TOSHKENT AXBOROT TEXNOLOGIYALARI UNIVERSITETI

DASTURIY INJINIRING FAKULTETI

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Mavzu: Teylor va Makloren qatorlari va ularning ba’zi misollarni yechishdagi tadbiqi.

Toshkent 2015

Teylor va Makloren qatorlari. Darajali qatorning taqribiy hisoblashlarga tatbiqi

Ma'lumki, [3, 7§, 95-97 b] $f(x)$ funksiya $x=a$ nuqta atrofida $(n+1)$-tartibgacha barcha hosilalarga ega bo'lsa, bu funksiya uchun quyidagi formula o'rinli edi:

$$f(x)=f(a)+\frac{x-a}{1!}f^{I}(a)+...+\frac{(x-a)^n}{n!}f^{(n)}(a)+R_n(x) \qquad \textbf{(1)}$$

Bunda $R_n(x)$ - qoldiq had bo'lib,

$$R_n(x)=\frac{(x-a)^{n+1}}{(n+1)!}f^{(n+1)}(a+\theta(x-a)), \quad 0<\theta<1 \qquad \textbf{(2)}$$

Agar $f(x)$ funksiya $x=a$ nuqta atrofida istalgan tartibli hosilalarga ega bo'lsa, (1) tenglikda $n\to\infty$ da limitga o'tsak, $\lim\limits_{n\to\infty n} R_n(x)=0$ bo'lganidan

$$f(x)=f(a)+\frac{f^{I}(a)}{1!}(x-a)+\frac{f^{II}(a)}{2!}(x-a)^2+...+\frac{f^{(n)}(a)}{n!}(x-a)^n+... \qquad \textbf{(3)}$$

Qator hosil bo'ladi va bu qatorga **Teylor qatori** deyiladi.

Xususiy holda $a=0$ bo'lsa,

$$f(x)=f(0)+\frac{f^{I}(0)}{1!}x+\frac{f^{II}(0)}{1!}x^2+....+\frac{f^{(n)}(0)}{1!}x^n+.... \qquad \textbf{(4)}$$

Qator hosil bo'ladi va bu qatorga **Makloren qatori** deyiladi.

Endi ko'p uchraydigan ba'zi elementar funksiyalarning darajali qatorlarga yoyilmalarini keltiramiz, [3, 129§, 188 b].

1. $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^n}{n!}+... \qquad -\infty<x<\infty$

2. $\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-...+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+... \qquad -\infty<x<\infty$

3. $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-...+(-1)^n\frac{x^{2n}}{(2n)!}+... \qquad -\infty<x<\infty$

4. $shx=x+\frac{x^3}{3!}+\frac{x^5}{5!}+... \qquad$ 5. $chx=1+\frac{x^2}{2!}+\frac{x^4}{4!}+...$

5. $arctgx=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+...+(-1)^{n+1}\frac{x^{2n-1}}{2n-1}$

6. $\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{}+...+(-1)^{n+1}\frac{x^4}{n}+... \qquad -1<x\le 1$

7. $tgx=x+\frac{x^3}{3}+\frac{x^5}{15}+\frac{17}{315}x^7+\frac{62}{2835}x^9+...$

Teylor va Makloren qatorlari. Darajali qatorning taqribiy hisoblashlarga tatbiqi

Ma'lumki, [3, 7§, 95-97 b] $f(x)$ funksiya $x = a$ nuqta atrofida $(n+1)$-tartibgacha barcha hosilalarga ega bo'lsa, bu funksiya uchun quyidagi formula o'rinli edi:

$$f(x) = f(a) + \frac{x-a}{1!} f'(a) + \ldots + \frac{(x-a)^n}{n!} f^{(n)}(a) + R_n(x) \qquad \textbf{(1)}$$

Bunda $R_n(x)$ - qoldiq had bo'lib,

$$R_n(x) = \frac{(x-a)^{n+1}}{(n+1)!} f^{(n+1)}(a + \theta(x-a)), \quad 0 < \theta < 1 \qquad \textbf{(2)}$$

Agar $f(x)$ funksiya $x = a$ nuqta atrofida istalgan tartibli hosilalarga ega bo'lsa, (1) tenglikda $n \to \infty$ da limitga o'tsak, $\lim\limits_{n\to\infty n} R_n(x) = 0$ bo'lganidan

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \ldots \qquad \textbf{(3)}$$

Qator hosil bo'ladi va bu qatorga **Teylor qatori** deyiladi.

Xususiy holda $a = 0$ bo'lsa,

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Qator hosil bo'ladi va bu qatorga **Makloren qatori** deyiladi.

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1. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots \qquad -\infty < x < \infty$

2. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots \qquad -\infty < x < \infty$

3. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots \qquad -\infty < x < \infty$

4. $shx = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$ 5. $chx = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$

5. $arctgx = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots + (-1)^{n+1} \frac{x^{2n-1}}{2n-1}$

6. $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots + (-1)^{n+1} \frac{x^4}{n} + \ldots \quad -1 < x \leq 1$

7. $tgx = x + \frac{x^3}{3} + \frac{x^5}{15} + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \ldots$

8. $\frac{x}{1+x^2} = x - x^3 + x^5 - x^7 + x^9 + \ldots$

9. $\arcsin x = x + \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1\cdot3\cdot x^3}{2\cdot4\cdot5} + \frac{1\cdot3\cdot5\cdot x^5}{2\cdot4\cdot6\cdot7} + \ldots$

10. $\ln(x+\sqrt{x^2+1}) = x - \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1\cdot3\cdot x^3}{2\cdot4\cdot5} - \frac{1\cdot3\cdot5\cdot x^7}{2\cdot4\cdot6\cdot7} + \ldots$

11. $\frac{1}{2}\ln\frac{1+x}{1-x} = x + \frac{x^5}{5} + \frac{x^7}{7} + \ldots$

12. $(1+x)^m = 1 + mx + \frac{m(m-1)}{1\cdot2}x^2 + \frac{m(m-1)(m-2)}{1\cdot2\cdot3}x^3 + \ldots \quad -1 < x < 1$

13. $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots$

14. $\frac{1}{(1+x)^2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - \ldots$

15. $\frac{1}{(1+x)^3} = 1 - \frac{2\cdot3}{2}x + \frac{3\cdot4}{2}x^2 - \frac{4\cdot5}{2}x^3 + \ldots$

16. $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - \ldots$

17. $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{2\cdot4}x^2 + \frac{1\cdot2}{2\cdot4\cdot6}x^3 - \frac{1\cdot3\cdot5}{2\cdot4\cdot6\cdot8}x^4 + \ldots$

18. $\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{1\cdot3}{2\cdot4}x^2 - \frac{1\cdot3\cdot5}{2\cdot4\cdot6}x^3 + \frac{1\cdot3\cdot5\cdot7}{2\cdot4\cdot6\cdot8}x^4 - \ldots$

19. $\frac{1}{\sqrt{1-x^2}} = 1 + \frac{1}{2}x^2 + \frac{1\cdot3}{2\cdot4}x^4 + \ldots + \frac{1\cdot3\cdot5}{2\cdot4\cdot6}x^4 + \frac{1\cdot3\cdot5\cdot7}{2\cdot4\cdot6\cdot8}x^8 + \ldots$

Endi darajali qatorni taqribiy hisoblashga tatbiqini o'rganamiz. Masalan, $x = x_0$ nuqtada $y = f(x)$ funksiyaning qiymatini aniq hisoblab bo'lmasa, darajali qator yordamida uning qiymatini istalgancha aniqlikda hisoblash mumkin. Agar $f(x)$ funksiya x ning darajalari bo'yicha qatorga yoyilgan bo'lsa, funksiyaning biror $x = x_0$ nuqtadagi qiymatini topish uchun:

1) $S_n = \sum_{k=1}^{n} a_n x_0^k$ topiladi; 2) $R_n = S - Sn$ qoldiq baholanadi.

Agar absolyut hato $\acute{a} = |A - a| \le \frac{1}{10^n}$ bo'lsa, a soni A ning $\frac{1}{10^n}$ gacha aniqlikdagi taqribiy qiymati bo'ladi. Xuddi shuningdek, aniq integralni taqriban hisoblash talab qilinsa, integral ostidagi funksiya darajali qatorga yoyiladi va so'ngra integrallanadi. Differensial tenglamalarni ham darajali qator yordamida taqribiy yechishni topish mumkin bo'ladi.

MISOLLAR:

1) $\ln 1{,}1$ ni 0,0001 gacha aniqlikda hisoblang.

2) $e^{0,1}$ ni 0,001 gacha aniqlikda hisoblang.

3) $\sqrt[3]{130}$ ni 0,0001 gacha aniqlikda taqribiy hisoblang.

8. $\frac{x}{1+x^2}=x-x^3+x^5-x^7+x^9+...$

9. $\arcsin x=x+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1\cdot3\cdot x^3}{2\cdot4\cdot5}+\frac{1\cdot3\cdot5\cdot x^5}{2\cdot4\cdot6\cdot7}+...$

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11. $\frac{1}{2}\ln\frac{1+x}{1-x}=x+\frac{x^5}{5}+\frac{x^7}{7}+...$

12. $(1+x)^m=1+mx+\frac{m(m-1)}{1-2}x^2+\frac{m(m-1)(m-2)}{1\cdot2\cdot3}x^3+...\quad -1<x<1$

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19. $\frac{1}{\sqrt{1-x^2}}=1+\frac{1}{2}x^2+\frac{1\cdot3}{2\cdot4}x^4+...+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}x^4+\frac{1\cdot3\cdot5\cdot7}{2\cdot4\cdot6\cdot8}x^8+...$

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5) $\frac{1}{\sqrt[5]{e}}$ ni 0,00001 gacha aniqlikda hisoblang.

6) $\cos 18^0$ ni 0,0001 gacha aniqlikda hisoblang.

7) $\ln 1.04$ ni 0, 0001 gacha aniqlikda hisoblang.

8) To'g'ri burchakli uchburchakni katetlari 1 sm va 5 sm. ga teng. Kichik katet qarshisidagi burchakni 0,001 radian aniqlikda hisoblang.

9) $\int_0^{1/2} \frac{1-\cos x}{x^2} dx$ integralni 0,0001 aniqlikgacha hisoblang.

10) $\int_0^{0,2} \frac{e^x - 1}{x} dx$ integralni 0,001 aniqlikgacha hisoblang.

Yechish: 1) 16§ dagi 6) formulaga asosan

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - ... + (-1)^{n-1}\frac{x^n}{n} + ..., \qquad -1 \le x \le 1$$

$x = 0,1$ desak

$$\ln 1,1 = \ln(1+0,1) = 0,1 - \frac{(0,1)^2}{2} + \frac{(0,1)^3}{3} - ... + (-1)^{n-1}\frac{(0,1)^n}{n} + ...$$

Hosil bo'lgan sonli qator ishorasi navbatlanuvchi qator bo'lib, $\frac{(0.1)^4}{4} = \frac{0,0001}{4} \prec 0.0001$ bo'lganidan

$$\ln 1.1 = 0,1 - \frac{0,01}{2} + \frac{0,001}{3} \approx 0,953, \text{ ya'ni } \ln 1,1 \approx 0,0953$$

2) $e^{0,1}$ ni 0,001 aniqlikda hisoblaymiz; 17 § dagi 1) formulaga asosan

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2} + ... + \frac{x^{n-1}}{(n-1)!} + Rn(x), \quad Rn(x) = \frac{x^n}{n!}l\xi, \; 0 \prec \xi \prec x$$

$x = 0,1$ deb olsak, $e\xi < e^{0,1} < e < 3$ bo'lganidan

$R_n(x) = \frac{3}{10^n n!} < 0,001$ tengsizlik $n = 3$ bo'lganda bajariladi, ya'ni e^x funksiyaning Maklaren formulasiga yoyilmasiga uchta hadni olish kifoya, ya'ni

$$e^{0,1} \approx 1 + 0,1 + \frac{(0,1)^2}{2!} = 1,1 + \frac{0,01}{2} \approx 1,005; \qquad e^{0,1} \approx 1,105$$

3) $\sqrt[3]{130}$ ni 0,0001 aniqlikda taqriban hisoblaymiz.

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + ... + \frac{m(m-1)...(m-(n-1))}{n!}x^n + ...,$$
$$-1 < x < 1$$

4) $\sqrt{e}$ ni 0,00001 gacha aniqlikda hisoblang.

5) $\frac{1}{\sqrt[5]{e}}$ ni 0.00001 gacha aniqlikda hisoblang.

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$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{n-1}\frac{x^n}{n} + \ldots, \qquad -1 \le x \le 1$$

$x = 0{,}1$ desak

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Hosil bo'lgan sonli qator ishorasi navbatlanuvchi qator bo'lib, $\frac{(0{,}1)^4}{4} = \frac{0{,}0001}{4} \prec 0{,}0001$ bo'lganidan

$$\ln 1{,}1 = 0{,}1 - \frac{0.01}{2} + \frac{0{,}001}{3} \approx 0{,}953, \text{ ya'ni } \ln 1{,}1 \approx 0{,}0953$$

2) $e^{0,1}$ ni 0,001 aniqlikda hisoblaymiz; 17 § dagi 1) formulaga asosan

$$e^x = 1 + \frac{x}{11} + \frac{x^2}{2} + \ldots + \frac{x^{n-1}}{(n-1)!} + Rn(x), \qquad Rn(x) = \frac{x^n}{n!} l\xi, \; 0 \prec \xi \prec x$$

$x = 0{,}1$ deb olsak, $e\xi < e^{0,1} < e < 3$ bo'lganidan

$R_n(x) = \frac{3}{10^n n!} < 0{,}001$ tengsizlik $n = 3$ bo'lganda bajariladi, ya'ni e^x funksiyaning Maklaren formulasiga yoyilmasiga uchta hadni olish kifoya, ya'ni

$$e^{0.1} \approx 1 + 0{,}1 + \frac{(0{,}1)^2}{2!} = 1{,}1 + \frac{0{,}01}{2} \approx 1{,}005; \qquad e^{0,1} \approx 1{,}105$$

3) $\sqrt[3]{130}$ ni 0,0001 aniqlikda taqriban hisoblaymiz.

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \ldots + \frac{m(m-1)\ldots(m-(n-1))}{n!}x^n + \ldots,$$

$-1 < x < 1$

$n = \frac{1}{3}$ deb $\sqrt[3]{130}$ ni quyidagicha ifodalaymiz:

$$\sqrt[3]{130} = \sqrt[3]{125+5} = \sqrt[3]{125(1+\frac{5}{125})} = 5(1+\frac{1}{25})^{\frac{1}{3}}$$

Bu xolda

$$(1+x)\ (1+x)^{\frac{1}{3}} = 1+\frac{1}{3}x+\frac{\frac{1}{3}(\frac{1}{3}-1)}{2!}x^2+\frac{\frac{1}{3}(\frac{1}{3}-2)}{3!}x^3+\ldots \text{ yoki}$$

$$(1+\frac{1}{25})^{\frac{1}{3}} = 1+\frac{1}{3\cdot 25}-\frac{1\cdot 2}{3^2\cdot 2!(25)^2}+\frac{1\cdot 2\cdot 5}{3^3\cdot 3!(25)^3}$$

Bu sonli qatorning to'rtinchi hadi

$\frac{1\cdot 2\cdot 5}{3^3\cdot 3!(25)^3} = \frac{1}{81\cdot 625} \prec 0{,}0001$ bo'lganidan $\sqrt[3]{1+\frac{1}{25}}$ ni 0,0001 aniqlikda hisoblash uchun qatorning uchta hadini olish kifoya ekan.

Haqiqatdan

$$\sqrt[3]{130} \approx 5(1+\frac{1}{25})^{\frac{1}{3}} = 5(1+\frac{1}{3\cdot 25}-\frac{1\cdot 2}{3^2\cdot 2!25^2}) \approx 5+0{,}0667-0{,}0009 = 5{,}0658;$$

ya'ni $\sqrt[3]{130} = 5{,}0658$

4) $\sqrt{e}$ 0,00001 aniqlikda hisoblaymiz:

$$e^x = 1+x+\frac{x^2}{2!}+\ldots+\frac{x^{n-1}}{(n-1)!}+\ldots, \qquad -\infty < x < \infty$$

$$\sqrt{e} = e^{\frac{1}{2}} = 1+\frac{1}{2}+\frac{1}{2!2^2}+\frac{1}{3!2^3}+\ldots,$$

$$R_n < \frac{x^n}{n!}\cdot\frac{x}{n+1-x}$$

$x = \frac{1}{2}$ desak, $R_n < \frac{1}{n!2n}\cdot\frac{1/2}{n+\frac{1}{2}} = \frac{1}{n!2n}\cdot\frac{1}{2n+1}$

n ni tanlash orqali $R_n < 0{,}00001$ tengsizlik bajarilishini aniqlaymiz. Masalan $n = 3$ bo'lsa,

$R_3 < \frac{1}{6\cdot 8\cdot 7} = \frac{1}{336}$. Agar $n = 5$ bo'lsa $R_5 < \frac{1}{42240}$ va $n = 6$ bo'lsa $R_6 < \frac{1}{64\cdot 720\cdot 13} < \frac{1}{100000}$. Demak, $\sqrt{e}$ ni 0,00001 aniqlikda hisoblash uchun $n = 6$ ta had olish kerak eakn, ya'ni

$n=\frac{1}{3}$ deb $\sqrt[3]{130}$ ni quyidagicha ifodalaymiz:

$$\sqrt[3]{130}=\sqrt[3]{125+5}=\sqrt[3]{125(1+\frac{5}{125})}=5(1+\frac{1}{25})^{\frac{1}{3}}$$

Bu xolda

$$(1+x)\ (1+x)^{\frac{1}{3}}=1+\frac{1}{3}x+\frac{\frac{1}{3}(\frac{1}{3}-1)}{2!}x^2+\frac{\frac{1}{3}(\frac{1}{3}-2)}{3!}x^3+\dots \text{ yoki}$$

$$(1+\frac{1}{25})^{\frac{1}{3}}=1+\frac{1}{3\cdot 25}-\frac{1\cdot 2}{3^2\cdot 2!(25)^2}+\frac{1\cdot 2\cdot 5}{3^3\cdot 3!(25)^3}-\dots$$

Bu sonli qatorning to'rtinchi hadi

$\frac{1\cdot 2\cdot 5}{3^3\cdot 3!(25)^3}=\frac{1}{81\cdot 625}\prec 0{,}0001$ bo'lganidan $\sqrt[3]{1+\frac{1}{25}}$ ni 0,0001 aniqlikda hisoblash uchun qatorning uchta hadini olish kifoya ekan.

Haqiqatdan

$$\sqrt[3]{130}\approx 5(1+\frac{1}{25})^{\frac{1}{3}}=5(1+\frac{1}{3\cdot 25}-\frac{1\cdot 2}{3^2\cdot 2!25^2})\approx 5+0{,}0667-0{,}0009=5.0658;$$

ya'ni $\sqrt[3]{130}=5.0658$

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$$e^x=1+x+\frac{x^2}{2!}+\dots+\frac{x^{n-1}}{(n-1)!}+\dots,\qquad -\infty<x<\infty$$

$$\sqrt{e}=e^{\frac{1}{2}}=1+\frac{1}{2}+\frac{1}{2!2^2}+\frac{1}{3!2^3}+\dots\ .$$

$$R_n<\frac{x^n}{n!}\cdot\frac{x}{n+1-x},$$

$x=\frac{1}{2}$ desak, $R_n<\frac{1}{n!2n}\cdot\frac{1/2}{n+\frac{1}{2}}=\frac{1}{n!2n}\cdot\frac{1}{2n+1}$

n ni tanlash orqali $R_n<0{,}00001$ tengsizlik bajarilishini aniqlaymiz. Masalan $n=3$ bo'lsa.

$R_3<\frac{1}{6\cdot 8\cdot 7}=\frac{1}{336}$. Agar $n=5$ bo'lsa $R_5<\frac{1}{42240}$ va $n=6$ bo'lsa $R_6<\frac{1}{64\cdot 720\cdot 13}<\frac{1}{100000}$. Demak, $\sqrt{e}$ ni 0,00001 aniqlikda hisoblash uchun $n=6$ ta had olish kerak eakn, ya'ni

$$\sqrt{e}=1+\frac{1}{2}+\frac{1}{2!\cdot2^2}+\frac{1}{3!\cdot2^3}+\frac{1}{4!\cdot2^4}+\frac{1}{5!\cdot2^5}+\frac{1}{6!\cdot2^6}=1+0{,}5+0{,}125+0{,}020833+0{,}002604+$$
$$+0{,}00026+0{,}000022=1{,}648719$$

Demak, $\sqrt{e}$ ning 0,00001 aniqlik bilan hisoblangan qiymati $\sqrt{e}\approx 1{,}64872$.

5) $\frac{1}{\sqrt[5]{e}}$ ning qiymatini 0,00001 aniqlikda taqriban hisoblaymiz;

$$\frac{1}{\sqrt[5]{e}}=e^{-\frac{1}{5}}=1-\frac{1}{5}+\frac{1}{2!5^2}-\frac{1}{3!5^3}+\dots$$

Hosil bo'lgan sonli qator ishorasi navbatlashuvchi qator bo'lib,

$R_6=\frac{1}{5!5^5}=\frac{1}{120\cdot3125}=\frac{1}{375000}<0.00001$ bo'lganidan $\frac{1}{\sqrt[5]{e}}$ ning qiymatini 0.00001 aniqlikda hisoblash uchun ishorasi navbatlashuvchi qatorning dastlabki beshta hadini olish kifoya ekan, ya'ni

$$\frac{1}{\sqrt[5]{e}}=1-\frac{1}{5}+\frac{1}{2!\cdot5^2}-\frac{1}{3!\cdot5^3}+\frac{1}{4!\cdot5^4}=1-0{,}2+0{,}02-0{,}00133+0{,}000067=0{,}818734.$$

6) $\cos18^0$ ni 0,0001 aniqlikda hisoblaymiz;

$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots$ bo'lganidan $x=18^0=\frac{\pi}{180^0}18^0=\frac{\pi}{10}$ bo'lganidan

$$\cos18^0=\cos\frac{\pi}{10}=1-\frac{1}{2!}\left(\frac{\pi}{10}\right)^2+\frac{1}{4!}\left(\frac{\pi}{10}\right)^4-\frac{1}{6!}\left(\frac{\pi}{10}\right)^6+\dots$$

$\frac{1}{6!}\left(\frac{\pi}{10}\right)^6=\frac{1}{720}\cdot0{,}0009584=0{,}0000013\prec0{,}0001$ bo'lganidan

$\cos18^0=\cos\frac{\pi}{10}$ ning qiymatini hisoblash uchun qator yoyilmasidan uch hadni olsak kifoya ekan.

$\frac{\pi}{10}=0{,}3142$, $\left(\frac{\pi}{10}\right)^2=0{,}0987$, $\left(\frac{\pi}{10}\right)^4=0{,}0097$ bo'lganidan

$\cos18^0\approx1-\frac{0.0987}{2}+\frac{1}{24}0{,}0097=0{,}9511$, ya'ni $\cos18^0\approx0.9511$

7) $\ln1.04$ ni 0,0001 aniqlikda hisoblaymiz.

1- misolda $\ln1{,}1$ ni taqriban hisoblagan edik.

$\ln(1+x)$ ni darajali qatorga yoyilmasini qo'llsak,

$\ln1{,}04=\ln()1+0{,}04=0{,}04-\frac{(0{,}04)^2}{2}+\frac{(0{,}04)^3}{3}-\frac{(0{,}04)^4}{4}+\dots$yoki

$$\sqrt{e}=1+\frac{1}{2}+\frac{1}{2!\cdot 2^2}+\frac{1}{3!\cdot 2^3}+\frac{1}{4!\cdot 2^4}+\frac{1}{5!\cdot 2^5}+\frac{1}{6!\cdot 2^6}=1+0,5+0,125+0,020833+0,002604+$$
$$+0.00026+0,000022=1,648719$$

Demak, $\sqrt{e}$ ning 0,00001 aniqlik bilan hisoblangan qiymati $\sqrt{e}\approx 1,64872$.

5) $\frac{1}{\sqrt[5]{e}}$ ning qiymatini 0,00001 aniqlikda taqriban hisoblaymiz;

$$\frac{1}{\sqrt[5]{e}}=e^{-\frac{1}{5}}=1-\frac{1}{5}+\frac{1}{2!5^2}-\frac{1}{3!5^3}+...$$

Hosil bo'lgan sonli qator ishorasi navbatlashuvchi qator bo'lib,
$R_6=\frac{1}{5!5^5}=\frac{1}{120\cdot 3125}=\frac{1}{375000}<0,00001$ bo'lganidan $\frac{1}{\sqrt[5]{e}}$ ning qiymatini 0.00001 aniqlikda hisoblash uchun ishorasi navbatlashuvchi qatorning dastlabki beshta hadini olish kifoya ekan, ya'ni

$$\frac{1}{\sqrt[5]{e}}=1-\frac{1}{5}+\frac{1}{2!\cdot 5^2}-\frac{1}{3!\cdot 5^3}+\frac{1}{4!\cdot 5^4}=1-0,2+0,02-0.00133+0,000067=0,818734.$$

6) $\cos 18^0$ ni 0,0001 aniqlikda hisoblaymiz;

$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+...$ bo'lganidan $x=18^0=\frac{\pi}{180^0}18^0=\frac{\pi}{10}$ bo'lganidan

$$\cos 18^0=\cos\frac{\pi}{10}=1-\frac{1}{2!}\left(\frac{\pi}{10}\right)^2+\frac{1}{4!}\left(\frac{\pi}{10}\right)^4-\frac{1}{6!}\left(\frac{\pi}{10}\right)^6+...$$

$\frac{1}{6!}\left(\frac{\pi}{10}\right)^6=\frac{1}{720}\cdot 0.0009584=0,0000013\prec 0.0001$ bo'lganidan

$\cos 18^0=\cos\frac{\pi}{10}$ ning qiymatini hisoblash uchun qator yoyilmasidan uch hadni olsak kifoya ekan.

$\frac{\pi}{10}=0.3142$, $\left(\frac{\pi}{10}\right)^2=0,0987$, $\left(\frac{\pi}{10}\right)^4=0,0097$ bo'lganidan

$\cos 18^0\approx 1-\frac{0,0987}{2}+\frac{1}{24}0,0097=0,9511$, ya'ni $\cos 18^0\approx 0,9511$

7) $\ln 1.04$ ni 0,0001 aniqlikda hisoblaymiz.

1- misolda $\ln 1.1$ ni taqriban hisoblagan edik.
$\ln(1+x)$ ni darajali qatorga yoyilmasini qo'llsak,

$\ln 1.04=\ln()1+0,04=0,04-\frac{(0,04)^2}{2}+\frac{(0,04)^3}{3}-\frac{(0,04)^4}{4}+...$ yoki

$$\ln 1{,}04 \approx 0{,}04 - 0{,}0008 + 0{,}000021 - 0{,}00000064 + \ldots$$

Bu yerda qatorning uchinchi hadi $0{,}000021 < 0{,}0001$ bo'lganidan qatorni dastlabki ikki hadi bilan chegaralanishi kifoya, ya'ni

$$\ln 1{,}04 \approx 0{,}04 - 0{,}0008 = 0{,}0392 .$$

8) To'g'ri uchburchakning katetlari 1 sm va 5 sm. ga teng. Kichik katet qarshisidagi burchakni 0,001 radian aniqlikda hisoblang.

Trigonometrik funksiyaning ta'rifiga asosan $tg\alpha = \frac{1}{5}$

yoki $\alpha = arctg\frac{1}{5} = arctg0{,}2$.

$$arctgx = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots \qquad x = \frac{1}{5} = 0{,}2 \text{ desak,}$$

$$arctg0{,}2 = 0{,}2 - \frac{(0{,}2)^3}{3} + \frac{(0{,}2)^5}{5} - \frac{(0{,}2)^7}{7} + \ldots$$

$\frac{(0{,}2)^5}{5} = 0{,}000064 < 0{,}001$ bo'lganidan, $arctg0{,}2$ ni qiymatini 0,001 aniqlikda hisoblash uchun qator yoyilmasida dastlabki ikki hadni olish kifoya qilar ekan, ya'ni

$$\alpha = arctg0{,}2 = 0{,}2 - \frac{0{,}008}{3} = 0{,}2 - 0{,}0026(6) = 0{,}197, \quad \alpha \approx 0{,}197$$

9) $\int\limits_0^{1/2} \frac{1 - \cos x}{x^2}$ integrali 0,0001 aniqlik bilan hisoblaymiz;

$$\frac{1-\cos x}{x^2} = \frac{1 - 1 + \frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} - \ldots}{x^2} = \frac{x^2\left(\frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} - \ldots\right)}{x^2} = \frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} - \ldots \text{ bo'lganidan}$$

$$\int\limits_0^{1/2} \frac{1-\cos x}{x^2}dx = \int\limits_0^{1/2}\left(\frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} - \ldots\right)dx = \left[\frac{1}{2!}x - \frac{x^3}{3\cdot 4!} + \frac{x^5}{5\cdot 6!} - \ldots\right]\Bigg|_0^{\frac{1}{2}} = \frac{1}{2!2} - \frac{1}{2^3\cdot 3\cdot 4!} + \frac{1}{2^5\cdot 5\cdot 6!} - \ldots \approx$$

$$\approx 0{,}25 - 0{,}0017 = 0{,}2483 .$$

Izoh. $\frac{1}{2^5\cdot 5\cdot 6!} = \frac{1}{32\cdot 5\cdot 720} = \frac{1}{115200} = 0{,}0000086 < 0{,}0001$ bo'lganidan hosil bo'lgan sonli qatorni dastlabki ikki hadi bilan chegaralandik, bundan tashqari darajali qatorni integrallashda darajali qatorni yaqinlashish oralig'ida integrallash mumkinligi haqidagi teoremadan foydalaniladi.

$\ln 1{,}04 \approx 0{,}04 - 0{,}0008 + 0{,}000021 - 0{,}00000064 + \ldots$

Bu yerda qatorning uchinchi hadi $0{,}000021 < 0{,}0001$ bo'lganidan qatorni dastlabki ikki hadi bilan chegaralanishi kifoya, ya'ni

$\ln 1{,}04 \approx 0{,}04 - 0{,}0008 = 0{,}0392$.

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Trigonometrik funksiyaning ta'rifiga asosan $tg\alpha = \frac{1}{5}$

yoki $\alpha = arctg\frac{1}{5} = arctg 0{,}2$.

$arctgx = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$. $\quad x = \frac{1}{5} = 0{,}2$ desak,

$$arctg 0{,}2 = 0{,}2 - \frac{(0{,}2)^3}{3} + \frac{(0{,}2)^5}{5} - \frac{(0{,}2)^7}{7} + \ldots$$

$\frac{(0{,}2)^5}{5} = 0{,}000064 < 0{,}001$ bo'lganidan, $arctg 0{,}2$ ni qiymatini 0,001 aniqlikda hisoblash uchun qator yoyilmasida dastlabki ikki hadni olish kifoya qilar ekan, ya'ni

$$\alpha = arctg 0{,}2 = 0{,}2 - \frac{0{,}008}{3} = 0{,}2 - 0{,}0026(6) = 0{,}197, \quad \alpha \approx 0{,}197$$

9) $\int\limits_0^{1/2} \frac{1 - \cos x}{x^2}$ integrali 0,0001 aniqlik bilan hisoblaymiz;

$$\frac{1-\cos x}{x^2} = \frac{1 - 1 + \frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} - \ldots}{x^2} = \frac{x^2\left(\frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} - \ldots\right)}{x^2} = \frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} - \ldots$$ bo'lganidan

$$\int\limits_0^{1/2} \frac{1-\cos x}{x^2}dx = \int\limits_0^{1/2}\left(\frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} - ..\right)dx = \left[\frac{1}{2!}x - \frac{x^3}{3\cdot 4!} + \frac{x^5}{5\cdot 6!} - ..\right]\Bigg|_0^{\frac{1}{2}} = \frac{1}{2!\cdot 2} - \frac{1}{2^3\cdot 3\cdot 4!} + \frac{1}{2^5\cdot 5\cdot 6!} - .. \approx$$

$\approx 0{,}25 - 0{,}0017 = 0{,}2483$.

Izoh. $\frac{1}{2^5\cdot 5\cdot 6!} = \frac{1}{32\cdot 5\cdot 720} = \frac{1}{115200} = 0{,}0000086 < 0{,}0001$ bo'lganidan hosil bo'lgan sonli qatorni dastlabki ikki hadi bilan chegaralandik, bundan tashqari darajali qatorni integrallashda darajali qatorni yaqinlashish oralig'ida integrallash mumkinligi haqidagi teoremadan foydalaniladi.

10) $\int_0^{0,2} \frac{e^x - 1}{x} dx$ integralni 0,001 aniqlikda hisoblaymiz.

$$e^x - 1 = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... - 1 = x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... = x\left[1 + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\right]$$ bo'lganidan

$$\int_0^{0,2} \frac{e^x - 1}{x} dx = \int_0^{0,2} \left[1 + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\right] dx = \left[x + \frac{x^3}{3 \cdot 2!} + \frac{x^4}{4 \cdot 3!} + ...\right]\Big|_0^{0,2} = 0,2 + \frac{(0,2)^3}{3 \cdot 2!} +$$

$$+ \frac{(0,2)^4}{4 \cdot 3!} + ... + .. = 0,2 + 0,0013(3) + 0,0000(6) + ...$$

$R_3(x) \approx 0.0000(6) < 0,001$ bo'lganidan bu yerda $\int_0^{0,2} \frac{e^x - 1}{x} dx$ ni 0,001 aniqlikda hisoblash uchun hosil bo'lgan sonli qatorning dastlabki ikki hadini olish kifoya, ya'ni

$$\int_0^{0,2} \frac{e^x - 1}{x} dx \approx 0,201(3).$$

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$e^x - 1 = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... - 1 = x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... = x\left[1 + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\right]$ bo'lganidan

$$\int_0^{0,2} \frac{e^x - 1}{x} dx = \int_0^{0,2} \left[1 + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\right] dx = \left[x + \frac{x^3}{3 \cdot 2!} + \frac{x^4}{4 \cdot 3!} + ...\right]\Big|_0^{0,2} = 0,2 + \frac{(0,2)^3}{3 \cdot 2!} +$$

$$+ \frac{(0,2)^4}{4 \cdot 3!} + ... + .. = 0,2 + 0,0013(3) + 0,0000(6) + ...$$

$R_2(x) \approx 0.0000(6) < 0,001$ bo'lganidan bu yerda $\int_0^{0,2} \frac{e^x - 1}{x} dx$ ni 0.001 aniqlikda hisoblash uchun hosil bo'lgan sonli qatorning dastlabki ikki hadini olish kifoya, ya'ni

$$\int_0^{0,2} \frac{e^x - 1}{x} dx \approx 0.201(3) .$$

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