O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

ALISHER NAVOIY NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

Ermamatova Zuxro Ermamatovna

Umumlashgan Koshi-Riman tenglamasi yechimini davom ettirish

Malakaviy bitiruv ishi

Ish ko'rib chiqildi va himoyaga ruxsat berildi

"Differensial tenglamalar"
kafedrasi mudiri v.b.: dots. Z.Malikov

Ilmiy rahbar: prof.T.Ishankulov

M.O'

Samarqand-2016 yil

MUNDARIJA

Kirish

1. **Masalaning qo'yilishi**. Kompleks o'zgaruvchili funksiyalar nazariyasi fanining umumiy kursidan ma'lum bo'lgan asosiy tushunchalardan biri hisoblangan bir qiymatli analitik funksiyalar uchun Koshining integral teoremasi va integral formulasidir. Koshining integral formulasini muhimligi shundan iboratki, bir qiymatli analitik funksiyaning sohani chegarasida berilgan qiymati bo'yicha uni shu soha ichkarisiga analitik davom ettiriladi. Bir qiymatli analitik funksiyaning haqiqiy va mavhum qismlari Koshi-Riman shartini qanoatlantiradi.

   Kompleks o'zgaruvchili funksiyalar nazariyasi fani matematikaning turli sohalarida keng qo'llaniladi. Shu jumladan, matematik fizika tenglamalari uchun qo'yilgan chegaraviy masalalarni yechishda ham muhim ahamiyat kasb etadi. Bir qiymatli analitik funksiyaning haqiqiy va mavhum qismlari qo'shma garmonik funksiyalardan iborat bo'lib, Laplas tenglamasining yechimidan iboratdir. Laplas tenglamasi uchun qo'yilgan Koshi masalasi nokorrekt ekanligidan foydalangan holda umumlashgan Koshi-Riman sistemasi uchun qo'yilgan Koshi masalasi yechimini topish ham muhim ahamiyat kasb etadi.

2.Mavzuning dolzarbligi. Matematik fizika tenglamalari, kvant fizikasi masalalarini yechishda Koshi-Riman sistemasi o'zining amaliy ahamiyati jihatidan muhim o'rin egallaydi. Tenglama yechimini topishda yechimning mavjudlik va yagonaligi, qo'yilgan chegaraviy shartlarni qanoatlantirishini tekshirish muhimdir. Tekislikda qaralayotgan Koshi-Riman sistemasi uchun qo'yilgan Koshi masalasining yechimini, ya'ni soha chegarasining bir qismida berilgan qiymati bo'yicha shu soha ichida yechimni tiklash muhimdir. Bunda Karleman formulasini tiklash qulay ko'rinishda ifodalanadi. Ushbu malakaviy bitiruv ishida uch o'lchovli fazoda Koshi-Riman sistemasi uchun qo'yilgan chegaraviy masalani yechishda kompleks o'zgaruvchili funksiyalar nazariyasi bilan bog'liqligi mavzuning dolzarbligini ifodalaydi.

3.Ishning maqsadi va vazifalai. Bitiruv malakaviy ishning maqsadi kompleks o'zgaruvchili funksiyalar nazariyasi tushunchalari asosida bir qiymatli analitik funksiyaning haqiqiy va mavhum qismlari orqali ifodalanuvchi Koshi-Riman sistemasi uchun qo'yilgan Koshi masalasi yechimini topish, ya'ni Karleman formulasini qurishdan iboratdir.

4.Ilmiy tadqiqot usullari. Kompleks o'zgaruvchili funksiyalar nazariyasi kursidan Koshining integral formulasi, o'rta qiymat haqidagi teorema, Koshi-Riman sistemasi yechimi, Karleman funksiyasi va formulasini qurish.

5.Ishning ilmiy ahamiyati. Bitifuv malakaviy ishda olingan natijalar ilmiy xarakterga ega bo'lib, kelgusida ko'p o'zgaruvchili bir jinsli va bir jinsli bo'lmagan Koshi-Riman sistemasi uchun qo'yilgan Koshi masalasining turli xil sohalarda yechishga yordam beradi.

6.Ishning amaliy ahamiyati. Bitifuv malakaviy ishda olingan natijalar matematik fizika tenglamari uchun qo'yilgan chegaraviy masalalarni yechish orqali, yani chegaraning bir qismida berilgan qiymati bo'yicha sohaning ichiga yechimni tiklash, analitik davom ettirish masalasi qaralgan. Bir qiymatli analitik funksiyaninig berilgan haqiqiy va mavhum qismlari bo'yicha o'zini tiklash muhim amaliy ahamiyatga egadir.

7.Ishning tuzilishi. Bitifuv malakaviy ishi kirish, 2 ta bob, 1-bobda 4-ta paragrapf, 2-bobda 3–ta paragrapf, xulosa qismi va foydalanilgan adabiyotlar ro'yxatidan iborat. Ushbu ish … matnli sahifan tashkil topgan. Har bir bob paragraflarga ajratilgan va ular o'zining nomerlanish hamda belgilanishiga ega.

8. Olingan natijaning qisqacha mazmuni. § 1.1. da bir qiymatli analitik funksiyanig sohaning chegarasida berilgan qiymati bo'yicha sohaga analitik davom ettirish masalasi, ya'ni Koshining integral formulasi o'rganilgan va bunga doir misollar yechilgan.

1.1-teorema *(Koshining integral formulasi)*. Agar $f(z)$ funksiya bir bog'lamli G sohada differensiallanuvchi bo'lib, D soha o'zining sodda yopiq silliq chegarasi ∂D bilan to'lasincha G sohaga qarashli bo'lsin, ya'ni $\overline{D} = D \cup \partial D \subset G$. U holda quyidagi

$$\frac{1}{2\pi i}\int_{\partial D}\frac{f(\varsigma)d\varsigma}{\varsigma - z} = \begin{cases} 0, & z \notin \overline{D}, \\ f(z), & z \in D \end{cases} \tag{1.1}$$

formula o'rinlidir.

§ 1.2. da bir jinsli Koshi-Riman sistemasi uchun Koshi masalasi yechimini ifodalashda Karleman formulasi o'rganilgan.

1.4-teorema (Karleman teoremasi): D soha yuqorida keltirilgan soha bo'lib, $f(z) \in A(D) \cap C(\overline{D})$, $\overline{D} = D \cup \partial D$ bo'lsin . U holda quyidagicha Karleman formulasi o'rinli:

$$f(z)=\frac{1}{2\pi i}\lim_{\sigma\to\infty}\int_S \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)}\frac{f(\xi)}{\xi-\mathrm{z}}d\xi,$$

bu yerda $\varphi_\sigma(\xi)=e^{-i\sigma\xi},\varphi_\sigma(z)=e^{-i\sigma z}$, σ-musbat parametr

§ 1.3. da bir jinsli bo'lmagan Koshi-Riman sistemasi

$$\frac{\partial f(z)}{\partial \bar{z}}=g(z), \qquad z\in D \qquad (1.5)$$

uchun Koshining integral formulasi isbotlangan, ya'ni

1.5-teorema **(Bir jinsli bo'lmagan Koshi-Riman sistemasi yechimining mavjudligi haqida)** Kompleks tekislikda bo'lakli silliq γ Jordan chizig'i bilan chegaralangan bir bog'lamli D soha berilgan bo'lsin. $f(\xi)\in C^1(D)\cap C(\overline{D})$ va $g(\xi)\in C'(\overline{D})$ bo'lsin . U holda (1.5) tenglamaning yechimi uchun quyidagi integral ko'rinish o'rinli:

$$f(\xi)=\frac{1}{2\pi i}\int_\gamma \frac{f(\xi)}{\xi-z}d\xi+\frac{1}{2\pi i}\iint_D \frac{g(\xi)}{\xi-z}d\xi\wedge d\bar{\xi};\ \ z\in D.$$

§ 1.4. da esa bir jinsli bo'lmagan Koshi-Riman sistemasi uchun Koshi masalasi yechimini ifodalashda Karleman formulasi o'rganilgan.

1.9-teorema. Agar $f(z)\in A(D)\cap C(\overline{D})\,,\overline{D}=D\cup\partial D$ bo'lib, $g(z)\in C^1(\overline{D})$,

$$|f(\xi)|_S=\varphi(\xi),\ \ \xi\in S \qquad (1.23)$$

shartni qanoatlantirsin. U holda quyidagi Karleman formulasi o'rinlidir

$$f(z)=\frac{1}{2\pi i}\lim_{\sigma\to\infty}\left(\int_S \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)}\frac{f(\xi)}{\xi-\mathrm{z}}d\xi+\iint_S \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)}\,g(\xi)d\xi\wedge d\bar{\xi}\right), \qquad (1.24)$$

bu yerda $\varphi_\sigma(\xi)=e^{\sigma\xi},\varphi_\sigma(z)=e^{\sigma z}$, σ-musbat parameter.

2-bob ishning asosiy qismini tashkik etadi. §2.1. Uch o'lcjovli fazoda Koshining integral formulasi

2.1-ta'rif. [4]. Uch komponentli $F(x)=(F_1(x),\ F_2(x),\ F_3(x))$ vector –funksiya D sohada potensial vector deyiladi, agar shu sohada

$$div\,F=0\,,\ rot\,F=0\,. \tag{2.1}$$

tenglamani qanoatlantirsa.

Potensial vector uchun Koshining integral formulasi o'rinlidir ([4], [25]):

$$\int\limits_{\partial D} M_0(y,x)F(y)dS_y=\begin{cases}0, & x\,\overline{\in}\,\overline{D},\\ F(x), & x\in D,\end{cases} \tag{2.2}$$

buyerda

$$M_0(x,\ y)=$$

$$=\frac{\partial\Phi_0(y,x)}{\partial x}\begin{pmatrix}\alpha(y_1-x_1)+\beta(y_2-x_2) & \beta(y_1-x_1)-\alpha(y_2-x_2) & \gamma(y_1-x_1)-\alpha(y_3-x_3)\\ \alpha(y_2-x_2)-\beta(y_1-x_1) & \alpha(y_1-x_1)+\beta(y_2-x_2) & \gamma(y_2-x_2)-\beta(y_3-x_3)\\ \alpha(y_3-x_3)-\gamma(y_1-x_1) & \beta(y_3-x_3)-\gamma(y_2-x_2) & \alpha(y_1-x_1)+\beta(y_2-x_2)\end{pmatrix}+ \tag{2.3}$$

$$+\begin{pmatrix}\gamma(y_3-x_3) & 0 & 0\\ 0 & \gamma(y_3-x_3) & 0\\ 0 & 0 & \gamma(y_3-x_3)\end{pmatrix},$$

$\Phi_0(y,x)=\dfrac{1}{4\pi r}$ - Laplastenglamasining fundamental yechimi.

§ 2.2. da Koshi-Riman sistemasi uchun chegralangan sohada Koshi masalasi uchun Adamar misoli va Karleman matrisasi qurilgan

2.2-ta'rif. (2.1), (2.8) masalaning Karlemn matritsasi deb quyidagi shartlarni qanoatlantiruvchi 3×3 $M_\sigma(y,x)$ matritsaga aytiladi:

1) $M_\sigma(y,x)=M_0(y,x)+G_\sigma(y,x)$, bu yerda σ – musbat sonli parametr, $G_\sigma(y,x)$ matritsa y o'zgaruvchi bo'yicha butun D sohada (2.1) sistemani qanoatlantiradi, $M_0(y,x)$ – (2.1) tenglamaning fundamental yechimlari matritsasi;

2) fiksirlangan $x\in D$ da $\int_T |M_\sigma(y,x)| dS_y \le \varepsilon(\sigma)$, $\sigma\to\infty$ da $\varepsilon(\sigma)\to 0$; bu yerda $|M_\sigma|$

$M=||M_{ij}||$ matritsaning yevklid normasi $|M_\sigma| = (\sum_{i,j=1}^{3} M_{ij}^2)^{\frac{1}{2}}$, xususiy holda F vector

uchun $|F| = (\sum_{i=1}^{3} F_i^2)^{\frac{1}{2}}$.

§ 2.3. da Koshi-Riman sistemasi uchun chegralangan sohada Koshi masalasining regulyarizatsiyasi

2.1-teorema [25]. $F(y)\in A(D)$ ∂D chegaranig T qismida (2.12) shartni qanoatlantirsin. U holda ixtiyoriy $x\in D$ va $\sigma > 0$ uchun

$$|F(x) - F_\sigma(x)| \le C(\sigma)\exp\left(-\sigma x_3^2\right), \tag{2.15}$$

tengsizlik o'rinlidir, bu yerda

$$C(\sigma) = \frac{\sqrt{7}}{\sqrt{\pi}}\left(\frac{2+\pi}{2\sqrt{\pi}} + \frac{1}{\sqrt{\sigma}}\right)$$

2.1-natija. Ixtiyoriy $x\in D$ uchun

$$\lim_{\sigma\to\infty} F_\sigma(x) = F(x) \tag{2.20}$$

tenglik o'rinlidir, D sohadan olingan kompaktda limit tekis bajariladi.

I-BOB. TEKISLIKDA UMUMLASHGAN KOSHI-RIMAN SISTEMASI YECHIMI UCHUN KARLEMAN FORMULASI

1.1-§. *Bir qiymatli analitik funksiyalar uchun Koshining integral formulasi*

Kompleks o'zgaruvchili funksiyalar nazariyasi umumiy kursida muhim o'rin egallagan Koshining integral teoremasidan Koshining integral formulasi kelib chiqadi.

1.1-teorema *(Koshining integral formulasi)*. Agar $f(z)$ funksiya bir bog'lamli G sohada differensiallanuvchi bo'lib, D soha o'zining sodda yopiq silliq chegarasi ∂D bilan to'lasincha G sohaga qarashli bo'lsin, ya'ni $\overline{D} = D \cup \partial D \subset G$. U holda quyidagi

$$\frac{1}{2\pi i}\int_{\partial D}\frac{f(\varsigma)d\varsigma}{\varsigma - z} = \begin{cases} 0, & z \notin \overline{D}, \\ f(z), & z \in D \end{cases} \tag{1.1}$$

formula o'rinlidir.

(1.1) formulaga Koshining integral formulasi deyiladi.

(1.11) formulada integral ostida turgan funksiya ikkita $f(\varsigma)$ va $\frac{1}{\varsigma - z}$ funksiyalar ko'paytmasidan iboratdir: $\varsigma = z$ nuqtada uzilishga ega bo'lgan $\frac{1}{\varsigma - z}$ funksiya Koshi integralining yadrosi, $f(\varsigma)$ esa zichligi deb ataladi.

1.1-eslatma. Agar $w = f(z)$ funksiya D sohada differensiallanuvchi va bo'lakli silliq Jordan egri chizig'idan iborat bo'lgan ∂D chegaragacha uzluksiz bo'lsa, u holda, D sohaga tegishli bo'lgan ixtiyoriy z uchun

$$f(z) = \frac{1}{2\pi i}\int_{\partial D}\frac{f(\varsigma)}{\varsigma - z}d\varsigma \tag{1.1'}$$

formula o'rinlidir. (1.1') formula ham xuddi (1.1) formula kabi isbotlanadi.

Koshining integral formulasi D soha ko'p bog'lamli bo'lganda ham o'rinlidir.

Amaliyotda Koshi integralining hisoblash uchun avval integral ostidagi funksiyani uzilish nuqtalari topilib, bu nuqtalarni shu sohaga qarashli yoki qarashli emasligi tekshiriladi. Agar uzilish nuqtalari sohaga qarashli bo'lsa, u holda, zichlik funksiyasi hisoblanadi. Aksincha, bu nuqtalar sohaga qarashli bo'lmasa integralning qiymati (Koshi teorema siga ko'ra) nolga teng bo'ladi.

1.1-misol. $\int\limits_{|z+i|=1} \sin^2 z \frac{dz}{z+2}$ integralni hisoblang.

Yechish. Integrallash chegarasi markazi $(0;-1)$, radiusi 1 ga teng bo'lgan aylanadan iborat. Zichlik funksiyasi $f(z)=\sin^2 z$ butun kompleks tekislikda differensiallanuvchi, yadrosi $\frac{1}{z+2}$ esa $z=-2$ nuqtadan tashqari hamma joyda differensiallanuvchi. $z=-2$ nuqta $|z+i|<1$ doiradan tashqarida joylashgan. $\varphi(z)=\frac{\sin^2 z}{z+1}$ funksiya $|z+i|<1$ doirada ikkita differensiallanuvchi funksiyaning nisbati sifatida differensiallanuvchi bo'ladi. Koshining integral teoremasiga ko'ra integralning qiymati nolga teng, ya'ni

$$\int\limits_{|z+i|=1} \sin^2 z \frac{dz}{z+2} = 0.$$

1.2-misol. $\int\limits_{|z+i|=3} \cos z \frac{dz}{z+2i}$ integralni hisoblang.

Yechish. Integrallash chegarasi markazi $(0;-1)$ nuqtada, radiusi 3 ga teng bo'lgan $|z+i|=3$ aylanadan iborat. Zichlik funksiyasi $f(z)=\cos z$ butun kompleks tekislikda differensiallanuvchi, yadrosi $\frac{1}{z+2i}$ esa $z=-2i$ nuqtadan tashqari hamma joyda differensiallanuvchi. $z=-2i$ nuqta $|z+i|<3$ doirada joylashgan. Koshining integral formulasiga ko'ra

$$\int\limits_{|z+i|=3} \cos z \frac{dz}{z+2i} = 2\pi i \cos(-2i) = 2\pi i \, ch2 .$$

1.3-misol. $\int\limits_{\partial D^+} \frac{z^2 e^z}{z^3+1} dz$ integralni hisoblang, bu yerda

$D^+ = \left\{ z : |z+1| \le \frac{1}{2} \right\}$.

Yechish. a) Birinchi navbatda ∂D^+ ni aniqlaymiz. $\partial D^+ = \left\{ z : |z+1| = \frac{1}{2} \right\}$ markazi

$(-1,0)$ nuqtada, radiusi $\frac{1}{2}$ ga teng bo'lgan aylana.

b) Integral ostidagi $F(z) = \frac{z^2 e^z}{z^3+1}$ funksiyaning uzilish nuqtalarini topamiz. Bu funksiya ikkita $f(z) = z^2 e^z$ va $g(z) = z^3 + 1$ funksiyalarning nisbati sifatida C kompleks tekislikda differensiallanuvchi. Demak, $F(z)$ funksiyaning uzilish nuqtalari $z^3 + 1 = 0$ tenglamaning ildizlaridan iborat bo'ladi. Ya'ni $(z+1)(z^2 - z + 1) = 0$ bundan $z+1=0,\ z^2 - z + 1 = 0$ tenglamalarning ildizlari $z_1 = -1,\ z_2 = \frac{1}{2} + i\frac{\sqrt{3}}{2},\ z_3 = \frac{1}{2} - i\frac{\sqrt{3}}{2}.$ Demak, $F(z)$ funksiya chegarasi $\partial D = \{z_1, z_2, z_3, \infty\}$ bo'lgan to'rt bog'lamli $D = \overline{C} \setminus \{z_1, z_2, z_3, \infty\}$ sohada differensiallanuvchi. Ildizlardan faqat $z_1 = -1$ D^+ sohaga tegishli, qolgan $z_2 = \frac{1}{2} + i\frac{\sqrt{3}}{2}, z_3 = \frac{1}{2} - i\frac{\sqrt{3}}{2}$ nuqtalar esa yopiq $\overline{D}^+$ sohaga tegishli emas.

v) b)- bo'limdan xulosa Koshi integral formulasining yadrosi - $\frac{1}{z+1}$, zichlik funksiyasi esa $f(z) = \frac{z^2 e^z}{z^2 - z + 1}$ ekan. (1.11) formulaga ko'ra integralning qiymati

$$2\pi i f(z_0) = 2\pi i f(-1) = 2\pi i \frac{(-1)^2 e^{-1}}{(-1)^2 - (-1) + 1} = \frac{2\pi i e^{-1}}{3} = \frac{2\pi i}{3e} \quad \text{bunda } z_0 = -1.$$

Demak,

$$\int_{\partial D^+} \frac{z^2 e^z\, dz}{z^3+1} = \int_{\partial D^+} \frac{z^2 e^z}{(z^2-z+1)} \frac{dz}{z-(-1)} = \frac{2\pi i}{3e}.$$

1.2-teorema (***o'rta qiymat haqidagi teorema***). Agar $w = f(z)$ funksiya $K = \{z : |z - z_0| < R\}$ doirada differensiallanuvchi va $\overline{K} = \{z : |z - z_0| \le R\}$ yopiq doirada uzluksiz bo'lsa, u holda bu funksiyaning doirani markazidagi qiymati aylana bo'yicha olingan o'rta arifmetik qiymatiga teng, ya'ni

$$f(z_0) = \frac{1}{2\pi} \int_0^{2\pi} f(z_0 + \mathrm{Re}^{i\varphi}) d\varphi \qquad (1.\ 2)$$

1.3-teorema (*garmonik funksiya uchun o'rta qiymat haqidagi teorema*).

$u(z) = u(x, y),\ \ z = x + iy$ funksiya $K = \{z : |z - z_0| < R\}$ doirada garmonik va $\overline{K} = \{z : |z - z_0| \le R\}$ yopiq doirada uzluksiz bo'lsin. U holda

$$u(z_0) = \frac{1}{2\pi} \int_0^{2\pi} u(z_0 + \mathrm{Re}^{i\varphi}) d\varphi\,. \qquad (1.3)$$

Koshining formulasidan Koshining integral teoremasi kelib chiqadi. Darhaqiqat

$$\int_{\partial D} f(\varsigma) d\varsigma = \int_{\partial D} \frac{(\varsigma - z) f(\varsigma)}{(\varsigma - z)} d\varsigma = \int_{\partial D} \frac{\varsigma\, f(\varsigma)}{\varsigma - z} d\varsigma -$$

$$- \int_{\partial D} \frac{zf(\varsigma)}{\varsigma - z} d\varsigma = 2\pi i z f(z) - 2\pi i z f(z) = 0 \,\bullet$$

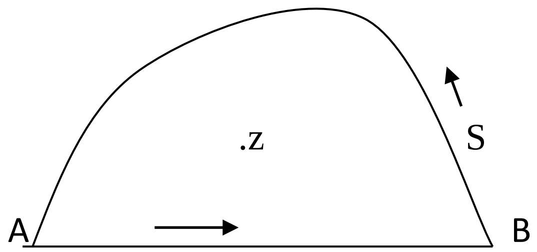
§1.2. Bir jinsli bo'lgan Koshi –Riman sistemasi uchun Karleman formulalari

1926-yilda buyuk matematik T.Karleman sohada berilgan analitik funksiyani uning chegarasining qismida berilgan qiymatlariga ko'ra tiklash masalasini yechdi.

Kompleks tekislikda bir bog'lamli chegaralangan D soha berilgan bo'lib, bu sohaning chegarasi quyidagicha tuzilgan:

$$\partial D = [A, B] \cup S$$

bu yerda S- yuqori yarim tekislikda joylashgan silliq yoy: $S \subset \{z: Jmz > 0\}$



Shu sohada bir jinsli Koshi-Riman sistemasi quyidagicha:

$$\frac{\partial f(z)}{\partial \bar{z}} = 0 \text{ ,} z \in D$$

Bu yerda $f(z) = u(x; y) + iv(x; y)$, $\frac{\partial f}{\partial z}$ esa quyidagicha aniqlanadi:

$$\frac{\partial f}{\partial z} = \frac{1}{2}\left[\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) + i\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]$$

Shu sistemaning yechimi uchun quyidagi integral formulasi o'rinli:

$$f(z) = \frac{1}{2\pi i}\int_{\partial D} \frac{f(\xi)}{\xi - \mathrm{z}} d\xi$$

Endi sistemaning yechimi uchun Karleman teoremasini isbot qilamiz:

1.4-teorema (Karleman teoremasi): D soha yuqorida keltirilgan soha bo'lib, $f(z) \in A(D) \cap C(\overline{D})$, $\overline{D} = D \cup \partial D$ bo'lsin . U holda quyidagicha Karleman formulasi o'rinli:

$$f(z) = \frac{1}{2\pi i} \lim_{\sigma \to \infty} \int_S \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)} \frac{f(\xi)}{\xi - z} d\xi,$$

bu yerda $\varphi_\sigma(\xi) = e^{-i\sigma\xi}, \varphi_\sigma(z) = e^{-i\sigma z}$, σ-musbat parametr

Isbot: Koshining integral formulasiga ko'ra:

$$f(z) = \frac{1}{2\pi i} \int_{\partial D} \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)} \frac{f(\xi)}{\xi - z} d\xi, \quad z \in D$$

ifoda o'rinli . D sohaning chegarasi bo'yicha olingan integralni ikkita integral yig'indisi ko'rinishda yozamiz:

$$\frac{1}{2\pi i} \int_{\partial D} e^{-i\sigma(\xi - z)} \frac{f(\xi)}{\xi - z} d\xi = \frac{1}{2\pi i} \left[\int_A^B e^{-i\sigma(\xi - z)} \frac{f(\xi)}{\xi - z} d\xi + \int_S e^{-i\sigma(\xi - z)} \frac{f(\xi - z)}{\xi - z} d\xi \right]$$

Oxirgi tenglikdan quyidagini hosil qilamiz:

$$f(z) - \int_S e^{-i\sigma(\xi - z)} \frac{f(\xi)}{\xi - z} d\xi = F(\sigma; z).$$

Bu yerda $\quad F(\sigma; z) = \int_A^B e^{-i\sigma(\xi - z)} \frac{f(\xi)}{\xi - z} d\xi$

endi $z \in D$ va $\sigma > 0$ bo'lganda $F(\sigma; z)$ ni baholaymiz.

1) $\left|e^{-i\sigma\xi}\right| = 1$ chunki ξ haqiqiy , σ musbat parameter, $A \leq \xi \leq B$
2) $\left|e^{i\sigma z}\right| = \left|e^{i\sigma(x+iy)}\right| = e^{Re(i\sigma x - \sigma y)} = e^{-y\sigma}$
3) $f(\xi) \in C(\overline{D})$, demak Veyershtrass teoremasiga ko'ra f(ξ) funksiya chegaralangan:$|f(\xi) \leq M < \infty|$

4) $\left|\xi - z\right| = \rho(z; [A; B]) > 0, z \in D$

Yuqorida keltirilgan mulohazalarga ko'ra $F(\sigma; z)$ ni baholaymiz:

$$|F(\sigma; z)| = \left|\int_A^B e^{-i\sigma(\xi - z)} \frac{f(\xi)}{\xi - z} d\xi\right| \leq \int_A^B \frac{\left|e^{-i\sigma\xi}\right| \left|e^{i\sigma z}\right| \left|f(\xi)\right|}{\left|\xi - z\right|} d\xi \leq \frac{(B - A)e^{-y\sigma}M}{\rho(z; [A; B])}$$

Oxirgi tengsizlikdan $\sigma \to \infty$ da limitga o'tsak $F(\sigma; z)$ nolga intiladi, chunki $e^{-\sigma y}$ $\xrightarrow[\sigma\to\infty]{} 0,\ y > 0.$

Demak,

$$f(z) = \frac{1}{2\pi i} \lim_{\sigma\to\infty} \int_S e^{-i\sigma(\xi - z)} \frac{f(\xi)}{\xi - z} d\xi \qquad (*)$$

ni hosil qildik. 1.4-t*eorema isbot bo'ldi*.

$f(z)$ funksiyani quyidagi ko'rinishda ham yozish mumkin:

$$f(z) = \frac{1}{2\pi i} \int_0^\infty \left(\int_S e^{-i\sigma(\xi - z)} f(\xi) d\xi\right) d\sigma + \frac{1}{2\pi i} \int_S \frac{f(\xi)}{\xi - z} d\xi \qquad (**)$$

(*) va (**) bir-biriga ekvivalentdir. Bu formula quyidagi formuladan kelib chiqadi:

$$\int_0^\infty \frac{\delta\psi(\sigma)}{\delta\sigma} d\sigma = \lim_{\sigma\to\infty} \psi(\zeta) - \psi(0)$$

$\psi(\zeta)$ deb [(**) formuladan]

$$\psi(\zeta) = \int_0^\infty e^{-i\sigma(\xi - z)} \frac{f(\xi)}{\xi - z}\ d\xi$$

belgilasak shu ekvivalent formula kelib chiqadi.

§1.3. Bir jinsli bo'lmagan Koshi –Riman sistemasi uchun

Koshining integral formulasi

Kompleks tekislikda chegaralangan bir bog'lamli D soha berilgan bo'lib, $\boldsymbol{z = x + iy}$ nuqta shu D sohaning ixtiyoriy nuqtasi bo'lsin. Shu sohada bir jinsli bo'lmagan Koshi –Riman sistemasi quyidagicha:

$$\frac{\partial \boldsymbol{f}(\boldsymbol{z})}{\partial \bar{\boldsymbol{z}}} = \boldsymbol{g}(\boldsymbol{z}), \qquad \boldsymbol{z} \in \boldsymbol{D} \qquad (\mathbf{1.5})$$

bu yerda

$$f(z) = f(x + iy) = u(x; y) + iv(x; y),$$

uning haqiqiy qismi $u(x; y)$ va mavhum qismi $v(x; y)$ hisoblanadi, ya'ni

$$u(x; y) = Ref(z); \qquad v(x; y) = Imf(z)$$

g(z) funksiyamiz esa quyidagicha:

$$g(z) = \frac{[\varphi(x;y) + iv(x;y)]}{2} \qquad (1.6)$$

aniqlangan. Uning haqiqiy va mavhum qismlari mos ravishda quyidagicha:

$$\varphi(x; y) = 2Reg(z); \qquad \psi(x; y) = 2Img(z)$$

$\frac{\partial f}{\partial \bar{z}}$ esa quyidagicha aniqlanadi:

$$\frac{\partial f}{\partial \bar{z}} = \frac{1}{2}\left[\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) + i\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] \qquad (1.7)$$

(1.6) va (1.7) ni (1.5) tenglamaga qo'ysak quyidagi tenglikni hosil qilamiz:

$$\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) + i\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \varphi(x; y) + i\psi(x; y). \qquad (1.8)$$

Oxirgi tenglikning o'ng va chap tarafidagi ifodalar bir –biriga teng bo'lishi uchun ularning haqiqiy va mavhum qismlai bir-biriga teng bo'lishi kerak. Natijada quyidagi sistemani hosil qilamiz:

$$\begin{cases} \dfrac{\partial u}{\partial x} - \dfrac{\partial v}{\partial y} = \varphi(x; y) \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} = \psi(x; y) \end{cases} \tag{1.5*}$$

(1.5) va (1.5*) bir-biriga ekvivalent. Agar g(z)$g(z) \equiv 0; z \in D$ bo'lsa, u holda (2.9) sistema quyidagi ko'rinishni oladi:

$$\frac{\partial f}{\partial \bar{z}} = 0,$$

ya'ni bir jinsli Koshi-Riman sistemasi hosil bo'ladi.

Bir jinsli bo'lmagan Koshi-Riman sistemasining yechimi yozilishdan oldin tashqi ko'paytma xossalarini keltiramiz:

1^0. $d\xi \wedge dz = -dz \wedge d\xi$

2^0. $d\xi \wedge d\xi = 0$

(A.Zorich, Matematicheskiy analiz . II tom, 197-203 betlar)

Kompleks tekislikda chegaralangan, bir bog'lamli D sohada bir jinsli bo'lmagan Koshi-Riman sistemasining umumiy yechimi quyidagi formula orqali aniqlanadi:

$$f(z) = \frac{1}{2\pi i}\int_{\partial D}\frac{f(\xi)}{\xi - z}d\xi + \frac{1}{2\pi i}\iint_{D}\frac{g(\xi)}{\xi - z}d\xi \wedge d\bar{\xi}; z \in D, \tag{1.9}$$

bu yerda $\partial D - D$ sohaning chegarasi (1.9) tenglikdagi

$$\frac{1}{2\pi i}\iint_{D}\frac{g(\xi)}{\xi - z}d\xi$$

integral Koshi tipidagi integral bo'lib, u shu D sohada analitik funksiyani ifodalaydi.

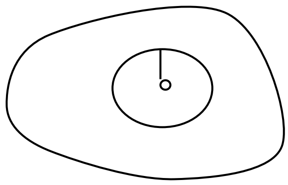
Quyidagicha teorema kuchga ega

1.5-teorema **(Bir jinsli bo'lmagan Koshi-Riman sistemasi yechimining mavjudligi haqida).** Kompleks tekislikda bo'lakli silliq γ Jordan chizig'i bilan chegaralangan bir bog'lamli D soha berilgan bo'lsin. $f(\xi) \in C^1(D) \cap C(\bar{D})$ va

$g(\xi)\in C'(\overline{D})$ bo'lsin . U holda (1.5) tenglamaning yechimi uchun quyidagi integral ko'rinish o'rinli:

$$f(\xi)=\frac{1}{2\pi i}\int_{\gamma}\frac{f(\xi)}{\xi-z}d\xi+\frac{1}{2\pi i}\iint_{D}\frac{g(\xi)}{\xi-z}d\xi\wedge d\bar{\xi};\quad z\in D.$$

Isbot:



∀∀

$\forall z\in D$ bo'lsin. D sohadan markazi z nuqtada radiusi r bo'lgan aylana olamiz va bu aylanani $\Delta(z;r)$ deb belgilaymiz: $\Delta(z;r)\subset D$

D_r deb quyidagicha ochiq sohani olamiz : $D_r=D\backslash\bar{\Delta}(z;r)$

Bu sohaning chegarasi: $\partial D_r=\gamma\backslash\gamma_r$, bu yerda $\gamma_r\backslash\Delta(z,r)$ aylananing chegarasi teoremani isbot qilishdan oldin biz quyidagicha lemmani isbot qilamiz.

1.1-lemma: Kompleks tekislikdagi D soha teoremaning isbotida keltirilgan soha bo'lsin. U holda shu sohada quyidagi tenglik o'rinli:

$$\iint_{D_r}\frac{\partial f}{\partial\bar{\zeta}}\frac{d\bar{\zeta}\wedge d\zeta}{\zeta-z}=\iint_{D_r}d(f(\zeta)\frac{d\zeta}{\zeta-z})$$

Isbot: Bizga ma'lumki differensiallanuvchi $F(z)$ funksiya uchun quyidagi tenglik o'rinli:

$$dF=\frac{\partial F}{\partial z}dz+\frac{\partial F}{\partial\bar{z}}d\bar{z}$$

Endi $f(\zeta)\frac{d\zeta}{\zeta-z}$ ifodaning differensialini topamiz:

$$d\left[f(\zeta)\frac{1}{\zeta-z}\right]=\frac{\partial}{\partial\zeta}\left(f(\zeta)\frac{1}{\zeta-z}\right)d\zeta+\frac{\partial}{\partial\bar{\zeta}}\left(f(\zeta)\frac{1}{\zeta-z}\right)d\bar{\zeta}$$

$$=\left[\frac{\partial f}{\partial\zeta}\frac{1}{\zeta-z}-f(\zeta)\frac{1}{(\zeta-z)^2}\right]d\zeta+\left[\frac{\partial f}{\partial\bar{\zeta}}\frac{1}{\zeta-z}+f\frac{\partial}{\partial\bar{\zeta}}\frac{1}{\zeta-z}\right]d\bar{\zeta}$$

$$=\frac{1}{\zeta-z}\frac{\partial f}{\partial\zeta}d\zeta-\frac{1}{(\zeta-z)^2}f(\zeta)d\zeta+\frac{\partial f}{\partial\bar{\zeta}}\frac{d\bar{\zeta}}{\zeta-z}$$

Demak,

$$\iint_{D_r} d(f(\zeta)\frac{1}{\zeta-z})d\zeta = \iint_{D}\left[\frac{1}{\zeta-z}\frac{\partial f}{\partial\zeta}-\frac{f(\zeta)}{(\zeta-z)^2}\right]d\zeta\Lambda d\zeta + \iint_{D_r}\frac{\partial f}{\partial\bar{\zeta}}\frac{d\bar{\zeta}\Lambda d\zeta}{\zeta-z}$$

chunki $d\zeta\Lambda d\zeta = 0.$

Shunday qilib quyidagini hosil qilamiz:

$$\iint_{D_r} d\left(f(\zeta)\frac{1}{\zeta-z}\right)d\zeta = \iint_{D_r}\frac{\partial f}{\partial\bar{\zeta}}\frac{d\bar{\zeta}\Lambda d\zeta}{\zeta-z}$$

Lemma isbot bo’ldi.

1.5-teoremaning isbotini davomi:

Yuqorida ko’rgan 1.1-lemmamizdan va Stoks formulasidan (Stoks formulasi:

$$\int_D d\omega = \int_{\partial D}\omega$$

$(A.Zorich\ .matematicheskiy\ analiz.II\ tom.246-249\ betlar)$ quyidagiga ega bo’lamiz:

$$\iint_{D_r}\frac{\partial f}{\partial\bar{\zeta}}\frac{d\bar{\zeta}\wedge d\zeta}{\zeta-z} = \iint_{D_r} d\left(f(\zeta)\frac{d\zeta}{\zeta-z}\right) = \int_{\gamma} f(\zeta)\frac{d\zeta}{\zeta-z} - \int_{\gamma_r} f(\zeta)\frac{d\zeta}{\zeta-z} \qquad (1.10)$$

(1.10) tenglikda $r\to 0$ da limitga o’tsak D_r bo’yicha ikki karrali integral D bo’yicha ikki karrali integralga yaqinlashadi, γ bo’yicha olingan integral o’zgarmaydi. γ_r bo’yicha olingan integralni tekshirish uchun parameterlar kiritamiz:

$$\zeta = z + re^{it};0\le t\le 2\pi;\quad d\zeta = rie^{it}dt$$

$$\int_{\gamma_r} f(\zeta)\frac{d\zeta}{\zeta-z} = \int_0^{2\pi}\frac{f(z+re^{it})rie^{it}dt}{re^{it}} = \int_0^{2\pi} f(z+re^{it})idt$$

Oxirgi tenglikda r→ 0 da limitga o’tsak quyidagini hosil qilamiz:

$$\lim_{r\to 0}\int_{\gamma_r} f(\zeta)\frac{d\zeta}{\zeta-z} = \int f(z)idt = 2\pi i f(z)$$

Demak, (1.10) tenglik quyidagi ko’rinishga keladi:

$$\iint_D \frac{\partial f}{\partial \bar{\zeta}} \frac{d\bar{\zeta}}{\zeta - z} = \int_\gamma f(\zeta) \frac{d\zeta}{\zeta - z} - 2\pi i f(z)$$

Oxirgi ifoda teorema shartidagiu integral ko'rinish

$$f(z) = \frac{1}{2\pi i} \int_\gamma \frac{f(\zeta)}{\zeta - z} + \frac{1}{2\pi i} \iint_D \frac{g(\zeta)}{\zeta - z} d\zeta \wedge d\bar{\zeta}, z \in D$$

bilan ustma-ust tushadi. 1.5-teorema isbot bo'ldi.

§1.4.Bir jinsli bo'lmagan Koshi –Riman sistemasi yechimini soha chegarasining bir qismi berilgan qiymati bo'yicha sohaga davom ettirish

Dirixle masalasi sohaning chegarasida beriladi. Koshi-Riman sistemasi uchun Dirixle masalasini qaraymiz:

$$\partial_{\bar{z}} f(z) = 0 \, ; z \in D \tag{1.11}$$

$$f \,/_{\partial D} = \varphi(\zeta) \tag{1.12}$$

Nokorrekt masala, ixtiyoriy φ uchun yechimi mavjud bo'lavermaydi. φ uzluksiz bo'lganda doim o'rinli bo'lavermaydi.

Bu ikkita analitik funksiyani ifodalaydi:

$\Phi^+(z) - \varphi(z)$ funksiya D sohaning ichkarisida yotganda

$\Phi^-(z) - \varphi(z)$ funksiya D sohaning tashqarisida bo'lganda

$$\Phi^+(z) - \Phi^-(z) = \varphi(\zeta)$$

***1.6-teorema**:* (1.11)-(1.12) Dirixle masalasi yechimga ega bo'lishi uchun

$\Phi^-(\boldsymbol{z}) \equiv \mathbf{0}$ bo'lishi zarur va yetarli.

Isbot: **Zarurligi*:*** (1.11)-(1.12) yechimiga ega. $\Phi^+(z)$ (1.11)-(1.12) ning yechimi, u holda

$$\Phi^+(z) = \frac{1}{2\pi i} \int_{\partial D} \frac{\varphi(\zeta)}{\zeta - z} d\zeta \, ; \; z \in D \; va \; z \in C(\bar{D})$$

$$\varphi(\zeta) = \frac{1}{2\pi i} \int_{\partial D} \frac{\varphi(\zeta)}{\zeta - z} d\zeta$$

ekanligidan $\Phi^-(z) \equiv 0$ bo'ladi.

(1.12) chegaradagi $\varphi(\zeta)$ funksiyaning D tashqarisidagi

Yetarliligi: $\Phi^{-}(z) \equiv 0$; $z \notin \overline{D}$ bundan $\Phi^{-}(\zeta) \equiv 0$; $\zeta \in \partial D$ keladi. Soxotskiy formulasidan $\Phi^{+}(z) = \varphi(\zeta)$; $\zeta \in \partial D$ bo'ladi.

Agar $\mathrm{w(z)} = \Phi^{+}(\mathrm{z})$ bo'lsa, teorema isbot bo'ldi. (1.12) chegaradagi $\varphi(\zeta)$funksiyaning D tashqarisidagi ∂D yechimi 0 ga teng bo'lsin.

1.7-teorema.: Agar φ(ζ) funksiya Gioldr shartini qanoatlantirsin.

Gioldr ko'rsatkichi 1ga teng. Bu masalani boshqacha aytganda Lipshist shartini $\varphi(\zeta) \in \mathrm{Lip}(\partial \mathrm{D})$ qanoatlantirsin. (1.11)-(1.12) Dirixle masalasi yechimga ega bo'lishi uchun quyidagi momentlar sharti bajarilishi zarur va yetarli:

$$\exists \varphi: \int_{\partial D} \varphi(\zeta) \cdot \zeta^{n} d\zeta = 0$$

Isbot: 1.6- teoremaga asosan (1.11)-(1.12) masalaning yechilishi uchun Koshi tipidagi integral Koshi integraliga aylanishi zarur.

$$\frac{1}{2\pi i} \int_{\partial D} \frac{\varphi(\zeta)}{\zeta - z} d\zeta\,; \quad \Phi^{-}(z) = 0; z \notin \overline{D}$$

$$\frac{1}{2\pi i} \int_{\partial D} \frac{\varphi(\zeta)}{\zeta - z} d\zeta \equiv 0$$

$$0 \equiv \frac{1}{2\pi i} \int_{\partial D} \frac{\varphi(\zeta)}{\zeta\left(1 - \frac{z}{\zeta}\right)} d\zeta = -\frac{1}{2\pi i} \int_{\partial D} \sum_{k=0}^{\infty} \frac{\zeta^{k}}{z^{k+1}} \varphi(\zeta) d\zeta$$

$$= \sum_{k=0}^{\infty} -\frac{1}{2\pi i} (z^{k+1})^{-1} \int_{\partial D} \zeta^{k} \varphi(\zeta) d\zeta \equiv 0$$

Funksiyaning darajali qatorga yoyishning yagonaligiga ko'ra koeffisentlarni 0ga tenglashtirib olamiz.

$$\int_{\partial D} \zeta^{k} \varphi(\zeta) d\zeta = 0\,; k = 0,1,2 \ldots$$

Teorema isbot bo'ldi.

Bir jinsli bo'lmagan Koshi-Riman sharti uchun Dirixle masalasi:

$$\partial_{\bar{z}} f(z) = g(z); \quad z \in \mathrm{D_{i}}, \tag{1.13}$$

$$f(z)/_{z \in \partial D} = \varphi(z), \tag{1.14}$$

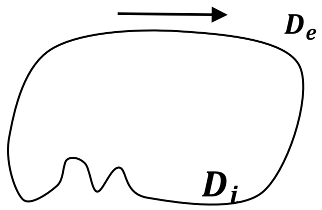
bu yerda $f(z)$ funksiya quyidagicha berilgan:

$$f(z)=\frac{1}{2\pi i}\int\limits_{\partial D}\frac{f(\zeta)}{\zeta-z}d\zeta-\frac{1}{\pi i}\iint\limits_{D}\frac{f_{\bar{\zeta}}(\zeta)}{\zeta-z}d\zeta$$

Borel –Pompey integral formulasidan topiladi.

1.8-teorema: $\boldsymbol{f\in L_p(D_i;C)}$ qiymatlarini C da qabul qiladi. $\boldsymbol{p>2}$; $\boldsymbol{\varphi\in C(L;C)}$ (1.13)-(1.14) masala faqat va faqat

$$\boldsymbol{\frac{1}{2\pi i}\int_L \zeta^n\varphi(\zeta)d\zeta-\frac{1}{\pi}\iint_{D_i} g(\zeta)\zeta^n d\xi d\eta=0\,;\quad n=0,1,2\,\dots} \qquad \textbf{(1.18)}$$



shart bajarilgandagina yechimga ega bo'ladi. Agar bu shart bajarilsa, u holda yagona yechim

$$f(z)=\frac{1}{2\pi i}\int\limits_{L}\frac{\varphi(\zeta)}{\zeta-z}d\zeta-\frac{1}{\pi i}\iint\limits_{D}\frac{g(\zeta)}{\zeta-z}d\zeta$$

formula orqali beriladi.

Isbot: *Zaruriyligi.* $f(z)\in C'(D)\cap C(\overline{D})$ bo'lganda quyidagi Borel-Pompey formulasi o'rinli:

$$\frac{1}{2\pi i}\int\limits_{L} f(\zeta)\frac{d\zeta}{\zeta-z}-\frac{1}{\pi}\iint\limits_{D_i}\frac{f_{\bar{\zeta}}(\zeta)}{\zeta-z}d\xi d\eta=\begin{cases} f(z);z\in D_i\\ 0;z\in D_e\end{cases} \qquad (1.19)$$

Koshi integral formulasiga ko'ra umumiyroq. Agar sohada analitik bo'lsa, ikki karrali integral 0 ga teng bo'lib, xususiy holdagi Koshi integral formulasiga keladi. (1.13),(1.14),(1.19) lardan quyidagi tenglikni yoza olamiz

$$\frac{1}{2\pi i}\int\limits_{L}\varphi(\zeta)\frac{d\zeta}{\zeta-z}-\frac{1}{\pi}\iint\limits_{D_i}g(\zeta)\frac{d\xi d\eta}{\zeta-z}=0;z\in D_e \qquad (1.20)$$

bu tenglikdan yetarli katta bo'lgan z lar uchun va $\left|\frac{\zeta}{z}\right|<1$ bo'lgani uchun z ni chiqarib olamiz. Bu esa cheksiz kamayuvchi geometrik progressiyaga keladi.

$$\sum_{n=0}^{\infty}\left[\frac{1}{2\pi i z^{n+1}}\int_L \varphi(\zeta)\zeta^n d\zeta-\frac{1}{\pi z^{n+1}}\iint_{D_i} g(\zeta)\zeta^n d\xi d\eta\right]=0 \qquad (1.21)$$

$Yetarliligi$: $\varphi(z) \in C(\partial D)$ funksiya (1.14) shartni qanoatlantiradi. (1.13) tenglamaning (1.11) shartni qanoatlantiruvchi yechimi borligini ko'rsatamiz. (1.19) tenglikning chap tomonidan iborat bo'lgan

$$\frac{1}{2\pi i}\int_L \varphi(\zeta)\frac{d\zeta}{\zeta-z} - \frac{1}{\pi}\iint_{D_i}\frac{g(\zeta)}{\zeta-z}d\xi d\eta. \qquad (*)$$

Bu ifoda ikkita funksiyani aniqlaydi. D_i sohada (1.13) tenglamaning yechimi bo'lgan $f_1(z)$ funksiyani aniqlaydi. D_e sohada esa analitik $f_2(z)$ funksiyani aniqlaydi. $\varphi \in L_p(D_i)$, $(p > 2)$ ekaligidan (*) da ikkinchi qo'shiluvchi ikki karrali integral butun kompleks tekislikning barcha nuqtalarida uzluksiz, D_i da analitik bo'ladi. Soxotskiy-Plemelya formulasiga ko'ra Koshi tipidagi integralning chegaraviy qiymati uchun

$$f_1(\zeta) - f_2(\zeta) = \varphi(\zeta), \zeta \in L. \qquad (1.22)$$

Ikkinchi tomondan etarlicha katta $z \in D_e$ (1.21) tenglik, yagonalik teoremaiga ko'ra (1.20) kelib chiqadi. Bundan esa $f_2(z) \equiv 0$. U holda $f_2(\zeta) \equiv 0, \zeta \in L$. Izlanayotfa Dirixle masalasining yechimi sifatida $f(z) = f_1(z)$ ni olsak, bu funksiya (1.13) tenglama va (1.14) shartni qanoatlantiradi.

D soha quyidagicha aniqlangan bo'lsin, ya'ni chegarasi $[-ai,\ ai]$ kesma va o'ng yarim tekislikda joylashgan silliq S yoydan iborat: $\partial D = [ai, -ai] \cup S$ bo'lsin.

1.9-teorema. Agar $f(z) \in A(D) \cap C(\overline{D})\,, \overline{D} = D \cup \partial D$ bo'lib, $g(z) \in C^1(\overline{D})$,

$$|f(\xi)|_S = \varphi(\xi),\ \ \xi \in S \qquad (1.23)$$

shartni qanoatlantirsin. U holda quyidagi Karleman formulasi o'rinlidir

$$f(z) = \frac{1}{2\pi i}\lim_{\sigma\to\infty}\left(\int\limits_S \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)}\frac{f(\xi)}{\xi - \mathrm{z}}d\xi + \iint\limits_S \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)}\,g(\xi)d\xi \wedge d\bar{\xi}\right), \qquad (1.24)$$

bu yerda $\varphi_\sigma(\xi) = e^{\sigma\xi}, \varphi_\sigma(z) = e^{\sigma z}$, σ-musbat parameter.

Isbot: D sohaning chegarasi bo'yicha olingan integralni ikkita integralning yig'indisi ko'rinishida tasvirlaymiz:

$$\frac{1}{2\pi i}\int_{\partial D}\frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)}\frac{f(\xi)}{\xi - z}\,d\xi =$$

$$= \frac{1}{2\pi i} \int_{-ai}^{ai} \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)} \frac{f(\xi)}{\xi - z} d\xi + \frac{1}{2\pi i} \int_S \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)} \frac{f(\xi)}{\xi - z}\, d\xi$$

.

Oxirgi tenglikning o'ng tomonidagi birinchi qo'shiluvchini $\mathcal{J}_\sigma(z)$ deb belgilaymiz

$$\mathcal{J}_\sigma(z) = \frac{1}{2\pi i} \int_{-ai}^{ai} \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)} \frac{f(\xi)}{\xi - z} d\xi,$$

$\xi = \zeta + i\eta,\ \zeta = 0$ bo'lganligi uchun $\varphi_\sigma(\xi) = e^{i\sigma\xi},\ \varphi_\sigma(i\eta) = e^{i\sigma\eta}$.

$|\varphi_\sigma(i\eta)| = |e^{i\sigma\eta}| = |\cos(\sigma\eta) + i\sin(\sigma\eta)| = \sqrt{cos^2(\sigma\eta) + cos^2(\sigma\eta)} = 1$

$$\mathcal{J}_\sigma(z) = \frac{1}{2\pi i} \int_{-ai}^{ai} \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)} \frac{f(\xi)}{\xi - z} d\xi =$$

$$= \frac{1}{2\pi i} \int_{-a}^{a} \frac{\varphi_\sigma(i\eta)}{\varphi_\sigma(z)} \frac{f(i\eta)}{i\eta - z} d(i\eta) = \frac{1}{2\pi} \int_{-a}^{a} \frac{\varphi_\sigma(i\eta)}{\varphi_\sigma(z)} \frac{f(i\eta)}{i\eta - z} d\eta$$

Endi $\mathcal{J}_\sigma(z)$ ni baholaymiz.

$$|\mathcal{J}_\sigma(z)| \le \frac{1}{2\pi} |e^{-\sigma z}| \cdot \left| \int_{-a}^{a} \frac{e^{i\sigma\eta}}{|i\eta - z|} \right| \le \frac{1}{2\pi} e^{-\sigma Rez} \cdot \frac{1}{|i\eta - z|}\ max|e^{i\sigma\eta}||f(i\eta)| \cdot 2a \le$$

$$\le \frac{e^{-\sigma Rez} \cdot M \cdot a}{\pi \cdot h(z)} \le \frac{e^{-\sigma\delta} \cdot M \cdot a}{\pi \cdot h(z)},$$

bu yerda $|e^{i\sigma\eta}| = 1, \max|f(\xi)| \le M,\ \min|i\eta - z| = h(z)$.

Oxirgi tengsizlikda $\sigma \to \infty$ da limitga o'tsak $\mathcal{J}_\sigma(z)$ nolga intiladi

$$\lim_{\sigma\to\infty} \mathcal{J}_\sigma(z) = 0.$$

2.10- teorema isbot bo'ldi.

(2.32) Karleman formulasini boshqacha ko'rinishda ham yozish mumkinligini ko'rsatamiz. (2.32) formulada quyidagicha belgilash olamiz:

$$\int_S \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)} \frac{f(\xi)}{\xi - z} d\xi + \iint_S \frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)} \frac{g(\xi)}{\xi - z} d\xi \wedge d\bar{\xi} = \psi(\sigma).$$

U holda Karleman formulasi quyidagi ko'rinishni oladi:

$$\frac{1}{2\pi i}\lim_{\sigma\to\sigma}\psi(\sigma)=f(z)$$

$\psi(0)$ ifoda esa quyidagichadir:

$$\psi(0)=\int_S\frac{\varphi(\xi)}{\xi-z}d\xi+\iint_D\frac{g(\xi)}{\xi-z}d\xi\wedge d\bar{\xi}.$$

Bizga ma'lumki

$$\int_0^\infty\psi'(\sigma)d\sigma=\psi(\sigma)_0^\infty=\lim_{\sigma\to\infty}\psi(\infty)-\psi(0),$$

$$\lim_{\sigma\to\infty}\psi(\infty)=\int_0^\infty\psi'(\sigma)d\sigma+\psi(0).$$

$$2\pi if(z)=\int_0^\infty\psi'(\sigma)d\sigma+\psi(0),$$

$$\psi(\sigma)=\int_S e^{\sigma(\xi-z)}\frac{f(\xi)}{\xi-\mathrm{z}}d\xi+\iint_S e^{\sigma(\xi-z)}\frac{g(\xi)}{\xi-z}d\xi\wedge d\bar{\xi}.$$

Oxirgi ifodaning hosilasini topamiz

$$\psi'(\sigma)=\int_S(\xi-z)e^{\sigma(\xi-z)}\frac{f(\xi)}{\xi-\mathrm{z}}d\xi+\iint_S(\xi-z)e^{\sigma(\xi-z)}\frac{g(\xi)}{\xi-z}d\xi\wedge d\bar{\xi}.$$

$$\psi'(\sigma)=\int_S e^{\sigma(\xi-z)}f(\xi)d\xi+\iint_S e^{\sigma(\xi-z)}g(\xi)\,d\xi\wedge d\bar{\xi}.$$

$$\int_0^\infty\psi'(\sigma)d\sigma=\int_0^\infty\left[\int_S\frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)}\frac{f(\xi)}{\xi-\mathrm{z}}d\xi+\iint_S\frac{\varphi_\sigma(\xi)}{\varphi_\sigma(z)}\frac{g(\xi)}{\xi-z}d\xi\wedge d\bar{\xi}\right]d\sigma.$$

$\mathcal{I}(\sigma)$ deb quyidagi ifodani belgilaymiz

$$\mathcal{I}(\sigma=\int_S\varphi_\sigma(\xi)f(\xi)d\xi+\iint_D\varphi_\sigma(\xi)g(\xi)d\xi\wedge d\bar{\xi}.$$

Demak,

$$2\pi if(z)=\int_0^\infty\frac{\mathcal{I}(\sigma)}{\varphi_\sigma(z)}d\sigma+\int_S\frac{\varphi(\xi)}{\xi-z}d\xi+\iint_D\frac{g(\xi)}{\xi-z}d\xi\wedge d\bar{\xi}.$$

Shunday qilib,

$$f(z)=\frac{1}{2\pi i}\left[\int_{0}^{\infty}\frac{\mathcal{J}(\sigma)}{\varphi_{\sigma}(z)}d\sigma+\int_{S}\frac{\varphi(\xi)}{\xi-z}d\xi+\iint_{D}\frac{g(\xi)}{\xi-z}d\xi\wedge d\bar{\xi}\right], \quad (1.25)$$

bu yerda

$$\mathcal{J}(\sigma=\int_{S}\varphi_{\sigma}(\xi)f(\xi)d\xi+\iint_{D}\varphi_{\sigma}(\xi)g(\xi)d\xi\wedge d\bar{\xi}.$$

Demak, biz (1.24) Karleman formulasiga ekvivalent bo'lgan (1.25) formulani keltirib chiqardik.

Xulosa qilib aytganda berilgan sohada bir jinsli bo'lmagan Koshi-Riman sistemasining yechimini shu soha chegarasining bir qismida berilgan qiymati bo'yicha tiklandi, ya'ni analitik davom ettirish masalasi yechildi.

II-BOB. UCH O'LCHOVLI FAZODADA KOSHI-RIMAN SISTEMASI YECHIMINI SOHA CHEGARASINING BIR QISMIDA BERILGAN QIYMATI BO'YICHA SOHAGA DAVOM ETTIRISH

§2.1. Uch o'lcjovli fazoda Koshining integral formulasi

R^3 uch o'lchovli haqiqiy o'zgaruvchining evklid fazosi bo'lib, $y=(y_1,y_2,y_3)$, $x=(x_1,x_2,x_3)\in R^3$, $y'=(y_1,y_2)$, $x'=(x_1,x_2)\in R^2$, $\alpha^2=|y'-x'|=(y_1-x_1)^2+(y_2-x_2)^2$, $r^2=|y-x|^2=\alpha^2+(y_3-x_3)^2$, $s=\alpha^2$, D -chegarasi chekli sondagi bo'lakli silliq sirtlardan tashkil topgan bir bog'lamli chegaralangan soha bo'lsin.

2.1-ta'rif. [4]. Uch komponentli $F(x)=(F_1(x),\ F_2(x),\ F_3(x))$ vector – funksiya D sohada potensial vector deyiladi, agar shu sohada

$$div\, F=0,\ rot\, F=0. \qquad (2.1)$$

tenglamani qanoatlantirsa.

Vektor analizdan ma'lum bo'lgan

$$divF(x)=\frac{\partial F_1(x)}{\partial x_1}+\frac{\partial F_2(x)}{\partial x_2}+\frac{\partial F_3(x)}{\partial x_3},$$

$$rot\, F=\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ F_1 & F_2 & F_3 \end{vmatrix}=$$

$$=(\frac{\partial F_3}{\partial x_2}-\frac{\partial F_2}{\partial x_3})i+(\frac{\partial F_3}{\partial x_1}-\frac{\partial F_1}{\partial x_3})j+(\frac{\partial F_2}{\partial x_1}-\frac{\partial F_1}{\partial x_2})k$$

ko'ra (2.1) tenglamani skalyar ko'rinishda quyidagicha ifodalash mumkin

$$\frac{\partial F_1}{\partial x_1}+\frac{\partial F_2}{\partial x_2}+\frac{\partial F_3}{\partial x_3}=0,$$

$$\frac{\partial F_3}{\partial x_2}-\frac{\partial F_2}{\partial x_3}=0,\ \frac{\partial F_3}{\partial x_1}-\frac{\partial F_1}{\partial x_3}=0,\quad \frac{\partial F_2}{\partial x_1}-\frac{\partial F_1}{\partial x_2}=0. \qquad (2.1')$$

Agar $x_3 = 0$ va $F_3 = 0$ bo'lsa, u holda (2.1') sistema R^2 tekislikdagi Koshi-Riman sistemasiga aylanadi. Demak, (2.1) sistema Koshi-Riman sistemasining R^3 fazodagi umumlashmasidir.

Potensial vector uchun Koshining integral formulasi o'rinlidir ([4], [25]):

$$\int_{\partial D} M_0(y,x)F(y)dS_y = \begin{cases} 0, & x \overline{\in} \overline{D}, \\ F(x), & x \in D, \end{cases} \tag{2.2}$$

buyerda

$$M_0(x, y) =$$

$$= \frac{\partial \Phi_0(y,x)}{\partial x}\begin{pmatrix} \alpha(y_1-x_1)+\beta(y_2-x_2) & \beta(y_1-x_1)-\alpha(y_2-x_2) & \gamma(y_1-x_1)-\alpha(y_3-x_3) \\ \alpha(y_2-x_2)-\beta(y_1-x_1) & \alpha(y_1-x_1)+\beta(y_2-x_2) & \gamma(y_2-x_2)-\beta(y_3-x_3) \\ \alpha(y_3-x_3)-\gamma(y_1-x_1) & \beta(y_3-x_3)-\gamma(y_2-x_2) & \alpha(y_1-x_1)+\beta(y_2-x_2) \end{pmatrix}+ \tag{2.3}$$

$$+\begin{pmatrix} \gamma(y_3-x_3) & 0 & 0 \\ 0 & \gamma(y_3-x_3) & 0 \\ 0 & 0 & \gamma(y_3-x_3) \end{pmatrix},$$

$\Phi_0(y,x) = \dfrac{1}{4\pi r}$ - Laplastenglamasining fundamental yechimi.

Gauss-Ostrogradskiyformulasidan

$$\iint_{\partial D} F \cdot n\, dS_y = \iiint_D divF dy$$

osongina ko'rish mumkinki Laplas tenglamasining ixtiyoriy regulyar yechimi uchun (2.2) Koshining integral formulasining qiymati nolga teng. Demak, Laplas tenglamasining fundamental yechimiga ixtiyoriy regulyar yechimni qo'shganda ham (2.2) formula o'rinlidir.

[25] ishda potensiyal vector uchun Koshining integral formulasi o'rinli bo'lishi ko'rsatilgan

$$\int_{\partial D} M(y,x)F(y)dS_y = \begin{cases} 0, & x \overline{\in} \overline{D}, \\ F(x), & x \in D, \end{cases} \tag{2.4}$$

bu yerda

$$M(x, y) =$$

$$
=\begin{pmatrix} \alpha(y_1-x_1)+\beta(y_2-x_2) & \beta(y_1-x_1)-\alpha(y_2-x_2) & \gamma(y_1-x_1)-\alpha(y_3-x_3) \\ \alpha(y_2-x_2)-\beta(y_1-x_1) & \alpha(y_1-x_1)+\beta(y_2-x_2) & \gamma(y_2-x_2)-\beta(y_3-x_3) \\ \alpha(y_3-x_3)-\gamma(y_1-x_1) & \beta(y_3-x_3)-\gamma(y_2-x_2) & \alpha(y_1-x_1)+\beta(y_2-x_2) \end{pmatrix} \times \frac{\partial \Phi(y,x)}{\partial x}+
$$

$$
+\begin{pmatrix} \gamma(y_3-x_3) & 0 & 0 \\ 0 & \gamma(y_3-x_3) & 0 \\ 0 & 0 & \gamma(y_3-x_3) \end{pmatrix}, \tag{2.5}
$$

$$
\Phi(y,x;k)=\left[-2\pi^2 K(x_3)\right]^{-1}\int_0^{\infty}\operatorname{Im}\left[\frac{K(w)}{w-x_3}\right]\frac{1}{\sqrt{u^2+\alpha^2}}\,du, \tag{2.6}
$$

Sh. Yarmuhamedovning [36] Laplas tenglamasini fundamental yechimi, $n(\alpha,\beta,\gamma)$ – ∂D sirtga y nuqtadagi tashqi normal, $r=|y-x|$, $K(w)$ – $w=u+iv$ (u, v –haqiqiy sonlar) kompleks o'zgaruvchining butun funksiyasi, w haqiqiy bo'lganda haqiqiy.

Agar $K(w)\equiv 1$ bo'lsa, u holda $\Phi(y,x)$ Laplas tenglamasining klassik fundamental yechimi, ya'ni ([19]. [33])

$$
\Phi(y,x)\equiv\Phi_0(y,x)=\frac{1}{4\pi r}. \tag{2.7}
$$

§2.2. Koshi-Riman sistemasi uchun chegralangan sohada Koshi masalasi uchun Adamar misoli va Karleman matrisasi

$D-R^3$ fazoda chegaralangan bir bo'g'lamli soha bo'lib, uning chegarasi ∂D $y_3=0$ tekislikning kompakt bog'lamli T qismi va $y_3\geq 0$ yarim tekislikda yotuvchi S Lyapunov silliq sirtining bo'lagidan iborat bo'lsin, $\overline{D}=D\cup\partial D$, $\partial D=S\cup T$, uch komponentli vector-funksiya $F(x)=(F_1(x),\ F_2(x),\ F_3(x))$ shu sohada birinchi tartibli uzluksiz hosilaga ega bo'lsin. Faraz qilamizki, nisbatan sohaning ixtiyoriy $x\in D$ nuqtasidan chiquvchi har bir nur S sirtni ko'pi bilan l nuqtada kesib o'tadi.

2.1 -masala. (2.1) sistema yechimining chegaraviy qiymati S sirtda ma'lum bo'lsin:

$$
F(y)=f(y), \quad y\in S. \tag{2.8}
$$

Berinlan $f(y)$ funksiyaga ko'ra D sohada $F(x)$ funksiyani tiklash talab qilinadi.

2.2 -masala. S sirtda uzluksiz $f(y)$ funksiya berilgan bo'lsin. (2.1) tenglamaning (2.8) shartni qanoatlantiruvchi yechimi mavjud bo'lishi uchun $f(y)$ funksiyaga qanday shart qo'yish kerakligini ko'rsating.

Ma'lumki Koshi-Riman sistemasi elliptik va elliptik tenglama uchun Koshi masalasi yechimi yagonadir [1.7], [1.18]. Biroq bu korrekt emas, ya'ni

1) ixtiyoriy berilganlar uchun yechim mavjud emas;

2) yechim Koshining berilganlaridan uzluksiz bog'liq emas. Quyidagi Adamar misoliga ([1], [22]) o'xshash misolda ko'rish mumkinki (2.1) sistema uchun Koshi masalasi korrekt emas.

2.1-misol. T $x_3 = 0$ tekislikning R^3 dagi bo'lagi bo'lsin

$$F^{(k)}{}_1(x) = \frac{\cos(kx_1)\sin(kx_2)sh\left(\sqrt{2}kx_3\right)}{k},$$

$$F_2^{(k)}(x) = \frac{\sin(kx_1)\cos(kx_2)sh\left(\sqrt{2}kx_3\right)}{k},$$

$$F_3^{(k)}(x) = \frac{\sqrt{2}\sin(kx_1)\,\sin(kx_2)ch\left(\sqrt{2}kx_3\right)}{k}.$$

$F^{(k)}(x) = (F_1^{(k)}(x), F_2^{(k)}(x), F_3^{(k)}(x))$ vektor-funksiya (2.1) sistemani D sohada qanoatlantirishini ko'rsatamiz.

$$\frac{\partial F^{(k)}{}_1(x)}{\partial x_1} = -\sin(kx_1)\sin(kx_2)sh\left(\sqrt{2}kx_3\right),$$

$$\frac{\partial F_2^{(k)}(x)}{\partial x_2} = -\sin(kx_1)\sin(kx_2)sh\left(\sqrt{2}kx_3\right),$$

$$\frac{\partial F_3^{(k)}(x)}{\partial x_3} = 2\sin(kx_1)\,\sin(kx_2)\,sh\left(\sqrt{2}kx_3\right).$$

Bundan

$$divF^{(k)} = \frac{\partial F_1^{(k)}}{\partial x_1} + \frac{\partial F_2^{(k)}}{\partial x_2} + \frac{\partial F_3^{(k)}}{\partial x_3} = 0.$$

$$\frac{\partial F_3^{(k)}(x)}{\partial x_2} = -\sqrt{2}\sin(kx_1)\,\cos(kx_2)ch\left(\sqrt{2}kx_3\right),$$

$$\frac{\partial F_2^{(k)}(x)}{\partial x_3} = \sqrt{2}\sin(kx_1)\,\cos(kx_2)\,ch\left(\sqrt{2}kx_3\right),$$

$$\frac{\partial F_3^{(k)}(x)}{\partial x_2} - \frac{\partial F_2^{(k)}(x)}{\partial x_3} = 0.$$

$$\frac{\partial F_3^{(k)}(x)}{\partial x_1} = \sqrt{2}\cos(kx_1)\sin(kx_2)ch\left(\sqrt{2}kx_3\right),$$

$$\frac{\partial F_1^{(k)}(x)}{\partial x_3} = \sqrt{2}\cos(kx_1)\sin(kx_2)\, ch\left(\sqrt{2}kx_3\right),$$

$$\frac{\partial F_1^{(k)}(x)}{\partial x_3} - \frac{\partial F_3^{(k)}(x)}{\partial x_1} = 0,$$

$$\frac{\partial F^{(k)}{}_2(x)}{\partial x_1} = \cos(kx_1)\cos(kx_2)\, sh\left(\sqrt{2}kx_3\right),$$

$$\frac{\partial F_1^{(k)}(x)}{\partial x_2} = \cos(kx_1)\cos(kx_2)\, sh\left(\sqrt{2}kx_3\right),$$

$$\frac{\partial F_2^{(k)}(x)}{\partial x_1} - \frac{\partial F_1^{(k)}(x)}{\partial x_2} = 0.$$

Shunday qilib, $F(x)$ vector-funksiya butun D sohada (2.1) tenglamani qanoatlantiradi.

Bundan tashqari $x_3 = 0$ uchun

$$\left|F^{(k)}(x)\right| \le \frac{\sqrt{2}}{k}.$$

Biroq $x_1 \neq 0,\ x_2 \neq 0$ va $x_3 > 0$ bo'lgan har bir $x = (x_1, x_2, x_3)$ nuqtada

$$\lim_{k\to\infty} F^{(k)}(x) = \infty.$$

Shunday qilib, (2.1), (2.8) Koshi masalasining yechimi berilganning kamgina o'zgarishiga nisbatan torg'un emasligini ko'rsatdik.

Asosiy maqsad giperbolik tenglamalar nazariyasida Koshi masalasi yechimini uchun B.Riman, V.Vol'ter, J.Adamarlar tomonidan olingan klassik formulalarga o'xshash aniq formulalarni hosil qilishdan iboratdir.

2.1-masalaning yechimi turg'un emasligi birinchi bo'lib Ж. Адамар [1] tomonidan aniqlangan. Amaliy masalalarda S sirtda $f(y)$ vektor-funksiyaning o'rniga mos holda $\delta > 0$ chetlanish bilan uning $f_\delta(y)$ yaqinlashishi beriladi va $f_\delta(y)$ bo'yicha D soha nuqtalarida oldindan berilgan aniqlikda yechimni qurish talab etiladi. Masalaning yechimi turg'un emasligidan, taqribiy yechimni qurish mumkin emas.

Turg'un yechimni qurish uchun qaralayotgan yechimlar sinfini aniqlashtirish kerak ([17], [32]). Bu eng avvalo aniq funksional fazolarda kompaktdir. Agar yechim tegishli bo'ladigan kompakt (kompaktni o'lchovi)ni ifodalovchi son ma'lum bo'lsa, musbat parameter $\sigma > 0$ dan bog'liq bo'lgan $F_\sigma(x) = F_\sigma(x, f_\delta)$ vector-funksiyalar (regulyarizatsiya) oilasini qurish haqida so'z ketadi.

[17] ga asoslangan holda

2.2-ta'rif. (2.1), (2.8) masalaning Karlemn matritsasi deb quyidagi shartlarni qanoatlantiruvchi 3×3 $M_\sigma(y,x)$ matritsaga aytiladi:

1) $M_\sigma(y,x) = M_0(y,x) + G_\sigma(y,x)$, bu yerda σ – musbat sonli parametr, $G_\sigma(y,x)$ matritsa y o'zgaruvchi bo'yicha butun D sohada (2.1) sistemani qanoatlantiradi, $M_0(y,x)$ – (2.1) tenglamaning fundamental yechimlari matritsasi;

2) fiksirlangan $x \in D$ da $\int_T |M_\sigma(y,x)| dS_y \le \varepsilon(\sigma)$, $\sigma \to \infty$ da $\varepsilon(\sigma) \to 0$; bu yerda $|M_\sigma|$ $M=\|M_{ij}\|$ matritsaning yevklid normasi $|M_\sigma| = (\sum_{i,j=1}^{3} M_{ij}^2)^{\frac{1}{2}}$, xususiy holda F vector uchun $|F| = (\sum_{i=1}^{3} F_i^2)^{\frac{1}{2}}$.

[36], [37] da isbotlangan

2.1-lemma. (2.6) formula bilan aniqlangan $\Phi_\sigma(y,x)$ funksiya

$$\Phi_\sigma(y,x) = \Phi_0(y,x) + g_\sigma(y,x) \tag{2.9}$$

ko'rinishda ifodalanadi, bu yerda $g_\sigma(y,x)$ – y, x ning barcha qiymatlarida aniqlangan va Laplas tenglamasini

$$\Delta g = 0, \quad y \in D, \quad \int_\Sigma \left(|\Phi_\sigma| + \left| \frac{\partial \Phi_\sigma}{\partial n} \right| \right) dS_y \le C(D)\sigma \exp(-\sigma x_3^2),$$

qanoatlantiruvchi funksiya, $C(D)$ – o'zgarmas.

(2.1) sistema uchun ham shunga o'xshahs lemma o'rinlidir.

2.2-lemma ([24]-[26]). (2.5) formula bilan aniqlangan $M_\sigma(y,x)$ matritsa (2.1), (2.8) masalaning Karleman matritsasi, ya'ni

$$M_\sigma(y,x) = M_0(y,x) + G_\sigma(y,x) \qquad (2.10)$$

ko'rish ega bo'lib, y, x ning barcha qiymatlarida aniqlangan $G_\sigma(y,x) = \|G_{ij\sigma}(y,x)\|_{3x3}$ - matrisa y o'zgaruvchi bo'yicha butun R^3 da (2.1) sistemani qanoatlantiradi.

Isbot. 2.1 lemma va 2.2 ta'rifdan (2.10) kelib chiqadi. $\Delta(\frac{\partial}{\partial y})g - H^2 g = 0$ ekanligidan, osongina ko'rish mukinki $G_\sigma(y, x; H)$ matritsa (2.1) tenglamani qanoatlantiradi, ya'ni y o'zgaruvchi bo'yicha $y = x$ .nuqta ham kiritilganda regulyar yechim bo'ladi.

(2.6) formuladan ko'rinadiki, T ($y_3 = 0$) da $\Phi_\sigma(y,x)$ funksiya va uning gradienti $\nabla\Phi_\sigma(y, x; H)$ $\sigma \to \infty$ da eksponensial tarzda barcha y_1, y_2 va $x \in R^3$, $x > 0$ nolga intiladi. Bundan (2.6) ga ko'ra $M_\sigma(y, x)$ matritsa $\sigma \to \infty$ da barcha y_1, y_2 va $x \in R^3$, $x > 0$ nolga intiladi. 2.2-ta'rifga ko'ra (2.5) formula bilan aniqlangan $M_\sigma(y,x)$ matritsa D soha va uning T п qismi uchun Karleman matritsasidan iboratdir. **Lemma isbot bo'ldi.**

§ 2.3. Koshi-Riman sistemasi uchun chegralangan sohada Koshi masalasining regulyarizatsiyasi

(2.6) formulada

$K(w) = e^{\sigma w^2}$, $w_1 = i\sqrt{u^2 + \alpha^2} + y_3$ deb olib

$$\mathrm{Im}\left[\frac{K(w)}{w}\right] = \frac{1}{2i}\left\{\frac{K(w)}{w} - \frac{K(\overline{w})}{\overline{w}}\right\} = \frac{\overline{w}K(w) - wK(\overline{w})}{2i(r^2+u^2)} =$$

$$= \frac{(y_3 - x_3)\,\mathrm{Im}\,K(w) - \sqrt{u^2+\alpha^2}\,\mathrm{Re}\,K(w)}{r^2+u^2},$$

almashtirishdan foydalansak, $\Phi_\sigma(y,x)$ ushbu ko'rinishni oladi:

$$2\pi^2\Phi_\sigma(y,x) = e^{-\sigma x_3^2 - \sigma\alpha^2 + \sigma y_3^2}\int_0^\infty \varphi_\sigma(y,x;u)e^{-\sigma\alpha^2}\frac{du}{u^2+r^2}, \qquad (2.11)$$

bu yerda

$$\varphi_\sigma(y,x;u)=\cos\tau\sqrt{u^2+\alpha^2}-(y_3-x_3)\frac{\sin\tau\sqrt{u^2+\alpha^2}}{\sqrt{u^2+\alpha^2}}, \quad \tau=2\sigma y_3. \quad (2.12)$$

Faraz qilaylik, $F(y)\in A(D)$ funksiya ∂D da chegaralangan

$$\left|F(y)\right|\le K, \qquad y\in T, \qquad (2.13)$$

K - berilgan musbat son.

Bu farazda Koshining integral formulasi o'rinlidir [4]

$$F(x)=\int_{\partial D} M_\sigma(y,x)F(y)dS_y, \quad x\in D. \qquad (2.14)$$

$$F_\sigma(x)=\int_S M_\sigma(y,x)F(y)dS_y, \quad x\in D$$

belgilab olamiz.

2.1-teorema [25]. $F(y)\in A(D)$ ∂D chegaranig T qismida (2.12) shartni qanoatlantirsin. U holda ixtiyoriy $x\in D$ va $\sigma>0$ uchun

$$\left|F(x)-F_\sigma(x)\right|\le C(\sigma)\exp\left(-\sigma x_3^2\right), \qquad (2.15)$$

tengsizlik o'rinlidir, bu yerda

$$C(\sigma)=\frac{\sqrt{7}}{\sqrt{\pi}}\left(\frac{2+\pi}{2\sqrt{\pi}}+\frac{1}{\sqrt{\sigma}}\right). \qquad (2.16)$$

Isbot. Koshining integral formulasi (2.14) dan ixtiyoriy $x\in D$ uchun

$$F(x)-F_\sigma(x)=J_\sigma(x), \quad J_\sigma(x)=\int_T M_\sigma(x,y)F(y)dS_y, \qquad (2.17)$$

(2.13) shartdan

$$\left|J_\sigma(x)\right|\le KT_\sigma(x), \qquad T_\sigma(x)=\int_T\left|M_\sigma(x,y;k)\right|dS_y, \qquad (2.18)$$

2.2-ta'rifga ko'ra (2.5) dan

$$\left|M_\sigma(x,y)\right|\le 2\sqrt{7}\left(\left|\Phi_\sigma(x,y)\right|+\left|grad\Phi_\sigma(x,y)\right|\right).$$

$\Phi_\sigma(x,y;k)$, $\frac{\partial}{\partial y_i}\Phi_\sigma(x,y;k)$ (2.11) formula bo'yicha hisoblab $y_3=0$ qo'yamiz. U holda (2.16) tenglik bilan aniqlangan $T_\sigma(x)$ uchun

$$T_\sigma(x)=\frac{\sqrt{7}}{\pi^2}e^{-\sigma x_3^2}\left[\int_T\left(\int_0^\infty\left(\frac{1}{u^2+\alpha^2+x_3^2}+\frac{2x_3\sigma}{u^2+\alpha^2+x_3^2}+\right.\right.\right.$$

$$+\frac{x_3}{(u^2+\alpha^2+x_3^2)^2}\Bigg)e^{-\sigma u^2}du\Bigg)e^{-\sigma\alpha^2}dy_1dy_2\Bigg] \tag{2.19}$$

Integralda T sohani R^2 fazo bilan almashtirib hosil bo'lgan takroriy integralni butun R^3 fazo bo'yicha olingan uch karrali integralga almashtiramiz va qutb koordinatalarini kiritib integralni ko'rinishini o'zgartirib, natijada

$$T_\sigma(x)\le\frac{2\sqrt{7}}{\pi}e^{-\sigma x_3^2}\left[\int_0^\infty\frac{2\sigma x_3}{t^2+x_3^2}t^2e^{-\sigma t^2}dt+\int_0^\infty\frac{x_3}{(t^2+x_3^2)^2}t^2e^{-\sigma t^2}dt\right]\le$$

$$\le\frac{\sqrt{7}e^{-\sigma x_3^2}}{\pi}\left[\frac{\sqrt{\pi}}{\sqrt{\sigma}}+\frac{2+\pi}{2}\right],\quad \frac{x_3 t}{t^2+x_3^2}\le 1\,.$$

Bundan 2.1-teoremaning tasdig'i kelib chiqadi.

2.1-natija. Ixtiyoriy $x\in D$ uchun

$$\lim_{\sigma\to\infty}F_\sigma(x)=F(x) \tag{2.20}$$

tenglik o'rinlidir, D sohadan olingan kompaktda limit tekis bajariladi.

Xulosa

Kompleks o'zgaruvchili funksiyalar nazariyasi fani o'zining ahamiyati jihatidan matematikaning barcha sohalarida asosiy o'rin egallaydi. Bu fanning haqiqiy o'zgaruvchili funksiyalar nazariyasi fanidan farqi ham Koshining integral teoremasi, Koshining integral formulasi, yagonalik teoremasi va analitik davom ettirish prisipi kabi muhim tushunchalar fanning mohiyatini yanada orttiradi.

Ushbu malakaviy bitiruv ishida kompleks o'zgaruvchili funksiyalar nazariyasining muhim tushunchalaridan biri hisoblangan Koshi-Riman sistemasi uchun qo'yilgan Koshi masalasi yechimini topish, ya'ni soha chegarasining bir qismida berilgan qiymati bo'yicha yechimni sohaga analitik davom ettirish masalasi o'rganilgan. Bunda Karleman formulasining o'rinli ekanligi ko'rsatilgan.

Xulosa qilib aytganda malakaviy bitiruv ishidagi natijalar texnika, fizika, mexanika sohasidagi masalalarini yechishda keng qo'llaniladi va bu ishni kelgusida yada kenroq sohada davom ettirish mumkin.

Foydalanilgan adabiyotlar:

1. Aramanovich I.G. i Levin V.I. Uravneniya matematicheskoy fiziki. Nauka, Moskva 1969 g.,str. 226-230, 267-271.

2. Bitsadze A.V Osnovi teorii analiticheskix funksiy kompleksnogo peremennogo. «Nauka» Gl. red. fiz. – mat. lit., M., 1984.

3. Bryuxov D.A. O novix obobsheniyax sistemi Koshi-Rimana s peremennimi koeffisentami // Seminar po mnogomernomy analizy (Seminar Vetushkina), 7 marta, 2012 g. GZ MGU.

4. Evgrafov M.A. Sbornik zadach po teorii analiticheskix funksiy. Izdatel’stvo «Nauka» Gl. red. fiz. – mat. lit., M., 1972

5. Lavrent’ev M.A., Shabat B.V. Metodi teorii funksiy kompleksnogo peremennogo: Uchebnoe posobie dlya universitetov. – M.: Nauka, Gl.red.fiz.-mat. lit., 1987. – 688 s.

6. Ishankulov T. O vozmojnosti obobshenno-analiticheskogo prodoljeniya v oblasti funksii, zadannix na kuske ee granitsi // Sibirskiy matematicheskiy jurnal. 2000, T.41, №6, s. 1350-1356.

7. Nikolenko V.N. Uravneniya matamaticheskoy fiziki. Izdatel’stvo Moskovskogo Universiteta 1981. 315s.

8. Nurimov T.N. Matematik fizika metodlari. O’qituvchi, 1988 y., 155-166 betlar.

9. Privalov.I.I “Granichniye svoystva analiticheskix funksiy” M.1950

10. Saloxiddinov M.S. Matematik fizika tenglamalari: Oliy o’quv yurtlarining talabalari uchun darslik. – T.: «O’zbekiston», 2002.- 448 b.

11. Saloxiddinov M.S., Islomov B.I. Matematik fizika tenglamalari fanidan masalalar to’plami. –T.: «MUMTOZ SO’Z», 2010.-372 b.

12. Sattorov E. Zadacha Koshi dlya obobshennoy sistemi Koshi-Rimana v prostnastve // Izvestiya VUZov. Matematika , 2010, №5, s. 32-40.

13. Sidorov Yu.V., Fedoryuk M.V., Shabunin M.I. Leksii po teorii funksiy kompleksnogo peremennogo: Ucheb. Dlya vuzov. – M.: Nauka, Gl. red. fiz. – mat. lit., 2001. – 480s.

14. Zorich A. Matematicheskiy analiz . 2002g., II tom, s.197-203.

15. Uss A.T. O kraevix zadachax dlya chetiryoxmernix analogov sistemi Koshi-Rimana s kompleksnomu koeffisentami // Vestnik Samarskogo gosudarstveennogo universiteta, 2001, bip. 12, s.10-16.

16.Internetresurslari:WWW.INTUIT.RU; http://www.mcmee.ru; http://lib.mexmat.ru;http://www.exponenta.ru; www.lib.homelinex.org/math/;www.eknigu.com/lib/Mathematics/; eknigu.com/info/M_Mathematics/MC

17. www.allmath.ru/highermath

A.Navoiy nomidagi SamDU mexanika-matematika fakul'teti matematika bo'limining

IV-kurs talabasi Ermamatova Zuxroning **"Bir o'lchamli Koshi-Riman sistemasi yechimini davom ettirish"** mavzusidagi malakaviy bitiruv ishiga

B A H O

Kompleks o'zgaruvchili funksiyalar nazariyasi o'zining qo'llanishi jihatidan haqiqiy o'zgaruvchili funksiyalar nazariyasiga qaraganda ko'proq imkoniyatga ega. Bir qiymatli analitik funksiyaning haqiqiy va mavhum qismlari Koshi-Riman shartini qanoatlantirishi, ular qo'shma garmonik funksiyalardan iborat ekanligi matematik fizikaning tekislikda qaraladigan chegaraviy masalalarini yechishda bir qator qulayliklarga egadir. Koshi-Riman sistemasi o'zining qo'llanishi jihatidan elliptik tenglamalarning eng sodda ko'rinishi bo'lsada, matematik fizika, kvant fizikasida muhim o'rin egallaydi. Shu nuqtai–nazardan by sistema uchun qo'yilgan chegaraviy masalalarni yechish qiziqarli va amaliy ahamiyatga egadir.

Ushbu malakaviy bitiruv ishi Koshi-Riman sistemasi uchun qo'yilgan Koshi masalasini yechishga bag'ishlangan bo'lib, kirish va ikkita bobdan iborat.

Birinchi bobning birinchi paragrafida bir qiymatli analitik funksiya tushunchasi, Koshi-Riman sharti, qo'shma garmonik funksiyalar o'rganilgan.

Birinchi bobning ikkinchi paragrafida Koshi-Riman sistemasining elliptikligi isbotlangan.

Birinchi bobning uchinchi paragrafida esa bir qiymatli analitik funksiyaning soha chegarasida berilgan qiymati bo'yicha sohaga analitik davom ettirish, ya'ni Koshining integral formulasi o'rganilgan va unga doir misollar yechib ko'rsatilgan.

Birinchi bobning to'rtinchi paragrafida Soxotskiy formulasi o'rganilgan.

Ikkinchi bobning birinchi paragrafida bir jinsli Koshi-Riman sistemasi uchun qo'yilgan Koshi masalasini yechishda Karleman formulasi, yechimning mavjudligini ifodalovchi Fok-Kuni teoremasi isbotlangan.

Ikkinchi bobning ikkinchi paragrafida bir jinsli bo'lmagan Koshi-Riman sistemasi uchun Koshining integral formulasi o'rganilgan.

Ikkinchi bobning uchinchi paragrafida bir jinsli Koshi-Riman sistemasi uchun chegaralangan sohada Karleman formulasi, beshinchi paragrafda esa bir jinsli bo'lmagan Koshi-Riman sistemasi uchun Karleman formulasi isbotlangan.

Ikkinchi bobning to'rtinchi paragrafida esa ikki o'zgarmas haqidagi teorema, regularizatsiya masalalari qaralgan.

Malakaviy bitiruv ishini bajarishda talaba M.Ermamatova o'quv yili davomida berilgan topshiriqlarni o'z vaqtida bajardi va o'ta mas'luyatli ekanligini namoyon etdi.

Ushbu malakaviy bitiruv ishi malakaviy bitiruv ishlari uchun qo'yilgan barcha talablarga javob beradi va uning bajaruvchisi Ermamatova Maftunaning bilimini "29" ball bilan baholash mumkin.

Fizika-matematika fanlari

doktori, professor **T.Ishankulov**