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REFERAT

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Mavzu: Ko`p hadli matritsalar

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Matritsalar

1.1.1-ta'rif: P maydonning mn ta a_{ij} sonlaridan tuzilgan

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . & . & \dots & . \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

ko'rinishidagi ifoda P maydon ustidagi matritsa deyiladi.

Matritsa elemetlarining gorizontal qatorlari uning satrlari deb,vertikal qatorlari ustunlari deb ataladi. Shunday qilib A matritsada m ta satr va n ta ustun bor. Matritsani tashkil etuvchi a_{ij} sonlarni uning elementlari deyiladi.Har bir a_{ij} elementning birinchi indeksi bu element turgan satrining nomerini ,ikkinchi indeksi esa ustunning nomerini bildiradi.Demak, a_{ij} element i-satr va j- ustunda turadi.Amatritsada satrlar soni ustunlar sonidan kichik,teng yoki katta (ya'ni $m < n, m = n, m > n$) bo'lishi mumkin.$m = n$ bo'lgan holda A ni n-tartibli kvadrat matritsa deyiladi.Kvadrat matritsada $a_{11}, a_{22,} \dots, a_{nn}$ elementlar matritsaning birinchi (bosh) diogonalini $a_{n1} a_{2n-1} \dots a_{n-1,2} a_{n-1,1}$ elementlar esa ikkinchi tashkil etadi.$m = n$ bo'lgan holda A ni to'g'ri burchakli matritsa deyiladi.To'g'ri burchakli matriysani m satrli va n ustunli matritsa yoki qisqaroq (m) matritsa deb ham aytiladi.

Xususiy holda, matritsa bir satrli va$n$ ustunli yoki m satrli va bir ustunli bo'lishi, ya'ni

$$A = (a_{11}, a_{12}, \dots, a_{1n})$$

yoki

$$A = \begin{pmatrix} a_{11} \\ a_{12} \\ . \\ a_{m1} \end{pmatrix}$$

ko'rinishidagi matritsani ifodalashi mumkin.

Ikkilasi ham m satrli va n ustunli A va B matritsalarni nomdosh (bir nomli) matritsalar deymiz.

Faqat nomdosh matritsalargina (tenglik sharti) teng bo'lishi mumkin . A va B matritsalarning tenglik sharti quyidagichadir. A ning har bir a_{ij} elementi B ning har bir b_{ij} elementiga teng bo'lgandagina bu ikki matritsa teng ya'ni $A = B$ bo'ladi. Demak, birorta i va j lar uchun $a_{ij} \neq b_{ij}$ bo'lsa A va B lar teng bo'lmagan matritsalarni tasvirlaydi. Matritsalarning teng emasligi $A \neq B$ kabi belgilanadi.

A matritsaning satrlari m ta n o'lchovli gorizontal

$$\begin{aligned} a_1 &= (a_{11}, a_{12}, \ldots., a_{1m}) \\ a_2 &= (a_{21}, a_{22}, \ldots, a_{2m}) \\ &\ldots\ldots\ldots \\ a_m &= (a_{m1}, a_{m2}, .., a_{mn}) \end{aligned}$$

vektorlarni, ustunlari esa n ta m o'lchovli vertikal vektorlarni tashkil etadi. Bu so'nggi vektorlarni gorizontal vektorlardan farq qilishi uchun

$$\begin{aligned} a^1 &= (a_{11}, a_{21}, \ldots, a_{m1}) \\ a^2 &= (a_{12}, a_{22}, \ldots, a_{m2}) \\ &\ldots\ldots. \\ a^n &= (a_{1n}, a_{2n}, \ldots, a_{mn}) \end{aligned}$$

ko'rinishda belgilaymiz.

1.1.2-ta'rif Quyidagi almashtirishlarga matritsani elementar almashtirishlar deyiladi.

1. Satrlarni ustunlar qilib va ustunlarni satrlar qilib yozish (matritsani transponirlash).

2. Ikki satr (yoki ustun)ni o'zaro o'rin almashtirish.

3. Satr (ustun) elementlarini noldan farqli songa ko'paytirish.

4. Satr (ustun) elementlarini istalgan songa ko'paytirib, boshqa satr (ustun) ning mos elementlariga qo'shish.

Blok matritsalar.

$A = \|a_{ij}\|$ matritsa gorizontal va vertikal chiziqlar yordamida to'g'ri burchakli alohida kataklarga ajratilgan bo'lib, ularning har biri kichikroq tartibli martitsalardan iborat bo'lib, dastlabki matritsaning bloki deyiladi.Bundayholatdadastlabki A matritsani elementlari $A_{\alpha\beta}$ bloklardan iborat yangi blok matritsa deb nomlanuvchi $A = \|A_{\alpha\beta}\|$ matritsa sifatida qarash imkoni bo'ladi.Ko'rsatilgan elementlarni katta lotin harflari bilan belgilaymiz,ularni ikkita indiks bilan nomerlab,birinchi indiks "blok" ustun nomerini ifodalaydi deb hisoblaymiz,bunday belgilash bu elementlarning umuman olganda qandaydir sonlar emas,balki matritsalar ekanini bildiradi.

Masalan:

$$A = \left\| \begin{array}{cccc|cc} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \end{array} \right\|$$

matritsani elementlari

$$A_{11} = \begin{Vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{Vmatrix} \qquad A_{12} = \begin{Vmatrix} a_{15} & a_{16} \\ a_{25} & a_{26} \end{Vmatrix}$$

$$A_{21} = \begin{Vmatrix} a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ a_{51} & a_{52} & a_{53} & a_{54} \end{Vmatrix} \qquad A_{22} = \begin{Vmatrix} a_{35} & a_{36} \\ a_{45} & a_{46} \\ a_{55} & a_{56} \end{Vmatrix}$$

bloklardan iborat bo'lgan

$$A = \begin{Vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{Vmatrix}$$

matritsa sifatida qarash mumkin.

Shunisi diqqatga sazovorki,blok matritsalar ustida bajariladigan asosiy amallar sonli matritsalar ustida bajariladigan amallar qoidalari bo'yicha amalga oshiriladi.

Haqiqatda $A=\|a_{ij}\|$ matritsa blok bo'lib, $A_{\alpha\beta}$ blok elementlardan iborat bo'lsa, u holda $\lambda A=\|\lambda a_{ij}\|$ matritsaga $\lambda A_{\alpha\beta}$ blok element mos keladi.

Agar A va B bir xil tartibli matritsalar bo'lib, va bir xil rarzda bloklarga ajratilgan bo'lsa, u holda A va B matrirsalarning yig'indisiga elementlari $A_{\alpha\beta}$ va $B_{\alpha\beta}$ lardan iborat bo'lgan $C_{\alpha\beta}=A_{\alpha\beta}+B_{\alpha\beta}$ matritsa mos keladi.

Endi A va B lar 2 shunday blok matritsalar bo'lsinki,har bir $A_{\alpha\beta}$ blokning ustunlari soni $B_{\beta\gamma}$ blokning satrlari soniga teng bo'lsin deylik.Demak, istalgan α,β,γ lar uchun $A_{\alpha\beta}\cdot B_{\beta\gamma}$ matritsalarning ko'paytmasi aniqlangan bo'ladi.U vaqtda $C=A\cdot B$ tenglik $C_{\alpha\beta}$ elementlari

$$C_{\alpha\gamma}=\sum_{\beta}A_{\alpha\beta}B_{\beta\gamma}$$

formula bilan aniqlangan matritsani ifodalaydi.

Bu formulani isbotlash uchun uning chap va o'ng tomonlarini odatdagi elementlari sonlardan iborat matritsalar ko'paytmasi ko'rinishida yozib olish yetarli.Blok matritsalarni qo'llashga doir misol sifatida kvadrat matritsalarning to'g'ri yig'indisi tushunchasiga to'xtalamiz.

Mos ravishta tartibli 2 ta kvadrat matritsalarning to'g'ri yig'indisi deb.Tartibli $C=\begin{Vmatrix}A0\\0B\end{Vmatrix}$ kvadrat blok matritsalarga aytiladi. Matritsalarning yig'indisini ko'rinishida belgilaymizMatritsalarning to'gri yig'indisi to'g'riligidan ko'rinadiki, umuman olganda bu yig'indi o'rin almashtirish xossasiga ega emas.Shunday bo'lsada to'g'ri yig'indining

$$(A\oplus B)\oplus C=A\oplus(B\oplus C)$$

assaotsiativlik xossasiga egaekanligini ko'rsatishimiz mumkin.

Blok matritsalar ustida amallar xossalari yordamida matritsalarning to'g'ri yig'indisi bilan matritsalarning odatdagi yig'indisi va ko'paytmasi amallari orasidagi bo'g'lanishni ifodalovchi ushbu formulalarni ham isbotlash mumkin:

$$(A_m\oplus A_n)+(B_m\oplus B_n)=(A_m+B_m)\oplus(A_n+B_n),$$

$A_m \oplus A_n) \cdot (B_m \oplus B_n) = A_m B_m \oplus A_{nB_n}$

Bu formulalarda mos ravishta m va n – tartibli kvadrat matritsalar A_n vaB_n lar esa n-tartibli kvadrat matritsalar.

Ko'p hadli matritsa tushunchasi.

1.3.1-ta'rif:Elementlari λ noma'lumning P sonlar maydonidagi ko'plaridan iborat bo'lgan n-tartibli

$$F(\lambda) = \left\| \begin{matrix} f_{11}(\lambda) & f_{12}(\lambda) & \cdot & f_{1n}(\lambda) \\ f_{21}(\lambda) & f_{22}(\lambda) & \cdot & f_{2n}(\lambda) \\ \cdot & \cdot & \cdot & \cdot \\ f_{n1}(\lambda) & f_{n2}(\lambda) & \cdot & f_{nn}(\lambda) \end{matrix} \right\| \qquad (1)$$

kvadrat matritsa ko'p hadli matritsa yoki, λ matritsa deb ataladi.

λ -matritsani qisqacha $F(\lambda), \phi(\lambda), \varphi(\lambda), \ldots$ bilan belgilaymiz.

λ –matritsaning darajasi deb matrisa tarkibiga kirgan $f_{ik}(\lambda)$ ko'phadlarning darajalaridan eng yuqorisiga aytiladi.

m- darajali λ - matritsani ko'p had shaklida quyidagich tasvirlash mumkin:

$$F(\lambda) = A_0(\lambda^m) + A_1(\lambda^{m-1}) + \ldots + A_{m-1}(\lambda) + A_m \qquad (2)$$

bunda A_i -faqat sonlardan tuzilgan $n -$ tartiblimatritsalardir.

Bu ko'phad, odatda, matritsali ko'phad deb ataladi.

Masalan,(1.3.1 misol) ushbu

$$F = \left\| \begin{matrix} \lambda^2 - \lambda & \lambda & 2\lambda^3 + \lambda^2 - 4\lambda + 7 \\ \lambda^2 & 5 & 8\lambda^2 + \lambda \\ -3\lambda^2 & \lambda^2 + 1 & 1 \end{matrix} \right\|$$

matritsani avval

$$F(\lambda) = \left\| \begin{matrix} 1\cdot\lambda^3 + 0\cdot\lambda^2 - 1\cdot\lambda + 0 & 0\cdot\lambda^3 + 0\cdot\lambda^2 + 1\cdot\lambda + 0 & 2\cdot\lambda^3 + 1\cdot\lambda^2 - 4\cdot\lambda + 7 \\ 0\cdot\lambda^3 + 1\cdot\lambda^2 + 2\cdot\lambda + 1 & 0\cdot\lambda^3 + 0\cdot\lambda^2 + 0\cdot\lambda + 5 & 0\cdot\lambda^3 + 8\cdot\lambda^2 + 1\cdot\lambda + 0 \\ 0\cdot\lambda^3 - 3\cdot\lambda^2 + 0\cdot\lambda + 0 & 0\cdot\lambda^3 + 1\cdot\lambda^2 + 0\cdot\lambda + 1 & 0\cdot\lambda^3 + 0\cdot\lambda^2 + 0\cdot\lambda + 1 \end{matrix} \right\|$$

ko’rinishda yozib,so’ngra matritsani qo’shish va sonlarga ko’paytirish qoidalaridan foydalanib, quyidagi matritsali ko’phadni hosil qilamiz:

$$F(\lambda) = A_0\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3$$

bunda

$$A_0 = \begin{Vmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{Vmatrix} \qquad A_1 = \begin{Vmatrix} 0 & 0 & 1 \\ 1 & 0 & 8 \\ -3 & 1 & 0 \end{Vmatrix}$$

$$A_2 = \begin{Vmatrix} -1 & 1 & -4 \\ 2 & 0 & 1 \\ 0 & 0 & 0 \end{Vmatrix} \qquad A_3 = \begin{Vmatrix} 0 & 0 & 7 \\ 1 & 5 & 0 \\ 0 & 1 & 1 \end{Vmatrix}$$

Aksincha,(1.3.2) ko’rinishida, ya’ni matritsali ko’phad shaklida yozilgan λ-matritsani har vaqt (1.3.1) ko’rinishiga keltira olamiz.

Masalan,(1.3.2- misol) ko’phad shaklida

$$F(\lambda) = \begin{Vmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \\ -1 & 4 & -6 \end{Vmatrix} \cdot \lambda^2 + \begin{Vmatrix} -1 & 1 & 1 \\ 0 & -2 & 0 \\ 5 & 6 & 7 \end{Vmatrix} \cdot \lambda + \begin{Vmatrix} 0 & -1 & 3 \\ 2 & 5 & 1 \\ 1 & 0 & 9 \end{Vmatrix}$$

matritsa ushbu

$$F(\lambda) = \begin{Vmatrix} \lambda^2 - \lambda & 3\lambda^2 + \lambda - 1 & \lambda + 3 \\ 2 & 2\lambda^2 - 2\lambda + 5 & \lambda^2 + 1 \\ -\lambda^2 + 5\lambda + 1 & 4\lambda^2 + 6\lambda & -6\lambda^2 + 7\lambda + 9 \end{Vmatrix}$$

matritsani ifodalaydi.

Ikkita matritsani teng bo’linishi shartidan darhol ikkita matritsali ko’phadning teng bo’lish sharti kelib chiqadi.

Ikkita

$$F(\lambda) = A_0\lambda^m + A_1\lambda^{m-1} + \ldots + A_{m-1}\lambda + A_m,$$

$$\Phi(\lambda) = B_0\lambda^m + B_1\lambda^{m-1} + \ldots + B_{m-1}\lambda + B_m$$

matritsali ko'phad faqat $B_0 = A_0$, $B_1 = A_1, \ldots, B_m = A_m$ bo'lgandagina teng deb hisoblanadi.

$\lambda -$ matritsa tushunchasiga butunlay boshqa tomondan qarash mumkin. P maydon ustida $n -$tartibli matritsaviy λ ko'phad deb koeffisientlari vazifasini elementlari P maydondan oligan bir xil $n -$tartibli kadrat matritsalar o'ynaydigan λ ning ko'phadiga aytamiz:uning umumiy ko'rinishi

$$A_0\lambda^k + A_1\lambda^{k-1} + \cdots + A_{k-1}\lambda + A_k$$

$\lambda -$ matritsaga matritsaviy ko'phad deb qarash, $\lambda -$ matritsalar uchun sonly ko'phadlarga bo'linish nazariyasiga o'xshash, lekin matritsalarni ko;paytirishning nokommutativligi va nolning bo'luvchilari mavjudligi sababli murakkablashgan bo'linish nazariyasini rivojlantirish imkonini beradi.Biz qoldiqli bo'lish algoritmi masalasi bilangina cheklanamiz.

λ -matritsaning normal diagonal shakli.

λ-matritsani elementar almashtirish haqida tushuncha beramiz.

2.1.1-ta'rif.

λ- matritsani elementar almashtirish deb,

1.Matritsani ikki yo'li (yoki ikki ustuni)ni bir biri bilan almashtirish

2.Yo'l (yoki ustun) elementini noldan farqli istalgan songa ko'paytirish hamda

3.Bir yo'l (yoki ustun) elementlarini istalgan $\varphi(\lambda)$ko'phadga ko'paytirib, boshqa yo'l (ustun)ning mos elementlariga qo'shishga aytiladi.

Bu elementar almashtirishlarni λmatrirsaga istalgan tartibda va har qancha narta tadbiq etish mumkin.

Elemetar almashtirishlar bajarilgandan keyin F(λ) matritsa o'zgarib, yangi $\Phi(\lambda)$matritsa hosil bo'ladi.Masalan:

$$F(\lambda)=\begin{Vmatrix} \lambda^3 & 2 & \lambda+1 \\ -\lambda^2 & \lambda & 4 \\ 0 & \lambda^2-3 & 5\lambda^4 \end{Vmatrix} \qquad (2.1.1)$$

matritsada 1- va 3- yo'lni almashtirsak 2-yo'lni 8-ko'paytirsak,so'ngra 2-ustunni λ^2ga ko'paytirib,1-ustunga qo'shsak,

$$\Phi(\lambda)=\begin{Vmatrix} \lambda^4-3\lambda^2 & \lambda^2-3 & 5\lambda^4 \\ -8\lambda^3+8\lambda^2 & -8\lambda & -32 \\ \lambda^3+2\lambda^2 & 2 & \lambda+1 \end{Vmatrix} \qquad (2.1.2)$$

matritsa hosil bo'ladi.

2.1.2-ta'rif: $F(\lambda)$matritsani chekli son marta elementar almashtirish natijasida hosil qilingan $\Phi(\lambda)$ matritsa $F(\lambda)$matritsaga ekvivalent matritsa deyiladi.

Masalan (2.1.1.) matritsa (2.1.2) matritsaga ekvivalentdir.

Bunda teskari almashtirish tushunchasini kiritish lozim.Agar $F(\lambda)$matritsada i- va k- yo'llar almashtirilib, $\Phi(\lambda)$ matritsa hosil qilingan bo'lsa, $\Phi(\lambda)$ matritsada i- va k- yo'llarni almashtirib yana $F(\lambda)$matritsaga qaytish mumkin.Mana shunday holda so'nggi almashyirishni avvalgiga nisbatan teskari almashtirish deyikadi.Shuningdek, $F(\lambda)$ matritsaning i-yo'lini $m\neq 0$ ko'paytirish bilan $\Phi(\lambda)$ matritsani hosil qilgan bo'lsak, $\Phi(\lambda)$ ning iyo'lini $\frac{1}{m}$ ga ko'paytirib $F(\lambda)$ matritsaga qaytamiz.Mana bu holda ham keyingi almshtirish avvalgiga teskari almashtirish deyiladi.Nihoyat, $F(\lambda)$ matritsaning i-yo'lini $\frac{1}{m}$ ga ko'paytirib,k-yo'liga qo'shish natijasida $\Phi(\lambda)$ hosil bo'lsa u holda $\Phi(\lambda)$ matritsadan $F(\lambda)$ matritsaga qaytish uchun bajariladigan teskari almashtirish $\Phi(\lambda)$ning i-yo'lini - $\varphi(\lambda)$ga ko'paytirib k- yo'liga qo'shishdan iborat bo'ladi.

Bu aytilganlarning hammasi ustunlal uchun ham o'rinli. Endi elementar almashtitishlar natijasida $F(\lambda)$matritsaga ekvivalent bo'lgan $\Phi(\lambda)$ matritsa hosil qilingan bo'lsa, $\Phi(\lambda)$ da bularga teskari (shu bilan birga elementar) almashtirishlarni bajarib $F(\lambda)$ matritsaga qaytish mumkiligidan, $F(\lambda)$matritsa o'z navbatida $\Phi(\lambda)$ matritsaga ekvivalent matritsa bo'ladi. Shuning uchun $F(\lambda)$ va $\Phi(\lambda)$ matritsalarni o'zaro ekvivalent yoki qisqacha ekvivalent matritsalar deyiladi.

Masalan, (2.14) matrisada 1-va 3- yo'llarni almashtirsak, 2-yo'lni $\frac{1}{8}$ gao'paytirsak va 2-ustunni $-\lambda^2$ga ko'paytirib 1- ustunga qo'shsak, yana (2.1.3) matritsa hosil bo'ladi.

λ-matritsaning invariant ko'paytuvchilari.

2.2.1-ta`rif. Agar λ-matritsa

$$\begin{Vmatrix} E_1(\lambda) & 0 & \cdot & 0 \\ 0 & E_2(\lambda) & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & E_n(\lambda) \end{Vmatrix}$$

diagonal shaklga ega bo'lib,bosh diagonaldagi $E_1(\lambda), E_2(\lambda), \dots E_n(\lambda)$ko'phadlarning bosh koeffisientlari 1 ga teng bo'lsa va har bir $E_{i+1}(\lambda)$ ko'phad o'zidan oldin turgan $E_1(\lambda)$ ko'phadga bo'linsa, u holda λmatritsaning bunday shakli normal diagonal shakl deyiladi.

Bu ta'rifdan ko'rinadiki, har bir $E_{i+1}(\lambda)$ ko'phad o'zidan oldin turgan hamma $E_i(\lambda), E_{i-1}(\lambda), \dots, E_1(\lambda)$ ko'phadlarga bo'linadi. Demak,biror $E_i(\lambda)$ ko'phad nolga teng ya'ni $E_i(\lambda) = 0$ bo'lsa , undan keying hamma $E_{i+1}(\lambda), \dots, E_n(\lambda)$ko'phadlar ham nolga teng bo'lishi lozim,chunki, masalan, $E_j(\lambda) \neq 0 (j > i)$ bo'lsa uning $E_i(\lambda)$ ko'phadga bo'linishi mumkin emas edi.Shuningdek , biror $E_i(\lambda)$ ko'phad 1 ga teng bo'lsa, $E_1(\lambda) = E_2(\lambda) = \cdots = E_{i-1}(\lambda) = 1$ bo'lishi kerak, chunki $E_k(\lambda) \neq 1 (k < i)$ shartda $E_i(\lambda)$ ko'phad $E_k(\lambda)$ ga bo'limas edi.

2.2.1-misol.

Quyidagi $\lambda -$ matritsalat normal diagonal shaklga ega:

1. $\begin{Vmatrix} \lambda & 0 & 0 \\ 0 & \lambda(\lambda-1) & 0 \\ 0 & 0 & \lambda^2(\lambda^3-1) \end{Vmatrix}$;

2. $\begin{Vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda^2-1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Vmatrix}$;

3. $\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \lambda-2 & 0 \\ 0 & 0 & 0 & (\lambda-2)^2 \end{Vmatrix}$;

Aytilganlarga asosan, birlik va nol matritsalar hamλ –normal diagonal matritsalarning xususiy holi bo’ladi.

Bu paragrafda berilgan $\lambda -$ matritsaga ekvivalent normal diagonal $\lambda -$matritsani toppish masalasi bilan shug’ullanamiz .

Berilgan $F(\lambda)$ matritsaga ekvivalent normal diagonal $\lambda -$ matritsani toppish protsessi,odatda, $F(\lambda)$ ni normal diagonal shaklga keltirish deb aytiladi.

2.2.1-teorema.Agar $F(\lambda)$ matritsaning $f_{11}(\lambda)$ birinchi elementi noldan farqli bo’lib,bu elementga $F(\lambda)$ ning biror $f_{ik}(\lambda)$ elementi bo’linmasa, u holda $F(\lambda)$matritsaga ekvivalent shunday $\Phi(\lambda)$ matritsa mavjudki, undagi birinchi $\varphi_{11}(\lambda)$elementning darajasi $f_{11}(\lambda)$ elementning darajasidan kichik bo`ladi.

2.2.1-isbot.1.Birinchi yo`lning $f_{1k}(\lambda)$ elementi $f_{11}(\lambda)$ ga bo`linmaydigan deb faraz qilaylik.U holda,

$$f_{1k}(\lambda) = f_{11}(\lambda)q_{11}(\lambda) + \varphi_{11}(\lambda) \quad (5)$$

tenglik hosil bo`ladi, bunda $\varphi_{11}(\lambda)$ −qoldiq bo`lib, uning darajasi $f_{11}(\lambda)$ elementning darajasidan kichik .Birinchi ustunni$[-q_{11}(\lambda)]$ ga ko`paytirib, $k$-chi ustundan ayirsak shunday matritsa hosil bo`ladiki ,unda $f_{ik}(\lambda)$element o`rnida $\varphi_{11}(\lambda)$element turadi[(5)tenglikka asosan]. Agar bu matritsaning birinchi k-ustunlarini almashtirsak, teoremada aytilgan $\Phi(\lambda)$ matritsa hosil bo'ladi.

$F(\lambda)$matritsaning$f_{11}(\lambda)$elementga bo'linmaydigan elementi 1-ustunda turgan holda ham xuddi shunday mulohaza qilinadi.

II.Endi 1- yo'l va 1- ustunning hamma elementlari $f_{11}(\lambda)$ ga bo'linib,qandaydir $f_{ik}(\lambda)$ element bo'limaydigan bo'lsin.Bu holni birinchi holga keltirish mumkin.Haqiqatan: $f_{i1}(\lambda)$ element $f_{11}(\lambda)$ ga bo'linadi,demak,$f_{i1}(\lambda) = f_{11}(\lambda)q_{11}(\lambda)$. Birinchi yo'lni $q_{11}(\lambda)$ ga ko'paytirib,i-yo'ldan ayirsak, bu yo'lning elementlari o'zgarib, $f_{i1}(\lambda)$element o'rnida nol hosil bo'ladi: $f_{ik}(\lambda)$ element esa

$$f'_{ik}(\lambda) = f_{ik}(\lambda) - q_{11}(\lambda)f_{ik}(\lambda)\quad(6)$$

element bilan almashinadi, ya'ni

$$F(\lambda) = \left\|\begin{matrix} f_{11}(\lambda) & \cdots & f_{1k}(\lambda) & \cdots & f_{1n}(\lambda) \\ . & . & . & . & . \\ f_{i1}(\lambda) & \cdots & f_{ik}(\lambda) & \cdots & f_{in}(\lambda) \\ . & . & . & . & . \end{matrix}\right\|$$

matritsa ushbu

$$F'(\lambda) = \left\|\begin{matrix} f_{11}(\lambda) & \cdots & f_{1k}(\lambda) & \cdots & f_{1n}(\lambda) \\ . & . & . & . & . \\ f_{i1}(\lambda) & \cdots & f'_{ik}(\lambda) & \cdots & f'_{in}(\lambda) \\ . & . & . & . & . \end{matrix}\right\|$$

matritsaga o'tadi. (6) element $f_{11}(\lambda)$ elementga bo'linmaydi.

$\lfloor -q_{11}(\lambda) f_{ik}(\lambda)$ ko'paytma $f_{11}(\lambda)$ elementga bo'linib, $f_{ik}(\lambda)$ esa bo'linmaganligisababli$\rfloor$ Endi ,$F'(\lambda)$matritsaning i-yo'lini 1-yo'liga qo'shib, quyidagi matritsani hosil qilamiz:

$$F''(\lambda) = \begin{Vmatrix} f_{11}(\lambda) & \cdots & f_{1k}(\lambda) & \cdots & f_{1n}(\lambda) \\ . & . & . & . & . \\ f_{i1}(\lambda) & \cdots & f''_{ik}(\lambda) & \cdots & f''_{in}(\lambda) \\ . & . & . & . & . \end{Vmatrix}$$

bunda

$$f''_{1k}(\lambda) = f_{1k}(\lambda) + f'_{ik}(\lambda)$$

element $f_{11}(\lambda)$ elementga bo'linmaydi. Shunday qilib,$F''(\lambda)$ matritsa birinchi holning talabiga javob beradi.Shu sababli, birinchi holdagi mulohazani takrorlab, teoremada aytilgan $\Phi(\lambda)$ matritsani hosil qilamiz.

2.2.1-eslatma.Agar $f_{11}(\lambda)$ elementga bo;='linmaydigan $f_{ik}(\lambda)$ elementning darajasi $f_{11}(\lambda)$ ning darajasidan kichik bo'lsa, $F(\lambda)$ matritsa yo'llarini o'zaro va ustunlarini o'zaro almashtirib, $f_{ik}(\lambda)$ elementni birinchi o'ringa keltirish bilan $\Phi(\lambda)$ matritsani hosil qilish mumkin.

2.2.2-teorema.Agar $F(\lambda)$matritsaning hamma elementlari biror ko'phadga bo'linsa , $F(\lambda)$ matritsada elementar almashtirishlar bajarishdan hosil bo'lgan $\Phi(\lambda)$ matritsaning ham hamma elementlari $f(\lambda)$ ko'phadga bo'linadi.

2.2.2-isbot. Agar $\Phi(\lambda)$ matritsa $F(\lambda)$ matritsaning ikki yo'lini (ikki ustunini) almashtirish yoki $F(\lambda)$ matritsanini biror yo'lini (ustunini) $m \neq 0$ songa ko'paytirish natijasida vujudga kelgan bo'lsa , $\Phi(\lambda)$ matritsadagi hamma elementlarning $f(\lambda)$ ko'phadga bo'linishi ravshan.

$F(\lambda)$matritsaning i- yo'lini biror $\varphi(\lambda)$ko'phadga ko'paytirib, k-yo'liga qo'shish bilan hosil qilingan $\Phi(\lambda)$ matritsaning k- yo'lidagi

$f_{kl}(\lambda) + \varphi(\lambda) f_{il}(\lambda)$

elementlari $f(\lambda)$ ko'phadga bo'linadi ($f_{kl}(\lambda)$ bo'ligani sababli).

Endi, $\Phi(\lambda)$ matritsaning boshqa yo'llari $F(\lambda)$ matritsaninig mos yo'llaridan farq qilmaganligi uchun.$\Phi(\lambda)$ matritsadagi barcha elementlar $f(\lambda)$ ko'phadga bo'linadi degan xulosaga kelamiz.

Teskarilanuvchi λ-matritsalar.

2.3.1. ta'rif. Agar $P(\lambda)$ matritsaga teskari $P^{-1}(\lambda)$ matritsa mavjud bo'lib,shu bilan birga $P^{-1}(\lambda)$ ham $\lambda -$ matritsadan iborat bo'lsa, $P(\lambda)$ teskarilanuvchi $\lambda -$ matritsa deyiladi.

2.3.1-teorema. $P(\lambda$)teskarilanuvchi $\lambda -$ matritsa bo'lishi uchun, $|P(\lambda)|$ determinant noldan farqli o'zgarmas songa teng bo'lishi zarur va yetarli.

2.3.1-isbot. 1. $(\lambda$)-teskarilanuvchi $\lambda -$ matritsa deb faraz qilaylik.Demak, mavjud va u $\lambda -$ matritsadan iborat.$P^{-1}(\lambda) = Q(\lambda)$ deb belgilaylik. $Q(\lambda)$ matritsa $P(\lambda)$ matritsaga teskari, ya'ni $P(\lambda) \cdot Q(\lambda) = E$ bo'lgani sababli $|P(\lambda) \cdot Q(\lambda)| = |P(\lambda)| \cdot |Q(\lambda)| = 1$ tenglik bajariladi.Endi, ikkita $|P(\lambda)|$ va $|Q(\lambda)|$ ko'phadning ko'paytmasi o'zgarmas 1 songa teng bo'lishi shuni ko'rsatadiki, bu ko'phadlarning o'zi noldan farqli o'zgarmas songa teng.Shunday qilib, $|P(\lambda)| = a \neq 0$

II. Aksincha $|P(\lambda)| = a \neq 0$ bo'lsin .Agar $|P(\lambda)|$ matritsadagi $p_{ik}(\lambda)$elementlarning algebraic to'ldiruvchilarini $P_{ik}(\lambda)$bilan belgilasak, u holda:

$$P^{-1}(\lambda) = \left\| \begin{matrix} \frac{P_{11}(\lambda)}{a} & \frac{P_{21}(\lambda)}{a} & \cdots & \frac{P_{n1}(\lambda)}{a} \\ . & . & . & . \\ \frac{P_{1n}(\lambda)}{a} & \frac{P_{2n}(\lambda)}{a} & \cdots & \frac{P_{nn}(\lambda)}{a} \end{matrix} \right\|.$$

Demak, teskari matritsa mavjud va u λ matritsadan iborat.

2.3.2-teorema. Teskarilanuvchi λ matritsaning ko'paytmasi ham teskarilanuvchi λ matritsadir.

Isbot. $P_1(\lambda)$, $P_2(\lambda)$, …, $P_m(\lambda)$-teskarilanuvchi matritsalar bo'lsin, u holda $|P_1(\lambda)| = a \neq 0$, $|P_2(\lambda)| = a \neq 0$, …, $|P_m(\lambda)| = a \neq 0$, Endi

$$|P_1(\lambda)P_2(\lambda)\dots P_m(\lambda)| = |P_1(\lambda)||P_2(\lambda)|\dots|P_m(\lambda)| = a_1 a_2 \dots a_m = a \neq 0$$

shartga asosan, berilgan matritsalar ko'paytmasi, haqiqatan teskarilanuvchi λ-matritsa ekanini ko'ramiz.

2.3.3-teorema. Teskarilanuvchi $P(\lambda)$ martitsaning normal diagonal shakli birlik matritsadan iborat.

2.3.3-isbot. Haqiqatan, $|P(\lambda)| = a \neq 0$ bo'lgani uchun $D_n(\lambda) = 1$.Ma'lumki, uchun $D_n(\lambda)$ ko'phad o'zidan oldinda turgan hamma $D_1(\lambda)$, $D_2(\lambda)$, …, $D_{n-1}(\lambda)$ ko'phadlarga bo'linadi. Shu sababli, $D_1(\lambda)$, $D_2(\lambda)$, …, $D_{n-1}(\lambda)=1$.Demak,

$E_1(\lambda)=D_1(\lambda)=1$,

$$E_2(\lambda) = \frac{D_2(\lambda)}{D_1(\lambda)} = 1, \dots, E_n(\lambda) = \frac{D_n(\lambda)}{D_{n-1}(\lambda)} = 1 \text{ va}$$

Misol.

Ushbu

$$F(\lambda) = \begin{Vmatrix} \lambda^2 - 2\lambda + 1 & \lambda \\ \lambda^4 - 3\lambda^3 + 3\lambda^2 - 2 & \lambda^2 - 2\lambda + 1 \end{Vmatrix}.$$

matritsa –teskarilanuvchi λ -matritsa, chunki

$|F(\lambda)| = 1.$

$F(\lambda)$ matritsaga teskari matritsa ushbu λ– matritsadan iborat:

$$F(\lambda) = \begin{Vmatrix} \lambda^2 - 2\lambda + 1 & -\lambda \\ -\lambda^4 + 3\lambda^3 - 3\lambda^2 + 2 & \lambda^2 - 2\lambda + 1 \end{Vmatrix}.$$

$F(\lambda)F^{-1}(\lambda)$=E bo'lishini tekshirib ko'ring.

2.3.1-natija. Hamma teskarilanuvchi –matritsalar o'zaro ekvivalent.

2.3.1-isbot. Istalgan ikkita teskarilanuvchi P(λ) va Q(λ) matritsaning har qaysisi E birlik matritsaga ekvivalent bo'lgani sababli P(λ) va Q(λ) o'zaro ekvivalent matritsadir.

2.3.4-teorema. $F(\lambda)$va $\Phi(\lambda)$matritsalar ekvivalent bo'lishi uchun ikkita teskarilanuvchi $\mathrm{F}(\lambda)$va $\mathrm{Q}(\lambda)$ matritsa mavjud bo'lib,

$$\Phi(\lambda) = P(\lambda) \cdot F(\lambda) \cdot Q(\lambda) \qquad (2.3.21)$$

Tenglik bajarilishi zarur va yetarli.

2.3.4-isbot. 1. $F(\lambda)$va $\Phi(\lambda)$ matritsalar ekvivalent deb faraz qilaylik.

(2.3.21) tenglikni qanoatlantiradigan teskarilanuvchi $\mathrm{F}(\lambda)$va $\mathrm{Q}(\lambda)$matritsalarning mavjudligini isbotlash uchun quyidagini ta'kidlab o'tamiz: $F(\lambda)$ matritsa ustida elementar almashtirishlar bajarish uni qandaydir teskarilanuvchi λ-matritsalarga chapdan yoki o'ngdan ko'paytirish bilan bir xil.Bunday matritsalar unimodulyar matritsalar (yoki elementar almashtirishlar matritsalari) deb ataladi.Haqiqatan:

$$F(\lambda) = \begin{Vmatrix} f_{11}(\lambda) & f_{12}(\lambda) & \dots & f_{1n}(\lambda) \\ f_{21}(\lambda) & f_{22}(\lambda) & \dots & f_{2n}(\lambda) \\ \cdot & \cdot & \cdot & \cdot \\ f_{n1}(\lambda) & f_{n2}(\lambda) & \dots & f_{nn}(\lambda) \end{Vmatrix}$$

matritsaning, masalan, $1-$ va $2-$ yo'llarini (yoki 1- va $2-$ ustunlarini) almashtirish $F(\lambda)$uchun matritsani chapdan (ustunlarini almashtirishda esa o'ngdan) ushbu:

$$\begin{Vmatrix} 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{Vmatrix} \qquad (2.3.22)$$

matritsaga ko'paytirish kifoya. Bu matritsa ham E birlik matritsadan $1-$va $2-$ yo'llarni (yoki, baribir, $1-$va $2-$ ustunlarni) almashtirish bilan hosil qilingan.

$F(\lambda)$ning, masalan, $2-$yo'lini (yoki ustunini) $m \neq 0$ ga ko'paytirish uchun, $F(\lambda)$matritsani chapdan (ustunini ko'paytirganda, o'ngdan):

$$\begin{Vmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & m & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{Vmatrix} \qquad (2.3.23)$$

matritsaga ko'paytirish lozim.Bunda ham biz hosil qilgan matritsa E matritsaning $2-$ yo'li (ustuni)ni $m \neq 0$ ga ko'paytirish bilan topiladi.

Nihoyat, $F(\lambda)$ ning $2-$ yo'lini .... ga ko'paytirib, 1-yo'liga qo'shish uchun $F(\lambda)$matritsani chapdan

$$\begin{Vmatrix} 1 & f(\lambda) & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{Vmatrix} \qquad (2.3.24)$$

matritsaga ko'paytirish kerak. Bu matritsa ham E matritsaning 2- yo'lini $f(\lambda)$ ga ko'paytirib, 1-yo'liga qo'shish bilan hosil qilinadi. Agar $F(\lambda)$matritsaning 2-ustunini $f(\lambda)$ga ko'paytirib,1-ustuniga qo'shish talab etilsa, u holda $F(\lambda)$matritsani o'ngdan

$$\begin{Vmatrix} 1 & 0 & \dots & 0 \\ f(\lambda) & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{Vmatrix} \qquad (2.3.25)$$

matritsaga ko'paytirish lozim.

$(2.3.22), (2.3.23), (2.3.24), (2.3.25)$ matritsalar teskarilanuvchi matritsalardir, chunki ularning determinantlari noldan farqli o'zgarnas sonlarga teng.

Shunday qilib, $F(\lambda)$matritsadan elementar almashtirishlar yordamida $\Phi(\lambda)$ matritsani hosil qilish uchun $F(\lambda)$matritsani chapdan va o'ngdan bir necha teskarilanuvchi λ-matritsalarga ko'paytirish lozimligi ma'lum bo'ldi.Teskarilanuvchi matritsalar ko'paytmasi yana teskarilanuvchi matritsa bo'lgani sababli $F(\lambda)$matritsaning chap tomonida turgan teskarilanuvchi matritsalar ko'paytmasini esa $Q(\lambda)$ bilan belgilasak, $(2.3.21)$ tenglik hosil bo'ladi.

Eslatma.Har bir teskarilanuvchi $R(\lambda)$ matritsa unimodulyar matritsalarningko'paytmasiga teng.Chindan ham, R(λ) matritsa E birlik matritsaga ekvivalent. Shu sababli, $\Phi(\lambda) = R(\lambda)$va $F(\lambda) = E$ bo'lgan xususiy holda, (2.3.21*)* tenglikdan

$$R(\lambda) = P(\lambda)Q(\lambda)$$

ekani keleb chiqadi, bu yerda P(λ)·Q(λ)unimodulyar matritsalarning ko'paytmasi