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**MOSLANGAN MANBALI UMUMIY TODA TENGLAMASINI TEZ
KAMAYUVCHI FUNKSIYALAR SINFIDA INTEGRALLASH**

**Mutaxassislik: 5A130101 – «Matematika(matematik mantiq,algebra va sonlar
nazariyasi)»**

D I S S E R T A T S I Y A

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KIRISH

Tadqiqot mavzusining dolzarbligi. Nochiziqli evolutsion tenglamalarga qo'yilgan Koshi masalalarining yechimlarini teskari masalalar usullaridan foydalanib topish masalasi matematika va fizikaning muhim masalalaridan biridir. S.S.Gardner, J.M.Grin, M.D.Kruskal va R.M.Miuralar nochiziqli evolyusion tenglamalarni sochilish nazariyasining teskari masalasi yordamida yechish usulining dastlabki kashfiyotchilari hisoblanadi. Ular KdF tenglamaga qo'yilgan Koshi masalasini o'z-o'ziga qo'shma bo'lgan Shturm-Liuivill operatori uchun sochilish nazariyasining teskari masalasi yordamida tez kamayuvchi funksiyalar sinfida yechdilar. Tez orada bu usul boshqa nochiziqli evolutsion tenglamalarga tadbiiq qilindi. Bu tenglamalar qatorida KdF tenglamasining diskret analogi bo'lgan Toda zanjiri ham bor edi. Keyingi vaqtlarda amaliy tadbiiqlardan kelib chiqqan holda nochiziqli evolutsion tenglamalarni manbalar bilan qarash muhim ahamiyat kasb etmoqda. Moslangan manbali Toda zanjiri tez kamayuvchi funksiyalar sinfida A. Cabada va G'. O'razboyevlar tomonidan o'rganilgan. Hozirgi kunda bu sohanning qattiq jismlar mexanikasi, plazma fizikasida va gidrodinamika masalalarida muhim tadbiiqlari topilganligi bois bu sohani o'rganishga bo'lgan qiziqish yanada ortmoqda.

Ilmiy izlanishning maqsad va vazifalari. Mazkur dissertatsiyada ushbu

$$\begin{cases} \dot{a}_n = a_n (G_{n+1,r+1} - G_{n,r+1}) + a_n \sum_{i=1}^N ((g_n^i)^2 - (g_{n+1}^i)^2), \\ \dot{b}_n = H_{n+1,r+1} - H_{n,r+1} - 2 \sum_{i=1}^N g_n^i (a_n g_{n+1}^i - a_{n-1} g_{n-1}^i), \\ a_{n-1} g_{n-1}^k + b_n g_n^k + a_n g_{n+1}^k = \lambda_k g_n^k, \quad k = 1, 2, \dots, N, \quad n \in \mathbb{Z}, \end{cases} \quad (1)$$

tenglamalar sistemasini qaraymiz. Bu yerda

$$G_{n,j}(t) = \sum_{s=0}^j c_{j-s} \langle \delta_n, L(t)^s \delta_n \rangle, \quad 0 \leq j \leq r+1,$$

$$H_{n,j}(t) = \sum_{s=0}^j 2a_n(t)c_{j-s} \langle \delta_{n+1}, L(t)^s \delta_n \rangle + c_j + 1, \quad 0 \leq j \leq r+1,$$

$$(L(t)y)_n \equiv a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n,$$

$$\langle \delta_m, \delta_n \rangle = \begin{cases} 0, & m \neq n \\ 1, & m = n, \end{cases}$$

c_1, c_2, \dots, c_{r+1} - oldindan berilgan ixtiyoriy xaqiqiy sonlar.

$\{a_n(t)\}_{-\infty}^{\infty}, \{b_n(t)\}_{-\infty}^{\infty}, \{g_n^k(t)\}_{-\infty}^{\infty}, k=1,2,\dots,N$ - noma'lum funksiyalar, $\{g_n^k(t)\}_{-\infty}^{\infty}$

funksiyalar $L(t)$ operatorning $\lambda_k = \frac{z_k + z_k^{-1}}{2}, k=1,2,\dots,N$ xos qiymatga mos

keluvchi ushbu

$$\sum_{n=-\infty}^{\infty} (g_n^k(t))^2 = A_k(t), \quad k=1,2,\dots,N, \quad (2)$$

shartni qanoatlantiruvchi xos funksiyasi, $A_k(t)$ oldindan berilgan musbat funksiya.

(1) tenglamalar sistemasini

$$a_n(0) = a_{n0}, \quad b_n(0) = b_{n0}, \quad n \in Z, \quad (3)$$

boshlang'ich shartlar bilan qaraymiz. Bu yerda $\{a_{n0}\}_{-\infty}^{\infty}, \{b_{n0}\}_{-\infty}^{\infty}$ vektorlar quyidagi

shartlarni qanoatlantiradi:

1. $a_{n0} > 0, \text{Im}b_{n0} = 0, n \in Z,$

2. $\sum_{n=-\infty}^{\infty} |n| \left(\left| a_{n0} - \frac{1}{2} \right| + |b_{n0}| \right) < \infty,$

3. $L(0)$ operator $[-1;1]$ kesma tashkarisida yotuvchi N ta

$\lambda_1(0), \lambda_2(0), \dots, \lambda_N(0)$ xos qiymatlarga ega.

(1)-(3) masalaning yechimini

$$a_n(t) > 0, \text{Im}b_n = 0, \quad n \in Z, \quad \sum_{n=-\infty}^{\infty} |n| \left(\left| a_n(t) - \frac{1}{2} \right| + |b_n(t)| \right) < \infty. \quad (4)$$

shartni qanoatlantiruvchi funksiyalar sinfida qaraymiz.

Ushbu magistrlik dissertatsiyasining asosiy maqsadi (1) masalaning $\{a_n(t)\}_{-\infty}^{\infty}$, $\{b_n(t)\}_{-\infty}^{\infty}$, $\{g_n^k(t)\}_{-\infty}^{\infty}$, $k=1,2,\dots,N$ yechimlariga $L(t)$ operator uchun sohilish nazariyasining teskari masalasi usulida tasvirlar olish.

Mavzuning o'rganilish darajasining qiyosiy tahlili. M.Toda [1], S.V.Manakov [2] va N.Flyashkalar [3] butun o'qda ikkinchi tartibli chekli ayirmali tenglamalar uchun sohilish nazariyasining to'g'ri va teskari masalalari yordamida Toda tenglamasini integrallashga muvaffaq bo'ldilar. Moslangan manbali Korteveg-de Friz tenglamasi V.K.Melnikov [4] va J.Leon, A.Latifi [5] hamda G'.O'.O'razboyev va A.B.Hasanovlar [6], [7] tomonidan o'rganilgan. Moslangan manbali Toda tenglamasi [8] ishda o'rganildi. Hozirgi kunda Shturm-Liuivill operatori uchun sohilish nazariyasining teskari masalasi usuli yordamida integrallanadigan barcha nochiziqli evalutsion tenglamalarni topishga bo'lgan qiziqish ortmoqda. Bu tenglamalar odatda umumiy Toda tenglamasi deb yuritiladi. Umumiy Toda tenglamasi birinchi bor K. Ueno va K. Takasakilar [9] tomonidan kiritilgan. Umumiy Toda tenglamasini Shturm-Liuivill operatori uchun sohilish nazariyasining teskari masalasi usuli yordamida integrallash mumkinligi [10] ishda ko'rsatilgan. Ammo moslangan manbali umumiy Toda tenglamasini tez kamayuvchi funksiyalar sinfida Shturm-Liuivill operatori sohilish nazariyasining teskari masalasi usuli yordamida integrallash masalasi hanuzgacha o'rganilmagan.

Tadqiqotning ilmiy yangiligi. Ushbu ishda moslangan manbali umumiy Toda tenglamasini tez kamayuvchi funksiyalar sinfida Shturm-Liuivill operatori sohilish nazariyasining teskari masalasi usuli yordamida integrallash mumkinligi ko'rsatilgan. Xususan, boshlang'ich shartlar aniq berilganda ikkinchi Toda tenglamasining yechimining aniq ko'rinishi yozilgan.

Tadqiqotning ob'ekti. Moslangan manbali umumiy Toda tenglamasi.

Tadqiqotning predmeti. Spektral analizning to'g'ri va teskari masalasi.

Tadqiqot usullari. Ushbu ishda oddiy differensial tenglamalar, matematik fizika tenglamalari, funksional analiz, differensial operatorlarning spektral nazariyasi predmetlari usullaridan foydalaniladi.

Ish tuzilishi va tarkibi: Magistrlik dissertatsiyasi kirish, 3 ta bob, xulosa va adabiyotlar ro'yxatidan iborat.

Tadqiqotning ilmiy ahamiyati. O'rganilgan natijalar amaliy ahamiyatga ega bo'lib, qattiq jismlar mexanikasi, plazma fizikasida va gidrodinamika masalalarida muhim tadbirlarga ega.

Bajarilgan ishning asosiy natijalari: Moslangan manbali umumiy Toda tenglamasiga qo'yilgan Koshi masalasi diskret Shturm-Liuville tenglamasiga qo'yilgan teskari masala usulidan foydalanib integrallangan.

Xulosa va takliflarning umumlashgan ifodasi:

Umumiy toda tenglamasini integral manba bilan ham qarash mumkin;

Olingan natijalarni Ablovits-ladik sistemasi uchun ham keltirib chiqarish mumkin.

I BOB.
TODA ZANJIRINI SOCHILISH NAZARIYASINING TESKARI
MASALASI USULIDA INTEGRALASH

**1.1-§ . Diskret Shturm-Liuivill tenglamasi uchun sochilish nazariyasining
berilganlarini aniqlash.**

Ushbu paragrafda Diskret Shturm-Liuivill tenglamasi sochilish nazariyasining to'g'ri va teskari spektral masalalari haqidagi zarur ma'lumotlarni keltiramiz.[11]-[12]

Quyidagi

$$Ly \equiv a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n, \quad n \in Z, \quad (1.1.1)$$

ikkinchi tartibli chekli ayirmali tenglamani qaraymiz. Bu yerda $\{y_n\}_{-\infty}^{\infty}$ - noma'lum funksiya, $\lambda = \frac{z + z^{-1}}{2}$ - spektral parametr va a_n, b_n koeffitsiyentlar

$$a_n > 0, \text{Im}b_n = 0, \quad n \in Z, \quad \sum_{n=-\infty}^{\infty} |n| \left(\left| a_n - \frac{1}{2} \right| + |b_n| \right) < \infty. \quad (1.1.2)$$

shartlarni qanoatlantiradi.

Bu paragrafda (1.1.1) tenglama uchun sochilish nazariyasining to'g'ri va teskari masalasi uchun kerak bo'ladigan muhim ma'lumotlar keltiriladi.

(1.1.2) shartlar bajarilganda (1.1.1) tenglamaning quyidagi

$$\begin{aligned} \varphi_n(z) &= z^n + o(1) \quad , \quad n \rightarrow \infty, \quad |z|=1, \\ \psi_n(z) &= z^{-n} + o(1) \quad , \quad n \rightarrow -\infty, \quad |z|=1. \end{aligned} \quad (1.1.3)$$

asimptotikaga ega bo'lgan Yost yechimlari mavjud va yagona bo'lib $|z| < 1$ doira ichiga analitik davom etadi.

$\varphi_n(z)$ funksiya uchun

$$\varphi_n(z) = \sum_{n'=n}^{\infty} K(n, n') z^{n'}, \quad (1.1.4)$$

tasvir o'rinli bo'lib, $K(n, n')$ koeffitsiyent z ga bog'liq emas va a_n, b_n

koeffitsiyentlar bilan

$$a_n = \frac{1}{2} \frac{K(n+1, n+1)}{K(n, n)}, \quad b_n = \frac{1}{2} \left(\frac{K(n, n+1)}{K(n, n)} - \frac{K(n-1, n)}{K(n-1, n-1)} \right). \quad (1.1.5)$$

tenglik yordamida bog‘langan.

$|z|=1$ da $\{\psi_n(z), \psi_n(z^{-1})\}$ funksiyalar jufti (1.1.1) tenglamaning fundamental yechimlar juftini tashkil qiladi. Bundan

$$\psi_n(z) = \alpha(z)\varphi_n(z^{-1}) + \beta(z)\varphi_n(z). \quad (1.1.6)$$

tenglik o‘rinli bo‘lishi kelib chiqadi.

Lemma 1.1.1 Ushbu $\{x_n(\lambda)\}_{-\infty}^{\infty}, \{y_n(\mu)\}_{-\infty}^{\infty}$ funksiyalar mos ravishda $Lx = \lambda x$ va $Ly = \mu y$ tenglamaning yechimi bo‘lsin. U holda quyidagi

$$(\mu - \lambda)x_n(\lambda)y_n(\mu) = W\{x_n(\lambda), y_n(\mu)\} - W\{x_{n-1}(\lambda), y_{n-1}(\mu)\}, \quad n \in Z.$$

ayniyat o‘rinli.

Isbot: Lemmaning shartiga ko‘ra ushbu

$$Lx = \lambda x$$

$$Ly = \mu y$$

tenglik o‘rinli bo‘ladi.

Ushbu

$$Ly \equiv a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1} = \mu y_n, \quad n \in Z$$

tenglamani x_n ga ko‘paytiramiz, xuddi shunday quyidagi

$$Lx \equiv a_{n-1}x_{n-1} + b_n x_n + a_n x_{n+1} = \lambda x_n, \quad n \in Z$$

tenglamani y_n ga ko‘paytiramiz.

Bundan

$$a_{n-1}y_{n-1}x_n + b_n y_n x_n + a_n y_{n+1} x_n = \mu y_n x_n, \quad n \in Z$$

$$a_{n-1}x_{n-1}y_n + b_n x_n y_n + a_n x_{n+1} y_n = \lambda x_n y_n, \quad n \in Z$$

tenglamalarni hosil qilamiz . Bu tengliklarni ayirib

$$(\mu - \lambda)x_n(\lambda)y_n(\mu) = W\{x_n(\lambda)y_n(\mu)\} - W\{x_{n-1}(\lambda)y_{n-1}(\mu)\}, \quad n \in \mathbb{Z}$$

tenglikni hosil qilamiz. *Lemma isbot bo'ldi.*

Natija 1.1.1. (1.1.1) tenglamaning ikkita yechimidan tashkil topgan

$$W\{\psi_n(z), \varphi_n(z)\} \equiv a_n(z)(\psi_n(z)\varphi_{n+1}(z) - \psi_{n+1}(z)\varphi_n(z))$$

Vronskiy detirminati n ga bog'lik emas.

Lemma 1.1.2 Ushbu

$$W\{\bar{\varphi}_n, \varphi_n\} = a_n \begin{vmatrix} z^{-n} & z^n \\ z^{-n-1} & z^{n+1} \end{vmatrix}$$

tenglik o'rinli.

Lemma 1.1.3 Ushbu

$$\alpha(z) = \frac{2}{z - z^{-1}} W\{\psi_n(z), \varphi_n(z)\} \quad (1.1.7)$$

tenglik o'rinli.

$$W\{\bar{\varphi}_n, \varphi_n\} = a_n \begin{vmatrix} z^{-n} & z^n \\ z^{-n-1} & z^{n+1} \end{vmatrix}.$$

Isbot. $|z|=1$ da $\{\psi_n(z), \psi_n(z^{-1})\}$ funksiyalar jufti (1.1.1)

tenglamaning fundamental yechimlar jufti bo'lgani uchun $|z|=1$ da (1.1.1)

tenglamaning ixtiyoriy yechimini

$$y(z) = c_1(z)\psi_n(z) + c_2(z)\varphi_n(z)$$

ko'rinishda yozish mumkin. Bu tenglik yordamida ushbu

$$W\{\psi_n, y\} = W\{\psi_n, c_1\psi_n + c_2\varphi_n\}$$

ko'rinishda yozish mumkin. Buni (1.6) tenglik yordamida

$$\begin{aligned} W\{\psi_n(z), c_1(z)\psi_n(z) + c_2(z)\varphi_n(z)\} &= \\ &= W\{\alpha(z)\varphi_n(z^{-1}) + \beta(z)\varphi_n(z), c_1(z)\psi_n(z) + c_2(z)\varphi_n(z)\} \end{aligned}$$

ko‘rinishda yozish mumkin. Vronskianning xossasiga ko‘ra

$$c_1(z)W\{\psi_n(z), \psi_n(z)\} + c_2(z)W\{\psi_n(z), \varphi_n(z)\} = \alpha(z)c_1(z)W\{\varphi_n(z^{-1}), \psi_n(z)\} +$$

$$+ \beta(z)c_1(z)W\{\varphi_n(z), \psi_n(z)\} + \alpha(z)c_2(z)W\{\varphi_n(z^{-1}), \varphi_n(z)\} + \beta(z)c_2(z)W\{\varphi_n(z), \varphi_n(z)\}$$

yoki

$$c_2(z)W\{\psi_n(z), \varphi_n(z)\} = \alpha(z)c_1(z)W\{\varphi_n(z^{-1}), \psi_n(z)\} +$$

$$+ \beta(z)c_1(z)W\{\varphi_n(z), \psi_n(z)\} + \alpha(z)c_2(z)W\{\varphi_n(z^{-1}), \varphi_n(z)\}$$

bo‘lishi kelib chiqadi. Oxirgi tenglikda c_1 va c_2 lar oldidagi

koeffitsiyentlarni tenglab

$$W\{\psi_n(z), \varphi_n(z)\} = \alpha(z)W\{\varphi_n(z^{-1}), \varphi_n(z)\}$$

yoki

$$\alpha(z) = \frac{W\{\psi_n(z), \varphi_n(z)\}}{W\{\varphi_n(z^{-1}), \varphi_n(z)\}}$$

bo‘lishini topamiz. Lemma 1.1.2 ga ko‘ra

$$\alpha(z) = \frac{2}{z - z^{-1}} W\{\psi_n(z), \varphi_n(z)\}$$

bo‘ladi. *Lemma isbot bo‘ldi.*

$\alpha(z)$ funksiya $|z| < 1$ doira ichiga analitik davom qiladi va u yerda cheklita

oddiy (karrasiz) z_1, z_2, \dots, z_N ildizga ega bo‘ladi va $\lambda_k = \frac{z_k + z_k^{-1}}{2}$, $k = 1, 2, \dots, N$

sonlar L operatorning xos qiymati bo‘ladi.

(1.1.6) tenglikka ko‘ra $z = z_k$ da

$$\psi_n^k = \beta_k \varphi_n^k, \quad k = 1, 2, \dots, N, \quad (1.1.8)$$

bo‘lishi kelib chiqadi. Bu yerda $\psi_n^k = \psi_n(z_k)$.

ξ_n^k funksiya L operatorning $\lambda_k = \frac{z_k + z_k^{-1}}{2}$ xos qiymatiga mos keluvchi va quyidagi

$$\sum_{n=-\infty}^{\infty} (\xi_n^k)^2 = 1,$$

shart bilan normallangan xos funksiyasi bo'lsin. U xolda $\xi_n^k = C_k \varphi_n^k$, $k = 1, 2, \dots, N$ bo'ladi. Ushbu

$$\dot{\alpha}(z_k) = -\frac{1}{z_k} \sum_{j=-\infty}^{\infty} \varphi_j(z_k) \psi_j(z_k),$$

tenglikdan quyidagiga ega bo'lamiz:

$$C_k^2 = -\frac{\beta_k}{z_k \dot{\alpha}(z_k)}, \quad k = 1, 2, \dots, N. \quad (1.1.9)$$

Ta'rif 1.1.1. $R(z) = \frac{\beta(z)}{\alpha(z)}$ funksiya sochilish funksiyasi deyiladi.

Ta'rif 1.1.2. Ushbu $\{R(z), z_1, z_2, \dots, z_N, C_1, C_2, \dots, C_N\}$ naborga (1.1.1) tenglamaning sochilish nazariyasining berilganlari deyiladi.

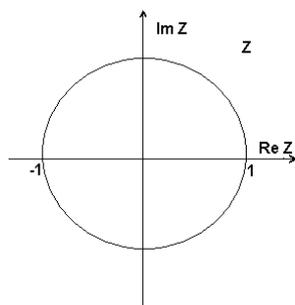
2.1-§. Diskret Shturm-Liuvill tenglamasi uchun Gelfand-Levitan-Marchenko integral tenglamasini keltirib chiqarish.

Bu paragrafda (1.1.1) tenglama uchun sochilish nazariyasining teskari masalasini yechishda muhim bo'ladigan Gelfand-Levitan-Marchenko integral tenglamasini keltirib chiqaramiz.

(1.1.6) tenglikni

$$\frac{\psi_n(z)}{\alpha(z)} = \varphi_n(z^{-1}) + R(z)\varphi_n(z), \quad |z|=1,$$

ko'rinishda yozib olamiz. Bu tenglikning ikkala tarafini ham z^{m-1} ($m \geq n$) ga ko'paytirib markazi z kompleks tekislikning koordinata boshida bo'lgan birlik aylana bo'yicha integrallaymiz (1-rasm).



1-rasm

U holda quyidagi

$$\frac{1}{2\pi i} \oint_{|z|=1} \frac{\psi_n(z)}{\alpha(z)} z^{m-1} dz = \frac{1}{2\pi i} \oint_{|z|=1} (\varphi_n(z^{-1}) + R(z)\varphi_n(z)) z^{m-1} dz. \quad (1.2.1)$$

tenglikka ega bo'lamiz. (1.2.1) tenglikni (1.1.4) tenglik va chegirma xaqidagi Koshi teoremasiga ko'ra

$$\frac{1}{2\pi i} \oint_{|z|=1} \varphi_n(z^{-1}) z^{m-1} dz = \frac{1}{2\pi i} \sum_{n'=n}^{\infty} K(n, n') \oint_{|z|=1} z^{-n'+m-1} dz = K(n, m), \quad (m \geq n). \quad (1.2.2)$$

ko‘rinishda yozamiz. Ushbu

$$\frac{1}{2\pi i} \oint_{|z|=1} R(z) \varphi_n(z) z^{m-1} dz = \frac{1}{2\pi i} \sum_{n'=n}^{\infty} K(n, n') \oint_{|z|=1} R(z) z^{n'+m-1} dz,$$

tenglikka ko‘ra (1.2.1) tenglikning o‘ng tarafini

$$RHS = \sum_{n'=n}^{\infty} K(n, n') F_c(n' + m) + K(n, m), \quad (1.2.3)$$

ko‘rinishda yozish mumkin. Bu yerda

$$F_c(m) = \frac{1}{2\pi i} \oint_{|z|=1} R(z) z^{m-1} dz.$$

(1.2.1) tenglikning chap tarafini

$$LFS = \frac{1}{2\pi i} \oint_{|z|=1} \frac{\psi_n(z)}{\alpha(z)} z^{m-1} dz = I_\alpha + I_0, \quad (1.2.4)$$

ko‘rinishda belgilaymiz. Bu yerda I_α - z_j , ($z_j \neq 0$), $j = 1, 2, \dots, N$ qutblardagi chegirma va I_0 - $z = 0$ qutbdagi chegirma. z_j , ($z_j \neq 0$), $j = 1, 2, \dots, N$ qutblardagi chegirma

$$\dot{\alpha}(z_k) = -\frac{1}{z_k} \sum_{j=-\infty}^{\infty} \varphi_j(z_k) \psi_j(z_k),$$

tenglik yordamida

$$I_\alpha = -\sum_{j=1}^N \frac{\psi_n(z_j) C_j^2}{\beta_j} z_j^m = -\sum_{j=1}^N \varphi_n(z_j) C_j^2 z_j^m = -\sum_{j=1}^N C_j^2 \sum_{n'=n}^{\infty} K(n, n') z_j^{m+n'}. \quad (1.2.5)$$

ko‘rinishda yozamiz.

Endi I_0 chegirmani hisoblash bilan shug‘ullanamiz.

Quyidagi

$$\varphi_n(z) = z^n f_n(z), \quad \psi_n(z) = z^{-n} g_n(z),$$

belgilashlarni kiritamiz. U holda (1.1.4)

$$f_n(0) = K(n, n). \quad (1.2.6)$$

bo'lishi kelib chiqadi. $z^n f_n(z)$ funksiya aniqlanishiga ko'ra u

$$a_{n-1} f_{n-1}(z) z^{-1} + b_n f_n(z) + a_n f_n(z) z = \frac{z + z^{-1}}{2} f_n(z),$$

tenglamani qanoatlantiradi. Demak, $z \rightarrow 0$ da

$$f_n(0) = 2a_{n-1} f_{n-1}(0)$$

bo'ladi. Xuddi shunday

$$g_n(0) = 2a_n g_{n+1}(0) \quad (1.2.7)$$

bo'lishini ko'rsatish mumkin. (1.1.7) tenglikni qo'llab va $f_n(z)$,

$g_n(z)$ funksiyalarning aniqlanishiga ko'ra

$$\frac{\psi_n(z)}{\alpha(z)} = \frac{g_n(z)(z^2 - 1)z^{-n}}{2a_n(g_n(z)f_{n+1}(z)z^2 - f_n(z)g_{n+1}(z))}, \quad (1.2.8)$$

tenglikka ega bo'lamiz. $n = m$ da $\frac{\psi_n(z)}{\alpha(z)} z^{m-1}$ funksiya $z = 0$ qutbga ega bo'ladi.

$n = m$ da $z = 0$ nuqtadagi chegirmani (1.2.6), (1.2.7) va (1.2.8) tengliklarni qo'llab

$$\operatorname{Res}_{z=0} \frac{\psi_n(z)}{\alpha(z)} z^{n-1} = \frac{g_n(0)}{2a_n f_n(0) g_{n+1}(0)} = \frac{1}{f_n(0)} = \frac{1}{K(n, n)}.$$

bo'lishini topamiz. Demak,

$$I_0 = \frac{1}{K(n, m)} \delta_{mn},$$

bo'ladi ekan. (1.2.4) tenglikka asosan,

$$LFS = \frac{1}{K(n,m)} \delta_{mn} - \sum_{n'=n}^{\infty} K(n,n') F_b(n'+m), \quad (m \geq n), \quad (1.2.9)$$

bo'ladi. Bu yerda δ_{ij} - Kroneker simvoli, F_b esa

$$F_b(m) = \sum_{j=1}^N C_j^2 z_j^m$$

tenglik bilan aniqlanadi. (1.2.1) tenglikka (1.2.3) va (1.2.9) tengliklarni qo'yib,

$$\chi(n,m) + F(n+m) + \sum_{n'=n+1}^{\infty} \chi(n,n') F(n'+m) = 0, \quad m > n,$$

$$(K(n,n))^{-2} = 1 + F(2n) + \sum_{n'=n+1}^{\infty} \chi(n,n') F(n'+n),$$

Gelfand-Levitan-Marchenko integral tenglamasini olamiz. Bu yerda

$$F(n) = \frac{1}{2\pi i} \oint_{|z|=1} R(z) z^{n-1} dz + \sum_{k=1}^N C_k^2 z_k^n,$$

$$\chi(n,m) = \frac{K(n,m)}{K(n,n)}.$$

Endi $\{a_n\}_{-\infty}^{\infty}, \{b_n\}_{-\infty}^{\infty}$ koeffitsiyentlar sochilish nazariyasining berilganlari orqali (1.1.5) tenglik yordamida aniqlanadi.

Xuddi $\varphi_n(z)$ funksiya kabi $\psi_n(z)$ funksiya uchun ham

$$\psi_n(z) = \sum_{n'=-\infty}^n L(n,n') z^{-n'}.$$

tasvir o'rinli bo'lishini ko'rsatish mumkin. Bu yerda $L(n,m)$ koeffitsiyentlar

$$\bar{\chi}(n,m) + \bar{F}(n+m) + \sum_{n'=-\infty}^{n-1} \bar{\chi}(n,n') \bar{F}(n'+m) = 0, \quad m < n,$$

$$(L(n,n))^{-2} = 1 + \bar{F}(2n) + \sum_{n'=-\infty}^{n-1} \bar{\chi}(n,n') \bar{F}(n'+n),$$

integral tenglamalar sistemasi orqali aniqlanadi. Bu yerda

$$\bar{F}(n) = \frac{1}{2\pi i} \oint_{|z|=1} \bar{R}(z) z^{-n-1} dz + \sum_{k=1}^N \bar{C}_k^2 z_k^{-n},$$

$$\bar{\chi}(n,m) = \frac{L(n,m)}{L(n,n)}, \quad \bar{R}(z) = -\frac{\beta(z^{-1})}{\alpha(z)}, \quad \bar{C}_k = \frac{C_k}{\beta_k}, \quad k=1,2,\dots,N.$$

Lemma 1.2.1 Ushbu

$$h_n^k = \left. \frac{d}{dz} (\psi_n(z) - \beta_k \varphi_n(z)) \right|_{z=z_k},$$

vektor funksiya $Ly = \lambda_k y$, $k=1,2,\dots,N$ tenglamaning yechimi bo'lib,

$$W\{h_n^k, \psi_n^k\} = \frac{\beta_k \dot{\alpha}(z_k)(z_k - z_k^{-1})}{2}. \quad (1.2.10)$$

tenglik o'rinli bo'ladi.

Isbot. Ushbu

$$h_n^k = \left. \frac{d}{dz} (\psi_n(z) - \beta_k \varphi_n(z)) \right|_{z=z_k} = \dot{\psi}(z_k) - \beta_k \dot{\varphi}(z_k)$$

funksiya

$$a_{n-1} y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n, \quad n \in Z$$

tenglama yechimi bo'lishini ko'rsatamiz. Oldingilardan foydalanib,

$$a_{n-1} (\dot{\psi}_{n-1}(z_k) - \beta_k \dot{\varphi}_{n-1}(z_k)) + b_n (\dot{\psi}_n(z_k) - \beta_k \dot{\varphi}_n(z_k)) + a_n (\dot{\psi}_{n+1}(z_k) - \beta_k \dot{\varphi}_{n+1}(z_k)) =$$

$$= \frac{z_k + z_k^{-1}}{2} (\dot{\psi}(z_k) - \beta_k \dot{\phi}_n(z_k))$$

yoki

$$a_{n-1} \dot{\psi}_{n-1}^k(z_k) + b_n \dot{\psi}_n^k(z_k) + a_n \dot{\psi}_{n+1}^k(z_k) - \\ - \beta_k \left(a_{n-1} \dot{\phi}_{n-1}^k(z_k) + b_n \dot{\phi}_n^k(z_k) + a_n \dot{\phi}_{n+1}^k(z_k) \right) = \frac{z_k + z_k^{-1}}{2} (\dot{\psi}_n(z_k) - \beta_k \dot{\phi}_n(z_k))$$

ko'rinishda yozamiz. Bu yerda $\dot{\psi}(z_k) = \left. \frac{d}{dz} \psi(z) \right|_{z=z_k}$. Bundan $\psi(z), \phi(z)$

funksiyalar (1.1.1) tenglamaning yechimi bo'lgani uchun h_n^k funksiya ham yechim bo'lishi kelib chiqadi.

(1.1.7) tenglikdan $|z| < 1$ va $n \rightarrow -\infty$

$$\phi_n(z) \rightarrow \alpha(z) z^n$$

bo'lishi kelib chiqadi. Bundan $n \rightarrow -\infty$ da

$$h_n^k \rightarrow -\beta_k \dot{\alpha}(z_k) z_k^n, \quad k = 1, 2, \dots, N.$$

bo'lishi. Natija 1 va (1.1.3) asimptotikaga ko'ra

$$W\{h_n^k, \psi_n^k\} = \frac{\beta_k \dot{\alpha}(z_k)(z_k - z_k^{-1})}{2}$$

bo'ladi.

1.3-§. Misollar

Misol 1. Sochilish nazariyasining berilganlari

$$\left\{ R(z,0) = 0, z_1(0) = \frac{\sqrt{2}}{2}, C_1(0) = \frac{\sqrt{2}}{2} \right\}$$

bo'lgan diskret Shturm-Liuvil operatorini quring.

Yechish. Bu holda

$$F(n) = \frac{1}{2} \left(\frac{\sqrt{2}}{2} \right)^n$$

bo'ladi. Bundan Gelfand-Levitan-Marchenko integral tenglamasining birinchi tenglamasiga ko'ra

$$\chi(n,m) + \frac{1}{2} \left(\frac{\sqrt{2}}{2} \right)^{n+m} + \sum_{n'=n+1}^{\infty} \chi(n,n') \frac{1}{2} \left(\frac{\sqrt{2}}{2} \right)^{n'+m} = 0, \quad m > n$$

bo'ladi. Quyidagicha

$$G_n = \left(\frac{\sqrt{2}}{2} \right)^n + \sum_{n'=n+1}^{\infty} \chi(n,n') \left(\frac{\sqrt{2}}{2} \right)^{n'}$$

belgilash kiritib, yuqoridagi tenglikka qo'yamiz.

Ushbu

$$\chi(n,m) = -\frac{1}{2} \left(\frac{\sqrt{2}}{2} \right)^m G_n$$

tenglikni ikkinchi tenglikka qo'yib,

$$G_n = \left(\frac{\sqrt{2}}{2} \right)^n + \sum_{n'=n+1}^{\infty} \left(-\frac{1}{2} \left(\frac{\sqrt{2}}{2} \right)^{2n'} G_n \right)$$

$$G_n \left(1 + \frac{1}{2} \sum_{n'=n+1}^{\infty} \left(\frac{\sqrt{2}}{2} \right)^{2n'} \right) = \left(\frac{\sqrt{2}}{2} \right)^n$$

$$G_n \left(1 + \frac{1}{2} \sum_{n'=n+1}^{\infty} \left(\frac{1}{2} \right)^{2n} \right) = \left(\frac{\sqrt{2}}{2} \right)^n$$

tenglik yordamida

$$\sum_{n'=n+1}^{\infty} \left(\frac{1}{2}\right)^{n'} = \left(\frac{1}{2}\right)^n$$

$$G_n = \frac{\left(\frac{\sqrt{2}}{2}\right)^n}{1 + \left(\frac{1}{2}\right)^{n+1}}$$

bo'lishini topamiz. Bundan

$$\chi(n, m) = \frac{K(n, m)}{K(n, n)} = \frac{-\frac{1}{2} \left(\frac{\sqrt{2}}{2}\right)^{n+m}}{1 + \left(\frac{1}{2}\right)^{n+1}}$$

tenglik hosil bo'ladi.

Endi Gelfand-Levitan-Marchenko integral tenglamasining ikkinchi tenglamasidan

$$\begin{aligned} (K(n, n))^{-2} &= 1 + \left(\frac{1}{2}\right)^{n+1} + \sum_{n'=n+1}^{\infty} \frac{-\frac{1}{2} \left(\frac{\sqrt{2}}{2}\right)^{n+m}}{1 + \left(\frac{1}{2}\right)^{n+1}} \cdot \frac{1}{2} \left(\frac{\sqrt{2}}{2}\right)^{n'+n} = \\ &= 1 + \left(\frac{1}{2}\right)^{n+1} - \frac{\left(\frac{1}{2}\right)^{n+2}}{1 + \left(\frac{1}{2}\right)^{n+1}} \sum_{n'=n+1}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1 + \left(\frac{1}{2}\right)^n}{1 + \left(\frac{1}{2}\right)^{n+1}} \\ &= \frac{1}{(K(n, n))^2} = \frac{1 + \left(\frac{1}{2}\right)^n}{1 + \left(\frac{1}{2}\right)^{n+1}} \end{aligned}$$

$K(n, n)$ koeffitsent quyidagi

$$K(n, n) = \sqrt{\frac{1 + \left(\frac{1}{2}\right)^{n+1}}{1 + \left(\frac{1}{2}\right)^n}}$$

tenglik yordamida topiladi. Bu orqali a_n , b_n koeffitsentlarni topamiz.

Ushbu

$$a_n = \frac{K(n+1, n+1)}{K(n, n)}$$

tenglikdan

$$\begin{aligned} a_n &= \frac{\frac{1}{2} \sqrt{\frac{1 + (\frac{1}{2})^{n+2}}{1 + (\frac{1}{2})^{n+1}}}}{\sqrt{\frac{1 + (\frac{1}{2})^{n+1}}{1 + (\frac{1}{2})^n}}} = \frac{1}{2} \frac{\sqrt{1 + (\frac{1}{2})^{n+2}} \cdot \sqrt{1 + (\frac{1}{2})^{n+1}}}{1 + (\frac{1}{2})^{n+1}} = \\ &= \frac{1}{2} \frac{\sqrt{1 + (\frac{1}{2})^{n+2}} \cdot \sqrt{1 + (\frac{1}{2})^{n+1}}}{1 + (\frac{1}{2})^{n+1}} = \frac{1}{2} \frac{\sqrt{(\frac{1}{2})^{2n+2} (2^{2n+2} + 5 \cdot 2^n + 1)}}{(\frac{1}{2})^{n+1} (2^{n+1} + 1)} = \\ &= \frac{1}{2} \frac{\sqrt{2^{2n+2} + 5 \cdot 2^n + 1}}{2^{n+1} + 1}, \\ a_n &= \frac{1}{2} \frac{\sqrt{2^{2n+2} + 5 \cdot 2^n + 1}}{2^{n+1} + 1} \end{aligned}$$

bo'lishi kelib chiqadi. Xuddi shunday b_n ko'effitsentni

$$b_n = \frac{1}{2} \left(\frac{K(n, n+1)}{K(n, n)} - \frac{K(n-1, n)}{K(n-1, n-1)} \right)$$

tenglik va

$$\chi(n, m) = \frac{K(n, m)}{K(n, n)} = \frac{-\frac{1}{2} \left(\frac{\sqrt{2}}{2} \right)^{n+m}}{1 + (\frac{1}{2})^{n+1}}$$

tenglikdan

$$\begin{aligned}
b_n &= \frac{1}{2} \left(\frac{-\left(\frac{1}{2}\right) \cdot \left(\frac{\sqrt{2}}{2}\right)^{2n+1}}{1 + \left(\frac{1}{2}\right)^{n+1}} - \frac{-\left(\frac{1}{2}\right) \cdot \left(\frac{\sqrt{2}}{2}\right)^{2n-1}}{1 + \left(\frac{1}{2}\right)^n} \right) = \\
&= \frac{1}{2} \cdot \frac{-\left(\frac{1}{2}\right) \cdot \left(\frac{\sqrt{2}}{2}\right)^{2n+1} \left(1 + \left(\frac{1}{2}\right)^n\right) + \left(\frac{1}{2}\right) \left(\frac{\sqrt{2}}{2}\right)^{2n-1} \left(1 + \left(\frac{1}{2}\right)^{n+1}\right)}{\left(1 + \left(\frac{1}{2}\right)^{n+1}\right) \left(1 + \left(\frac{1}{2}\right)^n\right)} = \\
&= \frac{1}{2} \cdot \frac{-\left(\frac{1}{2}\right)^{n+\frac{3}{2}} - \left(\frac{1}{2}\right)^{2n+\frac{3}{2}} + \left(\frac{1}{2}\right)^{n+\frac{1}{2}} + \left(\frac{1}{2}\right)^{2n+\frac{3}{2}}}{1 + 3 \cdot \left(\frac{1}{2}\right)^{n+1} + \left(\frac{1}{2}\right)^{2n+1}} = \\
&= \frac{1}{2} \cdot \frac{\left(\frac{1}{2}\right)^{n+\frac{1}{2}} \left(1 - \frac{1}{2}\right)}{\left(\frac{1}{2}\right)^{2n+1} \left(2^{2n+1} + 3 \cdot 2^n + 1\right)} = \frac{1}{2} \cdot \frac{2^n}{\sqrt{2} \left(2^{2n+1} + 3 \cdot 2^n + 1\right)}, \\
b_n &= \frac{1}{2} \cdot \frac{2^n}{\sqrt{2} \left(2^{2n+1} + 3 \cdot 2^n + 1\right)}
\end{aligned}$$

bo'lishi kelib chiqadi.

Xulosa

Mazkur bobda diskret Shturm-Liuivill operatori va uning tadbiqlari, Toda zanjirini sochilish nazariyasining teskari masalasi yordamida yechish usuli keltirilgan. Undan quyidagi natijalar olingan:

1. Diskret Shturm-Liuivill operatori spektral parametrlari o'rganildi;
2. Potensialli Toda zanjirining yechimi bo'lgan diskret Shturm-Liuivill operatori spektral parametrlarining t bo'yicha o'zgarish dinamikasi hisoblandi;
3. Bitta xos qiymatga mos keluvchi diskret Shturm-Liuivill operatori qurildi.

II BOB.

Ikkinchi Toda zanjirini integrallash.

2.1-§. Ikkinchi Toda zanjirini sochilish nazariyasini teskari masalasi usulida integrallash

Mazkur paragrafda ushbu

$$\begin{cases} \dot{a}_n = a_n(a_{n+1}^2 - a_{n-1}^2) + a_n(b_{n+1}^2 - b_n^2) \\ \dot{b}_n = 2a_n^2(b_{n+1} + b_n) - 2a_{n-1}^2(b_n + b_{n-1}), \quad n \in Z, \end{cases} \quad (2.1.1)$$

Toda zanjirini turidagi tenglamani sochilish nazariyasining teskari masalasi yordamida yechish usulini keltiramiz.

Quyidagi

$$\begin{cases} \dot{a}_n = a_n(a_{n+1}^2 - a_{n-1}^2) + a_n(b_{n+1}^2 - b_n^2), \\ \dot{b}_n = 2a_n^2(b_{n+1} + b_n) - 2a_{n-1}^2(b_n + b_{n-1}), \\ Lg_n^k = \lambda_k g_n^k, \quad k=1,2,\dots,N, \quad n \in Z, \end{cases} \quad (2.1.2)$$

tenglamalar sistemasini qaraymiz. Bu yerda $\{a_n(t)\}_{-\infty}^{\infty}$, $\{b_n(t)\}_{-\infty}^{\infty}$, – noma'lum funksiyalar, $\{g_n^k(t)\}_{-\infty}^{\infty}$ vektor-funksiya esa

$$L(t)y \equiv a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1},$$

operatorning $\lambda_k = \frac{z_k + z_k^{-1}}{2}$, $k=1,2,\dots,N$ xos qiymatga mos keluvchi xos vektori,

(2.1.2) tenglamalar sistemasini

$$a_n(0) = a_{n0}, \quad b_n(0) = b_{n0}, \quad n \in Z, \quad (2.1.3)$$

shartlar bilan qaraymiz. Bu yerda $\{a_{n0}\}_{-\infty}^{\infty}$, $\{b_{n0}\}_{-\infty}^{\infty}$ vektor-funksiyalar

1. $a_{n0} > 0$, $\text{Im}b_{n0} = 0$, $n \in Z$,

2. $\sum_{n=-\infty}^{\infty} |n| \left(\left| a_{n0} - \frac{1}{2} \right| + |b_{n0}| \right) < \infty$,

3. $L(0)$ operator $[-1;1]$ kesma tashqarisida yotuvchi N ta $\lambda_1(0), \lambda_2(0), \dots, \lambda_N(0)$ xos qiymatlarga ega.

(2.1.2)-(2.1.3) masalaning yechimini ushbu

$$a_n(t) > 0, \operatorname{Im} b_n = 0, n \in \mathbb{Z}, \sum_{n=-\infty}^{\infty} |n| \left(\left| a_n(t) - \frac{1}{2} \right| + |b_n(t)| \right) < \infty. \quad (2.1.4)$$

shartni qanoatlantiruvchi funksiyalar sinfida qaraymiz.

Ushbu paragrafning asosiy maqsadi (2.1.2) masalaning $\{a_n(t)\}_{-\infty}^{\infty}, \{b_n(t)\}_{-\infty}^{\infty}, \{g_n^k(t)\}_{-\infty}^{\infty}, k=1,2,\dots,N$ yechimlariga $L(t)$ operator uchun sohilish nazariyasining teskari masalasi usulida tasvirlar olish.

(2.1.2) tenglamaning Laks juftligini

$$(L\psi)_n \equiv a_{n-1}\psi_{n-1}(z) + b_n\psi_n(z) + a_n\psi_{n+1}(z) = \lambda\psi_n(z), \quad (2.1.5)$$

$$(B\psi)_n \equiv a_n a_{n+1} \psi_{n+2} + a_n (b_{n+1} + b_n) \psi_{n+1} - a_{n-1} (b_n + b_{n-1}) \psi_{n-1} - a_{n-1} a_{n-2} \psi_{n-2}, \quad (2.1.6)$$

ko‘rinishida izlaymiz.

Lemma 2.1.1. $L\psi_n = \lambda\psi_n(z)$ tenglamaning yetarlicha silliq yechimlar sinfida (2.1.1) chekli ayirmali tenglamalar sistemasi

$$\frac{dL}{dt} = -[L, B] \quad (2.1.7)$$

operatorli tenglik yordamida yozish mumkin. Bu yerda $[L, B] = LB - BL$ Puasson qavsi.

Ushbu

$$L\psi_n = \lambda\psi_n \quad (2.1.8)$$

tenglikni t bo‘yicha differensiallasak

$$\frac{dL}{dt} \psi_n + L \frac{d\psi_n}{dt} = \lambda \frac{d\psi_n}{dt}$$

tenglik hosil bo‘ladi. Buni (2.1.7) tenglik yordamida

$$-[L, B]\psi_n + L \frac{d\psi_n}{dt} = \lambda \frac{d\psi_n}{dt}$$

yoki

$$BL\psi_n - LB\psi_n + L\frac{d\psi_n}{dt} = \lambda\frac{d\psi_n}{dt}$$

ko‘rinishda yozish mumkin. Buni (2.1.8) tenglik yordamida

$$(L - \lambda)\left(\frac{d\psi_n}{dt} - B\psi_n\right) = 0 \quad (2.1.9)$$

ko‘rinishda yozish mumkin. (2.1.9) tenglikdan $\frac{d\psi_n}{dt} - B\psi_n$ funksiya ham (2.1.8)

tenglamaning yechimi bo‘lishi kelib chiqadi. Demak, bu yechimni xaqiqiy λ larda $\psi_n(z)$, $\varphi_n(z)$ yechimlar sistemasi orqali chiziqli ifodalanadi, ya’ni shunday $\tilde{\alpha}$ va $\tilde{\beta}$ sonlar mavjudki

$$\frac{d\psi_n}{dt} - B\psi_n = \tilde{\alpha}\psi_n(z) + \tilde{\beta}\varphi_n(z) \quad (2.1.10)$$

tenglik o‘rinli bo‘ladi. Biz $\tilde{\alpha}$ va $\tilde{\beta}$ sonlarni topishimiz kerak. Bunda $n \rightarrow -\infty$ da limitga o‘tsak

$$\frac{d\psi_n}{dt} - B\psi_n \rightarrow \frac{z^2 - z^{-2}}{4} z^{-n}$$

bo‘ladi. Bundan $n \rightarrow -\infty$ da

$$\tilde{\alpha} \rightarrow \frac{z^2 - z^{-2}}{4}, \quad \tilde{\beta} \rightarrow 0$$

bo‘lishi kelib chiqadi. Demak, (2.1.10) tenglikni

$$\frac{d\psi_n}{dt} - B\psi_n = \frac{z^2 - z^{-2}}{4} \psi_n \quad (2.1.11)$$

ko‘rinishda yozish mumkin.

Endi sochilish nazariyasi berilganlarining evolutsiyasini topish bilan shug‘ullanamiz.

(1.1.6) tenglik yordamida (2.1.11) tenglikni

$$\frac{d\alpha(z)}{dt} \varphi_n(z^{-1}) + \alpha(z) \frac{d\varphi_n(z^{-1})}{dt} + \frac{d\beta(z)}{dt} \varphi_n(z) + \beta(z) \frac{d\varphi_n(z)}{dt} -$$

$$-B(\alpha(z)\varphi_n(z^{-1}) + \beta(z)\varphi_n(z)) = \frac{z^2 - z^{-2}}{4}(\alpha(z)\varphi_n(z^{-1}) + \beta(z)\varphi_n(z))$$

ko‘rinishda yozish mumkin. Bunda $n \rightarrow \infty$ da limitga o‘tsak,

$$\frac{d\alpha}{dt} \cdot z^{-n} + \frac{d\beta}{dt} \cdot z^n - \frac{1}{4}(\alpha z^{-n-2} + \beta z^{n+2}) + \frac{1}{4}(\alpha z^{-n+2} + \beta z^{n-2}) = \frac{(z^2 - z^{-2})}{4}(\alpha z^{-n} + \beta z^n).$$

bo‘ladi. z^{-n} va z^n oldidagi koeffitsiyentlarni tenglasak,

$$\frac{d\alpha(z,t)}{dt} = 0, \quad \frac{d\beta(z,t)}{dt} = \frac{(z^2 - z^{-2})}{2} \beta(z,t). \quad (2.1.12)$$

bo‘lishi kelib chiqadi.

(2.1.12) tenglikdan $L(t)$ operatorning xos qiymatlari t ga bog‘liq emasligi kelib chiqadi, ya’ni

$$\frac{dz_k}{dt} = 0, \quad k = 1, 2, \dots, N.$$

Bundan tashqari (2.1.12) tengliklardan $|z|=1$ da

$$\frac{dR(z)}{dt} = \frac{d}{dt} \left(\frac{\beta(z)}{\alpha(z)} \right) = \frac{(z^2 - z^{-2})}{2} \frac{\beta(z)}{\alpha(z)} = \frac{(z^2 - z^{-2})}{2} R(z)$$

$$\frac{dR(z,t)}{dt} = \frac{(z^2 - z^{-2})}{2} R(z,t),$$

bo‘lishi kelib chiqadi.

Endi normallovchi o‘zgarmaslarning evolutsiyasini topish bilan shug‘ullanamiz. (2.1.11) tenglikda $z = z_k$ deymiz. $z = z_k$ da

$\psi_n^k = \beta_k \varphi_n^k$, $k = 1, 2, \dots, N$ bo‘lgani uchun

$$\frac{d\beta_k}{dt} \varphi_n^k - B\beta_k \varphi_n^k = \frac{z_k^2 - z_k^{-2}}{4} \beta_k \varphi_n^k \quad (2.1.13)$$

bo‘ladi. Bunda $n \rightarrow \infty$ da limitga o‘tsak,

$$\frac{d\beta_k}{dt} = \frac{z_k^2 - z_k^{-2}}{2} \beta_k, \quad k = 1, 2, \dots, N. \quad (2.1.14)$$

tenglik hosil bo‘ladi. Ushbu $C_k^2 = -\frac{\beta_k}{z_k \dot{\alpha}(z_k)}$, $k=1,2,\dots,N$ tenglikka ko‘ra (2.1.14)

tenglikni

$$\frac{dC_k^2}{dt} = -\frac{d\beta_k}{dt} \frac{1}{z_k \dot{\alpha}(z_k)} = \frac{z_k^2 - z_k^{-2}}{2} C_k^2, \quad k=1,2,\dots,N.$$

ko‘rinishda yozish mumkin. Demak, biz quyidagi teoremani isbotladik.

Teorema 2.1.1. Agar $a_n(t)$, $b_n(t)$, $g_n^k(t)$, $k=1,2,\dots,N$, $n \in Z$ funksiyalar (2.1.2)-(2.1.4) masalaning yechimi bo‘lsa, u holda

$$L(t)y \equiv a_{n-1}(t)y_{n-1} + b_n(t)y_n + a_n(t)y_{n+1}$$

operatorning sohilish nazariyasining berilganlari t bo‘yicha quyidagi ko‘rinishda o‘zgaradi.

$$R(z,t) = R(z,0) \exp\left\{\frac{z^2 - z^{-2}}{2} t\right\}, \quad |z|=1,$$

$$z_k(t) = z_k(0), \quad C_k^2(t) = C_k^2(0) \exp\left\{\frac{z_k^2 - z_k^{-2}}{2} t\right\}, \quad k=1,2,\dots,N.$$

2.2-§. Moslangan manbali ikkinchi Toda zanjirini integrallash.

Ushbu paragrafda

$$\begin{cases} \dot{a}_n = a_n(a_{n+1}^2 - a_{n-1}^2) + a_n(b_{n+1}^2 - b_n^2) + a_n \sum_{i=1}^N ((g_n^i)^2 - (g_{n+1}^i)^2), \\ \dot{b}_n = 2a_n^2(b_{n+1} + b_n) - 2a_{n-1}^2(b_n + b_{n-1}) - 2 \sum_{i=1}^N g_n^i (a_n g_{n+1}^i - a_{n-1} g_{n-1}^i), \\ a_{n-1} g_{n-1}^k + b_n g_n^k + a_n g_{n+1}^k = \lambda_k g_n^k, \quad k=1,2,\dots,N, \quad n \in Z, \end{cases} \quad (2.2.1)$$

tenglamalar sistemasini qaraymiz. Bu yerda

$\{a_n(t)\}_{-\infty}^{\infty}, \{b_n(t)\}_{-\infty}^{\infty}, \{g_n^k(t)\}_{-\infty}^{\infty}, k=1,2,\dots,N$ – noma'lum funksiyalar $\{g_n^k(t)\}_{-\infty}^{\infty}$

$$L(t)y \equiv a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1},$$

operatorning $\lambda_k = \frac{z_k + z_k^{-1}}{2}, k=1,2,\dots,N$ xos qiymatga mos keluvchi ushbu

$$\sum_{n=-\infty}^{\infty} (g_n^k(t))^2 = A_k(t), \quad k=1,2,\dots,N, \quad (2.2.2)$$

shartni qanoatlantiruvchi xos funksiyasi, $A_k(t)$ oldindan berilgan musbat funksiya.

(2.2.1) tenglamalar sistemasini

$$a_n(0) = a_{n_0}, \quad b_n(0) = b_{n_0}, \quad n \in Z, \quad (2.2.3)$$

boshlang'ich shartlar bilan qaraymiz. Bu yerda $\{a_{n_0}\}_{-\infty}^{\infty}, \{b_{n_0}\}_{-\infty}^{\infty}$ vektorlar quyidagi

shartlarni qanoatlantiradi:

1. $a_{n_0} > 0, \operatorname{Im} b_{n_0} = 0, \quad n \in Z,$

2. $\sum_{n=-\infty}^{\infty} |n| \left(\left| a_{n_0} - \frac{1}{2} \right| + |b_{n_0}| \right) < \infty,$

3. $L(0)$ operator $[-1;1]$ kesma tashkarisida yotuvchi N ta

$\lambda_1(0), \lambda_2(0), \dots, \lambda_N(0)$ xos qiymatlarga ega.

(2.2.1)-(2.2.3) masalaning yechimini

$$a_n(t) > 0, \operatorname{Im} b_n = 0, n \in \mathbb{Z}, \quad \sum_{n=-\infty}^{\infty} |n| \left(\left| a_n(t) - \frac{1}{2} \right| + |b_n(t)| \right) < \infty. \quad (2.2.4)$$

shartni qanoatlantiruvchi funksiyalar sinfida qaraymiz.

Ushbu paragrafning asosiy maqsadi (2.2.1) masalaning $\{a_n(t)\}_{-\infty}^{\infty}, \{b_n(t)\}_{-\infty}^{\infty}, \{g_n^k(t)\}_{-\infty}^{\infty}, k=1,2,\dots,N$ yechimlariga $L(t)$ operator uchun sochilish nazariyasining teskari masalasi usulida tasvirlar olish. (2.2.1)

tenglama uchun Laks juftligini

$$a_{n-1}\psi_{n-1}(z) + b_n\psi_n(z) + a_n\psi_{n+1}(z) = \lambda\psi_n(z), \quad (2.2.5)$$

$$\frac{d\psi_n(z)}{dt} = a_n a_{n+1} \psi_{n+2} + a_n (b_{n+1} + b_n) \psi_{n+1} - a_{n-1} (b_n + b_{n-1}) \psi_{n-1} - a_{n-1} a_{n-2} \psi_{n-2} + F_n. \quad (2.2.6)$$

ko‘rinishida izlaymiz. (2.2.5) tenglikni t bo‘yicha differensiallab va (2.2.1), (2.2.6) tengliklardan foydalanamiz, ya’ni

$$\dot{a}_{n-1}\psi_{n-1}(z) + a_{n-1}\dot{\psi}_{n-1}(z) + \dot{b}_n\psi_n(z) + b_n\dot{\psi}_n(z) + \dot{a}_n\psi_{n+1}(z) + a_n\dot{\psi}_{n+1}(z) = \lambda\dot{\psi}_n(z)$$

tenglamaga (2.2.1) tenglamalarni qo‘yamiz. Bundan esa

$$\begin{aligned} & a_{n-1}((a_n^2 - a_{n-2}^2) + a_{n-1}(b_n^2 - b_{n-1}^2))\psi_{n-1} + a_{n-1}(a_{n-1}a_n\psi_{n-1} + a_{n-1}(b_n + b_{n-1})\psi_n - \\ & - a_{n-2}(b_{n-1} + b_{n-2})\psi_{n-2} - a_{n-2}a_{n-3}\psi_{n-3}) + (a_n(a_{n+1}^2 - a_{n-1}^2) + a_n(b_{n+1}^2 - b_n^2))\psi_{n+1} + \\ & + a_n(a_{n+1}a_{n+2}\psi_{n+3} + a_{n+1}(b_{n+1} + b_{n+2})\psi_{n+2} - a_n(b_{n+1} + b_n)\psi_n - a_n a_{n-2}\psi_{n-1}) + \\ & + (2a_n^2(b_{n+1} + b_n) - 2a_{n-1}^2(b_n + b_{n-1}))\psi_n + b_n(a_n a_{n+1}\psi_{n+2} + a_n(b_{n+1} + b_n)\psi_{n+1} - \\ & - a_{n-1}(b_n + b_{n-1})\psi_{n-1} - a_{n-1}a_{n-2}\psi_{n-2}) = \lambda\psi_n \end{aligned}$$

bo‘lishi kelib chiqadi. Qavslarni ochib hisoblashlarni bajarsak,

$$-\lambda a_{n-1} a_{n-2} \psi_{n-2} + \lambda a_n a_{n+1} \psi_{n+2} - \lambda a_{n-1} b_{n-1} \psi_{n-1} - \lambda a_{n-1} b_n \psi_{n-1} + \lambda a_n b_{n+1} \psi_{n+1} + \lambda a_n b_n \psi_{n+1} = \lambda \psi_n$$

bo‘lishi kelib chiqadi. Bundan esa,

$$a_{n-1}F_{n-1} + b_n F_n + a_n F_{n+1} - \lambda F_n = a_{n-1} \sum_{i=1}^N ((g_n^i)^2 - (g_{n-1}^i)^2) \psi_{n-1} +$$

$$+ 2 \sum_{i=1}^N g_n^i (a_n g_{n+1}^i - a_{n-1} g_{n-1}^i) \psi_n + a_n \sum_{i=1}^N ((g_{n+1}^i)^2 - (g_n^i)^2) \psi_{n+1}. \quad (2.2.7)$$

bo'lishini topamiz. (2.2.7) tenglamaning yechimini

$$F_n = \alpha_n \psi_n(z) + \beta_n \varphi_n(z) \quad (2.2.8)$$

ko'rinishda izlaymiz. (2.2.6) tenglikka asosan, $n \rightarrow -\infty$ da

$$F_n \rightarrow \frac{z^2 - z^{-2}}{4} z^{-n}$$

bo'ladi. (2.2.8) dan $n \rightarrow -\infty$ da

$$\alpha_n \rightarrow \frac{z^2 - z^{-2}}{4}, \quad \beta_n \rightarrow 0 \quad (2.2.9)$$

bo'lishi kelib chiqadi. (2.2.8) ni (2.2.7) ga qo'yib va (2.2.5) tenglikdan foydalanib

$$\begin{aligned} & a_n (\alpha_{n+1} - \alpha_n) \psi_{n+1} + a_n (\beta_{n+1} - \beta_n) \varphi_{n+1} - a_{n-1} (\alpha_n - \alpha_{n-1}) \psi_{n-1} - a_{n-1} (\beta_n - \beta_{n-1}) \varphi_{n-1} = \\ & = a_{n-1} \sum_{i=1}^N ((g_n^i)^2 - (g_{n-1}^i)^2) \psi_{n-1} + 2 \sum_{i=1}^N g_n^i (a_n g_{n+1}^i - a_{n-1} g_{n-1}^i) \psi_n + \\ & \quad + a_n \sum_{i=1}^N ((g_{n+1}^i)^2 - (g_n^i)^2) \psi_{n+1}. \end{aligned} \quad (2.2.10)$$

bo'lishini topamiz.

Lemma 2.2.1. Ushbu

$$\begin{aligned} \alpha_{n+1} - \alpha_n &= \frac{1}{W\{\varphi_n, \psi_n\}} \sum_{i=1}^N ((\psi_n g_n^i + \psi_{n+1} g_{n+1}^i) W\{\varphi_n, g_n^i\} + (\varphi_n g_n^i + \varphi_{n+1} g_{n+1}^i) W\{\psi_n, g_n^i\}) \\ \beta_{n+1} - \beta_n &= -\frac{2}{W\{\varphi_n, \psi_n\}} \sum_{i=1}^N (\psi_n g_n^i + \psi_{n+1} g_{n+1}^i) W\{\psi_n, g_n^i\}. \end{aligned}$$

tengliklar bilan aniqlanuvchi α_n va β_n funksiyalar (2.2.10) tenglamaning yechimi bo'ladi.

Ushbu $\alpha(z) = \frac{2}{z - z^{-1}} W\{\psi_n(z), \varphi_n(z)\}$ tenglik hamda (2.2.9) formula va

lemma 2.2.1 dan foydalanib

$$\alpha_n = -\frac{2}{(z+z^{-1})\alpha(z)} \cdot \left[\sum_{j=-\infty}^{n-1} \sum_{i=1}^N (\psi_j g_j^i + \psi_{j+1} g_{j+1}^i) W\{\varphi_j, g_j^i\} + \right. \\ \left. + \sum_{j=-\infty}^{n-1} \sum_{i=1}^N (\varphi_j g_j^i + \varphi_{j+1} g_{j+1}^i) W\{\psi_j, g_j^i\} \right] + \frac{z^2 - z^{-2}}{4}, \\ \beta_n = \frac{4}{(z+z^{-1})\alpha(z)} \cdot \left[\sum_{j=-\infty}^{n-1} \sum_{i=1}^N (\psi_j g_j^i + \psi_{j+1} g_{j+1}^i) W\{\psi_j, g_j^i\} \right].$$

bo‘lishini topamiz. (2.2.8) tenglikka asosan (2.2.6) tenglikni

$$\frac{d\psi_n(z)}{dt} = a_n a_{n+1} \psi_{n+2} + a_n (b_{n+1} + b_n) \psi_{n+1} - a_{n-1} (b_n + b_{n-1}) \psi_{n-1} - a_{n-1} a_{n-2} \psi_{n-2} + \\ + \frac{(z^2 - z^{-2})}{4} \psi_n - \frac{2}{(z+z^{-1})\alpha(z)} \times \\ \times \sum_{j=-\infty}^{n-1} \sum_{i=1}^N [(\psi_j g_j^i + \psi_{j+1} g_{j+1}^i) W\{\varphi_j, g_j^i\} + (\varphi_j g_j^i + \varphi_{j+1} g_{j+1}^i) W\{\psi_j, g_j^i\}] \psi_n + \\ + \frac{4}{(z+z^{-1})\alpha(z)} \cdot \sum_{j=-\infty}^{n-1} \sum_{i=1}^N (\psi_j g_j^i + \psi_{j+1} g_{j+1}^i) W\{\psi_j, g_j^i\} \varphi_n. \quad (2.2.11)$$

ko‘rinishda yozish mumkin.

Lemma 2.2.2. Quyidagi

$$\sum_{j=-\infty}^{\infty} [(\psi_j g_j^i + \psi_{j+1} g_{j+1}^i) W\{\varphi_j, g_j^i\} + (\varphi_j g_j^i + \varphi_{j+1} g_{j+1}^i) W\{\psi_j, g_j^i\}] = 0, \\ \sum_{j=-\infty}^{\infty} (\psi_j g_j^i + \psi_{j+1} g_{j+1}^i) W\{\psi_j, g_j^i\} = 0, \quad i = 1, 2, \dots, N.$$

tengliklar o‘rinli.

Isbot. Qulaylik uchun $W_j = W\{\psi_j, g_j^i\}$ va $V_j = W\{\varphi_j, g_j^i\}$

(i fiksirlangan deb hisoblaymiz) belgilashlarni kiritamiz. Lemmaga asosan

$$\begin{aligned}
& \sum_{j=-\infty}^{\infty} [(\psi_j g_j^i + \psi_{j+1} g_{j+1}^i) V_j + (\varphi_j g_j^i + \varphi_{j+1} g_{j+1}^i) W_j] = \\
& = \sum_{j=-\infty}^{\infty} [(\psi_j g_j^i (V_j + V_{j-1}) + \varphi_j g_j^i (W_j + W_{j-1}))] = \\
& = \frac{1}{\lambda_i - \lambda} \sum_{j=-\infty}^{\infty} [(W_j - W_{j-1})(V_j + V_{j-1}) + (V_j - V_{j-1})(W_j + W_{j-1})] = \\
& = \frac{2}{\lambda_i - \lambda} \sum_{j=-\infty}^{\infty} [W_j V_j - W_{j-1} V_{j-1}] = 0.
\end{aligned}$$

bo'ladi. Bundan

$$\sum_{j=-\infty}^{\infty} (\psi_j g_j^i + \psi_{j+1} g_{j+1}^i) W_j = \frac{1}{\lambda_i - \lambda} \sum_{j=-\infty}^{\infty} (W_j^2 - W_{j-1}^2) = 0$$

bo'lishi kelib chiqadi. *Lemma isbot bo'ldi.*

(2.2.11) tenglikda (1.6) $(\psi_n(z) = \alpha(z)\varphi_n(z^{-1}) + \beta(z)\varphi_n(z))$ va lemma 2.2.2 ni qo'llab $n \rightarrow \infty$ da limitga o'tsak,

$$\frac{d\alpha}{dt} \cdot z^{-n} + \frac{d\beta}{dt} \cdot z^n = \frac{1}{4}(\alpha z^{-n-2} + \beta z^{n+2}) - \frac{1}{4}(\alpha z^{-n+2} + \beta z^{n-2}) + \frac{(z^2 - z^{-2})}{4}(\alpha z^{-n} + \beta z^n).$$

tenglik hosil bo'ladi. z^{-n} va z^n oldidagi koeffitsiyentlarni tenglab

$$\frac{d\alpha(z,t)}{dt} = 0, \quad \frac{d\beta(z,t)}{dt} = \frac{(z^2 - z^{-2})}{2} \beta(z,t) \quad (2.2.12)$$

bo'lishini topamiz. (2.2.12) ning birinchi tenglamasidan $L(t)$ operatorning xos qiymatlari t ga bog'lik emasligi, ya'ni

$$\frac{dz_k}{dt} = 0, \quad k = 1, 2, \dots, N.$$

bo'lishi kelib chiqadi. Bundan tashqari (2.2.12) dan sochilish funksiyasining

$$\frac{dR(z,t)}{dt} = \frac{(z^2 - z^{-2})}{2} R(z,t), \quad |z| = 1$$

t bo'yicha evolutsiyasini topamiz.

Endi normallovchi konstantalarning evolutsiyalarini topish bilan shug'ullanamiz. Xuddi uzluksiz spektrdagi kabi diskret spektr holida ham

$$\begin{aligned} \frac{d\psi_n^k}{dt} = & a_n a_{n+1} \psi_{n+2}^k + a_n (b_{n+1} + b_n) \psi_{n+1}^k - a_{n-1} (b_n + b_{n-1}) \psi_{n-1}^k - a_{n-1} a_{n-2} \psi_{n-2}^k + \\ & + \frac{(z_k^2 - z_k^{-2})}{4} \psi_n^k + \frac{1}{W\{h_n^k, \psi_n^k\}} \times \\ & \times \sum_{j=-\infty}^{n-1} \sum_{i=1}^N [(\psi_j^k g_j^i + \psi_{j+1}^k g_{j+1}^i) W\{h_j^k, g_j^i\} + (h_j^k g_j^i + h_{j+1}^k g_{j+1}^i) W\{\psi_j^k, g_j^i\}] \psi_n^k - \\ & - \frac{2}{W\{h_n^k, \psi_n^k\}} \cdot \sum_{j=-\infty}^{n-1} \sum_{i=1}^N (\psi_j^k g_j^i + \psi_{j+1}^k g_{j+1}^i) W\{\psi_j^k, g_j^i\} h_n^k, \quad k = 1, 2, \dots, N. \end{aligned} \quad (2.2.13)$$

tenglikni yozish mumkin. Quyidagi

$$\begin{aligned} \sum_{j=-\infty}^{\infty} [(\psi_j^k g_j^i + \psi_{j+1}^k g_{j+1}^i) W\{h_j^k, g_j^i\} + (h_j^k g_j^i + h_{j+1}^k g_{j+1}^i) W\{\psi_j^k, g_j^i\}] = 0, \quad k \neq i, \\ \sum_{j=-\infty}^{\infty} (\psi_j^k g_j^i + \psi_{j+1}^k g_{j+1}^i) W\{\psi_j^k, g_j^i\} = 0, \quad k \neq i. \end{aligned} \quad (2.2.14)$$

tengliklar o'rinli bo'lishini oson ko'rsatish mumkin.

$W\{\psi_j^k, g_j^k\} = 0$ bo'lgani uchun $k = i$ da

$$\sum_{j=-\infty}^{\infty} (\psi_j^k g_j^k + \psi_{j+1}^k g_{j+1}^k) W\{\psi_j^k, g_j^k\} = 0 \quad . \quad (2.2.15)$$

bo'ladi. Bundan tashqari, $g_j^k = b_k \cdot \psi_j^k$ tenglikdan

$$\begin{aligned} \sum_{j=-\infty}^{\infty} [(\psi_j^k g_j^k + \psi_{j+1}^k g_{j+1}^k) W\{h_j^k, g_j^k\} + (h_j^k g_j^k + h_{j+1}^k g_{j+1}^k) W\{\psi_j^k, g_j^k\}] = \\ = 2W\{h_n^k, \psi_n^k\} \sum_{j=-\infty}^{\infty} (g_j^k)^2. \end{aligned} \quad (2.2.16)$$

bo'lishini topamiz. (1.1.8), ya'ni $\psi_n^k = \beta_k \varphi_n^k$, $k = 1, 2, \dots, N$, tenglikni inobatga olib, (2.2.13) tenglikda $n \rightarrow \infty$ da limitga o'tamiz va (2.2.2), (2.2.14), (2.2.15),

(2.2.16) tengliklar yordamida

$$\frac{d\beta_k}{dt} = \left(\frac{z_k^2 - z_k^{-2}}{2} + 2A_k(t) \right) \beta_k, \quad k = 1, 2, \dots, N. \quad (2.2.17)$$

bo'lishini topamiz. (1.1.9) va (2.2.17) tenglikni inobatga olib

$$\frac{dC_k^2}{dt} = \left(\frac{z_k^2 - z_k^{-2}}{2} + 2A_k(t) \right) C_k^2, \quad k = 1, 2, \dots, N$$

bo'lishini topamiz.

Demak, biz quyidagi teoremani isbot qildik.

Teorema 2.2.1. Agar $a_n(t)$, $b_n(t)$, $g_n^k(t)$, $k = 1, 2, \dots, N$, $n \in Z$ funksiyalar (2.2.1)-(2.2.4) masalaning yechimlari bo'lsa, u holda

$$L(t)y \equiv a_{n-1}(t)y_{n-1} + b_n(t)y_n + a_n(t)y_{n+1}$$

operatorning sohilish nazariyasining berilganlari t bo'yicha quyidagicha o'zgaradi:

$$\frac{dR(z, t)}{dt} = \frac{(z^2 - z^{-2})}{2} R(z, t), \quad |z| = 1,$$

$$z_k(t) = z_k(0), \quad C_k^2(t) = C_k^2(0) \exp \left\{ \frac{z_k^2 - z_k^{-2}}{2} t + 2 \int_0^t A_k(\tau) d\tau \right\}, \quad k = 1, 2, \dots, N.$$

Olingan tengliklar $L(t)$ operatorni sohilish nazariyasining berilganlari orqali to'liq aniqlaydi.

2.3-§. Misollar

Misol 1. a_{n0} va b_{n0} lar

$$a_{n0} = \frac{1}{2} \frac{\sqrt{2^{2n+2} + 5 \cdot 2^n + 1}}{1 + 2^{n+1}},$$

$$b_{n0} = \frac{2^n}{\sqrt{2}(2^{2n+1} + 3 \cdot 2^n + 1)}, \quad n \in \mathbb{Z}.$$

bo'lsin. Bu holda I bobdagi misolga ko'ra

$$R(z,0) = 0, \quad z_1(0) = \frac{\sqrt{2}}{2}, \quad C_1(0) = \frac{\sqrt{2}}{2},$$

bo'ladi. Bundan teorema 2.1.1 ga ko'ra

$$R(z,t) = 0, \quad z_1(t) = \frac{\sqrt{2}}{2}, \quad C_1(t) = \frac{1}{2} e^{-\frac{3}{4}t},$$

bo'lishi kelib chiqadi. Sochilish nazariyasining teskari masalasi usulini qo'llasak,

$$F(n) = \frac{1}{2} e^{-\frac{3}{4}t} \cdot \left(\frac{\sqrt{2}}{2}\right)^n.$$

bo'ladi. Bundan Gelfand-Levitan-Marchenko integral tenglamasining birinchi tenglamasiga ko'ra

$$\chi(n,m) + \frac{1}{2} e^{-\frac{3}{4}t} \cdot \left(\frac{\sqrt{2}}{2}\right)^{n+m} + \sum_{n'=n+1}^{\infty} \chi(n,n') \frac{1}{2} e^{-\frac{3}{4}t} \cdot \left(\frac{\sqrt{2}}{2}\right)^{n'+m} = 0, \quad m > n$$

yoki

$$\chi(n,m) + \frac{1}{2} e^{-\frac{3}{4}t} \cdot \left(\frac{\sqrt{2}}{2}\right)^m G_n, \quad m > n$$

bo'ladi. Quyidagi

$$G_n = \left(\frac{\sqrt{2}}{2}\right)^n + \sum_{n'=n+1}^{\infty} \chi(n,n') \left(\frac{\sqrt{2}}{2}\right)^{n'}$$

belgilash kiritib, yuqoridagi tenglikka qo'yamiz.

Ushbu

$$\chi(n,m) = -\frac{1}{2} e^{-\frac{3}{4}t} \cdot \left(\frac{\sqrt{2}}{2}\right)^m G_n$$

tenglikni ikkinchi tenglikka qo'yib,

$$G_n = \left(\frac{\sqrt{2}}{2}\right)^n - \sum_{n'=n+1}^{\infty} \frac{1}{2} e^{-\frac{3}{4}t} \cdot \left(\frac{\sqrt{2}}{2}\right)^{2n'} G_n$$

$$G_n \left(1 + \frac{1}{2} e^{-\frac{3}{4}t} \cdot \sum_{n'=n+1}^{\infty} \left(\frac{\sqrt{2}}{2}\right)^{2n'}\right) = \left(\frac{\sqrt{2}}{2}\right)^n$$

$$\sum_{n'=n+1}^{\infty} \left(\frac{1}{2}\right)^{n'} = \left(\frac{1}{2}\right)^n$$

tenglik yordamida

$$G_n \left(1 + \left(\frac{1}{2}\right)^{n+1} e^{-\frac{3}{4}t}\right) = \left(\frac{\sqrt{2}}{2}\right)^n$$

$$G_n = \frac{\left(\frac{\sqrt{2}}{2}\right)^n}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{-\frac{3}{4}t}}$$

bo'lishini topamiz. Bundan

$$\chi(n, m) = \frac{K(n, m)}{K(n, n)} = \frac{-\frac{1}{2} \left(\frac{\sqrt{2}}{2}\right)^{n+m} \cdot e^{-\frac{3}{4}t}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{-\frac{3}{4}t}}$$

tenglik hosil bo'ladi.

Endi Gelfand-Levitan-Marchenko integral tenglamasining ikkinchi tenglamasidan

$$(K(n, n))^{-2} = 1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{\frac{\sqrt{2}}{2}t} + \sum_{n'=n+1}^{\infty} \frac{-\frac{1}{2} \left(\frac{\sqrt{2}}{2}\right)^{n+n'} \cdot e^{\frac{\sqrt{2}}{2}t}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{\frac{\sqrt{2}}{2}t}} \cdot \frac{1}{2} \left(\frac{\sqrt{2}}{2}\right)^{n'+n} \cdot e^{\frac{\sqrt{2}}{2}t} =$$

$$= 1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{\frac{3}{4}t} - \frac{\left(\frac{1}{2}\right)^{n+2} \cdot e^{\frac{3}{4}t}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{\frac{3}{4}t}} \sum_{n'=n+1}^{\infty} \left(\frac{1}{2}\right)^{n'} = \frac{1 + \left(\frac{1}{2}\right)^n \cdot e^{\frac{3}{4}t}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{\frac{3}{4}t}} =$$

$$\frac{1}{(K(n, n))^2} = \frac{1 + \left(\frac{1}{2}\right)^n \cdot e^{\frac{3}{4}t}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{\frac{3}{4}t}}$$

$K(n, n)$ koeffitsient quyidagicha

$$K(n,n) = \sqrt{\frac{1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{-\frac{3}{4}t}}{1 + \left(\frac{1}{2}\right)^n \cdot e^{-\frac{3}{4}t}}}$$

tenglik yordamida topiladi. Bu orqali a_n , b_n koeffitsientlarni topamiz.

$$a_n = \frac{K(n+1,n+1)}{K(n,n)}$$

tenglikdan

$$\begin{aligned} a_n &= \frac{1}{2} \sqrt{\frac{1 + \left(\frac{1}{2}\right)^{n+2} \cdot e^{-\frac{3}{4}t}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{-\frac{3}{4}t}}} \cdot \sqrt{\frac{1 + \left(\frac{1}{2}\right)^n \cdot e^{-\frac{3}{4}t}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{-\frac{3}{4}t}}} = \frac{1}{2} \frac{\sqrt{1 + \left(\frac{1}{2}\right)^{n+2} \cdot e^{-\frac{3}{4}t}} \cdot \sqrt{1 + \left(\frac{1}{2}\right)^n \cdot e^{-\frac{3}{4}t}}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{-\frac{3}{4}t}} = \\ &= \frac{1}{2} \frac{\sqrt{\left(\frac{1}{2}\right)^{2n+2} (2^{2n+2} + 5 \cdot 2^n \cdot e^{-\frac{3}{4}t} + e^{-\frac{3}{2}t})}}{\left(\frac{1}{2}\right)^{n+1} (2^{n+1} + e^{-\frac{3}{4}t})} = \\ &= \frac{1}{2} \frac{\sqrt{2^{2n+2} + 5 \cdot 2^n \cdot e^{-\frac{3}{4}t} + e^{-\frac{3}{2}t}}}{2^{n+1} + e^{-\frac{3}{4}t}} \\ a_n &= \frac{1}{2} \frac{\sqrt{2^{2n+2} + 5 \cdot 2^n \cdot e^{-\frac{3}{4}t} + e^{-\frac{3}{2}t}}}{2^{n+1} + e^{-\frac{3}{4}t}} \end{aligned}$$

Huddi shunday b_n koeffitsientni

$$\begin{aligned} b_n &= \frac{1}{2} \left(\frac{K(n,n+1)}{K(n,n)} - \frac{K(n-1,n)}{K(n-1,n-1)} \right), \\ \chi(n,m) &= \frac{K(n,m)}{K(n,n)} = \frac{-\frac{1}{2} \left(\frac{\sqrt{2}}{2}\right)^{n+m} \cdot e^{-\frac{3}{4}t}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{-\frac{3}{4}t}} \end{aligned}$$

tenglikdan foydalanib topamiz.

$$\begin{aligned}
b_n &= \frac{1}{2} \left(\frac{-\left(\frac{1}{2}\right) \cdot \left(\frac{\sqrt{2}}{2}\right)^{2n+1} \cdot e^{-\frac{3}{4}t}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot e^{-\frac{3}{4}t}} + \frac{\left(\frac{1}{2}\right) \cdot \left(\frac{\sqrt{2}}{2}\right)^{2n-1} \cdot e^{-\frac{3}{4}t}}{1 + \left(\frac{1}{2}\right)^n \cdot e^{-\frac{3}{4}t}} \right) = \\
&= \frac{1}{2} \cdot \frac{-\frac{1}{2} \cdot \left(\frac{\sqrt{2}}{2}\right)^{2n+1} \cdot e^{-\frac{3}{4}t} + \frac{1}{2} \left(\frac{\sqrt{2}}{2}\right)^{2n-1} \cdot e^{-\frac{3}{4}t}}{1 + 3 \cdot \left(\frac{1}{2}\right)^{n+1} \cdot e^{-\frac{3}{4}t} + \left(\frac{1}{2}\right)^{2n+1} \cdot e^{-\frac{3}{2}t}} = \\
&= \frac{1}{2} \frac{\left(\frac{1}{2}\right)^n \left(-\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{2}\right) \cdot e^{-\frac{3}{4}t}}{\left(\frac{1}{2}\right)^{2n+1} \left(2^{2n+1} + 3 \cdot 2^n \cdot e^{-\frac{3}{4}t} + e^{-\frac{3}{2}t}\right)} = \\
&= \frac{2^n \cdot e^{-\frac{3}{4}t}}{2\sqrt{2} \left(2^{2n+1} + 3 \cdot 2^n \cdot e^{-\frac{3}{4}t} + e^{-\frac{3}{2}t}\right)} \\
b_n &= \frac{2^n \cdot e^{-\frac{3}{4}t}}{2\sqrt{2} \left(2^{2n+1} + 3 \cdot 2^n \cdot e^{-\frac{3}{4}t} + e^{-\frac{3}{2}t}\right)}
\end{aligned}$$

tenglik yordamida topiladi

Xulosa

Mazkur bobda diskret Shturm-Liuivill operatori uchun moslangan manbali ikkinchi Toda zanjirini sohilish nazariyasining teskari masalasi yordamida integrallash usuli keltirilgan. Undan quyidagi natijalar olindi:

1. Oddiy xos qiymatga mos keluvchi diskret Shturm-Liuivill operatorining potentsiali moslangan manbali ikkinchi Toda zanjirining yechimi bo'lganda spektral parametrlarining t bo'yicha o'zgarish qonuni o'rganildi;

2. Bitta xos qiymatga mos keluvchi diskret Shturm-Liuivill operatori yordamida sohilish nazariyasining teskari masalasini qo'llab moslangan manbali ikkinchi Toda zanjiri integrallandi.

III BOB.

3.1-§. Umumiy Toda zanjirini sochilish nazariyasini teskari masalasi usulida integrallash

Ushbu paragrafda

$$\begin{cases} \dot{a}_n = a_n (G_{n+1,r+1} - G_{n,r+1}), \\ \dot{b}_n = H_{n+1,r+1} - H_{n,r+1}, \quad n \in Z, \end{cases} \quad (3.1.1)$$

tenglamalar sistemasini qaraymiz. Bu yerda

$$G_{n,j}(t) = \sum_{s=0}^j c_{j-s} \langle \delta_n, L(t)^s \delta_n \rangle, \quad 0 \leq j \leq r+1,$$

$$H_{n,j}(t) = \sum_{s=0}^j 2a_n(t)c_{j-s} \langle \delta_{n+1}, L(t)^s \delta_n \rangle + c_j + 1, \quad 0 \leq j \leq r+1,$$

$$(L(t)y)_n \equiv a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n,$$

$$\langle \delta_m, \delta_n \rangle = \begin{cases} 0, & m \neq n \\ 1, & m = n, \end{cases}$$

c_1, c_2, \dots, c_{r+1} - oldindan berilgan ixtiyoriy haqiqiy sonlar va $\lambda = \frac{z+z^{-1}}{2}$.

$\{a_n(t)\}_{-\infty}^{\infty}, \{b_n(t)\}_{-\infty}^{\infty}$, - noma'lum funksiyalar.

(3.1.1) tenglamalar sistemasini

$$a_n(0) = a_{n0}, \quad b_n(0) = b_{n0}, \quad n \in Z, \quad (3.1.2)$$

boshlang'ich shartlar bilan qaraymiz. Bu yerda $\{a_{n0}\}_{-\infty}^{\infty}, \{b_{n0}\}_{-\infty}^{\infty}$ vektorlar quyidagi shartlarni qanoatlantiradi:

1. $a_{n0} > 0, \text{Im} b_{n0} = 0, \quad n \in Z,$

$$2. \sum_{n=-\infty}^{\infty} |n| \left(\left| a_{n0} - \frac{1}{2} \right| + |b_{n0}| \right) < \infty,$$

3. $L(0)$ operator $[-1;1]$ kesma tashkarisida yotuvchi N ta

$\lambda_1(0), \lambda_2(0), \dots, \lambda_N(0)$ xos qiymatlarga ega.

(3.1.1)-(3.1.2) masalaning yechimini

$$a_n(t) > 0, \operatorname{Im} b_n = 0, n \in \mathbb{Z}, \sum_{n=-\infty}^{\infty} |n| \left(\left| a_n(t) - \frac{1}{2} \right| + |b_n(t)| \right) < \infty. \quad (3.1.3)$$

shartni qanoatlantiruvchi funksiyalar sinfida qaraymiz.

Ushbu paragrafning asosiy maqsadi (3.1.1) masalaning $\{a_n(t)\}_{-\infty}^{\infty}, \{b_n(t)\}_{-\infty}^{\infty}, k=1,2,\dots,N$ yechimlariga $L(t)$ operator uchun sochilish nazariyasining teskari masalasi usulida tasvirlar olish.

(3.1.1) tenglama uchun Laks juftligini

$$a_{n-1}\psi_{n-1}(z) + b_n\psi_n(z) + a_n\psi_{n+1}(z) = \lambda\psi_n(z), \quad (3.1.4)$$

$$\frac{d\psi_n(z)}{dt} = 2a_n\tilde{G}_{n,r}(z,t)\psi_{n+1}(z) - \tilde{H}_{n,r+1}(z,t)\psi_n(z), \quad (3.1.5)$$

ko'rishida izlaymiz. Bunda,

$$\tilde{G}_{n,r}(z,t) = \sum_{j=0}^r \lambda^j G_{n,r-j}(t), \quad \tilde{H}_{n,r}(z,t) = \lambda^{r+1} + \sum_{j=0}^r \lambda^j H_{n,r-j}(t) - G_{n,r+1}(t), \quad r \in N_0, t \in R.$$

(3.1.1) tenglamalar sistemasi

$$\frac{dL}{dt} = [P_{2r+2}(t), L(t)], \quad (3.1.6)$$

operatorli tenglik yordamida yozish mumkin. Bu yerda $[P_{2r+2}, L] = P_{2r+2}L - LP_{2r+2}$

Puasson qavsi, $P_{2r+2} = 2a_n\tilde{G}_{n,r}(z,t)S^+ - \tilde{H}_{n,r+1}(z,t)$, $S^{\pm}f_n = f_{n\pm 1}$,

$$\tilde{G}_{n,r}(z,t) = \sum_{j=0}^r \lambda^j G_{n,r-j}(t), \quad \tilde{H}_{n,r}(z,t) = \lambda^{r+1} + \sum_{j=0}^r \lambda^j H_{n,r-j}(t) - G_{n,r+1}(t), \quad r \in N_0, t \in R.$$

Endi sochilish nazariyasining berilganlarining evalutsiyasini keltirib chiqaramiz. Buning uchun ushbu

$$L\psi_n = \lambda\psi_n \quad (3.1.7)$$

tenglikni t bo'yicha differensiallasak,

$$\frac{dL}{dt}\psi_n + L\frac{d\psi_n}{dt} = \lambda\frac{d\psi_n}{dt}$$

tenglik hosil bo'ladi. Buni (3.1.6) tenglik yordamida

$$[P_{2r+2}, L]\psi_n + L \frac{d\psi_n}{dt} = \lambda \frac{d\psi_n}{dt}$$

yoki

$$P_{2r+2} L \psi_n - L P_{2r+2} \psi_n + L \frac{d\psi_n}{dt} = \lambda \frac{d\psi_n}{dt}.$$

(3.1.7) tenglik yordamida oxirgi tenglikni quyidagicha yozamiz:

$$\lambda P_{2r+2} \psi_n - L P_{2r+2} \psi_n + L \frac{d\psi_n}{dt} = \lambda \frac{d\psi_n}{dt},$$

Bundan esa, ushbu

$$(L - \lambda) \left(\frac{d\psi_n}{dt} - P_{2r+2} \psi_n \right) = 0 \quad (3.1.8)$$

tenglikga ega bo‘lamiz. (3.1.8) tenglikdan $\frac{d\psi_n}{dt} - P_{2r+2} \psi_n$ funksiya ham (3.1.7)

tenglamaning yechimi bo‘lishi kelib chiqadi. Demak, bu yechimni haqiqiy λ larda $\psi_n(z, t)$, $\varphi_n(z, t)$ yechimlar sistemasi orqali chiziqli ifodalanadi, ya’ni shunday $\tilde{\alpha}$ va $\tilde{\beta}$ sonlar mavjudki

$$\frac{d\psi_n}{dt} - P_{2r+2} \psi_n = \tilde{\alpha}(z, t) \psi_n(z, t) + \tilde{\beta}(z, t) \varphi_n(z, t)$$

yoki,

$$\frac{d\psi_n}{dt} - 2a_n \tilde{G}_{n,r}(z, t) \psi_{n+1}(z) + \tilde{H}_{n,r+1}(z, t) \psi_n(z) = \tilde{\alpha}(z, t) \psi_n(z, t) + \tilde{\beta}(z, t) \varphi_n(z, t) \quad (3.1.9)$$

tenglik o‘rinli bo‘ladi. Endi biz bu tenglikning $n \rightarrow -\infty$ dagi asimptotikasini o‘rganamiz:

$$\frac{d\psi_n}{dt} \rightarrow 0, n \rightarrow -\infty, \text{ haqiqatdan,}$$

$$\lim_{n \rightarrow -\infty} \lim_{\Delta t \rightarrow 0} \frac{\psi_n(z, t + \Delta t) - \psi_n(z, t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \lim_{n \rightarrow -\infty} \frac{\psi_n(z, t + \Delta t) - \psi_n(z, t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{z^{-n} - z^{-n}}{\Delta t} = 0.$$

$$\tilde{G}_{n,r}(z,t) \rightarrow \tilde{g}_r(z), n \rightarrow -\infty, \tilde{H}_{n,r+1}(z,t) \rightarrow \tilde{h}_{r+1}(z), n \rightarrow -\infty.$$

Bulardan quyidagiga ega bo‘lamiz:

$$\frac{d\psi_n}{dt} - P_{2r+2}\psi_n \rightarrow [\tilde{h}_{r+1}(z) - z^{-1}\tilde{g}_r(z)]z^{-n}$$

Bundan esa $n \rightarrow -\infty$ da

$$\alpha_n \rightarrow \tilde{h}_{r+1}(z) - z^{-1}\tilde{g}_r(z), \beta_n \rightarrow 0 \quad (3.1.10)$$

bo‘lishi kelib chiqadi. Demak, (3.1.9) tenglikni

$$\frac{d\psi_n}{dt} = 2a_n \tilde{G}_{n,r}(z,t)\psi_{n+1}(z) - \tilde{H}_{n,r+1}(z,t)\psi_n(z) + (\tilde{h}_{r+1}(z) - z^{-1}\tilde{g}_r(z))\psi_n \quad (3.1.11)$$

ko‘rinishida yozish mumkin.

Endi sochilish nazariyasining berilganlarining evalutsiyasini topish bilan shug‘ullanamiz.

(3.1.11) tenglikda (1.1.6) ni qo‘llab $n \rightarrow \infty$ da limitga o‘tsak,

$$\begin{aligned} \frac{d\alpha}{dt} \cdot z^{-n} + \frac{d\beta}{dt} \cdot z^n &= [z^{-1}\tilde{g}_r(z) - \tilde{h}_{r+1}(z)]\alpha(z)z^{-n} + [z\tilde{g}_r(z) - \tilde{h}_{r+1}(z)]\beta(z)z^n + \\ &+ [\tilde{h}_{r+1}(z) - z^{-1}\tilde{g}_r(z)](\alpha(z)z^{-n} + \beta(z)z^n) = (z - z^{-1})\tilde{g}_r(z)\beta(z)z^n. \end{aligned}$$

tenglik hosil bo‘ladi. z^{-n} va z^n oldidagi koeffitsientlarni tenglab

$$\frac{d\alpha(z,t)}{dt} = 0, \quad \frac{d\beta(z,t)}{dt} = (z - z^{-1})\tilde{g}_r(z)\beta(z,t) \quad (3.1.12)$$

bo‘lishini topamiz. (3.1.12) ning birinchi tenglamasidan $L(t)$ operatorning xos qiymatlari t ga bog‘liq emasligi, ya’ni

$$\frac{dz_k}{dt} = 0, \quad k = 1, 2, \dots, N.$$

bo‘lishi kelib chiqadi. Bundan tashqari (3.1.12) dan sochilish funksiyasining

$$\frac{dR(z,t)}{dt} = (z - z^{-1})\tilde{g}_r(z)R(z,t), \quad |z|=1 \quad (3.1.13)$$

t bo‘yicha evolutsiyasini topamiz.

Endi normallovchi konstantalarning evolutsiyalarini topish bilan

shugʻullanamiz. Xuddi uzluksiz spektrdagi kabi diskret spektr holdida ham

$$\frac{d\psi_n^k}{dt} = 2a_n \tilde{G}_{n,r}(z,t)\psi_{n+1}^k - \tilde{H}_{n,r+1}(z,t)\psi_n^k + [\tilde{h}_{r+1}(z) - z^{-1}\tilde{g}_r(z)]\psi_n^k. \quad (3.1.14)$$

tenglikni yozish mumkin. (1.8) yaʼni $\psi_n^k = \beta_k \varphi_n^k$, $k = 1, 2, \dots, N$, tenglikni inobatga olib, (3.1.14) tenglikda $n \rightarrow \infty$ da limitga oʻtamiz

$$\frac{d\beta_k}{dt} z_k^n = \tilde{g}_r(z_k)\beta_k z_k^{n+1} - \tilde{h}_{r+1}(z_k)\beta_k z_k^n + [\tilde{h}_{r+1}(z_k) - z_k^{-1}\tilde{g}_r(z_k)]\beta_k z_k^n$$

boʻlishni topamiz, bundan esa quyidagiga ega boʻlamiz:

$$\frac{d\beta_k}{dt} = (z_k - z_k^{-1})\tilde{g}_r(z_k)\beta_k, \quad k = 1, 2, \dots, N. \quad (3.1.15)$$

boʻlishini topamiz. (1.1.9) va (3.1.15) tenglikni inobatga olib

$$\frac{dC_k^2}{dt} = (z_k - z_k^{-1})\tilde{g}_r(z_k)C_k^2, \quad k = 1, 2, \dots, N.$$

boʻlishini topamiz.

Demak, biz quyidagi teoremani isbot qildik.

Teorema 3.1.1. Agar $a_n(t)$, $b_n(t)$, $n \in Z$ funksiyalar (3.1.1)-(3.1.3)

masalaning yechimlari boʻlsa, u holda

$$L(t)y \equiv a_{n-1}(t)y_{n-1} + b_n(t)y_n + a_n(t)y_{n+1}$$

operatorning sochilish nazariyasining berilganlari t boʻyicha quyidagicha oʻzgaradi:

$$R(z,t) = R(z,0) \exp\left\{\int_0^t (z - z^{-1})\tilde{g}_r(z)dt\right\}, \quad |z|=1,$$

$$z_k(t) = z_k(0), \quad C_k^2(t) = C_k^2(0) \exp\{(z_k - z_k^{-1})\tilde{g}_r(z_k)t\}, \quad k = 1, 2, \dots, N.$$

3.2-§. Moslangan manbali Umumiy Toda zanjirini integrallash

Ushbu paragrafda

$$\begin{cases} \dot{a}_n = a_n(G_{n+1,r+1} - G_{n,r+1}) + a_n \sum_{i=1}^N ((g_n^i)^2 - (g_{n+1}^i)^2), \\ \dot{b}_n = H_{n+1,r+1} - H_{n,r+1} - 2 \sum_{i=1}^N g_n^i (a_n g_{n+1}^i - a_{n-1} g_{n-1}^i), \\ a_{n-1} g_{n-1}^k + b_n g_n^k + a_n g_{n+1}^k = \lambda_k g_n^k, \quad k = 1, 2, \dots, N, \quad n \in Z, \end{cases} \quad (3.2.1)$$

tenglamalar sistemasini qaraymiz. Bu yerda

$$G_{n,j}(t) = \sum_{s=0}^j c_{j-s} \langle \delta_n, L(t)^s \delta_n \rangle, \quad 0 \leq j \leq r+1,$$

$$H_{n,j}(t) = \sum_{s=0}^j 2a_n(t) c_{j-s} \langle \delta_{n+1}, L(t)^s \delta_n \rangle + c_j + 1, \quad 0 \leq j \leq r+1,$$

$$(L(t)y)_n \equiv a_{n-1} y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n,$$

$$\langle \delta_m, \delta_n \rangle = \begin{cases} 0, & m \neq n \\ 1, & m = n, \end{cases}$$

c_1, c_2, \dots, c_{r+1} - oldindan berilgan ixtiyoriy xaqiqiy sonlar.

$\{a_n(t)\}_{-\infty}^{\infty}, \{b_n(t)\}_{-\infty}^{\infty}, \{g_n^k(t)\}_{-\infty}^{\infty}, k = 1, 2, \dots, N$ - noma'lum funksiyalar, $\{g_n^k(t)\}_{-\infty}^{\infty}$

funksiyalar $L(t)$ operatorning $\lambda_k = \frac{z_k + z_k^{-1}}{2}, k = 1, 2, \dots, N$ xos qiymatga mos

keluvchi ushbu

$$\sum_{n=-\infty}^{\infty} (g_n^k(t))^2 = A_k(t), \quad k = 1, 2, \dots, N, \quad (3.2.2)$$

shartni qanoatlantiruvchi xos funksiyasi, $A_k(t)$ oldindan berilgan musbat funksiya.

(3.2.1) tenglamalar sistemasini

$$a_n(0) = a_{n_0}, \quad b_n(0) = b_{n_0}, \quad n \in Z, \quad (3.2.3)$$

boshlang'ich shartlar bilan qaraymiz. Bu yerda $\{a_{n_0}\}_{-\infty}^{\infty}, \{b_{n_0}\}_{-\infty}^{\infty}$ vektorlar quyidagi

shartlarni qanoatlantiradi:

$$1. a_{n0} > 0, \text{Im}b_{n0} = 0, n \in Z,$$

$$2. \sum_{n=-\infty}^{\infty} |n| \left(\left| a_{n0} - \frac{1}{2} \right| + |b_{n0}| \right) < \infty,$$

3. $L(0)$ operator $[-1;1]$ kesma tashkarisida yotuvchi N ta $\lambda_1(0), \lambda_2(0), \dots, \lambda_N(0)$ xos qiymatlarga ega.

(3.2.1)-(3.2.3) masalaning yechimini

$$a_n(t) > 0, \text{Im}b_n = 0, n \in Z, \sum_{n=-\infty}^{\infty} |n| \left(\left| a_n(t) - \frac{1}{2} \right| + |b_n(t)| \right) < \infty. \quad (3.2.4)$$

shartni qanoatlantiruvchi funksiyalar sinfida qaraymiz.

Ushbu paragrafning asosiy maqsadi (3.2.1) masalaning $\{a_n(t)\}_{-\infty}^{\infty}, \{b_n(t)\}_{-\infty}^{\infty}, \{g_n^k(t)\}_{-\infty}^{\infty}, k=1,2,\dots,N$ yechimlariga $L(t)$ operator uchun sochilish nazariyasining teskari masalasi usulida tasvirlar olish.

(3.2.1) tenglama uchun Laks juftligini

$$a_{n-1}\psi_{n-1}(z) + b_n\psi_n(z) + a_n\psi_{n+1}(z) = \lambda\psi_n(z), \quad (3.2.5)$$

$$\frac{d\psi_n(z)}{dt} = 2a_n\tilde{G}_{n,r}(z,t)\psi_{n+1}(z) - \tilde{H}_{n,r+1}(z,t)\psi_n(z) + F_n, \quad (2.6)$$

ko'rinishida izlaymiz. Bunda,

$$\tilde{G}_{n,r}(z,t) = \sum_{j=0}^r \lambda^j G_{n,r-j}(t), \quad \tilde{H}_{n,r}(z,t) = \lambda^{r+1} + \sum_{j=0}^r \lambda^j H_{n,r-j}(t) - G_{n,r+1}(t), \quad r \in N_0, t \in R.$$

$\tilde{G}_{n,r}(z,t)$ va $\tilde{H}_{n,r}(z,t)$ lar yordamida (3.2.1) tenglamani quyidagicha yozish mumkin:

$$\begin{cases} \dot{a}_n = a_n[\tilde{H}_{n+1,r+1} + \tilde{H}_{n,r+1} - 2(\lambda - b_{n+1})G_{n+1,r}] + a_n \sum_{i=1}^N ((g_n^i)^2 - (g_{n+1}^i)^2), \\ \dot{b}_n = 2[a_n^2\tilde{G}_{n+1,r} - a_{n-1}^2\tilde{G}_{n-1,r} + (\lambda - b_n)^2\tilde{G}_{n,r} - (\lambda - b_n)\tilde{H}_{n,r+1}] - \\ \quad - 2\sum_{i=1}^N g_n^i(a_n g_{n+1}^i - a_{n-1} g_{n-1}^i), \\ a_{n-1}g_{n-1}^k + b_n g_n^k + a_n g_{n+1}^k = \lambda_k g_n^k, \quad k=1, 2, \dots, N, \quad n \in Z. \end{cases}$$

(3.2.5) tenglikni t bo'yicha differensiallab va (3.2.1), (3.2.6) tengliklardan foydalanamiz, ya'ni

$$\dot{a}_{n-1}\psi_{n-1}(z) + a_{n-1}\dot{\psi}_{n-1}(z) + \dot{b}_n\psi_n(z) + b_n\dot{\psi}_n(z) + \dot{a}_n\psi_{n+1}(z) + a_n\dot{\psi}_{n+1}(z) = \lambda\dot{\psi}_n(z)$$

Bundan quyidagi tenglik hosil bo'ladi.

$$\begin{aligned} & a_{n-1}(\tilde{H}_{n,r+1} + \tilde{H}_{n-1,r+1} - 2(\lambda - b_n)G_{n,r})\psi_n + a_{n-1}(2a_{n-1}\tilde{G}_{n-1,r}\psi_n - \tilde{H}_{n-1,r+1}\psi_{n-1}) + \\ & + 2(a_n^2\tilde{G}_{n+1,r} - a_{n-1}^2\tilde{G}_{n-1,r} + (\lambda - b_n)^2\tilde{G}_{n,r} - (\lambda - b_n)\tilde{H}_{n,r+1})\psi_n + b_n(2a_n\tilde{G}_{n,r}\psi_{n+1} - \tilde{H}_{n,r+1}\psi_n) + \\ & + a_{n-1}(\tilde{H}_{n+1,r+1} + \tilde{H}_{n,r+1} - 2(\lambda - b_{n+1})G_{n+1,r})\psi_{n+1} + a_n(2a_{n+1}\tilde{G}_{n+1,r}\psi_{n+2} - \tilde{H}_{n+1,r+1}\psi_{n+1}) = \lambda\dot{\psi}_n \end{aligned}$$

Qavslarni ochib hisoblashlarni bajarsak,

$$\begin{aligned} & a_{n-1}\tilde{H}_{n,r+1}\psi_{n-1} - 2(\lambda - b_n)\tilde{G}_{n,r}\psi_{n-1} + 2a_n^2\tilde{G}_{n+1,r}\psi_n + 2(\lambda - b_n)\tilde{G}_{n,r}\psi_n - \lambda H_{n,r+1}\psi_n + 2a_nb_n\tilde{G}_{n,r}\psi_{n+1} + \\ & + a_n\tilde{H}_{n,r+1}\psi_{n+1} - 2a_n(\lambda - b_{n+1})\tilde{G}_{n+1,r}\psi_{n+1} + 2(\lambda\psi_{n+1} - b_{n+1}\psi_{n+1} - a_n\psi_{n+1})\tilde{G}_{n+1,r} = \\ & = -\lambda H_{n,r+1}\psi_n - 2a_{n-1}(\lambda - b_n)\tilde{G}_{n,r}\psi_{n-1} + 2a_{n-1}(\lambda - b_n)\tilde{G}_{n,r}\psi_{n-1} + 2a_nb_n\tilde{G}_{n,r}\psi_{n+1} + \\ & + 2a_n(\lambda - b_n)\tilde{G}_{n,r}\psi_{n+1} = -\lambda H_{n,r+1}\psi_n - 2\lambda\tilde{G}_{n,r}a_{n-1}\psi_{n-1} + 2b_n\tilde{G}_{n,r}a_{n-1}\psi_{n-1} + \\ & + 2\lambda\tilde{G}_{n,r}a_{n-1}\psi_{n-1} - 2b_n\tilde{G}_{n,r}a_{n-1}\psi_{n-1} + 2\lambda\tilde{G}_{n,r}a_n\psi_{n+1} = \lambda(2a_n\tilde{G}_{n,r}\psi_{n+1} - \tilde{H}_{n,r+1}\psi_n) \end{aligned}$$

bo'ladi. Bundan esa

$$\begin{aligned} & a_{n-1}F_{n-1} + b_nF_n + a_nF_{n+1} - \lambda F_n = a_{n-1}\sum_{i=1}^N((g_n^i)^2 - (g_{n-1}^i)^2)\psi_{n-1} + \\ & + 2\sum_{i=1}^N g_n^i(a_n g_{n+1}^i - a_{n-1}g_{n-1}^i)\psi_n + a_n\sum_{i=1}^N((g_{n+1}^i)^2 - (g_n^i)^2)\psi_{n+1}. \end{aligned} \quad (3.2.7)$$

bo'lishini topamiz. (3.2.7) tenglamaning yechimini

$$F_n = \alpha_n\psi_n(z) + \beta_n\varphi_n(z) \quad (3.2.8)$$

ko'rinishda izlaymiz. (3.2.6) tenglikka asosan, $n \rightarrow -\infty$ da

$$F_n \rightarrow [\tilde{h}_{r+1}(z, 0) - z^{-1}\tilde{g}_r(z, 0)]z^{-n}$$

bo'ladi. (3.2.8) dan $n \rightarrow -\infty$ da

$$\alpha_n \rightarrow \tilde{h}_{r+1}(z) - z^{-1}\tilde{g}_r(z), \quad \beta_n \rightarrow 0 \quad (3.2.9)$$

bo'lishi kelib chiqadi. (3.2.8) ni (3.2.7) ga qo'yib va (3.2.5) tenglikdan foydalanib

$$\begin{aligned}
& \alpha_n(\alpha_{n+1} - \alpha_n)\psi_{n+1} + a_n(\beta_{n+1} - \beta_n)\varphi_{n+1} - a_{n-1}(\alpha_n - \alpha_{n-1})\psi_{n-1} - a_{n-1}(\beta_n - \beta_{n-1})\varphi_{n-1} = \\
& = a_{n-1} \sum_{i=1}^N ((g_n^i)^2 - (g_{n-1}^i)^2)\psi_{n-1} + 2 \sum_{i=1}^N g_n^i (a_n g_{n+1}^i - a_{n-1} g_{n-1}^i)\psi_n + \\
& \quad + a_n \sum_{i=1}^N ((g_{n+1}^i)^2 - (g_n^i)^2)\psi_{n+1}. \tag{3.2.10}
\end{aligned}$$

bo‘lishini topamiz.

Lemma 3.2.1. Ushbu

$$\begin{aligned}
\alpha_{n+1} - \alpha_n &= \frac{1}{W\{\varphi_n, \psi_n\}} \sum_{i=1}^N ((\psi_n g_n^i + \psi_{n+1} g_{n+1}^i)W\{\varphi_n, g_n^i\} + (\varphi_n g_n^i + \varphi_{n+1} g_{n+1}^i)W\{\psi_n, g_n^i\}) \\
\beta_{n+1} - \beta_n &= -\frac{2}{W\{\varphi_n, \psi_n\}} \sum_{i=1}^N (\psi_n g_n^i + \psi_{n+1} g_{n+1}^i)W\{\psi_n, g_n^i\}.
\end{aligned}$$

tengliklar bilan aniqlanuvchi α_n va β_n funksiyalar (3.2.10) tenglamaning yechimi bo‘ladi.

Ushbu $\alpha(z) = \frac{2}{z - z^{-1}} W\{\psi_n(z), \varphi_n(z)\}$ tenglik hamda (3.2.9) va lemma

3.2.1 dan foydalanib

$$\begin{aligned}
\alpha_n &= -\frac{2}{(z + z^{-1})\alpha(z)} \cdot \left[\sum_{j=-\infty}^{n-1} \sum_{i=1}^N (\psi_j g_j^i + \psi_{j+1} g_{j+1}^i)W\{\varphi_j, g_j^i\} + \right. \\
& \quad \left. + \sum_{j=-\infty}^{n-1} \sum_{i=1}^N (\varphi_j g_j^i + \varphi_{j+1} g_{j+1}^i)W\{\psi_j, g_j^i\} \right] + \tilde{h}_r(z) - z^{-1} \tilde{g}_r(z), \\
\beta_n &= \frac{4}{(z + z^{-1})\alpha(z)} \cdot \left[\sum_{j=-\infty}^{n-1} \sum_{i=1}^N (\psi_j g_j^i + \psi_{j+1} g_{j+1}^i)W\{\psi_j, g_j^i\} \right].
\end{aligned}$$

bo‘lishini topamiz. (3.2.8) tenglikga topilgan α_n va β_n larni qo‘ysak u holda

(3.2.6) tenglik quyidagi ko‘rinishga keladi:

$$\begin{aligned}
\frac{d\psi_n(z)}{dt} &= 2a_n \tilde{G}_{n,r}(z,t)\psi_{n+1}(z) - \tilde{H}_{n,r+1}(z,t)\psi_n(z) + \\
&+ [\tilde{h}_{r+1}(z) - z^{-1}\tilde{g}_r(z)]\psi_n - \frac{2}{(z+z^{-1})\alpha(z)} \times \\
&\times \sum_{j=-\infty}^{n-1} \sum_{i=1}^N [(\psi_j g_j^i + \psi_{j+1} g_{j+1}^i)W\{\varphi_j, g_j^i\} + (\varphi_j g_j^i + \varphi_{j+1} g_{j+1}^i)W\{\psi_j, g_j^i\}]\psi_n + \\
&+ \frac{4}{(z+z^{-1})\alpha(z)} \cdot \sum_{j=-\infty}^{n-1} \sum_{i=1}^N (\psi_j g_j^i + \psi_{j+1} g_{j+1}^i)W\{\psi_j, g_j^i\} \varphi_n. \quad (3.2.11)
\end{aligned}$$

Lemma 3.2.2. Quyidagi

$$\begin{aligned}
\sum_{j=-\infty}^{\infty} [(\psi_j g_j^i + \psi_{j+1} g_{j+1}^i)W\{\varphi_j, g_j^i\} + (\varphi_j g_j^i + \varphi_{j+1} g_{j+1}^i)W\{\psi_j, g_j^i\}] &= 0, \\
\sum_{j=-\infty}^{\infty} (\psi_j g_j^i + \psi_{j+1} g_{j+1}^i)W\{\psi_j, g_j^i\} &= 0, \quad i = 1, 2, \dots, N.
\end{aligned}$$

tengliklar o‘rinli.

Isbot. Qulaylik uchun $W_j = W\{\psi_j, g_j^i\}$ va $V_j = W\{\varphi_j, g_j^i\}$

(i fiksirlangan deb hisoblaymiz) belgilashlarni kiritamiz. Lemmaga asosan

$$\begin{aligned}
\sum_{j=-\infty}^{\infty} [(\psi_j g_j^i + \psi_{j+1} g_{j+1}^i)V_j + (\varphi_j g_j^i + \varphi_{j+1} g_{j+1}^i)W_j] &= \\
= \sum_{j=-\infty}^{\infty} [(\psi_j g_j^i(V_j + V_{j-1}) + \varphi_j g_j^i(W_j + W_{j-1}))] &= \\
= \frac{1}{\lambda_i - \lambda} \sum_{j=-\infty}^{\infty} [(W_j - W_{j-1})(V_j + V_{j-1}) + (V_j - V_{j-1})(W_j + W_{j-1})] &= \\
= \frac{2}{\lambda_i - \lambda} \sum_{j=-\infty}^{\infty} [W_j V_j - W_{j-1} V_{j-1}] &= 0.
\end{aligned}$$

bo‘ladi. Bundan

$$\sum_{j=-\infty}^{\infty} (\psi_j g_j^i + \psi_{j+1} g_{j+1}^i) W_j = \frac{1}{\lambda_i - \lambda} \sum_{j=-\infty}^{\infty} (W_j^2 - W_{j-1}^2) = 0$$

bo'lishi kelib chiqadi. *Lemma isbot bo'ldi.*

(3.2.11) tenglikda $\psi_n(z) = \alpha(z)\varphi_n(z^{-1}) + \beta(z)\varphi_n(z)$ va lemma 3.2.2 ni qo'llab $n \rightarrow \infty$ da limitga o'tsak,

$$\begin{aligned} \frac{d\alpha}{dt} \cdot z^{-n} + \frac{d\beta}{dt} \cdot z^n &= [z^{-1}\tilde{g}_r(z) - \tilde{h}_{r+1}(z)]\alpha(z)z^{-n} + [z\tilde{g}_r(z) - \tilde{h}_{r+1}(z)]\beta(z)z^n + \\ &+ [\tilde{h}_{r+1}(z) - z^{-1}\tilde{g}_r(z)](\alpha(z)z^{-n} + \beta(z)z^n) = (z - z^{-1})\tilde{g}_r(z)\beta(z)z^n. \end{aligned}$$

tenglik hosil bo'ladi. z^{-n} va z^n oldidagi koeffitsiyentlarni tenglab

$$\frac{d\alpha(z,t)}{dt} = 0, \quad \frac{d\beta(z,t)}{dt} = (z - z^{-1})\tilde{g}_r(z)\beta(z,t) \quad (3.2.12)$$

bo'lishini topamiz. (3.2.12) ning birinchi tenglamasidan $L(t)$ operatorning xos qiymatlari t ga bog'lik emasligi, ya'ni

$$\frac{dz_k}{dt} = 0, \quad k = 1, 2, \dots, N.$$

bo'lishi kelib chiqadi. Bundan tashqari (3.2.12) dan sochilish funksiyasining

$$\frac{dR(z,t)}{dt} = (z - z^{-1})\tilde{g}_r(z)R(z,t), \quad |z| = 1$$

t bo'yicha evolutsiyasini topamiz.

Endi normallovchi konstantalarning evolutsiyalarini topish bilan shug'ullanamiz. Xuddi uzluksiz spektrdagi kabi diskret spektr holida ham

$$\begin{aligned} \frac{d\psi_n^k}{dt} &= 2a_n \tilde{G}_{n,r}(z,t)\psi_{n+1}^k - \tilde{H}_{n,r+1}(z,t)\psi_n^k + \\ &+ [\tilde{h}_{r+1}(z) - z^{-1}\tilde{g}_r(z)]\psi_n^k + \frac{1}{W\{h_n^k, \psi_n^k\}} \times \end{aligned}$$

$$\times \sum_{j=-\infty}^{n-1} \sum_{i=1}^N [(\psi_j^k g_j^i + \psi_{j+1}^k g_{j+1}^i)W\{h_j^k, g_j^i\} + (h_j^k g_j^i + h_{j+1}^k g_{j+1}^i)W\{\psi_j^k, g_j^i\}]\psi_n^k -$$

$$-\frac{2}{W\{h_n^k, \psi_n^k\}} \cdot \sum_{j=-\infty}^{n-1} \sum_{i=1}^N (\psi_j^k g_j^i + \psi_{j+1}^k g_{j+1}^i) W\{\psi_j^k, g_j^i\} h_n^k, \quad k=1,2,\dots,N. \quad (3.2.13)$$

tenglikni yozish mumkin. Quyidagi

$$\sum_{j=-\infty}^{\infty} [(\psi_j^k g_j^i + \psi_{j+1}^k g_{j+1}^i) W\{h_j^k, g_j^i\} + (h_j^k g_j^i + h_{j+1}^k g_{j+1}^i) W\{\psi_j^k, g_j^i\}] = 0, \quad k \neq i, \quad (3.2.14)$$

$$\sum_{j=-\infty}^{\infty} (\psi_j^k g_j^i + \psi_{j+1}^k g_{j+1}^i) W\{\psi_j^k, g_j^i\} = 0, \quad k \neq i. \quad (3.2.14)$$

tengliklar o‘rinli bo‘lishini oson ko‘rsatish mumkin.

$W\{\psi_j^k, g_j^k\} = 0$ bo‘lgani uchun $k = i$ da

$$\sum_{j=-\infty}^{\infty} (\psi_j^k g_j^i + \psi_{j+1}^k g_{j+1}^i) W\{\psi_j^k, g_j^i\} = 0 \quad . \quad (3.2.15)$$

bo‘ladi. Bundan tashqari, $g_j^k = b_k \cdot \psi_j^k$ tenglikdan

$$\sum_{j=-\infty}^{\infty} [(\psi_j^k g_j^k + \psi_{j+1}^k g_{j+1}^k) W\{h_j^k, g_j^k\} + (h_j^k g_j^k + h_{j+1}^k g_{j+1}^k) W\{\psi_j^k, g_j^k\}] =$$

$$= 2W\{h_n^k, \psi_n^k\} \sum_{j=-\infty}^{\infty} (g_j^k)^2. \quad (3.2.16)$$

bo‘lishini topamiz. $\psi_n^k = \beta_k \varphi_n^k$, $k=1,2,\dots,N$, tenglikni inobatga olib, (3.2.13)

tenglikda $n \rightarrow \infty$ da limitga o‘tamiz va (3.2.2), (3.2.14), (3.2.15), (3.2.16)

tengliklar yordamida

$$\frac{d\beta_k}{dt} z_k^n = \tilde{g}_r(z_k) \beta_k z_k^{n+1} - \tilde{h}_{r+1}(z_k) \beta_k z_k^n +$$

$$+ [\tilde{h}_{r+1}(z_k) - z_k^{-1} \tilde{g}_r(z_k)] \beta_k z_k^n + \frac{1}{W\{h_n^k, \psi_n^k\}} 2W\{h_n^k, \psi_n^k\} \sum_{j=-\infty}^{\infty} (g_j^k)^2 \beta_k z_k^n$$

bo‘lishini topamiz, bundan esa quyidagiga ega bo‘lamiz:

$$\frac{d\beta_k}{dt} = [(z_K - z_k^{-1}) \tilde{g}_r(z_k) + 2A_k(t)] \beta_k, \quad k=1,2,\dots,N. \quad (3.2.17)$$

bo‘lishini topamiz. (3.2.9) va (3.2.17) tenglikni inobatga olib

$$\frac{dC_k^2}{dt} = [(z_k - z_k^{-1})\tilde{g}_r(z_k) + 2A_k(t)]C_k^2, \quad k = 1, 2, \dots, N$$

bo‘lishini topamiz.

Demak, biz quyidagi teoremani isbot qildik.

Teorema 3.2.1. Agar $a_n(t)$, $b_n(t)$, $g_n^k(t)$, $k = 1, 2, \dots, N$, $n \in Z$ funksiyalar (3.2.1)-(3.2.4) masalaning yechimlari bo‘lsa, u holda

$$L(t)y \equiv a_{n-1}(t)y_{n-1} + b_n(t)y_n + a_n(t)y_{n+1}$$

operatorning sochilish nazariyasining berilganlari t bo‘yicha quyidagicha o‘zgaradi:

$$R(z, t) = R(z, 0) \exp \left\{ \int_0^t (z - z^{-1}) \tilde{g}_r(z) dt \right\}, \quad |z| = 1,$$

$$z_k(t) = z_k(0), \quad C_k^2(t) = C_k^2(0) \exp \left\{ (z_k - z_k^{-1}) \tilde{g}_r(z_k) t + 2 \int_0^t A_k(\tau) d\tau \right\}, \quad k = 1, 2, \dots, N.$$

3.3-§. Misollar.

Misol. a_{n0} va b_{n0} lar

$$a_{n0} = \frac{1}{2} \frac{\sqrt{2^{2n+2} + 5 \cdot 2^n + 1}}{1 + 2^{n+1}},$$

$$b_{n0} = \frac{2^n}{\sqrt{2}(2^{2n+1} + 3 \cdot 2^n + 1)}, \quad n \in \mathbb{Z}.$$

bo'lsin. Bu holarda I bobdagi misolga ko'ra

$$R(z,0) = 0, \quad z_1(0) = \frac{\sqrt{2}}{2}, \quad C_1(0) = \frac{\sqrt{2}}{2},$$

bo'ladi. Bundan teoremaga ko'ra

$$R(z,t) = 0, \quad z_1(t) = \frac{\sqrt{2}}{2}, \quad C_1(t) = \frac{\sqrt{2}}{2} \exp\{\gamma(t)\}, \quad \text{bu yerda } \gamma(t) = \frac{\sqrt{2}}{4}t + \int_0^t A_1(\tau) d\tau.$$

bo'lishi kelib chiqadi.

Yechish. Bu holda

$$F(n) = C_1^2(t) \cdot z_1^n(t) = \left(\frac{\sqrt{2}}{2} e^\gamma\right)^2 \cdot \left(\frac{\sqrt{2}}{2}\right)^n = \exp\{2\gamma(t)\} \cdot \left(\frac{\sqrt{2}}{2}\right)^{n+2}$$

bo'ladi. Bundan Gelfand-Levitan-Marchenko integral tenglamasining birinchi tenglamasiga ko'ra

$$\chi(n,m) + \exp\{2\gamma(t)\} \cdot \left(\frac{\sqrt{2}}{2}\right)^{n+m+2} + \sum_{n'=n+1}^{\infty} \chi(n,n') \cdot \exp\{2\gamma(t)\} \cdot \left(\frac{\sqrt{2}}{2}\right)^{n'+m+2} = 0, \quad m > n$$

$$\chi(n,m) + \exp\{2\gamma(t)\} \cdot \left(\frac{\sqrt{2}}{2}\right)^{m+2} \left(\left(\frac{\sqrt{2}}{2}\right)^n + \sum_{n'=n+1}^{\infty} \chi(n,n') \cdot \left(\frac{\sqrt{2}}{2}\right)^{n'}\right) = 0$$

bo'ladi. Quyidagicha

$$G_n = \left(\frac{\sqrt{2}}{2}\right)^n + \sum_{n'=n+1}^{\infty} \chi(n,n') \cdot \left(\frac{\sqrt{2}}{2}\right)^{n'}$$

Belgilash kiritib, yuqoridagi tenglikka qo'yamiz.

Ushbu

$$\chi(n, m) = -\exp\{2\gamma(t)\} \cdot \left(\frac{\sqrt{2}}{2}\right)^{m+2} G_n$$

tenglikni ikkinchi tenglikka qo'yib,

$$G_n = \left(\frac{\sqrt{2}}{2}\right)^n - \sum_{n'=n+1}^{\infty} \left(\frac{\sqrt{2}}{2}\right)^{n'+2} \cdot \exp\{2\gamma(t)\} \cdot \left(\frac{\sqrt{2}}{2}\right)^{n'} G_n$$

$$G_n (1 + \exp\{2\gamma(t)\}) \cdot \sum_{n'=n+1}^{\infty} \left(\frac{\sqrt{2}}{2}\right)^{n'+2} \cdot \left(\frac{\sqrt{2}}{2}\right)^{n'} G_n = \left(\frac{\sqrt{2}}{2}\right)^n$$

$$G_n \left(1 + \frac{1}{2} \cdot \exp\{2\gamma(t)\}\right) \cdot \sum_{n'=n+1}^{\infty} \left(\frac{\sqrt{2}}{2}\right)^{2n'} = \left(\frac{\sqrt{2}}{2}\right)^n$$

$$\sum_{n'=n+1}^{\infty} \left(\frac{1}{2}\right)^{n'} = \left(\frac{1}{2}\right)^n$$

Tenglik yordamida

$$G_n = \frac{\left(\frac{\sqrt{2}}{2}\right)^n}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}}$$

bo'lishini topamiz. Bundan

$$\chi(n, m) = \frac{K(n, m)}{K(n, n)} = \frac{-\left(\frac{\sqrt{2}}{2}\right)^{n+m+2} \cdot \exp\{2\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}}$$

tenglik hosil bo'ladi.

Endi Gelfand-Levitan-Marchenko integral tenglamasining ikkinchi tenglamasidan

$$\begin{aligned}
(K(n,n))^{-2} &= 1 + \exp\{2\gamma(t)\} \cdot \left(\frac{\sqrt{2}}{2}\right)^{2n+2} + \\
&\quad \sum_{n'=n+1}^{\infty} \frac{-\left(\frac{\sqrt{2}}{2}\right)^{n+n'+2} \cdot \exp\{2\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}} \cdot \left(\frac{\sqrt{2}}{2}\right)^{n'+n+2} \cdot \exp\{2\gamma(t)\} = \\
&= 1 + \exp\{2\gamma(t)\} \cdot \left(\frac{\sqrt{2}}{2}\right)^{2n+2} - \frac{\left(\frac{1}{2}\right)^{n+2} \cdot \exp\{4\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}} \sum_{n'=n+1}^{\infty} \left(\frac{1}{2}\right)^{n'} = \\
&= 1 + \exp\{2\gamma(t)\} \cdot \left(\frac{\sqrt{2}}{2}\right)^{2n+2} - \frac{1 + \left(\frac{1}{2}\right)^{2n+2} \cdot \exp\{4\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}} = \frac{1 + \left(\frac{1}{2}\right)^n \cdot \exp\{2\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}} \\
\frac{1}{(K(n,n))^2} &= \frac{1 + \left(\frac{1}{2}\right)^n \cdot \exp\{2\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}}
\end{aligned}$$

$K(n,n)$ koeffitsient quyidagi

$$K(n,n) = \sqrt{\frac{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^n \cdot \exp\{2\gamma(t)\}}}$$

tenglik yordamida topiladi.

Ushbu

$$a_n = \frac{K(n+1, n+1)}{K(n,n)}$$

tenglikdan

$$\begin{aligned}
a_n(t) &= \frac{1}{2} \sqrt{\frac{1 + \left(\frac{1}{2}\right)^{n+2} \cdot \exp\{2\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}} \cdot \frac{1 + \left(\frac{1}{2}\right)^n \cdot \exp\{2\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}}} = \\
&= \frac{1}{2} \sqrt{\frac{1 + \left(\frac{1}{2}\right)^n \cdot \exp\{2\gamma(t)\} + \left(\frac{1}{2}\right)^{n+2} \cdot \exp\{2\gamma(t)\} + \left(\frac{1}{2}\right)^{2n+2} \cdot \exp\{4\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}}} = \\
&= \frac{1}{2} \sqrt{\frac{1 + 5 \cdot \left(\frac{1}{2}\right)^{n+2} \cdot \exp\{2\gamma(t)\} + \left(\frac{1}{2}\right)^{2n+2} \cdot \exp\{4\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}}} = \\
&= \frac{1}{2} \sqrt{\frac{\left(\frac{1}{2}\right)^{2n+2} (2^{2n+2} + 5 \cdot 2^n \cdot \exp\{2\gamma(t)\} + \exp\{4\gamma(t)\})}{\left(\frac{1}{2}\right)^{n+1} (2^{n+1} + \exp\{2\gamma(t)\})}}
\end{aligned}$$

yoki

$$a_n(t) = \frac{1}{2} \sqrt{\frac{2^{2n+2} + 5 \cdot 2^n \cdot \exp\{2\gamma(t)\} + \exp\{4\gamma(t)\}}{2^{n+1} + \exp\{2\gamma(t)\}}}$$

bo'lishi kelib chiqadi. Xuddi shunday b_n koeffitsientni

$$b_n = \frac{1}{2} \left(\frac{K(n, n+1)}{K(n, n)} - \frac{K(n-1, n)}{K(n-1, n-1)} \right)$$

tenglik va

$$\chi(n, m) = \frac{K(n, m)}{K(n, n)} = \frac{-\left(\frac{\sqrt{2}}{2}\right)^{2n+3} \cdot \exp\{2\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}}$$

tenglikdan

$$b_n(t) = \frac{1}{2} \left(\frac{-\left(\frac{\sqrt{2}}{2}\right)^{2n+3} \cdot \exp\{2\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\}} + \frac{\left(\frac{\sqrt{2}}{2}\right)^{2n+1} \cdot \exp\{2\gamma(t)\}}{1 + \left(\frac{1}{2}\right)^n \cdot \exp\{2\gamma(t)\}} \right) =$$

$$\begin{aligned}
&= \frac{1}{2} \cdot \frac{-\left(\frac{\sqrt{2}}{2}\right)^{2n+3} \cdot \exp\{2\gamma(t)\} + \left(\frac{\sqrt{2}}{2}\right)^{2n+1} \cdot \exp\{2\gamma(t)\}}{1 + 3 \cdot \left(\frac{1}{2}\right)^{n+1} \cdot \exp\{2\gamma(t)\} + \left(\frac{1}{2}\right)^{2n+1} \cdot \exp\{4\gamma(t)\}} = \\
&= \frac{1}{2} \cdot \frac{\left(\frac{\sqrt{2}}{2}\right)^{2n+1} \cdot \exp\{2\gamma(t)\} \left(1 - \left(\frac{\sqrt{2}}{2}\right)^2\right)}{\left(\frac{1}{2}\right)^{2n+1} (2^{2n+1} + 3 \cdot 2^n \cdot \exp\{2\gamma(t)\} + \exp\{4\gamma(t)\})} = \\
&= \frac{1}{2} \cdot \frac{2^n \cdot \exp\{2\gamma(t)\}}{\sqrt{2}(2^{2n+1} + 3 \cdot 2^n \cdot \exp\{2\gamma(t)\} + \exp\{4\gamma(t)\})}
\end{aligned}$$

yoki

$$b_n = \frac{1}{2} \cdot \frac{2^n \cdot \exp\{2\gamma(t)\}}{\sqrt{2}(2^{2n+1} + 3 \cdot 2^n \cdot \exp\{2\gamma(t)\} + \exp\{4\gamma(t)\})}$$

bo'lishi kelib chiqadi.

Demak,

$$a_n(t) = \frac{1}{2} \frac{\sqrt{2^{2n+2} + 5 \cdot 2^n \exp\{2\gamma(t)\} + \exp\{4\gamma(t)\}}}{2^{n+1} + \exp\{2\gamma(t)\}},$$

$$b_n(t) = \frac{2^n \exp\{2\gamma(t)\}}{\sqrt{2}(2^{2n+1} + 3 \cdot \exp\{2\gamma(t)\} \cdot 2^n + \exp\{4\gamma(t)\})},$$

$$g_n^1(t) = \frac{2^{\frac{n}{2}} \sqrt{A_1(t)}}{\sqrt{2^{2n+1} + 3 \exp\{2\gamma(t)\} \cdot 2^n + \exp\{4\gamma(t)\}}} \times$$

$$\times \left\{ \sum_{j=-\infty}^{\infty} \frac{2^j}{2^{2j+1} + 3 \exp\{2\gamma(t)\} \cdot 2^j + \exp\{4\gamma(t)\}} \right\}^{-\frac{1}{2}}, \quad n \in \mathbb{Z}$$

bo'lishini topamiz.

$$\sum_{j=-\infty}^{\infty} \frac{2^j}{2^{2^{j+1}} + 3\exp\{2\gamma(t)\} \cdot 2^j + \exp\{4\gamma(t)\}} =$$

$$= \lim_{K \rightarrow \infty} \sum_{j=-K}^K \left(\frac{1}{2^j + \exp\{2\gamma(t)\}} - \frac{1}{2^{j+1} + \exp\{2\gamma(t)\}} \right) = \frac{1}{\exp\{2\gamma(t)\}}$$

bo'lgani uchun

$$g_n^1(t) = \frac{2^{\frac{n}{2}} \sqrt{A_1(t)} \exp\{\gamma(t)\}}{\sqrt{2^{2^{n+1}} + 3\exp\{2\gamma(t)\} \cdot 2^n + \exp\{4\gamma(t)\}}}, \quad n \in \mathbb{Z}$$

bo'ladi.

Xulosa

Mazkur bobda diskret Shturm-Liuivill operatori uchun moslangan manbali umumiy Toda zanjirini sochilish nazariyasining teskari masalasi yordamida integrallash usuli o'rganildi. Undan quyidagi natijalar olindi:

1. Oddiy xos qiymatga mos keluvchi diskret Shturm-Liuivill operatorining potensiali moslangan manbali umumiy Toda zanjirining yechimi bo'lganda spektral parametrlarining t bo'yicha o'zgarish qonuni o'rganildi;

2. Bitta xos qiymatga mos keluvchi diskret Shturm-Liuivill operatori yordamida sochilish nazariyasining teskari masalasini qo'llab moslangan manbali umumiy Toda zanjiri integrallandi.

Xulosa.

Mazkur dissertatsiyada diskret Shturm-Liuivill operatori va uning tadbiqlari, umumiy Toda zanjirini tez kamayuvchi funksiyalar sinfida sochilish nazariyasining teskari masalasi yordamida yechish usuli keltirilgan. Undan quyidagi natijalar olingan:

1. Diskret Shturm-Liuivill operatori spektral parametrlari o'rganildi;
2. Potensialli Toda zanjirining yechimi bo'lgan diskret Shturm-Liuivill operatori spektral parametrlarining t bo'yicha o'zgarish dinamikasi hisoblandi;
3. Moslangan manbali ikkinchi Toda tenglamasi tez kamayuvchi funksiyalar sinfida sochilish nazariyasining teskari masalasi usulidan foydalanib integrallandi.
4. Sochilish nazariyasining teskari masalasi usulidan foydalanib, boshlang'ich shartlar aniq berilganda ikkinchi Toda tenglamasi yechimining aniq tasviri olindi.
5. Moslangan manbali umumiy Toda tenglamasi tez kamayuvchi funksiyalar sinfida sochilish nazariyasining teskari masalasi usulidan foydalanib integrallandi.

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