O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGIBUXORO DAVLAT UNIVERSITETI

Fizika –matematika fakulteti

“Matematik fizika va analiz” kafedrasi

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MAXSUS FUNKSIYALAR UCHUN FURE ALMASHTIRISHLARI

Mavzusidagi

Bitiruv malakaviy ishi

“Ish ko’rildi va himoyaga ruxsat berildi” **Ilmiy rahbar _____dots.Mamatova N.**

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Kafedra mudiri dots. Muxitdinov.R

Taqrizchi________ f-m.f.n.dots.X.M.Teshaev

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“Himoya qilishga ruxsat berildi”

Fakultet dekani prof.Sh.M.Mirzayev

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Mundarija

Kirish

> "Vatanimizning kelajagi, xalqimizning ertangi kuni, mamlakatimizning jahon hamjihatidagi obro'-e'tiboriavvalombor farzandlarimizning unib-o'sib, ulg'ayib, qanday inson bo'lib hayotga kirib borishiga bog'liqdir.Biz bunday o'tkir haqiqatni hech qachon unutmasligimiz kerak."
>
> I.A.Karimov

Biz farzandlarimizning nafaqat jismoniy va ma'naviy sog'lom o'sishi, balki ularning eng zamonaviy intelektual bilimlarga ega bo'lgan, uyg'un rivojlangan insonlar bo'lib, XXI asr talablariga to'liq javob beradigan barkamol avlod bo'lib voyaga yetishi uchun zarur barcha imkoniyat va sharoitlarni yaratishini o'z oldimizga maqsad qilib qo'yganmiz.

Shu kunlarda hukumatimiz tomanidan ushbu masala yuzasidan qabul qilingan Davlat dasturi ana shu ezgu maqsadga erishish yo'llariga jami davlat va nodavlat manbalari hisobga olingan holda mavjud barcha resurs va imkoniyatlarning safarbar etishni ko'zda tutadi kabi satrlarni o'qish mumkin.

Shuningdek, "Barkamol avlod" yili davlat dasturi to'g'risida O'zbekiston Respublikasi Prezidenti qarorida "ta'lim jarayonida yangi axborot komunikatsiya va pedagogic texnalogiyalarda, elektron dasturlar, multimediya vositalarini keng joriy etish orqali mamlakatimiz maktablarida, kasb-hunar kollejlari, litseylar va oliy o'quv yurtlariga o'qitish sifatini tubdan yaxshilash, ilm-fanni yanada rivijlantirish, iqtidorli va qobiliyatli yoshlarni ilmiy faoliyatiga keng jalb qilish, ularning o'z ijodiy va intelektual salohiyatini ro'yobga chiqarish uchun sharoit yaratishga doir kompleks chora-tadbirlarni ishlab chiqish "ko'zda tutilgan vazifalardan hisoblanar ekan, darslarda yangi pedagogik texnalogiyalardan foydalanish ta'lim samaradorligini oshirish

Bu muammoga hukumatimiz tomonidan alohida e'tibor berilmoqda. Yurtboshimiz I.A.Karimovning 2001 - yil may oyida Oliy Majlisning 5-sessiyasida so'zlagan nutqida axborot texnologiyalari va kompyuterlarni jamiyat hayotiga, kishilarning turmush tarziga, maktab va oliy o'quv yurtlariga jadallik bilan olib kirish g'oyasi ilgari surilgan edi. Prezidentimiz tashabbusi bilan O'zbekiston Respublikasi Vazirlar Mahkamasining 2001 yil 23 maydagi 230-sonli "2001-2005 yillarda kompyuter va axborot texnologiyalarini rivojlantirish ", shuningdek, "Internetning xalqaro axborot tizimlariga keng kirib borishini ta'minlash dasturini ishlab chiqishni tashkil etish chora-tadbirlari to'g'risida"gi qarorlari qabul qilindi.Oradan bir yil o'tgach, 2002 yil 30 mayda O'zbekiston Respublikasi Prezidentining "Kompyuterlashtirishni yanada rivojlantirish va axborot- kommunikatsiya texnologiyalarini joriy etish to'g'risida"gi farmoni va uning ijrosini amalga oshirish yuzasidan va axborot- kommunikatsiya texnologiyalari sohasida strategik ustuvorlikni amalga oshirishga doir amaliy chora-tadbirlarni ta'minlash maqsadida Vazirlar Mahkamasining2002 yil 6 iyundagi "2002-2010 yillarda kompyuterlashtirish axborot- kommunikatsiya texnologiyalarini rivojlantirish dasturi " to'g'risidagi qarori e'lon qilindi.

Prezidentimiz I. A. Karimovning " Yoshlarimiz bizdan ko'ra kuchli bilimli dono va albatta baxtli bo'lishlari kerak ", "O'zbekiston kelajagi yoshlar qo'lida" degan so'zlari va 2011-yilning "Kichik biznes va xususiy tadbirkorlik" deb belgilaganlari, yoshlarga va biznes va tadbirkorlikka bo'lgan e'tiborni va katta ishonchni ko'rsatadi. Biz yoshlar o'z navbatida bunday ishonchni oqlashga va komillik sari intilishga harakat qilamiz.

Yangi axborot – kommunikatsion texnologiyalari hozirgi vaqtda eng dolzarb mavzulardan biri bo'lib kelmoqda, sababi har bir sohani o'rganish, izlanish va tajriba orttirish uchun turli usullardan foydalanish kerak bo'ladi. Shuning uchun yangi axborot – kommunikatsion texnologiyalardan foydalanish maqsadga muvofiqdir.

Yurtboshimiz “Barkamol avlod – O`zbekiston taraqqiyotining poydevori” asrlarida quyidagi satrlarni o’qish mumkin: “Mamlakatimizni istiqlol yo’lidagi birinchi qadamlaridayoq, buyuk ma’naviyatimizni tiklash va yanada yuksaltirish, milliy

Jumladan, “Kadrlar tayyorlash milliy dasturi” da ta’kidlanganidek, - Ta’lim sohasini tubdan isloh qilish uni o’tmishdan qolgan mafkuraviy qarashlar va sarqitlardan to’la xolis etish, rivojlangan demakratik davlatlar darajasida yuksak ma’naviy va axloqiy talablarga javob beruvchi yuqori malakali kadrlar tayyorlash milliy tizimini yaratish” dan iborat. Ta’lim tizimini isloh qilish bilan bir qatorda, ta’lim jarayonining mazmunini, uning shakli tuzulmasi va unga qo’yilgan talablar ham yangicha ko’rinishga o’tmoqdaki, u endi o`zining oldingi, ya’ni sobiq tizim davridagi qotib qolgan ma’muriy, pedagogik tizimdan tubdan farq qilishi bilan xarakterlanadi.

Prezident Islom Abdug’aniyevich Karimov “barkamol avlod- O`zbekiston taraqqiyotining poydevori” mavzusidagi yana quyidagilarga to’xtalib o’tgan edi. Umumlashtirlgan holda kadrlar tayyorlash tizimining shakillanishi va faoliyat ko’rsatishning asosiy tamoillari, mening nazarimda quyidagi vazifalarni o`z ichiga oladi.

- Barcha xil va turdagi ta’lim muassasalarida yuqori malakali mutaxassislar tayyorlash uchun uzliksiz ta’lim, fan va ishlab chiqarish salohiyatidan samarali foydalanish;

- Davlat ta’lim standartlarini joriy etish va ularni faoliyat ko’rsatish mexanizmini ishlab chiqarish

- Ixtisosliklar, malaka darajasiga ko’ra mutaxassislarga bo’lgan umumdavlat va mintaqaviy talablarning istiqbolni aniqlash.

- Ta’lim tizimini tuzilishi va mazmun jihatdan isloh qilish uchun o’qituvchilarni va murabbiylarni qayta tayyorlash.

- Davlat va ijtimoiy muassasalarning kasbga yo’naltirish bo’yicha faoliyatini takomillashtirish. Bunda kasb tanlashning bozor ehtiyojlari va imkoniyatlarini e’tiborga olish zarur.

- O`quvchi yoshlarni Vatanga sadoqatli bo’lish ruhida tarbiyalash.

- Ta’limiy muassasalarni birinchi navbatda, umumta’lim maktablarini davlat tomonidan to’liq ta’minlashni meyorlarni oshirish va uning mexanizmini takomillashtirish;

- Kadrlar tayyorlash tizimi iste'molchilarni – muassasalar, banklar va boshqalarning imkoniyatlaridan, birinchi navbatda, o'rta maxsus kasb- hunar o`quv yurtlari va oliy o`quv yurtlarining moddiy va molyaviy bazasini mustahkamlash uchun mumkin qadar kengroq foydalanish;

shartlaridan biri. Istiqlol yo'lida qadam tashlab boarayotgan Vatanimizdagi mavjud ma'naviy madaniy omillarga ahamiyat berish bilan birga maorif, ta'lim tarbiya ishlaruga e'tibor kuchaytirilmoqda.

Ta'lim va tarbiya tizimini o`zgarmasdan turib ongni o`zgartirib bo'lmaydi. Ongni, tafakkurni o`zgartirmasdan turub esa biz ko`zlagan oliy maqsad "Ozod va obod jamiayatni barpo etib bo'lamaydi deydilar " deydilar Prezidentimiz I.A. Karimov Respublikamizda ta'limning yangi tizimini amalga oshirishda tariximizdagi ta'lim jarayonlarini o'rganib chiqib ta'limni isloh qilish dasturi tayyorlandi.

Barcha e'tibor ta'lim tizimini demakratik va insonparvarlik tamoillari asosida takomillashtirish, uning moddiy texnika bazasini zamon va davr talablari darajasida ko'tarish va O`zbekiston ma'rifiy salohiyatini kuchaytirish qaratiladi. Shu maqsadda 1992- yil 2- iyulda "Ta'lim to'g'risidagi " qonun qabul qilindi. Ma'lumki, O`zbekiston Respublikasi Prezidenti I. A. Karimovning Respublika Oliy Majlisi IX sessiyasida so`zlangan nutqi va "Ta'lim – tarbiya va kadrlar tayyorlash tizimini tubdan isloh qilish, barkamol avlodni voyaga etkazish to'g'risi" dagi Farmoni muhum ahamiyatga ega edi. Bu esa oliy ta'lim tizimida o'qitish sistemasini isloh qilishni taqizo etadi. Shuning uchun zamon talabiga mos o`quv qo'llanmalar yaratish milliy dasturini amalga oshirish vazifalarida biridir. Matematika, informatika, axborot texnoligiyalari o'sib borayotgan yosh avlodni kamol toptirishda o`quv fanlari sifatida keng imkoniyatlarga ega. Ular tafakkurni rivojlantirib, aqlni chaqxlaydi, fikrlashni tartibga soladi. Bu Prezidentimiz Islom Abdug'aniyevich Karimovning "O`zbekiston XXI asr bo'sag'asida : xavfsizlikka tahdid, barqarorlik shartlari va taraqqiyor kafolatlari" nomli kitoblarida keltirilgan quyidagi so`zlardan ham bilib olask bo'ladi: „Respublika quyidagi yo'nalishlar bo'yicha – jahon darajasidagi ilmiy maktablar yaratilgan bo'lib, ularda tadqiqotlar muvaffaqiyatli olib borilmoqda. Matematika, ehtimollar nazariyasi, tabiiy va ijtimoiy jarayonlarni matematik modellash, informatika va hisoblash texnikasisohasidagi tadqiqotlar shular jumlasidandir“, - degan fikrlarga mos keladi.

Bitiruv malakaviy ish mavzusining dolzarbligi: Asosan hisoblanishi odatdagi usul bilan murakkab bo’lgan integrallar alohida-alohida chuqur o’rganilgan. Integral tenglamalarni o’rganish muhim. Matematik fizikaning teskari masalalari integral tenglamalarga keladi. Integral almashtirishlar asosiy kursida to’liq o’rganilmaydi. Shu sababdan integral almashtirishlardan biri bo’lgan Furye almashtirishini o’rganish mumkin. Chunki integral almashtirishlar keng amaliy tadbiqqa ega. Shu tadbiqlardan biri masalan Fure almashtirishi yordamida texnikada ko’p qo’llaniladigan masalasi BMI da yechib ko’rsatilgan. Umuman aytganda

$$\int_0^{\infty} \frac{\cos x}{1+x^2} dx, \quad \int_0^{\infty} \frac{x \sin x}{1+x^2} dx$$

ko'rinishdagi integrallar va Parseval tengligidab foydalanib odatdagi usul bilan hisoblanishi murakkab bo’lgan xosmas integrallar hisoblangan.

Bitiruv malakaviy ishining maqsadi va vazifalari: Bitiruv malakaviy ishning maqsadi $L_1(-\pi,\pi)$ va $L_2(-\pi,\pi)$ sinflarda Fure almashtirishini keltirish va uning xossalarini o’rganishdan iborat. Har bir sinfda qaralayotgan $f(x)$ funksiyaning turli xil ko’rinishlarida Fure almashtirishi uchun formulalar keltirib chiqarish va ular yordamida misollar yechish. Bunday formulalar I bobning 1.2 rejasida keltirib o’tilgan.

Bitiruv malakaviy ishi tadqiqot ob’ekti va predmeti: Fure almashtirishi:

$$F(\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)\, e^{i\tau x} dx.$$

Fure almashtirishiga teskari amashtirish:

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F(\tau) e^{-i\tau x} d\tau.$$

Parseval tengligi:

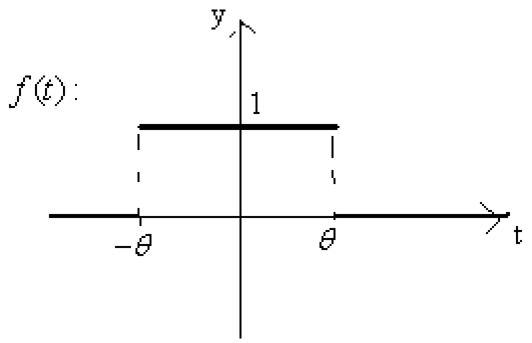
$\|f\| = \|F\|.$

Bitiruv malakaviy ishining yangiligi: Asosan funksiyalarning Fure almashtirishi o’rganilgan bo’lib, undan tashqari ba’zi maxsus funksiyaning Fure almashtirishi keltirib o’tilgan (II bobning 1.3 rejasida). Yuqorida keltirib o’tilgan integrallarni Fure almashtirishi yordamida oson hisoblash yo'llarini ham keltirib o'tilgan. Chunki bunday integrallar amaliyotda ko'p uchraydi va bu usul bunday integrallarni hisoblashda kam vaqt talab qiladi.

Bitiruv malakaviy ishining ilmiy ahamiyati: Bitiruv malakaviy ishi referativ xarakterga ega.

Bitiruv malakaviy ishining amaliy ahamiyati: Bitiruv malakaviy ishi amaliy ahamiyat ega bo'lib texnikaning ba'zi yo'nalishlarda qo'llaniladi. Masalan: To'g'ri to'rtburchakli implus to'g'risida quyidagi masalani keltirib o'tamiz: Quyidagi ko'rinishda aniqlangan juft $f(t)$ funksiya berilgan bo'lsin:

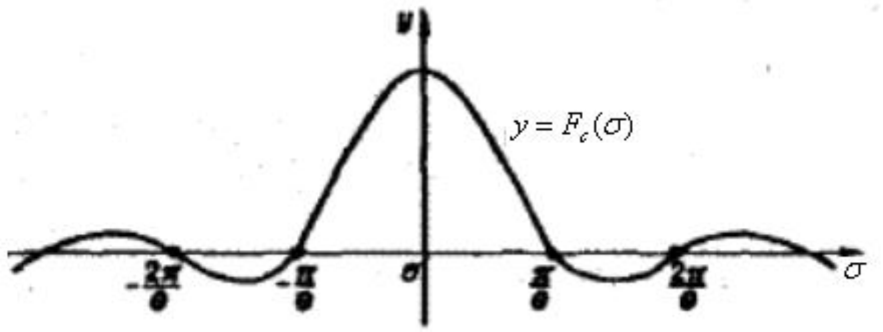
$$f(t)=\begin{cases}1, & agar\ |t|<\theta\\ 0, & agar\ |t|>\theta\end{cases}\qquad \theta>0-const.$$



Bu funksiya uchun Furening kosinus almashtirishi:

$$F_c(\sigma)=\sqrt{\frac{2}{\pi}}\int_0^{\theta}\cos\sigma t dt=\sqrt{\frac{2}{\pi}}\frac{\sin\sigma\theta}{\sigma}$$

bo'lib, bu funksiyaning grafigi



ko'rinishda bo'ladi. Yuqoridagi $f(x)$ funksiyani jismning odatdagi harakati deb olsak, keyingi $F_c(\sigma)$ funksiyaning grafigi sifatida to'qnashuv sodir bo'lgandan so'ng hosil bo'ladigan implus chizig'i deb qarasak bo'ladi.

Bitiruv malakaviy ishi tuzilishi va hajmi: Bitiruv malakaviy ishi kirish, ikki bob va har bir bobda uchtadan paragrof, ikkita xulosa, xotima va foydalanilgan adabiyotardan iborat bo'lib, kirish qismida ishda keltiriladigan misol va Fure almashtirishining texnikaga qo'llanilishi bo'yicha qisqacha ma'lumot keltirib o'tilgan. I bobda $L(-\pi,\pi)$ sinfda Fure almashtirishi. Fure almashtirishining xossalari, teskari Fure almashtirishi va bob bo'yicha xulosa keltirib o'tilgan. II bobda esa $L_2(-\pi,\pi)$ sinfda Fure almashtirishi. Plansheral nazariyasi va Bessel fuksiyasi uchun Furening kosinus almashtirishi va bob bo'yicha xulosa keltirib o'tilgan. Bitiruv malkaviy ishning so'ngida xotima va foydalanilgan adabiyotlar ro'yxati keltirib o'tilgan. Bitiruv malakaviy ishi __ betdan iborat.

I bob. $L(-\pi,\pi)$ sinfda Fure almashtirishi.Fure almashtirishining xossalari.

1.1. $L(-\pi,\pi)$ sinfda Fure almashtirishi.

1. **Fure qatorining kompleks ko'rinishi:** Faraz qilaylik $f(x)$ funksiya $L(-\pi;\pi)$ sinfda berilgan bo'lib $[-\pi;\pi]$ kesmada integirallanuvchi bo'lsin. Bu shartlar quyidagi Fure koeffitsientlarini mavjudligini taminlaydi

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)coskxdx, \quad k = 0,1,2,3, \dots \dots$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)sinkx\, dx, \qquad k = 1,2,3, \dots .$$

f(x) funksiyaga uning Fure qatorini mos qo'yamiz

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k coskx + b_k sinkx)$$

Eyler formulasidan foydalanib $sinkx, coskx$, trganametrik funksiyalarni ko'rsatkichli funksiyalar ko'rininshda yozamiz.

$$sinkx = \frac{e^{ikx} - e^{-ikx}}{2i}, \qquad coskx = \frac{e^{ikx} + e^{-ikx}}{2}; \ i = \sqrt{-1}$$

Quyidagi belgilashlarni kiritamiz.

$$a_{-k} = a_k, \qquad b_{-k} = b_k, \qquad k = 1,2, \dots$$

va

$$c_0 = \frac{a_0}{2}, c_k = \frac{a_k + ib_k}{2}, k \neq 0$$

bo'lsin $f(x)$ funksiyaning Fure qatorini ko'rsatkichli funksiya va c_k koeffitsiyentlari orqali qaytadan yozib olamiz.

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k coskx + b_k sinkx) = c_0 + \sum_{k=1}^{\infty}(a_k \frac{e^{ikx}+e^{-ikx}}{2}$$

$$+ b_k \frac{e^{ikx}-e^{-ikx}}{2i}) = c_0 + \sum_{k=1}^{\infty}(a_k \frac{e^{ikx}+e^{-ikx}}{2} - ib_k \frac{e^{ikx}-e^{-ikx}}{2})$$

$$= c_0 + \sum_{k=1}^{\infty}(\frac{a_{k-ib_k}}{2} * e^{ikx} + \frac{a_{k+ib_k}}{2} e^{-ikx})$$

$$= c_0 + \sum_{k=1}^{\infty}(c_k e^{ikx} + c_k e^{-ikx}) = \sum_{k=-\infty}^{\infty} c_k e^{-ikx};$$

Demak, $f(x)$ funksiyaning Fure qatorini quydagi sodda ko'rinishda yozamiz.

$$f(x) \backsim \sum_{k=-\infty}^{\infty} c_k e^{-ikx}$$

Faraz qilaylik f(x) funksiyaning Fure qatori tekis yaqinlashuvchi bo'lsin u holda matematik analiz kursidan ma'lumki,

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{-ikx} \qquad (1.1.1)$$

tenglik o'rinli bo'ladi.

(1.1.1)tenglikni chegaralangan $e^{i\mu k}$ funksiyaga ko'paytiramiz va bu qatorning tekis yaqinlashish sharti buzilmaydi. $e^{i\mu k}$ chegaralangan funksiyaga ko'paytirgandan so'ng hosil bo'lgan tenglikni $-\pi$ dan π ga integirallaymiz.

Demak, qator tekis yaqinlashuvchi, u holda uning integiralaganimizda Quydagiga ega bo'lamiz.

$$\int_{-\pi}^{\pi} f(x)e^{i\mu x}dx = \int_{-\pi}^{\pi}(\sum_{k=-\infty}^{+\infty} c_k e^{i(\mu-k)x})dx = (\sum_{k=-\infty}^{+\infty} c_k \int_{-\pi}^{\pi} e^{i(\mu-k)x}dx$$

oxirgi integiralni hisoblaymiz

$$\int_{-\pi}^{\pi} e^{i(\mu-k)x}dx = \frac{e^{i(\mu-k)x}}{i(\mu-k)}\Big|_{-\infty}^{+\infty} = \frac{e^{i(\mu-k)\pi} - e^{-i(\mu-k)\pi}}{i(\mu-k)} = \frac{2\sin(\mu-k)\pi}{\mu-k} = 0$$

Eslatib o'tamiz

$\mu \neq k, \mu = k$ bo'lsa $\int_{-\pi}^{\pi} e^{i(\mu-k)x}dx = 2\pi$

Demak ,quydagiga ega bo'ldik

$$\int_{-\pi}^{\pi} f(x)e^{i\mu x}dx = 2\pi c_{\mu}$$

bundan esa c_k koiftsentlarini quydagicha aniqlab olamiz

$$c_k = \frac{1}{2k}\int_{-\pi}^{\pi} f(x)e^{ikx}dx, k = 0 \pm 1 \pm 2..(1.1.2)$$

(1.1.2) L(-πμ, πμ)funksiyalar sinfi uchun Fure qatori

Faraz qilaylik f(x) funksiya $[-\pi\mu; \pi\mu]$ kesmada aniqlangan va L(-πμ, πμ) sinfga qarashli bo'lsin .

Quydagigicha almashtirish bajaramiz

X=u-μ bu yerda u o'zgaruvchi ko'rinib turibdiki $[-\pi; \pi]$ kesmada o'zgaradi bu holda

g(u)=f(uμ)=f(x)

Funksiya[-π;π]kesmada aniqlangan va L(-π;π)sinfga tegishli bo'ladi.

g(u) funksiya uchun Fure qatorini 1-paragrifda ko'rib o'tganimizdagidek aniqlab olamiz.

$$g(u) \sim \sum_{k=-\infty}^{+\infty} c_k\, e^{-iku} \quad c_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(u)e^{iku}\ du$$

c_k koeffitsentda $U = \frac{x}{\mu}$ almashtirish bajarilsa x o'zgaruvchiga qaytib keladi

$$c_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(u)e^{iku}\ du = \frac{1}{2\pi}\int_{-\pi\mu}^{\pi\mu} q\left(\frac{x}{\mu}\right)e^{\frac{ikx}{\mu}}\frac{1}{\mu}\, dx = \frac{1}{2\pi\mu}\int_{-\pi\mu}^{\pi\mu} f(x)\, e^{i\frac{kx}{\mu}}\, dx$$

Demak f(x)funksiya mos keluvchi Fure qatori

$$\sum_{k=-\infty}^{+\infty} c_k\, e^{-\frac{ikx}{\mu}}$$

Ko'rinishida ekan ,bu yerda C_k koeffisent

$$C_k = \frac{1}{2\pi\mu}\int_{-\pi\mu}^{\pi\mu} f(x)e^{\frac{ikx}{\mu}}dx \quad (1.1.3)$$

Formula bilan aniqlanadi.

(1.1.3) Funksiyaning Fure almashtirishiga olib kelish

Faraz qilaylik f(x)∈L (-πμ,πμ) bo'lib barcha μ>0 larda

$$f(x)=\sum_{k=-\infty}^{+\infty} c_k\, e^{-\frac{ikx}{\mu}}$$

tenglik o'rinli bo'lsin c_k koiffitsentini o'rniga (1.1.1)ifodani qo'yib. Quydagi

$$f(x)=\sum_{k=\infty}^{\infty} \frac{1}{2\pi\mu}\int_{-\pi\mu}^{\pi\mu} f(u)e^{-\frac{k}{\mu}(u-x)} \quad (1.1.4)$$

Formulaga ega bo'lamiz (1.1.4)tenglikni o'ng tomoni uning integrali yig'indisi ko'rinishda qarash mumkin .Buni tushuntirish uchun butun R da aniqlangan

$$\varphi_\mu(\varphi)=\frac{1}{2\pi}\int_{-\pi\mu}^{\pi\mu} f(u)e^{i\tau(u-x)}du \quad (1.1.4)$$

Funksiyani kiritamiz sonlar o'qini

$$0 \pm \frac{1}{\mu} \pm \frac{2}{\mu} \ldots..$$

nuqtalar yordamida uzunligi $\frac{1}{\mu}$ ga teng bo'lgan kesmalarga bo'lamiz .

Undan so'ng $\varphi_u(\delta)$funksiyani $\frac{k}{\mu}$ bo'linish nuqtalardagi qiymatni hisoblaymiz.

$$\varphi_\mu\left(\frac{k}{\mu}\right)= \frac{1}{2\pi}\int_{-\pi\mu}^{\pi\mu} f(u)e^{i\frac{k}{\mu}(u-k)}du \quad (1.1.5)$$

(1.1.3) ifodani har ikkala tomoni ni $[\frac{k}{\mu},\frac{k\mu}{\mu}]$ kesmaning uzunligiga ko'paytiramiz va k bo'yicha

$-\infty + \infty$gacha jamlaymiz(yig'indi tuzamiz)Bu jarayon mos ravishda aniq integiralni beradi.Demak biz quydagi munosabatga ega bo'ldik .

$$f(x) =\sum_{k=-\infty}^{+\infty} \frac{1}{2\pi\mu}\int_{-\pi\mu}^{\pi\mu} f(u)e^{i\frac{k}{\mu}(u-x)}du = \sum_{k=+\infty}^{+\infty} \varphi_\mu\left(\frac{k}{\mu}\right)\frac{1}{\mu}(1.1.6)$$

(1.1) ifodani $\mu \to \infty$ dagi limitini topaylik Ko'rinib turibdiki,

$$\mu \to \infty \text{ da } \left[\frac{k}{\mu},\frac{k+1}{\mu}\right]$$

kesmaning uzunligi nolga intiladi va bunda

integral yig'indiintegralga intiladi tenglikni limintga o'tsak bizga Furening integral formulasini beradi .

$$f(x)\int_{-\infty}^{+\infty}\psi_{\infty}(\sigma)\,d\sigma=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\sigma\int_{-\infty}^{+\infty}f\,(u)=\ e^{i\sigma(u-x)}\,du\ (1.1.5)$$

odatdagi usul bilan Fure funksiyasiga ega bo'ldik. Limitga o'tish usuli bilan ham hosil qilish mumkin.A ning birinchidan integral yig'indi cheksiz oraliqda, ikkinchidan integral ostidagi funksiya μ ga bog'liq.

L sinfdan olingan funksiyaning Fure almashtrishi

f (x) funksiya uchun Fure almashtrishining ko'rinishi va aniqlanishi.

Furening integral formulasini quyidagi ko'rinishda yozamiz. formulaga qarang.

$$F(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left\{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}f(u)e^{i\sigma u}du\right\}e^{-i\sigma x}d\sigma$$

$F(\sigma)$ funksiyani aniqlaymiz

$$F(\sigma)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}f(u)\,e^{i\sigma x}du$$

(I.2.1) Furening integral formulasi f(x) funksiya orqali yordamida $F(\sigma)$ funksiyani mos ravishda qo'yishdan hosil bo'lishdi. Fure almashtrilishi bilan faqat uning minus darajasiga farq qiladi.

$$f(x)\ =\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}F(\sigma)e^{-i\sigma x}d\sigma \qquad (I..)$$

(I.2.2) formulaga Fure almashtrish formulasi deyiladi.

$F(\sigma)$ funksiya (I.2.2) formula bilan aniqlanadi va u f(x) funksiyaning Fure almashtrilishi deyiladi.

Berilgan funksiyani biz kichik harf bilan, mos ravishda uning Fure almashtrilsh katta harf bilan belgilaymiz.

Shunday ekan f (x) funksiya f (x) funksiyaning Fure almashtrilishi deyiladi. Bu belgilashlar bilan birga Fure almashtrilishini operator ko'rinishi ham keltrib o'tamiz

Fure almashtirishning operatori yoki qisqacha Fure operatorni V bilan belgilaymiz va

$$F(\sigma)=Vf(x)$$

munosabatga ega bo'lamiz.

(I.2.3) formula Fure almashtrishtrishiga teskari almashtrishi deyiladi.f (x) funksiya F (σ) funksiyaning Fure almashtrishiga teskari almashtrish deyiladi. V^{-1} orqali Fure almashtirishiga teskari almashtrish operatorini belgilaymiz. Shunday qilib quyidagiga ega bo'ldik.

$f(x)V^{-1}F(\sigma);(I.2.1)$

Furening integral formulasini

$$f(x) = V^{-1}Vf(x)$$

ko'rinishida yozipsh mumkin.Eslatib o'tamiz Fure operatori chiziqli ya'ni V operatori chiziqlilik xossasini saqlaydi.

$$V\,[\alpha f(x) + \beta g(x)] = \alpha\, V\, f\,(x) + \beta V\, g(x)$$

bu esa integral xossasidan kelib chiqadi.

1.1.1ta'rif. Quyidagi

$$\varphi(a,b,x)\begin{Bmatrix} 1, x \epsilon (a,b) \\ 0, x \nexists (a,b) \end{Bmatrix}$$

$\text{moslik bog'lanish bilan aniqlangan funksiya } (a,b)\text{dagi xarakteristik}$ funksiya deyiladi .

1.1.1-teorema Agar f(x)ϵL bo'lsa,u holda f(x)funksiyaning Fure almashtirishi F(σ)=Vf(x)butun sonlar o'qida uzluksiz chegaralangan bo'ladi va

$\lim_{(\sigma)\to\infty} F(\sigma) \;\to$ Munosabatlarni qanoatlantiradi

Isbot.F(τ) chegaralangan ekanligi va quydagi bahodan ko'rinadi

$$|F(\sigma)| = \frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{+\infty} f(x)e^{i\sigma x}dx\right.$$

$$\left| \le \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left|f(x)e^{i\sigma x}\right|dx - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left|f(x)dx < \frac{1}{\sqrt{2\pi}}\right| \|f\| < \infty\right.$$

Bu yeda

$$\|f\| = \int_{-\infty}^{+\infty}|f(x)|dx = \infty\,.$$

ekanligidan foydalandik.

Teoremani qolgan tasdiqlarini intervaldagi xarakteristik funksiya uchun isbotlaymiz undan so'ng esa L dagi barcha pag'onali funksiyalar to'plamining to'liq xossasidan foydalanib umummiy xolga o'tkazamiz φ(a,b,x)funksiyaning Fure almashtirishini qaraymiz.

Ф(a,b,π)=

$$\frac{1}{2\pi}\int_{\infty}^{+\infty}\varphi(a,b,x)e^{i\sigma x}\,dx = \frac{1}{2\pi}\int_{-\infty}^{a}\varphi(a,b..x)e^{i\sigma x}dx + \frac{1}{2\pi}\int_{a}^{b}\varphi(a,b,x)\,e^{i\sigma x}dx + \frac{1}{\sqrt{2\pi}}$$

$$\int_{e}^{+\infty}\varphi(a,b,x)e^{i\sigma x}\,dx = \frac{1}{2\pi}\int_{a}^{b}\varphi(a,b,x)e^{i\sigma x}\,dx = \frac{1}{\sqrt{2\pi}}\int_{a}^{b}e^{i\sigma x}dx = \frac{e^{i\sigma x}}{\sqrt{2\pi\, i\tau}}$$

Φ(a,b,τ) funksiya butun sonlaro'qida aniqlanganligi uchun quyidagi.

$$|\partial(a,b,\sigma)|=\frac{1}{\sqrt{2\pi}}\left|\frac{e^{ib\sigma}-e^{ia\sigma}}{i\sigma}\right| \leq \frac{2}{\sqrt{2\pi\sigma}}$$

bahoga ega bo′lamiz bu bahodan

$$\lim_{|\sigma|\to\infty} \Phi(a,b,\sigma) = 0$$

ga ega bo′lamiz

Ixtiyoriy h(x)pag'onali funksiyani φ(a,b,x)funksiyaning chiziqli kombinatsiyasi ko'rinishda ko'rinish tasvirlash mumkin.

$$H(x)=\sum_{k=1}^{n} \alpha_k \varphi(a_k, b_k, x) \text{u holda } H(\tau)\text{uchun}$$

$$H(\tau)$$

$$=\cup\, h(x) \cup \left\{\sum_{k=1}^{k} \alpha_k \varphi(a_k, b_n; x)\right\} = \sum_{k=1}^{n} \alpha_k \cup \varphi(a_k b_k; x) = \sum_{k=1}^{n} \alpha_k \frac{e^{ib_k\sigma}-e^{ia_k\sigma}}{\sqrt{2\pi;\tau}}$$

Formulaga ega bo'lamiz.

1.2.Fure almashtirishning xossalari .

Furening sinus va kosinus almashtirishlari.

Faraz qilaylik haqiqiy qiymatli f(x)funksiya Lsinfdan olingan bo'lsin. f(x) funksiyaning Fure almashtirishini kosinus va sinus orqali formulasini yozamiz:

$$e^{i\sigma x} = cos\sigma x + isin\sigma x,$$

$$F(\sigma) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)cos\sigma x dx + \frac{i}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)sin\sigma x dx.$$

Agar qo'shmcha mulohazani kiritsak ,ya'ni f(x) juft (f(-x)=f(x)) funksiya deb qarasak ikkinchi yig'indi nolga aylanadi va biz f(x) funksiyaning cos almashtirishini olamiz:

$$F_c(\sigma) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)cos\sigma x dx$$

$$\int_{-a}^{a} f(x)dx = 2\int_{o}^{a} f(x)dx, \quad f(-x) = f(x)$$

formuladan foydalansak,

$$F_c(\sigma) = \sqrt{\frac{2}{\pi}} \int_o^{+\infty} f(x) cos\sigma x dx \qquad (1.2.1)$$

formulaga ega bo'lamiz. Xuddi shunday usulda

$$F_s(\sigma) = \sqrt{\frac{2}{\pi}} \int_o^{+\infty} f(x) sin\sigma x dx \qquad (1.2.2)$$

formulaga ega bo'lamiz. Odatda (1.2.1) va (1.2.2) formulalar $f(x)$ funksiya uchun mos ravishda Furening kosinus va Furening sinus almashtirishlari deyiladi.

Bizga ma'lumki ixtiyoriy haqiqiy qiymatli f(x)funksiyani ikkita,juft va toq funksiyalar yig'indisi ko'rinishida tasvirlash mumkin.

$$f(x) = \frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2} = \varphi_j(x) + \varphi_t(x).$$

Funksiyaning bunday ko'rinishini,uning Fure almashtirishida

$F(\sigma) = F_c(\sigma) + iF_s(\sigma)$

ko'rinishni oladi. Agar berilgan funksiya kompleks qiymatli bo'lsa, u holda uni

$$f(x) = Re\, f(x) + iImf(x)$$

ko'rinishda tasvirlaymiz. Bu funksiyani Fure almashtirishini topishda, Furening cos va sin almashtirishlaridan foydalanamiz.

Misol: Juft funksiyaning Fure almashtirishi, uning Fure cos almashtirishiga tengligiga isbotlang.

Isbot: Faraz qilaylik f(-x)=f(x)bo'lsin. Bu holda uning Fure almashtirishi.

$$F(\sigma) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{i\sigma x} dx$$
$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)(cos\sigma x + isin\sigma x) dx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) cos\sigma x dx$$
$$+ \frac{i}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) sinx dx < I_1 + iI_2$$

ko'rinishni oladi. Bunda I_1 va I_2 integiralni alohida hisoblaymiz:

$$I_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) cos\sigma x dx = \frac{1}{\sqrt{2\pi}} 2 \int_0^{+\infty} f(x) cos\sigma x dx = \sqrt{\frac{2}{\pi}} \int_0^{+\infty} f(x) cos\sigma x dx$$

$I_2 = I_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) sin\sigma x dx = \left| \begin{matrix} f(x) = f(-t) = f(t) \\ x = -t, dx = -dt, x_1 = -\infty \\ t_1 = +\infty, x_2 = +\infty, t_2 = -\infty \end{matrix} \right| = -$

$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t) sin\sigma(-t) dt$

$=$

$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t) sin\, \sigma t dt = -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(t) sin\, \sigma t dt = -I_2 \to I_2 = -I_2 + I_2 =$
$0 \to 2I_2 = 0$

$I_2 = 0,$

demak $f(-x) = f(x)$ funksiya uchun

$$I_2 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+} f(x) sin\sigma x dx = 0$$

Bundan esa

$$F_c(\sigma) = I_1 = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{+\infty} f(x) cos\sigma x dx$$

Furening kosinus almashtirishlarini olamiz.

Misol: $f(x) = \begin{cases} sin, x < a \\ 0, x \geq a \end{cases}$ funksiya uchun Furening sinus almashtirishlarini aniqlang .

Yechim:

$$F_s(\sigma) = \sqrt{\frac{2}{\pi}} \int_0^{+\infty} f(x) sin\sigma x dx = F_s(\sigma) \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} sinx sin\sigma x dx$$
$$= \frac{1}{\sqrt{2\pi}} \int_0^a [\cos(1-\sigma)\, x - \cos(1+\sigma)\, x] dx$$
$$= \frac{1}{\sqrt{2\pi}} \left[\frac{\sin(1-\sigma)\, x}{1-\sigma} - \frac{\sin(1+\sigma)\, x}{1+\sigma} \right] \Bigg|_o^a$$

$$= \frac{1}{\sqrt{2\pi}} = \left(\frac{\sin(1-\sigma)\, x}{1-\sigma} - \frac{\sin(1+\sigma)\, x}{1+\sigma} \right) = \frac{1}{\sqrt{2\pi}}$$

$$\left[\frac{sinacos\sigma a - sin\sigma acosa}{1-\sigma} - \frac{sinacos\sigma a + sinacosa}{1+\sigma} \right] = \frac{1}{\sqrt{2\pi}}$$

$$\left[\frac{sinacos\sigma a - sin\sigma acosa + \sigma(sinacos\sigma a - sin\sigma acosa) - sinacos\sigma a - sin\sigma acosa + \sigma(sinacos\sigma a + sin\sigma a}{1-\sigma^2}\right.$$

$$\frac{1}{\sqrt{2\pi}}\left[\frac{-2sin\sigma acosa + 2\sigma sinacos\sigma a}{1-\sigma^2}\right] = \sqrt{\frac{2}{\pi}}\,\frac{\sigma sinacos\sigma a - cosasin\sigma a}{1-\sigma^2} \quad \sigma \neq \pm 1$$

.

Fure almashtirishlarining xossalari.

Quyda keltiriladigan xossalar yordamida jadvalda beriladigan funksiyalarning Fure almashtirishini keltirib chiqarish mumkin.

1.xossa. Chiziqlilik xossasi: Agar

$f_1(x) \to F_1(\sigma)$ va $f_2(\sigma) \to F_2(\sigma)$

bo’lsa, u holda

$C_1 f_1(x) + C_2 f_2(x) \to C_1 F_1(\sigma) + C_2 F_2(\sigma)$

o’rinli.

Isbot: Faraz qilaylik $f_1(x)$ funksiyaning Fure almashtirishi $F_1(\sigma)$ va $f_2(x)$ funksiyaning Fure almashtirishi $F_2(\sigma)$ bo’lsin. U holda

$$F_1(\sigma) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f_1(x) e^{i\sigma x} dx \qquad (1.2.3)$$

$$F_2(\sigma) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f_2\, e^{i\sigma x} dx \qquad (1.2.4)$$

(1.2.3) va (1.2.4) ifodalarni mos ravishda c_1 va c_2 larga ko’paytirib ularni qo’shamiz.

$$C_1 F_1(\sigma) + C_2 F(\sigma) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} (C_1 f_1(x) + C_2 f_2(x))\, e^{i\tau x} dx$$

Bu esa xossani isbotlaydi.

2.xossa: Bir jinslilik xossasi.

Agar

$$f(x) \to F(\sigma)$$

bo'lsa, u holda

$$f(ax) \to \frac{1}{|a|} F\left(\frac{\sigma}{a}\right),\ a \neq o$$

bo'ladi.

Isbot: $f(ax)$ funksiyaning Fure almashtirishini yozamiz va $ax = t$ almashtirish bajaramiz.

$$F(ax) \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\sigma x} f(ax)dx = \frac{1}{|a|} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\frac{\sigma}{a}t} f(t)dt = \frac{1}{|a|} F\left(\frac{\sigma}{a}\right)$$

3.xossa: Agar $f(x) \to F(\sigma)$ bo'lsa, u holda $f(x-a) \to e^{i\sigma x}F(\sigma)$ bo'ladi.

Isbot:

$$F(x-a) \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\sigma x} f(x-a)dx = \begin{vmatrix} x-a=t \\ dx=dt \end{vmatrix}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\sigma(t+a)} f(t)dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\sigma a} e^{i\sigma t} f(t)dt = e^{i\sigma a} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\sigma t} f(t)dt = e^{i\sigma a}F(\sigma).$$

4.xossa: Agar $f(x) \to F(\sigma)$ bo'lsa u holda:

a) $f(x)cos\omega x \to \frac{1}{2}[F(\sigma-\omega)+F(\sigma+\omega)]$

b) $f(x)sin\omega x \to \frac{1}{2i}[F(\sigma-\omega)-F(\sigma+\omega)]$.

Isbot: a) cosωx ni Eyler formulasidan foydalanib yozamiz:

$$\text{f(x)cos}\omega\text{x} \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\sigma x} \text{ f(x) cos}\omega\text{xdx} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\sigma x}$$

$$\text{f(x)}\frac{e^{i\omega x}+e^{-i\omega x}}{2}dx =$$

$$\frac{1}{2}\left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \text{f(x)}e^{i(\sigma-\omega)x}dx + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \text{f(x)}e^{i(\sigma+\omega)x}dx\right] = \frac{1}{2}[F(\sigma-\omega)+F(\sigma+\omega)].$$

b) xossaning isboti ham shunga o'xshash isbotlanadi.

5.xossa. Agar $f(x) \to F(\sigma)$ bo'lsa u holda $f'(x) \to -i\sigma F(\sigma)$ bo'ladi.

Isbot: Fure almashtirishining aniqlanishiga ko'ra $f'(x)$ ning Fure almashtirishi

$$f'(x) \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\sigma x} f'(x)dx = \begin{vmatrix} u=e^{i\sigma x}, du=i\sigma e^{i\sigma x}dx \\ dv=f'(x)dx, v=f(x) \end{vmatrix} =$$

$$\frac{1}{\sqrt{2\pi}}\left(e^{i\sigma x}f(x)\Big|_{-\infty}^{+\infty} - i\sigma \int_{-\infty}^{+\infty} e^{i\sigma x}f(x)dx\right).$$

$f(x)$ funksiyani aniqlanishiga ko'ra butun sonlar o'qida absalut integirallanuvchi bo'lgani uchun $f(\pm\infty)=0$ bunda esa

$$f'(x) \to i\sigma \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\sigma x} f(x) dx = -i\sigma F(\sigma).$$

5.1xossa. Agar $f(x) \to \ F(\sigma)$ va $f(\pm\infty) = .. = f^{(k-1)}(\pm\infty) = 0$ bo’lsa u holda,

$$f^{(k)}(x) \to (i\sigma)^k F(\sigma)$$

bo’ladi.

6.xossa. O’zaro bir qiymatlilik xossasi.Furening kosinus va sinus almashtirishlari uchun o’zaro bir qiymatlilik xossasini keltirib o’tamiz.

Agar $f(x) \xrightarrow{c} F_c(\sigma)$ bo'lsa, u holda $F_c(x) \xrightarrow{c} f(\sigma)$ bo'ladi

Agar $f(x) \xrightarrow{s} F_s(\sigma)$ bo'lsa u holda $F_s(x) \xrightarrow{s} f(\sigma)$ bo'ladi.

1.3.Teskari Fure almashtirishi.

Furening teskari almashtirish ta’rifi.

Faraz qilaylik f(x)ϵL va uning L dagi Furealmashtirishi F(σ)bo'lsin. Umuman olganda F(σ) Lda yotmasligi ham mumkin.

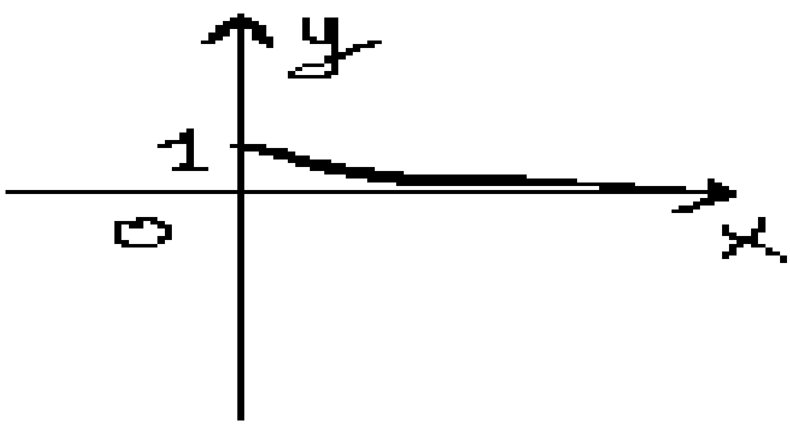
Misol: Quydagi f(x) funksiyani L sinfda Fure almashtirishni toping:

$$F(x)=\begin{cases} e^{-x}, x \geq 0 \\ 0, \ x < 0 \end{cases}$$

Yechim: Bu funksiyani Fure almashtirishini topish uchun (I.2.2)formuladan foydalanamiz ya’ni

$$F(\sigma) \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)\, e^{i\sigma x} dx (I.2.2)$$

Funksiyaning grafigini yasaymiz .



Chizma: 1.1.1 F(x)=$\begin{cases} e^{-x}, x \geq 0 \\ 0, \ x < 0 \end{cases}$ funksiyaning grafigi

Berligan funksiya barcha sonlar o'qida absalut integirallanuvchidir.

$$\int_{-\infty}^{+\infty} |f(x)|\, dx = \int_{-\infty}^{0} 0dx + \int_{0}^{+\infty} e^{-x}\, dx = -e^{-x}|_{-\infty}^{+\infty} = 1$$

F(x)funksiya butun sonlar o'qida bo'lakli silliq bo'ladi;

$$f'(x) = \begin{cases} 0, \ x\epsilon(-\infty; 0) \\ -e^{-x}, \ \ x\epsilon(0, +\infty) \end{cases}$$

(I.2.2) formuladan:

F(σ)=

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\, e^{i\sigma x} dx = \frac{1}{\sqrt{2\pi}}\int_{0}^{+\infty} e^{-x} * e^{i\sigma x}\, dx = \frac{1}{\sqrt{2\pi}}\int_{0}^{\infty} e^{(i\sigma-1)} dx = \frac{1}{\sqrt{2\pi}} * \frac{1}{i\sigma-1} e^{(i\sigma-1)x}\Big|_{o}^{\infty} = \frac{1}{\sqrt{2\pi}} * \frac{1}{(i\sigma-1)}$$

F(σ)funksiyaning modulini hisoblaymiz.

$$F(\sigma)=\frac{1}{\sqrt{2\pi|1-i\sigma|}} = \frac{1}{\sqrt{2F(1+\sigma^2)}} \infty \frac{1}{|\sigma|} = |\sigma|^{-1}$$

Demak, $|F(\sigma)|$ funksiya $|\sigma|^{-1}$ tartib bilan cheksizga intiladi va integirali mavjud bo'lmaydi. Shuning uchun bu Furening teskari almashtirishni to'g'ridan-to'g'ri qo'llay olmaymiz Furening teskari almashtirishini qo'llash uchun xosmas integiralning bo'sh qiymat formulasidan foydalanamiz Ma'lumki sonlar o'qida integirallanuvchi Q(σ) funksiya lim$\int_{n}^{n} Q(\sigma)d\sigma$ ko'rinishda aniqlanadi bu yerda N, N_1 lar bir − biridan bog'liqsiz ravishda ∞ ga intiladi.

Agar bu limit mavjud bo'lmasa, lim$\int_{-N}^{N} Q(\sigma)d\sigma$ limit mavjud bo'ladi. Bu holda lim$\int_{-\infty}^{+\infty} Q(\sigma)d\sigma$ xosmas integiral Koshining bo'sh qiymati manodagi integral deb yuritiladi;

Misol: Quyidagi Funksiyaning Fure almashtirishni toping

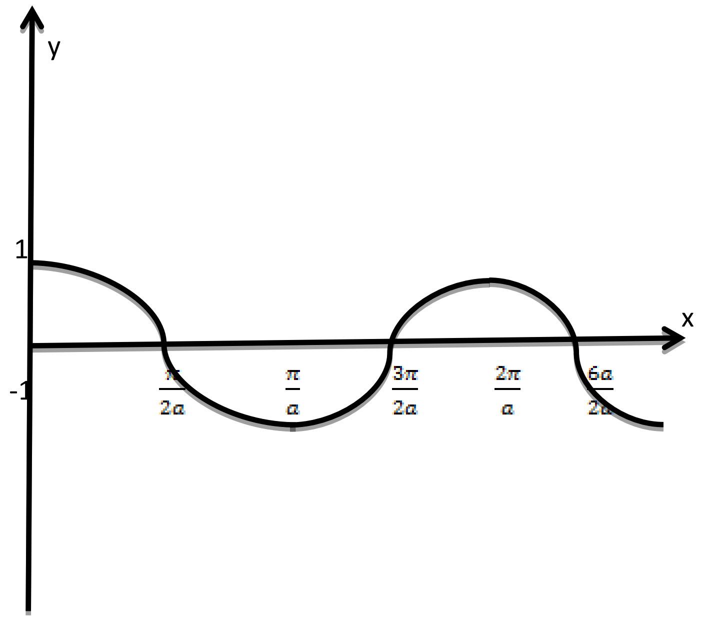
$$F(x)\begin{cases} cosax, \ |x| < a \\ 0, \ \ \ |x| \geq a \end{cases}$$

Yechim: Ko'rinib turibdiki f(x) fuksiya R da aniqlangan va absalyut integirallanivchi:

$$I=\int_{-\infty}^{+\infty}|f(x)|dx = \int_{-a}^{a}|cosax|dx = 2\int_{0}^{a}|cosax|dx$$

Hosil bo'lgan integiralni tahlil qilamiz.

Y(x)=cosax; a>0



Chizma-1.1.2 Y(x)=cosax; a>0 funksiya grafigi

1) Faraz qilaylik aϵ(0;$\frac{\pi}{2a}$)

I =$2\int_0^a|\text{cosax}|\text{dx} = 2\int_0^a \text{cosaxdx} = \frac{2}{a}\text{sinax}\,|_0^a = \frac{2}{a}sin^2$

Agar $a \to +0$ bo'lsa

$$\lim_{a\to+0}\frac{2sina^2}{a} = 2\lim_{a\to+0}\frac{sina^2}{a^2}a = 0$$

Chekli son

2) aϵ($\frac{\pi}{2a}$,$\frac{3\pi}{2a}$)

I=2

$$\int_0^a|\cos ax|dx = 2\int_0^{\frac{\pi}{2a}}\cos axdx = -2\int_{\frac{\pi}{2a}}^{a}\cos axdx = \frac{2}{a}\sin ax\,|_0^{\frac{\pi}{2a}} - \frac{2}{a}\sin ax\,|_{\frac{\pi}{2a}}^{\frac{a}{\pi}} = \frac{2}{a} - \frac{2}{a}(sina^2-1) = \frac{4}{a} - \frac{2}{a}sina^2 = \frac{2}{a}$$

$(2\text{-}\sin a^2)$

3) $a\epsilon(\frac{3\pi}{2a},\frac{5\pi}{2a})$

I=2

$$\int_0^a|\cos ax|dx = \int_0^{\frac{\pi}{2a}}\cos axdx - 2\int_{\frac{\pi}{2a}}^{\frac{3\pi}{2a}}\text{coax}dx + 2\int_{\frac{3\pi}{2a}}^{a}\cos axdx = \frac{2}{a} + \frac{4}{a} + \frac{2}{a}(\sin a^2 + 1) = \frac{8}{a} + \frac{2}{a}\sin a^2 = \frac{2}{a}(4 + \sin a^2)$$

a –chekli son bo'lganligi uchun integral chekli qiymat qabul qiladi.Bu esa f(x)funksiyaning butun sonlar o'qida absalut intgirallanuvchi ekanligi kelib chiqadi .

Endi Funksiyaning Fure almashtirishini topamiz:

F(σ)

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}f(x)e^{i\sigma x}dx = \frac{1}{\sqrt{2\pi}}\int_0^a cosaxe^{i\sigma x}dx = \frac{1}{\sqrt{2\pi}}\int_0^a\frac{e^{i\tau x}+e^{-i\sigma x}}{2} * e^{i\sigma x}dx = \frac{1}{2\sqrt{2\pi}}\int_0^a\left(e^{i(a+\sigma)x}+e^{i(a-\sigma)x}\right)dx =$$

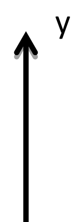
$$\frac{1}{2\sqrt{2\pi}}\left(\frac{1}{i(\sigma+a)}e^{i(a+\sigma)x} + \frac{1}{i(\tau-a)}e^{i(a-\sigma)x}\right)|_0^a = -\frac{i}{2\sqrt{2\pi}}\left(\frac{1}{a+\tau}\left(e^{i(a+\sigma)a}-1\right) + \frac{1}{\sigma-a}\left(e^{i(\sigma-a)a}-1\right)\right) = -\frac{i}{2\sqrt{2\pi}}\left(\frac{e^{i(a^2+\sigma a)}}{\sigma+a} + \frac{e^{i(a^2-\sigma a)}}{\sigma-a} - \frac{2\sigma}{\sigma^2-a^2}\right)$$

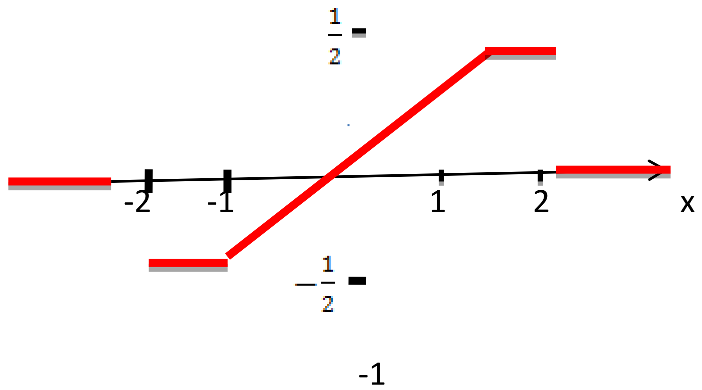
Misol: quyidagi funksiyaning Fure almashtirishini toping.

$$f(x)\begin{cases}\frac{x}{2}|x| \le 1\\ \frac{1}{2}, |<|x| \le 2\\ 0, |x| > 2\end{cases}$$

Yechim:Berilgan funksiyaning [-2;2]kesma tashqarisidan nolga teng ya'ni Fure funksiyasidir

Eslatma! Kesmaning tashqarisidagi barcha nuqtada nolga teng bo'lgan funksiyalarga limit funksiya deyiladi) y=f(x)funksiyani tekislikda tasvirlaymiz:





Chizma:1.1.3) y=f(x) funksiyani tekislikda tasviri

(I 2.2) integiraldan formuladan foydalanamiz:

$$F(\sigma)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)e^{i\sigma x}dx\frac{1}{\sqrt{2\pi}}\int_{-2}^{2} f(x)e^{i\sigma x}\,dx$$

f(x)funksiya siniq chiziqdan iborat bo′lganligi uchun$[-2;2]$kesmani

bo′laklarga bo'lib integirallaymiz .

$$[-2;2]=[-2;-1]\cup[-1;1]\cup[1;2]$$

$$F(\sigma)=\frac{1}{\sqrt{2\pi}}\int_{-2}^{-1}\frac{1}{2}e^{-i\sigma x}dx+\int_{-1}^{1}\frac{x}{2}e^{i\sigma x}dx+\int_{1}^{2}\frac{1}{2}e^{i\sigma x}dx)=$$

$$\frac{1}{\sqrt{2\pi}}(J_1+J_2+J_3)J_1+J_3=\int_{-2}^{-1}\frac{1}{2}e^{i\sigma x}dx+\int_{1}^{2}e^{i\sigma x}\,dx=$$
$$\frac{1}{2i\sigma}e^{i\sigma x}\int_{-2}^{-1}+\frac{1}{2i\sigma}e^{i\sigma x}\int_{1}^{2}\frac{1}{2i\sigma}\left(e^{-i\sigma}-e^{-i2\sigma}+e^{i2\pi}-e^{i\sigma}\right)=\frac{1}{2i\sigma}\left(e^{i2\pi}-e^{-i2\pi}-\left(e^{i\sigma}-e^{-i\sigma}\right)\right)=\frac{sin2\sigma-sin\sigma}{\sigma}$$

Keyingi integiralni hisoblashda Eyler formulasidan foydalanamiz.

$$J_2=\int_{-1}^{1}\frac{x}{2}e^{i\sigma x}dx=\frac{1}{2}\int_{-1}^{1}x(cos\sigma x+isin\sigma x)dx\frac{i}{2}\int_{-1}^{1}xsin\sigma xdx=>$$

$$\left|\begin{matrix}u=x,du=dx\\ sin\sigma xdx=dv,v=-\frac{1}{v}cos\sigma\end{matrix}\right|=\frac{i}{2}\left(-\frac{x}{\sigma}cos\sigma x\,|_{-1}^{1}+\frac{1}{\sigma}\int_{-1}^{1}cos\sigma xdx\right)$$
$$=\frac{i}{2\sigma}\left(-cos\sigma-cos\sigma+\frac{1}{\sigma^2}sin\sigma+\frac{1}{\tau^2}sin\sigma\right)$$
$$=\frac{i}{2\sigma}\left(1-2cos\sigma+\frac{2}{\sigma}sin\sigma\right)=i\frac{sin\sigma-\sigma cos\sigma}{\sigma^2}$$

Bu yerda

$$\int_{-1}^{1} \mathrm{x}\cos\sigma \mathrm{x}d\mathrm{x} = 0$$

tenglikdan foydalanib demak bergan funksiyaningFure almashtirishi.

$$F(\sigma)=\frac{1}{\sqrt{2\pi}}\left(\frac{sin2\sigma - sin\sigma}{\sigma} + i\,\frac{sin\sigma - \sigma cos\sigma}{\sigma^2}\right)$$

Biz o'rganayotgan masalalardagi f(x)funksiyaning Fure almashtirishi yaxshigina fizikaviy ahamiyatga ega.

F(σ)funksiyaF(x)ningspektralxarakteristikasi, $|F(\sigma)|$esa f(x)ning amplitudaviy spektori, argF(x)esa f(x)fazoviy spektori deyiladi.

Misol:f(x)funksiyaning amplitudaviy speklari toping bu yerda.

$$F(x)=\begin{cases} 1,0 < x \leq 1 \\ 0, R/(0,1] \end{cases}$$

Yechim:f(x)funksiyani Fure almashtirishini topish uchun (I,2.2) formuladan foydalanamiz.

$$F(\sigma)=$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)e^{i\sigma x}\,dx = \frac{1}{\sqrt{2\pi}}\int_{0}^{1} e^{i\sigma x}dx = \frac{1}{i\sqrt{2\pi\sigma}}e^{i\sigma x}\,|_0^1 = \frac{1}{i\sqrt{2\pi v}}\left(e^{i\sigma} - 1\right) = \frac{1}{i\sqrt{2\pi\sigma}}(cos\sigma - 1 + isin\sigma) \neq$$

Shuning uchun

$$|F(\sigma)| = \frac{1}{\sqrt{2\pi\sigma}}\sqrt{cos^2\sigma - 2cos\sigma + 1 + sin^2\sigma} = \frac{1}{\sqrt{2\pi\sigma}}\sqrt{8 - 2sin\sigma}$$

$$= \frac{\sqrt{2}}{\sqrt{\pi|\sigma|}}\sqrt{\frac{1 - cos\sigma}{2}}\sqrt{\frac{2}{\pi}} * \frac{1}{|\sigma|} * (sin\frac{\sigma}{2})$$

$$\left|F(\tau) = \sqrt{\frac{2}{\pi} * \frac{1}{|\sigma|}}\right| sin\frac{\pi}{2}$$

Amplitudaviy spektor

$$0 \leq |F(\alpha)| \leq \sqrt{\frac{2}{\pi}} * \frac{1}{|\sigma|}$$

tengsizlikni qanoatlantiradi.

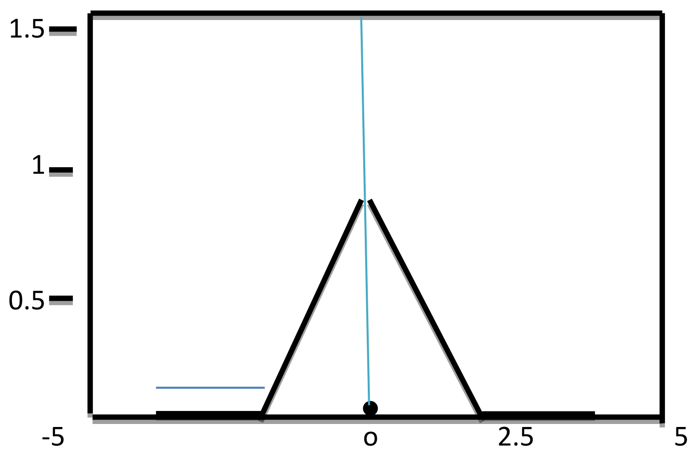
Endi ko’rib chiqadigan masalalarimiz bevosita fizika va informatika bilan bog’liqdir .

Misol: Uchburchak implusining Fure almashtirishini aniqlang va uning amplitudaviy spektorini yasang.

$$f(x)=\begin{cases} 1-\frac{2|x|}{\tau}, & |x|\le\frac{\tau}{2} \\ 0, & |x|>\frac{\tau}{2} \end{cases}$$

Yechim:Berilgan funksiyani $\tau=4$ bo’lgan holda grafigini yasaymiz ,hamda Fure almashtirishini topamiz:

Grafimiz τ=4 uchun



Chizma:1.1.4 funksiyani $\tau=4$ bo’lgan holda grafigi

F(σ)=

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \mathrm{f(x)e^{i\sigma x}dx} = \frac{1}{\sqrt{2\pi}}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\left(1-\frac{2|x|}{\tau}\right)e^{i\sigma x}dx = \frac{1}{\sqrt{2\pi}}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\left(1-\frac{2|x|}{\tau}\right)e^{i\sigma x}$$

(cosσx+isinσx)dx=

$$\frac{1}{\sqrt{2\pi}}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\left(1-\frac{2|x|}{\tau}\right)cos\sigma x dx+\frac{i}{\sqrt{2\pi}}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\left(1-\frac{2|x|}{\tau}\right)\sin\sigma x dx=\frac{i}{\sqrt{2\pi}}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\left(1-\frac{2|x|}{\tau}\right)\cos\sigma x dx=\sqrt{\frac{2}{\pi}}\int_{o}^{\frac{\tau}{2}}\left(1-\frac{2|x|}{\tau}\right)\cos\sigma x dx=\sqrt{\frac{2}{\pi}}\frac{1}{\sigma}\int_{o}^{\frac{\tau}{2}}\left(1-\frac{2|x|}{\tau}\right)d(\sin\sigma x)=$$

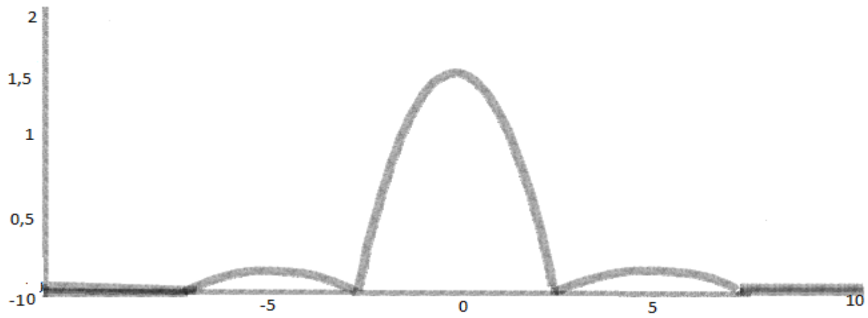
$$\sqrt{\frac{2}{\pi}}\frac{1}{\sigma}\int_{o}\left(\left(1-\frac{2|x|}{\tau}\right)sin\sigma x\int_{0}^{\frac{\tau}{2}}+\frac{2}{\pi}\int_{0}^{\frac{\tau}{2}}sin\sigma x dx\right)=$$

$$\sqrt{\frac{2}{\pi}}\frac{1}{\sigma}\left(-\frac{2}{\tau\sigma}cos\sigma x|\int_{0}^{\frac{\tau}{2}}\right)\sqrt{\frac{2}{\pi}}\frac{2}{\tau\sigma^{2}}\left(1-cos\frac{\sigma\tau}{2}\right)=\sqrt{\frac{2}{\pi}}\frac{4}{\tau\sigma^{2}}sin^{2}\left(\frac{\tau\sigma^{2}}{4}\right)=$$

$$\sqrt{\frac{2}{\pi}}\frac{\tau}{4}\frac{sin^{2}\left(\frac{\tau\sigma}{4}\right)}{\left(\frac{\tau\sigma}{4}\right)^{2}}=\sqrt{\frac{2}{\pi}}\frac{\tau}{4}\left(sin\left(\frac{\sigma\tau}{4}\right)\right)^{2}$$

$\tau=4$ da amplitudaviy spektorning signali

$$|F(\sigma)|=\sqrt{\frac{2}{\pi}}(sinc\sigma)^{2}$$



Chizma:1.1.5 $\tau=4$ da amplitudaviy spektorning signali

Fure almashtirishini teskarilash

1.3.1-ta'rif.f(x) funksiya x=$x_o$nuqtada Dini shartini qanoatlantiradi deyiladi,agar

$$\varphi(u)=\frac{f(x_{o+u})-f(x_o)}{u}$$

Funksiya $\delta>o$ larda L(-δ,δ)sinfga tegishli bo'lsa

1.3.1-teorema. Agar f(x)ϵL bo'lib va ba'zi x nuqtalarda Dini shartini qanoatlantirsa ,u holda bu nuqtalarda.

$$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} F(\sigma)\, e^{-i\sigma x} dx$$

tenglik o'rinli bo'lib, shuning bilan birgalikda qaralayotgan integral

bosh qiymat ma'nosida bo'ladi.

Isbot: Agar f(x)ϵL bo'lsa,u holda Riman–Lebeg teoremasiga ko'ra F(σ)=Vf(x)funksiya chegaralangan butun sonlar o'qida uzluksiz va

$$\lim_{|\sigma|\to\infty} F(\sigma) = o$$

munosabatni qanoatlantiradi.Quyidagi

$$f_N(x) = \frac{1}{\sqrt{2\pi}}\int_{-N}^{+N} F(\sigma) e^{-i\sigma x} d\sigma$$

Tenglik bilan aniqlanuvchi $f_N(x)$funksiyani qaraymiz, bu yerda N-ayrim musbat son . Bu tenglikka F(σ)funksiyaning qiymatini qo'yamiz va quyidagi munosabatni olamiz

$$f_N(x)=\frac{1}{2\pi}\int_{-N}^{N} d\sigma \int_{-\infty}^{+\infty} f(u)\, e^{i\sigma(u-x)} dx.$$

Bu yerda biz xosmas integralni σ bo'yicha integrallashimiz mumkin , yoki

$$\left|f(u)e^{i\sigma(u-x)}\right| \le |f(u)|$$

Baho o'rinli bo'lsa ,xosmas integral σ parameter bo'yicha tekis yaqinlashuvchi bo'ladi.Fubini teoremasidan foydalanib integrallash tartibini o'rgatamiz:

$$f_N(x) = \frac{1}{2\pi}\int_{-N}^{+N} d(\sigma)\int_{-\infty}^{+\infty} f(x)\, e^{-i\sigma(u-x)} du =$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} f(u)\, du \int_{-N}^{N} e^{i\sigma(u-x)} d\sigma = \int_{-\infty}^{+\infty} f(u) \frac{e^{i\sigma(u-x)}}{i\sigma(u-x)}\Bigg|_{\sigma=-N}^{\sigma=N} du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(u) \frac{e^{iN(u-x)} - e^{iN(u-x)}}{i\sigma(u-x)} du = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(u) \frac{sinN(u-x)}{u-x} du.$$

Hosil bo’lgan integralda t=u-x almashtirish bajarib ,uni quyidagi ko’rinishda yozamiz:

$$f_N(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(x+t)\frac{sinNt}{t}dt \qquad (1.3\ 1)$$

Xosmas integrallarni integrallash kursidan ma’lum bo’lgan

$$\int_0^{\infty}\frac{\sin\alpha x dx}{x} = \frac{\pi}{2}sign\alpha \qquad (1.3.2)$$

Formuladan foydalanamiz, bu yerda

$$sign\alpha = \begin{cases} 1, \alpha > 0 \\ 0, \alpha = 0 \\ -1, \alpha < 0 \end{cases}$$

(1.3.2) formuladan foydalanib quyidagi tenglikni olamiz

$$\int_{-\infty}^{+\infty}\frac{\sin Nt}{t} = \int_{-\infty}^{0}\frac{sinNt}{t}d + \int_0^{-\infty}\frac{sinNt}{t}dt = \int_0^{+\infty}\frac{sinNt}{t}dt + \int_0^{\infty}\frac{sinNt}{t}dt + \int_0^{\infty}\frac{sinNt}{t} -$$

$$2\int_0^{\infty}\frac{sinNt}{t}dt = 2\frac{\pi}{2}singnN = \pi$$

Agar bu ifodani π ga bo’lib f(x) ikkala tomonini f(x) ga ko’paytirsak , u holda f(x) ni quyidagi integral ko’rinishini oladi.

$$f(x) = \frac{1}{\pi}\int_{-\infty}^{+\infty} f(x)\frac{\sin Nt}{t}dt \qquad (1.3.3)$$

Keyingi bosqichda , x nuqtada Dini shartini qanoatlantiruvchi

$$\lim_{n\to\infty} f_N(x) = f(x)$$

Munosabatni isbotlashdan iborat bo’ladi. Agar shu munosabatni qura olsak, u holda x nuqtada bo’lib qiymat ma’nosida teskari fure almashtirish mavjud bo’lib,u f(n) bilan mos tushadi.(1.4.1)dan (1.4.3) ni ayiramiz.

$$f_N(x) - f(x) = \frac{1}{\pi}\int_{-\infty}^{+\infty}[f(x+t) - f(x)]\frac{\sin Nt}{t}dt$$

Va integralni 3 qismga ajratamiz.

$$f_N(x) - f(x) = \frac{1}{\pi}\int_{|t|\le A}\frac{f(x+t)+f(x)}{t}\sin Ntdt + \frac{1}{\pi}\int_{|t|>A}\frac{f(x+t)}{t}\sin Ntdt - \frac{f(x)}{\pi}\int_{|t|>A}\frac{\sin Nt}{t}dt. \qquad (1.3.4)$$

$\forall\, \varepsilon > 0$ni berilgan bo'lsin, u holda (1.3.4) tenglikdagi o'ng tomondan ikkinchi yig'indining moduli $\exists\, A\ (A > 1)$ larda $\frac{\varepsilon}{3}$ dan kichik qilib olish mumkin , yoki

$$|t \geq 1|$$

uchun

$$\left|\frac{\sin Nt}{t}\right| \leq 1$$

tengsizlik o'rinli bo'ladi, t argument bo'yicha esa $f(x+t) \epsilon L$ bo'ladi.(1.3.4) tenglikdagi o'ng tomonda joylashgan uchinchi ifodani esa modul jihatdan $\frac{\varepsilon}{3}$ dan kichik bo'lgan A sonlarni tanlash mumkin, chunki $N \geq 1$ da

$$\int_{-\infty}^{+\infty} \frac{\sin Nt}{t}\, dt$$

Xosmas integral tekis yaqinlashuvchi Dini shartidan quyidagi

$$\psi(x) = \begin{cases} \dfrac{f(x+t) - f(x)}{t}, & |t| \leq A \\ 0\ , & |t| > A \end{cases}$$

funksiya L sinfga tegishli bo'ladi, u holda Riman – Lebeg teoremasidan

$$\lim_{n\to\infty} \int_{-\infty}^{+\infty} \psi(t) \sin Nt\, dt$$

Munosabat ko'rinadi va n sonlar tanlab olish yordamida (1.3.4) tenglikni o'ng tomonidagi birinchi yig'indini modul jihatdan $\frac{\varepsilon}{3}$ dan kichik qilib olamiz, shunday qilib, $\forall \varepsilon > 0$ da $\exists\, N_0$ son topiladiki undan kata $\forall\, N > N_0$ lar uchun

$$|f_N(x) - f(x)| < \varepsilon$$

Shart bajariladi va uni bevosita

$$\lim_{n\to\infty} f_N(x) = f(x)$$

Munosabat bilan yozamiz.Teorema isbotlandi.

Quyida yana bitta Fure almashtirishini teskarilashga doir natijani isbotsiz keltirib o'tamiz.

1.3.2-ta'rif. F(x) funksiya [a,b] kesmada chegaralangan variatsiyaga ega deyiladi, agar [a,b] kesmani barcha mumkin bo'lgan

$a = x_0 < x_1 < x_2 < \dots < x_n = b$

bo'linishlarda

$$sup \sum_{k=0}^{n-1} |f(x_{k+1}) - f_{(k)}|$$

1.3.2-teorema. Agar

$$f(x) \epsilon L va \delta > 0, [\, x_0 - \delta, x_0 + \delta]$$

kesmada variatsiyasi chegaralangan bo'lsa, u holda

$$\frac{f(x_0+0)+f(x_0-0)}{2} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\sigma)e^{-ix_0} d\sigma,$$

Tenglik o'rinli bo'lib, shu bilan birga integral bosh qiymat ma'nosida bo'ladi.

Xususi holda, agar f(x) funksiya $x - x_0$ nuqtada uzluksiz bo'lsa, u holda tengklikning chap tomonini $f(x_0)$ ga almashtirish mumkin, ya'ni

$$f(x_0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\sigma)\, e^{-ix_0} d\sigma;$$

Misol: Agar $F(\sigma)$ funksiya f(x) funksiyaniong Fure almashtirishi, bo'lsa, u holda quyidagi funksiyalar uchun $F(\sigma)$ larni ko'rinishni aniqlang.

1) f(x)
2) f(x+a)
3) f(x+a)+f(x-a)
4) 2f(x)-f(x-a)-f(x+a)
5) f(ax+b)
6) e^{ibx}f(ax)
7)) f(x) axbx
8) f(ax+c) sin bx

Eslatma! a>0, b>0

1)f(x) : $\overline{f(x)}$

Yechim:

$$\phi(\sigma)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\, e^{i\sigma x}dx=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\, c^{-i\sigma x}dx=$$

$$\left\{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\, e^{i(-\sigma)x}dx\right\}=F(-\sigma)$$

Demak f(x) funksiyaning Fure almashtirishi

$$F(-\sigma)\ f(x)\xrightarrow{F}\overline{F(-\sigma)}$$

2)f ω: $f(x+a)$

Yechim :

$$\Phi(\sigma)=$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x+a)\, e^{i\sigma x}dx=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)\, e^{i\sigma(t-a)}dt=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)\, e^{i\sigma t}dt=e^{-i\sigma a}F(\sigma)$$

$$F(x+a)\xrightarrow{F} e^{-i\sigma x}F(\sigma).$$

3) f(x): f(x+a)+f (x-a)

Yechim:

$$\Phi(\sigma)=$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x+a)+f(x-a)\, e^{i\sigma x}dx=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x+a)\, e^{i\sigma x}dx+\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x-a)\, e^{i\sigma x}\, dx=e^{-i\sigma a}\, F(\sigma)+e^{i\sigma a}\, F(\sigma)=2\,\frac{e^{i\sigma a}+e^{-i\sigma a}}{2}.F(\sigma)=2cos(\sigma a)F(\sigma)$$

$$f(x+a)+f(x-a)\xrightarrow{F} 2cos(\sigma a)F(\sigma).$$

4) f(x) : 2 f(x)- f(x-a)-f(x+a)= 2 f(x) – (f(x-a) + f(x+a)).

Yechim : f (x) ning Fure almashtirilishi va 3), 5) lardan foydalanib,

$$\Phi\ (\sigma)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}(2f(x)-f(x-a)-f(x-a))e^{i\sigma x}dx=$$

$$= 2F(\sigma) - 2\cos\tau u\, F(\sigma) = 2(1-\cos(\sigma a))F(\sigma) = 4\frac{1-\cos(\sigma a)}{2}F\sigma$$

$$= 4\sin^2\frac{\sigma a}{2}F(\sigma)$$

Demak,

$$2f(x) - f(x-a) - f(x+a) \xrightarrow{F} 4\sin^2\frac{\sigma a}{2}F(\sigma)$$

5) f(x):f(ax+b)

<u>Yechim:</u>

$$\Phi(\sigma) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(ax+b)e^{i\sigma x}\,dx = \left| \begin{matrix} & ax+b=t \\ x = \dfrac{t-b}{a} & dx = \dfrac{1}{a}dt \end{matrix} \right|$$

$$= \frac{1}{a\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)e^{i\sigma\frac{t-b}{a}}\,dt = \frac{e^{i\sigma\frac{b}{a}}}{a\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)e^{i\frac{\sigma t}{a}}\,dt$$

$$= \frac{e^{i\sigma\frac{t-b}{a}}}{a}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(ax+b)e^{i\sigma x} = \frac{1}{a}e^{i\sigma\frac{b\sigma}{a}}F(\frac{\sigma}{a})$$

$$f(ax+b) \xrightarrow{F} \frac{1}{a}e^{i\tau\frac{b\tau}{a}}F(\frac{\sigma}{a})$$

6) $f(x){:}\, e^{ibx}$f(ax)

<u>Yechim:</u>

$$\Phi(\sigma) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{ibx} f(ax)e^{i\sigma x}dx = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{i(b+\sigma)x} f(ax)dx$$

$$= \left|x = \frac{a}{t}, dx = \frac{1}{a}dt\right| == \frac{1}{a\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{i(\frac{b+\sigma}{a})t}dt = \frac{1}{a}F(\frac{b+\sigma}{a})$$

$$e^{ibx}f(ax) \xrightarrow{F} \frac{1}{a}F(\frac{b+\sigma}{a})$$

7) f(x):f(x)cos bx

<u>Yechim:</u> $f(x)\cos bx$ni Eyler formulsidan foydalanib quyidagicha yozamiz.

$$f(x)\cos bx = f(x)\frac{e^{ibx}+e^{-ibx}}{2}$$

$$\Phi(\sigma)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\cos bx\, e^{i\sigma x}\,dx=$$
$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\frac{1}{2}\left(e^{ibx}+e^{-ibx}\right)e^{i\sigma x}dx=$$
$$=\frac{1}{2\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)e^{(b+\sigma)x}\,dx+\frac{1}{2\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)e^{(b-\sigma)x}\,dx=$$
$$=\frac{1}{2}F(b+\sigma)+\frac{1}{2}F(\sigma-b)=\frac{F(b+\sigma)+F(b-\sigma)}{2};$$

Demak,

$$f(x)\cos bx \overset{F}{\rightarrow} \frac{F(b+\sigma)+F(b-\sigma)}{2}$$

8) $f(x): f(ax+c)\sin bx$

<u>Yechim:</u> $f(ax+c)\sin bx$ni Eyler formulsidan foydalanib quyidagicha yozamiz.

$$f(ax+c)\sin bx=f(ax+c)\frac{e^{ibx}+e^{-ibx}}{2i}$$

$$\Phi(\sigma)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(ax+c)\sin bx\, e^{i\sigma x}\,dx$$
$$=\int_{-\infty}^{+\infty} f(ax+c)\left(\frac{e^{ibx}+e^{-ibx}}{2i}\right)e^{i\sigma x}dx=$$

$$\frac{1}{2i\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(ax+c)\left(e^{i(\sigma+bx)}+e^{i(\sigma-b)x}\right)dx=\left|\begin{matrix}ax+c=t\\ x=\frac{t-c}{a},\end{matrix}\quad dx=\frac{1}{a}dt\right|=$$
$$\frac{1}{2i\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)e^{i\frac{\sigma+b}{a}(t-c)x}\frac{1}{a}dt-\frac{1}{2i\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)e^{i\frac{\sigma-b}{a}(t-c)x}\frac{1}{a}dt=$$
$$\frac{1}{2ai\sqrt{2\pi}}\left[\int_{-\infty}^{+\infty} f(t)e^{i\frac{\sigma+b}{a}t}e^{-i\frac{\sigma+b}{a}c}dt-\int_{-\infty}^{+\infty} f(t)e^{i\frac{\sigma+b}{a}t}e^{-i\frac{\sigma+b}{a}c}dt\right]=$$
$$\frac{1}{2ai}e^{-i\frac{\sigma+b}{a}c}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)e^{i\frac{\sigma+b}{a}t}dte^{-i\frac{\sigma+b}{a}c}-$$
$$e^{-i\frac{\sigma+b}{a}c}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)e^{i\frac{\sigma+b}{a}t}dt=\frac{1}{2ai}\left[F\left(\frac{\sigma+b}{a}\right)e^{-i\frac{\sigma+b}{a}c}-F\left(\frac{\sigma-b}{a}\right)e^{-i\frac{\sigma+b}{a}c}\right]=$$
$$\frac{1}{2ai}\left[e^{-i\frac{\sigma+b}{a}c}F\left(\frac{\sigma+b}{a}\right)-e^{-i\frac{\sigma+b}{a}c}F\left(\frac{\sigma-b}{a}\right)\right]$$
;

Demak

$$f(ax+c)\sin bx\overset{F}{\rightarrow}\frac{1}{2ai}\left[e^{-i\frac{\sigma+b}{a}c}F\left(\frac{\sigma+b}{a}\right)-e^{-i\frac{\sigma+b}{a}c}F\left(\frac{\sigma-b}{a}\right)\right]$$

Xulosa

I-bob L($-\pi,\pi$)sinfda Fure almashtirishi . Fure almashtirishining xossalari deb nomlanadi I-bob 3 paragrifdan iborat L($-\pi,\pi$)sinfda Fure almashtirishi,bo'lib unda L($-\pi\mu,\pi\mu$)funksiyalar sinfi uchun Fure qatori va L sinfdan olingan funksiyaning Fure alamshtirishi haqida ma'lumot berilgan 2-paragrifda Fure almashtirishning xossalari Furening kosinus va sinus almashtirishlari haqida umumiy tushunchalar berilgan 3-paragrifda esa teskari Fure almashtirishlari haqida bir qancha misollar keltirilgan bo'lib unda Fure almashtirishini Teskarilashga doir misollar ,t'tif tioremalar ularning isbotlari keltirilgan .

II-bob L_2 sinfda Fure almashtirishi Plansherov nazariyasi Bessel funksiyasi uchun Fure almashtirishi.

2.1.L_2sinfda Fure almashtirishlari

.

Plansherel nazariyasi.

2.1.1 L_2sinf haqida

Butun sonlar o’qida aniqlangan va o’lchovli haqiqiy yoki komplekis qiymatli haqiqiy argumentli f(x)funksiya $\mathrm{L_2 sinfga\ qarashli\ bo'ladi,\ agar}$

$$\int_{-\infty}^{+\infty} |f(x)|^{2} dx < \infty$$

Tenglik o’rinli bo’lsa.Umuman olganda integiral chekli bo’lishi kerak.Aniqlangan $\mathrm{L_2\ sinf\ Lebeg\ integirali\ manosida\ tushuniladi.}$

Bu sinfdan olingan har bir f(x)funksiya$\mathrm{\|f\| son\ mos\ qo'yilgan}$ bo’lsin.

$$\|f\| = \left(\int_{-\infty}^{+\infty} |f(x)|^{2} dx < \infty\right)^{\frac{1}{2}},$$

Ko’rinadiki aniqlangan $\mathrm{\|f\| ga\ \ f(x)\ funksiyaning}$

$\mathrm{L_2\ sinfdagi\ normasi\ deyiladi}$.Normalshartlarni qanoatlantirishini osongina tekshirish mumkin.

2.1.1-ta’rif. $\mathrm{L_2\ sinfdan\ olingan\ \{f_n(x)\}\ n = 1,2,3,\ldots .\ funksiyalar\ ketma-ketligi}$ f(x)$\epsilon$$\mathrm{L_2 funksiyaga\ o'rta\ kvadiratik\ a(normal\ bo'yicha) yaqinlashdi}$ deyiladi,agar $\mathrm{lim_{n\to\infty}\ \|f_n\_f\| = 0\ bajarilsa.}$

2.1.2-ta’rif. $\mathrm{L_2}$ $\mathrm{sinfdan\ olingan\ \{f_n(x)\}}$ funksiyalar ketma-ketligi, o’ziga yaqinlashadi deyiladi, agar

$$\lim_{m,n\to\infty} \|f_n\_f\| = 0$$

$\mathrm{munosabat\ o'rinli\ bo'lsa.}$

Lemma:1 Agar $\mathrm{L_2 sinfdan\ olingan\ \{f_n(x)\}\ funksiyalar\ ketma-ketligi\ f(x)\epsilon L_2}$ funksiyaga o’rtacha kvadiratik ma’noda yaqinlashuvchi bo’lsa,u holda,u o’ziga yaqinlashadi.

Lemmaning isboti uchburchak tengsizlikidan kelib chiqadi ya’ni

$$\|f_n - f_m\| = \|f_n - f + f - f_m\| \le \|f_n - f\| + \|f_m - f\|.$$

Agar f(x),g(x)$\epsilon$$\mathrm{L_2\ bo'lsau\ holda\ ularning\ bu\ fazodagi\ skalyar\ ko'paytmasi}$

$$(f,g)=\int_{-\infty}^{+\infty} f(x)\, g(x) dx$$

ko’rinishda bo’ladi.Bu aniqlangan ifoda skalyar ko’paytmasining shartlarni qanoatlantiradi .

a) $\|f\|^{2} = (f, f);$

b) $(f,g)=(\overline{g, f});$

v) $(\alpha f,g)=\alpha(f,g),(f,\beta g)=\overline{\beta}\,(f,g),\forall\alpha\beta\sigma\tau;$

g) $(f+g,h)=(f,h)+(g,f);(f,g+h)=(f,g)+(f,h);$

d) $\sum_{k=1}^{n}\alpha_k f_k, \sum_{i=1}^{m}\beta_i y_i = \sum_{k=1}^{n}\sum_{i=1}^{m}\alpha_k \beta_i (f_k, g_k)$

Bu xossaning barchasi skalyar ko'paytma aniqlangan formula yordamida kelib chiqadi ,d)esa v)va g)dan kelib chiqadi.

L_2 da skalyar ko'paytma uchun bahoni qaraymiz. Bu bahoni Koshi-Bunyakeviskiy tangsizligi yordamida $|(f,g)| \le \|f\| * \|g\|, f(x), g(x)\epsilon L_2$ ko'rinihni oladi.Bu bahoga ekvivalent baho sifatida $(|f|,|g|)\le \|f\| * \|g\|$ ni olishimiz mumkin.

2.1.3-ta'rif. L_2 dan olingan $\{f_n(x)\}$ funksiyalar ketma-ketligi $f(x)\epsilon L_2$ ga kuchsiz ma'noda yaqinlashuvchi deyiladi,agar $\lim_{n\to\infty}(f_n,g) = (f,g)$ munosabat barcha $g(x)\epsilon L_2$lar uchun bajarilsa

2.1.1-teorema. O'rtacha kvadiratik yaqinlashishdan kuchsiz ma'noda yaqinlashish kelib chiqadi.

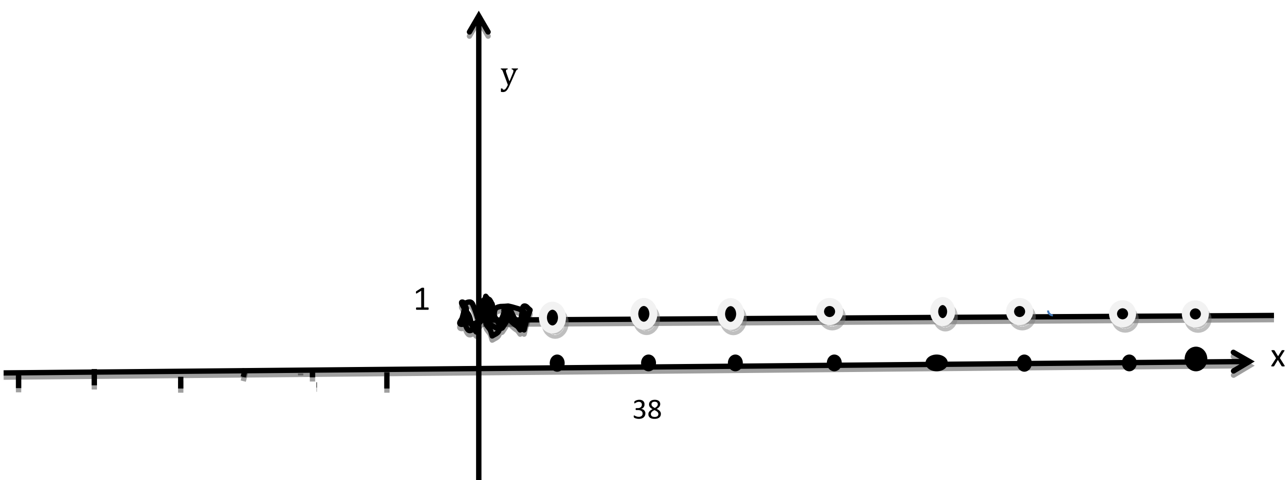
Isbot: Teoremani isbotlash uchun Koshi-Bunyakoviskiy tengsizligidan foydalanamiz.

$|(f_n,g)-(f,g)| = (f_n - f, g)\varepsilon\|f_n - f\|lg\ \| \to \lim_{n\to\infty}\|f_n - fl\| = 0$

ligidan $0\le \lim\limits_{n\to\infty}|(f_n,g)-(f,g)| = \lim\limits_{n\to\infty}\|f_n - f\| * \|g\| = 0 \to \lim\limits_{n\to\infty}(f_n,g) = (f,g)$

Teskari tasdiq umuman olganda o'rinli emas.Quyidagi L_2dan olingan $\{f_n(x)\}$ funksiyalar ketma-ketligini qaraymiz.

$$F_n(x) = \begin{cases} 1, x\epsilon(n,n-1) \\ 0, x\epsilon(n,n+1) \end{cases} \qquad ,n = 1,2,3,\dots\dots$$



-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

Chizma:1.1.6 $F_n(x) = \begin{cases} 1, x\epsilon(n, n-1) \\ 0, x\epsilon(n, n+1) \end{cases}$ funksiya tekislikdagi tasviri.

Bu funksiyalar ketma-ketligi,butun sonlar o'qida nolga tekis yaqinlashuvchi ekanligini ko'rsatamiz.

Faraz qilaylik $g(x)\epsilon L_2$ bo'lsin skalyar ko'paytmasi hisoblaymiz

$$|(f_{n,}g)| = |\int_{-\infty}^{+\infty} f_{n,}(x)\overline{g(x)}\,dx| = |\int_{n}^{n-a} f_n(x)\overline{g(x)}dx| \leq (\int_{n}^{n+1} 1^2\,dx)^{\frac{1}{2}}$$

$$(\int_{n}^{n+1}|g(x)|^{2}dx)^{\frac{1}{2}} = \left(\int_{n}^{n+1}|g(x)|^{2}\,dx\right)^{\frac{1}{2}}$$

Demak $g(x)\sigma L_2$, u holda oxirgi integiral $n\to\infty$ da nolga intiladiva kuchsiz ma'noda yaqinlashuvchi ekan .Biz qarayotgan funksiya o'rtacha kvadiratik ma'noda nolga yaqinlashmaydi,haqiqatdan ham.

$$\|f_n - 0\| = \|f_n\| = (\int_{-\infty}^{+\infty}|f_n(x)|^{2}\mathrm{dx})^{\frac{1}{2}} = 1 \quad \lim_{n\to\infty}\|f_n - 0\| = 1 \quad t > 0$$

L_2 sinfdan olingan funksiyalar uchun Fure almashtirishi.

Faraz qilaylik $f(x)\epsilon\, L_2$ bo'lsin.Bundan kelib chiqadi f(x) funksiya $\forall N > 0$ uchun

$$f(x)\epsilon L_2(-N,N) \quad \int_{-N}^{N}|f(x)|dn$$

Integiral uchun Koshi-Bunyokovskiy yengsizligini qo'llaymiz.

$$\int_{-N}^{N}|f(x)|dx \leq (\int_{-N}^{N}dx\int_{-N}^{N}|f(x)|^{2})^{\frac{1}{2}} < \infty.$$

Oxirgi tengsizlik f(x)funksiya L(-N,N)sinfga tegishli ekanligini ko'rsatadi.

Quydagi.

$$f_N(x) = \begin{cases} f(x), |x| \leq N \\ 0, |x| > N \end{cases}$$

Funksiya kiritamiz .Bu L(-N,N)sinfga tegishli bo'ladi va demak shuning uchun Fure almashtirishi mavjud.

$$F_N(\sigma) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} F_N(x)e^{i\sigma x}dx = \frac{1}{\sqrt{2\pi}}\int_{-N}^{N} f(x)e^{i\sigma x}\,dx.$$

Oxirgi integiralni N→ ∞ dagi limiti mavjud emasligini ko′rsatish

mumkin ya′ni.

$$\int_{-\infty}^{+\infty} f(x)e^{i\sigma x}\,dx$$

integral bosh qiymat ma'nosida uzoqlashuvchi .Ammo biz bu qisimda $\forall f(x)\sigma L_2$ funksiyauchun o′rtacha kvadiratik ma′noda

$$\lim_{n\to\infty} F_n(\sigma) = F(\sigma)$$

limit mavjud bo′ladi.

Yuqoridagi limit aniqlanishi bo'yicha f(x) funksiyaning Fure almashtirishi bo'ladi. Takidlash lozimki L sinfdan farqli ravishda Fure operatori f(x) funksiyani L_2dan chiqarib tashlamaydi, ya'ni F(σ)ϵL_2.

Lemma1. Agar f(x)va G(σ) funksiyalar Lsinfga qarashli bo'lsa , u holda (f,g)=(F,G) tenglik o'rinli bo'ladi.

Isbot: Lemma shartiga ko'ra f(x)ϵL, G(σ)ϵL.Demak , G(σ)funksiyaning teskari Fure almashtirishi ya'ni g(x) funksiya butun sonlar o'qida chegaralangan .SHubhasiz ,bu xossa $\overline{g(x)}$ funksiya uchun ham bor.SHuning uchun f(x) $\overline{g(x)}$ ko'paytma L sinfga qarashli bo'ladi.Demak, f(x) $\overline{g(x)}$ ko'paytmaning Fure almashtirishi mavjud va uni hisoblaymiz:

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\,\overline{g(x)}e^{iux}\mathrm{d}(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)e^{iux}\,\overline{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} G(\sigma)e^{-\iota\sigma x}}\mathrm{d}\sigma\mathrm{d}x=$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)e^{iux}\,dx\,\overline{\int_{-\infty}^{+\infty} G(\sigma)e^{-\iota\sigma x}}\mathrm{d}\sigma=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)e^{iux}\,dx\int_{-\infty}^{+\infty}\overline{G(\sigma)}e^{i\sigma x}\mathrm{d}\sigma=$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\overline{G(\sigma)}\,\mathrm{d}\sigma\int_{-\infty}^{+\infty} f(x)e^{i(\sigma+u)x}dx=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} F(\sigma+u)\,\overline{G(\sigma)}\mathrm{d}\sigma.$$

Skalyar ko'paytmani hisoblashda biz $g(x)$ funksiyani $\overline{V^{-1}G(\sigma)}$ funksiyaga almashtirdik va ma'lum bo'lgan Fubini teoremasidan faydalandik .Lemma shartidagi (f,g)=(F,G) tenglikni olish uchun U=0 ni qo'yamiz:

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\,\overline{g(x)}dx=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} F(\sigma)\,\overline{G(\sigma)}d\sigma \Rightarrow (f,g)=(F,G)$$

Xossa: Agar f(x)ϵL va F(σ)ϵL bo'lsa,u holda

$$\int_{-\infty}^{+\infty} |f(x)|^{2}\,dx = \int_{-\infty}^{+\infty} |F(\sigma)|^{2} d\sigma$$

Porseval tengligi o′rinli bo′ladi.

Xossani isbotlash uchun G(σ)=F(σ) olish yetarli.

Eslatma: Agar f(x),F(σ)ϵL bo'lsa, u holda f(x)ϵL_2 bo'ladi.Haqiqatdan ham f(x)=V^{-1}F(σ)funksiya chegaralangan,shuning uchun.

$|f(x)|^2 = f(x) * \overline{f(x)} \epsilon L$ yoki f(x)ϵL bo'lsa, $\overline{f(x)}$ chegaralangan bo'ladi.

Lemma2: Agar f(x)Finit funksiya va butun sonlar o'qida2-tartibli uzluksiz hosilaga ega bo'lsa,u holda uning F(**σ**)Fure almashtirish L sinfga tegishli bo'ladi

Isbot: Lemma shartiga ko'ra f ''(x)→Finit funksiya,va demak u L sinfdan olingan .Uning Fure almashtirishi butun sonlar o'qida chegaralangan V f''(x)funksiyani hisoblaymiz, buning uchun 2 marta bo'laklab integirallaymiz.

$$V f''(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f''(x)\, e^{i\sigma x}dx = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{i\sigma x}d f'(x) = \frac{1}{\sqrt{2\pi}}$$

$$[e^{i\tau x} f'(x)|_{-\infty}^{+\infty} - i\sigma\int_{-\infty}^{+\infty} f'(x)e^{i\sigma x}\, dx]=$$

$$-\frac{i\sigma}{\sqrt{2k}}\int_{-\infty}^{+\infty} e^{i\sigma x}df(x) = -\frac{i}{\sqrt{2k}}\left[e^{i\sigma x}f(x)|_{-\infty}^{+\infty} - i\sigma\int_{-\infty}^{+\infty} f(x)e^{i\sigma x}dx\right]$$

$$= -\frac{\sigma^2}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\, e^{i\sigma x}dx = -\sigma^2 F(\sigma)$$

f(x)va f'(x) Finit funksiya ekanligidan ,ikkita almashtirishda ham nolga aylandi.Quydagi bahoni olamiz.

$|\sigma^2 F(\sigma)| = |Vf''(x)| \leq c\ yoki |\delta\ F(\sigma)| \leq \frac{c}{|\sigma|^2}$.

Oxirgi,tengsizlik F(σ)funksiya $|\sigma|^{-2}$tartib bo'yichacheksiz kamayuvchi F(σ) uzluksiz u holda F(σ)ϵL Lemma isbotlandi.

Eslatma: Ko'rish mumkinki ,agar f(x)funksiya uchun Lemma sharti o'rinli bo'lsa ,u holda u uzliksiz hosilaga ega bo'ladi. Shuning uchun ,ixtiyoriy x qiymatda Fure almashtirishiga teskari almashtirish o'rinli bo'lib shuning bilan birga ma'noda ,yoki F(σ)funksiya butun sonlar o'qida uzluksiz va L sinfga qarashli bo'ladi .

2.2. Plansherel nazariyasi.

1.Unitar operator

Masalan ,VFure qatori Lsinfda aniqlangan shuning bilan birgalikda bu sinfdagi funksiyani Butun sonlar o'qida aniqlangan $|\delta| \to \infty$ da nolga aylanadigan uzliksiz funksiyalar sinfiga o'tkazish mumkin.

2.2.1-ta'rif. A operator chiziqli deyiladi ,agar ∀α, βgD va ∀ f, gϵL lar uchun.

$$A[\alpha f(x) + \beta g(x) = \alpha A f(x) + \beta g(x)]$$

tenglik o'rinli bo'lsa.

1.1.6-ta'rifdagi funksiyalar sinfi A operatorning aniqlanish sohasi hisoblanadi chiziqli bo'lganda ,ya'ni f(x)va g(x) funksiyalar ∀α,β sonlarda α f(x) β g(x) chiziqli kombinatsiyaga ega bo'lishi talab etiladi.

2.2.2-ta'rif. A: $L_2 \to L_2$ chiziqli operator unitar deyiladi,agar u normani saqlasaya'ni

$$\|Af\| = \|f\|.$$

Unitar operatorni aniqlanishini misol bilan tushuramiz.T_∂operator L_2 da aniqlangan bo'lib quydagi ko'rinishda bo'lsin.

$T_0 f(x) = e^{i\partial x} f(x)$

Bu yerda ∂ haqiqiy son.Bu operator chiziqli.

$$T_0[\alpha f(x) + \beta g(x)] = e^{i\partial x}[\alpha f(x) + \beta g(x)] =$$
$$e^{i\partial x} f(x) + \beta e^{i\partial x} g(x) = \alpha T_0 f(x) + \beta T_0 g(x).$$

Ma'lumki T_0 operator L_2 dan olingan barcha f(x)funksiyalar uchun aniqlangan ,shuning bilan birgalikda $\forall F(n) \epsilon L_2$ funksiya uchun$\exists f(x) \epsilon L_2$ ko'rsatilishi mumkinki F(x)= $T_0 f(x)$ o'rinli bo'ladi.DemakT_0f(x),ϵL_2 funksiyalar to'plami L_2 da ustma-ust tushadi.

Endi. $T_0 f(x)$,vaf(x) funksiyaning normalari bir xil ekanligi ko'rsatamiz.Haqiqatdan ham

$|T_0 f(x)| = |e^{i\partial x}| = |f(x)|,$

$T_0 f(x)$vaf(x)funksiyalarning modullari mosligidan ular normalarining mosligi kelib chiqadi.

Lemma.1 Agar A operator unitar bo'lsa ,u holda A^{-1} teskari operator mavjud va unitardir.

Isbot. F(x)→g(x)=Af(x)akslantirish o'zaro bir qiymatdir .Haqiqatdan ham ,agar Af_1(x)=Af_2 bo'lsa u holda A operatorning chiziqliylik va normaning saqlanishidir.

$$0 = \|Af_1(x) - Af_2\| = \|A(f_1 - f_2)\| = \|f_1 - f_2\|$$

tenglikka ega bo'lamiz.

Normaning shartidan $f_1(x) - f_2(x)$bo'ladi ya'ni A^{-1} operator mavjud.Demak Af(x)=g(x) operator teskarilanuvchi ekan, va biz qarayotgan A operator L_2 sinf L_2 sinfga o'tkazadi , bu holda uning A^{-1} teskarisi L_2 sinf L_2 sinfga o'tkazadi A^{-1}operatorning norma saqlashini oson ko'rsatiladi.

$\|A^{-1}g\| = \|f\| = \|Af\| = \|g\| \to \|A^{-1}g\| = \|g\|$

Lemma isbotlandi.

Keyingi keltirilgan teoremamizPlonshe tomonidan keltirilgan bo'lib , L_2 sinfda Fure almashtirishini tavsiflayiz.Teoremanioperator ko'rinishda beramiz.

2.2.1-teorema. L_2 da aniqlangan,quydagi

$$Vf(x)=F(\sigma)=\frac{1}{\sqrt{2\pi}}\frac{d}{d\sigma}\int_{-\infty}^{+\infty}\frac{e^{i\sigma x}-1}{ix}f(x)dx \quad (*)$$

Munosabat bilan berilgan V operator unitardir. Bu operatorga teskari operator.

$$V^{-1}F(\sigma)=\frac{1}{\sqrt{2\pi}}\frac{d}{d\tau}\int_{-\infty}^{+\infty}\frac{e^{i\sigma x}-1}{-i\sigma}F(\sigma)d\sigma \qquad (*,*)$$

formula bilan tasvirlanadi.V va V^{-1}operatorlar mos ravishda L_2 dagi Fure almashtirishioperator va Fure almashtirishuga teskari almashtirish operatorlari ya'ni ularni

$$\text{V f(x)=F(σ)=}\lim_{n\to\infty}\frac{1}{\sqrt{2k}}\int_{-N}^{N}F(\sigma)e^{i\tau x}dx \qquad (2.2.1)$$

$$V^{-1}\,F(\sigma)=f(x)\lim_{n\to\infty}\frac{1}{\sqrt{2k}}\int_{-N}^{N}F(\sigma)e^{i\tau x}dx \qquad (2.2.2)$$

Munosabat bilan aniqlash mumkin.

Isbot: Faraz qilaylik f(x)GL_2 bo'lsin. O'rtacha kvadiratik ma'noda f(x)funksiyaga yaqinlashuvchi ikkinchi tartibli uzluksiz hosilali Finit funksiyalar ketma-ketligini olamiz. Bunaqangi funksiyalar ketma ketligi L_2 da to'la.

Faraz qilaylik $F_n(x)$funksiyal ketma-ketligi (-K_n K_n)dan tashqarida integiral nolga aylansi α-1dagi 2-lemmaga ko'ra $f_n(x)$va F(τ)funksiyalar

$$\int_{-\infty}^{+\infty}|f_n(x)|^{\,2}dx=\int_{-\infty}^{+\infty}|F_n(\sigma)|^{\,2}dv \qquad (2.2.3)$$

Munosabat bilan berilgan.Fure almashtirishi chiziqli bo'lganligi uchun,

$$V[f_n(x)-f_m(x)]\ |F_n(\sigma)-F_m(\sigma)|^{\,2}d\sigma \qquad (2.2.4)$$

munosabat olamiz.

$\{f_n(x)\}$funksiyalar ketma-ketligi f(x)funksiyaga o'rtacha kvadiratik ma'noda yaqinlashuvchi shunday qilib u o'ziga yaqinlashuvchi,umuman aytganda (2.2.4) tenglikning chap qismi n,m→∞da nolga intiladi .Bundan esa tenglik o'ng tomoni ham nolga intilishi kelib chiqadi ya'ni $\{F_n(\sigma)\}$ketma-ketlik yaqinlashuvchi $\exists F_n(\sigma)\epsilon L_2$ funksiya mavjudki $\{F_n(\sigma)\}$ ketma-ketlik o'rta ma'noda F(σ)yaqinlashuvchi shunday qilib biz ushbu

$$\lim_{n\to\infty}\ F_n(\sigma)=F(\sigma)$$

Limitning mavjudligini ko'rsatdik Endi (x)formulani isbotlaymiz ,bunda faqat o'rta ma'noda limit mavjud bo'lgan $F(\sigma)$uchun qaraymiz $F_n(\sigma)$funksiya 0 dan σ gacha integirallaymiz:

$$\int_0^\sigma F_n(u)du=\int_0^\sigma\{\frac{1}{\sqrt{2\pi}}\int_{-K_n}^{K_n}f_n(x)e^{ixn}dx\{du=\frac{1}{\sqrt{2\pi}}\int_{-K_n}^{K_n}f_n(x)dx\int_0^\sigma e^{ixu}du$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-K_n}^{K_n}f_n(x)\frac{e^{ixu}}{ix}\int_0^\sigma dx=\frac{1}{\sqrt{2\pi}}\int_{-K_n}^{K_n}f_n(x)\frac{e^{ixu}-1}{ix}dx=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}f_n(x)\frac{e^{ixu}-1}{ix}dx. \quad (2.2.5)$$

Bu tenglik ikkala tomonini n→∞da limint mavjudligini ko'rsatamiz.Chap tomoniga qaraylik . $F_n(u)\epsilon L_2$ bundan esa . $F_n(u)\epsilon L_2\ (0,\sigma)$kelib chiqadi. Bundan tashqari

$$\left|\int_0^\sigma 1^2 du\right| = |\sigma| < \infty \text{ uchun } 1\epsilon L_2\ (0,\sigma)$$

$\{F_n(x)\}$funksiyalar ketma-ketligi L_2 da F(u)funksiyaga o'rta ma'noda yaqinlashuvchi shunday ekan L_2 (0,σ)da ham o'rta ma'noda yaqinlashuvchiligi kelib chiqadi,ya'ni

$$\lim_{n\to\infty}\int_0^\sigma F_n(u)du = \int_0^\sigma F(u)du$$

Munosabat o'rinli .

O'ng tomondagi limitning mavjudligi shunga o'xshab isbotlanadi.

f_n (x)ning f(x)ga o'rta ma'noda yaqinlashishdan kuchsiz ma'noda yaqinlashish kelib chiqadi.Quydagi

$$\left|\frac{e^{i\tau x}-1}{ix}\right|^2 \leq \frac{\left(\left|e^{i\tau x}\right|+1\right)^2}{|x|^2} = \frac{4}{x^2}$$

Bahodan va x ning barcha qiymatlarida

$$\frac{e^{i\tau x}-1}{ix}$$

funksiyaning uzluksizligidan, uning x argument bo'yicha L_2 sinfga qarashli bo'ladishunday qilib .

$$\lim_{n\to\infty}\int_{-\infty}^{+\infty} f_n(n)\frac{e^{i\tau x}-1}{ix}dx = \int_{-\infty}^{+\infty} f(x)\frac{e^{i\tau x}-1}{ix}dx$$

Tenglikka ega bo'lamiz (2.2.5)tenglikda ikkala tomonini n→∞ da limitga o'tsak

$$\int_0^\tau F(u)\,du = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\frac{e^{i\tau x}-1}{ix}dx \qquad (2.2.6)$$

Munosabatni olamiz.$F(u)\epsilon L_2$ ligidan $F(u)\epsilon L_2$ (-N,N)bo'ladibundan esa F(u)ϵL

(-N,N) Shunday qilib (2.2.6) formuladagi chap tomondagi integiralni yuqori chegarasi bo'yicha differensillash mumkin

$$\frac{d}{d\sigma}\int_0^\sigma F(u)du = F\sigma$$

$$F(\sigma)=\frac{1}{\sqrt{2\pi}}\frac{d}{d\sigma}\int_{-\infty}^{+\infty} f(x)\frac{e^{i\sigma x}-1}{ix}dx$$

bu yerda o'ng tomondagi hosila barcha nuqtalarda mavjud va (*) formula o'rinli ekan (* *)formula ham (*)ga o'xshash isbotlanadi.

$$f_n(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F_n(\sigma) e^{-i\sigma x} d\sigma$$

Tenglikni [0,x]kesma bo'yicha integirallaymiz:

$$\int_0^x f_n(u)\, du=$$

$$\frac{1}{\sqrt{2\pi}} \int_0^x du \int_{-\infty}^{+\infty} F_n(\sigma) e^{-i\sigma u} d\sigma = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F_n(\sigma) d\sigma \int_0^x e^{-i\sigma u} du =$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F_n(\sigma) = \frac{e^{i\sigma x}-1}{i\sigma} d\sigma$$

(2.2.5')

Integiral ostidagi

$$F_n(\sigma) e^{-i\sigma u}\ uchun \left|F_n(\sigma) e^{-i\sigma u}\right| \leq |F_n(\sigma)|$$

baho o'rinli va F(σ) funksiya L sinifiga qarashli Davom etamiz $\frac{e^{i\tau x}-1}{i\sigma}$ funksiya σ argument bo'yicha L_2 sinfga qarashli bo'ladi. Qolgan mulohazalar (x)ni alohida keltirilgan o'xshash ko'rsatiladi va

(* *)tenglikning to'griligi ko'rsatildi.V va V^{-1}unitar operatorlar normaning uzluksizligidan foydalanib (2.2.3) tenglikni limitga o'tkazamiz limitga o'tgandan so'ng Parseval tengligi deb ataluvchi tenglikka ega bo'lamiz.

$\|f\| = \|F\|$

(2.2.1)va (2.2.2)munosabatlarni qurib, F(σ)funksiya f(x)fuksiyaning Fure almashtirishini va f(x) funksiya F(σ) funksiyaning teskar Fure almashtirishi ekanligini ko'rsatib o'tamiz(2.2.1)formulani isbotlaymiz ((2.2.2)formula shunga o'xshash isbotlanadi).

Faraz qilaylik f(x)ϵ L_2 bo'lsin N$>$ o son olib

$$f_N(x)=\begin{cases} f(x), |x| < N \\ o, |x| \geq N \end{cases}$$

Funksiyani qaraymiz uLsinfga qarashli bo'ladi.(*)formula bo'yicha uni Fure almashtirishini aniqlaymiz.

$$F_N(\tau) = \frac{1}{\sqrt{2\pi}} \frac{\mathrm{d}}{\mathrm{d}\sigma} \int_{-\infty}^{+\infty} f_n(x) = \frac{e^{i\sigma x}-1}{ix} dx \frac{1}{\sqrt{2\pi}} \frac{\mathrm{d}}{\mathrm{d}\sigma} \int_{-N}^{N} f_n(x) = \frac{e^{i\sigma x}-1}{ix} dx$$

Integiral ostidagi funksiyani τ bo'yichadiferensiallaymiz

$$F_N(\sigma) \frac{1}{\sqrt{2\pi}} \int_{-N}^{N} f_n(x)\, e^{i\sigma x} dx$$

σ parametiri bo'yicha integiral tekis yaqinlashuvchi .Ikki tomondan ,agar F(σ)funksiya (x)formula yordamida f(x)bo'yicha aniqlangan bo'lsa ,u holda Parseval tengligidan.

$$\mathrm{F}=\|F - F_N\|^2 = \|f - f_N\|^2 = \int_{|x|\to N} |f(x)|^2 dx$$

Munosabatni olamiz.N→∞da o'ng tomondagi integiral nolga intiladi,demak

$$F_N(\sigma)\frac{1}{\sqrt{2\pi}}\int_{-N}^{N} f_n(x)\, e^{i\sigma x}dx$$

Funksiya o'rtacha kvadiratik ma'noda F(σ)ga yaqinlashadi .Bundan esa (2.2.1)formula kelib chiqadi Plonsherel teoremasi isbotlandi.

Umumlashgan Parsevol tengligi.

Faraz qilaylik f(x),g(x)ϵL_2bo'lsin, u holda $F(\sigma)$, $G(\sigma)\epsilon L_2$bo'ladi f(x)+ **λ** g(x) vaF(σ)+ c G(σ)funksiya uchun Parseval tengligini (***)tatadbiq qilamiz, bu yerda λ ixtiyoriy kompleks son

$$\|f+\lambda g\|^2=\|F+\lambda G\|^2.$$

Normaning kvadratini skalyar ko'paytma orqali yozamiz:

$$(f+\lambda g, f+\lambda g)=(F+\lambda G, F+\lambda G).$$

Skalyar ko'paytmani ochib yozamiz

$$(f,f+\lambda g)+(\lambda g,f+\lambda g)=(f,f)+(f,\lambda g)+(\lambda g,f)+(\lambda g,\lambda g)=\|f\|+\lambda(f,g)+\lambda(g,f)+|\lambda|^2\|g\|$$

Tenglikni ikkinchi tomoniniham skalyarko'paytmani xossalaridan foydalanibyozib chiqamiz.

$$(F+\lambda G, F+\lambda G)=\|F\|^2+\bar{\lambda}(F,G)+\lambda(F,G)+|\lambda|^2\|G\|^2$$

Perseval tengligidan foydalanib

$$\lambda(g,f)+\bar{\lambda}(f,g)=\lambda(G,F)+\bar{\lambda}(F,G)$$

munosabatga ega bo'lamiz.

Bu munosabatda oldin $\lambda=1$va $\lambda=i$ deb olib quydagi tenglikka ega bo'lamiz

$$(g,f)+(f,g)=(G,F)+(F,G) \qquad (2.2.7)$$

$$i(g,f)-i(f,g)=i(G,F)-i(F,G) \qquad (2.2.8)$$

(2.2.8) tenglikni iga qisqartirib (2.2.7) dan (2.2.8)ni ayiramiz va

$$(f,g)=(F,G)$$

ko'rinishdagi umumlashgan Parseval tengligini olamiz va uni

$$\int_{-\infty}^{+\infty} f(x)\overline{g(x)}\,dx=\int_{-\infty}^{+\infty} F(\sigma)\,\overline{G(\sigma)}\,d\sigma$$

ko'rinishda yozamiz

2.3. Bessel funksiyasi uchun Fure almashtirishlari.

Biz bu paragrofda to'g'ridan to'g'ri Fure almashtirishini qo'llashga murakkab bo'lgan funksiyaning Fure almashtirishini keltirib o'tamiz.

Avvalo Bessel funksiyalari haqida qisqacha ma'lumot keltirib o'taylik.

1. **Besselning I- va II-tur funksiyalari:**

$$y''+\frac{1}{x}y'+\left(1-\frac{v^2}{x^2}\right)y=0 \qquad (2.3.1)$$

tenglamaga Bessel tenglamasi deyiladi, uning yechimiga esa Bessel funksiyasi (yoki silindrik funksiya) deyiladi. (2.3.1) tenglamada $\nu -$sonli parametr bo'lib, tenglamaning indeksi deb ataladi.

Bessel tenglamasining yechimi

$$J_\nu(x)=\sum_{k=0}^{\infty}(-1)^k\frac{\left(\frac{x}{2}\right)^{\nu+2k}}{k!\Gamma(k+\nu+1)}=\left(\frac{x}{2}\right)^{\nu}\left[\frac{1}{\Gamma(\nu+1)}-\frac{\left(\frac{x}{2}\right)^{2}}{2!\Gamma(\nu+2)}+...\right] \qquad (2.3.2)$$

$$J_{-\nu}(x)=\sum_{k=0}^{\infty}(-1)^k\frac{\left(\frac{x}{2}\right)^{-\nu+2k}}{k!\Gamma(k-\nu+1)}=\left(\frac{x}{2}\right)^{-\nu}\left[\frac{1}{\Gamma(1-\nu)}-\frac{\left(\frac{x}{2}\right)^{2}}{2!\Gamma(2-\nu)}+...\right] \qquad (2.3.3)$$

ko'rinishda bo'lib, bular Besselning I-tur funksiyalari deyiladi.

Agar $\nu -$butun son bo'lmasa, u holda $J_\nu(x)$ va $J_{-\nu}(x)$ funksiyalar chiziqli bog'lanmagan bo'ladi va (2.3.1) tenglamaning yechimini

$$y(x)=c_1J_\nu(x)+c_2J_{-\nu}(x), \quad \nu\notin Z$$

ko'rinishda tasvirlash mumkin, bu yerda $c_1,c_2 -$lar ixtiyoriy o'zgarmas sonlar.

Besselning II-tur funksiyasi

$$Y_\nu(x)=\frac{J_\nu(x)\cos(\nu\pi)-J_{-\nu}(x)}{\sin(\nu\pi)}, \quad \nu\notin Z \qquad (2.3.4)$$

$$Y_n(x)=\lim_{\nu\to n}Y_\nu(x), \quad n\in Z \qquad (2.3.5)$$

formula ko'rinishda aniqlanadi.

$J_\nu(x)$ va $Y_\nu(x)$ funksiyalar chiziqli bog'lanmagan, shuning uchun Bessel tenglamasining umumiy yechimini

$$y(x)=c_1J_\nu(x)+c_2Y_\nu(x).$$

Besselning boshqa funksiyalari: III-tur Bessel funksiyasi yoki Xankel funksiyasi deb, quyidagi Besselning I- va II-tur funksiyalarning chiziqli kombinatsiyasiga aytiladi.

$$\begin{cases} H_\nu^{(1)}(x) = J_\nu(x) + iY_\nu(x) = i\dfrac{J_\nu(x)e^{-i\pi\nu} - J_{-\nu}(x)}{\sin(\pi\nu)} \\ H_\nu^{(2)}(x) = J_\nu(x) - iY_\nu(x) = -i\dfrac{J_\nu(x)e^{i\pi\nu} - J_{-\nu}(x)}{\sin(\pi\nu)} \end{cases} \tag{2.3.6}$$

bu yerda $\nu \notin Z$ va $n \in Z$ da

$$H_n^{(1)}(x) = \lim_{\nu\to n} H_\nu^{(1)}(x), \quad H_n^{(2)}(x) = \lim_{\nu\to n} H_\nu^{(2)}(x). \tag{2.3.7}$$

Quyidagi tenglamaga

$$x^2 y'' + xy' - \left(x^2 - \nu^2\right) y = 0 \tag{2.3.8}$$

tenglamaga Besselning modifitsirlangan (shakli o'zgartirilgan) tenglamasi deyiladi. Uning yechimiga

$$I_\nu(x) = \sum_{k=0}^{\infty} \frac{1}{k!\Gamma(\nu+k+1)} \left(\frac{x}{2}\right)^{\nu+2k} \tag{2.3.9}$$

esa Besselning I-tur modifitsirlanagan funksiyasi deyiladi, va

$$K_\nu(x) = \frac{\pi}{2} \frac{I_{-\nu}(x) - I_\nu(x)}{\sin(\pi\nu)}; \quad K_n(x) = \lim_{\nu\to n} K_\nu(x), \ \nu \notin Z, \ n \in Z \tag{2.3.10}$$

funksiyalarga Besselning II-tur modifitsirlangan funksiyasi deyiladi.

Bessel funksiyasi uchun Fure almashtirishi:

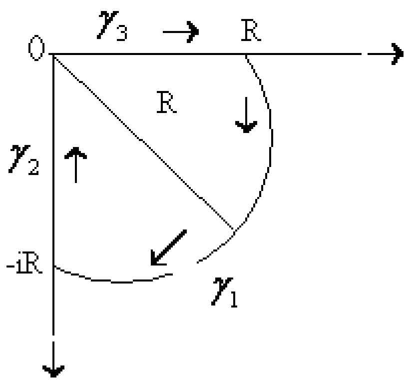
Bessel funksiyasida Furening kosinus va sinus almashtirishlarini olish uchun bizga gipergeometrik funksiyadan foydalanamiz.

Gipergeometrik qator deb darajali qarorga aytiladi va $F(a;b;c;z)$ kabi belgilanadi. Bu funksiya

$$F(a;b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n \cdot n!} \cdot z^n \tag{2.3.11}$$

ko'rinishda aniqlanadi, bu yerda $z-$kompleks o'zgaruvchi, $a,b,c-$kompleks parametrlar bo'lib

$$(c \neq 0,-1,-2,...)$$



$$\begin{cases} (a)_0 = 1 \\ (a)_n = \dfrac{\Gamma(a+n)}{\Gamma(a)} = a(a+1)(a+2)\cdot \ldots \cdot (a+n-1), \ n = 1,2,3 \end{cases}$$

(2.3.12)

ko'rinishda aniqlanadi va $(b)_n, (c)_n$ lar ham yuqoridagiga o'xshash aniqlanadi.

$|z|<1$ doirada gipergeometrik qatorning yig'indisi z kompleks o'zgaruvchili regulyar funksiya hisoblanadi.

Ko'plab elementar va maxsus funksiyalar gipergeometrik funksiyalar orqali ifodalanadi. Biz keying hisoblashlarda quyidagi munosabatlardan foydalanamiz:

$$F\left(\frac{1+\nu}{2};\frac{1-\nu}{2};\frac{1}{2};\sigma^2\right) = \frac{\cos(\nu \arcsin\sigma)}{\sqrt{1-\sigma^2}} \qquad (2.3.13)$$

$$F\left(1+\frac{\nu}{2};1-\frac{\nu}{2};\frac{3}{2};\sigma^2\right) = \frac{\sin(\nu \arcsin\sigma)}{\nu\sigma\sqrt{1-\sigma^2}}. \qquad (2.3.14)$$

Endi ishning asosiy qismiga o'tamiz. Besselning I-tur $I_\nu(x),\ 0<\nu<1$ funksiyasi uchun Furening kosinus almashtirishi

$$F_c(\sigma) = \begin{cases} \sqrt{\dfrac{2}{\pi}}\cdot\dfrac{\cos(\nu\arcsin\sigma)}{\sqrt{1-\sigma^2}}, & 0<\sigma<1 \\ -\sqrt{\dfrac{2}{\pi}}\cdot\sin\dfrac{\pi\nu}{2}\cdot\dfrac{(\sigma-\sqrt{\sigma^2-1})}{\sqrt{1-\sigma^2}}, & \sigma>1. \end{cases}$$

ko'rinishda bo'ladi.

Δ Isbot: Qaralayotgan holda Furening kosinus almashtirishi xosmas integral bo'lib, $\sigma=1$ da uzulishga ega. Shuning uchun $0<\sigma<1$ va $\sigma>1$ hollarini alohida qaraymiz. Qaralayotgan ikki holda ham kontur bo'yicha integralni qaraymiz.

Faraz qilaylik $0<\sigma<1$ bo'lsin. Quyidagi integralni qaraymiz

$$\oint_{\gamma_R} K_\nu(z)ch(\sigma z)dz$$

bu yerda $K_\nu(x)-$Besselning II-tur modifitsirlangan funksiyasi. Integralni γ_R kontur bo'yicha quyidagicha integrallaymiz(chizmadagi yo'nalish bo'yicha).

Integral ostidagi $K_\nu(z)$ va $ch(\sigma z)$ funksiyalar butun kompleks tekisligida regulyar, shuning uchun yopiq kontur bo'yicha olingan integral Koshi teoremasiga ko'ra nolga teng. γ_R konturni uch qisimga ajratamiz:

$$\int_{\gamma_1} K_\nu(z)ch(\sigma z)dz - i\int_R^0 K_\nu(-iy)\cos(\sigma y)dy + \int_0^R K_\nu(x)ch(\sigma x)dx = 0 \qquad (*)$$

bu yerda biz γ_2 chiziq bo'yicha integrallashda

$$z = -iy,\ (0 \le y \le R)$$

ko'rinishda, γ_3 chiziq bo'yicha integrallashda

$$z = x,\ (0 \le x \le R)$$

ko'rinishda oldik, va

$$ch(\sigma z) = \frac{e^{\sigma z} + e^{-\sigma z}}{2};\quad ch(-i\sigma y) = \frac{e^{-i\sigma y} + e^{i\sigma y}}{2} = \cos(\sigma y)$$

munosbatdan foydalandik. (*) integrallarni $R \to \infty$ dagi limitini hisoblaymiz.

1) $\int_{\gamma_1} K_\nu(z)ch(\sigma z)dz:\quad \gamma_1$

yoy bo'yicha integralni hisoblashda

$$z = R\mathrm{e}^{i\varphi},\ -\frac{\pi}{2} \le \varphi \le 0.$$

Shuning uchun

$$\left|\int_{\gamma_1} K_\nu(z)ch(\sigma z)dz\right| = \left|\int_{-\frac{\pi}{2}}^{0} K_\nu(Re^{i\varphi})ch(\sigma z)\cdot iRe^{i\varphi}d\varphi\right| \le R\int_{-\frac{\pi}{2}}^{0} \left|K_\nu(Re^{i\varphi})\right|\left|ch(\sigma Re^{i\varphi})\right|d\varphi$$

Integral ostidagi ifodalarni har birini baholaymiz:

$$\left|K_\nu(Re^{i\varphi})\right| < M_1 \cdot R^{-\frac{1}{2}} \cdot e^{-R\cos\varphi}$$

va

$$\left|ch(\sigma Re^{i\varphi})\right| < M_2 \cdot e^{\sigma R\cos\varphi}.$$

yuqoridagi baholarni [6] da uchratish mumkin. Shuning uchun qaralayotgan integral uchun

$$\left|\int_{\gamma_1} K_\nu(z)\cdot ch(\sigma z)dz\right| < M\cdot R^{\frac{1}{2}}\cdot\int_{-\frac{\pi}{2}}^{0} e^{-(1-\sigma)R\cos\varphi}d\varphi =$$

$$=\left|\psi=\frac{\pi}{2}+\varphi\right| = M\cdot R^{\frac{1}{2}}\cdot\int_0^{\frac{\pi}{2}} e^{-(1-\sigma)R\sin\psi}d\psi, \quad M = M_1\cdot M_2.$$

Bizga ma’lum bo’lgan

$$\frac{2\psi}{\pi}\le\sin\psi, \quad 0\le\psi\le\frac{\pi}{2}$$

tengsizlik va $0<\sigma<1$ shartdan

$$e^{-(1-\sigma)R\sin\psi}\le e^{-\frac{2}{\pi}(1-\sigma)R\cdot\psi}$$

ga ega bo’lamiz. Yuqoridagilardan foydalanib (*) tenglikning chap tomonidagi birinchi integralni $R\to\infty$ dagi limitini hisoblaymiz:

$$\left|\int_{\gamma_1} K_\nu(z)ch(\sigma z)dz\right| < M\cdot R^{\frac{1}{2}}\int_0^{\frac{\pi}{2}} e^{-\frac{2}{\pi}(1-\sigma)R\psi}d\psi < M\cdot R^{\frac{1}{2}}\int_0^{\infty} e^{-\frac{2}{\pi}(1-\sigma)R\psi}d\psi =$$

$$= M\cdot R^{\frac{1}{2}}\frac{1}{-\frac{2}{\pi}(1-\sigma)R}e^{-\frac{2}{\pi}(1-\sigma)R\psi}\Big|_0^\infty = \frac{\pi M}{2(1-\sigma)R^{1/2}}\underset{R\to\infty}{\to}0.$$

Shunday qilib $R\to\infty$ da (*) tenglikdan

$$-i\int_\infty^0 K_\nu(-iy)\cos(\sigma y)dy+\int_0^\infty K_\nu(x)ch(\sigma x)dx=0$$

ga ega bo’lamiz.

Agar

$$K_\nu(-iy)=\frac{i\pi}{2}e^{\frac{i\pi\nu}{2}}H_\nu^{(1)}(y)$$

munosabatni e’tiborga olsak, bu holda oxirgi tenglikni quyidagi ko’rinishda yoza olamiz:

$$\frac{\pi}{2}e^{\frac{i\pi\nu}{2}}\int_{\infty}^{0}H_{\nu}^{(1)}(y)\cos(\sigma y)dy+\int_{0}^{\infty}K_{\nu}(x)ch(\sigma x)dx=0$$

Tenglikning chap qismidagi birinchi integralda y ni x ga va integral chegarasini o’zgartiramiz

$$-\frac{\pi}{2}e^{\frac{i\pi\nu}{2}}\int_{0}^{\infty}H_{\nu}^{(1)}(x)\cos(\sigma x)dx=-\int_{0}^{\infty}K_{\nu}(x)ch(\sigma x)dx$$

$$\int_{0}^{\infty}H_{\nu}^{(1)}(x)\cos(\sigma x)dx=\frac{2}{\pi}e^{-\frac{i\pi\nu}{2}}\int_{0}^{\infty}K_{\nu}(x)ch(\sigma x)dx$$

yoki

$$\int_{0}^{\infty}H_{\nu}^{(1)}(x)\cos(\sigma x)dx=\frac{2}{\pi}\left(\cos\frac{\pi\nu}{2}-i\sin\frac{\pi\nu}{2}\right)\int_{0}^{\infty}K_{\nu}(x)ch(\sigma x)dx \qquad (2.3.15)$$

ko'rinishda yozamiz. $H_{\nu}^{(1)}(x)$ ni aniqlanishiga ko’ra

$$H_{\nu}^{(1)}(x)=I_{\nu}(x)+iY_{\nu}(x).$$

Oxirgi tenglining o’ng tomonini (2.3.15) ga qo’yamiz va har ikkala tomonini $\sqrt{\frac{2}{\pi}}$ ga ko’paytiramiz. Natijada

$$F_{c}(\sigma)=\sqrt{\frac{2}{\pi}}\int_{0}^{\infty}I_{\nu}(x)\cos(\sigma x)dx=\frac{2}{\pi}\sqrt{\frac{2}{\pi}}\cos\frac{\pi\nu}{2}\int_{0}^{\infty}K_{\nu}(x)ch(\sigma x)dx. \qquad (2.3.16)$$

O'ng tomondagi integralni hisoblaymiz:

$$ch(\sigma x)=\sum_{n=0}^{\infty}\frac{(\sigma x)^{2n}}{(2n)!}$$

ni e’tiborga olsak, u holda

$$\int_{0}^{\infty}K_{\nu}(x)ch(\sigma x)dx=\int_{0}^{\infty}K_{\nu}(x)\sum_{n=0}^{\infty}\frac{(\sigma x)^{2n}}{(2n)!}dx$$

yoki

$$\int_0^\infty K_\nu(x)ch(\sigma x)dx=\sum_{n=0}^\infty \frac{(\sigma)^{2n}}{(2n)!}\int_0^\infty x^{2n}K_\nu(x)dx$$

yoki o'ng tomondagi integralning qiymatini hisobga olib

$$\int_0^\infty K_\nu(x)ch(\sigma x)dx=\sum_{n=0}^\infty \Gamma\left(\frac{1+\nu}{2}+n\right)\Gamma\left(\frac{1-\nu}{2}+n\right)\frac{2^{2n-1}}{(2n)!}\sigma^{2n}.$$

Gamma funksiyaning xossasidan

$$\Gamma\left(\frac{1+\nu}{2}+n\right)=\left(\frac{1+\nu}{2}\right)_n\Gamma\left(\frac{1+\nu}{2}\right),\quad \Gamma\left(\frac{1-\nu}{2}+n\right)=\left(\frac{1-\nu}{2}\right)_n\Gamma\left(\frac{1-\nu}{2}\right)$$

$$\Gamma\left(\frac{1+\nu}{2}\right)\Gamma\left(\frac{1-\nu}{2}\right)=\frac{\pi}{\cos\frac{\pi\nu}{2}}.$$

Yana bir eslatmani keltirib o'tamiz

$$\frac{(2n)!}{2^{2n}}=\frac{1\cdot 2\cdot\ldots\cdot(2n-1)\cdot 2n}{2\cdot 2\cdot 2\cdot 2\cdot 2\ldots\cdot 2\cdot 2}=n!\frac{1}{2}\left(\frac{1}{2}+1\right)\cdot\ldots\cdot\left(\frac{1}{2}+n-1\right)=n!\left(\frac{1}{2}\right)_n,$$

ni e'tiborga olsak,moxirgi integral quyidagi ko'rinishni oladi:

$$\int_0^\infty K_\nu(x)ch(\sigma x)dx=\frac{\pi}{2\cos\frac{\pi\nu}{2}}\sum_{n=0}^\infty \frac{\left(\frac{1+\nu}{2}\right)_n\left(\frac{1-\nu}{2}\right)_n}{\left(\frac{1}{2}\right)_n n!}\sigma^{2n}=$$

$$=\frac{\pi}{2\cos\frac{\pi\nu}{2}}F\left(\frac{1+\nu}{2};\frac{1-\nu}{2};\frac{1}{2};\sigma^2\right)=[(2.3.13)]=\frac{\pi}{2\cos\frac{\pi\nu}{2}}\frac{\cos(\nu\arcsin\sigma)}{\sqrt{1-\sigma^2}}.$$

Oxirgi ifodani (2.3.16) ga qo'ysak Bessel funksiyasi uchun

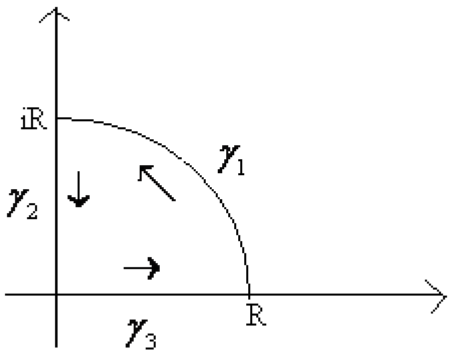
$$F_c(\sigma)=\sqrt{\frac{2}{\pi}}\frac{\cos(\nu\arcsin\sigma)}{\sqrt{1-\sigma^2}},\quad 0<\sigma<1$$

ko'rinishdagi Furening kosinus almashtirishini olamiz.▲

Faraz qilaylik $\sigma>1$ bo'lsin. Quyidagi

$$\oint_{\gamma_R} e^{-\sigma z}I_\nu(z)dz,$$

integralni qaraymiz, bu yerda γ_R kontur bo’yicha integralni



ko'rinishda olamiz. I_ν −Besselning I-tur modifitsirlangan funksiyasi. Qaralayotgan integralni ko’rsatilgan kontur bo’yicha yuqoridagi hisoblashlarni amalga oshirib

$$F_c(\sigma) = -\sqrt{\frac{2}{\pi}} \sin\frac{\pi\nu}{2} \frac{\left(\sigma - \sqrt{\sigma^2 - 1}\right)^\nu}{\sqrt{\sigma^2 - 1}}, \quad \sigma > 1$$

ifodani olamiz.

Quyidagi

$$\int_0^{+\infty} \frac{\cos x}{1+x^2} dx \qquad (1)$$

ko'rinishdagi integralning ikki xil usuldagi qiymatini keltirib chiqaramiz:

1) Fure almashtirishi yordamida $\int_0^{+\infty} \frac{\cos \sigma x}{1+\sigma^2} d\sigma$ integralni ko’rib chiqamiz.

Yechim: $f(x) = e^{-|x|}, \quad x \in (-\infty, +\infty)$ da funksiyaning Fure almashtirishini qaraylik.

$$F(\sigma) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-|x|} e^{-i\sigma x} dx = \frac{1}{\sqrt{2\pi}} \left(\int_{-\infty}^{0} e^{(1-i\sigma)x} dx + \int_0^{+\infty} e^{-(1+i\sigma)x} dx \right) =$$

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{1}{i\sigma - 1} e^{-(1-i\sigma)x} - \frac{1}{1+i\sigma} e^{-(1+i\sigma)x} \right)\Bigg|_0^{+\infty} = \frac{1}{\sqrt{2\pi}} \left(\frac{1}{1-i\sigma} + \frac{1}{1+i\sigma} \right) = \sqrt{\frac{2}{\pi}} \cdot \frac{1}{1+\sigma^2}$$

Demak, $f(x) = e^{-|x|}$ funksiyaning Fure almashtirishi $\sqrt{\frac{2}{\pi}} \cdot \frac{1}{1+\sigma^2}$ ko’rinishda ekan, ya’ni

$$e^{-|x|} \xrightarrow{F} \sqrt{\frac{2}{\pi}} \cdot \frac{1}{1+\sigma^2}, \quad x \in R, \ \sigma > 0.$$

Endi yuqoridagi natijaga ya’ni Fure almashtirishiga Teskari almashtirishni amalga oshiramiz.

$$e^{-|x|}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\sqrt{\frac{2}{\pi}}\cdot\frac{1}{1+\sigma^2}\cdot e^{i\sigma x}d\sigma=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{e^{i\sigma x}}{1+\sigma^2}d\sigma$$

yoki

$$\int_{-\infty}^{+\infty}\frac{e^{i\sigma x}}{1+\sigma^2}d\sigma=\pi e^{-|x|}$$

ko'rinishda bo’ladi. Endi oxirgi integralni Eyler formulasidan foydalanib ikkitaga ajratamiz,

$$\int_{-\infty}^{+\infty}\frac{e^{i\sigma x}}{1+\sigma^2}d\sigma=\int_{-\infty}^{+\infty}\frac{\cos\sigma x+i\sin\sigma x}{1+\sigma^2}d\sigma=\int_{-\infty}^{+\infty}\frac{\cos\sigma x}{1+\sigma^2}d\sigma+i\int_{-\infty}^{+\infty}\frac{\sin\sigma x}{1+\sigma^2}d\sigma=\int_{-\infty}^{+\infty}\frac{\cos\sigma x}{1+\sigma^2}d\sigma=\pi e^{-|x|}$$

bu yerda biz simmetrik oraliqda toq funksiyadan olingan integral nolga teng shartdan foydalandik, ya’ni

$$\int_{-\infty}^{+\infty}\frac{\sin\sigma x}{1+\sigma^2}d\sigma=0$$

Demak, (1) ko’rinishdagi integral

$$\int_{0}^{+\infty}\frac{\cos\sigma x}{1+\sigma^2}d\sigma=\frac{1}{2}\int_{-\infty}^{+\infty}\frac{\cos\sigma x}{1+\sigma^2}d\sigma=\frac{1}{2}\cdot\pi\cdot e^{-|x|}$$

Oxirgi integralda x ning o’rniga bir qo’yib, $\sigma\to x$ ga almashtirsak (1) integral kelib chiqadi, ya’ni

$$\int_{0}^{+\infty}\frac{\cos x}{1+x^2}dx=\frac{\pi}{2}\cdot e^{-1}\qquad(2)$$

2) Qoldiqlar nazariyasidan foydalanib (1) integralni (2) ko’rinishda ekanligini

keltirib chiqaramiz. Buning uchun juft funksiyalar uchun va kompleks analizdan ma’lum bo’lgan, quyidagi

$$F_c(\sigma)=\sqrt{\frac{2}{\pi}}\int_{0}^{+\infty}f(x)\cos\sigma x dx=\sqrt{2\pi}i\sum resf(z)e^{i\sigma z}\qquad(2.1)$$

formuladan foydalanamiz. Keling avval shu formulani keltirib chiqaraylik.

Isbot: Juft funksiyaning shartiga ko'ra $f(-x)=f(x).$

(2.1) formulani isbotlash uchun bizga kompleks analizdan ma'lum bo'lgan

$$\int_{-\infty}^{+\infty} f(x)e^{i\sigma x}dx = 2\pi i\sum resf(z)e^{i\sigma z}$$

formuladan foydalanamiz.

$$\int_{-\infty}^{+\infty} f(x)e^{i\sigma x}dx = \int_{-\infty}^{0} f(x)e^{i\sigma x}dx + \int_{0}^{+\infty} f(x)e^{i\sigma x}dx = \int_{0}^{+\infty} f(-x)e^{-i\sigma x}dx + \int_{0}^{+\infty} f(x)e^{i\sigma x}dx =$$

$$= \int_{0}^{+\infty} f(x)\left(e^{-i\sigma x}+e^{i\sigma x}\right)dx = 2\int_{0}^{+\infty} f(x)\cos\sigma x dx.$$

Bundan esa

$$F_c(\sigma) = \sqrt{\frac{2}{\pi}}\int_{0}^{+\infty} f(x)\cos\sigma x dx = \sqrt{\frac{2}{\pi}}\cdot\frac{1}{2}\cdot 2\pi i\sum resf(z)e^{i\sigma z} = \sqrt{2\pi}i\sum resf(z)e^{i\sigma z}$$

(2.1) formula kelib chiqadi. Endi (1) formulani keltirib chiqaramiz.

$$\int_{0}^{+\infty}\frac{\cos x}{1+x^2}dx$$

bu yerda $\qquad f(z)=\dfrac{1}{1+z^2}, \quad z_{1,2}=\pm i.$

$$\int_{0}^{+\infty}\frac{\cos x}{1+x^2}dx = \sqrt{2\pi}i\sum_{k=1}^{2}\underset{z=z_k}{res} f(z)e^{iz} = \left|\operatorname{Im} z_1>0,\ z_1=i\right| = \sqrt{2\pi}i\underset{z=z_1}{res} f(z)e^{iz} =$$

$$= \sqrt{2\pi}i\lim_{z\to i} f(z)e^{iz} = \sqrt{2\pi}i\lim_{z\to i}\frac{e^{iz}}{z+i} = \sqrt{2\pi}i\cdot\frac{e^{-1}}{2i} = \sqrt{\frac{\pi}{2}}\cdot e^{-1}$$

yoki

$$\sqrt{\frac{2}{\pi}}\int_0^{+\infty}\frac{\cos x}{1+x^2}dx=\sqrt{2\pi}i\lim_{z\to i}f(z)e^{iz}=\sqrt{2\pi}i\lim_{z\to i}\frac{e^{iz}}{z+i}=\sqrt{2\pi}i\cdot\frac{e^{-1}}{2i}=\sqrt{\frac{\pi}{2}}\cdot e^{-1}$$

$$\int_0^{+\infty}\frac{\cos x}{1+x^2}dx=\frac{\pi}{2}\cdot e^{-1}.$$

Yuqoridagiga o'xshash

$$\int_0^{\infty}\frac{x\sin x}{1+x^2}dx=\frac{\pi}{2}\cdot e^{-1}$$

ekanligi oson ko'rsatiladi.

Xulosa

II bob L_2 sinfda Fure almashtirishi.Plansherel nazaryasi ,Bessel funksiyasi uchun Fure almashtirishi deb nomlanadi.1-paragrifda L_2 sinfda Fure almashtirishi bo'yicha ta'rif teoremalar keltirilgan

2-paragrifda Plansherel nazaryasi bir qancha teorema ,Lemmalar va va ularning isbotlari keltirilgan

3-paragrifda Bessel funksiyasi uchun Fure almashtirishi bo'lib bunda Besselning I-II –tur funksiyalari haqida ma'lumotlar va Bessel funksiyasi uchun Fure almashtirishi haqida ma'lumotlar keltirilgan.

Xotima

Mazkur bitiruv malakaviy ishiningmavzusi:Maxsus funksiyalar uchun Fure almashtirishlari deb nomlanadi.Bu bituruv malakaviy ishi 2 ta bob paragrif 2 ta xulosa xotima va foydalanilgan adabiyotlardan iborat .I-bob 3 ta paragrifdan iborat bo'lib L$(-\pi,\pi)$ sinfda Fure alamashtirishi . Fure alamashtirishining xossalari deb nomlanadi.1-paragrifda L$(-\pi,\pi)$ sinfda Fure alamashtirishi yoritilgan bo'lib ,unda L$(-\pi\mu,\pi\mu)$funksiyalar sinfi uchun Fure qatori va E sinfdan olingan funksiyaning Fure almashtirishi haqida ma'lumot berilgan .2-paragrifda Fure almashtirishning xossalari .Furening kosinus va sinus almashtirishlari haqida umumiy tushunchalar berilgan .3- paragrifda teskari Fure almashtirishiga doir misollar keltirilgan.

II-bob ham 3 ta paragrifdan iborat II- bob L_2 sinfda Fure almashtirishi.Plansherel nazaryasi ,Bessel funksiyasi uchun Fure almashtirishi deb nomlanadi.1-paragrifda L_2 sinfda Fure almashtirishi 2-3- paragriflarda Plansherel nazaryasi va Bessel funksiyasi uchun Fure almashtirishlari haqida ma'lumot berilgan .Shuningdek ushbu bitiruv malakaviy ishi xotima va foydalaanilgan adabiyotlardan iborat bo'lib,

. Umuman aytganda

$$\int_0^\infty \frac{\cos x}{1+x^2}dx, \quad \int_0^\infty \frac{x\sin x}{1+x^2}dx$$

ko'rinishdagi integrallar va Parseval tengligidab foydalanib odatdagi usul bilan hisoblanishi murakkab bo'lgan xosmas integrallar hisoblangan.

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