O’ZBEKISTON RESPUBLIKASI

OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

NAMANGAN DAVLAT UNIVERSITETI

MATEMATIKA KAFEDRASI

ANALITIK GEOMETRIYA

Mavzu: Parabola va uning kanonik tenglamasi.

NAMANGAN 2016

Annotatsiya

- Ushbu taqdimot parabola va uning kanonik tenglamasi deb nomlanib unda, parabolaning kanonik tenglamasi keltirib chiqarilgan, parabolaning shakli, parabolani yasash keltirilgan. Parabola tenglamasiga doir tushunchalar misollar yechib ko`rsatilgan.

PARABOLA

Reja

1. Ta`rifi.
2. Kanonik tenglamasi.
3. Parabolaning shakli
4. Parabolani yasash.
5. $y = ax^2 + bx + c$ tenglama orqali berilgan parabola.
6. Misollar.

Mavzuning bayoni.

Ta`rif: Har bir nuqtasidan fokus deb ataluvchi berilgan nuqtagacha va direktrisa deb ataluvchi berilgan to`g`ri chiziqqacha bo`lgan masofa o`zaro teng bo`lgan tekislikdagi barcha nuqtalar to`plamiga parabola deb ataladi.

Parabola fokusini F va direktrisasini s orqali belgilaylik. F fokus s direktrisaga tegishli emas. $d(F,S)=p$ bo`lsin. F nuqtadan s to`g`ri chiziqqa perpendikulyar o`tkazamiz va uni abssissa o`qi OX uchun olamiz. $d(F,N)=p$ bo`lib, $N\in S$. FN kesmaning o`rtasi O nuqtadan OX ga perpendikulyar ordinata o`qi OY o`tkazamiz. OXY-dekart reper tashkil etadi. O`rnatilgan dekart reperda $F\left(\frac{P}{2},0\right)$, $N\left(-\frac{P}{2},0\right)$ koordinatalarga ega.

Direktrisaning tenglamasi $x=-\frac{p}{2}$. Fokus $F\left(\frac{P}{2},0\right)$ koordinatalarga ega. Parabola ta`rifiga ko`ra

$$d(F,M)=d(L,M) \qquad (1)$$

(1) tenglikni koordinatalar bo`yicha kanonik holga keltiraylik. $B=\{0,\vec{i},\vec{j}\}$ reperda M nuqta M($x$,$y$) koordinatalarga ega bo`lsin.

$$d(F,M)=\sqrt{\left(x-\frac{P}{2}\right)^2+y^2},\quad d(L,M)=\sqrt{\left(x+\frac{P}{2}\right)^2}=x+\frac{P}{2}.$$

$$\sqrt{\left(x-\frac{P}{2}\right)^2+y^2}=\left|x+\frac{P}{2}\right| \qquad (2)$$

(2) tenglamaning har ikki tomonini kvadratga ko\`taramiz:

$$x^2 - px + \frac{p^2}{4} = x^2 + px + \frac{x^2}{4} \Rightarrow y^2 = 2px \qquad (3)$$

(3) tenglama (1) tenglikdan kelib chiqadi. Endi (3) dan foydalanib, (1) tenglikni isbotlaymiz. Ixtiyoriy $M_1(x_1,y_1)$ nuqta (3) tenglamani qanoatlantirsin: $y_1^2 = 2px_1$.

$F\left(\frac{p}{2},0\right)$ nuqta va $s: x = -\frac{p}{2}$ to\`g\`ri chiziqni olaylik.

$$d(F,M) = \sqrt{\left(x_1 - \frac{P}{2}\right)^2 + y_1{}^2} = \sqrt{x_1{}^2 - Px_1 + \frac{P^2}{4} + 2Px_1} = \sqrt{x_1{}^2 + Px_1 + \frac{P^2}{4}} =$$

$$= \sqrt{\left(x_1 + \frac{P}{2}\right)^2} = \left|x_1 + \frac{P}{2}\right| = d(M_1,S) = d(L,M_1).$$

Ko\`ramizki, M_1 nuqta parabolaga tegishli. (3) ni parabolaning kanonik tenglamasi deyiladi. Parabola shaklini aniqlaylik.

1) Parabola ikkinchi tartibli chiziq.

2) $y^2 \geq 0$ va $p > 0 \Rightarrow x \geq 0$. Bundan parabolaning barcha nuqtalari $x \geq 0$ yarim tekislikka tegishliligi kelib chiqadi.

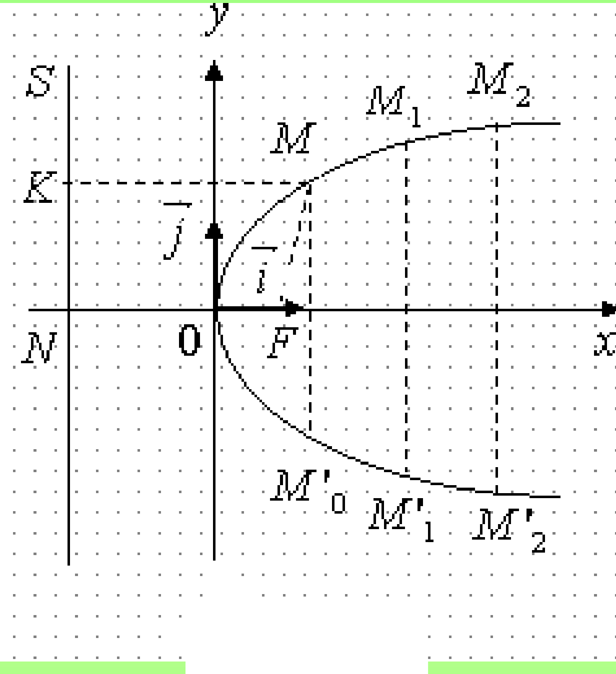
3) $x = 0,\ y = 0 \Rightarrow$ parabola koordinatalar boshidan o\`tadi. O(0,0) nuqtani parabolaning uchi deyiladi.

4) x ning har bir $x > 0$ qiymatiga y ning qarama-qarshi ishorali, ammo absolyut miqdorlari teng bo\`lgan ikki qiymati mos keladi. Xulosa shuki, parabola OX o\`qqa nisbatan simmetrik chiziq.

1) $y^2 = 2Px \Rightarrow y = \pm\sqrt{2Px} \Rightarrow x \to \infty \Rightarrow |y| \to \infty$. Yuqoidagi mulohazalar bo`yicha parabolaning shakli quyidagicha bo`lishi mumkin (43- chizma).

Agar parabola fokusi $F(-\frac{P}{2},0$ bo`lsa, u holda uning tenglamasi

$$y^2 = -2px \quad (4)$$



ko`rinishida bo`ladi. Agar fokus OY o`qining $E\left(0,\frac{p}{2}\right)$ nuqtasi bo`lsa, u holda parabolaning tenglamasi $x^2 = 2Py$ ko`rinishida bo`ladi.

Parabolani quyidagi usul bilan yasash mumkin.

$y^2 = 2px$ kanonik tenglamasi bilan berilgan parabolaning fokusi va direktrisasini yasaymiz. Buning uchun OX o`qda O nuqtadan chapda va o`ngda $d(N,O) = D(O,F) = \frac{p}{2}$ bo`lgan N va F nuqtalarni belgilaymiz. N nuqtadan OX ga perpendikulyar s to`g`ri chiziq o`tkazamiz. F fokusdan boshlab OX o`qqa perpendikulyar va har biri oldingisidan $P/2$ masofada turuvchi to`g`ri chiziqlarni o`tkazamiz. O`tkazilgan to`g`ri chiziqlarning har biridan direktrisagacha bo`lgan masofani radius qilib F markazli aylana chizamiz. Bu aylana mos to`g`ri chiziqni OX o`qqa simmetrik bo`lgan ikki nuqtada kesib o`tadi. Bu nuqtalar izlangan parabolaga tegishli bo`ladi. OX o`qqa perpendikulyar istalgancha to`g`ri chiziqlarni o`tkazish mumkin.

SHularning har birida ta`rifni qanoatlantiruvchi parabola nuqtalarini aniqlaymiz. Aniqlangan nuqtalarni tutashtirib parabola grafigini hosil qilamiz.

Tanlangan $B=\{0,\vec{i},\vec{j}\}$ reperda parabolaning tenglamasi

$$y=ax^2+bx+c \qquad (5)$$

ko`rinishida bo`lishi ham mumkin. (5) tenglamani o`ng tomonidan to`la kvadrat ajratamiz.

$$y=a\left(x^2+2\frac{b}{2a}x+\frac{b^2}{4a^2}\right)-\frac{b^2}{4a}+c$$

$$y=a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a}\Rightarrow y-\frac{4ac-b^2}{4a}=a\left(x+\frac{b}{2a}\right)^2 \quad (6)$$

Dekart koordinatalar boshini $O'\left(-\frac{b}{2a},\frac{4ac-b^2}{4a}\right)$ nuqtaga

$$\begin{cases} x+\dfrac{b}{2a}=X \\ y-\dfrac{4ac-b^2}{4a}=Y \end{cases} \qquad (7)$$

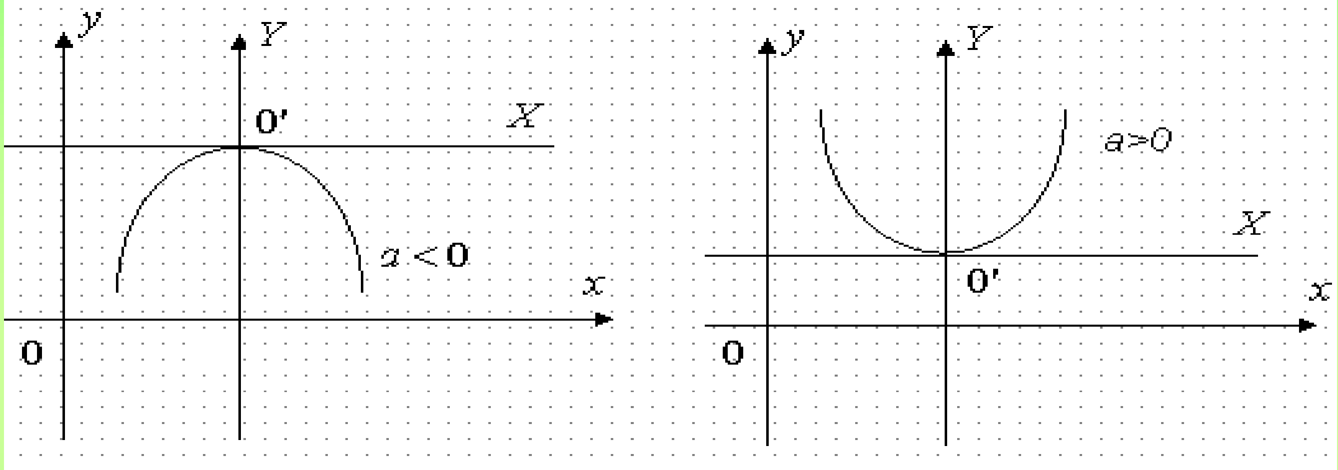
formula bo`yicha parallel ko`chiramiz. Yangi $B'=\{0',\vec{i}',\vec{j}'\}$ reperda (6) tenglama quyidagicha ko`rinishda bo`ladi:

$$Y=aX^2\Rightarrow X^2=\frac{1}{a}Y.$$

$q=\frac{1}{2|a|}$ belgilash kiritib,

$$X^2 = 2qY \quad (8)$$

tenglamaga ega bo`lamiz. (8) tenglama simmetriya o`qi (O'Y) va uchi $O'\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$ nuqtada bo`lgan parabolani aniqlaydi. Bunda $(O'Y)//(OY),\ (O'X)//(OX.)$



1-misol. $x^2 = -12y$ parabolada fokal radiusi $r = 9$ bo`lgan nuqta topilsin.

Yechish: $2q = -12 \Rightarrow q = -6.\ F(0,-3).\ r = \sqrt{x^2 + (y+3)^2} = 9 \Rightarrow x^2 + (y+3)^2 = 81.$

$$(y+3)^2 - 12y - 81 = 0 \Rightarrow y^2 - 6y - 72 - 0 \Rightarrow y_1 = -6,\ y_2 = 12.$$

Bu sonlarni $x^2 = -12y$ tenglamaga qo`ysak, $y_1 = -6$ da $x^2 = -12 \cdot (-6) = 72,$

1-misol. $x^2=-12y$ parabolada fokal radiusi $r=9$ bo`lgan nuqta topilsin.

Yechish: $2q=-12\Rightarrow q=-6.\ F(0,-3).\ r=\sqrt{x^2+(y+3)^2}=9\Rightarrow x^2+(y+3)^2=81$.

$$(y+3)^2-12y-81=0\Rightarrow y^2-6y-72-0\Rightarrow y_1=-6,\ y_2=12.$$

Bu sonlarni $x^2=-12y$ tenglamaga qo`ysak, $y_1=-6$ da $x^2=-12\cdot(-6)=72,$ $x_{1,2}=\pm 6\sqrt{2}.$, $y_2=12$ da tenglamani qanoatlantirmaydi, chunki $x^2=-12\cdot 12=-144.$ $x_{3,4}=\pm 12i$ (mavhum son)

Javob: $M_1\left(6\sqrt{2},-6\right),\ M_2\left(-6\sqrt{2},\ 6\right).$

2-misol: $y=4x^2-6x-\dfrac{3}{4}$ parabola tenglamasini kanonik ko`rinishga keltirilsin va parabola uchining koordinatalari aniqlansin.

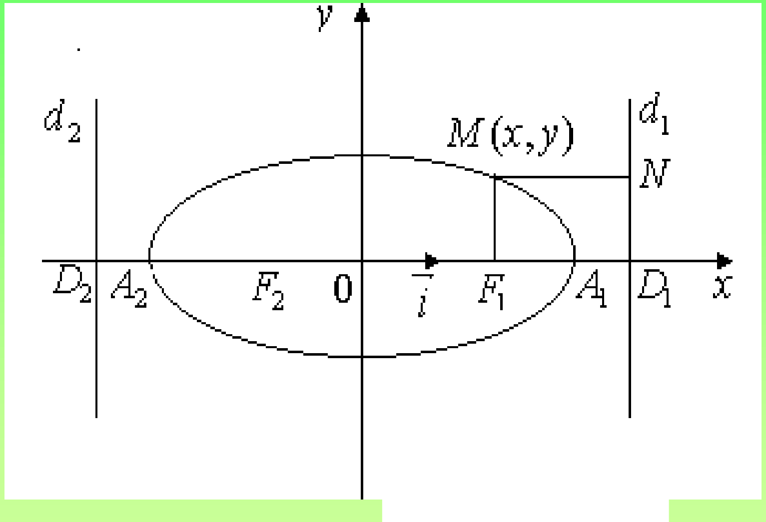
Yechish: $y=4\left(x^2-\dfrac{3}{2}x+\dfrac{9}{16}\right)-\dfrac{9}{4}-\dfrac{3}{4}\Rightarrow y=4\left(x-\dfrac{3}{4}\right)^2-3\Rightarrow y+3=4\left(x-\dfrac{3}{4}\right)^2.$

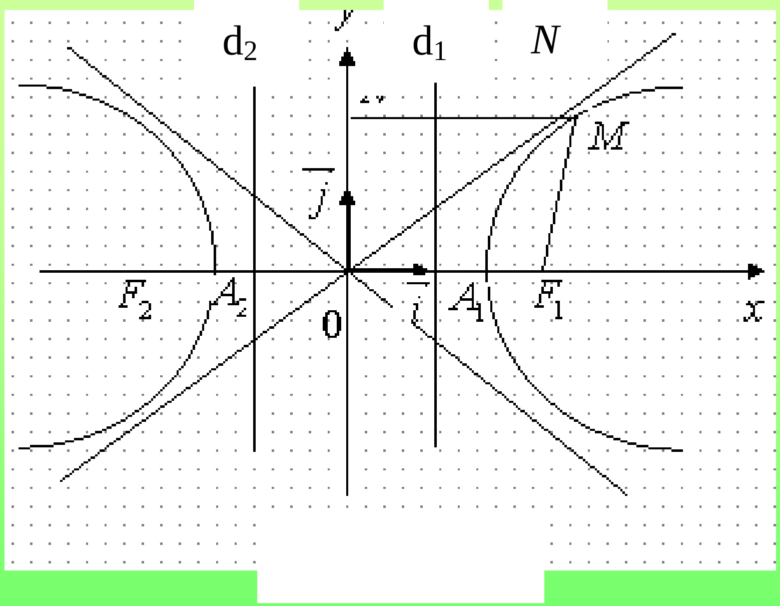
$y+3=Y,\ x-\dfrac{3}{4}=X$ belgilashlarni kiritsak, $Y=4X^2$. Parabolaning uchu $O'\left(\dfrac{3}{4},-3\right)$ nuqtada. $(O'Y)-$simmetriya o`qi.

Javob: $Y=4X^2,\ O'\left(\dfrac{3}{4},-3\right).$

ELLIPS VA GIPERBOLANING DIREKTRISALARI

Ta`rif: Ellips (giperbola) ning berilgan F fokusiga mos direktrisasi deb, uning fokal o`qilga perpendikulyar va markazidan shu F fokusi yotgan tomonda a/e masofada turuvchi to`g`ri chiziqqa aytiladi. Bu yerda *a*-ellips (giperbola) ning katta (haqiqiy) yarim o`qi, ye-ekssentrisiteti.





F_1 fokusga mos direktrisa d_1, F_2 fokusga mos direktrisa d_2 bo`lsin. $F_1(c,0)$ fokusga mos direktrisa tenglamasi $d_1 : x - \frac{a}{e} = 0$,

$F_2(-c,0)$, fokusga mos direktrisa tenglamasi esa $d_2 : x + \frac{a}{e} = 0$. Ellips uchun $a/_e > a$, giperbola uchun esa $a/_e < a$. Ko`ramizki, direktrisa Ellips (giperbola) bilan kesishmaydi.

Asosiy teorema: Ellips (giperbola) tekislikdagi shunday nuqtalar to`plamiki, bu nuqtalarning har biridan fokusgacha bo`lgan masofani o`sha nuqtadan shu fokusga mos direktrisagacha bo`lgan masofaga nisbati o`zgarmas miqdor bo`lib, Ellips (giperbola) ning ekssentrisiteti ye ga teng.

Isbot: Ellips (giperbola) ning $F_1(c,0)$ fokusi va shu fokusga mos direktrisasi $d_1 : x - \frac{a}{e} = 0$ ni olaylik. M(x,y) Ellips (giperbola)ning ixtiyoriy nuqtasi bo`lsin. M nuqtadan $F_1$ nuqtagacha bo`lgan r_1 masofa

$$r_1 = \rho(F_1, M) = |a - ex|. \qquad (1)$$

Endi ellips (giperbola) ning M nuqtasiga mos $N \in d_1$ nuqtaning koordinatalari $N\left(\frac{a}{e}, y\right)$ ni e`tiborga olsak,

$$\rho(M, N) = \rho(M, d_1) = \left|x - \frac{a}{e}\right| \qquad (2)$$

kelib chiqadi. U holda

$$\frac{\rho(F_1, M)}{\rho(M, d_1)} = \frac{|a - ex|}{\left|\frac{a}{e} - x\right|} = \frac{|a - ex| \cdot e}{|a - ex|} = e.$$

Ellips (giperbola) ga quyidagicha ta`rif berish mumkin.

Ta`rif: Har bir nuqtasidan biror F fokusgacha masofani shu fokusga mos direktrisagacha bo`lgan masofaga nisbati ekssentisitet (e) ga teng bo`lgan tekislikdagi nuqtalar to`plami $e<1\ (e>1)$ bo`lganda ellips (giperbola) deb ataladi.

1-misol: Ellipsning kichik o`qi $b=4$, direktrisalari $x=\pm 10$ tenglamalar orqali berilgan bo`lsa, uning kanonik tenglamasi yozilsin.

Yechish: $\frac{a^2}{c}=10,\ b^2=16 \Rightarrow a^2=10c,\ b^2=16.$

$$a^2-b^2=c^2 \Rightarrow 10c-16=c^2 \Rightarrow c^2-10c+16=0$$

$$c_{1,2}=\frac{10\pm\sqrt{100-64}}{2}=\frac{10\pm 6}{2} \Rightarrow c_1=8,\ c_2=2.\ a_1^2=80,\ a_2^2=20.$$

Ellips tenglamasi $\frac{x^2}{80}+\frac{y^2}{16}=1\ \left(\frac{x^2}{20}+\frac{y^2}{16}=1\right).$

2-misol: $16x^2-9y^2=144$ giperbola ekssentrisiteti va direktrisalarining tenglamalari aniqlansin.

Yechish: $\frac{x^2}{9}-\frac{y^2}{16}=1 \Rightarrow a=3,\ b=4.\ c^2=a^2+b^2=25;\ F_1(5,0),\ F_2(-5,0).$

$$e=\frac{c}{a}=\frac{5}{3}, x=\pm\frac{a}{e}=\pm\frac{9}{5}.$$

Javob: Giperbola ekssenrisiteti $e=\frac{5}{3}$, direktrisalari $S_1: x=\frac{9}{5}$, $S_2: x=-\frac{9}{5}$ tenglamalarga ega.

Foydalanilgan xorijiy adabiyotlar

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