ular uchun

$$a_2^1\left(a_2^2+b_2^2-c_2^2\right)+b_1^2\left(b_2^2+c_2^2-a_2^2\right)+c_2^2\left(c_2^2+a_2^2-b_2^2\right)\geq 16S_1S_2$$

tengsizlik o'rinli bo'lishini isbotlang.

2. Tomonlari a,b,c bo'lgan biror uchburchakning yarim primetri P ga teng bo'lsa, u holda ular uchun

$$\frac{1}{p-a}+\frac{1}{p-b}+\frac{1}{p-c}\geq\frac{9}{p}$$

tengsizlik o'rinli bo'lishini isbotlang.

Dilshod Atajanov (UrDU)

DIRAK TENGLAMASINI KELTIRIB CHIQARISHNING BITTA USULI HAQIDA

Аннотация. *Ushbu ishda Dirak tenglamasini keltirib chiqarishning sodda usuli keltirilgan. Relyativistik Grin funksiyasi yordamida Dirak tenglamasiga ekvivalent bo'lgan integral tenglama tuzilgan.*

Аннотация. *В работе предложен элементарный подход для получения уравнения Дирака. Построено эквивалентное интегральное уравнение к уравнению Дирака с помощью релятивистской функции Грина.*

Annotation. *In this paper, an elementary approach is offered to obtain Dirac's equation. The equivalent integral equation to the Dirac's equation is built by using relativistic Green's function.*

Kalit so'zlar: *Dirak operatori, to'lqin funksiyasi, spin, Shredinger tenglamasi.*

Ключевые слова: *оператор Дирака, волновая функция, спин, уравнения Шрёдингера.*

Key words: *The Dirac operator, wave function, spin, Schrodinger equation.*

Kvant mexanikasining fundamental tenglamalaridan biri bu Dirak tenglamasidir. Odatda, uni Shredinger tenglamasining relyativistik umumlashmasi deb qaraladi. Ma'lumki, Shredinger tenglamasi norelyativistik zarralarni vaqt bo'yicha holatini aniqlovchi to'lqin tenglamasi bo'lib, u 1926-yil avstraliyalik fizik E.Shredinger [1] tomonidan keltirib chiqarilgan

$$i\hbar\frac{\partial\psi(r,t)}{\partial t}=\frac{-\hbar^2}{2m}\nabla^2\psi(r,t)+V(r)\psi(r,t),\ r\in R^3$$

bu yerda: m – zarraning massasi, $2\pi\hbar$ Plank doimiysi. Shredinger bu tenglamani keltirib chiqarishda norelyativistik energiya tenglamasi $E=V+\frac{p^2}{2m}$ hamda energiya va impulsning operator $E\rightarrow i\hbar\partial_t, p\rightarrow -i\hbar\nabla$ ko'rinishidan foydalangan. Shredingerning bu natijasiga ko'ra, kvant mexanikasida yangi tushuncha, ya'ni $\psi(r,t)$ to'lqin funksiyasi tushunchasi paydo bo'ldi. Unga ko'ra, agar to'lqin funksiyasi $\psi(r,t)$ to'g'ri normallansa, $|\psi(r,t)|^2$ kattalik zarraning fazoni biror Ω sohasida bo'lish ehtimolini beradi.

Shundan keyin, relyativistik zarralar uchun ham to'lqin tenglamasini keltirib chiqarish mumkinmi, degan savolga javob topish masalasi o'sha davrning ko'plab olimlarining bu yo'nalishda ilmiy izlanishlar olib borishiga sababchi bo'ldi.[1 2 3]

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[1] M.Born, W.Heisenberg, P.Jordan (1925). "Zur Quantenmechanik II". Zeitschrift fr Physik 35 (89): 557615.

[2] E.Schrodinger. Quantization as an Eigenvalue Problem, Annalen der Physik, Vol. 79, 1926, 489.

[3] O.Klein. Quanten theorie und funfdimensionale Relativitats theorie, Zeitschrift fur Physik, 37 (1926).

Natijada 1926-yilda bir-biridan bexabar ravishda Klein[1] va Gardonlar[2] tomonidan dastlabki relyativistik to'lqin tenglamasi keltirib chiqarildi:

$$\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\psi(r,t) = \frac{m^2 c_0^2}{\hbar^2}\psi(r,t).$$

bu yerda: c_0 – yorug'lik tezligi, m –zarrani massasi. Hozirgi kunda bu tenglama Klein-Gardon tenglamasi deb yuritiladi va uning yechimi $\psi(r,t)$ spinsiz skalyar Bozonlar holatini ifodalaydi. Bu tehglamadan keyin 1927-yilda angliyalik fizik Dirak tomonidan relyativistik kvant nazariyasining yangi vaqtga nisbatan chiziqli bo'lgan "relyativistik to'lqin tenglamasi" keltirib chiqarildi.

Ushbu ishda Dirak tenglamasini keltirib chiqarishning sodda usulu keltirilgan va unga ekvivalent bo'lgan integral tenglama tuzilgan.

Eynshteyn nisbiylik nazariyasiga ko'ra, relyativistik energiya tenglamasi quyidagi

$$E = \sqrt{p^2c^2 + m^2c^4} \tag{1}$$

ko'rinishda yoziladi. Bu yerda $p = p(p_1, p_2, p_3)$ bo'lib, u zarraning impulsini bildiradi. Energiya va impulsning ushbu

$$p \to -i\hbar\nabla, \nabla = i\frac{\partial}{\partial x_1} + j\frac{\partial}{\partial x_2} + k\frac{\partial}{\partial x_3} and E \to i\hbar\frac{\partial}{\partial t}$$

operator ko'rinishini (1) tenglikka qo'yib, Shredinger nazariyasidan foydalansak, relyativistik zarralar uchun to'lqin tenglamasiga ega bo'lamiz:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi = \frac{m^2c^2}{\hbar^2}\psi. \tag{2}$$

Bu tenglama vaqtga nisbatan chiziqli emas, bizni qiziqtirgan relyativistik to'lqin tenglamasi vaqtga nisbatan chiziqli bo'lishi zarur, shu maqsadda (2) tenglikning chap tomonini quyidagicha ko'rinishda yozib olamiz:

$$\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} = \left(\alpha_1\partial_{x_1} + \alpha_2\partial_{x_2} + \alpha_3\partial_{x_3} + \frac{i}{c}\beta\partial_t\right)\left(\alpha_1\partial_{x_1} + \alpha_2\partial_{x_2} + \alpha_3\partial_{x_3} + \frac{i}{c}\beta\partial_t\right). \tag{3}$$

Bizning maqsad (3) tenglikni qanoatlantiruvchi α_i va β kattaliklarning mavjudligini ko'rsatishdir. Dirakkacha bo'lgan olimlar (3) tenglikni qanoatlantiruvchi kattaliklar haqiqiy va kompleks sonlar ichida, hatto 2x2 o'lchamli matritsalar ichida ham mavjud emasligini bilganlar. Dirakning xizmati shundaki, u bunday kattaliklarni 4x4 o'lchamli matritsalar ko'rinishida qidirgan, natijada ularning mavjudligini ko'rsatishga muvaffaq bo'lgan. (3) tenglikning bajarilishi uchun bu kattaliklardan ushbu

$$\beta^2 = I, \alpha_1^2 = \alpha_2^2 = \alpha_3^2 = I, \beta\alpha_i = 0, \alpha_i\alpha_j + \alpha_j\alpha_i = 0, i = 1,2,3; j = 1,2,3; i \neq j \quad 4)$$

shartlarning bajarilishi talab qilinadi. Bu shartlarni qanoatlantiruvchi matritsalar ushbu

$$\alpha_1 = \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & o \end{pmatrix}.$$

[1] O.Klein. Quanten theorie und funfdimensionale Relativitats theorie, Zeitschrift fur Physik, 37 (1926).

[2] W.Gordon. Der Comptonekt nach der Schrodingerschen Theorie, Zeitschrift fur Physik, 40 (1927).

$$\sigma_1 = \begin{pmatrix} o & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \beta = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}$$

ko'rinishda bo'lib, (3) tenglikning o'rinli ekanligini oddiy hisoblashlar yordamida ko'rsatish mumkin. Bu yerda I_2-2x2 o'lchamli birlik matritsa. Shundan keyin, (2) va (3) tengliklardan darhol quyidagi ayniyatni yoza olamiz:

$$\left(\alpha_1 \partial_{x_1} + \alpha_2 \partial_{x_2} + \alpha_3 \partial_{x_3} + \frac{i}{c} \beta \partial_t \right) \psi = \frac{mc}{\hbar} \psi . \tag{5}$$

Agar (5) tenglikning har ikkala tomoniga qaytadan bu matritsa operatorni qo'llasak, natijada (2) formulani hosil qilamiz, ya'ni

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \psi = \frac{m^2 c^2}{\hbar^2} \psi$$

bo'ladi. Shunday qilib, biz relyativistik zarralar holatini ifodalovchi, vaqtga nisbatan chiziqli bo'lgan ushbu

$$i\hbar \frac{\partial \psi(r,t)}{\partial t} = \left[\frac{\hbar c}{i} \left(\alpha_1 \frac{\partial}{\partial x_1} + \alpha_2 \frac{\partial}{\partial x_2} + \alpha_3 \frac{\partial}{\partial x_3} \right) + \beta mc^2 \right] \psi(r,t) \tag{6}$$

Dirak relyativistik to'lqin tenglamasini keltirib chiqardik. (6) tenglama hozirgi kunda Dirak tenglamasi deb yuritiladi va spinli zarralarning harakat tenglamasini ifodalaydi. (6) tenglamani ushbu

$$\hat{H}_0 \psi(r,t) = i\hbar \frac{\partial \psi(r,t)}{\partial t},$$

ko'rinishda ham yozish mumkin. Bu yerda

$$\hat{H}_0 = \frac{\hbar c}{i} \left(\alpha_1 \frac{\partial}{\partial x_1} + \alpha_2 \frac{\partial}{\partial x_2} + \alpha_3 \frac{\partial}{\partial x_3} \right) + \beta mc^2 = c\alpha p + \beta mc^2$$

$$p \to -i\hbar\nabla, \nabla = i\frac{\partial}{\partial x_1} + j\frac{\partial}{\partial x_2} + k\frac{\partial}{\partial x_3}, \; E \to i\hbar \frac{\partial}{\partial t}$$

$$\psi(r,t) = (\psi_1(r,t), \psi_2(r,t), \psi_3(r,t), \psi_4(r,t))^T;$$

$$\alpha = \alpha_1 i + \alpha_2 j + \alpha_3 k.$$

Dirak tenglamasi uchun spektral masalalarni o'rganishda ko'p hollarda statsionar Dirak tenglamasi bilan ishlashga to'g'ri keladi. (6) tenglamada ushbu

$$\psi(r,t) = \psi(r) e^{-\frac{iEt}{\hbar}} \tag{7}$$

almashtirishni bajarsak, natijada quyidagi

$$\hat{H}_0 \psi(r) = E\psi(r), \tag{8}$$

statsionar Dirak tenglamasiga ega bo'lamiz. Bu yerda E zarraning relyativistik energiyasi. Agar zarra biror skalyar $V(r)$ potensial maydon ta'sirida bo'lsa, u holda (8) tenglama quyidagicha yoziladi:

$$\hat{H} \psi(r,E) = E\psi(r,E), \tag{9}$$

bunda

$$\hat{H} = c\alpha p + \beta mc^2 + V(r) = \hat{H}_0 + V(r). \tag{10}$$

(10) tenglik yordamida aniqlangan ifoda, odatda, Dirak operatori deb yuritiladi.

Teorema. (9) tenglamaning yechimi quyidagicha

$$\psi(\mathbf{r}) = \psi_i(\mathbf{r}) + \int G(\mathbf{r}, \mathbf{r}'; E) V(\mathbf{r}') \psi(\mathbf{r}') d\mathbf{r}' \tag{11}$$

tasvirlanadi. Bunda $G(\mathbf{r}, \mathbf{r}'; E)$ funksiya Dirak operatorining Grin funksiyasi va

$\psi(\mathbf{r}) = \psi_i(\mathbf{r}) + \psi_s(\mathbf{r})$-to'lqin funksiyasi. To'lqin funksiyasi kiruvchi va chiquvchi to'lqinlar yig'indisidan iborat.

Isbot. Ma'lumki, Grin funksiyasi ushbu

$$\left(\hat{H}_0 - EI_4\right) G(\mathbf{r}, \mathbf{r}'; E) = -\delta(\mathbf{r} - \mathbf{r}') EI_4$$

tenglikdan aniqlanadi. Bunda $\delta(\mathbf{r})$ – delta Dirak funksiyasi. Grin funksiyasi uchun quyidagi

$$G(\mathbf{r}, \mathbf{r}'; E) = \frac{1}{2mc^2}\left(\hat{H}_0 + EI_4\right) g(\mathbf{r} \mid \mathbf{r}'; k) \tag{12}$$

tenglik bajarilishi ravshan. Bunda

$$g(\mathbf{r} \mid \mathbf{r}'; k) = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r} - \mathbf{r}'|}, \qquad \mathbf{r} \mid \mathbf{r}' = |\mathbf{r} - \mathbf{r}'|$$

nonrelativistik to'lqin tenglamasining Grin funksiyasi va u ushbu

$$\frac{\hbar^2}{2m}\left(\nabla^2 + k^2\right) I_4 g(\mathbf{r} \mid \mathbf{r}'; k) = -\delta(\mathbf{r} - \mathbf{r}') I_4$$

tenglikdan aniqlanadi. Relativistik Grin funksiyasining (12) ko'rinishdan (11) tasvir kelib chiqadi. Agar $\psi(\mathbf{r}) = \psi_i(\mathbf{r}) + \psi_s(\mathbf{r})$ ekanligini e'tiborga olsak, u holda (11) quyidagicha yozish mumkin:

$$\psi_s(\mathbf{r}) = \int G(\mathbf{r}, \mathbf{r}'; E) V(\mathbf{r}') \psi_i(\mathbf{r}') d\mathbf{r}' + \int G(\mathbf{r}, \mathbf{r}'; E) V(\mathbf{r}') \psi_s(\mathbf{r}') d\mathbf{r}'.$$

Teorema isbotlandi.