ON CONVERGENCE ALMOST EVERYWHERE OF SPECTRAL RESOLUTIONS OF ELLIPTIC DIFFERENTIAL OPERATORS

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The question of the validity of the Luzin conjecture for the spherical partial sums of the multiple Fourier integrals is open so far. But if we consider the Riesz means of the multiple Fourier integrals or in addition if we let $f = 0$ on an open set G, and investigate the convergence to zero almost everywhere on G (i.e. generalized localization principle), then there are many positive results. We fist remind some of these results and then study the generalized localization principle for compactly supported distributions and present sharp conditions for its fulfilment.

AMBARZUMYAN TYPE THEOREM FOR ENERGY-DEPENDENT STURM–LIOUVILLE EQUATIONS

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Sturm–Liouville spectral problems with potentials depending on the spectral parameter arise in various models of quantum and classical mechanics. For instance, to this form can be reduced the corresponding evolution equations (such as the Klein–Gordon equation [1, 2]) that are used to model interactions between colliding relativistic spinless particles. Another typical example is related to vibrations of mechanical systems in viscous media, see [3]. M. Jaulent and C. Jean in [4] studied the inverse scattering problems for energy-dependent Schrodinger operators on the line.

In 1929, Ambarzumyan investigated the Schrodinger operator with Neumann boundary conditions, and proved that if its spectrum consists of zero and infinitely many other square integers, then the potential is zero. From a historical viewpoint, the work of Ambarzumyan [5] in 1929 was thought of as the first paper in the theory of inverse spectral problems associated with Sturm-Liouville operators. Ambarzumyan's theorem was generalized in many directions [6- 10].

This paper presents an analog of Ambarzumyan's theorem to the energy-dependent Sturm-Liouville boundary problem.

We consider the boundary-value problem generated by the differential equation

$$-y'' + q(x)y + 2\lambda p(x)y = \lambda^2 y, \qquad 0 \le x \le \pi, \tag{1}$$

and two boundary conditions

$$y'(0) = hy(0), \quad y'(\pi) = -Hy(\pi), \qquad h, H \in R, \tag{2}$$

where λ is a spectral parameter and the real functions $q(x)$, $p(x) \in C^1[0, \pi]$.

We denote by μ_n, $n \in Z$, the spectrum of the problems (1) and (2). It is well known [11] that the sequence $\{\mu_n : n = 0, \pm1, \pm2, \pm3, ...\}$ satisfies the classical asymptotic form

$$\mu_n = n + c_0 + \frac{c_1}{n} + \frac{\gamma_n}{n},$$

where $\sum_n \gamma_n^2 < \infty$ and $c_0 = \frac{1}{\pi}\int_0^\pi p(x)dx$, $c_1 = \frac{1}{\pi}\left[h + H + \frac{1}{2}\int_0^\pi (q(x) + p^2(x))dx\right] dx$.

Let $\{\lambda_n : n = 0, \pm 1, \pm 2, ...\}$ and $\{n : n = 0, \pm 1, \pm 2, ...\}$ be spectrums of the problems

$$-y'' + q(x)y + 2\lambda p(x)y = \lambda^2 y, \quad 0 \le x \le \pi$$

$$y'(0) = 0, \quad y'(\pi) = 0,$$

and

$$-y'' = \lambda^2 y, \quad 0 \le x \le \pi$$

$$y'(0) = 0, \quad y'(\pi) = 0,$$

respectively.

The main results of this paper are as follows.

Theorem 1. If $\lambda_n = n, \ n \in Z$ then, $q(x) = p(x) = 0, \ \forall x \in [0, \pi]$.

Theorem 2. If $h + H \ge 0$ and $\mu_n = n, \ n \in Z$ then, $q(x) = p(x) = 0, \ \forall x \in [0, \pi]$.

REFERENCES

1. P. Jonas. On the spectral theory of operators associated with perturbed Klein–Gordon and wave type equations. J. Oper. Theory, 29(2):207–224, 1993.

2. B. Najman. Eigenvalues of the Klein–Gordon equation. Proc. Edinb. Math. Soc.(2), 26:181–190, 1983.

metricconverterProductID4. M3. M. Yamamoto. Inverse eigenvalue problem for a vibration of a string with viscous drag. J. Math. Anal. Appl., 152:20–34, 1990.

metricconverterProductID4. M4. M. Jaulent and C. Jean. The inverse problem for the one-dimensional Schrodinger equation with an energy-dependent potential. II. Ann. Inst. H. Poincarye Sect. A (N.S.), 25(2):119–137, 1976.

5. Ambarzumyan, V. A., (1929) Ьber eine frage der eigenwerttheorie, Zeitschrift fьr Physik, 53, 690-695.

6. Borg G. Eine Umkehrung der Sturm-Liouvilleschen Eigenwertaufgabe, Bestimmung der Differentialgleichung durch die Eigenwerte. Acta Math, 1946, 78: 1–96.

7. Carlson R. and Pivorachik V. N., (2007) Ambarzumian's theorem for trees, Electronic Journal of Differential Equations, 142, 1–9 .

8. Chern H. H., Law C. K. and Wang H. J., (2001) Extension of Ambarzumyans theorem to general boundary conditions, Journal of Mathematical Analysis and Applications, 263, 333-342.

9. Yang C.F. and Yang X.P., (2009) Some Ambarzumyan type theorems for Dirac operators, Inverse Problems, 25(9).

10. Yang C.F. and Yang X.P., (2011), Ambarzumyan's theorem for with eigenparameter in the boundary conditions, Acta Mathematica Scientia, 31(4), 1561-1568.

11. Gasymov M. G. and Guseinov G. Sh., (1981) Determination of a diffusion operator from the spectral data, Doklady Akademic Nauk Azerbaijan SSR, 37(2), 19-23.

THE PROBLEM OF INTEGRAL GEOMETRY WITH A WEIGHT FUNCTION OF A SPECIAL TYPE

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We study new problem of reconstruction of a function in a strip from their given integrals with known weight function along polygonal lines. We obtained two simply inversion formulas for the solution to the problem. We prove uniqueness and existence theorems for solutions and obtain stability estimates of a solution to the problem in Sobolev's spaces and thus show their weak ill-posedness. Then we consider integral geometry problems with perturbation. The uniqueness theorems are proved and stability estimates of solutions in Sobolev spaces are obtained. The existence theorem is proved.