

OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

TOSHKENT ARXITEKTURA QURILISH INSTITUTI

MATEMATIKA VA TABIIY FANLAR KAFEDRASI

REFERAT

MAVZU. DETERMINANTLAR

TOSHKENT 2016

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Reja:

1. Ikkinchi va uchunchi tartibli determinantlar
2. n - tartibli determinant tushunchasi
3. Determinantlarni xossalari va ularni hisoblash

Determinant tushunchasidan dastlab chiziqli tenglamalar sistemasini yechishda foydalanilgan bo'lib, keyinchalik determinantlar matematikaning bir qancha masalalarini yechishga, jumladan xos sonlarni topishga, differensial tenglamalarni yechishga, vektor hisobiga, keng tatbiq etildi ¹.

2.1. Ikkinchi va uchunchi tartibli determinantlar

Ikkinchi tartibli determinant

$$\det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21} \quad (1.2.1)$$

kabi belgilanadi va aniqlanadi.

$a_{11}, a_{12}, a_{21}, a_{22}$ sonlarga determinantning elementlari deyiladi. Bunda a_{11}, a_{12} 1-satr, a_{21}, a_{22} 2-satr, a_{11}, a_{21} 1-ustun va a_{12}, a_{22} 2-ustun elementlari hisoblanadi, ya'ni a_{ij} determinantning i -satr va j -ustunda joylashgan elementini ifodalaydi.

a_{11}, a_{22} elementlar joylashgan diagonalga determinantning bosh diagonal, a_{21}, a_{12} elementlar joylashgan diagonalga determinantning yordamchi diagonal deyiladi.

Shunday qilib, ikkinchi tartibli determinant bosh diagonal elementlari ko'paytmasidan yordamchi diagonal elementlari ko'paytmasini ayirilganiga teng:

2.1-misol.

determinantlarni

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} - \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

Berilgan
hisoblang.

$$1. \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix} = 3 \cdot 5 - (-2) \cdot 4 = 15 + 8 = 23;$$

¹ E.Kreyszig. Advancet engineering Matematics. Copyright. 2011, pp. 255-265

$$2. \begin{vmatrix} \operatorname{tg} \alpha & \sin \alpha \\ \sin \alpha & \operatorname{ctg} \alpha \end{vmatrix} = \operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha - \sin \alpha \sin \alpha = 1 - \sin^2 \alpha = \cos^2 \alpha.$$

Matritsaning muhim tavsiflaridan biri determinant hisoblanadi. Determinant faqat kvadrat matritsalar uchun kiritiladi.

A kvadrat matritsaning determinanti $\det A$ bilan belgilanadi. Masalan, $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ matritsaning determinanti $\det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ kabi aniqlanadi.

Bunda matritsani uning determinanti bilan adashtirmaslik kerak: matritsa – bu sonlar massivi; determinant – bu bitta son.

Uchinchi tartibli determinant

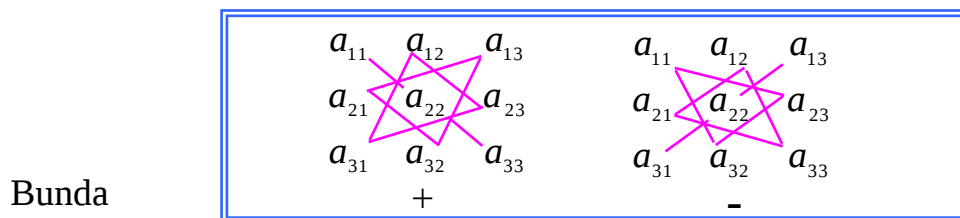
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32} \quad (1.2.2)$$

kabi belgilanadi va aniqlanadi.

Uchinchi tartibli determinant uchun satr, ustun, bosh diagonal, yordamchi diagonal tushunchalari ikkinchi tartibli determinantdagi kabi kiritiladi.

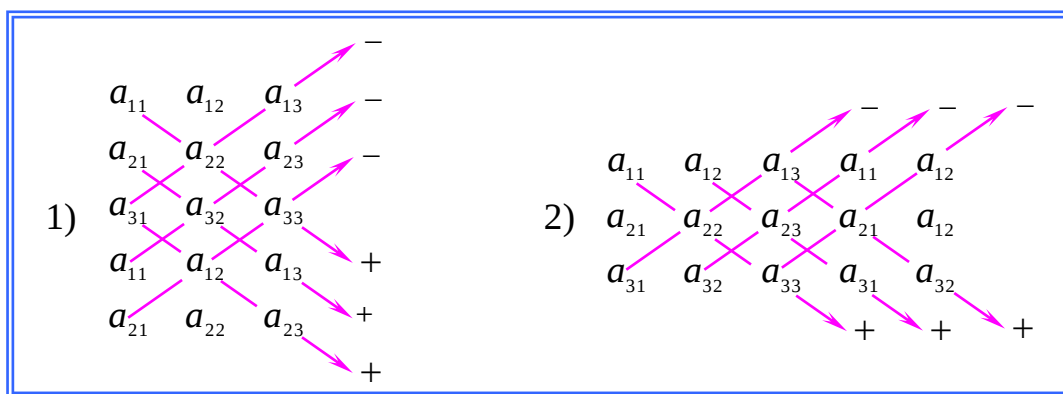
Uchinchi tartibli determinantlarni hisoblashda (1.2.2) tenglikning o'ng tomonidagi birhadlarni topishning yodda saqlash uchun oson bo'lgan qoidalaridan foydalaniladi.

«Uchburchak qoidasi» ushbu sxema bilan tasvirlanadi ²:



diagonallardagi yoki asoslari diagonallarga parallel bo'lgan uchburchaklar uchlaridagi elementlar uchta elementning ko'paytmasini hosil qiladi. Agar uchburchaklarning asoslari bosh diagonalga parallel bo'lsa, u holda elementlarning

² Lay, David C. Linear algebra and its applications. Copyright. 2012, pp.162-169



ko'paytmasi ishorasini saqlaydi. Agar uchburchaklarning asoslari yordamchi diagonalga parallel bo'lsa, u holda elementlarning ko'paytmasi teskari ishora bilan olinadi.

«Sarryus qoidalar» quyidagi sxemalar bilan ifodalanadi ³:

1-qoidada avval determinant tagiga uning birinchi ikkita satri yoziladi, 2-qoidada esa determinantning o'ng tomoniga uning birinchi ikkita ustuni yoziladi. Bunda diagonallardagi yoki diagonallarga parallel bo'lgan to'g'ri chiziqlardagi elementlar uchta ko'paytuvchini hosil qiladi. Agar to'g'ri chiziqlar bosh diagonalga parallel bo'lsa, u holda elementlarning ko'paytmasi ishorasini saqlaydi. Agar to'g'ri chiziqlar yordamchi diagonalga parallel bo'lsa, u holda elementlarning ko'paytmasi teskari ishora bilan olinadi.

2.2-misol. 1. $\det A = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix}$ determinantlarni uchburchak qoidasi bilan

hisoblang.

Yechish.

$$\begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix} \Rightarrow -8 + 1 + 27 = 20, \quad \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix} \Rightarrow 6 - 6 + 6 = 6, \quad \det A = 20 - 6 = 14.$$

³ E.Kreyszig. Advancet engineering Matematics. Copyright. 2011, pp. 257-259

2. $\det B = \begin{vmatrix} 1 & 5 & 3 \\ 3 & 1 & -2 \\ 2 & -4 & 1 \end{vmatrix}$ determinantni Sarryusning 1-qoidasi bilan hisoblang.

$$\begin{array}{ccc} 1 & 5 & 3^- \\ 3 & 1 & -2^- \\ 2 & -4 & 1^- \end{array} \Rightarrow \Delta_2 = 1 - 36 - 20 - 6 - 8 - 15 = -84.$$

$$\begin{array}{ccc} 1 & 5 & 3^+ \\ 3 & 1 & -2^+ \\ 2 & -4 & 1^+ \end{array}$$

3. $\det C = \begin{vmatrix} 3 & 4 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{vmatrix}$ determinantni Sarryusning 2-qoidasi bilan hisoblang.

$$\begin{array}{ccc} 3 & 4 & -1^- \\ 2 & 0 & 3^- \\ 3 & -1 & 2^- \end{array} \Rightarrow \Delta_3 = 0 + 36 + 2 - 0 + 9 - 16 = 31.$$

$$\begin{array}{ccc} 3 & 4 & -1^+ \\ 2 & 0 & 3^+ \\ 3 & -1 & 2^+ \end{array}$$

2.2. n - tartibli determinant tushunchasi

n - tartibli determinant

$$\det A = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

kabi belgilanadi va ma'lum qoida asosida hisoblanadi.

n - tartibli determinant har bir satr va har bir ustundan faqat bittadan olingan n ta elementning ko'paytmasidan tuzilgan $n!$ ta qo'shiluvchilar yig'indisidan iborat bo'ladi, bunda ko'paytmalar bir-biridan elementlarining tarkibi bilan farq qiladi va har bir ko'paytma oldiga inversiya tushunchasi asosida plyus yoki minus ishora qo'yiladi.

n -tartibli determinantni bu qoida asosida ifodalash etarlicha noqulaylikka ega. Shu sababli yuqori tartibli determinantlarni hisoblashda bir nechta ekvivalent

qoidalaridan foydalaniladi. Bunday qoidalaridan biri yuqori tartibli determinantlarni quyi tartibli determinantlar asosida hisoblash usuli hisoblanadi. Bu usulda determinant biror satr (yoki ustun) bo'yicha yoyiladi. Bunda quyi (ikkinchi va uchunchi) tartibli determinantlar yuqorida keltirilgan ta'riflar asosida topiladi.

n -tartibli determinantlarni yoyishda minor va algebraik to'ldiruvchi tushunchalaridan foydalaniladi.

n -tartibli determinant a_{ij} *elementining minori* deb, shu element joylashgan satr va ustunni o'chirishdan hosil bo'lgan $(n-1)$ - tartibli determinantga aytiladi va M_{ij} bilan belgilanadi.

Determinant a_{ij} *elementining* A_{ij} *algebraik to'ldiruvchisi* deb,

$$A_{ij} = (-1)^{i+j} M_{ij}$$

songa aytiladi.

Masalan, $\begin{vmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & -2 \end{vmatrix}$ determinantning $a_{21} = 2$ elementining minori va

algebraik to'ldiruvchisi quyidagicha topiladi:

$$\begin{vmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & -2 \end{vmatrix} \Rightarrow M_{21} = \begin{vmatrix} 3 & 2 \\ 2 & -2 \end{vmatrix} = -10, \quad A_{21} = (-1)^{2+1} M_{21} = 10.$$

2.3. Determinantning xossalari

Determinantning xossalarini uchinchi tartibli determinant uchun keltiramiz.

Bu xossalar ixtiyoriy n - tartibli determinant uchun ham o'rinli bo'ladi.

1-xossa. Transponirlash (barcha satrlarni mos ustunlar bilan almashtirish) natijasida determinantning qiymati o'zgarmaydi, ya'ni

$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} = \det A^T.$$

Isboti. Xossani isbotlash uchun tenglikning chap va o'ng tomonidagi determinantlarning qiymatlarini uchburchak qoidasi orqali yozib olish va olingan ifodalarning tengligiga ishonch hosil qilish kifoya.

1-xossa satr va ustunlarning teng huquqligini belgilab beradi. Boshqacha aytganda satrlar uchun isbotlangan xossalar ustunlar uchun ham o'rinli bo'ladi va aksincha. Shu sababli keyingi xossalarni ham satrlar va ham ustunlar uchun ifodalab, ularning isbotini faqat satrlar yoki faqat ustunlar uchun ko'rsatamiz.

2-xossa. Determinant ikkita satrining (ustunining) o'rinlari almashtirilsa, uning qiymati qarama-qarshi ishoraga o'zgaradi. Masalan,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = - \begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

Bu xossa ham 1-xossa kabi isbotlanadi.

3-xossa. Agar determinant ikkita bir xil satrga (ustunga) ega bo'lsa, u nolga teng bo'ladi.

Isboti. Haqiqatdan ham determinantda ikkita bir xil satrning o'rinlari almashtirilsa, uning qiymati o'zgarmaydi. Ikkinchi tomondan 2-xossaga ko'ra determinant qiymatining ishorasi o'zgaradi. Demak $\det A = -\det A$, yoki $2\det A = 0$. Bundan $\det A = 0$.

4-xossa. Determinantning biror satri (ustuni) elementlari λ songa ko'paytirilsa, determinant shu songa ko'payadi va aksincha determinant biror satr (ustun) elementlarining umumiy ko'paytuvchisini determinant belgisidan tashqatiga chiqarish mumkin. Masalan,

$$\begin{vmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \lambda \cdot \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

Isboti. Tenglikning chap tomondagi determinant hisoblanganida oltita qo'shiluvchining hammasida λ ko'paytuvchi qatnashadi.

Bu ko'paytuvchini qavsdan tashqariga chiqarib, qavslar ichidagi qo'shiluvchilardan determinant tuzilsa, tenglikning o'ng tomondagi ifoda hosil bo'ladi.

5-xossa. Agar determinant biror satrining (ustunining) barcha elementlari nolga teng bo'lsa, u nolga teng bo'ladi.

Xossaning *isboti* 4-xossadan $\lambda = 0$ da kelib chiqadi.

6-xossa. Agar determinantning ikki satri (ustuni) proporsional bo'lsa, u nolga teng bo'ladi. Masalan,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0.$$

Isboti. 4-xossaga ko'ra determinant ikkinchi satrining λ ko'paytuvchisini determinant belgisidan chiqarish mumkin. Natijada ikkita bir xil satrli determinant qoladi va u 3-xossaga ko'ra nolga teng bo'ladi.

7-xossa. Agar determinant biror satrining (ustunining) har bir elementi ikki qo'shiluvchining yig'indisidan iborat bo'lsa, bu determinant ikki determinant yig'indisiga teng bo'lib, ulardan birinchisining tegishli satri (ustuni) elementlari birinchi qo'shiluvchilardan, ikkinchisining tegishli satri (ustuni) elementlari ikkinchi qo'shiluvchilardan tashkil topadi.

Isboti. Determinant birinchi satrining har bir elementi ikkita qo'shiluvchi yig'indisidan iborat bo'lsin. U holda

$$\begin{vmatrix} a'_{11} + a''_{11} & a'_{12} + a''_{12} & a'_{13} + a''_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (a'_{11} + a''_{11})a_{22}a_{33} + (a'_{12} + a''_{12})a_{23}a_{31} + \\ + (a'_{13} + a''_{13})a_{21}a_{32} - (a'_{13} + a''_{13})a_{22}a_{31} - (a'_{12} + a''_{12})a_{21}a_{33} - (a'_{11} + a''_{11})a_{23}a_{32} = \\ = a'_{11}a_{22}a_{33} + a'_{12}a_{23}a_{31} + a'_{13}a_{21}a_{32} - a'_{13}a_{22}a_{31} - a'_{12}a_{21}a_{33} - a'_{11}a_{23}a_{32} + (a''_{11}a_{22}a_{33} + a''_{12}a_{23}a_{31} + \\ + a''_{13}a_{21}a_{32} - a''_{13}a_{22}a_{31} - a''_{12}a_{21}a_{33} - a''_{11}a_{23}a_{32}) = \begin{vmatrix} a'_{11} & a'_{12} & a'_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a''_{11} & a''_{12} & a''_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

8-xossa. Agar determinantning biror satri (ustuni) elementlariga boshqa satrining (ustunining) mos elementlarini biror songa ko‘paytirib qo‘shilsa, determinantning qiymati o‘zgarmaydi.

Isboti. $\det A$ determinantning ikkinchi satri elementlariga λ ga ko‘paytirilgan birinchi satrning mos elementlari qo‘shilgan bo‘lsin:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} + \lambda a_{11} & a_{22} + \lambda a_{12} & a_{23} + \lambda a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \lambda \cdot \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Qo‘shiluvchilardan birinchisi $\det A$ ga va ikkinchisi esa 3-xossaga ko‘ra nolga teng. Demak, yig‘indi $\det A$ ga teng.

9-xossa. Determinantning qiymati uning biror satri (ustuni) elementlari bilan shu elementlarga mos algebraik to‘ldiruvchilar ko‘paytmalarining yig‘indisiga teng bo‘ladi. Masalan, $\det A = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ yoki

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Isboti. Tenglikning o‘ng tomonida almashtirishlar bajaramiz:

$$\begin{aligned} & a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22}) = \\ & = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}. \end{aligned}$$

10-xossa. Determinant biror satri (ustuni) elementlari bilan boshqa satri (ustuni) mos elementlari algebraik to‘ldiruvchilari ko‘paytmalarining yig‘indisi nolga teng bo‘ladi. Masalan, $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13} = 0$.

Isboti. Determinantni 9-xossani qo‘llab, topamiz:

$$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

a_{11}, a_{12}, a_{13} mos ravishda a_{21}, a_{22}, a_{23} bilan bilan almashtirilsa, 3-xossaga ko'ra determinant nolga teng bo'ladi.

1-izoh. Determinantning xossalari asosida quyidagi teorema isbotlangan.

1-teorema. Bir xil tartibli A va B kvadrat matritsalar ko'paytmasining determinanti bu matritsalar determinantlarining ko'paytmasiga teng, ya'ni

$$\det(A \cdot B) = \det A \cdot \det B.$$

2.4. n -tartibli determinantlarni hisoblash

n - tartibli determinantni xossalar yordamida soddalashtirib, keyin tartibini pasaytirish yoki uchburchak ko'rinishga keltirish usullaridan biri bilan hisoblash mumkin.

Tartibini pasaytirish usuli

n -tartibli determinant 9-xossaga asosan biror satr yoki ustun bo'yicha yoyiysa, yoyilmada $(n-1)$ -tartibli algebraik to'ldiruvchilar hosil bo'ladi, ya'ni n -tartibli determinantni hisoblash tartibi bittaga past bo'lgan determinantlarni hisoblashga keltiriladi.

Umuman olganda quyidagi teoremlar o'rinli bo'ladi.

2-teorema. i satrining nomeri qanday bo'lishidan qat'iy nazar n -tartibli determinant uchun bu determinantni i -satr bo'yicha yoyilmasi deb ataluvchi

$$\det A = a_{i1}A_{i1} + a_{i2}A_{i2} + \dots + a_{in}A_{in}, \quad i = \overline{1, n} \quad (1.2.3)$$

formula o'rinli.

3-teorema. j ustunining nomeri qanday bo'lishidan qat'iy nazar n -tartibli determinant uchun bu determinantni j -ustun bo'yicha yoyilmasi deb ataluvchi

$$\det A = a_{1j}A_{1j} + a_{2j}A_{2j} + \dots + a_{nj}A_{nj}, \quad j = \overline{1, n} \quad (1.2.4)$$

formula o'rinli.

Determinantni biror satr yoki ustun bo'yicha yoyishga *Laplas yoyilmalari usuli* deyiladi ⁴.

⁴ Kenneth L. Kuttler-Elementary Linear Algebra [Lecture notes] (2015). pp.89-95

Laplas yoyilmalari usulida determinantning qaysi bir satrida (ustunida) nollar ko'p bo'lsa, u holda yoyishni shu satr (ustun) bo'yicha bajarish qulay bo'ladi. Bundan tashqari 8-xossani qo'llab, determinantning biror satrida (ustunida) bitta elementdan boshqa elementlarni nollarga keltirish mumkin. Bunda determinantning qiymati shu satrdagi (ustundagi) noldan farqli element bilan uning algebraik to'ldiruvchisining ko'paytmasidan iborat bo'ladi. Shunday qilib, n -tartibli determinant bitta $(n - 1)$ -tartibli determinantga keltirib, hisoblanadi.

2.3-misol.

$$\det A = \begin{vmatrix} 2 & -1 & 0 & 4 \\ 4 & 2 & -1 & 3 \\ -2 & 0 & 3 & -4 \\ 1 & 1 & 0 & -2 \end{vmatrix}$$

determinantni tartibini pasaytirish usuli bilan hisoblang.

Yechish. Bunda: 1) Ikkita elementi nolga teng bo'lgan uchinchi ustunni tanlaymiz va uning ikkinchi satrida joylashgan elementidan boshqa barcha elementlarini nolga aylantiramiz. Buning uchun ikkinchi satr elementlarini 3 ga ko'paytirib, uchinchi satrning mos elementlariga qo'shamiz va hosil bo'lgan determinantni uchinchi ustun elementlari bo'yicha yoyamiz;

2) Hosil qilingan uchinchi tartibli determinantda birinchi ustunning uchinchi satri elementidan yuqorida joylashgan elementlarini nolga aylantiramiz. Buning uchun avval uchinchi satrni (-2) ga ko'paytirib, birinchi satrga qo'shamiz, keyin uchinchi satrni (-10) ga ko'paytirib, ikkinchi satrga qo'shamiz, hosil bo'lgan determinantni birinchi ustun elementlari bo'yicha yoyamiz va hosil bo'lgan ikkinchi tartibli determinantni hisoblaymiz:

$$\det A = \begin{vmatrix} 2 & -1 & 0 & 4 \\ 4 & 2 & -1 & 3 \\ -2 & 0 & 3 & -4 \\ 1 & 1 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 0 & 4 \\ 4 & 2 & -1 & 3 \\ 10 & 6 & 0 & 5 \\ 1 & 1 & 0 & -2 \end{vmatrix} = (-1) \cdot (-1)^{2+3} \begin{vmatrix} 2 & -1 & 4 \\ 10 & 6 & 5 \\ 1 & 1 & -2 \end{vmatrix};$$

$$\det A = \begin{vmatrix} 0 & -3 & 8 \\ 0 & -4 & 25 \\ 1 & 1 & -2 \end{vmatrix} = \begin{vmatrix} -3 & 8 \\ -4 & 25 \end{vmatrix} = -75 + 32 = -43.$$

Uchburchak ko‘rinishga keltirish usuli

Bu usulda determinant xossalar yordamida soddalashtiriladi va uchburchak ko‘rinishga keltiriladi, ya’ni diagonalidan pastda (yuqorida) joylashgan barcha elementlari nolga aylantiriladi.

Bunda

$$\det A = (-1)^k \det U$$

bo‘ladi [3], bu yerda k – satrlarda va ustunlarda bajarilgan barcha o‘rin almashtirishlar soni; $\det U$ – berilgan determinantning uchburchak ko‘rinishi va uning qiymati quyidagi xossa asosida hisoblanadi:

Xossa. Uchburchak ko‘rinishidagi determinant bosh diagonalda joylashgan elementlarining ko‘paytmasiga teng ⁵.

2.4-misol.

$$\det A = \begin{vmatrix} 2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 2 \end{vmatrix}$$

determinantni ushburshak ko‘rinishga keltirib, hisoblang.

Yechish. Determinant ustida quyidagi soddalashtirishlarni bajaramiz:

- birinchi ustunni o‘zidan o‘ngda joylashgan ustunlar bilan ketma-ket $k = 3$ ta o‘rin almashtirib, to‘rtinchi ustunga o‘tkazamiz;

⁵ Kenneth L. Kuttler-Elementary Linear Algebra [Lecture notes] (2015). pp. 89-95

- birinchi ustunning birinchi satridan pastda joylashgan elementlarini nolga aylantiramiz;

- ikkinchi ustunning ikkinchi satridan pastda joylashgan elementlarini nolga aylantiramiz;

- uchinchi ustunning to'rtinchi satrida joylashgan elementini nolga aylantiramiz;

- $(-1)^k = (-1)^3 = -1$ ko'paytuvchi bilan hosil bo'lgan uchburchak ko'rinishdagi determinantning bosh diagonalda joylashgan elementlarini ko'paytiramiz.

$$\begin{aligned} \det A &= \begin{vmatrix} 2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 2 \end{vmatrix} = (-1)^3 \cdot \begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 2 & 1 & 1 \\ 1 & 0 & 2 & 0 \end{vmatrix} = (-1) \cdot \begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & -2 \end{vmatrix} = \\ &= (-1) \cdot \begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 2 & -2 \end{vmatrix} = (-1) \cdot \begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 8 \end{vmatrix} = (-1) \cdot 1 \cdot 1 \cdot 1 \cdot 8 = -8. \end{aligned}$$

2. Mashqlar

2.1. A matritsa $n \times n$ o'lchamli bo'lsin. $\det(\lambda A)$ da λ ni determinant belgisidan tashqariga chiqarish uchun formula keltirib chiqaring.

2.2. A kvadrat matritsa va $A^T A = I$ bo'lsin. $\det A = \pm 1$ bo'lishini ko'rsating.

2.3. $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ va $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ bo'lsin. $\det(A+B) = \det A + \det B$ faqat $a+d=0$

bo'lganida bajarilishini ko'rsating.

2.4. $A = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} 7 & 1 \\ 3 & 2 \end{pmatrix}$ bo'lsin. $\det(A \cdot B) = \det A \cdot \det B$ bo'lishiga ishonch hosil

qiling.

2.5. $A = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$ bo'lsin. $\det A^{1000}$ ni toping.

2.6. A va B matritsalar 3×3 o'lchamli, $\det A = -1$ va $\det B = 2$ bo'lsin. Topping:

- 1) $\det AB$; 2) $\det 5A$; 3) $\det A^T A$; 4) $\det B^3$.

2.7. A va B matritsalar 4×4 o'lchamli, $\det A = 4$ va $\det B = -3$ bo'lsin. Topping:

- 1) $\det AB$; 2) $\det B^5$; 3) $\det 2A$; 4) $\det IA$.

Ikkinchi tartibli determinantlarni hisoblang:

$$2.8. \begin{vmatrix} y & x-y \\ x & -x \end{vmatrix}.$$

$$2.9. \begin{vmatrix} 1 & a+b \\ b+1 & a+b \end{vmatrix}.$$

$$2.10. \begin{vmatrix} \sin^2 \alpha & \cos^2 \alpha \\ \sin^2 \beta & \cos^2 \beta \end{vmatrix}.$$

$$2.11. \begin{vmatrix} \operatorname{tg} \alpha + 1 & \operatorname{ctg} \alpha - 1 \\ \sin \alpha & \cos \alpha \end{vmatrix}.$$

Uchinchi tartibli determinantlarni uchburchak va Sarrius qoidalarini bilan hisoblang:

$$2.12. \begin{vmatrix} 5 & -1 & 1 \\ 4 & 0 & -3 \\ 2 & -3 & 1 \end{vmatrix}.$$

$$2.13. \begin{vmatrix} -2 & 0 & -4 \\ 3 & 1 & 1 \\ -1 & 2 & -3 \end{vmatrix}.$$

Uchinchi tartibli determinantlarni biror satr yoki ustun elementlari bo'yicha yoyib hisoblang:

$$2.14. \begin{vmatrix} 1 & b & 1 \\ b & b & 0 \\ b & 0 & -b \end{vmatrix}.$$

$$2.15. \begin{vmatrix} x & -1 & x \\ 1 & x & -1 \\ x & 1 & x \end{vmatrix}.$$

$$2.16. \begin{vmatrix} \sin \alpha & \sin \beta & 0 \\ \sin \alpha & 0 & \sin \gamma \\ 0 & \sin \beta & \sin \gamma \end{vmatrix}.$$

$$2.17. \begin{vmatrix} \operatorname{tg} \alpha & \operatorname{ctg} \beta & 0 \\ \operatorname{tg} \alpha & 0 & \operatorname{tg} \beta \\ 0 & \operatorname{ctg} \alpha & \operatorname{tg} \beta \end{vmatrix}.$$

Uchinchi tartibli determinantlarni xossalariidan foydalanib hisoblang:

$$2.18. \begin{vmatrix} 1 & c & ab \\ 1 & b & ca \\ 1 & a & bc \end{vmatrix}.$$

$$2.19. \begin{vmatrix} 1 & 1 & 1 \\ ax & ay & az \\ a^2+x^2 & a^2+y^2 & a^2+z^2 \end{vmatrix}.$$

$$2.20. \begin{vmatrix} a+b & b & b \\ b & a+b & b \\ b & b & a+b \end{vmatrix}.$$

$$2.21. \begin{vmatrix} x & x+y & x-y \\ x & x+z & x-2z \\ x & x & x \end{vmatrix}.$$

$$2.22. \begin{vmatrix} a & a^2+1 & (1+a)^2 \\ b & b^2+1 & (1+b)^2 \\ c & c^2+1 & (1+c)^2 \end{vmatrix}.$$

$$2.23. \begin{vmatrix} 1+\cos \alpha & 1 & 1+\sin \alpha \\ 1-\sin \alpha & 1 & 1-\cos \alpha \\ 1 & 1 & 1 \end{vmatrix}.$$

To'rtinchi tartibli determinantlarni hisoblang:

$$2.24. \begin{vmatrix} 1 & -1 & 2 & 2 \\ 3 & -1 & 5 & -2 \\ -2 & -3 & 0 & 2 \\ 0 & -2 & 4 & 1 \end{vmatrix}.$$

$$2.25. \begin{vmatrix} 1 & 1 & 3 & 2 \\ 2 & 0 & 0 & 8 \\ 3 & 0 & 0 & 2 \\ 4 & 4 & 7 & 5 \end{vmatrix}.$$

$$2.26. \begin{vmatrix} 5 & a & 2 & -1 \\ 4 & b & 4 & -3 \\ 2 & c & 3 & -2 \\ 4 & d & 5 & -4 \end{vmatrix}.$$

$$2.27. \begin{vmatrix} 3 & 2 & 2 & 2 \\ 9 & -8 & 5 & 10 \\ 5 & -8 & 5 & 8 \\ 6 & -5 & 4 & 7 \end{vmatrix}.$$

Adabiyotlar

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