O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

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Fizika-matematika fakulteti

“Matematika” *kafedrasi*

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« Yuqori tartibli determinantlar »

mavzusida

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BITIRUV MALAKAVIY ISHI

“ Ish ko’rildi va himoyaga ruxsat berildi ”

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“______” __________________2015 y

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Taqrizchi :

“_______” __________________2015y

«Himoya qilishga ruxsat berildi »

Fakultet deka prof. Sh.M. Mirzayev

«________»_______________2015 y

Buxoro – 2015

Mundarija.

"O'z oldiga qo'yilgan yuksak vazifa
va marralarga erishishda zamonaviy
bilim va tarbiya sohibi bo'lgan,
yangicha fikrlayotgan yoshlarimiz
bizning asosiy tayanchimiz va
suyanchimizdir".

I.A.Karimov.

Kirish.

Il'gor millat va rivijlangan davlat bo'lishning zarur shartlaridan biri aqliy va jismoniy, ma'daniy va ma'naviy, axloqiy, g'oyaviy – siyosiy va huquqiy juhatdan, har tomonlama yetuk, barkamol insonlarga ega bo'lishdir.

Ma'naviy – ma'rifiy jihatdan inson, inson irodasi mustahkam, e'tiqodi yuksak, vijdon amri bilan yashaydigan shaxs, barkamol avlod har qanday davlat, xalq va millatning eng katta boyligi, qudrati salohiyati manbaidir.

Mamlakatimiz prezidenti tomonidan ta'kidlab kelinayotganidek, "Har qaysi davlat, har qaysi millat nafaqat yerosti va yerusti boyliklari bilan, harbiy qudrati va ishlab chiqarish salohiyati bilan, balki birinchi navbatda o'zining yuksak madaniyati va ma'naviyati bilan kuchlidir".

Odamlari, fuqarolari bilimli – zakovatli, uddaburon, g'oyaviy siyosiy jiohatdan ziyrak va hushyor, tadbirkor, har tomonlama yetuk bo'lgan jamiyat har qanday islohotlarga qodir bo'ladi va har qanday muammo va qiyinchiliklarni yenga oladi. Oily rahbarimiz bu haqida shunday dedi: "Lo'nda qilib aytganda, bugungi kunda oldimizda qo'ygan buyuk maqsadlarimizga, ezgu niyatlartimizga erishishimizda jamiyatimizning yangilanishi, hayotimizning taraqqiyoti va istiqboli amalga oshirayotgan islohotlarimiz, rejalarimizning samarasi taqdiri bularning barchasi avvalambor zamon talablariga javob beradigan yuqori malakali, ongli mutaxassis kadrlar tayyorlash muammosi bilan chambarchas bog'liqligini barchamiz anglab yetmoqdamiz.

Shu bilan birga barchamiz yana bir haqiqatni anglab yetmoqdamiz. Faqatgina chinakam ma'rifatli odam inson qadrini, millat qadriyatlarini, bir so'z bilan aytganda, o'zligini aniqlash, erkin va ozod jamiyatda yashash, mustaqil davlatimizning jahon hamjamiyatida o'ziga munosib, fidoiylik bilan kurashishi mumkin".

Mustaqil va erkin fikrlayotgan, ongli yashaydigan, o'z haq – huquqlarini yaxshi taniydigan, o'z kuchi va aqliga ishonadigan, ma'naviy – axloqiy yetuk barkamol bo'lgan avlodni, mustaqil fikrlashga qodir, jasoratli, fidoiy va tashabbuskor kishilarni tarbiyalab yetkazadigan xalq va millat kelajakka ochiq ko'z, katta ishonch, umid va ixlos bilan qray oladi. Fuqarolarni ana shunday noyob xislat va fazilat sohiblari qilib shakllantirilgan davlatning istiqboli porloq bo'ladi.

Mamlakatimiz kuch qudrati aholining soni bilan emas, balki odamlarning intelektual salohiyati, zamonaviy bilim asoslarini texnika va texnologiyalarni, ko'plab xorijiy tillarni egallaganligi, g'oyaviy siyosiy yetukligi, huquqiy bilimlilik va madaniylik darajasi, irodasini mustahkamligi, inson e'tiqodining bitligi, o'z shaxsiy manfaatlarini shaxs, Vatan manfaatlari bilan uyg'un holda ko'ra olishi elim deb, yurtim deb yashashi, faoliyat ko'rsatishi bilan kuchlidir. O'zbekiston davlati hozirgi paytdao'z oldiga ana shunday oliy maqsadni qo'yib kelajak sari intilmoqda.

Prezident Islom Karimovning "O'zbekiston XXI – asrga intilmoqda" asosida belgilab berilgandek, "Erkin fuqaro ma'naviyatini, ozod shaxsni shakllantirish oldimizda turgan eng dolzarb vazifadir". Boshqacha qilib aytganda biz o'z haq – huquqlarini tayinlaydigan, o'z kuci va imkoniyatlariga tayanadigan, atrofida ro'y berayotgan voqea va hodisalarga mustaqil munosabat bilan yondashadigan, ayni zamonda shaxsiy manfaatlarini mamlakat va xalq manfaatlari bilan uyg'un holda ko'radigan erkin, har jihatdan barkamol insonlarni tarbiyalashimiz kerak.

Inson kamoloti millat va mamlakat taraqqiyotiga asos bo'ladi. Aql zakovatli yuksak ma'naviyatli kishilarni tarbiyalay olsakgina O'zbekiston davlat mustaqilligini barqaror bo'ladi, yurtimizda farovonlik va taraqqiyot qaror topadi, oldimizda turgan buyuk maqsadlarga erishamiz. "Oldimizda mustaqil davlat qurishdek murakkab va sharafli vazifa turgan bir paytda bu ma'naviy qadriyatlarning ahamiyati ming karra ortadi. Nega deganfa har qanday ulug' maqsadlarga yetishish, yangi jamiyat, faovon turmush qurish, inson zotiga munosib g’ozal hayot barpo etish, avvalo, shu jamiyat a,zolari bo’lgan komil odamlarga, kelajak barkamol avlodiga bog’liqdir”.

Kamolotga erishgan inson deganda mukammal tarbiyali, boshqalarga takabburlik qilmaydigan, hech kimga haqorat ko’zi bilan qaramaydigan, yolg’on so’zlamaydigan, rostgo’y, bag’rikeng, sahovatli, muruvvatli odam ko’z o’ngimizda gavdalanadi.

Kamolotga erishgan inson deganda mukammal tarbiyali, ma’naviyatli, inson va’dasiga vafo qiladi, omonatga xiyonat qilmaydi, birovlarni dog’da qoldirmaydi, so’zda sobit turadi, ig’vo, g’iybat, munofiqlikdan o’zini saqlaydi, yomon yo’llardan yaramas ishlardan, nojo’ya harakatdan o’zini tuyadi. Bunday inson ota – onasining hurmatini bajo keltiradi, ularni o’zidan rozi qilishga say – harakat qiladi, farzandlari, butun oila a’zolariga mehribonlik ko’rsatadi, qarindosh – urug’lridan aloqasini uzmaydi, mahalla ko’y, qishloqdoshlari, jamoasi, xullas kalom tevarak – atrofni o’rab olgan jamiki insonlar farovonligi haqida qayg’ uradi. Ularga mehr – muhabbatli bo’ladi, ulardan mehr – muhabbatini ayamaydi, muruvvatini ta’na qilmaydi, odamlarga yaxshilik qilish, ezgulik ko’rsatishni o’z vazifasi, ma’suliyati deb biladi.

Ma’naviy – axloqiy o’z nasl – nasabi, kechmisj tarixi, boy madaniy – ma’naviy merosi bilan farqlanish, milliy rasm – rusumlarni, oilaviy an’analarni avaylab – asrash va boyitish, ota – bobolarning muborak nomlariga dog’ tushurmaslikka intilish, o’z mehnati, aql – idrok tafakkuri, intelektual salohiyati bilan ajdodlari obro’yini tushurmaslikka harakat qilish, odamlarning hurmat – ehtiromiga sazovor bo’lish insonning kamoloyga yetganligining muhim belgilaridandir,

Ma’naviy barkamollikning ma’no – mazmuni aytib o’tilganlardangina iborat emas. “O’zbekistonga uning yeri, tabiatiga, bu yerda yashayotgan xalqlarga muhabbat, o’lkaning tarixi, madaniyat, an’analarini teran bilish” kamolotning namoyon bo’lishi hisoblanadi.

Bitiruv malakaviy ish mavzuning dolzarbligi:

Yuqori tartibli determinantlar matematikaning bir qancha bo`limlarida mavjud. Jumladan algebra sonlar nazariyasi, evolutsion algebra kabi bo`limlarda muhim ahamiyatga ega.

Bitiruv malakaviy ishining maqsadi:

Yuqori tartibli determinantlarning xossalarini o'rganish va ularni turli xil usullarda yechish.

Bitiruv malakaviy ishning vazifalari:

Yuqori tartibli determinantlarning ba'zi usullarda yechishga tadbiqlarini o'rganish.

Bitiruv malakaviy ishining o'rganilganlik darajasi:

Mazkur bitiruv malakaviy ishida turli xil yuqori tartibli determinantlar qatnashgan, misollar qarab chiqilgan va turli xil ko'rinishdagi misollar yechilgan, turli usullar o'rganilgan.

Bitiruv malakaviy ishining predmeti:

Bitiruv malakaviy ishini tayyorlash jarayonida determinantlarning asosiy xossalari o'rganilib, ularga doir misollar tahlil qilingan.

Bitiruv malakaviy ishining obyekti:

Bitiruv malakaviy ishining obyekti bu determinant tushunchasi, ikkinchi va uchinchi tartibli, yuqori tartibli determinantlar haqida ma'lumotlar. Ulardan foydalanish jarayonida turli xil ta'rif , tushuncha va tasdiqlardan foydalanilgan.

Mazkur bitiruv malakaviy ishida turli xil yuqori tartibli determinantlar qatnashgan, misollar qarab chiqilgan va turli xil ko'rinishdagi misollar yechilgan, turli usullar o'rganilgan.

Bitiruv malakaviy ishining ilmiy farazi:

Ikkinchi va uchinchi tartibli, yuqori tartibli chiziqli tenglamalar sistemalarini determinantlar yordamida yechish.

Bitiruv malakaviy ishining yangiligi:

Ushbu bitiruv malakaviy ishi referativ xarakterga ega bo'lib, mavjud ma'lumotlar yig'ib o'rganilgan.

Bitiruv malakaviy ishining amaliy ahamiyati:

Bitiruv malakaviy ishi mavzusi nazariy va amaliy ahamiyatga ega bo'lib, undan quyi kurs talabalari qo'shimcha manba sifatida foydalanishi mumkin.

Bitiruv malakaviy ishining metadalogik asosi:

Ish nazariy xarakterga ega bo'lib, olingan natijalar matematika, fizika va texnik masalalarni hal qilishga keng qo'llaniladi.

Bitiruv malakaviy ishining metodlari:

Bitiruv malakaviy ishida n-tartibli determinantlarga oid misollar ishlash.

Bitiruv malakaviy ishining tarkibi va hajmi:

Mazkur bitiruv malakaviy ishi kirish, ikki bob, besh paragraf, xulosa, xotima va foydalanilgan adabiyotlar ro'yxatidan iborat. Jami 54 bet

I ASOSIY TUSHUNCHA VA FAKTLAR.

1.1 Ikkinchi va uchinchi tartibli determinantlar va ularning xossalari.

Ikkinchi tartibli determinantlar.

To’rtta sondan iborat ushbu jadvalni qaraymiz va uni matritsa, aniqrog’i ikkinchi tartibli kvadrat matritsa deb ataymiz:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad (1.1.1)$$

$\Delta = a_{11}a_{22} - a_{12}a_{21}$ son (1.1.1) matritsaning determinanti yoki ikkinchi tartibli determinant (yoki matritsaning xarakteristikasi) deb ataladi. (1.1.1) – matritsa determinanti

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \qquad (1.1.2)$$

kabi belgilanadi.

Shunday qilib determinant uchun quyidagiga egamiz

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21} \qquad (1.1.3)$$

Matritsa bilan determinantni almashtirmaslik lozim. Matritsa sonlardan iborat jadval bo’lib, determinant esa shu jadvaldan (1.1.3) da ko’rsatilgan kabi hosil qilingan birgina sondir.

Determinantni tashkil qiladigan sonlar uning elementlari deb ataladi. Ikkinchi tartibli determinant ikkita satr va ikkita ustunga ega. Istalgan elementning belgilanishida birinchi indeks shu element turgan satr tartibini, ikkinchi indeks esa ustun tartibini ko’rsatadi. a_{11} va a_{12} lar birinchi satrni, a_{21} va a_{22} elementlari ikkinchi satrni tashkil etadi. a_{11}, a_{22} elementlar joylashgan diagonal determinantining bosh diagonali, a_{12} va a_{21} elementlar joylashgan diagonal esa yordamchi diagonal deb ataladi. Shunday qilib ikkinchi tartibli determinantni hisoblash uchun bosh diagonalda turgan elementlar ko’paytmasidan, yordamchi diagonalda turgan elementlar ko’paytmasini ayirish kerak.

1.1.1-Misol. $$\begin{vmatrix} 8 & -1 \\ 3 & 5 \end{vmatrix} = 8\cdot 5 - 3\cdot(-1) = 40 + 3 = 43,$$

1.1.2-Misol. $$\begin{vmatrix} \sin\varphi & \cos\varphi \\ -\cos\varphi & \sin\varphi \end{vmatrix} = \sin^2\varphi + \cos^2\varphi = 1.$$

Uchinchi tartibli determinant.

Uchinchi tartibli kvadrat matritsani, ya`ni 9 ta sondan iborat ushbu jadvalni qaraymiz.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad (1.1.4)$$

Bu matritsaning uchinchi tartibli determinanti deb quyidagi $\Delta = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{12}a_{21}a_{33}$ songa aytiladi va quyidagicha belgilanadi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

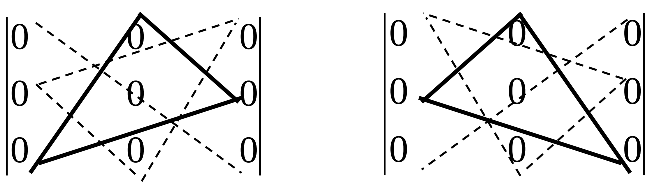
Shunday qilib,

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{33} + a_{21}a_{32}a_{13} - a_{31}a_{22}a_{13} - a_{22}a_{23}a_{11} - a_{12}a_{21}a_{33}.$$

3-chi tartibli determinantni hisoblash usullari:

a) Uchburchak usuli. Uchburchak usulida hisoblash sxemasi:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$



(1.1.5)

(+)ishora bilan (-)ishora bilan

Uchinchi tartibli determinantni aniqlagan (hisoblaydigan) son bosh diagonal elementlari ko'paytmasi va har bir bosh uchburchak uchlaridagi elemenetlari ko'paytmasidan tuzilgan uchta son yig'indisidan yordamchi diagonal elementlari ko'paytmasi va har biri yordamchi uchburchak uchlaridagi elementlari ko'paytmasidan tuzilgan uchta son yig'indisining ayirmasiga teng.

b) Sarrius usuli.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} a_{11}a_{12} \\ a_{21}a_{22} \\ a_{31}a_{32} \end{matrix} = \tag{1.1.6}$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{12}a_{21}a_{33}.$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{12}a_{21}a_{33}.$$

$$\begin{matrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$$

1.1.3-Misol.

a)

$$\begin{vmatrix} 2 & -2 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & 0 \end{vmatrix} \begin{matrix} 2 & -2 \\ 1 & 2 \\ 3 & 1 \end{matrix} = 2\cdot 2\cdot 0 + (-2)\cdot 1\cdot 3 + 3\cdot 1\cdot 1 - (3\cdot 2\cdot 3 + 1\cdot 1\cdot 2 + 0\cdot 1\cdot(-2)) =$$

$$= 0 - 6 + 3 - 18 - 2 - 0 = -23$$

b)

$$\begin{vmatrix} 2 & -2 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & 0 \end{vmatrix} = 2\cdot 2\cdot 0 + 1\cdot 1\cdot 3 + 3\cdot(-2)\cdot 1 - (3\cdot 2\cdot 3 + 2\cdot 1\cdot 1 + 1\cdot(-2)\cdot 0) =$$

$$= 0 + 3 - 6 - 18 - 2 - 0 = -23$$

Determinantning xossalari.

Determinantda mos satr va ustun elementlari o'rnini almashtirishga uni transponirlash deyiladi.

1.1.1-Xossa. Transponirlash natijasida determinantning qiymati o'zgarmaydi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} \qquad (1.1.7)$$

Isbot:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{23}a_{12}) - (a_{13}a_{22}a_{31} + a_{23}a_{32}a_{11} + a_{33}a_{12}a_{21})$$

$$\begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} = (a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{32}a_{21}) - (a_{31}a_{22}a_{13} + a_{32}a_{23}a_{11} + a_{33}a_{21}a_{12})$$

demak 1.1.1-xossa isbotlandi.

1.1.2-Xossa. Determinantda istalgan ikki satr yoki ikki ustunning o'rnini almashtirsak, uning qiymati o'z ishorasini o'zgartiradi, ammo absolyut qiymat o'zgarmaydi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{13} & a_{12} & a_{11} \\ a_{23} & a_{22} & a_{21} \\ a_{33} & a_{32} & a_{31} \end{vmatrix} \qquad (1.1.8)$$

Isbot:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{23}a_{12}) - (a_{13}a_{22}a_{31} + a_{23}a_{32}a_{11} + a_{33}a_{12}a_{21})$$

$$-\begin{vmatrix} a_{13} & a_{12} & a_{11} \\ a_{23} & a_{22} & a_{21} \\ a_{33} & a_{32} & a_{31} \end{vmatrix} = -[(a_{13}a_{22}a_{31} + a_{23}a_{32}a_{11} + a_{33}a_{21}a_{12}) - (a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{23}a_{12})] =$$

$$= (a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{23}a_{12}) - (a_{13}a_{22}a_{31} + a_{23}a_{32}a_{11} + a_{33}a_{12}a_{21})$$

1.1.2-xossa isbotlandi.

1.1.1-Natija. Ikkita satri yoki ustuni bir xil bo'lgan (yoki proporsional) determinantning qiymati nolga teng.

1.1.3-Xossa. Determinantning satri yoki ustunidagi elementlar umumiy λ ko'paytuvchiga ega bo'lsa, λ ni determinant belgisidan tashqariga chiqarish mumkin.

Isbot:

$$\begin{vmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (\lambda a_{11}a_{22}a_{33} + \lambda a_{21}a_{32}a_{13} + \lambda a_{31}a_{23}a_{12}) -$$

$$-(\lambda a_{13}a_{22}a_{31} + \lambda a_{23}a_{32}a_{11} + \lambda a_{33}a_{12}a_{21}) =$$

$$= \lambda[(a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{23}a_{12}) - (a_{13}a_{22}a_{31} + a_{23}a_{32}a_{11} + a_{33}a_{12}a_{21})] =$$

$$= \lambda \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

1.1.3-xossa isbotlandi.

1.1.2-Natija: Determinantning biror satri ustuni boshqa satri yoki ustuniga parallel bo'lsa bunday determinantning qiymati nolga teng.

1.1.4-Xossa. Agar determinantning biror qatorining har bir elementi ikki qo'shiluvchining yig'indisidan iborat bo'lsa, u holda bu determinant ikki determinant yig'indisidan

$$\begin{vmatrix} a_{11} + b_{11} & a_{12} & a_{13} \\ a_{21} + b_{21} & a_{22} & a_{23} \\ a_{31} + b_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & a_{12} & a_{13} \\ b_{21} & a_{22} & a_{23} \\ b_{31} & a_{32} & a_{33} \end{vmatrix} \qquad (1.1.9)$$

iborat bo'ladi.

Isbot:

Tenglikning chap va o'ng tarafini birinchi ustun bo'yicha yoyamiz

$$\begin{vmatrix} a_{11}+b_{11} & a_{12} & a_{13} \\ a_{21}+b_{21} & a_{22} & a_{23} \\ a_{31}+b_{31} & a_{32} & a_{33} \end{vmatrix} = (a_{11}+b_{11})\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - (a_{21}+b_{21})\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} +$$

$$+(a_{31}+b_{31})\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & a_{12} & a_{13} \\ b_{21} & a_{22} & a_{23} \\ b_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} +$$

$$+ b_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - b_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + b_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} =$$

$$= (a_{11}+b_{11})\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - (a_{21}+b_{21})\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + (a_{31}+b_{31})\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

1.1.4-xossa isbotlandi.

1.1.5-Xossa. Agar biror qator elementlariga boshqa parallel qatorning elementlari istalgan ko'paytuvchiga ko'paytirib qo'shilsa, determinant o'zgarmaydi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}+\lambda a_{12} & a_{12} & a_{13} \\ a_{21}+\lambda a_{22} & a_{22} & a_{23} \\ a_{31}+\lambda a_{32} & a_{32} & a_{33} \end{vmatrix} \qquad (1.1.10)$$

Isbot:

1.1.4-xossaga asosan isbotlnadi.

Algebraik to'ldiruvchilar va minorlar

1.1.1-Ta`rif. Determinant berilgan elementining minori deb, shu element turgan satr va ustunni bir vaqtda o'chirishdan hosil bo'lgan determinantga aytiladi.

Masalan, ushbu $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ determinant a_{12} turgan satr va ustunni o'chirish natijasida hosil bo'lgan $\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ 2-tartibli determinant a_{12} elementning minoridan iborat bo'ladi va M_{12} deb beriladi:

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \qquad (1.1.11)$$

Shunday qilib yuqorida tuzilgan uchinchi tartibli Δ determinantning har bir $a_{i,j}(i, j = 1, 2, 3)$ elementiga mos minori $M_{i,j}$ bo'lib, ular ikkinchi tartibli va hammasi 9 ta bo'ladi.

1.1.2-Ta`rif. Determinant biror elementning algebraik to'ldiruvchisi deb uning bu determinantda juft yoki toq joy egallaganiga bog'liq ravishda musbat yoki manfiy ishora bilan olingan minoriga aytiladi:

$$A_{i,j} = (-1)^{i+j} M_{i,j} \qquad (1.1.12)$$

Masalan, a_{22} elementning algebraik to'ldiruvchisi

$A_{22} = (-1)^{2+2}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} = (-1)^{2+2} M_{22} = M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$ son bo'ladi, chunki a_{22} juft joyda turibdi, a_{32} element algebraik to'ldiruvchisi $A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = -(a_{11}a_{23} - a_{13}a_{21})$ son bo'ladi, chunki a_{32} toq o'rinda turibdi.

1.2 Yuqori tartibli determinantlar va ularning xossalari va chiziqli tenglamalar sistemasini determinant (Kramer) usuli bilan yechish.

Yuqori tartibli determinantlar yoki n-tartibli determinantlar, deb quyidagi yig`indiga teng Δ songa aytiladi:

$$\Delta = \sum_j (-1)^{t(j)} a_{1j1} a_{2j2} \ldots a_{njn} \qquad \text{va}$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} \qquad (1.2.1)$$

ko`rinishda yoziladi, bu yerda, $j(j_1, j_2, \cdots j_n)$ - asosiy $(1,2,\cdots,n)$ o`rin almashtirishdan olinishi mumkin bo`lgan ixtiyoriy o`rin almashtirish, $t(j)$ – asosiydan j o`rin almashtirishga o`tishda transpozitsiyalar soni.

$(-1)^{t(j)} a_{1j1} a_{2j2} \dots a_{njn}$ ko`paytmaga determinantning hadi deyiladi. n–tartibli determinant $n^2$ haqiqiy son – elementlar orqali aniqlanadi va yi-g`indi $n!$ ta haddan iborat.

Minor va algebraik to`ldiruvchilar haqida tushuncha

n- tartibli $\Delta = |a_{ik}|$ determinant berilgan bo`lib, uning ixtiyoriy i-satrini va ixtiyoriy k-ustunini o`chiramiz. Qolgan ifoda $(n-1)$–tartibli determinant-ni tashkil etadi va a_{ik} elementning minori deyiladi. a_{ik} element minori M_{ik} yozuv bilan belgilanadi.

a_{ik} elementning algebraik to`ldiruvchisi yoki ad`yunkti deb,

$A_{ik} = (-1)^{i+k} M_{ik}$ kattalikka aytiladi.

Masalan, uchinchi tartibli $\Delta = |a_{ik}|$ determinantning a_{12} elementi minori M_{12} va algebraik to`ldiruvchisi A_{12} mos ravishda:

$$\mathrm{M}_{12} = \begin{vmatrix} \mathrm{a}_{21} & \mathrm{a}_{23} \\ \mathrm{a}_{31} & \mathrm{a}_{33} \end{vmatrix} = \mathrm{a}_{21}\mathrm{a}_{33} - \mathrm{a}_{23}\mathrm{a}_{31}, \quad \mathrm{A}_{12} = (-1)^{1+2}\mathrm{M}_{12} = -\mathrm{M}_{12} \qquad (1.2.2)$$

Determinantlarning xossalari

Ixtiyoriy n- tartibli determinant o`zining asosiy xossalaridan tashqari, qo`shimcha ravishda quyidagi xossalarga ham ega.

1.2.1-Xossa: Determinantning ixtiyoriy satri yoki ustuni elementlarining o`z algebraik to`ldiruvchilariga ko`paytmalarining yig`indisi uning kattaligiga teng:

$$\Delta = \sum_{k=1}^{n} a_{ik}A_{ik} \quad (i = 1,2,..., n) \quad (1.2.3) \qquad \Delta = \sum_{i=1}^{n} a_{ik}A_{ik} \quad (k = 1,2,..., n) \quad (1.2.4)$$

(1.2.3) yig\`indi n-tartibli determinantni i- satr elementlari bo\`yicha yoyish formulasi deyilsa, (1.2.4) yig\`indi k– ustun elementlari bo\`yicha yoyish formulasi deyiladi.

Isbot:

Uchinchi tartibli $\Delta = |a_{ik}|$ determinantni ikkinchi ustun elementlari bo\`yicha yoying.Uchinchi tartibli determinantni ikkinchi ustun elementlari bo\`yicha yoyish formulasini qo\`llaymiz, natijada

$$\sum_{i=1}^{3} a_{i2}A_{i2} = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} = -a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{22}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} -$$

$$-a_{32}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = -a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{22}(a_{11}a_{33} - a_{13}a_{31}) -$$

$$-a_{32}(a_{11}a_{23} - a_{13}a_{21}) = \Delta$$

1.2.2-Xossa: Determinant biror satri (yoki ustuni) elementlarining boshqa parallel satr (yoki ustun) mos elementlari algebraik to\`ldiruvchilariga ko\`paytmalarining yig\`indisi nolga teng:

Isbot:

Uchinchi tartibli determinantni olamiz

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32} = -a_{11}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} +$$

$$+a_{21}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} - a_{31}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} =$$

$$= -a_{11}(a_{21}a_{33} - a_{23}a_{31}) + a_{21}(a_{11}a_{33} - a_{13}a_{31}) - a_{31}(a_{11}a_{23} - a_{13}a_{21}) = 0$$

1.2.2-xossa isbotlandi.

$$\Delta = \sum_{k=1}^{n} a_{ik}A_{jk} = \sum_{k=1}^{n} a_{ki}A_{kj} = 0 \quad (i \neq j, \; i, j = 1,2,\ldots,n) \qquad (1.2.5)$$

Ushbu xossa determinantlarning 1.2.1- xossasi asosida isbotlanadi.

1.2.3-Xossa: n-tartibli aniq bir satrlari (ustunlari) bir-biridan farq qiluvchi, qolganlari esa aynan bir xil bo`lgan $\Delta_1$ va $\Delta_2$ determinantlar berilgan bo`lsin. Berilgan Δ_1 va Δ_2 determinantlarning yig`indisi ko`rsatilgan farqli satri (ustuni) mos elementlarining yig`indisidan iborat, umumiy satrlari (ustunlari) esa o`zgarmas qoladigan n-tartibli Δ determinantga teng.

Isbot:

Uchinchi ustunlari farqli, qolgan ustunlari aynan bir xil uchinchi tartibli determinantlar quyidagicha qo`shiladi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & b_{13} \\ a_{21} & a_{22} & b_{23} \\ a_{31} & a_{31} & b_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13}+b_{13} \\ a_{21} & a_{22} & a_{23}+b_{23} \\ a_{31} & a_{31} & a_{33}+b_{33} \end{vmatrix} \tag{1.2.6}$$

Ushbu xossa determinantning 1.1.4-xossasiga asosan isbotlanadi.

1.2.4-Xossa: Determinant kattaligi uning biror satri (ustuni) elementlariga boshqa parallel satr (ustun) mos elementlarini bir xil songa ko`paytirib qo`shganda o`zgarmaydi.

Yuqori tartibli determinantlarni hisoblashning ratsional usuli uning biror satri yoki ustunida keltirilgan xossa asosida nollar yig`ib, so`ngra shu satr yoki ustun bo`yicha yoyib hisoblashdir. Yuqori tartibli determinantni hisoblash masalasi ketma-ket ravishda quyi tartibli determinantlarni hisoblash bilan almashinadi.

Masalan:

$$\begin{vmatrix} 1 & -2 & 3 & 6 \\ 1 & 1 & 0 & 2 \\ -4 & 2 & -3 & 1 \\ -1 & 3 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 3 & 4 \\ 1 & 0 & 0 & 0 \\ -4 & 6 & -3 & 9 \\ -1 & 4 & 1 & 6 \end{vmatrix} = -1\begin{vmatrix} -3 & 3 & 4 \\ 6 & -3 & 9 \\ 4 & 1 & 6 \end{vmatrix} =$$

$$= -3\begin{vmatrix} -15 & 0 & -14 \\ 6 & 0 & 9 \\ 4 & 1 & 6 \end{vmatrix} = 3\begin{vmatrix} -15 & -14 \\ 6 & 9 \end{vmatrix} = 9\begin{vmatrix} -15 & -14 \\ 2 & 3 \end{vmatrix} = 9\cdot(-17) = -153$$

1.2.5-Xossa: n-tartibli berilgan $\Delta = |a_{ik}|$ va $\Delta = |b_{ik}|$ determinantlar ko`paytmasi n-tartibli $\Delta = |c_{ik}|$ determinantga teng va uning ixtiyoriy $c_{ik}$ elementi quyidagi formula bo`yicha hisoblanadi:

$$c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk} \qquad (1.2.7)$$

c_{ik} element Δ_1 determinant i-satri elementlarining Δ_2 determinant k- ustuni mos elementlariga ko`paytmalarining yig`indisiga teng.

Masalan:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = \begin{vmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{vmatrix} \qquad (1.2.8)$$

Laplas teoremasi

1.2.1-Teorema (Laplas teoremasi)

Determinant qiymati uning biror satri (yoki ustun) elementlarini bu elementlarning mos algebraik to'ldiruvchilariga ko'paytirilgan yig'indisiga teng.

Isbot: (1.2.2) determinantning ikkinchi ustuni uchun teoremaning tasdig'i quyidagicha $\Delta = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32}$ tenglikning to'g'riligidan iborat, ya`ni

$$\Delta = a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{21}a_{12}a_{33} - a_{31}a_{22}a_{13} - a_{11}a_{23}a_{32} =$$
$$= a_{12}\left(a_{31}a_{23} - a_{21}a_{33}\right) + a_{22}\left(a_{11}a_{33} - a_{31}a_{13}\right) + a_{32}\left(a_{21}a_{13} - a_{11}a_{23}\right) =$$
$$= a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{22}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} - a_{32}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = a_{12}(-1)^{1+2}M_{12} +$$
$$+ a_{22}(-1)^{2+2}M_{22} + a_{32}(-1)^{3+2}M_{32} = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32}.$$

1.2.1-Misol.

a)

$$\begin{vmatrix} 2 & -3 & 5 \\ 2 & 1 & 1 \\ 1 & 2 & 4 \end{vmatrix} = (-1)^{1+3}5\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + (-1)^{2+3}1\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} + (-1)^{3+3}4\begin{vmatrix} 2 & -3 \\ 2 & 1 \end{vmatrix} =$$

$$= 5(4-1) - (4+3) + 4(2+6) = 15 - 7 + 32 = 40.$$

b)

$$\begin{vmatrix} 2 & -3 & 5 \\ 2 & 1 & 1 \\ 1 & 2 & 4 \end{vmatrix} = (-1)^{1+1}2\begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} + (-1)^{1+2}(-3)\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} + (-1)^{1+3}5\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} =$$

$$= 2(4-2) - (-3)(8-1) + 5(4-1) = 4 + 21 + 15 = 40.$$

Chiziqli tenglamalar sistemasini determinant (Kramer) usuli bilan yechish.

Ikki noma`lumli ikkita chiziqli tenglamalar sistemasi.

Aytaylik bizga ushbu ikki noma`lumli chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \qquad (1.2.9)$$

Sistemaning yechimini topish uchun determinantlar nazariyasidan foydalanamiz. Chiziqli tenglamalar sistemasini yechish deganda, x va y sonlarning shunday to'plamini topish demakki, uni (1.2.9) tenglamadagi ayniyatga aylansin. Bu sonlar to'plamini sistemaning yechimi deb ataymiz. Kamida bitta yechimga ega bo'lgan sistema birgalikdagi sistema yoki aniq sistema deb ataladi. Cheksiz ko'p yechimga ega bo'lgan birgalikdagi sistema aniqmas sistema deb ataladi. Bitta ham yechimga ega bo'lmagan sistema birgalikda bo'lmagan sistema deb ataladi. Sistema koeffisiyentlaridan quyidagi determinantlarni tuzamiz va uni Δ bilan belgilaymiz:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

Unga bosh determinant deyiladi. So'ngra bu determinantda mos ravishda birinchi va ikkinchi ustunlarni ozod hadlar bilan almashtirib, Δ_x, Δ_y bilan belgilanadigan ushbu yordamchi determinantlarni tuzamiz.

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} \qquad \Delta_y = \begin{vmatrix} a_{12} & b_1 \\ a_{21} & b_2 \end{vmatrix}$$

Agar $\Delta \neq 0$ bo'lsa, (1) – sistemaning yechimini aniqlaydigan

$$x = \frac{\Delta_x}{\Delta} \quad y = \frac{\Delta_y}{\Delta} \qquad (1.2.10)$$

(1.2.10) formulani hosil qilamiz. Olingan bu qoida Kramer qoidasi deyiladi.

Bu yerda uch hol bo'lishi mumkin:

a) Agar $\Delta \neq 0$ bo'lsa, (1.2.9) sistema birgalikda bo'lib, birgina yechimga ega bo'ladi.

b) Agar $\Delta=0$, lekin Δ_x va Δ_y larning kamida bittasi nolga teng bo'lmasa, u holda (1.2.9) sistema birgalikda emas, ya`ni bitta ham yechimga ega bo'lmaydi.

v) Agar $\Delta = 0$ va $\Delta_x = 0,\ \Delta_y = 0$ bo'lsa, (1.2.9) – sistema aniqmas, ya`ni cheksiz ko'p yechimlarga ega bo'ladi.

1.2.2-Misol.

$$\begin{cases} 3x - y = 5 \\ x + 2y = 4 \end{cases}$$ sistema yechilsin.

Yechish.

$$\Delta = \begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix} = 7; \qquad \Delta_x = \begin{vmatrix} 5 & -1 \\ 4 & 2 \end{vmatrix} = 14; \qquad \Delta_y = \begin{vmatrix} 3 & 5 \\ 1 & 4 \end{vmatrix} = 7$$

(1.2.10) – formuladan $x = \dfrac{\Delta_x}{\Delta} = \dfrac{14}{7} = 2; \qquad y = \dfrac{\Delta_y}{\Delta} = \dfrac{7}{7} = 1.$

1.2.3-Misol.

$$\begin{cases} 3x + y = 2 \\ 6x + 2y = 3 \end{cases}$$ tenglamalar sistemasi yechilsin.

Yechish.

$$\Delta = \begin{vmatrix} 3 & 1 \\ 6 & 2 \end{vmatrix} = 0; \quad \Delta_x = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 1; \quad \Delta_y = \begin{vmatrix} 3 & 2 \\ 6 & 3 \end{vmatrix} = -3.$$

Sistema birgalikda emas, yechimlari yo'q.

1.2.4-Misol.

$$\begin{cases} 3x - y = 2 \\ 6x - 2y = 4 \end{cases}$$ tenglamalar sistemasi yechilsin.

Yechish.

$$\Delta = \begin{vmatrix} 3 & -1 \\ 6 & -2 \end{vmatrix} = 0; \qquad \Delta_x = \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix} = 0; \qquad \Delta_y = \begin{vmatrix} 3 & 2 \\ 6 & 4 \end{vmatrix} = 0.$$

Sistema aniqmas, cheksiz ko'p yechimlarga ega, ikkinchi tenglamani 2 ga qisqartirsak, sistema ushbu bitta tenglamaga keladi:

$$3x - y = 2$$

Noma`lum x ga ixtiyoriy qiymatlar berib y ning unga mos qiymatlari hosil qilinadi:

$x=0$ bo’lsa, u holda $y=-2$

$x=1$ bo’lsa, u holda $y=1$ va hokazo.

Uch noma`lumli uchta chiziqli tenglamalar sistemasini determinant usuli (Kramer) bilan yechish.

Uch noma`lumli uchta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x+a_{12}y+a_{13}z=b_1 \\ a_{21}x+a_{22}y+a_{23}z=b_2 \\ a_{31}x+a_{32}y+a_{33}z=b_3 \end{cases} \qquad (1.2.11)$$

berilgan bo’lsin. Bu sistemaning yechimini topish uchun quyidagi determinantlarni tuzamiz.

$$\Delta=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}; \quad \Delta_x=\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}; \quad \Delta_y=\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}; \quad \Delta=\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

Bu determinantlardan foydalanib (1.2.11) – sistemaning yechimlari $\Delta\neq 0$ bo’lganda quyidagi Kramer formulalaridan topiladi:

$$x=\frac{\Delta_x}{\Delta}; \qquad y=\frac{\Delta_y}{\Delta}; \qquad z=\frac{\Delta_z}{\Delta} \qquad (1.2.12)$$

a) Agar $\Delta\neq 0$ bo’lsa, sistema bitta yechimga ega bo’ladi.

b) Agar $\Delta=0$ va $\Delta_x,\Delta_y,\Delta_z$ determinantlardan kamida bittasi noldan farqli bo’lsa, u holda (1.2.11) sistema yechimga ega emas. Aniqlik uchun $\Delta_x=\Delta_y=0$ bo’lib, $\Delta_z\neq 0$ bo’sin. U holda (1.2.12) dan:

$$z=\frac{\Delta_z}{\Delta}\Rightarrow \Delta\cdot z=\Delta_z .$$

Ammo, oxirgi tenglikning o’ng tomoni noldan farqli $(\Delta_z\neq 0)$, chap tomoni esa nolga teng, buning bo’lishi mumkin emas. Demak, yechimga ega emas.

v) Agar $\Delta = 0$, $\Delta_x = 0$, $\Delta_y = 0$, $\Delta_z = 0$ (1.2.11) sistema cheksiz ko'p yechimga ega, yoki yechimga ega emas bo'ladi.

1.2.5-Misol.

$$\begin{cases} 2x - 3y + z = -5 \\ x + 2y - 4z = -9 \\ 5x - 4y + 6z = 5 \end{cases}$$

Yechish.

$$\Delta = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix} = 24 + 60 - 4 - 10 - 32 + 18 = 56;$$

$$\Delta_x = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix} = 24 + 60 - 4 - 10 - 32 + 18 = 56;$$

$$\Delta_y = \begin{vmatrix} 2 & -5 & 1 \\ 1 & -9 & -4 \\ 5 & 5 & 6 \end{vmatrix} = -60 + 60 + 36 - 10 + 80 - 162 = -56;$$

$$\Delta_z = \begin{vmatrix} 2 & -3 & -5 \\ 1 & 2 & -9 \\ 5 & -4 & 5 \end{vmatrix} = 20 + 135 + 20 + 50 - 72 + 15 = 16;$$

$$x = \frac{\Delta_x}{\Delta} = -1; \quad y = \frac{\Delta_y}{\Delta} = 2; \quad z = \frac{\Delta_z}{\Delta} = 3.$$

Tekshirish.

$$2 \cdot (-1) - 3 \cdot 2 + 3 = -2 - 6 + 3 = -5$$
$$-1 + 2 \cdot 2 - 4 \cdot 3 = -1 + 4 - 12 = -9$$
$$5 \cdot (-1) - 4 \cdot 2 + 6 \cdot 3 = -5 - 8 + 18 = 5$$

Demak, $\{-1;2;3\}$ yechim bo'ladi.

Uch noma`lumli uchta chiziqli tenglamalar sistemasining o'ng tomoni nolga teng bo'lsa, unga bir jinsli sistemasi deb ataladi: bir jinsli sistema doim nol (trivial) yechimga ega, ya`ni (0;0;0) uchlik doim yechim bo'ladi.

n noma`lumli n ta chiziqli tenglamalar sistemasini determinant (Kramer) usuli bilan yechish.

n nomalumli n ta chiziqli tenglamalar sistemasi quyidagi ko'rinishga ega:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = c_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = c_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = c_n \end{cases} \qquad (1.2.13)$$

Bu yerda a_{ik} sonlarga sistemaning koeffitsientlari, c_i -ozod hadlar, $x_1, x_2, \cdots, x_n$ - no'malumlar deyiladi.

1.2.1-Ta'rif. Agar (1.2.13) sistemaning har bir tenglamasidagi $x_1, x_2, \cdots, x_n$ no'malumlar o'rniga mos ravishda $\alpha_1, \alpha_2, \cdots, \alpha_n$ qiymatlar qo'yilganda sistemaning barcha tenglamalari ayniyatga aylansa, $\alpha_1, \alpha_2, \cdots, \alpha_n$ sonlar (1.2.13) sistemaning yechimi deyiladi.

Sistemaning yechimi mavjud bo'lish-bo'lmasligi quyidagi determinantga bog'liq:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} \qquad (1.2.14)$$

(1.2.14) determinant (1.2.13) sistemaning nomalumlari oldidagi koeffitsientlardan tuzilgan. Agar $\Delta \neq 0$ bo'lsa, sistema yagona yechimga ega bo'ladi va bu yechim

$$x_i = \frac{\Delta_{x_i}}{\Delta} (i = 1,\ n)$$

formulalar yordamida topiladi.

Bunda Δ_{x_i} determinant Δ determinantning birinchi ustun elementlarini (1.2.13) tenglamalar sistemasining ozod hadlari bilan almashtirishdan hosil qilinadi; Δ_{x_2} esa Δ determinantning ikkinchi ustun elementlarini ozod hadlar bilan almashtirishdan hosil bo'ladi; $\Delta_{x_3}, \cdots, \Delta_{x_n}$ lar ham shunga o'xshash hosil qilinadi.

(1.2.13) tenglamalar sistemasini yechishning bunday usuli ***determinant (yoki Kramer)*** usuli deyiladi. Demak (1.2.13) sistemani yechish uchun $(n+1)$ ta determinant tuzish va hisoblash kerak bo'ladi.

I Bobning Xulosasi.

Ishning birinchi bobida mavzuni bayon qilishda zarur bo\`ladigan ma\`lumotlar keltirilgan. Bu bob ikki paragrafdan iborat bo\`lib, birinchi paragrafda ikkinchi va uchinchi tartibli determinantlar, ularning xossalari va unga doir misollar, ikkinchi paragrafda Yuqori tartibli determinantlar va ularning xossalari va chiziqli tenglamalar sistemasini determinant (Kramer) usuli bilan yechish va unga doir misollar yechib ko'rsatilgan.

II YUQORI TARTIBLI DETERMINANTLARNI HISOBLASH USULLARI.

Determinantlarni hisoblash usuli birtadan tashqari satr (yoki ustun) nolga aylanadigan va keyinchalik tartibi pasayuvchi son elementli qatorlar berilgan tartibni harfli elementlar orqali hisoblashda ancha murakkablashadi.

Bu yo’l umumiy holda determinantning to’g’ridan -to’g’ri uni aniqlashni qo’llash orqali ifodaga keltiriladi. Bu usul harfli yoki son elementli va ixtiyoriy n-tartibli determinantlarni aniqlashda ayniqsa noqulay.

Bunday determinantlarni hisoblashning umumiy usuli mavjud emas (agar determinantni hisoblash uchun berilgan ifodasini hisobga olmasak). U yoki bu maxsus turkumlarni hisoblash uchun turli xil hisoblash usullari qo’llaniladi; ya’ni, determinantlarni ifodalar yordamida tartibini pasaytirish orqali . Biz bunday usullarning eng ko’p uchraydigan hollarini ko’rib chiqamiz, so’ngra o’zlashtirish maqsadida hisoblash usulini tanlab har bir usulni qo’llanilishi uchun misollar keltiramiz.

2.1 Uchburchak shakliga olib kelib hisoblash usuli.

Bu usulda determinantlarni almashtirish natijasida dioganalning bir tomonida yotgan barcha elementlar nolga teng bo’lgan ko’rinishga keltiriladi. Satr (yoki ustun) larni tartibini almashtirish usuli asosiy diagonal holatiga olib kelinadi. Keltirib chiqarilgan determinant asosiy dioganal element larining ko’paytmasiga teng.

2.1.1-Misol. n- tartibli determinantni hisoblang:

$$D = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 \\ 1 & 1 & 0 & \cdots & 1 \\ . & . & . & . & . \\ 1 & 1 & 1 & \cdots & 0 \end{vmatrix}.$$

Birinchi satrni qolgan barch satrlardan ayiramiz:

$$D=\begin{vmatrix}1 & 1 & 1 & \cdots & 1\\ 0 & -1 & 0 & \cdots & 0\\ 0 & 0 & -1 & \cdots & 0\\ . & . & . & . & .\\ 0 & 0 & 0 & \cdots & -1\end{vmatrix}=(-1)^{n-1}.$$

2.1.2-Misol. Determinantni hisoblang:

$$D=\begin{vmatrix}a_1 & x & x & \cdots & x\\ x & a_2 & x & \cdots & x\\ x & x & a_3 & \cdots & x\\ . & . & . & . & .\\ x & x & x & \cdots & a_n\end{vmatrix}.$$

Birinchi satrni qolgan barcha satrlardan ayiramiz:

$$D=\begin{vmatrix}a_1 & x & x & \cdots & x\\ x-a_1 & a_2-x & 0 & \cdots & 0\\ x-a_1 & 0 & a_3-x & \cdots & 0\\ . & . & . & . & .\\ x-a_1 & 0 & 0 & \cdots & a_n-x\end{vmatrix}.$$

Birinchi qatordan a_1-x, ni chiqaramiz, ikkinchi qatordan esa a_2-x, ni chiqaramiz,…, n-qatordan esa a_n-x ni chiqarib quyidagi ifodani hosil qilamiz:

$$D=(a_1-x)(a_2-x)\cdots(a_n-x)\begin{vmatrix}\frac{a_1}{a_1-x} & \frac{x}{a_2-x} & \frac{x}{a_3-x} & \cdots & \frac{x}{a_n-x}\\ -1 & 1 & 0 & \cdots & 0\\ -1 & 0 & 1 & \cdots & 0\\ . & . & . & . & .\\ -1 & 0 & 0 & \cdots & 1\end{vmatrix}.$$

$\frac{a_1}{a_1-x}=1+\frac{x}{a_1-x}$, almashtirib va barcha ustunlarni birinchi ustuga qo’shamiz.

$$D=(a_1-x)(a_2-x)\cdots(a_n-x)\begin{vmatrix} 1+\dfrac{x}{a_1-x}+\cdots+\dfrac{x}{a_n-x} & \dfrac{x}{a_2-x} & \dfrac{x}{a_3-x} & \cdots & \dfrac{x}{a_n-x} \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 1 \end{vmatrix}=$$

$$=x(a_1-x)(a_2-x)\cdots(a_n-x)\left(\frac{1}{x}+\frac{1}{a_1-x}+\frac{1}{a_2-x}+\cdots+\frac{1}{a_n-x}\right).$$

Endi uchburchak shakliga olib kelish usuli bilan yechiladigan misollarni ko’rib chiqamiz.

2.1.3-Misol.

$$\begin{vmatrix} 1 & 2 & 3 & \cdots & n \\ -1 & 0 & 3 & \cdots & n \\ -1 & -2 & 0 & \cdots & n \\ . & . & . & . & . \\ -1 & -2 & -3 & \cdots & 0 \end{vmatrix}$$

Birinchi satrni qolgan satrlarga qo'shib quyidagini olamiz

$$\begin{vmatrix} 1 & 2 & 3 & \cdots & n \\ 0 & 2 & 6 & \cdots & 2n \\ 0 & 0 & 3 & \cdots & 2n \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & n \end{vmatrix}$$

Bundan quyidagi kelib chiqadi

$$1\cdot\begin{vmatrix} 2 & 6 & 8 & \cdots & 2n \\ 0 & 3 & 8 & \cdots & 2n \\ 0 & 0 & 4 & \cdots & 2n \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & n \end{vmatrix}$$

Shu tariqa davom etamiz

$$1\cdot 2\cdot\begin{vmatrix} 3 & 8 & 10 & \cdots & 2n \\ 0 & 4 & 10 & \cdots & 2n \\ 0 & 0 & 5 & \cdots & 2n \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & n \end{vmatrix} =$$

$$=1\cdot 2\cdot 3\cdot \ldots\ldots\cdot n = n!$$

2.1.4-Misol

$$\begin{vmatrix} 1 & 2 & 3 & \cdots & n-2 & n-1 & n \\ 2 & 3 & 4 & \cdots & n-1 & n & n \\ 3 & 4 & 5 & \cdots & n & n & n \\ . & . & . & . & . & , & . \\ n & n & n & \cdots & n & n & n \end{vmatrix}$$

n-satrdan qolgan satrlarni ayirib quyidagini hosil qilamiz

$$\begin{vmatrix} n-1 & n-2 & n-3 & \cdots & 2 & 1 & 0 \\ n-2 & n-3 & n-4 & \cdots & 1 & 0 & 0 \\ n-3 & n-4 & n-5 & \cdots & 0 & 0 & 0 \\ . & . & . & . & . & . & . \\ n & n & n & \cdots & n & n & n \end{vmatrix}$$

Bundan quyidagi kelib chiqadi

$$n\cdot\begin{vmatrix} n-1 & n-2 & n-3 & \cdots & 3 & 2 & 1 \\ n-2 & n-3 & n-4 & \cdots & 2 & 1 & 0 \\ n-3 & n-4 & n-5 & \cdots & 1 & 0 & 0 \\ . & . & . & . & . & . & . \\ n & n & n & \cdots & n & n & n \end{vmatrix} = n\cdot(-1)^{\frac{n\cdot(n-1)}{2}}$$

2.1.5-Misol

$$\begin{vmatrix} x_1 & a_{12} & a_{13} & \cdots & a_{1n} \\ x_1 & x_2 & a_{23} & \cdots & a_{2n} \\ x_1 & x_2 & x_3 & \cdots & a_{3n} \\ . & . & . & . & . \\ x_1 & x_2 & x_3 & \cdots & x_n \end{vmatrix}$$

Bundan quyidagini olamiz

$$\begin{vmatrix} x_1 & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & x_2 - a_{12} & a_{23} - a_{13} & \cdots & a_{2n} - a_{1n} \\ 0 & x_2 - a_{12} & x_3 - a_{13} & \cdots & a_{3n} - a_{1n} \\ . & . & . & . & . \\ 0 & x_2 - a_{12} & x_3 - a_{13} & \cdots & x_n - a_{1n} \end{vmatrix}$$

Bu esa quyidagiga teng

$$x_1 \cdot \begin{vmatrix} x_2 - a_{12} & a_{23} - a_{13} & a_{24} - a_{14} & \cdots & a_{2n} - a_{1n} \\ x_2 - a_{12} & x_3 - a_{13} & a_{34} - a_{14} & \cdots & a_{3n} - a_{1n} \\ x_2 - a_{12} & x_3 - a_{13} & x_4 - a_{14} & \cdots & a_{4n} - a_{1n} \\ . & . & . & . & . \\ x_2 - a_{12} & x_3 - a_{13} & x_4 - a_{14} & \cdots & x_n - a_{1n} \end{vmatrix}$$

Birinchi satrni qoldirib qolgan satrlardan birinchi satrni ayirib o'z joyiga yozamiz

$$x_1 \cdot \begin{vmatrix} x_2 - a_{12} & a_{23} - a_{13} & a_{24} - a_{14} & \cdots & a_{2n} - a_{1n} \\ 0 & x_3 - a_{23} & a_{34} - a_{24} & \cdots & a_{3n} - a_{2n} \\ 0 & x_3 - a_{23} & x_4 - a_{24} & \cdots & a_{4n} - a_{2n} \\ . & . & . & . & . \\ 0 & x_3 - a_{23} & x_4 - a_{24} & \cdots & x_n - a_{2n} \end{vmatrix}$$

Bu esa quyidagiga teng

$$x_1\cdot(x_2-a_{12})\cdot\begin{vmatrix} x_3-a_{23} & a_{34}-a_{24} & a_{35}-a_{25} & \cdots & a_{3n}-a_{2n}\\ x_3-a_{23} & x_4-a_{24} & a_{45}-a_{25} & \cdots & a_{4n}-a_{2n}\\ x_3-a_{23} & x_4-a_{24} & x_5-a_{25} & \cdots & a_{5n}-a_{2n}\\ . & . & . & . & .\\ x_3-a_{23} & x_4-a_{24} & x_5-a_{25} & \cdots & x_n-a_{2n}\end{vmatrix}$$

Birinchi satrni qoldirib qolgan satrlardan birinchi satrni ayirib o'z joyiga yozamiz

$$x_1\cdot(x_2-a_{12})\cdot\begin{vmatrix} x_3-a_{23} & a_{34}-a_{24} & a_{35}-a_{25} & \cdots & a_{3n}-a_{2n}\\ 0 & x_4-a_{34} & a_{45}-a_{35} & \cdots & a_{4n}-a_{3n}\\ 0 & x_4-a_{34} & x_5-a_{35} & \cdots & a_{5n}-a_{3n}\\ . & . & . & . & .\\ 0 & x_4-a_{34} & x_5-a_{35} & \cdots & x_n-a_{3n}\end{vmatrix}$$

Shu tariqa davom etamiz va quyidagi ifodani olamiz

$$x_1\cdot(x_2-a_{12})\cdot(x_3-a_{23})\cdot(x_4-a_{34})\cdot......(x_n-a_{(n-1),n})$$

2.1.6-Misol

$$\begin{vmatrix} 1 & \cdots & 1 & 1 & 1\\ a_1 & \cdots & a_1 & a_1-b_1 & a_1\\ a_2 & \cdots & a_2-b_2 & a_2 & a_2\\ . & . & . & . & .\\ a_n-b_n & \cdots & a_n & a_n & a_n\end{vmatrix}$$

n- ustunni qoldirib qolgan ustunlarni ayirib quyidagini olamiz

$$\begin{vmatrix} 0 & \cdots & 0 & 0 & 1\\ 0 & \cdots & 0 & b_1 & a_1\\ 0 & \cdots & b_2 & 0 & a_2\\ . & . & . & . & .\\ b_n & \cdots & 0 & 0 & a_n\end{vmatrix}=b_1\cdot b_2\cdot\cdots\cdot b_n\cdot(-1)^{\frac{n\cdot(n-1)}{2}}$$

2.1.7-Misol

$$\begin{vmatrix} 3 & 2 & 2 & \cdots & 2 \\ 2 & 3 & 2 & \cdots & 2 \\ 2 & 2 & 3 & \cdots & 2 \\ . & . & . & . & . \\ 2 & 2 & 2 & \cdots & 3 \end{vmatrix}$$

Birinchi satrni qoldirib qolgan satrlardan ayirsak quyidagi kelib chiqadi

$$\begin{vmatrix} 3 & 2 & 2 & \cdots & 2 \\ 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ . & . & . & . & . \\ 1 & 0 & 0 & \cdots & -1 \end{vmatrix}$$

Endi birinchi ustunga qolgan ustunlarni qo'shamiz

$$\begin{vmatrix} 3+2(n-1) & 2 & 2 & \cdots & 2 \\ 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & -1 \end{vmatrix} = 2n+1$$

2.1.8-Misol

$$\begin{vmatrix} a_0 & a_1 & a_2 & \cdots & a_n \\ -x & x & 0 & \cdots & 0 \\ 0 & -x & x & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & x \end{vmatrix}$$

Birinchi ustunga qolgan ustunlarni yozib quyidagini olamiz

$$\begin{vmatrix} a_0+a_1+\cdots+a_n & a_1 & a_2 & \cdots & a_n \\ 0 & x & 0 & \cdots & 0 \\ 0 & -x & x & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & x \end{vmatrix}$$

Bu determinant $(n+1)\times(n+1)$ tartibli bo'ganligi uchun

$$(a_0+a_1+\cdots+a_n)\cdot\begin{vmatrix} x & 0 & 0 & \cdots & 0 \\ -x & x & 0 & \cdots & 0 \\ 0 & -x & x & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & x \end{vmatrix}$$

Ushbu ifoda kelib chiqadi va bu determinantning tartibi $n\times n$ ko'rinishiga ega bo'lib bundan quyidagi kelib chiqadi

$$(a_0+a_1+.....+a_n)\cdot\begin{vmatrix} x & 0 & 0 & \cdots & 0 \\ 0 & x & 0 & \cdots & 0 \\ 0 & -x & x & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & x \end{vmatrix}_n$$

Birinchi satr bo’yicha yoyib quyidagini olamiz

$$(a_0+a_1+\cdots+a_n)\cdot x\cdot\begin{vmatrix} x & 0 & 0 & \cdots & 0 \\ -x & x & 0 & \cdots & 0 \\ 0 & -x & x & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & x \end{vmatrix}_{n-1}$$

shu tariqa davom etsak quyidagi natija kelib chiqadi

$$(a_0+a_1+\cdots+a_n)\cdot x^2\cdot\begin{vmatrix} x & 0 & 0 & \cdots & 0\\ -x & x & 0 & \cdots & 0\\ 0 & -x & x & \cdots & 0\\ . & . & . & . & .\\ 0 & 0 & 0 & \cdots & x\end{vmatrix}=(a_0+a_1+\cdots+a_n)\cdot x^n$$

2.1.9-Misol.

$$\begin{vmatrix} a_1 & a_2 & a_3 & \cdots & a_n\\ -x_1 & x_2 & 0 & \cdots & 0\\ 0 & -x_2 & x_3 & \cdots & 0\\ . & . & . & . & .\\ 0 & 0 & 0 & \cdots & x_n\end{vmatrix}$$

Birinchi satrga qolgan satrlarni qo'shib birinchi satrga yozsak ushbu kelib chiqadi

$$\begin{vmatrix} a_1-x_1 & a_2 & a_3 & \cdots & a_n+x_n\\ -x_1 & x_2 & 0 & \cdots & 0\\ 0 & -x_2 & x_3 & \cdots & 0\\ . & . & . & . & .\\ 0 & 0 & 0 & \cdots & x_n\end{vmatrix}$$

Bu yerda birinchi ustun bo’yicha yoyamiz

$$(a_1-x_1)\cdot\begin{vmatrix} x_2 & 0 & 0 & \cdots & 0\\ -x_2 & x_3 & 0 & \cdots & 0\\ 0 & -x_3 & x_4 & \cdots & 0\\ . & . & . & . & .\\ 0 & 0 & 0 & \cdots & x_n\end{vmatrix}+x_1\begin{vmatrix} a_2 & a_3 & a_4 & \cdots & a_n+x_n\\ -x_2 & x_3 & 0 & \cdots & 0\\ 0 & -x_3 & x_4 & \cdots & 0\\ . & . & . & . & .\\ 0 & 0 & 0 & \cdots & x_n\end{vmatrix}$$

quyidagi natijaga erishamiz

$$(a_1-x_1)\cdot x_2\cdot x_3\cdot\cdots\cdot x_n+$$
$$+x_1\cdot(a_2-x_2)\cdot x_3\cdot\cdots\cdot x_n+x_1\cdot x_2\cdot(a_3-x_3)\cdot\cdots\cdot x_n+\cdots+x_1\cdot x_2\cdot\cdots\cdot(a_n-x_n)=$$
$$=x_1\cdot x_2\cdot\cdots\cdot x_n\left(\frac{a_1-x_1}{x_1}+\frac{a_2-x_2}{x_2}+\cdots+\frac{a_n-x_n}{x_n}\right)$$

2.1.10-Misol.

$$\begin{vmatrix} 1 & 2 & 3 & \cdots & n \\ 1 & x+1 & 3 & \cdots & n \\ 1 & 2 & x+1 & \cdots & n \\ . & . & . & . & . \\ 1 & 2 & 3 & \cdots & x+1 \end{vmatrix}$$

Birinchi satrni qolgan satrlardan ayirib quyidagini hosil qilamiz

$$\begin{vmatrix} 1 & 2 & 3 & \cdots & n \\ 0 & x-1 & 0 & \cdots & 0 \\ 0 & 0 & x-2 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & x-n+1 \end{vmatrix} = (x-1)\cdot(x-2)\cdot(x-3)\cdot\dots\cdot(x-n+1)$$

2.1.1-Misol.

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2-x & 1 & \cdots & 1 \\ 1 & 2 & 3-x & \cdots & 1 \\ . & . & . & . & . \\ 1 & 1 & 1 & \cdots & n+1-x \end{vmatrix}$$

Birinchi satrni qoldirib qolgan satrlarni birinchi satrdan ayiramiz va quyidagi kelib chiqadi

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & 1-x & 0 & \cdots & 0 \\ 0 & 0 & 2-x & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & n-x \end{vmatrix} = (1-x)\cdot(2-x)\cdot\dots\cdot(n-x) =$$

$$= (x-1)\cdot(x-2)\cdot\dots\cdot(x-n)\cdot(-1)^n$$

2.1.12-Misol

$$\begin{vmatrix} a_0 & a_1 & a_2 & \cdots & a_n \\ a_0 & x & a_2 & \cdots & a_n \\ a_0 & a_1 & x & \cdots & a_n \\ . & . & . & . & . \\ a_0 & a_1 & a_2 & \cdots & x \end{vmatrix}$$

Birinchi satrni qoldirib qolgan satrlardan birinchi satrni ayirib quyidagi ifodani olamiz

$$\begin{vmatrix} a_0 & a_1 & a_2 & \cdots & a_n \\ 0 & x-a_1 & 0 & \cdots & 0 \\ 0 & 0 & x-a_2 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & x-a_n \end{vmatrix}$$

Bu esa uchburchak ko'rinishiga keldi va bu quyidagi ifodaga teng

$$a_0 \cdot (x-a_1) \cdot (x-a_2) \cdot (x-a_3) \cdot \dots \cdot (x-a_n)$$

2.2 Chiziqli ko’paytuvchilarga ajratish usuli.

Determinant bir yoki bir nechta harflardan tashkil topgan harflardan tashkil topgan ko’phad ko’rinishida berilsa, almashtirish natijasida u chiziqli ko’paytuvchilarga ularning ko’paytmasiga ham bo’linishi aniqlanadi.

Determinantning ayrim hadlarini chiziqli ko’paytuvchining ko’paytma hadlari bilan solishtirsak, shu ko’paytuvchi determinantning ko’paytmadan xususiy holati topiladi va shu orqali determinantning ifodasi topiladi.

2.2.1-Misol. Determinantni hisoblang:

$$\begin{vmatrix} 0 & x & y & z \\ x & 0 & z & y \\ y & z & 0 & x \\ z & y & x & 0 \end{vmatrix}.$$

Agar birinichi ustunga qolgan barcha ustunlarga qolgan barcha ustunlarni qo’shsak determinant $x+y+z$ ga bo’linishi aniqlanadi; agar birinchi ustunga ikkinchi ustunni qo'shib uchinchi va to'rtinchi ustunlar ayirilsa, unda $y+z-x$ ko'paytuvchi kelib chiqadi; agar birinchi ustunga uchinchi ustunni qo'shib ikkinchi va to'rtinchi ustular ayirilsa, unda $x-y+z$ ko'paytuvchi kelib chiqadi; va nihoyat birinchi ustunga to'rtinchi ustunni qo'shib, ikkinchi va uchinchi ustunlar ayrilsa, unda $x+y-z$ ko'paytuvchi kelib chiqadi. x, y, z lar erkli o'zgaruvchi bo'lsa unda tortta ko'paytmalar o'zaro juft sodda degan xulosaga kelamiz. Bunda determinant ularning ko'paytmasiga bo'linadi $(x+y+z)(y+z-x)(x-y+z)(x+y-z)$.

Bu ko'paytuvchilarning tartibida z^4 hadi mavjud bo'lib, uning koeffitsienti -1 ga teng, determinantning ozi xuddi shu hdga ega bo'lib, z^4 ning koeffitsienti 1 ga teng.

Demak

$$D=-(x+y+z)(y+z-x)(x+z-y)(x+y-z)=x^4+y^4+z^4-2x^2y^2-2x^2z^2-2y^2z^2$$

2.2.2-Misol. n-tartibli Vandermond determinantni chiziqli ko'paytuvchilarga ajratish usuli bilan hisoblaymiz:

$$D_n=\begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{vmatrix}.$$

D_n ko'phadni $x_1 \cdots x_{n-1}$ ga bog'liq bo'lgan koeffitsientli erkli x_n ni ko'rib chiqsak, $x_n=x_1$, $x_n=x_2$,.... $x_n=x_{n-1}$ da nolda aylanishini ko'ramiz, shuning uchun x_n-x_1, x_n-x_2 ,.... x_n-x_{n-1} g abolish mumkin.

Bu barcha ko'paytuvchilar o'zaro sodda (chunki $x_1, x_2, \cdots x_n$ algebraik erkli).

Demak, D_n ularning ko'paytmasiga bo'linadi, ya'ni:

$$D_n=q(x_1,x_2,\cdots,x_n)=(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1}).$$

D_n ni so'ngi qator bo'yicha yoyib, u x_n ga nisbatan $n-1$ darajali ko'phad ekanligi ma'lum bo'ldi. Shu bilan birga x_n^{n-1} koeffitsient $x_1, x_2, \cdots x_{n-1}$ nomalumlardan D_{n-1} Vandermond determinantiga teng; chunki oxirgi tenglikning o'ng tarafidagi qavslar ko'paytmasi bir xil koeffitsientli x_n^{n-1} ni saqlaydi va $q(x_1, x_2, \cdots, x_n)$ ko'phad x_n ni saqlmaydi. Ikkala tomonda x_n ni saqlamaydi. Ikkala tomonda x_n^{n-1} koeffitsientlarni tenglashtirib, $D_{n-1} = q(x_1, x_2, \cdots, x_{n-1})$ ni keltirib chiqaramiz, bundan $D_n = D_{n-1}(x_n - x_1)(x_n - x_2)\cdots(x_n - x_{n-1})$ da n ni $n-1$ ga almashtirib bu tenglamani qo'llaganda:

$D_{n-1} = D_{n-2}(x_n - x_1)(x_n - x_2)\cdots(x_{n-1} - x_{n-2})$ ni hosil qilamiz.

D_{n-1} ifodani yuqoridagi D_n ifoda uchun qo'llaymiz, yuqoridagi fikrni qaytarib $x_2 - x_1$ ko'paytuvchiga ajratamiz, shundan keyin $D_1 = 1$ birinchi tartibli determinantga kelamiz.

Shunday qilib

$$D_n = (x_2 - x_1)(x_3 - x_2)\cdots(x_n - x_1)(x_n - x_2)\cdots(x_n - x_{n-1}) = \prod_{i \rhd j}(x_i - x_j).$$

2.2.3-Misol.

$$\begin{vmatrix} -x & a & b & c \\ a & -x & c & b \\ b & c & -x & a \\ c & b & a & -x \end{vmatrix}$$

Birinchi ustunga qolgan ustunlarni qo'shamiz

$$\begin{vmatrix} -x+a+b+c & a & b & c \\ -x+a+b+c & -x & c & b \\ -x+a+b+c & c & -x & a \\ -x+a+b+c & b & a & -x \end{vmatrix}$$

Birinchi satrni qolgan satrlardan ayirib quyidagini hosil qilamiz

$$\begin{vmatrix} -x+a+b+c & a & b & c \\ 0 & -x-a & c-b & b-c \\ 0 & c-a & -x-b & a-c \\ 0 & b-a & a-b & -x-c \end{vmatrix}$$

Birinchi ustun bo'yicha yoyamiz

$$(-x+a+b+c)\cdot\begin{vmatrix} -x-a & c-b & b-c \\ c-a & -x-b & a-c \\ b-a & a-b & -x-c \end{vmatrix}$$

birinchi ustunga qolgan ustunlarni qo'shamiz.

$$(-x+a+b+c)\cdot\begin{vmatrix} -x-a & 0 & b-c \\ c-a & -x-b+a-c & a-c \\ b-a & -x-c+a-b & -x-c \end{vmatrix}$$

Uchinchi satrdan uchinchi satrni ayiramiz

$$(-x+a+b+c)\cdot\begin{vmatrix} -x-a & 0 & b-c \\ c-a & -x-b+a-c & a-c \\ b-c & 0 & -x-a \end{vmatrix}$$

Ikkinchi ustun bo'yicha yoyamiz

$$(-x+a+b+c)\cdot(-x-b+a-c)\cdot\begin{vmatrix} -x-a & b-c \\ b-c & -x-a \end{vmatrix}$$

bu yerda birinchi ustunga ikkinchi ustunni qo'shamiz

$$(-x+a+b+c)\cdot(-x-b+a-c)\cdot\begin{vmatrix} -x-a+b-c & b-c \\ -x-a+b-c & -x-a \end{vmatrix}$$

Ikkinchi satrdan birinchi satrni ayirib ikkinchi satrga yozamiz

$$(-x+a+b+c)\cdot(-x-b+a-c)\cdot\begin{vmatrix}-x-a+b-c & b-c \\ 0 & -x-a-b+c\end{vmatrix}$$

Birinchi ustun bo'yicha yoyamiz va determinantimiz chiziqli ko'paytuvchilarga ajraladi

$$(x-a-b-c)\cdot(x-a+b+c)\cdot(x+a-b+c)\cdot(x+a+b-c)$$

2.2.4-Misol.

$$\begin{vmatrix}1 & 1 & 2 & 3 \\ 1 & 2-x^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9-x^2\end{vmatrix}$$

Ikkinchi satrdan birinchi satrni ayirib quyidagini hosil qilamiz

$$\begin{vmatrix}1 & 1 & 2 & 3 \\ 0 & 1-x^2 & 0 & 0 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9-x^2\end{vmatrix}$$

Ikkinchi satr bo'yicha yoyamiz

$$(1-x^2)\cdot\begin{vmatrix}1 & 2 & 3 \\ 2 & 1 & 5 \\ 2 & 1 & 9-x^2\end{vmatrix}$$

Uchichi satrdan ikkinchi satrni ayirib ushbuni olamiz

$$(1-x^2)\cdot\begin{vmatrix}1 & 2 & 3 \\ 2 & 1 & 5 \\ 0 & 0 & 4-x^2\end{vmatrix}$$

Determinantimiz chiziqli ko'paytuvchilarga ajlaldi

$$-3\cdot(1-x^2)\cdot(4-x^2)$$

2.2.5-Misol.

$$\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+z & 1 \\ 1 & 1 & 1 & 1-z \end{vmatrix}$$

Birinchi satrni qolgan satrlardan ayirib quyidagini olamiz

$$\begin{vmatrix} 1+x & 1 & 1 & 1 \\ -x & -x & 0 & 0 \\ -x & 0 & z & 0 \\ -x & 0 & 0 & -z \end{vmatrix}$$

Bu esa quyidgiga teng

$$x\cdot(-x)\cdot z\cdot(-z)\cdot\begin{vmatrix} \frac{1+x}{x} & -\frac{1}{x} & \frac{1}{z} & -\frac{1}{z} \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{vmatrix} = x^2\cdot z^2\cdot\begin{vmatrix} 1 & -\frac{1}{x} & \frac{1}{z} & -\frac{1}{z} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = x^2\cdot z^2$$

2.3 Rekurrent (rekurrent yoki o’shib boorish) usuli.

Bu usul dastroq tartibli shu turkumdagi determinantlar orqali satr yoki ustunga almashtirib yechish orqali determinantni ifodalashdan iborat. Shulardan kelib chiqqan tenglik rekurrent munosabat deyiladi.

Keyin bevosita determinantning umumiy holati bo’yicha rekurrent munosabatning o’ng tomoniga muvofiq pastroq tartibli determinantlar hisoblanadi. Yuqoriroq tartibli determinantlar rekurrent munosabatda ketma-ketlik bilan hisoblanadi. Agar ixtiyoriy n-tartibli determinantni hisoblash kerak bo’lsa, bir nechta past tartibli rekurrent munosabatni hisoblab, berilga ifodani umumiy ko’rinishini o’zgartirishga harakat qilinadi.

Keyinchalik ixtiyoriy n da bu ifodaning haqqoniyligini rekurrent munosabati va unda induksiya usuli orqali isbotlaymiz.

Umumiy ko’rinishini boshqa yo’ bilan hm olish mumkin. Buning uchun rekurrent munosabatga n-tartibli determinantni xuddi shu rekurrent munosabat n ni

$n-1$ ga almashtirib, determinantning $(n-1)$-tartibli ifodasi qo'yiladi, keyin determinantning $(n-2)$-tartibli ifodasi mos ravishda almashtiriladi, shu hol n-tartibli determinantning birinchi umumiy ko'rinishi ifodasi kelib chiqgunicha takrorlanadi ikkala usulni qo'shish ham mumkin, ya'ni ikkinchi usulni birinchi ifodani aniqlash uchun qo'llab, induksiya yo'li bilan n ning haqqoniyligini isbotlsh. Rekurrent munosabatli usul ko'rib chiqilayotgan usullardan eng kuchli usul hisoblanadi. Uni yordamida murakkabroq determinantlarni hisoblashda qo'llab ko'ramiz.

Determinantlarni rekurrent munosabatli usul orqali aniqlashdan oldin, misollar yechishning algoritmiga keltiruvchi xususiy holatini ko'rib chiqamiz. Rekurrent munosabat quyidagi ko'rinishga ega bo'lsin.

$$D_n = pD_{n-1} + qD_{n-2} \qquad n > 2\,, \qquad (2.3.1)$$

bu yerda p va q o'zgarmas (ya'ni n ga bog'liq bo'lgan kattalik).

$q=0$ da D_n geometrik progressiya hadi sifatida hisoblanadi: $D_n = p^{n-1}D_1$; bu yerda D_1 mazkur ko'rinishdagi birinchi tartibli determinanti.

Agar $q \neq 0$ va α, β kvadrat tenglamaning ildizlari bo'lsa $x^2 - px - q = 0$, unda $p = \alpha + \beta$, $q = -\alpha\beta$ va (2.3.1) tenglikni quyidagicha ifodalash mumkin:

$$D_n - \beta D_{n-1} = \alpha(D_{n-1} - \beta D_{n-2}) \qquad (2.3.2)$$

yoki

$$D_n - \alpha D_{n-1} = \beta(D_{n-1} - \alpha D_{n-2}) \qquad (2.3.3)$$

Avval $\alpha \neq \beta$ deb tasavvur qilaylik.

$(n-1)$-had uchun geometrik progressiya formulasi orqali (2.3.2) va (2.3.3) tengliklardan quyidagi kelib chiqadi:

$$D_n - \beta D_{n-1} = \alpha^{n-2}(D_2 - \beta D_1) \text{ va } D_n - \alpha D_{n-1} = \beta^{n-2}(D_2 - \alpha D_1)$$

bundan

$$D_n = \frac{\alpha^{n-1}(D_2 - \beta D_1) - \beta^{n-1}(D_2 - \alpha D_1)}{\alpha - \beta}$$

yoki

$$D_n = C_1\alpha^n + C_2\beta^n$$

bu yerda

$$C_1 = \frac{D_2 - \beta D_1}{\alpha(\alpha-\beta)}, \qquad C_2 = -\frac{D_2 - \alpha D_1}{\beta(\alpha-\beta)}. \qquad (2.3.4)$$

D_n ning oxirgi ko'rinishi yodda qolarli. U $n > 2$ uchun keltirib, bevosita $n = 1$ va $n = 2$ bo'lganda tekshiriladi. C_1 va C_2 larni (2.3.4) tenglikda keltirilgan ifodalardan emas, balki boshlang'ich shartlardan aniqlash ham mumkin.

$$D_1 = C_1\alpha + C_2\beta,$$

$$D_2 = C_1\alpha^2 + C_2\beta^2.$$

Endi esa $\alpha = \beta$ bo'lganda (2.3.2) va (2.3.3) tengliklar bir xil bo'lib qoladi

$$D_n - \alpha D_{n-1} = \alpha(D_{n-1} - \alpha D_{n-2})$$

bundan

$$D_n - \alpha D_{n-1} = A\alpha^{n-2}, \qquad (2.3.5)$$

$$A = D_2 - \alpha D_1.$$

Bu yerda n ni $n-1$ ga almashtiramiz, quyidagi kelib chiqadi:

$$D_{n-1} - \alpha D_{n-2} = A\alpha^{n-3}, \qquad (2.3.6)$$

bundan

$$D_{n-1} = \alpha D_{n-2} + A\alpha^{n-3}. \qquad (2.3.7)$$

(2.3.5) tenglikka yuqoridagi ifodalarni kiritamiz:

$$D_n = \alpha^2 D_{n-2} + 2A\alpha^{n-2} \qquad (2.3.8)$$

ni hosil qilamiz. Shu usulni bir necha marta qo'llab quyidagilarni hosil qilamiz:

$$D_n = \alpha^{n-1}D_1 + (n-1)A\alpha^{n-2} \qquad (2.3.9)$$

$$D_n = \alpha^n\left[(n-1)C_1 + C_2\right] \qquad (2.3.10)$$

bu yerda

$$C_1 = \frac{A}{\alpha^2}, \qquad C_2 = \frac{D_1}{\alpha} \qquad (\alpha \neq 0, q \neq 0) \qquad (2.3.11)$$

Endi rekurrent munosabatga keladigan bazi bir misollarni ko'rib chiqamiz.

2.3.1-Misol.

$$\begin{vmatrix} a_0 & a_1 & a_2 & \cdots & a_n \\ -y_1 & x_1 & 0 & \cdots & 0 \\ 0 & -y_2 & x_2 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & x_n \end{vmatrix}$$

Birinchi ustun bo'yicha yoyib quyidagini olamiz

$$a_0 \cdot \begin{vmatrix} x_1 & 0 & 0 & \cdots & 0 \\ -y_2 & x_2 & 0 & \cdots & 0 \\ 0 & -y_3 & x_3 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & x_n \end{vmatrix} + y_1 \cdot \begin{vmatrix} a_1 & a_2 & a_3 & \cdots & a_n \\ -y_2 & x_2 & 0 & \cdots & 0 \\ 0 & -y_3 & x_3 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & x_n \end{vmatrix}$$

Bu yerda rekurrent munosabat o'rnatamiz va determinantni metodika qoidasiga asosan hisoblaymiz

$$D_{n+1} = a_0 \cdot x_1 \cdot x_2 \cdot \cdots \cdot x_n + y_1 \cdot D_n$$
$$D_n = a_1 \cdot x_2 \cdot x_2 \cdot \cdots \cdot x_n + y_2 \cdot D_{n-1}$$
$$D_{n+1} = a_0 \cdot x_1 \cdot x_2 \cdot \cdots \cdot x_n + y_1 \cdot (a_1 \cdot x_2 \cdot x_2 \cdot \cdots \cdot x_n + y_2 \cdot D_{n-1})$$
$$\vdots$$
$$D_{n+1} = a_0 \cdot x_1 \cdot x_2 \cdot \cdots \cdot x_n + a_1 \cdot y_1 \cdot x_2 \cdot \cdots \cdot x_n + a_2 \cdot y_1 \cdot y_2 \cdot x_3 \cdot \cdots \cdot x_n +$$
$$+ \cdots + a_n \cdot y_1 \cdot y_2 \cdot \cdots \cdot y_n$$

2.3.2-Misol.

$$\begin{vmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & a_1 & 0 & \cdots & 0 \\ 1 & 0 & a_2 & \cdots & 0 \\ . & . & . & . & . \\ 1 & 0 & 0 & \cdots & a_n \end{vmatrix}$$

Ikkinchi satr bo'yicha yoyib quyidagini olamiz

$$-\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & a_2 & 0 & \cdots & 0 \\ 0 & 0 & a_3 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & a_n \end{vmatrix} + a_1 \cdot \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & a_2 & 0 & \cdots & 0 \\ 1 & 0 & a_3 & \cdots & 0 \\ . & . & . & . & . \\ 1 & 0 & 0 & \cdots & a_n \end{vmatrix}$$

Bu yerda rekurrent munosabat o’rnatib determinantni hisoblaymiz

$$D_{n+1} = a_1 \cdot D_n - a_2 \cdot a_3 \cdot \cdots \cdot a_n$$
$$D_n = a_2 \cdot D_{n-1} - a_3 \cdot a_4 \cdot \cdots \cdot a_n$$
$$D_{n-1} = a_3 \cdot D_{n-2} - a_4 \cdot a_5 \cdot \cdots \cdot a_n$$
$$D_{n-2} = a_4 \cdot D_{n-3} - a_5 \cdot a_6 \cdot \cdots \cdot a_n$$
$$\vdots$$
$$D_{n+1} = a_1 \cdot a_2 \cdot \cdots \cdot a_{n-1} - a_1 \cdot a_3 \cdot \cdots \cdot a_n - \cdots - a_2 \cdot a_3 \cdot \cdots \cdot a_n =$$
$$= -a_1 \cdot a_2 \cdot \cdots \cdot a_n \cdot \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \cdots + \frac{1}{a_n} \right)$$

2.3.3-Misol.

$$\begin{vmatrix} 2 & 1 & 0 & \cdots & 0 \\ 1 & 2 & 1 & \cdots & 0 \\ 0 & 1 & 2 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 2 \end{vmatrix}$$

Birinchi satr bo’yicha yoyib quyidagini olamiz

$$2 \cdot \begin{vmatrix} 2 & 1 & 0 & \cdots & 0 \\ 1 & 2 & 1 & \cdots & 0 \\ 0 & 1 & 2 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 2 \end{vmatrix} - 1 \cdot \begin{vmatrix} 2 & 1 & 0 & \cdots & 0 \\ 1 & 2 & 1 & \cdots & 0 \\ 0 & 1 & 2 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 2 \end{vmatrix}$$

Rekurrent munosabatni o’rnatamiz

$$D_n = 2D_{n-1} - D_{n-2}$$
$$x^2 - 2x + 1 = 0$$
$$(x-1)^2 = 0$$
$\alpha = \beta = 1$ ekanligidan
$$D_n = \alpha^n \cdot (C_1 + (n-1) \cdot C_2)$$
$$C_1 = \frac{D_1}{\alpha},$$
$$C_2 = \frac{A}{\alpha^2}$$
$$A = D_2 - \alpha \cdot D_1 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - 2 = 1$$
$$C_1 = 2 \qquad C_2 = 1 \qquad D_n = n + 1$$

2.3.4-Misol.

$$\begin{vmatrix} 3 & 2 & 0 & \cdots & 0 \\ 1 & 3 & 2 & \cdots & 0 \\ 0 & 1 & 3 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 3 \end{vmatrix}$$

Birinchi qator bo'yicha yoyib chiqib rekurent munosabatni topamiz

$$D_n = 3D_{n-1} - 2D_{n-2}$$

$$x^2 - 3x + 2 = 0$$

$$\alpha = 1$$

$$\beta = 2$$

$$D_n = C_1\alpha^n + C_2\beta^n$$

$$C_1 = \frac{D_2 - \beta D_1}{\alpha(\alpha-\beta)} = \frac{\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} - 2\cdot 3}{-1} = -1$$

$$C_2 = \frac{D_2 - \alpha D_1}{\beta(\beta-\alpha)} = 2$$

$$D_n = 2^{n+1} - 1$$

2.3.5-Misol.

$$\begin{vmatrix} 7 & 5 & 0 & \cdots & 0 \\ 2 & 7 & 5 & \cdots & 0 \\ 0 & 2 & 7 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 7 \end{vmatrix}$$

Birinchi qator bo'yicha yoyib chiqib rekurent munosabatni topamiz.

$$D_n = 7D_{n-1} - 10D_{n-2}$$

$$x^2 - 7x + 10 = 0$$

$$\alpha = 2$$

$$\beta = 5$$

$$D_n = C_1\alpha^n + C_2\beta^n$$

$$C_1 = \frac{D_2 - \beta D_1}{\alpha(\alpha-\beta)} = \frac{\begin{vmatrix} 7 & 5 \\ 1 & 7 \end{vmatrix} - 5\cdot 7}{-6} = -\frac{2}{3}$$

$$C_2 = \frac{D_2 - \alpha D_1}{\beta(\beta-\alpha)} = \frac{5}{3}$$

$$D_n = \frac{5^{n+1} - 2^{n+1}}{3}$$

2.3.6-Misol.

1-usul

$$\begin{vmatrix} 5 & 6 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 4 & 5 & 2 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 3 & 2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 3 & 2 & \cdots & 0 & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & \cdots & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 3 \end{vmatrix}$$

Bu n-tartibli determinantni quyidagiga keltiramiz.

$$5\cdot\begin{vmatrix}5&2&0&0&0&\cdots&0&0\\1&3&2&0&0&\cdots&0&0\\0&1&3&2&0&\cdots&0&0\\0&0&1&3&2&\cdots&0&0\\.&.&.&.&.&.&.&.\\0&0&0&0&0&\cdots&3&2\\0&0&0&0&0&\cdots&1&3\end{vmatrix}_{n-1}-6\cdot\begin{vmatrix}4&2&0&\cdots&0\\0&3&2&\cdots&0\\0&1&3&\cdots&0\\.&.&.&.&.\\0&0&0&\cdots&3\end{vmatrix}_{n-1}$$

Bundan quyidagini olamiz.

$$25\cdot\begin{vmatrix}3&2&0&\cdots&0\\1&3&2&\cdots&0\\0&1&3&\cdots&0\\.&.&.&.&.\\0&0&0&\cdots&3\end{vmatrix}_{n-2}-10\cdot\begin{vmatrix}3&2&0&\cdots&0\\1&3&2&\cdots&0\\0&1&3&\cdots&0\\.&.&.&.&.\\0&0&0&\cdots&3\end{vmatrix}_{n-3}-$$

$$-24\cdot\begin{vmatrix}3&2&0&\cdots&0\\1&3&2&\cdots&0\\0&1&3&\cdots&0\\.&.&.&.&.\\0&0&0&.....&3\end{vmatrix}_{n-2}$$

Birinchi qator bo'yicha yoyib chiqib rekurent munosabatni topildi.

$$\begin{vmatrix}3&2&0&\cdots&0\\1&3&2&\cdots&0\\0&1&3&\cdots&0\\.&.&.&.&.\\0&0&0&\cdots&3\end{vmatrix}=D_n$$

$$D_n = 3D_{n-1} - 2D_{n-2}$$
$$x^2 - 3x + 2 = 0$$
$$\alpha = 1$$
$$\beta = 2$$
$$D_n = C_1\alpha^n + C_2\beta^n$$
$$C_1 = \frac{D_2 - \beta D_1}{\alpha(\alpha-\beta)} = \frac{\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} - 2\cdot 3}{-1} = -1$$
$$C_2 = \frac{D_2 - \alpha D_1}{\beta(\beta-\alpha)} = 2$$
$$D_n = 2^{n+1} - 1$$
$$D_{n-1} = 2^n - 1$$
$$D_{n-2} = 2^{n-1} - 1$$
$$D_{n-3} = 2^{n-2} - 1$$

$$25\cdot D_{n-2} - 10\cdot D_{n-3} - 24\cdot D_{n-2} = D_{n-2} - 10\cdot D_{n-3}$$
$$= (2^{n-1} - 1) - 10\cdot(2^{n-2} - 1) = 9 - 2^{n+1}$$

2-usul

$$D_1 = 5$$
$$D_2 = \begin{vmatrix} 5 & 6 \\ 4 & 5 \end{vmatrix} = 1$$
$$\alpha = 1$$
$$\beta = 2$$
$$C_1 = \frac{D_2 - \beta D_1}{\alpha(\alpha-\beta)} = 9$$
$$C_2 = \frac{D_2 - \alpha D_1}{\beta(\beta-\alpha)} = -4$$

α va β $\begin{vmatrix} 3 & 2 & 0 & \cdots & 0 \\ 1 & 3 & 2 & \cdots & 0 \\ 0 & 1 & 3 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 3 \end{vmatrix}_{n-2}$ determinantnga tegishli shuning uchun

$D_n = C_1\alpha^{n-1} + C_2\beta^{n-1} = 9 - 2^{n+1}$ bo'ladi.

2.3.7-Misol.

$$\begin{vmatrix} 1 & 2 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 3 & 4 & 3 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 2 & 5 & 3 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 2 & 5 & 3 & \cdots & 0 & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & \cdots & 5 & 3 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 2 & 5 \end{vmatrix}$$

Bu determinantni rekurent ko'rinishiga keltirib olishimiz kerak.

$$D_n = -2 \cdot \begin{vmatrix} 5 & 3 & 0 & \cdots & 0 \\ 2 & 5 & 3 & \cdots & 0 \\ 0 & 2 & 5 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 5 \end{vmatrix}_{n-2} - 6 \cdot \begin{vmatrix} 5 & 3 & 0 & \cdots & 0 \\ 2 & 5 & 3 & \cdots & 0 \\ 0 & 2 & 5 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 5 \end{vmatrix}_{n-3}$$

rekurent usulga keltirib oldik

$$\begin{vmatrix} 5 & 3 & 0 & \cdots & 0 \\ 2 & 5 & 3 & \cdots & 0 \\ 0 & 2 & 5 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 5 \end{vmatrix}_{n-2} = 3^{n-1} - 2^{n-1}$$

$$\begin{vmatrix} 5 & 3 & 0 & \cdots & 0 \\ 2 & 5 & 3 & \cdots & 0 \\ 0 & 2 & 5 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 5 \end{vmatrix}_{n-3} = 3^{n-2} - 2^{n-2}$$

Bundan $D_n = 5 \cdot 2^{n-1} - 4 \cdot 3^{n-1}$ kelib chiqadi.

2-usul.

$$D_1 = 1$$

$$D_2 = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2$$

$$\alpha = 2$$

$$\beta = 3$$

$$C_1 = \frac{D_2 - \beta D_1}{\alpha(\alpha - \beta)} = \frac{5}{2}$$

$$C_2 = \frac{D_2 - \alpha D_1}{\beta(\beta - \alpha)} = -\frac{4}{3}$$

α va β $\begin{vmatrix} 5 & 3 & 0 & \cdots & 0 \\ 2 & 5 & 3 & \cdots & 0 \\ 0 & 2 & 5 & \cdots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \cdots & 5 \end{vmatrix}_{n-2}$ determinantnga tegishli shuning uchun

$D_n = C_1\alpha^{n-1} + C_2\beta^{n-1} = 5\cdot 2^{n-1} - 4\cdot 3^{n-1}$ bo'ladi.

2.3.8-Misol.

$$\begin{vmatrix} \alpha+\beta & \alpha\beta & 0 & 0 & \cdots & 0 \\ 1 & \alpha+\beta & \alpha\beta & 0 & \cdots & 0 \\ 0 & 1 & \alpha+\beta & \alpha\beta & \cdots & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \cdots & \alpha+\beta \end{vmatrix}$$

Birinchi qator bo'yicha yoyib chiqib rekurent munosabatni topamiz.

$$D_n = (\alpha+\beta)\cdot D_{n-1} - \alpha\beta\cdot \mathrm{D}_{\text{n-2}}$$

$$x^2 - (\alpha+\beta)x + \alpha\beta = 0$$

Bu yerda bu kvadrat tenglamaning ildizlari $x_1 = \alpha$ $x_2 = \beta$

$$D_n = C_1\alpha^n + C_2\beta^n = \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta}$$

II Bobning Xulosasi.

Mazkur bitiruv – malakaviy ishning ikkinchi bobida ba’zi misollarni yechishga tadbiqlari va ularning yechimlarini topish usullari yoritilgan.

Jumladan yuqori tartibli determinantlarni uchburchak ko'rinishiga olib kelib yechish yordamida topish usuli ko’rsatilgan bo’lsa $D_n = pD_{n-1} + qD_{n-2}$ ko’rinishdagi determinantlarni yechimlarini topish rekurrent munosabatlar yordamida amalga oshirilgan. Bu bobda shuningdek ba'zi bir misollar ikkixil usul yordamida ham yechib ko'rsatilgan**.**

Xotima.

Ishning birinchi bobida mavzuni bayon qilishda zarur bo`ladigan ma`lumotlar keltirilgan. Bu bob ikki paragrafdan iborat bo`lib, birinchi paragrafda ikkinchi va uchinchi tartibli determinantlar, ularning xossalari va unga doir misollar, ikkinchi paragrafda Yuqori tartibli determinantlar va ularning xossalari va chiziqli tenglamalar sistemasini determinant (Kramer) usuli bilan yechish va unga doir misollar yechib ko'rsatilgan. Mazkur bitiruv – malakaviy ishning ikkinchi bobida ba'zi misollarni yechishga tadbiqlari va ularning yechimlarini topish usullari yoritilgan.

Jumladan yuqori tartibli determinantlarni uchburchak ko'rinishiga olib kelib yechish yordamida topish usuli ko'rsatilgan bo'lsa $D_n = pD_{n-1} + qD_{n-2}$ ko'rinishdagi determinantlarni yechimlarini topish rekurrent munosabatlar yordamida amalga oshirilgan. Bu bobda shuningdek ba'zi bir misollar ikkixil usul yordamida ham yechib ko'rsatilgan.

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