O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

AL-XORAZMIY NOMLI URGANCH DAVLAT UNIVERSITETI

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Yuklangan hadli nochiziqli tenglamalarni davriy funksiyalar sinfida integrallash

5A130101-Matematika(yo‘nalishlar bo‘yicha)

Magistr akademik darajasini olish uchun yozilgan dissertatsiya

Ilmiy rahbar:

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Urganch – 2016 y.

M U N D A R I J A

Kirish

Mavzuning dolzarbligi. 1974-1976 yillarda S.P.Novikov, P.Laks, V.A.Marchenko, B.A.Dubrovin, V.B.Matveev, A.R.Its, G.Mak-Kin, E.Trubovitslarning ilmiy ishlarida KdF tenglamasining x oʻzgaruvchi boʻyicha davriy va chekli zonali yechimlarini tuzish usullari oʻrganilgan.

2010 yilda A.B.Hasanov va A.B.Yaxshimuratovlar moslangan manbali KdF tenglamasini davriy funksiyalar sinfida integrallashga muvoffaq boʻlishdi. 2011 yilda A.B.Yaxshimuratov moslangan manbali Shredinger nochiziqli tenglamasini davriy funksiyalar sinfida integralladi. 2013 yilda A.Cabada va A.B.Yaxshimuratovlar moslangan manbali Kaup sistemasini davriy funksiyalar sinfida integrallashdi.

Tadqiqotning maqsadi. Yuklangan hadli nochiziqli tenglamalarni davriy funksiyalar sinfida integrallash.

Tadqiqotning vazifasi. Nochiziqli evolyutsion tenglamalarni davriy funksiyalar sinfida yechish usulini tahlil qilib, uni yuklangan hadli nochiziqli tenglamalarni yechishga tatbiq qilish.

Tadqiqotning obyekti. Yuklangan hadli Korteveg-de Friz tenglamasi, Shredinger nochiziqli tenglamasi va Kaup sistemasi.

Tadqiqotning predmeti. Dubrovin-Trubovits differensial tenglamalar sistemasi.

Tadqiqotning usullari. Teskari spektral masala usuli.

Tadqiqotning natijalarining ilmiy jihatdan yangilik darajasi. Mazkur ishning asosiy natijalari yangi.

Tadqiqotning natijalarining amaliy ahamiyati va tatbiqi. Olingan natijalar va qoʻllanilgan usullar nochiziqli evolyutsion tenglamalarni integrallashga tatbiq qilinadi.

Ish tuzilishi va tarkibi. Dissertatsiya ishi kirish, uchta bob hamda adabiyotlar roʻyhatidan iborat. Har bir bob boʻlimlardan tuzilgan.

Bajarilgan ishning asosiy natijalari. 1) yuklangan hadli KdF tenglamasi davriy funksiyalar sinfida yechilgan; 2) yuklangan hadli NSh tenglamasi davriy funksiyalar sinfida yechilgan; 3) yuklangan hadli Kaup sistemasi davriy funksiyalar sinfida yechilgan

Xulosa va takliflarning qisqacha umumlashtirilgan ifodasi. Mazkur dissertatsiyada yuklangan hadli KdF, NSh tenglamalarini va Kaup sistemasini yechish usuli olingan. Qo‘llanilgan usullarni yuklangan hadli yuqori tartibli KdV, NSh tenglamalarini yechishga qo‘llashni taklif qilamiz.

I-BOB. YUKLANGAN HADLI KORTEVEG-DE FRIZ TENGLAMASINI DAVRIY FUNKSIYALAR SINFIDA INTEGRALLASH

Matematik fizikaning bir qator masalalari Shturm-Liuvill operatorining xos qiymatlarini va ortonormallangan xos funksiyalarini topishga keltiriladi. Jumladan, klassik matematik fizikaning asosiy tenglamalari hisoblangan tor tebranishi va issiqlik o‘tkazuvchanlik tenglamalarini Furye usuli bilan yechishda Shturm-Liuvill chegaraviy masalasining xos qiymatlarini, ortonormallangan xos funksiyalarini aniqlashga va ixtiyoriy funksiyani ular yordamida Furye qatoriga yoyishga to‘g‘ri keladi. Bu yo‘nalishdagi ilk natijalar D.Bernulli, J.Dalamber, L.Eyler, Liuvill va Shturmlar tomonidan olingan. Shturm-Liuvill operatori spektral nazariyasining asosiy g‘oyalari XX-asrda G.D.Birkgoff, G.Veyl, D.Gilbert, V.A.Steklov, E.Ch.Titchmarsh, N.Levinson, B.M.Levitan va boshqa olimlar tomonidan rivojlantirildi.

Shturm-Liuvill chegaraviy masalasining $\{\lambda_n\}_{n=0}^{\infty}$ xos qiymatlari va $\{\alpha_n\}_{n=0}^{\infty}$ normallovchi o‘zgarmaslariga yoki spektral funksiyasiga spektral xarakteristikalar deyiladi. Spektral xarakteristikalarni topish va ularning xossalarini o‘rganish masalasiga spektral analizning to‘g‘ri masalasi deyiladi.

Shturm-Liuvill chegaraviy masalasi xos funksiyalarining $L_2[0,\pi]$ fazoda to‘laligi haqidagi teoremani ilk bor XIX asrning oxirlarida rus matematigi V.A.Steklov isbotlashga muvaffaq bo‘lgan.

Spektral xarakteristikalar yordamida Shturm-Liuvill tenglamasi koeffitsientini va chegaraviy shartlarni aniqlash masalasiga spektral analizning teskari masalasi deyiladi.

Teskari masalalar nazariyasining rivojiga muhim turtki bo‘lgan ilk natija 1929 yilda taniqli astronom V.A.Ambartsumyan tomonidan olingan: *agar ushbu*

$$-y'' + q(x)y = \lambda y, \quad q(x) \in C[0,\pi],$$

$$y'(0) = 0, \ y'(\pi) = 0$$

Shturm-Liuvill chegaraviy masalasining xos qiymatlari $\lambda_n = n^2$, $n = 0, 1, 2, \dots$ *bo'lsa, u holda* $q(x) \equiv 0$ *bo'ladi.*

Ambartsumyanning bu natijasi muhim ekanliliga birinchi bo'lib, 1946 yilda Shved matematigi G.Borg e'tibor berdi, u Ambartsumyanning natijasi umumiy qoidadan istisno ekanligini ko'rsatib berdi, ya'ni faqat bitta spektr orqali Shturm-Liuvill chegaraviy masalasini bir qiymatli tiklab bo'lmasligini ko'rsatdi. G.Borg, faqat bitta chegaraviy sharti bilan farq qiluvchi ikkita Shturm-Liuvill masalasining spektrlari ma'lum bo'lsa, bu spektrlar orqali dastlabki masalalar bir qiymatli tiklanishini isbot qildi. Hozirda bu tasdiq, G.Borgning yagonalik teoremasi deb yuritiladi: *agar* $\lambda_0 < \lambda_1 < \dots < \lambda_n < \dots$ *sonlar ushbu*

$$-y'' + q(x)y = \lambda y, \quad q(x) \in C[0,\pi],$$

$$\begin{cases} y'(0) - hy(0) = 0, \ h \in R^1, \\ y'(\pi) + Hy(\pi) = 0, \ H \in R^1 \end{cases}$$

chegaraviy masalaning xos qiymatlari va $\mu_0 < \mu_1 < \dots < \mu_n < \dots$ *sonlar quyidagi*

$$-y'' + q(x)y = \lambda y, \quad q(x) \in C[0,\pi],$$

$$\begin{cases} y'(0) - hy(0) = 0, \ h \in R^1, \\ y'(\pi) + H_1 y(\pi) = 0, \ H \neq H_1, \ H_1 \in R^1 \end{cases}$$

chegaraviy masalaning xos qiymatlari bo'lsa, u holda λ_n *va* μ_n *xos qiymatlar ketma-ketligi orqali* $q(x)$ *haqiqiy funksiya va* h, H, H_1 *sonlar bir qiymatli aniqlanadi.*

Shturm-Liuvill masalasi chekli oraliqda qaralsa va $q(x)$ koeffisient bu oraliqda integrallanuvchi bo'lsa, bunday masalaga regulyar masala deyiladi, aksincha yoki interval chegaralanmagan bo'lsa, yoki $q(x)$ koeffisient bu oraliqda integrallanuvchi bo'lmasa, bunday Shturm-Liuvill masalasiga singulyar masala deyiladi.

Singulyar Shturm-Liuvill operatorlari uchun teskari masalalar sohasidagi ilk natija 1949 yilda A.N.Tixonov tomonidan olingan yagonalik teoremasidir, bu teoremada Shturm-Liuvill masalasi, “impedans” funksiyasi (ya’ni Veyl-Titchmarsh funksiyasi) bo‘yicha bir qiymatli tiklanishi ko‘rsatilgan: *quyidagi*

$$z'' + \lambda p^2(t)z = 0, \quad 0 < t < \infty$$

tenglamaning koeffitsienti $p(t)$ *bo‘lakli analitik funksiya bo‘lib, ushbu* $p(t) \geq p_0 > 0$ *shart bajarilsin. U holda* $p(t)$ *koeffitsient* $I(\lambda) = \dfrac{u'(0,\lambda)}{u(0,\lambda)}, \ \lambda < 0$ *funksiya orqali bir qiymatli aniqlanadi. Bu yerda* $u(t,\lambda)$ *orqali qaralayatgan tenglamaning ushbu* $\lim\limits_{t\to\infty} u(t,\lambda) = 0$ *shartni qanoatlantiruvchi yechimi belgilangan.*

Shuniyam aytib o‘tish lozimki, ushbu

$$y(x,\lambda) = \sqrt{p(t)} \cdot z(t,\lambda), \quad x = \int_0^t p(\xi)d\xi$$

almashtirishlar yordamida qaralayotgan tenglama Shturm-Liuvill tenglamasiga keltiriladi.

A.N.Tixonovning bu teoremasi, yer ichki qatlamlari elektrik hossalarini o‘rganish masalalarini matematik asoslashda muhim ahamiyatga ega bo‘lgan.

1950 yilda V.A.Marchenko spektral funksiya, ya’ni λ_n xos qiymatlar va α_n normallovchi o‘zgarmaslar ketma-ketligi Shturm-Liuvill chegaraviy masalasini yagona ravishda aniqlashini ko‘rsatib berdi.

V.A.Marchenkoning yagonalik teoremasi e’lon qilingandan so‘ng, spektral funksiya bo‘yicha Shturm-Liuvill operatorini tiklash masalasi dolzarb bo‘lib qoldi. Bu masalaning 1951 yilda I.M.Gelfand va B.M.Levitan tomonidan yechilishi, teskari spektral masalalar sohasidagi eng muhim bosqich bo‘ldi desak, mubolag‘a bo‘lmaydi. Bunda, almashtirish operatori deb ataluvchi operator hal qiluvchi rol o‘ynadi. Bu operator ikkita har xil Shturm-Liuvill tenglamasining yechimlarini o‘zaro bog‘laydi. Almashtirish operatorlari ilk bor B.M.Levitan va J.Delsartlarning ilmiy ishlarida vujudga keldi. Ixtiyoriy Shturm-Liuvill tenglamasi uchun almashtirish operatori A.Ya.Povzner tomonidan ishlab chiqildi. Spektral analizning teskari masalalarini yechishda almashtirish operatorlari I.M.Gelfand, B.M.Levitan va V.A.Marchenkolar tomonidan qo‘llanilgan. So‘ngra teskari masalani yechishning Gelfand-Levitan usuli B.M.Levitan, I.M.Gasimov va N.Levinson tomonidan mukammallashtirildi.

Hozirgi kunda, teskari masala yechishning bir nechta usullari bor. Bu usullar orasida Gelfand-Levitan usuli eng muhim ahamiyatga ega. Yuqorida aytganimizdek, bu usulda almashtirish operatori asosiy rolni o‘ynaydi. Usulning eng muxim bosqichi, almashtirish operatorining yadrosiga nisbatan olingan chiziqli integral tenglamadir. Bu integral tenglama teskari masalaning asosiy integral tenglamasi yoki Gelfand-Levitan integral tenglamasi deb yuritiladi.

1964 yilda I.M.Gasimov va B.M.Levitan ikki spektr yordamida Shturm-Liuvill chegaraviy masalasini qurish algoritmini ishlab chiqdilar.

1967 yilda K.Gardner, J.Grin, M.Kruskal, R.Miura zamonaviy matematik fizikaning asosiy tenglamalaridan biri bo‘lgan ushbu

$$q_t = q_{xxx} - 6qq_x$$

Korteveg-de Friz tenglamasiga qo‘yilgan Koshi masalasining yechimini Shturm-Liuvill operatoriga qo‘yilgan to‘g‘ri va teskari masalalardan foydalanib topishga muvoffaq bo‘lishdi. Natijada Shturm-Liuvill operatori uchun qo‘yilgan to‘g‘ri va teskari masalalarni o‘rganishga bo‘lgan qiziqish yanada ortdi. Tabiiyki, Korteveg-de Friz tenglamasini turli funksional sinflardagi yechimini topish masalasi, o‘z navbatida, Shturm-Liuvill operatori uchun turli funksional sinflarda qo‘yilgan teskari masalani o‘rganishni taqozo qiladi.

Davriy koeffitsientli Shturm-Liuvill operatorining spektri yarim o‘qdan chekli yoki cheksiz sondagi intervallarni chiqarib tashlash usuli bilan hosil qilinadi. Bu intervallarga lakunalar deyiladi. Agar lakunalar soni chekli bo‘lsa, Shturm-Liuvill operatorining koeffitsientiga chekli zonali potensial deyiladi. Bu holda Shturm-Liuvill operatorining spektri cheklita kesma va bitta nurning birlashmasidan iborat bo‘ladi. Umuman olganda, chekli zonali potensial davriy bo‘lishi shart emas.

1974 yilda S.P.Novikov chekli zonali potensiallarni boshqa usulda o‘rgangan va ularni Korteveg-de Friz tenglamasining yechimlarini topishga qo‘llagan.

Umumiy holda davriy potensialli Shturm-Liuvill operatori uchun teskari masala I.V.Stankevich, V.A.Marchenko, I.V.Ostrovskiy, X.P.Mak-Kin, E.Trubovits, X.Xoxshtadt tomonidan o‘rganilgan.

E.Trubovitsning tomonidan quyidagi fikr isbot qilingan: davriy potensiali Shturm-Liuvill operatoriga mos keluvchi lakunalar uzunligi eksponensial ravishda nolga intilishi uchun potensial haqiqiy analitik funksiya bo‘lishi zarur va yetarlidir.

X.Xoxshtadt tomonidan esa $\frac{\pi}{n}$ soni π davrli potensialning davri bo‘lishi uchun nomerlari n ga bo‘linmaydigan barcha lakunalar yopilgan bo‘lishi zarur va yetarliligini isbot qildi.

Bu teorema $n=2$ bo‘lgan holda G.Borg tomonidan 1946 yilda isbot qilingan. Shuning uchun bu teoremaga Borg teskari teoremasining umumlashmasi deyiladi.

Davriy koeffitsientli Shturm-Liuvill operatori uchun teskari spektral masala yechishda izlar formulasi va spektral parametrlarning evolyutsiyasi muhim o‘rin egallaydi.

1975 yilda B.A.Dubrovin tomonidan chekli zonali Shturm-Liuvill operatorining spektral parametrlari uchun differensial tenglamalar sistemasi keltirilib chiqarilgan, 1977 yilda esa E.Trubovits davriy potentsialli Shturm-Liuvill operatorining spektral parametrlari uchun shunga o‘xshash sistemani olishga muvaffaq bo‘ldi. Hozirgi kunda bu sistemaga Dubrovin-Trubovits sistemasi deyiladi.

S.P.Novikov, B.A.Dubrovin, V.B.Matveev, A.R.Its, V.A.Marchenko, B.M.Levitanlar Korteveg-de Friz tenglamasining yechimini chekli zonali, davriy va deyarli davriy funksiyalar sinflarida topishga muvoffaq bo‘ldilar.

Moslangan manbali Korteveg-de Friz tenglamasi tez kamayuvchi funksiyalar sinfida 1988 yilda V.K.Melnikov tomonidan, davriy funksiyalar sinfida esa 2010 yilda A.B.Hasanov va A.B.Yaxshimuratov tomonidan yechilgan.

Mazkur sissertatsiyaning I-bobida yuklangan hadli Korteveg-de Friz tenglamasining x o‘zgaruvchi bo‘yicha davriy bo‘lgan yechimini topish usuli o‘rganilgan. Bunda Shturm-Liuvill operatori uchun qo‘yilgan teskari spektral masalalar usuli qo‘llaniladi.

1-§. Lyapunov funksiyasining xossalari.

Butun o‘qda berilgan ushbu

$$-y'' + q(x)y = \lambda y,\ -\infty < x < \infty, \tag{1.1}$$

Shturm-Liuvill masalasini ko‘rib chiqamiz. Bu yerda $q(x)$ haqiqiy uzluksiz π-davrli funksiya. $c(x,\lambda)$ va $s(x,\lambda)$ orqali (1.1) tenglamaning ushbu

$$\begin{cases} c(0,\lambda) = 1 \\ c'(0,\lambda) = 0 \end{cases} \quad \text{va} \quad \begin{cases} s(0,\lambda) = 0 \\ s'(0,\lambda) = 1 \end{cases} \tag{1.2}$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini belgilaymiz. Soddalik uchun quyidagi beliglashlarni kiritib olamiz:

$$c := c(\pi,\lambda),\ c' := c'(\pi,\lambda),\ s := s(\pi,\lambda),\ s' := s'(\pi,\lambda).$$

1-xossa. a) $c(x,\lambda)$ va $s(x,\lambda)$ funksiyalar x o‘zgaruvchining har bir tayinlangan qiymatida λ parametrga nisbatan butun funksiyalardir.

b) $c(x,\lambda)s'(x,\lambda) - c'(x,\lambda)s(x,\lambda) = 1$ ayniyat bajariladi.

Teorema 1.1. Ushbu

$$\begin{cases} Ly \equiv -y'' + q(x)y = \lambda y \\ y(0) = y(\pi) \\ y'(0) = y'(\pi) \end{cases} \tag{1.3}$$

masalaning xos qiymatlari haqiqiydir. Bu masalaga davriy masala deyiladi.

Teorema 1.2. Davriy masalaning xos qiymatlari $\Delta(\lambda) - 2 = 0$ tenglamaning ildizlari bilan ustma-ust tushadi, bu yerda

$$\Delta(\lambda) = c(\pi,\lambda) + s'(\pi,\lambda).$$

$\Delta(\lambda)$ funksiyaga (1.1) masalaning Lyapunov funksiyasi yoki Xill diskriminanti deyiladi.

Teorema 1.1’. Ushbu

$$\begin{cases} Ly \equiv -y'' + q(x)y = \lambda y \\ y(0) = -y(\pi) \\ y'(0) = -y'(\pi) \end{cases} \tag{1.4}$$

masalaning xos qiymatlari haqiqiydir. Bu masalaga antidavriy masala deyiladi.

Teorema 1.2'. Antidavriy masalaning xos qiymatlari $\Delta(\lambda) + 2 = 0$ tenglamaning ildizlari bilan ustma-ust tushadi.

Natija. $\Delta(\lambda) - 2 = 0$ va $\Delta(\lambda) + 2 = 0$ tenglamalarning ildizlari haqiqiy.

Misol. (1.1) tenglamada $q(x) \equiv 0$ bo‘lsin. U holda

$$c(x,\lambda) = \cos\sqrt{\lambda}x, \quad s(x,\lambda) = \frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}$$

bo‘ladi. Bundan Lyapunov funksiyasi ushbu

$$\Delta(\lambda) = c(\pi,\lambda) + s'(\pi,\lambda) = \cos\sqrt{\lambda}\pi + \cos\sqrt{\lambda}\pi = 2\cos\sqrt{\lambda}\pi$$

tenglik bilan berilishi kelib chiqadi.

$\Delta(\lambda) - 2 = 0$ tenglamaning ildizlarini, ya’ni davriy masalaning xos qiymatlarini topamiz:

$$2\cos\sqrt{\lambda}\pi - 2 = 0,$$

$$\cos\sqrt{\lambda}\pi = 1,$$

$$\pi\sqrt{\lambda} = 2\pi n, \quad n = 0, 1, 2, \ldots ,$$

$$\lambda = (2n)^2, \quad n = 0, 1, 2, \ldots .$$

Demak, bu holda davriy masalaning $\lambda_0 = 0$ xos qiymati karrasiz, qolgan barcha xos qiymatlari ikki karrali ekan: $\lambda_{2n-1} = \lambda_{2n} = (2n)^2, \quad n = 1, 2, \ldots$. Bu xos qiymatlarga quyidagi ortonormallangan xos funksiyalar mos keladi:

$$y_0(x) = \frac{1}{\sqrt{\pi}}, \quad y_{2n-1}(x) = \sqrt{\frac{2}{\pi}}\cos 2nx, \; y_{2n}(x) = \sqrt{\frac{2}{\pi}}\sin 2nx, \; n = 1, 2, \ldots .$$

Endi esa, $\Delta(\lambda) + 2 = 0$ tenglamaning ildizlarini, ya’ni antidavriy masalani xos qiymtalarini topamiz:

$$2\cos\sqrt{\lambda}\pi + 2 = 0, \; \cos\sqrt{\lambda}\pi = -1,$$

$$\sqrt{\lambda}\pi = \pi + 2\pi n,$$
$$\lambda = (2n+1)^2,\ n = 0, 1, 2, \dots\ .$$

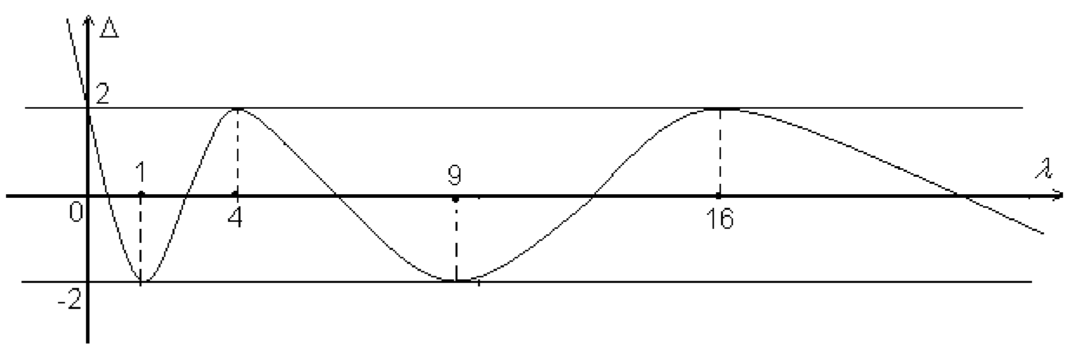
Bu holda antidavriy masalaning barcha xos qiymatlari ikki karrali bo‘ladi:

$$\mu_{2n} = \mu_{2n+1} = (2n+1)^2,\ n = 0, 1, 2, \dots\ .$$

Lyapunov funksiyasining ushbu

$$\Delta(\lambda) = 2\cos\sqrt{\lambda}\pi,\ \ \lambda \ge 0,$$
$$\Delta(\lambda) = 2\,\mathrm{ch}\sqrt{|\lambda|}\pi,\ \ \lambda < 0$$

ifodalaridan uning grafigi quyidagicha bo‘lishi ko‘rinadi:



(1-rasm)

Endi $\Delta(\lambda)$ funksiyaning asimptotikasini o‘rganamiz. Buning uchun avvalo, ushbu

$$\begin{cases} -c'' + q(x)c = \lambda c \\ c(0,\lambda) = 1 \\ c'(0,\lambda) = 0 \end{cases} \tag{1.5}$$

$$\begin{cases} -s'' + q(x)s = \lambda s \\ s(0,\lambda) = 0 \\ s'(0,\lambda) = 1 \end{cases} \tag{1.6}$$

masalalarga ekvivalent bo‘lgan integral tenglamalar tuzib olamiz. Ular quyidagicha bo‘ladi:

$$c(x,\lambda) = \cos\sqrt{\lambda}x + \int_0^x \frac{\sin\sqrt{\lambda}(x-t)}{\sqrt{\lambda}} q(t)c(t,\lambda)dt, \tag{1.7}$$

$$s(x,\lambda)=\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}+\int_0^x\frac{\sin\sqrt{\lambda}(x-t)}{\sqrt{\lambda}}q(t)s(t,\lambda)dt\,. \qquad (1.8)$$

Bulardan hosila olsak,

$$c'(x,\lambda)=-\sqrt{\lambda}\sin\sqrt{\lambda}x+\int_0^x\cos\sqrt{\lambda}(x-t)q(t)c(t,\lambda)dt\,, \qquad (1.9)$$

$$s'(x,\lambda)=\cos\sqrt{\lambda}x+\int_0^x\cos\sqrt{\lambda}(x-t)q(t)s(t,\lambda)dt \qquad (1.10)$$

kelib chiqadi.

Lemma 1.1. $\sqrt{\lambda}=k=\sigma+i\tau$ bo‘lsin, u holda quyidagi asimptotik formulalar o‘rinlidir:

1) $c(x,\lambda)=\underline{\underline{O}}(e^{|\tau|x}),\ \ |k|\to\infty,\ \ x\in[0,\pi]$,

2) $c(x,\lambda)=\cos kx+\underline{\underline{O}}\left(\dfrac{e^{|\tau|x}}{|k|}\right),\ |k|\to\infty,\ \ x\in[0,\pi]$,

3) $c'(x,\lambda)=-k\sin kx+\underline{\underline{O}}(e^{|\tau|x}),\ \ |k|\to\infty,\ \ x\in[0,\pi]$,

4) $s(x,\lambda)=\underline{\underline{O}}\left(\dfrac{e^{|\tau|\chi}}{|\chi|}\right),|\chi|\to\infty,\ \ x\in[0,\pi]$,

5) $s(x,\lambda)=\dfrac{\sin kx}{k}+\underline{\underline{O}}\left(\dfrac{e^{|\tau|x}}{|k|^2}\right),\ \ |k|\to\infty,\ \ x\in[0,\pi]$,

6) $s'(x,\lambda)=\cos kx+\underline{\underline{O}}\left(\dfrac{e^{|\tau|x}}{|k|}\right),\ \ |k|\to\infty,\ \ x\in[0,\pi]$.

Natija 1. Ushbu

$$\Delta(\lambda)=2\cos\sqrt{\lambda}\pi+\underline{\underline{O}}\left(\frac{e^{|\mathrm{Im}\sqrt{\lambda}|\pi}}{\sqrt{\lambda}}\right),|\lambda|\to\infty,$$

asimptotik tenglik o‘rinli.

Natija 2. Ushbu

$$\Delta(\lambda)=2\cos\sqrt{\lambda}\pi+\underline{\underline{O}}\left(\frac{1}{\sqrt{\lambda}}\right),\ \lambda\to+\infty$$

asimptotik formula o‘rinli. Demak, $+\infty$ da $\Delta(\lambda)$ funksiyaning qiymatlari $[-2,2]$ kesma atrofida bo‘ladi.

Natija 3. $\lambda < 0$ bo‘lsin, u holda $\sqrt{\lambda} = i\sqrt{|\lambda|}$ bo‘ladi va

$$\Delta(\lambda) = 2ch\sqrt{|\lambda|}\pi + \underline{\underline{O}}\left(\frac{e^{\sqrt{|\lambda|}\pi}}{\sqrt{|\lambda|}}\right), \lambda \to -\infty$$

asimptotik formula bajariladi.

Agar $\lambda = (2m+1)^2$ desak, $\Delta(\lambda) - 2 = -4 + \underline{\underline{O}}\left(\frac{1}{m}\right)$ bo‘ladi. Bu tenglikdan $\Delta(\lambda) - 2 = 0$ tenglama kamida bitta haqiqiy yechimga ega ekanligi ko‘rinadi. Bu tenglamaning eng kichik ildizini λ_0 orqali belgilaymiz.

Teorema 1.3. Ushbu

$$\frac{d\Delta(\lambda)}{d\lambda} = \int_0^{\pi}\left\{c'(\pi,\lambda)s^2(t,\lambda) + [c(\pi,\lambda) - s'(\pi,\lambda)]c(t,\lambda)s(t,\lambda) - s(\pi,\lambda)c^2(t,\lambda)\right\}dt$$

formula o‘rinli.

Teorema 1.4. A) λ son $\Delta(\lambda) = 2$ tenglamaning karrali ildizi bo‘lishi uchun $s(\pi,\lambda) = 0,\ s'(\pi,\lambda) = 1,\ c(\pi,\lambda) = 1,\ c'(\pi,\lambda) = 0$ tengliklar bajarilishi zarur va yetarlidir.

B) λ son $\Delta(\lambda) = -2$ tenglamaning karrali ildizi bo‘lishi uchun

$$s(\pi,\lambda) = 0,\ s'(\pi,\lambda) = -1,\ c(\pi,\lambda) = -1,\ c'(\pi,\lambda) = 0$$

tengliklar bajarilishi zarur va yetarlidir.

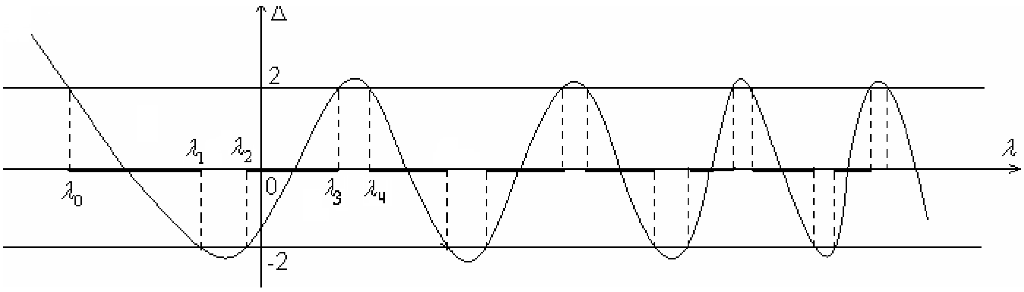
Teorema 1.5. A) Agar λ son $\Delta(\lambda) = 2$ tenglamaning karrali ildizi bo‘lsa, $\ddot{\Delta}(\lambda) < 0$ bo‘ladi. Xususan, bu nuqtada $\Delta(\lambda)$ funksiya lokal maksimumga erishadi.

B) Agar λ son $\Delta(\lambda) = -2$ tenglamaning ilidizi karrali bo‘lsa, $\ddot{\Delta}(\lambda) > 0$ bo‘ladi. Xususan, bu nuqtada $\Delta(\lambda)$ funksiya lokal minimumga erishadi.

Natija 1. $\Delta(\lambda) \pm 2 = 0$ tenglama ildizlarining karrasi ikkidan oshmaydi.

Natija 2. $\Delta(\lambda)-2=0$ tenglamaning eng kichik ildizi karrasizdir. Haqiqatdan ham, agar u karrali bo‘lsa, bu nuqtada lokal maksimum bo‘lar edi, natijada bundan kichik ildiz topilgan bo‘lar edi.

Natija 3. Yuqoridagi teoremalarni hisobga olib, Lyapunov funksiyasining grafigi quyidagicha bo‘lishini ko‘ramiz:



(2-rasm)

Ushbu

$$-y''+q(x+t)y=\lambda y,\ x\in R \tag{1.11}$$

tenglamani ko‘rib chiqamiz, bu yerda t haqiqiy parametr. (1.11) tenglamaning quyidagi $c(0,\lambda,t)=1$, $c'(0,\lambda,t)=0$, $s(0,\lambda,t)=0$, $s'(0,\lambda,t)=1$ boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini $c(x,\lambda,t)$ va $s(x,\lambda,t)$ orqali belgilaymiz. (1.11) tenglama uchun Lyapunov funksiyasi ushbu

$$\Delta(\lambda,t)=c(\pi,\lambda,t)+s'(\pi,\lambda,t)$$

formula bilan beriladi.

Teorema 1.6. Ushbu ayniyat o‘rinli $\Delta(\lambda,t)\equiv\Delta(\lambda)$, ya’ni Lyapunov funksiyasi t parametrga bog‘liq bo‘lmaydi.

Natija. Ma’lumki (1.11) masalaning spektri $-2\le\Delta(\lambda,t)\le 2$ tengsizlik bilan aniqlanadi. Teorema 1.6 ga ko‘ra ushbu tengsizlikning yechimi $-2\le\Delta(\lambda)\le 2$ tengsizlikning yechimi bilan ustma-ust tushadi. Demak, (1.11) masalaning spektri t parametrga bog‘liq emas ekan, uning spektri (1.1) masalaning spektri bilan ustma-ust tushar ekan.

2-§. Floke yechimlari va ularning xossalari.

Teorema 2.1. (Floke). A) $\Delta^2(\lambda)-4\neq 0$ bo‘lganda (1.1) tenglama quyidagi ko‘rinishdagi chiziqli erkli ikkita yechimga ega:

$$\psi_-(x,\lambda)=\rho_-^{\frac{x}{\pi}}\cdot p_-(x,\lambda),\quad \psi_+(x,\lambda)=\rho_+^{\frac{x}{\pi}}\cdot p_+(x,\lambda),$$

bu yerda $p_-(x,\lambda)$ va $p_+(x,\lambda)$ funksiyalar x bo‘yicha π davrli funksiyalar bo‘lib,

$$\rho_\pm=\frac{\Delta(\lambda)\mp\sqrt{\Delta^2(\lambda)-4}}{2};$$

B) Agar $\Delta(\lambda)=2$ bo‘lsa. (1.1) tenglamaning π davrli noldan farqli yechimi mavjud;

C) Agar $\Delta(\lambda)=-2$ bo‘lsa. (1.1) tenglamaning π antidavrli noldan farqli yechimi mavjud.

Isbot. (1.1) tenglamani biror ρ son uchun

$$y(x+\pi)=\rho y(x)$$

shartni qanoatlantiruvchi yechimini axtaramiz. Bu holda umumiy yechim

$$y(x)=C_1c(x,\lambda)+C_2s(x,\lambda)$$

bo‘ladi. $q(x)$ funksiya π davrli bo‘lgani uchun $c(x+\pi,\lambda)$ va $s(x+\pi,\lambda)$ funksiyalar ham yechim bo‘ladi. Bu yechimlarni yechimlar fundamental sistemasi bo‘yicha yoyish mumkin:

$$\begin{cases} c(x+\pi,\lambda)=c(\pi,\lambda)c(x,\lambda)+c'(\pi,\lambda)s(x,\lambda)\\ s(x+\pi,\lambda)=s(\pi,\lambda)c(x,\lambda)+s'(\pi,\lambda)s(x,\lambda).\end{cases}$$

Bundan

$$y(x+\pi)=C_1c(x+\pi,\lambda)+C_2s(x+\pi,\lambda)=$$

$$=[C_1c(\pi,\lambda)+C_2s(\pi,\lambda)]c(x,\lambda)+[C_1c'(\pi,\lambda)+C_2s'(\pi,\lambda)]s(x,\lambda)$$

kelib chiqadi. Ushbu $y(x+\pi)=\rho y(x)$ tenglikni ishlatsak,

$$\begin{cases} C_1(c(\pi,\lambda)-\rho)+C_2s(\pi,\lambda)=0\\ C_1c'(\pi,\lambda)+C_2(s'(\pi,\lambda)-\rho)=0\end{cases}$$

hosil bo‘ladi. Ohirgi tenglama noldan farqli yechimga ega bo‘lishi uchun uning asosiy determinanti nolga teng bo‘lishi zarur va yetarlidir:

$$\begin{vmatrix} c(\pi,\lambda)-\rho & s(\pi,\lambda) \\ c'(\pi,\lambda) & s'(\pi,\lambda)-\rho \end{vmatrix} = 0,$$

ya’ni

$$\rho^2 - \Delta(\lambda)\rho + 1 = 0.$$

Oxirgi kvadrat tenglamani yechamiz: $D = \Delta^2(\lambda) - 4$,

$$\rho_{\pm} = \frac{\Delta(\lambda) \mp \sqrt{\Delta^2(\lambda)-4}}{2}$$

A) $\Delta^2(\lambda) - 4 \neq 0$ bo‘lsin. Bu holda ikkita yechim mavjud $\rho_+ \neq \rho_-$. Bu yechimlarning har biriga (1.1) tenglamaning $\psi_+(x,\lambda)$ va $\psi_-(x,\lambda)$ yechimi mos keladi:

$$\psi_+(x+\pi,\lambda) \equiv \rho_+\psi_+(x,\lambda),\ \psi_-(x+\pi,\lambda) \equiv \rho_-\psi_-(x,\lambda).$$

Quyidagi funksiyalarni kiritamiz:

$$p_+(x,\lambda) = \rho_+^{-\frac{x}{\pi}}\psi_+(x,\lambda),\ p_-(x,\lambda) = \rho_-^{-\frac{x}{\pi}}\psi_-(x,\lambda).$$

Bu funksiyalar π davrli bo‘lishini ko‘rsatamiz:

$$p_+(x+\pi,\lambda) = \rho_+^{-\frac{x+\pi}{\pi}}\psi_+(x+\pi,\lambda) = \rho_+^{-\frac{x}{\pi}-1}\rho_+\psi_+(x,\lambda) = \rho_+^{-\frac{x}{\pi}}\psi_+(x,\lambda) = p_+(x,\lambda).$$

Demak,

$$\psi_-(x,\lambda) = \rho_-^{\frac{x}{\pi}} \cdot p_-(x,\lambda), \quad \psi_+(x,\lambda) = \rho_+^{\frac{x}{\pi}} \cdot p_+(x,\lambda),$$

bo‘lar ekan. $C_1 = 1$ bo‘lsin, u holda $C_2 = \dfrac{\rho - c(\pi,\lambda)}{s(\pi,\lambda)}$ bo‘ladi, ya’ni

$$C_2 = \frac{s'(\pi,\lambda) - c(\pi,\lambda) \mp \sqrt{\Delta^2(\lambda)-4}}{2s(\pi,\lambda)}.$$

Demak,

$$\psi_{\pm}(x,\lambda) = c(x,\lambda) + \frac{s'(\pi,\lambda) - c(\pi,\lambda) \mp \sqrt{\Delta^2(\lambda)-4}}{2s(\pi,\lambda)} \cdot s(x,\lambda). \qquad (2.1)$$

Bu yechimlarga Floke yechimlari deyiladi. Bu yechimlar chiziqli erkli bo‘ladi, chunki

$$\begin{vmatrix} \psi_+(0,\lambda) & \psi_-(0,\lambda) \\ \psi'_+(0,\lambda) & \psi'_-(0,\lambda) \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ m_+ & m_- \end{vmatrix} = m_- - m_+ = \frac{s' - c + \sqrt{\Delta^2 - 4}}{2s} -$$

$$- \frac{s' - c - \sqrt{\Delta^2 - 4}}{2s} = \frac{\sqrt{\Delta^2 - 4}}{s} \neq 0.$$

B) $\Delta(\lambda) = 2$ bo‘lsin. U holda

$$\rho^2 - 2\rho + 1 = 0,\ (\rho - 1)^2 = 0,\ \rho_1 = \rho_2 = 1,$$

ya’ni $\psi(x + \pi, \lambda) = \psi(x, \lambda)$ bo‘ladi. Demak, kamida bitta noldan farqli, π davrli yechim mavjud.

C) $\Delta(\lambda) = -2$ bo‘lsin. U holda

$$\rho^2 - 2\rho + 1 = 0,\ (\rho - 1)^2 = 0,\ \rho_+ = \rho_- = 1,$$

ya’ni $\psi(x + \pi, \lambda) = -\psi(x, \lambda)$. Demak, kamida bitta noldan farqli, π anti davrli yechim mavjud. Bu yechim 2π davrga ega. **Teorema 2.1 isbotlandi.**

Teorema 2. 2. A) Agar $|\Delta(\lambda)| > 2$ bo‘lsa,

$$\psi_-(x,\lambda) \in L_2(-\infty, 0),\ \psi_+(x,\lambda) \in L_2(0, \infty)$$

va

$$\psi_+(x,\lambda) \to \infty, x \to \infty,\ \psi_-(x,\lambda) \to \infty, x \to -\infty$$

bo‘ladi.

B) Agar $|\Delta(\lambda)| < 2$ bo‘lsa, $L_2(-\infty, 0)$ yoki $L_2(0, \infty)$ fazolarga qarashli bo‘ladigan noldan farqli yechimlar yo‘q, bu holda tenglamaning ixtiyoriy yechimi chegaralangan.

C) Agar $\Delta(\lambda) = \pm 2$ bo‘lsa, $L_2(-\infty, 0)$ yoki $L_2(0, \infty)$ fazolarga qarashli bo‘ladigan yechim yo‘q. Ammo kamida bitta chegaralangan yechim mavjud. λ soni $\Delta(\lambda) = \pm 2$ tenglamaning karrasiz ildizi bo‘lsa, faqat bitta chegaralangan yechim mavjud bo‘ladi, karrali yechimi bo‘lsa, ixtiyoriy yechim chegaralangan bo‘ladi.

Natija 1. Teorema 2.2 ushbu $\psi_+(x,\lambda)$ va $\psi_-(x,\lambda)$ yechimlar Veyl yechimi ekanligini hamda

$$m_{\pm}(\lambda)=\frac{s'(\pi,\lambda)-c(\pi,\lambda)\pm\sqrt{\Delta^2(\lambda)-4}}{2s(\pi,\lambda)}$$

funksiyalar Veyl-Titchmarsh funksiyalari ekanligini ko‘rsatadi.

Natija 2. Davriy potentsialli Shturm-Liuvill masalasining xos qiymati umuman yo‘q. Spektr uzluksiz bo‘lib,

$$E=\{\lambda\in R|-2\le\Delta(\lambda)\le 2\}$$

bo‘lar ekan. Bu to‘plam sanoqlita kesmalarning birlashmasidan iborat bo‘lishi ravshan:

$$E=[\lambda_0,\lambda_1]\cup[\lambda_2,\lambda_3]\cup...\cup[\lambda_{2n},\lambda_{2n+1}]\cup... \ .$$

Quyidagi

$$(-\infty,\lambda_0)\,,(\lambda_1,\lambda_2)\,,\,(\lambda_3,\lambda_4)\,,\,\ldots,\,(\lambda_{2n-1},\lambda_{2n})\,,\,\ldots$$

intervallarga lakunalar deyiladi. Ushbu $(-\infty,\lambda_0)$ intervalga esa trivial lakuna deyiladi.

3-§. Dubrovin-Trubovits sistemasi va izlar formulalari.

Butun o‘qda berilgan ushbu

$$Ly \equiv -y'' + q(x)y = \lambda y,\ -\infty < x < \infty, \tag{3.1}$$

Shturm-Liuvill masalasini ko‘rib chiqamiz. Bu yerda $q(x)$ haqiqiy uzluksiz π-davrli funksiya. $c(x,\lambda)$ va $s(x,\lambda)$ orqali (3.1) tenglamaning $c(0,\lambda)=1$, $c'(0,\lambda)=0$ va $s(0,\lambda)=0$, $s'(0,\lambda)=1$ boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini belgilaymiz. L operatorning spektri quyidagi to‘plamdan iborat bo‘ladi

$$E = \{\lambda \in R:\ -2 \le \Delta(\lambda) \le 2\} = R \setminus \left\{(-\infty,\lambda_0) \cup \bigcup_{n=1}^{\infty}(\lambda_{2n-1},\lambda_{2n})\right\}.$$

(3.1) tenglama uchun $[0,\pi]$ oraliqda qo‘yilgan Dirixle masalasining xos qiymatlarini ξ_n, $n \ge 1$ orqali belgilaymiz. Ular $s(\pi,\lambda)=0$ tenglamaning yechimlaridan iborat bo‘ladi hamda $\xi_n \in [\lambda_{2n-1},\lambda_{2n}]$, $n \ge 1$ munosabat bajariladi.

Ta’rif 1. Ushbu $\{\xi_n\}_{n=1}^{\infty}$ sonlar ketma-ketligi va $\sigma_n \equiv sign\{s'(\pi,\xi_n) - c(\pi,\xi_n)\} = \pm 1,\ n \ge 1$, ishoralar ketma-ketligiga L operatorning spektral parametrlari deyiladi.

Ta’rif 2. Spektrning chetki nuqtalari va spektral parametrlardan tashkil topgan $\{\lambda_n,\ n \ge 0,\ \xi_n,\ \sigma_n = \pm 1,\ n \ge 1\}$ to‘plamga L operatorning spektral berilganlari deyiladi.

Shturm-Liuvill operatorining spektral berilganlari orqali $q(x)$ potensialini topish masalasiga teskari spektral masala deyiladi.

Agar biz ushbu

$$L(\tau)y = -y'' + q(x+\tau)y = \lambda y,\ x \in R,\ \tau \in R,$$

siljigan argumentli Shturm-Liuvill tenglamasini qaraydigan bo‘lsak, uning spektri τ parametrga bo‘g‘liq bo‘lmaydi, ya’ni $\sigma(L(\tau)) = \sigma(L(0))$ bo‘ladi, ammo uning spektral parametrlari τ parametrga bog‘liq bo‘ladi: $\xi_n(\tau),\ \sigma_n(\tau),\ n \ge 1$. Bu spektral parametrlar quyidagi differensial tenglamalar sistemasini qanoatlantiradi

$$\frac{d\xi_n}{d\tau} = 2(-1)^{n-1}\sigma_n(\tau)h_n(\xi),\ n \geq 1. \tag{3.2}$$

Bu yerda

$$h_n(\xi) = \sqrt{(\xi_n - \lambda_{2n-1})(\lambda_{2n} - \xi_n)} \times \sqrt{(\xi_n - \lambda_0)\prod_{\substack{k=1\\k\neq n}}^{\infty}\frac{(\lambda_{2k-1} - \xi_n)(\lambda_{2k} - \xi_n)}{(\xi_k - \xi_n)^2}}.$$

Bu differensial tenglamalar sisitemasi chekli zonali potensiallar holida 1975 yilda B.A.Dubrovin tomonidan, davriy potensiallar holida 1977 yilda E.Trubovits tomonidan va cheksiz zonali deyarli davriy potensiallar holida B.M.Levitan tomonidan keltirib chiqarilgan.

Dubrovin-Trubovits tenglamalar sistemasi (3.2) va ushbu

$$q(\tau) = \lambda_0 + \sum_{k=1}^{\infty}(\lambda_{2k-1} + \lambda_{2k} - 2\xi_k(\tau)) \tag{3.3}$$

izlar formulasi birgalikda Shturm-Liuvill operatori uchun qo‘yilgan teskari masalani yechish usulini beradi. Bundan tashqari quyidagi izlar formulasi ham muhim o‘rin egallaydi:

$$q^2(\tau) - \frac{1}{2}q_{\tau\tau}(\tau) = \lambda_0^2 + \sum_{k=1}^{\infty}(\lambda_{2k-1}^2 + \lambda_{2k}^2 - 2\xi_k^2(\tau)).$$

1946 yilda shved matematigi G.Borg tomonidan quyidagi ajoyib teorema isbot qilingan. 1977 yilda X.Xoxshtadt bu teoremani sodda isbotini topishga muvaffaq bo‘ldi.

Teorema 3.1. (G.Borg). $\frac{\pi}{2}$ soni Xill tenglamasi $q(x)$ potensialining davri bo‘lishi uchun $\Delta(\lambda) + 2 = 0$ tenglamaning barcha ildizlari, ya’ni antidavriy chegaraviy masalaning barcha xos qiymatlari ikki karrali bo‘lishi zarur va yetarlidir.

Borg teoremasini lakunalar tilida ham bayon qilish mumkin: $\frac{\pi}{2}$ soni Xill tenglamasi $q(x)$ potensialining davri bo‘lishi uchun Xill tenglamasi barcha toq nomerli lakunalarining yopilishi zarur va yetarlidir.

Yuqorida bayon qilingan G.Borg teoremasi 1984 yilda X.Xoxshtadt tomonidan umumlashtirildi.

Teorema 3.2. (X.Xoxshtadt). $\frac{\pi}{n}$ soni Xill tenglamasi π davrli $q(x)$ potensialining davri bo'lishi uchun nomerlari n ga karrali bo'lmagan barcha lakunalarning yopilishi zarur va yetarli. Bu yerda $n \geq 2$ natural son.

1977 yilda Dubrovin-Trubovits sistemasi va izlar formulasi yordamida E.Trubovits Xill operatori lakunalari uzunliklari va $q(x)$ potensialning analitikligi orasidagi bog'lanish haqidagi quyidagi teoremani isbotlashga muvaffaq bo'ldi.

Teorema 3.3. (**E.Trubovits**) π - davrli haqiqiy $q(x)$ potensial analitik bo'lishi uchun Xill operatori lakunalarning uzunliklari eksponensial ravishda nolga intilishi, ya'ni shunday $a > 0$ va $b > 0$ musbat sonlar topilib

$$\lambda_{2n} - \lambda_{2n-1} < ae^{-bn}, \; n = 1, 2, 3, ...$$

tengsizlik bajarilishi zarur va yetarli.

4-§. Yuklangan hadli Korteveg-de Friz tenglamasini davriy funksiyalar sinfida integrallash

Quyidagi

$$q_t = q_{xxx} - 6qq_x + \gamma(t) \cdot q\big|_{x=0} \cdot q_x, \quad x \in R,\ t > 0 \tag{4.1}$$

yuklangan hadli Korteveg-de Friz tenglamasini ushbu

$$q(x,t)\big|_{t=0} = q_0(x) \tag{4.2}$$

boshlang‘ich shart bilan birga ko‘rib chiqamiz. Bu yerda $\gamma(t)$ - berilgan haqiqiy uzluksiz funksiya. (4.1) tenglamaning (4.2) boshlang‘ich shartni va ushbu

$$q(x,t) \in C_x^3(t>0) \cap C_t^1(t>0) \cap C(t \geq 0) \tag{4.3}$$

silliqlik shartlarini qanoatlantiruvchi, x o‘zgaruvchi bo‘yicha π davrli

$$q(x+\pi,t) = q(x,t) \tag{4.4}$$

haqiqiy $q = q(x,t)$ yechimini topish talab qilinadi.

(4.1)-(4.4) masalaning $q(x,t)$ yechimini topishga Shturm-Liuvill operatori uchun qo‘yilgan to‘g‘ri va teskari spektral masalani qo‘llaymiz.

Teorema 4.1. $q(x,t)$ funksiya (4.1)-(4.4) masalaning yechimi bo‘lsin. U holda ushbu

$$L(\tau,t)y \equiv -y'' + q(x+\tau,t)y = \lambda y, \quad x \in R \tag{4.5}$$

Shturm-Liuvill operatorining spektri τ va t parametrlarga bog‘liq bo‘lmaydi, ya’ni spektrning chetki nuqtalari λ_n, $n \geq 0$ lar τ va t parametrlarga bog‘liq bo‘lmaydi, $\xi_n(\tau,t)$, $n \geq 1$ spektral parametrlari esa quyidagi Dubrovin-Trubovits tenglamalar sistemasini qanoatlantiradi:

$$\frac{\partial \xi_n}{\partial t} = 2(-1)^n \sigma_n(\tau,t)[2q(\tau,t) - \gamma(t)q(0,t) + 4\xi_n]h_n(\xi), \quad n \geq 1. \tag{4.6}$$

Bu yerda

$$h_n(\xi) = \sqrt{(\xi_n - \lambda_{2n-1})(\lambda_{2n} - \xi_n)} \times \sqrt{(\xi_n - \lambda_0)\prod_{\substack{k=1 \\ k \neq n}}^{\infty} \frac{(\lambda_{2k-1} - \xi_n)(\lambda_{2k} - \xi_n)}{(\xi_k - \xi_n)^2}}$$

bo‘lib, $\sigma_n(\tau,t)$ ishoralar $\xi_n(\tau,t)$ spektral parametr o‘zining $[\lambda_{2n-1},\lambda_{2n}]$ lakunasi chetiga kelganida qarama-qarshi ishoraga o‘zgaradi. Bundan tashqari ushbu

$$\xi_n(\tau,t)\big|_{t=0}=\xi_n^0(\tau),\ \ \sigma_n(\tau,t)\big|_{t=0}=\sigma_n^0(\tau)\ ,\quad n\ge 1 \tag{4.7}$$

boshlang‘ich shartlar bajariladi. Bu yerda $\xi_n^0(\tau)$, $\sigma_n^0(\tau)$ $n\ge 1$ orqali $q_0(x+\tau)$ potensialga mos keluvchi spectral berilganlar belgilangan.

Isbot. (4.5) tenglama uchun qo‘yilgan

$$y(0)=0,\ \ y(\pi)=0$$

Dirixle masalasining $\xi_n=\xi_n(\tau,t)$, $n\ge 1$ xos qiymatlariga mos keluvchi ortonormollangan xos funksiyalarini $y_n=y_n(x,\tau,t)$, $n\ge 1$ orqali belgilaymiz.

Ushbu $(L(\tau,t)y_n,y_n)=\xi_n$ ayniyatni t bo‘yicha differensiallab hamda $L(\tau,t)$ operatorning simmetrikligidan foydalanib, quyidagiga ega bo‘lamiz:

$$\begin{aligned}\dot{\xi}_n&=(L(\tau,t)\dot{y}_n+q_t y_n,y_n)+(L(\tau,t)y_n,\dot{y}_n)=\\&=(\dot{y}_n,L(\tau,t)y_n)+(L(\tau,t)y_n,\dot{y}_n)+(q_t y_n,y_n)=\\&=\xi_n(\dot{(y_n,y_n)})+(q_t y_n,y_n)=\int_0^\pi q_t y_n^2dx.\end{aligned} \tag{4.8}$$

Ushbu

$$q_t(x+\tau,t)=q_{xxx}(x+\tau,t)-6q(x+\tau,t)q_x(x+\tau,t)+\gamma(t)\cdot q(0,t)\cdot q_x(x+\tau,t)$$

ayniyatdan foydalanib, (4.8) tenglikni quyidagi tarzda yozib olamiz

$$\dot{\xi}_n=\int_0^\pi[q_{xxx}-6qq_x+\gamma(t)\cdot q(0,t)\cdot q_x]y_n^2dx. \tag{4.9}$$

Integral ostidagi funksiyaning boshlang‘ichini y_n va y_n' ga nisbatan kvadratik forma ko‘rinishida izlaymiz:

$$(ay_n^2+by_ny_n'+cy_n'^2)'=[q_{xxx}-6qq_x+\gamma(t)q(0,t)q_x]y_n^2. \tag{4.10}$$

Bu yerda a, b, c koeffitsientlar y_n va y_n' ga bog‘liq emas. Agar (4.10) tenglikning chap tomonini hisoblab, $y_n''=(q-\xi_n)y_n$ ayniyatdan foydalansak, hamda mos koeffitsiyentlarni o‘zaro tenglasak, quyidagi tengliklar kelib chiqadi

$$b = -c',$$

$$a = \frac{1}{2}c'' - c\cdot(q-\xi_n),$$

$$\frac{1}{2}c''' - 2c'\cdot(q-\xi_n) - cq_x = q_{xxx} - 6qq_x + \gamma(t)q(0,t)q_x. \qquad (4.11)$$

Ushbu $c = 2q+\alpha$ funksiya (4.11) tenglikni qanoatlantirishi ravshan. Bunda $\alpha = 4\xi_n - \gamma(t)q(0,t)$. Bunga ko'ra

$$a = q_{xx} - [2q + 4\xi_n - \gamma(t)q(0,t)]\cdot(q-\xi_n),$$

$$b = -2q_x,$$

$$c = 2q + 4\xi_n - \gamma(t)q(0,t)$$

deb olsak, (4.10) tenglik bajariladi. Demak,

$$\dot{\xi}_n = (ay_n^2 + by_ny_n' + cy_n'^2)\Big|_0^\pi =$$

$$= [2q(\tau,t) - \gamma(t)q(0,t) + 4\xi_n][y_n'^2(\pi,\tau,t) - y_n'^2(0,\tau,t)]. \qquad (4.12)$$

(4.5) tenglamaning $s(0,\lambda,\tau,t)=0$, $s'(0,\lambda,\tau,t)=1$ boshlang'ich shartlarni qanoatlantiruvchi yechimini $s(x,\lambda,\tau,t)$ orqali belgilaymiz. U holda ushbu

$$y_n(x,\tau,t) = \frac{1}{c_n(\tau,t)}s(x,\xi_n,\tau,t)$$

tenglik o'rinli bo'ladi. Bu yerda

$$c_n^2(\tau,t) \equiv \int_0^\pi s^2(x,\xi_n,\tau,t)dx = s'(\pi,\xi_n,\tau,t)\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}.$$

Bunga muvofiq

$$y_n'^2(\pi,\tau,t) - y_n'^2(0,\tau,t) = \frac{1}{\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}}\left(s'(\pi,\xi_n,\tau,t) - \frac{1}{s'(\pi,\xi_n,\tau,t)}\right).$$

Bu yerga

$$s'(\pi,\xi_n,\tau,t) - \frac{1}{s'(\pi,\xi_n,\tau,t)} = \sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n)-4}$$

ifodani qo'ysak,

$$y_n'^2(\pi,\tau,t) - y_n'^2(0,\tau,t) = \frac{\sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n)-4}}{\dfrac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}}$$

kelib chiqadi. Bu yerda

$$\sigma_n(\tau,t) = sign\left\{ s'(\pi,\xi_n,\tau,t) - \frac{1}{s'(\pi,\xi_n,\tau,t)} \right\}.$$

Agar ushbu

$$\Delta^2(\lambda) - 4 = 4\pi^2(\lambda_0 - \lambda)\prod_{k=1}^{\infty}\frac{(\lambda_{2k-1}-\lambda)(\lambda_{2k}-\lambda)}{k^4},$$

$$s(\pi,\lambda,\tau,t) = \pi\prod_{k=1}^{\infty}\frac{\xi_k - \lambda}{k^2}$$

yoyilmalarni ishlatsak,

$$y_n'^2(\pi,\tau,t) - y_n'^2(0,\tau,t) = 2(-1)^n\sigma_n(\tau,t)h_n(\xi) \tag{4.13}$$

hosil bo‘ladi. Bu ifodani (4.12) tenglikka qo‘ysak, (4.6) sistema kelib chiqadi.

$L(\tau,t)$ operator spektrining chetki nuqtalari davriy va antidavriy masalalarning λ_n, $n \geq 0$ xos qiymatlaridan iborat ekanligi bizga yaxshi ma’lum. Shuning uchun davriy va antidavriy masalalarning λ_n, $n \geq 0$ xos qiymatlari t ga bog‘liq emasligini isbotlaymiz. $v_n(x,\tau,t)$, $n \geq 0$ orqali davriy yoki antidavriy masalaning normallangan hos funksiyalarini belgilasak, yuqoridagi usul bilan ushbu

$$\dot{\lambda}_n = 0, \; n \geq 0$$

tenglikni hosil qilamiz. Bu esa $L(\tau,t)$ operator spektrining chetki nuqtalari t parametrga bog‘liq emasligini bildiradi. **Teorema isbotlandi.**

Izoh 4.1. Ushbu

$$q(\tau,t) = \lambda_0 + \sum_{k=1}^{\infty}(\lambda_{2k-1} + \lambda_{2k} - 2\xi_k(\tau,t)) \tag{4.14}$$

izlar formulasi yordamida (4.6) sistemani “yopiq” ko‘rinishda yozish mumkin.

Natija 4.1. Yuqorida tuzilgan $q(\tau,t)$ funksiya (4.1) tenglamani qanoatlantirishini ko‘rsatamiz. Buning uchun ushbu

$$\frac{\partial \xi_n}{\partial \tau} = 2(-1)^{n-1}\sigma_n(\tau,t)h_n(\xi), \quad n \ge 1 \tag{4.15}$$

Dubrovin-Trubovits sistemasini hamda quyidagi izlar formulasini ham ishlatamiz

$$q^2(\tau,t) - \frac{1}{2}q_{\tau\tau}(\tau,t) = \lambda_0^2 + \sum_{k=1}^{\infty}(\lambda_{2k-1}^2 + \lambda_{2k}^2 - 2\xi_k^2(\tau,t)). \tag{4.16}$$

(4.6) va (4.16) Dubrovin-Trubovits sistemalariga ko‘ra

$$\frac{\partial \xi_k}{\partial t} = -[2q(\tau,t) - \gamma(t)q(0,t) + 4\xi_k]\frac{\partial \xi_k}{\partial \tau}, \quad k \ge 1. \tag{4.17}$$

(4.14) izlar formulasi hamda (4.17) tenglikdan foydalanib quyidagini keltirib chiqaramiz:

$$q_t = -2\sum_{k=1}^{\infty}\frac{\partial \xi_k}{\partial t} = 4q\sum_{k=1}^{\infty}\frac{\partial \xi_k}{\partial \tau} - 2\gamma(t)q(0,t)\sum_{k=1}^{\infty}\frac{\partial \xi_k}{\partial \tau} + 8\sum_{k=1}^{\infty}\xi_k\frac{\partial \xi_k}{\partial \tau}. \tag{4.18}$$

Agar (4.14) va (4.16) izlar formulalaridan τ bo‘yicha hosila olsak, ushbu

$$2\sum_{k=1}^{\infty}\frac{\partial \xi_k}{\partial \tau} = -q_\tau, \quad 4\sum_{k=1}^{\infty}\xi_k\frac{\partial \xi_k}{\partial \tau} = \frac{1}{2}q_{\tau\tau\tau} - 2qq_\tau$$

tengliklar kelib chiqadi. Bu ifodalarni (4.18) ga qo‘yamiz:

$$q_t = -2qq_\tau + \gamma(t)q(0,t)q_\tau + q_{\tau\tau\tau} - 4qq_\tau,$$

$$q_t = q_{\tau\tau\tau} - 6qq_\tau + \gamma(t)q(0,t)q_\tau.$$

Natija 4.2. Bu teorema (4.1)-(4.4) masalani yechish usulini beradi. Buning uchun avvalo ushbu

$$-y'' + q_0(x+\tau)y = \lambda y, \quad x \in R$$

tenglama uchun

$$\lambda_n, \ n \ge 0, \ \xi_n^0(\tau), \ \sigma_n^0(\tau), \ n \ge 1$$

spektral berilganlarni topib olamiz. So‘ngra (4.6)+(4.7) Koshi masalasiga $\tau = 0$ ni qo‘yib, hosil bo‘lgan masalaning $\xi_n(0,t)$, $n \ge 1$ yechimini topamiz. Shundan keyin, $\xi_n(0,t)$, $n \ge 1$ uchun topilgan ifodani (4.6) sistemaga qo‘yamiz hamda (4.6)+(4.7) Koshi masalasini ixtiyoriy τ da yechib $\xi_n(\tau,t)$, $n \ge 1$ larni aniqlaymiz. Bu ifodalarni (4.14) izlar formulasiga qo‘yib $q(\tau,t)$ yechimni topamiz.

Natija 4.3. Agar boshlang‘ich shartdagi $q_0(x)$ funksiya haqiqiy analitik funksiya bo‘lsa, u holda teorema 3.3 ga ko‘ra unga mos keluvchi lakunalar uzunliklari eksponensial ravishda nolga intiladi, bu lakunalar $q(x,t)$ funksiyaga ham mos keladi. Shuning uchun $q(x,t)$ yechim x o‘zgaruvchi bo‘yicha haqiqiy analitik funksiya bo‘ladi.

Natija 4.4. Agar boshlang‘ich shartdagi $q_0(x)$ funksiya $\frac{\pi}{n}$ davrga ega bo‘lsa, u holda Borg teskari teoremasining umumlashmasiga ko‘ra (teorema 3.2) nomerlari n ga karrali bo‘lmagan barcha lakunalar yo‘qoladi, bu lakunalar $q(x,t)$ koefitsiyentga ham mos keladi. Shuning uchun $q(x,t)$ yechim x o‘zgaruvchi bo‘yicha $\frac{\pi}{n}$ davrga ega bo‘ladi. Bu yerda $n \geq 2$ natural son va $(\lambda_{2k-1}, \lambda_{2k})$ lakunaning nomeri k.

I bob bo‘yicha xulosalar.

Ushbu bobda davriy koeffitsientli Shturm-Liuvill operatori uchun qo‘yilgan teskari spektral masala yordamida, yuklangan hadli Korteveg-de Friz tenglamasi uchun qo‘yilgan Koshi masalasi davriy funksiyalar sinifida yechilgan. Jumladan, Shturm-Liuvill operatori potensiali yuklangan hadli Korteveg-de Friz tenglamasining x o‘zgaruvchi bo‘yicha davriy yechimi bo‘lsa, u holda bu operatorning spektri t parametrga bog‘liq emasligi ko‘rsatilgan. Bu Shturm-Liuvill operatori spektral parametrlarining t parametr bo‘yicha evolyutsiyasi topilgan. Bundan tashqari, yechimning x bo‘yicha davri haqida va x bo‘yicha analitikligi haqida muhim natijalar olingan.

II-BOB. YUKLANGAN HADLI NOCHIZIQLI SHREDINGER TENGLAMASINI DAVRIY FUNKSIYALAR SINFIDA INTEGRALLASH

Quyidagi Dirak operatori

$$D = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \frac{d}{dx} + \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}, \qquad -\infty < x < \infty$$

matematika va fizikada juda keng qo‘llaniladi. Bu holda $u(x) = q(x) - ip(x)$ funksiyaga potensial deyiladi. Berilgan potensial bo‘yicha bu operatorning spektral harakteristikalarini topish masalasiga to‘g‘ri masala, aksincha spektral harakteristikalar orqali potensialni topish masalasiga teskari masala deyiladi.

K.Gardner, J.Grin, M.Kruskal, R.Miura larning bir qator ishlaridan keyin bu operatorlarga bo‘lgan qiziqish yanayam ortdi. Nochiziqli Shredinger

$$u_t = -iu_{xx} + 2iu|u|^2$$

va modifitsirlangan Korteveg-de Friz

$$u_t = 6u^2 u_x - u_{xxx}$$

tenglamalari uchun davriy boshlang‘ich shartli Koshi masalasini o‘rganish, davriy potensialli Dirak operatori uchun to‘g‘ri va teskari masalalarni o‘rganishga olib keladi. Bu holatda operatorlarning spektri zonali tuzilishga ega bo‘ladi.

Davriy koeffisientli operatorlar uchun teskari masala ancha murakkab, chunki spectral berilganlardagi ozgina o‘zgarish potensialning davriyligini buzishi mumkin. Shturm-Liuvill operatorining chekli zonali potensiali kvazidavriy funksiya bo‘lishi S.P.Novikov tomonidan isbot qilingan, A.R.Its va V.B.Matveev tomonidan esa chekli zonali potensiallar uchun yaqqol formula ham topilgan.

Davriy potensialli Dirak operatori uchun to‘g‘ri va teskari masalalar B.M.Levitan, M.Z.Zamonov, A.B.Hasanov, A.M.Ibragimov va boshqalar tomonidan o‘rganilgan.

Nochiziqli Shredinger tenglamasi, tez kamayuvchi funksiyalar sinfida, V.E.Zaxarov, Shabat A.B., Manakov S.V. lar tomonidan integrallangan. Davriy funksiyalar sinfida bu masala Its A.R., Kotlyarov V.P., Dubrovin B.A., Matveev V.B. va boshqalar tomonidan o‘rganilgan.

V.K.Mel’nikov, A.B.Hasanov, G.U.Urazboyev, A.A.Reyimberganovlarning hamda Hitoy matematiklari Y.B.Zeng, W.X.Ma, R.L.Lin, Y.Shao ishlarida moslangan manbali nochiziqli Sredinger tenglamasi tez kamayuvchi funksiyalar sinfida integrallangan, davriy funksiyalar sinfida esa bu masala A.B.Yaxshimuratov tomonidan o‘rganilgan.

Ushbu magistrlik dissertatsiyaning II-bobida yuklangan hadli nochiziqli Shredinger sistemasining x o‘zgaruvchi bo‘yicha davriy bo‘lgan yechimini topish usuli o‘rganilgan. Bunda Dirak operatori uchun qo‘yilgan teskari spektral masalalar usuli qo‘llanilgan.

1-§. Dirak sistemasi uchun Lyapunov funksiyasi va uning xossalari

Quyidagi Dirak sistemasini ko‘rib chiqamiz

$$Ly \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad x \in (-\infty, \infty). \qquad (1.1)$$

Bu yerda $p(x)$ va $q(x)$ haqiqiy uzluksiz π davrli funksiyalar, λ esa kompleks parametr. (1.1) tenglamaning ushbu

$$c(0,\lambda) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad s(0,\lambda) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini

$$c(x,\lambda) = \begin{pmatrix} c_1(x,\lambda) \\ c_2(x,\lambda) \end{pmatrix} \quad \text{va} \quad s(x,\lambda) = \begin{pmatrix} s_1(x,\lambda) \\ s_2(x,\lambda) \end{pmatrix}$$

orqali belgilaymiz.

$c(x,\lambda)$ va $s(x,\lambda)$ vektor-funksiyalar quyidagi integral tenglamalarni qanoatlantiradi:

$$c(x,\lambda)=\begin{pmatrix}1\\0\end{pmatrix}+\int_0^x\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}p(t)-\lambda & q(t)\\ q(t) & -p(t)-\lambda\end{pmatrix}c(t,\lambda)dt,$$

$$s(x,\lambda)=\begin{pmatrix}0\\1\end{pmatrix}+\int_0^x\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}p(t)-\lambda & q(t)\\ q(t) & -p(t)-\lambda\end{pmatrix}s(t,\lambda)dt\,.$$

Bu integral tenglamalardan $c(x,\lambda)$ va $s(x,\lambda)$ yechimlar mavjudligi, yagonaligi va x ning har bir fiksirlangan qiymatida λ parametrga nisbatan butun funksiya bo‘lishi kelib chiqadi. Bu yechimlar, (1.1) tenglamaning yechimlar fundamental sistemasini tashkil qiladi hamda bu yechimlar uchun quyidagi Vronskiy ayniyati bajariladi:

$$c_1(x,\lambda)s_2(x,\lambda)-c_2(x,\lambda)s_1(x,\lambda)=1\,.$$

Ta’rif 1. $\Delta(\lambda)=c_1(\pi,\lambda)+s_2(\pi,\lambda)$ funksiyaga Dirak operatori uchun Lyapunov funksiyasi yoki Xill diskriminanti deyiladi.

Teorema 1.1. a) $[0,\pi]$ kesmada Dirak sistemasi uchun qo‘yilgan davriy $y_1(0)=y_1(\pi)$, $y_2(0)=y_2(\pi)$ chegaraviy shartli masalaning xos qiymatlari haqiqiy bo‘ladi va ular $\Delta(\lambda)-2=0$ tenglamaning ildizlari bilan ustma-ust tushadi;

b) $[0,\pi]$ kesmada Dirak sistemasi uchun qo‘yilgan antidavriy $y_1(0)=-y_1(\pi)$, $y_2(0)=-y_2(\pi)$ chegaraviy shartli masalaning xos qiymatlari haqiqiy bo‘ladi va ular $\Delta(\lambda)+2=0$ tenglamaning ildizlari bilan ustma-ust tushadi.

Natija 1. Ushbu $\Delta(\lambda)-2=0$ va $\Delta(\lambda)+2=0$ tenlamalarning ildizlari haqiqiy bo‘ladi.

Misol. Agar $p(x)\equiv 0$, $q(x)\equiv 0$ bo‘lsa, u holda

$$c(x,\lambda)=\begin{pmatrix}\cos\lambda x\\ \sin\lambda x\end{pmatrix},\quad s(x,\lambda)=\begin{pmatrix}-\sin\lambda x\\ \cos\lambda x\end{pmatrix},\quad \Delta(\lambda)=2\cos\lambda\pi$$

bo‘ladi. $\Delta(\lambda)-2=0$ tenglamaning ildizlari $\lambda_{4n-1}=\lambda_{4n}=2n,\quad n\in Z$ bo‘ladi; $\Delta(\lambda)+2=0$ tenglamaning ildizlari $\lambda_{4n+1}=\lambda_{4n+2}=2n+1,\quad n\in Z$ bo‘ladi.

Ushbu

$$c(x,\lambda)=\begin{pmatrix}\cos\lambda x\\ \sin\lambda x\end{pmatrix}+\int_0^x\begin{pmatrix}\sin\lambda(x-t) & \cos\lambda(x-t)\\ -\cos\lambda(x-t) & \sin\lambda(x-t)\end{pmatrix}\begin{pmatrix}p(t) & q(t)\\ q(t) & -p(t)\end{pmatrix}c(t,\lambda)dt,$$

$$s(x,\lambda)=\begin{pmatrix}-\sin\lambda x\\ \cos\lambda x\end{pmatrix}+\int_0^x\begin{pmatrix}\sin\lambda(x-t) & \cos\lambda(x-t)\\ -\cos\lambda(x-t) & \sin\lambda(x-t)\end{pmatrix}\begin{pmatrix}p(t) & q(t)\\ q(t) & -p(t)\end{pmatrix}s(t,\lambda)dt$$

integral tenglamalar yordamida $c(x,\lambda)$ va $s(x,\lambda)$ yechimlarning katta $|\lambda|$ lardagi asimptotikasini o‘rganish mumkin. Bunda quyidagi asimptotik formulalar kelib chiqadi:

$$c(x,\lambda)=\begin{pmatrix}\cos\lambda x\\ \sin\lambda x\end{pmatrix}+\underline{\underline{O}}\left(\frac{e^{|\mathrm{Im}\lambda|x}}{\lambda}\right),\ |\lambda|\to\infty,\ x\in[0,\pi],$$

$$s(x,\lambda)=\begin{pmatrix}-\sin\lambda x\\ \cos\lambda x\end{pmatrix}+\underline{\underline{O}}\left(\frac{e^{|\mathrm{Im}\lambda|x}}{\lambda}\right),\ |\lambda|\to\infty,\ x\in[0,\pi].$$

Bulardan, hususan, Lyapunov funksiyasining haqiqiy λ lardagi asimptotikasi kelib chiqadi:

$$\Delta(\lambda)=2\cos\lambda\pi+\underline{\underline{O}}\left(\frac{1}{\lambda}\right),\qquad \lambda\to\infty.$$

Teorema 1.2. Lyapunov funksiyasining hosilasi uchun quyidagi formula o‘rinli:

$$\frac{d\Delta(\lambda)}{d\lambda}=-\int_0^\pi\{c_2(\pi,\lambda)s_1^2(t,\lambda)+[c_1(\pi,\lambda)-s_2(\pi,\lambda)]c_1(t,\lambda)s_1(t,\lambda)-s_1(\pi,\lambda)c_1^2(t,\lambda)+$$
$$+c_2(\pi,\lambda)s_2^2(t,\lambda)+[c_1(\pi,\lambda)-s_2(\pi,\lambda)]c_2(t,\lambda)s_2(t,\lambda)-s_1(\pi,\lambda)c_2^2(t,\lambda)\}dt. \tag{1.2}$$

Teorema 1.3. Agar haqiqiy λ son uchun ushbu $-2<\Delta(\lambda)<2$ tengsizlik bajarilsa, u holda bu son uchun quyidagi tengsizliklar ham o‘rinli bo‘ladi:

$$\frac{d\Delta(\lambda)}{d\lambda}\cdot c_2(\pi,\lambda)<0,\ \frac{d\Delta(\lambda)}{d\lambda}\cdot s_1(\pi,\lambda)>0.$$

Natija 1.2. Ushbu $-2<\Delta(\lambda)<2$ tasmada Lyapunov funksiyasing ekstremal qiymatlari yo‘q.

Teorema 1.4. a) λ son $\Delta(\lambda)=2$ tenglamaning ikki karrali ildizi bo‘lishi uchun quyidagi tengliklar bajarilishi zarur va yetarlidir:

$$s_1(\pi,\lambda)=0,\ s_2(\pi,\lambda)=1,\ c_1(\pi,\lambda)=1, c_2(\pi,\lambda)=0;$$

b) λ son $\Delta(\lambda)=-2$ tenglamaning ikki karrali ildizi bo‘lishi uchun quyidagi tengliklar bajarilishi zarur va yetarlidir:

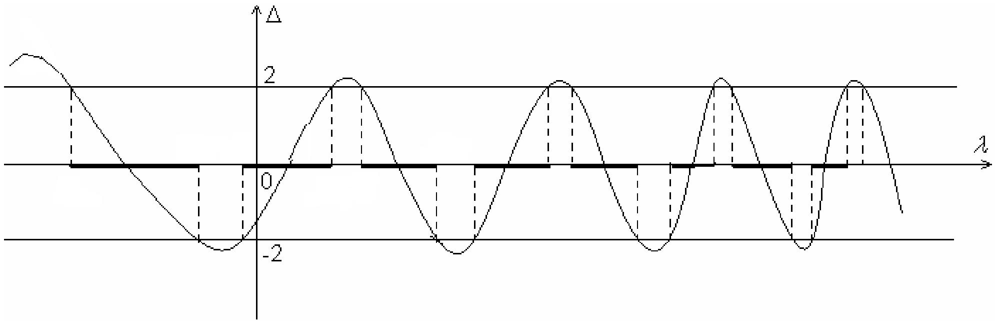
$$s_1(\pi,\lambda)=0,\ s_2(\pi,\lambda)=-1,\ c_1(\pi,\lambda)=-1,\ c_2(\pi,\lambda)=0.$$

Teorema 1.5. a) agar λ son $\Delta(\lambda)=2$ tenglamaning ikki karrali ildizi bo‘lsa, u holda $\ddot{\Delta}(\lambda)<0$ bo‘ladi. Hususan, bu nuqtada Lyapunov funksiyasi lokal maksimumga erishadi.

b) agar λ son $\Delta(\lambda)=-2$ tenglamaning ikki karrali ildizi bo‘lsa, u holda $\ddot{\Delta}(\lambda)>0$ bo‘ladi. Hususan, bu nuqtada Lyapunov funksiyasi lokal minimumga erishadi.

Natija 1.3. Ushbu $\Delta(\lambda)\pm 2=0$ tenglamalar ildizlarining karrasi ikkidan oshmaydi.

Yuqorida keltirilgan teoremalar va $\Delta(\lambda)$ Lyapunov funksiyasining asimptotikasiga ko‘ra, umuman olganda $\Delta(\lambda)$ funksiyaning grafigi quyidagi ko‘rinishda bo‘lishi kelib chiqadi:



(1-rasm)

$c(x+\pi,\lambda)$ va $s(x+\pi,\lambda)$ funksiyalar ham (1.1) tenglamani qanoatlantirishidan quyidagi lemma kelib chiqadi.

Lemma 1.1. (1.1) sistemaning $c(x,\lambda)$ va $s(x,\lambda)$ yechimlari uchun quyidagi ayniyatlar bajariladi:

$$c(x+\pi,\lambda)=c_1(\pi,\lambda)c(x,\lambda)+c_2(\pi,\lambda)s(x,\lambda),$$

$$s(x+\pi,\lambda)=s_1(\pi,\lambda)c(x,\lambda)+s_2(\pi,\lambda)s(x,\lambda).$$

Quyidagi Dirak sistemasini ko‘rib chiqamiz

$$L(\tau)y \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} p(x+\tau) & q(x+\tau) \\ q(x+\tau) & -p(x+\tau) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda \begin{pmatrix} y_1 \\ y_2 \end{pmatrix},\ x \in (-\infty, \infty), \tag{1.3}$$

bu yerda t haqiqiy parametr. $c(x,\lambda,\tau)$ va $s(x,\lambda,\tau)$ orqali (1.3) tenglamaning quyidagi boshlang‘ich shartlarni qanoatlanturuvchi yechimlarini belgilaymiz:

$$c(0,\lambda,\tau) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{va} \quad s(0,\lambda,\tau) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Lyapunov funksiyasining ta’rifiga ko‘ra $\Delta(\lambda,\tau) = c_1(\pi,\lambda,\tau) + s_2(\pi,\lambda,\tau)$.

Lemma 1.2. Quyidagi ayniyatlar o‘rinli:

$$c(x,\lambda,\tau) = s_2(\tau,\lambda)c(x+\tau,\lambda) - c_2(\tau,\lambda)s(x+\tau,\lambda),$$
$$s(x,\lambda,\tau) = -s_1(\tau,\lambda)c(x+\tau,\lambda) + c_1(\tau,\lambda)s(x+\tau,\lambda).$$

Isbot. $c(x+\tau,\lambda)$ va $s(x+\tau,\lambda)$ lar (1.3) tenglamaning fundamental yechimlar sistemasini tashkil etadi. Demak,

$$c(x,\lambda,\tau) = C_1 c(x+\tau,\lambda) + C_2 s(x+\tau,\lambda)$$

bo‘ladi. C_1 va C_2 larni boshlang‘ich shartlar yordamida topamiz:

$$C_1 = s_2(\tau,\lambda),\ C_2 = -c_2(\tau,\lambda).$$

Shuning uchun

$$c(x,\lambda,\tau) = s_2(\tau,\lambda)c_1(x+\tau,\lambda) - c_2(\tau,\lambda)s_1(x+\tau,\lambda).$$

Lemmadagi ikkinchi ayniyat ham shu tarzda isbot qilinadi. **Lemma isbotlandi.**

Teorema 1.6. Ushbu $\Delta(\lambda,\tau) \equiv \Delta(\lambda)$ ayniyat bajariladi, ya’ni Lyapunov funksiyasi τ parametrga bog‘liq emas.

Isbot. 1.2-lemmaga ko‘ra

$$\Delta(\lambda,\tau) = c_1(\pi,\lambda,\tau) + s_2(\pi,\lambda,\tau) =$$
$$= s_2(\tau,\lambda)c_1(\pi+\tau,\lambda) - c_2(\tau,\lambda)s_1(\pi+\tau,\lambda) -$$
$$- s_1(\tau,\lambda)c_2(\pi+\tau,\lambda) + c_1(\tau,\lambda)s_2(\pi+\tau,\lambda).$$

Endi 1.1-lemmadan foydalansak, quyidagiga ega bo‘lamiz

$$\Delta(\lambda,\tau) = s_2(\tau,\lambda)[c_1(\pi,\lambda)c_1(\tau,\lambda) + c_2(\pi,\lambda)s_1(\tau,\lambda)] -$$
$$- c_2(\tau,\lambda)[s_1(\pi,\lambda)c_1(\tau,\lambda) + s_2(\pi,\lambda)s_1(\tau,\lambda)] -$$

$$-s_1(\tau,\lambda)[c_1(\pi,\lambda)c_2(\tau,\lambda)+c_2(\pi,\lambda)s_2(\tau,\lambda)]+$$

$$+c_1(\tau,\lambda)[s_1(\pi,\lambda)c_2(\tau,\lambda)+s_2(\pi,\lambda)s_2(\tau,\lambda)].$$

Bu ifodani soddalashtirib Vronskiy ayniyatidan foydalanamiz

$$\Delta(\lambda,\tau)=c_1(\pi,\lambda)[c_1(\tau,\lambda)s_2(\tau,\lambda)-s_1(\tau,\lambda)c_2(\tau,\lambda)]+$$

$$+s_2(\pi,\lambda)[c_1(\tau,\lambda)s_2(\tau,\lambda)-s_1(\tau,\lambda)c_2(\tau,\lambda)]=$$

$$=c_1(\pi,\lambda)+s_2(\pi,\lambda)=\Delta(\lambda).$$

Teorema isbotlandi.

2-§. Dirak sistemasi uchun Floke yechimlari va ularning xossalari

Teorema 2.1. (Floke). a) agar $\Delta^2(\lambda)-4\neq 0$ bo‘lsa, u holda (1.1) tenglamaning quyidagi ko‘rinishdagi ikkita chiziqli erkli yechimi mavjud:

$$\psi_-(x,\lambda)=\rho_-^{\frac{x}{\pi}}\cdot p_-(x,\lambda),\ \ \psi_+(x,\lambda)=\rho_+^{\frac{x}{\pi}}\cdot p_+(x,\lambda),$$

bunda $p_-(x,\lambda)$ va $p_+(x,\lambda)$ funksiyalar x bo‘yicha π davrli bo‘lib,

$$\rho_\mp=\frac{\Delta(\lambda)\pm\sqrt{\Delta^2(\lambda)-4}}{2};$$

b) agar $\Delta(\lambda)=2$ bo‘lsa, u holda (1.1) tenglamaning π davrli yechimi mavjud bo‘ladi;

c) agar $\Delta(\lambda)=-2$ bo‘lsa, u holda (1.1) tenglamaning 2π davrli yechimi mavjud bo‘ladi.

Bu yerdagi ildiz analitik ildiz bo‘lib, uning quyidagi shartni qanoatlantiruvchi shohchasi olingan

$$\sqrt{\Delta^2(\lambda)-4}-2i\sin\lambda\pi=\overline{\overline{o}}\left(e^{|\mathrm{Im}\,\lambda|\pi}\right),\qquad \lambda\to i\infty.$$

Bunda $\sqrt{\Delta^2(\lambda)-4}$ funksiya $C\setminus E$ sohada analitik bo‘ladi, bu yerda E orqali ushbu $-2\leq\Delta(\lambda)\leq 2$ tengsizlikning yechimi belgilangan. E to‘plam bu funksiyaning uzilish nuqtalar to‘plamidir. $\psi_-(x,\lambda)$ va $\psi_+(x,\lambda)$ yechimlarga Floke yechimlari deyiladi.

Isbot. (1.1) tenglamani biror ρ uchun $y(x+\pi)=\rho y(x)$ shartni qanaotlantiradigan yechimlarini axtaramiz. (1.1) tenglamaning

$$y(x)=C_1c(x,\lambda)+C_2s(x,\lambda)$$

umumiy yechimiga 1.1-lemmani qo‘llaymiz:

$$y(x+\pi)=\left[C_1c_1(\pi,\lambda)_1+C_2s_1(\pi,\lambda)\right]c(x,\lambda)+\left[C_1c_2(\pi,\lambda)_1+C_2s_2(\pi,\lambda)\right]s(x,\lambda).$$

Ushbu $y(x+\pi)=\rho y(x)$ shartdan va $c(x,\lambda)$, $s(x,\lambda)$ chiziqli erkli ekanidan

$$\begin{cases} C_1c_1(\pi,\lambda)+C_2s_1(\pi,\lambda)=\rho C_1 \\ C_1c_2(\pi,\lambda)+C_2s_2(\pi,\lambda)=\rho C_2 \end{cases}$$

tenglamalar sistemasiga ega bo‘lamiz. Bu tenglamalar sistemasini noldan farqli yechimga ega bo‘lishi shartiga ko‘ra

$$\begin{vmatrix} c_1(\pi,\lambda)-\rho & s_1(\pi,\lambda) \\ c_2(\pi,\lambda) & s_2(\pi,\lambda)-\rho \end{vmatrix}=0,$$

$$(c_1-\rho)(s_2-\rho)-c_2s_1=0,$$

$$c_1s_2-\rho c_1-\rho s_2+\rho^2-c_2s_1=0,$$

$$\rho^2-\Delta(\lambda)\rho+1=0.$$

a) agar $\Delta^2(\lambda)-4\neq 0$ bo‘lsa , $\rho_+\neq\rho_-$ bo‘ladi, $C_1=1$ desak, $C_2=\dfrac{\rho_\pm-\theta_1}{s_1}$ bo‘ladi. (1.1) tenglamalar sistemasining yechimlari esa quyidagicha bo‘ladi:

$$\psi_\pm(x,\lambda)=c(x,\lambda)+\frac{s_2(\pi,\lambda)-c_1(\pi,\lambda)\pm\sqrt{\Delta^2(\lambda)-4}}{2s_1(\pi,\lambda)}s(x,\lambda).$$

Ushbu

$$p_\pm(x,\lambda)=\rho_+^{\frac{-x}{\pi}}\cdot\psi_\pm(x,\lambda)$$

funksiya π davrliligini ko‘rsatamiz:

$$p_\pm(x+\pi,\lambda)=\rho_\pm^{-(x+\pi)/\pi}\cdot\psi_\pm(x+\pi,\lambda)=\rho_\pm^{-(x+\pi)/\pi}\cdot\rho_\pm\cdot\psi_\pm(x,\lambda)=p_\pm(x,\lambda).$$

$\psi_{\pm}(x,\lambda)$ chiziqli erkli yechimlar ekanini ko‘rsatish uchun biror nuqtada ularning Vronskiani noldan farqliligini ko‘rsatish yetarli. Xususan, $x=0$ nuqtada

$$W\{\psi_+,\psi_-\}\big|_{x=0}=\begin{vmatrix}1 & 1\\ \dfrac{\rho_+-c_1}{s_1} & \dfrac{\rho_--c_1}{s_1}\end{vmatrix}=\frac{\rho_--\rho_+}{s_1}=\frac{\sqrt{\Delta^2(\lambda)-4}}{s_1}\neq 0.$$

b) Agar $\Delta(\lambda)=2$ bo‘lsa, $\rho_1=\rho_2=1$ bo‘ladi va

$$\psi(x,\lambda)=c(x,\lambda)+\frac{1-c_1(\pi,\lambda)}{s_1(\pi,\lambda)}\cdot s(x,\lambda)$$

π davrli yechim bo‘ladi.

c) Agar $\Delta(\lambda)=-2$ bo‘lsa , $\rho_1=\rho_2=-1$ bo‘lib,

$$\psi(x,\lambda)=c(x,\lambda)+\frac{-1-c_1(\pi,\lambda)}{s_1(\pi,\lambda)}\cdot s(x,\lambda)$$

2π davrli yechim bo‘ladi. **Teorema 2.1 isbotlandi.**

Teorema 2.2. a) agar $|\Delta(\lambda)|>2$ bo‘lsa, u holda $|\rho_-(\lambda)|>1, |\rho_+(\lambda)|<1$ bo‘ladi;

b) agar $|\Delta(\lambda)|\le 2$ bo‘lsa, u holda $|\rho_-(\lambda)|=|\rho_+(\lambda)|=1$ bo‘ladi.

Teorema 2.3. Floke yechimlari Veyl yechimlariga proporsional bo‘ladi va Veyl-Titchmarsh funksiyasi quyidagi formula bilan beriladi:

$$m^{\mp}(\lambda)=\frac{s_2(\pi,\lambda)-c_1(\pi,\lambda)\pm\sqrt{\Delta^2(\lambda)-4}}{2s_1(\pi,\lambda)}.$$

Natija 2.4. Davriy potensialli (1.1) Dirak operatori xos qiymatga ega emas. Uning spektri uzluksiz bo‘lib, u quyidagi to‘plamdan iborat:

$$E=\left\{\lambda\in R^1\,\middle|\,-2\le\Delta(\lambda)\le 2\right\},$$

ya’ni

$$E=R^1\setminus\bigcup_{n=-\infty}^{\infty}(\lambda_{2n-1},\lambda_{2n}) \tag{2.1}$$

Ushbu $(\lambda_{2n-1},\lambda_{2n})$, $n\in Z$ intervallarga lakunalar deyiladi.

Ushbu $s_1(\pi,\lambda)=0$ tenglamaning ildizlarini ξ_n, $n\in Z$ orqali belgilaymiz. Bu yerda Veyl-Titchmarsh funksiyasining mahrajidan kelib chiqdik. ξ_n, $n\in Z$ sonlar

(1.1) Dirak sistemasi uchun qo‘yilgan quyidagi Dirihle masalasining xos qiymatlari bilan ustma-ust tushadi

$$y_1(0)=0\,,\;\; y_1(\pi)=0\,.$$

Bundan tashqari $\xi_n \in [\lambda_{2n-1}, \lambda_{2n}]$, $n \in Z$ bo‘ladi.

Ta’rif 2.1. Usbu $\xi_n \in [\lambda_{2n-1}, \lambda_{2n}]$, $n \in Z$ sonlar va $\sigma_n = sign\{s_2(\pi,\xi_n) - c_1(\pi,\xi_n)\}$, $n \in Z$ ishoralarga (1.1) masalaning spektral parametrlari deyiladi. ξ_n o‘z lakunasining cheti bilan ustma-ust tushganida $\sigma_n = 1$ deb qabul qilamiz.

Ta’rif 2.2. Lakunalarning chetlari λ_n, $n \in Z$ hamda ξ_n, $\sigma_n = \pm 1$, $n \in Z$ spektral parametrlarga (1.1) masalaning spektral berilganlari deyiladi.

Ta’rif 2.3. (1.1) sistemaning spektral berilganlarini topish masalasiga to‘g‘ri masala deyiladi, aksincha, spektral berilganlar orqali (1.1) sistemaning koeffitsientlarini topish masalasiga teskari spektral masala deyiladi.

3-§. Dubrovin-Trubovits sistemasi va izlar formulalari

Quyidagi

$$L(\tau)y \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} p(x+\tau) & q(x+\tau) \\ q(x+\tau) & -p(x+\tau) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda \begin{pmatrix} y_1 \\ y_2 \end{pmatrix},\; x \in (-\infty, \infty)$$

masalaning Lyapunov funksiyasi τ parametrga bog‘liq emasligidan bu masalaning spektri ham τ parametrga bog‘liq emasligi kelib chiqadi. Demak, bu masalaning spektri ham (2.1) to‘plam bilan ustma-ust tushadi. Ammo, spektral parametrlar τ parametrga bog‘liq bo‘ladi, shuning uchun ularni $\xi_n(\tau), \sigma_n(\tau)$, $n \in Z$ orqali belgilaymiz. Spektral parametrlar τ ga bog‘liqlik qonuniyatini topish talab qilinadi. Ular quyidagi Dubrovin-Trubovits differensial tenglamalar sistemasi deb ataladigan sistemani qanoatlantiradi:

$$\frac{d\xi_n}{d\tau} = (-1)^{n-1}\sigma_n(\tau)\sqrt{(\xi_n - \lambda_{2n-1})(\lambda_{2n} - \xi_n)} \times$$

$$\times[2\xi_n + \sum_{k=-\infty}^{\infty}(\lambda_{2k-1} + \lambda_{2k} - 2\xi_k)] \times \sqrt{\prod_{\substack{k=-\infty,\\ k\neq n}}^{\infty} \frac{(\lambda_{2k-1} - \xi_n)(\lambda_{2k} - \xi_n)}{(\xi_k - \xi_n)^2}}, \quad n \in Z. \qquad (3.1)$$

(3.1) sistema quyidagi boshlang‘ich shartlar bilan qaraladi:

$$\xi_n(\tau)\big|_{\tau=0} = \xi_n(0), \quad n \in Z, \quad \sigma_n(\tau)\big|_{\tau=0} = \sigma_n(0), \quad n \in Z.$$

$\xi_n(\tau)$ spektral parametr o‘z lakunasining chetiga kelganidagina $\sigma_n(\tau)$ ishora qarama-qarshi ishoraga o‘zgaradi.

(1.1) Dirak sistemasi uchun quyidagi izlar formulalari deb ataladigan formulalar o‘rinli:

$$2p(\tau) = \sum_{k=-\infty}^{\infty}(\lambda_{2k-1} + \lambda_{2k} - 2\xi_k(\tau)),$$

$$q^2(\tau) + q'(\tau) = \sum_{k=-\infty}^{\infty}\left(\frac{\lambda_{2k-1}^2 + \lambda_{2k}^2}{2} - \xi_k^2(\tau)\right),$$

$$q(\tau) = \sum_{n=-\infty}^{\infty}(-1)^{n-1}\sigma_n(\tau)\sqrt{(\xi_n(\tau) - \lambda_{2n-1})(\lambda_{2n} - \xi_n(\tau))} \cdot \sqrt{\prod_{\substack{k=-\infty\\ k\neq n}}^{\infty}\frac{(\lambda_{2k-1} - \xi_n(\tau))(\lambda_{2k} - \xi_n(\tau))}{(\xi_k(\tau) - \xi_n(\tau))^2}}.$$

Natija. Agar bizga (1.1) masalaning spektral berilganlari ma’lum bo‘lsa, u holda bu boshlang‘ich shartlar bilan Dubrovin-Trubovits sistemasini yechib $\xi_n(\tau), \sigma_n(\tau)$, $n \in Z$ spektral parametrlarni topamiz. Bularni izlar formulalariga qo‘ysak, (1.1) masalanining koeffitsientlari kelib chiqadi.

[36] ishda Dubrovin-Trubovits sistemasi va izlar formulalarini qo‘llab, lakunalar uzunliklarining nolga intilish tartibi bilan $p(x)$ va $q(x)$ koeffitsientlarning analitikligini bog‘lovchi quyidagi ikkita teorema isbot qilingan.

Teorema 3.1. Agar $p(x)$, $q(x)$ π-davrli haqiqiy funksiyalar $C^2(-\infty, \infty)$ sinfga tegishli bo‘lsa va lakunalar uzunliklari $\lambda_{2n} - \lambda_{2n-1}$ eksponensial ravishda nolga intilsa, ya’ni $a > 0, b > 0$ o‘zgarmas sonlar topilib, n ning barcha butun qiymatida $\lambda_{2n} - \lambda_{2n-1} < ae^{-b|n|}$ bo‘lsa, u holda $p(x)$ va $q(x)$ funksiyalar haqiqiy o‘qda analitik funksiya bo‘ladi, ya’ni har bir nuqtaning biror atrofida yaqinlashuvchi darajali qatorga yoyiladi.

Teorema 3.2. Agar $p(x)$ va $q(x)$ haqiqiy analitik funksiyalar π-davrli bo‘lsa, u holda lakunalar uzunliklari $\lambda_{2n}-\lambda_{2n-1}$ eksponensial ravishda nolga intiladi.

[35] ishda Dirak operatori uchun Borg teskari teoremasining quyidagi analogi isbot qilingan.

Teorema 3.3. $\frac{\pi}{2}$ soni $p(x)$ va $q(x)$ koeffitsientlarning davri bo‘lishi uchun ($y(0)=-y(\pi)$) antidavriy masalaning barcha hos qiymatlari ikki karrali bo‘lishi zarur va yetarlidir.

4-§. Yuklangan hadli nochiziqli Shredinger tenglamasini davriy funksiyalar sinfida integrallash

Quyidagi yuklangan hadli Shredinger tenglamasini

$$u_t = 2iu|u|^2 - iu_{xx} + b(t)|u(0,t)|^2 u_x, \quad t>0, \ x\in R \tag{4.1}$$

ushbu

$$u(x,t)\big|_{t=0} = u_0(x) \tag{4.2}$$

boshlang‘ich shart va

$$u(x+\pi,t) = u(x,t) \tag{4.3}$$

x bo‘yicha davriylik sharti hamda ushbu

$$u(x,t)\in C_x^2(t>0)\cap C_t^1(t>0)\cap C(t\ge 0) \tag{4.4}$$

silliqlik sharti bilan birga ko‘rib chiqamiz. Bu yerda $b(t)$ - berilgan uzluksiz xaqiqiy funksiya.

Agar (4.1)-(4.4) masalada $u=-p+iq$ almashtirish bajarsak u quyidagi ko‘rinishni oladi:

$$\begin{cases} p_t = -q_{xx} + 2q(p^2+q^2) + b(t)[p^2(0,t)+q^2(0,t)]p_x \\ q_t = p_{xx} - 2p(p^2+q^2) + b(t)[p^2(0,t)+q^2(0,t)]q_x \end{cases}, \quad t>0, \ x\in R, \tag{4.5}$$

$$p(x,t)\big|_{t=0} = p_0(x)\,, \quad q(x,t)\big|_{t=0} = q_0(x)\,, \tag{4.6}$$

$$p(x+\pi,t) = p(x,t)\,, \quad q(x+\pi,t) = q(x,t)\,, \tag{4.7}$$

$$p(x,t), q(x,t) \in C_x^2(t>0) \cap C_t^1(t>0) \cap C(t \geq 0)\,. \tag{4.8}$$

Teorema 4.1. Agar $u(x,t) = -p(x,t) + iq(x,t)$ funksiya (4.1)-(4.4) masalaning yechimi bo‘lsa, u holda koeffitsiyentlari $p(x+\tau,t)$ va $q(x+\tau,t)$ bo‘lgan Dirak operatorining spektri τ va t parametrlarga bog‘liq bo‘lmaydi, spektral parametrlari $\xi_n(\tau,t)$, $n \in Z$ esa quyidagi Dubrovin-Trubovits sistemasini qanoatlantiradi:

$$\frac{\partial \xi_n}{\partial t} = 2(-1)^n \sigma_n(\tau,t) h_n(\xi) \times$$

$$\times \{q^2(\tau,t) + q_x(\tau,t) + [p(\tau,t) + \xi_n]^2 + \xi_n^2 - b(t)[p^2(0,t) + q^2(0,t)][p(\tau,t) + \xi_n]\}\,. \tag{4.9}$$

Bu yerda

$$h_n(\xi) = \sqrt{(\xi_n - \lambda_{2n-1})(\lambda_{2n} - \xi_n)} \cdot \sqrt{\prod_{\substack{k=-\infty,\\ k \neq n}}^{\infty} \frac{(\lambda_{2k-1} - \xi_n)(\lambda_{2k} - \xi_n)}{(\xi_k - \xi_n)^2}}\,.$$

Bunda $\sigma_n(\tau,t) = \pm 1$, $n \in Z$ ishoralar $\xi_n(\tau,t)$ spektral parametr $[\lambda_{2n-1}, \lambda_{2n}]$ o‘z lakunasining chetiga kelganida qarama-qarshi ishoraga o‘zgaradi. Bundan tashqari ushbu

$$\xi_n(\tau,t)\big|_{t=0} = \xi_n^0(\tau)\,, \; \sigma_n(\tau,t)\big|_{t=0} = \sigma_n^0(\tau)\,, \; n \in Z \tag{4.10}$$

boshlang‘ich shartlar ham bajariladi. Bu yerda $\xi_n^0(\tau)$, $\sigma_n^0(\tau)$, $n \in Z$ lar Dirak operatorining $p_0(x+\tau)$ va $q_0(x+\tau)$ koeffitsientlariga mos keluvchi spektral parametrlaridir.

Isbot. Ushbu

$$L(\tau,t)y \equiv B\frac{dy}{dx} + \Omega(x+\tau,t)y = \lambda y\,, \quad x \in R \tag{4.11}$$

Dirak sistemasi uchun qo‘yilgan $y_1(0)=0,\ y_1(\pi)=0$ Dirixle masalasining $\xi_n(\tau,t)$, $n\in Z$ xos qiymatlariga mos keluvchi ortonormallangan xos vektor-funksiyalarni $y_n(x,t,\tau)=\begin{pmatrix} y_{n,1}(x,t,\tau)\\ y_{n,2}(x,t,\tau)\end{pmatrix}$, $n\in Z$ orqali belgilaymiz. Bu yerda

$$B=\begin{pmatrix} 0 & 1\\ -1 & 0\end{pmatrix},\ \Omega(x,t)=\begin{pmatrix} p(x+\tau,t) & q(x+\tau,t)\\ q(x+\tau,t) & -p(x+\tau,t)\end{pmatrix},\ y=\begin{pmatrix} y_1(x)\\ y_2(x)\end{pmatrix}.$$

Ushbu

$$\xi_n=(L(\tau,t)y_n,y_n)$$

tenglikni t bo‘yicha differensiallab, $L(\tau,t)$ operatorning simmetrikligidan foydalanib, quyidagiga ega bo‘lamiz

$$\dot{\xi}_n=(\dot{\Omega}(x+\tau,t)y_n+L(\tau,t)\dot{y}_n,y_n)+(L(\tau,t)y_n,\dot{y}_n)=$$

$$=(\dot{\Omega}(x+\tau,t)y_n,y_n)+(\dot{y}_n,L(\tau,t)y_n)+(L(\tau,t)y_n,\dot{y}_n)=$$

$$=(\dot{\Omega}(x+\tau,t)y_n,y_n)+\xi_n\dot{((y_n,y_n))}=(\dot{\Omega}(x+\tau,t)y_n,y_n). \qquad (4.12)$$

Skalyar ko‘paytmaning

$$(y,z)=\int_0^{\pi}[y_1(x)\bar{z}_1(x)+y_2(x)\bar{z}_2(x)]dx,\quad y=\begin{pmatrix} y_1(x)\\ y_2(x)\end{pmatrix},\ z=\begin{pmatrix} z_1(x)\\ z_2(x)\end{pmatrix}$$

aniq ko‘rinishidan foydalangan holda (4.12) tenglikni quyidagicha yozamiz:

$$\dot{\xi}_n=\int_0^{\pi}[(p_t y_{n,1}+q_t y_{n,2})y_{n,1}+(q_t y_{n,1}-p_t y_{n,2})y_{n,2}]dx=$$

$$=\int_0^{\pi}[(y_{n,1}^2-y_{n,2}^2)p_t+2y_{n,1}y_{n,2}\,q_t]dx\,.$$

(4.5) ifodalarni bu tenglikka qo‘yganda ushbu

$$\dot{\xi}_n(t)=\int_0^{\pi}\{(y_{n,1}^2-y_{n,2}^2)[-q_{xx}+2q(p^2+q^2)+b(t)[p^2(0,t)+q^2(0,t)]p_x]+$$

$$+2y_{n,1}y_{n,2}[p_{xx}-2p(p^2+q^2)+b(t)[p^2(0,t)+q^2(0,t)]q_x]\}dx \qquad (4.13)$$

formula hosil bo‘ladi. Integral ostidagi funksiyaning boshlang‘ichini $y_{n,1}$ va $y_{n,2}$ ga nisbatan kvadratik forma ko‘rinishida izlaymiz, ya’ni

$$\{(c+b)y_{n,1}^2-2ay_{n,1}y_{n,2}+(c-b)y_{n,2}^2\}'=$$

$$= (y_{n,1}^2 - y_{n,2}^2)[-q_{xx} + 2q(p^2 + q^2) + b(t)[p^2(0,t) + q^2(0,t)]p_x] +$$
$$+ 2y_{n,1}y_{n,2}[p_{xx} - 2p(p^2 + q^2) + b(t)[p^2(0,t) + q^2(0,t)]q_x]. \tag{4.14}$$

Bu yerda $a = a(x,t,\tau,\xi_n)$, $b = b(x,t,\tau,\xi_n)$, $c = c(x,t,\tau,\xi_n)$ lar $y_{n,1}$ va $y_{n,2}$ ga bog‘liq emas. (4.11) tenglamadan quyidagi tenglik kelib chiqadi

$$\begin{cases} y'_{n,1} = q\, y_{n,1} - p\, y_{n,2} - \xi_n y_{n,2} \\ y'_{n,2} = \xi_n y_{n,1} - p\, y_{n,1} - q\, y_{n,2}. \end{cases} \tag{4.15}$$

(4.14) tenglik chap tomonidagi hosilalarni hisoblab, (4.15) ayniyatlardan foydalansak, quyidagi tenglik hosil bo‘ladi

$$y_{n,1}^2 \cdot [c' + b' + 2qb + 2qc + 2pa - 2\xi_n a] + y_{n,2}^2 \cdot [c' - b' + 2qb - 2qc + 2pa + 2\xi_n a] -$$
$$- 2y_{n,1}y_{n,2} \cdot [a' + 2pc + 2\xi_n b] =$$
$$= (y_{n,1}^2 - y_{n,2}^2)\{-q_{xx} + 2q(p^2 + q^2) + b(t)[p^2(0,t) + q^2(0,t)]p_x\} +$$
$$+ 2y_{n,1}y_{n,2}\{p_{xx} - 2p(p^2 + q^2) + b(t)[p^2(0,t) + q^2(0,t)]q_x\}. \tag{4.16}$$

Agar bu yerda mos koeffitsiyentlarni o‘zaro tenglasak, quyidagi tengliklar kelib chiqadi

$$c' + b' + 2qb + 2qc + 2pa - 2\xi_n a =$$
$$= -q_{xx} + 2q(p^2 + q^2) + b(t)[p^2(0,t) + q^2(0,t)]p_x, \tag{4.17}$$
$$c' - b' + 2qb - 2qc + 2pa + 2\xi_n a =$$
$$= q_{xx} - 2q(p^2 + q^2) - b(t)[p^2(0,t) + q^2(0,t)]p_x, \tag{4.18}$$
$$a' + 2pc + 2\xi_n b =$$
$$= -p_{xx} + 2p(p^2 + q^2) - b(t)[p^2(0,t) + q^2(0,t)]q_x. \tag{4.19}$$

(4.17) va (4.18) dan

$$c = -2\int_0^x [pa + qb]dx + d(t,\tau,\xi_n), \tag{4.20}$$

va

$$b' + 2qc - 2\xi_n a = -q_{xx} + 2q(p^2 + q^2) + b(t)[p^2(0,t) + q^2(0,t)]p_x \tag{4.21}$$

kelib chiqadi.

(4.19) va (4.21) tengliklarning o‘ng tomoni ξ_n ga bog‘liq bo‘lmagani uchun chap tomoni ham ξ_n ga bog‘liq bo‘lmasligi kerak. $a(x,t,\tau,\xi_n)$, $b(x,t,\tau,\xi_n)$, $c(x,t,\tau,\xi_n)$ va $d(t,\tau,\xi_n)$ larni ξ_n ga nisbatan kvadratik ko‘phad ko‘rinishida izlaymiz:

$$a(x,t,\tau,\xi_n) = a_0(x,t,\tau)\xi_n^2 + a_1(x,t,\tau)\xi_n + a_2(x,t,\tau), \qquad (4.22)$$

$$b(x,t,\tau,\xi_n) = b_0(x,t,\tau)\xi_n^2 + b_1(x,t,\tau)\xi_n + b_2(x,t,\tau), \qquad (4.23)$$

$$c(x,t,\tau,\xi_n) = c_0(x,t,\tau)\xi_n^2 + c_1(x,t,\tau)\xi_n + c_2(x,t,\tau), \qquad (4.24)$$

$$d(x,t,\tau,\xi_n) = d_0(x,t,\tau)\xi_n^2 + d_1(x,t,\tau)\xi_n + d_2(x,t,\tau). \qquad (4.25)$$

(4.22)-(4.25) ifodalarni (4.19) va (4.21) tengliklarga qo‘yamiz:

$$2b_0\xi_n^3 + [a_0' + 2pc_0 + 2b_1]\xi_n^2 + [a_1' + 2pc_1 + 2b_2]\xi_n + a_2' + 2pc_2 =$$
$$= -p_{xx} + 2p(p^2+q^2) - b(t)[p^2(0,t) + q^2(0,t)]q_x, \qquad (4.26)$$

$$-2a_0\xi_n^3 + [b_0' + 2qc_0 - 2a_1]\xi_n^2 + [b_1' + 2qc_1 - 2a_2]\xi_n + b_2' + 2qc_2 =$$
$$= -q_{xx} + 2q(p^2+q^2) + b(t)[p^2(0,t) + q^2(0,t)]p_x. \qquad (4.27)$$

Agar bu tengliklarda bir xil darajalar oldidagi koeffitsientlarni o‘zaro tenglasak, quyidagi munosabatlarni olamiz:

$$a_0 = 0,\ b_0 = 0,$$

$$a_1 = \frac{1}{2}b_0' + qc_0,\ \ b_1 = -\frac{1}{2}a_0' - pc_0,$$

$$a_2 = \frac{1}{2}b_1' + qc_1,\quad b_2 = -\frac{1}{2}a_1' - pc_1,$$

$$a_2' + 2pc_2 = -p_{xx} + 2p(p^2+q^2) - b(t)[p^2(0,t) + q^2(0,t)]q_x,$$

$$b_2' + 2qc_2 = -q_{xx} + 2q(p^2+q^2) + b(t)[p^2(0,t) + q^2(0,t)]p_x.$$

$$(4.28)$$

(4.22)-(4.25) ifodalarni (4.20) tenglikka qo‘ysak,

$$c_0 = -2\int_0^x [pa_0 + qb_0]dx + d_0,$$

$$c_1 = -2\int_0^x [pa_1 + qb_1]dx + d_1,$$

$$c_2 = -2\int_0^x [pa_2 + qb_2]dx + d_2$$

(4.29)

kelib chiqadi. (4.28) va (4.29) tengliklarga muvofiq

$$a_0 = 0,\ b_0 = 0,\ c_0 = 2,$$

$$a_1 = 2q,\ b_1 = -2p,\ c_1 = -b(t)[p^2(0,t) + q^2(0,t)],$$

$$a_2 = -p' - b(t)[p^2(0,t) + q^2(0,t)]q,\ \ b_2 = -q' + b(t)[p^2(0,t) + q^2(0,t)]p,$$

$$c_2 = p^2 + q^2.$$

Bularga ko‘ra

$$a = 2q\xi_n - p' - b(t)[p^2(0,t) + q^2(0,t)]q,$$

$$b = -2p\xi_n - q' + b(t)[p^2(0,t) + q^2(0,t)]p,$$

$$c = 2\xi_n^2 - b(t)[p^2(0,t) + q^2(0,t)]\xi_n + p^2 + q^2.$$

(4.30)

Shunday qilib, (4.13) va (4.14) tengliklarga asosan

$$\dot{\xi}_n = \{(c+b)y_{n,1}^2 - 2ay_{n,1}y_{n,2} + (c-b)y_{n,2}^2\}\Big|_0^\pi =$$

$$= [c(\pi,t,\tau,\xi_n) - b(\pi,t,\tau,\xi_n)]y_{n,2}^2(\pi,t,\tau) - [c(0,t,\tau,\xi_n) - b(0,t,\tau,\xi_n)]y_{n,2}^2(0,t,\tau).$$

(4.31)

Ushbu $a(x,t,\tau,\xi_n)$, $b(x,t,\tau,\xi_n)$, $c(x,t,\tau,\xi_n)$ funksiyalar x bo‘yicha π davrli ekanini hisobga olsak, (4.31) tenglik quyidagi ko‘rinishni oladi

$$\dot{\xi}_n = [c(0,t,\tau,\xi_n) - b(0,t,\tau,\xi_n)]\cdot[y_{n,2}^2(\pi,t,\tau) - y_{n,2}^2(0,t,\tau)]. \qquad (4.32)$$

Agar ushbu (4.30) tengliklardan foydalansak,

$$c(0,t,\tau,\xi_n) = 2\xi_n^2 - b(t)[p^2(0,t) + q^2(0,t)]\xi_n + p^2(\tau,t) + q^2(\tau,t),$$

$$b(0,t,\tau,\xi_n) = -2p(\tau,t)\xi_n - q'(\tau,t) + b(t)[p^2(0,t) + q^2(0,t)]p(\tau,t)$$

ifodalarni topamiz. Demak,

$$\dot{\xi}_n = \{2\xi_n^2 + 2p(\tau,t)\xi_n - b(t)[p^2(0,t) + q^2(0,t)][p(\tau,t) + \xi_n] +$$

$$+p^2(\tau,t)+q^2(\tau,t)+q'(\tau,t)\}\times[y_{n,2}^2(\pi,t,\tau)-y_{n,2}^2(0,t,\tau)]. \qquad (4.33)$$

(4.11) sistemaning

$$c(0,\lambda,t,\tau)=\begin{pmatrix}1\\0\end{pmatrix} \quad \text{va} \quad s(0,\lambda,t,\tau)=\begin{pmatrix}0\\1\end{pmatrix}$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini

$$c(x,\lambda,t,\tau)=\begin{pmatrix}c_1(x,\lambda,t,\tau)\\c_2(x,\lambda,t,\tau)\end{pmatrix} \quad \text{va} \quad s(x,\lambda,t,\tau)=\begin{pmatrix}s_1(x,\lambda,t,\tau)\\s_2(x,\lambda,t,\tau)\end{pmatrix}$$

orqali belgilaymiz. Quyidagi

$$s_1^2(x,\lambda,t,\tau)+s_2^2(x,\lambda,t,\tau)=s^T\cdot s=s^T\cdot(s+\lambda\frac{\partial s}{\partial\lambda})-\frac{\partial s^T}{\partial\lambda}\cdot(\lambda s)=$$

$$=s^T\cdot(B\cdot\frac{\partial s'}{\partial\lambda}+\Omega\cdot\frac{\partial s}{\partial\lambda})-\frac{\partial s^T}{\partial\lambda}\cdot(B\cdot s'+\Omega\cdot s)=$$

$$=s^T\cdot B\cdot\frac{\partial s'}{\partial\lambda}-\frac{\partial s^T}{\partial\lambda}\cdot B\cdot s'=\left(s_1\cdot\frac{\partial s_2}{\partial\lambda}-s_2\cdot\frac{\partial s_1}{\partial\lambda}\right)'$$

tenglikdan foydalanib, ushbu

$$\int_0^\pi[s_1^2+s_2^2]dx=s_1(\pi,\lambda,t,\tau)\cdot\frac{\partial s_2(\pi,\lambda,t,\tau)}{\partial\lambda}-s_2(\pi,\lambda,t,\tau)\cdot\frac{\partial s_1(\pi,\lambda,t,\tau)}{\partial\lambda}$$

ayniyatni keltirib chiqaramiz. Bundan foydalanib, Dirixle masalasi $s(x,\xi_n,t,\tau)$ xos funksiyasining normasini topamiz:

$$\alpha_n^2=\int_0^\pi[s_1^2+s_2^2]dx=-s_2(\pi,\xi_n,t,\tau)\cdot\frac{\partial s_1(\pi,\xi_n,t,\tau)}{\partial\lambda}. \qquad (4.34)$$

Ushbu tenglik

$$y_n(x,t,\tau)=\frac{1}{\alpha_n}s(x,\xi_n,t,\tau)$$

va (4.34) formulaga ko‘ra

$$y_{n,2}^2(\pi,t,\tau)-y_{n,2}^2(0,t,\tau)=\frac{s_2^2(\pi,\xi_n,t,\tau)-1}{\alpha_n^2}=-\frac{s_2(\pi,\xi_n,t,\tau)-\dfrac{1}{s_2(\pi,\xi_n,t,\tau)}}{\dfrac{\partial s_1(\pi,\xi_n,t,\tau)}{\partial\lambda}}. \qquad (4.35)$$

Ushbu

$$c_1(x,\lambda,t,\tau)s_2(x,\lambda,t,\tau)-c_2(x,\lambda,t,\tau)s_1(x,\lambda,t,\tau)=1$$

Vronskiy ayniyatida $x=\pi$ va $\lambda=\xi_n$ desak,

$$c_1(\pi,\xi_n,t,\tau)=\frac{1}{s_2(\pi,\xi_n,t,\tau)} \tag{4.36}$$

kelib chiqadi. Bu tenglikdan hamda ushbu

$$[c_1(\pi,\lambda,t,\tau)-s_2(\pi,\lambda,t,\tau)]^2=(\Delta^2(\lambda)-4)-4c_2(\pi,\lambda,t,\tau)s_1(\pi,\lambda,t,\tau)$$

ayniyantdan foydalanib, quyidagini hosil qilamiz

$$s_2(\pi,\xi_n,t,\tau)-\frac{1}{s_2(\pi,\xi_n,t,\tau)}=\sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n)-4}\,. \tag{4.37}$$

Bu yerda

$$\Delta(\lambda)=c_1(\pi,\lambda,t,\tau)+s_2(\pi,\lambda,t,\tau),\quad \sigma_n(\tau,t)=sign\{s_2(\pi,\xi_n,t,\tau)-c_1(\pi,\xi_n,t,\tau)\}.$$

Agar (4.37) ifodani (4.35) ga qo‘ysak, quyidagi tenglikni olamiz

$$y_{n,2}^2(\pi,t,\tau)-y_{n,2}^2(0,t,\tau)=-\frac{\sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n)-4}}{\frac{\partial s_1(\pi,\xi_n,t,\tau)}{\partial\lambda}}. \tag{4.38}$$

Ushbu

$$\Delta^2(\lambda)-4=-4\pi^2\prod_{k=-\infty}^{\infty}\frac{(\lambda-\lambda_{2k-1})(\lambda-\lambda_{2k})}{a_k^2},\quad s_1(\pi,\lambda,t,\tau)=\pi\prod_{k=-\infty}^{\infty}\frac{\xi_k-\lambda}{a_k},$$

yoyilmalardan ($a_0=1$ va $a_k=k$, $k\neq 0$) foydalanib, (4.38) ayniyatni quyidagi tarzda yozamiz:

$$y_{n,2}^2(\pi,t,\tau)-y_{n,2}^2(0,t,\tau)=$$

$$=2(-1)^n\sigma_n(\tau,t)\sqrt{(\xi_n-\lambda_{2n-1})(\lambda_{2n}-\xi_n)}\cdot\sqrt{\prod_{\substack{k=-\infty,\\ k\neq n}}^{\infty}\frac{(\lambda_{2k-1}-\xi_n)(\lambda_{2k}-\xi_n)}{(\xi_k-\xi_n)^2}}. \tag{4.39}$$

Bunda biz quyidagi tenglikdan ham foydalandik:

$$sign\left\{-\frac{\pi}{a_n}\prod_{\substack{k=-\infty\\ k\neq n}}^{\infty}\frac{\xi_k-\xi_n}{a_k}\right\}=(-1)^{n-1}.$$

Demak, (4.33) va (4.39) tengliklardan (4.9) kelib chiqadi.

Agar chegaraviy shartlarni $y(\pi) = y(0)$ davriy yoki $y(\pi) = -y(0)$ antidavriy shartlar bilan almashtirsak, (4.33) tenglamalar o‘rnida $\dot{\lambda}_n = 0$, $n \in Z$ tenglamalar hosil bo‘ladi. Demak, λ_n, $n \in Z$ davriy va antidavriy masalaning xos qiymatlari t parametrga bog‘liq emas ekan. **Teorema 4.1 isbotlandi.**

Izoh. 4.1. Ushbu izlar formulalari

$$p(\tau,t) = \sum_{k=-\infty}^{\infty} \left((\lambda_{2k-1} + \lambda_{2k})/2 - \xi_k(\tau,t) \right), \tag{4.40}$$

$$q^2(\tau,t) + q_\tau(\tau,t) = \sum_{k=-\infty}^{\infty} \left((\lambda_{2k-1}^2 + \lambda_{2k}^2)/2 - \xi_k^2(\tau,t) \right)$$

yordamida (4.9) sistemani yopiq ko‘rinishda yozish mumkin.

Natija 4.1. Yuqoridagi 4.1-teorema (4.1)-(4.4) masalani yechish usulini beradi. λ_n, $\xi_n(\tau,t)$, $\sigma_n(\tau,t)$, $n \in Z$ orqali $p(x+\tau,t)$ va $q(x+\tau,t)$ koeffitsiyentga mos keluvchi spektral berilganlarni belgilaymiz.

1) Avvalo $p_0(x+\tau)$ va $q_0(x+\tau)$ koeffitsientli Dirak operatorining λ_n, $\xi_n^0(\tau)$, $\sigma_n^0(\tau)$, $n \in Z$ spektral berilganlarini topamiz;

2) So‘ngra, (4.9) Dubrovin-Trubovits tenglamalar sistemasi uchun qo‘yilgan

$$\xi_n(\tau,t)\big|_{t=0} = \xi_n^0(\tau), \; \sigma_n(\tau,t)\big|_{t=0} = \sigma_n^0(\tau), \quad n \in Z \tag{4.41}$$

Koshi masalasida $\tau = 0$ deb olib, uning $\xi_n(0,t)$, $n \in Z$ yechimini topamiz;

3) Shundan keyin, $\xi_n(0,t)$, $n \in Z$ uchun topilgan ifodalarni (4.9) sistemaga qo‘yamiz hamda (4.9)+(4.41) Koshi masalasini ixtiyoriy τ da yechib $\xi_n(\tau,t)$, $\sigma_n(\tau,t)$, $n \in Z$ spektral parametrlarni aniqlaymiz.

4) Bu Koshi masalasining yechimini (4.40) va quyidagi

$$q(\tau,t) = \sum_{n=-\infty}^{\infty} (-1)^{n-1} \sigma_n(\tau,t) h_n(\xi)$$

izlar formulasiga qo‘yib, $p(x,t)$ va $q(x,t)$ funksiyalarni topamiz.

Natija 4.2. Agar boshlang‘ich shartlardagi $p_0(x)$ va $q_0(x)$ funksiyalar haqiqiy analitik funksiya bo‘lsa, u holda unga mos keluvchi lakunalar uzunliklari eksponensial ravishda nolga intiladi, bu lakunalar $p(x,t)$ va $q(x,t)$ funksiylaraga ham mos keladi. Shuning uchun $p(x,t)$ va $q(x,t)$ yechimlar x o'zgaruvchi bo'yicha haqiqiy analitik funksiya bo'ladi ([36]).

Natija 4.3. Agar boshlang'ich shartlardagi $p_0(x)$ va $q_0(x)$ funksiyalar $\frac{\pi}{2}$ davrga ega bo'lsa, u holda unga mos keluvchi barcha toq nomerli lakunalar yo'qoladi, bu lakunalar $p(x,t)$ va $q(x,t)$ koefitsiyentlarga ham mos keladi. Shuning uchun $p(x,t)$ va $q(x,t)$ yechimlar x o'zgaruvchi bo'yicha $\frac{\pi}{2}$ davrga ega bo'ladi ([35]).

II bob bo‘yicha xulosalar.

Ushbu bobda davriy koeffitsientli Dirak operatori uchun qo‘yilgan teskari spektral masala yordamida, Shredingerning yuklangan hadli nochiziqli tenglamasi uchun qo‘yilgan Koshi masalasi davriy funksiyalar sinifida yechilgan. Jumladan, Dirak operatori potensiali Shredinger yuklangan hadli nochiziqli tenglamasining x o‘zgaruvchi bo‘yicha davriy yechimi bo‘lsa, u holda bu operatorning spektri t parametrga bog‘liq emasligi ko‘rsatilgan. Bu Dirak operatori spektral parametrlarining t parametr bo‘yicha evolyutsiyasi topilgan. Bundan tashqari, yechimning x bo‘yicha davri haqida va x bo‘yicha analitikligi haqida muhim natijalar olingan.

III-BOB. DAVRIY FUNKSIYALAR SINFIDA KAUPNING YUKLANGAN HADLI SISTEMASINI INTEGRALLASH

D.J.Kaupning [23] ishida quyidagi

$$\begin{cases} \eta_\tau = \Phi_{xx} + \beta^2 \Phi_{xxxx} - \varepsilon \cdot (\Phi_x \eta)_x \\ \eta = \Phi_\tau + \dfrac{1}{2}\varepsilon \cdot \Phi_x^2 \end{cases}$$

sayoz suvda to‘lqinlarning tarqalishini ifodalovchi tenglamalar sistemasi tez kamayuvchi funksiyalar sinifida yechilgan. Shundan so‘ng, V.B.Matveev va I.V.Yavorning [43] ishida, bu sistemaning chekli zonali, ko‘p fazali, Riman teta-funksiyalari orqali ifodalanuvchi kompleks yechimlarini tuzish usuli olingan, bundan tashqari bu sistemening soliton yechimlari topilgan hamda ularning asimptotikalari o‘rganilgan. A.O.Smirnovning [56, 57] maqolalarida va Y.A.Mitropolskiy, N.N.Bogolyubov (ml.), A.K.Prikapatskiy va V.G.Somoylenko-larning monografiyasida ([42], 169-179 betlar) Kaup sistemasining chekli zonali haqiqiy yechimlari o‘rganilgan.

Agar Kaup sistemasida ushbu

$$\eta = \frac{4\beta^2}{\varepsilon}(q + p^2) + \frac{1}{\varepsilon}, \;\; \Phi_\tau = \frac{4\beta^2}{\varepsilon}(q + 3p^2) + \frac{1}{\varepsilon}, \;\; \Phi_x = -\frac{4\beta}{i\varepsilon} p, \;\; t = i\beta\tau,$$

almashtrishlarni ([43]) bajarsak, u quydagi oddiy ko‘rinishni oladi:

$$\begin{cases} p_t = -6pp_x - q_x \\ q_t = p_{xxx} - 4qp_x - 2pq_x. \end{cases}$$

Bu sistemaga ham Kaup sistemasi deyiladi.

Mazkur dissertatsiyaning III-bobida davriy koeffitsientli Shturm-Liuvill operatorlarining kvadratik dastasi uchun qo`yilgan teskari spektral masala yordamida, Kaupning yuklangan hadli sistemasi uchun qo`yilgan Koshi masalasi davriy funksiyalar sinfida yechilgan.

1-§. Davriy koeffitsientli Shturm-Liuvill operatorlarining kvadratik dastasi uchun teskari spektral masala.

Bu paragrafda davriy koeffitsientli Shturm-Liuvill operatorlarining kvadratik dastasi uchun qo‘yilgan teskari spektral masalaga oid kerakli ma’lumotlarni keltramiz ([16, 17, 9, 10]).

Quyidagi

$$T(\lambda)y \equiv -y'' + q(x)y + 2\lambda p(x)y - \lambda^2 y = 0\,,\ x \in R \qquad (1.1)$$

Shturm-Liuvill tenglamalarining kvadratik dastasini ko‘rib chiqamiz. Bunda $p(x)$ va $q(x)$ haqiqiy, uzluksiz, π davrli funksiyalar, λ esa kompleks parametr.

$c(x,\lambda)$ va $s(x,\lambda)$ orqali (1.1) tenglamaning ushbu $c(0,\lambda)=1$, $c'(0,\lambda)=0$, $s(0,\lambda)=0$, $s'(0,\lambda)=1$ boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini belgilaymiz.

Ushbu $\Delta(\lambda) = c(\pi,\lambda) + s'(\pi,\lambda)$ funksiyaga (1.1) tenglamaning Lyapunov funksiyasi yoki Xill diskriminanti deyiladi.

Teorema (Floke). Agar $\Delta^2(\lambda) - 4 \neq 0$ bo‘lsa, (1.1) tenglama quyidagi

$$\psi_-(x,\lambda) = \rho_-^{\frac{x}{\pi}} \cdot p_-(x,\lambda)\,, \qquad \psi_+(x,\lambda) = \rho_+^{\frac{x}{\pi}} \cdot p_+(x,\lambda)$$

ko‘rinishdagi ikkita chiziqli erkli yechimga ega. Bunda $p_-(x,\lambda)$ va $p_+(x,\lambda)$ funksiyalar x bo‘yicha π davrga ega va

$$\rho_\mp = \rho_\mp(\lambda) = \frac{\Delta(\lambda) \pm \sqrt{\Delta^2(\lambda) - 4}}{2}\,;$$

Agar $\Delta(\lambda) = 2$ bo‘lsa, (1.1) tenglama π davrli yechimga ega;

Agar $\Delta(\lambda) = -2$ bo‘lsa, (1.1) tenglama π antidavrli yechimga ega.

Agar $\psi_\pm(0,\lambda) = 1$ deb olsak, u holda

$$\psi_\pm(x,\lambda) = c(x,\lambda) + \frac{s'(\pi,\lambda) - c(\pi,\lambda) \mp \sqrt{\Delta^2(\lambda) - 4}}{2s(\pi,\lambda)} s(x,\lambda)$$

bo‘ladi.

Floke teoremasidagi $\psi_{\pm}(x,\lambda)$ yechimlarga Floke yechimlari deyiladi.

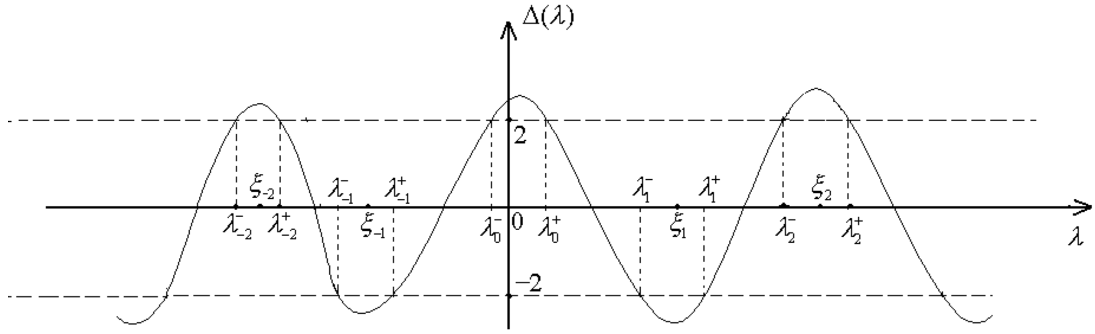
Malumki ([16]), agar $q \in L_2[0,\pi]$ va $p \in W_2^1[0,\pi]$ haqiyqiy, π davrli funksiyalar quyidagi shartlarni qanoatlantirsa: ushbu $y(x) \not\equiv 0$ va $y'(0)\bar{y}(0) - y'(\pi)\bar{y}(\pi) = 0$ shartlarni qanoatlantiruvchi $y(x) \in W_2^2[0,\pi]$ funksiyalar uchun

$$\int_0^{\pi} \left\{ \left| y'(x) \right|^2 + q(x) \left| y(x) \right|^2 \right\} dx > 0$$

tengsizlik bajarilsa, u holda (1.1) masalaning spektri quydagi to‘plamdan iborat bo‘ladi

$$\sigma(T) = \left\{ \lambda \in R \,\middle|\, -2 \le \Delta(\lambda) \le 2 \right\} = R \setminus \bigcup_{n=-\infty}^{\infty} (\lambda_{2n-1}, \lambda_{2n}).$$

Bu yerda $(\lambda_{2n-1}, \lambda_{2n})$, $n \in Z$ intervallarga lakunalar deyiladi. Nomerlash shunday kiritilganki, bunda $\lambda_{-1} < 0 < \lambda_0$ qo‘sh tengsizlik bajariladi.



(1-rasm)

ξ_n, $n \in Z \setminus \{0\}$ orqali $s(\pi,\lambda) = 0$ tenglamaning ildizlarini belgilaymiz. Bu holda ξ_n, $n \in Z \setminus \{0\}$ sonlar, (1.1) tenglama uchun qo‘yilgan Dirixle masalasining ($y(0) = y(\pi) = 0$) xos qiymatlari bilan ustma-ust tushadi va ushbu $\xi_n \in [\lambda_{2n-1}, \lambda_{2n}]$, $n \in Z \setminus \{0\}$ munosabatlar o‘rinli bo‘ladi.

Ta'rif 1. $\xi_n \in [\lambda_{2n-1}, \lambda_{2n}]$, $n \in Z \setminus \{0\}$ sonlarga va ushbu $\sigma_n = sign\{s'(\pi, \xi_n) - c(\pi, \xi_n)\}$, $n \in Z \setminus \{0\}$ ishoralarga (1.1) masalaning spektral parametrlari deyiladi.

Ta'rif 2. ξ_n, σ_n, $n \in Z \setminus \{0\}$ spektral parametrlar va spektrning chegaraviy nuqtalari λ_n, $n \in Z$ ga (1.1) masalaning spektral berilganlari deyiladi.

(1.1) masalaning spektral berilganlarini topish masalasiga to'g'ri spektral masala deyiladi. Aksincha, spektral berilganlar bo'yicha $p(x)$ va $q(x)$ koeffitsientlarni tiklash masalasiga teskari spektral masala deyiladi.

Shturm-Liuvill operatolarining kvadratik dastasini $p(x+\tau)$ va $q(x+\tau)$ koeffitsientlar bilan qarasak, uning spektri haqiqiy parametr τ ga bog'liq bo'lmaydi, ammo spektral parametrlari τ ga bog'liq bo'ladi, ularni $\xi_n(\tau)$, $\sigma_n(\tau)$, $n \in Z \setminus \{0\}$ orqali belgilaymiz. Bu spektral parametrlar quydagi Dubrovin-Trubovits sistemasini qanoatlantiradi:

$$\frac{d\xi_n}{d\tau} = 2(-1)^{n-1} sign(n) \cdot \sigma_n(\tau) \cdot \sqrt{(\xi_n - \lambda_{2n-1})(\lambda_{2n} - \xi_n)} \times$$

$$\times \sqrt{(\xi_n - \lambda_{-1})(\xi_n - \lambda_0) \prod_{k \neq n, 0} \frac{(\xi_n - \lambda_{2k-1})(\xi_n - \lambda_{2k})}{(\xi_n - \xi_k)^2}}, \quad n \in Z \setminus \{0\}.$$

Dubrovin-Trubovits sistemasi va ushbu

$$p(\tau) = \frac{\lambda_{-1} + \lambda_0}{2} + \sum_{0 \neq k = -\infty}^{\infty} \left(\frac{\lambda_{2k-1} + \lambda_{2k}}{2} - \xi_k(\tau) \right),$$

$$q(\tau) + 2p^2(\tau) = \frac{(\lambda_{-1})^2 + (\lambda_0)^2}{2} + \sum_{0 \neq k = -\infty}^{\infty} \left(\frac{(\lambda_{2k-1})^2 + (\lambda_{2k})^2}{2} - \xi_k^2(\tau) \right)$$

izlar formulalari birgalikda teskari spektral masalani yechishga imkon beradi. Bundan tashqari ulardan foydalanib, quyidagi teoremalarni isbot qilish mumkin.

Teorema 1. ([10]). Agar (1.1) tenglamaning $p(x)$ va $q(x)$ koeffitsientlari haqiqiy analitik funksiyalar bo'lsa, ya'ni haqiqiy o'qning har bir nuqtasida yaqinlashuvchi darajali qatorga yoyiluvchi bo'lsa, u holda (1.1) tenglamaga mos keluvchi lakunalarning uzunliklari eksponensial ravishda nolga intiladi, ya'ni shunday o‘zgarmas musbat sonlar $a>0$ va $b>0$ topilib, ushbu $\lambda_{2n}-\lambda_{2n-1}<ae^{-b|n|}$, $n\in Z$ tengsizlik o‘rinli bo‘ladi.

Teorema 2 ([10]). Agar (1.1) tenglamaga mos keluvchi lakunalar uzunliklari eksponensial ravishda nolga intilsa, u holda uning $p(x)$ va $q(x)$ koeffitsientlari haqiqiy analitik funksiyalar bo‘ladi.

Teorema 3. ([9]). Agar $p(x)$ va $q(x)$ koeffitsientlar $\frac{\pi}{2}$ davrga ega bo‘lsa, u holda (1.1) tenglamaga mos keluvchi toq nomerli barcha lakunalar yo‘qoladi. Aksincha, agar toq nomerli barcha lakunalar yo‘qolsa, $p(x)$ va $q(x)$ koeffitsientlar $\frac{\pi}{2}$ davrga ega bo‘ladi.

Bu yerda toq nomerli lakuna yo‘qoladi deganda uning chetki nuqtalari ustma-ust tushishi nazarda tutiladi. Bu hol antidavriy masalaning xos qiymati ikki karrali bo‘lganida sodir bo‘ladi.

2-§. Davriy funksiyalar sinfida Kaupning yuklangan hadli sistemasini integrallash

Kaupning yuklangan hadli sistemasini

$$\begin{cases} p_t=-6pp_x-q_x+\gamma(t)\cdot p\big|_{x=0}\cdot p_x \\ q_t=p_{xxx}-4qp_x-2pq_x+\gamma(t)\cdot p\big|_{x=0}\cdot q_x \end{cases} \tag{2.1}$$

ushbu

$$p(x,t)\big|_{t=0}=p_0(x),\ \ q(x,t)\big|_{t=0}=q_0(x) \tag{2.2}$$

boshlang‘ich shartlar bilan birga x bo‘yicha π davrli

$$p(x+\pi,t)\equiv p(x,t),\ q(x+\pi,t)\equiv q(x,t) \tag{2.3}$$

hamda ushbu

$$p(x,t),q(x,t)\in C_x^3(t>0)\cap C_t^1(t>0)\cap C(t\geq 0) \tag{2.4}$$

silliqlik shartlarini qanoatlantiruvchi haqiqiy funksiyalar sinfida ko‘rib chiqamiz. Bu yerda $\gamma(t)$ berilgan haqiqiy uzluksiz funksiya, $p_0(x), q_0(x) \in C^3(R)$ berilgan haqiqiy π davrli funksiyalar bo‘lib, $q_0(x) > 0$.

Teorema 2.1. Agar $p(x,t)$ va $q(x,t)$ funksiyalar juftligi (2.1)-(2.4) masalaning yechimi bo‘lsa, u holda koeffitsiyentlari $p(x+\tau,t)$ va $q(x+\tau,t)$ bo‘lgan Shturm-Liuvill operatorlari kvadratik dastasining spektri τ va t parametrlarga bog‘liq bo‘lmaydi, $\xi_n(\tau,t)$, $n \in Z \setminus \{0\}$ spektral parametrlari esa quyidagi Dubrovin-Trubovits sistemasini qanoatlantiradi:

$$\frac{\partial \xi_n}{\partial t} = 2(-1)^n \sigma_n(\tau,t) sign(n) \sqrt{(\xi_n - \lambda_{2n-1})(\lambda_{2n} - \xi_n)} \times$$
$$\times h_n(\xi) \times \{2p(\tau,t) + 2\xi_n(\tau,t) - \gamma(t) p(0,t)\}, \; n \in Z \setminus \{0\}. \qquad (2.5)$$

Bu yerda

$$h_n(\xi) = h_n(\dots, \xi_{-1}, \xi_1, \dots) = \sqrt{(\xi_n - \lambda_{-1})(\xi_n - \lambda_0) \prod_{k \neq n, 0} \frac{(\xi_n - \lambda_{2k-1})(\xi_n - \lambda_{2k})}{(\xi_n - \xi_k)^2}}.$$

Bunda $\sigma_n(\tau,t) = \pm 1$, $n \in Z \setminus \{0\}$ ishoralar $\xi_n(\tau,t)$ spektral parametr $[\lambda_{2n-1}, \lambda_{2n}]$ o‘z lakunasining chetiga kelganida qarama-qarshi ishoraga o‘zgaradi. Bundan tashqari ushbu

$$\xi_n(\tau,t)\big|_{t=0} = \xi_n^0(\tau), \;\; \sigma_n(\tau,t)\big|_{t=0} = \sigma_n^0(\tau), \quad n \in Z \setminus \{0\}$$

boshlang‘ich shartlar ham bajariladi. Bu yerda $\xi_n^0(\tau), \sigma_n^0(\tau)$, $n \in Z \setminus \{0\}$ lar $p_0(x+\tau)$ va $q_0(x+\tau)$ koeffitsientlarga mos keluvchi spektral parametrlardir.

Isbot. Ushbu

$$-y'' + q(x+\tau,t)y + 2\lambda\, p(x+\tau,t)y - \lambda^2 y = 0 \qquad (2.6)$$

Shturm-Liuvill tenglamalarining kvadratik dastasi uchun qo‘yilgan

$$y(0) = 0, \; y(\pi) = 0$$

Dirixle masalasining $\xi_n = \xi_n(\tau,t)$, $n \in Z \setminus \{0\}$ xos qiymatlariga mos keluvchi normallangan xos funksiyalarni $y_n(x,\tau,t)$, $n \in Z \setminus \{0\}$ orqali belgilaymiz.

Ushbu

$$-(y_n'',y_n)+(qy_n,y_n)+2\xi_n(py_n,y_n)-\xi_n^2=0$$

ayniyatni t bo‘yicha differensiallab, quyidagi tenglikka ega bo‘lamiz

$$-(\dot{y}_n'',y_n)-(y_n'',\dot{y}_n)+(q_ty_n+q\dot{y}_n,y_n)+(qy_n,\dot{y}_n)+$$

$$+2\dot{\xi}_n(py_n,y_n)+2\xi_n(p_ty_n+p\dot{y}_n,y_n)+2\xi_n(py_n,\dot{y}_n)-2\xi_n\dot{\xi}_n=0\,. \qquad (2.7)$$

Bu yerda $L_2(0,\pi)$ fazoning skalyar ko‘paytmasi ishlatildi.

Oxirgi tenglikni quyidagi tarzda yozib olamiz

$$\left(-\dot{y}_n''+q\dot{y}_n+2\xi_np\dot{y}_n,y_n\right)+\left(-y_n''+qy_n+2\xi_npy_n,\dot{y}_n\right)+$$

$$+\left(q_ty_n+2\xi_np_ty_n,y_n\right)+2\dot{\xi}_n(py_n,y_n)-2\xi_n\dot{\xi}_n=0\,,$$

$$2\dot{\xi}_n[\xi_n-(py_n,y_n)]=\left(q_ty_n+2\xi_np_ty_n,y_n\right),$$

ya’ni

$$2\dot{\xi}_n\left(\xi_n-\int_0^\pi py_n^2dx\right)=\int_0^\pi(q_t+2\xi_np_t)y_n^2dx\,. \qquad (2.8)$$

Ushbu

$$p_t(x+\tau,t)=-6p(x+\tau,t)p_x(x+\tau,t)-q_x(x+\tau,t)+\gamma(t)p(0,t)p_x(x+\tau,t)\,,$$

$$q_t(x+\tau,t)=p_{xxx}(x+\tau,t)-4q(x+\tau,t)p_x(x+\tau,t)-$$

$$-2p(x+\tau,t)q_x(x+\tau,t)+\gamma(t)p(0,t)q_x(x+\tau,t)$$

ayniyatlardan foydalanib, (2.8) tenglikni quyidagi tarzda yozib olamiz

$$2\dot{\xi}_n\left(\xi_n-\int_0^\pi py_n^2dx\right)=\int_0^\pi\{p_{xxx}-4qp_x-2pq_x+\gamma(t)p(0,t)q_x+$$

$$+2\xi_n[-6pp_x-q_x+\gamma(t)p(0,t)p_x]\}y_n^2dx\,. \qquad (2.9)$$

Integral ostidagi funksiyaning boshlang‘ichini y_n va y_n' ga nisbatan kvadratik forma ko‘rinishida izlaymiz, ya’ni

$$\{ay_n^2+by_ny_n'+cy_n'^2\}'=$$

$$=\{p_{xxx}-4qp_x-2pq_x+\gamma(t)p(0,t)q_x+2\xi_n[-6pp_x-q_x+\gamma(t)p(0,t)p_x]\}y_n^2. \quad (2.10)$$

Bu yerda $a=a(x,\tau,t,\xi_n)$, $b=b(x,\tau,t,\xi_n)$, $c=c(x,\tau,t,\xi_n)$ lar y_n va y_n' ga bog‘liq emas. (2.10) tenglik chap tomonidagi hosilalarni hisoblab, ushbu

$$y_n''=[q+2p\xi_n-\xi_n^2]y_n$$

ayniyatlardan foydalansak, quyidagi tenglik hosil bo‘ladi

$$(a' + bq + 2bp\xi_n - b\xi_n^2)y_n^2 + (2a + b' + 2cq + 4pc\xi_n - 2c\xi_n^2)y_n y_n' + (b + c')y_n'^2 =$$
$$= \{p_{xxx} - 4qp_x - 2pq_x + \gamma(t)p(0,t)q_x + 2\xi_n[-6pp_x - q_x + \gamma(t)p(0,t)p_x]\}y_n^2 .$$

(2.11)

Bunga ko‘ra

$$b = -c', \quad a = \frac{1}{2}c'' + c\cdot(\xi_n^2 - 2p\xi_n - q),$$

$$p_{xxx} - 4qp_x - 2pq_x + \gamma(t)p(0,t)q_x + 2\xi_n[-6pp_x - q_x + \gamma(t)p(0,t)p_x] =$$
$$= \frac{1}{2}c''' + 2c'\cdot(\xi_n^2 - 2p\xi_n - q) - c\cdot(2p'\xi_n + q'). \qquad (2.12)$$

Oxirgi tenglikning chap tomoni ξ_n ning chiziqli funksiyasi bo‘lgani uchun o‘ng tomoni ham ξ_n ning chiziqli funksiyasi bo‘lishi kerak. $c(x,\tau,t,\xi_n)$ ni ξ_n ga nisbatan 1-darajali ko‘phad ko‘rinishida izlaymiz:

$$c(x,\tau,t,\xi_n) = c_0(x,\tau,t)\xi_n + c_1(x,\tau,t). \qquad (2.13)$$

(2.13) ifodani (2.12) tenglikka qo‘ysak va ξ_n ning mos darajalari oldidagi koeffitsientlarni taqqoslasak, ushbu

$$c_0(x,\tau,t) = 2, \quad c_1(x,\tau,t) = 2p(x+\tau,t) - \gamma(t)p(0,t) \qquad (2.14)$$

tengliklarga ega bo‘lamiz.

(2.10) ayniyatga ko‘ra

$$2\dot{\xi}_n\left(\xi_n - \int_0^\pi p y_n^2 dx\right) = \left\{[\frac{1}{2}c'' + c\cdot(\xi_n^2 - 2p\xi_n - q)]y_n^2 - c'y_n y_n' + cy_n'^2\right\}\Bigg|_0^\pi =$$
$$= c(\pi,\tau,t,\xi_n)y_n'^2(\pi,\tau,t) - c(0,\tau,t,\xi_n)y_n'^2(0,\tau,t). \qquad (2.15)$$

Ushbu $c(x,\tau,t,\xi_n)$ funksiya x bo‘yicha π davrli ekanini hisobga olsak, (2.15) tenglik quyidagi ko‘rinishni oladi

$$2\dot{\xi}_n\left(\xi_n - \int_0^\pi p y_n^2 dx\right) = c(0,\tau,t,\xi_n)[y_n'^2(\pi,\tau,t) - y_n'^2(0,\tau,t)]. \qquad (2.16)$$

Bu yerda ushbu

$$c(x,\tau,t,\xi_n) = 2\xi_n + 2p(x+\tau,t) - \gamma(t)p(0,t)$$

ifodadan foydalansak, quyidagi

$$2\dot{\xi}_n\left(\xi_n-\int_0^\pi py_n^2dx\right)=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}[y_n'^2(\pi,\tau,t)-y_n'^2(0,\tau,t)] \quad (2.17)$$

tenglik kelib chiqadi.

Agar [11] adabiotning 56-betidagi ushbu

$$2\int_0^\pi[\lambda-p(x+\tau,t)]s^2(x,\lambda,\tau,t)dx=s'(\pi,\lambda,\tau,t)\frac{\partial s(\pi,\lambda,\tau,t)}{\partial\lambda}-s(\pi,\lambda,\tau,t)\frac{\partial s'(\pi,\lambda,\tau,t)}{\partial\lambda}$$

formuladan foydalansak, quyidagi tenglikni olamiz:

$$2\xi_n\alpha_n^2-2\int_0^\pi p(x+\tau,t)s^2(x,\xi_n,\tau,t)dx=s'(\pi,\xi_n,\tau,t)\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}. \quad (2.18)$$

Bu yerda

$$\alpha_n^2=\int_0^\pi s^2(x,\xi_n(t),\tau,t)dx.$$

Ushbu

$$y_n(x,\tau,t)=\frac{1}{\alpha_n}s(x,\xi_n(t),\tau,t)$$

ifodani (2.17) formulaga qo‘yib, (2.18) tenglikdan foydalanamiz:

$$2\dot{\xi}_n\left(\xi_n\alpha_n^2-\int_0^\pi ps^2(x,\xi_n,\tau,t)dx\right)=$$

$$=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}\cdot[s'^2(\pi,\xi_n,\tau,t)-1],$$

$$\dot{\xi}_ns'(\pi,\xi_n,\tau,t)\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}\cdot[s'^2(\pi,\xi_n,\tau,t)-1],$$

$$\dot{\xi}_n\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}\cdot\left(s'(\pi,\xi_n,\tau,t)-\frac{1}{s'(x,\xi_n,\tau,t)}\right). \quad (2.19)$$

Ushbu

$$c(x,\lambda,\tau,t)s'(x,\lambda,\tau,t)-c'(x,\lambda,\tau,t)s(x,\lambda,\tau,t)=1$$

Vronskiy ayniyatida $x=\pi$ va $\lambda=\xi_n$ desak,

$$c(\pi,\xi_n,\tau,t)=\frac{1}{s'(\pi,\xi_n,\tau,t)} \quad (2.20)$$

kelib chiqadi. Bu tenglikdan hamda ushbu

$$[c(\pi,\lambda,\tau,t)-s'(\pi,\lambda,\tau,t)]^2=(\Delta^2(\lambda)-4)-4c'(\pi,\lambda,\tau,t)s(\pi,\lambda,\tau,t)$$

ayniyantdan foydalanib, quyidagini hosil qilamiz

$$s'(\pi,\xi_n,\tau,t)-\frac{1}{s'(\pi,\xi_n,\tau,t)}=\sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n)-4}\,. \tag{2.21}$$

Bu yerda

$$\Delta(\lambda)=c(\pi,\lambda,t)+s'(\pi,\lambda,t),\ \sigma_n(\tau,t)=sign\left\{s'(\pi,\xi_n,\tau,t)-\frac{1}{s'(\pi,\xi_n,\tau,t)}\right\}.$$

Agar (2.21) ifodani (2.19) ga qo‘ysak, quyidagi tenglikni olamiz

$$\dot{\xi}_n=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}\cdot\frac{\sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n)-4}}{\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}}. \tag{2.22}$$

Ushbu

$$\Delta^2(\lambda)-4=-4\pi^2(\lambda-\lambda_{-1})(\lambda-\lambda_0)\prod_{0\neq k=-\infty}^{\infty}\frac{(\lambda-\lambda_{2k-1})(\lambda-\lambda_{2k})}{k^2},$$

$$s(\pi,\lambda,\tau,t)=\pi\prod_{0\neq k=-\infty}^{\infty}\frac{\xi_k-\lambda}{k}$$

yoyilmalardan foydalanib, (2.22) ayniyatni quyidagi tarzda yozamiz:

$$\dot{\xi}_n=2(-1)^n\sigma_n(\tau,t)sign(n)\cdot\sqrt{(\xi_n-\lambda_{2n-1})(\lambda_{2n}-\xi_n)}\times h_n(\xi)\times$$

$$\times\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}. \tag{2.23}$$

Bunda biz quyidagi tenglikdan ham foydalandik:

$$sign\left\{-\frac{\pi}{n}\prod_{k\neq n,0}\frac{\xi_k-\xi_n}{k}\right\}=(-1)^n sign(n).$$

Demak, (2.5) tenglik kelib chiqdi.

Agar chegaraviy shartlarni davriy yoki antidavriy shartlar bilan almashtirsak, (2.17) tenglamalar o‘rnida $\dot{\lambda}_n=0$, $n\in Z$ tenglamalar hosil bo‘ladi. Demak, λ_n, $n\in Z$ davriy va antidavriy masalaning xos qiymatlari t parametrga bog‘liq emas ekan. **Teorema 2.1 isbotlandi.**

Izox 2.1. Ushbu izlar formulasi

$$p(\tau,t)=\frac{\lambda_{-1}+\lambda_0}{2}+\sum_{0\neq k=-\infty}^{\infty}\left(\frac{\lambda_{2k-1}+\lambda_{2k}}{2}-\xi_k(\tau,t)\right) \tag{2.24}$$

yordamida (2.5) sistemani “yopiq” ko‘rinishda yozish mumkin.

Natija 2.1. Yuqoridagi 2.1-teorema (2.1)-(2.4) masalani yechish usulini beradi:

1) Avvalo $p_0(x+\tau)$ va $q_0(x+\tau)$ koeffitsientli Shturm-Liuvill tenglamalarining kvadratik dastasi uchun λ_n, $n\in Z$, $\xi_n^0(\tau)$, $\sigma_n^0(\tau)$, $n\in Z\setminus\{0\}$ spektral berilganlarini topamiz;

2) So‘ngra, (2.5)+(2.6) Koshi masalasini $\tau=0$ bo‘lganida yechib, $\xi_n(0,t)$, $n\in Z\setminus\{0\}$ spektral parametrlarni topamiz hamda (2.24) formula yordamida $p(0,t)$ ni aniqlaymiz;

3) Shundan so‘ng, (2.5)+(2.6) Koshi masalasini τ parametrning ixtiyoriy qiymatida yechib, $\xi_n(\tau,t)$, $\sigma_n(\tau,t)$, $n\in Z\setminus\{0\}$ spektral parametrlarni topamiz;

4) Bu yechimlarni (2.24) va quyidagi

$$q(\tau,t)+2p^2(\tau,t)=\frac{(\lambda_{-1})^2+(\lambda_0)^2}{2}+\sum_{0\neq k=-\infty}^{\infty}\left(\frac{(\lambda_{2k-1})^2+(\lambda_{2k})^2}{2}-\xi_k^2(\tau,t)\right)$$

izlar formulasiga qo‘yib, $p(x,t)$ va $q(x,t)$ funksiyalarni aniqlaymiz.

Izox 2.2. Yuqoridagi usul yordamida tuzilgan $p(\tau,t)$, $q(\tau,t)$ funksiyalar (2.1) sistemani qanoatlantirishini ko‘rsatamiz. Buning uchun Dubrovinning quyidagi sistemasidan

$$\frac{\partial\xi_n}{\partial\tau}=2(-1)^{n-1}sign(n)\sigma_n(\tau,t)h_n(\xi)\sqrt{(\xi_n-\lambda_{2n-1})(\lambda_{2n}-\xi_n)},\ n\in Z\setminus\{0\} \tag{2.25}$$

va ushbu

$$-\frac{3}{4}p_{\tau\tau}(\tau,t)+4p^3(\tau,t)+3p(\tau,t)q(\tau,t)=$$
$$=\frac{(\lambda_{-1})^3+(\lambda_0)^3}{2}+\sum_{0\neq k=-\infty}^{\infty}\left(\frac{(\lambda_{2k-1})^3+(\lambda_{2k})^3}{2}-\xi_k^3(\tau,t)\right) \tag{2.26}$$

izlar formulasidan ham foydalanamiz ([8]). (2.5) va (2.9) sistemalarga ko‘ra

$$\frac{\partial \xi_k}{\partial t} = -\{2p + 2\xi_k - \gamma(t)p(0,t)\}\frac{\partial \xi_k}{\partial \tau}, \; k \in Z \setminus \{0\}. \tag{2.27}$$

Agar (2.7) izlar formulasini t bo‘yicha differensiallab, (2.11) ayniyatlarni e’tiborga olsak, ushbu

$$p_t = -\sum_{0\neq k=-\infty}^{\infty} \frac{\partial \xi_k}{\partial t} = 2p\sum_{0\neq k=-\infty}^{\infty} \frac{\partial \xi_k}{\partial \tau} + 2\sum_{0\neq k=-\infty}^{\infty} \xi_k \frac{\partial \xi_k}{\partial \tau} - \gamma(t)p(0,t)\sum_{0\neq k=-\infty}^{\infty} \frac{\partial \xi_k}{\partial \tau} \tag{2.28}$$

tenglik kelib chiqadi. (2.7) va (2.8) izlar formulalaridan τ bo‘yicha hosila olamiz:

$$\sum_{0\neq k=-\infty}^{\infty} \frac{\partial \xi_k}{\partial \tau} = -p_\tau, \quad 2\sum_{0\neq k=-\infty}^{\infty} \xi_k \frac{\partial \xi_k}{\partial \tau} = -4pp_\tau - q_\tau. \tag{2.29}$$

Bu ifodalarni (2.28) tenglikka qo‘yib, ushbu

$$p_t = -6pp_\tau - q_\tau + \gamma(t)p(0,t)p_\tau \tag{2.30}$$

ayniyatni olamiz.

Endi (2.24) izlar formulasini t bo‘yicha differensiallaymiz va (2.27) ayniyatlarni ishlatamiz

$$q_t = -4pp_t - 2\sum_{0\neq k=-\infty}^{\infty} \xi_k \frac{\partial \xi_k}{\partial t} =$$

$$= -4pp_t + 4p\sum_{0\neq k=-\infty}^{\infty} \xi_k \frac{\partial \xi_k}{\partial \tau} + 4\sum_{0\neq k=-\infty}^{\infty} \xi_k^2 \frac{\partial \xi_k}{\partial \tau} - 2\gamma(t)p(0,t)\sum_{0\neq k=-\infty}^{\infty} \xi_k \frac{\partial \xi_k}{\partial \tau}.$$

Bu tenglikka (2.29) ifodalarni qo‘ysak, hamda (2.26) izlar formulasidan foydalansak, ushbu

$$q_t = -4pp_t + 2p(-4pp_\tau - q_\tau) + (p_{\tau\tau\tau} - 16p^2 p_\tau - 4p_\tau q - 4pq_\tau) -$$
$$- \gamma(t)p(0,t)(-4pp_\tau - q_\tau)$$

ayniyat kelib chiqadi. Bu yerga (2.30) ifodani qo‘ysak, u quyidagi ko‘rinishni oladi

$$q_t = p_{\tau\tau\tau} - 4qp_\tau - 2pq_\tau + \gamma(t)p(0,t)q_\tau.$$

Demak, tuzilgan $p(\tau,t)$, $q(\tau,t)$ funksiyalar (2.1) sistemani qanoatlantirar ekan.

Natija 2.2. Agar boshlang‘ich shartlardagi $p_0(x)$ va $q_0(x)$ funksiyalar haqiqiy analitik funksiya bo‘lsa, u holda unga mos keluvchi lakunalar uzunliklari eksponensial ravishda nolga intiladi, bu lakunalar $p(x,t)$ va $q(x,t)$ funksiylaraga ham mos keladi. Shuning uchun $p(x,t)$ va $q(x,t)$ yechimlar x o‘zgaruvchi bo‘yicha haqiqiy analitik funksiya bo‘ladi ([10]).

Natija 2.3. Agar boshlang‘ich shartlardagi $p_0(x)$ va $q_0(x)$ funksiyalar $\frac{\pi}{2}$ davrga ega bo‘lsa, u holda unga mos keluvchi barcha toq nomerli lakunalar yo‘qoladi, bu lakunalar $p(x,t)$ va $q(x,t)$ koefitsiyentlarga ham mos keladi. Shuning uchun $p(x,t)$ va $q(x,t)$ yechimlar x o‘zgaruvchi bo‘yicha $\frac{\pi}{2}$ davrga ega bo‘ladi ([9]).

III bob bo‘yicha xulosalar.

Mazkur bobda, davriy koeffitsientli Shturm-Liuvill operatorlarining kvadratik dastasi uchun qo‘yilgan teskari spektral masala yordamida, Kaupning yuklangan hadli sistemasi uchun qo‘yilgan Koshi masalasi davriy funksiyalar sinifida yechilgan.

Xulosa

Mazkur dissertatsiyaning I-bobida davriy koeffitsientli Shturm-Liuvill operatori uchun qo‘yilgan teskari spektral masala yordamida, yuklangan hadli Korteveg-de Friz tenglamasi uchun qo‘yilgan Koshi masalasi davriy funksiyalar sinifida yechilgan. Jumladan, Shturm-Liuvill operatori potensiali yuklangan hadli Korteveg-de Friz tenglamasining x o‘zgaruvchi bo‘yicha davriy yechimi bo‘lsa, u holda bu operatorning spektri t parametrga bog‘liq emasligi ko‘rsatilgan. Bu Shturm-Liuvill operatori spektral parametrlarining t parametr bo‘yicha evolyutsiyasi topilgan. Bundan tashqari, yechimning x bo‘yicha davri haqida va x bo‘yicha analitikligi haqida muhim natijalar olingan.

II-bobda davriy koeffitsientli Dirak operatori uchun qo‘yilgan teskari spektral masala yordamida, Shredingerning yuklangan hadli nochiziqli tenglamasi uchun qo‘yilgan Koshi masalasi davriy funksiyalar sinifida yechilgan. Jumladan, Dirak operatori potensiali Shredinger yuklangan hadli nochiziqli tenglamasining x o‘zgaruvchi bo‘yicha davriy yechimi bo‘lsa, u holda bu operatorning spektri t parametrga bog‘liq emasligi ko‘rsatilgan. Bu Dirak operatori spektral parametrlarining t parametr bo‘yicha evolyutsiyasi topilgan. Bundan tashqari, yechimning x bo‘yicha davri haqida va x bo‘yicha analitikligi haqida muhim natijalar olingan.

III-bobda davriy koeffitsientli Shturm-Liuvill operatorlarining kvadratik dastasi uchun qo‘yilgan teskari spektral masala yordamida, Kaupning yuklangan hadli sistemasi uchun qo‘yilgan Koshi masalasi davriy funksiyalar sinifida yechilgan.

Foydalanilgan adabiyotlar

1. Ахмедиев Н.Н., Корнеев В.И. Модуляционная неустойчивость и периодические решения нелинейного уравнения Шредингера. // ТМФ, 1986, т. 69, № 2, с. 189-194

2. Ахмедиев Н.Н., Елеонский В.М., Кулагин Н.Е. Точные решения первого порядка нелинейного уравнения Шредингера. // ТМФ, 72, № 2, 1987, с. 183-196.

3. Alfimov G.L., Its A.R., Kulagin N.E. Modulation instability of solutions of the nonlinear Schrödinger equation. //Theoret. Mat. Fiz., 84:2, 163-172 (1990).

4. Yakhshimuratov A. The Nonlinear Schrodinger Equation with a Self-consistent Source in the Class of Periodic Functions. // Mathematical Physics, Analysis and Geometry, (2011) 14, pp.153-169.

5. Смирнов А.О. Вещественные конечнозонные регулярные решения уравнения Каупа-Буссинеска. Теорет. мат.физ., **66**:1, (1986), 30-46.

6. Смирнов А.О. Матричный аналог теоремы Аппеля и редукции многомерных тэта-функций Римана. Мат. сб., **133**(**175**):3(7), (1987), 382-391.

7. Хасанов А.Б., Яхшимуратов А.Б. Об уравнении Кортевега-де Фриза с самосогласованным источником в классе периодических функций. // «Теоретическая и математическая физика», 2010 г., т. 164, № 2, стр. 214-221.

8. Яхшимуратов А.Б. Интегрирование уравнения Кортевега-де Фриза со специальным свободным членом в классе периодических функций. // Уфимский матем. журнал. 2011, т. 3, № 4, с. 144-150.

9. Яхшимуратов А.Б. Аналог обратной теоремы Борга для квадратичного пучка операторов Штурма-Лиувилля. Вестник Елецкого государственного университета им. И.А.Бунина, серия «Математика. Компьютерная математика», **8**:1, (2005), 121-126.

10. Бабажанов Б.А., Хасанов А.Б., Яхшимуратов А.Б. Об обратной задаче для квадратичного пучка операторов Штурма-Лиувилля с

периодическим потенциалом. Дифференциальные уравнения, **41**:3, (2005), 298-305.

11. Белоколос Е.Д., Бобенко А.И., Матвеев В.Б., Энольский В.З. Алгеброгеометрические принципы суперпозиции конечнозонных решений интегрируемых нелинейных уравнений. // УМН, 1986, т. 41, № 2, с. 3-42.

12. Бабич М.В., Бобенко А.И., Матвеев В.Б. Решения нелинейных уравнений, интегрируемых методом обратной задачи, в тэта-функциях Якоби и симметрии алгебраических кривых. // Изв. АН СССР, сер. матем. 1985, т. 49, № 3, с. 511-529.

13. Боголюбов Н.Н.(мл.), Прикарпатский А.К., Курбатов А.М., Самойленко В.Г. Нелинейная модель типа Шредингера: законы сохранения, гамильтонова структура и полная интегрируемость. // ТМФ, 1985, т. 65, № 2, с. 271-284.

14. Bikbaev R.F., Its A.R. Algebrogeometric solutions of a boundary-value problem for the nonlinear Schrödinger equation. // Mat. Zametki, 45:5, 3-9 (1989).

15. Matveev V.B., Yavor M.I. Solutions presque periodiques et a N-solitons de l'equation hydrodynamique nonlineaire de Kaup. Ann. Inst. Henri Poincare, Sect. A , **31**, (1979), 25-41.

16. Гусейнов Г.Ш. Обратные спектральные задачи для квадратичного пучка операторов Штурма-Лиувилля на конечном интервале. Сб. Спектральная теория операторов и ее приложения, вып. 7, Изд-во «Элм», Баку, (1986), 51-101.

17. Гусейнов Г.Ш. Спектр и разложения по собственным функциям квадратичного пучка операторов Штурма-Лиувилля с периодическими коэффициентами. Сб. Спектральная теория операторов и ее приложения, вып. 6, Изд-во «Элм», Баку, (1985), 56-97.

18. Гусейнов Г.Ш. О квадратичном пучке операторов Штурма-Лиувилля с периодическими коэффициентами. Вестн. Моск. ун-та. Сер. 1, мат., мех., № 3, (1984), 14-21.

19. Grinevich P.G., Taimanov I.A. Spectral conservation laws for periodic nonlinear equations of the Melnikov type. //Amer. Math. Soc. Transl. Ser. 2, 224, 125-138 (2008).

20. Дубровин Б.А., Новиков С.П. Периодический и условно периодический аналоги многосолитонных решений уравнения Кортевега-де Фриза. // ЖЭТФ, 1974, т. 67, № 12, с. 2131-2143.

21. Дубровин Б.А. Периодическая задача для уравнения Кортевега-де Фриза в классе конечнозонных потенциалов. // Функц. анализ и прил., 1975, т. 9, № 3, с. 41-51.

22. Додд Р., Эйлбек Дж., Гиббон Дж., Моррис Х. Солитоны и нелинейные волновые уравнения. - Москва: Мир, 1988, 694 с.

23. Kaup D.J. A higher-order water-wave equation and the method for solving it. Prog. Theor. Phys., **54**, (1975), 396-408.

24. Захаров В.Е., Шабат А.Б. Точная теория двумерной само-фокусировки и одномерной автомодуляции волн в нелинейных средах. //ЖЭТФ, т. 61, № 1, с. 118-134 (1971).

25. Захаров В.Е., Шабат А.Б. О взаимодействии солитонов в устойчивой среде. //ЖЭТФ, т. 64, № 5, с. 1627-1639 (1973).

26. Захаров В.Е., Манаков С.В. Полная интегрируемость нелинейного уравнения Шредингера. //ТМФ, т. 19, № 3, с. 332-343 (1974).

27. Захаров В.Е., Манаков С.В., Новиков С.П., Питаевский Л.П. Теория солитонов: метод обратной задачи. - Москва: Наука, 1980, 320 с.

28. Zeng Y.B., Ma W.X., Lin R.L. Integration of the soliton hierarchy with self-consistent sources. // J. Math. Phys., 41:8, 5453-5489 (2000).

29. Итс А.Р., Рыбин А.В., Салль М.А. К вопросу о точном интегрировании нелинейного уравнения Шредингера. //ТМФ, 74:1, 29-45 (1988).

30. Итс А.Р. Асимптотика решений нелинейного уравнения Шредингера и изомонодромные деформации систем линейных дифференциальных уравнений. // Докл. АН СССР, 1981, т. 261, № 1, с. 14-18.

31. Итс А.Р., Котляров В.П. Явные формулы для решений нелинейного уравнения Шредингера. // Докл. АН УССР, сер. А, 1976, № 11, с. 965-968.

32. Итс А.Р. Обращение гиперэллиптических интегралов и интегрирование нелинейных дифференциальных уравнений. // Вестник ЛГУ, 1976, № 7, с. 39-46.

33. Итс А.Р., Матвеев В.Б. Операторы Шредингера с конечнозонным спектром и N-солитонные решения уравнения Кортевега-де Фриза. // Теор. и мат. физика, 1975, т. 23, № 1, с. 51-68.

34. Левитан Б.М., Саргсян И.С. Операторы Штурма-Лиувилля и Дирака. М.: “Наука” 1988 г.

35. Lax P. Periodic Solutions of the KdV equation. // Comm. Pure Appl. Math., 1975, v. 28, p. 141-188.

36. Lax P. Periodic solutions of the KdV equations. // Lecture in Appl. Math. AMS, 1974, v. 15, p. 85-96.

37. Leon J., Latifi A. Solution of an initial-boundary value problem for coupled nonlinear waves // J. Phys. A: Math. Gen., 1990, v. 23, p. 1385-1403.

38. Марченко В.А. Периодическая задача Кортевега-де Фриза. // Мат. сб. 1974, т. 95, № 3, с. 331-356.

39. McKean H.P., Trubowitz E. Hill’s operator and Hyperelliptic Function Theory in the Presence of infinitely Many Branch Points. // Comm. Pure Appl. Math., 1976, v. 29, p. 143-226.

40. Мельников В.К. Метод интегрирования уравнения Кортевега-де Вриса с самосогласованным источником. Дубна, ОИЯИ, 1988. Препринт № 2-88-11/798, 6 с.

41. Mel’nikov V.K. Integration of the nonlinear Schrodinger equation with a source. // Inverse Probl., 1992, v. 8, p. 133-147.

42. Митропольский Ю.А., Боголюбов Н.Н.(мл.), Прикарпатский А.К., Самойленко В.Г. Интегрируемые динамические системы: спектральные и дифференциально-геометрические аспекты. Наукова думка, Киев (1987).

43. Matveev, V.B., Yavor, M.I.: Solutions presque periodiques et a N-solitons de l'equation hydrodynamique nonlineaire de Kaup. //Ann. Inst. Henri Poincare, Sect. A , 31, 25-41 (1979).

44. Mel'nikov V.K. Integration of the Nonlinear Schrödinger Equation with a Self-Consistent Source. //Commun. Math. Phys. 137, 359-381 (1991).

45. Shao Y., Zeng Y. The solutions of the NLS equations with self-consistent sources. //J. Phys. A: Math. Gen. 38, 2441-2467 (2005).

46. Мисюра Т.В. Характеристика спектров периодической и антипериодической краевых задач, порождаемых операцией Дирака I.// сб. "Теория функций, функц. анализ и их приложения", Харьков, 1978, вып. 30, с. 90-101.

47. Мисюра Т.В. Характеристика спектров периодической и антипериодической краевых задач, порождаемых операцией Дирака II.// сб. "Теория функций, функц. анализ и их приложения", Харьков, 1979, вып. 31 стр. 102-109.

48. Мисюра Т.В. Конечно-зонные операторы Дирака. // Теор. функц. функ. ан. и прил. № 33, 107-111 (1980).

49. Мисюра Т.В. Аппроксимация периодического потенциала оператора Дирака конечно-зонными потенциалами. // Теор. функц. функ. ан. и прил. № 36, 55-65 (1981).

50. Misyura T.V. An asymptotic formula for Weyl solutions of the Dirac equations. //J. Math. Sci. **77**:1, 2941-2954 (1995).

51. Марченко В.А. Операторы Штурма-Лиувилля и их приложения. – Киев: Наукова думка, 1977, 332 с.

52. Новиков С.П. Периодическая задача Кортевега-де Фриза I. // Функц. анализ и прил., 1974, т. 8, № 3, с. 54-66.

53. Orlov A.Yu., Schulman E.I. Additional symmetries for integrable equations and conformal algebra representation. // Lett. Math. Phys., 1986, 12, 3, p. 171-179.

54. Станкевич И.В. Об одной задаче спектрального анализа для уравнения Хилла.// ДАН СССР, 1970, т. 192, № 1, с. 34-37.

55. Satsuma J., Yajima N. Initial value problems of one-dimensional self-modulation of nonlinear waves in dispersive media.// Progr. Theor. Phys. Suppl. **55**, 284-306 (1974)

56. Смирнов А.О. Эллиптические решения нелинейного уравнения Шредингера и модифицированного уравнения Кортевега-де Фриза.// Математический сборник, 1994, т. 185, № 8, с. 103-114.

57. Смирнов А.О. Эллиптические по t решения нелинейного уравнения Шредингера.// ТМФ, 1996, т. 107, № 2, с. 188-200.

58. Станкевич И.В. Об одной задаче спектрального анализа для уравнения Хилла.// ДАН СССР, 1970, т. 192, № 1, с. 34-37.

59. Trubowitz E. The inverse problem for periodic potentials.// Comm. Pure. Appl. Math., 1977, v. 30, p. 321-337.

60. Тахтаджян Л.А., Фаддеев Л.Д. Гамильтонов подход в теории солитонов. - Москва: Наука, 1986, 528 с.

61. Уразбоев Г.У., Хасанов А.Б. Интегрирование уравнения Кортевега-де Фриза с самосогласованным источником при начальных данных типа «ступеньки». // Теор. и мат. физика. 2001, т. 129, № 1, с. 38-54.

62. Хасанов А.Б., Уразбоев Г.У. Интегрирование общего уравнения КдФ с правой частью в классе быстроубывающих функций. // Узб. матем. журнал. 2003, № 2, с. 53-59.

63. Хасанов А.Б., Яхшимуратов А.Б. Об уравнении Кортевега-де Фриза с самосогласованным источником в классе периодических функций. // Теор. и мат. физика. 2010, т. 164, № 2, с. 214-221.

64. Hochstadt H. A Generalization of Borg's Inverse Theorem for Hill's Equations. // Journal of math. analysis and applications, 102, 1984, p. 599-605.

65. Хасанов А.Б., Яхшимуратов А.Б. Аналог обратной теоремы Г.Борга для оператора Дирака. // Узбекский математический журнал, 2000, № 3, с. 40-

66. Хасанов А.Б., Ибрагимов А.М. Об обратной задаче для оператора Дирака с периодическим потенциалом. // Узбекский математический журнал, 2001, № 3-4, с. 48-55.

67. Яхшимуратов А.Б., Матякубов М.М. Интегрирование уравнения Кортевега-де Фриза с нагруженным членом в классе периодических функций. «Известия высших учебных заведений. Математика», (2016), № 2, 87-92.

68. Hasanov A.B. Xill tenglamasi uchun teskari masalalar va ularning tatbiqlari. – 2013 yil. Toshkent, “FAN” nashryoti, 384 bet.

69. Яхшимуратов А.Б., Купалова К.К. Об одной теореме единственности решения обратной спектральной задачи Штурма-Лиувилля, Материалы Республиканской научной конференции «Математическая физика и родственные проблемы современного анализа», 26-27 ноября 2015 г., г.Бухара, с. 299.

70. Yaxshimuratov A., Matyoqubov M., Ko`palova K. Davriy funksiyalar sinfida Kaupning yuklangan hadli sistemasi uchun Koshi masalasi. “Ilm sarchashmalari” jurnali, 2016 yil, 5-son, 3-8 betlar.