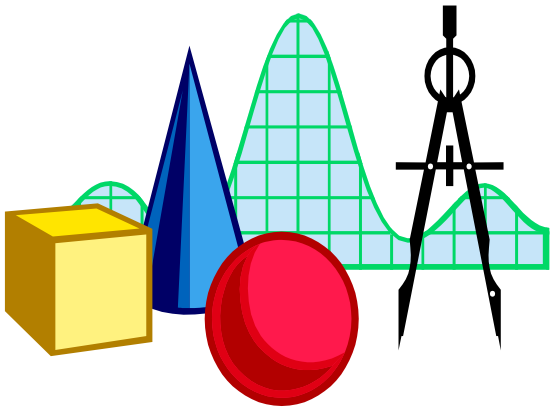
O`ZBEKISTON RESPUBLIKASI OLIY VA O`RTA MAXSUS TA`LIM VAZIRLIGI
MIRZO ULUG'BEK NOMIDAGI SAMARQAND DAVLAT ARXITEKTURA – QURILISH INSTITUTI

«OLIY MATEMATIKA VA FIZIKA»
kafedrasi

"OLIY MATEMATIKA" FANINING "MAYDONLAR NAZARIYASI ELEMENTLARI" BO'LIMIGA DOIR MUSTAQIL ISHLAR TOPSHIRIQLARI VA ULARNI BAJARISH BO'YICHA

USLUBIY KO'RSATMA



SAMARQAND - 2016

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“OLIY MATEMATIKA VA FIZIKA ”
KAFEDRASI

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t.f.n., dotsent A. T. Quldashev
_____
2016 yil “____”_____

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340000 –“ Arxitektura va qurilish” ta’lim sohasining barcha ta’lim yo’nalishlarida o’qiydigan 2-bosqich talabalari uchun mo’ljallangan

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UDK. 517
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Uslubiy ko‘rsatma “Oliy matematika” fanining “Maydonlar nazariyasi elementlari” bo‘limi bo‘yicha mustaqil ishlar topshiriqlarini bajarishga bag‘ishlangan bo‘lib, u 340000 – “Arxitektura va qurilish” ta‘lim sohasining barcha ta’lim yo‘nalishlarida o’qiydigan talabalar uchun mo‘ljallangan.

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«Oliy matematika va fizika» kafedrasi majlisida (“__”_____ 2016 yil. Bayonnoma № ___) ko‘rib chiqilgan va ma’qullangan.

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K I R I S H

Mazkur “Oliy matematika fanining “Maydonlar nazariyasi elementlari” bo‘limiga doir mustaqil ishlar topshiriqlari va ularni bajarish bo‘yicha uslubiy ko‘rsatma” ikkinchi bosqich, uchinchi semestrida fanning «Maydonlar nazariyasi elementlari» bo‘limiga doir nazariy savollar va amaliy mustaqil ishlar topshiriqlarini, ularni bajarish bo‘yicha nazariy ma’lumotlarni o‘z ichiga olgan. Unda topshiriqlarni bajarish bo‘yicha yetarlicha misol-masalalar izohlar bilan yechib ko‘rsatilgan. Uslubiy korsatmada nazariy tushunchalar, nisbatan sodda bayon qilingan bo‘lib, ularning fizik ma’nosini ochib beruvchi misol-masalalar ham keltirilgan.

O‘ylaymizki, uslubiy ko’rsatma talabalarga “Maydonlar nazariyasi elementlari” bo‘limi bo‘yicha mustaqil ishlar topshiriqlarini bajarishda foydali bo‘ladi.

Foydalanilgan shartli belgilar:

Δ – misol yoki masalani yechishning boshlanishi;

▲ – misol yoki masalani yechishning tugashi.

1. Maydon tushunchasi. Skalyar va vektor maydonlar

Maydonlar nazariyasi fizika, matematika va mexanikaning bir qismi bo‘lib, unda skalyar, vektor va tenzor maydonlari o‘rganiladi.

Fizika, elektrotexnika, mexanika va matematikaning bir qancha masalalari skalyar, vektor va tenzor maydonlarini qo‘llashga olib keladi. Masalan, butun olam tortishish kuchlari, magnit va elektr kuchlari manbagacha bo‘lgan masofaning kvadratiga teskari proporsional holda o‘zgaradi; eritmada hosil bo‘lgan diffuziya har xil muhitlarda issiqlik tarqalish qonuniga mutanosib ravishda kechadi; magnit kuch chiziqlari suyuqlikning biror to‘siqdan oqib o‘tishiga mos keladi. Maydon nazariyasining asosiy tushunchalari gradiyent, oqim, potensial, divergensiya, rotor, sirkulyatsiya va shunga o‘xshashlardir.

Matematikada maydon fazodagi biror chekli yoki cheksiz V soha bo‘lib, uning har bir nuqtasida qandaydir miqdor aniqlangan bo‘ladi. Agar sohaning har bir M nuqtasida biror $u = u(M)$ son (skalyar) aniqlangan bo‘lsa, soha shu *skalyarning maydoni* deyiladi va $u = u(M)$ *skalyar funksiya* berilgan bo‘ladi. Agar bu sohaning har bir M nuqtasiga biror $\vec{a} = \vec{a}(M)$ vektor mos kelsa, bu maydon *vektor maydon* deyiladi, bunda $\vec{a} = \vec{a}(M)$ *vektor-funksiya* berilgan bo‘ladi.

Skalyar maydonga temperatura maydoni, havo bosimi maydoni, elektr potensiali maydoni, vektor maydonga esa og‘irlik kuchi maydoni, suyuqlik tezligi maydoni, magnit maydoni, elektr toki maydoni misol bo‘la oladi.

Agar maydon vaqtga bog‘liq bo‘lmasa – *statsionar (barqaror) maydon*, aks holda – *nostatsionar maydon* deyiladi. Biz quyida, asosan, statsionar maydonlarni qaraymiz.

Maydonni ifodalovchi V soha uch o‘lchovli bo‘lsa, maydon uchta x, y, z o‘zgaruvchiga bog‘liq bo‘ladi. Unda u skalyarning maydonini

$$u = u(x, y, z) \text{ yoki } u = u(M) \text{ yoki } u = u(\vec{r}) \qquad (1.1)$$

deb yozish mumkin, chunki maydonning har bir $M(x, y, z)$ nuqtasi uchta koordinata bilan aniqlanadi, bunda $\vec{r}$ M nuqtaning radius-vektori. Shu kabi, $\vec{a}$ vektorning maydonini quyidagicha yozish mumkin:

$\vec{a} = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$ yoki $\vec{a} = \vec{a}(M)$ yoki $\vec{a} = \vec{a}(\vec{r})$ (1.2)

Agar V ikki o‘lchovli (tekis) soha bo‘lsa, maydonning har bir nuqtasi ikkita x, y erkli o‘zgaruvchiga bog‘liq bo‘ladi:

$$u = u(x,y), \qquad \vec{a} = P(x,y)\vec{i} + Q(x,y)\vec{j}.$$

Agar P, Q, R o‘zgarmas sonlar bo‘lsa, $\vec{a}\{P, Q, R\}$ maydon *bir jinsli vektor maydon* yoki *o‘zgarmas vektor maydon* deyiladi. Masalan, og‘irlik kuchi o‘zgarmas maydondir, chunki $P = 0, Q = 0, R = -mg$ bo‘lganidan, uning maydonini ifodalovchi $\vec{a}\{0, 0, -mg\}$ vektor o‘zgarmas bo‘ladi, bu erda m – jism massasi, g – erkin tushish tezlanishi.

1.1 - misol. $u = \sqrt{1 - x^2 - y^2 - z^2}$ qanday maydon tashkil etishini aniqlang.

Δ Ushbu

$$u^2 = 1 - x^2 - y^2 - z^2;\ x^2 + y^2 + z^2 = 1 - u^2,\ (0 \le u \le 1)$$

tenglikdan ko‘rinadiki, berilgan funksiya markazi koordinatalar boshida, radiusi 1 ga teng bo‘lgan sharning skalyar maydonini aniqlaydi. ▲

1.2-misol. Oz o‘qi atrofida soat strelkasiga teskari yo‘nalishda $\vec{\omega}$ burchak tezlik bilan aylanayotgan moddiy nuqtaning $\vec{\mathrm{v}}$ chiziqli tezlik maydonini aniqlang.

Δ. Oz o‘qida yotgan $\vec{\omega}$ burchak tezlik yuqoriga yo‘nalgan bo‘lsin:

$$\vec{\omega} = \vec{\omega}\{0, 0, \omega\};\ \vec{\omega} = \omega\vec{k}.$$

M nuqtaning radius-vektori $\vec{r} = \vec{r}(x,y,z)$ ni o‘tkazamiz (1 – rasm).

Aylanayotgan M nuqtadan Oz o‘qigacha masofani ρ deb belgilasak, fizika kursidan ma’lumki, $\mathrm{v} = \omega\rho$. $\rho = r\sin\varphi$ bo‘lganidan, $\mathrm{v} = \omega r\sin\varphi$ yoki

$$|\vec{\mathrm{v}}| = |\vec{\omega} \times \vec{r}|.$$

$\vec{\mathrm{v}}$ vektorning yo‘nalishi nuqta aylanishi yo‘nalishi bo‘yicha bo‘lib, $\vec{\omega} \times \vec{r}$ vektor ko‘paytmaga mos keladi, chunki $\vec{\mathrm{v}} \perp \vec{r}, \vec{\omega} \times \vec{r}$, $\vec{\omega}, \vec{r}, \vec{\mathrm{v}}$ o‘ng bog‘lamni tashkil etadi. Demak,

$$\vec{\mathrm{v}} = \vec{\omega} \times \vec{r}.$$

Vektor ko‘paytmaning koordinatalar orqali ifodasiga muvofiq:

$$\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 0 & \omega\\ x & y & z\end{vmatrix}=-\omega y\vec{i}+\omega x\vec{j} \quad \text{yoki} \quad \vec{v}=\{-\omega y;\ \omega x;\ 0\}.$$

Ko‘rinadiki, qo‘zg‘almas o‘q atrofida aylanuvchi jismning chiziqli tezligi $\vec{v}$ tekis vektor maydonni tashkil etadi. ▲.

2. Skalyar maydon

2.1. Sath sirtlari va chiziqlari

$u=u(x;\ y;\ z)$ skalyar maydonning *sath sirtlari* deb, u miqdor o‘zgarmas qiymat qabul qiladigan

$$u(x;\ y;\ z)=C,\ \ (C=const)$$

sirtlarga aytiladi. C ga turli $C_1, C_2, \ldots, C_n$ qiymatlar berib, turli sirtlarni hosil qilish mumkin. Masalan:

1. $u=\sqrt{x^2+y^2+z^2}$ skalyar maydonning sath sirtlari

$$\sqrt{x^2+y^2+z^2}=C;\ \ x^2+y^2+z^2=C^2,\ (\mathrm{C}=\mathrm{const})$$

tenglama bilan aniqlanuvchi sirtlar, ya’ni markazi koordinatalar boshida, radiusi C ga teng sferalar (konsentrik sferalar)dir (2-rasm). Ularni koordinata boshiga joylashtirilgan zaryadning elektr maydoni hosil qilgan sath sirtlari deyish mumkin. Fizika kursidan ma’lumki, bunday sirtlar *ekvipotensial sirtlar* deyiladi.

2. $u=x^2+y^2-z$ skalyar maydonning sath sirtlari

$$x^2+y^2-z=C,\ (\mathrm{C}=\mathrm{const});\ \ z=x^2+y^2+C$$

tenglama bilan aniqlanuvchi sirtlar – uchlari Oz o‘qida yotgan elliptik paraboloidlardir (3-rasm).

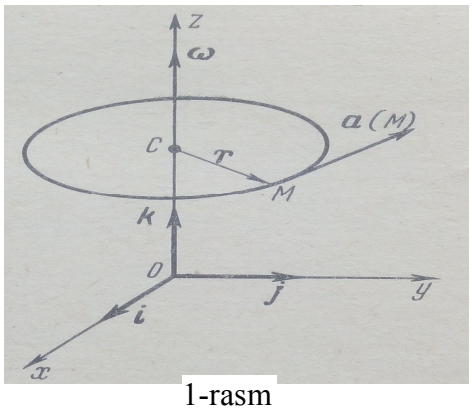
$u=u(x;\ y)$ tekis skalyar maydonning *sath chiziqlari* deb u miqdor o‘zgarmas qiymat qabul qiladigan

$$u(\mathrm{x};\ \mathrm{y})=\mathrm{C},\ (\mathrm{C}=\mathrm{const})$$

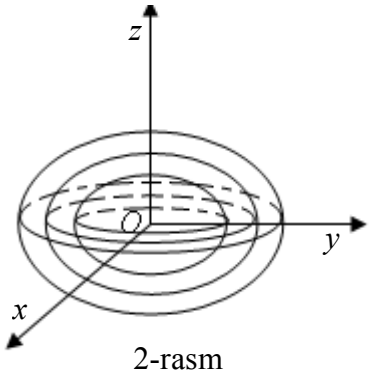
chiziqlarga aytiladi. Masalan, $u=\dfrac{x^2}{9}+\dfrac{y^2}{4}$ skalyar maydon uchun sath chiziqlari

$$\frac{x^2}{9}+\frac{y^2}{4}=C \quad \text{yoki} \quad \frac{x^2}{(3\sqrt{C})^2}+\frac{y^2}{(2\sqrt{C})^2}=1, \ (\mathrm{C}=\mathrm{const}, \ \mathrm{C}>0)$$

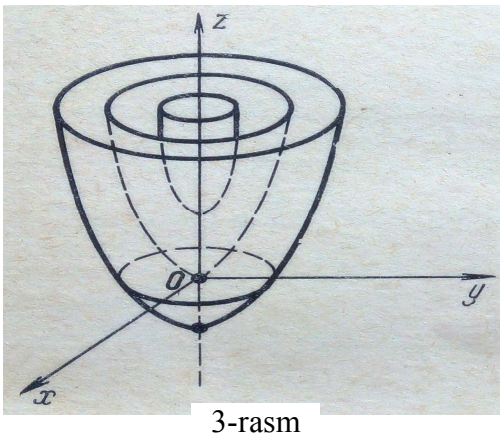
tenglama bilan aniqlanadigan, koordinata o‘qlariga nisbatan simmetrik, yarim o‘qlari $3\sqrt{C}$, $2\sqrt{C}$ bo‘lgan ellipslardan iborat (4-rasm).



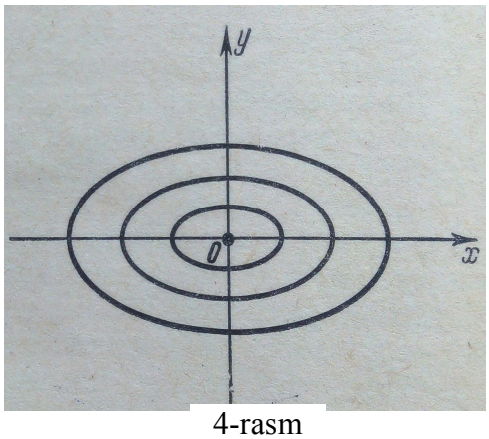
1-rasm



2-rasm



3-rasm



4-rasm

2.2. Yo‘nalish bo‘yicha hosila

Maydonning berilgan yo‘nalish bo‘yicha o‘zgarish tezligini xarakterlash uchun yo‘nalish bo‘yicha hosila tushunchasi kiritiladi.

$u=u(x;y;z)$ - skalyar maydon, $M(x;y;z)$ - shu maydondagi nuqta bo‘lsin. Bu maydonning boshi $M(x;y;z)$ nuqtada bo‘lgan $\vec{l}\{l_x;\, l_y;\, l_z\}$ vektor yo‘nalishi bo‘yicha o‘zgarish tezligini qaraymiz. Bu vektorning yo‘naltiruvchi kosinuslari

$$\cos\alpha=\frac{l_x}{l}, \ \cos\beta=\frac{l_y}{l}, \ \cos\gamma=\frac{l_z}{l}, \ \left(l=\sqrt{l_x^2+l_y^2+l_z^2}\right).$$

bo‘ladi. M nuqta $\vec{l}$ vektor bo‘ylab $M_1(x+\Delta x;\, y+\Delta y;\, z+\Delta z)$ nuqtaga o‘tishdagi orttirmani Δu desak,

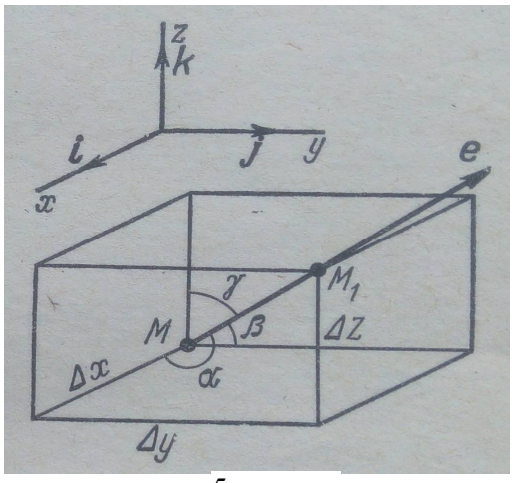
$$\Delta u = u(M_1) - u(M) = u(x + \Delta x;\ y + \Delta y;\ z + \Delta z) - u(x;\ y;\ z)$$

bo‘ladi (5 - rasm). Bu holda

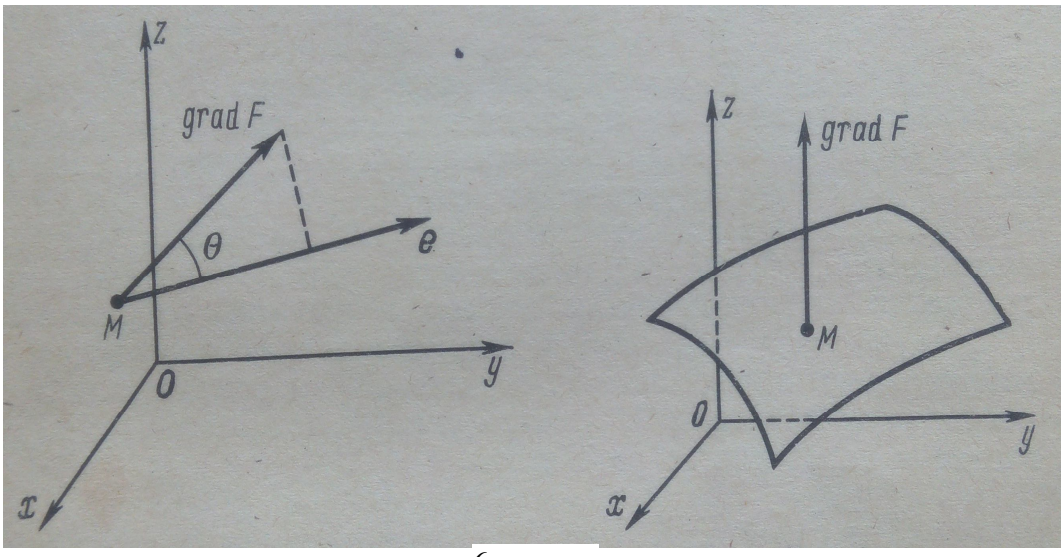
$$\Delta l = \left|\overrightarrow{MM_1}\right| = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} \ .$$

$u = u(M)$ funksiyaning $\vec{l}$ *yo‘nalish bo‘yicha hosilasi* deb quyidagiga aytiladi:

$$\frac{du}{d\vec{l}} = \lim_{\Delta l \to 0} \frac{\Delta u}{\Delta l} = \lim_{M_1 \to M} \frac{u(M_1) - u(M)}{\left|\overrightarrow{MM_1}\right|} \ . \qquad (2.1)$$



5-rasm



6-rasm

Maydon M nuqtasidagi yo‘nalish bo‘yicha hosila *maydon M nuqtasining shu yo‘nalish bo‘yicha tezligini* ifodalaydi. Agar $\frac{du}{d\vec{l}} > 0$ bo‘lsa, $u = u(x;y;z)$ funksiya $\vec{l}$ yo‘nalish bo‘yicha o‘sadi, $\frac{du}{d\vec{l}} < 0$ bo‘lsa – kamayadi. $\left|\frac{du}{d\vec{l}}\right|$ miqdor $u = u(x;y;z)$ funksiyaning *M nuqtadagi $\vec{l}$ yo‘nalish bo‘yicha oniy tezligi* ham deyiladi. Bu miqdor

qancha katta bo‘lsa, $u = u(x; y; z)$ funksiya shuncha tez o‘zgaradi. Bu - *yo‘nalish bo‘yicha hosilaning fizik ma’nosini* aniqlaydi.

Yo‘nalish bo‘yicha hosila uchun formula keltirib chiqaramiz. Buning uchun $u = u(x; y; z)$ funksiyaning M nuqtadagi orttirmasini quyidagicha yozamiz:

$$\Delta u = \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z + \alpha_1\Delta x + \alpha_2\Delta y + \alpha_3\Delta z,$$

bu erda $\alpha_1, \alpha_2, \alpha_3$ lar $\Delta l \to 0$ da cheksiz kichik miqdorlar.

$$\Delta x = \Delta l\cos\alpha,\ \Delta y = \Delta l\cos\beta,\ \Delta z = \Delta l\cos\gamma$$

bo‘lgani uchun

$$\frac{\Delta u}{\Delta l} = \frac{\partial u}{\partial x}\cos\alpha + \frac{\partial u}{\partial y}\cos\beta + \frac{\partial u}{\partial z}\cos\gamma + \alpha_1\cos\alpha + \alpha_2\cos\beta + \alpha_3\cos\gamma\,.$$

Yo‘nalish bo‘yicha hosilaning ta’rifiga asosan:

$$\frac{\Delta u}{\Delta l} = \frac{\partial u}{\partial x}\cos\alpha + \frac{\partial u}{\partial y}\cos\beta + \frac{\partial u}{\partial z}\cos\gamma + \alpha_1\cos\alpha + \alpha_2\cos\beta + \alpha_3\cos\gamma$$

$$\frac{du}{d\vec{l}} = \lim_{\Delta l\to 0}\frac{\Delta u}{\Delta l} = \frac{\partial u}{\partial x}\cos\alpha + \frac{\partial u}{\partial y}\cos\beta + \frac{\partial u}{\partial z}\cos\gamma\,,$$

ya’ni

$$\frac{du}{d\vec{l}} = \frac{\partial u}{\partial x}\cos\alpha + \frac{\partial u}{\partial y}\cos\beta + \frac{\partial u}{\partial z}\cos\gamma\,. \qquad (2.2)$$

(2.2) – *yo‘nalish bo‘yicha hosilani hisoblash formulasi*. Uni vektor shaklda

$$\frac{du}{d\vec{l}} = \left\{\frac{\partial u}{\partial x}; \frac{\partial u}{\partial y}; \frac{\partial u}{\partial z}\right\}\cdot\{\cos\alpha;\ \cos\beta;\ \cos\gamma\}$$

deb yozish mumkin.

Tekis maydon uchun $u = u(x;\ y),\ \cos\beta = \cos(\frac{\pi}{2} - \alpha) = \sin\alpha,\ \cos\gamma = 0$ va (2.2) formula quyidagi ko‘rinishni oladi:

$$\frac{du}{d\vec{l}} = \frac{\partial u}{\partial x}\cos\alpha + \frac{\partial u}{\partial y}\sin\alpha\,. \qquad (2.3)$$

2.1-misol. $M(0; 1; 2)$ va $M_1(2; 3; 3)$ nuqtalar berilgan. $u = x^2 + y^2 - 4yz$ funksiyaning M nuqtadagi $\vec{l} = \overrightarrow{MM_1}$ yo‘nalish bo‘yicha hosilasini toping.

Δ. $\vec{l} = \overrightarrow{MM_1}$ vektor va uning yo‘naltiruvchi kosinuslarini topamiz:

$$\vec{l} = \overrightarrow{MM_1}\{2-0;\ 3-1;\ 3-2\} = \overrightarrow{MM_1}\{2;\ 2;\ 1\};\ l = \left|\vec{l}\right| = \left|\overrightarrow{MM_1}\right| = \sqrt{2^2+2^2+1^2} = \sqrt{9} = 3;$$

$$\cos\alpha = \frac{l_x}{l} = \frac{2}{3};\ \cos\beta = \frac{l_y}{l} = \frac{2}{3};\ \cos\gamma = \frac{l_z}{l} = \frac{1}{3}.$$

Funksiyaning xususiy hosilalarini topamiz:

$$\frac{\partial u}{\partial x} = 2x;\ \frac{\partial u}{\partial y} = 2y - 4z;\ \frac{\partial u}{\partial z} = -4y.$$

Ularning $M(0;\ 1;\ 2)$ nuqtadagi qiymatini hisoblaymiz:

$$\left.\frac{\partial u}{\partial x}\right|_M = 2\cdot 0 = 0;\ \left.\frac{\partial u}{\partial y}\right|_M = 2\cdot 1 - 4\cdot 2 = -6;\ \left.\frac{\partial u}{\partial z}\right|_M = -4\cdot 1 = -4.$$

(2.2) formuladan foydalanib yo‘nalish bo‘yicha hosilani topamiz:

$$\left.\frac{du}{d\vec{l}}\right|_M = \left.\frac{\partial u}{\partial x}\right|_M \cos\alpha + \left.\frac{\partial u}{\partial y}\right|_M \cos\beta + \left.\frac{\partial u}{\partial z}\right|_M \cos\gamma = 0\cdot\frac{2}{3} - 6\cdot\frac{2}{3} - 4\cdot\frac{1}{3} = -5\frac{1}{3}.$$

$\frac{du}{d\vec{l}} < 0$ bo‘lgani uchun $u = x^2 + y^2 - 4yz$ funksiyaning qiymatlari $\vec{l} = \overrightarrow{MM_1}$ yo‘nalish bo‘yicha kamayadi. ▲.

2.2 - misol. $u = x^2y - y^2z + z^2x$ skalyar maydonning $M(1,-1,2)$ nuqtadagi $\vec{l} = 2\vec{i} - 5\vec{j} + 4\vec{k}$ vektor yo’nalishi bo’yicha hosilasini toping.

Δ. u funksiyaning $M(1,-1,2)$ nuqtadagi xususiy hosilalari:

$$\left.\frac{\partial u}{\partial x}\right|_M = \left(2xy + z^2\right)_M = 2\cdot 1\cdot(-1) + 2^2 = 2;$$

$$\left.\frac{\partial u}{\partial y}\right|_M = \left(x^2 - 2yz\right)_M = 1^2 - 2\cdot(-1)\cdot 2 = 5;$$

$$\left.\frac{\partial u}{\partial z}\right|_M = \left(-y^2 + 2xz\right)_M = -(-1)^2 + 2\cdot 1\cdot 2 = 3.$$

$\vec{l}$ vektor bilan yo’nalishi bir xil bo’lgan $\vec{l}^{\,0}$ birlik vektor $\vec{l}^{\,0} = \frac{\vec{l}}{\left|\vec{l}\right|}$ ga teng, bu yerda $\left|\vec{l}\right| = \sqrt{2^2 + (-5)^2 + 4^2} = 3\sqrt{5}$. Demak, $\vec{l}^{\,0} = \frac{2}{3\sqrt{5}}\vec{i} - \frac{5}{3\sqrt{5}}\vec{j} + \frac{4}{3\sqrt{5}}\vec{k}$.

Shunday qilib, $\frac{\partial u}{\partial l} = 2\cdot\frac{2}{3\sqrt{5}} - 5\cdot\frac{5}{3\sqrt{5}} + 3\cdot\frac{4}{3\sqrt{5}} = -\frac{3}{\sqrt{5}}$. ▲.

2.3. Skalyar maydon gradiyenti

Biror sohada aniqlangan, differensiallanuvchi $u = u(x; y; z)$ *skalyar maydonning gradiyenti* deb, $gradu$ kabi belgilanuvchi

$$gradu = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k}. \qquad (2.4)$$

vektor maydonga aytiladi. Ikki vektor skalyar ko‘paytmasidan foydalanib, yo‘nalish bo‘yicha hosilani hisoblash formulasi

$$\frac{du}{d\vec{l}} = \frac{\partial u}{\partial x}\cos\alpha + \frac{\partial u}{\partial y}\cos\beta + \frac{\partial u}{\partial z}\cos\gamma$$

ni

$$\frac{du}{d\vec{l}} = gradu \cdot \vec{l}_0 \quad \text{yoki} \quad \frac{du}{d\vec{l}} = \left|gradu\right| \cdot \cos\varphi \qquad (2.5)$$

deb yozish mumkin, bu erda $\vec{l}_0 -$ $\vec{l}$ vektor yo‘nalishidagi birlik vektor: $\vec{l}_0 = \vec{l} / \left|\vec{l}\right|$. , $\varphi -$ $gradu$ va $\vec{l}$ vektorlar orasidagi burchak (6-rasm).

(2.5) formuladan ko‘rinadiki, yo‘nalish bo‘yicha hosila $\varphi = 0$ da, ya’ni $\vec{l}$ vektor yo‘nalishi gradiyent yo‘nalishi bilan ustma-ust tushganida eng katta qiymatga ega bo‘ladi. Demak, *gradient skalyar maydonning eng tez o‘zgarish yo‘nalishini aniqlaydi.* $u = u(x; y; z)$ funksiyaning eng katta o‘zgarish tezligi

$$\left|gradu\right| = \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2} \qquad (2.6)$$

formula bilan aniqlanadi.

Gradientning asosiy xossalari

1. Gradient berilgan nuqtadan o‘tuvchi sath sirtiga normal yo‘nalgan bo‘ladi (7-rasm).
2. $grad(u + \mathrm{v}) = gradu + grad\,\mathrm{v}$.
3. $grad(\mathrm{C}\cdot u) = C \cdot gradu, \quad C = const.$
4. $grad(u \cdot \mathrm{v}) = u \cdot grad\mathrm{v} + \mathrm{v} \cdot gradu.$
5. $grad\left(\frac{u}{\mathrm{v}}\right) = \frac{\mathrm{v} \cdot gradu - u \cdot grad\mathrm{v}}{\mathrm{v}^2}.$

6. $gradf(u) = \dfrac{\partial f}{\partial u} \cdot gradu$.

2.3-misol. $u = \dfrac{x}{y} + \dfrac{y}{z} + \dfrac{z}{x}$ funksiyaning $A(-1;1;-1)$ nuqtadagi eng tez o‘zgarish tezligini toping.

Δ. Berilgan funksiyaning eng tez o‘zgarish tezligini gradiyentnng moduliga teng bo’lganligi uchun, uni (2.6) formula bilan topamiz:

$$\frac{\partial u}{\partial x} = \left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x}\right)'_x = \frac{1}{y} - \frac{z}{x^2}; \quad \left.\frac{\partial u}{\partial x}\right|_M = \frac{1}{1} + \frac{1}{1^2} = 2;$$

$$\frac{\partial u}{\partial y} = \left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x}\right)'_y = -\frac{x}{y^2} + \frac{1}{z}; \quad \left.\frac{\partial u}{\partial y}\right|_M = 1 - 1 = 0;$$

$$\frac{\partial u}{\partial z} = \left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x}\right)'_z = -\frac{y}{z^2} + \frac{1}{z}; \quad \left.\frac{\partial u}{\partial z}\right|_M = 1 + 1 = 2.$$

$$gradu\big|_M = \left.\frac{\partial u}{\partial x}\right|_M \vec{i} + \left.\frac{\partial u}{\partial y}\right|_M \vec{j} + \left.\frac{\partial u}{\partial z}\right|_M \vec{k} = 2 \cdot \vec{i} + 0 \cdot \vec{j} + 2 \cdot \vec{k} = 2\vec{i} + 2\vec{k};$$

$$\left|gradu\big|_M\right| = \left|2\vec{i} + 2\vec{k}\right| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}. \ \blacktriangle$$

2.4 - misol. $u = (x - y)z^2$ funksiyaning $M(0,-1,4)$ nuqtadagi eng katta o’zgarishi kattaligi va yo’nalishini toping.

Δ. u funksiyaning $M(1,-1,2)$ nuqtadagi xususiy hosilalari:

$$\left.\frac{\partial u}{\partial x}\right|_M = z^2\big|_M = 4^2 = 16;$$

$$\left.\frac{\partial u}{\partial y}\right|_M = -z^2\big|_M = -4^2 = -16;$$

$$\left.\frac{\partial u}{\partial z}\right|_M = 2(x - y)z\big|_M = 2(0 + 1) \cdot 4 = 8.$$

$gradu = 16\vec{i} - 16\vec{j} + 8\vec{k}$, $|gradu| = \sqrt{16^2 + (-16)^2 + 8^2} = \sqrt{576} = 24$. ▲.

2. 5 - misol. $u = x^2 - 6y^2 + 6z^2$ va $v = \dfrac{2}{xyz}$ skalyar maydonlarning $M(1,2,3)$ nuqtadagi gradiyentlari orasidagi burchakni toping:

Δ u va v funksiyalarning $M(1,2,3)$ nuqtadagi xususiy hosilalarini topamiz:

$$\left.\frac{\partial u}{\partial x}\right|_M = 2x|_M = 2\cdot 1 = 2;$$

$$\left.\frac{\partial u}{\partial y}\right|_M = -12y|_M = -12\cdot 2 = -24;$$

$$\left.\frac{\partial u}{\partial z}\right|_M = 12z|_M = 12\cdot 3 = 36.$$

$gradu = 2\vec{i} - 24\vec{j} + 36\vec{k}$, $|gradu| = \sqrt{2^2 + (-24)^2 + 36^2} = \sqrt{1876}$.

$$\left.\frac{\partial v}{\partial x}\right|_M = \left.\frac{-2}{x^2yz}\right|_M = \frac{-2}{1^2\cdot 2\cdot 3} = \frac{-1}{3};$$

$$\left.\frac{\partial v}{\partial y}\right|_M = \left.\frac{-2}{xy^2z}\right|_M = \frac{-2}{1\cdot 2^2\cdot 3} = \frac{-1}{6};$$

$$\left.\frac{\partial v}{\partial z}\right|_M = \left.\frac{-2}{xyz^2}\right|_M = \frac{-2}{1\cdot 2\cdot 3^2} = \frac{-1}{9}.$$

$$gradv = -\frac{1}{3}\vec{i} - \frac{1}{6}\vec{j} - \frac{1}{9}\vec{k},\ |gradv| = \sqrt{\left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{6}\right)^2 + \left(\frac{-1}{9}\right)^2} = \frac{7}{18}.$$

$$\cos\alpha = \frac{-2\cdot\frac{1}{3} + 24\cdot\frac{1}{6} - 36\cdot\frac{1}{9}}{\sqrt{1876}\cdot\frac{7}{18}} = -\frac{12}{7\sqrt{1876}}.\ \alpha = arccoc\left(-\frac{12}{7\sqrt{1876}}\right).\ \blacktriangle.$$

3. Vektor maydon

3.1. Vektor chiziqlari

Agar fazodagi (tekislikdagi) chekli yoki cheksiz sohaning har bir $M(x; y; z)$ nuqtasida ma'lum

$$\vec{a} = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}, \qquad (3.1)$$

qisqacha, $\vec{a} = \vec{a}(M)$ yoki $\vec{a} = \vec{a}\{P; Q; R\}$ vektor berilgan bo'lsa, bu sohada $\vec{a} = \vec{a}(M)$ yoki $\vec{a} = \vec{a}\{P; Q; R\}$ *vektor maydon* berilgan deyiladi. Demak, vektor maydon shunday bir sohaki, uning har bir nuqtasiga aniq bir vektor mos keladi va aksincha, bir vektorga maydondagi bitta nuqta mos keladi.

Vektor maydondagi *vektor chiziqlari* deb shunday chiziqlarga aytiladiki, ularning istalgan nuqtasida o'tkazilgan urinmaning yo'nalishi shu nuqtadagi maydonning yo'nalishi bilan bir xil bo'ladi. Masalan, oqayotgan suyuqlik zarrachalarining tezlik maydoni vektor chiziqlari bo'ladi (u *oqim chiziqlari* deyiladi);

magnit maydonida shimoliy qutbdan janubiy qutbga harakat qiluvchi kuch chiziqlari vektor chiziqlari bo‘ladi.

(3.1) tenglik bilan berilgan vektor maydonning vektor chiziqlari

$$\frac{dx}{P(x,y,z)}=\frac{dy}{Q(x,y,z)}=\frac{dz}{R(x,y,z)} \tag{3.2}$$

differensial tenglamadan topiladi. Haqiqatan, vektor chizig‘i istalgan (x,y,z) nuqtasining radius-vektorini $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ desak, vektor chiziqlari ta’rifiga ko‘ra, uning differensiali $d\vec{r}=dx\vec{i}+dy\vec{j}+dz\vec{k}$ vektor chizig‘iga urinma bo‘ylab yo‘nalgan, ya’ni bu nuqtada $d\vec{r}$ va $\vec{a}$ kolleniar. Ikki vektorning kolleniarlik shartidan (3.2) tenglik kelib chiqadi.

Vektor maydonda vektor chizig‘i bo‘lmagan yopiq chiziq berilgan bo‘lsin. Bu chiziq orqali o‘tuvchi barcha vektor chiziqlari biror silindrsimon sirt tashkil etadi. Bu sirt *vektor naychasi yoki vektor trubkasi* deyiladi (8-rasm).

3.1-misol. O‘zgarmas ω burchak tezlik bilan aylanayotgan jismning chiziqli tezligi maydoni vektor chiziqlari topilsin.

Δ. Ilgari yechilgan 1.2 - misolda topilgan ediki,

$$\vec{\mathrm{v}}=-\omega y\vec{i}+\omega x\vec{j}\ .$$

(3.2) tenglamaga asosan,

$$\frac{dx}{-\omega y}=\frac{dy}{\omega x}=\frac{dz}{0}\quad\text{yoki}\quad\begin{cases}\omega x dx=-\omega y dy,\\ 0\cdot dy=\omega x dz.\end{cases}$$

Sistemaning birinchi tenlamasidan:

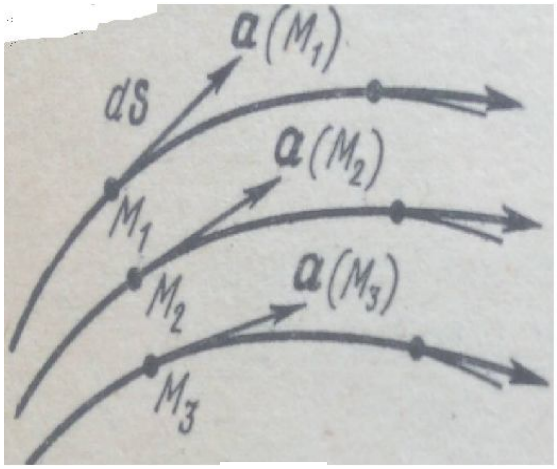
$$\int x dx=-\int y dy;\ \ \frac{x^2}{2}+\frac{y^2}{2}=C^2;\ \ x^2+y^2=C_1^2,\ \ (C_1^2=2\mathrm{C}^2).$$

Ikkinchi tenglamasidan: $dz=0,\ z=C_2.$

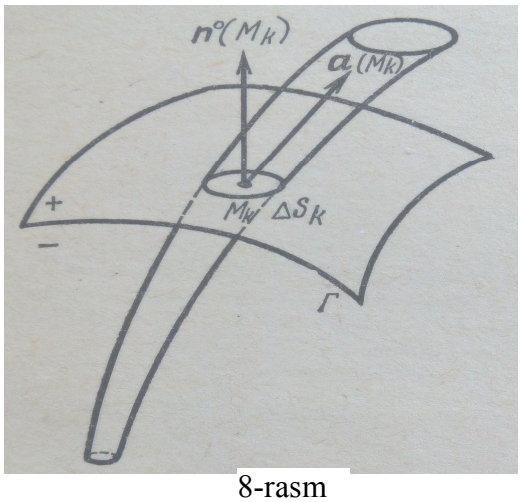
Demak, berilgan vektor maydonning vektor chiziqlari tenglamasi:

$$\begin{cases}x^2+y^2=C_1^2,\\ z=C_2.\end{cases}$$

Bu erda C_1, C_2 - ixtiyoriy o‘zgarmas sonlar, $C_1 \geq 0$; $x^2 + y^2 = C_1^2$ - markazi Oz o‘qida joylashgan aylanalar bo‘lib, ular Oz o‘qiga perpendikulyar bo‘lgan



7-rasm



8-rasm

tekisliklarda yotadi.

3.2. Vektor maydon oqimi

Vektor maydon

$$\vec{a}(M) = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k} \qquad (3.3)$$

tenglik bilan berilgan bo‘lsin. $\vec{a} = \vec{a}(M)$ ni biror S sirt orqali statsionar holatda oqib o‘tayotgan oqayotgan suyuqlikning tezlik vektori deb qaraylik. Vaqt birligi ichida bu sirt orqali oqib o‘tayotgan suyuqlik miqdorini hisoblaymiz. S sirtning biror tomonini olamiz. Shu tomon normalining birlik vektori $\vec{n}\{\cos\alpha; \cos\beta; \cos\beta\}$ bo‘lsin. Sirtni biror usulda m ta $\Delta S_1, \Delta S_2, .., \Delta S_m$ elementar sirtchalarga bo‘lamiz. Har bir sirtchadan bittadan ixtiyoriy $M_i (i = 1,2,..., m)$ nuqta olib, bu nuqtalarda $\vec{a}(M_1), \vec{a}(M_2), ..., \vec{a}(M_m)$ larni hisoblaymiz (9-rasm). ΔS_i sirtchani tekis yuza, $\vec{a} = \vec{a}(M)$ vektorning modulini shu yuza ustida o‘zgarmas va $\vec{a}(M_i)$ ga teng deb faraz qilamiz. U holda vaqt birligida ΔS_i orqali oqib o‘tayotgan suyuqlik miqdori Π_i taqriban:

$$\Pi_i \approx H_i \cdot \Delta S_i$$

bo‘ladi, bu erda H_i – yasovchisi $\vec{a}(M_i)$ bo‘lgan silindrning balandligi. $\vec{n}_i = \vec{n}(M_i)$ normal vektor va H_i bir xil yo‘nalgan bo‘lganligi uchun H_i $\vec{a}(M_i)$ vektorning

normal vektorga proyeksiyasi bo‘ladi: $H_i = np_{\vec{n}_i}\vec{a}(M_i) = \vec{a}(M_i)\cdot\vec{n}_i$, bu erda nuqta skalyar ko‘paytmani bildiradi.

Demak, vaqt birligida ΔS_i orqali oqib o‘tayotgan suyuqlik miqdori taqriban

$$\Pi_i \approx \vec{a}(M_i)\cdot\vec{n}_i\Delta S_i,$$

S sirt orqali oqib o‘tayotgan suyuqlik miqdori Π esa taqriban

$$\Pi_m = \sum_{i=1}^{m}\Pi_i \approx \sum_{i=1}^{m}\vec{a}(M_i)\cdot\vec{n}_i\,\Delta S_i$$

bo‘ladi. Bu tenglikda elementar sirtchalar diametrlarining eng kattasi d_i nolga intilganda limitga o‘tib, S sirt orqali oqib o‘tayotgan suyuqlik miqdorining haqiqiy qiymatini hosil qilamiz:

$$\Pi = \lim_{\max d_i\to 0}\sum_{i=1}^{m}\vec{a}(M_i)\cdot\vec{n}_i\,\Delta S_i\,. \tag{3.4}$$

Ma’lumki, bu tenglik o‘ng tomonidagi ifoda S sirt bo‘yicha olingan sirt integralidir:

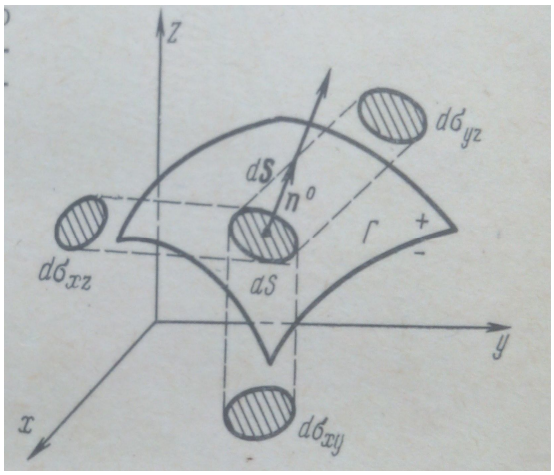
$$\lim_{\max d_i\to 0}\sum_{i=1}^{m}\vec{a}(M_i)\cdot\vec{n}_i\,\Delta S_i = \iint_S(\vec{a}\cdot\vec{n})\,ds,$$

bu yerda nuqta ikki vektorning skalyar kopaytmasini bildiradi. Demak,

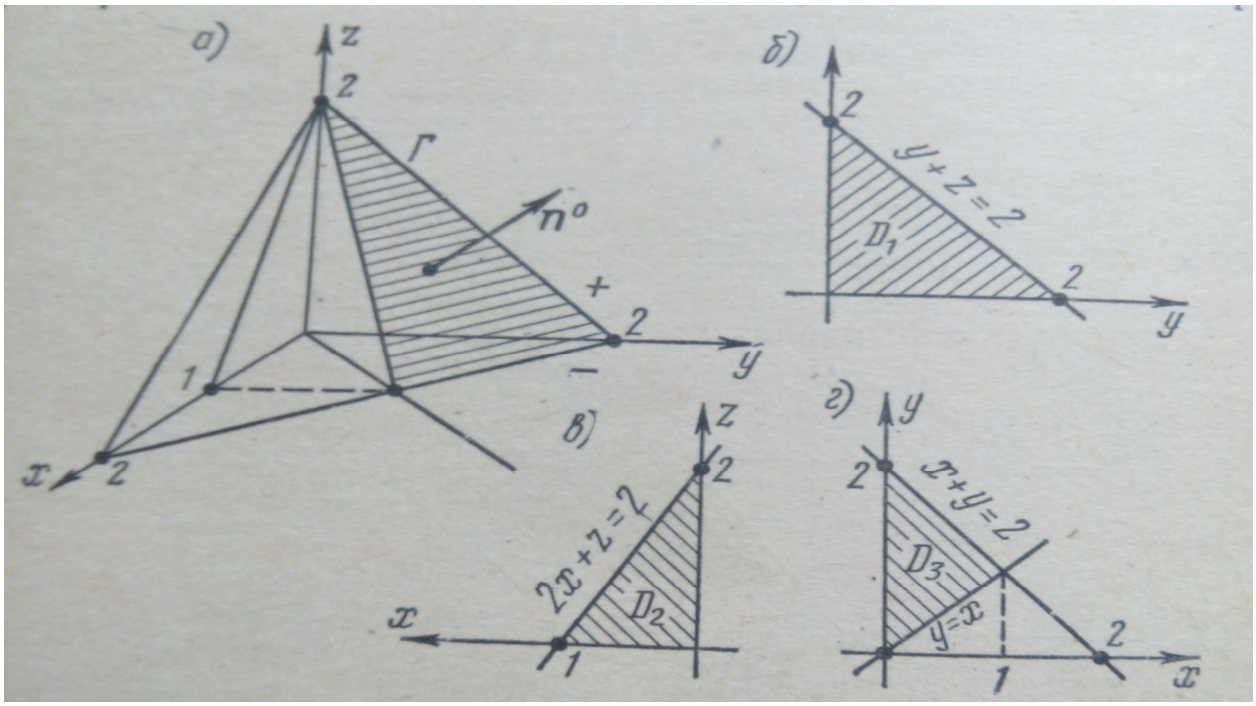
$$\Pi = \iint_S(\vec{a}\cdot\vec{n})\,ds\,. \tag{3.5}$$

$\vec{a}\cdot\vec{n} = \left|\vec{a}\right| np_{\vec{n}}\vec{a} = a_n$ bo‘lgani uchun bu formulani quyidagicha yozish mumkin:

$$\Pi = \iint_S a_n ds\,, \tag{3.6}$$



9-rasm



10-rasm

bu erda a_n $\vec{a}$ vektorning $\vec{n}$ normal vektor yo'nalishiga proyeksiyasi. $\vec{a}=\vec{a}\{P;\,Q;\,R\}$, $\vec{n}=\vec{n}\{\cos\alpha;\,\cos\beta;\,\cos\beta\}$ bo'lgani uchun (3.5) formulani koordinatalar orqali quyidagicha yozish mumkin:

$$\Pi=\iint_S\left(P\cos\alpha+Q\cos\beta+R\cos\gamma\right)ds. \tag{3.7}$$

$\cos\alpha\,ds=dydz$, $\cos\beta ds=dxdz$, $\cos\gamma ds=dxdy$ ekanidan foydalanib,

$$\Pi=\iint_S Pdydz+Qdxdz+Rdxdy \tag{3.8}$$

kabi yozish ham mumkin.

Demak, vektor oqimi Π skalyar miqdor bo'lib, u S sirt orqali vaqt birligida oqib o'tayotgan suyuqlikning hajmiga teng bo'ladi. Bu – *vektor maydon oqimining fizik ma'nosi*.

3.2-misol. $3x+6y-2z-6=0$ tekislikning koordinata o'qlari bilan kesishishidan hosil bo'lgan uchburchakning yuqori qismi bo'yicha $\vec{a}=z\vec{i}-x\vec{j}+y\vec{k}$ vektor maydoni oqimi (vektor oqimi)ni toping.

Δ. Berilgan tekislik tenglamasini quyidagicha yozamiz:

$$\frac{x}{2}+\frac{y}{1}-\frac{z}{3}=1.$$

U koordinata o‘qlarini $A(2;\,0;\,0)$, $B(0;\,1;\,0)$, $C(0;\,0;\,-3)$ nuqtalarda kesib o‘tadi (10 - rasm). $\vec{a}$ vektorning koordinatalari $P=z,\ Q=-x,\ R=y$ bo‘lganidan vektor maydon oqimini (3.8) formula bilan hisoblash mumkin:

$$\Pi=\iint\limits_S zdydz-xdxdz+ydxdy=\iint\limits_S zdydz-\iint\limits_S xdxdz+\iint\limits_S ydxdy=\Pi_1-\Pi_2+\Pi_3. \quad (3.9)$$

Bu yerdagi

$$\Pi_1=\iint\limits_S zdydz,\ \Pi_2=\iint\limits_S xdxdz,\ \Pi_3=\iint\limits_S ydxdy$$

ikki karrali integrallarni hisoblaymiz:

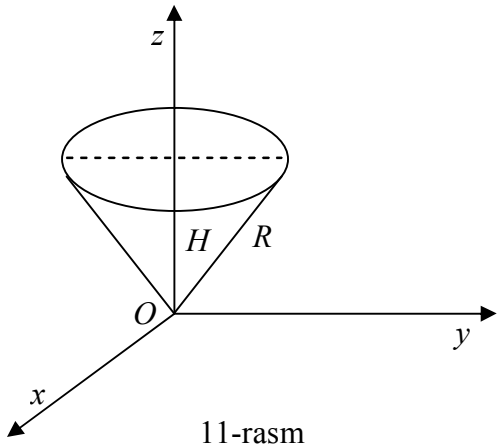
$$\Pi_1=\iint\limits_S zdydz=-\iint\limits_{BOC} zdydz=-\int\limits_0^1 dy\int\limits_{3y-3}^0 zdz=-\int\limits_0^1 dy\left(\frac{z^2}{2}\Big|_{3y-3}^0\right)=\frac{1}{2}\int\limits_0^1(3y-3)^2\,dy=\frac{3}{2};$$

$$\Pi_2=-\iint\limits_S xdxdz=\iint\limits_{AOC} xdxdz=\int\limits_0^2 xdx\int\limits_{\frac{3x-6}{2}}^0 dz=\int\limits_0^2\frac{3x-6}{2}xdx=\frac{3}{2}\int\limits_0^2 x(x-2)dx=-2;$$

$$\Pi_3=\iint\limits_S ydxdy=\iint\limits_{AOB} ydxdy=\int\limits_0^2 dx\int\limits_0^{1-\frac{x}{2}} ydy=\frac{1}{3}.$$

Demak, $\Pi=\frac{3}{2}+2+\frac{1}{3}=3\frac{5}{6}$. ▲.

3.3 - misol. Uchi koordinatalar boshida, asosining radiusi R va balandligi H ga teng bo‘lgan to‘g‘ri konusning tashqi sirti bo‘ylab $\vec{r}$ radius-vektorning oqimini toping (11 - rasm).



11-rasm

Δ. Konusning tashqi sirti bo‘yicha vektor oqimi konusning yon sirti bo‘yicha va asosi bo‘yicha vektor oqimlari yig‘indisiga teng:

$$\Pi=\iint\limits_S r_n ds=\iint\limits_{\text{ён сирти}} r_n ds+\iint\limits_{acoc} r_n ds=\Pi_1+\Pi_2.$$

Konus yon sirti uchun: $np_n\vec{r}=r_n=0;\ \Pi_1=0.$

Konus asosi uchun: $np_{\vec{n}}\vec{r}=r_n=H;\ \Pi_2=\iint\limits_{acoc} Hds=H\iint\limits_{acoc} ds=H\cdot S_{acoc}=\pi R^2H.$

Shuning uchun, $\Pi = \pi R^2 H$. ▲.

3.3. Vektor maydon divergensiyasi. Ostrogradskiy-Gauss formulasi

Vektor maydonga oid muhim tushunchalardan biri divergensiyadir.

$$\vec{a}(M) = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$$

vektor maydonning divergensiyasi deb $div\vec{a}(\mathrm{M})$ kabi belgilanuvchi va

$$div\vec{a}(\mathrm{M}) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \tag{3.6}$$

tenglik bilan aniqlanuvchi skalyar miqdorga aytiladi. Maydonning aniq nuqtasi ko‘rsatilmasa, divergensiya

$$div\vec{a} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \tag{3.7}$$

kabi yoziladi.

Divergensiyaning xossalari

1. Agar $\vec{a}\{P; Q; R\}$ vektor o‘zgarmas: $P = Q = R = const$ bo‘lsa, $div\vec{a} = 0$.
2. O‘zgarmas miqdorni divergensiya ishorasidan tashqariga chiqarib yozish mumkin:

$$div(\mathrm{C}\cdot\vec{a}) = \mathrm{C}\cdot div\vec{a},\ \ C = const.$$

3. $div(\vec{a} + \vec{b}) = div\vec{a} + div\vec{b}.$
4. Agar $u-$ skalyar, $\vec{a}-$vektor funksiyalar differensiallanuvchi bo‘lsa:

$$div(u\vec{a}) = udiv\vec{a} + \vec{a}\cdot gradu,$$

bu erda nuqta ikki vektorning skalyar ko‘paytmasini bildiradi.

Isboti. $\vec{a} = P\vec{i} + Q\vec{j} + R\vec{k}$ desak, $u\vec{a} = uP\vec{i} + uQ\vec{j} + uR\vec{k}.$

$$div(u\vec{a}) = div(uP\vec{i} + uQ\vec{j} + uR\vec{k}) = \frac{\partial(uP)}{\partial x} + \frac{\partial(uQ)}{\partial y} + \frac{\partial(uR)}{\partial z} =$$

$$= u\frac{\partial P}{\partial x} + P\frac{\partial u}{\partial x} + u\frac{\partial Q}{\partial y} + Q\frac{\partial u}{\partial y} + u\frac{\partial R}{\partial z} + R\frac{\partial u}{\partial z} = u\left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right) +$$

$$+ P\frac{\partial u}{\partial x} + Q\frac{\partial u}{\partial y} + R\frac{\partial u}{\partial z} = udiv\vec{a} + (P\vec{i} + Q\vec{j} + R\vec{k})\cdot\left(\frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \vec{k}\frac{\partial u}{\partial z}\right) =$$

$= udiv\vec{a} + \vec{a} \cdot gradu.$

3.4–misol. $\vec{a} = 2x^2y^2\vec{i} - yz\vec{j} + 3z^2\vec{k}$ vektor maydonning $M(1;-2;1)$ nuqtadagi divergensiyasini hisoblang.

$\Delta.\quad \vec{a} = P\vec{i} + Q\vec{j} + R\vec{k} = 2xy^2\vec{i} - yz\vec{j} + 3z^2\vec{k};\ P = 2xy^2;\ Q = -yz;\ R = 3z^2.$

$$\frac{\partial P}{\partial x} = P_x' = \left(2xy^2\right)_x' = 2y^2;\ \frac{\partial Q}{\partial y} = Q_y' = \left(-yz\right)_y' = -z;\ \frac{\partial R}{\partial z} = R_z' = \left(3z^2\right)_z' = 6z;$$

$$\left.\frac{\partial P}{\partial x}\right|_M = 2y^2 = 2\cdot(-2)^2 = 8;\ \left.\frac{\partial Q}{\partial y}\right|_M = -z = -1;\ \left.\frac{\partial R}{\partial z}\right|_M = 6z = 6$$

bo‘lganligi uchun (3.7) va (3.6) formulalarga ko‘ra

$$div\vec{a} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 2y^2 - z + 6z = 2y^2 + 5z.$$

$$div\vec{a}(M) = \left.\frac{\partial P}{\partial x}\right|_M + \left.\frac{\partial Q}{\partial y}\right|_M + \left.\frac{\partial R}{\partial z}\right|_M = 8 - 1 + 6 = 13.\ \blacktriangle.$$

Fazoda S yopiq sirt bilan chegarangan ω sohada $\vec{a}$ vektor maydon berilgan bo‘lsin. Bu maydon divergensiyasidan ω soha bo‘yicha olingan uch o‘lchovli integralni shu soha bilan chegaralangan S sirt bo‘yicha olingan sirt integrali orqali ifodalash mumkin.

Ostrogradskiy-Gauss teoremasi. $\vec{a}$ vektor maydon divergensiyasidan ω soha bo‘yicha olingan uch o‘lchovli integral bu vektor maydonning shu soha bilan chegaralangan yopiq sirt bo‘yicha vektor oqimiga teng:

$$\iiint_{\omega}\left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right)d\omega = \iint_S Pdydz + Qdxdz + Rdxdy \qquad (3.8)$$

yoki qisqacha

$$\iiint_{\omega} div\vec{a}\,d\omega = \iint_S a_n ds\,. \qquad (3.9)$$

(3.8) - skalyar shakldagi, (3.9) – vektor shakldagi *Ostrogradskiy-Gauss formulasi* deyiladi.

Divergensiyaning invariant ta’rifi. Divergensiyaning yuqoridagi ta’rifi Dekart koordinatalar sistemaga bog’liq. Ostrogradskiy – Gauss teoremasidan foydalanib, divergensiyaning invariant, ya’ni koordinata sistemasiga bog’liq bo’lmagan ta’rifini

berish mumkin. M - $\vec{a}$ vektor maydondagi istalgan ichki nuqta, σ_M uni o'z ichiga oluvchi istalgan yopiq sirt, v_M - bu sirt bilan chegaralangan jismning hajmi bo'lsin.

Ta'rif. $\vec{a}$ vektor maydonning M nuqtadagi divergensiyasi (yoki tafovuti, uzoqlashishi) $div\vec{a}(M)$ deb, $\vec{a}$ maydon σ_M sirt orqali o'tuvchi oqimining bu sirt bilan chegaralangan jism hajmi v_M ga nisbatining $v_M \to 0$ dagi limitiga aytiladi:

$$div\vec{a}(M) = \lim_{v_M \to 0} \frac{\iint_{\sigma_M} \vec{a} \cdot d\sigma}{v_M} . \qquad (3.10)$$

(3.10) tenglik (3.9) formulaga o'rta qiymat haqidagi teoremani qo'llash natijasida kelib chiqadi.

3.4. Vektor maydon sirkulyatsiyasi

Vektor maydon $\vec{a} = P(x; y; z)\vec{i} + Q(x; y; z)\vec{j} + R(x; y; z)\vec{k}$ tenglik bilan berilgan bo‘lsin.

Bu maydonda biror L yopiq chiziq olib, unda aniq bir yo‘nalishni tanlaymiz. L chiziqda (konturda) biror M nuqta olamiz, uning radius-vektori $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ bo‘lsin. Uning differensiali $d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$ kontur yo‘nalishi bo‘yicha urinma bo‘ylab yo‘nalgan bo‘ladi va bunda $\left|d\vec{r}\right|$ egri chiziq yoyi differensialining uzunligiga teng:

$$\left|d\vec{r}\right| = dl = \sqrt{(dx)^2 + (dy)^2 + (dz)^2} .$$

$\vec{a}$ vektor maydonning L yopiq chiziq (kontur) bo‘yicha *sirkulyatsiyasi* deb uning $d\vec{r}$ vektorga skalyar ko‘paytmasidan olingan egri chiziqli integralga aytiladi. U

$$C = \oint_L \vec{a} \cdot \overrightarrow{dr} . \qquad (3.11)$$

Skalyar ko‘paytmaning xossasiga asosan,

$$\vec{a} \cdot d\vec{r} = \left|d\vec{r}\right| np_{d\vec{r}}\vec{a} = a_\tau dl = Pdx + Qdy + Rdz,$$

bunda a_τ $\vec{a}$ vektorning τ urinmaga proyeksiyasi. Shuning uchun vektor maydon sirkulyaysiyasini

$$C = \oint_L a_\tau dl \qquad (3.12)$$

yoki

$$C = \oint_L Pdx + Qdy + Rdz \qquad (3.13)$$

ko‘rinishda ham yozish mumkin.

(3.13) formula quyidagi *fizik ma’noga* ega: agar egri chiziq $\vec{a}\{P; Q; R\}$ kuch maydonida joylashgan bo‘lsa, sirkulyatsiya kuch maydonining moddiy nuqtani L chiziq bo‘yicha harakatida bajargan ishi bo‘ladi.

Yopiq vektor chiziqlari sirkulyatsiya noldan farqli bo‘lib, uning ishorasi $\vec{a}$ vektorning yo‘nalishi vektor chiziqlari bilan bir xil bo‘lsa – musbat, teskari bo‘lsa – manfiy bo‘ladi.

3.5–misol. Aylanish o‘qiga perpendikulyar bo‘lgan jism aylanish jismning yopiq egri chiziq bo‘yicha chiziqli tezligi $\vec{v} = -\omega y\vec{i} + \omega x\vec{j}$ bo‘lgan vektor maydonning sirkulyasiyasini toping.

Δ. (3.13) formuladan foydalanib topamiz. $P = -\omega y;\ Q = \omega x;\ R = 0$ bo‘lganidan

$$C = \oint_L (-\omega ydx + \omega xdy) = \omega \oint_L (xdy - ydx) = 2\omega \cdot \frac{1}{2}\oint_L (xdy - ydx) = 2\omega S;\ C = 2\omega S.$$

Bu erda

$$S = \frac{1}{2}\oint_L (xdy - ydx)$$

L chiziq bilan chegaralangan sirtning yuzi. ▲.

3.6–misol. $\vec{a} = (x - 2z)\vec{i} + (x + 3y + z)\vec{j} + (5x + y)\vec{k}$ vektor maydonning uchlari $A(1; 0; 0)$, $B(0; 1; 0)$, $C(0; 0; 1)$ nuqtalarda yotgan uchburchakning konturi $ABCA$ bo‘yicha sirkulyatsiyasini toping (12 - rasm).

Δ. (3.12) formulaga asosan, $P = x - 2z,\ Q = x + 3y + z,\ R = 5x + y;$

$$C = \oint_{ABCA} (x - 2z)dx + (x + 3y + z)dy + (5x + y)dz =$$

$$= \int_{AB} (x - 2z)dx + (x + 3y + z)dy + (5x + y)dz +$$

$$+\int_{BC}(x-2z)dx+(x+3y+z)dy+(5x+y)dz+$$

$$+\int_{CA}(x-2z)dx+(x+3y+z)dy+(5x+y)dz.$$

AB to‘g‘ri chiziqning kesmalar bo‘yicha tenglamasi $x+y=1,\ z=0$. Bundan $y=1-x,\ dy=-dx,\ dz=0$ va AB kesma uchun $1\le x\le 0$ bo‘lganidan

$$\int_{AB}(x-2z)dx+(x+3y+z)dy+(5x+y)dz=\int_{1}^{0}(x-0)dx+(x+3-3x+0)(-dx)+0=$$

$$=3\int_{1}^{0}(x-1)dx=3\left(\frac{x^2}{2}-x\right)\Big|_{1}^{0}=\frac{3}{2}.$$

BC to‘g‘ri chiziqning kesmalar bo‘yicha tenglamasi $y+z=1,\ x=0$. Bunda $z=1-y,\ dz=-dy,\ dx=0,$ BC kesma uchun $1\le y\le 0$. Shuning uchun

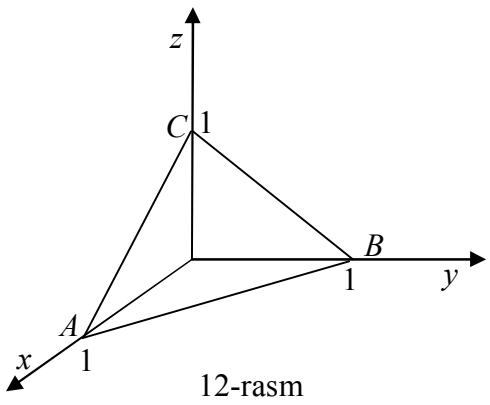
$$\int_{BC}(x-2z)dx+(x+3y+z)dy+(5x+y)dz=$$

$$=\int_{1}^{0}(0-2+2y)\cdot 0+(0+3y+1-y)dy+(0+y)(-dy)=-\frac{3}{2}.$$

CA to‘g‘ri chiziq tenglamasi $x+z=1,\ y=0,$ bundan $z=1-x,\ dz=-dx,\ dy=0,$ CA kesma uchun $0\le x\le 1$ bo‘lganidan

$$\int_{CA}(x-2z)dx+(x+3y+z)dy+(5x+y)dz=$$

$$=\int_{0}^{1}(x-2+2x)dx+0+(5x+0)(-dx)=-3.$$



12-rasm

Demak,

$$C=\oint_{ABCA}(x-2z)dx+(x+3y+z)dy+(5x+y)dz=\frac{3}{2}+\left(-\frac{3}{2}\right)+(-3)=-3.\ \blacktriangle.$$

3.5. Vektor maydon rotori (uyurmasi). Stoks formulasi

$\vec{a}=P(x,y,z)\vec{i}+Q(x,y,z)\vec{j}+R(x,y,z)\vec{k}$ vektor maydonning *rotori* (yoki *uyurmasi* yoki *vixri*) deb $rot\vec{a}$ kabi belgilanuvchi, quyidagi tenglik bilan aniqlanuvchi vektor maydonga aytiladi:

$$rot\vec{a} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k} \tag{3.14}$$

Rotor(uyurma)ni simvolik ravishda determinant yordamida quyidagicha yozish mumkin:

$$rot\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}. \tag{3.15}$$

Rotorning xossalari

1. O‘zgarmas vektorning rotori nolga teng: $rot\vec{a} = 0.$
2. O‘zgarmas ko‘paytuvchini rotor belgisidan tashqariga chiqarish mumkin:

$$rot(\mathrm{C}\cdot\vec{a}) = C \cdot rot\vec{a}, \quad C = const.$$

3. Ikki vektor yig‘indisining ular rotorlarining yig‘indisiga teng:

$$rot(\vec{a} + \vec{b}) = rot\vec{a} + rot\vec{b}.$$

4. Agar $u -$ skalyar funksiya, $\vec{a} -$ vektor funksiya bo‘lsa, quyidagi o‘rinli:

$$rot(u\cdot\vec{a}) = u \cdot rot\vec{a} + gradu \times rot\vec{a},$$

bu erda $\times -$ vektor ko‘paytma belgisi. Bu xossalarni rotorning ta’rifidan, ya’ni (3.14) tenglikdan foydalanib isbotlash qiyin emas.

Yopiq kontur bo‘yicha vektor maydon sirkulyatsiyasini shu kontur bilan chegaralangan ikki o‘lchovli integral orqali ifodalash mumkin. Bu quyidagi Stoks teoremadan foydalanib bajariladi.

Stoks teoremasi. Vektor maydonning yopiq kontur bo‘yicha sirkulyatsiyasi maydon rotori(uyurmasi)ning shu kontur bilan chegaralangan yuza bo‘yicha vektor oqimiga teng.

Teoremaga ko‘ra $\vec{a} = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$ vektor maydonning L yopiq kontur bo‘yicha sirkulyasiyasi uchun quyidagi formula o‘rinli:

$$\oint_L \vec{a}\cdot d\vec{r} = \iint_S (rot\vec{a}\cdot\vec{n})ds, \tag{3.16}$$

yoki

$$\oint_L a_\tau dl = \iint_S rot_{\vec{n}} \vec{a}\, ds \qquad (3.17)$$

bu erda $\vec{n}$ – sirt tashqi normalining birlik vektori, $rot_{\vec{n}} \vec{a}$ – rotorning tashqi normal yo‘nalishiga proyeksiyasi. Formulaning koordinatalar orqali yozilishi:

$$\oint_L Pdx + Qdy + Rdz = \iint_S \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) dydz + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) dxdz + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy. \qquad (3.18)$$

(3.16) - (3.18) – *Stoks formulasi* deyiladi. Aniqrog‘i, (3.16) va (3.17) - Stoks formulasining vektor ko‘rinishi, (3.18) – skalyar ko‘rinishidir.

Vektor maydon *rotorining fizik ma’nosini* qaraymiz. Biror qattiq jism Oz o‘qi atrofida ω burchak tezlik bilan aylanayotgan bo‘lsin. Jism chiziqli tezligining rotorini aniqlaymiz. *1.2 – misol*da bunday jismning chiziqli tezligi $\vec{v} = -\omega y\vec{i} + \omega x\vec{j}$ ekani ko‘rsatilgan edi. Rotorning ta’rifiga ko‘ra:

$$rot\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\omega y & \omega x & 0 \end{vmatrix} = -\frac{\partial(\omega x)}{\partial z}\vec{i} - \frac{\partial(\omega y)}{\partial z}\vec{j} + \frac{\partial(\omega x)}{\partial x}\vec{k} + \frac{\partial(\omega y)}{\partial y}\vec{k} = 2\omega\vec{k} = 2\vec{\omega}.$$

Maydonning bu rotori aylanish o‘qi (Oz o‘qi)ga parallel yo‘nalgan bo‘lib, uning moduli ikkilangan burchak tezlik qiymatiga teng. Demak, chiziqli tezlik maydonining rotori aylanayotgan qattiq jismning burchak tezligini ifodalaydi.

4. Gamilton operatori. Vektor differensial amallar

4.1. Birinchi tartibli vektor differensial amallar. Gamilton operatori.

$u = u(x; y; z)$ skalyar va $\vec{a} = P(x; y; z)\vec{i} + Q(x; y; z)\vec{j} + R(x; y; z)\vec{k}$ vektor maydonlar ustida asosiy differensial amallarga gradient $gradu$, divergensiya $div\vec{a}$ va rotor $rot\vec{a}$ kiradi. Ular *birinchi tartibli vektor differensial amallar (operatsiyalar)* deyiladi.

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

vektorga *Gamilton operatori* yoki *nabla-operator* deyiladi (∇ – “nabla” deb o‘qiladi). Bunday vektorning operator deyilishiga sabab, u faqat skalyar yoki vektor funksiyaga ta’sir ettirilgandan keyin ma’noga ega bo‘ladi.

$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ larning $u,\ P, Q,\ R$ larga ko‘paytmasi ulardan olingan xususiy hosilalarni ifodalaydi. Skalyar yoki vektor miqdorlarni $\vec{\nabla}$ operatorga ko‘paytirish vektorlar algebrasi qoidalari asosida bajariladi.

Gamilton operatori $\vec{\nabla}$ ning $u = u(x; y; z)$ skalyarga ko‘paytmasi, $\vec{a} = P(x; y; z)\vec{i} + Q(x; y; z)\vec{j} + R(x; y; z)\vec{k}$ vektorga skalyar va vektor ko‘paytmalari quyidagicha:

1. $\vec{\nabla}u = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)u = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k} = gradu.$

2. $\vec{\nabla}\cdot\vec{a} = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\cdot\left(P\vec{i} + Q\vec{j} + R\vec{k}\right) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = \mathrm{div}\,\vec{a}.$

3. $\vec{\nabla}\times\vec{a} = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\times\left(P\vec{i} + Q\vec{j} + R\vec{k}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = rot\,\vec{a}.$

4.2. Ikkinchi tartibli vektor differensial amallar

Skalyar yoki vektor maydonga Gamilton operatorini qo‘llasak, yangi maydon hosil bo‘ladi va ularga yana bu operatorni qo‘llash mumkin. Natijada *ikkinchi tartibli vektor differensial amallar* hosil bo‘ladi. Ulardan ba’zilarini qaraymiz.

1. $divgradu = \vec{\nabla}(\vec{\nabla}u) = \vec{\nabla}^2 u = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)u = \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right).$

$\vec{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}-$ *Laplas operatori* deyiladi va Δ bilan belgilanadi: $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. Demak, $divgradu = \vec{\nabla}^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}.$

2. $rotgradu = \vec{\nabla} \times (\vec{\nabla} u) = (\vec{\nabla} \times \vec{\nabla}) u = 0,$ chunki ayni bir vektorning o‘z-o‘ziga vektor ko‘paytmasi nolga teng.

3. $graddiv\vec{a} = \vec{\nabla}(\vec{\nabla} \cdot \vec{a}) = \frac{\partial}{\partial x}\left(div\vec{a}\right)\vec{i} + \frac{\partial}{\partial y}\left(div\vec{a}\right)\vec{j} + \frac{\partial}{\partial z}\left(div\vec{a}\right)\vec{k} =$

$$= \left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 Q}{\partial y \partial x} + \frac{\partial^2 R}{\partial z \partial x}\right)\vec{i} + \left(\frac{\partial^2 P}{\partial y \partial x} + \frac{\partial^2 Q}{\partial y^2} + \frac{\partial^2 R}{\partial y \partial z}\right)\vec{j} + \left(\frac{\partial^2 P}{\partial x \partial z} + \frac{\partial^2 Q}{\partial y \partial z} + \frac{\partial^2 R}{\partial z^2}\right)\vec{k}.$$

4. $divrot\vec{a} = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{a}) = 0,$ chunki uch vektor aralash ko‘paytmasida ikkita bir xil ko‘paytuvchi qatnashsa, aralash ko‘paytma nolga teng bo‘ladi.

5. $rotrot\vec{a} = \vec{\nabla} \times (\vec{\nabla} \times \vec{a}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{a}) - (\vec{\nabla} \cdot \vec{\nabla})\vec{a} = graddiv\vec{a} - \Delta\vec{a}.$

5. Eng sodda vektor maydonlar va ularning xossalari

Eng sodda vektor maydonlar deb

$$div\vec{a} = 0 \tag{5.1}$$

yoki

$$rot\vec{a} = 0 \tag{5.2}$$

yoki

$$div\vec{a} = 0 \qquad \text{va} \qquad rot\vec{a} = 0 \tag{5.3}$$

tengliklar o‘rinli bo‘lgan maydonlarga aytiladi.

Solenoidal (naychasimon) maydon.

Solenoidal maydon deb har bir nuqtasida divergensiyasi nolga teng, ya’ni (5.1) tenglik o‘rinli bo‘lgan maydonga aytiladi. Solenoidal so‘zi lotincha “solen” – naycha (trubka) so‘zidan olingan, shuning uchun *naychasimon* yoki *trubkasimon* maydon ham deyiladi. Bunday deyilishining ma’nosini qaraymiz. Bu maydonda biror S_0 yuza olib uni chegaralab turgan chiziqning har bir nuqtasi orqali vektor chiziqlari o‘tkazamiz. Hosil bo‘lgan naysimon sirt *vektor naychasi* deyiladi. Agar maydon suyuqlik oqimi tezligi desak, suyuqlik bu naycha bo‘ylab, uning devorini kesib o‘tmasdan oqadi. Bu naychaning S_0 va yana biror S_1 kesim bilan chegaralangan

qismidan iborat sohani qaraymiz (13 - rasm). $div\vec{a}=0$ bo‘lgani uchun istalgan yopiq sirt, jumladan, bu sohani chegaralab turgan sirt bo‘yicha vektor oqimi nolga teng:

$$\iint\limits_{S} a_n d\sigma + \iint\limits_{S_0} a_n d\sigma + \iint\limits_{S_1} a_n d\sigma = \iiint\limits_{\tau} div\vec{a}\, d\tau = 0,$$

bu erda $S-$naychaning yon sirti, $\vec{n}-$tashqi normal. Yon sirt uchun tashqi normal maydon vektoriga perpendikulyar bo‘lganida

$$\iint\limits_{S} a_n d\sigma = 0.$$

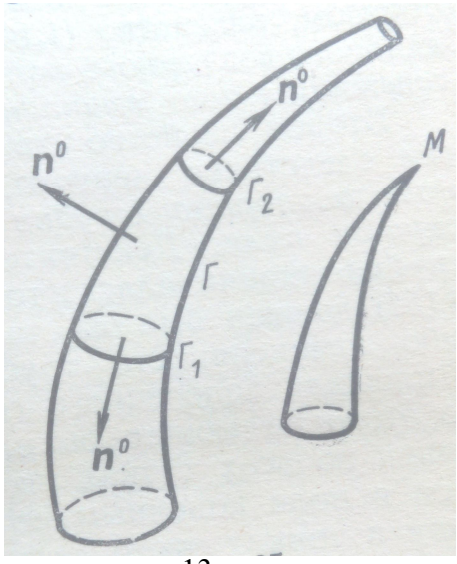
Bundan

$$\iint\limits_{S_1} a_n d\sigma = -\iint\limits_{S_0} a_n d\sigma.$$

S_0 sirtga normal yo‘nalishini tashqidan ichki $\vec{n}'$ ga o‘zgartirsak:

$$\iint\limits_{S_1} a_n d\sigma = \iint\limits_{S_0} a_{n'} d\sigma.$$

Demak, vektor naychasining istalgan kesimi orqali vektor oqimi bir xil, ya’ni *manba yoki qurdumga ega bo‘lmagan maydon naychasining istalgan kesimi orqali bir xil miqdorda suyuqlik oqib o‘tadi.*



13-rasm

$div rot\vec{a}=0$ tenglikdan kelib chiqadiki, istalgan vektor maydon uyurmasining maydoni naychasimon maydondir. Bu tasdiqning teskari ham o‘rinli har qanday naychasimon maydon biror vektor maydonning uyurmasi maydoni bo‘ladi, ya’ni $div\vec{a}=0$ bo‘lsa, $\vec{a}=rot\vec{b}$ ni qanoatlantiruvchi vektor mavjud. $\vec{b}$ vektor berilgan maydonning *vektor-potensiali* deyiladi.

Potensial maydon. Potensiallik shartlari.

Agar

$$\vec{a}(M)=P(x,y,z)\vec{i}+Q(x,y,z)\vec{j}+R(x,y,z)\vec{k}$$

vektor maydon uyurmasi ω sohaning hamma nuqtalarida nolga teng bo'lsa, bu maydon shu sohada potensial (yoki gradiyentli, yoki uyurmasiz) maydon deyiladi.

Potensial maydonning ta'rifiga ko'ra maydonning har bir nuqtasi uchun

$$rot\vec{a} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) = \vec{0} \qquad (5.5)$$

bo'ladi, ya'ni quyidagi ayniyatlar o'rinli bo'ladi:

$$\frac{\partial R}{\partial y} = \frac{\partial Q}{\partial z},\ \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x},\ \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}. \qquad (5.6)$$

Ta'rif. Gradiyenti $\vec{u}(x,y,z)$ skalyar maydonni vujudga keltiruvchi $u(x,y,z)$ skalyar funksiya shu vektor maydonning potensial funksiyasi (yoki potensiali) deyiladi.

Shunday qilib, potensial maydon

$$grad\vec{u} = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k} = \vec{a}$$

munosabat bilan ifodalanadi, bunda

$$\vec{a} = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$$

bo'lib, shu bilan birga $rot\vec{a} = \vec{0}$ yoki $rot\ gradu = \vec{0}$.

Misol. $\vec{a} = 2xy\vec{i} + \left(x^2 - 2yz\right)\vec{j} - y^2\vec{k}$ maydon potensial maydon bo'lishi yoki bo'lmasligini tekshiring.

Δ. $P = 2xy$, $Q = x^2 - 2yz$, $R = -y^2$ bo'lgani uchun bu yerdan xususiy hosilalarni topamiz:

$$\frac{\partial P}{\partial y} = 2x,\ \frac{\partial Q}{\partial x} = 2x,\ \frac{\partial R}{\partial x} = 0,$$

$$\frac{\partial P}{\partial z} = 0,\ \frac{\partial Q}{\partial z} = -2y,\ \frac{\partial R}{\partial y} = -2y.$$

Quyidagilar ravshan,

$$\frac{\partial R}{\partial y} = \frac{\partial Q}{\partial z} = -2y,\ \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x} = 0,\ \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} = 2x,$$

ya'ni (5.6) shart bajariladi, shuning uchun berilgan maydon potensial maydondir. ▲.

Potensial maydon holida chiziqli integralni hisoblash

Agar ω fazoviy soha bir bog'lamli bo'lsa, u holda potensial maydondagi chiziqli integral integrallash yo'liga bog'liq bo'lmasdan, balki shu yo'lning boshlang'ich A hamda oxirgi B nuqtalarining koordinatalariga bog'liq bo'ladi va $u(x,y,z)$ funksiyaning shu nuqtalardagi orttirmasiga teng bo'ladi, ya'ni

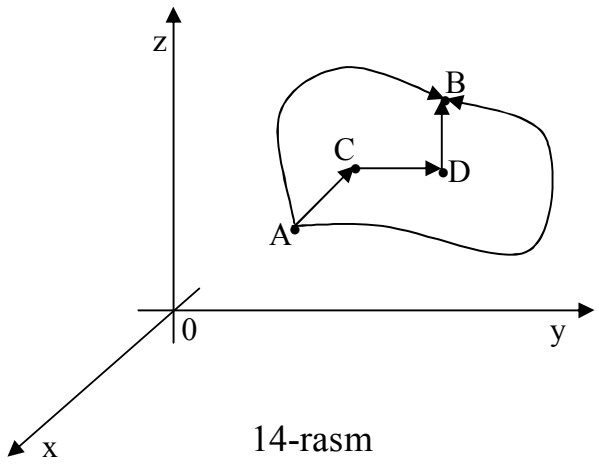
$$\int_{AB} P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = u(x_B,y_B,z_B) - u(x_A,y_A,z_A), \quad (5.7)$$

bu yerda AB yo'l - $A(x_A,y_A,z_A)$ nuqtadan $B(x_B,y_B,z_B)$ nuqtagacha ixtiyoriy integrallash yo'li. Odatda bunday yo'l tarzida $ACDB$ siniq chiziq olinadi, uning AC, CD va DB bo'ginlari koordinatalar o'qiga parallel. Bu holda potensialni hisoblash formulasi quyidagi ko'rinishga ega bo'ladi:

$$\begin{aligned} u(x,y,z) &= \int_a^b P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = \\ &= \int_{x_0}^{x} P(x,y_0,z_0)dx + \int_{y_0}^{y} Q(x,y,z_0)dy + \int_{z_0}^{z} Q(x,y,z)dz \end{aligned} \quad (5.8)$$

bunda $A(x_0,y_o,z_0)$, $B(x,y_o,z_0)$, $D(x,y,z_0)$, $B(x,y,z)$, $\overrightarrow{AC} = (x-x_0)\vec{i}$, $\overrightarrow{CD} = (y-y_0)\vec{j}$, $\overrightarrow{DB} = (z-z_0)\vec{k}$.

Agar potensial maydon kuch maydoni bo'lsa, u holda bunday maydonda nuqtani ko'chirishda bajarilgan ish maydonning bir A nuqtasidan ikkinchi B nuqtasiga ko'chirish yo'liga bog'liq bo'lmaydi va (5.7) formula bo'yicha hisoblanishi mumkin.



14-rasm

Potensial vektor maydonda bir bog'lamli sohada yotgan har qanday L yopiq egri chiziq bo'yicha sirkulyatsiya nolga teng. Kuch maydoni uchun bu maydon kuchlarining har qanday L yopiq egri chiziq bo'yicha bajargan ishi nolga teng bo'ladi.

Misol. Ushbu

$$\vec{a} = 2xy\vec{i} + \left(x^2 - 2yz\right)\vec{j} - y^2\vec{k}$$

maydonning potensialini toping.

Δ. Bu vektorning maydoni potensial ekanini ko'rsatgan edik.

$u(x,y,z)$ potensialni (5.8) formula bo'yicha topamiz:

$$u(x,y,z) = \int_{x_0}^{x} 2xy_0\,dx + \int_{y_0}^{y}\left(x^2 - 2yz_0\right)dy - \int_{z_0}^{z} y^2\,dz =$$

$$= y_0x^2\Big|_{x_0}^{x} + \left(x^2y - z_0y^2\right)\Big|_{y_0}^{y} - y^2z\Big|_{z_0}^{z} =$$

$$= y_0x^2 - y_0x_0^2 + x^2y - z_0y^2 - x^2y_0 + z_0y_0^2 - y^2z + y^2z_0 = x^2y - y^2z + z_0y_0^2 - y_0x_0^2.$$ ▲.

Misol. $\vec{a} = (4x - 2yz)\vec{i} + (2xz - 4y)\vec{j} + 4xyz\vec{k}$ vektor maydon solenoidlimi, potensialmi, garmonikmi ekanini aniqlang:

Δ. Quyidagilarga egamiz: $P = 4x - 2yz$, $Q = 2xz - 4y$, $R = 4xyz$.

$$div\vec{a} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 4 - 4 + 4xy = 4xy \neq 0,$$

$\vec{a}$ – solenoidli maydon emas.

$$rot\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 4x - 2yz & 2xz - 4y & 4xyz \end{vmatrix} =$$

$$= (4xz - 2x)\vec{i} + (-2y - 4)\vec{j} + (2z - 2z)\vec{k} \neq 0,$$

$\vec{a}$ – potensial maydon emas.

$div\vec{a} = 0$ va $rot\vec{a} = 0$ shartlar bir vaqtda bajarilsa, maydon garmonik bo'ladi. Qaralayotgan holatda shartlar bajarilmayapti, shuning uchun maydon garmonik emas. ▲.

NAZORAT UCHUN SAVOLLAR

1. Maydon nima va uning qanday turlari bor?
2. Skalyar maydon deb nimaga aytiladi? Misol keltiring.
3. Sath sirtlari va sath chiziqlari nima?
4. Yo'nalish bo'ycha hosila formulasini keltirib chiqaring.

5. Skalyar maydon gradiyenti nima? Ununing ma’nosini tushuntiring?
6. Gradiyentning xossalarini ayting va isbotlang.
7. Vektor maydon deb nimaga aytiladi?
8. Vektor chizig’i nima va uning differensial tenglamasi qanday yoziladi?
9. Vektor naychalari nima?
10. Suyuqlikning tezligi maydoni uchun vektor chizig’i va vektor naychasining ma’nosi nima?
11. Sirt orqali o’tuvchi vektor oq’imi deb nimaga aytiladi?
12. Suyuqlikning tezligi maydoni uchun vektor oqimining fizik ma’nosi qanday?
13. Divergensiyaning ta’rifini ayting.
14. Ostrogradskiy-Gauss teoremasini ayting. Ostrogradskiy-Gauss formulasining vektor va skalyar ko’rinishini yozing.
15. Divergensiyaning invariant (koordinata sistemasiga bog’liq bo’lmagan) ta’rifini ayting.
16. Chiziqli integral nima? Vektor maydonning sirkulyatsiyasi deb nimaga aytiladi?
17. Vektor maydon rotori (uyurmasi) deb nimaga aytiladi? Uning xossalarini ayting.
18. Vektor maydon rotori (uyurmasi)ning fizik ma’nosi qanday?
19. Stoks teoremasini ayting.
20. Stoks formulasining vektor va skalyar ko’rinishini yozing.
21. Vektor maydonning qanday turlarini bor?
22. Qanday maydon solenoidal (solenoidli) maydon deyiladi? Uning xossalarini ayting.
23. Qanday maydon potensial maydon maydon deyiladi? Maydonning potensiallik sharti qanday?
24. Potensial deb nimaga aytiladi va u qanday hisoblanadi ?
25. Gamilton operatori (nabla-operator) nima?
26. Gamilton operatori (nabla-operator) yordamida amallar bajarish qoidalarini tushuntiring.

AMALIY MUSTAQIL ISHLAR TOPSHIRIQLARI

1 - TOPSHIRIQ

$u = u(x, y, z)$ skalyar maydonning M nuqtadagi $\vec{l}$ vektor yo'nalishi bo'yicha hosilasini toping:

1.1. $u = \left(x^2 + y^2 - z^2\right)^{3/2}$, $\vec{l} = \vec{i} - \vec{j} + \vec{k}$, $M(1,1,1)$.

1.2. $u = x + \ln\left(z^2 + y^2\right)$, $\vec{l} = -2\vec{i} - \vec{j} - \vec{k}$, $M(2,1,1)$.

1.3. $u = x^2 y + \sqrt{xy + z^2}$, $\vec{l} = 2\vec{j} - 2\vec{k}$, $M(1,5,-2)$.

1.4. $u = y\ln\left(1 + x^2\right) + arctgz$, $\vec{l} = 2\vec{i} - 3\vec{j} - 2\vec{k}$, $M(0,1,1)$.

1.5. $u = x(\ln y + arctgz)$, $\vec{l} = 8\vec{i} + 4\vec{j} + 8\vec{k}$, $M(-2,1,-1)$.

1.6. $u = \ln\left(3 - x^2\right) - xy^2 z$, $\vec{l} = -\vec{i} + 2\vec{j} - 2\vec{k}$, $M(1,3,2)$.

1.7. $u = \sin(x + 2y) - \sqrt{xyz}$, $\vec{l} = 4\vec{i} + 3\vec{j}$, $M\left(\frac{\pi}{2}, \frac{3\pi}{2}, 3\right)$.

1.8. $u = x^2 y^2 z + \ln(z - 1)$, $\vec{l} = 5\vec{i} - 6\vec{j} + 2\sqrt{5}\vec{k}$, $M(1,1,2)$.

1.9. $u = x^3 - \sqrt{y^2 + z^2}$, $\vec{l} = \vec{j} - \vec{k}$, $M(1,-3,4)$.

1.10. $u = \frac{\sqrt{x}}{y} + \frac{yz}{x + \sqrt{y}}$, $\vec{l} = 2\vec{i} + \vec{k}$, $M(4,1,-2)$.

1.11. $u = \sqrt{xy} - \sqrt{9 - z^2}$, $\vec{l} = -2\vec{i} + 2\vec{j} - \vec{k}$, $M(1,1,0)$.

1.12. $u = 2\sqrt{x + y} - yarctgz$, $\vec{l} = 4\vec{i} - 3\vec{k}$, $M(3,-2,1)$.

1.13. $u = z^2 - 2arctg(x - y)$, $\vec{l} = \vec{i} + 2\vec{j} - 2\vec{k}$, $M(1,2,-1)$.

1.14. $u = \ln\left(x^2 + y^2\right) - xyz$, $\vec{l} = \vec{i} - \vec{j} + 5\vec{k}$, $M(1,-1,2)$.

1.15. $u = xy - \frac{x}{z}$, $\vec{l} = 5\vec{i} + \vec{j} - \vec{k}$, $M(-4,3,-1)$.

1.16. $u = \ln\left(x - \sqrt{y^2 + z^2}\right)$, $\vec{l} = \vec{i} + 5\vec{j} - \vec{k}$, $M(1,-3,4)$.

1.17. $u = x^2 + arctg(y + z)$, $\vec{l} = 4\vec{j} - 5\vec{k}$, $M(2,1,1)$.

1.18. $u = x^2y - y^2z - z^2x$, $\vec{l} = 2\vec{i} + 5\vec{j} - 3\vec{k}$, $M(1,-1,3)$.

1.19. $u = \ln(xy + yz + 2xz)$, $\vec{l} = 4\vec{i} - 2\vec{j} - 2\vec{k}$, $M(-2,4,-1)$.

1.20. $u = 5x^2yz - xy^2z - yz^2$, $\vec{l} = 6\vec{i} - 4\vec{j} + 8\vec{k}$, $M(1,1,1)$.

1.21. $u = \dfrac{10}{2x^2 + y^2 + z^2 + 1}$, $\vec{l} = 2\vec{i} - \vec{j} + 4\vec{k}$, $M(-2,3,3)$.

1.22. $u = \dfrac{4x}{x^2 + y^2 + z^2}$, $\vec{l} = -6\vec{i} - 2\vec{k}$, $M(2,2,4)$.

1.23. $u = 6x^2yz - xy^2z + 2yz$, $\vec{l} = 2\vec{i} - 2\vec{j} + 2\vec{k}$, $M(1,-4,4)$.

1.24. $u = x^2 + 2xy^2 - 5xyz$, $\vec{l} = 3\vec{i} - \vec{j} + 6\vec{k}$, $M(3,1,-6)$.

1.25. $u = \sqrt{2 + x^2 + y^2 + z^2}$, $\vec{l} = 4\vec{i} + \vec{j}$, $M(2,2,2)$.

1.26. $u = \ln(2x^3 + y^3 + z + 1)$, $\vec{l} = -6\vec{i} + 2\vec{j} + 3\vec{k}$, $M(2,3,0)$.

1.27. $u = \dfrac{x}{y} - \dfrac{y}{z} + \dfrac{z}{x}$, $\vec{l} = 5\vec{i} + 2\vec{j} - \vec{k}$, $M(-2,1,1)$.

1.28. $u = 2x^2 + y^2 + z^2 - 4xyz$, $\vec{l} = 2\vec{i} + 2\vec{j}$, $M(2,-2,4)$.

1.29. $u = \ln(x^2 + 2y^2 + 2z^2)$, $\vec{l} = 2\vec{i} + \vec{j} - \vec{k}$, $M(-2,4,2)$.

1.30. $u = \dfrac{2x}{z} - \dfrac{y}{z} + \dfrac{x}{z}$, $\vec{l} = -6\vec{i} + 2\vec{j} - \vec{k}$, $M(3,3,3)$.

2- TOPSHIRIQ

$u = u(x, y, z)$ funksiyaning M nuqtadagi eng katta o'zgarishi kattaligi va yo'nalishini toping:

2.1. $u = xyz$, $M(1,1,-2)$.

2.2. $u = xy^2z$, $M(1,-2,-2)$.

2.3. $u = x^2y^2z$, $M(-1,1,3)$.

2.4. $u = xy^2z^2$, $M(-3,1,1)$.

2.5. $u = x^2y - y^2z$, $M(0,-3,2)$.

2.6. $u = xy + xz$, $M(-1,2,5)$.

2.7. $u = xyz$, $M(2,2,-2)$.

2.8. $u = x^2yz$, $M(4,0,4)$.

2.9. $u = xyz^2$, $M(3,0,3)$.

2.10. $u = x^2yz^2$, $M(8,1,-1)$.

2.11. $u = y^2z + x^2$, $M(0,1,2)$.

2.12. $u = x(y+z)$, $M(1,1,8)$.

2.13. $u = x^2yz$, $M(1,-2,4)$.

2.14. $u = xyz^2$, $M(4,1,4)$.

2.15. $u = 4x^2yz$, $M(-5,1,0)$.

2.16. $u = (x-y)z^2$, $M(1,-2,4)$.

2.17. $u = 2x^2(y^2 - z)$, $M(2,1,3)$.

2.18. $u = 2x^2(y^2 + 2z)$, $M(4,0,2)$.

2.19. $u = x(2y^2 + 2z^2)$, $M(2,-4,2)$.

2.20. $u = 2x^2z - 3y^2$, $M(2,2,-4)$.

2.21. $u = x^2y - 3z$, $M(-1,1,2)$.

2.22. $u = 2y(x-z)$, $M(0,2,-4)$.

2.23. $u = 4x^2y^2z$, $M(2,0,8)$.

2.24. $u = (x+5z)y^2$, $M(3,3,3)$.

2.25. $u = (x^2 - z)y^2$, $M(-2,1,1)$.

2.26. $u = (x^2 + 6y)z^3$, $M(4,2,0)$.

2.27. $u = 4x^2 + 3y^2 - 2z^2$, $M(1,1,0)$.

2.28. $u = xz^2 - 4y$, $M(2,3,1)$.

2.29. $u = xy^2 - z^2$, $M(-2,1,2)$.

2.30. $u = 2z(x-y)$, $M(-2,2,2)$.

3-TOPSHIRIQ

$u = u(x, y, z)$ va $v = v(x, y, z)$ skalyar maydonlar gradiyentlari orasidagi burchakni toping:

3.1. $u = \frac{x^3}{2} - 6y^3 + 3\sqrt{6}z^3$, $v = \frac{yz^2}{x^2}$, $M\left(\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right)$.

3.2. $u = \frac{4\sqrt{6}}{x} + \frac{6}{9y} + \frac{3}{z}$, $v = x^2 yz^3$, $M\left(2, \frac{1}{3}, \frac{\sqrt{3}}{2}\right)$.

3.3. $u = 9\sqrt{2}x^3 - \frac{y^3}{2\sqrt{2}} + \frac{4z^3}{\sqrt{3}}$, $v = \frac{z^3}{xy^2}$, $M\left(\frac{1}{3}, 2, \sqrt{\frac{3}{2}}\right)$.

3.4. $u = \frac{x^3}{2} - 6y^3 + 3\sqrt{6}z^3$, $v = \frac{yz^2}{x^2}$, $M\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, 1\right)$.

3.5. $u = \frac{3}{x} - \frac{4}{y} - \frac{1}{\sqrt{6}z}$, $v = \frac{z}{x^3 y^2}$, $M\left(1, 2, \frac{1}{\sqrt{6}}\right)$.

3.6. $u = \frac{\sqrt{6}}{2x} - \frac{\sqrt{6}}{2y} - \frac{2}{3z}$, $v = \frac{yz^2}{x}$, $M\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right)$.

3.7. $u = \frac{x^3}{2} - 8y^3 + 3z^3$, $v = \frac{x^2}{yz^2}$, $M\left(\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right)$.

3.8. $u = 3\sqrt{2}x^2 - \frac{y^2}{\sqrt{2}} + 3\sqrt{2}z^2$, $v = \frac{z^2}{xy^2}$, $M\left(\frac{1}{3}, 2, \sqrt{\frac{2}{3}}\right)$.

3.9. $u = 3\sqrt{2}x^2 + \frac{y}{\sqrt{2}} + 3\sqrt{2}z^2$, $v = \frac{xy^2}{z^2}$, $M\left(\frac{1}{3}, 2, \sqrt{\frac{2}{3}}\right)$.

3.10. $u = \frac{3}{x} - \frac{6}{y} - \frac{1}{\sqrt{6}z}$, $v = \frac{x^3 y^2}{z}$, $M\left(1, 2, \frac{1}{\sqrt{6}}\right)$.

3.11. $u = -\frac{4\sqrt{2}}{x} - \frac{\sqrt{2}}{9y} + \frac{1}{\sqrt{3}z}$, $v = \frac{1}{x^2 yz}$, $M\left(2, \frac{1}{3}, \frac{1}{\sqrt{6}}\right)$.

3.12. $u = \frac{8}{x} + \frac{2}{y} - \frac{3\sqrt{3}}{2\sqrt{2}z}$, $v = \frac{x^2}{y^2 z^3}$, $M\left(\sqrt{2}, \sqrt{2}, \frac{\sqrt{3}}{2}\right)$.

3.13. $u = x^2 - 9y^2 + 6z^2$, $v = xyz$, $M\left(1, \frac{1}{3}, \frac{1}{\sqrt{6}}\right)$.

3.14. $u=\frac{4}{x}+\frac{5}{2y}-\frac{\sqrt{6}}{4z},\ v=\frac{y^3}{x^2z},\ M\left(\frac{\sqrt{2}}{\sqrt{3}},\frac{\sqrt{3}}{\sqrt{2}},\frac{1}{2}\right).$

3.15. $u=\frac{x^2}{2}-y^3+\sqrt{6}z^3,\ v=\frac{y^4z^2}{x^2},\ M\left(\sqrt{2},\frac{1}{\sqrt{2}},\frac{1}{\sqrt{3}}\right).$

3.16. $u=\frac{\sqrt{6}}{2x}+\frac{\sqrt{6}}{2y}+\frac{2}{3z},\ v=\frac{2x}{yz^2},\ M\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\frac{1}{\sqrt{3}}\right).$

3.17. $u=\frac{6}{x}+\frac{4}{y}+\frac{3\sqrt{3}}{2\sqrt{2}z},\ v=\frac{y^2z^3}{2x},\ M\left(\frac{1}{\sqrt{2}},\sqrt{2},\frac{\sqrt{3}}{2}\right).$

3.18. $u=\frac{1}{\sqrt{2}x}+\frac{2\sqrt{2}}{y}+\frac{3\sqrt{3}}{z},\ v=\frac{y^2z^3}{3x},\ M\left(\frac{1}{\sqrt{2}},\sqrt{2},\frac{\sqrt{3}}{2}\right).$

3.19. $u=6\sqrt{6}x^3+6\sqrt{6}y^3+z^3,\ v=\frac{3yz^2}{x},\ M\left(\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},2\right).$

3.20. $u=2x^2-2y^2+4z^3,\ v=\frac{yz^2}{x},\ M\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{2}}\right).$

3.21. $u=\frac{x^2}{\sqrt{2}}+\frac{y^2}{\sqrt{2}}+\sqrt{2}z^2,\ v=\frac{3z^2}{x^2y^2},\ M\left(\frac{4}{3},4,\frac{\sqrt{2}}{3}\right).$

3.22. $u=\frac{x^3}{\sqrt{3}}+\frac{y^3}{\sqrt{3}}+\frac{z^3}{\sqrt{2}},\ v=\frac{2x^2}{y^2z^3},\ M\left(\sqrt{3},\sqrt{3},\frac{\sqrt{3}}{2}\right).$

3.23. $u=\frac{3}{2}x^3+6y^3-z^2,\ v=x^2y^2z,\ M\left(2,\frac{1}{5},\frac{2}{3}\right).$

3.24. $u=9\sqrt{2}x^3+\frac{y^3}{2\sqrt{2}}+\frac{z^3}{3},\ v=\frac{x^2y}{z^3},\ M\left(\frac{1}{4},2,1\right).$

3.25. $u=\sqrt{2}x^2+\frac{3y^3}{2}+4\sqrt{2}z^2,\ v=\frac{2}{xy^2z},\ M\left(1,\frac{2}{3},\frac{1}{5}\right).$

3.26. $u=\frac{1}{\sqrt{2}x}+\frac{2\sqrt{2}}{y}+\frac{3\sqrt{3}}{2z},\ v=xyz^2,\ M\left(4,\frac{1}{3},2\right).$

3.27. $u=\frac{2\sqrt{2}}{x}-\frac{\sqrt{2}}{9y}+\frac{2}{z},\ v=\frac{3}{xyz^2},\ M\left(6,\frac{1}{2},\frac{1}{4}\right).$

3.28. $u=6x^2+8y^2+z^2,\ v=\frac{5}{xyz},\ M\left(1,\frac{1}{4},\frac{1}{3}\right).$

3.29. $u=\frac{x^3}{\sqrt{2}}+\frac{y^3}{\sqrt{2}}+\frac{6z^3}{\sqrt{3}}$, $v=\frac{yz^3}{x^2}$, $M\left(\sqrt{2},\sqrt{2},\frac{\sqrt{3}}{2}\right)$.

3.30. $u=\frac{3x^3}{\sqrt{2}}+\frac{2\sqrt{2}y^3}{3}+6\sqrt{3}z^3$, $v=\frac{x^2z^2}{y^3}$, $M\left(\sqrt{\frac{2}{3}},\sqrt{\frac{3}{2}},\frac{1}{3}\right)$.

4 – TOPSHIRIQ

$\vec{a}$ vektor maydon solenoidlimi (1-11- variantlar), potensialmi (12-25-variantlar), garmonikmi (26-30-variantlar) ekanini aniqlang:

4.1. $\vec{a}=(y+z)\vec{i}+xy\vec{j}-xz\vec{k}$

4.2. $\vec{a}=x^2y\vec{i}-2xy^2\vec{j}+2xyz\vec{k}$

4.3. $\vec{a}=(2yz-2x)\vec{i}+(xz-2y)\vec{j}+xy\vec{k}$

4.4. $\vec{a}=(x^2-y^2)\vec{i}-3xy\vec{j}+(y^2+z^2)\vec{k}$

4.5. $\vec{a}=2xyz\vec{i}-y(yz+1)\vec{j}+z\vec{k}$

4.6. $\vec{a}=(2x-3y)\vec{i}+2xy\vec{j}-z^2\vec{k}$

4.7. $\vec{a}=(x^2-y^2)\vec{i}-(y^2-z^2)\vec{j}+(z^2-x^2)\vec{k}$

4.8. $\vec{a}=yz\vec{i}+(x-y)\vec{j}+z^2\vec{k}$

4.9. $\vec{a}=(y+z)\vec{i}+(x+z)\vec{j}+(x+y)\vec{k}$

4.10. $\vec{a}=3x^2y\vec{i}-2xy^2\vec{j}-2xyz\vec{k}$

4.11. $\vec{a}=(x+y)\vec{i}-2(y+z)\vec{j}+(z-x)\vec{k}$

4.12. $\vec{a}=(yz-2x)\vec{i}+(xz+zy)\vec{j}+xy\vec{k}$

4.13. $\vec{a}=yz\vec{i}+xz\vec{j}+xy\vec{k}$

4.14. $\vec{a}=6xy\vec{i}+(3x^2-2y)\vec{j}+z\vec{k}$

4.15. $\vec{a}=(2x-yz)\vec{i}+(2x-xy)\vec{j}+yz\vec{k}$

4.16. $\vec{a}=(y-z)\vec{i}+3xyz\vec{j}+(z-x)\vec{k}$

4.17. $\vec{a} = (y - z)\vec{i} + (x + z)\vec{j} + (x^2 - y^2)\vec{k}$

4.18. $\vec{a} = (x + y)\vec{i} - 2xz\vec{j} - 3(y + z)\vec{k}$

4.19. $\vec{a} = z^2\vec{i} + (xz + y)\vec{j} + x^2y\vec{k}$

4.20. $\vec{a} = xy(3x - 4y)\vec{i} + x^2(x - 4y)\vec{j} + 3z^2\vec{k}$

4.21. $\vec{a} = 6x^2\vec{i} + 3\cos(3x + 2z)\vec{j} + \cos(3y + 2z)\vec{k}$

4.22. $\vec{a} = (x + y)\vec{i} + (z - y)\vec{j} + 2(x + z)\vec{k}$

4.23. $\vec{a} = 3(x - z)\vec{i} + (x^2 - y^2)\vec{j} + 3z\vec{k}$

4.24. $\vec{a} = (2x - yz)\vec{i} + (xz - 2y)\vec{j} + 2xyz\vec{k}$

4.25. $\vec{a} = 3x^2\vec{i} + 4(x - y)\vec{j} + (x - z)\vec{k}$

4.26. $\vec{a} = x^2z\vec{i} + y^2\vec{j} - xz^2\vec{k}$

4.27. $\vec{a} = (x + y)\vec{i} + (y + z)\vec{j} + (x + z)\vec{k}$

4.28. $\vec{a} = \frac{x}{y}\vec{i} + \frac{y}{z}\vec{j} + \frac{z}{x}\vec{k}$

4.29. $\vec{a} = yz\vec{i} + xz\vec{j} + xy\vec{k}$

4.30. $\vec{a} = (y - z)\vec{i} + (z - x)\vec{j} + (x - y)\vec{k}$

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