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“Tabiiy va umumkasbiy fanlar” kafedrasi

“OLIY MATEMATIKA, EXTIMOLLAR NAZARIYASI VA MATEMATIK STATISTIKA”

fanidan

Referat

Mavzu: 1- VA 2-TUR EGRI CHIZIQLI INTEGRALLARNING TADBIQI. 1- VA 2-TUR SIRT INTEGRALLARI. XOSSALARI VA HISOBLASH USULLARI

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REJA

1. **1- va 2-tur egri chiziqli integrallarning tadbiqi.**
2. **1- va 2-tur sirt integrallari.**
3. **Xossalari va hisoblash usullari.**

Tayanch ibora va tushunchalar

Karralli integrallar, integral yig'indi, ikki o'lchovli integral, yuz elementi, ikki o'lchovli integral, integrallash sohasi, 1-tur egri chiziqli integrallar, 2-tur egri chiziqli integrallar.

1. Birinchi va ikkinchi tur egri chiziqli integrallarning tadbiqi.

Birinchi tur egri chiziqli integrallar yordamida egri chiziq yoyining uzunligini, moddiy yoy massasini, silindrik sirt yuzini hisoblash mumkin.

a) $\int_{Ab} dl = l_{AB}$ bu yerda l_{AB} *AB* yoy uzunligi

b) $\int_{Ab} f(x,y,z)dl = m$ bu yerda *m* *AB* yoy moddiy massasi, $f(x,y,z) - ¥$ bu yoyning chiziqli zichligi.

c) $\int_{Ab} f(x,y)dl = S$ bu yerda *S* – yasovchilari *Oz* o'qqa parallel va *AB* yoy nuqtalaridan o'tuvchi, pastdan bu yoy bilan, yuqoridan silindr sirtning $z = f(x,y)$ $(f(x,y) > 0)$ sirt bilan kesishish chizig'i bilan, yon tamondan esa *A* va *B* nuqtalardan *Oz* o'qqa parallel o'tgan chiziqlar bilan chegaralangan silindrik sirtning yuzi.

Ikkinchi tur egri chiziqli integrallar yordamida shaklning yuzini, kuch ishini, funksiyaning uning ma'lum to'liq differensiali bo'yicha topish mumkin.

a) $\int_{AB} P(x,y)dx + Q(x,y)dy = A$, bunda *A* $\vec{F} = P(x,y)\vec{i} + Q(x,y)\vec{i}$ kuch bajargan ish.

b) $\frac{1}{2}\oint_L (xdy - ydx) = S$, bunda *S* – yopiq L kontur bilan chegaralangan soha yuzi

Agar L D sohaning chegarasi bo'lsa va $P(x,y), Q(x,y)$ funksiyalar yopiq D sohada o'zlarining xusuisy hosilalari bilan birgalikda uzliksiz bo'lsalar, u holda ushbu Grin formulasi o'rinlidir:

$$\oint_L P(x,y)dx + Q(x,y)dy = \iint_D \left(\frac{\partial Q}{\partial x} + \frac{\partial P}{\partial y}\right)dxdy \quad (1)$$

Bu yerda L konturni aylanib chiqish ushun shunday tanlanadiki, D soha chap tamonda qoladi (musbat yo’nalishda).

Agar biror D sohada Grin formulasi shartlari o’rinli bo’lsa, u holda quydagi shartlar teng kuchlidir:

a) $\oint_l Pdx + Qdy = 0$, nubda l D sohada joylashgan ixtiyoriy yopiq kontur.

b) $\int_{AB} Pdx + Qdy$ integral A va B nuqtalarni tutashtiruvchi integrallash yo’liga bog’liq emas.

c) $Pdx + Qdy = du(x, y)$, bunda $du(x, y)$ $u(x, y)$ funksiyaning to’liq differensiali.

d) D sohaning hamma nuqtalarida $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$

Agar $du(x, y) = P(x, y)dx + Q(x, y)dy$ bo’lsa, u holda

$$u(x, y) = \int_{x_0}^{x} P(x, y)dx + Q(x_0, y)dy$$

yoki

$$u(x, y) = \int_{x_0}^{x} P(x, y_0)dx + Q(x, y)dy$$

formulalar o’rinli.

2. 1-va 2- tur sirt integrallari.

Birinchi tur sirt integralining tarifi *f(x,y,z)* funktsiya (S) sirtda ((S)$\subset$ R3) berilgan bulsin. Bu sirtning P bulinishini va bu bulinishning xar bir (Sk)bulagida (k=1,2,3,….n)ixtieriy ($\text{¶}_k, \text{¦}_k, \text{S}_k$)nuktani olaylik.Berilgan funktsiyaning ($\text{¶}_k, \text{¦}_k, \text{S}_k$)nuktadagi l kiymatini (Sk) ning S_k yuziga kupaytirib kuydagi yigindini tuzamiz

$$\text{¢} = \sum_{k=1}^{n} f(\text{¶}_k, \text{¦}_k, \text{S}_k)\, S_k$$

<u>**1-tarif.**</u> Ushbu

$$\text{¢} = \sum_{k=1}^{n} f(\text{¶}_k, \text{¦}_k, \text{S}_k)\, S_k \qquad (2)$$

yigindi f(x,y,z) funktsiyaning integral yigindisi eki Riman yigindisi deb ataladi

(S) sirtning shunday

$$P_{1,}P_2,.....P_m,... \qquad (3)$$

bulinishlarini karaymizki ularning mos diametrlaridan tashkil topgan.

$$\text{a}_{p_1}, \text{a}_{p_2}, \text{a}_{p_3}......\text{a}_{p_m},.....$$

ketma ketlik nolga intilsa . $.\text{a}_{p_m} \to 0$ Bunday $P_m\,(m = 1,2,....)$ bulinishlarga nisbatan f(x,y,z) funktsiyaning integral yigindilarini tuzamiz.Natijada S sirtning (3) bulinishlariga mos integral yigindilar kiymatlaridan iborat kuyidagi ketma ketlik xosil buladi.

$$\pm_1, \pm_2,.......\pm_m,......$$

<u>2-tarif</u>. Agar (S) sirtning xar kanday (3) bulinishlari ketma ketligi {P_m} olinganda xam unga mos integral yigindi kiymatlaridan iborat {$\pm_m$} ketma ketlik ($¶_k, ¦_k, \text{s}_k$) nuktalarni tanlab olinishiga boglik bulmagan xolda xama vakt bitta I songa intilsa bu I$\pm$ yigindining limiti deb ataladi va u

$$\lim_{\text{a}_p \to 0} \pm = \lim_{\text{a}_p \to 0} \sum_{k=1}^{n} f(¶_k, ¦_k, \text{s}_k)\, S_k = \text{I}$$

kabi belgilanadi. Integral yigindining limitini kuyidagicha xam tariflash mumkin

Agar $\forall £$ >0 olinganda xam shunday$¢$ >0 topilsaki (S) sirtning diametri $\text{a}_p < ¢$ bulgan xar kanday bulinishi xamda xar bir (S_k)bulakdan olingan ixtieriy ($¶_k, ¦_k, \text{s}_k$)Lar uchun

$$|¢ - I| < £$$

tengsizlik bajarilsa u xolda I soni $\pm$ yigindining limiti deb ataladi

Agar $\text{a}_p \to 0$ f(x,y,z) funktsiyaning integral yigindisi $\pm$ chekli limitga ega bulsa f(x,y,z) funktsiya (s) sirt buyicha integrallanuvchi (Riman manosida integrallanuvchi) funktsiya deb ataladi.Bu yigindining chekli limiti I esa , f(x,y,z) funktsiyaning birinchi tur sirt integrali deyiladi va u

$$\iint_{(s)} f(x,y,z)\,ds$$

kabi belgilanadi.

Demak,

$$\iint_{(s)} f(x,y,z)\,ds = \lim_{\text{a}_p \to 0} \pm = \lim_{\text{a}_p \to 0} \sum_{k=1}^{n} f(¶_k, ¦_k, \text{s}_k)\, S_k$$

Endi birinchi tur sirt integralining mavjud bulishini taminlaydigan shartni topish bilan shugullanamiz.

Faraz kilaylik R3 fazodagi (S) sirt

Z=z(x,y) tenglama bilan berilgan bulsin .Bunda Z=z(x,y) funktsiya chegaralangan epik (D) soxada ((D))⊂ R2) uzluksiz va $z'_x(x,y), z'_y(x,y)$ xosilalarga ega xamda bu xosilalar xam(D).da uzluksiz.

1-teorema. Agar f(x,y,z) funktsiya (S) irtda berilgan va uzluksiz bulsa u xolda bu funktsiyaning (S) sirt buyicha birinchi tur sirt integrali

$$\iint\limits_{(s)} f(x,y,z)ds$$

mavjud va

$$\iint\limits_{(s)} f(x,y,z)ds = \iint\limits_{(D)} f(x,y,z,(x,y))\sqrt{1+z'^2_x(x,y)} + z'^2_y(x,y)\,\text{dxdy/}$$

buladi

3.Birinchi tur sirt integrallarining xossalari. Yuqorida keltirilgan teorema uzluksiz funktsiyalar birinchi tur sirt integrallarining ikki karali Riman integrallariga kelishini kursatadi. Binobarin bu sirt integrallar xam ikki karali Riman integrallari xossalsri kabi xossalarga ega buladi.

3. Birinchi tur sirt integrallarini xisoblash. Yuqorida keltirilgan teorema funktsiya birinchi tur sirt integralining mavjudligini tasdiklabgina kolmasdan uni xisoblash yulini xam kursatadi. Demak birinchi tur sirt integrallar ikki karali Riman integraliga keltirib xisoblanadi

$$\iint\limits_{(S)} f(x,y,z)ds = \iint\limits_{(D)} f(x,y,z(x,y))\sqrt{1+z'^2_x(x,y)+z'^2_y(x,y)}dxdy$$

$$\iint\limits_{(S)} f(x,y,z)ds = \iint\limits_{(D)} f(x(y,z),y,z)\sqrt{1+x'^2_y(y,z)+x'^2_z(y,z)}dydz$$

$$\iint\limits_{(S)} f(x,y,z)ds = \iint\limits_{(D)} f(x,y,(z,x),z)\sqrt{(1+y'^2_z(z,x)+y'^2_x(z,x)}dzdx$$

Misol. Ushbu

$$I=\iint\limits_{(S)} (x+y+z)ds$$

Integralni karaylik. Bunda (S)-$x^2+y^2+z^2=r^2$ sferaning z=0 tekislikning yukorida joylashgan kismi.

Ravshanki.(S)sirt

z=$\sqrt{r^2-x^2-y^2}$

Tenglama bilan aniklangan bulib, bu sirtda berilgan f(x,y,z)=x+y+z

funktsiya uzluksizdir. 1 teoremaga ko'ra

$$I=\iint_{(D)}(x+y+\sqrt{r^2-x^2-y^2})\sqrt{1+z_x'^2(x,y)+z_y'^2(x,y)}\,dxdy$$

buladi. Bunda (D)={(x,y)$\in$ R$^2: x^2+y^2\le r^2$}

endi bu tenglikning ung tamonidagi ikki karali integralni xisoblaymiz.

$$z_x'(x,y)=-\frac{x}{\sqrt{r^2-x^2-y^2}},\qquad z_y'(x,y)=-\frac{y}{\sqrt{r^2-x^2-y^2}},$$

$$\sqrt{1+z_x'^2(x,y)+z_y'^2(x,y)}=\frac{r}{\sqrt{r^2-x^2-y^2}}$$

demak

$$I=\iint_{(D)}(x+y+\sqrt{r^2-x^2-y^2})\sqrt{1+z_x'^2(x,y)+z_y'^2(x,y)}\,dxdy=r\iint_{(D)}\left(\frac{x+y}{\sqrt{r^2-x^2-y^2}}+1\right)dxdy$$

keyingi integralda uzgaruvchilarni almashtiramiz.

X= ᵒ cos ¨ , y= ° sin ¨

natijada

I=r

$$\int_0^{2®}(\int_0^r\left[\frac{°(\cos¨+\sin¨)}{\sqrt{r^2-°^2}}+1\right]°d°)d¨ = r\int_0^{2®}(\int_0^r \frac{°(\cos¨+\sin¨)}{\sqrt{r^2-°^2}}°d°)d¨ + r\int_0^{2®}(\int_0^r °d°)d¨ = r\int_0^{2®}(\cos¨ +$$

$$\sin¨)d¨\int_0^r\frac{°^2d°}{\sqrt{r^2-°^2}} + r2®\frac{r^2}{2} = ®r^3$$

demak berilgan integral

$$\iint\limits_{(S)}(x+y+z)ds = ®r^3$$

bo'ladi.

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