O‘ZBEKISTON RESPUBLIKASI

QISHLOQ VA SUV XO‘JALIGI VAZIRLIGI

TOSHKENT IRRIGATSIYA VA MELIORATSIYA INSTITUTI

“NAZARIY VA QURILISH MEXANIKASI”

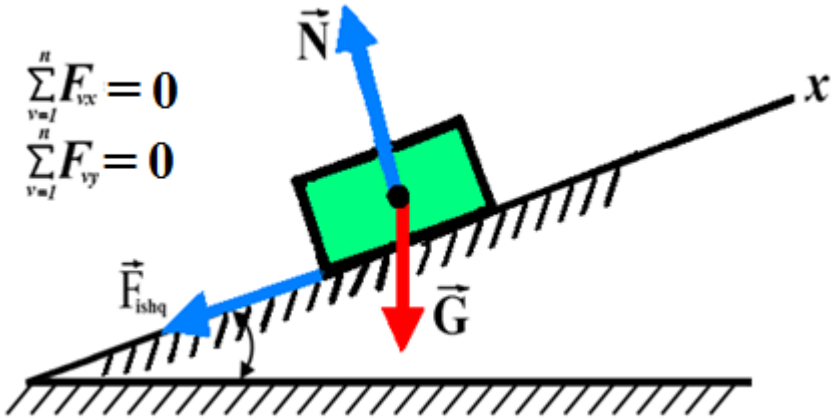
kafedrasi

“NAZARIY MEXANIKA”

fanining hisob-grafik

ishlarini EHM yordamida bajarish uchun

uslubiy ko‘rsatma



Toshkent – 2015

Ushbu uslubiy ko‘rsatma institut Ilmiy- uslubiy Kengashining 9 yanvar 2015 yilda bo’lib o‘tgan 4 -sonli majlisi qarori bilan tasdiqlangan.

Ushbu uslubiy ko‘rsatmada nazariy mexanikaning statika,kinematika va dinamika bo‘limidagi hisob-grafik ishlarini Mathcad matematik paketi yordamida qanday hal etilishi bayon etilgan.

Ko‘rsatmada nazariy mexanika bo‘limlari bo‘yicha talabalar bajaradigan uchta hisob-grafik ishi ham ko‘rsatilgan bo‘lib, ulardan birinchisi statika, ikkinchisi kinematika, uchinchisi dinamikadan iborat. Shuningdek bunday hisob-grafik ishlarini bajarish namunalari ham keltirilgan.

Mazkur uslubiy ko‘rsatma oliy texnika o‘quv yurtlarining talabalari hamda suv xo‘jaligi va melioratsiya ishlarini mexanizatsiyalashtirish, texnologik jarayonlar va ishlab chiqarishni avtomatlashtirish hamda boshqarish, suv xo‘jaligi qurilishi,elektr energetika ixtisosliklari bo‘yicha ta’lim oluvchi talabalar uchun mo’ljallangan.

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Kirish

Nazariy mexanika texnikaning barcha sohalari bo‘yicha muxandislar tayyorlashda muhim o‘rinni egallaydi. Muxandislik ishlarida muammoli va amaliyotga tatbiq etiladigan masalalarni hal etishda nazariy mexanika fani fundamental predmetlardan biri hisoblanib, u konkret masalalarni yechishda matematika faniga asoslanadi.

Hozirgi zamon texnikasi va kompyuter texnologiyasining rivojlanishi mexanika masalalarni matematik Matehematica, Maple, Matlab, Mathcad paketlar yordamida hal etishni ko‘ndalang vazifa qilib qo‘ymoqda.

Taqdim etilayotgan uslubiy ko‘rsatma mexanikaning hisob-grafik ishlarini Mathcad matematik paket yordamida yechish uchun ko‘nikmalar hosil qiladi.

Shuningdek unda Mathcad yordamida statika, kinematika va dinamikadan hisob-grafik ishlarini bajarish uchun namunalar keltirilgan hamda topshiriqlar ko‘rsatilgan.

1– topshiriq (statika)

1–jadvalda ko‘rsatilgan konstruksiya berilgan kuchlar ta’sirida muvozanatda turadi. $\vec{P}$ kuchning qo‘yilish nuqtasi (d masofa), og‘ish burchagi α va $\vec{P}$ kuch miqdori o‘zgarishi mumkin.

Mazkur topshiriqda quyidagilar talab qilinadi:

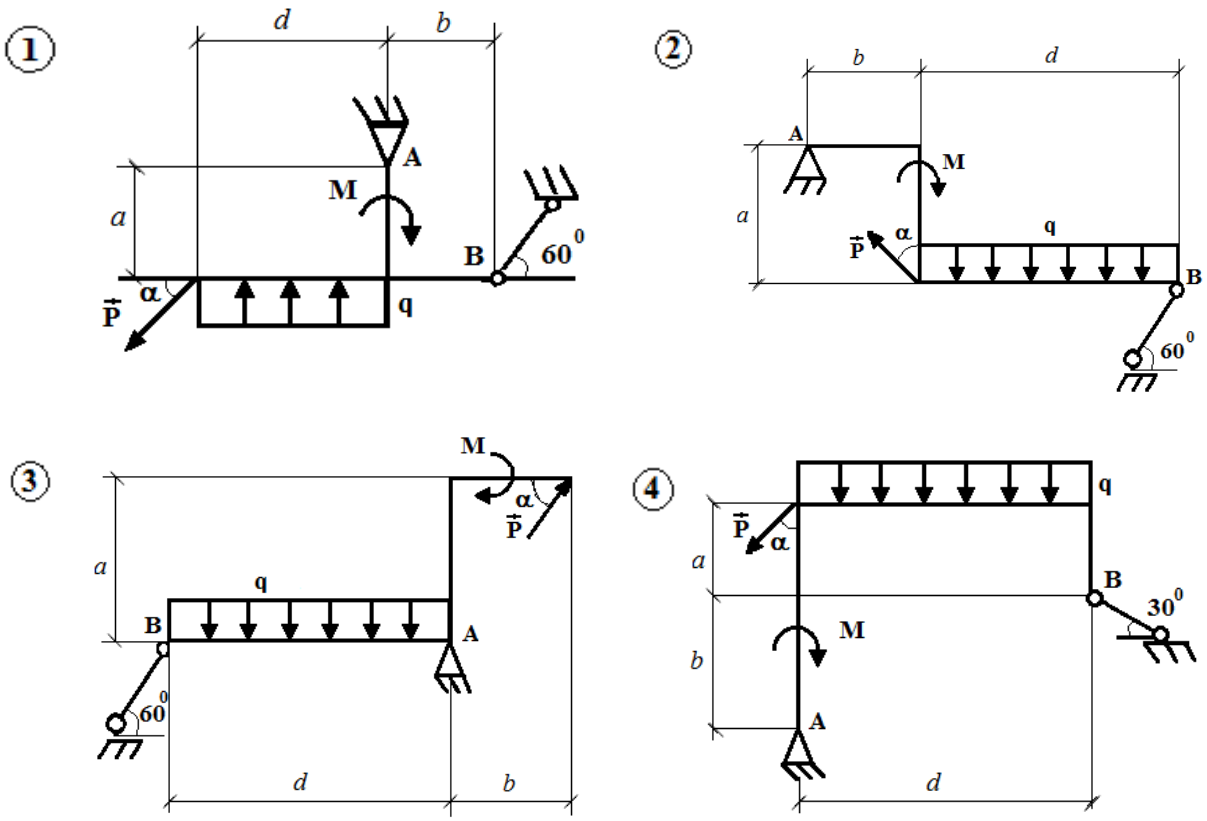
1) $d = d_1$ $(0{,}5m \le d \le 5m)$ parametrni shunday qiymati topilsinki, bunda R_B reaksiya eng kichik modulga ega bo‘lsin. Shu hol uchun konstruksiyaga qo‘yilgan bog‘lanish reaksiyalari topilsin.

2) $\vec{R}_B$ reaksiya kuchining moduli 1-jadvalda ko‘rsatilgan R_{Bm} dan oshib ketmasligi uchun P kuch qanday oraliqda $(P_1 \le P \le P_2)$ o‘zgarishi aniqlansin. Bunda $\alpha = \alpha_0, d = d_1, P \ge 0$ bajariladi.

3) $R_B \le R_{Bm}$ bo‘lishi uchun α burchak qanday oraliqda $(\alpha_1 \le \alpha \le \alpha_2)$ o‘zgarishi topilsin. Bunda $P = P_0 = const, \alpha \in [0, \pi], d = d_1$.

Topshiriqni bajarish uchun zarur miqdorlar 2-jadvalda ko‘rsatilgan.

1-jadval



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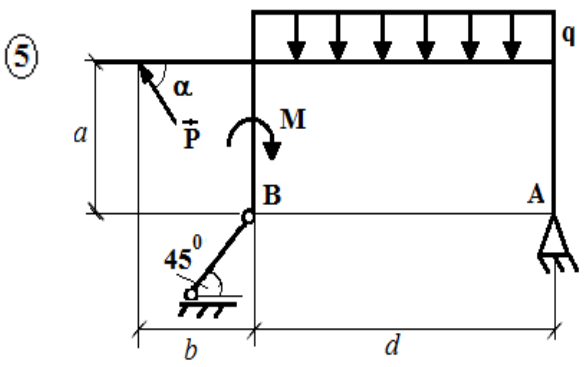

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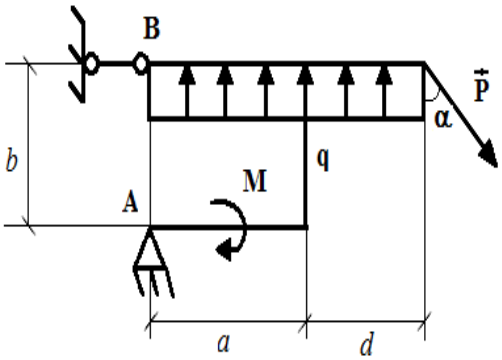

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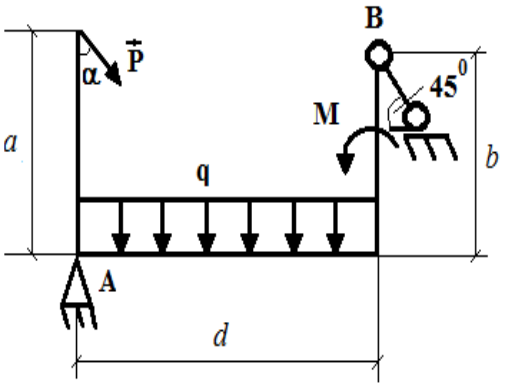

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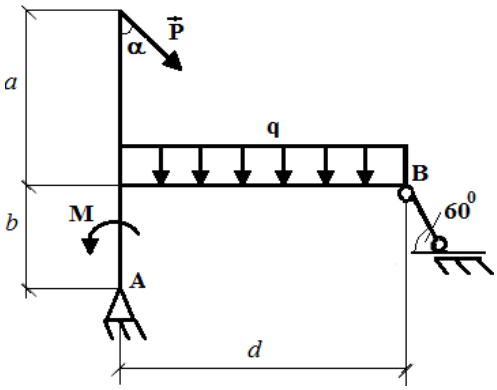

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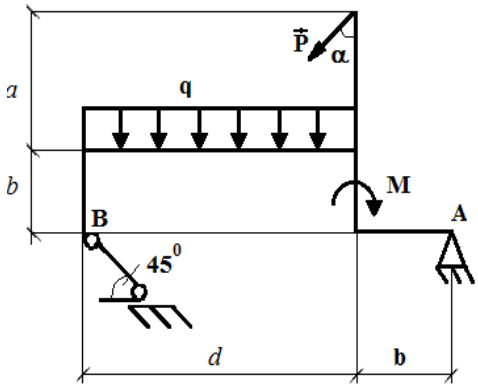

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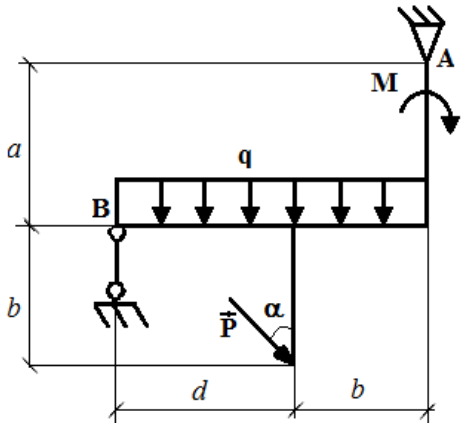

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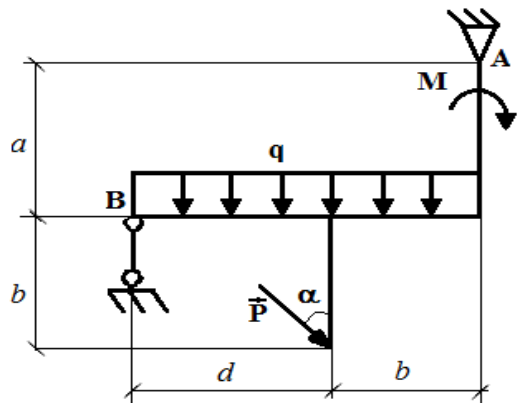

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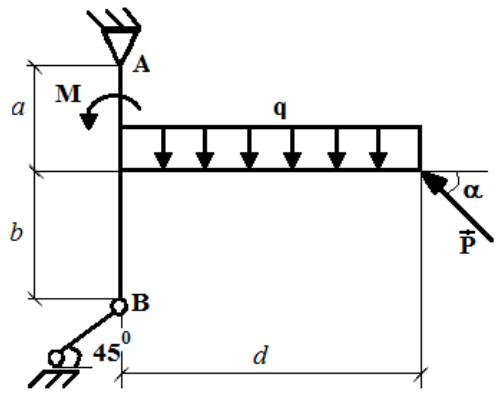

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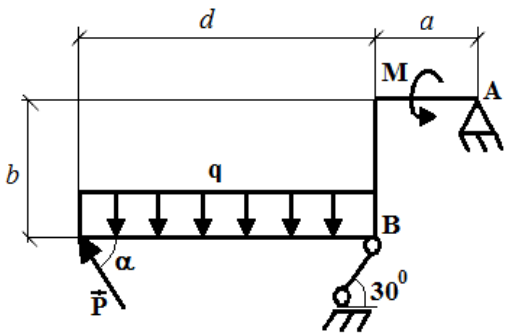

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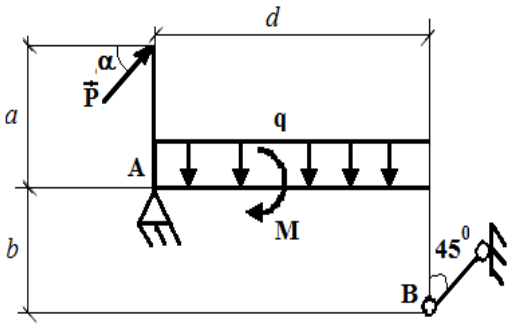

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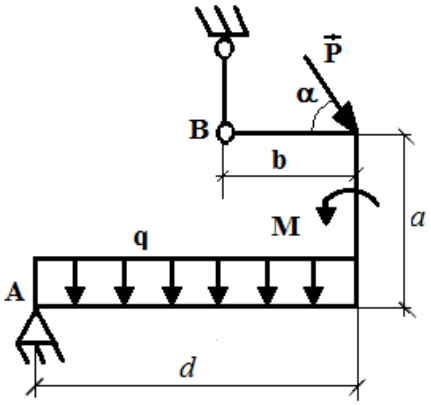

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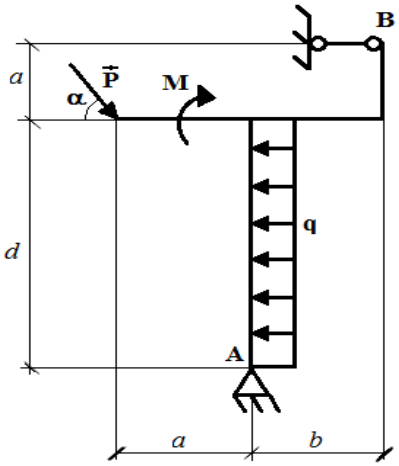

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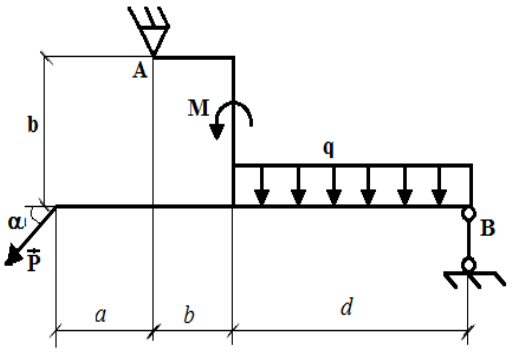

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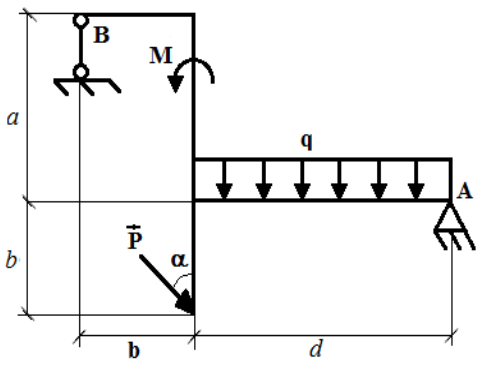

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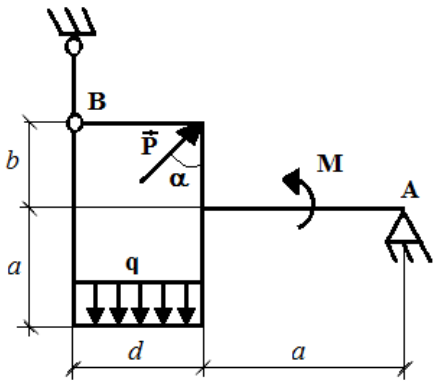

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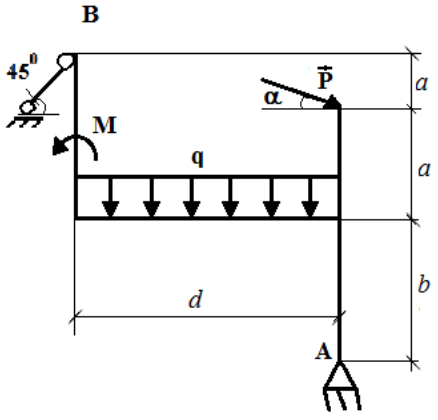

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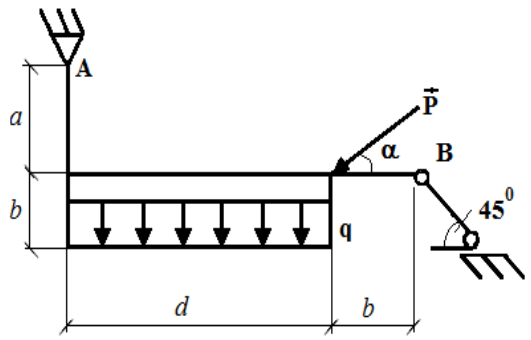

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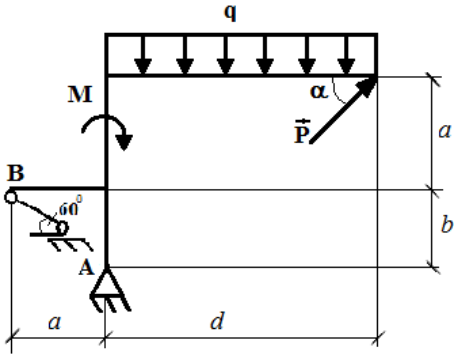

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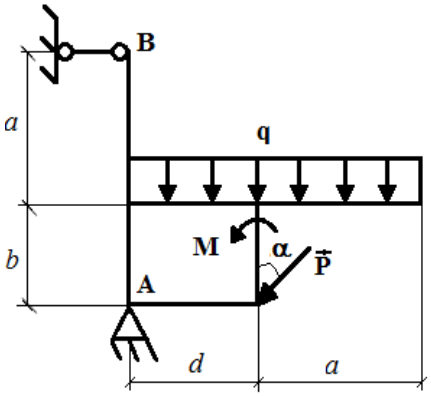

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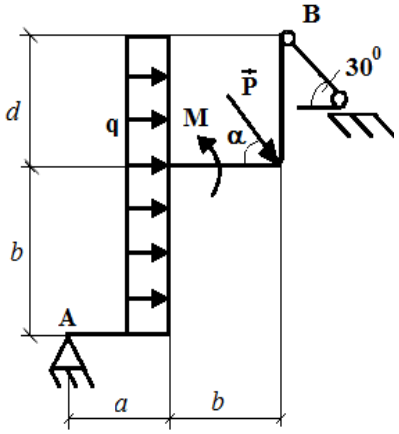

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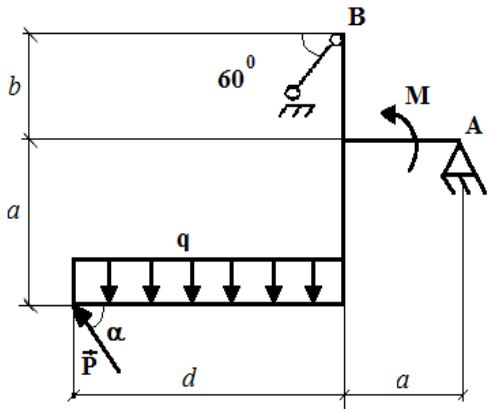

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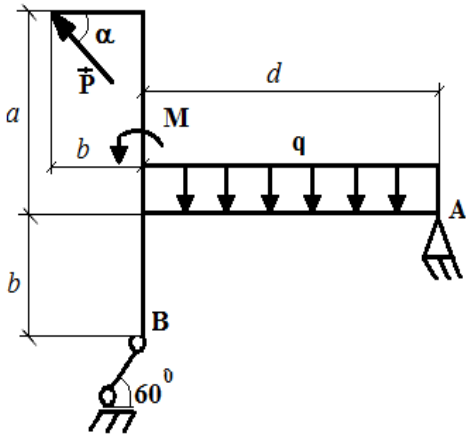

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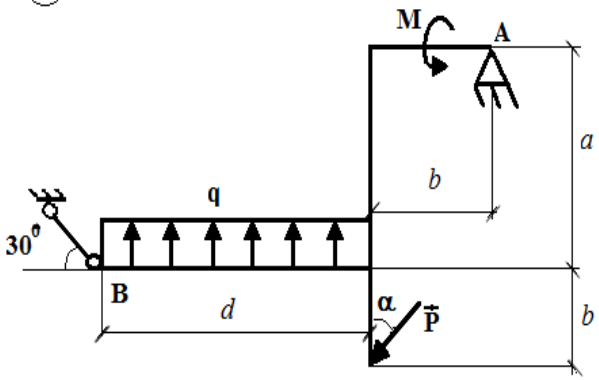

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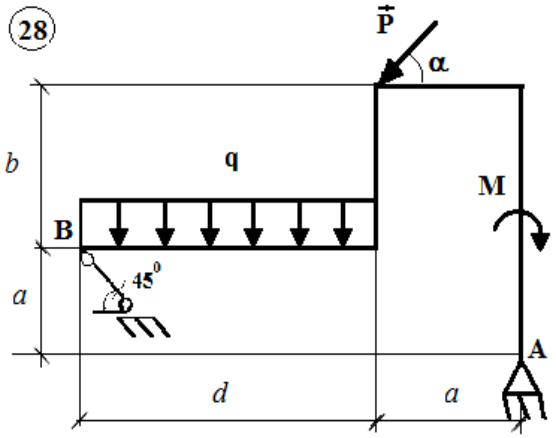

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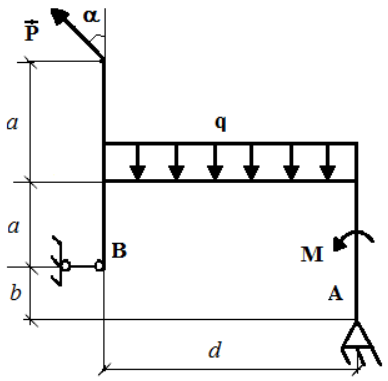

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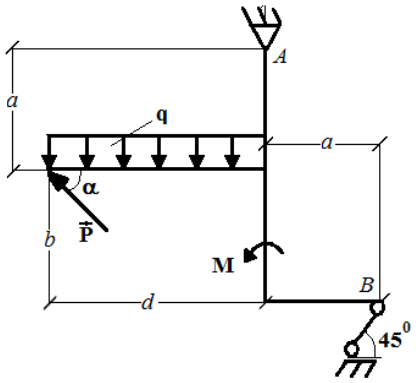
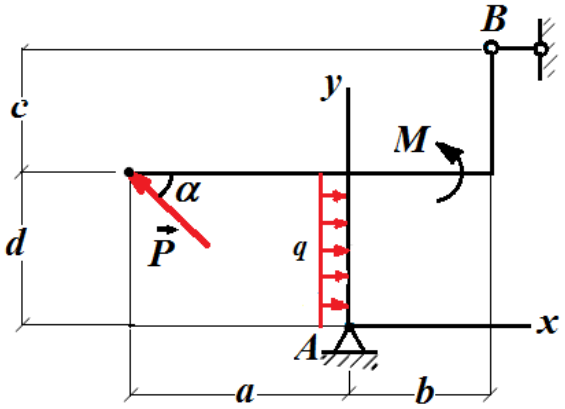


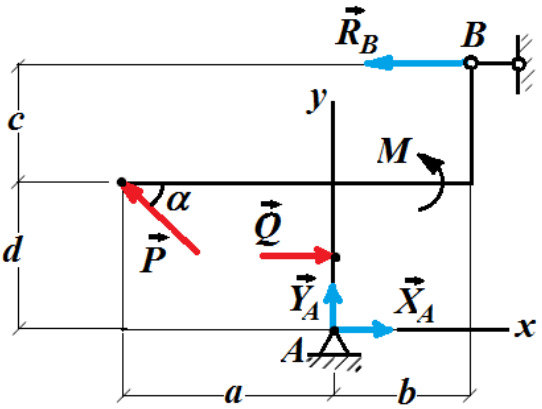
2-jadval

Variant №	P_0, kN	$q, kN/m$	M, kNm	$a.m$	b, m	$\alpha, grad$	R_{Bm}, kN
1	12	4	5	2	1	30	10
2	10	8	50	2	1,5	30	20
3	40	4	50	2	4	45	20
4	10	10	60	2	2	60	20
5	15	12	20	2,1	1,8	45	25
6	20	4	10	1	2,5	60	10
7	20	4	12	2	1,5	60	12
8	20	7	25	2	1,5	60	14
9	10	5	15	3	2	30	6
10	20	10	20	1,5	0,5	30	15
11	22	6	18	0,2	2	45	10
12	35	8	19	0,2	2	30	12
13	25	10	15	0,6	5	30	18
14	40	12	5	0,9	3	60	11
15	22	8	15	0,9	0,2	30	20
16	21	4	2	1,1	3	60	10
17	51	8	40	1,1	1	60	10
18	21	4	20	2,1	1	30	10
19	20	4	20	2	1,1	30	10
20	60	8	10	0,5	0,6	60	13
21	30	8	13	0,6	0,7	60	15
22	10	8	13	1,6	1,5	60	15
23	40	6	33	0,7	2,5	30	14
24	10	6	27	1,2	1	60	13
25	20	10	20	0,2	1	60	12
26	5,5	3	3	7	0,5	60	10
27	40	5	4	2,8	1,7	45	16
28	15	2	60	3	0,5	30	10
29	40	5	40	0,5	1,5	60	20
30	40	18	40	0,5	1,5	45	10

Topshiriqni bajarish namunasi. 1-rasmda ko‘rsatilgan konstruksiya uchun quyidagilar berilgan: $P_0 = 20kN, q = 8\,kN/m, M = 5kNm, \alpha_0 = 60^0, a = 3m,$ $b = 2m, c = 2m, R_{Bm} = 10kN.$



1-rasm



2-rasm

1) d masofa qanday bo‘lganda R_B reaksiya kuchi minimum bo‘lishi aniqlansin;

2) P ning o‘zgarish chegarasi topilsin;

3) α ning o‘zgarish chegarasi aniqlansin.

Yechish. Masalani hal etish uchun 1-rasmdagi konstruksiya muvozanatini tekshiramiz. Sanoq sistemasini 2-rasmdagidek tanlaymiz. Konstruksiyaga qo‘yilgan bog‘lanishlarni $\vec{X}_A, \vec{Y}_A, \vec{R}_B$ reaksiya kuchlari bilan almashtiramiz. Intensivligi q bo‘lgan yukning teng ta’sir etuvchisi $Q = q \cdot d$ bo‘lib, u A nuqtadan $0{,}5d$ masofada yuqoriga qo‘yilgan va Ax o‘qiga parallel yo‘nalgan (2-rasm).

1) $\boldsymbol{R_B}$ **reaksiya kuchining minimum qiymatini aniqlash.** Buning uchun konstruksiyaga qo‘yilgan kuchlarni muvozanat tenglamalarini tuzamiz:

$$\sum m_B\left(\vec{F}_v\right) = 0; \quad Pd\cos\alpha - Pa\sin\alpha - q(d^2/2) + R_B(c+d) + M = 0, \qquad (1)$$

$$\sum F_{vx} = 0; \qquad -P\cos\alpha + X_A - R_B = 0, \qquad (2)$$

$$\sum F_{vy} = 0; \qquad P\sin\alpha + Y_A = 0. \qquad (3)$$

(1) dan:

$$R_B = \frac{-Pd\cos\alpha + Pa\sin\alpha + q(d^2/2) - M}{c+d}.$$

Son qiymatlarni qo‘ysak:

$$R_B = (1/d + 2) \cdot (4d^2 - 10d + 46{,}96) \quad (4)$$

R_B ni minimum qiymatini aniqlash uchun (4) dan d bo‘yicha birinchi va ikkinchi tartibli hosila olamiz:

$$(R_B)' = \frac{(8d-10)(d+2)-4d^2+10d-46{,}96}{(d+2)^2} = \frac{4d^2+16d-66{,}96}{(d+2)^2} \quad (5)$$

$$(R_B)'' = \frac{(8d+16)(d+2)^2-2(d+2)\left(4d^2+10d-46{,}96\right)}{(d+2)^4} =$$

$$= (165{,}92d + 331{,}84) \cdot (d+2)^3 \quad (6)$$

(5) ni nolga tenglashtirib kritik nuqtalarni topamiz:

$$4d^2 + 16d - 66{,}96 = 0;\ d_1 = 2{,}554m, d_2 = -6{,}554m.$$

bu yerda d ning manfiy qiymatini hisobga olib bo‘lmaydi.

d_1 ni (6) ga qo‘ysak, $(R_B)'' > 0$ kelib chiqadi. Shuning uchun $d = 2{,}554m$ bo‘lganda R_B minimum bo‘ladi, ya’ni $R_B = 10{,}443kN$. Buni (2) va (3) ga qo‘yib, $X_A = 0{,}005kN, Y_A = -17{,}321kN$ larni hosil qilamiz.

2) ***P* ning o‘zgarish chegarasini topish**, ya’ni $\alpha = \alpha_0, d = d_1, P \geq 0$ bo‘lganda R_B reaksiya kuchining $R_{Bm} = 10kN$dan oshib ketmasligini aniqlash.

$R_B \leq R_{Bm}$ bo‘lishi uchun quyidagi shart bajarilishi kerak:

$$|R_B| = |(-Pd\cos\alpha) + Pa\sin\alpha + q(d^2/2) - M)/(c+d)| \leq R_{Bm}. \quad (7)$$

(7) tengsizlikni yechamiz:

$$(-Pd\cos\alpha) + Pa\sin\alpha + q(d^2/2) - M)/(c+d) \leq R_{Bm},$$
$$-(-Pd\cos\alpha) + Pa\sin\alpha + q(d^2/2) - M)/(c+d) \leq R_{Bm}.$$

Masala shartidagi berilganlarni hisobga olsak:

$$-P \cdot 0{,}5 \cdot 2{,}554 + P \cdot 0{,}866 \cdot 3 + 0{,}5 \cdot 8 \cdot 2{,}554^2 - 5 \leq 45{,}54,$$

$$P \cdot 0{,}5 \cdot 2{,}554 - P \cdot 0{,}866 \cdot 3 - 0{,}5 \cdot 8 \cdot 2{,}554^2 + 5 \leq 45{,}54.$$

bu yerdan $P \leq 18{,}507kN, P \geq -50{,}44kN$ kelib chiqadi. Lekin masala shartiga asosan $P > 0$.

Demak, $0 \leq P \leq 18{,}507$ bo‘lganda $R_B \leq R_{Bm}$ bajariladi.

3) ***α* ning o‘zgarish chegarasini aniqlash.**

Masala shartiga ko‘ra:

$$P = P_0,\ d = d_1, 0 \leq \alpha \leq \pi\ ;$$

$$R_B \le R_{Bm}.$$

(7) dan foydalanib α ni aniqlaymiz. Son qiymatlarni (7) ga qo‘ysak:

$$-0{,}561 \cdot 20\cos\alpha + 0{,}659 \cdot 20\sin\alpha + 4{,}631 \le 10,$$

$$0{,}561 \cdot 20\cos\alpha - 0{,}659 \cdot 20\sin\alpha - 4{,}631 \le 10.$$

bu yerdan

$$-0{,}561\cos\alpha + 0{,}659\sin\alpha \le 0{,}268,$$

$$0{,}561\cos\alpha + 0{,}659\sin\alpha \le 0{,}7315 \qquad (8)$$

kelib chiqadi.

(8) sistemani yechish uchun quyidagi belgilashni kiritamiz:

$$\begin{cases} A\sin\beta = -0{,}561 \\ A\cos\beta = 0{,}659 \end{cases} \qquad (9)$$

(9) ni yechsak:

$$A = \sqrt{0{,}659^2 + 0{,}561^2} = 0{,}865,$$

$$\beta = arc\tan(-0{,}561/0{,}659) = -40^0 24'.$$

(9) ni (8) ga qo‘yamiz:

$$A\sin(\alpha + \beta) \le 0{,}268,$$

$$A\cos(\alpha + \beta) \le -0{,}7315$$

yoki

$$0{,}865\sin(\alpha - 40^0 24') \le 0{,}268,$$

$$0{,}865\sin(\alpha - 40^0 24') \le -0{,}7315$$

bu yerdan $\alpha_1 \le 58^0 35', \alpha_2 \le -17^0 21'$ kelib chiqadi.

Demak, $\alpha \ge 0,$ ekanligini hisobga olsak, $0 \le \alpha \le 58^0 35'$ bo‘lib,

$$R_B \le R_{Bm} = 10kN$$

bo‘ladi.

Endi masalani *Mathcad* matematik faket yordamida qanday yechilishini ko‘rib chiqamiz.

***Mathcad* matematik paket yordamida yechilishi**.

1-konstruksiya uchun berilganlarni yozib olamiz:

$P0 := 20 \quad q := 8 \quad M := 5 \quad \alpha 0 := \frac{\pi}{3} \quad a := 3 \quad b := 2 \quad c := 2 \quad RbMax := 10$

(1) tenglamadagi *Rb* (R_B) ni *d* parametr orqali ifodasi quyidagicha yoziladi:

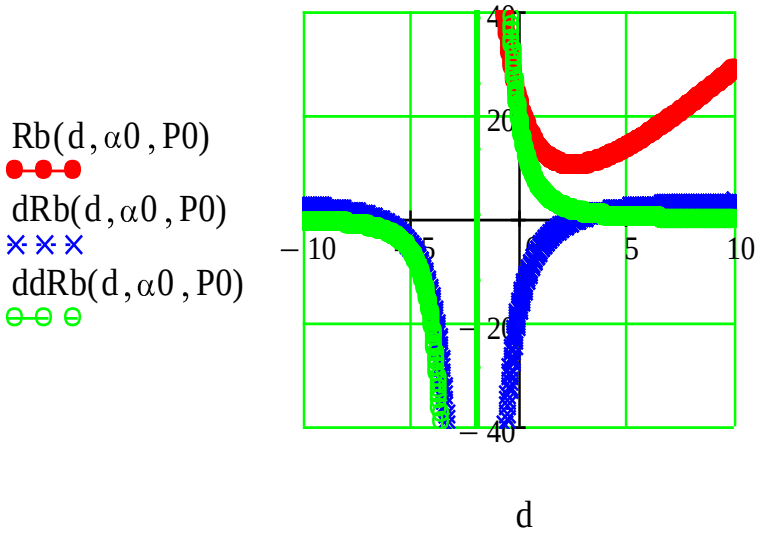
$$Rb(d,\alpha,P) := \frac{P\cdot(a\cdot\sin(\alpha) - d\cdot\cos(\alpha)) + q\cdot 0.5\cdot d^2 - M}{c + d}$$

Bu funksiyani ekstremumini aniqlash uchun d bo‘yicha birinchi va ikkinchi tartibli hosilalarni hisoblaymiz:

$$dRb(x,\alpha,P) := \frac{d}{dx} Rb(x,\alpha,P)$$

$$ddRb(x,\alpha,P) := \frac{d}{dx} dRb(x,\alpha,P)$$

Rb(d), dRb(d) va *ddRb(d*) lar grafigi quyidagicha:



Grafikdan ko‘rinib turibdiki d ning biror qiymatida *Rb* minimum qiymtga erishadi. d parametrning ana shu qiymatini aniqlaymiz:

$d\_min := root(dRb(d, \alpha 0, P0), d, 1, 4) \qquad d\_min = 2.554$

d ga quyidagi aniqlikni kiritamiz:

$$d0 := d\_min$$

(1) sistemani yechish uchun matrisa usulidan foydalanamiz:

$$A := \begin{pmatrix} c + d0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B := \begin{pmatrix} P0 \cdot \sin(\alpha 0)a - P0 \cdot \cos(\alpha 0) \cdot d0 + 0.5 \cdot q \cdot d0^2 - M \\ P0 \cdot \cos(\alpha 0) - q \cdot d0 \\ -P0 \cdot \sin(\alpha 0) \end{pmatrix}$$

$$R := A^{-1} \cdot B$$

$$R^T \text{ simplify} \rightarrow \begin{pmatrix} 10.433286807987295574 & 6.9995783297404587882e\text{-}19 & -10 \cdot \sqrt{3} \end{pmatrix}$$

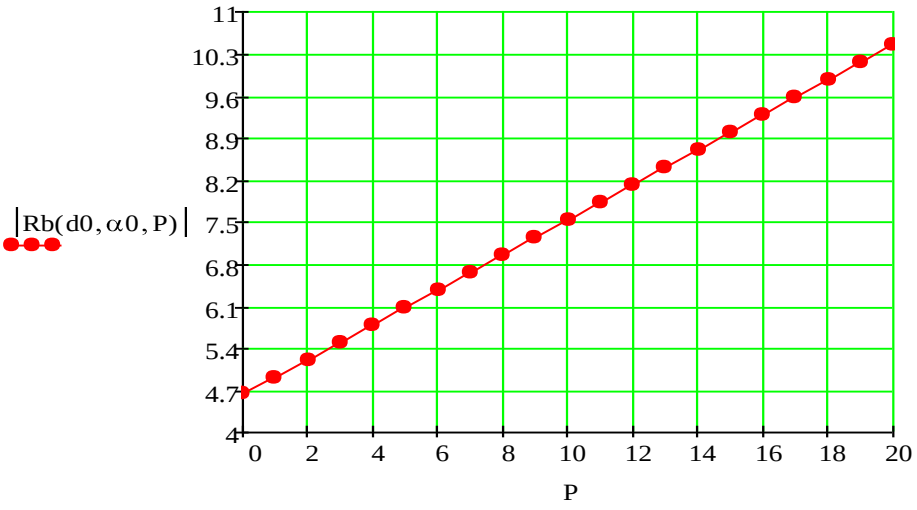
$$R^T = \begin{pmatrix} 10.433 & 8.18 \times 10^{-10} & -17.321 \end{pmatrix} \qquad R = \begin{pmatrix} 10.433 \\ 8.18 \times 10^{-10} \\ -17.321 \end{pmatrix}$$

Bu yerda R^T=[Rb, Xa, Ya] ($R^T = [R_B, X_A, Y_A]$) taynch reaksiyalarining matrisasi; *T*- transponirlangan matrisa. Demak:

$$Rb = 10.43 \qquad Xa = 0 \qquad Ya = -17.321 \qquad P := 0 .. 18.506$$

Endi $|Rb| \le Rbm$ ($|R_B| \le R_{Bm}$) qanoatlantiradigan P ning qiymatini aniqlaymiz va *Rb(P)* funksiya grafigini chizamiz.

$$P := 0 \ldots 20$$



$$\text{root}(Rb(d0, \alpha 0, P) - RbMax, P, 15, 20) = 18.506$$

$$R^T = \begin{pmatrix} 10.433 & 8.18 \times 10^{-10} & -17.321 \end{pmatrix}$$

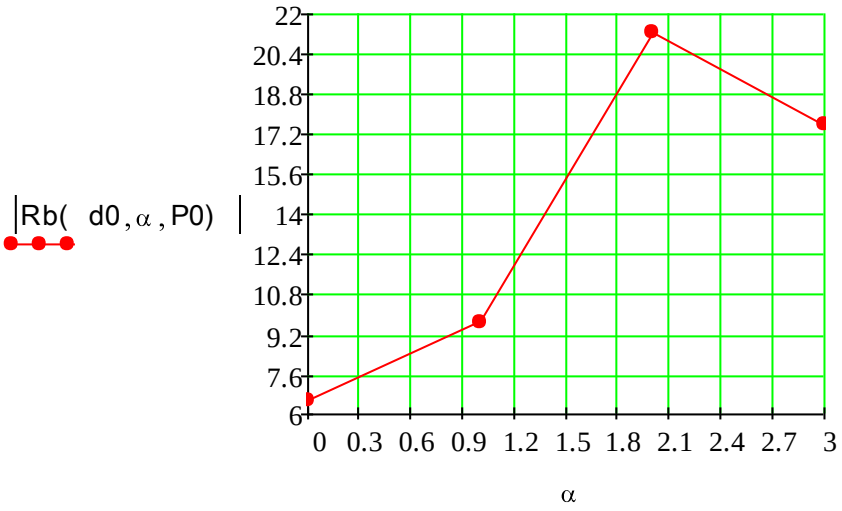
$$R^T \text{ simplify} \rightarrow \begin{pmatrix} 10.433286807987295574 & 6.9995783297404587882e\text{-}19 & -10 \cdot \sqrt{3} \end{pmatrix}$$

$$Rb = 10.433 \qquad Xa = 0 \qquad Ya = -17.321 \qquad P := 0 .. 18.506 \qquad \alpha := 0 .. 3.14$$

$\alpha 1 := 0$ $\alpha 2 := \text{root}(Rb(d0, \alpha, P0) - RbMax, \alpha, 0.9, 1.2)$ $\alpha := 0..1.021$

$\alpha 1 = 0$ $\alpha 2 = 1.021$ $\alpha 2 := \frac{\alpha 2 \cdot 180}{\pi}$ $\alpha 2 = 58.484$

Shunday qilib, $|Rb| \leq Rbm$ bajarilganda $\alpha \in [0, 58{,}484^0]$ oraliqda bo‘ladi.



Mathcad matematik paket yordamida bundan ham murakkabroq masalalarni yechish mumkin.

1 – masala. *Rb=RbMin, d=2m* bo`lganda *P=Popt* va $\alpha = \alpha\ opt$; $5 \leq P \leq 30kN$ va $0 \leq \alpha \leq 2\pi$ da P bilan Rb qanday o‘zgarishi aniqlansin. Quyidagilar berilgan:

$a := 3$ $b := 2$ $q := 8$ $M := 5$ $d := 2$ $0 \leq P \leq 30$ $0 \leq \alpha \leq 2\pi$

Rb reaksiya kuchining moduli quyidagicha:

$$Rb(\alpha, P) := \left| \frac{1}{c+d} \cdot \left(M - 0.5 \cdot q \cdot d^2 + P \cdot d \cdot \cos(\alpha) - P \cdot a \cdot \sin(\alpha) \right) \right|$$

Yechish:

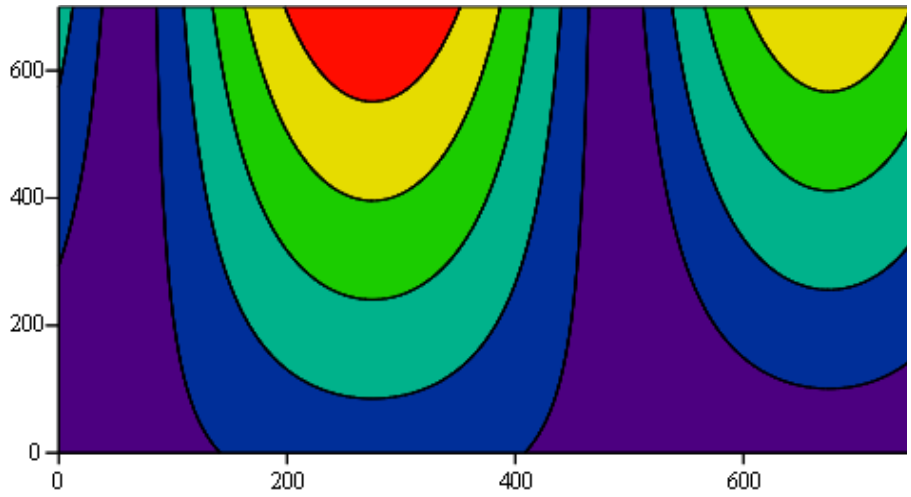
$n := 800$ $m := 700$ $\Delta\alpha := \frac{2 \cdot \pi}{n}$ $\Delta P := \frac{25}{m}$

$i := 0..n$ $j := 0..m$ $\Delta\alpha = 7.854 \times 10^{-3}$ $\Delta P = 0.036$

$H(i,j) = Rb(\alpha \cdot i, P \cdot j)$ belgilashni kiritib (α, P) maydonida *Rb* grafigini yasaymiz .

Bu yerda:

$$H_{i,j} := \left| \frac{1}{c+d} \cdot \left[M - 0.5 \cdot q \cdot d^2 + (5 + \Delta P \cdot j) \cdot (d \cdot \cos(\Delta\alpha \cdot i) - a \cdot \sin(\Delta\alpha \cdot i)) \right] \right|$$



H

$$L := \left| \begin{array}{l} ker \leftarrow F(0,0) \\ \text{for } i \in 0..n \\ \quad \text{for } j \in 0..m \\ \quad\quad Bob \leftarrow F(i,j) \\ \quad \text{if } Bob \le ker \\ \quad\quad Z \leftarrow 1000 \cdot i + j \\ \quad\quad ker \leftarrow Bob \\ Z \end{array} \right.$$

$L = 6.27 \times 10^4 \quad im := floor\left(\frac{L}{1000}\right)$

$jm := L - 1000 \cdot im$

$im = 62 \quad jm = 700$

Parametrlar optimal qiymati bunday bo‘ladi:

$\alpha opt := \Delta\alpha \cdot im \quad Popt := 5 + \Delta P \cdot jm$

$\alpha opt = 0.487$

$\frac{\alpha opt}{deg} = 27.9 \quad Popt = 30 \quad F(im, jm) = 0.022$

Bu yerda im va jm L=1000im+jm formula yordamida F(i,j) funksiyani minimumini ta’minlovchi miqdordan iborat.

Bu holda

F(im, jm)=0,022; Rb=0,022kN

bo‘ladi.

2 − masala. $\alpha = \pi/3, P = Popt, d = dopt$ bo‘lganda Ra=RaMin. P kuch $5 \le P \le 30kN$ oraliqda, d parametr $1 \le d \le 3m$. P va d o‘zgarishi bilan Ra ning qanday o‘zgarishi aniqlansin.

Yechish. (1) muvozanat tenglamalardan Xa (X_A) ва Ya (Y_A) ni topib olamiz:

$$Xa = \frac{-1}{c+d}\cdot[M + q\cdot d\cdot(\ c + 0.5\cdot d) \ - P\cdot c\cdot\cos(\ \alpha) \ - P\cdot a\cdot\sin(\ \alpha)\ ]$$

$$Ya = -P\cdot\sin(\ \alpha)$$

Bu yerdan:

$$Ra = \sqrt{\left[\frac{1}{c+d}\cdot[M + q\cdot d\cdot(\ c + 0.5\cdot d) \ - P\cdot c\cdot\cos(\ \alpha) \ - P\cdot a\cdot\sin(\ \alpha)\ ]\right]^2 + (\ P\cdot\sin(\ \alpha)\ )^2}$$

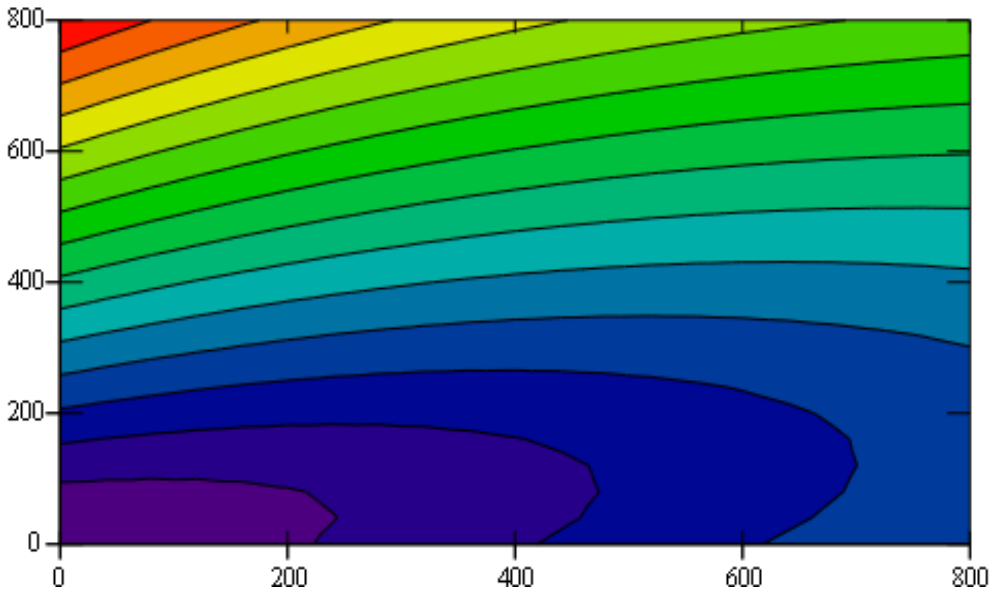
$$n := 800 \qquad m := 700 \qquad \Delta d := \frac{2}{n} \qquad \Delta P := \frac{25}{m} \qquad i := 0..n \qquad j := 0..m$$

Yuqoridagilarni e’tiborga olib, reaksiyalarni hisoblaymiz:

$$X(\ i,j)\ := \frac{M + q\cdot(\ \Delta d\cdot i + 1)\cdot[c + 0.5\cdot(\ \Delta d\cdot i + 1)\ ] - (\ 5 + \Delta P\cdot j)\cdot(\ c\cdot\cos(\ \alpha) + a\cdot\sin(\ \alpha)\ )}{c + (\ \Delta d\cdot i + 1)}$$

$$Y(\ i,j)\ := (\ 5 + \Delta P\cdot j)\cdot\sin(\ \alpha)\ , \ S(\ i,j)\ := \sqrt{X(\ i,j)^{\ 2} + Y(\ i,j)^{\ 2}}$$

(d, P) maydonida Ra ning konturli grafigini chizamiz: Absissa o‘qi bo‘yicha *d* ni, ordinata o‘qida esa *P* ni qo‘yamiz.



S

F(i,j) funksiya minumini topish uchun quyidagi programmani tuzamiz:

$$F(\ i,j)\ := \sqrt{X(\ i,j)^{\ 2} + Y(\ i,j)^{\ 2}}$$

$$L := \left| \begin{array}{l} ker \leftarrow F(0,0) \\ \text{for } i \in 0..n \\ \quad \left| \begin{array}{l} \text{for } j \in 0..m \\ \quad Bob \leftarrow F(i,j) \\ \text{if } Bob \le ker \\ \quad \left| \begin{array}{l} Z \leftarrow 1000 \cdot i + j \\ ker \leftarrow Bob \end{array} \right. \end{array} \right. \\ Z \end{array} \right. \qquad L=0$$

im va jm L=1000im+jm formula yordamida F(i,j) funksiyani minimumini ta'minlovchi miqdordan iborat.

$$im := floor\left(\frac{L}{1000}\right) \qquad jm := L - 1000 \cdot im \qquad Popt = 5 \qquad im = 0 \qquad jm = 0$$

Parametrlarning optimal qiymati quyidagicha:

$$dopt := 1 + \Delta d \cdot im \qquad Popt := 5 + \Delta P \cdot jm \qquad dopt = 1 \qquad Popt = 5$$

Yuqoridagilarga asosan Ra minimum qiymati bunday bo‘ladi:

$$F(im,jm) = 4.92 \qquad Ra := F(im,jm) \qquad Ra = 4.92 \ kN$$

Shu bilan masala to‘la - to‘kis hal bo‘ldi.

2-topshiriq (kinematika)

Moddiy nuqta harakati koordinata usulida berilganda tezlik va tezlanishni aniqlash

Moddiy nuqta harakati $[t_0, t_1]$ oraliqda

$$x = x(t), y = y(t)$$

tenglamalar bilan berilgan.

Nuqtaning trayektoriya tenglamasi tuzilsin, *t=1* sekund bo‘lganda trayektoriyadagi holati, tezligi, to‘la,urinma va normal tezlanishi hamda egrilik radiusi aniqlansin va ular yo`nalishi trayektoriyada ko`rsatilsin (3-jadval).

3-jadval

№	$x = x(t)$, m	$y = y(t)$, m	t_0 , s	t_2 , s	t_1 , s
1	$2\,e^{-t} + 2$	e^{-t}	0	2	1
2	$2\sin(\pi t) + 1$	$\sin(2\pi t) - 2$	0	1,2	1/3
3	$3\,tg(\pi t) + 2$	$ctg(\pi t) + 3$	0,1	1,5	1/3
4	$4\sin^2 t - 2$	$2\cos t + 1$	0	2	π/6
5	$\sin(\pi t) + \cos(\pi t)$	$\sin(\pi t) - \cos(\pi t)$	0	3	1/6
6	$3\sin t$	$2\cos(2t)$	0	2,5	π/6
7	$e^1 + e^{-1}$	$e^1 - e^{-1}$	0	0,5	0,5
8	$2\cos(2t)$	$3\sin^2 t + 1$	0	2	π/6
9	$(1/\cos t) + \cos t$	$(1/\cos t) - \cos t$	0,1	1,5	π/6
10	$2\cos^2 t$	$2\sin(2t) - 1$	0	2,5	π/6
11	$4/\cos^2 t$	$2\,tg^2 t$	0,1	2	π/3
12	$4\cos^2 t$	$\cos(2t) + 2$	0	2,4	π/6
13	$ctg^2 t$	$2/\sin^2 t$	0,1	1,5	π/6
14	$3\cos^2(\pi t)$	$2\cos^2(2\pi t)$	0	3	1/6
15	$3\sin(\pi t/2)$	$6\sin^2(\pi t/4)$	0	1,2	1/3
16	$2\sin^2 t$	$3\cos^2(2t)$	0	2,2	π/3
17	$2\,tg^2 t + 3$	$4\,tg^2 t - 2$	0,1	1,6	π/4
18	$4\cos^2(\pi t/2)$	$2\sin(\pi t)$	0	1,8	1/6
19	$2\sin^2 t$	$\sin^2(2t)$	0	1,5	π/6
20	$4t\,e^{-1}$	$2e^1/t$	0,1	2	1
21	$2\,t^2$	$t^4 + 2\,t^2 + 1$	0	2	1
22	$\sin(\pi t^2/3) + 1$	$\sin^2(\pi t^2/3) - 1$	0	2,5	1
23	$4\cos(\pi t^2/3)$	$2/\cos(\pi t^2/3)$	0	3	1
24	$t - 1$	$t^2 - t - 1$	0	4	2
25	$2\,e^{-1} - 2$	$(4/e^{2t}) + 1$	0	1,5	0,5
26	$\cos(\pi t^2/6) - 1$	$2\cos^2(\pi t^2/6) + 1$	0	2	1
27	$2\,tg(\pi t/3)$	$3 - tg^2(\pi t/3)$	0,1	2	1
28	$\ln t + 1$	$2\ln(2t + 3)$	0,1	1,5	e^{-1}
29	$\sqrt{t^2 + 1}$	$2\,t^2 + 3$	0	2,4	1
30	$\ln(t^2) + 1$	$\ln(t + 2) - 2$	0,1	2	e^{-1}

Topshiriqni bajarish namunasi

Masala. Moddiy nuqta $[0{,}1;\ 2]$ oraliqda

$$x = \sqrt{t^2 + t + 1}\ m,\ \ y = \sqrt{2t^2 + 2t}\ m; \qquad (10)$$

tenglamalarga ko`ra harakat qiladi.

Nuqtaning trayektoriya tenglamasi tuzilsin, $t_0 = 0{,}1; t_1 = 0{,}2;\ t_2 =$ 2 sekund bo‘lganda trayektoriyadagi holati, tezligi, to‘la, urinma va normal tezlanishi hamda egrilik radiusi aniqlansin va ular yo`nalishi trayektoriyada ko`rsatilsin.

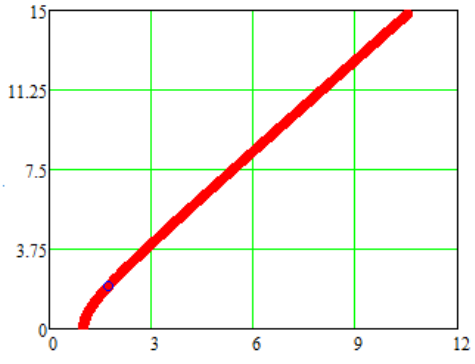
Yechish. Trayektoriya tenglamasini topish uchun (1) dan vaqtni yo‘qatamiz. Buning uchun (1) ni ikkala tomonini kvadratga ko’tarib, hosil bo’lgan ifodaning birinchisini 2 ga ko’paytirib, so’ngra ikkinchisini ayiramiz:

$$\begin{cases} 2x^2 = 2t^2 + 2t + 2 \\ y^2 = 2t^2 + 2t \end{cases}$$
$$2x^2 - y^2 = 2$$

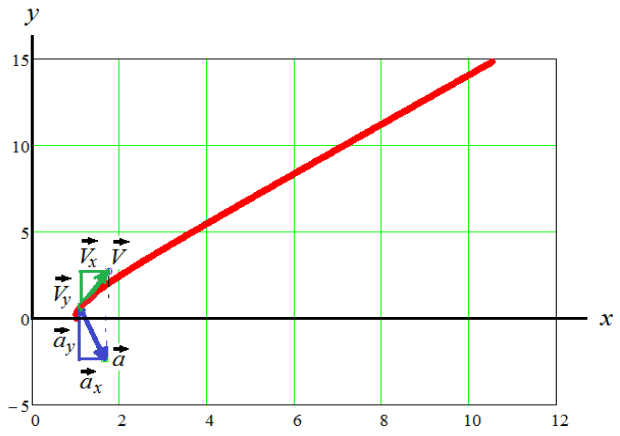
Bu yerdan:

$$y = \sqrt{2x^2 - 2} \qquad (11)$$

(2) nuqta trayektoriyasini tenglamasidan iborat. Uni grafigi 3-rasmdagidek bo’ladi.



3-rasm



4-rasm

Tezlik va tezlanish vektorlarini mos ravishda quyidagi formulalardan aniqlaymiz:

$$\vec{V} = \vec{\imath} \cdot V_x + \vec{\jmath} \cdot V_y \qquad (12)$$

$$\vec{a} = \vec{\imath} \cdot a_x + \vec{j} \cdot a_y \qquad (13)$$

bu yerda $\vec{\imath}, \vec{j} - x, y$ o`qlarining birlik yo`naltiruvchi vektorlari; V_x , V_y tezlik vektorining, a_x, a_y esa tezlanish vektorining koordinata o`qlaridagi proyeksiyalaridan iborat.

Nuqta tezligini aniqlash uchun (10) dan vaqt bo`yicha hosila olamiz:

$$V_x = \frac{2t+1}{2\sqrt{t^2+t+1}}, \qquad V_y = \frac{2t+1}{\sqrt{2t^2+2t}}. \qquad (14)$$

Nuqta tezlanishini aniqlash uchun (14) dan vaqt bo`yicha hosila olamiz:

$$a_x = \frac{4\sqrt{t^2+t+1} - \frac{(2t+1)^2}{\sqrt{t^2+t+1}}}{4(t^2+t+1)}, a_y = \frac{2\sqrt{2t^2+2t} - \frac{(2t+1)^2}{\sqrt{2t^2+2t}}}{2t^2+2t}. \quad (15)$$

Tezlik va tezlanish moduli quyidagi formulalardan aniqlanadi:

$$V = \sqrt{{V_x}^2 + {V_y}^2}, a = \sqrt{{a_x}^2 + {a_y}^2} \qquad (16)$$

Nuqta tezlanishining tezlik yo'nalishidagi proyeksiyasi (urinma tezlanish) quyidagicha topiladi:

$$a_V = \frac{dV}{dt}, a_\tau = |a_V| = \left|\frac{dV}{dt}\right|$$

yoki

$$a_\tau = \frac{V_x a_x + V_y a_y}{V} \qquad (17)$$

Izoh. Agar tezlikni berilgan vaqtdagi moduli nolga teng bo'lib qolsa, (17) noaniq bo'ladi $(0/0)$, urinma tezlanishni aniqlab bo'lmaydi. Bu holda u bunday topiladi:

$$a_\tau = \frac{x + y(dy/dx)}{\sqrt{1 + (dy/dx)^2}} \qquad (18)$$

dV/dt hosila oldidagi ishora musbat (+) bo'lsa, nuqta harakati tezlanuvchi bo'lib, $\vec{a}_V$ bilan $\vec{V}$ yo'nalishi bir xil bo'ladi; manfiy ($-$) bo'lganda harakat sekinlanuvchi bo'lib, $\vec{a}_\tau$ bilan $\vec{V}$ yo'nalishi qarama-qarshi bo'ladi.

Normal tezlanish quyidagi formula bilan ifodalanadi:

$$a_n = \sqrt{a^2 - a_\tau^2} \tag{19}$$

Chiziqning egrilik radiusi quyidagi formuladan topiladi:

$$\rho = V^2/a_n \tag{20}$$

$t_1 = 0{,}2$ sekundni (1) - (11) ga qo‘ysak,

$$x = 1{,}114m, \qquad y = 0{,}693m;$$

$$V_x = 0{,}629\ m/s\,, V_y = 2{,}021\ m/s\,, V = 2{,}116\ m/s\,;$$

$$a_x = 0{,}543\ m/s^2\,, a_y = -3{,}007\ m/s^2\,, a = 3{,}056\ m/s^2\,;$$

$$a_\tau = -2{,}71\,m/s^2\,, a_n = 1{,}412\ \ m/s^2\,;\ \rho = 3{,}172\,m$$

kelib chiqadi. Berilgan vaqt uchun tezlik va tezlanish vektorlarining yo‘nalishi 4-rasmda ko‘rsatilgan.

Endi tezlik va tezlanishni qutb koordinatalarida qanday aniqlanishini ko‘rib chiqamiz. Berilgan paytda nuqta tezligi radial va ko‘dalang (transversal) tuzuvchilarining geometrik yig‘indisidan iborat: $\vec{V} = \vec{V}_r + \vec{V}_p$

bu yerda $\vec{V}_r$ $(\vec{V}_{rad})$ – radial, $\vec{V}_p$ $(\vec{V}_{tr})$ – ko‘ndalang tezlik bo‘lib, ular quyidagicha:

$$\vec{V}_r = \vec{r} \cdot \frac{\vec{V} \cdot \vec{r}}{r^2}, \qquad \vec{V}_p = \frac{\vec{r} \times \vec{V} \times \vec{r}}{r^2}\ ;\ r^2 = \sqrt{x^2 + y^2}\,. \tag{21}$$

(21) ni Dekart koordinata o‘qlariga proyeksiyalasak:

$$V_{rx} = x \cdot \frac{V_x x + V_y y}{r^2}, V_{ry} = y \cdot \frac{V_x x + V_y y}{r^2};\quad V_r = \sqrt{{V_{rx}}^2 + {V_{ry}}^2}; \tag{22}$$

$$V_{px} = \frac{-x \cdot y \cdot V_y + y^2 \cdot V_x}{r^2}, V_{py} = \frac{x^2 \cdot V_y - x \cdot y \cdot V_x}{r^2};\ V_p$$

$$= \sqrt{{V_{px}}^2 + {V_{py}}^2}. \tag{23}$$

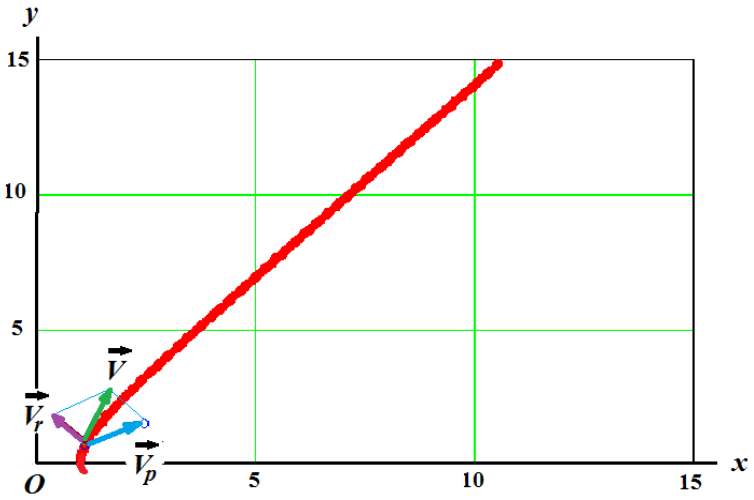
$$V = \sqrt{{V_r}^2 + {V_p}^2} \tag{24}$$

(22) - (24) ga son qiymatlarni qo‘ysak,

$$V_{rx} = 1{,}360\,m/s\,, V_{ry} = 0{,}846\,m/s\,, V_r = 1{,}601\,m/s\,; V_{px} = -0{,}731\,m/s^2,$$

$$V_{py} = 1{,}175\,m/s^2 \quad V_p = 1{,}384\,m/s^2\,;\ V = 2{,}116\,m/s^2$$

kelib chiqadi. Bu tezliklar yo‘nalishlari 5-rasmda ko‘rsatilganidek bo‘ladi.



5-rasm

Nuqta tezlanishini ham uning radial va ko‘ndalang (transversal) tuzuvchilarining geometrik yig‘indisidan iborat deb qaraymiz:

$$\vec{a} = \vec{a}_r + \vec{a}_p$$

bu yerda $\vec{a}_r$ – radial, $\vec{a}_p$ – ko‘ndalang tezlanish bo‘lib, ular quyidagicha:

$$\vec{a}_r = \vec{r}\frac{\vec{a}\vec{r}}{r^2}, \qquad \vec{a}_p = \frac{\vec{r}\times\vec{a}\times\vec{r}}{r^2} \tag{25}$$

(25) ni Dekart koordinata o‘qlariga proyeksiyalasak:

$$a_{rx} = x\cdot\frac{a_x x + a_y y}{r^2}, a_{ry} = y\cdot\frac{a_x x + a_y y}{r^2}; \quad a_r = \sqrt{{a_{rx}}^2 + {a_{ry}}^2}; \tag{26}$$

$$a_{px} = \frac{-x\cdot y\cdot a_y + y^2\cdot a_x}{r^2}, a_{py} = \frac{x^2\cdot a_y - x\cdot y\cdot a_x}{r^2}; \tag{27}$$

$$a_p = \sqrt{{a_{px}}^2 + {a_{py}}^2}; a = \sqrt{a_r^2 + a_p^2}. \tag{28}$$

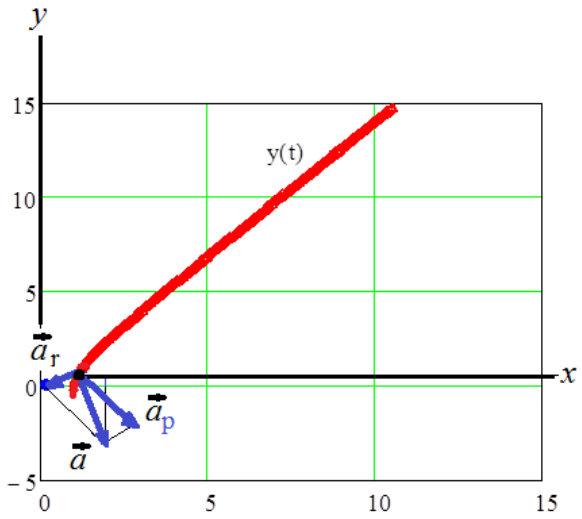
(26) - (28) ga son qiymatlarni qo‘ysak,

$$a_{rx} = -0{,}957\, m/s^2\,, a_{ry} = -0{,}596\, m/s^2\,, a_r = 1{,}127\, m/s^2\,;$$

$$a_{px} = 1{,}5\, m/s^2\,, \qquad a_{py} = -2{,}411\, m/s^2\,, \qquad a_p = 2{,}84\, m/s^2\,;$$

$$a = 3{,}058\, m/s^2$$

kelib chiqadi. Yuqoridagi tezllanishlar yo‘nalishlari 6-rasmda ko‘rsatilganidek bo‘ladi.



6-rasm

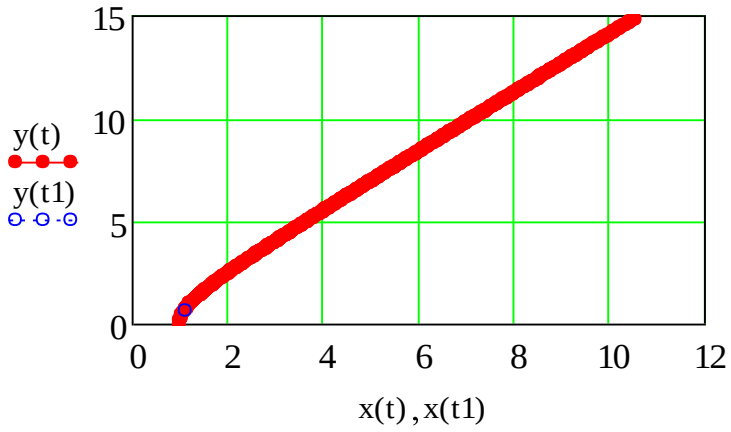
Tekshirilayotgan masalani *Mathcad* paketi yordamida yechilganini uning programma oynasidan ko‘chiramiz.

Berilgan:

$x(t) := \sqrt{t^2 + t + 1}$ $\quad y(t) := \sqrt{2 \cdot t^2 + 2 \cdot t}$ $\quad t1 := 0.2$

$x(t1) = 1.114$ $\quad y(t1) = 0.693$ $\quad r(t) := \sqrt{x(t)^2 + y(t)^2}$ $\quad r(t1) = 1.311$

Nuqta trayektoriyasini chizamiz



Tezlik va tezlanishlar quyidagicha aniqlanadi:

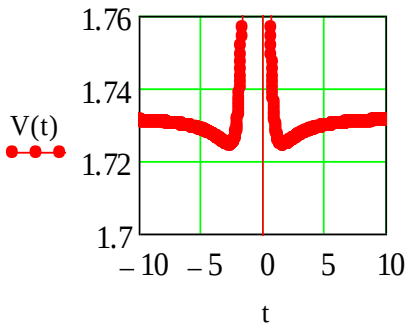
$Vx(t) := \frac{d}{dt} x(t)$ $\quad Vy(t) := \frac{d}{dt} y(t)$ $\quad V(t) := \sqrt{Vx(t)^2 + Vy(t)^2}$

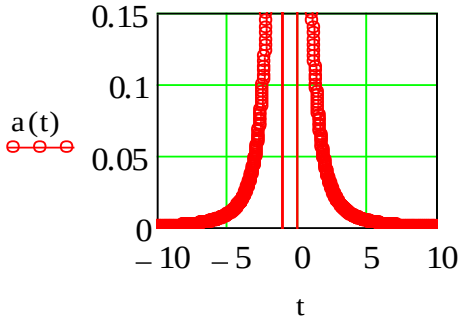
$ax(t) := \frac{d}{dt} Vx(t)$ $\quad ay(t) := \frac{d}{dt} Vy(t)$ $\quad a(t) := \sqrt{ax(t)^2 + ay(t)^2}$

$Vx(t1) = 0.629$ $Vy(t1) = 2.021$ $V(t1) = 2.116$

$ax(t1) = 0.543$ $ay(t1) = -3.007$ $a(t1) = 3.056$

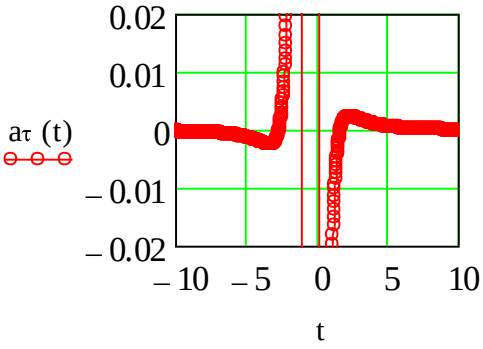
Urinma va normal tezlanishlari hamda egrilik radiusi bunday topiladi:

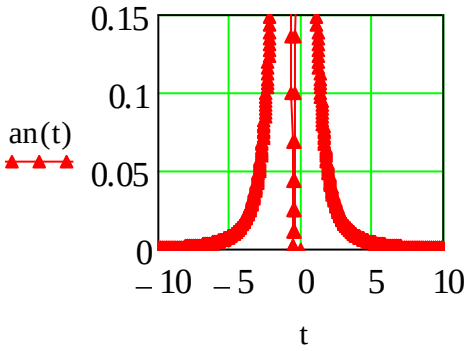




$$a_\tau(t) := \frac{Vx(t)\cdot ax(t) + Vy(t)\cdot ay(t)}{V(t)} \qquad a_\tau(t1) = -2.71 \qquad an(t) := \sqrt{a(t)^2 - a_\tau(t)^2}$$

$an(t1) = 1.412$ $\rho(t) := \frac{V(t)^2}{an(t)}$ $\rho(t1) = 3.172$





Tezlik va tezlanishni radial hamda transversal tuzuvchilaridan foydalanib hisoblaymiz:

$$Vradx(t) := x(t)\cdot\frac{(Vx(t)\cdot x(t) + Vy(t)\cdot y(t))}{r(t)^2} \qquad Vradx(t1) = 1.36$$

$$Vrady(t) := y(t)\cdot\frac{(Vx(t)\cdot x(t) + Vy(t)\cdot y(t))}{r(t)^2} \qquad Vrady(t1) = 0.846$$

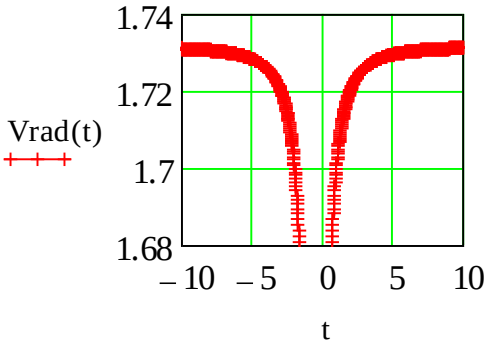
$$Vtrx(t) := \frac{-x(t)\cdot y(t)\cdot Vy(t) + y(t)^2\cdot Vx(t)}{r(t)^2} \qquad Vtrx(t1) = -0.731$$

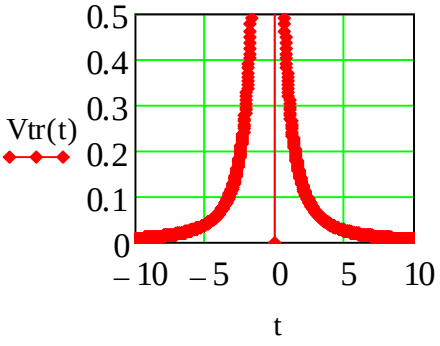
$$Vtry(t) := \frac{x(t)^2 \cdot Vy(t) - x(t) \cdot y(t) \cdot Vx(t)}{r(t)^2} \qquad Vtry(t1) = 1.175$$

$$Vrad(t) := \sqrt{Vradx(t)^2 + Vrady(t)^2} \qquad Vrad(t1) = 1.601$$

$$Vtr(t) := \sqrt{Vtrx(t)^2 + Vtry(t)^2} \qquad Vtr(t1) = 1.384$$

$$V(t) := \sqrt{Vrad(t)^2 + Vtr(t)^2} \qquad V(t1) = 2.116$$





$$aradx(t) := x(t) \cdot \frac{(ax(t) \cdot x(t) + ay(t) \cdot y(t))}{r(t)^2} \qquad aradx(t1) = -0.957$$

$$arady(t) := y(t) \cdot \frac{(ax(t) \cdot x(t) + ay(t) \cdot y(t))}{r(t)^2} \qquad arady(t1) = -0.596$$

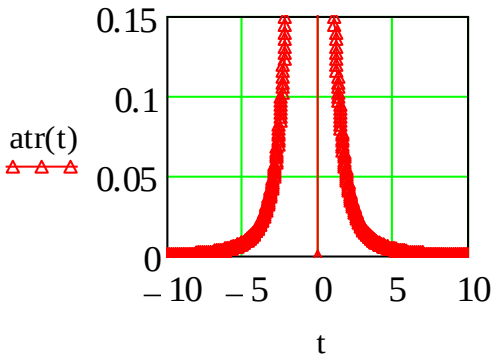
$$arad(t) := \sqrt{aradx(t)^2 + arady(t)^2} \qquad arad(t1) = 1.127$$

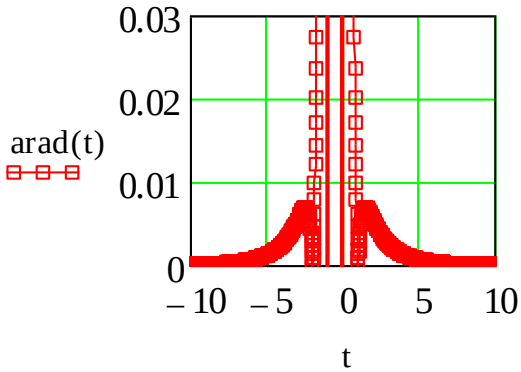
$$atrx(t) := \frac{-x(t) \cdot y(t) \cdot ay(t) + y(t)^2 \cdot ax(t)}{r(t)^2} \qquad atrx(t1) = 1.5$$

$$atry(t) := \frac{x(t)^2 \cdot ay(t) - x(t) \cdot y(t) \cdot ax(t)}{r(t)^2} \qquad atry(t1) = -2.411$$

$$atr(t) := \sqrt{atrx(t)^2 + atry(t)^2} \qquad atr(t1) = 2.84$$

$$a(t) := \sqrt{arad(t)^2 + atr(t)^2} \qquad a(t1) = 3.056$$



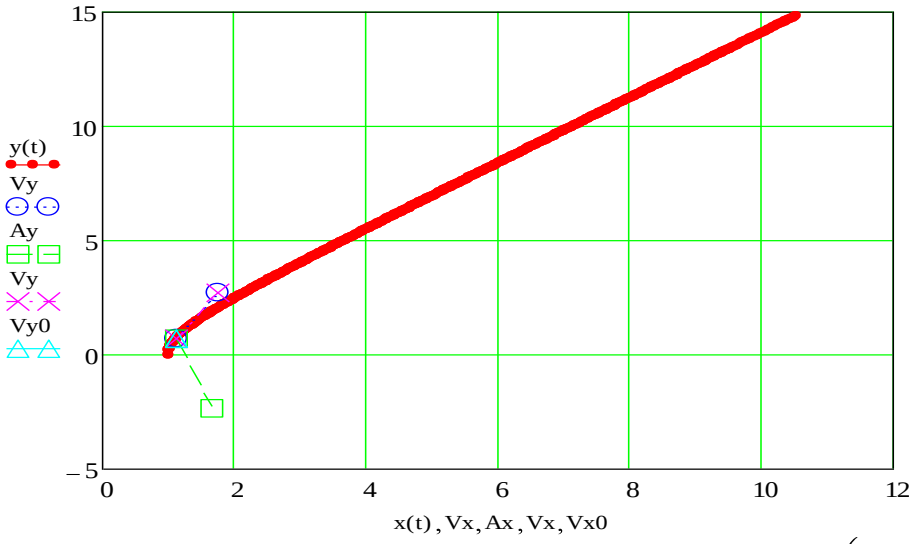


Tezlik va tezlanish ,shuningdek radial va transversal tezlik hamda tezlanishlar yo‘nalishini aniqlash uchun quyidagi programmani tuzib, rasmda tasvirlaymiz:

$$x0 := x(t1) \qquad y0 := y(t1) \qquad Vx := \begin{pmatrix} x0 \\ x0 + Vx(t1) \end{pmatrix}$$

$$Vy := \begin{pmatrix} y0 \\ y0 + Vy(t1) \end{pmatrix} \qquad Vx0 := \begin{pmatrix} x0 \\ x0 \end{pmatrix} \qquad Vy0 := \begin{pmatrix} y0 \\ y0 \end{pmatrix}$$

$$Ax := \begin{pmatrix} x0 \\ x0 + ax(t1) \end{pmatrix} \qquad Ay := \begin{pmatrix} y0 \\ y0 + ay(t1) \end{pmatrix} \qquad A0 := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$



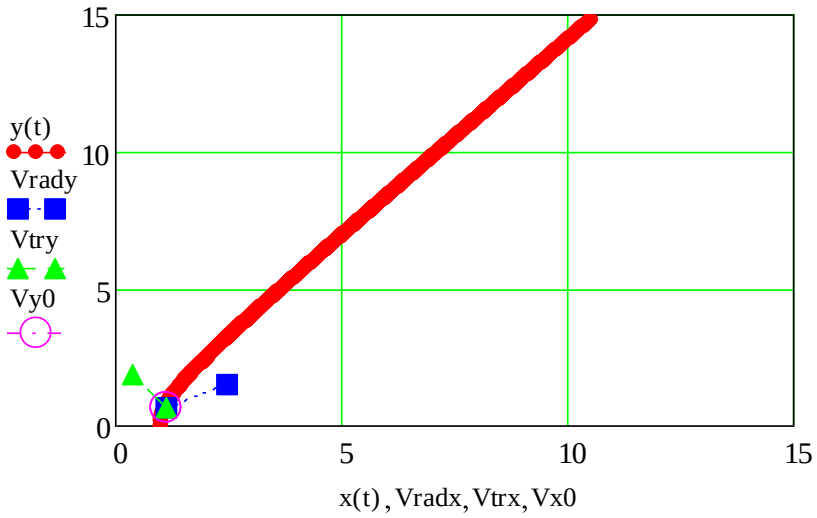
$$x0 := x(t1) \qquad y0 := y(t1) \qquad Vradx := \begin{pmatrix} x0 \\ x0 + Vradx(t1) \end{pmatrix}$$

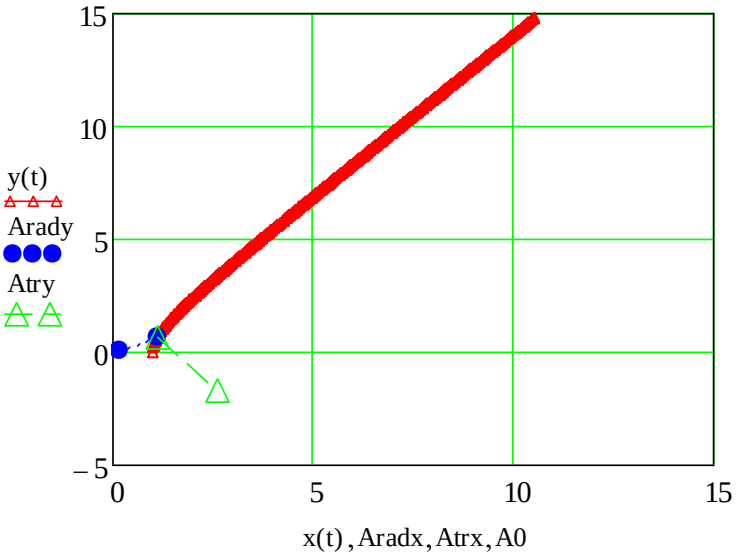
$$Vrady := \begin{pmatrix} y0 \\ y0 + Vrady(t1) \end{pmatrix} \qquad Vx0 := \begin{pmatrix} x0 \\ x0 \end{pmatrix} \qquad Vy0 \begin{pmatrix} y0 \\ y0 \end{pmatrix}$$

$$Aradx := \begin{pmatrix} x0 \\ x0 + aradx(t1) \end{pmatrix} \qquad Arady := \begin{pmatrix} y0 \\ y0 + arady(t1) \end{pmatrix} \qquad A0 := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$Vtrx := \begin{pmatrix} x0 \\ x0 + Vtrx(t1) \end{pmatrix} \qquad Vtry := \begin{pmatrix} y0 \\ y0 + Vtry(t1) \end{pmatrix}$$

$$Atrx := \begin{pmatrix} x0 \\ x0 + atrx(t1) \end{pmatrix} \qquad Atry := \begin{pmatrix} y0 \\ y0 + atry(t1) \end{pmatrix}$$





3- topshiriq (dinamika)

Moddiy nuqta harakatini xarakterlovchi parametrlarni tekshirish.

Moddiy nuqta

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + r(t)\vec{k} \qquad (29)$$

qonunga ko'ra harakatlanadi. Unga aktiv kuch

$$\vec{F}(t) = F_{1x}(t)\vec{i} + F_{1y}(t)\vec{j} + F_{1z}(t)\vec{k}$$

va moddiy nuqta tezligiga proporsional bo'lgan qarshilik kuchi

$$\vec{R}(t) = -\alpha\vec{V}(t)$$

ta’sir qiladi.Bu erda α-proporsionallik koeffisienti.

$t = t_k$ vaqtda quyidagi parametrlarni:

1) Moddiy nuqtaning berilgan harakatini ta’minlovchi $\vec{F}_2$ kuchni vaqt orqali ifodasi,

2) Vaqtning 0 va t_k oralig’idagi $\vec{F}_1(t)$, $\vec{F}_2(t)$ va $\vec{R}(t)$ larning bajargan ishi;

3) $\vec{F}_1(t)$, $\vec{F}_2(t)$ va $\vec{R}(t)$ lar impulsini vaqt orqali ifodasi hamda ularning $(0, t_k)$ intervaldagi gafigi;

4) Moddiy nuqta tezligi qanday $t = t_1$ da V_1 bo’lishi, mazkur vaqtda kuchlar impulslarining koordinata o’qlaridagi proeksiyasi hamda nuqta harakat miqdorining modulini aniqlash kerak.

Topshiriqning hamma variantlarida $m = 3kg$, $F_{1x}(t) = ay(t)$, $F_{1y}(t) = bz(t)$, $F_{1z}(t) = cx(t)$, $a = 2N/m$, $b = 0{,}5N/m$, $c = 0{,}8N/m$ deb qabul qilinsin.

Topshiriqni bajarish uchun zarur parametrlar 4-jadvaldan olinsin. Javoblari 5-jadvalda keltirilgan.

4-jadval

Variant №	a_1, m	a_2,m	b_1, m	b_2, m	c_1, m	c_2, m	p_1	p_2	h_1	h_2	λ_0	λ_1
1	5,5	2,3	5	3	2	2	1	3	2	3	3	0,7
2	3	2,3	5	3	2	1,2	1	2	1	3	3	0,7
3	3	2	5	3	2,5	1,2	2	3	2	3	3	0,7
4	4	5	3	1	2,5	1	1	3	2	2	3	0,7
5	4	2	3	1	2,8	0,8	4	1	5	2	2	0,7
6	2,7	2	1,3	1	2,8	0,8	2	1	5	2,5	3	0,7
7	-1,7	6	1,3	5	3	1,5	2	-2	2	4	3	1,7
8	3,7	4	3,3	5	3,4	1	2,7	3	2,2	3,5	4	1,2
9	3,7	2	7,3	5	2,4	1	2	1,5	2	-1,5	4	1,2
10	-2,7	2	4,3	-2	2,8	1,6	3	1,5	-4	-1,5	4	1,6
11	4,7	-2	3	-2	3,4	1,2	4	1,5	-4	-1	3,7	2,2

12	6	3	5	1,8	2,4	1,2	4	2,5	2	1,5	1,7	0,5
13	3	3	4	2,8	2,4	1,2	4	2,5	3,3	-1,5	4,7	1,8
14	6	-3	2	6	2,6	1,8	-2	4	3	2	5	3,8
15	1	-3	7	2	2,6	1,4	4	-5	2	2,5	2,7	2,5
16	6	4	3	2	3,6	1,2	3	-3	1,5	2,5	3,5	1,5
17	6	-4	2	2	3	1,5	3	2	1,3	3,5	4,5	2,5
18	2	5	5	2	3,2	1,4	2,5	1,8	1,7	3	4	3,5
19	-3	4,2	6	5	4	0,6	3,5	2	-2,2	2	4,8	1,5
20	-4	3,2	2	-3	4	1,2	3	2	2	4	3,8	2,5
21	5	2	4	3	5	1,5	2,5	2	3	2	3,6	1,5
22	-2	3,2	-3	1,4	3	1	3,5	2	3	-4	4,6	2,5
23	4	3,2	2	3,4	3	1,4	3	2	1	3	3,2	1,5
24	5	4,2	-2	1,4	4	2,4	2	1,5	1	-3	3,8	2,6
25	2	-4,2	-3	3,4	4,5	2,2	4	1	2	-1	3,5	1,6
26	-6	-4	2	4	4	1,2	-2	3	3	-4	3,3	2,6
27	4	-2	2	4	3,5	1,6	3	4	3	2	3,8	1,6
28	2	-3	5	4	3,8	1,2	1	3,2	3	-2,4	3,2	2,2
29	-5	-2	3	1	3,1	1,4	2	3,6	3,5	-2,1	3,8	1,7
30	4	5	3,4	2	3,6	1,5	3	4,5	2,5	3	4,3	2,3

5-jadval

Variant №	Javoblar
1	1.1,1.2, 2.1, 2.2, 3.3, 3.4, 4.3
2	1.3, 1.4, 2.4, 2.5, 3.1, 3.5, 4.2
3	1.5, 2.2, 2.3, 3.3, 3.5, 4.1
4	1.3, 1.4, 2.4, 2.5, 3.1, 3.2, 4.2
5	1.1, 1.5, 2.1, 2.5, 3.2, 3.3, 4.3
6	1.3, 1.4, 2.4, 2.5, 3.3, 3.5, 4.1
7	1.2, 1.5, 2.2, 2.6, 3.1, 3.3, 4.3
8	1.2, 1.3, 2.2, 2.4, 3.3, 3.4, 4.2

9	1.3, 1.4, 2.5, 2.6, 3.3, 3.5, 4.3
10	1.5, 2.2, 2.3, 3.1, 3.4, 4.1
11	1.2, 1.4, 2.2, 2.5, 3.3, 3.4, 4.2
12	1.3, 1.4, 2.1, 2.4, 3.1, 3.2, 4.3
13	1.1, 1.2, 2.5, 2.6, 3.3, 3.4, 4.1
14	1.5, 2.2, 2.3, 3.1, 3.5, 4.2
15	1.2, 1.3, 2.4, 2.5, 3.3, 3.5, 4.3
16	1.3, 1.4, 2.1, 2.2, 3.1, 3.2, 4.2
17	1.1, 1.3, 2.2, 2.6, 3.3, 3.4, 4.3
18	1.2, 1.5, 2.2, 2.6, 3.1, 3.5, 4.1
19	1.3, 1.4, 2.2, 2.4, 3.3, 3.4, 4.2
20	1.1, 1.2, 2.4, 2.5, 3.3, 3.5, 4.3
21	1.5, 2.5, 2.6, 3.1, 3.2, 4.1
22	1.5, 2.5, 2.6, 3.1, 3.2, 4.1
23	1.2, 1.3, 2.2, 2.3, 3.3, 3.4, 4.2
24	1.1, 1.5, 2.2, 2.3, 3.1, 3.5, 4.3
25	1.1, 1.5, 2.2, 2.3, 3.1, 3.5, 4.3
26	1.3, 1.4, 2.1, 2.2, 3.4, 3.5, 4.3
27	1.1, 1.2, 2.1, 2.5, 3.2, 3.4, 4.1
28	1.3, 1.4, 2.3, 2.6, 3.3, 3.4, 4.2
29	1.2, 1.5, 2.3, 2.4, 3.2, 3.4, 4.3
30	1.3, 1.4, 2.4, 2.5, 3.2, 3.3, 4.2

3-topshiriqni bajarish uchun namuna:

Berilgan:

$x(t) = 2\cos \pi t\ (m)$; $y(t) = 4\sin \pi t\ (m)$; $z(t) = 0{,}5t(m)$;

$\mu = 0{,}4\ N\,s/m$; $m = 3$; t_k=1,5c; V_1=10m/s (30)

Yechish. 1. $\vec{F}_2 = \vec{F}_2(t)$ ni va $t = t_2$ momentdagi $\vec{F}_2$ kuch moduli va yonalishini aniqlaymiz.

(29) dan foydalanib nuqta tezligi va tezlanishini hisoblaymiz.

Nuqta tezlik vektori va tezlanish vektori mos ravishda quyidagicha bo'ladi:

$$\vec{V}(t) = V_x(t)\vec{\imath} + V_y(t)\vec{\jmath} + V_z(t)\vec{k};\ \vec{a}(t) = a_x(t)\vec{\imath} + a_y(t)\vec{\jmath} + a_z(t)\vec{k} \quad (31)$$

bu yerda $\vec{\imath}, \vec{\jmath}, \vec{k}$ - x, y, z o'qlarining birlik vektorlari; $V_x(t), V_y(t), V_z(t) - \vec{V}(t)$ ning $a_x(t), a_y(t), a_z(t) - \vec{a}(t)$ ning proyeksiyalari. Bularni aniqlash uchun (30) dan vaqt bo'yicha birinchi va ikkinchi tartibli hosilalar olamiz.

$$V_x(t) = \dot{x}(t) = -2\pi\, sin(\pi t)\,, V_y(t) = \dot{y}(t) = 4\pi\, cos(\pi t),$$

$$V_z(t) = \dot{z}(t) = 0{,}5 \tag{32}$$

$$a_x(t) = -2\pi^2\, cos(\pi t)\,, a_y(t) = -4\pi^2\, sin(\pi t)\,, a_z(t) = 0 \tag{33}$$

Tezlik moduli bunday topiladi:

$$V(t) = \sqrt{V_x(t)^2 + V_y(t)^2 + V_z(t)^2} \tag{34}$$

biz bilamizki masala shartiga ko'ra moddiy nuqtaga $\vec{F}_1(t), \vec{F}_2(t)$ va $\vec{R}(t)$ kuchlar ta’sir qiladi.Moddiy nuqtaning differensial tenglamalari quyidagicha bo’ladi.

$$\begin{aligned} m\ddot{x}(t) &= F_{1x}(t) + F_{2x}(t) + R_x, \\ m\ddot{y}(t) &= F_{1y}(t) + F_{2y}(t) + R_y, \\ m\ddot{z}(t) &= F_{1z}(t) + F_{2z}(t) + R_z. \end{aligned} \tag{35}$$

(35) dan:

$$\begin{aligned} F_{2x}(t) &= m\ddot{x}(t) - F_{1x}(t) - R_x = -2m\pi^2 cos\pi t - 4asin\pi t - 2\mu\pi sin\pi t, \\ F_{2y}(t) &= m\ddot{y}(t) - F_{1y}(t) - R_y = -4m\pi^2 sin\pi t - b\cdot 0{,}5t + 4\mu\pi cos\pi t, \\ F_{2z}(t) &= m\ddot{z}(t) - F_{1z}(t) - R_z = -2ccos\pi t + 0{,}5\mu. \end{aligned} \tag{36}$$

kelib chiqadi.

$\vec{F}_2(t)$ kuch moduli:

$$F_2(t) = \sqrt{F_{2x}(t)^2 + F_{2y}(t)^2 + F_{2z}(t)^2} \tag{37}$$

$\vec{F}_2(t)$kuchning yonaltiruvchi kosinuslari:

$$cos\ (\vec{F}_2\hat{}\,, \vec{\imath}) = \frac{F_{2x}(t)}{F_2\,(t\,)},\ cos\ (\vec{F}_2\hat{}\,, \vec{\jmath}) = \frac{F_{2y}(t)}{F_2\,(t\,)}, cos\ (\vec{F}_2\hat{}\,, \vec{k}) = \frac{F_{2z}(t)}{F_2\,(t\,)} \tag{38}$$

$t = t_k$ bo’lganda (36)-(38) quyidagicha bo’ladi:

$F_{2x}(t_k)$=10,513N, $F_{2y}\,(t_k)$=118,060N, $F_{2z}(t_k)$=0,2N ; $F_2(t_k)$=118,52N

$$cos(\vec{F}_2\hat{}\,, \vec{\imath}) = 0{,}089, cos(\vec{F}_2\hat{}\,, \vec{\jmath}) = 0{,}996, cos(\vec{F}_2\hat{}\,, \vec{k})=0{,}002$$

2. $\vec{F}_1(t), \vec{F}_2(t)$ va $\vec{R}(t)$ kuchlarning $(0, t_k)$ intervaldagi bajargan ishlarni hisoblash.

$\vec{F}_1(t), \vec{F}_2(t)$ va $\vec{R}(t)$ bajargan ishi:

$$A_{F_1}(t) = \int_0^t \vec{F}_1(t)\cdot\vec{V}(t)dt = \int_0^t \Big(F_{1x}(t)V_x(t) + F_{1y}(t)V_y(t) + F_{1z}(t)V_z(t)\Big)\,dt,$$

$$A_{F_2}(t) = \int_0^t \vec{F}_2(t)\cdot\vec{V}(t)dt = \int_0^t \Big(F_{2x}(t)V_x(t) + F_{2y}(t)V_y(t) + F_{2z}(t)V_z(t)\Big)\,dt,$$

$$A_R(t) = \int_0^t \vec{R}(t) \cdot \vec{V}(t)dt = \int_0^t \Big(R_x(t)V_x(t) + R_y(t)V_y(t)\Big)dt +$$

$$+ \int_0^t R_z(t)V_z(t)dt \qquad (39)$$

$t = t_k$ bo'lganda:

$$A_{F_1}(t_k) = -39{,}772J, A_{F_2}(t_k) = -78{,}513J, A_R(t_k) = -59{,}368J.$$

3. $\vec{F}_1(t)$, $\vec{F}_2(t)$ va $\vec{R}(t)$ impulslarini vaqt orqali ifodasi va $(0, t_k)$ intervaldagi ularning grafigini aniqlash.

$\vec{F}_1(t), \vec{F}_2(t)$ va $\vec{R}(t)$ impulslarining koordinata o'qlaridagi proyeksiyalari mos ravishda quyidagicha bo'ladi:

$$S_{1x}(t) = \int_0^t F_{1x}(t)dt = 4a\int_0^t sin\pi dt = (4a/\pi)(1 - cos\pi t), \qquad (40)$$

$$S_{1y}(t) = \int_0^t F_{1y}(t)dt = 0{,}5b\int_0^t tdt = bt^2/4 \qquad (41)$$

$$S_{1z}(t) = \int_0^t F_1(t)dt = 2c\int_0^t cos\pi t dt = 2\frac{c}{\pi}sin\pi t; \qquad (42)$$

$$S_{2x}(t) = \int_0^t F_{2x}(t)dt = \int_0^t(-2m\pi^2 cos\pi t - 4asin\pi t - 2a\pi sin\pi t)dt =$$

$$= 2m\pi sin\pi t + (4a\pi + 2a)(1 - cos\pi t), \qquad (43)$$

$$S_{2y}(t) = \int_0^t F_{2y}(t)dt = \int_0^t(-4m\pi^2 sin\pi t - 0{,}5bt + 4a\pi cos\pi t)dt =$$

$$= -2m\pi(1 - cos\pi t) - \frac{bt^2}{4} + 4asin\pi t, \qquad (44)$$

$$S_{2z}(t) = \int_0^t F_{2z}(t)dt = \int_0^t(-2ccos\pi t + 0{,}5a)dt = -\frac{2c}{\pi} + 0{,}5at; \qquad (45)$$

$$S_{Rx} = \int_0^t R_x(t)dt = 2\mu\pi\int_0^t \sin\pi t\, dt = 2\mu(1 - \cos\pi t), \qquad (46)$$

$$S_{Ry} = \int_0^t R_y(t) = -4\mu\pi\int_0^t \cos\pi t\, dt = -4\mu\pi \sin\pi t, \qquad (47)$$

$$S_{Rz}(t) = \int_0^t R_z(t)dt = -0{,}2\int_0^t dt = -0{,}2t \qquad (48)$$

Mazkur impulslarning grafigida *Matchad* yordamida keyinroq ko'rib chiqamiz.

4. Moddiy nuqta tezlik moduli tezligi V_1 ga erishadigan eng yaqin moment $t = t_1$ ni va shu momentdagi kuchlar impulslarining miqdori, nuqta harakat miqdorining modulini aniqlash.

$t = t_1$ ni aniqlash uchun (34) formuladan foydalanamiz:

$$V_1^2 = 4\pi^2 \sin^2 \pi t + 16\pi^2 \cos^2 \pi t + 0{,}5^2$$

bu yerdan

$$t_1 = \frac{1}{\pi} arc\cos\left(\frac{V_1^2 - 0{,}5^2 - 4\pi^2}{12\pi^2}\right) = 0{,}247s.$$

kelib chiqadi.

$t = t_1$ bo'lganda tezlik proyeksiyasi, moduli va nuqta harakat miqdori quyidagicha bo'ladi:

$$V_{1x} = -4{,}403\, m/s\,, V_{1y} = 8{,}964\, m/s\,, V_{1z} = 0{,}5\, m/s\,;$$

$$V = \sqrt{V_{1x}^2 + V_{1y}^2 + V_{1z}^2}\,, V = 9{,}999\, m/s\,, mV = 30\, kg \cdot m/s$$

$t = t_1$ ni (40 - 48) ga qo'ysak,

$$S_{1x} = 0{,}73Ns, S_{1y} = 0{,}007Ns, S_{1z} = 0{,}357Ns\,;$$

$$S_{2x} = -14{,}169Ns, S_{2y} = -9{,}692Ns, S_{2z} = -0{,}307Ns;$$

$$S_{Rx} = 0{,}229Ns, S_{Ry} == 1{,}121Ns, S_{Rz} = -0{,}049Ns$$

kelib chiqadi.

Endi masalani ***Matchad*** matematik paket yordamida qanday yechilishini ko'rib chiqamiz.

Berilgan:

$tk := \frac{3}{2}$ $m := 3$ $b := 2$ $c := 0.5$ $d := 0.8$ $\mu := 0.4$

$x(t) := 2\cdot\cos(\pi\cdot t)$ $y(t) := 4\cdot\sin(\pi\cdot t)$ $z(t) := 0.5t$ $V1 := 10$

Moddiy nuqta tezligining proyeksiyalari va tezlik moduli mos ravishda quyidagicha:

$Vx(t) := \frac{d}{dt}x(t)$ $Vy(t) := \frac{d}{dt}y(t)$ $Vz(t) := \frac{d}{dt}z(t)$ $V(t) := \sqrt{Vx(t)^2 + Vy(t)^2 + Vz(t)^2}$

$Vx(t) \rightarrow -2\cdot\pi\cdot\sin(\pi\cdot t)$ $Vy(t) \rightarrow 4\cdot\pi\cdot\cos(\pi\cdot t)$ $Vz(t) \rightarrow 0.5$ $V(tk) = 6.303$

Moddiy nuqta tezlanishining proyeksiyalari va tezlanish moduli mos ravishda bunday bo`ladi:

$$ax(t) := \frac{d}{dt}Vx(t) \qquad ay(t) := \frac{d}{dt}Vy(t) \qquad az(t) := \frac{d}{dt}Vz(t)$$

$$a(t) := \sqrt{ax(t)^2 + ay(t)^2 + az(t)^2} \qquad a(tk) = 39.478$$

$\rightarrow$
F1(t) ning proyeksiyalari va ularning bajargan ishi mos ravishda quyidagicha topiladi:

$$b := 2 \qquad c := 0.5 \qquad d := 0.8 \qquad \mu := 0.4$$

$$F1x(t) := b \cdot y(t) \qquad F1y(t) := c \cdot z(t) \qquad F1z(t) := d \cdot x(t)$$

$$A1(t) := \int_0^t (F1x(t) \cdot Vx(t) + F1y(t) \cdot Vy(t) + F1z(t) \cdot Vz(t))\, dt \qquad A1(tk) = -39.772$$

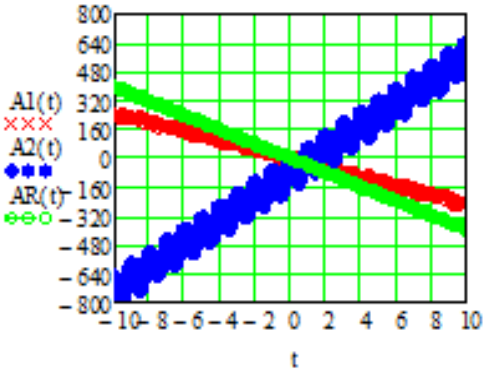
$\rightarrow$
F2(t) ning proyeksiyalari va ularning bajargan ishi mos ravishda quyidagicha topiladi:

$$F2x(t) := m \cdot ax(t) - F1x(t) + \mu \cdot Vx(t) \qquad F2y(t) := m \cdot ay(t) - F1y(t) + \mu \cdot Vy(t)$$

$$F2z(t) := m \cdot az(t) - F1z(t) + \mu \cdot Vz(t)$$

$$AR(t) := \int_0^t (Rx(t) \cdot Vx(t) + Ry(t) \cdot Vy(t) + Rz(t) \cdot Vz(t))\, dt$$

$$A2(t) := \int_0^t (F2x(t) \cdot Vx(t) + F2y(t) \cdot Vy(t) + F2z(t) \cdot Vz(t))\, dt \qquad A2(tk) = -78.513$$



Endi t1 vaqtni hisoblaymiz:

$$V1 := 10 \qquad f(t) := V1^2 - V(t)^2 \qquad t1 := root(f(t), t, 0, 0.3) \qquad t1 = 0.247$$

Moddiy nuqtaning harakat miqdori bunday bo`ladi:

$$m \cdot V(t1) = 30$$

$\vec{F1}(t)$ va $\vec{F2}(t)$ kuchlar impulslarining proyeksiyasi mos ravishda quyidagicha:

$$S1x(t) := \int_0^t F1x(t)\,dt \qquad S1y(t) := \int_0^t F1y(t)\,dt \qquad S1z(t) := \int_0^t F1z(t)\,dt \qquad S1(t) := \sqrt{S1x(t)^2 + S1y(t)^2 + S1z(t)^2}$$

$$S1x(t1) = 0.73 \qquad S1y(t1) = 7.636 \times 10^{-3} \qquad S1z(t1) = 0.357 \qquad S1(t1) = 0.813$$

$$S2x(t) := \int_0^t F2x(t)\,dt \qquad S2y(t) := \int_0^t F2y(t)\,dt \qquad S2z(t) := \int_0^t F2z(t)\,dt$$

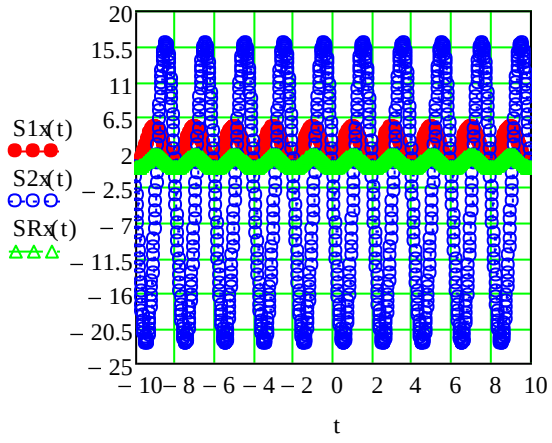
$$S2x(t1) = -14.169 \qquad S2y(t1) = -9.692 \qquad S2z(t1) = -0.307 \qquad S2(t) := \sqrt{S2x(t)^2 + S2y(t)^2 + S2z(t)^2}$$

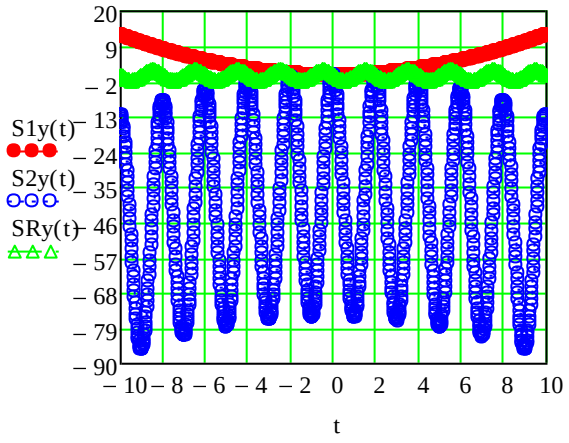
$$SRx(t) := \int_0^t Rx(t)\,dt \qquad SRy(t) := \int_0^t Ry(t)\,dt \qquad SRz(t) := \int_0^t Rz(t)\,dt \qquad S2(t1) = 17.169$$

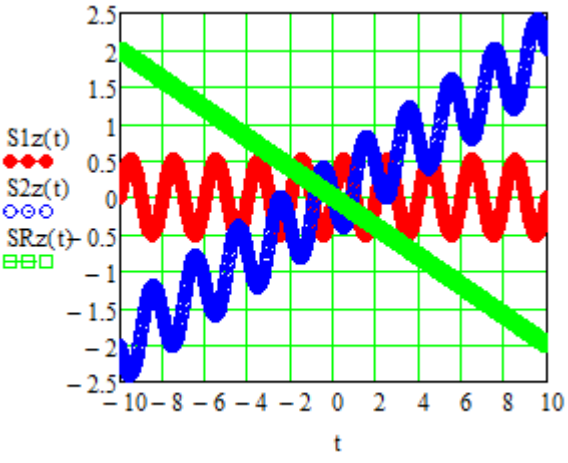
$$SRx(t1) = 0.229 \qquad SRy(t1) = -1.121 \qquad SRz(t1) = -0.049 \qquad SR(t) := \sqrt{SRx(t)^2 + SRy(t)^2 + SRz(t)^2}$$

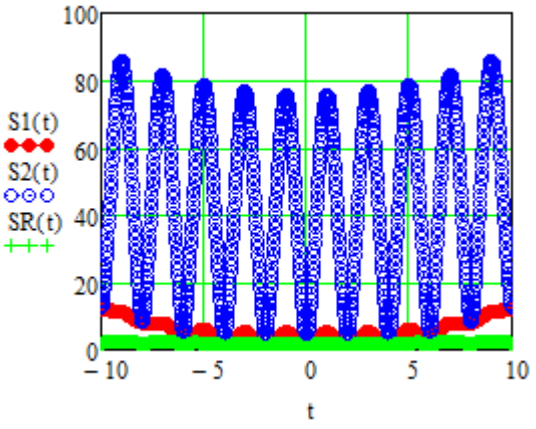
$$SR(t1) = 1.146$$

Yuqoridagi kuch impulslarning grafiklari quyidagicha bo`ladi:









NAZARIY MEXANIKADAN HISOB-GRAFIK ISHLARINI BAJARISHGA TAVSIYA VA UMUMIY TALABLAR

Talabalar topshiriqni bajarish uchun avval nazariyada berilgan mavzularni chuqur tahlil qilishi, nazariyada va amaliyotda hal etilgan misol hamda masalalarni o‘zlashtirishi lozim. Topshiriqni bajarishda rasmni mashtab tanlab qalamda chizishi va masala shartini yozib olishi kerak. Rasamda barcha burchaklar, jismlar soni va ularning joylashishi masala shartiga mos kelishi lozim.

Chizma aniq ko‘zga tashlanishi, uning o‘lchamlari aniq ko‘rsatilishi shart. Masala yechilganda qisqa tushuntirish berilishi, qanday formulalar yoki teoremalar qo‘llanishi, u yoki bu natijalar qayerdan kelib chiqqanligi mufassal tushuntirilishi, hisoblashlarning borishi to‘liq ko‘rsatilgan bo‘lishi shart.

Topshiriq A4 formatdagi oq qog‘ozda bajarilib, taqriz uchun joy (polya) qoldiriladi. Yuqoridagi talablarga javob bermaydigan ishlar tekshirilmasdan tuzatish uchun qaytarib beriladi. Talabalarga topshiriq va topshiriq variantlarini o‘zlarining amaliyot ustozlari belgilab beradi.

Variant bo‘yicha bajarilmagan mustaqil va hisob-grafik ishlari himoya qilishga qo‘yilmaydi. Hisob-grafik ishlari o‘quv grafigi bo‘yicha himoyaga qo‘yiladi. Talaba topshiriqni himoya qilayotganda amaliy mashg‘ulotda hal etiladigan tipik masala yechishni hamda topshiriqqa mos kelgan nazariy savollarga javob bera olishi kerak. Talabalar taqriz qiluvchiga bir yo‘la bir nechta hisob-grafik ishlarni olib kelishi tavsiya etilmaydi, chunki taqriz qilish kechikadi. Natijada talaba kamchiligini o‘z vaqtida bilib va uni tuzata olmaydi.

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Hisob-grafik ishlarining titul varag‘ini rasmiylashtirish namunasai

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MUSTAQIL (HISOB-GRAFIK) ISHI №

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TOSHKENT – 2015

Movlonov To‘lqin Movlonovich
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“NAZARIY MEXANIKA” fanining hisob-grafik ishlarini EHM yordamida bajarish uchun uslubiy ko‘rsatma

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