O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

BUXORO DAVLAT UNIVERSITETI

Fizika –matematika fakulteti

“Matematika” kafedrasi

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BITIRUV MALAKAVIY ISHI

“Ish ko’rildi va himoya qilishga ruxsat berildi”

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Kirish.

“Yangi avlodni tarbiyalash –
xalqni tarbiyalash demakdir”
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Mustaqil fikrlaydigan, zamonaviy ilm-fan va kasb-hunarlarni puxta egallagan, o’z yurti, o’z xalqiga fidoyi, biz boshlagan ishlarni davom ettirishga qodir bo’lgan, har tomonlama sog’lom avlodni yengib bo’ladimi? Bugun biz o’z oldimizga qo’ygan yuksak maqsadlarga yetishda ana shu navqiron avlodlarimiz hal qiluvchi kuch bo’lib maydonga chiqayotgani barchamizga g’urur va iftixor bag’ishlaydi. Bugungi kunda katta umid, orzu – maqsadlar bilan, Vatanim taraqqiyotiga hissa qo’shaman, deb belini mahkam bog’lab maydonga chiqayotgan, azmu shijoatli yoshlarimizni har tomonlama qo’llab-quvvatlash barchamizning nafaqat vazifamiz, balki burchimizga aylanishi kerak.

Yurtimizda ta’lim sohasida o’quv muassasalarini qurish, rekonstruksiya qilish va kapital ta’mirlash borasidagi ishlar izchil davom ettirilmoqda. 2014 yilda hammasi bo’lib 540 dan ziyod obyekt, shu jumladan, 380 ta maktab, 160 dan ortiq kasb-hunar kolleji va akademik litsey tubdan yangilandi. Mazkur maqsadlar uchun qariyb 550 milliard so’m sarflandi. Ushbu mablag’larning 120 milliard so’mdan ortig’i ta’lim muassasalarini o’quv, laboratoriya va ishlab chiqarish uskunalari, kompyuter va multimediya vositalari bilan ta’minlash uchun ajratildi.

2011-2016 yillarda oliy ta’lim muassasalarining moddiy-texnik bazasini modernizatsiya qilish dasturi doirasida 19 ta oliy ta’lim muassasasida qurilish, rekonstruksiya qilish, kapital ta’mirlash va jihozlash bo’yicha qiymati 230 milliard so’mlik ishlar bajarildi. Andijon davlat universiteti, Buxoro muhandislik-texnologiya instituti, O’zbekiston Milliy universitetida yangi o’quv binolari barpo etildi.

2014 yilda bir qator yangi institut va fakultetlar, o’quv-ilmiy markazlar ochildi. Toshkent tibbiyot akademiyasining stomatologiya fakulteti va hududlardagi tibbiyot oliy o’quv yurtlari negizida Andijon, Buxoro, Samarqand, va Nukus shaharlarida filiallariga ega bo'lgan Toshkent davlat stomatologiya instituti tashkil etildi.

O'tgan yili oliy ta'lim muassasalariga qabul qilishning takomillashtirilgan tizimi joriy etildi, ya'ni test sinovlariga informatika bo'yicha savollar kiritildi, mutaxassislikka oid fanlar bo'yicha baholash ballari oshirildi.Bu esa oliy o'quv yurtlariga tanlagan kasbini hisobga olgan holda, eng yaxshi tayyorgarlik ko'rgan abituriyentlarni saralab olish imkonini beradi.

Ta'lim- tarbiya va tibbiyot muassasalarini yanada rivojlantirish, ularning moddiy – texnik bazasini mustahkamlash va bugungi kun talablari asosida jihozlash darajasini oshirish, ijtimoiy infratuzilma obyektlarini jadal rivojlantirish, aholini sifatli ichimlik suvi va zamonaviy sanitariya tozalash tizimlari bilan ta'minlash biz uchun ustuvor yo'nalish hisoblanadi.

Jismoniy tarbiya va sport bilan muntazam shug'ullanish aholimiz, avvalo yosh avlodimizning sog'lig'ini mustahkamlashda muhim o'rin tutishiini inobatga olib, bu sohaga alohida e'tibor qaratmoqdamiz. Bu borada sport inshoatlarini har tomonlama rivojlantirish va ularni zamonaviy inventarlar bilan jihozlash uchun barcha zarur sharoitlar yaratilmoqda.

Mustaqillik yillarida yurtimizda sog'lom va barkamol avlodni voyaga yetkazish yo'lida amalga oshirgan ulkan, tom ma'noda tarixiy ishlarimiz bugungi kunda mamlakatimizni jadal taraqqiy toptirish, boshqalar havas qilayotgan yuksak marralarga erishishda hal qiluvchi omilga aylanmoqda.

"Biz ta'lim – tarbiya sohasidagi keng ko'lamli islohatlarimizni, albatta, oxiriga yetkazishimiz, bunday ta'lim dargohlarini nafaqat qurish, ta'mirlash va jihozlash, balki farzandlarimizni zamon talablari asosida kamol toptirish uchun eng yuksak malakali o'qituvchi va domlalarni jalb etish, ularning bilim va tajribasini muntazam oshirib borishga eng muhim, g'oyat mas'uliyatli vazifa deb qarashimiz zarur" – deb aytganlar yurtboshimiz Islom Karimov.

Prezidentimiz Islom Karimov yoshlarga qarata shunday deydilar: "Avvalombor, bilimga intilish, ikkinchidan, hech tortinmasdan boshqalardan o'rganish kerak.

Hech qachon o'qish-o'rganishdan, izlanishdan uyalmanglar.Odamzod bor ekan, umrining oxirigacha o'rganadi.Men hamma narsani bilaman, deb boshqalarni pisand qilmasdan yuradigan odamning hech qachon biri ikki bo'lmaydi."

Yurtboshimizning "Ona yurtimiz baxtu iqboli va buyuk kelajagi yo'lida xizmat qilish – eng oliy saodatdir" asarlarida quyidagi satrlarni o'qishimiz mumkin:

"Jismoniy va ma'naviy jihatdan sog'lom avlod bizning eng katta boyligimiz.Yaqin qo'shnilarimiz bo'lgan ayrim davlatlarda yoshlarning 50-60 foizi savodsiz.Ertaga ulardan nima kutish mumkin?Ular ertangi kunini nimaning hisobidan ko'radi shu ma'noda, O'zbekistonning eng katta boyligi nima desa, hech ikkilamasdan aytishga xalqimiz – eng katta boyligimiz bizning yoshlarimiz, bolalarimiz.Bugungi avlod biz bunday 18 yil oldin qabul qilgan Kadrlar tayyorlash milliy dasturining mevasi bo'lib, o'shapaytlar biz maqsad qilgan "portlash effekti" ro'y bermoqda.

Jumladan, "Kadrlar tayyorlash milliy dasturi"da ta'kidlanganidek, - Ta'lim sohasini tubdan isloh qilish, uni o'tmishdan qolgan mavkuraviy qarashlar va sarqitlardan to'la xolos etish, rivojlangan demokratik davlatlar darajasida yuksak ma'naviy va axloqiy talablarga javob beruvchi yuqori malakali kadrlar tayyorlash milliy tizimni yaratishdan iborat."

Ta'lim tizimini isloh qilish bilan bir qatordda ta'lim jarayonining mazmunini, uning shakli tuzilmasi va unga qo'yilgan talablar ham yangicha ko'rinishda bo'lmoqdaki, u endi o'zining oldingi, ya'ni Sobiq tizim davridagi qotib qolgan ma'naviy, pedagogic tizimdan tubdan farq qilishi bilan xarakterlanadi. Prezidentimiz Islom Abdug'aniyevich Karimov: "Ona yurtimiz baxtu iqboli va buyuk kelajagi yo'lida xizmat qilish – eng oliy saodatdir" asarlarida quyidagilarga to'xtalib o'tganlar: "Zamonaviy bilim va tafakkurga, sog'lom dunyoqarashga ega, rivojlangan davlatlardagi tengdoshlari bilan bellashishga tayor bo'lgan, ko'zi yonib turadigan navqiron avlodimiz katta ishonch bilan hayotga dadil kirib kelayotgani barchamizni quvontiradi. Bu dunyoda har qaysi ota-ona, avvalambor, bolalarini o'stirish, voyaga yetkazish, uyli-joyli qilishdek ezgu niyatlar bilan yashaydi.Ayniqsa, o'zbek xalqining bolajonligi, o'z bolasini jonidan ham yaxshi ko'rishi, bu boradagi fidoyiligi har qanday tahsinga sazovordir.Bugungi kunda yurtimizda ertangi kunimizni o'z qo'limiz bilan quramiz, degan yoshlarni ko'rganimda, menda xuddi qanot paydo bo'lgandek bo'ladi.Biz bunyod etayotgan bu go'zal hayotni ertaga kimning qo'liga topshiramiz? Albatta, hech kimdan kam bo'lmaydigan yoshlarimiz, farzandlarimizning qo'liga! O'zbekistonda yangi avlod paydo bo'layotganini dunyo tan olyapti. Bunday avlod bilan ertaga, albatta, maqsadlarimizga yetamiz."

Prezidentimiz Islom Abdug'aniyevich Karimov: "Tarixiy xotirasiz kelajak yo'q" asarida komil insonni tarbiyalash borasida quyidagi fikrlarni keltiradi: "Komil inson tarbiyasini davlat siyosatining ustuvor sohasi deb e'lon qilganmiz. Komil inson deganda biz, avvalo, ongi yuksak, mustaqil fikrlay oladigan, xulq –atvori bilan o'zgalarga ibrat bo'ladigan bilimli, ma'rifatli kishilarni tushunamiz.Ongli, bilimli odamni oldi – qochdi gaplar bilan aldab bo'lmaydi.U har bir narsani aql, mantiq tarozisiga solib ko'radi.O'z fikr – o'yi, xulosasini mantiq asosida qurgan kishi yetuk odam bo'ladi".

"Xalqaro anjumanlarda manman degan olimlar bilan xohlang, iqtisodiyot, xohlang siyosat , xohlang tarix, ma'naviyat sohalari bo'lsin, bemalol bahslasha oladigan bilimdon, zukko ma'rifatli odamlar kerak".

"Jamiyat taraqqiyotning asosi, uni muqarrar halokatdan qutqarib qoladigan yagona kuch – ma'rifatdir".

"Aql-zakovatli, yuksak ma'naviyatli kishilarni tarbiyalay olsakkina, oldimizga qo'ygan maqsadlarga erisha olamiz, yurtimizda farovonlik va taraqqiyot qaror topadi"

"Ma'naviyatni tiklash, tug'ilib o'sgan yurtida o'zini boshqalardan kamsitmay boshini baland ko'tarib yurish uchun insonga albatta tarixiy xotira kerak".

"Kimbo'lishdanqat'iynazar, jamiyatningharbira'zosio'zo'tmishiniyaxshibilsa, bundayodamlarniyo'ldan urish, har xil aqidalar ta'siriga olish mumkin emas."

Oliy majlis XII sessiasida so'zlagan “Xalqimiz jipsligi tinchlik va taraqqiyot garovi” nutqida yurtboshimiz yangi jamiyat barpo etishda ta'lim tizimining o'rni haqida to'xtalib quyidagi fikrlarni aytadilar:

“Eng avvalo bizning ta'lim tizimiga bo'lgan munosabatimizni ham tubdan o'zgartirish kerak.Ta'lim islohoti bilan demokratik o'zgarishlar, yangi jamiyat barpo etish yo'lidan dadil yetaklovchi, barchamizni harakatlantiruvchi ichki kuch bo'lmog'i zarur.Har birimizga besh barmoqday, ta'lim-tarbiya tizimini o'zgartirmasdan turib, odamlar ongini, demakki, ularning turmush tarzini ham tubdan o'zgartirish mumkin emas”.

Ta'lim tizimini isloh etish vazifalari muvaffaqiyatli hal etilsa, ijtimoiy siyosiy iqlim keskin o'zgaradi, odamlar ongida demokratik qadriyatlar qaror topadi, inson jamiyatidagi o'rnini ongli ravishda o'zi belgilaydi.

Jamiyatni yangilash, erkin demokratik davlatni shakllantirish, taraqqiyot va ravnaq yo'lidagi barcha sa'y-harakatlarimizni biz aytgan mana shu tamoyil asosida tashkil etishimiz kerak.

Bugungi kunda hayotimizda tom ma'noda tarixiy ahamiyatga ega bo'lgan, dunyo jamoatchiligi haqli ravishda e'tirof etayotgan ta'lim – tarbiya sohasidagi keng ko'lamli islohatlarimiz qanday katta natijalar berayotgani haqida har qancha faxrlanib, g'ururlanib gapirsak arziydi, albatta.

Haqiqatdan ham, bundan 18 yil oldin boshlangan kadrlar tayyorlash milliy dasturi va 2004 yilda uning tarkibiy qismi sifatida qabul qilingan “Maktab ta'limini rivojlantirish umummilliy davlat daasturi ijtimoiy hayotimizda tub burilish yasadi.

Bu dasturlar doirasida yurtimizda ming-minglab litsey va kollejlar, maktablar qurish va ularni rekonstruktsiya qilish, eng yuksak talablar asosida jihozlashdek keng ko'lamli vazifani qo'yganimizda, bu ishlarning qanday ulkan, aytish mumkinki, beqiyos maqsadlarni ko'zda tutishini kamdan – kam odam o'ziga tasavvur qilardi.

Kelajagimizning mustahkam poydevoriga aylanib borayotgan ta'lim sohasidagi buyuk dasturlarimizni o'z vaqtida, uzoqni ko'zlab ishlab chiqqanimiz va amalga oshirganimiz naqadar to’g’ri bo’lganini bugun mamlakatimiz erishgan yuksak marra va natijalar yaqqol namoyon etmoqda.

Agar biz o’z vaqtida uzoqni ko’zlab, katta umid bilan hayotda kirib kelayotgan yoshlarimizning chuqur bilim va kassb-hunar egallashi uchun zamin yaratmasak, ularni zamon talab qiladigan mutaxassis kadrlar etib tayyorlamasak, bugungi kunda butun dunyoni qamrab olgan moliyaviy – iqtisodiy inqiroz davrida yurtimizda tinchlikni saqlab, iqtisodiyotimizning barqaror o’sish sur’atlarini ta’minlashga, ayni shunday og’ir sharoitda xalqimiz hayotining tobora yuksalishiga erisha olarmidik?

Yo’q, albatta.

Hammamimizga chuqur bir haqiqat ayon bo’lishi kerak – biz yurtimizning ertangi rivoji yo’lida qanday chuqur o’ylangan dasturlarni tuzmaylik, bu rejalarni bajarish uchun qanday moddiy baza va imkoniyatlarni yaratmaylik, buning uchun qancha ko’p sarmoya safarbar etmaylik, ularning barchasini amalga oshiradigan, ro’yobga chiqaradigan qudratli biro mil borki, u ham bo’lsa, yuqori malakali ish kuchi va yurtimizning ertangi kuni, taraqqiyoti uchun mas’uliyatni o’z zimmasiga olishga qodir bo’lgan yetuk mutaxassis yoshlarimiz, desak, o’ylaymanki, hech qanday xato bo’lmaydi.

Bugun hech kimga sir emaski, biz yashayotgan XXI asr – intelektual boylik hukmronlik qiladigan asr.

Kimki bu haqiqatni o’z vaqtida anglab olmasa, intellektual bilim, intellektual boylikka intilish har qaysi millat va davlat uchun kundalik hayot mazmuniga aylanmasa – bunday davlat jahon taraqqiyoti yo’lidan chetda qolib ketishi muqarrar. Buni chuqur anglab olgan davlat, bunday xulosani chiqargan , xalqaro hamjamiyat va taraqqiy topgan mamlakatlar qatoriga ko’tarilish uchun harakat qilayotgan jamiyat, birinchi navbatda, bugun unib-o’sib kelayotgan farzandlarimizning har tomonlama barkamol avlod bo’lib hayotga kirib borishini o’zi uchun eng ulug’, kerak bo’lsa, eng muqaddas maqsad, deb biladi.

Yuqorida aytilgan, keltirilgan fikrlardan quyidagi xulosalarga kelishimiz mimkin. Demakki, kelajakda ozod va obod vatan, erkin va farovon hayot barpo etish, yurtimizni jahon hamjamiyatidagi mavqeini yanada oshirish va mustahkamlash o’sib kelayotgan yosh avlodni qay darajada ilmiy intellektual salohiyatlarini oshirishga xizmat qiladi.

<u>Bitiruv malakaviy ishining dolzarbligi.</u>

Davlat test markazi tomonidan e’lon qilingan mavzuga oid testlarning muhim jihatlarini ochish, masalalar shartlarini tahlil qilish. Masalalarning qulay, tushunarli, eng kam vaqt sarflanadigan yechimlarini topish, tahlil qilish. Test markazi tomonidan e’lon qilinayotgan testlarning davlat ta’lim standartlariga qay darajada to’g’ri kelishini, o’quv darslar rejalarida bu mavzular uchun ajratilgan dars soatlarining yetarli yoki kam ekanligini ko’rsatish. Testlarga tanqidiy yondashish. Bitiruv malakaviy ishida tanlangan mavzu misolida ushbu qirralarni ochib berish muhim va dolzarbdir.

<u>Tadqiqot ob’yekti va predmeti.</u>

Mavzuga oid geometriya darsliklari, o’quv qo’llanmalar, test materiallari, jismlar kombinatsiyalariga oid formulalar, tasdiqlar va tushunchalar o’rganildi, tadqiq qilindi.

<u>Bitiruv malakaviy ishining maqsadi va vazifalari.</u>

Bitiruv malakaviy ishining maqsadi va vazifalari abituriyentlarga bir qarashda qiyindek tuyulgan jismlarning kombinatsiyasiga oid testlarni yechishning bir nechta usullarini ko’rsatishdan, qulay usulni tanlash malakasini shakllantirishdan, formulalarni ustalik bilan qo’llab jismlarning kombinatsiyasiga oid masalalarni yechishdagi malakasini oshirishdan iborat.

<u>Bitiruv malakaviy ishining yangiligi.</u>

Mavzuga oid masala va testlarni to’plash ularni tahlil, tadbiq etish va ularning muhim qirralarini ochishdan, testlarning o’ziga xos qulay usullarini ko’rsatishdan, BMI ni akademik litsey va kasb – hunar kolleji bitiruvchilari va abituriyentlar misolida tatbiq etishdan iboratdir.

Bitiruv malakaviy ishining qo'llanilishi.

Akademik litsey va kasb – hunar kolleji talabalariga darslar o'tishda, abituriyentlarni oliy o'quv yurtlariga kirish imtihonlariga tayyorlashda testlar bilan ishlashni o'rganishda qo'llaniladi.

Bitiruv malakaviy ishining strukturasi va hajmi.

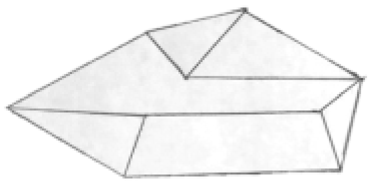
Bitiruv malakaviy ish kirish qismi, ikkita bob, sakkizta paragraf, xulosa, xotima va foydalanilgan adabiyot ro'yxatidan iborat bo'lib, jami 83 betdan iborat.

I Bob. Fazoviy jismlar. Asosiy tushuncha va ta'riflar.

1.1.Ko'pyoqlar haqida asosiy tushunchalar va tasdiqlar. Prizma. Parallelepiped. Piramida.

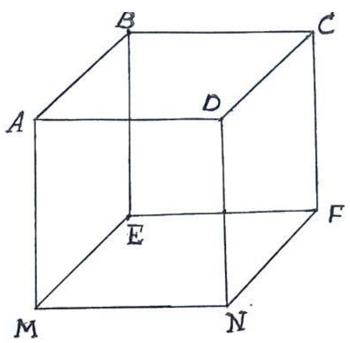
Ma'lumki, geometriya kursida tekislikdagi figuralar va fazoviy jismlar o'rganiladi. Biz sirtda fazoviy jismlar – ko'pyoqlarga tegishli ma'lumotlarni keltiramiz.

1.1.1-ta'rif. Sirti chekli miqdordagi yassi tekisliklardan iborat jism ko'pyoq deyiladi.



1.1.1-chizma

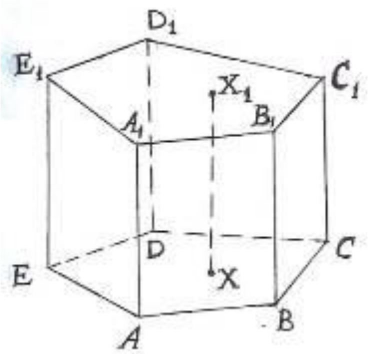
Ko'pyoqlar berilishi jihatidan ikki turga bo'linadi: muntazam va nomuntazam ko'pyoqlar. Ko'pyoq o'zini chegaralovchi tekisliklarning har biridan bir tomonda yotsa, u qavariqdir. Qavariq ko'pyoqning sirti bilan bunday tekislikning umumiy qismi yoq deyiladi. Qavariq ko'pyoqning yoqlari yassi qavariq ko'pburchaklardan iborat. Ko'pyoq yoqlarining tomonlari uning qirralari, uchlari esa ko'pyoqning uchlari deyiladi. Masalan, kub qavariq ko'pyoqdir, uning sirti oltita kvadratdan-yoqlardan tashkil topgan (1.1.2-chizma). ABCD, BEFC,… .Bu kvadratlar kubning yoqlaridir. Bu kvadratlarning AB,BC,BE,… tomonlari kubning qirralari bo'ladi. Kvadratlarning A.B,C,D,E,…uchlari kubning uchlari bo'ladi. Kubda oltita yoq, o'n ikkita qirra va sakkizta uch bor.



1.1.2-chizma

***Prizma*.**

Prizma ikkita parallel tekislik orasiga joylashgan barcha parallel to’g’ri chiziqlar kesmalaridan tuzilgan ko’pyoq bo’lib, bu kesmalar shu tekisliklardan birida yotgan yassi ko’pburchakni kesib o’tadi (1.1.3-chizma). Ko’pburchaklar prizmaning asoslari deyiladi, mos uchlarni tutashtiruvchi kesmalar esa prizmaning yon qirralari deyiladi. Parallel ko’chirish harakat bo’lgani uchun prizmaning asoslari teng bo’ladi. Parallel ko’chirishda tekislik parallel tekislikka o’tgani uchun prizmada asoslar parallel tekisliklarda yotadi. Parallel ko’chirishda nuqtalar parallel to’g’ri chiziqlar bo’ylab ayni bir xil masofaga siljigani uchun prizmada yon qirralari parallel va o’zaro teng. Prizmaning sirti asoslaridan va yon sirtidan iborat. Yon sirti parallelogrmmlardan iborat. Prizma asoslarining tekisliklari orasidagi masofa prizmaning balandligi deyiladi. Prizmaning bitta yog’iga tegishli bo’lmagan ikki uchini tutashtiruvchi kesma prizmaning diagonali deyiladi. Agar prizmaning asosi *n* burchakli bo’lsa, u n burchakli prizma deyiladi.

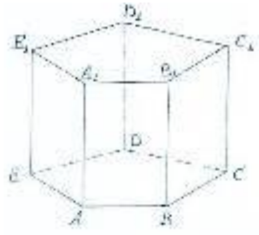


1.1.3-chizma

1.1.3-chizmada beshburchakli prizma tasvirlangan.Uning asoslari-$ABCDE, A_1B_1C_1D_1E_1$beshburchaklardir. XX_1-asoslarining mosnuqtalarini tutashtiruvchi kesma. Prizmaningyon qirralari-AA_1, BB_1,…,EE_1 kesmalaridir. Prizmaning yon yoqlari-ABB_1A_1 ,BCC_1B_1,…parallelogrammlardir.

To'g'ri va muntazam prizma.

1.1.2-ta'rif. Agar prizmaning yon qirrasi asoslariga perpendikulyar bo'lsa, bunday prizma to'g'ri prizma deyiladi. Aks holda og'ma prizma deyiladi.To'g'ri prizmaning yon yoqlari to'g'ri to'rtburchaklardir (1.1.4-chizma).



1.1.4-chizma

Agar to'g'ri prizmaning asoslari muntazam ko'pburchaklar bo'lsa, bunday prizma muntazam prizma deyiladi. Prizmaning yon sirti deb yon yoqlari yuzlarining yig'indisiga aytiladi. Prizmaning to'la sirti yon sirti bilan asoslari yuzlarining yig'indisiga teng.

Quyidagi tasdiqlar o'rinli:

1.Prizma yon sirtining yuzi uning perpendikulyar kesimi bilan yon qirrasining ko'paytmasiga teng:

$$S_{yon} = P_{perp.kes.} \cdot l \,, \qquad (1.1.1)$$

bu yerda $l = AA_1$

To'g'ri prizmaning yon sirti asosining perimetri P_{as} bilan yon qirrasi uzunligi l ning ko'paytmasiga teng:

$$S_{yon} = P_{as} \cdot l \,, \qquad (1.1.2)$$

2.Prizma to'la sirtining yuzi

$$S_{to'la} = S_{yon} + 2S_{asos}, \qquad (1.1.3)$$

formula orqali hisoblanadi.

3.Prizmaning hajmi uning asosi yuzi bilan balandligi ko'paytmasiga teng:

$$V = S_{asos} \cdot h \,, \qquad (1.1.4)$$

bu yerda h – prizma balandligi.

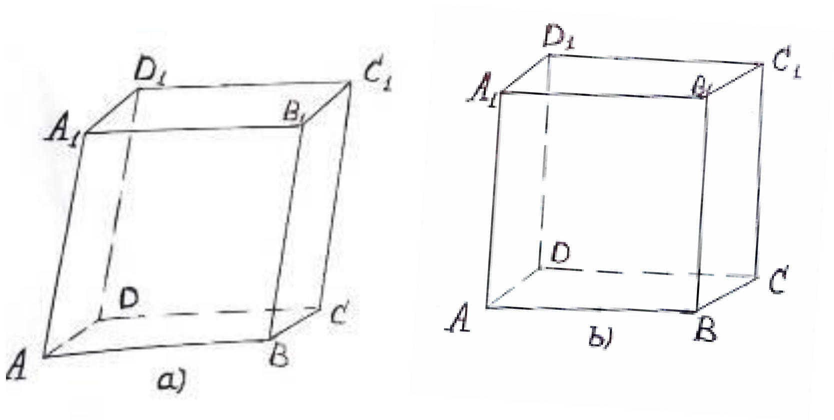
4.Og'ma prizma shunday prizmaga tengdoshki, uning asosi og'ma prizmaning perpendikulyar kesimiga, balandligi esa og'ma prizmaning yon qirrasiga tengdir.

Parallelepiped.

1.1.3-ta'rif. Prizmaning asosi parallelogramm bo'lsa, bunday prizma parallelepiped deyiladi.

Parallepipedning hamma yoqlari parallelogrammlardir. Agar prizmaning yon yoqlari ham parallelogrammlardan iborat bo'lsa, u og'ma parallelepiped, yon yoqlari asoslarga perpendikulyar bo'lsa, parallelepiped to'g'ribo'ladi, hamma yoqlari to'g'ri to'rtburchaklardan iborat parallelepiped to'g'ri burchaklidir.

1.1.5-a chizmada og'ma parallelepiped, 1.1.5-b-chizmada to'g'ri parallelepiped tasvirlangan.



1.1.5-chizma

Quyidagi tasdiqlar o'rinli:

1.Parallelepipedning qarama-qarshi tomonlari teng va parallel.

2.Parallelepipedning diagonallari bitta nuqtada kesishadi va kesishish nuqtasida teng ikkiga bo'linadi.

3.Parallelepiped diagonallari kvadratlarining yig'indisi uning hamma qirralari kvadratlarining yig'indisiga teng.

4.To'g'ri burchakli parallelepiped istalgan diagonalining kvadrati uning uchta o'lchovi kvadratlari yig'indisiga teng.

Parallelepipedning perpendikulyar kesimi uning yon qirrasiga perpendikulyar o'tkazilgan tekislik va parallelepipedning kesishishidan hosil bo'lgan kesimdir.

5.Og'ma parallelepipedning yon sirti perpendikulyar kesimning perimetri bilan yon qirrasining ko'paytmasiga teng:

$$S_{yon} = P_{per.kes.} \cdot l \, , \qquad (1.1.5)$$

6.To’g’ri parallelepipedning yon sirti uning asosi perimetri bilan yon qirrasining ko’paytmasiga teng:

$$S_{yon} = P_{asos} \cdot H \, , \qquad (1.1.6)$$

7.Parallelepiped to’la sirtining yuzi quyidagi formula orqali hisoblanadi:

$$S_{to'la} = S_{yon} + 2S_{asos} \, , \qquad (1.1.7)$$

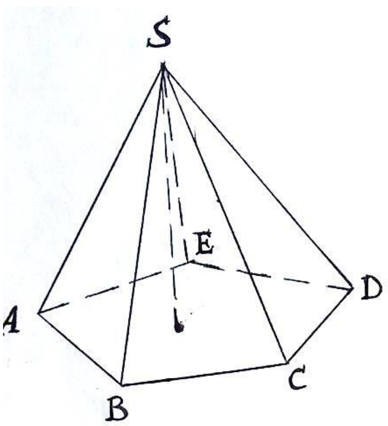
8.Parallelepipedning hajmi-uning asosi yuzi va balandligining ko’paytmasiga teng:

$$V_{par-d} = S_{asos} \cdot H \, , \qquad (1.1.8)$$

Piramida.

1.1.4-ta’rif. Piramida deb shunday ko’pyoqqa aytiladiki, u yassi ko’pburchak piramida asosidan, asos tekisligida yotmagan nuqta – piramida uchidan va uchni asosining nuqtalari bilan tutashtiruvchi hamma kesmalardan iborat (1.1.6-chizma). Piramidaning uchini asosining uchlari bilan tutashtiruvchi kesmalar piramidaning yon qirralari deyiladi.

Piramidaning sirti asosidan va yon yoqlaridan iborat. Har bir yon yoq uchburchak.Uning uchlaridan biri piramidaning uchi bo’ladi, qarshisidagi tomoni esa piramida asosining tomoni bo’ladi.Piramidaning uchidan asos tekisligiga tushirilgan perpendikulyar piramidaning balandligi deyiladi.Piramidaning asosi n burchakdan iborat bo’lsa, u n burchakli piramida deyiladi.Uchburchakli piramida tetraedrdeb ham ataladi.



1.1.6-chizma

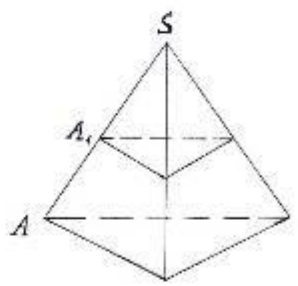
1.1.6-chizmada: S – piramaidaning uchi; ABCDE – piramidaning asosi; ΔSAB, ΔSBC, ΔSCD, ΔSDE, ΔSEA – piramidaning yon yoqlari; SA, SB, SC, SD, SE – piramidaning yon qirralari; SO – piramidaning balandligidir. Muntazam piramida – asosi muntazam ko’pburchak bo’lib, balandligi asosning markazidan o’tadigan piramidadir.Muntazam piramidaning o’qi uning balandligi yotgan to’g’ri chiziqdan iborat. Apofema – muntazam piramida yon yog’ining uchidan o’tkazilgan balandlikdir.

Kesik piramida.

1.1.1-teorema. Piramidaning asosiga parallel va uni kesib o’tadigan tekislik shu piramidaga o’xshash piramida ajratadi.

Isboti.Faraz qilaylik, S – piramidaning uchi, A – asosining uchi, A_1 – kesuvchi tekislikning SA yon qirra bilan kesishish nuqtasi (1.1.7-chizma). Piramidani S uchiga nisbatan

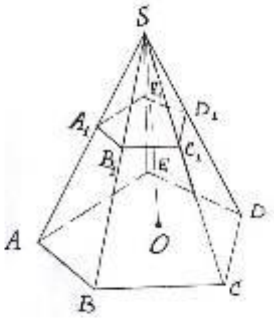
$$k = \frac{SA_1}{SA}$$



1.1.7-chizma

gomotetiya koeffitsiyenti bilan gomotetik almashtiramiz. Bunday gomotetiyada asos tekisligi A_1 nuqta orqali o’tuvchi parallel tekislikka o’tadi, ya’ni kesuvchi tekislikka o’tadi, demak, butun piramida bu tekislik kesib ajratgan qismga o’tadi.Gomotetiya o’xshashlik koeffitsiyenti bo’lganligi uchun piramidaning kesib ajratilgan qismi berilgan piramidaga o’xshash piramida bo’ladi.Teorema isbotlandi.

Yuqoridagi teoremaga ko’ra piramida asosining tekisligiga parallel bo’lgan va piramidaning yon qirralarini kesib o’tuvchi tekislik piramidadan unga o’xshash piramida ajratadi. Ajratilgan bo'lakning ikkinchi qismi ham ko'pyoq bo'lib, kesik piramida deyiladi (1.1.8-chizma).Kesik piramidaning parallel tekisliklarda yotgan yoqlari piramidaning asoslari deyiladi, qolgan yoqlari esa yon yoqlari deyiladi. Kesik piramidaning asoslari o'xshash ko'pburchaklardan, yon yoqlari esa trapetsiyalardan iborat.



1.1.8-chizma

Muntazamko'pyoqlar.

Agar qavariq ko'pyoq yoqlarining tomonlari soni bir xil bo'lgan muntazam ko'pburchakdan iborat bo'lsa va shu bilan birga ko'pyoqning har bir uchida bir xil miqdordagi qirralar uchrashsa, bunday qavariq ko'pyoq muntazam ko'pyoq deyiladi. Muntazam qavariq ko'pyoqlarning besh turi bor: muntazam tetraedr, kub, oktaedr, dodekaedr, ikosaedr. Muntazam tetraedrning yoqlari muntazam uchburchaklardan iborat; har bir uchida uchtadan qirra birlashadi.Tetraedr hamma qirralari teng bo'lgan uchburchakli piramidadan iborat. Kubning hamma yoqlari kvadratlardan iborat; har bir uchida uchta qirra birlashadi. Kub qirralari teng bo'lgan to'g'ri burchakli parallelepipeddir. Oktaedrning yoqlari muntazam uchburchaklar bo'lib, tetraedrdan farqi shundaki, uning har bir uchida to'rttadan qirra birlashadi.Dodekaedrning yoqlari muntazam beshburchaklardan iborat. Uning har bir uchida uchtadan qirra birlashadi. Ikosaedrning yoqlari muntazam uchburchaklardan iborat bo'lib, tetraedr va oktaedrdan farqi shundaki, uning har bir uchida beshtadan qirra birlashadi.

Quyidagi xossalar va tasdiqlar o'rinli.

1.Piramidaning asosiga parallel va uni kesib o’tadigan tekislik o’tkazilgan bo’lsa, piramidaning asosi va kesim yuzlarining nisbati piramida uchidan asoslargacha bo’lgan mos masofalar kvadratlarining nisbatiga teng:

$$\frac{S_{as}}{S_{kes}} = \frac{H^2}{h^2}, \qquad (1.1.9)$$

2.Muntazam piramidaning yon sirti asosining perimetri va apofemasi ko’paytmasining yarmiga teng:

$$S_{yon} = \frac{1}{2} P_{as} \cdot l, \qquad (1.1.10)$$

bu yerda l – apofema, P_{as} - asosning perimetri.

3.Piramidaning hajmi asosining yuzi bilan balandligi ko’paytmasining uchdan biriga teng:

$$V_{pir.} = \frac{1}{3} S_{as} \cdot H\ , \qquad (1.1.11)$$

bu yerdaS_{as}- asosi yuzi, H – balandligi

4.Muntazam kesik piramidaning yon sirti – uning asoslari perimetrlari yig’indisining yarmi bilan apofemasining ko’paytmasiga teng:

$$S_{yon} = \frac{P_1 + P_2}{2} \cdot l\ , \qquad (1.1.12)$$

bu yerdaP_1 – quyi asos perimetri, P_2 – yuqori asos perimetri, l – apofema.

5.Muntazam bo’lmagan kesik piramidaning yon sirti uning yon yoqlari yuzlarining yig’indisiga teng.

6.Agar kesik piramida asoslarining yuzlari mos ravishda, S_1 va S_2, balandligi h bo’lsa (1.1.9-chizma), kesik piramidaning hajmi

$$V = \frac{1}{3} H\left(S_1 + S_2 + \sqrt{S_1 S_2}\right), \qquad (1.1.13)$$

formula orqali hisoblanadi.

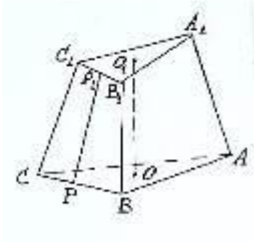
7.O’xshash bo’lgan piramidalar hajmlarining nisbati ularning mos chiziqli o’lchovlari kublarining nisbatiga teng:

$$\frac{V}{V_1} = \frac{H^3}{h^3} = \frac{a^3}{{a_1}^3}, \qquad (1.1.14)$$

8.Piramida to’la sirtining yuzi

$$S_{to'la} = S_{yon} + S_{asos}, \qquad (1.1.15)$$

formula orqali hisoblanadi.



1.1.9-chizma

1.2. Aylanish jismlari haqida asosiy tushunchalar va tasdiqlar. Silindr. Konus. Shar. Sfera.

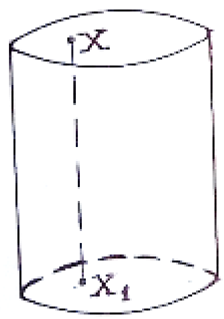
1.2.1-ta'rif. Aylanma jism deb shunday jismga aytiladiki, bu jism biror to'g'ri chiziqqa perpendikulyar bo'lgan tekisliklar bilan markazi shu to'g'ri chiziqda yotgan doiralar bo'yicha kesishadi.

Silindr, konus, sharlar aylanma jismlarga taaluqli jismlardir. To'g'ri to'rtburchakning bir tomoni atrofida aylanishi natijasida silindr hosil qilinadi va shunga o'xshash to'g'ri burchakli uchburchakning biror kateti atrofida aylanishidan konus yoki yarim doiraning diametric atrofida aylanishidan shar hosil qilish mumkin ekanligi ravshandir.

Silindr.

1.2.2-ta'rif. Parallel ko'chirish bilan ustma-ust joylashadigan va bitta tekislikda yotmaydigan ikki doiradan va bu doiralarning mos nuqtalarini tutashtiruvchi hamma parallel to'g'ri chiziq kesmalaridan tashkil topgan jism silindr (to'g'rirog'I doiraviy silindr) deyiladi (1.2.1-chizma). Doiralar silindrning asoslari deyiladi, doira aylanalari mos nuqtalarini tutashtiruvchi kesmalar silindrning yasovchilari deyiladi.

Parallel ko'chirish harakat bo'lgani uchun silindrning asoslari teng bo'ladi.Parallel ko'chirishda tekislik parallel tekislikka (yoki o'ziga) o'tgani uchun silindrning asoslari parallel tekisliklarda yotadi. Parallel ko'chirishda nuqtalar parallel (yoki ustma-ust tushuvchi) to'g'ri chiziqlar bo'yicha ayni bir xil masofaga ko'chgani uchun silindrning yasovchilari o'zaro parallel va teng bo'ladi. Silindrning sirti asoslaridan va yon sirtidan tashkil topadi. Yon sirt yasovchilardan tuzilgan. Silindrning yasovchilari asos tekisliklariga perpendikulyar bo'lsa, bunday silindr to'g'ri silindr deyiladi. Silindr asosining radiusi silindrning radiusi deyiladi.Silindr asoslarining tekisliklari orasidagi masofa silindrning balandligi deyiladi. Asoslarining markazidan o'tuvchi to'g'ri chiziq silindrning o'qi deyiladi. Bu o'q yasovchilarga parallel bo'ladi.



1.2.1-chizma

Quyidagi tasdiqlar o'rinli.

1.Agar silindr asosining radiusi $OA = R$, yasovchisi $AA_1 = H$ (1.2.3-chizma), silindr yon sirtining yuzi

$$S_{yon} = 2\pi R \cdot H, \qquad (1.2.1)$$

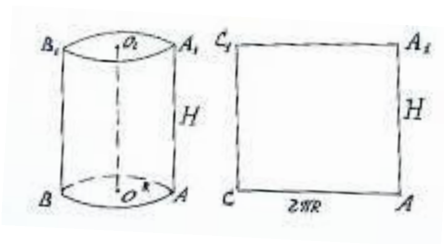
asos aylanasi uzunligi bilan balandligining ko'paytmasiga teng.

2.Silindr to'la sirtining yuzi uning yon sirti yuzi va asoslari yuzlarining yig'indisiga teng:

$$S_{to'la} = S_{yon} + 2S_{asos} = 2\pi R \cdot H + 2\pi R^2 = 2\pi R(H + R)\,, \qquad (1.2.2)$$

3.Silindrning hajmi asosining yuzi bilan balandligining ko'paytmasiga teng:

$$V_s = S_{asos} \cdot H = \pi R^2 \cdot H\,, \qquad (1.2.3)$$

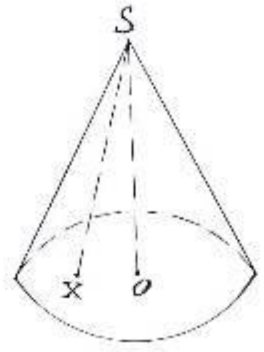


1.2.2-chizma

Konus.

1.2.3-ta'rif. Konus (aniqrog'i, doiraviy konus) deb shunday jismga aytiladiki, u doira – konus asosidan, shu doira tekisligida yotmagan nuqta– konusning uchidan va konusning uchini asosining hamma nuqtalari bilan tutashtiruvchi kesmalardan iborat bo'ladi (1.2.3-chizma). Konus uchini asos aylanasi nuqtalari bilan tutashtiruvchi kesmalar konusning yasovchilari deyiladi.

1.2.4-ta'rif. Konusning sirti asosidan va yon sirtidan iborat. Konusning uchi bilan asos aylanasining markazini tutashtiruvchi to'g'ri chiziq asos tekisligiga perpendikulyar bo'lsa, bunday konus to'g'ri konus deyiladi. Konusning uchidan uning asosiga tushirilgan perpendikulyar konusning balandligi deyiladi.To'g'ri konus balandligining asosi asos markazi bilan ustma-ust tushadi. To'g'ri doiraviy konusning balandligidan o'tuvchi to'g'ri chiziq uning o'qi deyiladi.



1.2.3-chizma

Agar konusning yon sirtini bitta yasovchi bo'yicha kesib, tekislikka yoysak, konusning yoyilmasini hosil qilamiz.Yasovchisi, asosining radiusi R bo'lgan konusning yoyilmasi radiusi va yoy uzunligi $2\pi R$ bo'lgan doiraviy sektordir, uning yuzi konus yon sirtining yuziga teng.

Quyidagi tasdiqlar o'rinli.

1.Konus yon sirtining yuzi:

$$S_{yon} = \pi \cdot R \cdot l, \qquad (1.2.4)$$

bu yerda l – konusning yasovchisi, R – konus asosining radiusi.

2.Konus to'la sirtining yuzi:

$$S_{to'la} = S_{yon} + S_{asos}, \qquad (1.2.5)$$

ya'ni

$$S_{to'la} = \pi Rl + \pi R^2 = \pi R(l + R), \quad (1.2.6)$$

3.Konusning hajmi:

$$V_k = \frac{1}{3}\pi R^2 \cdot H, \qquad (1.2.7)$$

bu yerda H – konusning balandligi.

Konusning asosiga parallel va u bilan kesishadigan tekislik o'tkazilganda tekislik konusni doira bo'ylab kesadi. Kesik konus – konusning asosi va unga parallel tekislik bilan kesilgan qismidir (1.2.6-chizma). Chizmada: AA_1 - kesik konusning yasovchisi, $OO_1 = H$ - kesik konusning balandligi, $OA = R$va$O_1A_1 = r$ kesik konus asoslarining radiuslari.

4.Kesik konus yon sirtining yuzi:

$$S_{yon} = \frac{P_1 + P_2}{2} \cdot l, \qquad (1.2.8)$$

yoki

$$P_1 = 2\pi R, \quad P_2 = 2\pi r \qquad (1.2.9)$$

bo'lishini hisobga olsak,

$$S_{yon} = \pi(R + r)l, \qquad (1.2.10)$$

5.Kesik konus to'la sirtining yuzi:

$$S_{to'la} = S_{yon} + S_{yu.as} + S_{q.as}, \qquad (1.2.11)$$

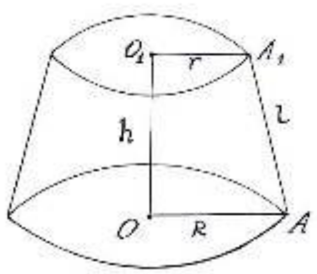
yoki

$$S_{to'la} = \pi(R + r)l + \pi R^2 + \pi r^2, \qquad (1.2.12)$$

6.Kesik konusning hajmi:

$$V_{k.k} = \frac{1}{3}\pi h(R^2 + Rr + r^2), \qquad (1.2.13)$$

formula bo'yicha hisoblanadi.



1.2.4-chizma

7.O’xshash bo’lgan konuslar hajmlarining nisbati ularning mos chiziqli o’lchovlari kublarining nisbatiga teng:

$$\frac{V}{V_1} = \frac{H^3}{h^3} = \frac{R^3}{r^3}, \qquad (1.2.14)$$

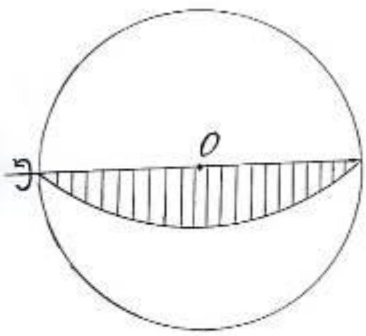
Shar.

1.2.5-ta’rif. Fazoning berilgan nuqtadan berilgan masofadan katta bo’lmagan uzoqlikda yotgan hamma nuqtalaridan iborat jism shar deyiladi. Berilgan nuqta sharning markazi, berilgan masofa esa sharning radiusi deyiladi. Sharning chegarasi shar sirti yoki sfera deyiladi.

Shunday qilib, sharning markazidan radiusga teng masofa qadar uzoqlashgan hamma nuqtalar sferaning nuqtalaridir.

1.2.6-ta’rif. Shar markazini shar sirtining nuqtasi bilan tutashtiruvchi istalgan kesma ham radius deyiladi. Shar sirtining ikki nuqtasini tutashtituvchi va sharning markazidan o’tuvchi kesma diametr deyiladi. Istalgan diametrning oxirlari sharning diametral qarama-qarshi nuqtalari deyiladi.

Silindr va konus kabi shar ham aylanish jismidir. U yarim doirani uning diametri atrofida aylantirish natijasida hosil qilinadi (1.2.5-chizma).



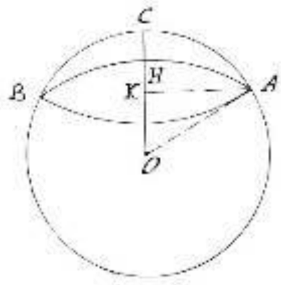
1.2.5-chizma

Agar sfera markazining koordinatalari (a;b;c), radiusi R bo’lsa, uning tenglamasi

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2, \qquad (1.2.14)$$

ko’rinishda yoziladi.

Shar segmenti sharning sharni kesuvchi tekislik bilan chegaralangan qismidan iborat.1.2.6-chizmada $AKBCA$ shar segmentidir.Kesuvchi tekislik sharni ikkita segmentga ajratadi. Agar shar segmenti asosidagi nuqtalarni shar markazi bilan tutashtirsak, konus hosil bo'ladi va uning sirti shar segmenti bilan birgalikda shar sektorini tashkil qiladi. Agar shar segmenti yarimshardan katta bo'lsa, sharning shu konus chiqarib tashlangan qismi shar sektoridir (1.2.6-chizmada $AOBCA$ – shar sektori).Sharning uni kesuvchi parallel tekisliklar bilan chegaralangan qismi shar kesmalaridir.



1.2.6-chizma

Shar segmentining balandligi segment asosining markazidan asosga o'tkazilgan perpendikulyarning shar sirti bilan kesishish nuqtasigacha bo'lgan masofadir.

Quyidagi tasdiqlar o'rinli.

1.Sferaning radiusi R bo'lsa, uning sirti

$$S = 4\pi R^2 , \qquad (1.2.15)$$

formula bo'yicha hisoblanadi.

2.Agar berilgan shardagi segmentning balandligi H, sferaning radiusi R ga teng bo'lsa, shar segmenti yon sirtining yuzi

$$S = 2\pi RH, (1.2.16)$$

formuladan topiladi.

3.Shar sektori sirtining yuzi

$$S_{sekt.} = S_{segm.} + S_{konus} , \qquad (1.2.17)$$

formula bo'yicha hisoblanadi.

4.Sharning radiusi R bo'lsa, uning hajmi

$$V_{t.} = \frac{4}{3}\pi R^3, \qquad (1.2.18)$$

formula bo'yicha hisoblanadi.

5.Sharning radiusi R, shar segmentining balandligi H bo'lsa, shar segmentining hajmi

$$V_{segm.} = \pi H^2 \left(R - \frac{1}{3}H\right), \qquad (1.2.19)$$

shar sektorining hajmi esa

$$V_{sekt.} = \frac{2}{3}\pi R^2 H, \qquad (1.2.20)$$

formula bo'yicha hisoblanadi.

1.3. Jismlarning kombinatsiyalariga oid asosiy tushunchalar va tasdiqlar.

Prizma va shar kombinatsiyasi haqida.

1.Agar prizma to'g'ri bo'lib, uning asosiga aylanani tashqi chizish mumkin bo'lsa, prizmaga sferani tashqi chizish mumkin.

2.Agar: 1) prizmaning perpendikulyar kesimiga aylanani ichki chizish mumkin bo'lsa;

2)Prizmaning balandligi aylananing diametriga teng bo'lsa, prizmaga sferani ichki chizish mumkin.

3. Balandligi H ga teng bo'lgan prizmaga r radiusli shar ichki chizilgan bo'lsa, quyidagi munosabat o'rinli bo'ladi, ya'ni ichki chizilgan shar diametri prizma balandligiga teng:

$$H = 2r, \qquad (1.3.1)$$

4. Katta diagonali dgateng bo'lgan parallelepipedga R radiusli shar tashqi chizilgan bo'lsa, quyidagi munosabat o'rinli:

$$d = 2R, \qquad (1.3.2)$$

5. Hajmi V ga, to'la sirti S ga teng bo'lgan ko'pyoqqa r radiusli shar ichki chizilgan bo'lsa, u holda quyidagi munosabat o'rinli:

$$V = \frac{1}{3}Sr, \qquad (1.3.3)$$

6. Kubga tashqi chizilgan shar radiusi quyidagiga teng:

$$R = \frac{a\sqrt{3}}{2}, \qquad (1.3.4)$$

7. Kubga ichki chizilgan shar radiusi quyidagiga teng:

$$r = \frac{a}{2}\ , \qquad (1.3.5)$$

bu yerda a – kub qirrasi.

Piramida va shar kombinatsiyasi haqida.

1.Agar piramida asosiga aylana tashqi chizish mumkin bo’lsa, piramidaga tashqi sfera chizish mumkin.

2.Agar piramidaning asosiga aylanani ichki chizish mumkin bo’lsa va piramidaning balandligi o’sha aylana markazidan o’tsa, piramidaga sferani ichki chizish mumkin.

Demak, muntazam piramidaga sferani doimo ichki chizish mumkin.

3.Asosiga ichki chizilgan aylana radiusi r_1 ga, asosidagi ikki yoqli burchakα ga teng bo’lgan muntazam piramidaga r radiusli shar ichki chizilgan bo’lsa, quyidagi munosabat o’rinli bo’ladi:

$$r = \frac{\sin\alpha}{1+\cos\alpha} r_1, \qquad (1.3.6)$$

4.Balandligi H ga, yon qirrasi L ga teng bo’lgan muntazam piramidaga R radiusli shar tashqi chizilgan bo’lsa,berilganlar orasida quyidagicha munosabat o’rnatish mumkin:

$$L^2 = 2HR, \qquad (1.3.7)$$

Muntazam uchburchakli piramida tetraedr uchun tashqi chizilgan shar radiusi

$$R = \frac{a\sqrt{6}}{4}\ , \qquad (1.3.8)$$

Ichki chizilgan shar radiusi

$$r = \frac{a\sqrt{6}}{12}, \qquad (1.3.9)$$

formula bilan hisoblanadi, bu yerda a – tetraedr qirrasining uzunligi.

Silindr va shar kombinatsiyasi haqida.

1.Ixtiyoriy silindrga sferani tashqi chizish mumkin.Tashqi chizilgan sferaning markazi silindr o'q kesimiga tashqi chizilgan aylananing markazidir.

2.Agar silindrning balandligi uning asosi diametriga teng bo'lsa, silindrga sferani ichki chizish mumkin, ya'ni silindrga shar ichki chizilgan bo'lsa, silindrning o'qi orqali o'tuvchi tekislikda kvadratga ichki chizilgan doira hosil bo'ladi. Bunda silindrning o'q kesimi kvadrat va sharning radiusi doiraning radiusiga teng bo'ladi. Silindrning balandligi H, asosining radiusi R, ichki chizilgan shar radiusi r bo'lsa, u holda quyidagi munosabat o'rinli bo'ladi:

$$H = 2r, \qquad R = r \quad (1.3.10)$$

3.Diagonali d ga teng bo'lgan silindrga R radiusli shar tashqi chizilgan bo'lsa, quyidagi munosabat o'rinli bo'ladi:

$$d = 2R, \qquad (1.3.11)$$

Konus va shar kombinatsiyasi haqida.

1.Ixtiyoriy konusga sferani tashqi chizish mumkin. Tashqi chizilgan sferaning markazi konusning o'q kesimiga tashqi chizilgan aylananing markazidir.

2.Har qanday konusga sferani ichki chizish mumkin.

3.Asosining radiusi R ga, yasovchisi bilan asos tekisligi orasidagi burchakα ga teng bo'lgan konusga r radiusli shar ichki chizilgan bo'lsa, quyidagi munosabat o'rinli bo'ladi:

$$r = \frac{\sin \alpha}{1+\cos \alpha} \cdot R, \qquad (1.3.12)$$

4.Balandligi H ga, yasovchisi L ga teng bo'lgan konusga R_1 radiusli shar tashqi chizilgan bo'lsa, quyidagi munosabat o'rinli bo'ladi:

$$L^2 = 2HR_1, \qquad (1.3.13)$$

5.Asoslarining radiuslari R ga, balandligi H ga, yasovchisi L ga teng bo'lgan kesik konusga r_1 radiusli shar ichki chizilgan bo'lsa, u holda quyidagi munosabat o'rinli:

$$H = 2r_1 , \qquad (1.3.14)$$

va

$$L = R + r \,, \qquad (1.3.15)$$

6.Radiusi R gateng bo’lgan sharga balandligi h ga asosining radiusi r gat eng bo’lgan konus ichki chizilgan bo’lsa, quyidagi munosabat o’rinli bo’ladi:

$$R = \frac{r^2 + h^2}{2h}, \qquad (1.3.16)$$

7.Balandligi h ga yasovchisi l ga asosining radiusi R ga teng bo’lgan konusga r radiusli shar ichki chizilgan bo’lsa, quyidagi munosabat o’rinli bo’ladi:

$$r = \frac{R \cdot h}{l + R}, \qquad (1.3.17)$$

Xulosa.

Bu bobda stereometrik shakl figuralar, ularning yasalishi haqida aytilgan silindr, konus, shar, piramida, prizma sirtlari hajmlari va boshqa parametrlarini topishi formulalari jismlarning chizmalari bilan bobning birinchi, ikkinchi qismida aytilgan.

Bobning uchinchi qismida esa jismlar kombinatsiyasi turlari va jismlarning turli holatdagi kombinatsiyalari uchun bir nechta formulalar keltirilgan.

Bu bob BMI ning asosiy qismi bo’lgan ikkinchi bob uchun nazariy zamin tayyorlagan. Bu yerda asosiy formulalar bilan birga turli almashtirishlar yordamida hosil qilingan formulalar ham keltirilgan.

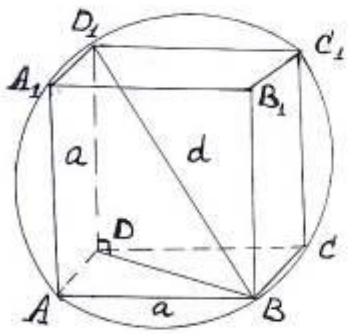
Almashtirishlar yordamida hosil qilingan formulalarning keltirib chiqarishning usullari ba’zilari uchun ko’rib chiqilgan, ya’ni ikkinchi bobda shu formulalarni keltirib chiqarilishi bilan bog’liq bo’lgan masalalar ko’rilgan.

II Bob.Jismlarning kombinatsiyalariga oid masalalar.

2.1. Prizma va shar kombinatsiyalariga oid testlarni yechish usullari.

2.1.1. ([10], 96-1-53) To’la sirtining yuzi 72 ga teng bo’lgan kubga tashqi chizilgan sharning radiusini toping (2.1.1-chizma).

A)3 B)6 C)$3\sqrt{3}$ D)$2\sqrt{3}$ E)4

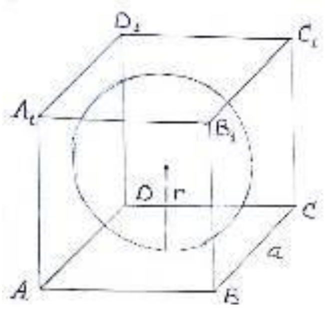


2.1.1-chizma

Yechilishi: Kub to'la sirtining formulasi quyidagicha: $S = 6a^2$. Bundan $6a^2 = 72$, $a^2 = 12$, $a = 2\sqrt{3}$ bo'ladi. 2.1.1-chizmaga ko'ra, $2R = d$ (d-kub diagonali, R-shar radiusi). Pifagor teoremasiga ko'ra: $(2R)^2 = a^2 + (a\sqrt{2})^2 = 3a^2$ bo'ladi. $R = \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{2} \cdot 2\sqrt{3} = 3$. Binobarin, to'g'ri javob: A)3

2.1.2. ([10], 97-8-56) Hajmi 125 bo'lgan kubga ichki chizilgan shar sirtining yuzini toping (2.1.2-chizma).

A)125π B)25π C)24.5π D)105π E)25.5π

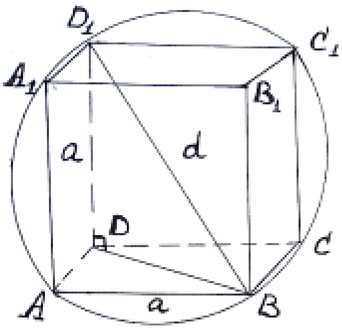


2.1.2-chizma

Yechilishi: Kub hajmini topish formulasi quyidagicha: $V = a^3$. Bundan esa $a^3 = 125$, $a = 5$ bo'ladi. $r = \frac{a}{2} = \frac{5}{2} = 2.5$. Shar sirtining yuzi: $S = 4\pi r^2 = 25\pi$. Binobarin, to'g'ri javob: B)25π

2.1.3. ([10], 98-12-95) Ikkita qo'shni yoqlarning markazlari orasidagi masofa $2\sqrt{2}$ ga teng bo'lgan kubga tashqi chizilgan shar sirtining yuzini toping (2.1.3-chizma).

A)28π B)36π C)48π D)$18\sqrt{2}\pi$ E)$12\sqrt{3}\pi$



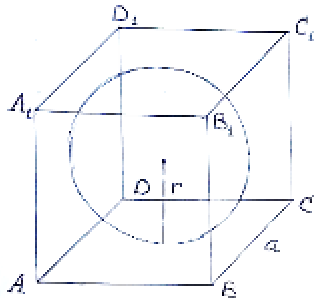
2.1.3-chizma

Yechilishi: 2.1.3-chizmadanko'rinibturibdiki, Pifagorteoremasigako'ra:$(2\sqrt{2})^2 = \frac{a^2}{4} + \frac{a^2}{4} = \frac{a^2}{2}$bo'ladi.

Bundanesa$\frac{a^2}{2} = 8, a^2 = 16, a = 4$ bo'ladi.Kubning diagonali:$d^2 = a^2 + (\sqrt{2}a)^2 == 3a^2$,$d^2 = 3 \cdot 16 = 48$,$d = 4\sqrt{3}$ ga teng. $d = 2R$ ekanligidan $2R = 4\sqrt{3}$, $R = 2\sqrt{3}$ bo'ladi. Shar sirtining yuzi: $S = 4\pi R^2 = 48\pi$. Binobarin, to'g'ri javob: C)48π

2.1.4. ([10], 96-6-60) Kubning qirrasi 6 ga teng.Kubga ichki chizilgan sharning hajmini toping (2.1.4-chizma).

A)12π B)36 π C)27 π D)18 π E)26 π



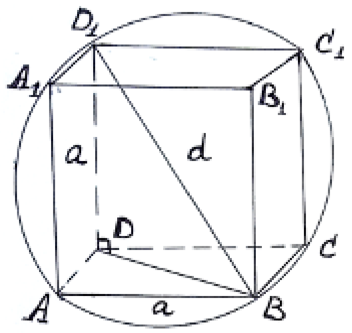
2.1.4-chizma

Yechilishi: 2.1.4-chizmaga ko'ra, kubning tomoni unga ichki chizilgan sharning diametriga teng: $a = d = 2r, r = \frac{a}{2} = \frac{6}{2} = 3$. Shar hajmi esa$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 27 = 36\pi$bo'ladi.

Binobarin, to'g'ri javob: B)36 π

2.1.5. ([10], 00-5-59) Radiusi 5 ga teng bo'lgan sharga balandligi 8 ga teng to'rtburchakli muntazam prizma ichki chizilgan.Prizmaning hajmini toping (2.1.5-chizma).

A)136 B)144 C)196 D)172 E)184



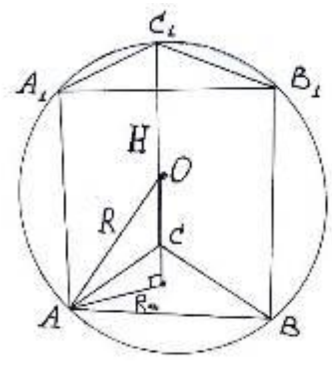
2.1.5-chizma

Yechilishi: Shar radiusi 5 ga teng bo'lsa, u holda kubning diagonali 10 ga teng bo'ladi. 2.1.5-chizmaga ko'ra, prizma asosining diagonalini topamiz: $10^2 = 8^2 + +2a^2$, $2a^2 = 36$, $a^2 = 18$, $a = 3\sqrt{2}$. Endi prizma hajmini topamiz: $V = S_{asos} \cdot h = a^2 \cdot h = 18 \cdot 8 = 144$.Demak, to'g'ri javob: B)144

2.1.6. ([10], 02-10-39) Radiusi R ga teng sharga balandligi H ga teng bo'lgan uchburchakli muntazam prizma ichki chizilgan.Prizmaning hajmini toping (2.1.6-chizma).

A)$\frac{3\sqrt{3}H}{16}(4R^2 - H^2)$ B)$\frac{3\sqrt{3}}{8}(2R^2 - H^2)$ C)$\sqrt{3}H(4R^2 - H^2)$

D)$\frac{3\sqrt{2}}{16}H(4R^2 - H^2)$ E)$\frac{3H}{16}(4R^2 - H^2)$



2.1.6-chizma

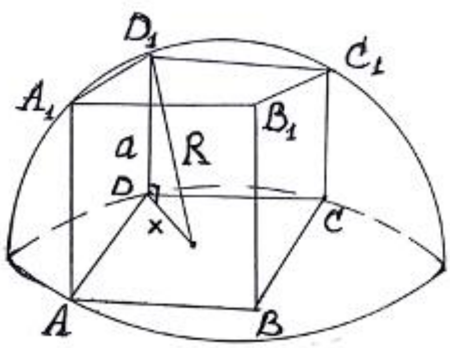
Yechilishi: Prizma asosi muntazam uchburchakdan iborat. Bundan $R_a = \frac{a}{\sqrt{3}}$ bo'ladi. Pifagor teoremasini qo'llasak: $R^2 = \frac{H^2}{2} + {R_a}^2, R^2 - \frac{H^2}{4} = \frac{a^2}{3}$,

$a^2 = \frac{3(4R^2-H^2)}{4}$ga teng bo'ladi. Prizmaning hajmi: $V_p = S_{asos} \cdot H = \frac{\sqrt{3}}{4} a^2 \cdot H == \frac{\sqrt{3}}{4} \cdot \frac{3(4R^2-H^2)}{4} \cdot H = \frac{3\sqrt{3}}{16} H(4R^2 - H^2)$ bo'ladi.

Demak, to'g'ri javob: A)$\frac{3\sqrt{3}}{16} H(4R^2 - H^2)$

2.1.7. ([10], 02-10-79) Radiusi $\sqrt{\frac{3}{2}}$ ga teng yarim sharga kub ichki chizilgan bo'lib, uning to'rtta uchi yarim shar asosida, qolgan to'rttasi shar sirtida yotadi.Kubning hajmini toping (2.1.7-chizma).

A)1 B)$\frac{3}{2}\sqrt{\frac{3}{2}}$ C)2.5 D)2 E)1,5



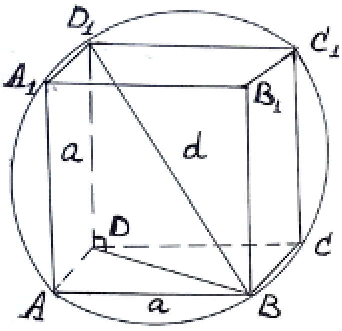
2.1.7- chizma

Yechilishi: 2.1.7- chizmaga asosan: $2x=a\sqrt{2}$, $x=\frac{a}{\sqrt{2}}$ bo'ladi. Pifagor teoremasiga ko'ra: $R^2=a^2+(\frac{a}{\sqrt{2}})^2=\frac{3a^2}{2}$, $a^2= \frac{2R^2}{3}=\frac{2}{3} \cdot \frac{3}{2}$ =1; a=1 ga teng bo'ladi. Endi kubning hajmini topamiz: $V=a^3=1$.

Binobarin, to'g'ri javob: A) 1

2.1.8. ([10], 02-11-61) Qirrasining uzunligi 8 ga teng bo'lgan kubning barcha uchlaridan o'tuvchi sferaning radiusini toping (2.1.8-chizma).

A) $3\sqrt{3}$ B) $4\sqrt{3}$ C) $5\sqrt{3}$ D) $6\sqrt{3}$ E) $8\sqrt{3}$



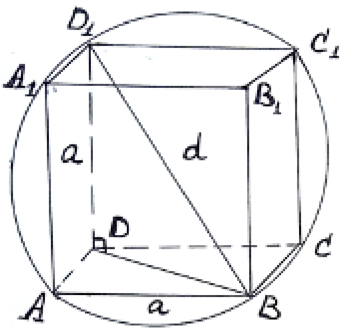
2.1.8-chizma

Yechilishi:Bizga ma'lumki, kubning diagonali $d^2=a^2+(a\sqrt{2})^2=3a^2$, $d=\sqrt{3}a$ ga teng. Masala shartida kub qirrasining uzunligi 8 ga tengligini e'tiborga olsak: $d=8\sqrt{3}$ bo'ladi. Bundan $2R=d$, $R=\frac{d}{2}$, $R=4\sqrt{3}$ ga teng.

Demak, to'g'ri javob: B) $4\sqrt{3}$

2.1.9. ([10], 03-1-45) Kubga tashqi chizilgan sharning hajmi $10\frac{2}{3}\pi$ ga teng. Kubning diagonaliga tegishli bo'lmagan uchlaridan diagonaligacha bo'lgan masofani toping (2.1.9-chizma).

A) $\frac{4\sqrt{2}}{3}$ B) $\frac{3\sqrt{2}}{4}$ C) $\frac{4\sqrt{2}}{9}$ D) $\frac{3\sqrt{2}}{8}$ E) $\frac{2\sqrt{2}}{9}$



2.1.9-chizma

Yechilishi: *1-usul.* Sharning hajmini topish formulasidan foydalansak: $V=\frac{4}{3}\pi R^3$, $\frac{4}{3}\pi R^3 = 10\frac{2}{3}\pi \Rightarrow R=2$ bo'ladi. $d=\sqrt{3}a$, $a=\frac{d}{\sqrt{3}}=\frac{4}{\sqrt{3}}$. Endi to'g'ri burchakli uchburchak uchun katetlarning gepotinuzadagi proyektsiyalarini topamiz: $a^2=c_a\cdot c$, $b^2=c_b\cdot c$. Bu yerda $c_a+c_b=c$ ga teng. $\frac{16}{3}=c_a\cdot 4$, $c=\frac{4}{3}$, $\frac{32}{3}=c_b\cdot 4$, $c_b=\frac{8}{3}$. Bundan $h=\sqrt{c_a\cdot c_b}=\sqrt{\frac{4}{3}\cdot\frac{8}{3}}=\frac{4\sqrt{2}}{3}$ bo'ladi.

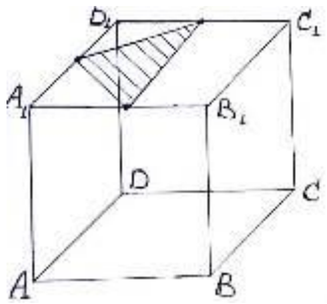
Demak, to'g'ri javob: A)$\frac{4\sqrt{2}}{3}$

2-usul. Yuqorida 2.1.9-chizmaga asosan d=4, a=$\frac{4}{\sqrt{3}}$ ekanligini topdik. Endi h ni topamiz. 2.1.9-chizmadagi to'g'ri burchakli uchburchak uchun h=$\frac{a\cdot b}{c}$=$\frac{\frac{4}{\sqrt{3}}\cdot 4\sqrt{\frac{2}{3}}}{4}$= $\frac{4\sqrt{2}}{3}$ bo'ladi.

Binobarin, to'g'ri javob: A)$\frac{4\sqrt{2}}{3}$

2.1.10. ([10], 03-2-53) Kubning bir uchidan chiqqan uchta qirralarning o'rtalari orqali o'tkazilgan kesimning yuzi $16\sqrt{3}$ ga teng.Shu kubga ichki chizilgan shar sirtining yuzinihisoblang (2.1.10 – chizma).

A)96π B) 256π C) 144π D) 102π E)128π



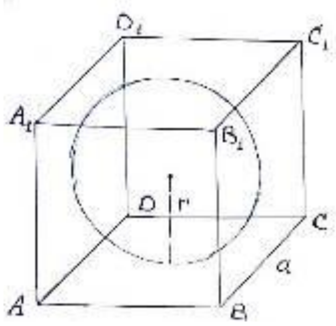
2.1.10- chizma

Yechilishi: 2.1.10- chizmaga asosan, hosil bo'lgan kesim teng tomonli uchburchak ekan. Teng tomonli uchburchakning tomonini topamiz: x^2=$\frac{a^2}{4}+\frac{a^2}{4}=\frac{a^2}{2}$ => x=$\frac{a}{\sqrt{2}}$ ga teng ekan. Hosil bo'lgan kesimning yuzi $16\sqrt{3}$ ga teng ekanligidan foydalanamiz: S=$\frac{\sqrt{3}}{4}x^2$=$\frac{\sqrt{3}}{4}\cdot\frac{a^2}{2}$=$16\sqrt{3}$ => a^2=128 => a=$8\sqrt{2}$. Shu kubga ichki chizilgan sharning radiusi r=$\frac{a}{2}=\frac{8\sqrt{2}}{2}=4\sqrt{2}$ ga teng bo'ladi. Bundan esa shar sirtining yuzini topamiz: S=4 $\pi r^2 = 4\pi\cdot 32=128\,\pi$

Demak, to'g'ri javob: E)128π

2.1.11. ([10], 03-4-53) Kubga ichki chizilgan sharning hajmi $85\frac{1}{3}\pi$ ga teng. Shu kubning to'la sirtini toping (2.1.11 – chizma).

A)382 B) 386 C) 385 D) 384 E)388



2.1.11 – chizma

Yechilishi: Masala shartiga ko’ra kubga ichki chizilgan sharning hajmi V=$85\frac{1}{3}$ ga teng. Bundan esa $\frac{4}{3}\pi r^3 = 85\frac{1}{3}\pi$ => r³=64 =>r=4 bo’ladi. 2.1.11 – chizmaga ko’ra, a=2r=8 ga teng. Endi kubning to’la sirtini topamiz: $S_{to'la}$= 6a^2=6·64=384

Demak, to’g’ri javob:D) 384

2.1.12. ([10], 03-10-58) Kubga tashqi chizilgan sharning hajmi unga ichki chizilgan sharning hajmidan necha marta katta?

A) 8 B) 4 C) $4\sqrt{2}$ D) $4\sqrt{3}$ E)$3\sqrt{3}$

Yechilishi: Kubga tashqi chizilgan sharning radiusi R=$\frac{\sqrt{3}}{2}a$ ga teng. Unga ichki chizilgan sharning radiusi esa r=$\frac{a}{2}$ ga teng.$V_{tashqi\ ch.sh}$=$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi \cdot \frac{3\sqrt{3}}{8}a^3 == \frac{\sqrt{3}\pi a^3}{2}$ga, $V_{ichki\ ch.sh}$= $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot \frac{a^3}{8} = \frac{\pi a^3}{6}$ bo’ladi. Bulardan foydalanib $\frac{V_{tashqi\ ch.sh}}{V_{ichki\ ch.sh}} = \frac{\frac{\sqrt{3}\pi a^3}{2}}{\frac{\pi a^3}{6}}$=3$\sqrt{3}$

Demak, to’g’ri javob:E) $3\sqrt{3}$

2.1.13. ([10], 03-10-59) To’g’ri prizmaning hajmi 40 ga, unga ichki chizilgan sharning hajmi $\frac{32}{3}\pi$ ga teng. Prizmaning yon sirtini toping.

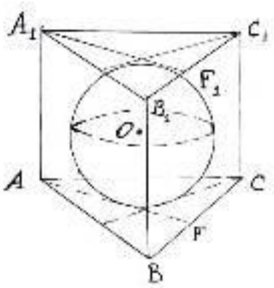
A)40 B)16 C)24 D)20 E)30

Yechilishi: Masala shartiga ko’ra, V_{prizma}=40, V_{shar}=$\frac{32}{3}\pi$ ga teng. V_{shar}=$\frac{4}{3}\pi r^3 == \frac{32}{3}\pi,\ r^3 = 8,\ r = 2,$ V_p=S_a·h=40; S_a=10, S_a=$\frac{Pr}{2} = \frac{P\cdot 2}{2} = 10$ p=10. Endi prizmaning yon sirtini hisoblaymiz: S_{yon}=P·h=10·4=40

Demak, to’g’ri javob: A) 40

2.1.14. ([6], 15.2.3-masala)Radiusi R bo’lgan sharga uchburchakli muntazam prizma tashqi chizilgan.Prizma to’la sirtining yuzini toping(2.1.13 – chizma).

A) $24R^2\sqrt{3}$ B) $18R^2\sqrt{3}$ C) $16R^2\sqrt{3}$ D) $20R^2$ E)$24\sqrt{3}R^2$



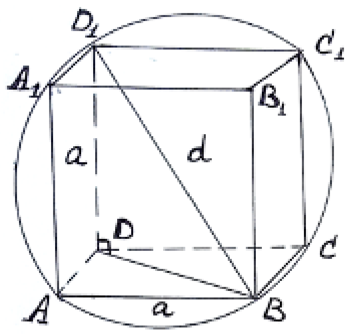
2.1.13 – chizma

Yechilishi: Shar ichki chizilgan bo’lganligidan, uning diametri prizmaning balandligiga teng: H=2R. Agar prizma asosining tomoni AB=a, balandligi H bo’lsa, to’la sirti $S_{t.pr}=3a\cdot H+2S_{as}$, $S_{as}=\frac{a^2\sqrt{3}}{4}$formula bo’yicha hisoblanadi. Sharning markazidan prizmaning asosiga parallel tekislik o’tkazamiz va kesimda prizmaning asosi ΔABC ga teng bo’lgan uchburchakni hamda unga ichki chizilgan R radiusli aylanani hosil qilamiz. Muntazam ΔABC ga ichki chizilgan aylananing O markazi uchburchak bissektrisalarning kesishish nuqtasidir. Shuning uchun, ΔOAK- to’g’ri burchakli (OK $\perp$AB), AOK=30^0, AK=$\frac{a}{2}$ va $AK=OK\cdot ctg30^0$yoki$\frac{a}{2}=R\cdot ctg30^0$, $a=2R\sqrt{3}$u holda $S_{yon}=3\cdot 2R\sqrt{3}\cdot 2R=12R^2\sqrt{3}$; $S_{as}=\frac{1}{4}(2R\sqrt{3})^2\cdot\sqrt{3}$=$3R^2\sqrt{3}$va $S_{t.pr}=S_{yon}+2S_{as}=12R^2\sqrt{3}++6R^2\sqrt{3}$=$18R^2\sqrt{3}$

Binobarin, to’g’ri javob: B) $18R^2\sqrt{3}$

2.1.15. ([6], 15.4.10-masala)To’rt burchakli muntazam prizma asosining tomoni 6 sm, yon qirrasi 17 sm bo’lsa, unga tashqi chizilgan sharning radiusini toping (2.1.14 – chizma).

A) 9,5 B) 8 C) 10 D) 8,5 E)12 sm



2.1.14 – chizma

Yechilishi: 2.1.14 – chizmaga asosan prizma asosining diaganali $d^2 = 6^2 + 6^2 == 72$, d=$6\sqrt{2}$ ga teng. Endi shar radiusini topamiz: $(2R)^2 = 17^2 + (6\sqrt{2})^2$; $4R^2 = 289 + 72 = 361$; 2R=19; R=9,5

Demak, to'g'ri javob: A) 9,5

2.1.16. ([6], 15.4.11-masala) To'rt burchakli parallelepipedning o'lchamlari nisbati 2:3:6 kabi, to'la sirtining yuzi 1152 sm^2 bo'lsa, unga tashqi chizilgan sharning radiusi topilsin. A) 16 B) 14 C) 12 D) 15 E) 10 sm

Yechilishi: Parallelepiped asosanining diaganalini topamiz: $d^2 = (2x)^2 + (3x)^2 = 13x^2$, d=$\sqrt{13}x$.
$S_t = 2S_a + S_y = 2 \cdot 2x \cdot 3x + 2 \cdot (3x + 2x) \cdot 6x$; $S_t = 12x^2 + 60x^2 = 72x^2$; $72x^2 = 1152$; $x^2 = 16$; $x = 4$. Endi shar radiusini topamiz: $(2R)^2 = (6x)^2 + (\sqrt{13}x)^2 = 49x^2$; R=$\frac{7}{2} \cdot 4 = 14$

Demak, to'g'ri javob: B) 14

2.1.17. ([6], 15.4.17-masala) Olti burchakli muntazam prizma asosining tomoni 4 dm, prizmaning balandligi 15 dm bo'lsa, unga tashqi chizilgan sharning radiusini toping.

A) 10 B) 9,5 C) 9 D) 8,5 E) 8 dm

Yechilishi: Masala shartiga ko'ra, prizma asosining tomoni 4 ga teng. U holdaasosining katta diaganali 8 ga teng bo'ladi. Endi prizmaga tashqi chizilgan shar radiusini topamiz: $(2R)^2 = 8^2 + 15^2 = 289$; 2R=17; R=8,5

Demak, to'g'ri javob: D) 8,5

2.2. Piramida va shar kombinatsiyalariga oid testlarni yechish usullari.

2.2.1. ([10], (98-12-71) Piramidaning to'la sirti 60 ga, unga ichki chizilgan sharning radiusi 5ga teng. Piramidaning hajmini toping.

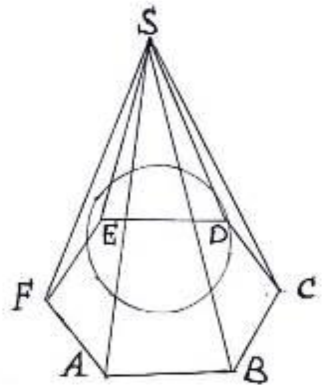
A) 120 B) 80 C) 90 D) 100 E)150

Yechilishi: Masala shartida piramidaning to'la sirti va unga ichki chizilgan sharning radiusi berilgan. Bulardan foydalanib piramidaning hajmini hisoblaymiz: $V = \frac{1}{3}S_{to'la} \cdot r = \frac{1}{3} \cdot 60 \cdot 5 = 100$ ga teng.

Demak, to'g'ri javob:D) 100

2.2.2. ([10], 99-5-48) Muntazam oltiburchakli piramidaning apofemasi 5 ga, uning asosiga tashqi chizilgan doiraning yuzi12πga teng. Shu piramidaga ichki chizilgan sharning radiusini toping (2.2.1 – chizma).

A) 3 B) 3,2 C) 1,5 D) 2,5 E)2,4



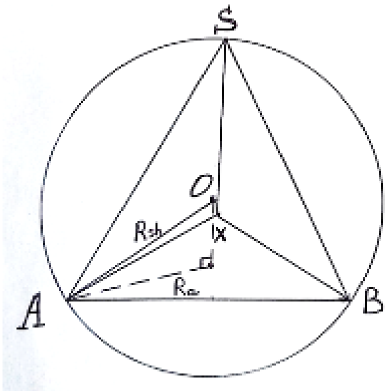
2.2.1 – chizma

Yechilishi: Masala shartiga ko'ra: S_d=12π ga teng. Bundan esa $\pi R^2 = 12\pi$ => R=$2\sqrt{3}$ bo'ladi. R=a=$2\sqrt{3}$. Quyidagi formuladan piramidaga ichki chizilgan shar radiusini topamiz: $r = \frac{a}{2tg\frac{180°}{n}}$, bu masalada n=6 ga teng ekanligidan $r = \frac{a}{2tg30°} = \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = 3$

Demak, to'g'ri javob:A) 3

2.2.3. ([10], 99-5-50) Muntazam tetraedrning qirrasi 1 ga teng. Shu tetraedrga tashqi chizilgan sharning radiusini toping (2.2.2 – chizma).

A)$\frac{2\sqrt{2}}{3}$ B) $\frac{\sqrt{6}}{4}$ C) $\frac{3\sqrt{2}}{8}$ D) $\frac{11\sqrt{2}}{24}$ E)$\frac{2\sqrt{3}}{5}$



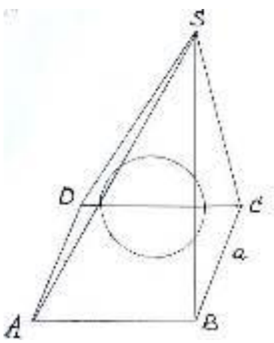
2.2.2 – chizma

Yechilishi:2.2.2 – chizmaga asosan, $H-R_{sh}=x$. Tetraedrning asosida tashqi chizilgan aylaning radiusi $R_a=\frac{a}{\sqrt{3}}$ga teng. Tetraedrning qirrasi 1 ga teng ekanligidan $R_a=\frac{1}{\sqrt{3}}$ bo'ladi.$R_{sh}^2=(H-R_{sh})^2+\frac{1}{3}$, $2R_{sh}H=H^2+\frac{1}{3}$, $2R_{sh}H--H^2=\frac{1}{3}$, R_{sh}=$\frac{\frac{1}{3}+H^2}{2H}$. Ikkinchi tomondan $H^2+\frac{1}{3}=1$, $H=\sqrt{\frac{2}{3}}$ bo'ladi. R_{sh}=$\frac{\frac{1}{3}+\frac{2}{3}}{2\sqrt{\frac{2}{3}}}=\frac{\sqrt{3}}{2\sqrt{2}}=\frac{\sqrt{6}}{4}$

Demak, to'g'ri javob:B) $\frac{\sqrt{6}}{4}$

2.2.4. (([10], 00-6-49) Muntazam to'rtburchakli piramida asosining tomoni 12 ga, unga ichki chizilgan sharning radiusi 3 ga teng.Piramidaning yon sirtini toping (2.2.3-chizma).

A)240 B) 120 C) 480 D) 360 E)280



2.2.3-chizma

Yechilishi: Piramidaningyonsirti$S_{yon}=\frac{Pl}{2}$formuladantopiladi.

$S_{yon}=\frac{Pl}{2}==\frac{4al}{2}=2al, S_{yon}=2\cdot 12l=24l$,

piramidagaichkichizilgansharningradiusi

$r=\frac{2S_{\Delta}}{a+b+c}$formuladantopilishibizgama’lum.$3=\frac{2\cdot h\cdot 12}{\frac{2}{2l+12}}$, $\frac{2h}{l+6}=1$, $2h-6=l$.

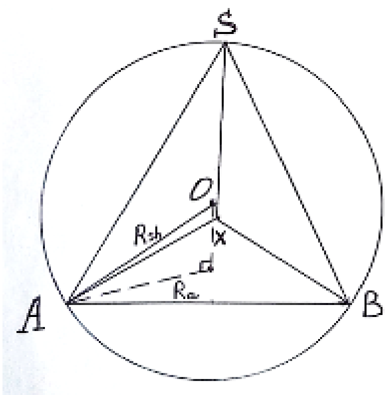
Ikkinchitomondanh^2= l^2-36 bo’ladi. h^2=(2h-6)^2-36,h^2=4h^2-24h+36-36,

3h^2-24h=0, h=8, l=10.S_{yon}=24·10=240

Binobarin, to’g’ri javob:A)240

2.2.5. ([10], 00-6-49) Uchburchakli muntazam piramidaga tashqi chizilgan sharning markazi uning balandligini 6 va 3 ga teng bo’lgan qismlariga ajratadi. Piramidaning hajmini toping (2.2.4- chizma).

A)$\frac{25\sqrt{3}}{4}$ B) $\frac{81\sqrt{3}}{2}$ C) $\frac{729\sqrt{3}}{4}$ D) $\frac{81\sqrt{3}}{4}$ E)$\frac{243\sqrt{3}}{4}$



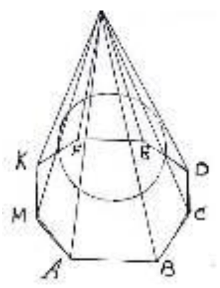
2.2.4- chizma

Yechilishi: 2.2.4- chizmagako’ra, $R_{sh}=6$bo’ladi. Endi$R_a$ni topamiz.$R_a=\frac{a}{2\sin 60^0}$, $R_a=\frac{a}{\sqrt{3}}$ekanligidan foydalanamiz. Pifagor teoremasiga ko’ra, $R_a^2+3^2=6^2$; $\frac{a^2}{3}+9=36$, $\frac{a^2}{3}=27$, $a^2=81, a=9$. Endi piramidaning hajmini topamiz: $V_{pir}=\frac{1}{3}S_{asos}\cdot H==\frac{1}{3}\cdot\frac{\sqrt{3}}{4}a^2\cdot H=\frac{\sqrt{3}}{12}\cdot 81\cdot 9=\frac{243\sqrt{3}}{4}$

Demak, to’g’ri javob:E)$\frac{243\sqrt{3}}{4}$

2.2.6. ([10], 00-9-52) Muntazam sakkizburchakli piramidaning apofemasi 10 ga, uning asosiga ichki chizilgan doiraning yuzi 36π ga teng.Shu piramidaga ichki chizilgan sharning radiusini toping (2.2.5- chizma).

A)1 B) 2 C) 3 D) 4 E)5



2.2.5- chizma

Yechilishi: Berilganpiramidaningasosigaichkichizilgandoiraningyuzi 36π gatengligidanfoydalanib shar radiusinitopamiz: $\pi r^2=36\pi$, $r^2=36, r=6$. Piramidaga ichki chizilgan shar radiusi $r_{shar}=\frac{2S_\Delta}{a+b+c}=\frac{2\cdot\frac{8\cdot 12}{2}}{32}=3$

Binobarin, to'g'ri javob:C) 3

2.2.7. ([10], 98-4-12) Piramidaning hajmi 25 ga, unga ichki chizilgan sharning radiusi 1,5 ga teng. Piramidaning to'la sirtini toping.

A)20 B) 15 C) 25 D) 30 E)50

Yechilishi: Masala shartida piramidaning hajmi va unga ichki chizilgan sharning radiusi berilgan. Shulardan foydalanib piramidaning to'la sirtini topamiz: $V=\frac{1}{3}S_{to'la}\cdot r_{sh}$,$S_{to'la}=\frac{3V}{r_{sh}}$=$\frac{3\cdot 25}{1{,}5}=50$

Demak, to'g'ri javob:E)50

2.2.8. ([10], 02-11-62) Muntazam oltiburchakli piramidaning to'la sirti 2000 ga, hajmi 4800 ga teng. Shu piramidaga ichki chizilgan sharning radiusini toping.

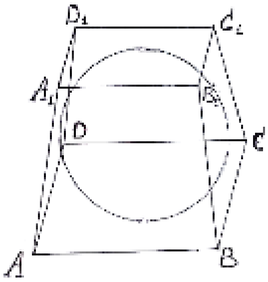
A)4 B) $4{,}5$ C) 7 D) $7{,}2$ E)8

Yechilishi: Masala shartida piramidaning to'la sirti va hajmi berilgan. Shulardan foydalanib piramidaga ichki chizilgan sharning radiusini topamiz: $V=\frac{1}{3}S_{to'la}\times\times r_{sh}, r_{sh}=\frac{3V}{S_{to'la}}$=$\frac{3\cdot 4800}{2000}=7{,}2$

Demak, to'g'ri javob:D) $7{,}2$

2.2.9.([10], 03-2-56) Muntazam to'rtburchakli kesik piramida kichik asosining yuzi 50 ga, katta asosining yuzi 200 ga teng. Piramidaga ichki chizilgan sfera sirtining yuzini toping (2.2.6- chizma).

A)96π B) 125π C) 120π D) 100π E)144π



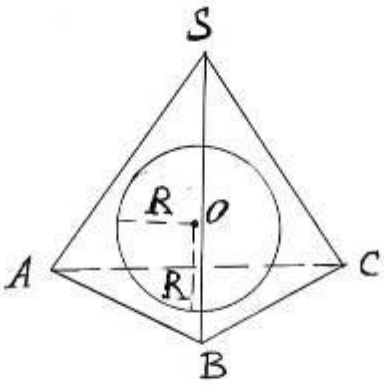
2.2.6- chizma

Yechilishi: Masala shartiga ko'ra, S_{as1}=50, $S_{as2} = 200$ ya'ni $a^2 = 50, a = 5\sqrt{2},\ b^2$=200, b=10$\sqrt{2}$ ga teng. Kesik piramidaning yon yoqlari teng yonli trapetsiyadan iborat ekanligidan foydalanib apofemasini topamiz: $2l = 5\sqrt{2} + 10\sqrt{2} = 15\sqrt{2},\ \ l = 7{,}5\sqrt{2}$.Sfera radiusi esa $(2r)^2 = l^2 - \frac{25}{4} \cdot 2 = \frac{225}{4} \cdot 2 - \frac{25}{4} \cdot 2 = 100,\ 4r^2 = 100, r^2 = 25, r = 5$ ga teng bo'ladi. Endi sfera sirtining yuzini hisoblaymiz: $S_{sfera} = 4\pi r^2 = 4\pi \cdot 25 = 100\pi$.

Demak, to'g'ri javob: D) 100π

2.2.10. ([6], 15.2.6-masala) To'la sirtning yuzi S bo'lgan piramidaga R radiusli sfera ichki chizilgan.Piramidaning hajmi hisoblansin (2.2.7-chizma).

A)$\frac{1}{2}SR$ B) $\frac{1}{8}S^2R$ C) $\frac{R}{4}(S^2 + R^2)$ D)$\frac{S}{4}(S^2 + R^2)$ E)$\frac{1}{3}SR$



2.2.7- chizma

Yechilishi: Masalaniuchburchakli piramida uchun yechamiz. Faraz qilaylik, piramida yoqlarining yuzlari, mos ravishda, $S_{ABS} = S_1$, $S_{BSC} = S_2, S_{ASC} == S_3$, $S_{ABS} = S_4$bo'lsin. Ichki chizilgan sharning O markazidan piramidaning yoqlariga radiuslaro'tkazamiz va bu radiuslar mos yoqlarga perpendikulyar bo'ladi. O markazni piramidaning uchlari bilan tutashtirsak, u to'rtta OSAB, OSBC, OSAC, OABC piramidaga ajraladi. Natijada berilgan piramidaning hajmi shu to'rtta piramida hajmlarining yig'indisiga teng bo'ladi: $V = V_1 + V_2 + V_3 + +V_4$ya'ni

$V = \frac{1}{3}S_1R + \frac{1}{3}S_2R + \frac{1}{3}S_3R + \frac{1}{3}S_4R;\ V = \frac{1}{3}R(S_1 + S_2 + S_3 + S_4).\ S_1 + +S_2 + S_3 + S_4 = S_t = S$

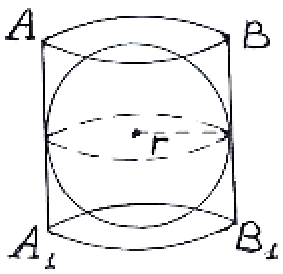
. Shuning uchun piramidaning hajmi $V = \frac{1}{3}S \cdot R$ bo'ladi.

Demak, to'g'ri javob: E) $\frac{1}{3}S \cdot R$

2.3. Silindr va shar kombinatsiyalariga oid testlarni yechish usullari.

2.3.1. ([10], 97-12-58) O'q kesimi kvadratdan iborat silindrga ichki chizilgan sharning hajmi $\frac{9\pi}{16}$ ga teng. Silindrning yon sirtini toping (2.3.1.- chizma).

A)$\frac{3\pi}{4}$ B) $\frac{7\pi}{4}$ C) $\frac{9\pi}{4}$ D) $\frac{5\pi}{4}$ E)3π



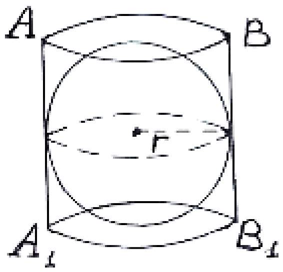
2.3.1.- chizma

Yechilishi: Masala shartida silindrga ichki chizilgan sharning hajmi berilgan. $V_{shar} = \frac{9\pi}{16} = \frac{4}{3}\pi r^3$. Bundan esa r=$\frac{3}{4}$ ekanligi kelib chiqadi. 2.3.1-chizmaga asosan, R=r ga $S_{yon} = 2\pi RH = 2\pi \cdot \frac{3}{4} \cdot 2 \cdot \frac{3}{4} = \frac{9\pi}{4}$

Binobarin, to'g'ri javob:C) $\frac{9\pi}{4}$

2.3.2. ([10], 98-2-58) Teng tomonli silindrga radiusi 3 ga teng bo'lgan shar ichki chizilgan. Silindr va shar sirtlari orasidagi jismning hajmini toping (2.3.2- chizma).

A)27π B) 24π C) 18π D) 12π E)21π



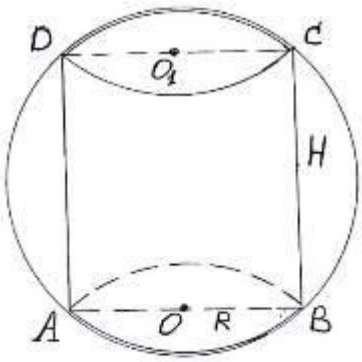
2.3.2- chizma

Yechilishi: 2.3.2- chizmaga ko'ra $r = R$ ga teng. Sharning hajmi esa $V_{shar} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 27 = 36\pi$, Silindrning hajmi$V_{silindr} = S_{asos} \cdot H = \pi R^2 \cdot H = \pi R^2 \cdot 2R = 2\pi \cdot 27 = 54\pi$ ga teng bo'ladi$V_{jism} = V_{silindr} - V_{shar} = 54\pi - 36\pi =$
$= 18\pi$

Demak, to'g'ri javob: C) 18π

2.3.3. ([10], 98-5-53) Radiusi 1 ga teng bo'lgan sferaga ichki chizilgan eng kata hajmli silindrning balandligini aniqlang (2.3.3- chizma).

A)$\frac{3\sqrt{3}}{4}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{2\sqrt{3}}{3}$ D) $\frac{\sqrt{2}}{3}$ E)$\frac{2\sqrt{5}}{3}$



2.3.3- chizma

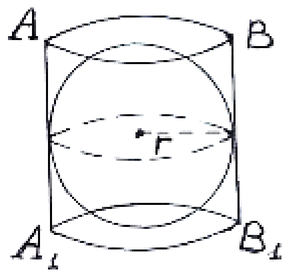
Yechilishi: Pifagorteoremasigako'ra, $(2r)^2 + H^2 = (2R)^2$, $4r^2 + H^2 = 4R^2$, R=1 ekanligidan$r^2 = \frac{4-H^2}{4}$bo'ladi. Silindrning hajmi esa $V_{silindr} = S_{asos} \cdot H = \pi r^2 \cdot H = \pi\left(\frac{4-H^2}{4}\right) \cdot H = \frac{\pi}{4}(4H - H^3)$bo'ladi.

$V_{sil.\max}(H) = \frac{\pi}{4}(4 - 3H^2) = 0,\ \ H^2 = \frac{4}{3},\ \ H = \frac{2\sqrt{3}}{3}$

Binobarin, to'g'ri javob:C)$\frac{2\sqrt{3}}{3}$

2.3.4. ([10], 98-6-57) Teng tomonli silindrga shar ichki chizilgan. Agar sharning hajmi $10\frac{2}{3}\pi$ ga teng bo'lsa, silindrning yon sirtini toping (2.3.4- chizma).

A)12π B) 13π C) 16π D) 15π E)17π



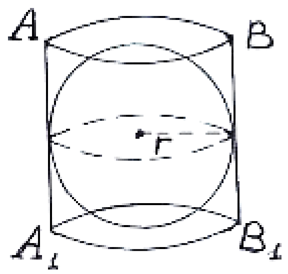
2.3.4- chizma

Yechilishi: Masala shartiga ko'ra, $V_{sh} = 10\frac{2}{3}\pi$ga teng. Bundan esa $\frac{4\pi}{3}r^3$=$\frac{32}{3}\pi$, r^3=8,$r = 2$. 2.3.4- chizmaga ko'ra, $R = r$. Endi silindrning yon sirtini topamiz: $S_{yon} = 2\pi RH = 2\pi r \cdot 2r = 4\pi r^2$. $S_{yon} = 4\pi r^2 = 4\pi \cdot 4 = 16\pi$

Binobarin, to'g'ri javob:C) 16π

2.3.5. ([10], 00-7-48) Silindrga shar ichki chizilgan. Silindrning hajmi 16π ga teng bo'lsa, sharning hajmini toping (2.3.5- chizma).

A)$\frac{32\pi}{3}$ B) $\frac{16\pi}{3}$ C) $\frac{64\pi}{3}$ D) $10\frac{1}{3}\pi$ E)16π



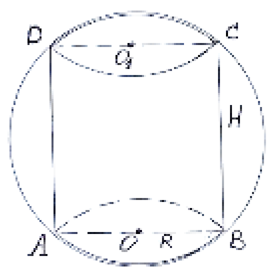
2.3.5- chizma

Yechilishi: 2.3.5- chizmaga ko'ra, $H = 2r$. $r=\frac{H}{2}$ ga teng. Ikkinchi tomondan $4R = 2H,\ H = 2R,\ R = r$ ga teng.$V_{silindr} = 16\pi$, $\pi R^2 \cdot H = 16\pi,\ \pi \cdot r^2 \cdot 2r{=}2\pi r^3 = 16\pi,\ r^3 = 8,\ r = 2$. Endi sharning hajmini topamiz: $V_{shar} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 8 = \frac{32\pi}{3}$

Demak, to'g'ri javob:A)$\frac{32\pi}{3}$

2.3.6.([10], 01-9-20) Radiusi 6 sm bo'lgan metal shardan eng kata hajmli silindr yo'nilgan. Bu silindr asosining radiusi nechaga teng? (2.3.6- chizma)

A)$2\sqrt{6}$ B) 3 C) $\sqrt{6}$ D) 4 E)$14\sqrt{2}$



2.3.6- chizma

Yechilishi: 2.3.6- chizmaga asosan, $(2R)^2 + H^2 = 144, H^2 = 144 - 4R^2, H = \sqrt{144 - 4R^2} = 2\sqrt{36 - R^2}$.

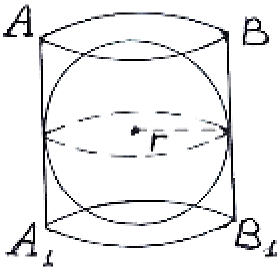
$V_{silindr} = S_{asos} \cdot H = \pi R^2 \cdot H,\ V_{silindr} = \pi R^2 \cdot 2\sqrt{36 - R^2} = 2\pi R^2\sqrt{36 - R^2}$.

$V_{sil.max}(R) = 2\pi\left(2R\sqrt{36-R^2} - \frac{R^2\cdot 2R}{2\sqrt{36-R^2}}\right) = 0,\ 2R(36-R^2)$-$R^3 = 0,$ 72-$2R^2 - R^2 = 0, 3R^2 = 72,\ R^2 = 24,\ R = 2\sqrt{6}$

Demak, to'g'ri javob:A)$2\sqrt{6}$

2.3.7. ([10], 02-3-70) Hajmi 432π ga teng bo'lgan silindrning o'q kesimi kvadratdan iborat.Silindrga ichki chizilgan shar sirtining yuzini toping (2.3.7-chizma).

A)120π B) 134π C) 144π D) 150π E)124π



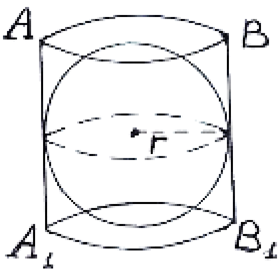
2.3.7- chizma

Yechilishi: Masala shartiga ko'ra, $V_{silindr} = 432\pi$ ga teng. Bundan esa$\pi R^2 \cdot H == 432\pi$, $\pi R^2 \cdot 2R = 432\pi,$ $R^3 = 216,\ R = 6$ bo'ladi. $R = r,\ r = 6.$ Endi shar sirtining yuzini hisoblaymiz: $S_{shar} = 4\pi r^2 = 4\pi \cdot 36 = 144\pi$

Demak, to'g'ri javob:C)144π

2.3.8. ([10], 03-1-46) Teng tomonli silindr shakldagi g'o'ladan eng katta hajmdagi shar yo'nib olindi.G'o'laning qancha foizi chiqindiga ketgan? (2.3.8- chizma)

A)25 B) 30 C) $33\frac{1}{3}$ D) $37\frac{2}{5}$ E)$32\frac{2}{3}$



2.3.8- chizma

Yechilishi: 2.3.8- chizmaga ko'ra: $R = r$. $V_{silindr} = \pi R^2 \cdot H = \pi R^2 \cdot 2R = 2\pi R^3$, $V_{shar} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$.$V_1 = V_{silindr} - V_{shar} = 2\pi R^3 - \frac{4}{3}\pi R^3 = \frac{2}{3}\pi R^3$,

$\frac{V_1 \cdot 100}{V_{silindr}} = \frac{\frac{2}{3}\pi R^3 \cdot 100}{2\pi R^3} = \frac{100}{3} = 33\frac{1}{3}$

Demak, to'g'ri javob: C)$33\frac{1}{3}$

2.3.9. ([6], 15.4.40-masala)Silindrning balandligi h va asosining radiusi r. Silindrga tashqi chizilgan sfera sirtining yuzini hisoblang.

A)$\pi(2r^2 - h^2)$ B) $\pi(4r^2 - 2h^2)$ C) $\pi(4r^2 + h^2)$

D) $\pi(2r^2 + 2h^2)$ E)$\pi(2r^2 + 4h^2)$

Yechilishi: Berilganiga ko'ra shar radiusini topamiz: $(2R)^2 = h^2 + (2r)^2 = h^2 + 4r^2$; $2R = \sqrt{h^2 + 4r^2}$; $R = \frac{\sqrt{h^2+4r^2}}{2}$. Endi sfera sirtining yuzini topamiz:$S_{sf} = 4\pi R^2 = 4\pi \cdot \left(\frac{h^2+4r^2}{4}\right) = \pi(4r^2 + h^2)$

Demak, to'g'ri javob: C)$\pi(4r^2 + h^2)$

2.3.10. ([5], 11.184-masala) Silindr va shar berilgan. Silindr asosining va shar katta doirasining radiuslari teng. Silindr to'la sirtining shar sirtiga bo'lgan nisbati m:n kabi. Ularning hajmlari nisbatini toping.

A)$\frac{6m-3n}{4n}$ B) $\frac{3m}{4n}$ C) $\frac{2m-n}{5}$ D) $\frac{5m}{6n}$ E)$\frac{3n-m}{7}$

Yechilishi: Masala shartiga ko'ra $\frac{S_{t.s}}{S_{sh}} = \frac{m}{n}$ ga teng. $S_{t.s} = 2\pi R_s(R_s + H)$; $S_{sh} = 4\pi R_{sh}^2$; $R_s = R_{sh}$. $\frac{2\pi R_s(R_s+H)}{4\pi R_{sh}^2} = \frac{m}{n}$; $R_s + Hn = 2R_s m$; $H = \frac{R_s(2m-n)}{n}$. Endi ularning hajmlari nisbatini topamiz:$\frac{V_s}{V_{sh}} = \frac{\pi R_s^2 \cdot H}{\frac{4}{3}\pi R_s^3} = \frac{3H}{4R_s} = \frac{3(2m-n)}{4n}$; $\frac{V_s}{V_{sh}} = \frac{6m-3n}{4n}$

Demak, to'g'ri javob: A)$\frac{6m-3n}{4n}$

2.3.11. ([5], 11.080masala) O'q kesimi kvadrat bo'lgan silindrga uchlari silindr o'qining o'rtasida bo'lgan ikkita konus yasalgan. Agar silindr balandligi 2p ga teng bo'lsa, konuslarning to'la sirtlari yig'indisi va hajmlari yig'indisini toping.

A)$2\pi p^2(\sqrt{3} + 1)$; $\frac{2}{5}\pi p^3$ B) $2\pi p^2(\sqrt{2} + 1)$; $\frac{2}{3}\pi p^3$ C) $3\pi p^2(\sqrt{2} + 1)$; $\frac{2}{3}\pi p^3$

D) $4\pi p^2;\ \frac{2}{7}\pi p^3$ E)$2\pi p^2; 4\pi p^3$)

Yechilishi: Masala shartiga asosan, H=2p ga teng. Ushbu konuslar to'la sirtlari va hajmlari tengligidan, ya'ni $V_1 = V_2$va $S_1 = S_2$. $V_1 + V_2 = 2V$; $S_1 + S_2 = 2S$ bo'ladi.$S = \pi R(l + R) = \pi R(\sqrt{2}R + R)$;$S = \pi R^2(\sqrt{2} + 1) = \pi p^2(\sqrt{2} + 1)$.

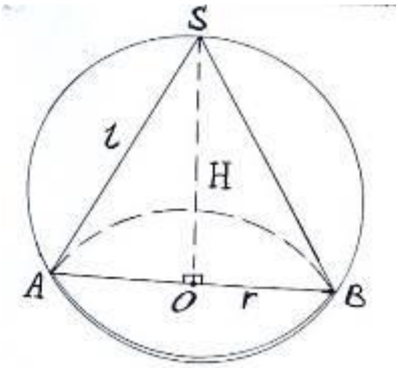
$S_1 + S_2 = 2\pi p^2(\sqrt{2} + 1)$. $V = \frac{1}{3}\pi R^2 \cdot H_k = \frac{1}{3}\pi p^3$; $V_1 + V_2 = \frac{2}{3}\pi p^3$

Binobarin, to'g'ri javob: B) $2\pi p^2(\sqrt{2} + 1);\ \frac{2}{3}\pi p^3$

2.4. Konus va shar kombinatsiyalariga oid testlarni yechish usullari.

2.4.1. ([10], 96-7-52) Sharga ichki chizilgan konusning balandligi 3 ga, asosining radiusi $3\sqrt{3}$ ga teng. Sharning radiusini toping. (2.4.1- chizma)

A)5 B) 6 C) $4\sqrt{3}$ D) $5\sqrt{2}$ E)5,6



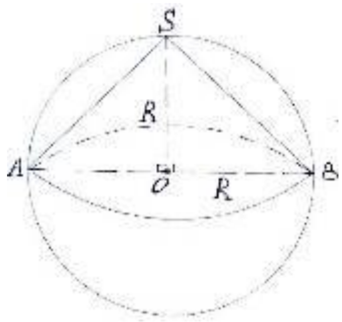
2.4.1- chizma

Yechilishi: To'g'ri burchakli ΔSOA dan, Pifagor teoremasiga ko'ra, konusning yasovchisi $l^2 = H^2 + r^2$, $l^2 = 9 + 27 = 36$, $l = 6$ ga teng. Endi sharning radiusini topamiz: R=$\frac{abc}{4S_\Delta}$=$\frac{6\cdot6\cdot6\sqrt{3}}{4\cdot\frac{3\cdot6\sqrt{3}}{2}}$=6

Demak, to'g'ri javob:B)6

2.4.2. ([10], 97-1-41) Sharga ichki chizilgan konusning asosi sharning eng katta doirasidan iborat. Sharning hajmi konusning hajmidan necha marta katta? (2.4.2- chizma)

A)2 B) 4 C) 3 D) 1,5 E)2,5



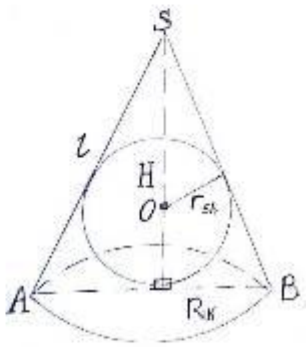
2.4.2- chizma

Yechilishi: Ma'lumki, konusning hajmi $V_k = \frac{1}{3} S_{asos} \cdot H$va shar hajmi $V_{sh} = \frac{4}{3}\pi R^3$ formula bo'yicha hisoblanadi. $V_k = \frac{1}{3}\pi R^2 \cdot R = \frac{1}{3}\pi R^3$. $\frac{V_{sh}}{V_k} = \frac{\frac{4}{3}\pi R^3}{\frac{1}{3}\pi R^3} = 4$

Demak, to'g'ri javob: B)4

2.4.3. ([10], 97-3-52) Konusningbalandligi 6 ga, yasovchisi 10 ga teng. Konusga ichki chizilgan sharning radiusini toping (2.4.3- chizma).

A)3 B) $2\frac{2}{3}$ C) 4 D) $3\sqrt{3}$ E)$2\sqrt{2}$



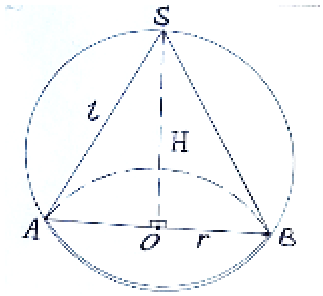
2.4.3 – chizma

Yechilishi: 2.4.3-chizmaga ko'ra, konus asosining radiusi ${R_k}^2 = l^2 - h^2 = 10^2 - 6^2 = 64,\ R_k = 8$ ga teng bo'ladi. Konusga ichki chizilgan sharning radiusi $r_{sh} = \frac{2S_\Delta}{a+b+c} = \frac{2\cdot\frac{6\cdot 16}{2}}{36} = \frac{8}{3} = 2\frac{2}{3}$

Demak, to'g'ri javob: B)$2\frac{2}{3}$

2.4.4.([10], 97-6-41) Sharga konus ichki chizilgan.Konusning yasovchisi asosining diametriga teng. Shar hajmining konus hajmiga nisbatini toping (2.4.4- chizma)

A)$32:9$ B) $8:3$ C) $16:9$ D) $27:4$ E)$9:4$



2.4.4- chizma

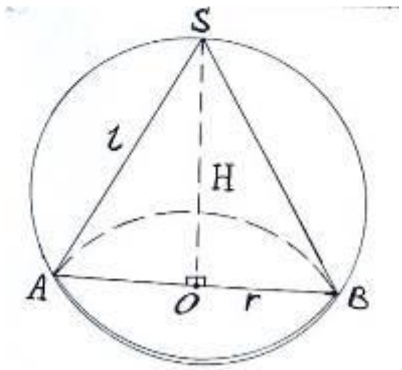
Yechilishi: To'g'ri burchakli ΔSOB dan:$H^2 = l^2 - r^2$, $H^2 = (2r)^2 - r^2 = 3r^2$, $H = \sqrt{3}r$. Shar radiusini topamiz: $R_{sh} = \frac{abc}{4s_\Delta} = \frac{2r\cdot 2r\cdot 2r}{4\cdot\frac{\sqrt{3}}{4}\cdot 4r^2} = \frac{2r}{\sqrt{3}}$,

$$\frac{V_{sh}}{V_k} = \frac{\frac{4}{3}\pi R_{sh}^3}{\frac{1}{3}\pi r^2\cdot h} = \frac{\frac{4}{3}\pi\frac{8r^3}{3\sqrt{3}}}{\frac{1}{3}\pi r^2\cdot\sqrt{3}r} = \frac{32}{9} = 32:9$$

Demak, to'g'ri javob:A)32:9

2.4.5. ([10], 97-7-52) Balandligi 3 ga, yasovchisi 6 ga teng bo'lgan konusga tashqi chizilgan sharning radiusini toping (2.4.5- chizma).

A)$3\sqrt{3}$ B) 5 C) 6 D) $4\sqrt{2}$ E)4,5



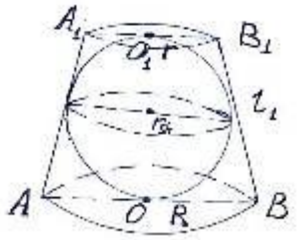
2.4.5- chizma

Yechilishi: To'g'ri burchakli ΔSOB dan, Pifagor teoremasiga asosan: $l^2 = H^2 + +r^2$, $r^2 = l^2 - H^2 = 36 - 9 = 27$, $r = 3\sqrt{3}$. Endi shar radiusini topamiz: R=$\frac{abc}{4S_\Delta}$=$\frac{6\cdot 6\cdot 6\sqrt{3}}{4\cdot\frac{3\cdot 6\sqrt{3}}{2}}$=6

Binobarin, to'g'ri javob:C)6

2.4.6. ([10], 97-9-79) Sharga tashqi chizilgan kesik konusning yasovchilari o'rtalaridan o'tuvchi tekislik bilan shu kesik konus hosil qilgan kesimning yuzi 4π ga teng. Kesik konusning yasovchisini toping (2.4.6- chizma).

A)2 B) 4 C) 3 D) 5 E)6



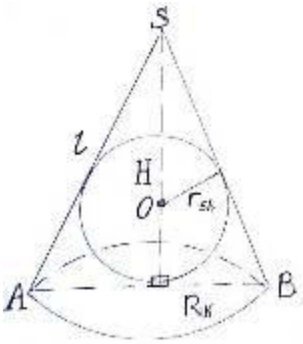
2.4.6- chizma

Yechilishi: Kesik konusning o'q kesimi teng yonli trapetsiyadan iborat bo'lgani uchun 2r+2R=$2l_1$, $r + R = l_1$. Kesim yuzi $\pi r_{sh}^2 = 4\pi$,$r_{sh} = 2$. $\frac{2r+2R}{2} = 2r_{sh}$, $2r + R = 2r_{sh}$,l_1 =$2r_{sh} = 2 \cdot 2 = 4$

Binobarin, to'g'ri javob:B)4

2.4.7. ([10], 97-10-52) Yasovchisi 10 ga asosining radiusi 6 ga teng bo'lgan konusga ichki chizilgan sharning radiusini toping (2.4.7- chizma)

A)3 B) 4 C) $3\sqrt{3}$ D) $2\sqrt{2}$ E)$3\frac{1}{3}$



2.4.7- chizma

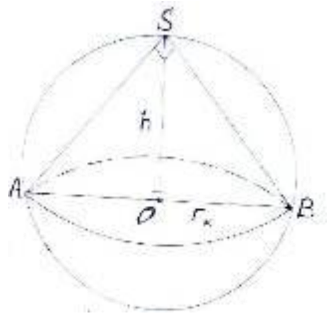
Yechilishi: 2.4.7- chizmaga asosan: $H^2 = l^2 - {R_k}^2$=$10^2 - 6^2 = 64$, $H = 8$.

$r_{sh} = \frac{2S_\Delta}{a+b+c} = \frac{2 \cdot \frac{8 \cdot 12}{2}}{32}$=3.

Binobarin, to'g'ri javob:A)3

2.4.8. ([10], 97-11-41) Sharga ichki chizilgan konusning o’q kesimi teng yonli to’g’riburchakli uchburchakdan iborat. Konusning hajmi shar hajmining qanday qismini tashkil etadi? (2.4.8- chizma)

A)0,25 B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) $\frac{3}{7}$ E)0,75



2.4.8- chizma

Yechilishi: 2.4.8- chizmaga asosan:

$h = \frac{ab}{c} = \frac{\sqrt{2}r_k \cdot \sqrt{2}r_k}{2r_k} = r_k$. $R_{sh} = \frac{abc}{4S_\Delta} = \frac{\sqrt{2}r_k \cdot \sqrt{2}r_k \cdot 2r_k}{4 \cdot \frac{\sqrt{2}r_k \cdot \sqrt{2}r_k}{2r_k}} =$

r_k. $\frac{V_k}{V_{sh}} = \frac{\frac{1}{3}S_{as} \cdot h}{\frac{4}{3}\pi R_{sh}^3} = \frac{\frac{1}{3}\pi r_k^2 \cdot h}{\frac{4}{3}\pi R_{sh}^3} = \frac{r_k^2 \cdot r_k}{4r_k^3} = \frac{1}{4} = 0{,}25$

Binobarin, to’g’ri javob: A)0,25

2.4.9. ([10], 99-2-55) Kesik konusga shar ichki chizilgan.Konusning ustki asosining yuzi 36π ga, ostki asosining yuzi esa $64\,\pi$ ga teng. Shar sirtining yuzini toping.

A) 172π B)100π C) 144π D) 156π E)192π

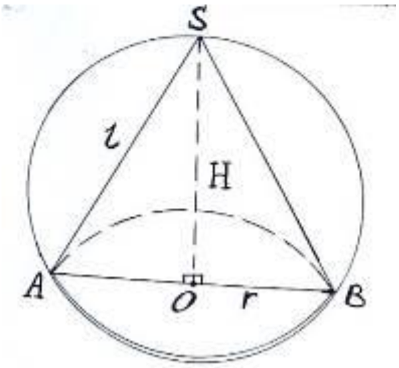
Yechilishi: Masala shartiga ko’ra, $\pi r^2 = 36\pi$, $\pi R^2 = 64\pi$. Shu sababli r=6 va R=8 bo’ladi. Kesik konusning o’q kesimi teng yonli trapetsiyadan iborat ekanligidan 2r+2R=$2l_1$, r+R=l_1, $l_1 = 6 + 8 = 14$.

$h^2 = l_1^2 - (\frac{2R-2r}{2})^2 = 196 - 4 = 192$, $h = \sqrt{192}$. $2r_{sh} = h$, $r_{sh} = \frac{\sqrt{192}}{2}$. Endi shar sirtining yuzini hisoblaymiz: $S_{shar} = 4\pi r_{sh}^2 = 4\pi \cdot \frac{192}{4} = 192\pi$

Binobarin, to’g’ri javob:E)192π

2.4.10. ([10], 99-2-55) Radiusi 5 ga teng bo’lgan sharga ichki chizilgan konusning balandligi 4 ga teng.Konusning hajmini toping (2.4.9-chizma).

A) 28π B)18π C) 24π D) 32π E)16π



2.4.9-chizma

Yechilishi: Berilganiga ko'ra $R=5,\ H=4$ ga teng. $R=\frac{abc}{4S_{\Delta}}=\frac{l\cdot l\cdot 2r}{4\cdot\frac{4\cdot 2r}{2}}=\frac{l^2}{8}$,

$R=\frac{l^2}{8}=5.\ \ l^2=40.\ l^2=H^2+r^2=16+r^2,\ 16+r^2=40,\ r^2=24,$

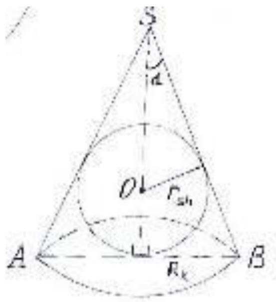
$r=2\sqrt{3}.$ Endi konus hajmini topamiz:

$V_k=\frac{1}{3}S_{asos}\cdot H=\frac{1}{3}\pi R_k^2\cdot H=\frac{1}{3}\pi\cdot 24\cdot 4=32\pi$

Demak, to'g'ri javob: E) 32π

2.4.11. ([10], 99-3-49) Radiusi 2 ga teng shar konusga ichki chizilgan, Konus yasovchisi va balandligi orasidagi burchak 30°ga teng. Konusning yon sirtining yuzini toping (2.4.10-chizma).

A) 24π B)4π C) 16π D) 18π E)20π



2.4.10-chizma

Yechilishi: 2.4.10-chizmaga ko'ra $SB=l$ deb olamiz va $\frac{R_k}{l}=sin\alpha,\ \ R_k=lsin\alpha,\ \frac{h}{l}=cos\alpha,\ h=lcos\alpha. r_{sh}=\frac{2S_{\Delta}}{a+b+c}=\frac{2llsin2\alpha/2}{2l+2R_k}=$ $\frac{l^2sin2\alpha}{2(l+R_k)}=\frac{l^2sin2\alpha}{2(l+lsin\alpha)}=\frac{lsin2\alpha}{2(1+sin\alpha)}$

, $l = \frac{2r_{sh}(1+sin\alpha)}{sin2\alpha} = \frac{2\cdot2(1+\frac{1}{2})}{\frac{\sqrt{3}}{2}} = 4\sqrt{3}$, $R_k = 4\sqrt{3}\cdot\frac{1}{2} = 2\sqrt{3}$.

$S_{yon} = \pi R_k \cdot l = \pi\cdot 2\sqrt{3}\cdot 4\sqrt{3} = 24\pi$

Binobarin, to'g'ri javob:A) 24π

2.4.12. ([10], 99-4-48) Balandligi 6 ga yasovchisi 10 ga teng konusga ichki chizilgan sharning sirtini toping.

A) $\frac{32\pi}{3}$ B)$\frac{64\pi}{3}$ C) $\frac{256\pi}{9}$ D)$\frac{64\pi}{9}$ E)$\frac{128\pi}{9}$

Yechilishi: Pifagor teoremasiga ko'ra, l^2=$R_k^2 + h^2$,$R_k^2 = l^2 - h^2 = 10^2 - 6^2 = 64$,$R_k$=8.

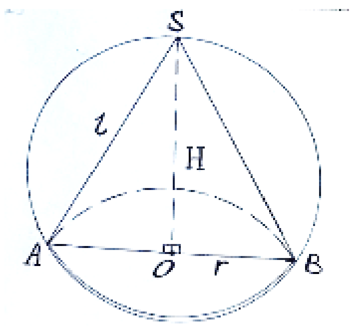
$r_{sh} = \frac{2S_\Delta}{a+b+c} = \frac{2\cdot\frac{h\cdot 2R_k}{2}}{2l+2R_k} = \frac{2\cdot6\cdot8}{2\cdot(10+8)} = \frac{48}{18} = \frac{8}{3}$. Endi sirtini topamiz:

$S_{sh} = 4\pi r_{sh}^2 = 4\pi\frac{64}{9} = \frac{256}{9}\pi$

Binobarin, to'g'ri javob:C)$\frac{256\pi}{9}$

2.4.13. ([10], 99-9-46) Radiusi 10 ga teng bo'lgan sferaga balandligi 18 ga teng bo'lgan konus ichki chizilgan.Konusning hajmini toping (2.4.11-chizma).

A) 210π B)216π C) 220π D) 228π E)204π



2.4.11-chizma

Yechilishi: $R = \frac{abc}{4S_\Delta} = \frac{l\cdot l\cdot 2r}{4\cdot\frac{h\cdot 2r}{2}} = \frac{l^2}{2h}$, $l^2 = 2h\cdot R = 2\cdot 18\cdot 10 = 360$; $l = 6\sqrt{10}$.

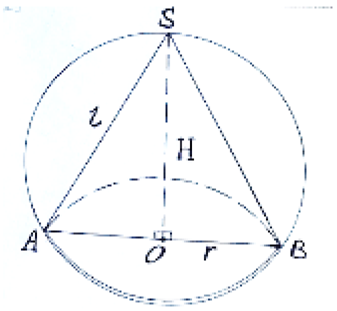
Pifagor teoremasiga ko'ra, $r^2 + H^2 = l^2$, $r^2 = l^2 - H^2 = 360 - 324 = 36$, $r == 6$,

$V_k = \frac{1}{3}S_{asos}\cdot H = \frac{1}{3}\pi r^2\cdot H = \frac{1}{3}\pi\cdot 36\cdot 18 = 216\pi$

Demak, to'g'ri javob:B) 216π

2.4.14. ([10], 00-3-84) Sharga balandligi asosining diametriga teng bo'lgan konus ichki chizilgan. Agar konusning asosiy yuzi 2,4 ga teng bo'lsa shar sirtining yuzini toping (2.4.12-chizma).

A) 9π B)6 C) $12{,}5\pi$ D) 15 E)10π



2.4.12-chizma

Yechilishi: To'g'ri burchakli ΔSOB dan: $l^2 = H^2 + r^2$,$l^2 = (2r)^2 + r^2 = 5r^2$ va $l = \sqrt{5}r$. Berilganiga ko'ra, $S_{asos} = 2{,}4$, $\pi r^2 = 2{,}4$,

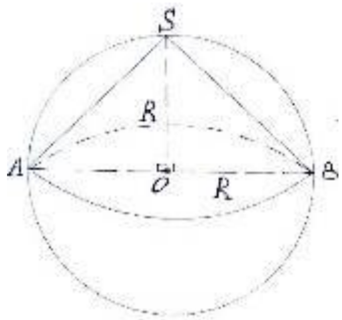
$r = \sqrt{\frac{2{,}4}{\pi}}$; $R = \frac{abc}{4S_\Delta} = \frac{l^2 \cdot 2r}{4 \cdot \frac{2r \cdot 2r}{2}} = \frac{l^2}{4r} = \frac{5r^2}{4r} = \frac{5}{4}r$,

$R = \frac{5}{4}\sqrt{\frac{2{,}4}{\pi}}$, $S_{shar} = 4\pi R^2 = 4\pi \cdot \frac{25}{16} \cdot \frac{2{,}4}{\pi} = 15$

Demak, to'g'ri javob:D) 15

2.4.15. ([10], 00-9-7) Sharga ichki chizilgan konusning asosi sharning katta doirasiga teng.Konus o'q kesimining yuzi 9 ga teng sharning hajmini toping (2.4.13-chizma).

A) 30π B)32π C) 42π D) 36π E)48π



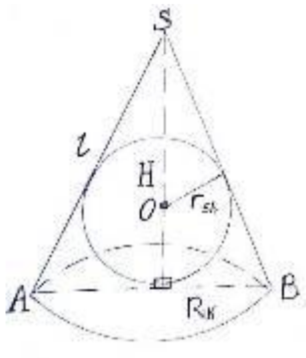
2.4.13-chizma

Yechilishi: Berilganiga ko'ra: $S_{k.o'qkesim} = 9 \cdot \frac{R \cdot 2R}{2} = 9$, $R^2 = 9,\ R = 3.\ V_{sh} == \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \cdot 27 = 36\pi$

Binobarin, to'g'ri javob: D)36π

2.4.16. ([10], 01-2-50) O'q kesimi teng tomonli uchburchakdan iborat konusga diametri D ga teng sfera ichki chizilgan.Konusning to'la sirtini toping (2.4.14-chizma).

A)$\frac{3}{2}\pi D^2$ B)$\frac{5}{2}\pi D^2$ C) $\frac{3}{4}\pi D^2$ D)$\frac{5}{4}\pi D^2$ E)$\frac{9}{4}\pi D^2$



2.4.14-chizma

Yechilishi: 2.4.14-chizmaga asosan: ${R_k}^2 + H^2 = 4{R_k}^2,\ H^2 = 3R^2, H = \sqrt{3}R_k$. Berilganiga ko'ra D=$2r_{sf}$, ikkinchi tomondan

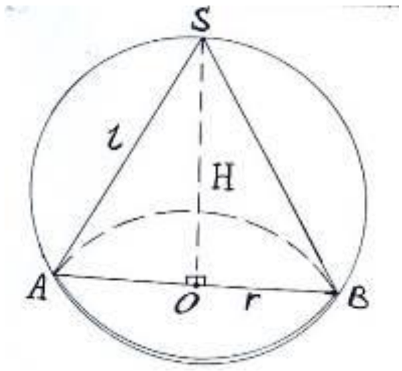
$r_{sf} = \frac{2S_\Delta}{a+b+c} = \frac{2\cdot\frac{H\cdot 2R_k}{2}}{6R_k} = \frac{H}{3}.\ r_{sf} = \frac{H}{3} = \frac{\sqrt{3}R_k}{3} = \frac{R_k}{\sqrt{3}};\ \frac{D}{2} = \frac{R_k}{\sqrt{3}}; R_k = \frac{\sqrt{3}}{2}D.$

$S_{to'la} = S_{asos} + S_{yon} = \pi R_k^2 + \pi R_k l = \pi R_k^2 + \pi R_k \cdot 2R_k = 3\pi R_k^2 = 3\pi \cdot \frac{3}{4}D^2 = \frac{9}{4}D^2$

Demak, to'g'ri javob:E)$\frac{9}{4}D^2$

2.4.17. ([10], 01-3-29) Sharga konus shunday ichki chizilganki konusning yasovchisi asosining diametriga teng.Konusning to'la sirti shar sirti yuzining necha foizini tashkil etadi?(2.4.15-chizma).

A)62 B) 56,25 C)54,5 D)60,75 E)48



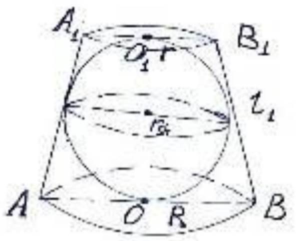
2.4.15-chizma

Yechilishi: To'g'ri burchakli ΔSOBdan: $H^2 = l^2 - r^2 = 4r^2 - r^2 = 3r^2$va $H = \sqrt{3}R$. Shar radiusini hisoblaymiz: $R = \frac{abc}{4S_\Delta} = \frac{2r\cdot 2r\cdot 2r}{4\cdot\frac{h\cdot 2r}{2}} = \frac{2r^2}{h} = \frac{2r^2}{\sqrt{3}r} = \frac{2r}{\sqrt{3}}$; $S_{k.to'la} = S_{asos} + S_{yon} = \pi r^2 + \pi r l = \pi r^2 + \pi r\cdot 2r = 3\pi r^2$. $S_{shar} = 4\pi R^2 = 4\pi\cdot\frac{4\pi r^2}{3} = \frac{16\pi}{3}$. $\frac{S_{k.to'la}\cdot 100\%}{S_{shar}} = \frac{3\pi r^2\cdot 100\%}{\frac{16\pi}{3}r^2} = 56{,}25$

Demak, to'g'ri javob: B) $56{,}25$

2.4.18. ([10], 01-4-16) Kesik konusning yon sirti 10π ga, to'la sirti 18π ga teng.Konusning to'la sirti unga ichki chizilgan shar sirtidan qancha ko'p?(2.4.16-chizma).

A)14π B) 6π C)8π D)10π E)12π



2.4.16-chizma

Yechilishi: Kesik konusning o'q kesimi teng yonli trapetsiyadan iborat bo'lgani uchun 2r+2R=2l_1, r+R=l_1, h=2

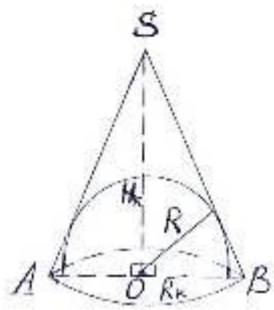
r_{sh}. $l_1^2 = h^2 + (R-r)^2 = 4r_{sh}^2 + (R-r)^2 = (R+r)^2.4r_{sh}^2 = r^2 + 2Rr + r^2 - R^2 + 2Rr - r^2 = 4Rr.\ r_{sh}^2 = Rr$

. Berilganiga ko'ra: $S_{k.to'la} = S_{asos1} + S_{asos2} + S_{yon} = \pi r^2 + \pi R^2 + 10\pi = 18\pi,\ r^2 + R^2 = 8. S_{k.yon} = \pi(R+r)l_1 = \pi(R+r)(R+r) = \pi(R+r)^2 = 10\pi. (R+r)^2 = 10,\ r^2 + 2Rr + r^2 = 10,\ \ 8 + 2Rr = 10,\ 2Rr = 2, Rr = 1,\ r_{sh}^2 = 1.\ S_{shar} = 4\pi r_{sh}^2 = 4\pi.\ S_{k.to'la} - S_{shar} = 18\pi - 4\pi = 14\pi$

Demak, to'g'ri javob: A) 14π

2.4.19. ([10], 01-4-33) R radiusli yarim sharga asosining markazi bilan ustma-ust tushadigan konus tashqi chizilgan.Konusning balandligi qanday bo'lganida, uning hajmi eng kichik bo'ladi? (2.4.17-chizma).

A)$\sqrt{3}$ R B)$\frac{\sqrt{3}}{2}R$ C)$\sqrt{2}R$ D)$\frac{\sqrt{2}}{2}R$ E)$\frac{1}{\sqrt{3}}R$



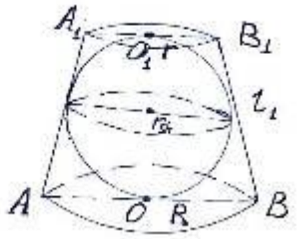
2.4.17-chizma

Yechilishi: 2.4.17-chizmaga asosan: $l^2 = H_k^2 + R_k^2$. R= $\frac{H_k \cdot R_k}{l} = \frac{H_k \cdot R_k}{\sqrt{H_k^2 + R_k^2}}$, $R \cdot \sqrt{H_k^2 + R_k^2} = H_k \cdot R_k,\ R^2 \cdot (H_k^2 + R_k^2) = H_k^2 R_k^2,\ R^2 \cdot H_k^2 + R^2 \cdot R_k^2 = H_k^2 R_k^2,\ R_k^2 = \frac{R^2 \cdot H_k^2}{(H_k^2 - R^2)}, V_k = \frac{1}{3} S_{asos} \cdot H_k = \frac{1}{3}\pi R_k^2 \cdot H_k = \frac{1}{3}\pi \frac{R^2 \cdot H_k^2}{(H_k^2 - R^2)} \cdot H_k = \frac{\pi}{3} \cdot \frac{R^2 \cdot H_k^3}{H_k^2 - R^2}. V_{k\ min}(H) = \frac{1}{3}\pi R^2 \cdot \frac{3H_k^2(H_k^2 - R^2) - 2H_k^4}{(H_k^2 - R^2)^2} = 0,\ 3H_k^4 - 3H_k^2 R^2 - 2H_k^4 = 0,\ \ H_k^4 - 3H_k^2 R^2 = 0, H_k^2(H_k^2 - 3R^2) = 0. H_k^2 = 3R^2, H_k = \sqrt{3}R$

Demak, to'g'ri javob: A) $\sqrt{3}R$

2.4.20. ([10], 01-10-47) Kesik konus asoslarining yuzlari 9πva 25π ga teng. Agar bu konusga sharni ichki chizish mumkin bo'lsa, konusning yon sirtini toping? (2.4.18-chizma).

A)80π B) 36π C)54π D)64π E)144π



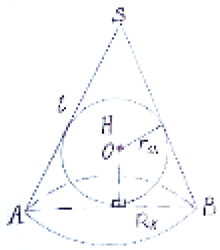
2.4.18-chizma

Yechilishi: Berilganiga ko'ra: $S_1 = 9\pi,\ S_2 = 25\pi,\ \pi r^2 = 9\pi, r = 3; \pi R^2 = 25\pi R = 5$. Kesik konusning o'q kesimi teng yonli trapetsiyadan iborat bo'lgani uchun: $2l_1 = 2r + 2R,\ l_1 = r + R, l_1 = 3 + 5 = 8$. $S_{kyon} = \pi \cdot 8 \cdot 8 = 64\pi$

Demak, to'g'ri javob: D) 64π

2.4.21. ([10], 02-5-53) Yasovchisi 5 ga asosining diametri 6 ga teng bo'lgan konusga ichki chizilgan shar sirtining yuzini toping (2.4.19-chizma).

A)16π B) $\frac{64}{11}\pi$ C)9π D)$\frac{71}{9}\pi$ E)8π



2.4.19-chizma

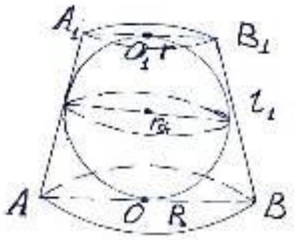
Yechilishi: Berilganiga ko'ra: $d = 6, 2R_k = \text{d},\ 2R_k = 6, R_k = 3$

Pifagor teoremasiga ko'ra: $l^2 = H^2 + R_k^2,\ H^2 = l^2 - R_k^2 = 25 - 9 = 16,\ H = 4$. $r_{sh} = \frac{2S_\Delta}{a+b+c} = \frac{2 \cdot \frac{h \cdot 2R_k}{2}}{2l + 2R_k} = \frac{h \cdot 2R_k}{l + R_k}$; $r_{sh} = \frac{4 \cdot 3}{5+3} = \frac{12}{8} = \frac{3}{2} = 1{,}5$. $S_{shar} = 4\pi r_{sh}^2 = 4\pi \frac{9}{4} = 9\pi$

Demak, to'g'ri javob: C) 9π

2.4.22. ([10], 02-6-52) Kesik konusga shar ichki chizilgan agar kesik konus asoslarining yuzlari πva 4π ga teng bo'lsa shu konus yon sirtining yuzini toping (2.4.20-chizma).

A)6π B) 7π C)8π D)9π E)10π



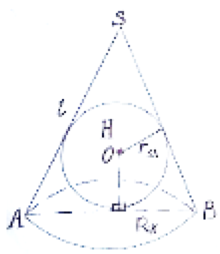
2.4.20-chizma

Yechilishi: Masala shartiga ko'ra: $S_1 = \pi \;\; va\, S_2 = 4\pi$; $\pi r^2 = \pi$; $r = 1, \pi R^2 = 4\pi$; $R = 2$. $2l_1 = 2r + 2R,\; l_1 = r + R$; $l_1 = 1 + 2 = 3$. $S_{k.yon} = \pi(r+R)l_1 == \pi \cdot 3 \cdot 3 = 9\pi$

Binobarin, to'g'ri javob: D) 9π

2.4.23 ([10], 02-8-35) Radiusi 4 ga teng bo'lgan sharga balandligi 9 ga teng bo'lgan konus tashqi chizilgan.Konus asosining radiusini toping (2.4.21-chizma).

A)12 B) 9 C)10 D)8 E)14



2.4.21-chizma

Yechilishi: Berilganiga ko'ra: $r_{sh} = 4$. $r_{sh} = \frac{2S_\Delta}{a+b+c} = \frac{2\cdot\frac{H\cdot 2R_k}{2}}{2l+2R_k} = \frac{H\cdot R_k}{l+R_k} = 4$;

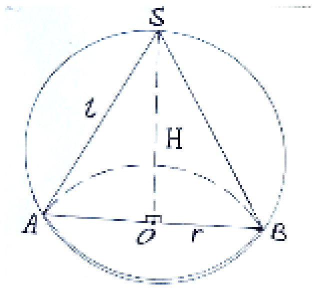
$H \cdot R_k = 4(l + R_k)$. $9 \cdot R_k = 4l + 4R_k$; $5R_k = 4l,\; l = \frac{5}{4}R_k$2.4.21-chizmaga asosan:

$l^2 = H^2 + R_k^2, H^2 = l^2 - R_k^2 = \frac{25}{16}R_k^2 - R_k^2 = \frac{9R_k^2}{16}. H = \frac{3}{4}R_k, R_k = \frac{4}{3} \cdot 9 = 12.$

Demak, to'g'ri javob: A) 12

2.4.24. ([10], 02-9-60) Teng tomonli konusga shar tashqi chizilgan shar sirtining konusning to'la sirtiga nisbatini toping (2.4.22-chizma).

A)$9:4$ B) $7:4$ C)$16:9$ D)$4:3$ E)$9:5$



2.4.22-chizma

Yechilishi:To'g'ri burchakli ΔSOB dan: $H^2 = l^2 - r^2 = 4r^2 - r^2 = 3r^2$,

$H = \sqrt{3}r$. Konusning o'q kesimi teng tomonli uchburchakligidan

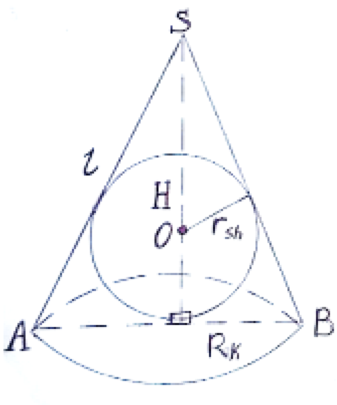
$R = \frac{2}{\sqrt{3}}r$. $S_{k.to'la} = S_{asos} + S_{yon} = \pi r^2 + \pi rl = \pi r^2 + \pi r \cdot 2r =$

$3\pi r^2$. $\frac{S_{shar}}{S_{k.to;la}} = \frac{4\pi R^2}{3\pi r^2} = \frac{4\frac{4}{3}r^2}{3\pi r^2} = 16:9$

Demak, to'g'ri javob: C) $16:9$

2.4.25. ([10], 03-2-16) Konus asosining yuzi 9π ga, yon sirtining yuzi 15π ga teng.Shu konusga ichki chizilgan sferaning radiusini toping (2.4.23-chizma).

A)$1{,}5$ B) $1{,}8$ C)2 D)$2{,}4$ E)$2{,}5$



2.4.23-chizma

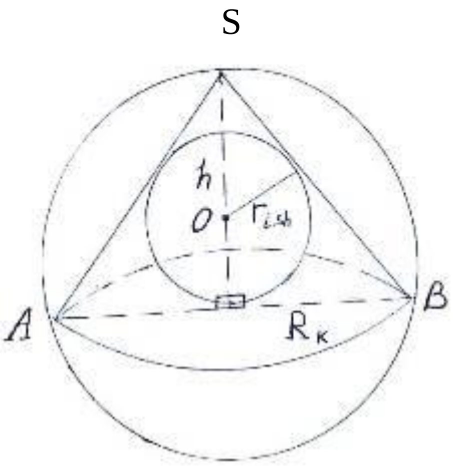
Yechilishi: Masala shartiga ko'ra: $S_{asos} = 9\pi$,$\pi R_k^2 = 9\pi$, $R_k = 3$; $S_{k.yon} = 15\pi$,

$\pi R_k \cdot l = 15\pi,\ 3 \cdot l = 15,\ l = 5.\ H^2 + R_k^2 = l^2,\ h^2 = l^2 - R_k^2 = 25 - 9 = 16.\ r_{sf} = \frac{h \cdot R_k}{l + R_k} = \frac{4 \cdot 3}{5+3} = \frac{12}{8} = \frac{3}{2} = 1{,}5$

Demak, to'g'ri javob: A) $1{,}5$

2.4.26. ([10], 03-2-55) Teng tomonli konusga ichki va tashqi shar chizildi. Ichki chizilgan shar hajmi tashqi chizilgan shar hajmining necha foizini tashkil etadi? (2.4.24-chizma)

A)10 B) $12{,}5$ C)20 D)25 E)$22{,}5$



2.4.24-chizma

Yechilishi: 2.4.24-chizmaga asosan, $SB = l$ deb olamiz va $l^2 = {R_k}^2 + h^2; 4R_k^2 == h^2 + R_k^2, h^2 = 3R_k^2,\ R = \sqrt{3}R_k,\ R_{t.sh} = \frac{2R_k}{\sqrt{3}}.\ r_{i.sh} = \frac{R_k}{\sqrt{3}}.$

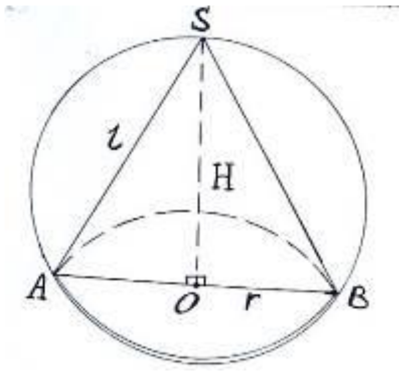
$V_{i.sh} = \frac{4}{3}\pi r_{i.sh}^3 = \frac{4}{3}\pi \frac{R_k^3}{3\sqrt{3}} == \frac{4\pi R_k^3}{9\sqrt{3}}.\quad V_{t.sh} = \frac{4}{3}\pi R_{t.sh}^3 = \frac{4}{3}\pi \frac{8R_k^3}{3\sqrt{3}} = \frac{32\pi R_k^3}{9\sqrt{3}}$

$$\frac{V_{i.sh} \cdot 100}{V_{t.sh}} = \frac{\frac{4\pi R_k^3}{9\sqrt{3}} \cdot 100}{\frac{32\pi R_k^3}{9\sqrt{3}}} = 100 \cdot \frac{1}{8} = 12{,}5$$

Demak, to'g'ri javob: A) $12{,}5$

2.4.27. ([10], 03-6-82) Radiusi R ga teng bo'lgan shar ichiga chizilgan eng katta hajmga ega bo'lgan konusning balandligini toping (2.4.25-chizma).

A)$\frac{2}{3}R$ B) $\frac{1}{3}R$ C)$1\frac{2}{3}R$ D)$\frac{\sqrt{3}}{2}R$ E)$1\frac{1}{3}R$



2.4.25-chizma

Yechilishi: To'g'ri burchakli ΔSOB dan: $l^2 = H^2 + r^2,\ R = \frac{abc}{4S_\Delta} = \frac{l \cdot l \cdot 2r}{4 \cdot \frac{H \cdot 2r}{2}} = \frac{H^2 + r^2}{2H}; 2R \cdot H = H^2 + r^2 . r^2 = 2HR - H^2.$

$V_k = \frac{1}{3}\pi r^2 \cdot H = \frac{1}{3}\pi(2HR - H^2)H == \frac{1}{3}\pi(2H^2R - H^3).$

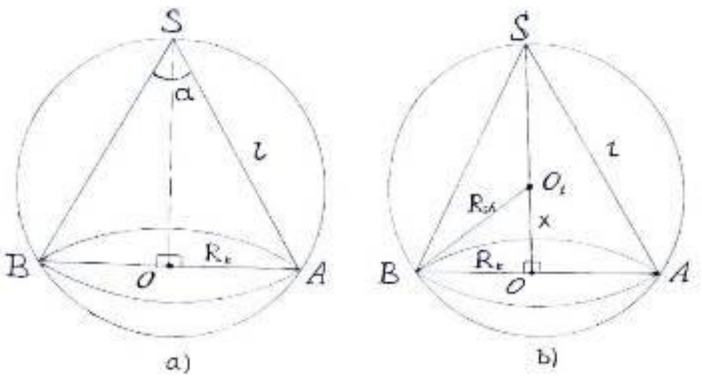
$V'_{k,max}(H) = \frac{1}{3}\pi(4HR - 3H^2) = 0. \frac{1}{3}\pi(4HR - 3H^2) = 0.$

$4HR - 3H^2 = 0.\ H(4R - 3H) = 0,\ H = \frac{4}{3}R = 1\frac{1}{3}R$

Demak, to'g'ri javob: E) $1\frac{1}{3}R$

2.4.28. ([10], 03-7-82) Radiusi 1 ga teng bo'lgan sharga yasovchisi $\sqrt{3}$ ga teng bo'lgan konus ichki chizildi.Shu konus o'q kesimining uchidagi burchakning kattaligini toping (2.4.26-chizma).

A) $90°$ B) $30°$ C) $45°$ D) $60°$ E) $arccos\frac{2}{3}$



2.4.26-chizma

Yechilishi: *1-usul:* Masala shartiga ko'ra: $R_{sh} = 1,\ l = \sqrt{3}$ ga teng 2.4.26-a chizmaga ko'ra: $\frac{R_k}{l} = sin\frac{\alpha}{2}$, $R_k = \sqrt{3}sin\frac{\alpha}{2}$.

$R_{sh} = \frac{abc}{4S_\Delta} = \frac{l \cdot l \cdot 2R_k}{4 \cdot \frac{l \cdot l \cdot sin\alpha}{2}} = \frac{R_k}{sin\alpha} = 1; R_k = sin\alpha = 2sin\frac{\alpha}{2}cos\frac{\alpha}{2} =,$

$R_k = \sqrt{3} \cdot sin\frac{\alpha}{2}; cos\frac{\alpha}{2} = \frac{\sqrt{3}}{2}, \frac{\alpha}{2} = 30°,\ \alpha = 60°$

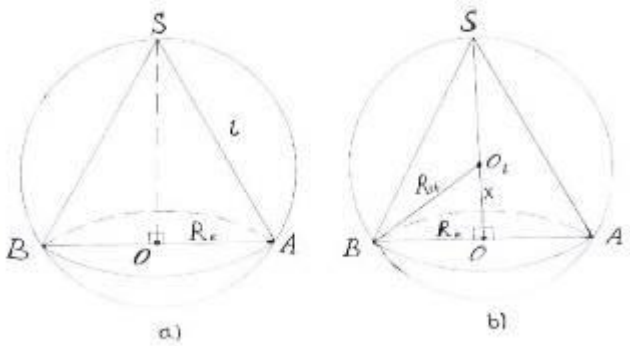
Demak, to'g'ri javob: D) $60°$

2-usul: 2.4.26-b chizmaga ko'ra: $x^2 + R_k^2 = R_{sh}^2,$ $x^2 + R_k^2 = 1,$ $R_k^2 + (R_{sh} + x)^2 = l^2,$ $R_k^2 + (1 + x)^2 = 3.$ $R_k^2 + 1 + 2x + x^2 = 3,$ 2x=1, $x = \frac{1}{2}.$ $R_k^2 = 1 - x^2 = 1 - \frac{1}{4} = \frac{3}{4}, R_k = \frac{\sqrt{3}}{2}.$ $\frac{R_k}{l} = sin\frac{\alpha}{2};\ sin\frac{\alpha}{2} = \frac{1}{2},$ $\frac{\alpha}{2} = 30°, \alpha = 60°$

Demak, to'g'ri javob: D) $60°$

2.4.29. ([10], 03-9-56) Sferaga balandligi asosining diametriga teng bo'lgan konus ichki chizilgan.Agar sfera sirtining yuzi 125 ga teng bo'lsa, konus asosining yuzini toping (2.4.27-chizma).

A)10 B) 10π C)15 D)20 E)20π



2.4.27-chizma

Yechilishi: *1-usul:* Berilganiga ko'ra: $S_{sfera} = 125,\ 4\pi R_s^2 = 125. R_s^2 = \frac{125}{4\pi},$ $R_s = \frac{5\sqrt{5}}{2\sqrt{\pi}}.$ 2.4.27-a chizmaga ko'ra: $l^2 = 4R_k^2 + R_k^2 = 5R_k^2.$ $R_s = \frac{abc}{4S_\Delta} = \frac{l \cdot l \cdot 2R_k}{4 \cdot \frac{l \cdot l \cdot 2R_k}{2}} == \frac{l^2}{4R_k^2} = \frac{5R_k^2}{4R_k} = \frac{5}{4}R_k;$ $\frac{5\sqrt{5}}{2\sqrt{\pi}} =$ $\frac{5}{4}R_k,$ $R_k = \frac{2\sqrt{5}}{\sqrt{\pi}}$

.$S_{k.asos} = \pi R_k^2 = \pi \cdot \frac{4 \cdot 5}{\pi} = 20$

Demak, to'g'ri javob: D) 20

2-usul: Berilganiga ko'ra: $S_{sfera} = 125,\ 4\pi R_s^2 = 125. R_s^2 = \frac{125}{4\pi}$, 2.4.27-b chizmaga ko'ra: $x^2 + R_k^2 = R_s^2$, $x^2 = R_s^2 - R_k^2$, $x = \sqrt{R_s^2 - R_k^2}. l^2 = 4R_k^2 + R_k^2 == 5R_k^2$, $l^2 = (R_s + x)^2 + R_k^2$. $5R_k^2 = (R_s + x)^2 + R_k^2$, $4R_k^2 = (R_s + x)^2$, $2R_k = \left(R_s + \sqrt{R_s^2 - R_k^2}\right)$, $2R_k - R_s = \sqrt{R_s^2 - R_k^2}$. $R_s^2 - R_k^2 = (2R_k - R_s)^2$,

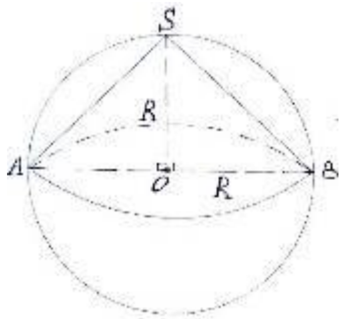
$R_s^2 - R_k^2 = 4R_k^2 - 4R_kR_s + R_s^2$, $5R_k^2 - 4R_kR_s = 0$, $R_k = \frac{4}{5}R_s$.

$S_{k.asos} = \pi R_k^2 = \pi \cdot \frac{16}{25} \cdot \frac{125}{4\pi} = 20$

Demak, to'g'ri javob: D) 20

2.4.30.([10], Var.-07) Sharga ichki chizilgan konusning asosi sharning katta doirasiga teng. Konus o'q kesimining yuzi 36 ga teng.Sharning hajmini toping (2.4.28-chizma).

A)144π B) 432π C)288π D)334π E)400π



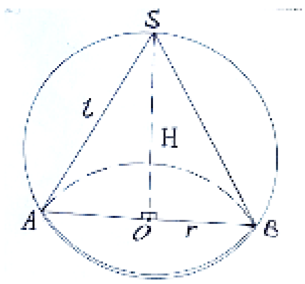
2.4.28-chizma

Yechilishi: Masala shartiga ko'ra: $S_{k.o'qkesim} = 36$, $\frac{R \cdot 2R}{2} = 36$, $R^2 = 36$, $R = 6$. Endi shar hajmini topamiz:$V_{sh} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \cdot 216 = 288\pi$

Demak, to'g'ri javob: C) 288π

2.4.31. ([10], Var.-07) Radiusi 15 ga teng bo'lgan sharga ichki chizilgan konusning balandligi 12 ga teng. Konusning hajmini toping (2.4.29-chizma).

A)486π B) 756π C)864π D)672π E)700π



2.4.29-chizma

Yechilishi: Shar radiusini hisoblaymiz: $R=\frac{H^2+r^2}{2H}=\frac{l^2}{2H}=\frac{l^2}{24}$, $\frac{l^2}{24}=15.$

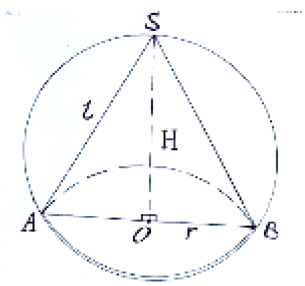
$l^2 == H^2+r^2; l^2=144+r^2;\ r^2=l^2-144=360-144=216.$

$V_k=\frac{1}{3}S_{asos}\cdot H=\frac{1}{3}\pi r^2, H=\frac{1}{3}\pi\cdot 216\cdot 12=864\pi$

Demak, to’g’ri javob: C) 864π

2.4.32. ([10], Var.-07) Radiusi 6 ga teng sharga ichki chizilgan konus yasovchisi va balandligining orasidagi burchak 30° ga teng. Konus yon sirtining yuzini toping (2.4.30-chizma).

A)96π B) 48C)216π D)72π E)54π



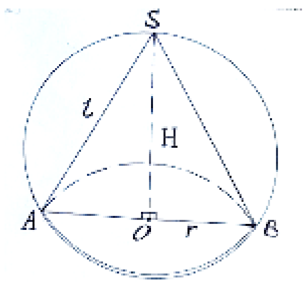
2.4.30-chizma

Yechilishi: 2.4.30-chizmaga ko’ra $\frac{r}{l}=sin\alpha$, $\frac{r}{l}=sin30^{\circ}, r=\frac{l}{2}$, $\frac{H}{l}=cos\alpha$, $\frac{H}{l}=cos30^{\circ}$, $H=\frac{\sqrt{3}}{2}l$, $R=\frac{abc}{4S_{\Delta}}=\frac{l\cdot l\cdot 2r}{4\cdot\frac{H\cdot 2r}{2}}=\frac{l^2}{4H}=6.$ $\frac{l^2}{4\cdot\frac{\sqrt{3}}{2}l}=6;$ $l=6\sqrt{3}$, $r=3\sqrt{3}$. $S_{k.yon}=\pi r\cdot l=\pi\cdot 3\sqrt{3}\cdot 6\sqrt{3}=54\pi$

Demak, to’g’ri javob: C) 54π

2.4.33. ([10], Var.-07) Balandligi $\sqrt{3}$ ga, yasovchisi $2\sqrt{3}$ ga teng bo’lgan konusga tashqi chizilgan sharning radiusini toping (2.4.31-chizma).

A)2 B) $2\sqrt{3}$ C)$3\sqrt{3}$ D)$3\sqrt{2}$ E)$4\sqrt{3}$



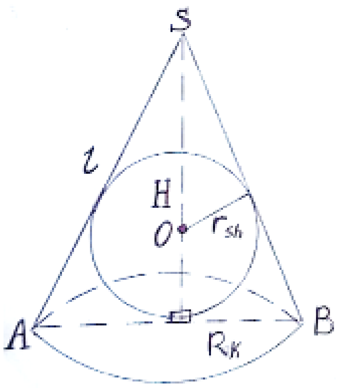
2.4.31-chizma

Yechilishi: 2.4.31-chizmaga ko'ra $l^2 = H^2 + r^2,\ r^2 = l^2 - H^2 = 12 - 3 = 9,\ r = 3$. Endi shar radiusini hisoblaymiz: $R = \frac{abc}{4S_\Delta} = \frac{l \cdot l \cdot 2r}{4 \cdot \frac{H \cdot 2r}{2}} = \frac{l^2}{2H},\ \ R = \frac{l^2}{2H} = \frac{12}{2\sqrt{3}} = 2\sqrt{3}$

Demak, to'g'ri javob: B) $2\sqrt{3}$

2.4.34. ([10], Var.-07) Konusning yasovchisi 20 ga, asosining diametri 24 ga teng. Unga ichki chizilgan shar sirtining yuzini toping (2.4.32-chizma).

A)156π B) 169π C)289π D)144π E)200π



2.4.32-chizma

Yechilishi: Berilganiga ko'ra: $d = 24,\ 2R_k = 24,\ R_k = 12$. 2.4.32-chizmaga ko'ra: $l^2 = R_k^2 + H^2, H^2 = l^2 - R_k^2 = 400 - 144 = 256,\ H = 16$.

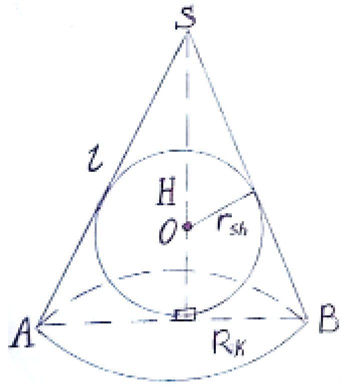
$$r_{sh} = \frac{2S_\Delta}{a+b+c} = \frac{2 \cdot \frac{H \cdot 2R_k}{2}}{2l + 2R_k} = \frac{H \cdot R_k}{l + R_k} = \frac{16 \cdot 12}{20+12} = \frac{16 \cdot 12}{32} = 6.$$

$$S_{sh} = 4\pi r_{sh}^2 = 4\pi \cdot 36 = 144\pi$$

Demak, to'g'ri javob: D) 144π

2.4.35. ([10], Var.-07) Yasovchisi 15 ga, asosining radiusi 9 ga teng bo'lgan konusga ichki chizilgan sharning radiusini toping (2.4.33-chizma).

A)6 B) 4,5C)$3\sqrt{2}$ D)$4,5\sqrt{3}$ E)$5\sqrt{2}$



2.4.33-chizma

Yechilishi: 2.4.33-chizmaga ko'ra: $l^2 = H^2 + R_k^2$, $H^2 = l^2 - R_k^2$=225-81=144, $H = 12$. $r_{sh} = \frac{2S_\Delta}{a+b+c} = \frac{2\cdot\frac{H\cdot 2R_k}{2}}{2l+2R_k} = \frac{H\cdot R_k}{l+R_k} = \frac{12\cdot 9}{15+9} = \frac{12\cdot 9}{24} = 4,5$.

Demak, to'g'ri javob: B) 4,5

2.4.36. ([5], 11.128-masala) r radiusli sharga yasovchisi *l* ga teng bo'lgan kesik konus tashqi chizilgan. Uning yon sirtini va hajmini toping.

A)$\pi l^2; \frac{2}{3}\pi r(l^2 - r^2)$ B) $2\pi l^2; \frac{1}{3}\pi r(l^2 - r^2)$ C)$\pi l^2; \frac{4}{3}\pi r(l^2 - r^2)$

D)$\pi l^2; \frac{1}{9}\pi r(l^2 - r^2)$ E)$\pi l^2; \frac{4}{3}\pi r(4l^2 - r^2)$

Yechilishi: Kesik konusning asoslari radiuslarida R_1vaR_2 bilan belgilasak, u holda $R_1 + R_2 = l$ bo'ladi. Pifagor teoremasiga asosan $l^2 = H^2 + (R_2 - R_1)^2$; $H^2 = (R_1 + R_2)^2 - (R_2 - R_1)^2 = 4R_1R_2$; $H = 2\sqrt{R_1R_2}$. 2r=H ekanligidan $R_1R_2 = r^2$ ga teng. $S_y = \pi(R_1 + R_2)l = \pi l^2$. $V_k = \frac{\pi}{3}H(R_1^2 + R_1R_2 + R_2^2) = \frac{2\pi}{3}r((R_1 + R_2)^2 - R_1R_2)$; $V_k = \frac{2\pi}{3}r(l^2 - r^2)$

Binobarin, to'g'ri javob: A)$\pi l^2; \frac{2}{3}\pi r(l^2 - r^2)$

2.4.37. ([5], 12.239-masala) Konusga ichki chizilgan shar sirtining konus asosining yuziga nisbati k ga teng. Konusning yasovchisi bilan uning asos tekisligi orasidagi burchakning kosinusini va k ning yo'l qo'yiladigan qiymatlarini toping.

A)$\frac{6-k}{6+k}, 0 < k < 6$ B) $\frac{5-k}{5+k}, 0 < k < 5$C)$\frac{4-k}{4+k}, 0 < k < 4$

D)$\frac{k}{5+k}, k > 0$ E)$\frac{k}{7+k}, k > 0$

Yechilishi: Masala shartiga asosan: $\frac{S_{sh}}{S_{k.a}} = k$; $k = \frac{4\pi r^2}{\pi R^2} = \frac{4r^2}{R^2}$; $\frac{r}{R} = tg\frac{\alpha}{2}$; $tg\frac{\alpha}{2} = \frac{\sqrt{k}}{2}$.
$cos\frac{\alpha}{2} = \frac{1}{\sqrt{1+tg^2\frac{\alpha}{2}}}$; $cos\frac{\alpha}{2} = \frac{2}{\sqrt{4+k}}$; $cos\alpha = 2cos^2\frac{\alpha}{2} - 1 = \frac{4-k}{4+k}, 0 < k < 4$

Demak, to'g'ri javob: C)$\frac{4-k}{4+k}, 0 < k < 4$

2.4.38. ([6], 15.4.41-masala) Konusning yasovchisi l va asosi tekisligi bilan α burchak tashkil qiladi.Konusga tashqi chizilgan sharning hajmini hisoblang.

A)$\frac{\pi l^3 sin\alpha}{cos^3\alpha}$ B) $\frac{\pi l^3 cos\alpha}{4sin^2\alpha}$ C) $\frac{\pi l^3}{6sin^3\alpha}$ D)$\frac{\pi l^3 cos\alpha}{6sin^2\alpha}$ E)$\frac{\pi l^3 sin\alpha}{4cos^2\alpha}$

Yechilishi: Masala shartiga ko'ra konusning yasovchisi l va asosi tekisligi bilan α burchak hosil qilishi berilgan. Balandligi $H = lsin\alpha$ bo'ladi.$R_{sh}^2 = (H - R_{sh})^2 + R_k^2$,$R_{sh} = \frac{H^2+R_k^2}{2H} = \frac{l^2}{2H}$; $R_{sh} = \frac{l}{2sin\alpha}$. Endi shar hajmini topamiz: $V_{sh} = \frac{4}{3}\pi R_{sh}^3 = \frac{4}{3}\pi \cdot \frac{l^3}{8sin^3\alpha} = \frac{\pi l^3}{6sin^3\alpha}$

Binobarin, to'g'ri javob: C)$\frac{\pi l^3}{6sin^3\alpha}$

2.4.39. ([6], 15.4.38-masala) Konusning o'q kesimi muntazam uchburchakdan iborat. Konusga tashqi va ichki chizilgan sferalarning nisbatini toping.

A)$6:5$ B) $3:1$ C)$4:5$ D)$2:3$ E)$4:1$

Yechilishi: Pifagor teoremasiga ko'ra $H^2 + R^2 = l^2$; $H^2 + R^2 = 4R^2$; $H = \sqrt{3}R$ ga teng bo'ladi. $r_s = \frac{2R}{2\sqrt{3}} = \frac{R}{\sqrt{3}}$; $R_s = \frac{abc}{4S_\Delta} = \frac{8R^3}{4\cdot\frac{\sqrt{3}R\cdot 2R}{2}} = \frac{2}{\sqrt{3}}R$; $\frac{S_{sf1}}{S_{sf2}} = \frac{4\pi\cdot\frac{4}{3}}{4\pi\cdot\frac{1}{3}} = 4:1$

Demak, to'g'ri javob: E)$4:1$

2.4.40. ([5], 11.223-masala) Konus balandligini konusga tashqi chizilgan shar radiusiga nisbati q ga teng. Bu jismlarning hajmlari nisbatini toping.

A)$\frac{q^2(2-q)}{4}, q < 2$ B) $\frac{q^2(2+q)}{4}$ C)$\frac{q(2-q)}{3}, q < 2$ D)$\frac{q^2(4-q)}{5}, q < 4$
E)$\frac{q^2(4-q)}{4}, q < 6$

Yechilishi: Masala shartiga asosan, $\frac{H}{R} = q$ ga teng. $R = \frac{H^2+r^2}{2H}$; $R = \frac{H}{q}$; $\frac{H}{q} = \frac{H^2+r^2}{2H}$; $2H^2 = (H^2 + r^2)q$; $r^2 = \frac{2H^2-qH^2}{q} = \frac{H^2(2-q)}{q}$. Endi bu jismlarning hajmlari nisbatini topamiz: $\frac{V_k}{V_{sh}} = \frac{\frac{1}{3}\pi r^2 \cdot H}{\frac{4}{3}\pi R^3} = \frac{r^2 \cdot H}{4R^3}$; $\frac{V_k}{V_{sh}} = \frac{H^3(2-q)q^3}{4qH^3} = \frac{q^2(2-q)}{4}, q < 2$

Binobarin, to'g'ri javob: A)$\frac{q^2(2-q)}{4}, q < 2$

2.4.41.([7], 293-masala) Balandligi h asos aylanasining radiusi r bo'lgan konusga ichki chizilgan shar hajmini toping.

A)$\frac{4\pi r^3 h^3}{3(r+\sqrt{r^2+h^2})^3}$ B) $\frac{2\pi r^3 h^3}{3(r+\sqrt{r^2+h^2})^3}$ C) $\frac{\pi r^3 h^3}{3(r+\sqrt{r^2+h^2})^3}$ D)$\frac{5\pi r^3 h^3}{3(r+\sqrt{r^2+h^2})^3}$
E)$\frac{\pi r^3 h^3}{(r+\sqrt{r^2+h^2})^3}$

Yechilishi: Masala shartiga ko'ra konusning balandligi h va asos aylanasining radiusi r berilgan. Konus yasovchisini topamiz.$l^2 = h^2 + r^2$, $l = \sqrt{h^2 + r^2}$; $r_{sh} = \frac{hr}{l+r}$; $r_{sh} = \frac{h \cdot r}{r+\sqrt{h^2+r^2}}$; $V_{sh} = \frac{4}{3}\pi r_{sh}^3 = \frac{4\pi r^3 h^3}{3(r+\sqrt{r^2+h^2})^3}$.

Demak, to'g'ri javob: A)$\frac{4\pi r^3 h^3}{3(r+\sqrt{r^2+h^2})^3}$

2.4.42. ([7], 299-masala) Kesik konusga shar ichki chizilgan. Kesik konus hajmining shar hajmiga nisbati 13:6. Konus yasovchisining asos tekisligi bilan tashkil etgan burchagini toping.

A)$30°$ B) $60°$ C)$90°$D)$75°$ E)$45°$

Yechilishi: Kesik konusning o'q kesimida teng yonli trapetsiya va unga ichki chizilgan aylana hosil bo'ladi.R-sharning, R_1–ustki asosning, R_2 –ostki asosning radiuslari bo'lsin. Masala shartiga ko'ra $\frac{V_k}{V_{sh}} = \frac{13}{6}$ ga teng. $V_k = \frac{1}{3}\pi H(R_1^2 + R_1R_2 + +R_2^2)$; $V_{sh} = \frac{4}{3}\pi R^3$. $R_1 + R_2 = l$, $R^2 = R_1R_2$. $V_k = \frac{2}{3}\pi R((R_1 + R_2)^2 - R_1R_2) = \frac{2}{3}\pi R(l^2 - R^2)$. $\frac{\frac{2}{3}\pi R(l^2-R^2)}{\frac{4}{3}\pi R^3} = \frac{13}{6}$;

$3l^2 - 3R^2 = 13R^2$; $16R^2 = 3l^2$; $R = \frac{\sqrt{3}}{4}l$; $\frac{2R}{l} = sin\alpha$; $sin\alpha = \frac{\sqrt{3}}{2}$;$\alpha = 60°$

Demak, to’g’ri javob: B) $60°$

2.4.43. ([7], 313-masala) Konusning balandligi unga ichki chizilgan shar radiusidan to’rt marta katta. Konusning yasovchisi b ga teng. Konusning yon sirti va unga tashqi chizilgan sharning radiusini toping.

A)$\frac{\pi b^2}{4}$; $\frac{3\sqrt{2}}{8}b$ B) $\frac{\pi b^2}{3}$; $\frac{3\sqrt{3}}{8}b$ C)$\frac{\pi b^2}{3}$; $\frac{3\sqrt{2}}{8}b$ D)$\frac{\pi b^2}{5}$; $\frac{4\sqrt{2}}{3}b$ E)$\frac{\pi b^2}{3}$; $\frac{4\sqrt{3}}{8}b$

Yechilishi: Masala shartiga asosan $H = 4r$ ga teng. $r = \frac{h \cdot R_k}{R_k + b}$; $\frac{h \cdot R_k}{R_k + b} = \frac{h}{4}$; $4R_k = R_k + b$; $R_k = \frac{b}{3}$; $b^2 = h^2 + R_k^2 = b^2 - \frac{b^2}{9} = \frac{8b^2}{9}$; $h = \frac{2\sqrt{2}}{3}b$. Konusga tashqi chizilgan shar radiusini topamiz: $R = \frac{h^2 + R_k^2}{2h} = \frac{b^2}{2h}$; $R = \frac{b^2}{2 \cdot \frac{2\sqrt{2}}{3}b} = \frac{3\sqrt{2}}{8}b$.

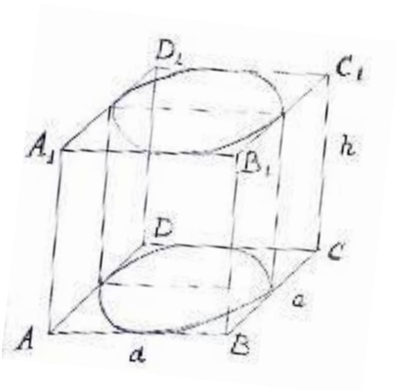
$S_{yon} = \pi R_k b = \frac{\pi b^2}{3}$

Demak, to’g’ri javob: C)$\frac{\pi b^2}{3}$; $\frac{3\sqrt{2}}{8}b$

2.5. Turli kombinatsiyalarga oid testlarni yechish usullari.

2.5.1. ([10], 97-4-59) Silindr va unga tashqi chizilgan muntazam to’rtburchakli parallelepipedning balandligi 3 ga teng. Parallelepipedning balandligi 3 ga teng.Parallelepiped asosining tomoni 4 ga teng.Silindrning hajmini toping (2.5.1-chizma).

A)10π B) 12π C)16π D)20π E)8π



2.5.1-chizma

Yechilishi: 2.5.1-chizmaga ko’ra: $2r_s = a,\ 2r_s = 4, r_s = 2. V_s = S_{asos} \cdot h = \pi r_s^2 \cdot h = 4 \cdot 3\pi = 12\pi$

Demak, to’g’ri javob: B) 12π

2.5.2.([10], 98-9-54) Uchburchakli muntazam prizmaga tashqi chizilgan silindr yon sirti yuzining unga ichki chizilgan silindrning yon sirti yuziga nisbatini toping.

A)3 B) 2 C)1,5 D)2,5 E)1,2

Yechilishi: Prizmaning asosi muntazam uchburchak bo’lgani uchun

$$R = \frac{a}{2\sin 60^0}, R = \frac{a}{\sqrt{3}}, r = \frac{a}{2tg60^0}, r = \frac{a}{2\sqrt{3}}. \frac{S_{yon.t}}{S_{yon\,i\,ch.}} = \frac{2\pi Rh}{2\pi rh} = \frac{R}{r} = \frac{\frac{a}{\sqrt{3}}}{\frac{a}{2\sqrt{3}}} = 2$$

Demak, to’g’ri javob: B) 2

2.5.3. ([10], 00-1-57) Muntazam to’rtburchakli prizmaga silindr ichki chizilgan. Silindr hajmining prizma hajmiga nisbatini toping.

A)$\frac{\pi}{3}$ B) $\frac{\pi}{5}$ C) $\frac{\pi}{4}$ D)$\frac{2\pi}{3}$ E)$\frac{3\pi}{4}$

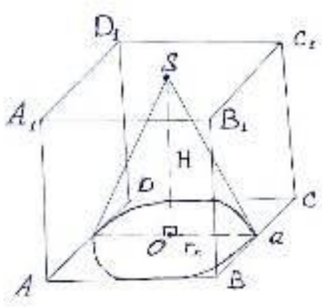
Yechilishi: $2r_s = a, r_s = \frac{a}{2}. V_s = V_{asos} \cdot H = \pi r_s^2 \cdot H, V_{priz} = S_{asos} \cdot H = a^2 \cdot H.$

$$\frac{V_s}{V_{priz}} = \frac{\pi r_s^2 \cdot H}{a^2 \cdot H} = \frac{\pi \cdot \frac{a^2}{4}}{a^2} = \frac{\pi}{4}$$

Demak, to’g’ri javob: C) $\frac{\pi}{4}$

2.5.4. ([10], 98-6-58) Qirrasi 12 ga teng kubga konus ichki chizilgan. Agar konusning asosi kubning pastki asosida chizilgan bo’lsa, uchi esa yuqoridagi asosining markazida yotsa, konusning hajmini toping (2.5.2 – chizma).

A)120π B) 132π C)126π D)156π E)144π



2.5.2 – chizma

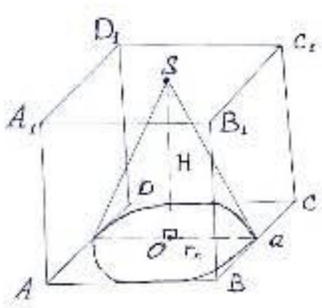
Yechilishi:2.5.2 – chizmaga ko'ra $a = 2r_k$, $r_k = \frac{a}{2} = \frac{12}{2} = 6$.

$V_s = \frac{1}{3}S_{asos} \cdot H = \frac{1}{3}\pi r_k^2 \cdot H = \frac{1}{3}\pi \cdot 36 \cdot 12 = 144\pi$

Demak, to'g'ri javob: E) 144π

2.5.5. ([10], 00-7-49) Muntazam to'rtburchakli prizmaga konus ichki chizilgan. Konusning asosi prizmaning ostki asosida, uchi esa prizma ustki asosining markazida yotadi. Prizma hajmining konus hajmiga nisbatini toping (2.5.3 – chizma).

A)$\frac{8}{\pi}$ B) $\frac{9}{\pi}$ C) $\frac{12}{\pi}$ D)$\frac{10}{\pi}$ E)$\frac{7}{\pi}$



2.5.3 – chizma

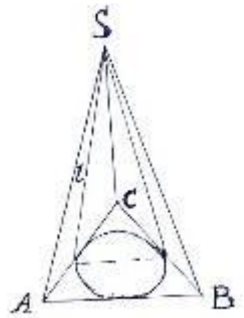
Yechilishi: 2.5.3 – chizmaga asosan: $2r_k = a, r_k = \frac{a}{2}. V_{prizma} = S_{asos} \cdot H = a^2H.$

$V_k = \frac{1}{3}S_{asos} \cdot H = \frac{1}{3}\pi r_k^2 \cdot H = \frac{1}{3}\pi \cdot \frac{a^2}{4} \cdot H = \frac{\pi a^2 H}{12} . \frac{V_{prizma}}{V_{konus}} = \frac{a^2 H}{\frac{\pi a^2}{12} \cdot H} = \frac{12}{\pi}$

Binobarin, to'g'ri javob: C) $\frac{12}{\pi}$

2.5.6. ([10], 00-1-50) Muntazam uchburchakli piramidaga konus ichki chizilgan. Agar piramidaning yon yoqlari bilan asosi 60° li burchak hosil qilib, piramidaning asosiga ichki chizilgan aylananing radiusi 16 ga teng bo'lsa, konusning yon sirtini toping (2.5.4 – chizma).

A)524π B) 512π C)536π D)514π E)518π



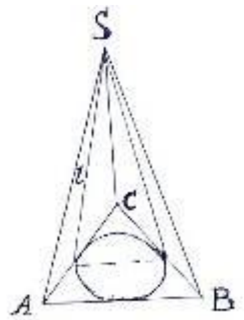
2.5.4 – chizma

Yechilishi: 2.5.4 – chizmaga ko'ra: $\frac{r}{l}=cos\alpha$ =>$r=lcos\alpha$=> $r=lcos60^{\circ}=\frac{l}{2}$ =>$l=2r=32$. $S_{k\ yon}=\pi rl=\pi\cdot 16\cdot 32=512\pi$

Binobarin, to'g'ri javob: B) 512π

2.5.7. ([10], 01-1-60) Konus asosining radiusi 2 ga, o'q kesimining uchidagi burchagi 60° ga teng. Shu konusga tashqi chizilgan muntazam uchburchakli piramidaning hajmini toping (2.5.5 – chizma).

A)12 B) $12\sqrt{3}$ C)$18\sqrt{3}$ D)6 E)24



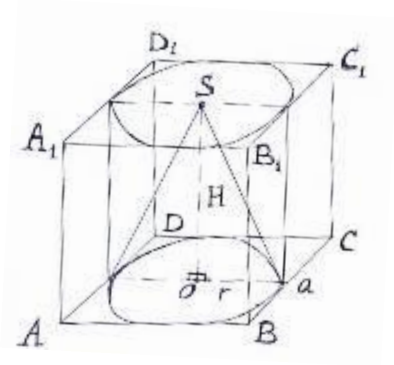
2.5.5 – chizma

Yechilishi: 2.5.5 – chizmaga ko'ra: $\frac{r}{l}=sin\frac{\alpha}{2}$ =>$l=\frac{r}{sin\frac{\alpha}{2}}=\frac{2}{\frac{1}{2}}=4$. Pifagor teoremasiga ko'ra: $l^2=H^2+r^2$, $H^2=l^2-r^2=16-4=12$ =>$H=2\sqrt{3}$. $V_{pr.}=\frac{1}{3}S_{asos}\cdot H=\frac{1}{3}\cdot\frac{\sqrt{3}}{4}\cdot 48\cdot 2\sqrt{3}=24$

Demak, to'g'ri javob: E) 24

2.5.8. ([10], 02-6-49) Kubga ichki chizilgan silindrning hajmi unga ichki chizilgan konusning hajmidan π ga ortiq. Kubning hajmini toping (2.5.6 – chizma).

A)4 B) 6 C)8 D)12 E)27



2.5.6 – chizma

Yechilishi: 2.5.6 – chizmaga asosan: $2r = a$ $=> r = \frac{a}{2}$. $V_s = S_{asos} \cdot H = \pi r^2 \cdot H = \pi \frac{a^2}{4} \cdot a = \frac{\pi a^3}{4}$;

$V_k = \frac{1}{3} S_{asos} \cdot H = \frac{1}{3}\pi r^2 H = \frac{1}{3}\pi \frac{a^3}{4} = \frac{\pi}{12} a^3$,

$V_s - V_k = \pi \cdot \frac{\pi a^3}{4} - \frac{\pi a^3}{12} = \pi => \frac{2a^3}{12} = 1 => a^3 = 6$, $V_{kub} = a^3 = 6$

Demak, to'g'ri javob: B) 6

2.5.9. ([10], 03-1-44) Kubga ichki chizilgan silindrning hajmi 2π ga teng. Shu kubga tashqi chizilgan sferaning yuzini toping.

A)12π B) 18π C)20π D)24π E)27π

Yechilishi: Berilganiga ko'ra: $V_s = 2\pi => \pi r^2 \cdot H = 2\pi => r^2 \cdot H = 2$. $2r = a$ $=> r = \frac{a}{2}$, H=a. $r^2 \cdot H = \frac{a^2}{4} \cdot a = 2$. $=> a^3 = 8 =>$a=2.

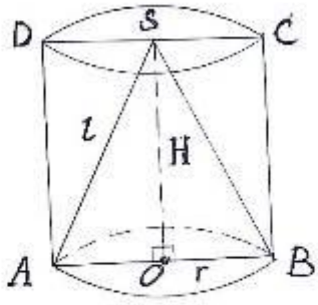
$d^2 = (a\sqrt{2})^2 + a^2 = 3a^2 = 3 \cdot 4 = 12 => d = 2\sqrt{3}$, $R_{sf} = \sqrt{3}$,

$S_{sf} = 4\pi R_{sf}^2 = 4\pi \cdot 3 = 12\pi$

Demak, to'g'ri javob: A) 12π

2.5.10. ([10], 03-4-54) Asosining radiusi 6 ga teng bo'lgan silindrga konus ichki chizilgan. Konusning asosi silindrning asosi bilan, uchi esa silindr ustki asosining markazi bilan ustma-ust tushadi. Konusning yon sirti 60π ga teng silindrning yon sirtini toping (2.5.7-chizma).

A)92π B) 94π C)96π D)98π E)90π



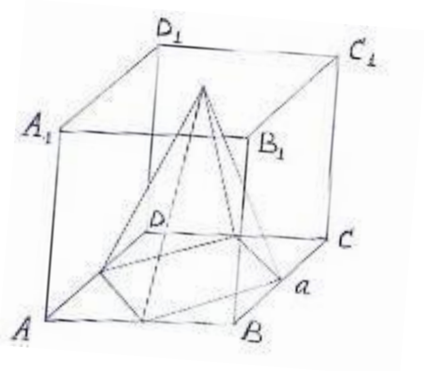
2.5.7-chizma

Yechilishi: Berilganiga ko'ra: $S_{k.yon} = 60\pi => \pi r \cdot l = 60\pi => r \cdot l = 60$, $6l = 60$ $=> l = 10$.To'g'ri burchakli ΔSOB dan: $H^2 = l^2 - r^2 == 10^2 - 6^2 = 64 =>$
$H = 8$. $S_{s.yon} = 2\pi r \cdot H = 2\pi \cdot 6 \cdot 8 = 96\pi$

Demak, to'g'ri javob: A) 96π

2.5.11. ([10], 03-7-83) Kubning ostki asosidagi tomonlarining o'rtalari ketma-ket tutashtirildi. Hosil bo'lgan to'rtburchakning uchlari kub ustki asosining markazi bilan tutashtirildi. Agar kubning qirrasi a ga teng bo'lsa, hosil bo'lgan piramidaning to'la sirtini toping (2.5.8-chizma).

A)$\frac{2a^2}{3}$ B) $3a^2$ C)$1{,}5a^2$ D)$2a^2$ E)$\frac{2a^2\sqrt{3}}{3}$



2.5.8-chizma

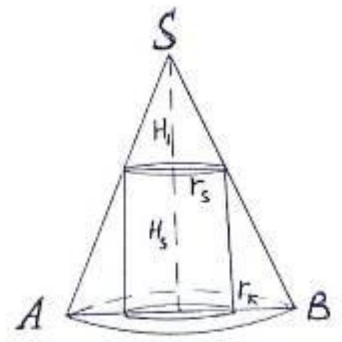
Yechilishi: 2.5.8-chizmaga ko'ra, hosil bo'lgan piramida asosining tomoni $\frac{a}{\sqrt{2}}$ ga teng. Uning apofemasi: $l^2 = h^2 + \frac{a^2}{8} = a^2 + \frac{a^2}{8} = \frac{9a^2}{8}$, $=> l = \frac{3a}{2\sqrt{2}}$ga teng.

$S_{p.to'la} = S_{asos} + S_{yon} = \frac{a^2}{2} + \frac{pl}{2} = \frac{a^2}{2} + \frac{4al}{2\sqrt{2}} = \frac{a^2}{2} + \frac{2a}{\sqrt{2}} \cdot \frac{30}{2\sqrt{2}} = \frac{a^2}{2} + \frac{3a^2}{2} = 2a^2$

Demak, to’g’ri javob: D) $2a^2$

2.5.12. ([10], 03-11-50) Balandligi 16 ga, asosining radiusi 12 ga teng bo’lgan konusga balandligi 10 ga teng bo’lgan silindr ichki chizilgan. Silindrning radiusini toping (2.5.9-chizma).

A)$4{,}5$ B) 4 C)$4{,}8$ D)$4{,}2$ E)5



2.5.9-chizma

Yechilishi: 2.5.9-chizmaga ko’ra: $H_1 + H_s = H$ =>$H_1 = H - H_s = 16 - 10 = 6$.

$\frac{H_1}{H} = \frac{r_s}{r_k}$=>$r_s = \frac{H_1 \cdot r_k}{H} = \frac{6 \cdot 12}{16} = 4{,}5$

Demak, to’g’ri javob: A) $4{,}5$

2.5.13.([7], 347-masala) Shar, o’q kesimi kvadrat bo’lgan silindr va konus berilgan. Silindr va konus bir xil asosga ega, ularning balandliklari esa shar diametriga teng. Silindr, shar va konus hajmlari qanday nisbatda bo’ladi?

A)$2:3:5$ B) $3:2:1$ C)$4:3:1$ D)$1:3:2$ E)$2:3:5$

Yechilishi: Masala shartiga ko’ra: $R_s = R_k = R_{sh} = R, H = 2R$ga teng.$V_{sh} = \frac{4}{3}\pi R^3$; $V_k = \frac{1}{3}\pi R^2 \cdot H = \frac{2}{3}\pi R^3$, $V_s = \pi R^2 \cdot H = 2\pi R^3$.

$V_s : V_{sh} : V_k = 3:2:1$

Demak, to’g’ri javob: B) $3:2:1$

2.5.14. ([7], 105-masala) Konusning yasovchisi bilan asosi tekisligi orasidagi burchak 30° ga, konusning yon sirti $3\pi\sqrt{3}$ ga teng. Bu konusga ichki chizilgan oltiburchakli muntazam piramidaning hajmini toping.

A)$\frac{27\sqrt{3}}{8}$ B) $\frac{32\sqrt{2}}{9}$ C)$\frac{28\sqrt{2}}{3}$ D)$\frac{27\sqrt{2}}{8}$ E)$\frac{32\sqrt{5}}{9}$

Yechilishi: $H = l\sin 30°$; $H = \frac{l}{2}$. $R = l\cos 30°$; $R = \frac{\sqrt{3}}{2}l$. $S_{yon} = 3\pi\sqrt{3}$; $\pi R l = 3\pi\sqrt{3}$; $\frac{\sqrt{3}}{2}l \cdot l = 3\sqrt{3}$, $l^2 = 6$; $l = \sqrt{6}$.

$$V_p = \frac{1}{3}S_a \cdot H = \frac{1}{3} \cdot \frac{6\sqrt{3}}{4}R^2 \cdot H = \frac{\sqrt{3}}{2} \cdot \frac{3}{4} \cdot 6 \cdot \frac{\sqrt{6}}{2} = \frac{27\sqrt{2}}{8}$$

Demak, to'g'ri javob: D)$\frac{27\sqrt{2}}{8}$

Xulosa

Bitiruv malakaviy ishimning bu bobida jismlarning kombinatsiyasiga oid davlat test markazi tomonidan 1996-2008-yillarda e'lon qilingan testlar, M.I. Skanavining umumiy tahriri ostida Matematikadan masalalar to'plami, T.Tolaganov, A.Normatov Matematikadan praktikum, geometriya darsliklaridagi va geometriyadan masalalar to'plamidagi masalalar to'plandi. Bu masala ichidan qiziqarlilari tanlab olindi. Masalalar, testlar yechilib, tahlil va tadqiq qilindi. Yechimlar ichidan eng tushunarli, qulay, kam vaqt sarflab yechiladigan ya'ni yechishga tez boriladigan optimal yechimlar ko'rsatildi.

Masalaning qaysi birini qanday usul bilan yechsak, o'quvchilarga tushunarli, ko'p vaqt sarflasa ham o'quvchilarning tasavvur qilishlari uchun oson va qaysi usullar testda qulay ekanligini tahlil qilib, masalalarningbir necha yechish usullari ko'rib chiqildi. Chunki bitta usulda yechganda ko'p vaqt ketsa, boshqa usulda yechsak kamroq vaqt ketishi mumkin.

Bitiruv malakaviy ishim asosan testlar bilan ishlash haqida bo'lgani uchun, asosan testlar ko'rilib, testlar bilan ularning yechilish usullari ko'rsatildi.

Har bir masala yoki testda abiturient qanday yondashadi va qaysi usulda yechadi va testda qaysi usul bilan yondashish kerakligini, testlarni yechganda asosan nimalarga e'tibor berish kerakligi haqida to'xtaldik. Shu o'rinda bir narsani aytib o'tish kerakki, ayrim masalalar chizma chizish talab etmasada, aksariyat masalalarda yechimni toppish chizmani qay darajada aniq va to'g'ri tasavvur qilib chizishga bog'liq. Chunki, shunday masalalar borki, agar bu chizmani to'g'ri va aniq chiza olsak, yechimning o’zi ko’rinib turadi. Ba’zilarida esa chizmani chizmay, parametrlarning o’zaro bog’liqligini tiklay olish kerak xolos. Biz yana testlarning qiyinchilik darajasi va darsliklar va o’quv qo’llanmalarga qay darajada mosligini ham tahlil qildik va shu narsalarni nazarda tutib ish tutdik.

Xotima

1. Ko’pyoqlar, prizma, parallelepiped, piramida ta’riflari, ularning sirtlari, hajmlarining va boshqa parametrlarining o’zaro bog’liqlik formulalari, ularni hisoblash formulalari keltirildi.

2. Aylanish jismlari silindr, konus, shar, sfera ta’riflari, ularning yasalishi, sirtlari, hajmlarining va boshqa parametrlariningo’zaro bog’liqlik formulalari, ularni hisoblash formulalari keltirildi.
3. Jismlarning turli kombinatsiyalari, kombinatsiyada bo’lganda jismlar parametrlarining o’zaro bog’liqlik formulalari, ularni hisoblash formulalari keltirilgan. Keltirilgan formulalarning ko’pi asosiy formulalar bo’lsada, ba’zi almashtirishlar yordamida hosil qilingan formulalar ham keltirilgan, ammo keltirib chiqarilgan formulalarning ayrimlarini isboti masalalar yechish davomida keltirildi.
4. Jismlarning kombinatsiyasiga oid davlat test markazi tomonidan 1996-2008-yillarda e’lon qilingan testlar, M.I. Skanavining umumiy tahriri ostida Matematikadan masalalar to’plami, T.Tolaganov, A.Normatov Matematikadan praktikum, geometriya darsliklari va o’quv qo’llanmalaridagi, geometriyadan masalalar to’plamidagi shu mavzuga oid masalalar to’plandi. To’plangan masalalrning eng yaxshilari tanlab olinib, BMI ga kiritildi.
5. Masalalar, testlar yechilib, tahlil va tadqiq qilindi. Yechimlar ichidan eng qulay, tushunarli, yechimga tez olib boradigan optimallari ko’rsatildi.
6. Masalalar bir necha usulda yechilib, qaysi usul o’quvchilarga darslar o’tishda tushuntirishga qulay vao’quvchilarga osonroq, bolalarning tasavvur qilish qobiliyatini o’stirishga xizmat qiladi va qaysilari testlarda qo’llash uchun qulay ekanligini tahlil qilinib, tavsiyalar beriladi.
7. Har bir masalaga atroflicha yondashilib, bu masala yoki testga abiturient qanday yondashadi va qaysi usulda yechadi, testlarga qaysi usullarni qo’llash kerak va qo’llay oladi, shu haqda ham mushohada qilindi.
8. Geometrik masalalarni yechganda, testlar bilan ishlaganda asosan nimalarga e’tibor qilish kerak. Shu o’rinda bir narsani aytib o’tishimiz kerakki, ayrim masalalar chizma chizishni talab etmaydi. Faqat jismlar parametrlari orasidagi o’zaro bog’liqlikni ko’ra olish kerak. Aksariyat masalalar esa

chizma chizishni talab etadi. Agar chizmasini aniq va to'g'ri chiza olsa, javobni o'zi chizmada ko'rinadi.

9. Testlarning qiyinchilik darajasi,o'quvchilarning bilim darajasiga qanchalik mos kelishini, darslarda mulohazali masalalar ko'rilganligi yoki yo'q ekanligini tahlil qildik. Jismlarning kombinatsiyaga oid masalalarni hosila yordamida yechiladigan turlari darsliklarda umuman keltirilmagan, hatto nazariyasi ham tushuntirilmagan. Lekin bu turdagi masalalarni testlarda uchratishimiz mumkin.
10. Endilikda jismlarning kombinatsiyasiga oid masalalrni yechish akademik litsey va kasb – hunar kollejlarining 3-bosqich talabalariga o'tilyapti. Bu vaqtda ular algebra va analiz fanidan hosila tadbiqini ko'rib chiqqan bo'ladilar. Shu sababli hosila yordamida yechiladigan jismlarning kombinatsiyasiga oid masalalarni yechishni akademik litsey va kasb – hunar kollejlari o'quvchilariga kiritilishini taklif etmoqchiman. Chunki, stereometriyaning hayotdagi asosiy tadbiqni asosan hosila yordamida ko'rsatilishi mumkin. Fanlarning hayotdagi tatbiqini ko'rsatishi, o'quvchi uchun qiziqarli bo'ladi deb o'ylashga haqlimiz.
11. Akademik litsey va kasb – hunar kollejlarida bu mavzularga kam (8 soat) dars soatlari ajratilganligi uchun o'quvchilar bu mavzularni ancha murakkab bo'lganligi sababli to'liq mohiyatini anglab tushunib yetmaydilar. Shu sababli repititerlarga ehtiyoj tug'iladi. Dars rejalariga streometriya, xususan jismlarning kombinatsiyasiga oid mavzular uchun ko'proq dars soatlarining ajratilishini taklif etmoqchiman.

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