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5130100 «Matematika» ta'lim yo’nalishi bo’yicha bakalavr

darajasini olish uchun

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,, Potensiallar va ularning issiqlik tarqalish tenglamasini uchun

***chegaraviy masalalarni yechishga tadbiqi*”**

mavzusida

Bitiruv malakaviy ishi

“Ish ko’rildi va himoyaga ruxsat berildi”

Ilmiy rahbar prof. Durdiyev D.Q. ”______”_________________2016y.

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“Himoya qilishga ruxsat berildi”

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Asosiy vazifamiz-jamiyatimizni isloh etish va demokratlashtirish, mamlakatimizni modernizatsiya qilish jarayonlarini yangi bosqichga ko'tarishdan iborat. (I.A.Karimov)

Oldimizga turgan ulkan vazifalarni, amalga oshirayotgan islohotlarimizni yanada chuqurlashtirish va mantiqiy yakuniga yetkazish bizuchun oldin ham oson bo'lmagan, bugun ham katta sinovdir. Lekin shu borada ishonchim komil, mustaqil, ozod va erkin demokratik davlat va farovon jamiyat qurish, hech kimga qaram bo'lmasdan yalinmasdan yashash imkoniyati xalqimizga yuz yillarda bir marta beriladi.

Ayni shu tarixiy imkoniyatni qo'limizdan chiqarish-bu nafaqat bugungi avlodimiz, balki nafaqat avlodlar oldida ham xiyonat bo'lur edi.

Bunday imkoniyatni biz hech qachon hech kimga bermaymiz. Bunday xomxayol hech kimni tushiga ham kirmagan. Biz boshlagan yo'limizni oxiriga yetkazamiz. Barchamizga ma'lum, 2015-yilni yurtimizda " Keksalarni e'zozlash yili deb e'lon qilingan edi.

Bizning bunday qarorga kelishimizdan asosiy maqsad-bugungi farovon hayotgaerishishda munosib hissa qo'shgan keksa avlod vakillarining izzat hurmatini joyiga qo'yish, muxtaram faxriylarimizning ruhini, ko'nglini ko'tarish, ular uchun qo'shimcha qulayliklar yaratish yo'lida olib borayotgan ishlarimizning samarasini oshirishdan iborat bo'lib, ayni shunday muhim vazifalarni o'z oldimizga qo'ygan edik.

“Keksalarni e’zozlash yili” Davlat dasturi doirasida amalga oshirgan ishlarimiz haqida yana uzoq gapirish, ko’plab raqam va misollarni olib kelish mumkin. Lekin muxtasar ifoda qiladigan bo’lsak, dastur bo’yicha barcha manbalar hisobidan 2 trillion 246 milliard so’m va 225 million AQSH dollaridan ziyod mablag’ sarflangan. Bu boradagi ishlarimizning ko’lami va miqyosidan yaqqol dalolat beradi deb o’ylayman.

Ma’lumki, bu yorug’ olamda hayot bor ekan, oila bor. Oila bor ekan, farzand deb atalmish bebaho ne’mat bor. Har qaysi ongli inson jondan aziz farzandining baxtini, kamolini o’ylab, hamisha ezgu orzu intilishlar bilan yashaydi, o’z zurriyotining har tomonlama sog’lom va barkamol bo’lishini istaydi.

Sog’lom bola, sog’lom avlod orzusi ajdodlarimizdan bizga o’tib kelayotgan, qon-qonimizga singib ketgan azaliy intilishdir. Agar ota-bobolarimizning turmush va tafakkur tarziga nazar solsak, ular insonning nasl-nasabiga, yeti pushtining tozaligi, avlodning sog’ligiga juda kata e’tibor berganini ko’ramiz.

Sog’lom bola sog’lom va ahil oilaning mevasi bo’lib, faqatgini sog’lom onadan sog’lom bola tug’iladi. Asrlar davomida o’z tasdig’ini topgan bu haqiqatni tushuntirib, izohlab o’tirishning hojati yo’q. Ana shunday muhim hayotiy qadriyatni jamiyatda yanada chuqur qaror toptirish, sog’lom va barkamol avlodni tarbiyalash borasidagi ishlarimizni yanada yuksak bosqichga ko’tarish maqsadida 2016-yil **“Sog’lom ona va bola yili”** deb e’lon qilindi.

Barchamizga yaxshi ma’lumki, har bir inson avvalo Ollohning marhamati bilan , ona zotining o’z bolasini yuragi ostida asrab, to’qqiz oy davomida tortgan mashaqqatlari tufayli dunyoga keladi. Shu bois hammamiz Yaratganning buyuk mo’jizasi bo’lgan aziz ayollarimiz timsolida hayot abadiyligini ta’minlab kelayotgan tabarruk zotlarni ko’ramiz.

Yer yuzida hayot paydo bo'libdiki, odamzod mo'tabar ona siymosi oldida hamisha ta'zim qiladi. Muborak hadislardagi "Jannat –onalarning oyog'i ostidadir" degan so'zlar ham ana shu yuksak mehr va ehtiromning yorqin ifodasidir.

Ayollarga sharqona ehtirom va e'zoz ko'rsatish –biz uchun ota-bobolarimizdan qolgan ibratli me'ros bo'lib, biz bunday an'analarimizga doim sodiq bo'lib yashaymiz. Biz mustaqillikka erishganimizdan so'ng sog'lom va barkamol avlodni voyaga yetkazish, 1-navbatda, oila, onalik va bolalikni himoya qilish masalasini o'zimiz uchun eng muhim, ustuvor vazifa sifatida belgilab oldik.

Bu sohada zarur huquqiy me'yoriy bazani takommillashtirish, millat genofondini yaxshilash, aholining hayot darajasi va sifatini oshirish borasida qanday ulkan ishlarni amalga oshirganimiz barchamizga yaxshi ma'lum.

Ana shu yo'nalishdagi keng ko'lamli ishlarimizning davomi haqida gapirganda, birgina misol keltirish mumkin. Bugungi kunda Toshkent shahrida Janubiy Koreya davlatining beg'araz yordami bilan qiymati 130 million 600 ming dollar bo'lgan ko'p tarmoqli zamonaviy bolalar shifoxonasining qurilishi boshlandi.

Bu Majmuaning -Mustaqil davlatlar hamdo'stligi mamlakatlari hududida yagona bo'lgan, to'rtinchi, ya'ni eng yuqori darajaga ega noyob tibbiyot maskanining yaqin kelgusida foydalanishga topshirilishi sog'lom avlod tarbiyasi yo'lida ulkan qadam bo'lishi shubhasizdir.

"Sog'lom ona va bola yili" Davlat dasturini ishlab chiqishda, avvalo, yurtimizda oila, onalik va bolalikni himoya qilish bo'yicha shakllangan huquqiy-me'yoriy bazani takomillashtirish masalasi e'tiborimiz markazida bo'lishi lozim.

Ayniqsa, olis va chekka hududlarda murakkab iqlim sharoitida yashayotgan aholi, avvalo, ayollar uchun ijtimoiy,maishiy, tibbiy nuqtai nazardan zarur sharoitlarni yaratish, qishloq joylarda namunaviy loyihalar asosidagi uy- joylar, ijtimoiy infratuzilma tarmoqlarini barpo etish, aholini toza ichimlik suvi, tabiiy gaz bilan ta’minlash, xizmat ko’rsatish darajasini yuksaltirish borasidagi ishlarimizni izchil davom ettirishimiz lozim.

Mavzuning dolzarbligi: Bitiruv malakaviy ishim matematik-fizika fanida muhim o’rin tutgan bo’lib, zamonaviy matematika masalalarini o’rganishda issiqlik potensiallarini integral tenglama uchun boshlang’ich, boshlang’ich- chegaraviy masalalarni o’rganish muhim hisoblanadi.

Bitiruv malakaviy ishning maqsadi: Yuqorida ishning dolzarbligi qismida bayon qilingan mulohazalar ishning maqsadini aniqlab beradi va ular quyidagilardan iborat.

Oddiy qatlam potensiali va uni tadbiq etish.

Ikkilangan qatlam potensiali va uni tadbiq etish.

Issiqlik potensiallari uchun boshlang’ich va chegaraviy shartlarni qo’yilishini o’rganish va matematika -fizika tenglamalarini yechishda qo’llash.

Bitiruv malakaviy ishning vazifasi: Bu bitiruv malakaviy ishimizning vazifasi shundan iboratki, issiqlik o’tkazuvchanlik tenglamasini uchun chegaraviy masalalarga integral tenglamani yechishga keltirishni o’rganish.

Bitiruv malakaviy ishning o’rganilganlik darajasi: Qo’yilgan mavzu to’liq o’rganilgan.

Bitiruv malakaviy ishining predmeti: issiqlik potensiallari uchun boshlang’ich va chegaraviy masala, integral tenglama.

Bitiruv malakaviy ishning obyekti: Potensiallarni xossalarini, issiqlik potensiallariga qo’yilgan boshlang’ich va chegaraviy masalasining yechimini xossalarini **o’rganish.**

Bitiruv malakaviy ishning ilmiy farazi: Ushbu B.M.I.refarativ xarakterga ega bo’lib,issiqlik potensiallari uchun boshlang’ich va chegaraviy masalasi yechimi o’rganilgan.

Bitiruv malakaviy ishning yangiligi: Bitiruv malakaviy ish reforativ xarakterga ega bo'lib, integral tenglamalar uchun boshlang'ich va chegaraviy masalasining xususiyatlari issiqlik potensiallari misolida o'rganilgan**.**

Bitiruv malakaviy ishning amaliy ahamiyati: Ushbu bitiruv malakaviy ishidan matematika va amaliy matematika yuqori kurs talabalari va ushbu soha magistirlari o'zlarining sohalarida qo'llaydilar.

Bitiruv malakaviy ishning metodologik asosi: Matematik analiz, integral tenglamalar,matematika fizika masalalari usullari qo'llaniladi.

Bitiruv malakaviy ishning metodlari:potensialllarning turlari,integral tenglama turlari.

Bitiruv malakaviy ishning tarkibi va hajmi: Mazkur bitiruv malakaviy ish kirish qismi, 2 ta bob, har bir bobda 2 tadan 4 ta paragraf, xulosa, xotima va foydalanilgan adabiyotlardan iborat.

I.bob.Potensiallar nazariyasi

I.1.Potensiallar tushunchasi va ularning fizik ma’nosi. Agar D bo’laklari silliq S sirt bilan chegaralangan soha bo’lib ,$u(x)$ funksiya $C^2(D)\cap C^1(\overline{D})$ sinfga tegishli bo’lsa ,

C^2sinf funksiyalarning va garmonik funksiyalarning integral ifodasini keltirib o’tamiz.

D sohaning o’zgaruvchi nuqtasini ξ orqali belgilab olamiz.

u(ξ) funksiya $C^2(D)\cap C(\overline{\overline{D}})^1$sinfga tegishli bo’lsin. D sohaning ixtiyoriy x nuqtasini olamiz va bu nuqtani markaz qilib ε radiusli $Q_\varepsilon$shar chizamiz, S_ε

sharning sirti bo’lsin. ε radiusni shunday kichik qilib olamizki, Q_ε

shar D sohada to’la yotsin. $D-Q_\varepsilon$niD^ε orqali belgilaymiz.Ravshanki,D^εsohada $u(x)$ va $E(x,$ξ) funksiyalar C^2 $(D^\varepsilon)(C^1(D^\varepsilon))$ sinfga tegishli. D^εsohada bu funksiyalarga Grin formulalasini qo’llaymiz:

$$\int_{D^\varepsilon}[E(x,\xi)\Delta u-u\Delta E(x,\xi)]d\xi=\int_S\left[E(x,\xi)\frac{\partial u}{\partial n}-u\frac{\partial E(x,\xi)}{\partial n}\right]dS$$

$$+\int_S\left[E(x,\xi)\frac{\partial u}{\partial n}-u\frac{\partial E(x,\xi)}{\partial n}\right]d_\xi S_\varepsilon$$

Bu yerda differensial belgisidagi ξ indeks integrallash ξ bo’yicha bajarilayotganini bildiradi.

Ma’lumki, ΔE=0.Avvalo n>2 bo’lsin. S_ε sferada $r=|x-\xi|=\varepsilon$, n-normal D^ε sohaga tashqi bo’lganligi sababli ε radiusga qarama-qarshi yo’nalgan. Shuning uchun

$$\frac{\partial E}{\partial n}=-\frac{\partial}{\partial r}\left[\frac{1}{r^{n-2}}\right]_{r=\varepsilon}=\frac{n-2}{\varepsilon^{n-1}}.$$

Birlik sferani S_1 orqali belgilasak, ma'lumki, $dS_\varepsilon = \varepsilon^{n-1} dS_1$, ξ-x=θε almashtirishni bajarsak,

ξ∈S_ε bo'lganda, θ∈S_1 bo'ladi, shu sababli avvalgi formulani quyidagi ko'rinishda yozib olamiz:

$$\int_{D^\xi} \frac{\Delta u(\xi)}{r^{n-2}} d\xi = \int_S \left[\frac{1}{r^{n-2}} \frac{\partial u(\xi)}{\partial n} - u(\xi) \frac{\partial}{\partial n} \frac{1}{r^{n-2}}\right] d_\xi S +$$

$$\int_{S_1} [\varepsilon \frac{\partial u}{\partial n} - (n-2) u(x+\theta\varepsilon)] dS_1 \; (\mathrm{I}.1)$$

Ma'lumki, $\lim_{\varepsilon\to 0} \int_{D^\varepsilon} \frac{\Delta u(\xi)}{r^{n-2}} d\xi = \int_D \frac{\Delta u(\xi)}{r^{n-2}} d\xi$

(I.1) formulaning o'ng tomonidagi birinchi integral ε ga bog'liq emas.

U(ξ)∈C^1 ($\bar{D}$)

Bo'lgani uchun $|\frac{\partial u}{\partial \xi_i}| \le M$, $M - const,$

$\left|\frac{\partial u}{\partial n}\right| = \left|\sum_{i=1}^n \frac{\partial u}{\partial \xi_i} \cos(\mathrm{n}, \xi_i)\right| \le n\, M$

Bundan darhol

$$\left|\varepsilon \int_{S_1} \frac{\partial u}{\partial n} dS_1\right| \le \varepsilon n M |s_1| \underset{\varepsilon\to 0}{\longrightarrow} 0,$$

$|S_1|$- birlik sferaning yuzi .

$$\lim_{\varepsilon\to 0} [-(n-2) \int_{S_1} u(x+\theta\varepsilon) dS_1] = -(n-2)\; u(x)|S_1|.$$

Demak,(I.1) tenglikdan ushbu

$u(x)$=$\frac{1}{(n-2)|S_1|}\int_S \left(\frac{1}{r^{n-2}}\frac{\partial u(\xi)}{\partial n}-u(\xi)\frac{\partial}{\partial n}\frac{1}{r^{n-2}}\right)d_\xi S-\frac{1}{(n-2)|S_1|}\int_D \frac{\Delta u(\xi)}{r^{n-2}}d\xi$ (I. 2)

formula hosil bo'ladi.Eslatib o'tamiz,birlik S_1sfera sirtning yuzi $|S_1|=\frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}$, $n=2$

bo'lganda (I.2) formula o'z ma'nosini yo'qotadi. Bu holda $E(x,\xi)$=ln$\frac{1}{r}$ ekanligini e'tiborga olib, avvalgi hisoblashlarni qaytarsak,

$u(x)$=$\frac{1}{2\pi}\int_S \left[\frac{\partial u(\xi)}{\partial n}\ln\frac{1}{r}-u(\xi)\frac{\partial}{\partial n}\ln\frac{1}{r}\right]-d_\xi S-\frac{1}{2\pi}\int_S \ln\frac{1}{r}\Delta u\,(\xi)d\xi$ (I.3)

formulaga ega bo'lamiz.Agar x nuqta D sohadan tashqarida yotgan bo'lsa,

$\int_D \frac{\Delta u(\xi)}{r^{n-2}}\xi=\int_s \left(\frac{1}{r^{n-2}}\frac{\partial u(\xi)}{\partial n}-u(\xi)\frac{\partial}{\partial n}\frac{1}{r^{n-2}}\right)d_\xi$S, n> 2,

$\int_D ln\frac{1}{r}\Delta u(\xi)=\int_s \left(ln\frac{1}{r}\frac{\partial u}{\partial n}-u(\xi)\frac{\partial}{\partial n}\,ln\frac{1}{r}\right)d_\xi S,$ n=2.

formulalar hosil bo'ladi.

Endi $u(\xi)$ funksiya (I.2) va (I.3) formulalarni chiqarishdagi shartdan tashqari D sohada garmonik ham bo'lsin. Bu holda (I.2) ,(I.3) formulalarda $\Delta u=0$ bo'ladi, natijada garmonik funksiyalarning quyidagi integral ifodasiga ega bo'lamiz:

$u(x)=\frac{1}{(n-2)|S_1|}\int_s \left[\frac{1}{r^{n-2}}\frac{\partial u(\xi)}{\partial n}-u(\xi)\frac{\partial}{\partial n}\frac{1}{r^{n-2}}\right]d_\xi$S, n> 2 , **(I.4)**

$u(x)=\frac{1}{2\pi}\int_s \left[ln\frac{1}{r}\frac{\partial u(\xi)}{\partial n}-u(\xi)\frac{\partial}{\partial n}\,ln\frac{1}{r}\right]d_\xi S$ n=2 **(I.5)**

I.1-teorema. Biror sohada garmonik bo'lgan $u(x)$ funksiya shu sohada barcha tartibli hosilalarga ega bo'ladi.

Isbot: $u(x)$ funksiya Dsohada garmonik bo'lsin ,D da to'la yotuvchi , ya'ni o'zining chegarasi bilan birga D_1 sohani olamiz. D_1ni shunday tanlab olamizki, uning chegarasi

Γ₁ bo'laklari silliq sirtdan iborat bo'lsin . Ravshanki ,$u(x) \in C^2$ $(\overline{D_1})$ va D_1 sohaga (4) formulani qo'llaymiz:

$$u(x) = \frac{1}{(n-2)|S_1|} \int_{G_1} \left[\frac{1}{r^{n-2}} \frac{\partial u(\xi)}{\partial n} - u(\xi) \frac{\partial}{\partial n} \frac{1}{r^{n-2}}\right] d_\xi \Gamma_1, \quad \text{(I.6)}$$

$x \epsilon D_1$

$x$nuqtaning atrofida (I.6) integral ostidagi funksiya x va ξ o'zgaruvchilar bo'yicha uzluksiz va x nuqtaning barcha $x_1, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, x_n$ koordinatalari bo'yicha barcha tartibli hosilalarga ega. Parametrga bog'liq bo'lgan integrallarni differensiallash haqidagi teoremaga asosan , $u(x)$ funksiya x nuqtada $x_1, \ldots\ldots\ldots, x_n$ lar bo'yicha barcha tartibli hosilalarga ega va bu hosilalarni (I.6)formulada integral ostida differensiallash natijasida hosil qilish mumkin.

Grin formulalari. D bo'laklari silliq$S$ sirt bilan chegaralangan E^nfazodagi soha bo'lib, $u(x)$va$v(x)$funksiyalar$C^2(D) \cap C^1(\overline{D})$sinfga tegishli bo'lsin. Dsoha bo'yicha quyidagi

$$v\Delta u = \sum_{i=1}^{n} \left[\frac{\partial}{\partial x_i}\left(v \frac{\partial u}{\partial x_i}\right) - \frac{\partial v}{\partial x_i} \frac{\partial u}{\partial x_i}\right],$$

$$v\Delta u - u\Delta v = \sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(v \frac{\partial u}{\partial x_i} - u \frac{\partial v}{\partial x_i}\right).$$

Ayniyatlarni integrallab va Gauss-Ostragradskiy formulasini qo'llab,

$$\int_D v\Delta u dx = -\int_D \sum_{i=1}^{n} \frac{\partial u}{\partial x_i} \frac{\partial v}{\partial x_i} dx + \int_S v \frac{\partial u}{\partial n} ds \qquad (\mathrm{I.7})$$

$$\int_D (v\Delta u - u\Delta v)dx = \int_S \left(v\frac{\partial u}{\partial n} - u\frac{\partial v}{\partial n}\right)ds \qquad (I.8)$$

Formulalarni hosil qilamiz, bunda $n-S$ ga o'tkazilgan tashqi normal, (I.7) ni Grinning birinchi, (I.8) ni esa ikkinchi formulasi deb yuritiladi. Agar $u(x)$va $v(x)$funksiyalarda garmonik bo'lsa, u holda (I.7) va (I.8) formulalar quyidagi ko'rinishga ega bo'ladi:

$$\int_S v\frac{\partial u}{\partial n}ds = \sum_{i=1}^n \int_D \frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_i}dx \quad (I.9)$$

$$\int_S \left(v\frac{\partial u}{\partial n} - u\frac{\partial v}{\partial n}\right)ds = 0 \qquad (I.10)$$

(I.9) va (I.10) formulalarga asosan garmonik funksiyalarning qator sodda xossalari kelib chiqadi.

I.1-xossa. Agar D sohada garmonik bo'lgan $u(x)$ funksiya $D\cup S$ da o'zining birinchi tartibli hosilalari bilan birga uzluksiz bo'lib, D sohaning chegarasi Sda nolga teng bo'lsa ,u holda barcha $x\in D\cup S$ lar uchun $u(x)$=0 bo'ladi.(**garmonik funksiyaning yagonalikxossasi**.)

Agar (I.9) tenglikda $u(x)$=$v(x)$ desak, undan bu xossa darrov kelib chiqadi.

Haqiqatan, $y\in S$ da $u(y)$=0 bo'lgani uchun (I.9) dan

$$\sum_{i=1}^n \int_D \left(\frac{\partial u}{\partial x_i}\right)^2 dx = \int_S u(y)\frac{\partial u}{\partial n}ds \qquad (I.11)$$

yoki$\sum_{i=1}^n \int_D \left(\frac{\partial u}{\partial x_i}\right)$dx=0 kelib chiqadi.

Demak,$\frac{\partial u}{\partial x_i} = 0, i = 1, \ldots. n, x\in D,$ ya'ni barcha $x\in D$ lar uchun $u(x)$=const.Bundan $y\in S$,$u(y)$=0 bo'lgani sababli, yopiq $D\cup S$ sohada $u(x)$ning uzluksizligidan barcha $x\in D\cup S$ lar uchun $u(x)$=0

I.2-xossa. Agar D sohada garmonik, $D\cup S$da birinchi tartibli hosilalari bilan uzluksiz bo'lgan $u(x)$ funksiyaning $\frac{\partial u}{\partial n}$ normal hosilasi D ning chegarasi S da nolga teng bo'lsa, barcha $x\in D$ nuqtalar uchun u=const bo'ladi.

Bu xossa barcha $y\in S$lar uchun $\frac{\partial u}{\partial n}=0$ bo'lgani sababli,(I.11) tenglikdan darhol kelib chiqadi.

I.3-xossa. Dsohada garmonik, $D\cup S$ da o'zining birinchi tartibli hosilalari bilan uzluksiz bo'lgan $u(x)$ funksiyaning $\frac{\partial u}{\partial n}$ normal hosilasidan S bo'yicha olingan integral nolga teng.

Haqiqatan, (I.9) formulada $v(x)$=1, $x\in D$desak,

$\int_S \frac{\partial u}{\partial n}dS$=0 hosil bo'ladi.

$u(x)$funksiya (I.2) yoki (I.3) formula bilan ifodalanadi. Bu integral ifoda maxsus ko'rinishga ega bo'gan va matematik fizikada muhim rol o'ynaydigan uchta integral operatorni kiritishga imkon beardi . (I.2) formulada $\Delta u(\xi),\ u(\xi)\ \ va\ \frac{\partial u(\xi)}{\partial n}$ funksiyalarni ixtiyoriy $\rho(\xi),\ \mu(\xi)\ va\ \ \sigma(\xi)$ funksiyalar bilan almashtiramiz. Natijada, x ga parametr sifatida bog'liq bo'lgan uchta

$$u(x)=\int_D \frac{\rho(\xi)}{r^{n-2}}d\xi\ ,\quad v(x)=\int_s \mu(\xi)\frac{\partial}{\partial n}\frac{1}{r^{n-2}}d_\xi S,$$

$$\omega(x)=\int_s \frac{\sigma(\xi)}{r^{n-2}}d_\xi S,\qquad r=|x-\xi|$$

integralga ega bo'lamiz. $u(x)$ **hajm potensiali** yoki **Nyuton potensiali**, $v(x)$**ikkilanganqatlam potensiali**, $w(x)$ esa **oddiy qatlam potensiali** deyiladi. $\rho(\xi),\ \mu(\xi)\ va\ \ \sigma(\xi)$funksiyalar bu potensiallarning zichligi deb aytiladi.

n=2 bo'lgan holda ,(I.3) formuladan yuqoridagi potensiallar o'rniga logarifmik potensiallar deb ataluvchi quyidagi integrallar hosil bo'ladi.

$u(x)=\int_D \ \rho(\xi)\ln\frac{1}{r}d\xi$, $\quad v(x)=\int_s \ \mu(\xi)\frac{\partial}{\partial n}\ln\frac{1}{r}d_\xi S,$

$\omega(x)=\int_s \ \sigma(\xi)\ln\frac{1}{r}d_\xi S,$

Hajm va oddiy qatlam potensiallari juda sodda fizik, ma'noga egadir. Faraz qilaylik, uch o'lchovli fazoning biror x_0 nuqtasida nuqtaviy elektrik q zaryad joylashgan bo'lsin. Fizikadan ma'lumki ,bu zaryad elektrostatik maydon hosil qiladi, bu maydonning x_0 nuqtadan farqli ixtiyoriy x nuqtadagi kuchlanishi E ushbu

$\bar{E}=kq\frac{\bar{r}}{r^3}\bar{r}=|x-x_0|$

formula bilan aniqlanadi.,bunda k- proporsionallik koeffitsienti, u tanlab olingan birlik sistemasiga bog'liq bo'ladi. Avvalgi formula proyeksiyalarda quyidagicha yoziladi.

$E_{x_1}=kq(\frac{x_1}{r^3}-\frac{x_{01}}{r^3})$, $E_{x_2}\ =kq(\frac{x_2}{r^3}-\frac{x_{02}}{r^3})$,$E_{x_3}=kq(\frac{x_3}{r^3}-\frac{x_{03}}{r^3})$,

Bu tengliklarning o'ng tomoni teskari ishorasi bilan uchun

$u(x)=kq\frac{1}{r}+const$

funksiyalarning hosilalariga teng ,ya'ni $\bar{E}$ =- grad u; u

U berilgan elektrostatik maydonning *potensiali* deyiladi. Odatda,$x\to\infty$ da $u(x)\to 0$ da *const*=0 deb hisoblanadi. Matematikada soddalik uchun k=1 deb hisoblanadi.

Shunday qilib,miqdori q ga teng nuqtaviy zaryad $u=\frac{q}{r}$ potensial hosil bo'ladi.

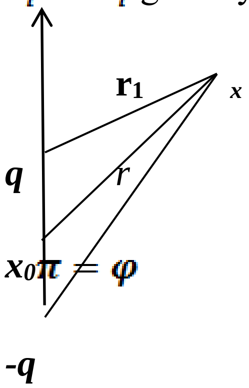
Ma'lumki,bir nechta nuqtaviy zaryadlarning umumiy potensialini topish uchun ularning potensiallarini qo'shish kerak. Shu sababli, uzluksiz taqsimlangan zaryadlar hosil qilgan potensialni yig'indining limiti sifatida , ya'ni integral sifatida topiladi.

Agar zaryad hajm zichligi ρ bo'lgan V hajm bo'yicha tarqalgan bo'lsa, bu zaryad hosil qilgan potensial hajm potensialiga teng bo'ladi. Agarda zaryad S sirt bo'yicha tarqalgan bo’lsa, potensial oddiy qatlam potensialidan iborat bo’ladi.

Endi ikkilangan qatlam potensialining fizik ma’nosini tekshiramiz. Faraz qilaylik, ikkita $q$va $-q$ zaryadlar l o’qda bir-biridan ε masofada joylashgan bo’lsin.

Shu bilan birga bu zaryadlar $x_0(x_{01}, x_{02}, x_{03})$ nuqtaga intilayotgan bo’lib,

-q dan q gacha yo’nalish l ning musbat yo’nalishi bilan bir xil bo’lsin.



r2 ***(I.1-chizma).***

$x_0$nuqtadan farqli ixtiyoriy nuqtadagi potensial bir-biriga teng bo’lishiga intilayotgan ikki miqdorning ayirmasidan iborat bo’ladi.

Shu sababli, tekshirilayotgan potensial nolga teng. Agar harakat vaqtida q zaryad shunday o’ zgarsaki,

$q\varepsilon = p = const$

bo’lib qolsa, potensial quyidagi limitga teng bo’ladi:

$$u(x) = \lim_{\varepsilon\to 0} q\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \lim_{\varepsilon\to 0} \frac{p}{\varepsilon}\frac{r_2 - r_1}{r_1 r_2} = \lim_{\varepsilon\to 0} \frac{p}{\varepsilon}\frac{r_2^2 - r_1^2}{(r_2 + r_1) r_1 r_2}$$

r_1 va $r_2$masofalarni chizmaga asosan darhol hisoblashimiz mumkin:

$$r_2{}^2 = r^2 + \frac{\varepsilon^2}{4} - r\varepsilon\cos(\pi - \varphi) = r^2 + \frac{\varepsilon^2}{4} + r\varepsilon\cos\varphi$$

$$r^2{}_1 = r^2 + \frac{\varepsilon^2}{4} - r\varepsilon\cos\varphi,\ r_2{}^2 - r^2{}_1 = 2r\cos\varphi.$$

Demak, $u(x) = \lim_{\varepsilon\to 0} \frac{p}{\varepsilon}\frac{2r\varepsilon\cos\varphi}{(r_1 + r_2) r_1 r_2} = p\frac{\cos\varphi}{r^2}$

Ushbu

$$\frac{\cos\varphi}{r^2} = -\frac{\partial}{\partial l}\left(\frac{1}{r}\right) \qquad (I.12)$$

tenglikning to'g'riligiga ishonch hosil qilish qiyin emas.

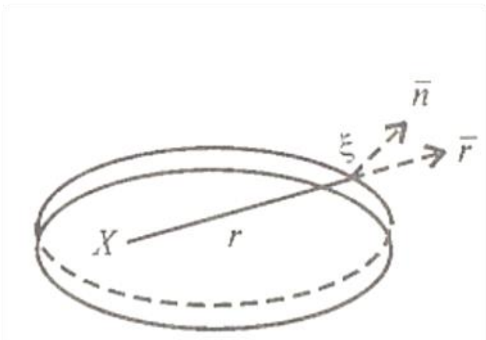
Haqiqatan ham

$$\frac{\partial}{\partial l}\left(\frac{1}{r}\right) = -\frac{x_1 - x_{01}}{r^3}\cos(l, x_1) - \frac{x_2 - x_{02}}{r^3}\cos(l, x_2) - \frac{x_3 - x_{03}}{r^3}\cos(l, x_3)$$
$$= -\frac{1}{r^2}\left[\cos(r, x_1)\cos(l, x_1) + \cos(r, x_2)\cos(l, x_2) + \cos(r, x_3)\cos(l, x_3)\right]$$
$$= -\frac{\cos\varphi}{r^2}$$

Fizikada zaryadlarning limitik joylashishi ***dipol*** deyiladi, p***miqdor moment***, l o'qni ***dipolning o'qi*** deb aytiladi.

Faraz qilaylik, aniq yo'nalishga ega bo'lgan S sirt berilgan bo'lsin, ya'ni bu sirtda ichki va tashqi tomonlar aniq ko'rsatilgan. Bu sirtda moment zichligi $\mu(\xi)$, $\xi \in S$ sirt berilgan bo'lsin. Bundan tashqari har bir ξ nuqtada dipol o'qining yo'nalishi S sirtning shu nuqtadagi normali (ichki yoki tashqi) yo'nalishi bilan bir xil bo'lsin (I.***2-chizma***). Bu dipol hosil qilgan potensial ikkilangan qatlam potensialidan iborat bo'ladi:

$$v(x) = \int_s \frac{\mu(\xi)\cos\varphi}{r^2}dS = -\int_{s'} \mu(\xi)\frac{\partial}{\partial n\xi}\frac{1}{r}dS.$$



(I.***2-chizma***).

I.2. Parametrga bog'liq bo'lgan xosmas integrallar.

Yuqorida kiritilgan potensiallarning nazariyasini bayon qilishda parametrga bog'liq bo'lgan xosmas integrallarga duch kelamiz. Shu sababli bu integrallar nazariyasining asosiy tushunchalarini eslatib o'tamiz. E^nfazodagi chegaralangan D sohada $F(x,\xi)$ funksiya x nuqtadan boshqa barcha nuqtalarda uzliksiz bo'lib x ning atrofida chegaralanmagan bo'lsin. Bu holda

$$I(x)=\int_D F(x,\xi)d\xi \qquad (I.13)$$

Integral x ga parametr sifatida bog'liq bo'lganxosmas integraldir. Avvalo xususiy Ya'ni

$$F(x,\xi)=\frac{1}{r^\alpha},\ r=|x-\xi|$$

bo'lgan holni ko'ramiz.

Markazi x nuqtada, radiusi R ga teng va D sohani o'z ichiga oluvchi sharni Q_R orqali belgilab olamiz.U holda

$$\int_D \frac{d\xi}{r^\alpha} \leq \int_{Q_R} \frac{d\xi}{r^\alpha}.$$

Ma'lumki, $d\xi = dr dS_r = r^{n-1} dr dS_1$.

Bunga asosan

$$\int_{Q_R} \frac{d\xi}{r^\alpha} = \int_{S_1}\left(\int_0^R r^{n-\alpha-1}\,dr\right) dS_1 = \begin{cases} \frac{|S_1|}{n-\alpha} r^{n-\alpha}\Big|_0^R, & n \neq \alpha \\ |S_1| \ln r\Big|_0^R, & n=\alpha \end{cases}$$

Demak, $\alpha < n$ bo'lganda yuqoridagi integral mavjud va$\frac{|S_1|}{n-\alpha}R^{n-\alpha}$ ga teng, $\alpha \geq 0$ bo'lganda esa u mavjud emas. Agar $F(x,\xi)$ funksiya ushbu

$$|F(x,\xi)| \leq \frac{C}{r^\alpha},\ \alpha < n$$

tengsizlikni qanoatlantirsa,(I.13) xosmas integral absolyut yaqinlashuvchi bo'ladi.

Agar ixtiyoriy kichik ε> 0 uchun shunday $\delta(\varepsilon)$ ko'rsatish mumkin bo'lib, $|x - x_0| < \delta(\varepsilon)$ tengsizlikni qanoatlantiruvchi x nuqtalar uchun va x_0 nuqtani o'z ichiga olgan hamda diametr$d \leq \delta(\varepsilon)$ bo'gan ixtiyoriy D_δ soha uchun

$$\left|\int_{D_\delta} F(x,\xi)d\varepsilon\right| < \varepsilon$$

tengsizlik bajarilsa, (I.13) integrak x_0 nuqtada *tekis yaqinlashuvchi* deyiladi.

I.2-teorema. Agar (I.13) integral x_0 nuqtada tekis yaqinlashuvchi bo'lsa, $I(x)$ funksiya shu nuqtada uzluksiz bo'ladi.

Isbot: Ixtiyoriy kichik ε> 0 son uchun, x_0 nuqtani o'z ichiga olgan va tekis yaqinlashuvchanlik ta'rifidagi D_δ sohani olib,(I.13) integralni

$$I(x)=\int_{D_\delta} F(x,\xi) + \int_{D-D_\delta} F(x,\xi)d\xi = I_1(x) + I_2(x)$$

ko'rinishida yozib olamiz. Bu holda

$$I(x) - I(x_0)=I_1(x) + I_2(x) - I_1(x_0) - I_2(x_0).$$

Bundan

$$|I(x) - I(x_0)| = |I_1(x)| + |I_1(x_0)| - |I_2(x) - I_2(x_0)|.$$

x nuqtaD_δ sohaning ichida yotgan bo'lsin. U holda, (I.13) integral tekis yaqinlashuvchi bo'lgani uchun x_0 nuqtada

$$|I_1(x)| < \frac{\varepsilon}{3}, |I_1(x_0)| < \frac{\varepsilon}{3}$$

tengsizliklar o'rinli bo'ladi. $I_2(x)$da integrallash D-D_δ soha bo'yicha bajarilayotgani, x_0 nuqtada va uning biror atrofida uzluksiz bo'ladi, ya'ni

$$|I_2(x_0) - I_2(x_0)| < \frac{\varepsilon}{3}.$$

Demak, $|I(x) - I(x_0| < \varepsilon$

Bundan, (I.13)integralning x_0 nuqtada uzluksizligi kelib chiqadi.(I.13)integral o'rniga umumiyroq

$$I(x)=\int_D F(x,\xi)\varphi(\xi)\,d\xi$$

integralni tekshirish mumkin, bu yerda $F(x,\xi)$ avvalgi funksiya, $\varphi(x,\xi)$ esa D sohada chegaralangan funksiya. Bu integral uchun ham isbotlangan teorema o'z kuchini saqlab qoladi.

I.3.Hajm potensiali.

Ushbu $u(x)=\int_D \frac{\rho(\xi)}{r^{n-2}}d\xi$(I.14)

hajm potensialini tekshiramiz. $\rho(\xi)$zichlik$D$ sohada integrallanuvchi funksiya bo'lsin. Agar x nuqta D sohadan tashqarida yotsa $(r \neq 0)$,(I.14) integral xos integral bo'ladi.

Bu holda $u(x)$ funksiya uzluksiz va barcha tartibli hosilalarga ega.

Bu hosilalarni integral belgisi ostida differensiallash natijasida hosil qilish mumkin,hamda D sohadan tashqarida $u(x)$ funksiya Laplas tenglamasini qanoatlantiradi, ya'ni$\Delta u = 0$.

Endi, x nuqta ixtiyoriy yo'nalish bo'yicha cheksizlikka intilganda

$$|u(x)| < \frac{C}{|x|^{n-2}}, \qquad C\text{-const}$$

tengsizlik o'rinli bo'lishini ko'rsatamiz. Yetarli katta$|x|$ lar uchun $|x| > 2|\xi|$ deb olishimiz mumkin. Bunga asosan,

$r = |x-\xi| \geq |x|-|\xi|$.Yoki $r > \frac{|x|}{2}$

Demak, $$|u(x)| \leq \int_D \frac{|\rho(\xi)|}{r^{n-2}}d\xi < \frac{2^{n-2}}{|x|^{n-2}}\int_D |\rho(\xi)|d\xi = \frac{C}{|x|^{n-2}}$$

Bunda $\quad C=2^{n-2}\int_D |\rho(\xi)|d\xi.$

Shunday qilib, $u(x)$ hajm potensiali D sohadan tashqarida garmonik funksiya ekan.

I.3-teorema. Agar $\rho(\xi)$ zichlik D sohada chegaralangan va integrallanuvchi funksiya bo'lsa, u holda $u(x)$ potensial va uning birinchi tartibli hosilalari barcha fazoda uzluksiz bo'ladi va bu hosilalarni integral belgisi ostida differensiallash natijasida hosil qilish mumkin.

Isbot. Avvalo, (I.14) integral va uni x_i o'zgaruvchi bo'yicha normal differensiallash natijasida hosil bo'lgan

$$X_i(x) = -(n-2)\int_D \rho(\xi)\frac{x_i-\xi_i}{r^n}d\xi \qquad \text{(I.15)}$$

integral ixtiyoriy x_0 nuqtada tekis yaqinlashuvchi ekanligini ko'rsatamiz.

D soha ichidax_0 nuqtada tekis yaqinlashuvchi ekanligini ko'rsatamiz.

D soha ichida x_0 nuqtani o'z ichiga olganD_δ sohani olamiz. Markazi x_0 nuqtada va radiusi δ ga teng bo'lgan sharni Q_δ orqali belgilab olsak, $|\rho(\xi)| < A$,

A-const bo'lgani uchun ushbu

$$\left|\int_{D_\delta}\frac{\rho(\xi)}{r^{n-2}}d\xi\right| < A\int_{D_\delta}\frac{d\xi}{r^{n-2}}$$

tengsizlikka ega bo'lamiz. Agar $Q_{2\delta}$ orqali markazi x nuqtada va radiusi 2δ ga teng bo'lgan sharni belgilasak, quyidagi tengsizlikni hosil qilamiz:

$$A\int_{Q_\delta(x_0)}\frac{d\xi}{r^{n-2}} < A\int_{Q_{2\delta}(x)}\frac{d\xi}{r^{n-2}} = A\int_{S_1}\left(\int_0^{2\delta} r^{n-1}r^{-n+2}dr\right)dS_1 = 2A|S_1|\delta^2.$$

Ixtiyoriy kichik $\varepsilon > 0$ uchun $\delta = \sqrt{\frac{\varepsilon}{2A|S_1|}}$ qilib olsak (x_0 nuqtaning tanlab olinishiga bog'liq emas),

$$\left| \int_{D_\delta} \frac{\rho(\xi)}{r^{n-2}} d\xi \right| < \varepsilon$$

bo'ladi. Bu tengsizlik esa (I.14) integralning D sohaning ixtiyoriy x_0 nuqtasida tekis yaqinlashuvchi ekanligini ko'rsatadi.

Xuddi shunga o'xshash

$$\left| \int_{D_\delta} \rho(\xi) \frac{x_i - \xi_i}{r^n} d\xi \right| \leq A \int_{Q_\delta(x_0)} \frac{d\xi}{r^{n-1}} < A \int_{Q_{2\delta(x)}} \frac{d\xi}{r^{n-1}} = A|S_1|2\delta < \varepsilon,$$

Agar $\delta < \delta(\varepsilon) = \frac{\varepsilon}{2A|s_1|}$

Bunda $X_i(x)$ning ham tekis yaqinlashuvchanligi kelib chiqadi. (I.14) va(I.15) integrallarning tekis yaqinlashuvchanligi ρ(ξ) zichlikning chegaralanganligini talab qilib isbotlanganligi uchun, bu integrallarρ(ξ) funksiyaning uzilish nuqtalarida ham uzluksiz bo'ladi. Dsoha chegarasi nuqtalarini sohadan tashqarida nolga teng bo'lgan ρ(ξ) zichlikning uzilish nuqtalari deb qarash mumkin.

Shunday qilib, $u(x)$ potensial va $X_i(x)$funksiya barcha E^nfazoda uzluksiz bo'ladi.

Endi, $u(x)$funksiyaning xususiy hosilasi $X_i(x)$ funksiya tengligini ko'rsatamiz.

x nuqtaning x_i koordinatasiga Δx orttirma berish natijasida hosil bo'lgan nuqtani x' orqali, x' va ξ nuqtalar orasidagi masofani r_1 orqali belgilab olamiz, ya'ni

$r_1 = |x - \xi|$. Ushbu

$$I(x) = \frac{u(x') - u(x)}{\Delta x_i} - X_i(x) = \frac{1}{\Delta x_i} \int_D \rho(\xi) \left(\frac{1}{r_1^{n-2}} - \frac{1}{r^{n-2}} \right) d\xi - (n-2) \int_D \rho(\xi) \frac{x_i - \xi_i}{r^n} d\xi$$

Ayirmani tekshiramiz va uni Δx_i bilan birga cheksiz kichik bo'lishini ko'rsatamiz.

x va x'nuqtalarni o'z ichiga olgan, D ning qismi bo'lgan |x-ξ|< δ shartni Q_δ orqali, D ning qismi bo'lgan |x-ξ|< δshartni $Q_\delta$orqali, D ning qolgan qismini D_1 orqali belgilab olamiz. U holda

$$U(x)=\int_{Q_\delta}\frac{\rho(\xi)}{r^n}d\xi+\int_{D_1}\frac{\rho(\xi)}{r^{n-2}}d\xi=u_1+u_2,$$

$$X_i(x)=-(n-2)\int_{Q_\delta}\rho(\xi)\frac{x_i-\xi_i}{r^n}d\xi-(n-2)\int_{D_1}\rho(\xi)\frac{x_i-\xi_i}{r^n}d\xi=X_i^{\,1}+X_i^{\,2}.$$

Bularga asosan$I(x)=\frac{u_1(x')-u_1(x)}{\Delta x_i}-X_i^{\,1}+\frac{u_2(x')-u_2(x)}{\Delta x_i}-X_i^{\,2}$.

Bu ifodani baholaymiz:

$$|I(x)|\le\left|\frac{u_1(x')-u_1(x)}{\Delta x_i}\right|-\left|X_i^{\,1}\right|+\left|\frac{u_2(x')-u_2(x)}{\Delta x_i}-X_i^{\,2}\right|$$

$x$nuqta D_1 sohaga tegishli bo'lmagani uchun

$$\lim_{\Delta x_i\to 0}\frac{u_2(x')-u_2(x)}{\Delta x_i}=X_i^{\,2}=\int\limits_{D_1}\rho(\xi)\frac{\partial}{\partial x_i}\frac{1}{r^{n-2}}d\xi=-(n-2)\int\limits_{D_1}\rho(\xi)\frac{x_i-\xi_i}{r^n}d\xi.$$

Avvalgi tengsizlikdan birinchi ifodani baholaymiz. Uni $I_1(x)$orqali belgilab olsak, ya'ni

$$|I_1(x)|\le\left|\frac{u_1(x')-u_1(x)}{\Delta x_i}\right|\le\frac{1}{|\Delta x_i|}\int\limits_{Q_\delta}|\rho(\xi)|\left|\frac{1}{r_1^{\,n-2}}-\frac{1}{r^{n-2}}\right|d\xi.$$

Endi $|r_1-r|\le\Delta x_i$ tengsizlikni, hamda

$r^{n-2}-r_1^{\,n-2}=(r-r_1)(r^{n-3}+r^{n-4}r_1+\cdots\ldots+r_1^{\,n-3})$

tenglikni e'tiborga olib, avvalgi tengsizlikni

$$|I_1(x)| < A \int_{Q_\delta(x)} \left(\frac{1}{{r_1}^{n-2} r} + \frac{1}{{r_1}^{n-3} r^2} + \cdots \ldots + \frac{1}{r^{n-2} r_1}\right) d\xi$$

ko'rinishda yozib olamiz. $Q_\delta$sohani ikkiga ajratamiz: $r > r_1$ bo'lgan

qismini$Q_\delta{}^1$, $r < r_1$ bo'lganini esa $Q_\delta{}^2$orqali belgilab olamiz, u holda

$$|I_1(x)| < A(n-2) \int_{Q_\delta^1} \frac{d\xi}{r_1^{n-1}} + A(n-2) \int_{Q_\delta^2} \frac{d\xi}{r^{n-1}}$$

Demak,

$$\int_{Q_\delta^1} \frac{d\xi}{{r_1}^{n-1}} < \int_{Q_{2\delta}(x')} \frac{d\xi}{{r_1}^{n-1}} = 2\delta|S_1|, \qquad \int_{Q_\delta^2} \frac{d\xi}{r^{n-1}} < \int_{Q_\delta(x)} \frac{d\xi}{r^{n-1}} = \delta|S_1|.$$

Xuddi shunga o'xshash

$$|X_i{}^1(x)| \leq (n-2) \int_{Q_\delta} |\rho(\xi)| \left|\frac{x_i - \xi_i}{r^n}\right| d\xi < A(n-2) \int_{Q_\delta} \frac{d\xi}{r^{n-1}} = A(n-2)|S_1|\delta.$$

Shunday qilib,

$$|I_1(x)| - \left|X_i{}^1(x)\right| < 2A(n-2)|S_1|\delta + A(n-2)|S_1|\delta + A(n-2)|S_1|\delta =$$

$$4\, A(n-2)|S_1|\delta$$

Ixtiyoriy ε>0 uchun δ ni $\delta < \frac{\varepsilon}{8A(n-2)|S_1|}$

Tengsizlikni qanoatlantiradigan qilib tanlab olsak,

$$\left|\frac{u_1(x') - u_1(x)}{\Delta x_i} - X_i{}^1(x)\right| < \frac{\varepsilon}{2}.$$

$x$vax'nuqtalar D sohaga tegishli bo'lmagani uchun, yuqoridagiga asosan, $\varepsilon > 0$ uchun shunday $\delta_1 > 0$ son topiladiki, $|\Delta x_i| < \delta_1$ bo'lganda

$$\left|\frac{u_2(x')-u_2(x)}{\Delta x_i}-X_i{}^2(x)\right|<\frac{\varepsilon}{2}.$$

tengsizlik bajariladi. Agar δ_2=min(δ,δ_1) bo'lsa, $|\mathrm{I}(\varepsilon)|<\varepsilon$ bo'ladi, ya'ni

$\lim_{\Delta x_i\to 0}\frac{u_2(x')-u_2(x)}{\Delta x_i}=X_i(x)$yoki$\frac{\partial u}{\partial x_i}=-(n-2)\int_D \rho(\xi)\frac{x_i-\xi_i}{r^n}d\xi.$

I.4-teorema. Agar $\rho(\xi)$ zichlik funksiya D sohada chegaralangan birinchi tartibli uzluksiz hosilalarga ega bo'lsa,$u(x)$ hajm potensiali D da ikkinchi tartibli uzluksiz hosilalarga ega bo'ladi hamda ushbu

$$\Delta u(x)=-(n-2)|S_1|\rho(x),\qquad n>2$$

$$\Delta u(x)=-2\pi\rho(x),\qquad n=2$$

Puasson tenglamasini qanoatlantiradi.

Isbot.$\frac{\partial}{x_i}\frac{1}{r^{n-2}}=-\frac{\partial}{\partial\xi_i}\frac{\partial}{r^{n-2}}$bo'lganligi sababli

$$\frac{\partial u}{\partial x_i}=\int\limits_D \rho(\xi)\frac{\partial}{\partial x_i}\frac{1}{r^{n-2}}d\xi=-\int\limits_D \rho(\xi)\frac{\partial}{\partial\xi_i}\frac{1}{r^{n-2}}d\xi=$$

$$=-\int_D\left\{\frac{\partial}{\partial\xi_i}\left[\delta(\xi)\frac{1}{r^{n-1}}\right]-\frac{\partial}{\partial\xi_i}\frac{1}{r^{n-2}}\right\}d\xi=\int_D\frac{\partial}{\partial\xi_i}\frac{1}{r^{n-2}}d\xi-\int_S\frac{\rho(\xi)}{r^{n-2}}\cos(n,\xi_i)dS,\qquad \text{(I.16)}$$

Bu erda S sirt D sohaning chegarasi ,n esa ξ nuqtada S ga o'tkazilgan tashqi normal. (I.16) tenglikning o'ng tomonidagi ikkinchi qo'shiluvchi $x\epsilon D$ bo'lganda x_i o'zgaruvchilar bo'yicha birinchi tartibli uzluksiz hosilalarga ega , bu hosilalarni integral belgisi ostida differensiallash natijasida hosil qilish mumkin.$\frac{\partial\rho}{\partial\xi_i}$funksiya$D$ sohada chegaralangan va uzluksiz bo'lgani uchun I.3-teoremaga asosan (I.16) tenglikdagi birinchi qo'shiluvchi ham uzluksiz hosilalarga ega. Endi, (I.14) hajm potensialining Puasson tenglamasini qanoatlantirishini ko'rsatamiz.Shu maqsadda $u(x)$funksiyaning ikkinchi tartibli hosilalarini hisoblaymiz. (I.16)ga asosan

$$\frac{\partial^2 u}{\partial x_i{}^2} = \int\limits_D \frac{\partial \rho}{\partial \xi_i}\frac{\partial}{\partial x_i}\frac{1}{r^{n-2}}d\xi - \int\limits_S \rho(\xi)\frac{\partial}{\partial x_i}\frac{1}{r^{n-2}}\cos(n,\xi_1)\,dS$$

$$= \int\limits_S \rho(\xi)\frac{\partial}{\partial \xi_i}\frac{1}{r^{n-2}}\cos(n,\xi_1)\,dS - \int\limits_D \frac{\partial \rho}{\partial \xi_i}\frac{\partial}{\partial \xi_i}\frac{1}{r^{n-2}}d\xi$$

Demak, $x \in D$ bo'lganda

$$\Delta u(x) = \int\limits_S \rho(\xi)\sum_{i=1}^{n}\frac{\partial}{\partial \xi_i}\frac{1}{r^{n-2}}\cos(n,\xi_1)\,dS = \int\limits_D \sum_{i=1}^{n}\frac{\partial \rho}{\partial \xi_i}\frac{\partial}{\partial \xi_i}\frac{1}{r^{n-2}}d\xi.$$

$x$nuqtani markaz qilib, ε> 0 radiusli D sohada to'la

yotuvchi $Q_\varepsilon(x)$: $|x-\xi| < \varepsilon$

shar chizamiz, bu sharning chegarasi S_ε bo'lsin. Dsohaning shardan tashqari qismini D_ε orqali belgilaymiz, ya'ni $D_\varepsilon = D - Q_\varepsilon$.

U holda, $\Delta u = \int_S \rho(\xi)\frac{\partial}{\partial n}\frac{1}{r^{n-2}}dS - \lim_{\varepsilon\to 0}\int_{D_\varepsilon}\sum_{i=1}^{n}\frac{\partial}{\partial \xi_i}\frac{\partial \rho}{\partial \xi_i}\frac{1}{r^{n-2}}d\xi$ (I.17)

Bu yerdagi ikkinchi integral ostidagi funksiyani quyidagi ko'rinishda yozib olamiz:

$$\frac{\partial \rho}{\partial \xi_i}\frac{\partial}{\partial \xi_i}\frac{1}{r^{n-2}} = \frac{\partial}{\partial \xi_i}\left(\rho\frac{\partial}{\partial \xi_i}\frac{1}{r^{n-2}}\right) - \rho(\xi)\frac{\partial^2}{\partial \xi_i{}^2}\frac{1}{r^{n-2}}.$$

Bunga asosan,

$$\int\limits_{D_\varepsilon}\sum_{i=1}^{n}\frac{\partial}{\partial \xi_i}\frac{\partial \rho}{\partial \xi_i}\frac{1}{r^{n-2}}d\xi$$

$$= \int\limits_S \rho(\xi)\sum_{i=1}^{n}\frac{\partial}{\partial \xi_i}\frac{1}{r^{n-2}}\cos(n,\xi_i)\,dS$$

$$+ \int\limits_{S_\varepsilon} \rho(\xi)\sum_{i=1}^{n}\frac{\partial}{\partial \xi_i}\frac{1}{r^{n-2}}\cos(n,\xi_i)\,dS_\varepsilon - \int\limits_{D_\varepsilon} \rho(\xi)\sum_{i=1}^{n}\frac{\partial^2}{\partial \xi_i{}^2}\frac{1}{r^{n-2}}d\xi.$$

$x \neq \xi$ bo'lganda,

$$\sum_{i=1}^{n} \frac{\partial^2}{\partial {\xi_i}^2} \frac{1}{r^{n-2}} = 0$$

bo'lgani uchun, ushbu

$$\int_{D_\varepsilon} \sum_{i=1}^{n} \frac{\partial \rho}{\partial \xi_i} \frac{\partial}{\partial \xi_i} \frac{1}{r^{n-2}} d\xi = \int_S \rho(\xi) \frac{\partial}{\partial n} \frac{1}{r^{n-2}} dS + \int_{S_\varepsilon} \rho(\xi) \frac{\partial}{\partial n} \frac{1}{r^{n-2}} dS_\varepsilon \quad \text{(I.18)}$$

tenglikka ega bo'lamiz. (I.17) va(I.18) tengliklarga asosan,

$x \in D$ bo'lganda

$$\Delta u = -\lim_{\varepsilon \to 0} \int\limits_{S_\varepsilon} \rho(\xi) \frac{\partial}{\partial n} \frac{1}{r^{n-2}} dS_\varepsilon$$

yo'nalgani uchun

$$\frac{\partial}{\partial n} \frac{1}{r^{n-2}} = -\frac{\partial}{\partial r} \frac{1}{r^{n-2}}$$

$$\Delta u = -(n-2) \lim_{\varepsilon \to 0} \int\limits_{S_\varepsilon} \frac{\rho(\xi) - \rho(x)}{\varepsilon^{n-1}} dS_\varepsilon - (n-2)\rho(x) \lim_{\varepsilon \to 0} \int\limits_{S_\varepsilon} \frac{d\xi}{\varepsilon^{n-1}}$$

Bu ifodada $\xi = x + \varepsilon\theta$ almashtirish bajarsak, $\xi \in S_\varepsilon$ bo'lganda $\theta \in S_1$ bo'ladi va $d\xi = \varepsilon^{n-1} d\theta$. Shu sababli

$$\Delta u = -(n-2) \lim_{\varepsilon \to 0} \int_{S_1} [\rho(x + \varepsilon\theta) - \rho(x)] dS_1 - (n-2)\rho(x) \int_{S_1} dS_1$$
$$= -(n-2)|S_1|\rho(x)$$

I.4- teoremani isbotlashda qo'llanilgan barcha amallar va formulalar D sohaning chegarasi S yetarli silliq sirt bo'lgandagina o'rinli bo'ladi .

Agar $u(x)$ funksiyani

$$u(x) = \int_{D_R} \frac{\rho(X)}{r^{n-2}} d\xi + \int_{Q_R} \frac{\rho(X)}{r^{n-2}} d\xi \quad x \in D \text{ (I.19)}$$

ko'rinishida yozib olsak ,bunda $Q_R: |\xi - x| \leq R$shar D sohada yotadi, $D_R$sohaning Q_R shardan tashqari qismidir , S sirtga qo'yilgan shartlarning hojati bo'lmaydi. (I.19) ifodaning o'ng tomonidagi birinchi qo'shiluvchi Q_R sharning ichida garmonik funksiyadir, ikkinchi qo'shiluvchiga esa yuqorida keltirilgan mulohazalar to'g'ri bo'ladi .Xuddi shunga o'xshash ,n=2 bo'lgan holda

I.4- teorema isbotlanadi .

Isbotlangan teoremaga asosan , D sohada

$$\Delta u = f(x) \qquad \text{(I.20)}$$

Puasson tenglamasi uchun qo'yilgan

$$u|_S = \varphi$$

Dirixle masalasini Laplas tenglamasi uchun Dirixle masalasini yechishga darhol olib kelinadi.

Haqiqatdan ham , agar $f(x)$ funksiya D sohada chegaralangan va birinchi tartibli uzluksiz hosilalarga ega bo'lsa ,

$$u_0(x) = -\frac{1}{(n-2)|S_1|} \int_D \frac{f(\xi)}{r^{n-2}} d\xi$$

Funksiya (I.20) tenglamani yechimidan iborat bo'ladi . U holda (I.20) tenglamani yechimini $u(x) = u_0(x) + v(x)$ ko'rinishda izlasak , $v(x)$ funksiya $\Delta v = 0$Laplas tenglamasi va$v(x)|_S = \varphi - u_0(x)|_S = \psi$ chegaraviy shartni qanoatlantiradi. Shu bilan birga berilgan φ funksiya uzluksiz bo'lsa ,ψ ham uzluksiz funksiya bo'ladi .

I.1.Lyapunov sirtlari.

Oddiy va ikkilangan qatlam potensiallarining xossalarini o'rganishda S ning Lyapunov sirti bo'lishi talab qilinadi. Shu munosabat bilan bu sirt tushunchasini kiritamiz.

Quyidagi shartlarni qanoatlantiruvchi S sirt Lyapunov sirti deyiladi:

1) *S sirtning har bir nuqtasida urinma tekislik mavjud, demak aniq normal mavjuddir;*
2) *Shunday o'zgamas* $d > 0$*son mavjudki S ning ixtiyoriy* x *nuqtasini markaz qilib ,d radiusli yoki d dan kichik radiusli sfera chizsak ,*x *nuqtada o'tkazilgan normalga parallel to'g'ri chiziqlar sferaning ichida yotgan S ning qismini bittadan ortiq nuqtada kesishmaydi;*
3) *S sirtning ixtiyoriy* x*va* ξ *nuqtalarida o'tgan normallar orasidagi burchak* θ *bo'lib ,* $r = |x - \xi|$ *bo'lsa , shunday a va* α *musbat sonlar mavjud bo'ladiki*

$$\theta \leq a r^{\alpha} \quad (0 < \alpha \leq 1) \qquad \textbf{\textit{(I.1.1)}}$$

tengsizlik bajariladi .

n=2 bo'lgan holda SniLyapunov egri chizig'ideyiladi

Bu ta'rifdan Lyapunov sirtlari silliqligi $C^{(1,\alpha)}$ bo'lgan sirtlar sinfiga tegishli ekanligi kelib chiqadi. Bunga biz keyinchalik ishonch hosil qilamiz . Biz bundan keyin d ni yetarli kichik qilib olib ,

$$a d^{\alpha} < 1 \qquad \textbf{(I.1.2)}$$

tengsizlik bajariladi deb faraz qilamiz , ya'ni S ning x nuqtasini markaz qilib , d radiusli $S_d(x)$ sfera chizsak , x nuqtadagi normal bilan S ning $S_d(x)$ ichida yotgan ixtiyoriy ξ nuqtadagi normal orasidagi θ burchak $\frac{\pi}{2}$ dan kichik bo'ladi.

I.1.1-shart.Lyapunov sirtining har bir nuqtasida mahalliy $\xi_1, \xi_2, \dots \dots \dots, \xi_n$koordinata sistemasini tuzishga imkon beradi

ya'ni bu shunday dekart koordinatalar sistemasiki, koordinata boshi$x$nuqtada ξ_1, ξ_2…………..,ξ_n, o'qlar S ga x nuqtada urinma bo'lgan $(n-1)$ o'lchovli tekislikda joylashgan ξ_n o'q esa S ga x nuqtada o'tkazilgan normal bo'yicha yo'naltirilgan.

*I.1.1-shart*shu narsani ko'rsatadiki ,S sirtning $S_d(x)$ sfera ichiga tushgan qismini tenglamasini mahalliy koordinata sistemasida

$\xi_n = f(\xi_1, \xi_2 \dots\dots\dots \xi_{n-1})$**(I.1.3)**

ko'rinishda yozish mumkin.

Mahalliy koordinata sistemasining koordinata boshi bo'lgan x nuqtada $\xi_n = 0$ tekislik S sirtga urinma bo'lgani uchun

$$f(0,0, \dots \dots \dots \dots \dots \dots \dots ,0) = 0, \qquad \frac{\partial}{\partial \xi_i} f(0,0, \dots \dots \dots \dots \dots \dots \dots ,0) = 0,$$
$$i = 1,2,3, \dots \dots \dots \dots . . n-1$$

bo'ladi.

Endi $f(\xi_1, \xi_2, \dots \dots \dots \dots \dots , \xi_n)$funksiya va uning birinchi tartibli hosilalarini $S_d(x)$ ichida kichiklik tartibini aniqlaymiz. Sning $S_d(x)$ ichida yotgan qismiga tegishli ξ nuqtani olamiz va bu nuqtaga o'tkazilgan normalning yo'nalishini v orqali belgilaymiz .U holda ,

$$\cos(v, \xi_n) = \cos(v, n) \cos\theta = 1 - 2\sin^2\frac{\theta}{2} \geq 1 - \frac{\theta^2}{2}$$

Bundan ,**(I.1.1)** ga asosan ,

$$\cos(v, \xi_1) \geq 1 - \tfrac{1}{2} a^2 r^{2\alpha}. \qquad \textbf{(I.1.4)}$$

(I.1.2) ga ko'ra $a^2 r^{2\alpha} \leq a^2 d^{2\alpha} \leq 1$bo'lgani uchun

$$\cos(v,\xi_n) \geq \frac{1}{2}$$

(**I.1.3**) tenglamaga asosan

$$\cos(v,\xi_n) = \frac{1}{\sqrt{1+\sum_{i=1}^{n-1}\left(\frac{\partial f}{\partial \xi_i}\right)^2}},$$

$$\cos(v,\xi_i) = -\frac{\frac{\partial f}{\partial \xi_i}}{\sqrt{1+\sum_{i=1}^{n-1}\left(\frac{\partial f}{\partial \xi_i}\right)^2}},\textbf{(I.1.5)}$$

Bunda ,**(I.1.2)** va (**I.1.4**) ni e'tiborga olib ,

$$\sqrt{1+\sum_{i=1}^{n-1}\left(\frac{\partial f}{\partial \xi_i}\right)^2} = \frac{1}{\cos(v,\xi_n} \leq \frac{1}{1-\frac{1}{2}a^2r^{2\alpha}} = 1+\frac{\frac{1}{2}a^2r^{2\alpha}}{1-\frac{1}{2}a^2r^{2\alpha}}$$

$$< 1+\frac{\frac{1}{2}a^2r^{2\alpha}}{1-\frac{1}{2}a^2r^{2\alpha}} < 1+a^2r^{2\alpha}$$

tengsizlikni hosil qilamiz ,yoki har ikki tomonini kvadratga ko'tarib

$$\sum_{i=1}^{n-1}\left(\frac{\partial f}{\partial \xi_1}\right)^2 < 2\,a^2r^{2\alpha} + a^4r^{4\alpha}$$

tengsizlikka ega bo'lamiz . $r \leq d$ bo'lganligi uchun , (**I.1.2**) ga asosan

$$\sum_{i=1}^{n-1}\left(\frac{\partial f}{\partial \xi_i}\right)^2 < 3a^2\,r^{2\alpha}$$

Demak ,

$$\left|\frac{\partial f}{\partial \xi_i}\right| < \sqrt{3}ar^{\alpha}\textbf{(I.1.6)}$$

(**I.1.5**) formulaga muvofiq

$$|\cos(v,\xi_i)| \leq \left|\frac{\partial f}{\partial \xi_i}\right| < \sqrt{3}ar^{\alpha} i = 1,2,3,\dots\dots\dots\dots..n-1 \qquad \textbf{(I.1.7)}$$

Ushbu

$$\rho^2 = \sum_{i=1}^{n-1} \xi_i^2$$

belgilashni kiritamiz , u holda

$$r^2 = \rho^2 + \xi_i^2 \textbf{(I.1.8)}$$

(**I.1.6**) formuladaξ_i,S ga o'tkazilgan ixtiyoriy yo'nalishni ,xususiy holda ρ bo'yicha yo'nalishni bildirishi mumkin.

U holda

$$\left|\frac{\partial f}{\partial \xi_i}\right| < \sqrt{3}ar^{\alpha} < \sqrt{3}ar^{\alpha} \leq \sqrt{3} \qquad \textbf{(I.1.9)}$$

Bundan

$$|f| = \int_0^{\rho} \frac{\partial f}{\partial \rho} d\rho \leq \int_0^{\rho} \left|\frac{\partial f}{\partial \rho}\right| d\rho < \sqrt{3}\rho \textbf{(I.1.10)}$$

Demak ,

$$|\xi_n| < \sqrt{3}\rho \textbf{(I.1.11)}$$

Bu bahoni yana ham yaxshilash mumkin. (**I.1.6**) va (**I.1.11**) tengsizliklarga asosan $r \leq 2\rho$ bo'ladi.

Bunga muvofiq**(I.1.9)** tengsizlik quyidagicha yoziladi .

$$\left|\frac{\partial f}{\partial \xi_i}\right| < \sqrt{3}2^{\alpha} a\rho^{\alpha}$$

Shunday qilib ,oxirgi tengsizlikka ko'ra (**I.1.10**) ushbu

$|f| = |\xi_n| < a_1 \rho^{n+1}$, $\qquad a_1 = \frac{2^{\alpha}\sqrt{3}\,a}{\alpha+1}$;

Ko’rinishiga keladi . $\rho \leq r$bo’lgani uchun

$|\xi_n| < a_1 r^{a+1}$**(I.1.12)**

tengsizlikni hosil qilamiz .

Mahalliy koordina sistemasida koordinata boshidan ξ nuqtagacha bo’lgan masofa r bo’lgani uchun quyidagi tengsizlikni yozamiz :

$$\cos(v,r) = \sum_{i=1}^{n} \cos(v,\xi_i)\cos(r,\xi_i)$$
$$= \sum_{i=1}^{n} \frac{\xi_i}{r}\cos(v,\xi_i) = \sum_{i=1}^{n-1} \frac{\xi_1}{r}\cos(v,\xi_i) + \frac{\xi_n}{r}\cos(v,\xi_n)$$

(I.1.7)va **(I.1.12**) tengsizlarga asosan ,

$\frac{|\xi_i|}{r} = |\cos(r,\xi_i)| \leq 1, \qquad |\cos(v,\xi_n| \leq 1$

bo’lgani uchun ,avvalgi tenglikdan

$|\cos(v,r| \leq a_2 r^{a}$, $\qquad a_{2=}\sqrt{3}\,a(n-1) + a_1$**(I.1.13)**

tengsizlikka ega bo’lamiz.

7.Oddiy qatlam potensiali. Ushbu

$\omega(x) = \int_S \frac{\sigma(\xi)}{r^{n-2}} d_\xi S$ **(I.1.14)**

Oddiy qatlam potensiali $\sigma(x)$ zichlik integrallanuvchi bo'ganda $x \in S$ nuqtalarda garmonik funksiya bo'lib, cheksizlikda$|x|^{-(n-2)}$ kabi nolga intilishiga xuddi ikkilanlangan qatlam potensialiga o'xshash, ishonch hosil qilish qiyin emas.

Agar S yopiq Lyapunov sirti bo'lib, $\sigma(x)$zichlik integrallanuvchi va chegaralangan bo'lsa, oddiy qatlam potensiali barcha E^n fazoda uzluksiz bo'ladi.

$\omega(x)$funksiyaning x$\in S$ da uzluksizligi ravshan bo'lgani uchun, $x \in S$ nuqtalarda uning uzluksizligi ko'rsatamiz.Buning uchun**(I.1.14)** integralning S sirt nuqtalarida tekis yaqinlashuvchi bo'lishini isbotlash kifoyadir.

Shu maqsadda$x$ nuqtani markaz qilib, η $< d$ (d-Lyapunov sirti ta'rifidagi son) radiusli $S_\eta$sfera chizamiz. S sirtning bu sfera ichidagi qismini S_η^1, tashqarisidagi qismini esa, S_η^2 orqali belgilab olamiz.

y- E^nfazoning ixtiyoriy nuqtasi bo'lsin. η$< d$bo'lgani uchunS_η^1 da x nuqtani markaz qilib $\xi_1, \dots \dots, \xi_n$ mahalliy koordinatalar sistemasini tuzish mumkin.

$y$nuqtaning S ga$x$ nuqtada o'tkazilgan urinma tekislikdagi, ya'ni$\xi_\eta = 0$ tekislikdagi proyeksiyasi y' bo'lsin. y'ning mahalliy koordinatalari $(y_1, y_2, \dots \dots, y_{n-1}, 0)$, bo'ladi. $\rho^2 = \sum_{i=1}^{n-1}(\xi_k - y_k)^2$belgilashni kiritamiz.

$\rho - \xi$va y nuqtalarni birlashtiruvchi kesmaning $\xi_\eta = 0$ tekislikdagi proyeksiyasining uzunligidir. Ravshanki,$\rho \leq |\xi - y|$.S_η^1sirtning$\xi_\eta = 0$ tekislikdagi proyeksiyasini D_η^1 desak,

$$d\xi_1, \dots \dots, d\xi_{n-1} = \cos(\nu, \xi_\eta) d_\xi S$$

formulani e'tiborga olib, $|\sigma(\xi)| \leq M = const$, $\cos(\nu, \xi_\eta) \geq \frac{1}{2}$ tengsizliklarga asosan

$$|w_1(y)| = \left| \int\limits_{s_\eta^1} \frac{\sigma(\xi)}{|\xi - y|^{n-2}} d_\xi S \right| \leq 2M \int\limits_{D_\eta^1} \frac{d\xi_1, \dots \dots, d\xi_{n-1}}{\rho^{n-2}}$$

tengsizlikni hosil qilamiz. Endi y nuqtani $x$ga shunday yaqin qilib olamizki, $|y - x| < \frac{\eta}{2}$ bo'lsin. Agar $\xi \in S_\eta^1$ bo'lsa,

$$\rho \leq |\xi - y| = |\xi - x + x - y| \leq |\xi - x| + |x - y| < \frac{3\eta}{2}.$$

Bu tengsizlik D_η^1 sohaning (n-1) o'lchovli $\rho < \frac{3\eta}{2}$ sharda to'la yotishini ko'rsatadi. Demak,

$$|w_1(y)| \leq 2M \int_{\rho < \frac{3\eta}{2}} \frac{d\xi_1, \dots \dots, d\xi_{n-1}}{\rho^{n-2}} \quad \textbf{(I.1.15)}$$

E^{n-1}fazoda markazi y' nuqtada bo'lgan sferik koordinatalarni kiritamiz. U holda $d\xi_1, \dots \dots, d\xi_{n-1} = \rho^{n-2} d\rho dS_1$, bu yerda dS_1 orqali E^{n-1} fazodagi S_1 birlik sfera yuzining elementi belgilangan. U holda, **(I.1.15)** tengsizlik

$$|w_1(y)| \leq 2M \int\limits_0^{\frac{3\eta}{2}} d\rho \int\limits_{S_1} dS_1 = 3M\eta |S_1|$$

ko'rinishida yoziladi. Bu baho x nuqta S sirtning qayerida yotishiga bog'liq emas. $\varepsilon > 0$-berilgan son bo'lsin. η sonni shunday tanlaymizki, $\eta = \frac{\varepsilon}{3M|S_1|}$ bo'lsin. Bu holda $|y - x| < \frac{\varepsilon}{3M|S_1|}$ bo'lsa,

$$|w_1(y)| = \left| \int_{s_\eta^1} \frac{\sigma(\xi)}{|\xi - y|^{n-2}} d_\xi S \right| < \varepsilon \qquad \textbf{(I.1.16)}$$

tengsizlik o'rinli bo'ladi. Ravshanki, **(I.1.16)**tengsizlik $y = x$ bo'lganda ham to'g'ri bo'ladi. **(I.1.16)** tengsizlik **(I.1.14)** integralning x nuqtada tekis yaqinlashuvchanligini bildiradi. Demak, 2- teoremaga asosan $w(x)$ funksiya $x \in S$ da uzluksiz bo'ladi.

Oddiy qatlam potensialining normal hosilasi. Avvalgiday S ni yopiq Lyapunov sirti deb hisoblaymiz. x-E^nfazoning ixtiyoriy nuqtasi, n esa x nuqtadan o'tuvchi S sirtning tashqi normali bo'lsin.

Agar $x \in S$ bo'lsa, u holda (I.1.14) potensialning n normal yo'nalishi bo'yicha hosilasini to'g'ridan-to'g'ri integral belgisi ostida differensiallash bilan hisoblash mumkin:

$$\frac{\partial w(x)}{\partial n} = \int_S \sigma(\xi) \frac{\partial}{\partial n} \frac{1}{r^{n-2}} d_\xi S.$$

Yuqoridagiga o'xshagan hisoblashlarni bajarsak, quyidagi formulani hosil qilamiz:

$$\frac{\partial}{\partial n} \frac{1}{r^{n-2}} = \frac{n-2}{r^{n-1}} \cos(r, n) \quad (I.1.17)$$

Bunga asosan

$$\frac{\partial w(x)}{\partial n} = (n-2) \int_S \sigma(\xi) \frac{\cos(r,n)}{r^{n-1}} d_\xi S \qquad (I.1.18)$$

$x \in S$ bo'lsin. Agar $\sigma(\xi)$ zichlik integrallanuvchi va chegaralangan, $|\sigma(\xi)| \leq M = const$ bo'lsa, (I.1.18) integral yaqinlashuvchi bo'lishini isbotlaymiz. SsirtningS_η,η< d sfera ichida yotuvchi S_η^1 qismini ajratib olamiz. Ushbu

$\int_{S_\eta^1} \sigma(\xi) \frac{\cos(r,n)}{r^{n-1}} d_\xi S$integralning yaqinlashuvchi bo'lishini ko'rsatish yetarlidir.

Markazi $x$nuqtada bo'lgan mahalliy koordinatalar sistemasini kiritib, oxirgi integralni

$$\int_{S_\eta^1} \sigma(\xi) \frac{\cos(r,n)}{r^{n-1}} d_\xi S = \int_{D_\eta^1} \sigma(\xi) \frac{\cos(r,n) d\xi_1, \dots \dots, d\xi_{n-1}}{r^{n-1} \cos(\nu, \xi_n)}$$

ko'rinishida yozib olamiz. Bu integral ostidagi funksiyani baholaymiz:

$$\left|\sigma(\xi)\frac{\cos(r,n)}{r^{n-1}\cos(\nu,\xi_n)}\right| \leq \frac{2M}{\rho^{n-1}}|\cos(r,n)|, \rho^2 = \sum_{i=1}^{n-1}\xi_i{}^2.$$

I.1-dagi $|\cos(r,n)| \leq a_2 r^{\alpha}$, r$\leq 2\rho$ tengsizliklarga asosan, oldingi tengsizlik quyidagi ko'rinishda yoziladi:

$$\left|\sigma(\xi)\frac{\cos(r,n)}{r^{n-1}\cos(\nu,\xi_n)}\right| \leq \frac{2^{\alpha+1}a_2 M}{\rho^{n-1-\alpha}}.$$

Bu tengsizlik esa, (I.1.18) integralning yaqinlashuvchanligini bildiradi. (I.1.18) integralning $x \in S$ nuqtadagi qiymati oddiy qatlam potensialining normal hosilasining *to'g'ri qiymati* deyiladi va $\frac{\partial w}{\partial n}$ orqali belgilanadi. S sirtning ichidan yoki tashqarisidan $x' \to x \in S$ dagi $\frac{\partial w}{\partial n}$ ning limit qiymatlarini (agar ular mavjud bo'lsa), ya'ni

$$\lim_{x'\to x}\frac{\partial w(x')}{\partial n}$$

ni $\frac{\partial w(x)}{\partial n_i}, \frac{\partial w(x)}{\partial n_e}$ orqali belgilaymiz.

Agar bu limitga intilish x ga nisbatan tekis bo'lsa, u holda bu limit qiymatlar *to'g'ri normal hosilalar* deyiladi.

I.1.teorema. *Agar S yopiq Lyapunov sirti (egri chizig'I bo'lib),* $\sigma(x)$ *zichlik S da uzluksizfunksiya bo'lsa, oddiy qatlam potensiali S da uning ichida ham tashqarisida ham to'g'ri normal hosilalarga ega bo'ladi va bu hosilalar quyidagi formulalar orqali ifodalanadi:*

$$\begin{cases} \frac{\partial w(x)}{\partial n_i} = \frac{n-2}{2}|S_1|\sigma(x) + \frac{\partial \overline{w}}{\partial n}, \\ \frac{\partial w(x)}{\partial n_e} = -\frac{n-2}{2}|S_1|\sigma(x) + \frac{\partial \overline{w}}{\partial n}, \end{cases} \qquad n > 2; \qquad \text{(I.1.19)}$$

$$\begin{cases} \frac{\partial w}{\partial n_i} = \pi\sigma(x) + \frac{\partial \overline{w}}{\partial n}, \\ \frac{\partial w}{\partial n_e} = -\pi\sigma(x) + \frac{\partial \overline{w}}{\partial n}, \end{cases} \qquad n = 2 \qquad \text{(I.1.19')}$$

Bu teoremani isbotlash uchun zichligi $\sigma(x)$ bo'lgan

$$v(x) = \int_S \sigma(\xi) \frac{\partial}{\partial v} \frac{1}{r^{n-1}} d_\xi S$$

Ikkilangan qatlam potensialini kiritib, ushbu

$$F(x) = \frac{\partial w(x)}{\partial n} + v(x) = \int_S \sigma(\xi) \left(\frac{\partial}{\partial n} \frac{1}{r^{n-2}} + \frac{\partial}{\partial v} \frac{1}{r^{n-2}}\right) d_\xi S$$

yig'indini tekshiramiz. x nuqta normal bo'yicha harakat qilib, S sirtni kesib o'tganda $F(x)$ yig'indining uzluksiz o'zgarishini ko'rsatish qiyin emas. $x_0 \in S$ bo'lib, n-x_0 nuqtada S ga o'tkazilgan normal , x esa n normaldagi S dan tashqarida yoki ichkarida yotuvchi ixtiyoriy nuqta bo'lsin.

$x_0$nuqtani markaz qilib, $\eta < d$ radiusli sfera chizamiz. Ssirtning bu sfera ichidagi qismini, xuddi yuqoridagidek, S_η^1 tashqaridagi qismini orqali belgilaymiz. U holda

F(x)=$\int_{S_\eta^1} \sigma(\xi)\left(\frac{\partial}{\partial n}\frac{1}{r^{n-2}} + \frac{\partial}{\partial v}\frac{1}{r^{n-2}}\right) d_\xi S$+$\int_{S_\eta^2} \sigma(\xi)\left(\frac{\partial}{\partial n}\frac{1}{r^{n-2}} + \frac{\partial}{\partial v}\frac{1}{r^{n-2}}\right.$

$\left.\right) d_\xi S = F_1(x) + F_2(x).$ (I.1.20)

Quyidagi ravshan bo'lgan ayniyatlarga egamiz:

$$\frac{\partial}{\partial n}\frac{1}{r^{n-2}} = \frac{n-2}{r^{n-1}} \sum_{i=1}^{n} \frac{\xi_i - x_i}{r} \cos(n, \xi_i).$$

$$\frac{\partial}{\partial v}\frac{1}{r^{n-2}} = -\frac{n-2}{r^{n-1}}\sum_{i=1}^{n}\frac{\xi_i - x_i}{r}\cos(v,\xi_i).$$

Koordinata boshi x_0 nuqtada bo;lgan mahalliy koordinatalar sistemasini kiritamiz. Bu sistemada x=($x_1 \dots \dots \dots, x_n$) nuqta n normalda yotganligi uchun uning koordinatalari (0,……..0,x_n) bo'ladi, ya'ni 1≤ $i \le n-1$ da $x_i = 0$, $\cos(n,\xi_i)$=0, $\cos(n,\xi_n) = 1$. U holda

$$\frac{\partial}{\partial n}\frac{1}{r^{n-2}} + \frac{\partial}{\partial v}\frac{1}{r^{n-2}} = \frac{n-2}{r^{n-1}}\frac{\xi_n - x_n}{r}[1-\cos(v,\xi_n)] - \frac{n-2}{r^{n-1}}\sum_{i=1}^{n}\frac{\xi_i}{r}\cos(v_1,\xi_i).$$

Agar $r_0 = |\xi - x_0|$ desak, (I.1.7) ga asosan

$|\cos(v,\xi_i)| \le \sqrt{3}ar^{\alpha}$, i=1,2,……….,n-1.

So'ngra (I.1.4) ga ko'ra

$$\cos(v,\xi_n) \ge 1 - \frac{a^2 r_0^{2\alpha}}{2}.$$

Bundan,

$$1-\cos(v,\xi_n) \le \frac{a^2 r_0^{2\alpha}}{2} < \frac{a r_0^{\alpha}}{2}.$$

Demak,

$$\left|\frac{\partial}{\partial n}\frac{1}{r^{n-2}} + \frac{\partial}{\partial v}\frac{1}{r^{n-2}}\right| \le \frac{c r_0^{\alpha}}{r^{n-1}}, \text{c-const.} \qquad \text{(I.1.21)}$$

Ushbu

$$\rho^2 = \sum_{i=1}^{n-1}\xi_i{}^2$$

Belgilashni kiritib yuqoridagidek S_η^1 sirtning x_0 nuqtada o'tkazilgan urinma tekislikdagi proyeksiyasini D_η^1 orqali belgilaymiz. Ma'lumki, $r_0 \leq 2\rho$.Ikkinchi tomondan

$$r^2 = \sum_{i=1}^{n}(\xi_i - x_i)^2 = \sum_{i=1}^{n-1}\xi_i{}^2 + (\xi_n - x_n)^2 \geq \rho^2.$$

Bundan, $\rho \leq r$.Endi (I.1.21) dan

$$\left|\frac{\partial}{\partial n}\frac{1}{r^{n-2}} + \frac{\partial}{\partial v}\frac{1}{r^{n-2}}\right| \leq \frac{2^\alpha c}{\rho^{n-1-\alpha}}$$

Tengsizlik kelib chiqadi.

$|\sigma(\xi)| \leq M$bo'lsin, u holda

$$|F_1(x)| \leq 2^\alpha cM \int_{S_\eta^1} \frac{d_\xi S}{\rho^{n-1-\alpha}} = 2^\alpha cM \int_{D_\eta^1} \frac{d\xi_1, \dots\dots, d\xi_{n-1}}{\rho^{n-1-\alpha}\cos(v, \xi_n)} \leq 2^{\alpha+1} cM \int_{D_\eta^1} \frac{d\xi_1, \dots\dots, d\xi_{n-1}}{\rho^{n-1-\alpha}}.$$

D_η^1sohada$\rho \leq r \leq \eta$ tengsizlik o'rinli bo'lgani uchun, D_η^1 butunlay $\rho \leq \eta$ sharda yotadi. Shunday qilib,

$$|F_1(x)| \leq 2^{\alpha+1} cM \int_{\rho \leq n} \frac{d\xi_1, \dots \dots, d\xi_{n-1}}{\rho^{n-1-\alpha}}$$

$$= 2^{\alpha+1} cM \int_{S_1} dS_1 \int_0^\eta \rho^{\alpha-1} d\rho = \frac{2^{\alpha+1} cM(S_1)}{\alpha}\eta^\alpha$$

Bu baho, x_0 nuqtada S ga o'tkazilgan normaldagi x nuqtaning ixtiyoriy holatida o'rinlidir, shu bilan birga x nuqta x_0 bilan ustma-ust tushishi ham mumkin.

Bundan, agar$\varepsilon > 0$ berilgan bo'lsa, η ni

$$\eta < \left[\frac{\alpha\varepsilon}{3 \times 2^{\alpha+1} cM|S_1|}\right]^{\frac{1}{\alpha}}$$

tengsizliklarni qanoatlantiradigan qilib olsak,

$|F_1(x)| < \frac{\varepsilon}{3}$(I.1.22)

tengsizlik bajariladi. (I.1.20) ga asosan

F(x)-F(x_0)=$F_1(x)$-$F_1(x_0) + F_2(x) - F_2(x_0)$.

Bundan

$$|\mathrm{F}(\mathrm{x}) - \mathrm{F}(x_0)| \leq |F_1(x)| + |F_1(x_0)| + |F_2(x) - F_2(x_0)|$$

Yoki (I.1.22) ga binoan

$$|\mathrm{F}(\mathrm{x}) - \mathrm{F}(x_0)| \leq \frac{2\varepsilon}{3} + |F_2(x) - F_2(x_0)|. \qquad \text{(I.1.23)}$$

$F_2(x)$ integralda integrallash S_η^2 sirt bo'yicha bajarilayotgani uchun, x_0 nuqta esa S_η^1 da yotganligi sababli $F_2(x)$ funksiya x_0 nuqtada va uning biror atrofida uzluksiz bo'ladi. Shunday qilib, x_0 nuqtaga yetarli yaqin bo'lgan x nuqtalar uchun

$$|F_2(x) - F_2(x_0)| < \frac{\varepsilon}{3}.$$

(I.1.23) ga binoan

$$|\mathrm{F}(\mathrm{x}) - \mathrm{F}(x_0)| < \varepsilon.$$

Demak,

$$\lim_{x \to x_0} F(x) = F(x_0).$$

Shunday qilib,

F(x)=$\frac{\partial w(x)}{\partial n} + v(x)$

Yig'indi x nuqta S sirtni kesib o'tganda uzluksiz ekan. U holda, bu yig'indining limit qiymatlari va to'g'ri qiymatlari ustma-ust tushadi:

$$\frac{\partial w(x_0)}{\partial n_i} + v_i(x_0) = \frac{\partial w(x_0)}{\partial n_e} + v_e(x_0) = \frac{\partial \overline{w}(x_0)}{\partial n} + \bar{v}(x_0).$$

Bundan, $\begin{cases} v_i(x_0) = -\frac{n-2}{2}|S_1|\mu(x_0) + \bar{v}(x_0), \\ v_e(x_0) = \frac{n-2}{2}|S_1|\mu(x_0) + \bar{v}(x_0), \end{cases}$ $n > 2$;ga asosan

$$\frac{\partial w(x_0)}{\partial n_i} = \bar{v}(x_0) - v_i(x_0) + \frac{\partial \overline{w}(x_0)}{\partial n} = \frac{(n-2)|S_1|}{2}\sigma(x_0) + \frac{\partial \overline{w}(x_0)}{\partial n},$$

$$\frac{\partial w(x_0)}{\partial n_e} = \bar{v}(x_0) - v_e(x_0) + \frac{\partial \overline{w}(x_0)}{\partial n} = -\frac{(n-2)|S_1|}{2}\sigma(x_0) + \frac{\partial \overline{w}(x_0)}{\partial n}.$$

Bu ikki tenglikni birini ikkinchisidan ayirsak, oddiy qatlam potensialining zichligini uning normal hosilalari limit qiymatlari bilan bo'g'lovchi formula kelib chiqadi:

$$\frac{\partial w(x_0)}{\partial n_i} - \frac{\partial w(x_0)}{\partial n_e} = (n-2)|S_1|\sigma(x_0).$$

Endi $w(x)$oddiy qatlam potensialining S sirtda to'g'ri normal hosilalarga ega ekanligini ko'rsatish qoldi. Shu maqsadda

$$\frac{\partial w(x)}{\partial n} = \left[\frac{\partial w(x)}{\partial n} + v(x)\right] - v(x) = F(x) - v(x) \text{(I.1.24)}$$

tenglikni tekshiramiz. x nuqta normal bo'yicha S ichidan yoki tashqarisidan $x_0 \in S$ nuqtaga intilganda $F(x)$ o'zining $F(x_0)$ limit qiymatiga tekis intilishiga ishonch hosil qilamiz. (I.1.24) formulaga asosan bu xossaga$\frac{\partial w(x)}{\partial n}$ham egabo'ladi. Bu esa, ta'rifga ko'ra, oddiy qatlam potensiali S ning ichida ham, tashqarisida ham to'g'ri normal hosilaga ega ekanligini bildiradi.

I.2.Ikkilangan qatlam potensiali va uning xossalari.

Gauss integrali. Ikkilangan qatlam potensialining zichligi birga teng bo'lgan holda, u ya'ni

$$v_0(x) = \int_S \frac{\partial}{\partial n}\frac{1}{r^{n-2}}dS \quad (I.2.1)$$

integral **Gauss integrali** deyiladi.

Agar S yopiq Lyapunov sirti bo'lsa, Gauss integralining qiymatlari ushbu formula bilan aniqlanadi:

$$v_0(x) = \begin{cases} -(n-2)|S_1| = -\frac{2(n-2)\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})} \\ 0, \quad x \text{ nuqta } S \text{ ning tashqarisida yotganda,} \\ -\frac{n-2}{2}|S_1| = -\frac{(n-2)\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}, \quad x \in S; \ n \leq 2; \end{cases} \quad (I.2.2)$$

$$\int_S \frac{\partial}{\partial n}\ln\frac{1}{r}dS = \begin{cases} -2\pi, \quad x \text{ nuqta } S \text{ ning ichida yotganda} \\ 0, \quad x \text{ nuqta } S \text{ ning tashqarisida yotganda,} \\ -\pi, \quad x \in S; \quad n = 2. \end{cases} \quad (I.2.2')$$

(I.2.2) formuladagi birinchi ikkita tenglik ixtiyoriy bo'laklari silliq yopiq S sirt uchun ham to'g'ri bo'ladi. Haqiqatan ham, S shunday sirt bo'lib, x nuqta S ning ichida yotsin. Bu nuqtani markaz qilib, ε radiusli S ning ichida yotuvchi $S_\varepsilon(x)$ sfera chizamiz (I.2.1-chizma). Sva$S_\varepsilon(x)$ sirtlar bilan chegaralangan sohada $\frac{1}{r^{n-2}}$ garmonik funksiya bo'lgani uchun

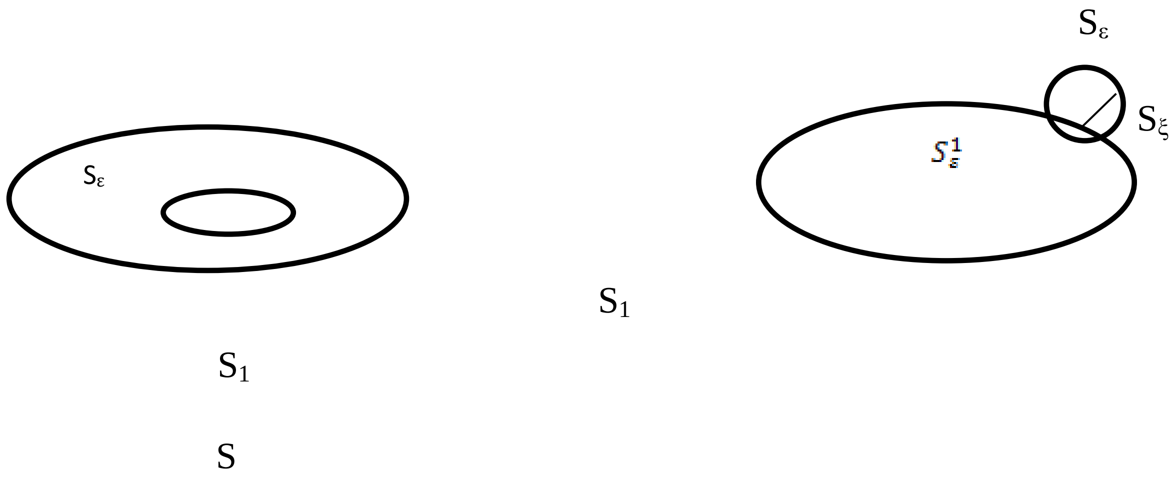
$$\int_S \frac{\partial}{\partial n}\frac{1}{r^{n-2}}dS + \int_{S_\varepsilon(x)} \frac{\partial}{\partial n}\frac{1}{r^{n-2}}dS_\varepsilon = 0$$

$S_\varepsilon$da

$$\frac{\partial}{\partial n}\frac{1}{r^{n-2}} = \frac{\partial}{\partial r}\frac{1}{r^{n-2}}\bigg|_{r\,=\,\varepsilon} = \frac{n-2}{\varepsilon^{n-1}}$$

Demak,

$$\int\limits_{S_\varepsilon} \frac{\partial}{\partial n}\frac{1}{r^{n-2}} dS_\varepsilon = \frac{n-2}{\varepsilon^{n-1}}\int\limits_{S_\varepsilon} d\,S_\varepsilon = (n-2)\frac{|S_\varepsilon|}{\varepsilon^{n-1}} = (n-2)|S_1|.$$



Agar x nuqta S sirtdan tashqarida yotgan bo'lsa, $\frac{1}{r^{n-2}}$ funksiya Sning ichida garmonik bo'ladi, u holda

$\int_S \frac{\partial}{\partial n}\frac{1}{r^{n-2}}$dS=0.

Endi Syopiq Lyapunov sirti bo'lib, x∈S bo'lsin. $x$nuqtani markaz qilib yetarli kichik ε,0< $\varepsilon < d$, radiusli $S_\varepsilon(x)$ sfera chizamiz.SsirtningS_ε

sferadan tashqarida yotgan qismini S_1 orqali, S_ε sferaning S ichidagi qismini $S_\varepsilon{}^1$ orqali belgilab olamiz (I.2.2)-chizma. Xosmas integralning ta'rifiga asosan

$\lim_{\varepsilon\to 0}\int_{S_1} \frac{\partial}{\partial n}\frac{1}{r^{n-2}} dS_1 = \int_S \frac{\partial}{\partial n}\frac{1}{r^{n-2}} dS.$ (I.2.3)

$x$nuqtaS_1 va S_ε^1 sirtlar bilan chegaralangan sohadan tashqarida yotgani uchun, bu sohada $\frac{1}{r^{n-2}}$ garmonik funksiya bo'ladi. U holda

$$\int\limits_{S_1} \frac{\partial}{\partial n}\frac{1}{r^{n-2}}dS_1 + \int\limits_{S_\varepsilon^1} \frac{\partial}{\partial n}\frac{1}{r^{n-2}}dS_\varepsilon^1 = 0.$$

Demak, (I.2.3) ga asosan

$$\int_{S_1} \frac{\partial}{\partial n}\frac{1}{r^{n-2}}dS_1 = -\lim_{\varepsilon\to 0}\int_{S_\varepsilon^1} \frac{\partial}{\partial n}\frac{1}{r^{n-2}}dS_\varepsilon^1 \quad (I.2.4)$$

S_ε^1bo'yicha olingan integralning qiymatini hisoblaymiz. S_ε^1da

$$\frac{\partial}{\partial n}\frac{1}{r^{n-2}} = -\frac{\partial}{\partial r}\frac{1}{r^{n-2}}\bigg|_{r=\varepsilon} = \frac{n-2}{\varepsilon^{n-1}}$$

bo'lgani uchun

$$\int\limits_{S_\varepsilon^1} \frac{\partial}{\partial n}\frac{1}{r^{n-2}}dS_\varepsilon^1 = \frac{n-2}{\varepsilon^{n-1}}\int\limits_{S_\varepsilon^1} dS_\varepsilon^1 = (n-2)\frac{|S_\varepsilon^1|}{\varepsilon^{n-1}}$$

ε yetarli kichik bo'lganda S_ε^1 sirt urinma tekislikka yopishgan yarim sferaga yaqin bo'ladi. Shu sababli

$$\lim_{\varepsilon\to 0}\int\limits_{S_\varepsilon^1} \frac{\partial}{\partial n}\frac{1}{r^{n-2}}dS_\varepsilon^1 = (n-2)\lim_{\varepsilon\to 0}\frac{|S_\varepsilon^1|}{\varepsilon^{n-1}} = (n-2)\lim_{\varepsilon\to 0}\frac{\varepsilon^{n-1}|S_1|}{2\varepsilon^{n-1}} = \frac{n-2}{2}|S_1|$$

Bu mulohazalardan to'laroq ishonch hosil qilish uchun n=3 bo'lgan holda oldingi integralni hisoblaymiz. Markazi x nuqtada bo'lgan sferik koordinatalarni kiritamiz:

$$\frac{\partial}{\partial n}\frac{1}{r}\bigg|_{S_\varepsilon^1} = \frac{1}{\varepsilon^2}, \quad dS_\varepsilon^1 = \varepsilon^2 \sin\theta\, d\theta\, d\varphi.$$

U holda

$$\int_{S_\varepsilon^1} \frac{\partial}{\partial n}\frac{1}{r} dS_\varepsilon^1 =$$
$$\int_0^{2\pi}\int_0^{\theta(\varphi)} \sin\theta' d\theta' d\varphi = \int_0^{2\pi}[1-\cos\theta(\varphi)]d\varphi = 2\pi - \int_0^{2\pi}\cos\theta(\varphi)d\varphi$$
(I.2.5)

Ushbu $\lim_{\varepsilon\to 0}\int_0^{2\pi}\cos\theta(\varphi)d\varphi = 0$ (I.2.6)

tenglik bajarilishini ko'rsatamiz. Shu maqsadda markazi x nuqtada bo'lgan mahalliy ξ_1,ξ_2,ξ_3 koordinata sistemasini kiritamiz. ξ_3 o'qni x nuqtada S sirtga o'tkazilgan normal bo'yicha yo'naltiramiz, $\xi_1\xi_2$ tekislik sifatida x nuqtada S ga o'tkazilgan urinma tekislikni olamiz.(ε,$\varphi,\theta(\varphi)$) nuqtalar S_ε sferaning S Lyapunov sirti bilan kesishgan chizig'ida yotishni eslatib o'tamiz. Bu holda (I.1.13) ga asosan

$$|\cos\theta(\varphi)| \le a_2\varepsilon^\alpha$$

tenglik o'rinlidir.

Bunga asosan $\varepsilon\to 0$ da $\cos\theta(\varphi)$ ning nolga tekis, ya'ni x ga bog'liq bo'lmay, intilishi kelib chiqadi. Bundan darhol (I.2.6) tenglikning to'g'riligiga ishonch hosil qilamiz.

Demak, (I.2.5) formulaga asosan

$$\lim_{\varepsilon\to 0}\int_{S_\varepsilon^1} \frac{\partial}{\partial n}\frac{1}{r} dS_\varepsilon^1 = 2\pi$$

yoki (I.2.4) ga binoan

$$\lim_{\varepsilon\to 0}\int_S \frac{\partial}{\partial n}\frac{1}{r} dS = -2\pi$$

Ikkilangan qatlam potensiali.

Ushbu

$$\nu(x) = \int_S \mu(\xi)\frac{\partial}{\partial n}\frac{1}{r^{n-2}}d_\xi S \qquad \text{(I.2.7)}$$

Ikkilangan qatlam potensialini tekshiramiz. *Agar* $\mu(\xi)$ *zichlik* S *daintegrallanuvchi bo'lsa,* $\nu(x)$ *potensial* S *bilan umumiy nuqtaga ega bo'lmagan ixtiyoriy chekli yoki cheksiz sohada garmonik funksiya bo'ladi.*

Haqiqatan ham, $x \bar{\in} S$ da ikkilangan qatlam potensiali barcha tartibli hosilalarga ega va bu hosilalarni integral ostida differensiallab hisoblash mumkin. Bundan darhol $x \neq \xi$ da $\frac{1}{r^{n-2}}$ garmonik funksiya bo'lgani uchun,

$\nu(x)$ning ham garmonikligi kelib chiqadi. x nuqta S sirtning tashqarisida yotgan holda , $\nu(x)$ning cheksiz uzoqlashgan nuqtadagi xarakterini aniqlaymiz.

Shu maqsadda , $\nu(x)$ ni baholaymiz:

$$|\nu(x)| \leq \int_S |\mu(\xi)| \left|\frac{\partial}{\partial n}\frac{1}{r^{n-2}}\right| d_\xi S \leq$$

$$\leq (n-2)\int_S |\mu(\xi)| \sum_{i=1}^{n} \left|\frac{x_i - \xi_i}{r^n}\right| |\cos(n, x_i)| d_\xi S$$

Ushbu

$$|r| = |x - \xi| \geq |x| - |\xi|, r \geq \frac{|x|}{2}, |x_i - \xi_i| \leq r |\cos(n, x_i)| \leq 1$$

tengsizliklarga asosan

$$|\nu(x)| \leq \frac{2^{n-1} n(n-2)}{|x|^{n-1}} \int_S |\mu(\xi)| d_\xi S$$

tengsizlikni hosil qilamiz. $\mu(\xi)$zichlik integrallanuvchi bo'lgani uchun oldingi tengsizlikning o'ng tomonidagi integral cheklidir.

Shunday qilib, ikkilangan qatlam potensiali S sirtdan tashqari barcha E^n fazoda garmonik bo'lib, cheksizda $|x|^{-(n-1)}$ kabi nolga intiladi. Gauss integralidan ko'rinadiki, umuman aytganda ikkilangan qatlam potensiali x nuqta S sirtni kesib o'tganda uzilishga ega.

$x_0 \in S$bo'sin.x nuqta S ning ichida yotib, x_0 nuqtaga intilgandagi $\nu(x)$ ning qiymatini $\nu_i(x_0)$ orqali, x nuqta S dan tashqarida yotib, x_0 ga intilgandagi qiymatini $\nu_e(x_0)$ orqali belgilaymiz. $\nu(x)$ ning $x_0 \in S$ nuqtadagi qiymati bu *potensialning to'g'ri qiymati* deyiladi, va u $\bar{\nu}(x_0)$ orqali belgilanadi.

I.2.1.teorema. Agar S yopiq Lyapunov sirtni bo'lib, $\mu(\xi)$ zichlik S da uzluksiz bo'lsa, ikkilangan qatlam potensiali $\nu(x)$ uchun quyidagi limit munosabat o'rinlidir:

$$\begin{cases} \nu_i(x_0) = -\frac{n-2}{2}|S_1|\mu(x_0) + \bar{\nu}(x_0), \\ \nu_e(x_0) = \frac{n-2}{2}|S_1|\mu(x_0) + \bar{\nu}(x_0), \end{cases} \quad n > 2; \qquad \text{(I.2.8)}$$

$$\begin{cases} \nu_i(x_0) = -\pi\mu(x_0) + \bar{\nu}(x_0), \\ \nu_e(x_0) = \pi\mu(x_0) + \bar{\nu}(x_0), \end{cases} \quad n = 2. \qquad \text{(I.2.8')}$$

Isbot.$\nu(x)$funksiyani quyidagi

$$\nu(x) = \int_S [\mu(\xi) - \mu(x_0)] \frac{\partial}{\partial n} \frac{1}{r^{n-2}} d_\xi S + \mu(x_0) \int_S \frac{\partial}{\partial n} \frac{1}{r^{n-2}} d_\xi S = \nu_1(x) + \mu(x_0)\nu_0(x).$$
(I.2.9)

ko'rinishda yozib olamiz.

$\nu_1(x)$potensialningx_0 nuqtada uzluksiz ekanini ko'rsatamiz. Shu maqsaddax_0 nuqtani markaz qilib, η radiusli sfera chizamiz. Ssirtning bu sfera ichidagi qismini S_1 orqali, tashqarisidagi S_2 orqali belgilab olamiz. U holda

$$v_1(x) = \int_{S_1} [\mu(\xi) - \mu(x_0)] \frac{\partial}{\partial n} \frac{1}{r^{n-2}} d_\xi S_1 + \int_{S_2} [\mu(\xi) - \mu(x_0)] \frac{\partial}{\partial n} \frac{1}{r^{n-2}} d_\xi S_2$$
$$= v_1^1(x) + v_1^2.$$

Bunga asosan

$$|v_1(x) - \overline{v_1}(x_0)| \leq |v_1^1(x)| + \left|\overline{v_1^1}(x_0)\right| + \left|v_1^2(x) - \overline{v_1^2}(x_0)\right|. \qquad \text{(I.2.10)}$$

$\mu(\xi)$funksiya uzluksiz bo'lgani uchun η ni shunday tanlaymizki, $|\xi - x_0| < \eta$

bo'lganda,

$$|\mu(\xi) - \mu(x_0)| < \frac{\varepsilon}{3C}$$

bo'lsin bu yerda

$$\left|\int_S \frac{\partial}{\partial n} \frac{1}{r^{n-2}} d_\xi S\right| \leq C.$$

Bu holda, ixtiyoriy x$\in E^n$ uchun

$$|v_1^1(x)| \leq \int_{S_1} |\mu(\xi) - \mu(x_0)| \left|\frac{\partial}{\partial n} \frac{1}{r^{n-2}}\right| d_\xi S_1 < \frac{\varepsilon}{3C} \int_{S_1} \frac{\partial}{\partial n} \frac{1}{r^{n-2}} d_\xi S_1 \leq \frac{\varepsilon}{3C} \int_S \frac{\partial}{\partial n} \frac{1}{r^{n-2}} d_\xi S \leq \frac{\varepsilon}{3}$$
(I.2.11)

Xususiy holda

$$\left|\overline{v_1^1}(x_0)\right| < \frac{\varepsilon}{3}. \qquad \text{(I.2.12)}$$

$v_1^2(x)$potensial integralS_2 bo'yicha bajarilyapti,x_0 nuqta S_1 da yotadi.Shuning uchun uzluksiz, ya'ni shunday $\delta > 0$ son mavjudki,$|x - x_0| < \delta$ bo'ganda

$$|v_1^2(x) - v_1^2(x_0)| = \left|v_1^2(x) - \overline{v_1^2}(x_0)\right| < \frac{\varepsilon}{3}. \qquad \text{(I.2.13)}$$

(I.2.10)-(I.2.13) munosabatlarga asosan, agar $|x - x_0| < \delta$ bo'lsa,

$$|v_1(x) - \overline{v_1}(x_0)| < \varepsilon \qquad \text{(I.2.14)}$$

Bo'ladi ya'nix_0 nuqtada esa v_1(x) potensial uzluksiz. Shunday ekan v_1(x) potensialning limit qiymatlari va to'g'ri qiymati x_0 nuqtada ustma-ust tushadi, ya'ni

$$v_{1i}(x_0) = v_{1e}(x_0) = \overline{v_1}(x_0). \qquad \text{(I.2.15)}$$

(I.2.2) formulaga asosan

$$v_{0i}(x_0) = -(n-2)|S_1|, \;\; v_{0e}(x_0)=0, \; \overline{v_0}(x_0) = -\frac{n-2}{2}|S_1|.$$

(I.2.9) va (I.2.15) formulalardan $v_i(x_0)$va$v_e(x_0)$ limit qiymatlarning mavjudligi kelib chiqadi, shu bilan birga

$$v_i(x_0)=\overline{v_1}(x_0) + \mu(x_0)\, v_{0i}(x_0) = \overline{v_1}(x_0) - (n-2)|S_1|\mu(x_0), \qquad \text{(I.2.16)}$$

$$v_e(x_0)=\overline{v_1}(x_0) + \mu(x_0)v_{0e}(x_0) = \overline{v_1}(x_0).$$

So'ngra

$$\overline{v_1}(x_0) = \int_S [\mu(\xi) - \mu(x_0)]\frac{\partial}{\partial n}\frac{1}{r^{n-2}} d_\xi S = \bar{v}(x_0) + \frac{(n-2)|S_1|}{2}\mu(x_0)\,. \quad \text{(I.2.17)}$$

(I.2.16)va (I.2.17) munosabatlardan darhol (I.2.8) formulalar kelib chiqadi.

<u>I.2.1-izoh.</u>$x \in S^p$bo'lganda (I.2.9) dan

$$\bar{v}(x) = \overline{v_1}(x) - \frac{n-2}{2}|S_1|\mu(x_0).$$

Endi x nuqtani S bo'ylab x_0 ga yo'naltiramiz, $v_1(x)$ uzluksiz bo'lgani uchun

$$\lim_{x\to x_0} \bar{v}(x) = \bar{v_1}(x_0) - \frac{n-2}{2}|S_1|\mu(x_0).$$

(I.2.9) ga ko'ra

$$\bar{v}(x_0) = \bar{v_1}(x_0) - \frac{n-2}{2}|S_1|\mu(x_0).$$

Demak,

$$\lim_{x\to x_0} \bar{v}(x) = \bar{v}(x_0)$$

Ya'ni $\mu(x)$ zichlik Sda uzluksiz bo'lsa, $\nu(x)$potensial ham Sda uzluksiz bo'lar ekan.

I.2.2-izoh. Ikkilangan qatlam potensiali $\nu(x)$ o'zining limit qiymatlariga sirtning ichida ham, tashqarisida ham tekis intiladi. Haqiqatan ham, $\mu(x)$ zichlik S da uzluksiz, demak, S yopiq sirt bo'lgani uchun tekis uzluksiz bo'ladi. Bu holda$h$ radiusni x_0 nuqtaning S dagi joylashganiga bog'liq bo'lmagan holda tanlab olish mumkin. $\nu_1^2(x)$potensial aslida ikkita $x \in E^n$ va $x_0 \in S$ nuqtaning funksiyasidir. Agar h radius tayin bo'lsa, $x_0 \in S$, $|x - x_0| < \frac{1}{2}h$ munosabatlar bilan aniqlangan chegaralangan yopiq to'plamda $\nu_1^2(x)$ uzluksiz,demak, tekis uzluksiz bo'ladi. Shuning uchun ham (I.2.14) tengsizlikda ishtirok etuvchi δ ni faqat ε ga bog'liq qilib olish mumkin.Bu esa, $x_0 \in S$ nuqtaga nisbatan $x \to x_0$ da $\nu_1(x)$ ning $\nu_1(x_0)$ ga tekis intilishini ko'rsatadi. Gauss integrali bo'lgan $\nu_0(x)$ funksiya S sirt ichida ham, tashqarisida o'zgarmas bo'lganligi sababli, agar S ning ichidan yoki tashqarisidan $x \to x_0$da, $\nu_0(x)$ o'zining limit qiymatlariga tekis intiladi. (I.2.9) formuladan bu xossaga ν(x) ning ham ega ekanligi kelib chiqadi.

II bob.Issiqlik o'tkazuvchanlik tenglamasi uchun chegaraviy masalalarni yechishga tadbiqi.

2.1. Issiqlik potensiallari.

Issiqlik o'tkazuvchanlik tengl.amasining istalgan yechimi

$$u(x,t) = \int\limits_{AB} G_0 u d\xi - \int\limits_{AP} G_0 u d\xi + \int\limits_{BQ} G_0 u d\xi + a^2 \int\limits_{BQ+PA} \left(G_0 \frac{\partial u}{\partial \xi} - u \frac{G_0}{\partial \xi} \right) d\tau.$$

kabi tasvirlanadi.

Bu yig'indidagi alohida qo'shiluvchilarni o'rganamiz va ularning har biri issiqlik o'tkazuvchanlik tenglamasini qanoatlantirishini isbotlaymiz.

Haqiqatan ham, birinchi qo'shiluvchi Puasson formulasi bo'ladi.

PABQ sohaning ichki nuqtalari uchun

$$V = a^2 \int\limits_{AP} G_0 v d\tau = \frac{a^2}{2\sqrt{\pi}} \int\limits_0^t \frac{1}{\sqrt{a^2(t-\tau)}} e^{-\frac{[x-\chi_1(\tau)]^2}{4a^2(t-\tau)}} v(\tau) d\tau \qquad (\xi = \chi_1(\tau)),$$

$$W = 2a^2 \int\limits_{AP} \frac{\partial G_0}{\partial \xi} \mu d\tau = \frac{1}{2a\sqrt{\pi}} \int\limits_0^t \frac{x - \chi_1(\tau)}{(t-\tau)^{\frac{3}{2}}} e^{-\frac{[x-\chi_1(\tau)]^2}{4a^2(t-\tau)}} \mu(\tau) d\tau$$

Integrallar issiqlik o'tkazuvchanlik tenglamasini qanoatlantirishini isbotlaymiz.

Vva$W$ funksiyalar issiqlik potensiallari deyiladi. (mos ravishda sodda va ikki qatlamli).

V va W funksiyalarning hosilalari integral belgisi ostida differensiallash yordamida hisoblanadi, t bo'yicha differensiallashda ishtirok etayotgan o'rniga qo'yish nolga teng. $x \neq \chi_1(t)$bo'lgani uchun

$$G_0(x,t,\chi_1(\tau),\tau)\mu(\tau)\Big|_{\tau=t}=\frac{1}{2\sqrt{\pi}}\frac{1}{\sqrt{a^2(t-\tau)}}e^{-\frac{[x-\chi_1(\tau)]^2}{4a^2(t-\tau)}}\mu(\tau)\Bigg|_{\tau=t}=0$$

o'rinli.Shunday qilib, x,t parametrlar bo'yicha differensiallash G_0 funksiyaga taaluqli bo'lib, G_0 issiqlik o'tkazuvchanlik tenglamasi yechimi bo'ladi. Qolgan qo'shiluvchilar shunga o'xshash o'rganiladi.

Endi V va W funksiyalarning AP $(x=\chi_1(t))$ chiziqdagi holatini o'rganamiz.

Vintegral (x,t) nuqta AP chiziq bo'yicha o'tganda uzluksiz bo'ladi, chunki bu integral tekis yaqinlashuvchi. Wfunksiya AP chiziq orqali o'tishda uzilishga ega bo'lib,

$$W|_{x=\chi_1(t)+0}=W|_{x=\chi_1(t)}+\mu(t),$$

$$W|_{x=\chi_1(t)-0}=W|_{x=\chi_1(t)}-\mu(t).$$

bo'lishini ko'rsatamiz. Bu isbot $\chi_1(t)$ va $\mu(t)$ funksiyalar differensiallanuvchi deb faraz qilinib amalga oshiriladi.

Avval W funksiyani $\mu(t)=\mu_0$zichlik o'zgarmas holda qaraymiz:

$$W^0(x,t)=\frac{1}{2a\sqrt{\pi}}\int_{t_0}^{t}\frac{x-\chi_1(\tau)}{(t-\tau)^{\frac{3}{2}}}e^{-\frac{[x-\chi_1(\tau)]^2}{4a^2(t-\tau)}}\mu_0\,d\tau$$

va qo'shimcha

$$\widetilde{V^0}(x,t)=\frac{1}{2a\sqrt{\pi}}\int_{t_0}^{t}\frac{2\chi'(\tau)}{\sqrt{t-\tau}}e^{-\frac{[x-\chi_1(\tau)]^2}{4a^2(t-\tau)}}\mu_0\,d\tau,$$

Integrallar yuqoridagi eslatmalarga ko'ra AP yoy nuqtalarida uzluksiz bo'ladi.

$W^0-\widetilde{V^0}$ayirma bevosita hisoblanadi.

$W^0(x,t) - \widetilde{V^0}(x,t)$=

$$\frac{1}{2a\sqrt{\pi}}\int_{t_0}^{t} e^{-\frac{[x-\chi_1(\tau)]^2}{4a^2(t-\tau)}}\left[\frac{x-\chi_1(\tau)}{(t-\tau)^{\frac{3}{2}}} - \frac{2\chi'(\tau)}{\sqrt{t-\tau}}\right]\mu_0 d\tau == \mu_0\frac{2}{\sqrt{\pi}}\int_{\frac{x-\chi_1(t_0)}{2\sqrt{a^2(t-t_0)}}}^{a_0} e^{-a^2}\, da\left(a = \frac{x-\chi_1(\tau)}{2\sqrt{a^2(t-t_0)}}\right),$$

Bu yerda

agar$x > \chi_1(t)$ bo'lsa, $a_0 = +\infty$,

agar$x = \chi_1(t) = x_0$bo'lsa, $a_0 = 0$,

agar$x < \chi_1(t)$ bo'lsa, $a_0 = -\infty$,

$x \to x_0 \pm 0$ bo'lganda

$$[W^0(x_0 \pm 0, t) - W^0(x_0, t)]\text{-}\left[\widetilde{V^0}(x_0 \pm 0, t) - \widetilde{V^0}(x_0, t)\right] = \mu_0\frac{2}{\sqrt{\pi}}\int_0^{\pm\infty} e^{-a^2}\, da = \pm\mu_0.$$

ni hosil qilamiz. $\widetilde{V}$ning uzluksizligiga ko'ra

$\widetilde{V}(x_0 \pm 0, t)$- $\widetilde{V}(x_0, t)$=0.

o'rinli.Shunday qilib,

$W^0(x_0 \pm 0, t) = W^0(x_0, t) \pm \mu_0.$

agar$\mu(t)$ o'zgarmas bo'lmasa, u holda

$W(x,t) = W^0(x_0,t) - \psi(x,t)$,

Bu yerda

$$\psi(x,t) = \frac{a^2}{2\sqrt{\pi}}\int_{t_0}^{t}\frac{x-\chi_1(\tau)}{[a^2(t-\tau)]^{\frac{3}{2}}} e^{-\frac{[x-\chi_1(\tau)]^2}{4a^2(t-\tau)}}[\mu(t) - \mu(\tau)]\, d\tau.$$

$\mu(t)$ funksiya differensiallanuvchiligi haqidagi farazimizga ko'ra, bu integral xuddi V funksiya kabi $\tau = t$ da maxsuslikka ega, tekis yaqinlashuvchi va AP egri chiziqda uzluksiz funksiya bo'ladi.

Shunday qilib, $W(x,t)$ ning $x = x_0 \pm 0$ dagi limiti

$$W(x_0 \pm 0, t) = W^0(x_0, t) \pm \mu,$$

ga teng. Shuni isbotlash talab qilingan edi.

$\frac{\partial V}{\partial x}(x,t)$hosila$W(x,t)$ funksiyaga o'xshab $x = x_0$ da uzilishga ega. Bu hosila

$$\frac{\partial V}{\partial x} = -\frac{1}{2a\sqrt{\pi}}\frac{1}{2}\int_0^t \frac{x - \chi_1(\tau)}{(t-\tau)^{\frac{3}{2}}} e^{-\frac{[x-\chi_1(\tau)]^2}{4a^2(t-\tau)}} \nu(\tau)d\tau$$

ga teng va zichligi

$$\mu(t) = \frac{\nu(t)}{2}.$$

bo'lsa, $W(x,t)$ga teng. Bundan

$$\frac{\partial V}{\partial x}(x_0 \pm 0, t) = \frac{\partial V}{\partial x}(x_0, t) \pm \frac{\nu(t)}{2}$$

kelib chiqadi, bu yerda

$$\frac{\partial V}{\partial x}(x_0, t) = -\frac{1}{2a\sqrt{\pi}}\frac{1}{2}\int_0^t \frac{x_0 - \chi_1(\tau)}{(t-\tau)^{\frac{3}{2}}} e^{-\frac{[x_0-\chi_1(\tau)]^2}{4a^2(t-\tau)}} \nu(\tau)d\tau$$

integral V ning x_0 nuqtadagi chap va o'ng hosilalarining yig'indisining yarmiga teng:

$$\frac{1}{2}\left[\frac{\partial V}{\partial x}(x_0 + 0, t) + \frac{\partial V}{\partial x}(x_0 - 0, t)\right].$$

$V(x,t)$funksiyax_0 nuqtaning o'zida hosilaga ega emas.

Shu bilan potensiallarning AP bo'ylab o'rganishni yakunlaymiz. Potensiallarning BQ chiziq bo'yicha xossalari shunga o'xshash o'rganiladi.

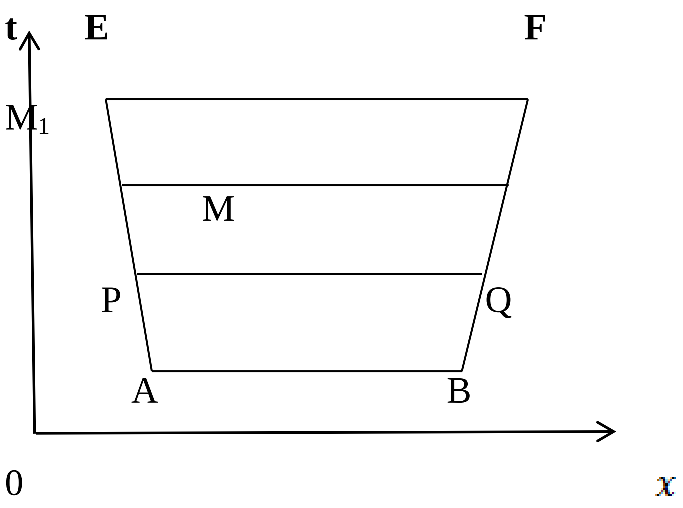
II.2.Issiqlik o'tkazuvchanlik yenglamasi uchun chegaraviy masalalarni integral tenglamalarni yechishga keltirish.

Issiqlik o'tkazuvchanlik tenglamasi uchun vaqt o'tishi bilan almashinuvchi chegaraga ega soha uchun chegaraviy masala qo'yish mumkin.

Soddalik uchun bu masalani bitta geometrik o'zgaruvchili

$\mathcal{L}(u)=a^2u_{xx}-u_t=0, \quad (\text{II}.\,2.1)$

tenglama uchun qaraymiz, lekin quyida bayon qilingan barcha mulohazani ko'p o'zgaruvchili holga o'tkazish mumkin.



II 1.1 rasm

AB va EF xarakteristika hamda

$x=\mathcal{X}_1(t) \quad (AE-uchun)$va

$x=\mathcal{X}_2(t) \quad (BF-uchun)$

tenglama bilan aniqlangan chiziqlar bilan chegaralangan BAEF sohani qaraymiz.

(II.2.1) issiqlik o'tkazuvchanlik tenglamasining

$$\begin{cases} u = \varphi(x) \quad AB \quad da \\ u|_{x=x_1(t)} = \mu_1(t), \quad u|_{x=x_2(t)} = \mu_2(t). \end{cases} \text{(II.2.2)}$$

Boshlang'ich va chegaraviy shartni qanoatlantiruvchi yechimini aniqlash bu soha uchun **birinchi chegaraviy masala** bo'ladi.

Maksimal qiymat prinsipiga ko'ra bu masala bittadan ko'p uzluksiz yechimga ega bo'lolmaydi. Boshqa chegaraviy masala ham shunga o'xshash qo'yiladi.

(II.2.1) tenglama uchun Grin formulasini va bu masala yechimi uchun integral tasvirini olamiz.

$\mathcal{M}$(v)=$a^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial v}{\partial t}$; (II. 2.3)

Operatorni qaraymiz va

$\boldsymbol{\psi\mathcal{L}(\varphi) - \varphi\mathcal{M}(\psi) = a^2}(\psi\varphi_x - \varphi\psi_x)_x$-$(\varphi\psi)_t$. Ifodani PABQ soha bo'yicha integrallab, va Grin formulasidan foydalanib,

$$\iint [\psi\boldsymbol{\mathcal{L}(\varphi)} - \boldsymbol{\varphi\mathcal{M}(\psi)}]dxdt = \oint \left[\varphi\psi dx + a^2(\psi\frac{\partial\varphi}{\partial x} - \varphi\frac{\partial\psi}{\partial x})dt\right],$$

ni hosil qilamiz, bu yerda $\varphi(x,t)$ va $\psi(x,t)$-ixtiyoriy, yetarli marta differensiallanuvchi funksiyalar. Bunda o'ng integral PABQ yopiq kontur bo'yicha olinadi. Agar $\mathcal{L}(\varphi) = 0$ va $\mathcal{M}(\psi) = 0$ bo'lsa, u holda

$$\int_{PQ} \varphi\psi dx = \int_{AB} \varphi\psi dx + \int_{BQ} \left[\varphi\psi dx + a^2(\psi\frac{\partial\varphi}{\partial x} - \varphi\frac{\partial\psi}{\partial x})dt\right] - \int_{AP} \left[\varphi\psi dx + a^2(\psi\frac{\partial\varphi}{\partial x} - \varphi\frac{\partial\psi}{\partial x})dt\right]. \qquad \text{(II. 2.4)}$$

ni hosil qilamiz. Faraz qilamiz, $\varphi(x,t) = u(x,t) - \mathcal{L}(u) = 0$ issiqlik o'tkazuvchanlik tenglamasining biror yechimi bo'lsin, $\psi = G_0(x,t,\xi,\tau)$ esa to'g'ri chiziqdagi manba formulasi bo'lib,

$$G_0(x,t,\xi,\tau) = \frac{1}{2\sqrt{\pi a^2(t-\tau)}} e^{-\frac{(x-\xi)^2}{4a^2(t-\tau)}}, \qquad \text{(II. 2.5)}$$

Ko'p hollarda issiqlik o'tkazuvchanlik tenglamasining fundamental yechimi deyiladi. $G_0(x,t,\xi,\tau)$ funksiya x,t bo'yicha $\mathcal{L}(G_0)=0$ tenglamani va ξ,τ bo'yicha

$\mathcal{M}(G_0)=0$ qo'shma tenglamani qanoatlantiradi.

$\mathcal{M}(x,t)$-BAEF soha ichidagi fikserlangan nuqta bo'lib, unda $u(x,t)$ funksiya qiymatini aniqlaymiz, $\mathcal{M}_1$ esa, $x, t+h$ koordinataga ega nuqta, bu yerda $h > 0$.

$\mathcal{M}$ nuqta orqali PQ xarakteristika o'tkazib, (II.2.4) formulada x ni ξ ga, t ni τ ga almashtirib va ABQP sohaga qo'llab hamda

$$\varphi = u(\xi,\tau) \text{va} (\xi,\tau) = G_0(x,t+h,\xi,\tau), \qquad \text{(II. 1.6)}$$

formulalarga qo'llab:

$$\int_{PQ} \frac{e^{-\frac{(x-\xi)^2}{4a^2h}}}{2\sqrt{\pi a^2 h)}} u(\xi,t)d\xi$$
$$= \int_{PABQ} \left[u(\xi,\tau)G_0(x,t+h,\xi,\tau)d\xi + a^2\left(G_0\frac{\partial u}{\partial \xi} - u\frac{\partial G_0}{\partial \xi}\right)d\tau \right]. \qquad \text{(II. 2.7)}$$

ni hosil qilamiz. $h \to 0$ da limitga o'tib, $G_0(x,t+h,\xi,\tau)$ va $\frac{\partial G_0}{\partial \xi}$ funksiyalarning PABQ da h bo'yicha uzluksizligini hamda

$$\lim_{h\to 0} \int_{PQ} \frac{e^{-\frac{(x-\xi)^2}{4a^2h}}}{2\sqrt{\pi a^2 h)}} u(\xi,t)d\xi = u(x,t), \qquad \text{(II. 2.8)}$$

tenglikni inobatga olib, agar (x,t)-PQ kesmada yotsa, issiqlik o'tkazuvchanlik tenglamasining ixtiyoriy yechimi ko'rinishini beruvchi

$$u(x,t) = \int\limits_{PABQ} u(\xi,\tau)G_0(x,t,\xi,\tau)d\xi + \int\limits_{BQ+PA} a^2\left(G_0\frac{\partial u}{\partial \xi} - u\frac{\partial G_0}{\partial \xi}\right)d\tau, \qquad \text{(II.2.9)}$$

asosiy integral formulani hosil qilamiz. Uni yana bir bor batafsil yozamiz:

$$u(x,t) = \int\limits_{PABQ} \frac{e^{-\frac{(x-\xi)^2}{4a^2(t-\tau)}}}{2\sqrt{\pi a^2(t-\tau)}}u(\xi,\tau)d\xi$$
$$+ a^2 \int\limits_{BQ+PA} \frac{e^{-\frac{(x-\xi)^2}{4a^2(t-\tau)}}}{2\sqrt{\pi a^2(t-\tau)}}\frac{\partial u}{\partial \xi}d\tau$$
$$- a^2 \int\limits_{BQ+PA} u(\xi,\tau)\frac{\partial}{\partial \xi}\left(\frac{e^{-\frac{(x-\xi)^2}{4a^2(t-\tau)}}}{2\sqrt{\pi a^2(t-\tau)}}\right)d\tau. \qquad \text{(II.2.9')}$$

Bu formula chegaraviy masala yechimini bermaydi, chunki o'ng qismini hisoblashda nafaqat u ning qiymatini, balki $\frac{\partial u}{\partial \xi}$ ning AE va BF yoydagi qiymatini bilish kerak.

Manba formulasini kiritishda Laplas tenglamasi uchun bajarilgan almashtirishga o'xshash almashtirish yordamida bu formuladan $\frac{\partial u}{\partial \xi}$ ni yo'qotish mumkin.

Faraz qilamiz v - PQ da nolga aylanuvchi biror yechimi bo'lsin, $\mathcal{M}(v)=0$ qo'shma tenglamaning, u esa $\mathcal{L}(u)=0$ issiqlik o'tkazuvchanlik tenglamaning yechimi bo'lsin. v va u funksiyalarga PABQ soha uchun (II.2.4) formulani qo'llab

$$0=\int_{PABQ}\left[u(\xi,\tau)v(\xi,\tau)d\xi + a^2\left(v\frac{\partial u}{\partial \xi} - u\frac{\partial v}{\partial \xi}\right)d\tau\right]. \qquad \text{(II.2.10)}$$

ni hosil qilamiz (II.1.9) dan (II.1.10) tenglamani ayirib,

$$u(x,t)=\int\limits_{PABQ}\left[u(\xi,\tau)G(x,t,\xi,\tau)d\xi+a^2\left(G\frac{\partial u}{\partial\xi}-u\frac{\partial G}{\partial\xi}\right)d\tau\right],\qquad (\mathrm{II}.2.11)$$

ni hosil qilamiz, bu yerda

$$G(x,t,\xi,\tau)=G_0(x,t,\xi,\tau)-v.\qquad (\mathrm{II}.2.12)$$

Agar ν funksiyani PA va BQ da G=0 beradigan qilib tanlasak, u holda $u(x,t)$uchun

$$u(x,t)=\int\limits_{AB}u(\xi,\tau)G(x,t,\xi,\tau)d\xi+a^2\int\limits_{AP}u\frac{\partial G}{\partial\xi}d\tau-a^2\int\limits_{BQ}u\frac{\partial G}{\partial\xi}d\tau.\qquad (\mathrm{II}.2.13)$$

ko'rinishdagi integral tasvirini hosil qilamiz. (II.2.13) formula (II.2.1)-(II.2.2) chegaraviy masala yechimini beradi, uning shartlarida u funksiya AP va BQ da hamda AB to'g'ri chiziqda beriladi.

Gfunksiyani batafsil o'rganamiz. U (II.2.12) tasvir yordamida aniqlanadi, bu yerda ν(ξ,τ) funksiya quyidagi shartlar yordamida tavsiflanadi:

$\mathrm{II}.\mathbf{2}.\mathbf{1^0}$**.**ν($\xi$,$\tau$) funksiya PABQ sohada aniqlangan va $\tau<t$ uchun $\mathcal{M}$(ν)=0 qo'shma tenglamani qanoatlantiradi.

$\mathrm{II}.\mathbf{2}.\mathbf{2^0}$.PQ da, ya'ni $\tau=t$ da ν=0.

$\mathrm{II}.\mathbf{2}.\mathbf{3^0}$.PA va QB da $v(\xi,\tau)=-G_0(x,t,\xi,\tau)$.

Bu shartlarga ko'ra ν funksiya x,t parametrlarga bog'liq bo'lib, ν=ν(x,t,ξ,τ) va uni yechish uchun $\mathcal{L}$(u)=0 tenglama uchun (II.2.2) ko'rinishdagi chegaraviy masala yechimga ekvivalent bo'lgan $\mathcal{M}$(ν)=0 tenglama uchun chegaraviy masalani yechish kerak.Bunga ishonch hosil qilish uchun τishorasini o'zgartirish kerak. Shunday qilib, (II.2.2) chegaraviy masala yechimini beruvchi (II.2.11) formula yordamida $u(x,t)$ funksiyani tasvirlashda asosiy qiyinchilik $\nu(x,t,\xi,\tau)$ ni topishdan iborat.

Quyidagi shartlar bilan aniqlanuvchi $\bar{v}(x,t,\xi,\tau)$ funksiyani qaraymiz:

II. $\mathbf{2.4^0}$. $\bar{v}(x,t,\xi,\tau)$ funksiya $t>\tau$ uchun PABQ sohada aniqlangan bo'lib, x,t ning funksiyasi sifatida $\mathcal{L}(\nu)=0$ issiqlik o'tkazuvchanlik tenglamasini qanoatlantiradi.

II. $\mathbf{2.5^0}$. AB da $t=\tau$ da $\bar{v}=0$.

II. $\mathbf{2.6^0}$. AP va BQ da $\bar{v}=-G_0$.

$\nu(x,t,\xi,\tau)=\bar{v}(x,t,\xi,\tau)$ bo'lishini isbotlaymiz.

$\bar{G}(x,t,\xi,\tau)=G_0+\bar{v}$ funksiyani qaraymiz. $\mathcal{M}(u)=0$ tenglamaning istalgan $\bar{u}$ yechimi uchun (II.2.9) ga o'xshash

$$\bar{u}(\xi,\tau)=\int_{BQPA}\bar{u}G_0dx+a^2\left(\bar{u}\frac{\partial G_0}{\partial x}-G_0\frac{\partial\bar{u}}{\partial x}\right)dt,\qquad (\text{II}.2.9'')$$

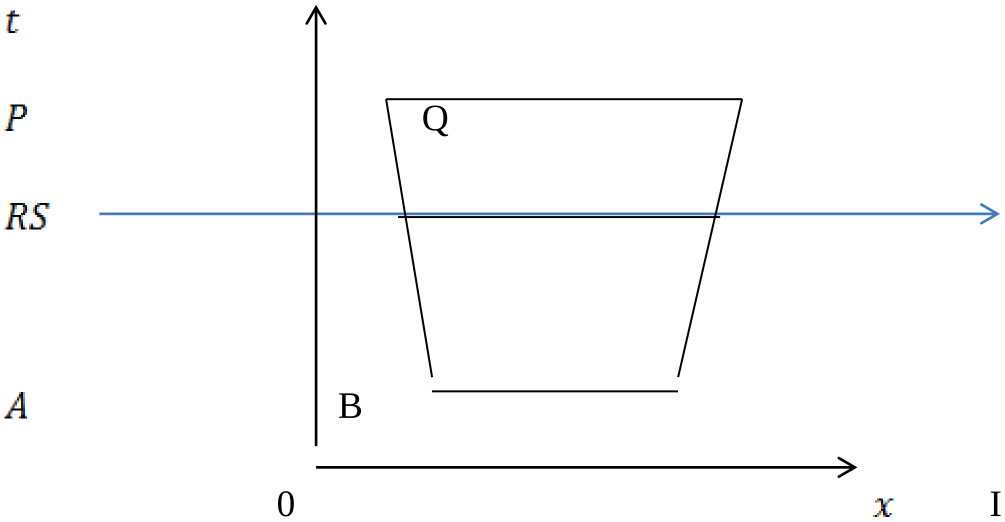
formula o'rinli, bundan tashqari (II.2.13) ga o'xshash

$$\bar{u}(\xi,\tau)=\int_{PQ}\bar{u}\bar{G}dx+a^2\int_{BQ}\bar{u}\frac{\partial\bar{G}}{\partial x}dt-a^2\int_{AP}\bar{u}\frac{\partial\overline{G_0}}{\partial x}dt.\qquad (\text{II}.2.13')$$

formula o'rinli. Bu formulani (II.2.9) va (II.2.13) formulalardan τ ning ishorasini almashtirishdan hosil qilinishi mumkin. Bunda $\mathcal{M}=0$ tenglama $\mathcal{L}=0$ tenglamaga o'tadi.

PQRS sohaga va bu sohadagi $\mathcal{L}(u)=0$ tenglamaning $u(x,t)=\bar{G}(x,t,\xi,\tau)$ uzluksiz yechimiga (II.2.13) formulani qo'llab,

$\bar{G}(x,t,\xi,\tau) \;=\int_{RS} \bar{G}(x',\theta,\xi,\tau)G(x,t,x',\theta)\, \mathrm{d}x',$



I I.2:2

ni hosil qilamiz, bu yerda II.2.6^0 xossaga ko'ra RP va SQ bo'yicha integrallar nolga teng, bu yerda RS-θ ordinataga mos t ning kesmasi va $t > \theta > \tau$.

ARSB sohaga va bu sohada $\mathcal{M}(u)$=0 tenglamaning $\bar{u}(\xi,\tau) = G(x,t,\xi,\tau)$ uzluksiz yechimga $(13')$ formulani qo'llab

$G(x,t,\xi,\tau)=\int_{RS} \bar{G}(x',\theta,\xi,\tau)G(x,t,x',\theta)\, \mathrm{d}x',$

ni hosil qilamiz. Chunki BS va AR bo'yicha integral nolga teng. Bu formulalarni taqqoslab

$G(x,t,\xi,\tau)= \bar{G}(x,t,\xi,\tau)$

ga ega bo'lamiz. Bu tenglikka ko'ra$G$ funksiya x,t ning funksiyasi sifatida

t=τ va x=ξ da maxsuslikka ega bo'lib, manba formulasi uchun t=τ va $x \neq \xi$ da nolga teng bo'lib, G funksiya APBQ ichida $\mathcal{L}_{x,t}(G) = 0$ tenglamani qanoatlantiradi, va AP va BQ da nolga aylanadi. Bunday funksiyaAPQB sohadagi issiqlik o'tkazuvchanlik tenglamasi uchun nuqtali manbaning ta'sir funksiyasi deyiladi.

Shunday qilib, issiqlik o'tkazuvchanlik tenglamasining ixtiyoriy yechimi manba formulasi yordamida (II.2.13) formula kabi tasvirlanadi.

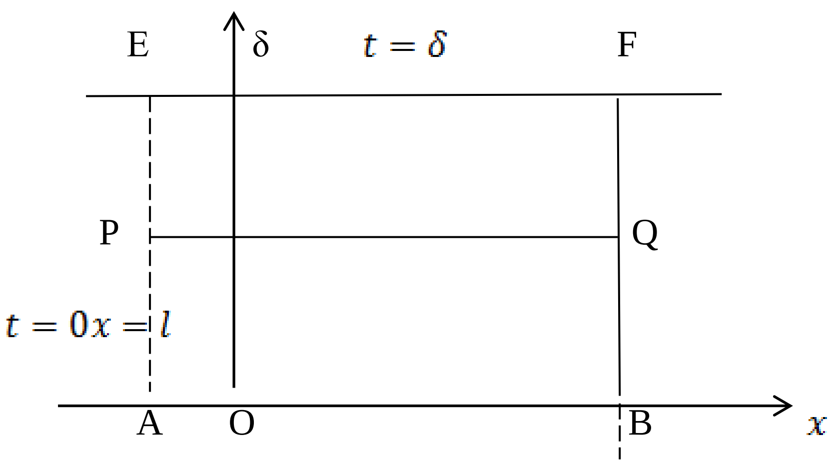
Agar $\mathcal{L}(u)=f(x,t)$ bir jinsli bo'lmagan tenglama berilgan bo'lsa, u holda (II.2.13) formulaning o'ng tomoniga

$$\iint_S G(x,t,\xi,\tau)\mathrm{f}(\xi,\tau)\mathrm{d}\xi d\tau$$

qo'shiluvchini qo'shish kerak.

II.2.1. Chegaraviy masala yechimi. Yuqorida hosil qilingan (II.2.13)-formula oxiri qo'zg'aluvchi chegaralangan kesma uchun chegaraviy masala yechimini beradi. Agar AB kesma oxirlari qo'zg'almas bo'lsa, u holda AE va BF yoylar t o'qga parallel to'g'ri chiziqlar kesmalariga almashtiriladi.

Bu holda S soha koordinata o'qlariga parallel tomonlarga ega to'g'ri to'rtburchak shaklida bo'ladi. (II.2.11) umumiy formuladan limitga o'tish orqali Puasson formulasini hosil qilish mumkin, u cheksiz to'g'ri chiziqqa berilgan boshlang'ich shartli issiqlik o'tkazuvchanlik tenglamasi yechimi bo'ladi.



Faraz qilamiz, A va B nuqtalardan o'tuvchi t=0 va t=δ xarakteristikalar bilan chegaralangan yo'lakda u va u_x funksiyalar

$$|u(x,t)|e^{-Kx^2} < N$$

va

$$\left|\frac{\partial u}{\partial x}\right| e^{-Kx^2} < N, \qquad \textbf{(A)}$$

tengsizlikni qanoatlantirsin, bu yerda $K > 0$ va $N > 0$ lar biror sonlar. BQ yoyni $x = l$ to'g'ri chiziq kesmasi bilan almashtiramiz, bunda l-biror son bo'lib, kelgusida yetarlicha kattalashtiramiz. Bunda biz (II.2.9) formulani

$$u(x,t) = \int\limits_{PABQ} \frac{e^{-\frac{(x-\xi)^2}{4a^2(t-\tau)}}}{2\sqrt{\pi a^2(t-\tau)}}\left[u(\xi,\tau)d\xi + a^2\frac{\partial u}{\partial \xi}d\tau - u(\xi,\tau)\frac{x-\xi}{2(t-\tau)}d\tau\right].$$

kabi yozib olib foydalanamiz.

BQ kesma bo'yicha integralni qaraymiz:

$$\int\limits_{BQ} \frac{e^{-\frac{(x-l)^2}{4a^2(t-\tau)}}}{2\sqrt{\pi a^2(t-\tau)}}\left[a^2\left(\frac{\partial u}{\partial \xi}\right)\xi = l - u(l,\tau)\frac{x-l}{2(t-\tau)}\right]d\tau$$
$$= \int\limits_0^t a^2\left(\frac{\partial u}{\partial \xi}\right)\xi = l\,\frac{e^{-\frac{(x-l)^2}{4a^2(t-\tau)}}}{2\sqrt{\pi a^2(t-\tau)}}d\tau - \int\limits_0^t u(l,\tau)\frac{e^{-\frac{(x-l)^2}{4a^2(t-\tau)}}}{4\sqrt{\pi a^2(t-\tau)}}\frac{x-l}{t-\tau}d\tau$$
$$= I_1 + I_2$$

va$l \to \infty$ da nolga intilishini isbotlaymiz.

$l$ning yetarlichakatta qiymatlarida I_1 integralni baholaymiz:

$|I_1| \leq \frac{Na^2}{2\sqrt{\pi a^2}}e^{Kl^2 - \frac{l^2}{16a^2\delta}}\int_0^t \frac{dt}{\sqrt{t-\tau}}$ (agar$x < \frac{l}{2}$ va $(t-\tau) < \delta$ bo'lsa).

$l \to \infty$da$|I_1| \to 0$ bo'ladi, chunki K fikserlangan son, δ esa yetarlicha kichik qilib tanlanishi mumkin. Masalan,

$K < \frac{1}{16a^2\delta}$. kabi.

$l \to \infty$ bo'lganda $|I_2| \to 0$bo'lishi shunga o'xshash isbotlanadi.

$u(x,t)$ funksiya va uning $\frac{\partial u}{\partial x}$hosilasi uchun A tengsizlik manfiy x larda ham bajarilsa, AE egri chiziq sifatida $x = -l$ to'g'ri chiziq kesmasini olish mumkin va bayon qilingan mulohazalarni takrorlab limitga o'tib (9) formuladagi PA bo'yicha integral 0 ga intilishini ko'rsatish mumkin. Natijada

$$u(x,t) = \frac{1}{2\sqrt{\pi a^2}} \int_{-\infty}^{+\infty} \frac{e^{\frac{-(x-\xi)^2}{4a^2t}}}{\sqrt{(t)}} u(\xi, 0) d\xi.$$

Puasson formulasini hosil qilamiz.

yarim chegaralangan sohani qarab va $G(x,t,\xi,\tau)$ manba funksiya uchun (A) tengsizlik bajarilsin deb faraz qilib,

$$u(x,t) = -\int_{PA} a^2\mu(\tau) \frac{\partial G}{\partial \xi}\Big|_{\xi = x_A} d\tau + \int_{x_A}^{\infty} \varphi(\xi) G(x,t,\xi,0) d\xi, \qquad \text{(II.2.14)}$$

ni hosil qilamiz, bu yerda

$\mu(t) = u(x_A, t)$va $\varphi(x) = u(x, 0)$.

$x \geq 0$ yarim cheksiz to'g'ri chiziqlar uchun manba funksiyasi akslantirish usuli bilan hosil qilinadi va

$$G(x,t,\xi,\tau) = \frac{1}{2\sqrt{\pi a^2(t-\tau)}} \left[e^{-\frac{(x-\xi)^2}{4a^2(t-\tau)}} - e^{-\frac{(x+\xi)^2}{4a^2(t-\tau)}} \right],$$

ga teng, chunki u (II.2.12) ko'rinishda tasvirlanadi, x, t o'zgaruvchi bo'yicha issiqlik o'tkazuvchanlik tenglamasini qanoatlantiradi va x=0 da nolga aylanadi:

$$G(0,t,\xi,\tau) = 0.$$

Ushbu

$$\frac{\partial G}{\partial \xi}\bigg|_{\xi = 0} = \frac{x}{2\sqrt{\pi}} \frac{1}{[a^2(t-\tau)]^{\frac{3}{2}}} e^{-\frac{(x)^2}{4a^2(t-\tau)}}$$

hosilani hisoblaymiz va uning qiymatini (II.2.14) ga qo'yib

$$u(x,t) = \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{1}{\sqrt{a^2 t}} \left[e^{-\frac{(x-\xi)^2}{4a^2 t}} - e^{-\frac{(x+\xi)^2}{4a^2 t}} \right] \varphi(\xi) d\xi$$
$$+ \frac{1}{2\sqrt{\pi}} \int_0^t \frac{a^2 x}{[a^2(t-\tau)]^{\frac{3}{2}}} e^{-\frac{(x)^2}{4a^2(t-\tau)}} \mu(\tau) d\tau, \qquad (\text{II}.2.15)$$

formulani hosil qilamiz, u

$$u_t = a^2 u_{xx} \quad (0 < x < \infty, \quad t > 0)$$

tenglamani va

$$u(x,0) = \varphi(x),$$

$$u(0,t) = \mu(t) \qquad (0 < x < \infty)$$

II.2.2. Kesma uchun manba funksiyasi.

Chegaralangan $0 < x < l$ kesmadagi issiqlik o'tkazuvchanlik tenglamasi yechimi (II.2.11) formula bilan beriladi, u PA va BQ yoylarni to'g'ri chiziq kesmalarga almashtirib hamda koordinata boshini A nuqtaga ko'chirgach quyidagi ko'rinishga ega bo'ladi:

$$G(x,t,\xi,\tau)=a^2\int_0^t \frac{\partial G}{\partial \xi}\Big|_{\xi=0}\mu_1(\tau)d\tau - a^2\int_0^t \frac{\partial G}{\partial \xi}\Big|_{\xi=l}\mu_2(\tau)d\tau+\int_0^t G(x,t,\xi,0)\varphi(\xi)d\xi$$

Bu yerda

$$\mu_1(t)=u(0,t),\qquad \mu_2(t)=u(l,t),\qquad \varphi(x)=u(x,0).$$

Kesma uchun $G(x,t,\xi,\tau)$ manba funksiyasi akslantirish usuli bilan qurilishi mumkin. Musbat manbalarni$2nl+\xi$ va manfiy manbalarni $2nl-\xi$ nuqtalarga joylashtirib, manba funksiyasini

$$u(x,t)=\sum_{n=-\infty}^{\infty}[G_0(x,t,2nl+\xi,\tau)-G_0(x,t,2nl-\xi,\tau)], \qquad (\text{II}.2.16)$$

qator yordamida tasvirlaymiz, bu yerda

$$G_0(x,t,\xi,\tau)=\frac{1}{2\sqrt{\pi a^2(t-\tau)}}e^{-\frac{(x-\xi)^2}{4a^2(t-\tau)}}$$

chegaralanmagan to'g'ri chiziq uchun manba funksiyasi.

Qatorning yaqinlashishi,$G|_{x=0}$=0 va $G|_{x=l}$=0 chegaraviy shartlarning bajarilishi qiyinchiliksiz tekshiriladi.

Manba funksiyasining boshqa tasviri ham o'rinli:

$$G(x,t,\xi,\tau)=\frac{2}{l}\sum_{n=1}^{\infty}e^{-\frac{n^2\pi^2}{l^2}a^2(t-\tau)}\sin\frac{\pi n}{l}x\sin\frac{\pi n}{l}\xi. \qquad (\text{II}.2.17)$$

ikkala tasvirning ekvivalentligini isbotlaymiz.

(II.2.17) formulani $G(x,t,\xi,\tau)$ funksiyaning (0,l) oraliqda sinuslar bo'yicha Furye qatoriga yoyilmasi sifatida qarash mumkin:

$$G(x,t,\xi,\tau)=\sum_{n=1}^{\infty} G_n(x,t,\tau)\sin\frac{\pi n}{l}\xi. \qquad (\text{II}.2.18)$$

Gfunksiyaning (II.2.16) qator orqali aniqlanuvchi G_n Furye koeffisientlarini hisoblaymiz:

$$G_n=\frac{2}{l}\int_0^l G(x,t,\xi,\tau)\sin\frac{\pi n}{l}\xi d\xi$$
$$=\frac{2}{l}\sum_{n=-\infty}^{\infty}\left\{\int_0^l G_0(x,t,2nl+\xi_1,\tau)\sin\frac{\pi n}{l}\xi_1 d\xi_1-\int_0^l G_0(x,t,2nl\right.$$
$$\left.-\xi_2,\tau)\sin\frac{\pi n}{l}\xi_2 d\xi_2\right\}.$$

integrallashning yangi

$\xi'=2nl+\xi_1$va$\xi''=2nl-\xi_2$,

almashtirishni kiritib,

$$G_n=\frac{2}{l}\sum_{n=-\infty}^{\infty}\left\{\int_{2nl}^{(2n+1)l} G_0(x,t,\xi',\tau)\sin\frac{\pi n}{l}\xi' d\xi'\right.$$
$$\left.+\int_{(2n-1)l}^{2nl} G_0(x,t,\xi'',\tau)\sin\frac{\pi n}{l}\xi'' d\xi''\right\},$$

ni hosil qilamiz. Bundan

$$G_n=\frac{2}{l}\int_{-\infty}^{+\infty} G_0(x,t,\xi,\tau)\sin\frac{\pi n}{l}\xi d\xi=\frac{2}{l}\int_{-\infty}^{+\infty}\frac{1}{2\sqrt{\pi a^2(t-\tau)}}e^{-\frac{(x-\xi)^2}{4a^2(t-\tau)}}\sin\frac{\pi n}{l}\xi d\xi.$$

kelib chiqadi.

$\lambda=\frac{\xi-x}{2\sqrt{a^2(t-\tau)}}$

belgilashni kiritamiz. U holda

$$d\lambda = \frac{d\xi}{2\sqrt{a^2(t-\tau)}};$$

$$\sin\frac{\pi n}{l}\xi = \sin\frac{\pi n}{l}\left(x + 2\sqrt{a^2(t-\tau)}\lambda\right) = \sin\frac{\pi n}{l}x\cos\frac{2\pi n}{l}\sqrt{a^2(t-\tau)}\lambda +$$

$$\cos\frac{\pi n}{l}x\sin\frac{2\pi n}{l}\sqrt{a^2(t-\tau)}\lambda;$$

$$G_n = \frac{2}{l}\sin\frac{\pi n}{l}\frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} e^{-\lambda^2}\cos\frac{2\pi n}{l}\sqrt{a^2(t-\tau)}\lambda$$

$$d\lambda + +\frac{2}{l}\cos\frac{\pi n}{l}x\frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} e^{-\lambda^2}\sin\frac{2\pi n}{l}\sqrt{a^2(t-\tau)}\lambda d\lambda.$$

ikkinchi integral nolga teng, chunki integral belgisi ostida koordinata boshiga nisbatan toq funksiya turibdi. Birinchi integral

$$I(\alpha,\beta) = \int_{-\infty}^{+\infty} e^{-\alpha\lambda^2}\cos\beta\lambda d\lambda,$$

integralning xususiy holi bo'lib,

$$I(\alpha,\beta) = \sqrt{\frac{\pi}{\alpha}}e^{-\frac{\beta^2}{4\alpha^2}}.$$

ga tengdir. Bizning holda $\alpha = 1, \beta = \frac{2\pi n}{l}\sqrt{a^2(t-\tau)}$,

shu sababli $I = \sqrt{\pi}e^{-\frac{\pi^2 n^2}{l^2}a^2(t-\tau)}$

$$G_n = \frac{2}{l}e^{-\frac{\pi^2 n^2}{l^2}a^2(t-\tau)}\sin\frac{\pi n}{l}x.$$

G_n Furye koeffisientlari uchun topilgan ifodani (II.2.18) formulaga qo'yib, G manba funksiya uchun (II.2.17) ikkiinchi tasvirni hosil qilamiz. Shunday qilib, 2ta turli (II.2.17) va (II.2.18) tasvirlarning ekvivalentligi isbotlandi.

II.2.3. Chegaraviy masalalarni yechish. Issiqlik potensiallari chegaraviy masalalarni yechish uchun qulay apparat hisoblanadi.

Dastlab, 1-chegaraviy masalani $x > \chi_1(t)$ yarim chegaralangan soha uchun qaraymiz:

$u_t = a^2 u_{xx}$,tenglamaning$x > \chi_1(t),\ t \geq t_0$ sohadagi quyidagi

$u(x, t_0) = \varphi(x), \quad x \geq \chi_1(t_0);$

$u(\chi_1(t); t) = \mu(t), \qquad\qquad t \geq t_0$

shartlarni qanoatlantiruvchi yechimi topilsin.

Umumiylikdan chekinmagan holda $\varphi(x)$=0 deb olish mumkin, yoki nol bo'lgan holga keltirish mumkin.

Faraz qilamiz, boshlang'ich shart nol bo'lgan holga keltirilgan. Yechimni quyidagi ko'rinishda ifodalaymiz:

$$u(x,t) = \frac{1}{2a^2} W(x,t) = \int_0^t \frac{\partial G_0}{\partial \xi}(x, t, \chi_1(\tau), \tau)\bar{\mu}(\tau)d\tau =$$

$$\frac{1}{4\sqrt{\pi}} \int_{t_0}^t \frac{x - \chi_1(\tau)}{[a^2(t-\tau)]^{\frac{3}{2}}} e^{-\frac{[x-\chi_1(\tau)]^2}{4a^2(t-\tau)}} \bar{\mu}(\tau)d\tau;$$

Ushbu funksiya tenglamani $x > \chi_1(t)$ da qanoatlantiradi, cheksizlikda chegaralangan va $\bar{\mu}(t)$ funksiya tanlanishiga bog'liq bo'lmagan holda nol boshlang'ich shartga ega. U $x = \chi_1(t)$da uzilishga ega va uning

$x = \chi_1(t)$+0 dagi qiymati$\mu(t)$ ga teng bo'lishi kerak:

$$\frac{\bar{\mu}(t)}{2a^2} + \frac{1}{4\sqrt{\pi}} \int_{t_0}^{t} \frac{\chi_1(t) - \chi_1(\tau)}{[a^2(t-\tau)]^{\frac{3}{2}}} e^{-\frac{[\chi_1(t)-\chi_1(\tau)]^2}{4a^2(t-\tau)}} \bar{\mu}(\tau)d\tau = \mu(t).$$

Bu munosabat $u(x,t)$ izlanayotgan yechimni aniqlovchi $\bar{u}(t)$ funksiyani topishi uchun 2-tur Volterra integral tenglamasining yechimi bo'ladi.

Agar $x = \chi_1(t)$funksiya differensiallanuvchi funksiya orqali aniqlansa, umumiy nazariyaga ko'ra yechimning mavjudligi hamisha ma'noga ega.

Agar soha chegarasi o'zgarmas bo'lsa $\chi_1(t)$=x_0 bu tenglama sodda bo'ladi.

Bu holda integral nolga aylanadi va

$\bar{\mu}(t)$=$2a^2\mu(t)$,

bo'ladi, chunki izlanayotgan yechim

$$u(x,t) = \frac{1}{2a\sqrt{\pi}}\int_0^t \frac{x-x_0}{(t-\tau)^{\frac{3}{2}}} e^{-\frac{(x-x_0)^2}{4a^2(t-\tau)}}\mu(\tau)d\tau.$$

ko'rinishga ega .

Ikkinchi va uchinchi chegaraviy masalalar potensial yordamida shunga o'xshash yechiladi.

$u(x,0) = \varphi(x)\chi_1(0) < x < \chi_2(0),$

$u[\chi_1(t);t] = \mu_1(t),\qquad\qquad u[\chi_2(t);t] = \mu_2(t) \quad (t > 0)$

ko'rinishdagi qo'shimcha shartni olib, chegaralangan sohada chegaraviy masalani qaraymiz. Boshlang'ich qiymat nolga keltirilgan deb faraz qilib,

$\varphi(x) = 0$, yechimni

$$u(x,t) = \frac{1}{2a^2}(W_1+W_2)=\int_0^t \frac{\partial G_0}{\partial \xi}(x,t,\chi_1(\tau),\tau)\widetilde{\mu_1}(\tau)d\tau + \int_0^t \frac{\partial G_0}{\partial \xi}(x,t,\chi_2(\tau),\tau)\widetilde{\mu_2}(\tau)d\tau.$$

ko'rinishida tasvirlaymiz. Bu funksiya tenglamani va $\widetilde{\mu_1}(t)$ va $\widetilde{\mu_2}(t)$ funksiyalarni istalgan tanlashda nol boshlang'ich shartni qanoatlantiradi.

$x = \chi_1(t)$ +0 va $x = \chi_2(t)$ -0 dagi limitik qiymati $\mu_1(t)$ va $\mu_2(t)$ ga teng bo'lishi kerak va

$$\frac{\mu_1(t)}{2a^2}+\frac{1}{4\sqrt{\pi}}\int_0^t\frac{\chi_1(t)-\chi_1(\tau)}{[a^2(t-\tau)]^{\frac{3}{2}}}e^{-\frac{[\chi_1(t)-\chi_1(\tau)]^2}{4a^2(t-\tau)}}\widetilde{\mu_1}(\tau)d\tau$$
$$+\frac{1}{4\sqrt{\pi}}\int_0^t\frac{\chi_1(t)-\chi_2(\tau)}{[a^2(t-\tau)]^{\frac{3}{2}}}e^{-\frac{[\chi_1(t)-\chi_2(\tau)]^2}{4a^2(t-\tau)}}\widetilde{\mu_2}(\tau)d\tau=\mu_1(t);$$

$$\frac{-\widetilde{\mu_2}(t)}{2a^2}+\frac{1}{4\sqrt{\pi}}\int_0^t\frac{\chi_2(t)-\chi_1(\tau)}{[a^2(t-\tau)]^{\frac{3}{2}}}e^{-\frac{[\chi_2(t)-\chi_1(\tau)]^2}{4a^2(t-\tau)}}\widetilde{\mu_1}(\tau)d\tau$$
$$+\frac{1}{4\sqrt{\pi}}\int_0^t\frac{\chi_2(t)-\chi_2(\tau)}{[a^2(t-\tau)]^{\frac{3}{2}}}e^{-\frac{[\chi_2(t)-\chi_2(\tau)]^2}{4a^2(t-\tau)}}\widetilde{\mu_2}(\tau)d\tau=\mu_2(t);$$

tenglamani beradi. Bu sistema hamisha yechimga ega bo’ladigan Volterra turidagi integral tenglamalar sistemasi bo’ladi.

Xulosa.

Bitiruv malakaviy ishning birinchi bobi potensiallar nazariyasi deb nomlanib, unda potensiallarning tushunchasi va ularning fizik ma'nosi, Lyapunov sirtlari, hajm potensiali, oddiy va ikkilangan qatlam potensiali ta'riflari ularga doir misollar bilan birgalikda va xossalari isboti bilan keltirib o'tilgan.Bob ikki qismdan iborat.

Birinchi qismda oddiy qatlam potensiali va uning xossalari haqida ma'lumot berilgan. Bunda oddiy qatlam potensialining ta'rifi, uning normal hosilasi, Lyapunov sirtlari shartlari, ularga doir teoremalarning isboti ham aniq yoritilgan.

Ikkinchi qismda esa ikkilangan qatlam potensiali va uning xususiy holi bo'lgan Gauss integrali haqida batafsil yoritib berilgan.Bunda ularning ta'riflari va ularga doir teoremalarning isboti ko'rsatib o'tilgan.

Ikkinchi bobnibiz "Issiqlik o'tkazuvchanlik tenglamasi uchun chegaraviy masalalarni yechishga tadbiqi" deb nomladik. Bu bob ham ikki qismdan iborat bo'lib, bu qismlar bitiruv malakaviy ishning mohiyatini ochishga xizmat qiladi.Birinchi qismda "Issiqlik potensiallari"ning PABQ sohada oddiy va ikkilangan qatlamning issiqlik potensiallarikeltirib chiqarilgan.

Bobning ikkinchi qismida bitiruv malakaviy ishning asosiy qismi yoritib berilgan ya'ni issiqlik o'tkazuvchanlik tenglamasi uchun chegaraviy masalalarni integral tenglamalarni yechishga keltirish bayon etilgan.

Bunda biz ikkinchi qismda 1-2-3-chegaraviy masalalarning qo'yilishini o'rganish va ularning Volterra turidagi integral tenglamalar sistemasiga kelishi bayon qilingan.

XOTIMA

Mazkur bitiruv malakaviy ishim “Potensiallar va ularning issiqlik tarqalish tenglamasi uchun chegaraviy masalalarni yechishga tadbiqi” deb nomlanib kirish qismi, 2 ta bob, 4 ta paragraf, xulosa, xotima va adabiyotlar ro’yxatidan iborat.

Birinchi bob bo’lib, ikki paragrafdan iborat.Bu bobda potensial tushunchasi va uning fizik ma’nosi, hajm potensiali , Lyapunov sirtlari va biz uchun asosiysi bo’lgan oddiy va ikkilangan qatlam potensiali haqida umumiy ma’lumotlar keltirilgan. Hajm potensiali, oddiy va ikkilangan qatlam potensialiga doir ta’rif va teoremalar batafsil yoritib berilgan.

Ikkinchi bob malakaviy bitiruv ishining asosiy qismi bo’lib, ,,Issiqlik o’tkazuvchanlik tenglamasi uchun chegaraviy masalalarni yechishga tadbiqi’’deb nomlangan.Bu bobda quyidagi ishlar amalga oshirilgan.

Chegaralangan 0$< x < l$ kesmadagi issiqlik o’tkazuvchanlik tenglamasining yechimi

$$u(x,t) = \int_{PABQ} \left[u(\xi,\tau)G(x,t,\xi,\tau)d\xi + a^2\left(G\frac{\partial u}{\partial \xi} - u\frac{\partial u}{\partial \xi}\right)\right]$$

Ko’rinishida bo’lishini o’rgandik va

1-chegaraviy masalani$x > \chi_1(t)$ yarim chegaralangan soha uchun qaraymiz:

$u_t = a^2 u_{xx}$,tenglamaning$x > \chi_1(t)$, $t \geq t_0$ sohadagi quyidagi

$u(x, t_0) = \varphi(x), \quad x \geq \chi_1(t_0);$

$u(\chi_1(t); t) = \mu(t), \qquad\qquad t \geq t_0$

shartlarni qanoatlantiruvchi yechimini o'rgandik va yechim

$$u(x,t) = \frac{1}{2a^2} W(x,t) = \int_0^t \frac{\partial G_0}{\partial \xi}(x, t, \chi_1(\tau), \tau)\bar{\mu}(\tau) d\tau =$$

$$\frac{1}{4\sqrt{\pi}} \int_{t_0}^t \frac{x - \chi_1(\tau)}{[a^2(t-\tau)]^{\frac{3}{2}}} e^{-\frac{[x-\chi_1(\tau)]^2}{4a^2(t-\tau)}} \bar{\mu}(\tau) d\tau;$$

Bo'ladi. Bu yechim 2-tur Volterra integral tenglamasining yechimi bo'ladi.

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