frac{(r_i/2)^{2m+1}}{m!(m+1)!};\quad a_{r11}=\sum_{m=0}^{\infty}\left[\lambda_2^m+\lambda_1qP_m\right]\frac{(r_1/2)^{2m+1}}{m!(m+1)!};$$

$$d_{i2}=\sum_{m=0}^{\infty}\lambda_1\left[2q_2\frac{\partial^2}{\partial z^2}P_m+(1-q_1)\lambda_2^m\right]\frac{(r_i/2)^{2m+1}}{m!(m+1)!};\qquad a_{r21}=\sum_{m=0}^{\infty}\frac{\partial}{\partial z}\lambda_1q_1P_m\frac{(r_1/2)^{2m+1}}{m!(m+1)!};$$

$$d_{i3}=r_1\sum_{m=0}^{\infty}\frac{\partial}{\partial z}\left[2\lambda_2P_{m+1}q_2+\lambda_2^{m+1}\right]N_0(r_i)\frac{(r_i/2)^{2m+1}}{m!(m+1)!}+\frac{r_1}{r_i}\frac{\partial}{\partial z};\quad a_{z11}=\sum_{m=0}^{\infty}\left[\lambda_1^m+\lambda_1q_1P_m\right]\frac{(r_1/2)^{2m}}{(m!)^2};$$

$$d_{i4}=r_1\sum_{m=0}^{\infty}\left[2q_2\frac{\partial^2}{\partial z^2}+\lambda_2^{m+1}\right]N_0(r_i)\frac{(r_i/2)^{2m+1}}{m!(m+1)!}+\frac{r_1}{r_i};\qquad a_{z21}=\sum_{m=0}^{\infty}q_1P_m\frac{\partial}{\partial z}\frac{(r_1/2)^{2m}}{(m!)^2};$$

$$a_{r31}=\sum_{m=0}^{\infty}N_0(r_1)\left[\lambda_2^{m+1}+\lambda_2q_2P_{m+1}\right]\frac{(r_1/2)^{2m+1}}{m!(m+1)!}+\frac{1}{r_1};\qquad a_{z31}=\sum_{m=0}^{\infty}N_1(r_1)\left[\lambda_1^m+\lambda_1q_2P_m\right]\frac{(r_1/2)^{2m}}{(m!)^2};$$

$$a_{r41}=\sum_{m=0}^{\infty}N_{1,n}(r_1)q_2\frac{\partial}{\partial z}P_{m+1}\frac{(r_1/2)^{2m+1}}{m!(m+1)!};\quad a_{z41}=\sum_{m=0}^{\infty}N_1(r_1)q_2\frac{\partial}{\partial z}P_m\lambda_2\frac{(r_1/2)^{2m}}{(m!)^2};$$

$$P_m=\sum_{k=0}^{m}\lambda_1^{(m-k-1)}\lambda_2^k;\ \lambda_1^m=\left(\frac{1}{a^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)^m;\ \lambda_2^m=\left(\frac{1}{b^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)^m.$$

Chiqarilgan (2.57) tenglamalar sistemasi ichiga qovushoq siqiluvchi suyuqlik to’ldirilgan doiraviy elastik qatlamning bo’ylama – radial tebranishlari umumiy tenglamalaridan iboratdir. Ulardagi asosiy izlanuvchi funksiyalar elastik qatlamning qovushoq siqiluvchi suyuqlik bilan kontaktda bo’lgan ichki sirti nuqtalari ko’rishlarining bosh qiymatlaridan iborat. Bu tenglamalar tarkibi (2.58) formulalar bilan aniqlanuvchi λ_1^n va λ_2^n operatorlarining ko’rinishlariga mos ravishda, izlanuvchi funksiyalarning *z* koordinata va *t* vaqt bo’yicha istalgan tartibdagi hosilalari kiruvchi hadlar kiradi. Shuningdek chiqarilgan tenglamalar o’z strukturalarida qatlamning tashqi sirtiga qo’yilgan tashqi zo’riqishlari to’g’ri hisobga oladi, hamda ular tarkibi qatlam ichidagi suyuqlik reaksiyasi ham kiradi.

§ 2.4. Ko'chishlar, kuchlanishlar va tezliklar maydonlarini aniqlash

Endi qaralayotgan gidroelastik sistemaning kuchlanganlik–deformatsiyalanganlik holatini hisoblash algoritimini keltiramiz. Buning uchun σ_{ij} va P_{ij} kuchlanish tenzorlari, qatlam nuqtalarining ko'chish vektori hamda suyuqlik zarrachalari tezlik vektori komponentalarini (2.57) tenglamalarning yechimlari orqali ifodalsh kifoya.

Avvalo U_r va U_z ko'chishlar va σ_{rr}, σ_{rz}, σ_{zz} va $\sigma_{\theta\theta}$ kuchlanishlarni $U_{r,k}$ va $U_{z,k}$ $(k = 0,1)$ - izlanuvchi funksiyalar orqali ifodalaymiz ko'chishlarni ushbu funksiyalar orqali ifodalash uchun, (2.46) tenglamalarni teskari almashtirish yetarli

$$U_r(r,z,t) = a_{r1}U_{r,0} + a_{r2}U_{z,0} - r_1 a_{r3}U_{r,1} - r_1 a_{r4}U_{z,1};$$

$$U_z(r,z,t) = a_{z1}U_{z,0} + a_{z2}U_{r,0} - r_1 a_{z3}U_{z,1} - r_1 a_{z4}U_{r,1}, \tag{2.59}$$

bu yerda

$$a_{ij} = a_{ij1}\Big|_{r_1=r}, \quad \lambda_1^m = \left(\frac{1}{a^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)^m; \quad \lambda_2^m = \left(\frac{1}{b^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)^m; \quad q_1 = -\frac{\lambda+\mu}{\mu};$$

$$q_2 = -\frac{\lambda+\mu}{\lambda+2\mu}; \quad P_m = \sum \lambda_1^{(m-k-1)}\lambda_2^k; \quad P_0 = 0; \quad P_1 = 1; \quad P_2 = \lambda_1^2 + \lambda^2.$$

silindirik qatlam nuqatalidagi kuchlanish tenzori kompanentalari aniqlash uchun (1.52) formulalardan foydalanamiz. Avvalo Φ va Ψ_2 potensial funksiyalarning (1.57) tasviridan foydalanib (1.52) formulalarni almashtiramiz. Shuningdek σ_{rr} va σ_{rz} kuchlanishlarning (1.63) tasvirlaridan hamda σ_{zz} va $\sigma_{\theta\theta}$ kuchlanishlarning quyidagi tasvirlaridan foydalanamiz

$$[\sigma_{zz}, \sigma_{\theta\theta}] = \int_0^\infty \left.\begin{matrix} \sin kz \\ -\cos kz \end{matrix}\right\} dk \int_{(l)} \left[\sigma_{zz}^{(0)}, \sigma_{\theta\theta}^{(0)}\right] e^{pt} dp \tag{2.60}$$

ko'rsatilgan tasvirlarni (1.52) formulalarga qo'llab, kuchlanish tenzorining almashtirilgan $\sigma_{ij}^{(0)}$ –komponentalarini almashtirilgan $\Phi^{(k)}$ va $\Psi_2^{(0)}$ potensial funksiyalar orqali ifodalaymiz. Olingan ifodalarga (2.28) umumiy yechimlarni qo'yib quyidagilarga ega bo'lamiz

$$\sigma_{rr}^{(0)}=\left[2\mu\alpha^2+\left(\alpha^2-k^2\right)\lambda\right]\left[I_0(\alpha r)A_1+K_0(\alpha r)A_2\right]-\frac{2\alpha}{r}\mu\left[I_1(\alpha r)A_1-K_1(\alpha r)A_2\right]-$$

$$-2\mu k\beta^2\left[I_0(\beta r)B_1+K_0(\beta r)B_2\right]+\frac{2k\beta}{r}\mu\left[I_1(\beta r)B_1-K_1(\beta r)B_2\right];$$

$$\sigma_{rz}^{(0)}=\mu\left\{2\alpha k\left[I_1(\alpha r)A_1-K_1(\alpha r)A_2\right]-\beta\left(\beta^2+k^2\right)\left[I_1(\beta r)B_1-K_1(\beta r)B_2\right]\right\}; \qquad (2.61)$$

$$\sigma_{zz}^{(0)}=\left[\lambda\left(\alpha^2-k^2\right)-2\mu k^2\right]\left[A_1I_0(\alpha r)+A_2K_0(\alpha r)\right]-2\mu k\beta^2\left[B_1I_0(\beta r)+B_2K_0(\beta r)\right];$$

$$\sigma_{rr}^{(0)}=\frac{\lambda+2\mu}{r}\alpha\left[A_1I_1(\alpha r)+A_2K_1(\alpha r)\right]-\lambda k^2\left[A_1I_0(\alpha r)+A_2K_0(\alpha r)\right]+$$

$$+\frac{2\mu k}{r}\beta\left[B_1I_1(\beta r)-B_2K_1(\beta r)\right]\ .$$

Ushbu ifodalar tarkibiga kiruvchi Bessel modifitsirlangan funksiayalari o’rniga ularning darajali qatorga yoyilamalarini qo’yib, olingan ifodalardagi A_i va B_i o’zgarmaslar o’rniga ularning (2.45) sistemadan topilgan qiymatlarini qo’yamiz va yuqoridagi ma’lum protsedura bo’yicha z va t o’zgaruvchilarga o’tib kuchlanish tenzori komponentalari uchun quyidagi formulalarni olamiz.

$$\sigma_{rr}^{(0)}=\mu\left[c_{r0}U_{r,0}+c_{z0}\frac{\partial U_{z,0}}{\partial z}-c_{r1}U_{r,1}+c_{z1}\frac{\partial U_{z,1}}{\partial z}\right];$$

$$\sigma_{rz}^{(0)}=\mu\left[d_{r0}\frac{\partial U_{r,0}}{\partial z}+d_{z0}U_{z,0}-d_{r1}\frac{\partial U_{r,1}}{\partial z}-d_{z1}U_{z,1}\right];$$

$$\sigma_{zz}=\mu\left\{\sum_{m=0}^{\infty}\left[\left[(1-q_1)\lambda_1^m-2P_mq_1\lambda_1+2\lambda_2^m\right]U_{r,0}+\frac{\partial}{\partial z}\left[(1-q_1)\lambda_1^m-2\lambda_1P_mq_1\right]U_{z,0}-\right.\right.$$

$$\left.\left.-r_1N_1(z)\left[\left[\lambda_1^m-2\left(P_{m+1}q_2+\lambda_2^m\right)\right]\lambda_2U_{r,1}+\frac{\partial}{\partial t}\left(\lambda_1^m-2q_2P_{m+1}\right)U_{z,1}\right]\right]\frac{(r/2)^{2m}}{(m!)^2}\right\},$$

$$
\begin{aligned}
\sigma_{\theta\theta} = \mu \Bigg\{ \sum_{m=0}^{\infty} \Bigg[\Bigg[-q_1(m+1)\lambda_1^m + q_1 P_m \frac{\partial^2}{\partial z^2} + \lambda_1^m m \Bigg] U_{r,0} + \frac{\partial}{\partial z}\left[\lambda_1 q_1 P_m - (m+1)(q_1+1)\lambda_1^m\right] U_{z,0} + \\
+ r_1 \Bigg[\Bigg[(m+1)\lambda_1^m \left(-q_2 \frac{\partial^2}{\partial z^2} + q_2\lambda_2^2 + \lambda_1^2 \right) N_2(r) - \left(\beta^2 P_{m+1} q_2 + \lambda_2^{m+1}\right) N_1(r) \Bigg] U_{r,1} - \qquad (2.62) \\
- \left[P_{m+1} q_2 N_1(r) - (m+1)(2q_2 - 1)\lambda_1^m N_2(r) \frac{\partial}{\partial z} U_{z,1} \right] \Bigg] \frac{(r/2)^{2m}}{m!(m+1)!} \Bigg] - \frac{2r_1}{r^2} U_{r,1} \Bigg\};
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{zz} = \mu \Bigg\{ \sum_{m=0}^{\infty} \Bigg[\left[(1-q_1)\lambda_1^m - 2P_m q_1 \lambda_1 + 2\lambda_2^m\right] U_{r,0} + \frac{\partial}{\partial z}\left[(1-q_1)\lambda_1^m - 2\lambda_1 P_m q_1\right] U_{z,0} - \\
- r_1 N_1(z) \Bigg[\left[\lambda_1^m - 2\left(P_{m+1} q_2 + \lambda_2^m\right)\right] \lambda_2 U_{r,1} + \frac{\partial}{\partial t}\left(\lambda_1^m - 2q_2 P_{m+1}\right) U_{z,1} \Bigg] \Bigg] \frac{(r/2)^{2m}}{(m!)^2} \Bigg\},
\end{aligned}
$$

bu yerda $c_{ij} = c_{ijk} /_{r_k = r} \qquad d_{ij} = d_{ijk} /_{r_k = r}$.

Hosil qilingan (2.59) va (2.62) formulalar bilan silindirik qatlam nuqtalaridagi kuchlanishlar va ko'chishlarni to'la aniqlash mumkin, agar bo'ylama va radial ko'chishlarning bosh qismlari aniqlangan bo'lsa. Bundan tashqari ushbu formulalar suyuqlik saqlangan qatlam chetlarida berilgan chegaraviy shartlarni shakillantirishga (yozish) imkon beradi. Bu yerda gidroelastik sistemaning kuchlangan–deformatsiyalangan holati qaralayotgan sistemaning ixtiyoriy kesimida aniqlanishi zarurligini takidlaymiz. Demak, bunda *z* koordinat fiksirlangan va *r* – radial koordinata hamda *t* – vaqt bo'yicha olingan talab etilgan aniqlikda aniqlanish talab etiladi.

Suyuqlik zarrachalari tezliklari komponentalarini aniqlash uchun (2.29) yechimlarni V_r va V_z komponentalarning (2.35) ifodalariga qo'yib

$$
V_r^{(0)} = p[c\delta I_1(\delta r) - Dk\gamma I_1(\gamma r)], \;\; V_z^{(0)} = p[kcI_0(\delta r) - D\gamma^2 I_0(\gamma r)]. \qquad (2.63)
$$

formulalarga ega bo'lamiz. Bu formulalarda *C* va *D* o'zgarmalar o'rniga ularning (2.39) ifodalarini qo'ysak tezlik komponentalri uchun modifitsirlangan Bessel funksiyalari orqali ifodalangan formulalarni olamiz. Bu formulalar juda murakkab bo'lganligi uchun Bessel funksiyalarining darajali qatorlarga yoyilmalarida birinchi hadlar bilan chegaralanamiz. Shunday qilib

$$V_r^0 = \frac{r}{r_1}\frac{\partial}{\partial t}U_r^{(0)}, \quad V_z^0 = \frac{\partial}{\partial t}U_z^{(0)}. \tag{2.64}$$

formulalarga ega bo'lamiz.

Xuddi shunga o'xshash ishlarni suyuqlik nuqtalaridagi P_{ij} kuchlanishlar uchun ham bajarib, ular uchun quyidagi ifodalarga ega bo'lamiz.

$$\begin{gathered}
P_{rr} = \left[\frac{2\rho_0'}{3r_1}\left(3a_0^2 + 4\nu'\frac{\partial}{\partial t}\right) + \frac{\mu'}{r_1}\right]U_r - \frac{\rho_0'}{3}\frac{\partial}{\partial z}\left(3a_0^2 + 4\nu'\frac{\partial}{\partial t}\right)U_z;\\
P_{\theta\theta} = \left[\frac{2\rho_0'}{3r_1}\left(3a_0^2 + 4\nu'\frac{\partial}{\partial t}\right) + \frac{\mu'}{r_1}\right]U_r - \frac{\rho_0'}{3}\frac{\partial}{\partial z}\left(3a_0^2 + 4\nu'\frac{\partial}{\partial t}\right)U_z;\\
P_{zz} = \frac{2\rho_0'}{3r_1}\left(3a_0^2 + 4\nu'\frac{\partial}{\partial t}\right)U_r - \frac{\partial}{\partial z}\left(\frac{\rho_0'}{3}\left(3a_0^2 + 4\nu'\frac{\partial}{\partial t}\right) + \mu'\frac{\partial}{\partial t}\right)U_z; \qquad (2.65)\\
P_{rz} = \frac{r\rho_0'}{6}\left(3\frac{\partial^2}{\partial t^2} - \left(3a_0^2 + 4\nu'\frac{\partial}{\partial t}\right)\frac{\partial^2}{\partial z^2}\right)U_z + \frac{2r\mu'}{r_1}\frac{\partial^2}{\partial t\partial z}U_r.
\end{gathered}$$

Shunday qilib olingan (2.59), (2.62), (2.64) va (2.65) formulalar qaralayotgan gidroelastik sistemaning ixtiyoriy kesimidagi kuchlanganlik – deformatsiyalangan holatini vaqtning istalgan payti uchun, *r* va *z* koordinatalar bo'yicha istalgan aniqlikda hisoblashga imkon yaratadi.

ASOSIY XULOSALAR

1. Ichida qovushoq siqiluvchi suyuqlik saqlangan doiraviy silindrik elastik qobiqning o'qqa nisbatan simmetrik, nostatsionar tebranishlari umumiy va taqribiy tenglamalarini keltirib chiqarish metodikasi o'rganib chiqildi. Ushbu metod silindrik qobiqni uch o'lchovli elastik jism deb qarab va bu jism uch o'lchovli masalalarni aniq qo'yish va bunday masalaga Fur'e va Laplas almashtirishlarini qo'llab, almashtirilgan tenglamalarining umumiy yechimlaridan foydalanishga asoslangan. Olingan umumiy yechimlarda qatnashuvchi Besselning modifitsirlangan funksiyalarining darajali qatorlarga standart yoyilmalari qo'llaniladi. Xuddi shu umumiy yechimlar orqali kuchlanish tenzori va ko'chish vektorining hamma tashkil etuvchilari ifodalanadi;

2. Ichiga qovishoq siqiluvchi suyuqlik to'ldirilgan doiraviy silindrik qatlamning buralma tebranish umumiy va taqribiy tenglamalaridan foydalanib ichidagi qovishoq siqiluvchi suyuqlik bilan ozaro nostatsionar ta'sirlashuvchi doiraviy silindrik elastik qobiqning buralma va bo'ylama-radial tebranishlari umumiy va taqribiy tenglamalari keltirib chiqarildi. Olingan umumiy tenglamalardan u yoki bu yaqinlashish bilan chegaralangan holda klassik tenglamalar yoki ularning har xil aniqlashtirilmalarini olish mumkin;

3. Asosiy natijaviy tenglamalar hamda kuchlanishlar (qobiq va suyuqlik nuqtalaridagi), ko'chishlar va suyuqlik zarrachalari tezlik vektorining tashkil etuvchilari qobiqning suyuqlik bilan kontaktda bo'lgan sirti bo'ylama-radial ko'chishlarining bosh qismlari orqali ifodalangan. Ta'kidlash lozimki, ushbu sirt o'rniga boshqa ixtiyoriy sirt ham qaralishi mumkin edi. Xususiy holda qobiqning ichki radiusini nolga intiltirsak ushbu sirt doiraviy sterjenning simmetriya o'qiga o'tadi.

4. O'rganib chiqilgan usul, tebranish tenglamalalari bilan bir qatorda kuchlanish tenzorlari (qobiq va suyuqlik uchun), ko'chish va suyuqlik zarrachalari tezlik vektorlarning tashkil etuvchilarini talab etilgan aniqlikda hisoblashga yaroqli algoritm yaratishga imkon beradi. Bunda olingan formulalar amaliy masalalarni yechishda zarur bo'ladigan chegaraviy, dinamik va kinematik kontakt shartlarini formulirovka qilishga muhim o'rin tutadilar. Bu formulalar kontakt shartlarini aniq qo'yish imkonini beradilar.

5. Ishda bajarilgan yana bir muhim faktor-bu qovishoq siqiluvchi suyuqlikning silindrik qobiq buralma va bo'ylama-radial tebranishlariga reaksiyasi aniqlangan. Bunga ko'ra qovushoq siqiluvchi suyuqlikning silindrik qobiq bo'ylama-radial tebranishlariga reaksiyasi birinchi yaqinlashishda qobiq nuqtalarining tangensial yo'nalishidagi tezlanishiga to'g'ri proporsional bo'ladi. Ma'lumki klassik mexanikada ushbu reaksiya tezlikning kvadratiga to'g'ri proporsional bo'ladi.

6. Umumiy va aniqlashtirilgan tenglamalardan kelib chiquvchi xususiy va limitik hollar tahlil qilindi. Xususiy hollarda doiraviy silindrik qobiqning va doiraviy silindrik yupqa devorli qobiqning qovishoq siqiluvchi suyuqlik bilan o'zaro ta'sirlashuvini hisobga oluvchi tenglamalar olindi. Shunindek suyuqlik bo'lmagan hol uchun ham buralma bo'ylama-radial taqribiy tenglamalari keltirib chiqarilgan.

7. Amaliy masalalarni yechishda chegaraviy shartlarni to'g'ri qatlam uchun oxirgi paragrafda kuchlanishlar va ko'chishlar hamda tezliklar uchun chiqarilgan formulalarda, tebranish tenglamalari qanday yaqinlashish (nolinchi, birinchi, va

h.k) bilan chegaralangan holda chiqarilgan bo’lsa, shunday yaqinlashish bilan chegaralanish zarur, yoki undan yuqoriroq yaqinlashish bilan chegaralanish zarur bo’ladi.

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