O`zbekiston Respublikasi  Oliy va o`rta
maxsus ta`lim vazirligi

Namangan
muhandislik–pedagogika instituti

Oliy matematika kafedrasi

BIQ ta'lim yo'nalishi talabalari uchun III semestrda o'tiladigan mavzular bo'yicha
MASALALAR   TO`PLAMI

NAMANGAN - 2015

Oliy matematika kafedrasi
uslubiy seminarida ko`rib chiqilib,
institut kengashiga tavsiya etilgan.
1-sonli majlis bayoni 28.08. 2015 y

Namangan muhandislik-
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etilgan.
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Kirish

Ushbu masalalar to'plami oliy texnika o'quv yurti o'qituvchi va talabalari oliy matematika fanidan amaliy mashg`ulot darslarida foydalanish uchun mo`ljallangan. Unda keltirilgan mavzular BIQ (Bino va inshootlar qurilishi) yo'nalshining amaldagi ishchi dasturiga mos bo'lib, 3-semestrda foydalanish mumkin.

To'plam oliy matematikaning aniq integral, qatorlar, ko'p o'zgaruvchili funksiyalar, karrali integrallar bo'limlarini o`z ichiga qamrab olgan.

To'plamni yozishda hozirga qadar rus va o`zbek tillarida chop etilgan adabiyotlardan foydalanildi, hamda mualliflar to`plagan tajribaga asoslanib oddiydan murakkabga ketma-ketligini saqlashga harakat qilindi. Kitob o`quv dasturiga moslab yozilgan bo`lib, har bir mavzuga kerakli formulalar keltirilib, na`muna tarzida masalalarning yechilishi keltirilgan. Mustaqil ishlash va uy vazifasi uchun masalalar va ularning javoblari keltirilgan.

Mualliflar to'plamni ilmiy va uslubiy jihatdan yaxshilash uchun bildiriladigan takliflarni mamnuniyat bilan qabul qiladilar.

Mualliflar.

Aniq integral.

Aniq integralning xossalari, Nyuton - Leybnis formulasi.

Aniq integral xossalari

1) $\int_a^b f(x)dx = -\int_b^a f(x)$

2) $\int_a^a f(x)dx = 0$

3) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \quad c \in (a,b)$

4) $\int_a^b \left(f_1(x) \pm f_2(x)\right)dx = \int_a^b f_1(x)dx \pm \int_a^b f_2(x)dx$

5) $\int_a^b Cf(x)dx = C\int_a^b f(x)dx$

6) $\int_a^b f(x)dx = F(x)\ \Big|_a^b = F(b) - F(a)$

1-misol. $\int_0^1 x^2 dx;$ integralni ta`rif yordamida hisoblang.

Yechish: $f(x) = x^2.\ \ a = 0,\ \ b = 1;\ [0;1]$ kesmani n ta bo`lakka bo`lamiz.

$$\Delta x_k = (b-a)/n = 1/n, \quad \xi_k = x_k \qquad x_0 = 0,\ x_1 = \frac{1}{n};\ x_2 = \frac{2}{n};\cdots;\ x_{n-1} = \frac{n-1}{n};\ x_n = \frac{n}{n} = 1$$

$$f(\varsigma_1) = \left(\frac{1}{n}\right)^2;\ f(\varsigma_2) = \left(\frac{2}{n}\right)^2;\dots,\ f(\varsigma_n) = \left(\frac{n}{n}\right)^2;\ f(\varsigma_k)\cdot\Delta x_k = \left(\frac{k}{n}\right)^2\cdot\frac{1}{n} \quad \text{u holda}$$

$$\int_0^1 x^2 dx = \lim_{n\to\infty}\frac{1^2+2^2+3^2+\cdots+n^2}{n^2} = \lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3} = \lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6} = \frac{1}{3}$$

2-misol. $\int_{-3}^{2}(2x+3)dx$ integralni hisoblang.

Yechish:

$$\int_{-3}^{2}(2x+3)dx = \int_{-3}^{2}2xdx + \int_{-3}^{2}3dx = x^2\Big|_{-3}^{2} + 3x\Big|_{-3}^{2} = (2^2-(-3)^2)+(3\cdot2-3\cdot(-3)) = -5+15 = 10$$

3-misol. $\int_{\pi/6}^{\pi/4}\frac{4dx}{\cos^2 x}$ integralni hisoblang

Yechish: $$\int_{\pi/6}^{\pi/4}\frac{4dx}{\cos^2 x} = 4\int_{\pi/6}^{\pi/4}\frac{dx}{\cos^2 x} = 4\cdot tgx\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = 4\cdot\left[tg\frac{\pi}{4} - tg\frac{\pi}{6}\right] = 4\cdot\left(1-\frac{\sqrt{3}}{3}\right)$$

Darsda yechish uchun misollar.
Quyidagi integrallarni hisoblang.

1. $\int_1^3 x^3 dx$ j. 20
2. $\int_1^4 \sqrt{x}dx$ j. $\frac{14}{3}$
3. $\int_1^2\left(x^2+\frac{1}{x^4}\right)dx$ j. $\frac{63}{24}$
4. $\int_0^{\pi/4}\sin 4xdx$ j. $\frac{1}{2}$
5. $\int_0^3 e^{\frac{x}{3}}dx$ j. 3(e-1)
6. $\int_a^{a\sqrt{3}}\frac{dx}{a^2+x^2}$ j. $\frac{\pi}{12a}$

7. $\int_0^2 |1-x|dx$ j. 1 8. $\int_1^4 \frac{x+1}{\sqrt{x}}dx$ j. $6\frac{2}{3}$ 9. $\int_{-1}^7 \frac{dx}{\sqrt{3x+4}}$ j. $\frac{8}{3}$

10. $\int_0^1 \frac{dz}{(2z+1)^3}$ j. $\frac{2}{9}$ 11. $\int_0^1 x\sqrt{1+x}dx$ j. $\frac{4\left(\sqrt{2}+1\right)}{15}$

12. $\int_{\pi/8}^{\pi/4} ctg^2 xdx$ j. $\frac{1}{2}-\frac{\pi}{8}$ 13. $\int_0^{\pi} \cos\frac{x}{2}\cos\frac{3x}{2}dx$ j.0

Mustaqil uy vazifasi uchun misollar

14. $\int_0^3 \left(3^{1-x}+\left(\frac{1}{3}\right)^{2x-1}\right)dx$ j. $\frac{1066}{243\ln 3}$ 15. $\int_{-2}^2 \frac{x^3+x^2+x+1}{x^2+1}dx$ j. 4

16. $\int_{-3}^2 (5x+2)dx$ j. $-2\frac{1}{2}$ 17. $\int_1^c \frac{x^2+1}{2x^2}dx$ j. $\frac{c^2-1}{2c}$

18. 1) $\int_1^3 |2-x|dx$ j. 1 2) $\int_0^1 x(1-x)^2\,dx$ j. $\frac{1}{12}$

Aniq integralni o`zgaruvchini almashtirib integrallash

Aniq integralni hisoblashda o`zgaruvchini almashtirish usuli.

$$\int_a^b f(x)dx=\int_{\alpha}^{\beta} f(\varphi(t))\varphi'(t)dt \quad \text{bu yerda} \quad a=\varphi(\alpha), b=\varphi(\beta)$$

$$\int_a^b f[\varphi(x)]\varphi'(x)\,dx=\left\|\begin{matrix} t=\varphi(x), & \alpha=\varphi(a) \\ dt=\varphi'(x)\,dx, & \beta=\varphi(b)\end{matrix}\right\|=\int_{\alpha}^{\beta} f(t)\,dt$$

1-misol. $\int_0^1 \frac{x^2dx}{(x+1)^2}$ integralni hisoblang.

Yechish:

$$\int_0^1 \frac{x^2dx}{(x+1)^2}=\left|\begin{matrix} t=x+1, \ x=1;\ t=2 \\ x=t-1, \ x=0;\ t=1 \\ dx=dt\end{matrix}\right|=\int_1^2 \frac{(t-1)^2}{t^2}dt=\int_1^2 \frac{t^2-2t+1}{t^2}dt=\int_1^2 dt-2\int_1^2 \frac{dt}{t}+\int_1^2 \frac{dt}{t^2}=t\Big|_1^2-2\ln t\Big|_1^2-\frac{1}{t}\Big|_1^2=$$

$$=2-1-2\ln 2-\frac{1}{2}+1=\frac{3}{2}+2\ln 2$$

2-misol. $\int_0^{\pi/2} \cos^2 x\cdot \sin xdx$ integralni hisoblang.

Yechish: $$\int_0^{\pi/2} \cos^2 x\cdot\sin xdx=\left|\begin{matrix} t=\cos x, dt=-\sin xdx \\ x=0, \quad t=1. \\ x=\frac{\pi}{2}; \quad t=0\end{matrix}\right|=-\int_1^0 t^2dt=\frac{1}{3}t^3\Big|_0^1=\frac{1}{3}$$

Darsda yechish uchun misollar.
Quyidagi integrallarni hisoblang.

19. $\int_1^8 \frac{xdx}{\sqrt{3x-1}}$ j:8

20. $\int_0^1 \frac{x^2dx}{\sqrt{4-x^2}}$ j. $\frac{\pi}{3}-\frac{\sqrt{3}}{2}$

21. $\int_0^{\sqrt{a}} x^2\sqrt{a^2-x^2}dx$ j. $\frac{\pi a^2}{16}$

22. $\int_0^{\pi/2} x^2\sqrt{9-x^2}dx$ j. $\frac{81}{8}\pi$

23. $\int_1^3 x^3\sqrt{x^2-1}dx$ j. $\frac{464\sqrt{2}}{15}$

24. $\int_3^8 \frac{xdx}{\sqrt{x+1}}$ j. $\frac{32}{3}$

25. $\int_{\frac{\sqrt{2}}{2}}^1 \frac{\sqrt{1-x^2}dx}{x^2}$ j. $1-\frac{\pi}{4}$

26. $\int_1^2 \frac{dx}{2x-1}$ j. $\frac{1}{2}\ln 3$

27. $\int_0^{\frac{\pi}{2}} \frac{cosxdx}{1+\sin^2 x}$ j. $\frac{\pi}{4}$

28. $\int_{\sqrt{3}}^{\sqrt{8}} x\sqrt{1+x^2}dx$ j. $\frac{19}{3}$

29. $\int_1^e \frac{\cos(\ln x)}{x}dx$ $j:\ \sin 1$

30. $\int_0^{\pi} \sin\left(\frac{\pi}{6}-2x\right)dx$

31. $\int_0^{0.5} e^{\sin\pi x}\cos\pi x dx$

Mustaqil uy vazifasi uchun misollar

32. $\int_1^e \frac{dx}{x(1+\ln^2 x)}$ $j:\ \frac{\pi}{4}$

33. $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}dx$ j. $e-\sqrt{e}$

34. $\int_0^1 \frac{xdx}{1+x^4}$ j. $\frac{\pi}{8}$

35. $\int_0^1 e^{x+e^x}dx$ j. e^e-e

36. $\int_1^3 \frac{\sqrt{x}}{x+1}dx$ j. $2\sqrt{3}-2-\frac{\pi}{6}$

37. $\int_{-\frac{\pi}{12}}^0 \sin\left(\frac{\pi}{4}-3x\right)dx$ j. $\frac{\sqrt{2}}{6}$

38. $\int_{25}^{196} \frac{dx}{x-4\sqrt{x}}$ j. $2\ln 10$

39. $\int_4^{25} \frac{dx}{\sqrt{x}-1}$ j. $6+4\ln 2$

Aniq integralni bo`laklab integrallash

Bo`laklab integrallash formulasi. $\int_a^b udv=(u\cdot v)\Big|_a^b-\int_a^b vdu$

1-misol. $\int_e^{e^2} x\ln xdx$ integralni hisoblang.

$$\int_e^{e^2} x\ln x dx = \left| \begin{matrix} u = \ln x, du = \frac{1}{x}dx \\ dv = xdx, v = \frac{x^2}{2} \end{matrix} \right| = \frac{x^2}{2}\ln x \Big|_e^{e^2} - \int_e^{e^2} \frac{x^2}{2}\cdot\frac{dx}{x} = e^4 - \frac{e^2}{2} - \frac{1}{2}\frac{x^2}{2}\Big|_e^{e^2} = e^4 - \frac{e^2}{2} - \frac{e^4}{4} + \frac{e^2}{4} =$$

$$= \frac{3}{4}e^4 - \frac{e^2}{4} = \frac{e^2}{4}\left(3e^2 - 1\right)$$

2-misol $\int_0^1 xe^x dx$ integralni hisoblang.

Yechish: $$\int_0^1 xe^x dx = \left| \begin{matrix} u = x, du = dx \\ dv = e^x dx, v = e^x \end{matrix} \right| = xe^x \Big|_0^1 - \int_0^1 e^x dx = e - e^x \Big|_0^1 = e - e + 1 = 1$$

3-misol $\int_{-a}^{a} x\cos\frac{x}{a}dx$ integralni hisoblang.

Yechish:

$$\int_{-a}^{a} x\cos\frac{x}{a}dx = \left| \begin{matrix} u = x, du = dx \\ dv = \cos\frac{x}{a}dx, v = a\sin\frac{x}{a} \end{matrix} \right| = x\cdot a\cdot\sin\frac{x}{a}\Big|_{-a}^{a} - a\int_{-a}^{a}\sin\frac{x}{a}dx = a^2\sin 1 - (-a)a\sin(-1) -$$

$$- a\cdot a\left(\cos\frac{x}{a}\right)\Big|_{-a}^{a} = a^2\sin 1 - a^2\sin 1 + a^2\cos 1 + a^2\cos(-1) = 0$$

4 misol. $I = \int_0^{\pi/2} e^x \sin xdx$ integralni hisoblang.

Yechish:

$$I = \int_0^{\pi/2} e^x \sin xdx = \left| \begin{matrix} u = e^x, dv = \sin xdx \\ du = e^x dx, v = -\cos x \end{matrix} \right| = -e^x\cos x \Big|_0^{\pi/2} + \int_0^{\pi/2} e^x\cos xdx = \left| \begin{matrix} u = e^x, dv = \cos xdx \\ du = e^x dx, v = \sin x \end{matrix} \right| =$$

$$= -e^x\cos x\Big|_0^{\pi/2} + e^x\sin x\Big|_0^{\pi/2} - \int_0^{\pi/2} e^x\sin xdx = 1 + e^{\pi/2} - \int_0^{\pi/2} e^x \sin xdx$$

$$I = 1 + e^{\pi/2} - I$$

$$I = \frac{1}{2}(1 + e^{\pi/2})$$

Darsda yechish uchun misollar.

Aniq integrallarni bo`laklab integrallash yordamida hisoblang.

40. $\int_0^1 arctgxdx$ j. $\frac{\pi}{4} - \frac{1}{2}\ln 2$

41. $\int_0^1 xe^{3x}dx$ j. $\frac{2e^3+1}{9}$

42. $\int_{-1}^1 xarctgxdx$ j. $\frac{\pi}{2} - 1$

47. $\int_1^4 \frac{\ln x}{\sqrt{x}}dx$ j. $4\left(\ln 4 - 1\right)$

48. $\int_0^1 \ln(x+1)dx$ j. $2\ln 2 - 1$

49. $\int_1^e x^2\ln xdx$ j. $\frac{2e^3+1}{9}$

43. $\int_0^{\pi/2} e^x \cos x dx$ j. $\dfrac{\left(e^{\frac{\pi}{2}}-1\right)}{2}$

44. $\int_1^e \sin(\ln x)dx$ j: $\dfrac{e(\sin 1-\cos 1)}{2}$

45. $\int_{-1}^0 (2x+3)e^{-x}dx$ j. $3e-5$

46. $\int_0^1 xarctgxdx$ j. $\dfrac{\pi-2}{4}$

50. $\int_0^1 x\sin xdx$ j. π

51. $\int_1^e \sqrt[4]{x}\ln xdx$ j. $\dfrac{4\left(\sqrt[4]{e^5}+4\right)}{25}$

52. $\int_0^{\frac{\pi}{2}} (x-1)\cos xdx$ j. $\dfrac{\pi}{2}-2$

Mustaqil uy vazifasi uchun misollar

53. $\int_1^e \cos\ln xdx$ j. $\dfrac{e^{\frac{\pi}{2}}-1}{2}$

54. $\int_0^{\pi/2} x\cos xdx$ j. $\dfrac{\pi}{2}-1$

55. $\int_1^e \ln^2 xdx$ j. $e-2$

56. $\int_1^e x\ln^2 xdx$ j. $\dfrac{1}{4}\left(e^2-1\right)$

57. $\int_0^{\pi/2} e^{2x}\cos xdx$ j. $\dfrac{\pi^2-8}{32}$

58. $\int_{-1}^0 \arccos xdx$ j. $\pi-1$

Aniq integral yordamida yuzalarni hisoblash.

1-misol. $y=4x-x^2, y=0$ chiziqlar bilan chegaralangan soha yuzini toping.

Yechish: $y=4x-x^2$ va $y=0$ tenglamalarni birgalikda yechib, parabolani $0X$ o`qi bilan kesishish nuqtasini topamiz.
O(0;0) va M(4;0) nuqtalarda kesishadi.

$$S=\int_0^4 (4x-x^2)dx=\left[2x^2-\frac{1}{3}x^3\right]\Bigg|_0^4=\frac{32}{3} \quad \text{(kv.bir)}$$

Darsda yechish uchun misollar.

Quyidagi chiziqlar bilan chegaralangan sohani yuzini toping.

59. $y=-x^2, x+y+2=0$ j. 4,5

60. $x=12\cos t+5\sin t, y=5\cos t-12\sin t$ j. $169\cdot\pi$

61. $y=16/x^2, y=17-x^2$ (I - CHorak) j. 18

62. $x=a\cos^3 t, y=a\sin^3 t$ (astroida) j. $\dfrac{3\pi a^2}{8}$

63. $y^2=4x^3, y=2x^2$ j. $\dfrac{2}{15}$

64. $xy=20, x^2+y^2=41$ (I - CHorak) j. $\dfrac{41}{2}\arcsin\dfrac{9}{41}+20\ln(0,8)$

65. $y=\sin x, y=\cos x, x=0$ j. $\sqrt{2}-1$

Mustaqil uy vazifasi uchun misollar

Quyidagi chiziqlar bilan chegaralangan sohani yuzini toping

66. $y = 0.25x^2$, $y = 3x - 0.5x^2$ j. 8

67. $xy = 4\sqrt{2}, x^2 - 6x + y^2 = 0, y = 0, x = 4$ j. $\frac{9\pi}{4} - \sqrt{2} + 4\sqrt{2}\ln 2 - \frac{9}{2}\arcsin\left(\frac{1}{3}\right)$

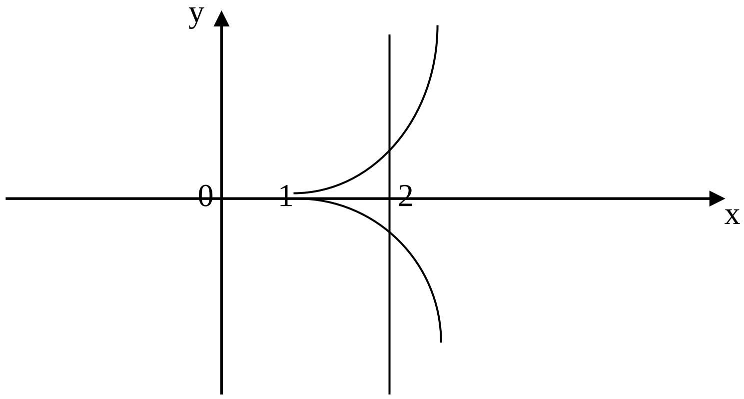
68. $y = \frac{x^2}{2}, \quad x = 1, \quad x = 3$ parabola va to`g`ri chiziqlar va OX o`qi bilan chegaralangan soha yuzini hisoblang. j. $\frac{13}{3}$

69. $x = 2 - y - y^2$ egri chiziq va ordinatalar o`qi bilan chegaralangan figuraning yuzini hisoblang. j. $\frac{9}{2}$

Aniq integral yordamida jism hajmini hisoblang

1-masala $y^2 = (x-1)^3$ va $x = 2$ chiziqlar bilan chegaralangan figurani OX o`qi atrofida aylantirishdan hosil bo`lgan jismning hajmini hisoblang.

Yechish.



$$V = \pi\int_1^2 y^2 dx = \pi\int_1^2 (x-1)^3 dx = \frac{1}{4}\pi(x-1)^4\Big|_1^2 = \frac{1}{4}\pi \text{ (kub bir.)}$$

2-masala. $y = Sin\,x$ sinusoidaning yarim to`lqini OX o`qining $[0,\pi]$ kesmasi bilan chegaralangan figuraning OY o`qi atrofida aylanishidan hosil bo`lgan jismning hajmini hisoblang.

Yechish

$$V = 2\pi\int_0^\pi xy\,dx = 2\pi\int_0^\pi xSin\,x\,dx = \begin{vmatrix} u = x, & du = dx \\ dv = Sin\,x, & v = -Cos\,x \end{vmatrix} = 2\pi\left(-xCos\,x\Big|_0^\pi + \int_0^\pi Cos\,xdx\right) =$$

$$= 2\pi\left(-\pi\cos\pi + Sin\,x\Big|_0^\pi\right) = 2\pi^2 \ (\kappa ub\ bir.)$$

3-masala $Y=x^2$ va $y^2=8x$ parabolalar bilan chegaralangan figurani 0y o`qi atrofida aylantirishdan hosil bo`lgan jism hajmini hisoblang.

Yechish: $y=x^2$ va $y^2=8x$ tenglamalarni birgalikda yechib kesishish nuqtalarini topamiz. x=0 va x=2

$$V_y = 2\pi\int_a^b x(y_2 - y_1)dx = 2\pi\int_0^2 x\left(\sqrt{8x} - x^2\right)dx = 2\pi\int_0^2\left(\sqrt{8}x^{\frac{3}{2}} - x^3\right)dx = 2\pi\left(\sqrt{8}\frac{2}{5}x^{\frac{5}{2}} - \frac{x^4}{4}\right)\Bigg|_0^2 =$$

$$=2\pi\left(\frac{2\sqrt{8\cdot 32}}{5}-4\right)=2\pi\left(\frac{2\cdot 16}{5}-4\right)=2\pi\frac{12}{5}=\frac{24\pi}{5}$$ kub birlik

Darsda yechish uchun misollar.

Quyidagi chiziqlar bilan chegaralangan figuralarni koordinata o`qlari atrofida aylantirishidan hosil bo`lgan jismlarning hajmini toping.

70. $xy=9$, $y=3$, $y=9$ $0y$ o`q atrofida j. 18π

71. $y=10-x^2$, $y=x^2+2$ $0y$ o`q atrofida j. $0{,}16\pi$

72. $y=4-x^2, 2x+y-4=0$ $0x$ o`q atrofida j. $6{,}4\pi$

73. $x^2+y^2=a^2$ $0x$ o`q atrofida j. $\frac{4}{3}\pi a^3$

74. $y=\cos\left(x-\frac{\pi}{3}\right)$, $x=0$, $y=0$, $(x>0)$ $0x$ o`q atrofida j. $\frac{\pi}{4}\left(\frac{5\pi}{3}+\frac{\sqrt{3}}{2}\right)$

75. $x^2-y^2=4$, $y=2$ 0y o`q atrofida j. $\frac{32\pi}{3}$

76. $y=\frac{1}{4}x^2$ egri chiziqning OY o`qi atrofida $y=1$ $y=5$ chegaralarda aylanishidan. j. 48π

77. $y=x^3$ kubik parabolaning OY o`qi atrofida $y=1$ $y=8$ j. $18{,}6\pi$

Mustaqil uy vazifasi uchun misollar

Quyidagi chiziqlar bilan chegaralangan figuralarni koordinata o`qlari atrofida aylantirishidan hosil bo`lgan jismlarning hajmini toping.

78. $y=\cos\left(x-\frac{\pi}{3}\right)$, $x=0$, $y=0$, $(x>0)$ $0x$ o`q atrofida j. $\frac{\pi}{4}\left(\frac{5\pi}{3}+\frac{\sqrt{3}}{2}\right)$

79. $x^2-y^2=4$, y=2 $0y$ o`q atrofida j. $\frac{32\pi}{3}$

80. $x=\sqrt{y}$ $y=1$ $y=4$ $0y$ o`q atrofida j. $\frac{15\pi}{2}$

81. $y=3x^2$ parabola $x=0$ $x=2$ $0x$ o`q atrofida j. $57{,}6\pi$

82. $y=3x$ $y=2$ $y=4$ $0y$ o`q atrofida j. $\frac{56\pi}{27}$

83. $y=\frac{1}{2}x$ $x=4$ $x=6$ abssissalar o`qi bilan chegaralangan trapesiyaning OX o`qi atrofida j. $\frac{38\pi}{3}$

Yoy uzunligi va sirt yuzalarni hisoblash

Yoy uzunligi $l=\int_a^b\sqrt{1+y'^2}dx$ formula bilan hisoblanadi. Bunda y=f(x) funksiya [a,b] da uzluksiz va 1- tartibli uzluksiz hosilaga ega bo’lgan funksiya.

Agar funksiya $x=x(t),\ y=y(t)$ parametrik usulda berilgan bo’lsa, $l=\int_\alpha^\beta\sqrt{x'^2+y'^2}dt$ formula bilan hisoblanadi.

Agar funksiya $\rho=\rho(\varphi)$ qutb koordinatalar sistemasida berilgan bo'lsa, $l=\int_{\alpha}^{\beta}\sqrt{\rho^2+\rho'^2}d\varphi$ formula bilan hisoblanadi.

[a,b] kesmada aniqlangan, silliq, y=f(x) funksiya grafigini Ox o'qi atrofida aylantirilishidan hosil bo'lgan sirt yuzi $S_x=2\pi\int_a^b y\sqrt{1+y'^2}dx$ formula bilan hisoblanadi.

1-masala $y^2=x^3$ yarim kubik parabolaning koordinatalar boshidan A(4;8) nuqtagacha bo`lgan yoyi uzunligini toping.

Yechish: $y^2=x^3,\ \ y=x^{\frac{3}{2}},\ \ y'=\frac{3}{2}x^{\frac{1}{2}}$

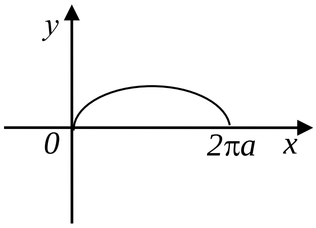
$$L=\int_0^4\sqrt{1+\left(\frac{3}{2}x^{\frac{1}{2}}\right)^2}\ dx=\int_0^4\sqrt{1+\frac{9}{4}x}\ dx=\frac{4}{9}\cdot\frac{2}{3}\left(1+\frac{9}{4}x\right)^{\frac{3}{2}}\Bigg|_0^4=\frac{8}{27}\left(10\sqrt{10}-1\right)$$ (uzunlik birligi)

2-masala y=a(t-sint), y=a(1-cost) sikloidaning bir arki uzunligini toping.

Yechish: x'=*a*(1-cos*t*); y'=*a*sin*t*

$$L=\int_0^{2\pi}\sqrt{a^2(1-\cos t)^2+a^2\sin^2 t}\ dt=a\int_0^{2\pi}\sqrt{2-2\cos t}\ dt=$$

$$=2a\int_0^{2\pi}\sin\frac{t}{2}dt=-4a\cos\frac{t}{2}\Bigg|_0^{2\pi}=8a$$ (uzunlik birlik)



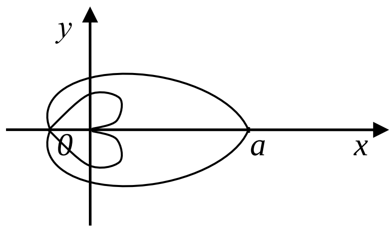
3- masala $r=a\sin^4\frac{\varphi}{4}$ yopik egri chiziq uzunligini toping.

Yechish: Berilgan funksiya juft funksiya. SHu sababli berilgan egri chiziq qutb o`qiga nisbatan simmetrik.

Nuqta butun egri chiziqni burchak 0 dan 2π gacha o`zgarganda chizadi.

$r'=a\sin^3\frac{\varphi}{4}\cdot\cos\frac{\varphi}{4}$,

demak,

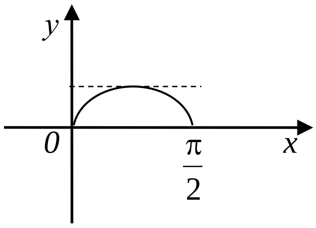


$$\frac{L}{2}=\int_0^{2\pi}\sqrt{a^2\sin^8\frac{\varphi}{4}+a^2\sin^6\frac{\varphi}{4}\cdot\cos^2\frac{\varphi}{4}}d\varphi=a\int_0^{2\pi}\sin^3\frac{\varphi}{4}d\varphi=-4a\int_0^{2\pi}\sin^2\frac{\varphi}{4}\cdot d\left(\cos\frac{\varphi}{4}\right)=$$

$$=-4a\int_0^{2\pi}\left(1-\cos^2\frac{\varphi}{4}\right)d\left(\cos\frac{\varphi}{4}\right)=-4a\left(\cos\frac{\varphi}{4}-\frac{\cos^3\frac{\varphi}{4}}{3}\right)\Bigg|_0^{2\pi}=-4a\left(1-\frac{1}{3}\right)=\frac{8}{3}a$$

demak, $L=\frac{16}{3}a$ (uzunlik birlik)

4- masala. $y = \sin 2x, \quad 0 \le x \le \pi/2$ sinusoida yoyining 0x o`qi atrofida aylantirishdan hosil bo`lgan jism sirtining yuzini hisoblang.



Yechish:

$$y = \sin 2x, \qquad y' = 2\cos 2x$$

$$S_x = 2\pi \int_0^{\frac{\pi}{2}} \sin 2x \cdot \sqrt{1 + 4\cos^2 2x}\,dx = \left| \begin{array}{ll} 2\cos 2x = t & x = 0, \quad t = 2 \\ -4\sin 2x dx = dt & x = \pi/2. \quad t = -2 \\ \sin 2x dx = \left(-\frac{1}{4}\right)dt & \end{array} \right|$$

$$= 2\pi \int_2^{-2} \sqrt{1+t^2}\left(-\frac{1}{4}\right)dt = \frac{\pi}{2}\int_{-2}^{2}\sqrt{1+t^2}\,dt = \frac{\pi}{2}\left[\frac{t}{2}\sqrt{1+t^2} + \frac{1}{2}\ln\left(t + \sqrt{1+t^2}\right)\right]\Bigg|_{-2}^{2} =$$

$$= \frac{\pi}{2}\left(2\sqrt{5} + \frac{1}{2}\ln\frac{\sqrt{5}+2}{\sqrt{5}-2}\right) = \frac{\pi}{2}\left[2\sqrt{5} + \ln\left(\sqrt{5}+2\right)\right] \quad \text{(kv.bir)}$$

Darsda yechish uchun misollar.

Quyidagi egri chiziqlar yoylari uzunliklarini hisoblang.

84. $y=\ln(\sin x)$, $\quad \frac{\pi}{3} \le x \le \frac{\pi}{2}$ $\qquad$ j. $\frac{1}{2}\ln 3$

85. $x = \frac{1}{3}t^3 - t, \quad y = t^2 + 2, \quad 0 \le t \le 3$ $\qquad$ j. $\sqrt{2}(\pi - 1)$

86. $x = e^t \cos t., \quad y = e^t \sin t \qquad t = 0 \quad t = \ln \pi$ $\qquad$ j. 12

Egri chiziq yoyining 0x o`qi atrofida aylanishdan hosil bo`lgan jism sirti yuzini hisoblang.

87. $y=x^3, \quad 0 \le x \le \frac{1}{2}$ $\qquad$ j. $\frac{61\pi}{1728}$

88. $x=t-\sin t$, $\quad y=1-\cos t$ $\quad$ (bir arkasi) $\quad$ j. $\frac{64\pi}{3}$

89. $y = \frac{64}{x^2+16}, \quad x^2 = 8y$ $\qquad$ 0x o`q atrofida $\quad$ j. $16\pi(5\pi + \frac{8}{5})$

Mustaqil uy vazifasi uchun misollar

Egri chiziq yoyining 0x o`qi atrofida aylanishdan hosil bo`lgan jism sirti yuzini hisoblang.

90. $y^2=x, \quad x^2=y$ $\qquad$ 0y o`q atrofida $\quad$ j. $0{,}3\pi$

91. $y = \frac{x^2}{2}, \qquad y = \frac{x^3}{8}$ $\qquad$ j. $\frac{4\pi}{35}$

Aniq integralning mexanika va fizika masasalarini yechishga tadbiqi.
Egri chiziq va texnik shaklning statik momenti

U=f(x), ($a \le x \le b$) egri chiziqni statik momenti

$$M_x = \int_a^b y dL \qquad : \qquad M_y = \int_a^b x dL$$

formula bilan hisoblanadi.

Inersiya momenti esa

$$I_x = \int_a^b y^2 dL \qquad I_y = \int_a^b x^2 dL$$

formulalar bilan xisoblanadi. Bu yerda dL $= \sqrt{1+\left(y\ \right)^2}$ dx

$y = f(x),\quad y = o,\quad x = a,$ va $x = b$ chiziqlar bilan chegaralangan egri chiziqli trapesiyaning statik va inersiya momentlari

$$M_x = \frac{1}{2}\int_a^b y^2 dx, \quad , \qquad M_y = \int_a^b xydx.$$

$$I_x = \frac{1}{3}\int_a^b y^3 dx, \qquad I_y = \int_a^b x^2 ydx.$$

formula bilan xisoblanadi.

2. Og`irlik markazi koordinatalarini hisoblash

U= $f(x)\quad (a \le x \le b)$ egri chiziqning koordinatalari og`irlik markazini

$$\bar{x} = \frac{i}{L}\int_a^b xdL \quad ; \quad \bar{y} = \frac{i}{L}\int_a^b ydL$$

formulalar bilan hisoblanadi. Bu yerda $dL = \sqrt{1+\left(y'\right)^2}dx$

L – yoy uzunligi egri chiziqli trapesiya og`irlik markazining koordinatalari.

$$\bar{x} = \frac{1}{S}\int_a^b xydx. \qquad \bar{y} = \frac{1}{2S}\int_a^b y^2 dx.$$

formula bilan hisoblanadi. Bu yerda S berilgan trapesiya yuzi.

1-masala. $y = \sqrt{r^2 - x^2}\quad (-r \le x \le r)$ Sharning aylanasi Ox o`qiga nisbatan statik moment va inersiyasini toping.

Yechish: Statik momentini $M_x = \int_a^b ydL$ formula bilan hisoblaymiz.

$$dL = \sqrt{1+(y')^2}dx \qquad y^1 = -\frac{x}{\sqrt{r^2 - x^2}}$$

$$M_x = \int_{-r}^{r} \sqrt{r^2 - x^2}.\sqrt{1+\frac{x^2}{r^2 - x^2}}dx = r\int_{-r}^{r} dx = 2r^2$$

Endi inersiya momentini topamiz.

$$I_x = \int_{-r}^{b} y^2 dL = \int_{-r}^{r}\left(r^2 - x^2\right)\cdot\sqrt{1+\frac{x^2}{r^2 - x^2}}dx = r\cdot\int_{-r}^{r}\sqrt{r^2 - x^2}dx = 2r\int_0^r\sqrt{r^2 - x^2}dx = \left|\begin{matrix} x = rS\text{int} \\ dx = rCostdt \\ x = 0; t = 0 \\ x = r; t = \pi/2 \end{matrix}\right| =$$

$$2r\int_0^{\pi/2}\sqrt{r^2 - r^2\sin^2 t}\cdot r\cos tdt = r^3\int_0^{\pi/2}(1+\cos 2t)dt = r^3\left[t + \frac{1}{2}\sin 2t\right]\Bigg|_0^{\pi/2} = \frac{\pi r^3}{2};$$

2- masala x=a cost, y=b sint tenglamalar bilan berilgan ellips yuzini 0y o`qiga nisbatan inersiya momentini toping.

Yechish: $Iy = \int_{-a}^{a} x^2 ds$,

bu yerda $ds = 2ydx$ $ds = 2b\sin t \cdot a(-\sin t)dt = -2ab\sin^2 t\, dt.$

$$I_y = 2\int_{\frac{\pi}{2}}^{0} a^2\cos^2 t\left(-2ab\sin^2 t\right)dt = -4a^3b\int_{\frac{\pi}{2}}^{0}\sin^2 t\cos^2 t dt = \frac{1}{2}a^3b\int_{0}^{\frac{\pi}{2}}(1-\cos 4t)dt = \frac{\pi a^3 b}{4};$$

3- masala x=a cost, y=b sint tenglama bilan berilgan ellipsning og`irlik markazi koordinatalarini toping (1-chorakda)

Yechish: x = o dan a gacha o`zgarganda t= $\pi/2$ dan o gacha o`zgaradi.

$$X = \frac{1}{S}\int_0^a xydx = \frac{1}{S}\int_{\frac{\pi}{2}}^{0} a\cos t\cdot b\sin t(-a\sin t)dt = \frac{a^2b}{S}\int_0^{\pi/2}\sin^2 t\cos t dt = \frac{a^2b}{3S}\sin^3 t\Big|_0^{\pi/2} = \frac{a^2b}{3S}$$

Ellipsning yuzini $S = \pi ab$ ga teng ekanligini hisobga olsak,

u holda $\bar{x} = \frac{4a}{3\pi}$ bo`ladi.

$$\bar{y} = \frac{1}{2s}\int_0^a y^2dx = \frac{1}{2s}\int_{\frac{\pi}{2}}^{0} b^2\sin^2 t(-a\sin t)dt = \frac{2ab^2}{\pi ab}\int_{\pi/2}^{0}\left(1-\cos^2 t\right)d(\cos t) =$$

$$= \frac{2b}{\pi}\left[\cos t - \frac{1}{3}\cos^3 t\right]\Bigg|_{\pi/2}^{0} = \frac{4b}{3\pi}$$

Darsda yechish uchun misollar.

92. x=a $\cos^3$t, y=a $\sin^3$t tenglama bilan berilgan astroidani statik momenti va inersiya momentini toping (2 chorak) j: $M_x = M_y = (\frac{3}{5})a^2, \quad I_x = I_y = (\frac{3}{8})a^3$

93 . y=4-x^2 va y=3 chiziqlar bilan chegarlangan parabolik segmentning Ox o`qiga nisbatan inersiya momentini toping. j: $I_x = \frac{628}{105}$

94. $y = \sqrt{r^2 - x^2}$ egri chiziq va Ox o`qi bilan chegaralangan yarim aylanani og`irlik markazi koordinatalarini toping. j: $\bar{x} = 0,\ \bar{y} = \frac{2r}{\pi}$

95. x=0, $x = \frac{\pi}{2}$, y=0, y=sinx chiziqlar bilan chegaralangan sohani og`irlik markazi koordinatalarini toping. . j: $\bar{x} = 1,\ \bar{y} = \frac{\pi}{8}$

96. 1) y=4-x^2 parabola va y=o to'g'ri chiziq bilan chegaralangan sohani og`irlik markazi koordinatalarini toping. . j: $\bar{x} = 0\ \ \bar{y} = \frac{8}{5}$

2) Agar prujinaga 1 N kuch qo'yilganda 1 sm ga cho'zilishi ma'lum bo'lsa, 4 smga cho'zish uchun qancha ish bajarish kerak bo'ladi?

Mustaqil uy vazifasi uchun misollar

97. x=a $\cos^3$t, y=a $\sin^3$t astroida yoyini og`irlik markazining koordinatalirini toping (1 - choragida). j: $\bar{x}=0,\ \bar{y}=\frac{2}{5}$

98. $y=2x-x^2$, y=0 chiziqlar bilan chegaralangan sohani og`irlik markazining koordinatalarini toping. j: $\bar{x}=1,\ \bar{y}=\frac{2}{5}$

99. x=0, $x=\frac{\pi}{2}$, y=0, y=cosx chiziqlar bilan berilgan sohani og`irlik markazining koordinatalarini toping.

Xosmas integrallarni hisoblash

$\int_a^{+\infty} f(x)dx \quad \int_{-\infty}^{b} f(x)dx \quad \int_{-\infty}^{+\infty} f(x)dx$ - ko`rinishidagi integrallar chegarasi cheksiz xosmas integrallar deyiladi.

$$\int_a^{+\infty} f(x)dx=\lim_{b\to+\infty}\int_a^b f(x)dx, \qquad \int_{-\infty}^{b} f(x)dx=\lim_{a\to-\infty}\int_a^b f(x)dx$$

$$\int_{-\infty}^{+\infty} f(x)dx=\int_{-\infty}^{c} f(x)dx+\int_{c}^{+\infty} f(x)dx$$

ko`rinishida hisoblanadi.

Agar limit mavjud va chekli bo`lsa xosmas integral yaqinlashuvchi, aks xolda uzoqlashuvchi deyiladi.

1-misol. $\int_a^{+\infty}\cos x dx$ xosmas integralni hisoblang.

Yechish: $\int_a^{+\infty}\cos x dx=\lim_{b\to+\infty}\int_0^b\cos x dx=\lim_{x\to+\infty}\sin x/_0^b=\lim_{x\to+\infty}(\sin b-\sin 0)=\lim_{b\to+\infty}\sin b$

bunday limit mavjud emas . Demak, xosmas integral uzoqlashuvchi.

2-misol. $\int_{-\infty}^{-1}\frac{dx}{x^2}$ xosmas integralni hisoblang.

Yechish: $\lim_{a\to-\infty}\int_a^{-1}\frac{dx}{x^2}=\lim_{a\to-\infty}\left[-\frac{1}{x}\right]\Big|_a^{-1}=\lim_{a\to-\infty}\left(1+\frac{1}{a}\right)=1$

limit chekli, demak, xosmas integral yaqinlashuvchi.

3-misol $\int_{-\infty}^{+\infty}\frac{dx}{1+x^2}$ xosmas integralni hisoblang.

Yechish: Integral ostidagi funksiya juft bo`lganligi sababli

$$\int_{-\infty}^{+\infty}\frac{dx}{1+x^2}=2\int_0^{+\infty}\frac{dx}{1+x^2}$$

$$\int_0^{+\infty}\frac{dx}{1+x^2}=\lim_{b\to+\infty}\int_0^b\frac{dx}{1+x^2}=\lim_{b\to+\infty}arctgx\Big|_0^b=\lim_{b\to+\infty}arctgb=\frac{\pi}{2}$$

Demak,

$$\int_{-\infty}^{+\infty} \frac{dx}{1-x^2} = 2 \cdot \frac{\pi}{2} = \pi$$

Xosmas integral chekli limitga ega. Demak, integral yaqinlashuvchi.

4-misol. $\int_0^{+\infty} xe^{-x^2}dx$ xosmas integralni hisoblang.

Yechish:

$$\int_0^{+\infty} xe^{-x^2}dx = \lim_{b \to +\infty} \int_0^b xe^{-x^2}dx = \lim_{b \to +\infty} \left[-\frac{1}{2}e^{-x^2} \right]_0^b = \lim_{b \to +\infty} \left(\frac{1}{2} - \frac{e^{-b^2}}{2} \right) = \frac{1}{2}$$

Limit chekli, demak, xosmas integral yaqinlashuvchi

Darsda yechish uchun misollar

Xosmas integrallarni hisoblang va yaqinlashuvchilikka tekshiring..

100. $\int_0^{+\infty} \frac{arctgx}{1+x^2}dx$ j: $\frac{\pi^2}{8}$

101. $\int_{-\infty}^{0} \frac{dx}{4+x^2}$ j: $\frac{\pi}{4}$

102. $\int_0^2 \frac{x^5}{4-x^2}dx$ j: $\frac{256}{15}$

103. $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$ j: π

104. $\int_{-1}^1 \frac{dx}{x^2}$ j: $+\infty$

105. $\int_0^1 x\ln^2 xdx$ j: $\frac{1}{4}$

Mustaqil uy vazifasi uchun misollar

106. $\int_{-\infty}^{+\infty} \frac{dx}{x^2+x+1}dx$ j: $\frac{2\pi}{\sqrt{3}}$

107. $\int_1^{+\infty} x\cos xdx$ j: uzoqlashuvchi

108. $\int_1^{+\infty} \frac{arctgxdx}{x^2}$ j: $\frac{\pi}{4}+\frac{\ln 2}{2}$

109. $\int_0^{+\infty} e^{-x^2}dx$ j: yaqinlashuvchi

Aniq integralni taqribiy hisoblash.

1. To`g`ri to`rtburchak formulasi.

$y = f(x)$ funksiya $[a,b]$ kesmada uzluksiz bo`lsa, u holda

$$\int_a^b f(x)dx \approx \frac{b-a}{n}(y_0 + y_1 + ... + y_{n-1}) = \frac{b-a}{n}\sum_{i=0}^{n-1} y_i \qquad (1)$$

$$\int_a^b f(x)dx \approx \frac{b-a}{n}(y_1 + y_2 + ... + y_{n-1}) = \frac{b-a}{n}\sum_{i=1}^{n} y_i \qquad (2)$$

(1) formula (ichki) va (2) formula (tashqi) lar o`rinli bo`ladi. Taqribiy hisoblashning absolyut xatoligi $R_1 = M_1\frac{(b-a)^2}{4n}$ (3) dan katta emas $M_1 = \max\limits_{[a,b]}|f'(x)|$; $h = \Delta x = \frac{b-a}{n}$ bo`lak uzunligi.

2. Trapesiya formulasi.

$$\int_a^b f(x)dx \approx \frac{b-a}{n}\left(\frac{y_0+y_n}{2}+y_1+y_2+...+y_{n-1}\right)$$ absolyut xatoligi $R_2 = M_2\frac{(b-a)^3}{12n^2}$ dan

$M_2 = \max\limits_{[a,b]}|f''(x)|$

3. Simpson formulasi.

$$\int_a^b f(x)dx \approx \frac{b-a}{6m}(y_0+y_{2m}+4(y_1+y_3+...y_{2n-1})+2(y_2+y_4+...+y_{2m-2})) \qquad (4)$$

absolyut xatoligi $R_3 = M_3\frac{(b-a)^5}{2880n^4}$ $\quad h=\Delta x=\frac{b-a}{2m}$ dan katta emas.

$M_3 = \max\limits_{[a,b]}|f^{IV}(x)|$

110-misol. $J=\int_0^1\frac{dx}{1+x}$ integralni taqribiy qiymatini [0;1] kesmada to`g`ri to`rtburchak formulasi bo`yicha toping.

Yechish: Avval integralni aniq qiymatini N`yuton-Leybnis formulasi bo`yicha hisoblaymiz.

$$\int_0^1\frac{dx}{1+x}=\ln|1+\alpha|\Big|_0^1=\ln 2\approx 0.69315.$$

[0;1] kesmani $\Delta x=\frac{1-0}{10}=0.1$ qadam bilan teng 10 bo`lakka ajratamiz va har bir nuqtada $f(x)=\frac{1}{1+x}$ funksiyani qiymatini hisoblab quyidagi jadvalni tuzamiz.

I	0	1	2	3	4	5	6	7	8	9	10
x_i	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y_i	1	0.9091	0.8333	0.7692	0.7143	0.6667	0.6250	0.5882	0.555	0.526	0.500

endi $h=10,\quad \Delta x=\frac{1-0}{10}=0.1$ bo`yicha (1) formulaga qo`yib hisoblaymiz

$J\approx 0.1(1+0.9091+...+0.5263)=0.71877$ (2) formula bo`yicha

$J\approx 0.1(0.9091+0.8333+...+0.5)=0.66877;$

Endi xatoligini hisoblaymiz:

$$f(x)=\frac{1}{1+x} \quad \text{va} \quad f'(x)=-\frac{1}{(x+1)^2}$$

$M_1=\max|f'(x)|=\max\left|-\frac{1}{(1+x)^2}\right|\le 1$ demak $R_1=\frac{M_1(b-a)^2}{4n}=\frac{1}{4\cdot 10}=0.025$ dan ortmaydi.

111-misol. $\int_0^2\sin(x^2)dx$ integralni trapesiyalar formulasi yordamida hisoblang.

Yechish: $n=10,\quad \Delta x=\frac{b-a}{n}=\frac{2-0}{10}=0.2$

Quyidagi jadvalni to`ldiramiz.

i	0	1	2	3	4	5	6	7	8	9	10
x_i	0	0.2	0.4	0.6	0.8	1,0	1,2	1,4	1,6	1,8	2,0

y_i	0	0,004	0,1593	0,3523	0,5972	0,8415	0,9915	0,9249	0,5487	0,3427	0,1576

trapesiya formulasiga asosan

$$\int_0^2 \sin(x^2)dx \approx 0.2\ (\ \frac{0+0.1576}{2} + 0.04 + 0.1593 + 0.3523 + 0.5972 + 0.8415 + 0.9915 + 0.9249 + 0.5487 + 0.3427\ ) = 1.11722$$

endi absolyut xatoni topamiz:

$$f'(x) = (\sin(x^2))' = 2x\cos(x^2)$$

$$f''(x) = 2\cos(x^2) - 4x^2\sin(x^2) \qquad R_2 = \frac{M_2(b-a)^3}{12n^2} = \frac{2\cdot 8}{12\cdot 100} = \frac{4}{300} = 0.013$$

$$M_2 = \max\left|2\cos(x^2) - 4x^2\sin(x^2)\right| \le 2$$

dan katta emas.

112-misol. $\int_0^1 e^{-x^2}dx$ integralni parabolalar formulasi yordamida taqribiy hisoblang.

Yechish: $n = 10,\quad h = \Delta x = \frac{1-0}{10} = \frac{1}{10}.$

Quyidagi jadvalni to`ldiramiz.

I	0	1	2	3	4	5	6	7	8	9	10
x_i	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y_i	1	0.99005	0.96079	0.91393	0.85214	0.77680	0.69768	0.4263	0.52729	0.44486	0.36788

Simpson formulasiga asosan:

$$\int_0^1 e^{-x^2}dx \approx \frac{1}{30}[(1.0000 + 0.3678) + 4(0.99005 + 0.91393 + 0.77680 + 0.4263 + 0.44486) +$$

$$+ 2(0.96079 + 0.85214 + 0.69769 + 0.52729)] = \frac{1}{30}(1.3678 + 6.0758 + 14.9610) \approx 074682$$

Endi absolyut xatoni topamiz.

$$f(x) = e^{-x^2}$$

$$f'(x) = -2xe^{-x^2}$$

$$f''(x) = -2e^{-x^2} + 4x^2e^{-x^2}$$

$$f'''(x) = 4xe^{-x^2} + 8xe^{-x^2} - 8x^3 = 12xe^{-x^2} - 8x^3e^{-x^2}$$

$$f^{IV}(x) = 12e^{-x^2} - 24x^2e^{-x^2} - 24x^2e^{-x^2} + 16x^4e^{-x^2} = 12e^{-x^2} - 48x^2e^{-x^2} + 16x^4e^{-x^2} =$$

$$= 4e^{-x^2}(4x^4 - 12x^2 + 3) \le 12\ ,$$

Demak, $R_3 = \frac{M_3(b-a)^5}{2880\cdot n^4} = \frac{12\cdot 1}{2880\cdot 10000} = \frac{12}{28800000} \approx 0.4\cdot 10^{-6}$ dan katta emas.

Darsda yechish uchun misollar

Quyidagi integralni to`g`ri to`rtburchak formulasi yordamida hisoblang.

113. $\int_0^{2\pi} x\sin x dx$ J: -6,2832

Quyidagi integrallarni trapesiya formulasi yordamida hisoblang.

114. $\int_0^1 \frac{dx}{1+x^3},\ (n = 12)$ J: 0,83566 115. $\int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{1}{4}\sin^2 x}dx,\ (n = 6)$ j: 1,4675

Quyidagi integrallarni Simpson formulasi yordamida hisoblang.

116. $\int_1^9 \sqrt{x}dx \quad (n=4)$ j: 17,333 117. $\int_0^{\pi} \sqrt{3+\cos x}dx \quad (n=6)$ j: 5,4024

Mustaqil uy vazifasi uchun misollar

Simpson formulasi yordamida hisoblang.

118. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{x}dx, \quad (n=10)$ j: 1,37039 119. $\int_0^1 \frac{xdx}{\ln(1+x)}, \quad (n=6)$ j: 0,2288

Sonli qatorlar. Qatorlar yig'indisini topish.

1-misol. $\sum_{n=0}^{\infty}\frac{1}{2^n}=1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^n}+\dots$ qatorni tekshiring va yig'indisini toping.

Yechish:Bu qator birinchi hadi $a_1=1$ va maxraji $q=\frac{1}{2}$ bo'lgan cheksiz kamayuvchi geometrik progressiyadir. Geometrik progressiyani dastlabki n ta hadining yig'indisi

$$A_n=\frac{a_1(1-q^n)}{1-q}$$ formulaga ko'ra $$A_n=\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}=2\left[1-\left(\frac{1}{2}\right)^n\right]$$ Bundan

$$A=\lim_{n\to\infty}A_n=\lim_{n\to\infty}2\left[1-\left(\frac{1}{2}\right)^n\right]=2$$

Demak, qator yaqinlashuvchi bo'lib, uning yig'indisi 2 ga teng, ya'ni:

$$\sum_{n=0}^{\infty}\frac{1}{2^n}=1+\frac{1}{2}+\frac{1}{2^2}+\dots+\frac{1}{2^2}+\dots=2$$

2-misol. $\sum_{n=1}^{\infty}\frac{4}{4n^2-1}=\frac{4}{3}+\frac{4}{15}+\frac{4}{35}+\dots+\frac{4}{4n^2-1}+\dots$

qatorning A_n qismiy yig'indisini va A yig'indisini toping.

Yechish. qatorning umumiy hadini

$$a_n=\frac{4}{4n^2-1}=\frac{4}{(2n-1)(2n+1)}=\frac{2}{2n-1}-\frac{2}{2n+1} \qquad \forall n\in N$$

sodda kasrlar ayirmasi ko'rinishda yozish mumkin ekanligini ehtiborga olib, berilgan qatorni

$$\left(2-\frac{2}{3}\right)+\left(\frac{2}{3}-\frac{2}{5}\right)+\frac{2}{5}-\dots-\frac{2}{2n-1}+\left(\frac{2}{2n-1}-\frac{2}{2n+1}\right)+\dots$$

shaklda yozish mumkin. qavslarni ochib, soddalashtirganimizdan so'ng, bu qatorning dastlabki n ta hadining qismiy yig'indisi quyidagicha bo'ladi:

$$A_n=2-\frac{2}{2n+1}$$ Demak, $$A=\lim_{n\to\infty}A_n=\lim_{n\to\infty}\left(2-\frac{2}{2n+1}\right)=2$$

Ya'ni, qator yaqinlashadi va uning yig'indisi 2 ga teng.

Darsda yechish uchun misollar

Berilgan qatorlar uchun yaqinlashishning zaruriy sharti bajariladimi?

120. $\frac{2}{3}+\frac{4}{9}+\frac{6}{27}+\frac{8}{81}+...;$ 121. $2+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+...+\frac{n+1}{n}+...;$

122. $1\frac{2}{1}+\frac{5}{8}+\frac{10}{27}+\frac{17}{64}+...+\frac{n^2+1}{n^3}+...;$ 123. $\sum_{n=1}^{\infty}(n^2+9)\cdot arcsin\frac{1}{n^2+5}$

124 . $\sum_{n=1}^{\infty}\left(\frac{n-1}{n+1}\right)^n$

Berilgan qatorlarni yaqinlashuvchi ekanini ko'rsating va yig'indisini toping.

125. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+...+\frac{1}{n\cdot(n+1)}+...;$

126. $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+...+\frac{1}{(2n-1)\cdot(2n+1)}+...;$

127. $\sum_{n=1}^{\infty}\frac{2n+1}{n^2(1+n)^2}$ 128. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}$

Mustaqil uy vazifasi uchun misollar

Berilgan qatorlar uchun yaqinlashishning zaruriy sharti bajariladimi?

129. $\frac{1}{1\cdot 3\cdot 5}+\frac{1}{3\cdot 5\cdot 7}+\frac{1}{5\cdot 7\cdot 9}+...;$ 130. $\sum_{n=1}^{\infty}\frac{n}{(2n-1)^2\cdot(2n+1)^2}$

131. $\sum_{n=1}^{\infty}\frac{1}{(2n+1)(2n+3)(2n+5)}$

Berilgan qatorlarni yaqinlashuvchi ekanini ko'rsating va yig'indisini toping.

132. $\sum_{n=1}^{\infty}\frac{1}{(2n+1)(2n+3)(2n+5)}$ 133. $\sum_{n=1}^{\infty}\frac{1}{(3n+1)\cdot(3n-2)}$

134. $\sum_{n=1}^{\infty}\frac{n}{(2n-1)^2\cdot(2n+1)^2}$

Musbat hadli qatorlar yaqinlashishining taqqoslash va Dalamber alomati

Agar $$\sum_{n=1}^{\infty}a_n=a_1+a_2+......+a_n+......$$

qatorning barcha hadlari manfiy bo'lmasa, yahni $a_n\geq 0,\quad n\in N$ bo'lsa, uni musbat hadli qator deb ataymiz.

1-misol. Ushbu $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+......+\frac{1}{n(n+1)}+$ qatorning yaqinlashishi tekshirilsin.

Yechish. Har qanday n uchun berilgan qatorning hadlari $\frac{4}{1\cdot 3}+\frac{4}{3\cdot 5}+\frac{4}{5\cdot 7}+......+\frac{4}{(2n-1)(2n+1)}+......$ qatorning mos hadlaridan kichik,

ya'ni $\frac{1}{n(n+1)}<\frac{4}{(2n-1)(2n+1)},\qquad \forall n\in N$

Ammo, keyingi qator yaqinlashadi Demak berilgan qator ham yaqinlashadi.

Dalamber alomati. Agar musbat hadli qator uchun $U_n = \frac{a_{n+1}}{a_n}$

nisbat $n \to \infty$ da l (chekli) limitga ega bo'lsa, ya'ni

$\lim\limits_{n\to\infty} U_n = \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = l$ bo'lsa, u holda: $l < 1$ bo'lganda, qator yaqinlashuvchi; $l > 1$ bo'lganda, qator uzoqlashuvchi bo'ladi.

Agar $l = 1$ bo'lsa, qatorning yaqinlashishi yoki uzoqlashishi haqidagi savolga, boshqa alomatlar yordamida javob topish mumkin.

2-misol. Ushbu

$$\sum_{n=1}^{\infty} \frac{2n-1}{\left(\sqrt{2}\right)^n} = \frac{1}{\sqrt{2}} + \frac{3}{2} + \frac{5}{2\sqrt{2}} + \ldots\ldots + \frac{2n-1}{\left(\sqrt{2}\right)^n} + \ldots\ldots$$

qatorning yaqinlashishi tekshirilsin.

Yechish. Bunda $Q_n = \frac{2n-1}{\left(\sqrt{2}\right)^n}$, $Q_{n+1} = \frac{2n+1}{\left(\sqrt{2}\right)^{n+1}}$ shu sababli

$$\frac{Q_{n+1}}{Q_n} = \frac{2n+1}{\left(\sqrt{2}\right)^{n+1}} : \frac{2n-1}{\left(\sqrt{2}\right)^n} = \frac{2n+1}{\sqrt{2}(2n-1)}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2n+1}{\sqrt{2(2n-1)}} = \frac{1}{\sqrt{2}} \lim_{n\to\infty} \frac{2n+1}{2n-1} = \frac{1}{\sqrt{2}} < 1$$

Demak, qator yaqinlashadi.

3-misol. Ushbu

$$\sum_{n=1}^{\infty} \frac{3^n n!}{n^n} = 3 + \frac{3^2 \cdot 2!}{2^2} + \frac{3^3 \cdot 3!}{3^3} + \frac{3^4 \cdot 4!}{4^4} + \ldots\ldots + \frac{3^n n!}{n^n} + \ldots$$

qatorning yaqinlashishi tekshirilsin.

$$a_n = \frac{3^n \cdot n!}{\mathrm{n}^{\mathrm{n}}}, \qquad a_{n+1} = \frac{3^{(n+1)}(n+1)!}{(n+1)^{n+1}}, \qquad \frac{a_{n+1}}{a_n} = \frac{3 \cdot n^n}{(n+1)^n},$$

Yechish.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{3 \cdot n^n}{(n+1)^n} = 3 \lim_{n\to\infty} \frac{1}{\left(1 - \frac{1}{n}\right)^n} = \frac{3}{e} > 1, \qquad (e \approx 2{,}71)$$

Demak, qator uzoqlashadi, uning umumiy hadi $a_n \to \infty$

Eslatma. Dalamber alomati faqat $\lim\limits_{n\to\infty} \left(\frac{a_{n+1}}{a_n}\right)$ mavjud va birdan farqli bo'lgan holda berilgan musbat hadli qator yaqinlashadimi yoki yo'qmi degan savolga javob beradi.

Darsda yechish uchun misollar

Berilgan qatorlarni taqqoslash alomatlaridan foydalanib, yaqinlashishi tekshirilsin:

135. $\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 2^3} + \frac{1}{5 \cdot 2^5} + \ldots + \frac{1}{(2n-1)2^{2n-1}} + \ldots$

136. $\frac{1}{2\cdot 5}+\frac{1}{3\cdot 6}+\frac{1}{4\cdot 7}+...+\frac{1}{(n+1)(n+4)}+...$

137. $\frac{2}{3}+\frac{3}{8}+\frac{4}{15}+\frac{5}{24}...+\frac{n+1}{(n+2)n}+...$

138. $\frac{1}{1+5}+\frac{1}{1+5^2}+\frac{1}{1+5^3}+...+\frac{1}{1+5^n}+...$

139. $\sin\frac{\pi}{2}+\sin\frac{\pi}{4}+\sin\frac{\pi}{8}+...+\sin\frac{\pi}{2^n}+...$

Berilgan qatorlarni Dalamber alomatidan foydalanib yaqinlashishi tekshirilsin:

140. $tg\frac{\pi}{4}+2tg\frac{\pi}{8}+3tg\frac{\pi}{16}+...+n\cdot tg\frac{\pi}{2^{n+1}}+...$

141. $\frac{2}{1}+\frac{2\cdot 5}{1\cdot 5}+\frac{2\cdot 5\cdot 8}{1\cdot 5\cdot 9}+...+\frac{2\cdot 5\cdot 8...(3n-1)}{1\cdot 5\cdot 9...(4n-3)}+...;$

142. $sin\frac{\pi}{2}+4\,sin\frac{\pi}{4}+9\,sin\frac{\pi}{8}+...+n^2\cdot sin\frac{\pi}{2^n}+...$

143. $\frac{1}{3}+\frac{1\cdot 3}{3\cdot 6}+\frac{1\cdot 3\cdot 5}{27\cdot 6}+...+\frac{1\cdot 3\cdot 5...(2n-1)}{3^n\cdot n!}+...;$

144. $\frac{1}{\sqrt{3}}+\frac{3}{(\sqrt{3})^2}+\frac{5}{(\sqrt{3})^3}+...+\frac{2n-1}{(\sqrt{3})^n}+...;$

145. $\sum_{n=1}^{\infty}\frac{3^n}{(2n+1)\cdot 2^n}$ 146. $\sum_{n=1}^{\infty}\frac{(2n+1)!}{(3n+4)\cdot 3^n}$

Mustaqil uy vazifasi uchun misollar

Berilgan qatorlarni taqqoslash alomatlaridan foydalanib, yaqinlashishi tekshirilsin:

147. $\frac{1}{2}+\frac{1}{5}+\frac{1}{8}+\frac{1}{11}+...+\frac{1}{3n-1}+...$

148. $\frac{1}{\ln 2}+\frac{1}{\ln 3}+\frac{1}{\ln 4}+\frac{1}{\ln 5}...+\frac{1}{\ln(n+1)}+...$

Berilgan qatorlarni Dalamber alomatidan foydalanib yaqinlashishi tekshirilsin:

149. $\sum_{n=1}^{\infty}\frac{3^n}{n\cdot 2^n}$ 150. $\sum_{n=1}^{\infty}\frac{1}{(5n-4)\cdot(4n-1)}$ 151. $\sum_{n=1}^{\infty}\frac{n!}{5^n}$

152. $\sum_{n=3}^{\infty}\frac{7^{3n}}{(2n-5)!}$ 153. $\sum_{n=1}^{\infty}\frac{2n-1}{2^n}$

Musbat hadli qatorlar yaqinlashishining Koshi va integral alomati.

Koshi teoremasi. Agar musbat hadli qator uchun $U_n=\sqrt[n]{a_n}$ miqdor $n\to\infty$ da l chekli limitga ega bo'lsa, ya'ni

$$\lim_{n\to\infty}U_n=\lim_{n\to\infty}\sqrt[n]{a_n}=l$$

bo'lsa, u holda:

1) $l<1$ bo'lgan holda qator yaqinlashuvchi ;
2) $l>1$ bo'lgan holda qator uzoqlashuvchi bo'ladi.

Agar $\sqrt[n]{a_n}<1$ bo'lib, $\lim\limits_{n\to\infty}\sqrt[n]{a_n}=1$ bo'lsa, shubhali hol deyiladi, chunki bu holda qator yo yaqinlashuvchi, yo uzoqlashuvchi bo'lib, masalani aniqlash uchun qo'shimcha tekshirish kerak bo'ladi.

1-misol. Ushbu $\sum\limits_{n=1}^{\infty}\left(\frac{n+1}{2n-1}\right)^n=2+1+\frac{4}{5}+\frac{5}{7}+\ldots\ldots+\left(\frac{n+1}{2n-1}\right)^n+\ldots\ldots$ qatorning yaqinlashishi tekshirilsin.

Yechish. Koshi alomatidan foydalanamiz:

$$\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\left(\frac{n+1}{2n-1}\right)^n}=\lim_{n\to\infty}\frac{n+1}{2n-1}=\frac{1}{2}<1$$

Demak, berilgan qator yaqinlashuvchi bo'ladi.

2-misol. $\lim\limits_{n\to\infty}\frac{a^n}{n!}=0$ ekanligini isbotlang.

Yechish. Bu yerda $f(n)=\left(a^n/n!\right)$ $f(n+1)=\left\lfloor a^{n+1}/(n+1)!\right\rfloor$.

$\sum\limits_{n=1}^{\infty}\frac{a^n}{n!}$ qatorni tuzamiz va Dalamber alomati yordamida yaqinlashishini tekshiramiz:

$$\lim_{n\to\infty}\frac{f(n+1)}{f(n)}=\lim_{n\to\infty}\frac{a^{n+1}n!}{(n+1)!\,a^n}=\lim_{n\to\infty}\frac{a}{n+1}=0<1$$

qator yaqinlashuvchi. Demak,

$\lim\limits_{n\to\infty}\left(a^n/n!\right)=0$.

Bundan $\lim\limits_{n\to\infty}\sqrt[n]{a_n}=0<1$, ya'ni Koshi alomatiga ko'ra $\sum\limits_{n=1}^{\infty}a_n$ qator yaqinlashuvchi

Demak, $\lim\limits_{n\to\infty}\frac{n^{n^2}}{[(3n)!]^n}=0$

qator yaqinlashishining integral alomati

Ushbu $\sum\limits_{n=1}^{\infty}a_n=a_1+a_2+a_3+\ldots\ldots+a_n+$ qatorning hadlari musbat, lekin o'suvchi bo'lmasin, ya'ni

$a_1\geq a_2\geq a_3\geq\ldots\ldots\geq a_n\geq\ldots\ldots>0$ deb faraz qalaylik.

Agar $x\geq 1$ lar uchun aniqlangan $f(x)$ funksiya uzluksiz, musbat va monoton kamayuvchi bo'lib,

$f(1)=a_1,\quad f(2)=a_2,\ \ldots\ldots\ldots, f(n)=a_n,\ldots\ldots$

bo'lsa, qatorning yaqinlashishi uchun xosmas integralning

$\int\limits_1^{\infty}f(x)dx$ yaqinlashuvchi bo'lishi zarur va yetarli.

3-misol. Ushbu $\sum_{n=1}^{\infty}\frac{1}{n^m}=\frac{1}{1^m}+\frac{1}{2^m}+\frac{1}{3^m}+........+\frac{1}{n^m}+$

qatorning yaqinlashishi tekshirilsin.

<u>Yechish</u>. $f(x)=\frac{1}{x^m}$ deb olamiz Bu funktsiya teoremaning hamma shartlarini qanoatlantiradi. quyidagi integralni qaraymiz:

$$\int_1^N\frac{dx}{x^m}=\begin{cases} m\neq 1 \quad bo'lganda \quad \left.\frac{x^{1-m}}{1-m}\right|_1^N=-\frac{1-N^{1-m}}{1-m}, \\ m=1 \quad bo'lganda \quad \ln x\left.\right|_1^N=\ln N \end{cases}$$

$N\to\infty$ da $m>1$ bo'lsa $\int_1^{\infty}\frac{dx}{x^m}=\frac{1}{m-1}$

Ya'ni, xosmas integral chekli shuning uchun berilgan qator yaqinlashadi; $m\leq 1$ bo'lsa, $\int_1^{\infty}\frac{dx}{x^m}=\infty$ bo'ladi. Ya'ni integral cheksiz. qator uzoqlashadi.

Darsda yechish uchun misollar

Koshi alomatiga asosan quyidagi qatorlarning yaqinlashishi tekshirilsin:

154. $\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+...+\left(\frac{n}{2n+1}\right)^n+...$

155. $arcsin1+arcsin^2\frac{1}{2}+arcsin^3\frac{1}{3}+...+arcsin^n\frac{1}{n}+...$

156. $\frac{1}{ln2}+\frac{1}{ln^2 3}+\frac{1}{ln^3 4}+\frac{1}{ln^4 5}+...+\frac{1}{ln^n(n+1)}+...$

157. $arctg1+arctg^2\frac{1}{2}+arctg^3\frac{1}{3}+...+arctg^n\frac{1}{n}+...$

158. $\frac{1}{2}+\left(\frac{2}{5}\right)^3+\left(\frac{3}{8}\right)^5+...+\left(\frac{n}{3n-1}\right)^{2n-1}+...;$

159. $\sum_{n=1}^{\infty}3^{n+1}\cdot\left(\frac{n+2}{n+3}\right)^{n^2};$

Integral alomati bilan quyidagi qatorlarning yaqinlashishi tekshirilsin:

160. $\frac{1}{2\ln^2 2}+\frac{1}{3\ln^2 3}+\frac{1}{4\ln^2 4}+...;$

165. $\sum_{n=2}^{\infty}\frac{n}{\sqrt{n^2-1}};$

161. $\frac{e^{-\sqrt{1}}}{\sqrt{1}}+\frac{e^{-\sqrt{2}}}{\sqrt{2}}+\frac{e^{-\sqrt{3}}}{\sqrt{3}}+...;$

162. $\sum_{n=1}^{\infty}\left(\frac{1+n}{1+n^2}\right)^2$

164. $\sum_{n=1}^{\infty}\frac{2n-3}{n(n+1)};$

163. $\sum_{n=1}^{\infty}\frac{n}{1+n^2}$

166. $\sum_{n=3}^{\infty} \frac{1}{n^4-9}$

Mustaqil uy vazifasi uchun misollar

Koshi alomatiga asosan quyidagi qatorlarning yaqinlashishi tekshirilsin:

167. $arctg1 + arctg^2 \frac{1}{2} + arctg^3 \frac{1}{3} + ... + arctg^n \frac{1}{n} + ...$

168. $\frac{1}{2} + \left(\frac{2}{5}\right)^3 + \left(\frac{3}{8}\right)^5 + ... + \left(\frac{n}{3n-1}\right)^{2n-1} + ...;$

169. $\sum_{n=1}^{\infty} \left(\frac{n-1}{n+1}\right)^{n^2+4n+5}$; 170. $\sum_{n=1}^{\infty} 3^{-n} \cdot \left(\frac{n+1}{n}\right)^{n^2}$; 171. $\sum_{n=1}^{\infty} \left(\frac{an}{n+2}\right)^n$; $(a>0)$

Integral alomati bilan quyidagi qatorlarning yaqinlashishi tekshirilsin:

172. $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$;

173. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(n+1)}$;

174. $\sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{(2n-3)^2}}$;

175. $\sum_{n=1}^{\infty} \frac{1}{(n+1) \cdot ln(n+1)}$;

176. $\sum_{n=2}^{\infty} \frac{1}{n \cdot ln^3(n)}$;

Ishorasi navbatlashuvchi qatorlar. Leybnis teoremasi.

Ishoralari navbatlashuvchi qatorlar deb, hadlari navbat bilan musbat va manfiy ishoraga ega bo’lgan qatorlarga aytiladi.

$a_1 - a_2 + a_3 - a_4 + \ldots\ldots - (-1)^{n-1} a_n + \ldots\ldots$

kabi yozish qulaydir (bu yerda hamma $a_n > 0$ deb hisoblanadi).

Leybnits teoremasi. Agar ishoralari navbatlashuvchi qatorning hadlari absolyut qiymatlari bo’yicha monoton kamayuvchi bo’lsa

$a_1 > a_2 > a_3 > \ldots\ldots > a_n > \ldots\ldots$

va $n \to \infty$ da qatorning n - hadi a_n nolga intilsa, ya’ni $\lim_{n\to\infty} a_n = 0$ bo’lsa, u holda qator yaqinlashuvchi bo’ladi va uning yig’indisi musbat bo’lib birinchi haddan katta bo’lmaydi. Agar ishorasi almashinuvchi qator yaqinlashsa, lekin uni hadlarining absolyut qiymatlaridan tuzilgan qator uzoqlashsa, u holda berilgan ishorasi almashinuvchi qator shartli yaqinlashuvchi qator deb ataladi.

1-misol. Leybnits alomatidan foydalanib, ushbu qatorning

$$\frac{2}{4} - \frac{3}{7} + \frac{4}{12} - \frac{5}{19} + \ldots\ldots\ldots + (-1)^{n-1} \frac{n+1}{n^2+3}$$

yaqinlashishini tekshiring.

Yechish. Berilgan qatorning hadlari absolyut qiymati bo’yicha monoton kamayadi :

$$\frac{2}{4} > \frac{3}{7} > \frac{4}{12} > \frac{5}{19} > \ldots\ldots$$

va uning umumiy hadi esa $n \to \infty$ da nolga intiladi, ya'ni :

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+1}{n^2+3} = 0.$$

Demak, qator Leybnits alomatiga ko'ra yaqinlashadi.

2 - misol. Ushbu $1{,}1 - 1{,}01 + 1{,}001 - 1{,}0001 + \ldots$ qatorning yaqinlashishi tekshirilsin.

Yechish. qatorning hadlari absolyut qiymatlari bo'yicha kamayuvchi:

$1{,}1 > 1{,}01 > 1{,}001 > 1{,}0001 > \ldots\ldots,,$

Lekin, $n \to \infty$ da nolga intilmaydi:

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(1 + \frac{1}{10^n}\right) = 1 \neq 0$$

Demak, qator uzoqlashadi chunki Leybnits teoremasining ikkinchi sharti (ya'ni qator yaqinlashishining zaruriy sharti) bajarilmadi.

Darsda yechish uchun misollar

Berilgan qatorlarni yaqinlashishi aniqlansin:

177. $1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \frac{1}{9^2} - \ldots + \frac{(-1)^{n+1}}{(2n-1)^2} \ldots;$

178. $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots + \frac{(-1)^{n+1}}{2n-1} \ldots;$

179. $1 - \frac{1}{2} + \frac{1}{5} - \frac{1}{10} + \frac{1}{17} - \ldots + (-1)^n \frac{1}{n^2+1} + \ldots;$

180. $1 - \frac{2}{7} + \frac{3}{13} - \ldots + (-1)^{n-1} \frac{n}{6n-5} + \ldots;$

181. $\sqrt{\frac{1}{101}} - \sqrt{\frac{1}{201}} + \sqrt{\frac{1}{301}} - \sqrt{\frac{1}{401}} + \ldots + (-1)^{n-1}\sqrt{\frac{1}{100n+1}} + \ldots$

182. $1 - \frac{1}{2} + \frac{1}{5} - \frac{1}{10} + \frac{1}{17} - \ldots + (-1)^n \frac{1}{n^2+1} + \ldots;$

183. $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} \cdots\cdots (-1)^{n-1}\frac{1}{\sqrt{n}} + \cdots$

184. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots + (-1)^{n-1}\frac{1}{n} + \ldots;$

Mustaqil uy vazifasi uchun misollar

Berilgan qatorlarni yaqinlashishi aniqlansin:

185. $\frac{\ln 2}{2} - \frac{\ln 3}{3} + \frac{\ln 4}{4} - \frac{\ln 5}{5} - \ldots + (-1)^n \frac{\ln n}{n} + \ldots;$

186. $1 - \frac{2!}{2^2} + \frac{3!}{3^3} - \frac{4!}{4^4} + \frac{5!}{5^5} - \ldots + (-1)^n \frac{n!}{n^n} + \ldots;$

187. $1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \frac{1}{9^2} - \ldots + \frac{(-1)^{n+1}}{(2n-1)^2} \ldots;.$

188. $0{,}1-\frac{0{,}1^2}{2}+\frac{0{,}1^3}{3}-\frac{0{,}1^4}{4}+\frac{0{,}1^5}{5}-...+\frac{(-1)^{n+1}0{,}1^n}{n}+...;$

Funksional qatorlar

Ta'rif. Hadlari o'zgaruvchi x ning biror X sohada aniqlangan funktsiyalaridan iborat bo'lgan

$$\sum_{n=1}^{\infty}U_n(x)=U_1(x)+U_2(x)+......+U_n(x)+...... \qquad (1)$$

qator funktsional qator deyiladi.
x ga aniq son qiymatlar berib, cheksiz ko'p turli sonli qatorlarni hosil qilamiz. Bular yaqinlashuvchi va uzoqlashuvchi bo'lishi mumkin.
x ning (1) funktsional qator yaqinlashadigan qiymatlar to'plami, shu qatorning yaqinlashish sohasi deyiladi.
Ta'rif. Agar har qanday $\varepsilon>0$ uchun x ga bog'liq bo'lmagan shunday nomer N mavjud bo'lsaki, $n\succ N$ bo'lganda X sohaning hamma x lari uchun bir vaqtda

$$|S_n(x)-S(x)|<\varepsilon \quad \text{yoki} \quad |r_n(x)|<\varepsilon \qquad (2)$$

tengsizlik o'rinli bo'lsa, u holda, qator X sohada tekis yaqinlashuvchi deyiladi.
Agar X sohada yaqinlashuvchi bo'lgan (1) funktsional qator aytilgan xossaga ega bo'lmasa, u holda bu qatorga notekis yaqinlashuvchi qator deyiladi.
1-misol.

$$\sum_{n=1}^{\infty}x^{n-1}=1+x+x^2+......+x^n$$ funktsional qatorni qaraymiz.

Yechish. Bu qator x ning $(-1;\ 1)$ intervaldagi hamma qiymatlarida, ya'ni x ning $|x|<1$ shartni qanoatlantiruvchi hamma qiymatlarida yaqinlashadi va yig'indisi $\frac{1}{1-x}$ ga teng, chunki bu maxraji x ga teng bo'lgan kamayuvchi geometrik progressiyadir. SHunday qilib berilgan qator $(-1;\ 1)$ intervalda

$$\frac{1}{1-x}=1+x+x^2+x^3+......+x^n+......$$

qatorning yig'indisi bo'lgan $S(x)=\frac{1}{1-x}$ funksiyani aniqlaydi.

2-misol. Ushbu $$\sum_{n=1}^{\infty}\frac{1}{(n+x)(n+x+1)}$$

funktsional qatorning yaqinlashish sohasi va yig'indisini toping.

Yechish. $S_n(x)=\frac{1}{(n+x)(n+x+1)}$ funktsiya $x=-n$ va $x=-(n+1)$ nuqtalarda aniqlanmagan (bu nuqtalar funktsiyani uzulish nuqtalari), shuning uchun biz qatorni $x\neq -n \quad (n=1,\ 2,\ 3,......)$ nuqtalarda tekshiramiz.

$$S_n(x)=\frac{1}{n+x}-\frac{1}{n+x+1}$$

ekanligini e'tiborga olib,qatorning n -qismiy yig'indisini quyidagi ko'rinishda yozamiz:

$$S_n(x)=\frac{1}{(1+x)(2+x)}+\frac{1}{(2+x)(3+x)}+\ldots\ldots\frac{1}{(n+x)(n+x+1)}=$$

$$=\left(\frac{1}{1+x}-\frac{1}{2+x}\right)+\left(\frac{1}{2+x}-\frac{1}{3+x}\right)+\left(\frac{1}{3+x}-\frac{1}{4+x}\right)+\ldots\ldots+$$

$$+\left(\frac{1}{n-1+x}-\frac{1}{n+x}\right)+\left(\frac{1}{n+x}-\frac{1}{n+x+1}\right)=\frac{1}{1+x}-\frac{1}{n+x+1}$$

Demak, $\lim\limits_{n\to\infty} S_n(x)$ mavjud va

$$S(x)=\lim_{n\to\infty}S_n(x)=\lim_{n\to\infty}\left(\frac{1}{1+x}-\frac{1}{n+x+1}\right)=\frac{1}{1+x},$$

Ya'ni, qator x ning $x\neq -n \quad (n=1,\ 2,\ 3,\ldots)$ shartni qanoatlantiruvchi barcha qiymatlarida yaqinlashadi va yig'indisi $S(x)=\dfrac{1}{1+x}$ funktsiyadan iborat bo'ladi.

Darsda yechish uchun misollar

189. Ushbu $\dfrac{4-x}{7x+2}+\dfrac{1}{3}\left(\dfrac{4-x}{7x+2}\right)^2+\dfrac{1}{5}\left(\dfrac{4-x}{7x+2}\right)^3+\ldots;$ funktsional qatorning $x=0$ va $x=1$ nuqtalarda yaqinlashishi tekshirilsin.

190. Ushbu $\dfrac{1}{1+x^2}+\dfrac{1}{1+x^4}+\dfrac{1}{1+x^6}+..;$ funktsional qatorning yaqinlashishi tekshirilsin va yaqinlashish sohasi topilsin. J: $|x|>1$

191. Ushbu $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{x^{2n}+n};$ funktsional qatorning $(-\infty,\infty)$oraliqda tekis yaqinlashishi ko'rsatilsin.

192. Ushbu $x+\dfrac{x^2}{2}+\dfrac{x^3}{4}+\dfrac{x^4}{8}+\ldots,$ funktsional qatorning $(-2;2)$ intervalda yaqinlashishi isbot qilinsin.

Berilgan funktsional qatorlarning yaqinlashish intervalini toping.

193. $x+\dfrac{x^2}{\sqrt{2}}+\dfrac{x^3}{\sqrt{3}}+\dfrac{x^4}{\sqrt{4}}+\dfrac{x^5}{\sqrt{5}}+\ldots+\dfrac{x^n}{\sqrt{n}}+\ldots$

194. $\dfrac{x}{1+x}+\dfrac{x^2}{1+x^4}+\dfrac{x^3}{1+x^6}+\dfrac{x^4}{1+x^8}+\dfrac{x^5}{1+x^{10}}+\ldots+\dfrac{x^n}{1+x^{2n}}+\ldots$

195. $\dfrac{1}{1+x}+\dfrac{1}{1+x^2}+\dfrac{1}{1+x^3}+\dfrac{1}{1+x^4}+\ldots+\dfrac{1}{1+x^n}+\ldots$

196. $\dfrac{\cos x}{e^x}+\dfrac{\cos 2x}{e^{2x}}+\dfrac{\cos 3x}{e^{3x}}+\dfrac{\cos 4x}{e^{4x}}+\dfrac{\cos 5x}{e^{5x}}+\ldots+\dfrac{\cos nx}{e^{nx}}+..$

197. $\dfrac{\sin x}{1}+\dfrac{\sin 2x}{2^2}+\dfrac{\sin 3x}{3^2}+\dfrac{\sin 4x}{4^2}+\dfrac{\sin 5x}{5^2}+\ldots+\dfrac{\sin nx}{n^2}+..$

198. $e^{-x}+e^{-4x}+e^{-9x}+e^{-16x}+\ldots+e^{-n^2x}+\ldots$

Mustaqil uy vazifasi uchun misollar

Berilgan funktsional qatorlarning yaqinlashish intervalini toping.

199. $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+...+(-1)^{n+1}\frac{x^{2n-1}}{(2n-1)!}+...,$

200. $(x+8)+\frac{(x+8)^2}{2^2}+\frac{(x+8)^3}{3^2}+\frac{(x+8)^4}{4^2}+...+\frac{(x+8)^n}{n^2}+...$

201. $\ln x+\ln^2 x+\ln^3 x+\ln^4 x+...+\ln^n x+...$

202. $x+x^4+x^9+x^{16}+...+x^{n^2}+...$

203. $tg\,x+\frac{tg^2x}{2^2}+\frac{tg^3x}{3^2}+\frac{tg^4x}{4^2}+\frac{tg^5x}{5^2}+...+\frac{tg^nx}{n^2}+...$

Darajali qatorlar. Yaqinlashish intervali.

$$\sum_{n=0}^{\infty} a_n x^n = a_0+a_1x+a_2x^2+a_3x^3+...+a_nx^n+... \quad (1)$$

ko'rinishdagi funktsional qatorga darajali qator deyiladi. Bunda a_i $(i=\overline{0,\infty})$ lar o'zgarmas sonlar bo'lib, darajali qatorning koeffitsientlari deyiladi.

x argumentga tayin bir qiymat berish natijasida (1) darajali qator sonli qatorga aylanadi va bu sonli qator yaqinlashuvchi yoki uzoqlashuvchi bo'lishi mumkin.

(1) darajali qator yaqinlashadigan barcha $x=\bar{x}$ nuqtalardan tashkil topgan $X=\{\bar{x}\}$ to'plamga darajali qatorning yaqinlashish sohasi deyiladi.

1-teorema. (Abel teoremasi) Agar (1) darajali qator 0 dan farqli $x=\bar{x}$ qiymat uchun yaqinlashuvchi bo'lsa, u holda bu qator $|x|<|\bar{x}|$ tengsizlikni qanoatlantiruvchi istalgan x uchun absolyut yaqinlashuvchi bo'ladi; agar qator $x=\bar{x}$ qiymat uchun uzoqlashuvchi bo'lsa, u holda bu qator $|x|>|\bar{x}|$ tengsizlikni qanoatlantiruvchi istalgan x da uzoqlashuvchi bo'ladi.

$\bar{x}=R$ desak, bu R songa darajali qatorning yaqinlashish radiusi deyiladi. $(-R;\ R)$ interval esa yaqinlashish intervali deyiladi.

Darajali qatorning yaqinlashish radiusi R ni aniqlash uchun Dalamber belgisidan foydalanib, quyidagiga ega bo'lamiz :

$$R=\lim_{n\to\infty}\frac{|a_n|}{|a_{n+1}|}$$

Darajali qatorning yaqinlashish radiusi R ni aniqlash uchun shunga o'xshash Koshi alomatidan ham foydalansak, u holda

$$R=\frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{a_n}}$$

1-misol. Ushbu

$$\sum_{n=1}^{\infty}\frac{3^n x^n}{2^n\sqrt{n}}=\frac{3x}{2}+\frac{3^2x^2}{2^2\sqrt{2}}+\frac{3^3x^3}{2^3\sqrt{3}}+...$$

darajali qatorning yaqinlashish intervali topilsin.

Yechish. Dalamber belgisini tatbiq etamiz:

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\left|\frac{3^n\cdot 2^{n+1}\sqrt{n+1}}{3^{n+1}2^n\sqrt{n}}\right|=\lim_{n\to\infty}\left|\frac{2}{3}\sqrt{\frac{n+1}{n}}\right|=\frac{2}{3}\lim_{n\to\infty}\sqrt{\frac{1}{1+\frac{1}{n}}}=\frac{2}{3}$$

Demak, $R=\frac{2}{3}$ bo'ladi. Bundan $(-\frac{2}{3};\frac{2}{3})$ interval berilgan qator uchun yaqinlashish intervali bo'ladi.

Endi intervalning chetki nuqtalarida (ya'ni $x=-\frac{2}{3}$ va $x=\frac{2}{3}$da) berilgan qatorning yaqinlashishini, tekshiramiz.
Buning uchun har bir chegarada yakka-yakka holda tekshiramiz.

a) $x=-\frac{2}{3}$ bo'lganda $\sum_{n=1}^{\infty}\frac{3^n\left(-\frac{2}{3}\right)^n}{2^n\sqrt{n}}=1+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}-...$ ishoralari navbatlashuvchi sonli qator hosil bo'ladi. qator uchun Leybnits teoremasining ikkala sharti ham bajariladi, yahni

1) $1>\frac{1}{\sqrt{2}}>\frac{1}{\sqrt{3}}>\frac{1}{\sqrt{4}}>...;$

2) $\lim_{n\to\infty}\frac{1}{\sqrt{n}}=0$

va demak, berilgan qator $x=-\frac{2}{3}$ nuqtada yaqinlashuvchi. b) $x=\frac{2}{3}$ bo'lganda

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}=1+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}-... \qquad (5.8)$$

musbat hadli sonli qator hosil bo'ladi. (5.8) qator uzoqlashuvchi.
Demak, berilgan darajali qator uchun yaqinlashish intervali $\left[-\frac{2}{3};\frac{2}{3}\right[$ yoki $-\frac{2}{3}\le x<\frac{2}{3}$ bo'ladi.

2-misol. $$x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+...+\frac{x^n}{n!} \qquad (5.9)$$

qatorning yaqinlashish intervali topilsin.
Yechish. Berilgan qatorga Dalamber belgisini tatbiq etsak,

$$\lim_{n\to\infty}\left|\frac{x^{n+1}n!}{x^n(n+1)!}\right|=|x|\lim_{n\to\infty}\frac{1}{n+1}=0<1$$

limitning x ga bog'liq emasligi va birdan kichikligi tufayli qator x ning barcha qiymatlarida yaqinlashadi, yahni qatorning yaqinlashish radiusi $R=\infty$ bo'lib, yaqinlashish intervali $]-\infty,+\infty[$dan ibrat.

Darsda yechish uchun misollar

Berilgan darajali qatorlarning yaqinlashish intervali aniqlansin va intervalining chegaralarida ham qatorni yaqinlashishi tekshirilsin:

204 $1+\frac{1}{3\cdot 2}x+\frac{1}{3^2\cdot 3}x^2+\frac{1}{3^2\cdot 4}x^3+...\frac{1}{3^n\cdot(n+1)}x^n+;$

205. $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - ... + (-1)^n \frac{x^n}{n} + ...$

206. $x + \frac{x^2}{20} + \frac{x^3}{300} + \frac{x^4}{4000} + ... + \frac{x^n}{n \cdot 10^{n-1}} + ...$

207. $1 + 2x^2 + 4x^4 + ... + 2^{n-1} x^{2(n-1)} + ...$

208. $x + 4x^2 + 27x^3 + 256x^4 ... + (n \cdot x)^n + ...$

209. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{(2n)!};$

210. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{(n+1) \cdot 7^{n-1}};$

211. $\sum_{n=1}^{\infty} \frac{x^n n}{(n+1) 2^n};$

212. $\sum_{n=1}^{\infty} \left(\left(\frac{n+1}{n} \right)^n x \right)^n;$

213. $\sum_{n=1}^{\infty} \frac{(x-2)^n n}{(2n-1)2^n};$

214. $\sum_{n=1}^{\infty} \frac{x^n}{(n+1)^2 e^{n-1}};$

Mustaqil uy vazifasi uchun misollar

Berilgan darajali qatorlarning yaqinlashish intervali aniqlansin va intervalining chegaralarida ham qatorni yaqinlashishi tekshirilsin:

215. $x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!} ... + \frac{x^{2n-1}}{(2n-1) \cdot (2n-1)!} + ...$

216. $1 + x + 2x^2 + 6x^3 + 24x^n ... + n! \cdot x^n + ...$

217. $\sum_{n=1}^{\infty} \frac{2^n \cdot n!}{(2n)!} x^{2n}$

218. $x + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \frac{x^4}{4^2} + \frac{x^5}{5^2} + ... + \frac{x^n}{n^2} + ...$

Ba'zi elementar funksiyalarni darajali qatorga yoyish.

Agar $f(x)$ funktsiya $x = a$ nuqtani o'z ichiga olgan biror intervalda $(n+1)$- tartibli hamma hosilalarga ega bo'lsa, u holda $f(x)$ funksiya uchun qoldiq hadli Teylor formulasi quyidagi ko'rinishda bo'ladi:

$$f(x) = f(a) + = \frac{(x-a)}{1!} f'(a) + ... + \frac{(x-a)^n}{n!} f^{(n)}(a) + R_n(x) \quad (1)$$

bundagi $R_n(x)$ qoldiq had, quyidagi

$$R_n(x) = \frac{(x-a)^{n+1}}{(n+1)!} f^{(n+1)}\{a + \upsilon(x-a)\} \quad (0 < \upsilon < 1) \quad (2)$$

Lagranj formulasi bilan baholanadi.

Agar $f(x)$ funktsiya $x = a$ nuqta atrofida istalgan tartibli hosilaga ega bo'lsa, Teylor formulasidagi n sonini istagancha kattalashtirib olish mumkin. Bunday holda n sonining cheksiz o'sishi bilan ko'phad quyidagi

$$f(x) \approx f(a) - \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + ... + \frac{(x-a)^n}{n!} f^{(n)}(a) + ... \quad (3)$$

cheksiz qatorga aylanadi.

Bu qator, ya'ni Teylorning formal ravishda yozilgan qatori o'zining yaqinlashish intervalida $f(x)$ dan $\lim_{n\to\infty} R_n(x)$ miqdor bilan farq qiluvchi funktsiyadir, chunki:

$$R_n(x)=\left\{f(x)-f(a)-\frac{(x-a)}{1!}f'(a)-\frac{(x-a)^2}{2!}f''(a)-...-\frac{(x-a)^n}{n!}f^{(n)}(a)\right\}$$

Agar Teylor formulasida n soni cheksizlikka intilganda $(n\to\infty)$, $R_n(x)$ qoldiq had x ning barcha qiymatlarida nolga intilsa, ya'ni $\lim_{n\to\infty} R_n(x)=0$ bo'lsa, u holda bu formuladan $f(x)$ funktsiyaga yaqinlashuvchi quyidagi

$$f(x)=f(a)+\frac{(x-a)}{1!}f'(a)+\frac{(x-a)^2}{2!}f''(a)+...+\frac{(x-a)^n}{n!}f^{(n)}(a)+... \quad (4)$$

cheksiz qator hosil bo'ladi. (4) qator $f(x)$ funktsiyaning $x=a$ nuqtadagi Teylor qatori deyiladi.

Agar Teylor qatorida $a=0$ deb olsak, u holda Teylor qatorining xususiy holi bo'lgan Makloren qatorini hosil qilamiz:

$$f(x)=f(0)+\frac{x}{1!}f'(0)+\frac{x^2}{2!}f''(0)+...+\frac{x^n}{n!}f^{(n)}(0)+... \quad (5)$$

Matematik analizning ko'pchilik masalalarida berilgan $f(x)$ funktsiyani Teylor yoki Makloren qatoriga yoyishga to'g'ri keladi. Ko'p hollarda bu masala quyidagi taxminiy plan yordamida yechiladi:

1. $f(x)$ funktsiyaning ketma-ket hosilalari topiladi;
2. $f(a)$ va $f^{(n)}(a)$ sonlar hisoblanadi $(n=1, 2, 3...)$;
3. $f(x)$ funktsiya uchun Teylor qatori formal ravishda yoziladi, ya'ni

$$f(x)\approx f(a)-\frac{(x-a)}{1!}f'(a)+\frac{(x-a)^2}{2!}f''(a)+...+\frac{(x-a)^n}{n!}f^{(n)}(a)+...$$

Ba'zi elementar funktsiyalarni darajali qatorga yoyilmasi:

1. $\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-...+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+...;$
2. $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-...+(-1)^n\frac{x^{2n}}{(2n)!}+...;$
3. $\ln(1+x)=x-\frac{x^2}{2!}+\frac{x^3}{3!}+...+(-1)^{n-1}\frac{x^n}{n}+...;$
4. $e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^{n-1}}{(n-1)!}+...;$
5. $arctgx=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+...+(-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!}+...; \quad (|x|<1)$
6. $\arcsin x=x-\frac{x^3}{2\cdot 3}+\frac{1\cdot 3x^5}{2\cdot 4\cdot 5}+\frac{1\cdot 3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7}+...(|x|<1)$
7. $\frac{1}{1-x}=1+x+x^2+...x^n+..., (|x|<1)$

8. $\frac{1}{1+x}=1-x+x^2-x^3+...+(-1)^{n-1}x^{n-1}+...,\ (|x|<1)$

9. $\sqrt{1+x}=1+\frac{1}{2}x-\frac{1}{2\cdot 4}x^2+...+(-1)^n\frac{1\cdot 3\cdot 5...(2n-5)x^{n-2}}{2\cdot 4\cdot 6\cdot 8...(2n-2)}+x$

10. $\frac{1}{\sqrt{1+x}}=1-\frac{1}{2}x-\frac{1\cdot 3}{2\cdot 4}x^2-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3+...,\ (-1<x<1)$

11. $(1+x)^m=1\_mx+\frac{m(m-1)}{2!}x^2+\frac{m(m-1)(m-2)}{3!}x^3+...$

$...+\frac{m(m-1)(m-2)...(m-n+1)}{n!}x^n+...$ binomial qator

12. $(arctgx)'_x=\frac{1}{1+x^2}=1+x^2+x^4+...+(-1)^{n-1}x^{2n-2}+...,\ (|x|<1)$

13. $(\arcsin x)'_x=\frac{1}{\sqrt{1-x^2}}=1+\frac{1}{2}x^2+...+\frac{(2n-1)!!}{2n!!}x^{2n}+...,\ (|x|<1)$

1-misol. $f(x)=\ln\sqrt[3]{\frac{1+2x}{1-x}}$ Funktsiyani x ning darajalari bo'yicha qatorga yoyilsin.

Yechish. Bizga ma'lumki $ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n},\quad (-1\langle x\langle 1)$

shuning uchun

$$ln(1+2x)=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^nx^n}{n},\quad (-\frac{1}{2}\langle x\langle\frac{1}{2})$$

$$ln(1-x)=-\sum_{n=1}^{\infty}\frac{x^n}{n},\quad (-1\langle x\langle 1)$$

bundan,

$$\ln\sqrt[3]{\frac{1+2x}{1-x}}=\frac{1}{3}[\ln(1+2x)-\ln(1-x)]=\frac{1}{3}\left[\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^nx^n}{n}-\sum_{n=1}^{\infty}\frac{x^n}{n}\right]=$$

$$=\frac{1}{3}\left[\sum_{n=1}^{\infty}(1+(-1)^{n+1}\cdot 2^n)\cdot\frac{x^n}{n}\right]=\frac{1}{3}\left(3x-\frac{3}{2}x^2+\frac{9}{3}\cdot x^3-\frac{15}{4}x^4+...\right)=$$

$$=x-\frac{1}{2}x^2+x^3-\frac{5}{4}x^4+...,\qquad \left(-\frac{1}{2}<x<\frac{1}{2}\right)$$

Darsda yechish uchun misollar

Berilgan funktsiyalarni Teylor qatoriga yoying.

219. $f(x)=e^{-x^2}$

220. $f(x)=e^{5x}$

221 . $f(x)=ln(1+4x)$

222. $f(x)=\sqrt{1+x^3}$

223. $f(x)=(1+x)\cdot e^x$

224. $f(x)=\dfrac{3}{(1-x)(1+2x)}$ funktsiyani x ning darajalari bo'yicha qatorga yoyilsin

225. $f(x)=\ln x$ funktsiyani $(x-1)$ ning darajalari bo'yicha qatorga yoyilsin.

226. $f(x)=x^3-3x$ funktsiya $(x+1)$ ning darajalari bo'yicha qatorga yoyilsin.

227. $f(x)=ln\left(x+\sqrt{1+x}\right)$ funktsiya x ning darajalari bo'yicha qatorga yoyilsin. (Ko'rsatma $ln\left(x+\sqrt{1+x}\right)=\int\limits_0^x \dfrac{dt}{\sqrt{1+t^2}}$ formuladan foydalaning).

Mustaqil uy vazifasi uchun misollar

Berilgan funktsiyalarni $x=0$ nuqta atrofida Teylor qatoriga yoying.

228. $f(x)=e^{3x}$ 229. $f(x)=\ln(1-2x)$ 230. $f(x)=\ln(1-\dfrac{x}{3})$

231. $f(x)=\sqrt{1+x^3}$

Fur'e qatorlari

Dirixle sharti. Agar $y=f(x)$ funksiya (a,b) oraliqda chegaralangan va bo`lakli – monoton bulsa , u holda bu funksiya (a,b) oraliqda Dirixle shartlarini qanoatlantiradi deyiladi.

Agar $y=f(x)$ funksiya uzunligi 2π ga teng $(-\pi;\pi)$ oraliqda Dirixle shartlarini qanoatlantirsa, u xolda bu oraliqning $f(x)$ uzluksiz bo`lgan har qanday $x$ nuqtasida funksiyani Fur'e trigonometrik qatoriga yoyish mumkin, ya`ni

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos nx+b_n\sin nx\right),$$

bu yerda a_n,b_n - Fur'e koeffisientlari bo`lib, ular quyidagi formulalar bo'yicha hisoblanadi

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx,\ b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx$$

Darsda yechish uchun misollar

Quyidagi funksiyalarni berilgan oraliqda Fur'e qatoriga yoying

232. $f(x)=x;\ [-\pi;\pi]$. 233. $f(x)=e^x;[-\pi;\pi]$. 234. $f(x)=x^3;\ [-\pi;\pi]$.

235. $f(x)=\begin{cases}-1\ ,\ -\pi\le x\le 0\\ 1\ ,\ \ 0\le x\le\pi\end{cases}$. 236. $f(x)=1;\ [-\pi;\pi]$.

Quyidagi tengliklarni isbotlang.

237. 1) $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n}{n^2}=\dfrac{\pi^2}{12}$ 2) $\sum\limits_{n=1}^{\infty}\dfrac{1}{n^2}=\dfrac{\pi^2}{6}$ 3) $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{2n-1}=\dfrac{\pi}{4}$

Mustaqil uy vazifasi uchun misollar

Quyidagi funksiyalarni berilgan oraliqda Fur'e qatoriga yoying.

238. $f(x)=\pi+x;\ [-\pi;\pi]$ 239. $f(x)=\begin{cases}-2x\ ,-\pi\le x\le 0\\ 3x\ \ ,0\le x\le\pi\end{cases},\ T=2\pi$

Quyidagi tengliklarni isbotlang.

240. 1) $\sum_{n=1}^{\infty}\frac{1}{4n^2-1}=\frac{1}{2}$ 2) $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$

Ko`p o`zgaruvchili funksiya va uning aniqlanish sohasi.

1-Misol Ushbu $z=\frac{1}{R^2-x^2-y^2}$ funksiyaning aniqlanish sohasini toping.

yechish: Funksiyaning aniqlanish sohasi $\frac{1}{R^2-x^2-y^2}$ ifoda aniqlangan nuqtalar to`plami ya`ni $R^2-x^2-y^2\neq 0$ yoki $x^2+y^2\neq R^2$ bajariladigan nuqtalar to`plami bo`ladi. Bu to`plamga tekislikning $x^2+y^2=R^2$ aylana nuqtalaridan tashqari hamma nuqtalari tegishli bo`ladi.

2. Misol. Ushbu $U=\sqrt{R^2-x^2-y^2-z^2}$ funksiyaning aniqlanish sohasini toping.

Yechish: Funksiya $R^2-x^2-y^2-z^2\geq 0$ yoki $x^2+y^2+z^2\leq R^2$ Demak, aniqlanish sohasi shar.

Darsda yechish uchun misollar

Funksiyalarning aniqlanish sohasini toping.

241. $z=\frac{4}{x^2+y^2}$ j: (0,0) nuqtadan tashqaridagi nuqtalar

242. $z=\sqrt{a^2-x^2-y^2}$ j: $x^2+y^2\leq a^2$

243. $z=\sqrt{xy}$ j: $xy\geq 0$

244. $z=\frac{1}{\sqrt{1-x^2-y^2}}$ j: $x^2+y^2<1$

245. $z=\frac{xy}{y-x}$ j: . $y=x$ to`g`ri chiziqdan tashqaridagi nuqtalar

246. $z=\ln(y^2-4x+8)$ j: $y^2>48-8$

247. $z=\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}$ j: $\frac{x^2}{a^2}+\frac{y^2}{b^2}\leq 1$

248. $Z=\sqrt{4-x^2-y^2}$ j: $x^2+y^2\geq 4$

249. $z=\ln(x+2y)$ j: $x+2y>0$

250. $Z=\frac{1}{xy-2}$ j: $xy\neq 2$

251. $Z=x^2y-\sqrt{y}$ j: $y\geq 0$

Mustaqil uy vazifasi uchun misollar

Funksiyalarning aniqlanish sohasini toping

252. $z=\sqrt{x+y}$ - $\sqrt{x-y}$ j: Kordinata burchaklarining bissektrissalari tashkil etgan o`ng vertikal burchakning ichki qismi (bissektrissalar ham kirgan holda)

253. $z=\frac{4}{x+y}$ j: $y=-x$ to`g`ri chiziqdan tashqari tekislikning barcha nuqtalari.

254. $z=x+\sqrt{x^2-y^2}$ j: $|y|\leq|x|$ burchak ichidagi va tomonlaridagi nuqtalar

255. $Z=\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}$ j: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning ichki nuqtalari va ellipsda yotgan nuqtalar to`plami.

256. $z=\arccos\frac{y-6}{x}$ j: $y\geq -x+6$ $y\leq x+6$

Funksiyaning birinchi tartibli xususiy hosilalari

1-Misol. Ushbu $z=\sqrt{x^2+y^2}$ funksiyaning xususiy hosilalarini toping.

Yechish: y ni o`zgarmas deb hisoblab funksiyaning, $\frac{\partial z}{\partial x}$ xususiy hosilasini topamiz.

$\frac{\partial z}{\partial x}=\frac{1}{2\sqrt{x^2+y^2}}\cdot 2x=\frac{x}{\sqrt{x^2+y^2}}$ x ni o`zgarmas deb

$$\frac{\partial z}{\partial y}=\frac{1}{2\sqrt{x^2+y^2}}\cdot 2y=\frac{y}{\sqrt{x^2+y^2}}$$

ni topamiz.

2- Misol: $U=xyz$ funksiyaning xususiy hosilalarini toping.

Yechish: y va z ni o`zgarmas deb hisoblab, uning $\frac{\partial U}{\partial x}$ xususiy hosilasini topamiz. $\frac{\partial U}{\partial x}=yz$

x va z ni o`zgarmas deb hisoblab, $\frac{\partial U}{\partial y}$ hosilasini topamiz. $\frac{\partial U}{\partial y}=xz$

x va y ni o`zgarmas deb hisoblab, $\frac{\partial U}{\partial z}$ hosilasini topamiz. $\frac{\partial U}{\partial z}=xy$

Darsda yechish uchun misollar

Funksiyaning xususiy hosilalarini toping.

257. $z=x-y$ J: $\frac{\partial z}{\partial x}=1,\ \frac{\partial z}{\partial y}=-1$

258. $z=x^3y-y^3x$ J: $\frac{\partial z}{\partial x}=3x^2y-y^3,\frac{\partial z}{\partial y}=x^3-3y^2x$

259. $z=\frac{U}{V}+\frac{V}{U}$ J: $\frac{\partial z}{\partial u}=\frac{1}{V}-\frac{V}{U^2},\ \frac{\partial z}{\partial V}=-\frac{U}{V^2}+\frac{1}{U}$

260. $z=\frac{x^3+y^3}{x^2+y^2}$ J: $\frac{\partial z}{\partial x}=\frac{y}{x^2+y^2},\frac{\partial z}{\partial y}=-\frac{x}{x^2+y^2}$

261. 1) $z=(5x^2y-y^3+7)^3$ J: $.\frac{\partial z}{\partial x}=30xy(5x^2y-y^3+7)^2$ $\frac{\partial z}{\partial y}=3(5x^2y-y^3+7)^2(5x^2-3y^2)$

2) $z=x^2-2xy+y^2-x+2y$ sirtga M(1,1,1) nuqtada o'tkazilgan urinma va normal tenglamasini tuzing. J: $x-2y+z=0$, $\frac{x-1}{-1}=\frac{y-1}{2}=\frac{z-1}{-1}$

262. $z=arctg\,\frac{x}{y}$ J: $\frac{\partial z}{\partial x}=\frac{y}{x^2+y^2}$; $\frac{\partial z}{\partial x}=\frac{x}{x^2+y^2}$;

263. 1) $u=x^y$ $\frac{x}{y}\cdot\frac{\partial u}{\partial x}+\frac{1}{\ell nx}\cdot\frac{\partial u}{\partial y}=2y;$ ekanini isbot qiling.

2) $f(\mathrm{x,y})=\mathrm{x}^y$ $x=\frac{u}{v},\ y=uv$ $f_u'(\mathrm{x,y})$, $f_v'(\mathrm{x,y})$ ni toping

Mustaqil uy vazifasi uchun misollar

Funksiyaning xususiy hosilalarini toping.

264. $z=e^{\frac{x}{y}}\ell ny$ $x\frac{\partial u}{\partial x}+y\cdot\frac{\partial z}{\partial y}=\frac{z}{\ell ny};$ ekanini isbot qiling.

265. $z=\ell n(x+\sqrt{x^2+y^2})$ 266. $z=\dfrac{1}{arctg\dfrac{y}{x}}$

267. $z=\ell ntg\dfrac{c}{y}$ 268. $z=\ell n(x+\ell ny)$ 269. $z=arctg\dfrac{x+y}{1-xy}$ $z'_x, z'_y - ?$

270. $z=\ln(x^2+y^2)$ sirtga M(1,0,0) nuqtada o'tkazilgan urinma va normal tenglamasini tuzing.

J: $z-2x+2=0$, $\dfrac{x-1}{2}=\dfrac{y}{0}=\dfrac{z}{-1}$

Ko`p o`zgaruvchili funksiyaning to`liq differensiali

$dz=\dfrac{\partial z}{\partial x}dx+\dfrac{\partial z}{\partial x}dy$ ifoda $z=F(x,y)$ funksiyaning to`liq differensiali deb ataladi.

1-misol. $z=\sqrt{x^2+y^2}$ funksiyaning to`liq differensialini toping.

$$\frac{\partial z}{\partial x}=\frac{1}{2\sqrt{x^2+y^2}}\cdot 2x=\frac{x}{\sqrt{x^2+y^2}} \qquad \frac{\partial z}{\partial y}=\frac{1}{2\sqrt{x^2+y^2}}\cdot 2y=\frac{y}{\sqrt{x^2+y^2}}$$

$$dz=\frac{xdx+ydy}{\sqrt{x^2+y^2}}$$

2-misol. $u=x^5-y^4-5z^3$ funksiyaning to`liq differensialini toping.

$\dfrac{\partial u}{\partial x}=5x^4$ x bo`yicha xususiy hosila, $\dfrac{\partial u}{\partial y}=-4y^3$ $y$ bo`yicha xususiy hosila.

$\dfrac{\partial u}{\partial z}=-15z^2$ z bo`yicha xususiy hosila.

$$du=5x^4dx-4y^3dy-15z^2dz$$

3-misol. $z=arctg\dfrac{x+y}{x-y}$ funksiyaning to`liq differensialini toping.

$$\frac{\partial z}{\partial x}=\frac{1}{1+\left(\dfrac{x+y}{x-y}\right)^2}\cdot\frac{1(x-y)-1(x+y)}{(x-y)^2}=\frac{(x-y)^2}{(x-y)^2-(x+y)^2}\cdot$$

$$\cdot\left(\frac{-2y}{(x-y)^2}\right)=\frac{-2y}{2x^2+2y^2}=\frac{-y}{x^2+y^2}$$

$$\frac{\partial z}{\partial y}=\frac{1}{1+\left(\dfrac{x+y}{x-y}\right)^2}\cdot\frac{1(x-y)-(-1)(x+y)}{(x-y)^2}=\frac{x}{x^2+y^2}$$

$$dz=-\frac{y}{x^2+y^2}dx+\frac{x}{x^2+y^2}dy=\frac{xdy-ydx}{x^2+y^2}$$

Darsda yechish uchun misollar

To`liq differensialini toping.

271. $z = x^2 y$ J: $dz = 2xydx + x^2 dy$

272. $z = \frac{x+y}{x-y}$ J: $dz = \frac{2(xdy - ydx)}{(x-y)^2}$

273. $z = \frac{x^2 + y^2}{x^2 - y^2}$ J: $dz = \frac{4xy(xdy - ydx)}{(x^2 - y^2)^2}$

274. $z = arctg(xy)$ J: $\frac{xdy + ydx}{1 + x^2 y^2}$

275. $u = x^{yz}$ J: $du = x^{zy-1}(yzdx + zx\ell nxdy + xy\ell nxdz;$

276. $z = \sin(xy)$ J: $dz = (xdy + ydx)\cos(xy);$

277. $Z = \frac{xy}{x-y}$ J: $\frac{(2xy - y^2)dx + (x^2 - 2xy)dy}{(x-y)^2}$

278. $Z = e^{\frac{s}{t}}$ J: $e^{\frac{s}{t}} \cdot \frac{1}{t^2}(-sdt + ds)$

279. $U = \sqrt{x^2 + y^2 + z^2}$ J: $\frac{xdx + ydy + zdz}{\sqrt{x^2 + y^2 + z^2}}$

Mustaqil uy vazifasi uchun misollar

280. $Z = \sqrt{x^2 + y^2}$ 281. $z = \arcsin\frac{x}{y}$ 282. $Z = \frac{x}{y} - \frac{y}{x}$

283. $S = x \cdot \ln t$ 284. $u = \sqrt{1 - x^2 - y^2 - z^2}$ 285. $z = arctg\frac{y}{x}$

286. $u = \cos\sqrt{1 + x^2 + y^2 + z^2}$

Yuqori tartibli hosilalar.

$$\frac{\partial\left(\frac{\partial F}{\partial x}\right)}{\partial x} = \frac{\partial^2 F}{\partial x^2}; \qquad \frac{\partial\left(\frac{\partial F}{\partial x}\right)}{\partial y} = \frac{\partial^2 F}{\partial x \partial y};$$

$$\frac{\partial\left(\frac{\partial F}{\partial y}\right)}{\partial x} = \frac{\partial^2 F}{\partial x \partial y}; \qquad \frac{\partial\left(\frac{\partial F}{\partial y}\right)}{\partial y} = \frac{\partial^2 F}{\partial y^2};$$

Uchinchi tartibli va boshqa yuqori tartibli xususiy hosilalar ham belgilanadi. Hosila olish tartibi bilangina farqlanuvchi aralash hosilalar uzluksiz bo`lsalar, ular o`zaro teng bo`ladi.

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial^2 F}{\partial y \partial x}; \qquad \frac{\partial^3 F}{\partial x^2 \partial y} = \frac{\partial^3 F}{\partial x \partial y \partial x} = \frac{\partial^3 F}{\partial y \partial x^2}$$

1-misol.

$z = x^3 + xy^2 - 5xy^3 + y^5$ funksiya berilgan $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ ekanligini ko`rsating.

Yechish: $\frac{\partial z}{\partial x}=3x^2+y^2-5y^3$ $\qquad \frac{\partial z}{\partial y}=2xy-15xy^2+5y^4$

$\frac{\partial^2 z}{\partial x\partial y}=2y-15y^2$ $\qquad \frac{\partial^2 z}{\partial y\partial x}=2y-15y^2$ demak, $\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}$

2-misol:

$z=\ell n(x+\sqrt{x^2+y^2})$ funksiya berilgan $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y\partial x}, \frac{\partial^2 z}{\partial y^2}$ larni toping.

Yechish:

$$\frac{\partial z}{\partial x}=\frac{1}{x+\sqrt{x^2+y^2}}\cdot\left(1+\frac{1}{2\sqrt{x^2+y^2}}\cdot 2x\right)=\frac{1}{x+\sqrt{x^2+y^2}}\cdot\left(\frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}}\right)=\frac{1}{\sqrt{x^2+y^2}}$$

$$\frac{\partial^2 z}{\partial x^2}=\frac{1}{x^2+y^2}\cdot\frac{1}{2\sqrt{x^2+y^2}}\cdot 2x=-\frac{1}{\sqrt{(x^2+y^2)^3}}$$

$$\frac{\partial^2 z}{\partial x\partial y}=-\frac{1}{x^2+y^2}\cdot\frac{1}{2\sqrt{x^2+y^2}}\cdot 2x=-\frac{y}{\sqrt{(x^2+y^2)^3}}$$

$$\frac{\partial^2 z}{\partial y^2}=\frac{1}{x+\sqrt{x^2+y^2}}\cdot\frac{1}{2\sqrt{x^2+y^2}}\cdot 2y=\frac{y}{\left(\left(x+\sqrt{x^2+y^2}\right)\cdot\sqrt{x^2+y^2}\right)}$$

$$\frac{\partial^2 z}{\partial y^2}=\frac{\left(x+\sqrt{x^2+y^2}\right)\cdot\sqrt{x^2+y^2}-y\left(\frac{2y\sqrt{x^2+y^2}}{2\sqrt{x^2+y^2}}\cdot+\frac{\left(x+\sqrt{x^2+y^2}\right)}{2\sqrt{x^2+y^2}}\cdot 2y\right)}{\left(x+\sqrt{x^2+y^2}\right)^2\left(x^2+y^2\right)}=$$

$$=\frac{\left(x+\sqrt{x^2+y^2}\right)\left(x^2+y^2\right)-y^2\sqrt{x^2+y^2}-y^2\left(x+\sqrt{x^2+y^2}\right)}{\sqrt{\left(x^2+y^2\right)^3}\left(x+\sqrt{x^2+y^2}\right)^2}=$$

$$=\frac{x^3+xy^2+x^2\sqrt{x^2+y^2}-y^2\sqrt{x^2+y^2}}{\sqrt{(x^2+y^2)^3}\cdot\left(x+\sqrt{x^2+y^2}\right)^2}=\frac{x^3+(x^2-y^2)\sqrt{x^2+y^2}}{\sqrt{(x^2+y^2)^3}\left(x+\sqrt{x^2+y^2}\right)^2}$$

Darsda yechish uchun misollar.

287. $z=\frac{x^2}{1-2y}$ ikkinchi tartibli xususiy hosilalar topilsin.

J: $\frac{\partial^2 z}{\partial x^2}=\frac{2}{1-2y}$ $\frac{\partial^2 z}{\partial x\partial y}=\frac{4x}{(1-2y)^2}$ $\frac{\partial^2 z}{\partial y^2}=\frac{8x^2}{(1-2y)^3}$

288. $z=\frac{xy}{x-y}$ funksiya berilgan $\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=\frac{2}{x-y}$ ekanligi isbot qilinsin.

289. 1) $z=x^y$ funksiya berilgan $\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}$ ekanligi isbot qilinsin.

2) $f(x,y)=x^y$ $x=\frac{u}{v}$, $y=uv$ $f''_{uu}(x,y)$, $f''_{vv}(x,y)$, $f''_{uv}(x,y)$, df, $d^2 f$ larni toping.

290. $z=\frac{1}{3}\sqrt{(x^2+y^2)^3}$ funksiya berilgan $\frac{\partial^2 z}{\partial x^2},\frac{\partial^2 z}{\partial x\partial y},\frac{\partial^2 z}{\partial y^2}$ larni toping.

J: $\frac{\partial^2 z}{\partial x^2}=\frac{2x^2+y^2}{\sqrt{x^2+y^2}}$ $\frac{\partial^2 z}{\partial y^2}=\frac{x^2+2y^2}{\sqrt{x^2+y^2}}$ $\frac{\partial^2 z}{\partial x\partial y}=\frac{xy}{\sqrt{x^2+y^2}}$

291. $z=\ell n(x^2+y^2)$ $\frac{\partial^3 z}{\partial x\partial y^2}=?$ J: $\frac{\partial^3 z}{\partial x\partial y^3}=\frac{4x(3y^2-x^2)}{(x^2+y^2)^3}$

Mustaqil uy vazifasi uchun misollar

292. $z=\sin^2(ax+by)$ ikkinchi tartibli xususiy hosilalarini toping

293. $z=\frac{x-y}{x+y}$ ikkinchi tartibli xususiy hosilalarni toping.

294. $z=y^{\ell nx}$ ikkinchi tartibli xususiy hosilalarni toping

295. $u=\sqrt{x^2+y^2+z^2-2xz}$ $\frac{\partial^2 u}{\partial y\partial z}=?$

Ko`p o`zgaruvchili funksiyaning ekstremumlari.

Ekstremumning zaruriy sharti $z=F(x,y)$ funksiya faqat $\frac{\partial F}{\partial x}=0$ va $\frac{\partial F}{\partial y}=0$ nuqtalardagina ekstremumga ega bo`lishi mumkin. Bu nuqtalar kritik nuqtalar deyiladi.

yetarli shartlar. $\frac{\partial^2 F}{\partial x^2}=A$ $\frac{\partial^2 F}{\partial x\partial y}=B$ va $\frac{\partial^2 F}{\partial y^2}=C$ orqali belgilaymiz.

$\Delta=AC-B^2>0$ bo`lsa ekstremum mavjud bo`lib;
$A>0$ bo`lsa, minimum nuqta bo`ladi.
$A<0$ bo`lsa, maksimum nuqta bo`ladi.
$\Delta<0$ bo`lsa, ekstremum nuqta bo`lmaydi.
$\Delta=0$ bo`lsa, u holda ekstremum bo`lishi ham, bo`lmasligi ham mumkin.

1-misol.

$z=x^3+xy^2-5xy^3+y^5$ funksiya berilgan $\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}$ ekanligini ko`rsating.

Yechish: $\frac{\partial z}{\partial x}=3x^2+y^2-5y^3$ $\frac{\partial z}{\partial y}=2xy-15xy^2+5y^4$

$\frac{\partial^2 z}{\partial x\partial y}=2y-15y^2$ $\frac{\partial^2 z}{\partial y\partial x}=2y-15y^2$ demak, $\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}$

2-misol:

$z = \ell n(x + \sqrt{x^2 + y^2})$ funksiya berilgan $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y \partial x}, \frac{\partial^2 z}{\partial y^2}$ larni toping.

Yechish:

$$\frac{\partial z}{\partial x} = \frac{1}{x + \sqrt{x^2 + y^2}} \cdot \left(1 + \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x\right) = \frac{1}{x + \sqrt{x^2 + y^2}} \cdot \left(\frac{\sqrt{x^2 + y^2} + x}{\sqrt{x^2 + y^2}}\right) = \frac{1}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{x^2 + y^2} \cdot \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = -\frac{1}{\sqrt{(x^2 + y^2)^3}}$$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{1}{x^2 + y^2} \cdot \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = -\frac{y}{\sqrt{(x^2 + y^2)^3}}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{1}{x + \sqrt{x^2 + y^2}} \cdot \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\left(\left(x + \sqrt{x^2 + y^2}\right)\left(\cdot \sqrt{x^2 + y^2}\right)\right)}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\left(x + \sqrt{x^2 + y^2}\right) \cdot \sqrt{x^2 + y^2} - y\left(\frac{2y\sqrt{x^2 + y^2}}{2\sqrt{x^2 + y^2}} \cdot + \frac{\left(x + \sqrt{x^2 + y^2}\right)}{2\sqrt{x^2 + y^2}} \cdot 2y\right)}{\left(x + \sqrt{x^2 + y^2}\right)^2 \left(x^2 + y^2\right)} =$$

$$= \frac{\left(x + \sqrt{x^2 + y^2}\right)\left(x^2 + y^2\right) - y^2\sqrt{x^2 + y^2} - y^2\left(x + \sqrt{x^2 + y^2}\right)}{\sqrt{\left(x^2 + y^2\right)^3}\ \left(x + \sqrt{x^2 + y^2}\right)^2} =$$

$$= \frac{x^3 + xy^2 + x^2\sqrt{x^2 + y^2} - y^2\sqrt{x^2 + y^2}}{\sqrt{(x^2 + y^2)^3}\ \left(x + \sqrt{x^2 + y^2}\right)^2} = \frac{x^3 + (x^2 - y^2)\sqrt{x^2 + y^2}}{\sqrt{(x^2 + y^2)^3}\ \left(x + \sqrt{x^2 + y^2}\right)^2}$$

3-misol:

$z = x^2 - xy + y^2 + 9x - 6y + 20$ funksiyaning ekstremumini toping.

Yechish: $\frac{\partial z}{\partial x} = 2x - y + 9$ $\quad \frac{\partial z}{\partial x} = -x + 2y - 6$

$$\begin{cases} \frac{\partial z}{\partial x} = 0 \\ \frac{\partial z}{\partial y} = 0 \end{cases} \quad \begin{cases} 2x - y + 9 = 0 \\ -x + 2y - 6 = 0 \end{cases} \Rightarrow \begin{cases} 2x - y = -9 \\ -x + 2y = 6 \end{cases} \Rightarrow \begin{cases} 2x - y = -9 \\ 3y = 3 \end{cases} \quad y = 1, \quad x = -4$$

$$\frac{\partial^2 z}{\partial x^2} = 2 \qquad \frac{\partial^2 z}{\partial y^2} = 2 \qquad \frac{\partial^2 z}{\partial x \partial y} = -1 \qquad A = 2\ \ B = -1 \qquad C = 2$$

$$\Delta = AC - B^2 = 4 - 1 = 3 > 0$$

$\Delta > 0$ demak ekstremum mavjud. $A > 0$ bo\`lgani uchun minimum nuqta.

$z_{\min}(x = -4, y = 1) = 16 + 4 + 1 - 36 - 6 + 20 = -1$. Demak $z_{\min} = -1$

Darsda yechish uchun misollar.

296. $z = 4(x-y) - x^2 - y^2$ funksiya berilgan ekstremumini toping. J: .(2;-2)

297. $z = x^2 + xy + y^2 + x - y - 1$ ekstremumini toping. J: (-1;1)

298. $x = y = \frac{a}{\sqrt[3]{3}}$ nuqta $z = x^2 + xy + y^2 + \frac{a^3}{x} + \frac{a^2}{y}$ funksiyaning minimum nuqtasi ekanligini ko`rsating.

299. $z = y\sqrt{x} - y^2 - x + 6y$ ekstremumini toping. J: $z\min = 9$ $x = 0, y = 3$

Mustaqil uy vazifasi uchun misollar

300. $z = 3x + 6y - x^2 - xy - y^2$ 301. $z = x^2 + y^2 - 2x - 4\sqrt{xy} - 2y + 8$

302. $z = 2x^3 - xy^2 + 5x^2 + y^2$ 303. $z = 3x^2 - 2x\sqrt{y} + y - 8x + 8$

Ikki o`lchovli integral va uning yordamida yuza hisoblash.

$$S = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \sum\sum \Delta x \Delta y = \iint_{(S)} dxdy = \int_a^b dx \int_{y_1(x)}^{y_2(x)} dy$$

Agar, (S) soha $h \le y \le \ell$, $x_1(y) \le x \le x_2(y)$ tengsizliklar bilan aniqlangan bo`lsa u holda

$$S = \iint_{(S)} dxdy = \int_h^{\ell} dy \int_{x_1(y)}^{x_2(y)} dx$$

1-misol: $\int_{-2}^{1} \left(\int_x^{2-x^2} dy \right) dx$ ni hisoblang.

Yechish: $\int_{-2}^{1} \left(\int_x^{2-x^2} dy \right) dx = \int_{-2}^{1} (2 - x^2 - x) dx = \left[2x - \frac{x^3}{3} - \frac{x^2}{2} \right]\Bigg|_{-2}^{1} = \frac{9}{2}$

2-misol: $x = 4y - y^2$ va $x + y = 6$ chiziqlar bilan chegaralangan figuraning yuzi hisoblansin.

Yechish: $x = 4y - y^2$ va $x + y = 6$ chiziqlarni kesishish nuqtasini topamiz.

$\begin{cases} x = 4y - y^2 \\ x + y = 6 \end{cases}$ ni yechib A(4;2) va B(3;3) nuqtalarni topamiz.

Natijada:

$$S = \int_D dxdy = \int_2^3 dy \int_{6-y}^{4y-y^2} dx = \int_2^3 x \Bigg|_{6-y}^{4y-y^2} dy = \int_2^3 (-y^2 + 5y - 6) dy = \left[-\frac{1}{3} y^3 + \frac{5}{2} y^2 - 6y \right]_2^3 = \frac{1}{6}$$

$$\cdot \int_0^{\frac{\pi}{6}} \left(\frac{4}{3} \cos^2 \theta - 1 \right) d\theta = \int_0^{\frac{\pi}{6}} \left(\frac{2}{3} + \frac{2}{3} \cos 2\theta - 1 \right) d\theta$$

Darsda yechish uchun misollar

304. Hisoblang. $\int_{0}^{2\pi} cos^2 xdx \int_{0}^{a} ydy$ $j: \dfrac{\pi a^2}{2}$ 305. Hisoblang. $\int_{1}^{3} dx \int_{x^3}^{x} (x-y)dy$ $j: 112\dfrac{8}{105}$

306. 1) $\iint_{D} y\ln xdxdy$ D soha quyidagi chiziqlar bilan chegaralangan. $xy=1, \quad y=\sqrt{x} \quad x=2$ J: $5(2\ln 2-1)/8$

2) $\iint_{D} (x^2+y^2)dxdy$ $D=\{(x,y)\in R^2 : 1\le xy \le 2, 0\le x \le 2y \le 4x\}$ D soha bo'yicha 2 karrali integralni hisoblang J: $63/16$

Quyida berilgan chiziqlar bilan chegaralangan
figuralarning yuzini hisoblang

307. $x=y^2-2y \quad x+y=0 \quad J: \dfrac{1}{6}$ 308. $y^2=4x-x^2 \quad y^2=2x \quad J: 2\pi-16/3$

309. 1) $y=2-x \quad y^2=4x+4 \quad J: \dfrac{64}{3}$ 2) $\rho=1, \rho=\dfrac{2}{\sqrt{3}}\cos\varphi$ $j: \dfrac{3\sqrt{3}-\pi}{18}$

310. $3x^2=25x \quad 5x^2=9y \quad J: 5$

311. $x^2+y^2=8$, $x=0, \ y=0, \ z=0, x+y+z=4$ berilgan sirtlar bilan chegaralangan jism hajmini toping. J: $8\pi-\dfrac{32\sqrt{2}}{3}$

Mustaqil uy vazifasi uchun misollar

Ikki o`lchovli integralni hisoblang

312. $\iint_{D} xydxdy \quad (0\le x\le 1, \quad 0\le y\le 2)$ j: 1

313. 1) $\iint_{D} e^{x+y}dxdy \quad (0\le x\le 1, \quad 0\le y\le 1)$ $j: (e-1)^2$

2) $\iint_{D} \sin\sqrt{(x^2+y^2)}dxdy$ $D=\{(x,y)\in R^2 : \pi^2\le x^2+y^2\le 4\pi^2\}$ D soha bo'yicha 2 karrali integralni hisoblang J: $-6\pi^2$

Quyida berilgan chiziqlar bilan chegaralangan
figuralarning yuzini hisoblang

314. $y^2+2y-3x+1=0$
$3x-3y-7=0$ $J: 125/18$

315. $y=4x-x^2 \quad y=2x^2-5x \quad J: 27/2$

316. $x=2y^2, \ y=0, \ z=0, x+2y+z=4$ berilgan sirtlar bilan chegaralangan jism hajmini toping. J: $\dfrac{17}{5}$

Uch o`lchovli integral

Agar $T=\{(x,y,z)\in R^3: x_1\le x\le x_2, y_1(x)\le y\le y_2(x), z_1(x,y)\le z\le z_2(x,y)\}$ bo'lsa,T soha bo'yicha olingan 3 karrali integral $\iiint_T f(x,y,z)\,dxdydz=\int_{x_1}^{x_2}dx\int_{y_1(x)}^{y_2(x)}dy\int_{z_1(x,y)}^{z_2(x,y)}f(x,y,z)dz$ formula bilan hisoblanadi.

O'zgaruvchini almashtirish usuli. $x=x(u,v,w), y=y(u,v,w), z=x(u,v,w)$ bo'lib. $f(x,y,z)$ funksiya berilgan bo'lsin. U holda

$\iiint_T f(x,y,z)\,dxdydz=\iiint_\Delta f(x(u,v,w),y(u,v,w),z(u,v,w))\,|J|\,dudvdw$ bo'ladi.

Bunda J – Yakobian $J=\begin{vmatrix} x'_u & x'_v & x'_w\\ y'_u & y'_v & y'_w\\ z'_u & z'_v & z'_w\end{vmatrix}\ne 0$ formula bilan aniqlanadi.

Xususan, silindirik koordinatalar sistemasida $x=\rho\cos\varphi, y=\rho\sin\varphi, z=z\ , J=\rho$,

sferik koordinatalar sistemasida esa $x=\rho\sin\theta\cos\varphi, y=\rho\sin\theta\sin\varphi, z=\rho\cos\theta\ , J=\rho^2\sin\theta$, $(0\le\rho\prec+\infty, 0\le\varphi\le 2\pi, 0\le\theta\le\pi)$ bo'ladi.

1-misol. $\iiint_T (x^2+y^2)\,dxdydz \quad T:\ x^2+y^2+z^2\le r^2$ sharning yuqori yarmi bo'yicha integralni hisoblang.

Yechish. Sferik koodinatalar sistemasiga o'tsak, $0\le\rho\le r, 0\le\varphi\le 2\pi, 0\le\theta\le\pi/2$ bo'lib,

$$\iiint_T (x^2+y^2)\,dxdydz=\iiint_T \rho^4\sin^3\theta\, d\rho\, d\varphi\, d\theta=\int_0^r p^4dp\int_0^{\pi/2}\sin^3\theta d\theta\int_0^{2\pi}d\varphi=$$

$$=2\pi\int_0^r p^4dp\int_0^{\pi/2}(\sin^2\theta-1)d\cos\theta=2\pi\int_0^r p^4dp[\frac{1}{3}\cos^3\theta-\cos\theta]_0^{\frac{\pi}{2}}=\frac{4}{15}\pi r^5$$

Darsda yechish uchun misollar

317. $\iiint_T (x^2+y^2+z^2)\,dxdydz \quad T:\ 0\le x\le a,\ 0\le y\le b,\ 0\le z\le c \quad j: \dfrac{abc(a^2+b^2+c^2)}{3}$

318. $\iiint_T xyzdxdydz \quad T:\ x^2+y^2+z^2=1 \quad x=0, y=0, z=0 \quad j: \dfrac{1}{48}$

319. $\iiint_T xy^2z^3dxdydz \quad T:\ z=xy, y=x, x=1, z=0 \quad j: \dfrac{1}{364}$

320. $\iiint_T zdxdydz \quad T:\ x^2+y^2=z^2,\ z=2\ \ j: 4\pi$

Mustaqil uy vazifasi uchun misollar

321. $\iiint_T (x^2+y+z^2)^3\,dxdydz \quad T:\ x^2+y^2=1, y=0, y=1\ \ j: \dfrac{3\pi}{2}$

322. $\iiint_T dxdydz \quad T:\ x^2+y^2+z^2\le r^2 \quad j: \dfrac{4\pi r^3}{3}$

323. $\iiint_T z\sqrt{x^2+y^2}\,dxdydz \quad T:\ x^2+y^2=2x, y=0, z=0, z=a \quad j: \dfrac{8a^2}{9}$

Egri chiziqli integrallar

Aniq integral tushunchasini integrallash sohasi tekislikda yotuvchi qandaydir egri chiziqni bir qismi bo'lgan hol uchun umumlashtiramiz.

Bunday turdagi integrallar egri chiziqli integrallar deb ataladi.

Egri chiziqli integrallar ikki turda bo'ladi: birinchi va ikkinchi tur egri chiziqli integrallar. Tushunishga qulaylik tug'dirish maqsadida ta'rifni tekislikda yotgan egri chiziq uchun beramiz.

$$\sum_{i=1}^{n} f(M_i^*)\Delta l_i$$

yig'indini tuzamiz, bu yerda Δl_i $M_{i-1}M_i$ yoy uzunligi, M_i^* $M_{i-1}M_i$ yoyning ixtiyoriy nuqtasi.

Ta'rif. Agar integral yig'indi $\Delta l_i \to 0$ da J limitga ega bo'lsa, u holda bu limit AB egri chiziq bo'yicha f(x,y) funktsiyadan olingan I-tur egri chiziqli integral deb ataladi va quyidagicha belgilanadi.

$$J = \int_{AB} f(M)dl$$

$$J = \int_{AB} f(x,y)dl$$

I-tur egri chiziqli integralni aniq integralga keltirish mumkin. AB egri chiziqda parametr sifatida A nuqtadan boshlanadigan yoy uzunligi l ni olsak, egri chiziqli $x = x(l), y = y(l)$ parametrik ko'rinishini hosil qilamiz.

U holda I-tur egri chiziqli integral

$$\int_{AB} f(xy)dl = \int_{0}^{l} f(x(l), y(l))dl$$

AB egri chiziq $x = \varphi(t),\ y = \psi(t)$ parametrik tenglamalar bilan berilgan bo'lsin. U holda I-tur egri chiziqli integral quyidagi formula bo'yicha hisoblanadi.

$$\int_{AB} f(x,y)dl = \int_{\alpha}^{\beta} f[\varphi(t),\psi(t)]\sqrt{(\varphi'(t))^2 + (\psi'(t))^2}\,dt \qquad (1)$$

Agar AB egri chiziq $y = y(x)$, tenglama bilan berilgan bo'lsa $(a \le x \le b)$, u holda I-tur egri chiziqli integral

$$\int_{AB} f(x,y)dl = \int_{a}^{b} f(x, y(x))\sqrt{1 + [y'(x)]^2}\,dx \qquad (2)$$

ko'rinishni oladi.

II - tur egri chiziqli integral

$$\int_{AB} P(x,y)dx + \int_{AB} Q(x,y)dy$$

yig'indi ikkinchi tur egri chiziqli integral deb ataladi va

$$\int_{AB} P(x,y)dx + Q(x,y)dy$$

ko'rinishda belgilanadi.

Faraz qilaylik, AB egri chiziq $x=\varphi(t), \quad y=\varphi(t), \quad \alpha\le t\le\beta$ parametrik tenglamalar bilan berilgan bo'lsin, bu yerda $\varphi(t)$ *ва* $\psi(t)$ uzluksiz $\varphi'(t)$ *ва* $\psi'(t)$ hosilalarga ega bo'lgan funktsiyalar.

U holda quyidagi formulalar o'rinli.

$$\int\limits_{AB} P(x,y)dx=\int\limits_{\alpha}^{\beta} P[\varphi(t),\psi(t)]\varphi(t)$$

$$\int\limits_{AB} Q(x,y)dx=\int\limits_{\alpha}^{\beta} Q[\varphi(t),\psi(t)]\varphi(t)$$

$$\int\limits_{AB} P(x,y)dx+Q(x,y)dy=\int\limits_{\alpha}^{\beta}\{P[\varphi(t),\psi(t)]\varphi(t)+Q[\varphi(t),\psi(t)]\psi(t)\}dt \qquad (3)$$

1-misol. $\int\limits_{c}(x^2+y^2)dl$ egri chiziqli integralni $x=a\cos t, \quad y=a\sin t \quad o\le t\le 2\pi$ parametrik formulalar bilan berilgan S aylana bo'yicha hisoblang.

yechish: Integralni (1) formula bo'yicha hisoblaymiz. $\varphi'(t)=-a\sin t, \quad \psi'(t)=\cos t$ bo'lganligi uchun

$$\int\limits_{c}(x^2+y^2)dl=\int\limits_{o}^{2\pi}(a^2cos^2t+a^2\sin^2 t)\sqrt{a^2\sin^2 t+a^2cos^2t}\,dt=$$

$$\int\limits_{o}^{2\pi}a^2(cos^2t+\sin^2 t)\sqrt{a^2(\sin^2 t+a^2cos^2t)}\,dt=\int\limits_{o}^{2\pi}a^2\cdot a\,dt=2\pi a^3.$$

2-misol. $\int\limits_{c}(x-y)de$ egri chiziqli integralni hisoblang, bu yerda s to'g'ri chiziqning A(0;0) nuqtadan B (4;3) nuqtagacha bo'lgan qismi:

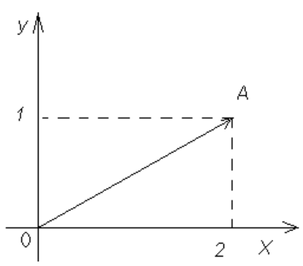
Yechish: Avval A(0;0) va B(4;3) nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini tuzib olamiz.

$A(x_1y_1)$ *va* $B(x_2y_2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini topish $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ formulasiga asosan $\frac{x-0}{4-0}=\frac{y-0}{3-0}$ bu yerdan $y=\frac{3}{4}x$ $\quad y'=\frac{3}{4}$ bo'lgani uchun (2) formulaga asosan

$$\int\limits_{c}(x-y)dl=\int\limits_{0}^{4}(x-\frac{3}{4}x)\sqrt{1+\frac{9}{16}}\,dx=\frac{5}{16}\int\limits_{0}^{4}xdx=\frac{5}{32}x^2\Big|_0^4=\frac{5}{2}.$$

3-misol. $\int\limits_{L}2xydx-x^2dy$ egri chiziqli integral hisoblang, bu yerda L O(0,0) va A(2,1) nuqtalarni birlashtiruvchi to'g'ri chiziq kesmasi.

Yechish.



OA kesmani ko'ramiz. Integrallash yo'li tenglamasini tuzamiz. Buning uchun ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasiga $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ ga ko'ra OA to'g'ri chiziq tenglamasini tuzamiz.

$OA: \frac{x-0}{2-0}=\frac{y-0}{1-0} \Rightarrow y=\frac{1}{2}x, \quad 0\le x\le 2$ formulaga asosan

$$\int_L 2xydx - x^2dy = \int_0^2 2x\frac{1}{2}xdx - x^2(\frac{1}{2}x)'dx = \int (x^2-\frac{1}{2}x^2)dx = \frac{1}{2}\int_0^2 x^2dx = \frac{1}{2}\cdot\frac{x^3}{3}\Big|_0^2 = \frac{4}{3}.$$

Darsda yechish uchun misollar

Quyidagi egri chiziqli integrallar hisoblansin.

324. $\int_K (x-y)ds$, $K: A(0,0), B(4,3)$ nuqtalarni tutashtiruvchi kesma.

325. $\int_{AB} \frac{yds}{\sqrt{x}}$, $AB: y=\frac{4}{9}x^3$ $A(3,2\sqrt{3})$, B$(8,\frac{32\sqrt{2}}{3})$ yarimkubik parabola bo'yicha.

326. $\int_L (x^2+y^2)dx + ydy$ egri chiziqli integralni hisoblang. Bu yerda L $y=e^x$ egri chiziqning A(0;1) nuqtalardan B(1;e) bo'lgan yoyini hisoblang.

327. $\int y^2dx + 2xy\,dy$ integral $x=a\cos t_1$ $y=a\sin t$ aylana bo'yicha.

328. $\int ydx - xdy$ integral $x=\cos t_1$ $y=a\sin t$ ellips yoyi bo'yicha.

329. $\int\left(\frac{x}{x^2+y^2}dx - \frac{y}{x^2+y^2}dy\right)$ integral markazi koordinatalar boshida bo'lgan aylana bo'yicha.

330. $\int\frac{ydx+xdy}{x^2y^2}$ integral $y=x$ to'g'ri chiziqning $x=1$ dan $x=2$ gacha kesmasi bo'yicha.

331. $\int yzdx + xzdy + xydz$ integral t= 0 dan 2π gacha o'zgarganida
$x=a\cos t$ $y=a\sin t$, $z=rt$ chizig'ining yoyi bo'yicha.

332. $\int xdy - y\,dx$ integral $x=a\cos^3 t_1$ $y=a\sin^3 t$ astroidaning yoyi bo'yicha. .

Mustaqil yechish uchun misollar

333. $\int_K (x+2y)ds$, $K: A(0,0), B(1,3)$ nuqtalarni tutashtiruvchi kesma.

334. $\int_{AB} \frac{y^2ds}{\sqrt{x}}$, $AB: y=\frac{4}{9}x^3$ $A(3,2\sqrt{3})$, B$(8,\frac{32\sqrt{2}}{3})$ yarimkubik parabola bo'yicha.

335. $\int xdy - yxd$ integral $x=\frac{3at}{1+t^3}$; $y=\frac{3at^2}{1+t^3}$ Dekart yaprog'ining sirtmog'i bo'yicha.

336. $\int xdy - ydx$ integral $x=a(t-\sin t)$, $y=a(1-\cos t)$ $(0\le t\le 2\pi)$ egri chiziq bo'yicha.

337. $\int_{AB} (x^2-y^2)dx + xydy$ bu yerda AB: A(1;1), B(3;4) nuqtalardan o'tuvchi to'g'ri chiziq.

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