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MATEMATIKA VA TABIIY FANLAR KAFEDRASI

REFERAT

MAVZU. MATRITSALAR

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1.Matritsalar

Matritsa tushuncha sifatida XYIII-XIX asrlar davomida shakllantirildi va ishlab chiqildi. Daslabki vaqtlarda matritsa geometrik ob’ektlarni almashtirish va chiziqli tenglamalarni yechish bilan bog‘liq holda rivojlantirildi. Hozirgi vaqtda matritsalar matematikaning kuchli tatbiqiy vositalaridan biri hisoblanadi.

Matritsalar sonlar, funksiyalar va matematik belgilarning katta massivlarini yagona ob’ekt sifatida qarash va bunday massivlarni o‘z ichiga olgan masalalarni qisqa ko‘rinishda yozish va yechish imkonini beradi.

Matritsalar matematika, texnika va iqtisodiyotning turli sohalarida keng qo‘llaniladi. Masalan, ulardan matematikada algebraik va differensial tenglamalar sistemasini yechishda, kvant nazariyasida fizik kattaliklarni oldindan aytishda, internet tarmog‘ida ma’lumotlarni shifrlashda foydalaniladi.

Sonlarni joylashtirishda «Matritsa» tushunchasi *1850 yilda James Joseph Sylvester* tomonidan kiritilgan[1].

Ushbu bandda matritsalar nazariyasining asosiy tushunchalari bilan tanishamiz va uning ayrim tatbiqlarini o‘rganamiz. Bunda muhim tushuncha va qoidalar misollar yordamida mustahkamlanadi, qat’iy tasdiqlarni isbotlashda intuiktiv yondashishdan foydalaniladi.

1.1. Matritsa va uning turlari

Matritsani o‘rganishdan oldin ikkita sodda misolni ko‘rib chiqamiz.

[1] Lay, David C. Linear algebra and is applications. Copyright. 2012, pp. 92-112

1. $a_1x_1+a_2x_2+a_3x_3+a_4x_4=0$ chiziqli tenglama berilgan bo‘lsin. Bu tenglama a_1,a_2,a_3,a_4 koeffitsientlardan va x_1,x_2,x_3,x_4 noma’lumlardan tashkil topgan bo‘lib, u $\{a_1,a_2,a_3,a_4\}$ koeffitsiyentlar massivi bilan to‘liq aniqlanadi.

Shu kabi

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{pmatrix}$$

koeffitsiyentlar massivi besh noma’lumli

$$\begin{cases} a_{11}x_1+a_{12}x_2+a_{13}x_3+a_{14}x_4+a_{15}x_5=0, \\ a_{21}x_1+a_{22}x_2+a_{23}x_3+a_{24}x_4+a_{25}x_5=0 \end{cases}$$

ikkita chiziqli tenglamalar sistemasini aniqlaydi. Sistemada koeffitsiyentlar qulaylik uchun ikkita indeks bilan yozilgan bo‘lib, ulardan birinchisi sistema tenglamasining tartib raqamini, ikkinchisi esa o‘zgaruvchining tartib raqamini bildiradi. Berilgan sistemaning har ikkala tomonini biror songa ko‘paytiraylik yoki tenglamalardan birini ikkinchisiga qo‘shaylik. Bunda qo‘shish va ko‘paytirish amalda massiv ustida bajariladi.

2. Uch o‘lchovli fazoda vektor o‘zining tartiblangan uchta koordinatasi bilan beriladi: $\vec{a}=\{a_1;a_2;a_3\}$. Bunda vektorlar ustida chiziqli amallar koordinatalar ustida amallarga keltiriladi.

Shunday qilib, bir qancha masalalarni yechishda alohida kattaliklar bilan emas, balki ularning tartiblangan to‘plamlari (massivi) bilan ish ko‘rishga to‘g‘ri keladi.

Matritsa – bu sonlar (elementlar) massivining satr hamda ustunlarda berilgan va kichik qavslarga olingan to‘g‘ri burchakli jadvalidir[2]. Shuningdek, matritsaning elementlari algebraik belgilardan yoki matematik funksiyalardan iborat bo‘lishi mumkin.

Matritsaning o‘lchami uning satrlari soni va ustunlari soni bilan aniqlanadi. Matritsaning o‘lchamini ifodalash uchun $m\times n$ belgi ishlatiladi. Bu belgi

[2] Lay, David C. Linear algebra and is applications. Copyright. 2012, pp. 92-112

matritsaning m ta satr va n ta ustundan tashkil topganini bildiradi. Matritsaning o'zi lotin alifbosining bosh harflaridan biri bilan belgilanadi va uning elementlari jadvali kichik qavsga olinadi.

Masalan,

3×2 *o'lchamli matritsa*	2×3 *o'lchamli matritsa*	2×2 *o'lchamli matritsa*
$A=\begin{pmatrix} 2 & 5 \\ 0 & 7 \\ 3 & 1 \end{pmatrix}$	$B=\begin{pmatrix} -1 & 4 & 7 \\ 2 & 5 & 6 \end{pmatrix}$	$C=\begin{pmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{pmatrix}$

A matritsaning i-satr va j-ustunda joylashgan elementi a_{ij} bilan belgilanadi.

$A=(a_{ij})$, $(i=\overline{1,m}, j=\overline{1,n})$ yoki $A=\|a_{ij}\|$, $(i=\overline{1,m}, j=\overline{1,n})$ yozuv A matritsa a_{ij} elementlardan tashkil topganini bildiradi:

$$A=(a_{ij})=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}; \quad A=\|a_{ij}\|=\left\|\begin{matrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{matrix}\right\|.$$

$1\times n$ o'lchamli $A=(a_{11} \quad a_{12} \quad \dots \quad a_{1n})$ matritsaga *satr matritsa* yoki *satr-vektor* deyiladi.

$m\times 1$ o'lchamli $A=\begin{pmatrix} a_{11} \\ a_{21} \\ \dots \\ a_{m1} \end{pmatrix}$ matritsaga *ustun matritsa* yoki *ustun-vektor* deyiladi.

$n\times n$ o'lchamli maritsaga n- *tartibli kvadrat matritsa* deyiladi.

Kvadrat matritsaning chap yuqori burchagidan o'ng quyi burchagiga yo'nalgan $a_{11}, a_{22}, \dots, a_{nn}$ elementlaridan tuzilgan diagonaliga uning *bosh diagonali*, o'nq yuqori burchagidan chap quyi burchagiga yo'nalgan $a_{1n}, a_{2(n-1)}, \dots, a_{n1}$ elementlardan tuzilgan diagonaliga uning *yordamchi diagonali* deyiladi.

Bosh diagonalidan yuqorida (pastda) joylashgan barcha elementlari nolga teng bo'lgan

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix} \quad \left(A'=\begin{pmatrix} a_{11} & 0 & \dots & 0 \\ a_{21} & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \right)$$

matritsaga *yuqoridan uchburchak (quyidan uchburchak) matritsa* deyiladi.

Bosh diagonalda joylashmagan barcha elementlari nolga teng bo‘lgan

$$A=\begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix}$$

matritsaga *diagonal matritsa* deyiladi.

Barcha elementlari birga teng bo‘lgan diagonal matritsaga *birlik matritsa* deyiladi va I harfi bilan belgilanadi.

Barcha elementlari nolga teng bo‘lgan ixtiyoriy o‘lchamdagi matritsaga *nol matritsa* deyiladi va O harfi bilan belgilanadi.

A matritsada barcha satrlarni mos ustunlar bilan almashtirish natijasida hosil qilingan A^T matritsaga A matritsaning *transponirlangan matritsasi* deyiladi: $(a_{ij})^T=(a_{ji})$. Agar $A=A^T$ bo‘lsa, A matritsaga *simmetrik matritsa* deyiladi.

1.2. Matritsalar ustida arifmetik amallar

Maritsalarning tengligi

Bir xil o‘lchamli $A=(a_{ij})$ va $B=(b_{ij})$ matritsalarning barcha mos elementlari teng, ya’ni $a_{ij}=b_{ij}$ bo‘lsa, bu matritsalarga *teng matritsalar* deyiladi va $A=B$ deb yoziladi.

$$\boxed{\begin{gathered} A=B \quad \Leftrightarrow \quad a_{ij}=b_{ij} \\ \text{bacha } i=\overline{1,m},\ \ j=\overline{1,n} \text{ uchun} \end{gathered}}$$

Matritsani songa ko‘paytirish

1- ta'rif. $A=(a_{ij})$ *matritsaning* λ *songa ko'paytmasi* deb, elementlari $c_{ij}=\lambda a_{ij}$ kabi aniqlanadigan $C=\lambda A$ matritsaga aytiladi.

$$\boxed{C=\lambda A \quad \Leftrightarrow \quad c_{ij}=\lambda a_{ij}\,.}$$

1.1-misol. $A=\begin{pmatrix} 2 & -1 & 0 \\ 3 & 4 & -1 \end{pmatrix}$ bo'lsin. $3A$ ni toping.

Yechish.

$$3A=3\cdot\begin{pmatrix} 2 & -1 & 0 \\ 3 & 4 & -1 \end{pmatrix}=\begin{pmatrix} 3\cdot 2 & 3\cdot(-1) & 3\cdot 0 \\ 3\cdot 3 & 3\cdot 4 & 3\cdot(-1) \end{pmatrix}=\begin{pmatrix} 6 & -3 & 0 \\ 9 & 12 & -3 \end{pmatrix}.$$

Matritsalarni qo'shish va ayirish

Matritsalarni qo'shish va ayirish amallari *bir xil o'lchamli matritsalar* uchun kiritiladi. Bunda yig'indi matrisa qo'shiluvchi matritsalar bilan bir xil o'lchamga ega bo'ladi.

2-ta'rif. $A=(a_{ij})$ va $B=(b_{ij})$ *matritsalarning yig'indisi* deb, elementlari $c_{ij}=a_{ij}+b_{ij}$ kabi aniqlanadigan $C=A+B$ matritsaga aytiladi.

$$\boxed{C=A+B \quad \Leftrightarrow \quad c_{ij}=a_{ij}+b_{ij}\,.}$$

1.2-misol. $A=\begin{pmatrix} 1 & -1 & 4 \\ 3 & 0 & 1 \end{pmatrix}$ va $B=\begin{pmatrix} 2 & 3 & 2 \\ 1 & 0 & -2 \end{pmatrix}$ bo'lsin. $A+B$ ni toping.

Yechish.

$$A+B=\begin{pmatrix} 1 & -1 & 4 \\ 3 & 0 & 1 \end{pmatrix}+\begin{pmatrix} 2 & 3 & 2 \\ 1 & 0 & -2 \end{pmatrix}=\begin{pmatrix} 1+2 & -1+3 & 4+2 \\ 3+1 & 0+0 & 1+(-2) \end{pmatrix}=\begin{pmatrix} 3 & 2 & 6 \\ 4 & 0 & -1 \end{pmatrix}.$$

$-A=(-1)\cdot A$ matritsa A *matritsaga qarama-qarshi matritsa* deb ataladi.

3-ta'rif. $A=(a_{ij})$ va $B=(b_{ij})$ *matritsalarning ayirmasi* deb $C=A-B=A+(-B)$ matritsaga aytiladi. Bunda C matritsaning elementlari $c_{ij}=a_{ij}+(-b_{ij})=a_{ij}-b_{ij}$ kabi topiladi.

$$\boxed{C = A - B \quad \Leftrightarrow \quad c_{ij} = a_{ij} - b_{ij}.}$$

1.3-misol. $A = \begin{pmatrix} 2 & -3 & 2 \\ 2 & -1 & 4 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & -1 \end{pmatrix}$ bo‘lsin. $A - B$ ni toping.

Yechish.

$$A - B = \begin{pmatrix} 2 & -3 & 2 \\ 2 & -1 & 4 \end{pmatrix} - \begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 2-1 & -3-3 & 2-2 \\ 2-2 & -1-1 & 4-(-1) \end{pmatrix} = \begin{pmatrix} 1 & -6 & 0 \\ 0 & -2 & 5 \end{pmatrix}.$$

Matritsalar ustida chiziqli amallar quyidagi xossalarga ega [3].

A, B, C, O matritsalar $m \times n$ o‘lchsamli va λ, μ - skalyar sonlar bo‘lsa, u holda:

1^o. $A + B = B + A;$

2^o. $(A + B) + C = A + (B + C);$

3^o. $A + O = A;$

4^o. $A + (-A) = O;$

5^o. $\lambda(A + B) = \lambda A + \lambda B;$

6^o. $(\lambda + \mu)A = \lambda A + \mu A;$

7^o. $\mu(\lambda A) = \lambda(\mu A) = (\lambda\mu)A;$

8^o. $1 \cdot A = A;$

9^o. $(A + B)^T = A^T + B^T;$

10^o. $(\lambda A)^T = \lambda A^T;$

11^o. $A + C = B$ bo‘lsa, $C = B - A$ bo‘ladi;

12^o. $\lambda A = O$ bo‘lsa, $\lambda = 0$ yoki $A = O$ bo‘ladi;

13^o. $\lambda A = \lambda B$ va $\lambda \neq 0$ bo‘lsa, $A = B$ bo‘ladi.

Isboti. $1^o - 4^o$ xossalarning isboti bevosita 2-ta’rifdan kelib chiqadi.

5^o-xossani qaraymiz. A va B bir xil o‘lchamli matritsalar bo‘lsin.

U holda 1 va 2-ta’riflarga ko‘ra istalgan i, j da birinchidan

$$A + B = (a_{ij} + b_{ij}) \quad \text{yoki} \quad \lambda(A + B) = \lambda(a_{ij} + b_{ij}) = \lambda(a_{ij}) + \lambda(b_{ij})$$

va ikkinchidan

$$\lambda(a_{ij}) + \lambda(b_{ij}) = \lambda A + \lambda B$$

bo‘ladi. Oxirgi ikkita tenglikdan $\lambda(A + B) = \lambda A + \lambda B$ bo‘lishi kelib chiqadi [4].

Qolgan xossalar shu kabi isbotlanadi.

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[3] Lay, David C. Linear algebra and is applications. Copyright. 2012, pp. 92-112

[4] E.Kreyszig. Advancet engineering Matematics. Copyright. 2011, pp. 255-265

Matritsalarni ko‘paytirish

$A-$satr martitsa va $B-$ustun matritsa bir xil sondagi elementlarga ega bo‘lsin deylik. Bunda A satrning B ustunga ko‘paytmasi quyidagicha aniqlanadi:

$$AB=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \end{pmatrix}\cdot\begin{pmatrix} b_{11} \\ b_{12} \\ \dots \\ b_{1n} \end{pmatrix}=a_{11}b_{11}+a_{12}b_{12}+\ldots+a_{1n}b_{1n},$$

ya’ni ko‘paytma matritsalarning mos elementlari ko‘paytmalarining yig‘indisiga teng bo‘ladi[5].

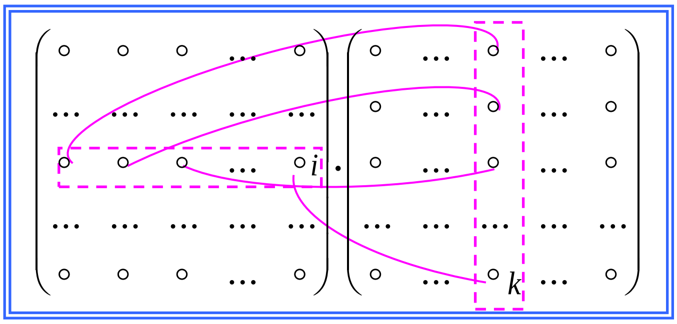
Matritsalarni ko‘paytirishning bu qoidasi *satrni ustunga ko‘paytirish qoidasi* deb yuritiladi.

Ikki matritsani ko‘paytirish amali *moslashtirilgan matritsalar* uchun kiritiladi. A matritsaning ustunlari soni B matritsaning satrlari soniga teng bo‘lsa, A va B *matritsalar moslashtirilgan* deyiladi.

4-ta’rif. $m\times p$ o‘lchamli $A=(a_{ij})$ *matritsaning* $p\times n$ o‘lchamli $B=(b_{jk})$ *matritsaga ko‘paytmasi* AB deb, c_{ik} elementi A matritsaning i-satrini B matritsaning j-ustuniga satrni ustunga ko‘paytirish qoidasi bilan, ya’ni

$$c_{ik}=a_{i1}b_{1k}+a_{i2}b_{2k}+\ldots+a_{ip}b_{pk}=\sum_{r=1}^{p}a_{ir}b_{rk},\ i=1,\ldots,m,\ k=1,\ldots,n$$

(qo‘shiluvchlari quyidagi sxemada keltirilgan) kabi aniqlanadigan $m\times n$ o‘lchamli $C=(c_{ik})$ matritsaga aytiladi.



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[5] E.Kreyszig. Advancet engineering Matematics. Copyright. 2011, pp. 255-265

1.4-misol. Berilgan matritsalarni ko‘paytiring.

1. $\begin{pmatrix} 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2\cdot 3 + 1\cdot 4 \end{pmatrix} = (10);$

2. $\begin{pmatrix} 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 4 \end{pmatrix} = \begin{pmatrix} 2\cdot 3 & 2\cdot 4 \\ -1\cdot 3 & -1\cdot 4 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ -3 & -4 \end{pmatrix};$

3. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 1\cdot 5 + 2\cdot 7 & 1\cdot 6 + 2\cdot 8 \\ 3\cdot 5 + 4\cdot 7 & 3\cdot 6 + 4\cdot 8 \end{pmatrix} = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix};$

4. $\begin{pmatrix} 2 & 1 \\ -3 & 4 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 & 4 \\ -1 & 0 & 3 \end{pmatrix} =$

$$= \begin{pmatrix} 2\cdot 3 + 1\cdot(-1) & 2\cdot 2 + 1\cdot 0 & 2\cdot 4 + 1\cdot 3 \\ -3\cdot 3 + 4\cdot(-1) & -3\cdot 2 + 4\cdot 0 & -3\cdot 4 + 4\cdot 3 \\ 0\cdot 3 + 2\cdot(-1) & 0\cdot 2 + 2\cdot 0 & 0\cdot 4 + 2\cdot 3 \end{pmatrix} = \begin{pmatrix} 5 & 4 & 11 \\ -13 & -6 & 0 \\ -2 & 0 & 6 \end{pmatrix}.$$

Agar A matritsaning satrlarini $A_1, A_2, \ldots, A_m$ bilan va B matritsaning ustularini $B_1, B_2, \ldots, B_n$ bilan belgilansa, u holda matritsalarni ko‘paytirish qoidasini quyidagi ko‘rinishda yozish mumkin:

$$C = AB = \begin{pmatrix} A_1 \\ A_2 \\ \ldots \\ A_m \end{pmatrix} \cdot \begin{pmatrix} B_1 & B_2 & \ldots & B_n \end{pmatrix} = \begin{pmatrix} A_1B_1 & A_1B_2 & \ldots & A_1B_n \\ A_2B_1 & A_2B_2 & \ldots & A_2B_n \\ \ldots & \ldots & \ldots & \ldots \\ A_mB_1 & A_mB_2 & \ldots & A_mB_n \end{pmatrix}.$$

Matritsalarni ko‘paytirishda A^2 yozuv ikkita bir xil matritsani ko‘paytmasini bildiradi: $A^2 = A\cdot A$. Shu kabi $A^3 = A\cdot A\cdot A \cdot \ldots \cdot A^n = \underbrace{A\cdot A\cdot \ldots \cdot A}_{n\ marta}$.

1.5-misol. $f(x) = 2x - x^2 + 5$ va $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$ bo‘lsin. $f(A)$ ni toping.

Yechish. Matritsa ko‘rinishdagi $f(A)$ funksiyaga o‘tishda λ sonli qo‘shiluvchi λI ko‘paytma bilan almashtiriladi, bu yerda I - birlik matritsa.

$$f(A) = 2A - A^2 + 5I = 2\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} + 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$=\begin{pmatrix}2 & -2\\ 0 & 4\end{pmatrix}-\begin{pmatrix}1 & -3\\ 0 & 4\end{pmatrix}+\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}=\begin{pmatrix}6 & 1\\ 0 & 5\end{pmatrix}.$$

Umuman olganda matritsalarni ko'paytirish nokommutativ, ya'ni $AB \neq BA$. Masalan, $1\times n$ o'lchamli A matritsaning $n\times 1$ o'lchamli B matritsaga AB ko'paytmasi sondan, ya'ni 1×1 o'lchamli matritsadan iborat bo'lsa, BA ko'paytmasi n- tartibli kvadrat matritsa bo'ladi.

Bir xil tartibli A va B kvadrat matritsalar uchun $AB = BA$ bo'lsa, A va B matritsalarga *kommutativ matritsalar*, $AB - BA$ ayirmaga *kommutator* deyiladi.

1.6-misol. $A=\begin{pmatrix}2 & 1\\ 0 & 3\end{pmatrix}$ va $B=\begin{pmatrix}3 & 5\\ 4 & -1\end{pmatrix}$ matritsalarning kommutatorini toping.

Yechish.

$$AB=\begin{pmatrix}2 & 1\\ 0 & 3\end{pmatrix}\cdot\begin{pmatrix}3 & 5\\ 4 & -1\end{pmatrix}=\begin{pmatrix}2\cdot 3+1\cdot 4 & 2\cdot 5+1\cdot(-1)\\ 0\cdot 3+3\cdot 4 & 0\cdot 5+3\cdot(-1)\end{pmatrix}=\begin{pmatrix}10 & 9\\ 12 & -3\end{pmatrix},$$

$$BA=\begin{pmatrix}3 & 5\\ 4 & -1\end{pmatrix}\cdot\begin{pmatrix}2 & 1\\ 0 & 3\end{pmatrix}=\begin{pmatrix}3\cdot 2+5\cdot 0 & 3\cdot 1+5\cdot 3\\ 4\cdot 2+(-1)\cdot 0 & 4\cdot 1+(-1)\cdot 3\end{pmatrix}=\begin{pmatrix}6 & 18\\ 8 & 1\end{pmatrix},$$

$$AB-BA=\begin{pmatrix}10 & 9\\ 12 & -3\end{pmatrix}-\begin{pmatrix}6 & 18\\ 8 & 1\end{pmatrix}=\begin{pmatrix}4 & -9\\ 4 & -4\end{pmatrix}.$$

Matritsalarni ko'paytirish amali ushbu xossalarga bo'ysunadi [6].

1^o. A matritsa $m\times n$ o'lchamli va B,C matritsalar $n\times p$ o'lchamli bo'lsa, $A(B+C)=AB+AC$ bo'ladi;

2^o. A matritsa $m\times n$ o'lchamli va B,C matritsalar $n\times p$ o'lchamli bo'lsa, $A(B+C)=AB+AC$ bo'ladi;

3^o. A,B,C matritsalar mos ravishda $m\times n, n\times p, p\times q$ o'lchamli bo'lsa, $A(BC)=(AB)C$ bo'ladi;

4^o. (4) A,B,I,O moslashtirilgan matritsalar va λ,μ skalyar sonlar bo'lsa, u holda:

1) $(\lambda A)(\mu B)=(\lambda\mu)(AB)$; 2) $A(\lambda B)=(\lambda A)B=\lambda(AB)$;

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[6] Lay, David C. Linear algebra and is applications. Copyright. 2012, pp. 92-112

3) $AI = IA = A$; 4) $AO = OA = O$;

5) $(AB)^T = B^T A^T$.

5^o. $A, I, O - n$- tartibli kvadrat matritsalar va p, q manfiy bo‘lmagan butun sonlar bo‘lsa, u holda:

1) $A^p A^q = A^{p+q}$; 2) $(A^p)^q = (A)^{pq}$;

3) $A^1 = A$; 4) $A^0 = I$.

Isboti. Xossalardan ayrimlarini ta’riflar yordamida isbotlaymiz va ayrimlarining to‘g‘riligiga misollarni yechish orqali ishonch hosil qilamiz.

2^o-xossani qaraylik. $A = (a_{ij})$ matritsa $m \times n$ o‘lchamli va $B = (b_{ij})$, $C = (c_{ij})$ matritsalar $n \times p$ o‘lchamli bo‘lsin. U holda 2 va 3-ta’riflarga ko‘ra istalgan i, j da birinchidan $B + C = (b_{ij} + c_{ij})$ yoki

$$A(B+C) = \sum_{k=1}^{n} a_{ik}(b_{kj} + c_{kj}) = \sum_{k=1}^{n}(a_{ik}b_{kj} + a_{ik}c_{kj}) = \sum_{k=1}^{n} a_{ik}b_{kj} + \sum_{k=1}^{n} a_{ik}c_{kj}$$

va ikkinchidan

$$\sum_{k=1}^{n} a_{ik}b_{kj} + \sum_{k=1}^{n} a_{ik}c_{kj} = AC + BC$$

bo‘ladi. Oxirgi ikkita tenglikdan $A(B+C) = AB + AC$ bo‘lishi kelib chiqadi [7].

6^o- xossani qaraylik. $A = (a_{ij})$ va $B = (b_{ij})$ bo‘lsin. Bundan $A^T = (a'_{ij})$ va $B^T = (b'_{ij})$ bo‘ladi, bu yerda $a'_{ij} = a_{ji}$, $b'_{ij} = b_{ji}$. U holda 3-ta’rifga ko‘ra istalgan i, j da birinchidan

$$AB = \sum_{k=1}^{n} a_{ik}b_{kj} \text{ yoki } (AB)^T = \sum_{k=1}^{n} a_{jk}b_{ki}$$

va ikkinchidan

$$\sum_{k=1}^{n} a_{jk}b_{ki} = \sum_{k=1}^{n} b_{ki}a_{jk} = \sum_{k=1}^{n} b'_{ik}a'_{ik} = B^T A^T$$

bo‘ladi. Bundan $(AB)^T = B^T A^T$ bo‘lishi kelib chiqadi [8].

3^o-xossani to‘g‘riligiga misol yechish orqali ishonch hosil qilamiz.

$A = (1 \quad 2)$, $B = \begin{pmatrix} 3 & -1 \\ 0 & 4 \end{pmatrix}$, $C = \begin{pmatrix} -2 & 4 & 1 \\ 5 & 0 & 2 \end{pmatrix}$ bo‘lsin.

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[7] E.Kreyszig. Advancet engineering Matematics. Copyright. 2011, pp. 255-265

[8] E.Kreyszig. Advancet engineering Matematics. Copyright. 2011, pp. 255-265

U holda

$$BC=\begin{pmatrix}3 & -1\\ 0 & 4\end{pmatrix}\cdot\begin{pmatrix}-2 & 4 & 1\\ 5 & 0 & 2\end{pmatrix}=\begin{pmatrix}-11 & 12 & 1\\ 20 & 0 & 8\end{pmatrix},$$

$$A(BC)=\begin{pmatrix}1 & 2\end{pmatrix}\cdot\begin{pmatrix}-11 & 12 & 1\\ 20 & 0 & 8\end{pmatrix}=\begin{pmatrix}29 & 12 & 17\end{pmatrix},$$

$$AB=\begin{pmatrix}1 & 2\end{pmatrix}\cdot\begin{pmatrix}3 & -1\\ 0 & 4\end{pmatrix}=\begin{pmatrix}3 & 7\end{pmatrix},$$

$$(AB)C=\begin{pmatrix}3 & 7\end{pmatrix}\cdot\begin{pmatrix}-2 & 4 & 1\\ 5 & 0 & 2\end{pmatrix}=\begin{pmatrix}29 & 12 & 17\end{pmatrix}.$$

1. Mashqlar

Demak, $A(BC)=(AB)C$.

1.1. A kvadrat matritsa bo‘lsin. $A+A^T$ simmetrik matritsa bo‘lishini ko‘rsating.

1.2. $A=\begin{pmatrix}3\\4\\5\end{pmatrix}$ matritsani $X=\begin{pmatrix}1\\0\\0\end{pmatrix}$, $Y=\begin{pmatrix}0\\1\\0\end{pmatrix}$ va $Z=\begin{pmatrix}0\\0\\1\end{pmatrix}$ matritsalarning chiziqli kombinatsiyasi ko‘rinishida ifodalang.

1.3. $a\begin{pmatrix}2\\3\end{pmatrix}+b\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}10\\5\end{pmatrix}$ bo‘lsa, a va b ni toping.

1.4. Matritsa 30 ta elementga ega bo‘lsa, u qanday tartiblarda berilishi mumkin?

1.5-1.1.8 masqlarda A, B matritsalar va λ, μ sonlar berilgan. $\lambda A+\mu B$ matritsani toping:

1.5. $A=\begin{pmatrix}1 & -1 & -1\\ 2 & -3 & 0\end{pmatrix}$, $B=\begin{pmatrix}2 & 3 & -1\\ -1 & 0 & 2\end{pmatrix}$, $\lambda=-1,\ \mu=2.$

1.6. $A=\begin{pmatrix}0 & -3\\ -2 & 1\\ 1 & 4\end{pmatrix}$, $B=\begin{pmatrix}-1 & 2\\ 3 & -1\\ 2 & -5\end{pmatrix}$, $\lambda=2,\ \mu=-3.$

1.7. $A=\begin{pmatrix}2 & -1 & 0\\ -1 & 3 & -2\\ 2 & 3 & 1\end{pmatrix}$, $B=\begin{pmatrix}-3 & 1 & 1\\ 0 & -1 & 0\\ -4 & -3 & 2\end{pmatrix}$, $\lambda=-3,\ \mu=-2.$

1.8. $A=\begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix}$, $B=E$, $\lambda=1$, $\mu=-\nu$.

1.9. A va B moslashtirilgan matritsalar bo‘lsin. Quyidagilarni ko‘rsating:
(a) agar A matritsa satr matritsa bo‘lsa, u holda AB satr matritsa bo‘ladi;
(b) agar B matritsa ustun matritsa bo‘lsa, u holda AB ustun matritsa bo‘ladi.

1.10. $\begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}\cdot\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}=\begin{pmatrix} x & 0 \\ 9 & 0 \end{pmatrix}$ bo‘lsa, x va y ni toping.

1.11. Agar A matritsa 3×3 o‘lchamli va C esa 5×5 o‘lchamli bo‘lsa, ABC ko‘paytma ma’noga ega bo‘lishi uchun B matritsa qanday o‘lchamda bo‘lishi kerak?

1.12. $A=\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ matritsa berilgan. AB ko‘paytmani nol matritsaga aylantiruvchi B matritsani toping.

1.13-1.1.16 mashqlarda A va B matritsalar berilgan. AB matritsani toping:

1.13. $A=\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 1 & -1 & -2 \\ 3 & 2 & 0 \end{pmatrix}$.

1.14. $A=\begin{pmatrix} 2 & 1 \\ 0 & -1 \\ 3 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 4 & -2 \\ 2 & 3 \end{pmatrix}$.

1.15. $A=\begin{pmatrix} 1 & 1 & 4 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$, $B=\begin{pmatrix} -1 & 3 \\ 0 & -1 \\ 2 & 1 \end{pmatrix}$.

1.16. $A=\begin{pmatrix} 1 & -1 & 2 \\ -2 & 0 & 3 \\ 1 & -1 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 4 & 0 & -2 \\ 2 & -1 & 0 \\ 0 & -1 & 3 \end{pmatrix}$.

1.17. $A=\begin{pmatrix} 2 & -2 \\ 2 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 4 \\ -2 & 5 \end{pmatrix}$, $C=B-3I$ bo‘lsa, $(AB)C$ matritsani toping.

1.18. $A=\begin{pmatrix} 3 & -1 \\ 2 & 4 \end{pmatrix}$, $B=\begin{pmatrix} 4 & 5 \\ 2 & 6 \end{pmatrix}$, $C=\begin{pmatrix} -1 & 4 \\ 5 & 3 \end{pmatrix}$ bo‘lsa, $A(BC)$ matritsani toping.

1.19. $A=\begin{pmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 0 & 1 \\ 3 & 2 \\ -2 & 0 \end{pmatrix}$ matritsalar berilgan. AB, $B^T B$, A^2 matritsalarni toping.

1.20. $A=\begin{pmatrix}1 & -2\\ 0 & 3\end{pmatrix}$ va $f(x)=3x^2+5x-4$ bo‘lsin. $f(A)$ ni toping.

1.21. $A=\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$ bo‘lsa, A^{20} ni toping.

1.22. Agar $A^2=I$ va A matritsa 2×2 o‘lchamli bo‘lsa, A ni toping.

Adabiyotlar

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