O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS

TA’LIM VAZIRLIGI

URGANCH DAVLAT UNIVERSITETI

Qo‘l yozma huquqida

UDK 517.958

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CHEKLI O’LCHAMLI MOSLANGAN MANBALI TODA ZANJIRINI INTEGRALLASH.

D I S S E R T A T S I Y A

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Urganch – 2017

MUNDARIJA

KIRISH

Tadqiqot mavzusining dolzarbligi. Zamonaviy matematik-fizikaning har xil sohalaridagi ko‘plab tadqiqotlar nochiziqli to‘lqin nazariyasini o‘rganishga bag‘ishlangan. Bunday tenglamalardan biri Korteveg-de Friz tenglamasidir.

1967 yilda G.Gardner, J.Grin, M. Kruskal, R.Miuralar zamonaviy matematik-fizikaning asosiy tenglamalardan biri bo‘lgan ushbu

$$u_t - 6uu_x + u_{xxx} = 0, \qquad u(x,0) = u_0(x), \; x \in R^1$$

Korteveg-de Friz tenglamasiga qo‘yilgan Koshi masalasining yechimini Shturm-Liuvill operatorining to‘g‘ri va teskari spektral masalalaridan foydalanib topishga muvaffaq bo‘lishdi. Keyinchalik P. Laks bu usulning umumiy xarakterga ega ekanligini ko‘rsatdi.

Toda tenglamasi nochiziqli evalutsion tenglamalardan biri bo’lib, qattiq jismlar mexanikasida muhim amaliy tadbiqlarga ega. Hozirgi kunda moslangan manbali nochiziqli evalutsion tenglamalarning muhim amaliy tadbiqlari topilganligi bois bunday nochiziqli evolutsion tenglamalarni umumiy ko`rinishlarini topish va ularni integrallash masalalariga bo`lgan qiziqish yanada ortmoqda.

Ilmiy izlanishning maqsad va vazifalari. Ushbu ish umumiy davriy Toda tenglamasiga qo‘ yilgan Koshi masalasini $m = 2$ bo‘ lgan holda teskari masalalar usulidan foydalanib yechish algoritmini keltirib chiqarishga bag’ishlangan.

Quyidagi

$$\begin{cases} \dot{a}_n = a_n(b_{n+1} - b_n) + a_n(a_{n+1}^2 - a_{n-1}^2) + a_n(b_{n+1}^2 - b_n^2) + a_n \sum\limits_{i=1}^{2N} \tilde{\theta}_{N+1}(\lambda_i, t) \left[(f_{n+1}^i)^2 - (f_n^i)^2 \right], \\ \dot{b}_n = 2(a_n^2 - a_{n-1}^2) - 2a_{n-1}^2(b_n + b_{n-1}) + 2a_n^2(b_{n+1} + b_n) - 2\sum\limits_{i=1}^{2N} \tilde{\theta}_{N+1}(\lambda_i, t) f_n^i (a_n f_{n+1}^i - a_{n-1} f_{n-1}^i), \\ a_{n-1} f_{n-1}^i + b_n f_n^i + a_n f_{n+1}^i = \lambda_i f_n^i, \\ a_{n+N} = a_n, \;\; b_{n+N} = b_n, \;\; (f_{n+N}^i)^2 = (f_n^i)^2, \;\; i = 1, 2, \ldots, 2N, \;\; n \in Z, \end{cases}$$

(0.1)

moslangan manbali N - davrli Toda turidagi tenglamani ushbu

$$a_n(0)=a_n^0,\ b_n(0)=b_n^0,\quad n\in Z\,, \tag{0.2}$$

boshlang'ich shartlar bilan qaraymiz. Bu erda $a_n^0,\ b_n^0,\ \ n\in Z$ sonlar N-davrli haqiqiy sonlar ketma-ketligi bo'lib $\{a_n(t)\}_{-\infty}^{\infty}$, $\{b_n(t)\}_{-\infty}^{\infty}$, $\{f_n^1(t)\}_{-\infty}^{\infty}$, $\{f_n^2(t)\}_{-\infty}^{\infty}$, $\dots$, $\{f_n^{2N}(t)\}_{-\infty}^{\infty}$ – noma'lum funksiyalar, hamda $\{f_n^i(t)\}_{-\infty}^{\infty}$ funksiyalar ushbu

$$(L(t)y)_n \equiv a_{n-1}y_{n-1}+b_ny_n+a_ny_{n+1}=\lambda y_n\,, \tag{0.3}$$

Diskret Xill tenglamasining λ_i xos qiymatlariga mos keluvchi va quyidagi

$$f_1^i(t)=1,\quad i=1,2,\dots,2N\,. \tag{0.4}$$

shart bilan normallangan Floke yechimlaridir. λ_i xos qiymatlar

$$\Delta^2(\lambda)-4=0$$

tenglamani ildizlari. Bu erda $\Delta(\lambda)=\theta_N(\lambda,t)+\varphi_{N+1}(\lambda,t)$, hamda $\theta_n(\lambda,t)$, $n\in Z$ va $\varphi_n(\lambda,t)$, $n\in Z$ lar (0.3) tenglamani quyidagi

$$\theta_0(\lambda,t)=1,\ \theta_1(\lambda,t)=0,\quad \varphi_0(\lambda,t)=0,\ \varphi_1(\lambda,t)=1$$

boshlang'ich shartlarni qanoatlantiruvchi yechimlaridir. (0.1) tenglamadagi $\tilde{\theta}_{N+1}(\lambda,t)$ funksiya ushbu $\tilde{\theta}_{N+1}(\lambda,t)=\prod_{j=1}^{N-1}(\lambda-\mu_j(t))$ tenglikdan aniqlanadi, bunda $\mu_1(t),\mu_2(t),\dots,\mu_{N-1}(t)$ lar $\theta_{N+1}(\lambda,t)=0$ tenglamani ildizlari.

Ushbu magistrlik dissertetsiyasining asosiy maqsadi-(0.1)-(0.4) masalaning yechimini teskari masalalar usulidan foydalanib topish algoritmini berishdan iborat.

Mavzuning o' rganilish darajasining qiyosiy tahlili: Zarrachani to' g' ri chiziqli o' zaro eksponensial ta'sirini ifodalovchi ushbu

$$\frac{\partial^2 u_n}{\partial t^2}=\exp(u_{n-1}-u_n)-\exp(u_n-u_{n+1})\,,\quad n\in Z\,,$$

Toda[1] zanjiri Flashka [2] o‘ zgaruvchilari orqali quyidagicha

$$\begin{cases} \dot{a}_n = a_n(b_n - b_{n+1}), \\ \dot{b}_n = 2(a_{n-1}^2 - a_n^2), \quad n \in Z \end{cases}$$

ko‘ rinishga ega bo‘ ladi. Toda tenglamasi birinchi bor B.M. Toda tomonidan kiritilgan. Shundan keyin Flashka diskret Xill tenglamasiga qo‘ yilgan teskari masala usuli yordamida Toda tenglamasiga qo‘ yilgan Koshi masalasining yechimini topishga muyassar bo‘ lgan. Xuddi shunday natija Flashkani natijasidan bexabar ravishda Manakov[3] tomonidan ham olingan. Davriy Toda tenglamasi [4]-[6] ishlarda o‘ rganilgan. Shundan keyin diskret Xill tenglamasiga qo‘ yilgan teskari masalalar usuli yordamida integrallanadigan barcha nochiziqli evalutsion tenglamalarni topishga bo‘ lgan qiziqish ortgan. Bu tenglamalar odatda umumiy Toda tenglamasi deb yuritiladi. Umumiy Toda tenglamasi birinchi bor K. Ueno va K. Takasakilar[7] tomonidan kiritilgan. Umumiy Toda tenglamasining diskret Xill tenglamasiga qo‘ yilgan teskari masalalar usulida integrallash mumkinligi [8] ishlarda ko‘ rsatilgan. Moslangan manbali nochiziqli tenglamalar [9]-[14] ishlarda o‘rganilgan.

Tadqiqotning ilmiy yangiligi: Ushbu ishda diskret Xill tenglamasiga qo‘ yilgan teskari masalalar usuli yordamida integrallanadigan yangi nochiziqli Toda tenglamasi turidagi evalutsion tenglama keltirib chiqarilgan va uni integrallashning yangicha usuli berilgan. Diskret Xill tenglamasiga qo‘ yilgan teskari masalalar usulidan foydalanib (0.1)-(0.4) masalaning yechimi uchun aniq tasvir olingan.

Tadqiqot predmeti va ob’ekti. Ushbu ishda oddiy differensial tenglamalar, matematik fizika tenglamalari, funksional analiz, differensial operatorlarning spektral nazariyasi predmetlari usullaridan foydalaniladi. Tatqiqotning ob’ekti: moslangan manbali II-tur Toda tenglamasi.

Tadqiqotning ilmiy ahamiyati. O‘ rganilgan natijalar amaliy ahamiyatga ega bo‘ lib, qattiq jismlar mexanikasi, kristallografiya va geologiya-razvedka masalalarida muhim tadbiqlarga ega.

I-BOB. Diskret Xill tenglamasi uchun spektral masala

1.1-§. Diskret Xill tenglamasi. Asosiy tushunchalar

Ushbu

$$(ly)_n \equiv a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n \,, \tag{1.1.1}$$

$$a_{n+N} = a_n \,, \quad b_{n+N} = b_n \,, \quad n \in Z \,,$$

diskret Xill tenglamasini qaraymiz. Bu yerda λ parametr, N biror natural son. $\theta_n(\lambda)$, $n \in Z$ va $\varphi_n(\lambda)$, $n \in Z$ orqali (1.1.1) tenglamaning quyidagi

$$\theta_0(\lambda) = 1, \quad \theta_1(\lambda) = 0, \qquad \varphi_0(\lambda) = 0, \;\; \varphi_1(\lambda) = 1 \tag{1.1.2}$$

boshlang‘ ish shartlarni qanoatlantiruvchi yechimlarini belgilaymiz.

Ta’rif 1.1. Ushbu

$$\Delta(\lambda) = \theta_N(\lambda) + \varphi_{N+1}(\lambda) \tag{1.1.3}$$

funksiyaga Xill diskriminanti yoki Lyapunov funksiyasi deyiladi.

$\lambda_1, \lambda_2, \ldots, \lambda_{2N}$ sonlar orqali ushbu

$$\Delta^2(\lambda) - 4 = 0$$

tenglamaning ildizlarini belgilaymiz.

$\mu_1, \mu_2, \ldots, \mu_{N-1}$ sonlar orqali esa ushbu

$$\theta_{N+1}(\lambda) = 0$$

tenglamaning ildizlarini belgilaymiz. Quyidagi

$$\Delta^2(\lambda) - 4 = \left(\prod_{j=1}^{N} a_j \right)^{-2} \prod_{j=1}^{2N} (\lambda - \lambda_j) \,,$$

$$\theta_{N+1}(\lambda) = -a_0 \left(\prod_{j=1}^{N} a_j \right)^{-1} \prod_{j=1}^{N-1} (\lambda - \mu_j)$$

tengliklarni bajarilishi ravshan. Ushbu

$$\sigma_j = sign\left[\theta_N(\mu_j) - \frac{1}{\theta_N(\mu_j)}\right], \quad j = 1, 2, \ldots, N-1$$

belgilashni kiritamiz.

Ta'rif 1.2. μ_j, $j = 1, 2, \ldots, N-1$ sonlar va σ_j, $j = 1, 2, \ldots, N-1$ ishoralar ketma-ketligiga diskret Xill tenglamasini spektral parametrlari deyiladi.

Ta'rif 1.3. $\{\mu_j, \sigma_j\}_{j=1}^{N-1}$ spektral parametrlar va λ_i, $i = 1, 2, \ldots, 2N$ sonlar ketma-ketligiga diskret Xill tenglamasini spektral berilganlari deyiladi.

Spektral berilganlarni topish va ularning xossalarini oʻ rganishga diskret Xill tenglamasi uchun qoʻ yilgan toʻ gʻ ri masala deyiladi. Aksincha, spektral berilganlar yordamida diskret Xill tenglamasini koeffitsientlari a_n, b_n larni topish masalasiga diskret Xill tenglamasi uchun qoʻ yilgan teskari masala deyiladi.

Lemma 1.1. $\theta_n(\lambda)$ va $\varphi_n(\lambda)$ yechim mavjud va yagona boʻ lib, ular uchun ushbu

$$\theta_n(\lambda) = -a_0\left(\prod_{j=1}^{n-1} a_j\right)^{-1}\left\{\lambda^{n-2} - \left(\sum_{j=2}^{n-1} b_j\right)\lambda^{n-3} + \left[\frac{1}{2}\left(\sum_{j=2}^{n-1} b_j\right)^2 - \frac{1}{2}\sum_{j=2}^{n-1} b_j^2 - \sum_{j=2}^{n-2} a_j^2\right]\lambda^{n-4} + \ldots\right\} \tag{1.1.4}$$

$$\varphi_n(\lambda) = \left(\prod_{j=1}^{n-1} a_j\right)^{-1}\left\{\lambda^{n-1} - \left(\sum_{j=1}^{n-1} b_j\right)\lambda^{n-2} + \left[\frac{1}{2}\left(\sum_{j=1}^{n-1} b_j\right)^2 - \frac{1}{2}\sum_{j=1}^{n-1} b_j^2 - \sum_{j=1}^{n-2} a_j^2\right]\lambda^{n-3} + \ldots\right\} \tag{1.1.5}$$

tasvirlar oʻ rinli.

Isbot:

$$a_{n-1}\theta_{n-1} + b_n\theta_n + a_n\theta_{n+1} = \lambda\theta_n, \quad n \in Z \tag{1.1.6}$$

tenglikda $n = 1$ desak, va (1.1.2) boshlangʻ ich shartlarni inobatga olsak

$$a_0 + a_1\theta_2 = 0, \tag{1.1.7}$$

kelib chiqadi. Bunga koʻ ra $\theta_2(\lambda) = -\dfrac{a_0}{a_1}$ boʻ ladi (1.1.10) da $n = 2$ desak va boshlangʻ ich shartlardan foydalansak

$$b_2\theta_2 + a_2\theta_3 = \lambda\theta_2, \tag{1.1.8}$$

kelib chiqadi (1.1.7) va (1.1.8) dan

$$\theta_3 = -\frac{\lambda - b_2}{a_2}\cdot\theta_2 = -\frac{(\lambda - b_2)a_0}{a_1\cdot a_2} \tag{1.1.9}$$

kelib chiqadi. (1.1.6) da $n = 3$ desak

$$a_2\theta_2 + b_3\theta_3 + a_3\theta_4 = \lambda\theta_3 \tag{1.1.10}$$

kelib chiqadi.(1.1.7), (1.1.9) va (1.1.10) dan

$$\begin{aligned}\theta_4 &= -\frac{\lambda - b_3}{a_3}\cdot\theta_3 - \frac{a_2}{a_3}\cdot\theta_2 = -\frac{(\lambda - b_3)(\lambda - b_2)\cdot a_0}{a_1\cdot a_2\cdot a_3} + \frac{a_2\cdot a_0}{a_3\cdot a_1} = \\ &- \frac{a_0}{a_1\cdot a_2\cdot a_3}[\lambda^2 - (b_2 + b_3)\lambda + b_2\cdot b_3 - a_2^2];\end{aligned} \tag{1.1.11}$$

kelib chiqadi. $a_n \neq 0$, $n \in Z$ shartdan θ_n yechimning mavjud va yagona boʻ lishi ham shu tarzda koʻ rsatiladi, umuman $n \geq 2$ boʻ lganda shu jarayonni davom qildirish natijasida (1.1.4) va (1.1.5) tengliklar oʻ rinli boʻ lishi kelib chiqadi.

Lemma 1.2. Ushbu

$$\Delta^2(\lambda) - 4 = \left(\prod_{j=1}^{N} a_j\right)^{-2}\left\{\lambda^{2N} - 2\left(\sum_{j=1}^{N} b_j\right)\lambda^{2N-1} + \left[2\left(\sum_{j=1}^{N} b_j\right)^2 - \sum_{j=1}^{N} b_j^2 - 2\sum_{j=1}^{N} a_j^2\right]\lambda^{2N-2} + \ldots\right\} \tag{1.1.12}$$

tenglik o‘rinli.

Isbot: Teoremani isboti Lyapunov funksiyasini ko‘rinishi va bazis yechimlar uchun olingan (1.1.4)-(1.1.5) tasvirlardan to‘g‘ridan- to‘g‘ri hisoblashlar yordamida kelib chiqadi.

Ta’rif 1.4. Ushbu

$$W = a_n \cdot \left[\theta_n(\lambda)\cdot\varphi_{n+1}(\lambda) - \theta_{n+1}(\lambda)\cdot\varphi_n(\lambda)\right] \tag{1.1.13}$$

ifodaga $\theta_n(\lambda), \varphi_n(\lambda)$ yechimlarning Vronskiy determinanti deyiladi.

Lemma 1.3. $\theta_n(\lambda)$ va $\varphi_n(\lambda)$ larning W Vronskiy determinanti n ga ham, λ ga ham bog‘liq emas bo‘lib, ushbu

$$W = a_n\left[\theta_n(\lambda)\varphi_{n+1}(\lambda) - \theta_{n+1}(\lambda)\varphi_n(\lambda)\right] = a_0 \tag{1.1.14}$$

bunga ko‘ra $\det M = \left[\theta_N(\lambda)\varphi_{N+1}(\lambda) - \theta_{N+1}(\lambda)\varphi_N(\lambda)\right] = 1$ tenglik o‘rinli.

Isbot: $\theta_n(\lambda)$ va $\varphi_n(\lambda)$ larni (1.1) ga qo‘ysak

$$\begin{cases} a_{n-1}\theta_{n-1} + b_n\theta_n + a_n\theta_{n+1} = \lambda\theta_n \\ a_{n-1}\varphi_{n-1} + b_n\varphi_n + a_n\varphi_{n+1} = \lambda\varphi_n \end{cases} \tag{1.1.15}$$

tenglik o‘rinli bo‘ladi.Bundan esa

$$\begin{cases} a_{n-1}\theta_{n-1}\varphi_n + b_n\theta_n\varphi_n + a_n\theta_{n+1}\varphi_n = \lambda\theta_n\varphi_n \\ a_{n-1}\varphi_{n-1}\theta_n + b_n\varphi_n\theta_n + a_n\varphi_{n+1}\theta_n = \lambda\varphi_n\theta_n \end{cases},$$

$$\begin{cases} a_{n-1}\theta_{n-1}\varphi_n + a_n\theta_{n+1}\varphi_n = (\lambda - b_n)\theta_n\varphi_n \\ a_{n-1}\varphi_{n-1}\theta_n + a_n\varphi_{n+1}\theta_n = (\lambda - b_n)\varphi_n\theta_n \end{cases}$$

tehgliklar kelib chiqadi. Pastdagi sistemada tengliklarni bir-biridan ayirib

$$a_n(\theta_{n+1}\varphi_n - \varphi_{n+1}\theta_n) = a_{n-1}(\varphi_{n-1}\theta_n - \varphi_n a_{n-1} \tag{1.1.16}$$

tenglikni hosil qilamiz.Quyidagi bizga ma’lum munosabatlarni inobatga olib,

$$\begin{cases}\theta_{N+n}(\lambda)=\theta_n(\lambda)\\ \varphi_{N+n}(\lambda)=\varphi_n(\lambda)\end{cases},\quad \begin{cases}\theta_N=\theta_0\\ \varphi_N=\varphi_0\end{cases} \tag{1.1.17}$$

va (1.1.4) tengliklardan

$$\varphi_1\theta_0-\varphi_0\theta_1=1 \tag{1.1.18}$$

ekani kelib chiqadi, (1.1.2),(1.1.16) va (1.1.18) tengliklardan foydalansak

$$W=a_n\left[\theta_n(\lambda)\varphi_{n+1}(\lambda)-\theta_{n+1}(\lambda)\varphi_n(\lambda)\right]=a_{n+N}\left[\theta_{n+N}\varphi_{n+N+1}-\theta_{n+N+1}\varphi_{n+N}\right]=$$
$$=a_0\left[\theta_0\varphi_1-\varphi_0\theta_1\right]=a_0$$

tenglik o‘ rinli bo‘ lishi kelib chiqadi. Bunda $\det M=\theta_N\varphi_{N+1}-\theta_{N+1}\varphi_N=1$ ekani ravshan**. Lemma 1.3** isbotlandi.

1.2-§. Diskret Xill tenglamasi uchun teskari spektral masala

Ushbu

$$(L\varphi)_n \equiv a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n, \quad n \in Z \qquad (1.2.1)$$

tenglamani koeffisientlarini $k \in Z$ butun songa siljitish natijasida hosil qilingan quyidagi tenglamani koʻ rib chiqamiz:

$$a_{n-1+k}y_{n-1} + b_{n+k}y_n + a_{n+k}y_{n+1} = \lambda y_n, \; n \in Z. \qquad (1.2.2)$$

$\theta_{n+k}(\lambda)$ va $\varphi_{n+k}(\lambda)$ lar (1.2.2) ning yechimi boʻ ladi. Haqiqatan: Ushbu

$$a_{n-1}\theta_{n-1} + b_n\theta_n + a_n\theta_{n+1} = \lambda\theta_n,$$

tenglama koeffisientlarini $k \in Z$ butun songa siljitsak

$$a_{n-1+k}\theta_{n-1+k} + b_{n+k}\theta_{n+k} + a_{n+k}\theta_{n+1+k} = \lambda\theta_{n+k}$$

boʻ ladi. Oxirgi tenglikda quyidagi

$$y_n = \theta_{n+k},$$

belgilashni kiritsak, u holda

$$a_{n-1+k}y_{n-1} + b_{n+k}y_n + a_{n+k}y_{n+1} = \lambda y_n.$$

kelib chiqadi. (1.2.2) tenglamaning quyidagi boshlangʻ ich shartlarni qanoatlantiruvchi yechimlarini $u_{n,k}(\lambda)$ va $v_{n,k}(\lambda)$ orqali belgilaymiz:

$$\begin{cases} u_{0,k}(\lambda) = 1 \\ u_{1,k}(\lambda) = 0 \end{cases}, \quad \begin{cases} v_{0,k}(\lambda) = 0 \\ v_{1,k}(\lambda) = 1 \end{cases} \qquad (1.2.3)$$

Teorema 1.1. (1.2.2) tenglamaning Lyapunov funksiyasi $\tilde{\Delta}(\lambda)$ bo‘ lsa, u holda $\tilde{\Delta}(\lambda)=\Delta(\lambda)$ bo‘ ladi.

Isbot: $\theta_{n+k}(\lambda)$ va $\varphi_{n+k}(\lambda)$ yechimlar chiziqli erkli bo‘ lgani uchun

$$u_{n,k}(\lambda)=A_1\theta_{n+k}(\lambda)+A_2\varphi_{n+k}(\lambda), \tag{1.2.4}$$

$$v_{n,k}(\lambda)=B_1\theta_{n+k}(\lambda)+B_2\varphi_{n+k}(\lambda), \tag{1.2.5}$$

(1.2.4) dan

$$\begin{cases} A_1\theta_k(\lambda)+A_2\varphi_k(\lambda)=1 \\ A_1\theta_{k+1}(\lambda)+A_2\varphi_{k+1}(\lambda)=0 \end{cases} \tag{1.2.6}$$

kelib chiqadi. (1.2.6) sistemani yechamiz:

$$\begin{vmatrix} \theta_k(\lambda) & \varphi_k(\lambda) \\ \theta_{k+1}(\lambda) & \varphi_{k+1}(\lambda) \end{vmatrix}=\frac{a_0}{a_k}, \quad \begin{vmatrix} 1 & \varphi_k(\lambda) \\ 0 & \varphi_{k+1}(\lambda) \end{vmatrix}=\varphi_{k+1}(\lambda), \quad \begin{vmatrix} \theta_k(\lambda) & 1 \\ \theta_{k+1}(\lambda) & 0 \end{vmatrix}=-\theta_{k+1}(\lambda)$$

bo‘ lgani uchun

$$A_1=\frac{a_k}{a_0}\varphi_k(\lambda),\ A_2=-\frac{a_k}{a_0}\theta_k(\lambda)$$

bo‘ ladi. Bunga ko‘ ra

$$u_{n,k}(\lambda)=\frac{a_k}{a_0}\cdot\left[\varphi_{k+1}(\lambda)\theta_{n+k}(\lambda)-\theta_{k+1}(\lambda)\varphi_{n+k}(\lambda)\right] \tag{1.2.7}$$

bo‘ ladi. Xuddi shu tarzda quyidagi tenglik kelib chiqadi:

$$v_{n,k}(\lambda)=\frac{a_k}{a_0}\cdot\left[-\varphi_k(\lambda)\theta_{n+k}(\lambda)+\theta_k(\lambda)\varphi_{n+k}(\lambda)\right], \tag{1.2.8}$$

Ushbu

$$\theta_{N+k}(\lambda)=\frac{a_0}{a_k}\cdot\left[\varphi_N(\lambda)\theta_k(\lambda)+\theta_{N+1}(\lambda)\varphi_k(\lambda)\right] \tag{1.2.9}$$

$$\varphi_{N+k}(\lambda)=\frac{a_0}{a_k}\cdot\left[\varphi_N(\lambda)\theta_k(\lambda)+\varphi_{N+1}(\lambda)\varphi_k(\lambda)\right] \tag{1.2.10}$$

tengliklar o‘rinli bo‘lishi ravshan. (1.2.6)-(1.2.10) larga muvofiq,

$$\begin{aligned} u_{N,k}(\lambda)&=\varphi_{k+1}(\lambda)\left[\theta_N(\lambda)\theta_k(\lambda)+\theta_{N+1}(\lambda)\varphi_k(\lambda)\right]-\\ &-\theta_{k+1}(\lambda)\left[\varphi_N(\lambda)\theta_k(\lambda)+\varphi_{N+1}(\lambda)\varphi_k(\lambda)\right] \end{aligned} \tag{1.2.11}$$

$$\begin{aligned} v_{N,k}(\lambda)&=-\varphi_k(\lambda)\left[\theta_N(\lambda)\theta_k(\lambda)+\theta_{N+1}(\lambda)\varphi_k(\lambda)\right]+\\ &+\theta_k(\lambda)\left[\varphi_N(\lambda)\theta_k(\lambda)+\varphi_{N+1}(\lambda)\varphi_k(\lambda)\right] \end{aligned} \tag{1.2.12}$$

(1.2.7) ga ko‘ra

$$\begin{aligned} v_{N+1,k}(\lambda)&=-\varphi_k(\lambda)\theta_{N+k+1}(\lambda)+\theta_k(\lambda)\varphi_{N+k+1}(\lambda)=\\ &=-\varphi_k(\lambda)[\theta_N(\lambda)\theta_{k+1}(\lambda)+\theta_{N+1}(\lambda)\varphi_{k+1}(\lambda)]+\\ &+\theta_k(\lambda)[\varphi_N(\lambda)\theta_{k+1}(\lambda)+\varphi_{N+1}(\lambda)\varphi_{k+1}(\lambda)] \end{aligned} \tag{1.2.13}$$

(1.2.11) va (1.2.13) ga ko‘ra

$$\begin{gathered} \tilde{\Delta}(\lambda)=u_{N,k}(\lambda)+v_{N+1,k}(\lambda)=\\ =\theta_N(\lambda)[\theta_k(\lambda)\varphi_{k+1}(\lambda)-\theta_{k+1}(\lambda)\varphi_k(\lambda)]+\varphi_{N+1}(\lambda)[\theta_k(\lambda)\varphi_{k+1}(\lambda)-\\ -\theta_{k+1}(\lambda)\varphi_k(\lambda)]=\theta_N(\lambda)+\varphi_{N+1}(\lambda)=\Delta(\lambda) \end{gathered}$$

teorema 1.1 isbot bo‘ldi.

Natija: (1.2.1) tenglamani spektri k ga bog‘liq emas.

Lemma 1.4. Ushbu tengliklar o‘rinli:

$$b_1=\frac{\lambda_1+\lambda_{2N}}{2}+\frac{1}{2}\sum_{j=1}^{N-1}\left(\lambda_{2j}+\lambda_{2j+1}-2\mu_j\right), \tag{1.2.14}$$

$$\begin{gathered} a_0^2=\frac{\lambda_1^2+\lambda_{2N}^2}{8}+\frac{1}{8}\sum_{j=1}^{N-1}\left(\lambda_{2j}^2+\lambda_{2j+1}^2-2\mu_j^2\right)-\\ -\frac{1}{4}\left[\frac{\lambda_1+\lambda_{2N}}{2}+\frac{1}{2}\sum_{j=1}^{N-1}\left(\lambda_{2j}+\lambda_{2j+1}-2\mu_j\right)\right]^2-\frac{1}{2}\sum_{j=1}^{N-1}\frac{\sigma_j\sqrt{\prod_{i=1}^{2N}(\mu_j-\lambda_i)}}{\prod_{\substack{i=1\\ i\neq j}}^{N-1}(\mu_j-\mu_i)}. \end{gathered} \tag{1.2.15}$$

Lemma 1.5. Ushbu tengliklar o‘rinli:

$$b_{k+1} = \frac{\lambda_1 + \lambda_{2N}}{2} + \frac{1}{2}\sum_{j=1}^{N-1}\left(\lambda_{2j} + \lambda_{2j+1} - 2\mu_{j,k}\right), \tag{1.2.16}$$

$$a_k^2 = \frac{\lambda_1^2 + \lambda_{2N}^2}{8} + \frac{1}{8}\sum_{j=1}^{N-1}\left(\lambda_{2j}^2 + \lambda_{2j+1}^2 - 2\mu_{j,k}^2\right) -$$

$$-\frac{1}{4}\left[\frac{\lambda_1 + \lambda_{2N}}{2} + \frac{1}{2}\sum_{j=1}^{N-1}\left(\lambda_{2j} + \lambda_{2j+1} - 2\mu_{j,k}\right)\right]^2 - \frac{1}{2}\sum_{j=1}^{N-1}\frac{\sigma_{j,k}\sqrt{\prod_{i=1}^{2N}(\mu_{j,k} - \lambda_i)}}{\prod_{\substack{i=1\\ i\neq j}}^{N-1}(\mu_{j,k} - \mu_{i,k})}, \tag{1.2.17}$$

Bu erda $\mu_{j,k}$, $j = \overline{1, N-1}$ sonlar $u_{N+1,k}(\lambda) = 0$ tenglamani ildizlari.

Teorema 1.2.(Floke).

a) Agar $\Delta(\lambda) - 4 \neq 0$ bo‘lsa, (1.1.1) tenglama quyidagi chiziqli erkli ikkita yechimga ega:

$$\psi_n^+(\lambda) = (\rho_+(\lambda))^{\frac{n}{N}} \cdot P_n^+, \qquad P_{n+N}^+(\lambda) \equiv P_n^+(\lambda), \tag{1.2.18}$$

$$\psi_n^-(\lambda) = (\rho_-(\lambda))^{\frac{n}{N}} \cdot P_n^-, \qquad P_{n+N}^-(\lambda) \equiv P_n^-(\lambda) \tag{1.2.19}$$

b) Agar $\Delta(\lambda) - 2 = 0$ bo‘lsa ,(1.1.1) tenglamaning davri N bo‘lgan $\psi_n(\lambda)$ yechimi mavjud:

$$\psi_{n+N}(\lambda) \equiv \psi_n(\lambda),\ n \in Z$$

c) Agar $\Delta(\lambda) + 2 = 0$ bo‘lsa, (1.1.1) tenglamaning davri $2N$ bo‘lgan $\psi_n(\lambda)$ yechimi mavjud:

$$\psi_{n+2N}(\lambda) \equiv \psi_n(\lambda),\ n \in Z\ ;$$

Isbot: (1.1.1) masalani biror ρ son uchun $y_{n+N}(\lambda) \equiv \rho y_n(\lambda), n \in Z$ shartni qanoatlantiradigan yechimlarini axtaramiz:

$y_n(\lambda) = c_1\theta_n(\lambda) + c_2\varphi_n(\lambda)$ - bu (1.1.1) ning umumiy yechimi. (1.1.2) shartga ko‘ ra $\theta_{n+N}(\lambda), \varphi_{n+N}(\lambda)$ larni chiziqli kombinatsiyasi ham (1.1.1) ni umumiy yechimi bo‘ ladi. (1.2.6) va (1.2.7) tengliklardan va

$$y_{n+N}(\lambda) = c_1 \cdot \theta_{n+N}(\lambda) + c_2\varphi_{n+N}(\lambda) \qquad (1.2.20)$$

munosabat o‘ rinli ekanidan hamda $y_{n+N} = \rho y_n$ shartda $n = 0$ va $n = 1$ desak, u holda

$$\begin{cases} C_1\theta_N + C_2\varphi_N = \rho C_1 \\ C_1\theta_{N+1} + C_2\varphi_{N+1} = \rho C_2 \end{cases}, \quad \begin{cases} C_1(\theta_N - \rho) + C_2\varphi_N = 0 \\ C_1\theta_{N+1} + C_2(\varphi_{N+1} - \rho) = 0 \end{cases} \qquad (1.2.21)$$

tenglamalar sistemasini hosil qilamiz. Bu (1.2.21) bir jinsli tenglamalar sistemasi noldan farqli yechimga ega bo‘ lishi uchun

$$\begin{vmatrix} \theta_N - \rho & \varphi_N \\ \theta_{N+1} & \varphi_{N+1} - \rho \end{vmatrix} = 0 \qquad (1.2.22)$$

tenglik bajarilishi kerak. Bundan lemma 1.3 ga ko‘ ra va (1.1.3) tenglikdan $\rho^2 - \Delta(\lambda)\rho + 1 = 0$ bo‘ lishi kelib chiqadi. Bu kvadrat tenglamani yechib

$$D = \Delta^2(\lambda) - 4,$$

$$\rho_+(\lambda) = \frac{\Delta(\lambda) + \sqrt{\Delta^2(\lambda) - 4}}{2},$$

$$\rho_-(\lambda) = \frac{\Delta(\lambda) - \sqrt{\Delta^2(\lambda) - 4}}{2} \qquad (1.2.23)$$

tengliklarga ega bo‘ lamiz.

a) $\Delta^2(\lambda)-4\neq 0$ demak ushbu tenglamani $\rho_+(\lambda),\ \rho_-(\lambda)$ ikkita yechimi mavjud. Bularni har biriga bittadan (1.1) tenglamaning yechimi mos keladi. Ularni $\psi_n^+(\lambda),\ \psi_n^-(\lambda)$ deb belgilaymiz, $\psi_{n+N}^+(\lambda)=\rho_+\psi_n^+,\quad \psi_{n+N}^-(\lambda)=\rho_-\psi_n^-$.

Ushbu $P_n^+(\lambda),\ P_n^-(\lambda)$ funksiyalarni kiritamiz.

$$P_{n+N}^+(\lambda)=\rho_+^{-\frac{n}{N}}\psi_n^+,\quad P_n^-(\lambda)=\rho_-^{-\frac{n}{N}}\psi_n^- \tag{1.2.24}$$

(1.2.24) funksiyalar N davrli ekanini ko‘ rsatamiz

$$P_{n+N}^+(\lambda)=\rho_+^{-\frac{n+N}{N}}\psi_{n+N}^+=\rho_+^{-\frac{n}{N}}\rho_+^{-1}\rho_+^1\psi_n^+=\rho_+^{-\frac{n}{N}}\psi_n^+$$
$$P_{n+N}^-(\lambda)=\rho_-^{-\frac{n+N}{N}}\psi_{n+N}^-=\rho_-^{-\frac{n}{N}}\rho_-^{-1}\rho_-^1\psi_n^-=\rho_-^{-\frac{n}{N}}\psi_n^-$$

Demak (1.2.24) tengliklardan (1.2.18) va (1.2.19) kelib chiqadi.

$C_1=1$ deylik , u holda $C_2=\dfrac{\rho-\theta_N}{\varphi_N}$ ya’ni

$$C_2=\frac{\Delta(\lambda)-2\theta_N\pm\sqrt{\Delta^2(\lambda)-4}}{2\varphi_N}=\frac{\varphi_{N+1}-\theta_N\pm\sqrt{\Delta^2(\lambda)-4}}{2\varphi_N} \tag{1.2.25}$$

munosabat o‘ rinli. Demak

$$y_n^{\pm}(\lambda)=C_1\theta_n(\lambda)+C_2\varphi_n(\lambda)=\theta_n(\lambda)+\frac{\varphi_{N+1}(\lambda)-\theta_N(\lambda)\pm\sqrt{\Delta^2(\lambda)-4}}{2\varphi_N(\lambda)}\varphi_n(\lambda)$$

(1.2.26)

(1.2.26) tenglik bilan aniqlanadigan yechimlar Floke yechimlari deyiladi. Bu yechimlar chiziqli erkli ekanini tekshirib ko‘ rish qiyin emas. Bu tenglikdagi

$$m^{\pm}(\lambda)=\frac{\varphi_{N+1}(\lambda)-\theta_N(\lambda)\pm\sqrt{\Delta^2(\lambda)-4}}{2\varphi_N(\lambda)}$$

funksiyaga Veyl-Titchmarsh funksiyasi deyiladi.

b) $\Delta(\lambda)=2$ bo‘lsa $\rho^2-\Delta(\lambda)\rho+1=0$ tenglama $\rho_+=\rho_-=1$ yechimga ega bo‘ladi, ya’ni (1.1) ni $y_{n+N}=1\cdot y_n$ shartni qanoatlantiruvchi yechimi mavjud.

c) $\Delta(\lambda)=-2$ bo‘lsa, $\rho^2-\Delta(\lambda)\rho+1=0$ tenglama $\rho_+=\rho_-=-1$ yechimga ega bo‘ladi, ya’ni (1.1.1) tenglamaning $y_{n+N}(\lambda)=-1\cdot y_n(\lambda)$ shartni qanoatlanti-

-ruvchi yechimi mavjud. Bu yechimning davri $2N$ ga teng

$$y_{n+2N}(\lambda)=y_{n+N+N}(\lambda)=-y_{n+N}(\lambda)=-(-y_n(\lambda))=y_n(\lambda)$$

teorema 1.2 isbotlandi.

Ta’rif 1.5. λ o‘qning (1.1.1) tenglamaning chegaralangan noldan farqli yechimi mavjud bo‘ladigan qismiga spektr deyiladi.

Demak, spektr $E=\{\lambda\in R:-2\le\Delta\lambda\le 2\}$, $R\setminus E$- to‘plamga lakunalar deyiladi.

Endi (1.2.1) tenglamani koefitsientlarini $k\in Z$ butun songa siljitish natijasida hosil qilingan ushbu

$$a_{n-1+k}y_{n-1}+b_{n+k}y_n+a_{n+k}y_{n+1}=\lambda y_n,\ \ n\in Z \tag{1.2.27}$$

tenglamani ko‘rib chiqamiz. (1.2.27) tenglamaning quyidagi

$$\begin{cases}\theta_{0,k}(\lambda)=1\\ \theta_{1,k}(\lambda)=0\end{cases},\qquad \begin{cases}\varphi_{0,k}(\lambda)=0\\ \varphi_{1,k}(\lambda)=1\end{cases}$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimlarini $\theta_{n,k}(\lambda)$ va $\varphi_{n,k}(\lambda)$ orqali belgilaymiz.

$\mu_{1,k},\ \mu_{2,k},\ ...,\ \mu_{N-1,k}$ sonlar orqali ushbu

$$\theta_{N+1,k}(\lambda)=0$$

tenglamaning ildizlarini belgilaymiz.

$$\sigma_{j,k} = sign\left[\theta_{N,k}(\mu_{j,k}) - \frac{1}{\theta_{N,k}(\mu_{j,k})}\right], \quad j = 1, 2, \dots, N-1$$

bo‘ lsin.

Quyidagi belgilashlarni kiritamiz:

$$R(\lambda) = \prod_{j=1}^{2N} (\lambda - \lambda_j) \ , \tag{1.2.28}$$

$$S_k(\lambda) = \prod_{j=1}^{N-1} (\lambda - \mu_{j,k}), \tag{1.2.29}$$

$$Q_k(\lambda) = \frac{\varphi_{N+1,k}(\lambda) - \theta_{N,k}(\lambda)}{\prod_{i=1}^{N} a_n}. \tag{1.2.30}$$

Ushbu munosabat bajariladi:

$$Q_k(\lambda) = (\lambda - b_{k+1}) S_k(\lambda) + S_k(\lambda) \sum_{j=1}^{N-1} \frac{Q_k(\mu_{j,k})}{S_k'(\mu_{j,k})(\lambda - \mu_{j,k})}, \tag{1.2.31}$$

bunda

$$Q_k(\mu_{j,k}) = \tau_{j,k} \sqrt{R(\mu_{j,k})} \ , \tag{1.2.32}$$

va $\tau_{j,k} = sign\, Q_k(\mu_{j,k})$,

$$Q_k^2(\lambda) = R(\lambda) + 4a_k^2 S_{k-1}(\lambda) S_k(\lambda) \ , \tag{1.2.33}$$

$$S_{k+1}(\lambda) = \frac{a_k^2}{a_{k+1}} S_{k-1}(\lambda) - \frac{(\lambda - b_{k+1})}{a_{k+1}^2} Q_k(\lambda) + \frac{(\lambda - b_{k+1})^2}{a_{k+1}^2} S_k(\lambda), \tag{1.2.34}$$

$$Q_{k+1}(\lambda) = 2(\lambda - b_{k+1}) S_k(\lambda) - Q_k(\lambda). \tag{1.2.35}$$

(1.2.31)-(1.2.35) tengliklar diskret Xill tenglamasi uchun teskari masala yechish algoritmini beradi:

Bizga $\{\mu_j, \sigma_j\}_{j=1}^{N-1}$ spektral parametrlar va $\lambda_1, \lambda_2, ..., \lambda_{2N}$ sonlar berilgan. b_k va a_k, $k \in Z$ larni topish algoritmini keltiramiz:

Ushbu

$$b_1 = \frac{\lambda_1 + \lambda_{2N}}{2} + \frac{1}{2}\sum_{j=1}^{N-1}(\lambda_{2j} + \lambda_{2j+1} - 2\mu_j),$$

$$a_0^2 = \frac{\lambda_1^2 + \lambda_{2N}^2}{8} + \frac{1}{8}\sum_{j=1}^{N-1}(\lambda_{2j}^2 + \lambda_{2j+1}^2 - 2\mu_j^2) -$$

$$-\frac{1}{4}\left[\frac{\lambda_1 + \lambda_{2N}}{2} + \frac{1}{2}\sum_{j=1}^{N-1}(\lambda_{2j} + \lambda_{2j+1} - 2\mu_j)\right]^2 - \frac{1}{2}\sum_{j=1}^{N-1}\frac{\sigma_j\sqrt{\prod_{i=1}^{2N}(\mu_j - \lambda_i)}}{\prod_{\substack{i=1\\ i\neq j}}^{N-1}(\mu_j - \mu_i)},$$

$$a_1^2 = \frac{\lambda_1^2 + \lambda_{2N}^2}{8} + \frac{1}{8}\sum_{j=1}^{N-1}(\lambda_{2j}^2 + \lambda_{2j+1}^2 - 2\mu_j^2) -$$

$$-\frac{1}{4}\left[\frac{\lambda_1 + \lambda_{2N}}{2} + \frac{1}{2}\sum_{j=1}^{N-1}(\lambda_{2j} + \lambda_{2j+1} - 2\mu_j)\right]^2 + \frac{1}{2}\sum_{j=1}^{N-1}\frac{\sigma_j\sqrt{\prod_{i=1}^{2N}(\mu_j - \lambda_i)}}{\prod_{\substack{i=1\\ i\neq j}}^{N-1}(\mu_j - \mu_i)}.$$

formulalardan b_1, a_0, a_1 larni topamiz.

1. $\sigma_{j,0} = \sigma_j$, $\mu_{j,0} = \mu_j$, $j = 1, 2, ..., N-1$ bo‘ lsin.
2. (1.2.28), (1.2.29) va (1.2.32) munosabatlardan $R(\lambda)$, $S_0(\lambda)$ va $Q_0(\mu_{j,0})$ larni aniqlaymiz.
3. (1.2.31) tenglikdan $Q_0(\lambda)$ ni topamiz.
4. (1.2.33) dan $S_{-1}(\lambda)$ ni topamiz.
5. $S_1(\lambda)$ ko‘ phadni (1.2.34) orqali aniqlaymiz va uning ildizlari $\{\mu_{j,1}\}_{j=1}^{N-1}$ sonlarni topamiz.
6. (1.2.35) yordamida $Q_1(\lambda)$ ifodani tiklaymiz va uning ishorasi $\tau_{j,1} = sign\, Q_1(\mu_{j,1})$ ni aniqlaymiz.

7. Ushbu

$$b_{k+1} = \frac{\lambda_1 + \lambda_{2N}}{2} + \frac{1}{2}\sum_{j=1}^{N-1}\left(\lambda_{2j} + \lambda_{2j+1} - 2\mu_{j,k}\right),$$

$$a_k^2 = \frac{\lambda_1^2 + \lambda_{2N}^2}{8} + \frac{1}{8}\sum_{j=1}^{N-1}\left(\lambda_{2j}^2 + \lambda_{2j+1}^2 - 2\mu_{j,k}^2\right) -$$

$$-\frac{1}{4}\left[\frac{\lambda_1 + \lambda_{2N}}{2} + \frac{1}{2}\sum_{j=1}^{N-1}\left(\lambda_{2j} + \lambda_{2j+1} - 2\mu_{j,k}\right)\right]^2 - \frac{1}{2}\sum_{j=1}^{N-1}\frac{\sigma_{j,k}\sqrt{\prod_{i=1}^{2N}(\mu_{j,k} - \lambda_i)}}{\prod_{\substack{i=1\\ i\neq j}}^{N-1}(\mu_{j,k} - \mu_{i,k})},$$

tengliklarda $k=1$ ekanligini e’tiborga olib, b_2 va a_2 larni topamiz.

8. Ushbu jarayonni davom qildirish natijasida barcha b_k va a_k, $k \in Z$ sonlarni aniqlaymiz.

I bob bo‘yicha xulosalar

Ushbu bobda Diskret Xill tenglamasi uchun to`g`ri va teskari spektral masala o’rganilgan. Bunda diskret Xill tenglamasi haqidagi asosiy tushunchalar berilgan bo`lib teskari masala yechishda muhim ahamiyatga ega bo`lgan izlar formulasi keltirib chiqarilgan hamda diskret Xill tenglamasi uchun qo`yilgan teskari masalani yechishning effektiv usuli keltirilgan.

II-BOB.
II-Davriy Toda tenglamasini integrallash

2.1-§ II- davriy Toda tenglamasini manbasizholda integrallash

Ushbu paragrafda umumiy davriy Toda tenglamasiga qo‘ yilgan Koshi masalasini $m=2$ bo‘ lgan holda teskari masalalar usulidan foydalanib yechish algoritmini beramiz.

Quyidagi

$$\begin{cases} \dot{a}_n = a_n(b_{n+1}-b_n) + a_n(a_{n+1}^2 - a_{n-1}^2) + a_n(b_{n+1}^2 - b_n^2), \\ \dot{b}_n = 2(a_n^2 - a_{n-1}^2) - 2a_{n-1}^2(b_n + b_{n-1}) + 2a_n^2(b_{n+1}+b_n), \\ a_{n+N} = a_n, \ \ b_{n+N} = b_n, \quad n \in Z, \end{cases} \tag{2.1.1}$$

N - davrli Toda tenglamasini ushbu

$$a_n(0) = a_n^0, \ b_n(0) = b_n^0, \quad n \in Z , \tag{2.1.2}$$

boshlang‘ ich shartlar bilan qaraymiz. Bu erda $a_n^0, \ b_n^0, \ \ n \in Z$ sonlar N-davrli haqiqiy sonlar ketma-ketligi bo‘ lib $\{a_n(t)\}_{-\infty}^{\infty}, \ \ \{b_n(t)\}_{-\infty}^{\infty},$ – noma’lum funksiyalar, hamda ushbu

$$(L(t)y)_n \equiv a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n , \tag{2.1.3}$$

Diskret Xill tenglamasining λ_i xos qiymatlariga mos keluvchi va quyidagi

$$f_1^i(t) = 1, \quad i = 1, 2, ..., 2N . \tag{2.1.4}$$

shart bilan normallangan Floke yechimlaridir. λ_i xos qiymatlar

$$\Delta^2(\lambda) - 4 = 0$$

tenglamani ildizlari. Bu erda $\Delta(\lambda) = \theta_N(\lambda,t) + \varphi_{N+1}(\lambda,t)$, hamda $\theta_n(\lambda,t), \ n \in Z$ va $\varphi_n(\lambda,t), \ n \in Z$ lar (2.1.3) tenglamani quyidagi

$$\theta_0(\lambda,t) = 1, \ \theta_1(\lambda,t) = 0, \quad \varphi_0(\lambda,t) = 0, \ \varphi_1(\lambda,t) = 1$$

boshlang‘ ich shartlarni qanoatlantiruvchi yechimlaridir. (2.1.1) tenglamadagi $\tilde{\theta}_{N+1}(\lambda,t)$ funksiya ushbu $\tilde{\theta}_{N+1}(\lambda,t)=\prod_{j=1}^{N-1}(\lambda-\mu_j(t))$ tenglikdan aniqlanadi, bunda $\mu_1(t),\mu_2(t),\dots,\mu_{N-1}(t)$ lar $\theta_{N+1}(\lambda,t)=0$ tenglamani ildizlari.

Mazkur ishning asosiy natijasi quyidagi teoremadan iborat.

Teorema 3.1. Agar $a_n(t)$, $b_n(t)$, $n\in Z$ funksiyalar (2.1.1)-(2.1.4) masalaning yechimlari bo‘ lsa, u holda (2.1.3) diskret Xill tenglamasining spektri t parametrga bog‘ liq bo‘ lmaydi va $\mu_j(t)$, $j=1,2,\dots,N-1$ spektral parametrlar ushbu

$$\dot{\mu}_j(t)=-2\frac{\sigma_j(t)\cdot\sqrt{\prod_{k=1}^{2N}(\mu_j(t)-\lambda_k)}}{\prod_{\substack{k=1\\k\neq j}}^{N-1}(\mu_j(t)-\mu_k(t))}\cdot\left[1+b_1(t)+\mu_j(t)+\sum_{i=1}^{2N}\frac{\tilde{\theta}_{N+1}(\lambda_i,t)}{\lambda_i-\mu_j(t)}\right],$$

(2.1.5)

tenglamalar sistemasini qanoatlantiradi. Bu erda

$$b_1(t)=\frac{\lambda_1+\lambda_{2N}}{2}+\frac{1}{2}\sum_{k=1}^{N-1}\left(\lambda_{2k}+\lambda_{2k+1}-2\mu_k(t)\right).$$

Isbot: $y^j(t)=(y_0^j(t),y_1^j(t),\dots,y_N^j(t))^T$, $j=1,2,\dots,N-1$ orqali ushbu

$$\begin{cases}(L(t)y)_n\equiv a_{n-1}y_{n-1}+b_ny_n+a_ny_{n+1}=\lambda y_n,\ 1\le n\le N\\ y_1=0,\ y_{N+1}=0.\end{cases}$$

chegaraviy masalaning $\lambda=\mu_j(t)$, $j=1,2,\dots,N-1$ xos qiymatlarga mos keluvchi ortonormal xos vektor-funksiyalarini belgilaymiz. [11] ishdagi natijaga ko‘ ra

$$\dot{\mu}_j(t)=\sum_{n=1}^{N}(2\dot{a}_n(t)y_n^jy_{n+1}^j+\dot{b}_n(t)(y_n^j)^2)$$

bo‘ ladi. Oxirgi tenglikni (2.1.1) dan foydalanib quyidagicha

$$\dot{\mu}_j(t)=\sum_{n=1}^{N}2[a_n(b_{n+1}-b_n)+a_n(a_{n+1}^2-a_{n-1}^2)+a_n(b_{n+1}^2-b_n^2)]y_n^j y_{n+1}^j+$$

$$+\sum_{n=1}^{N}[2(a_n^2-a_{n-1}^2)+2a_n^2(b_{n+1}+b_n)-2a_{n-1}^2(b_n+b_{n-1})](y_n^j)^2+$$

yozamiz. Qulaylik uchun quyidagi belgilashni kiritamiz:

$$H_n=2[a_n(b_{n+1}-b_n)+a_n(a_{n+1}^2-a_{n-1}^2)+a_n(b_{n+1}^2-b_n^2)]y_n^j y_{n+1}^j+$$

$$+[2(a_n^2-a_{n-1}^2)+2a_n^2(b_{n+1}+b_n)-2a_{n-1}^2(b_n+b_{n-1})](y_n^j)^2.$$

Ushbu $u_{n+1}-u_n=H_n$ shartni qanoatlantiruvchi u_n ketma-ketlikni topamiz. u_n ni quyidagi

$$u_n=A_n(y_n^j)^2+2B_n y_n^j y_{n+1}^j+C_n(y_{n+1}^j)^2, \tag{2.1.6}$$

ko‘ rinishda izlaymiz. Bunda $A_n=A_n(t,\mu_j),\ B_n=B_n(t,\mu_j)$ va $C_n=C_n(t,\mu_j)$ hozircha noma’lum koeffitsientlar. Agar ushbu

$$y_{n+2}^j=\frac{1}{a_{n+1}}[(\mu_j-b_{n+1})y_{n+1}^j-a_n y_n^j]$$

tenglikni e’tiborga olsak, u holda quyidagiga ega bo‘ lamiz

$$(A_{n+1}-C_n)(y_{n+1}^j)^2-A_n(y_n^j)^2-2B_n y_n^j y_{n+1}^j+\frac{2B_{n+1}}{a_{n+1}}y_{n+1}^j[(\mu_j-b_{n+1})y_{n+1}^j-a_n y_n^j]+$$

$$+\frac{C_{n+1}}{a_{n+1}^2}(\mu_j-b_{n+1})^2(y_{n+1}^j)^2-\frac{2C_{n+1}}{a_{n+1}^2}a_n(\mu_j-b_{n+1})y_n^j y_{n+1}^j+\frac{C_{n+1}}{a_{n+1}^2}a_n^2(y_n^j)^2=H_n. \tag{2.1.7}$$

(2.1.7) formuladan mazkur

$$-B_n-\frac{a_n}{a_{n+1}}B_{n+1}-\frac{a_n(\mu_j-b_{n+1})}{a_{n+1}^2}C_{n+1}=a_n(b_{n+1}-b_n)+a_n(a_{n+1}^2-a_{n-1}^2)+a_n(b_{n+1}^2-b_n^2), \tag{2.1.8}$$

$$-C_{n-1}+\frac{2(\mu_j-b_n)}{a_n}B_n+\frac{(\mu_j-b_n)^2}{a_n^2}C_n+\frac{a_n^2}{a_{n+1}^2}C_{n+1}=2(a_n^2-a_{n-1}^2)-2a_{n-1}^2(b_n+b_{n-1})+2a_n^2(b_{n+1}+b_n). \tag{2.1.9}$$

tengliklar kelib chiqadi. Ravshanki, ushbu

$$C_n=2a_n^2(b_n+\mu_j+1),\quad B_n=a_n(b_n^2+b_n+a_{n-1}^2-a_n^2-\mu_j-\mu_j^2)$$

ketma-ketliklar (2.1.8) va (2.1.9) sistemalarning yechimi bo‘ ladi. (2.1.6) ga asosan quyidagi tenglikga ega bo‘ lamiz:

$$\dot{\mu}_j(t) = \sum_{n=1}^{N}\{2[a_n(a_{n+1}^2 - a_{n-1}^2) + a_n(b_{n+1}^2 - b_n^2)]y_n^j y_{n+1}^j + [2a_n^2(b_{n+1}+b_n) - 2a_{n-1}^2(b_n + b_{n-1})](y_n^j)^2\} =$$

$$= C_{N+1}(y_{N+2}^j)^2 - C_1(y_2^j)^2 = 2a_0^2(\mu_j(t) + b_1(t) + 1)[(y_N^j)^2 - (y_0^j)^2]. \qquad (2.1.10)$$

$$\dot{\mu}_j(t) = 2a_0^2(\mu_j(t) + b_1(t) + 1)[(y_N^j)^2 - (y_0^j)^2] \qquad (2.1.12)$$

tenglikni hosil qilamiz. Ushbu

$$\left\|\theta^j\right\|^2 = \sum_{n=1}^{N}(\theta_n^j)^2 = a_N\theta_N^j(\theta_{N+1}^j)'\Big|_{\lambda=\mu_j},\ (\theta^j)' = \frac{d\theta^j}{d\lambda},$$

$$(y_0^j)^2 = \frac{(\theta_0^j)^2}{\left\|\theta^j\right\|^2},\ (y_N^j)^2 = \frac{(\theta_N^j)^2}{\left\|\theta^j\right\|^2},$$

tengliklarni e’tiborga olsak (2.1.12) ni quyidagicha yoza olamiz:

$$\dot{\mu}_j(t) = 2a_0\left(\theta_N^j(\mu_j(t),t) - \frac{1}{\theta_N^j(\mu_j(t),t)}\right)\frac{1}{(\theta_{N+1}^j)'\Big|_{\lambda=\mu_j(t)}}\cdot 1 + \mu_j(t) + b_1(t)\ . \qquad (2.1.13)$$

Ravshanki

$$\theta_N^j(\mu_j(t),t) - \frac{1}{\theta_N^j(\mu_j(t),t)} = \sigma_j(t)\sqrt{\Delta^2(\mu_j(t)) - 4}, \qquad (2.1.14)$$

$$\theta'_{N+1}(\lambda)\Big|_{\lambda=\mu_j(t)} = -a_0\left(\prod_{k=1}^{N}a_k\right)^{-1}\prod_{\substack{k=1\\k\neq j}}^{N-1}(\mu_j(t) - \mu_k(t)), \qquad (2.1.15)$$

bunda $\sigma_j(t) = sign\left(\theta_N^j(\mu_j(t),t) - \varphi_{N+1}^j(\mu_j(t),t)\right),\ j = 1, 2, ..., N-1.$

(2.1.14) va (2.1.15) ni (2.1.13) ga qo‘ yib (2.1.5) tenglikga ega bo‘ lamiz.

$$a_{n-1} g_{n-1}^k + b_n g_n^k + a_n g_{n+1}^k = \lambda_k g_n^k .$$

Oxirgi tenglikni t bo‘yicha differensiallab g_n^k ga ko‘paytiramiz va n bo‘yicha yig‘amiz va quyidagiga ega bo‘lamiz:

$$\frac{d\lambda_k}{dt} = \sum_{n=1}^{N} \left\{ 2\dot{a}_n(t) g_n^k g_{n+1}^k + \dot{b}_n(t) \left(g_n^k \right)^2 \right\}. \qquad (2.1.16)$$

(2.1.16) da (2.1.1) ning ko‘rinishidan foydalansak, natijada (2.1.16) ushbu ko‘rinishga keladi:

$$\dot{\lambda}_k(t) = 2\sum_{n=1}^{N} [a_n(a_{n+1}^2 - a_{n-1}^2) + a_n(b_{n+1}^2 - b_n^2)] g_n^k g_{n+1}^k \qquad (2.1.17)$$

(2.1.12) formulani keltirib chiqarishdagi usulni (2.1.17) ga qo‘llasak $\dot{\lambda}_k(t) = 0$ ekanligiga ishonch hosil qilamiz. **Teorema isbotlandi.**

Izoh. Bu teorema (2.1.1)-(2.1.4) masalani yechish usulini beradi. Buning uchun oldin $\{a_n^0\}$ va $\{b_n^0\}$ lar yordamida λ_i, $\mu_j(0)$, $\sigma_j(0)$ spektral berilganlarni topamiz. Keyin, teskari masalani yechish algoritmidan foydalanib $\mu_{j,k}(0)$, $\sigma_{j,k}(0)$ larni topamiz. Yuqorida isbotlangan teoremani qo‘llab $\mu_{j,k}(t)$, $\sigma_{j,k}(t)$ larni topamiz. Izlar formulasini qo‘llab $a_k(t)$, $b_k(t)$ larni topamiz.

2.2-§ Moslangan manbali II-davriy Toda tenglamasini integrallash

Ushbu paragrafda moslangan manbali umumiy davriy Toda tenglamasiga qo’yilgan Koshi masalasini $m=2$ bo’lgan holda teskari masalalar usulidan foydalanib yechish algoritmini beramiz.

Quyidagi

$$\begin{cases} \dot{a}_n = a_n(b_{n+1}-b_n) + a_n(a_{n+1}^2 - a_{n-1}^2) + a_n(b_{n+1}^2 - b_n^2) + a_n \sum_{i=1}^{2N} \tilde{\theta}_{N+1}(\lambda_i, t) \left[(f_{n+1}^i)^2 - (f_n^i)^2 \right], \\ \dot{b}_n = 2(a_n^2 - a_{n-1}^2) - 2a_{n-1}^2(b_n + b_{n-1}) + 2a_n^2(b_{n+1}+b_n) - 2\sum_{i=1}^{2N} \tilde{\theta}_{N+1}(\lambda_i, t) f_n^i (a_n f_{n+1}^i - a_{n-1} f_{n-1}^i), \\ a_{n-1} f_{n-1}^i + b_n f_n^i + a_n f_{n+1}^i = \lambda_i f_n^i, \\ a_{n+N} = a_n, \;\; b_{n+N} = b_n, \;\; (f_{n+N}^i)^2 = (f_n^i)^2, \;\; i = 1, 2, \ldots, 2N, \;\; n \in Z, \end{cases}$$

(2.2.1)

moslangan manbali N - davrli Toda tenglamasini ushbu

$$a_n(0) = a_n^0, \;\; b_n(0) = b_n^0, \quad n \in Z, \tag{2.2.2}$$

boshlang’ich shartlar bilan qaraymiz. Bu erda $a_n^0,\ b_n^0,\ \ n \in Z$ sonlar N-davrli haqiqiy sonlar ketma-ketligi bo’lib $\{a_n(t)\}_{-\infty}^{\infty}$, $\{b_n(t)\}_{-\infty}^{\infty}$, $\{f_n^1(t)\}_{-\infty}^{\infty}$, $\{f_n^2(t)\}_{-\infty}^{\infty}$, $\ldots$, $\{f_n^{2N}(t)\}_{-\infty}^{\infty}$ – noma’lum funksiyalar, hamda $\{f_n^i(t)\}_{-\infty}^{\infty}$ funksiyalar ushbu

$$(L(t)y)_n \equiv a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n, \tag{2.2.3}$$

Diskret Xill tenglamasining λ_i xos qiymatlariga mos keluvchi va quyidagi

$$f_1^i(t) = 1, \quad i = 1, 2, \ldots, 2N. \tag{2.2.4}$$

shart bilan normallangan Floke yechimlaridir. λ_i xos qiymatlar

$$\Delta^2(\lambda) - 4 = 0$$

tenglamani ildizlari. Bu erda $\Delta(\lambda) = \theta_N(\lambda,t) + \varphi_{N+1}(\lambda,t)$, hamda $\theta_n(\lambda,t)$, $n \in Z$ va $\varphi_n(\lambda,t)$, $n \in Z$ lar (2.2.3) tenglamani quyidagi

$$\theta_0(\lambda,t) = 1, \;\; \theta_1(\lambda,t) = 0, \quad \varphi_0(\lambda,t) = 0, \;\; \varphi_1(\lambda,t) = 1$$

boshlang'ich shartlarni qanoatlantiruvchi yechimlaridir. (2.2.1) tenglamadagi $\tilde{\theta}_{N+1}(\lambda,t)$ funksiya ushbu $\tilde{\theta}_{N+1}(\lambda,t)=\prod_{j=1}^{N-1}(\lambda-\mu_j(t))$ tenglikdan aniqlanadi, bunda $\mu_1(t),\mu_2(t),\ldots,\mu_{N-1}(t)$ lar $\theta_{N+1}(\lambda,t)=0$ tenglamani ildizlari.

Mazkur ishning asosiy natijasi quyidagi teoremadan iborat.

Teorema 3.1. Agar $a_n(t)$, $b_n(t)$, $f_i^{\,j}(t)$, $n\in Z$ funksiyalar (2.2.1)-(2.2.4) masalaning yechimlari bo'lsa, u holda (2.2.3) diskret Xill tenglamasining spektri t parametrga bog'liq bo'lmaydi va $\mu_j(t)$, $j=1,2,\ldots,N-1$ spektral parametrlar ushbu

$$\dot{\mu}_j(t)=-2\frac{\sigma_j(t)\cdot\sqrt{\prod_{k=1}^{2N}(\mu_j(t)-\lambda_k)}}{\prod_{\substack{k=1\\k\neq j}}^{N-1}(\mu_j(t)-\mu_k(t))}\cdot\left[1+b_1(t)+\mu_j(t)+\sum_{i=1}^{2N}\frac{\tilde{\theta}_{N+1}(\lambda_i,t)}{\lambda_i-\mu_j(t)}\right],$$

(2.2.5)

tenglamalar sistemasini qanoatlantiradi. Bu erda

$$b_1(t)=\frac{\lambda_1+\lambda_{2N}}{2}+\frac{1}{2}\sum_{k=1}^{N-1}\left(\lambda_{2k}+\lambda_{2k+1}-2\mu_k(t)\right).$$

Isbot: $y^j(t)=(y_0^j(t),y_1^j(t),\ldots,y_N^j(t))^T$, $j=1,2,\ldots,N-1$ orqali ushbu

$$\begin{cases}(L(t)y)_n\equiv a_{n-1}y_{n-1}+b_ny_n+a_ny_{n+1}=\lambda y_n,\ 1\le n\le N\\ y_1=0,\ y_{N+1}=0.\end{cases}$$

chegaraviy masalaning $\lambda=\mu_j(t)$, $j=1,2,\ldots,N-1$ xos qiymatlarga mos keluvchi ortonormal xos vektor-funksiyalarini belgilaymiz. [11] ishdagi natijaga ko'ra

$$\dot{\mu}_j(t)=\sum_{n=1}^{N}(2\dot{a}_n(t)y_n^jy_{n+1}^j+\dot{b}_n(t)(y_n^j)^2)$$

bo'ladi. Oxirgi tenglikni (2.2.1) dan foydalanib quyidagicha

$$\dot{\mu}_j(t)=\sum_{n=1}^{N}2[a_n(b_{n+1}-b_n)+a_n(a_{n+1}^2-a_{n-1}^2)+a_n(b_{n+1}^2-b_n^2)]y_n^j y_{n+1}^j+$$

$$+\sum_{n=1}^{N}[2(a_n^2-a_{n-1}^2)+2a_n^2(b_{n+1}+b_n)-2a_{n-1}^2(b_n+b_{n-1})](y_n^j)^2+$$

$$+\sum_{n=1}^{N}\left\{\sum_{i=1}^{2N}[2a_n\tilde{\theta}_{N+1}(\lambda_i,t)((f_{n+1}^i)^2-(f_n^i)^2)y_n^j y_{n+1}^j]\right\}+$$

$$+\sum_{n=1}^{N}\left\{\sum_{i=1}^{2N}[2a_n\tilde{\theta}_{N+1}(\lambda_i,t)(f_n^i f_{n+1}^i)(y_n^j)^2-2a_{n-1}\tilde{\theta}_{N+1}(\lambda_i,t)(f_{n-1}^i f_n^i)(y_n^j)^2]\right\}$$

yozamiz. Qulaylik uchun quyidagi belgilashni kiritamiz:

$$F_i^j=\tilde{\theta}_{N+1}(\lambda_i,t)\sum_{n=1}^{N}\{2a_n[(f_{n+1}^j)^2-(f_n^j)^2]y_n^j y_{n+1}^j+$$

$$+2a_n(f_n^i f_{n+1}^i)(y_n^j)^2-2a_{n-1}(f_{n-1}^i f_n^i)(y_n^j)^2\},$$

$$H_n=2[a_n(b_{n+1}-b_n)+a_n(a_{n+1}^2-a_{n-1}^2)+a_n(b_{n+1}^2-b_n^2)]y_n^j y_{n+1}^j+$$

$$+[2(a_n^2-a_{n-1}^2)+2a_n^2(b_{n+1}+b_n)-2a_{n-1}^2(b_n+b_{n-1})](y_n^j)^2.$$

Ushbu $u_{n+1}-u_n=H_n$ shartni qanoatlantiruvchi u_n ketma-ketlikni topamiz. u_n ni quyidagi

$$u_n=A_n(y_n^j)^2+2B_n y_n^j y_{n+1}^j+C_n(y_{n+1}^j)^2, \tag{2.2.6}$$

ko’rinishda izlaymiz. Bunda $A_n=A_n(t,\mu_j),\ B_n=B_n(t,\mu_j)$ va $C_n=C_n(t,\mu_j)$ hozircha noma’lum koeffitsientlar. Agar ushbu

$$y_{n+2}^j=\frac{1}{a_{n+1}}[(\mu_j-b_{n+1})y_{n+1}^j-a_n y_n^j]$$

tenglikni e’tiborga olsak, u holda quyidagiga ega bo’lamiz

$$(A_{n+1}-C_n)(y_{n+1}^j)^2-A_n(y_n^j)^2-2B_n y_n^j y_{n+1}^j+\frac{2B_{n+1}}{a_{n+1}}y_{n+1}^j[(\mu_j-b_{n+1})y_{n+1}^j-a_n y_n^j]+$$

$$+\frac{C_{n+1}}{a_{n+1}^2}(\mu_j-b_{n+1})^2(y_{n+1}^j)^2-\frac{2C_{n+1}}{a_{n+1}^2}a_n(\mu_j-b_{n+1})y_n^j y_{n+1}^j+\frac{C_{n+1}}{a_{n+1}^2}a_n^2(y_n^j)^2=H_n. \tag{2.2.7}$$

(3.7) formuladan mazkur

$$-B_n-\frac{a_n}{a_{n+1}}B_{n+1}-\frac{a_n(\mu_j-b_{n+1})}{a_{n+1}^2}C_{n+1}=a_n(b_{n+1}-b_n)+a_n(a_{n+1}^2-a_{n-1}^2)+a_n(b_{n+1}^2-b_n^2),$$

(2.2.8)

$$-C_{n-1}+\frac{2(\mu_j-b_n)}{a_n}B_n+\frac{(\mu_j-b_n)^2}{a_n^2}C_n+\frac{a_n^2}{a_{n+1}^2}C_{n+1}=2(a_n^2-a_{n-1}^2)-2a_{n-1}^2(b_n+b_{n-1})+2a_n^2(b_{n+1}+b_n)$$

.

(2.2.9)

tengliklar kelib chiqadi. Ravshanki, ushbu

$$C_n=2a_n^2(b_n+\mu_j+1),\quad B_n=a_n(b_n^2+b_n+a_{n-1}^2-a_n^2-\mu_j-\mu_j^2)$$

ketma-ketliklar (2.2.8) va (2.2.9) sistemalarning yechimi bo'ladi. (2.2.6) ga asosan quyidagi tenglikga ega bo'lamiz:

$$\dot{\mu}_j(t)=\sum_{n=1}^{N}\{2[a_n(a_{n+1}^2-a_{n-1}^2)+a_n(b_{n+1}^2-b_n^2)]y_n^j y_{n+1}^j+[2a_n^2(b_{n+1}+b_n)-2a_{n-1}^2(b_n+b_{n-1})](y_n^j)^2\}+$$

$$+\sum_{i=1}^{2N}F_i^j=C_{N+1}(y_{N+2}^j)^2-C_1(y_2^j)^2+\sum_{i=1}^{2N}F_i^j=2a_0^2(\mu_j(t)+b_1(t)+1)[(y_N^j)^2-(y_0^j)^2]+\sum_{i=1}^{2N}F_i^j\ .$$

(2.2.10)

F_i^j funksiyaning ko'rinishidan quyidagilar kelib chiqadi

$$F_i^j=\tilde{\theta}_{N+1}(\lambda_i,t)\sum_{n=1}^{N}[a_n f_{n+1}^i y_{n+1}^j(y_n^j f_{n+1}^i-y_{n+1}^j f_n^i)+2a_n f_n^i y_n^j(y_n^j f_{n+1}^i-f_n^i y_{n+1}^j)]=$$

$$=\tilde{\theta}_{N+1}(\lambda_i,t)\left[2\sum_{n=1}^{N}f_{n+1}^i y_{n+1}^j T_n+2\sum_{n=0}^{N}f_{n+1}^i y_{n+1}^j T_{n+1}\right]=\tilde{\theta}_{N+1}(\lambda_i,t)\left[2\sum_{n=1}^{N}f_{n+1}^i y_{n+1}^j(T_n+T_{n+1})\right]=$$

$$=2\tilde{\theta}_{N+1}(\lambda_i,t)\sum_{n=1}^{N}\frac{1}{\lambda_i-\mu_j(t)}(T_{n+1}-T_n)(T_{n+1}+T_n)=\frac{2\tilde{\theta}_{N+1}(\lambda_i,t)}{\lambda_i-\mu_j(t)}(T_{N+1}^2-T_1^2),$$

bunda $T_n=a_n(y_n^j f_{n+1}^i-y_{n+1}^j f_n^i)$. Shunday qilib, ushbu

$$F_i^j=\frac{2\tilde{\theta}_{N+1}(\lambda_i,t)a_0^2(f_1^i)^2}{\lambda_i-\mu_j(t)}[(y_N^j)^2-(y_0^j)^2] \qquad (2.2.11)$$

tenglikga ega bo'ldik. (2.2.11) formulani (2.2.10) ga qo'ysak, quyidagi

$$\dot{\mu}_j(t) = 2a_0^2[(y_N^j)^2 - (y_0^j)^2]\left\{1 + \mu_j(t) + b_1(t) + \sum_{i=1}^{2N} \frac{\tilde{\theta}_{N+1}(\lambda_i, t)}{\lambda_i - \mu_j(t)}\right\} \qquad (2.2.12)$$

tenglikni hosil qilamiz. Ushbu

$$\left\|\theta^j\right\|^2 = \sum_{n=1}^{N} (\theta_n^j)^2 = a_N \theta_N^j (\theta_{N+1}^j)'\Big|_{\lambda=\mu_j}, \ (\theta^j)' = \frac{d\theta^j}{d\lambda},$$

$$(y_0^j)^2 = \frac{(\theta_0^j)^2}{\left\|\theta^j\right\|^2}, \ (y_N^j)^2 = \frac{(\theta_N^j)^2}{\left\|\theta^j\right\|^2},$$

tengliklarni e’tiborga olsak (2.2.12) ni quyidagicha yoza olamiz:

$$\dot{\mu}_j(t) = 2a_0\left(\theta_N^j(\mu_j(t), t) - \frac{1}{\theta_N^j(\mu_j(t), t)}\right)\frac{1}{(\theta_{N+1}^j)'\Big|_{\lambda=\mu_j(t)}} \cdot \left\{1 + \mu_j(t) + b_1(t) + \sum_{i=1}^{2N} \frac{\tilde{\theta}_{N+1}(\lambda_i, t)}{\lambda_i - \mu_j(t)}\right\}. \qquad (2.2.13)$$

Ravshanki

$$\theta_N^j(\mu_j(t), t) - \frac{1}{\theta_N^j(\mu_j(t), t)} = \sigma_j(t)\sqrt{\Delta^2(\mu_j(t)) - 4}, \qquad (2.2.14)$$

$$\theta'_{N+1}(\lambda)\Big|_{\lambda=\mu_j(t)} = -a_0\left(\prod_{k=1}^{N} a_k\right)^{-1} \prod_{\substack{k=1 \\ k\neq j}}^{N-1} (\mu_j(t) - \mu_k(t)), \qquad (2.2.15)$$

bunda $\sigma_j(t) = sign\left(\theta_N^j(\mu_j(t), t) - \varphi_{N+1}^j(\mu_j(t), t)\right), \ j = 1, 2, ..., N-1.$

(2.2.14) va (2.2.15) ni (2.2.13) ga qo’yib (2.2.5) tenglikga ega bo’lamiz.

Endi $\lambda_k(t)$ xos qiymatlarni t parametrga bog’liq emasligini ko’rsatamiz. $g_n^k(t)$ - funksiyalar $L(t)$ operatorning $\lambda_k(t)$, $k = 1, 2, ..., 2N$ xos qiymatlarga mos keluvchi normallangan xos funksiyalari bo’lsin, ya’ni

$$a_{n-1}g_{n-1}^k + b_n g_n^k + a_n g_{n+1}^k = \lambda_k g_n^k.$$

Oxirgi tenglikni t bo’yicha differensiallab g_n^k ga ko’paytiramiz va n bo’yicha yig’amiz va quyidagiga ega bo’lamiz:

$$\frac{d\lambda_k}{dt} = \sum_{n=1}^{N} \left(2\dot{a}_n(t) g_n^k g_{n+1}^k + \dot{b}_n(t) \left(g_n^k\right)^2\right). \tag{2.2.16}$$

(2.2.16) da (2.2.1) ning ko'rinishidan foydalansak, natijada (2.2.16) ushbu ko'rinishga keladi:

$$\dot{\lambda}_k(t) = 2\sum_{n=1}^{N} [a_n(a_{n+1}^2 - a_{n-1}^2) + a_n(b_{n+1}^2 - b_n^2)] g_n^k g_{n+1}^k +$$

$$+\sum_{n=1}^{N} [2a_n^2(b_{n+1} + b_n) - 2a_{n-1}^2(b_n + b_{n-1})](g_n^k)^2 + \sum_{i=1}^{2N} F_i^{\,j} \,. \tag{2.2.17}$$

(2.2.12) formulani keltirib chiqarishdagi usulni (2.2.17) ga qo'llasak $\dot{\lambda}_k(t) = 0$ ekanligiga ishonch hosil qilamiz. **Teorema isbotlandi.**

Izoh. Bu teorema (2.2.1)-(2.2.4) masalani yechish usulini beradi. Buning uchun oldin $\{a_n^0\}$ va $\{b_n^0\}$ lar yordamida λ_i, $\mu_j(0)$, $\sigma_j(0)$ spektral berilganlarni topamiz. Keyin, teskari masalani yechish algoritmidan foydalanib $\mu_{j,k}(0)$, $\sigma_{j,k}(0)$ larni topamiz. Yuqorida isbotlangan teoremani qo'llab $\mu_{j,k}(t)$, $\sigma_{j,k}(t)$ larni topamiz. Izlar formulasini qo'llab $a_k(t)$, $b_k(t)$ larni topamiz.

2.3-§ Misollar

Yuqorida isbotlangan teoremani qo‘ llanilishini aniq misol yordamida ko‘ rsatamiz. (2.2.1)-(2.2.2) masalani ushbu

$$(a_n^0)^2 = \frac{5}{2} - (-1)^n \frac{3}{2}, \quad b_n^0 = 0,\ n \in Z$$

boshlang‘ ich shartlarda qaraymiz. Bu holda

$$N = 2,\ \lambda_1 = -3,\ \lambda_2 = -1,\ \lambda_3 = 1,\ \lambda_4 = 3,\quad \mu_1(0) = 0,\ \sigma_1(0) = 1.$$

Yuqorida keltirilgan izohdagi ketma-ketlikni qo‘ llab, quyidagiga ega bo‘ lamiz

$$a_n^2(t) = \frac{5}{2} - \frac{1}{2}\mu^2(t) - (-1)^n \frac{\sigma(t)}{2}\sqrt{(\mu^2(t)-1)(\mu^2(t)-9)}\,,$$

$$b_n(t) = (-1)^n \mu(t),\ n \in Z,$$

$$f_0^k(t) = \frac{\lambda_k^2 - \mu^2(t) - \sigma(t)\sqrt{(\mu^2(t)-9)(\mu^2(t)-1)}}{2a_0(t)(\lambda_k - \mu(t))}\,,\ \ f_1^k(t) = 1,\ k = 1,2,3,4,$$

bu erda $\mu(t)$ ushbu

$$\frac{d\mu(t)}{dt} = 10\sigma(t)\sqrt{(1-\mu^2(t))(9-\mu^2(t))}\,,$$

tenglama va quyidagi

$$\mu(0) = 0,\ \sigma(0) = 1$$

boshlang‘ ich shartlardan aniqlanadi. $\mu(t)$ harakatlanib o‘ z lakunasining, ya’ni $[-1, 1]$ kesmaning oxiriga kelganda $\sigma(t)$ funksiya o‘ z ishorasini qarama-qarshisiga o‘ zgartiradi. Ushbu $\mu(t) = \sin x(t)$ almashtirishni kiritamiz. $sign\sigma(t) \cdot sign\,(\cos x(t)) = \sigma(0)$ ekanligini e’tiborga olsak, quyidagiga ega bo‘ lamiz:

$$a_n^2(t) = \frac{5}{2} - \frac{1}{2}\sin^2 x(t) - (-1)^n \frac{3}{2}\cos x(t)\sqrt{1 - \left(\frac{1}{3}\right)^2 \sin^2 x(t)}\,,$$

$$b_n(t) = (-1)^n \sin x(t),\quad n \in Z,$$

$$f_0^k(t)=\frac{\lambda_k^2-\sin^2 x(t)-\cos x(t)\sqrt{9-\sin^2 x(t)}}{2a_0(t)(\lambda_k-\sin x(t))},\quad f_1^k(t)=1,\ k=1,2,3,4,$$

Bu yerda $x(t)$ funksiya quyidagi

$$\dot{x}(t)=30\sqrt{1-\left(\frac{1}{3}\right)^2\sin^2 x(t)}\ ,$$

$$x(0)=0.$$

Koshi masalasining yechimi. Ma'lumki ([15] ga qarang)

$$x(t)=am\left(30t,\frac{1}{3}\right).$$

Bu yerda am Yakobi amplituda funksiyasi. Shunday qilib, quyidagilarga ega boʻ lamiz:

$$a_n(t)=\sqrt{\frac{5}{2}-\frac{1}{2}sn^2\left(30t,\frac{1}{3}\right)-(-1)^n\frac{3}{2}cn\left(30t,\frac{1}{3}\right)dn\left(30t,\frac{1}{3}\right)},$$

$$b_n(t)=(-1)^n sn\left(30t,\frac{1}{3}\right),$$

$$f_0^k(t)=\frac{\lambda_k^2-sn^2\left(30t,\frac{1}{3}\right)-3cn\left(30t,\frac{1}{3}\right)dn\left(30t,\frac{1}{3}\right)}{2a_0(t)\left[\lambda_k-sn\left(30t,\frac{1}{3}\right)\right]},\quad f_1^k(t)=1,\ k=1,2,3,4,$$

sn, cn va dn lar Yakobi elliptik funksiyalari.

II bob bo‘yicha xulosalar

Ushbu bobda spektral parametrning vaqt bo’yicha o’zgarish dinamikasini ifodalovchi muxtor differensial tenglamalmar sistemasi keltirib chiqarilgan va II- davriy Toda tenglamasini manbali va manbasiz hollarda yechish algoritmi berilgan. Bir zonali holda yechim uchun aniq tasvir olingan va bu misollar orqali ko’rsatib berilgan.

III-BOB. INTEGRALLANUVCHI TENGLAMALAR SISTEMALARI

3.1-§ Simmetriyalar va almashtirishlar gruppalari

Simmetriya o'zi nima? Masalan, aylananing simmetriyasinin ko'rib chiqamiz. O'z-o'zidan ma'lumki aylanani

(1) o'z markaziga nisbatan burish

yoki

(2) diametrning aksiga nisbatan

burilsa yana aylananing o'zi bo'ladi.

Buni aniq matematik tilda qanday ifodalaymiz? (x, y) koordinatalar tekisligida aylana quyidagi tenglamani qanoatlantiruvchi nuqtalar to'plamidan iborat bo'ladi.

$$x^2 + y^2 = r^2. \tag{3.1.1}$$

Tekisliklarni burish quyidagicha chiziqli almashtirish orqali amalga oshiriladi

$$\begin{matrix} x' \\ y' \end{matrix} = \begin{matrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{matrix} \begin{matrix} x \\ y \end{matrix}. \tag{3.1.2}$$

aksi bilan burish ham xuddi tekislikni burish kabi chiziqli almashtirish bo'ladi

$$\begin{matrix} x' \\ y' \end{matrix} = \begin{matrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{matrix} \begin{matrix} x \\ y \end{matrix}. \tag{3.1.3}$$

Quyidagi fo'rmula bilan berilgan almashtirish (1.1) tenglama shartlarini qanoatlantirsa,bu formula aylana simmetriyasi bo'ladi

$$\begin{matrix} x' \\ y' \end{matrix} = \begin{matrix} a & b \\ c & d \end{matrix} \begin{matrix} x \\ y \end{matrix} \tag{3.1.4}$$

Boshqacha aytganda (x, y) nuqtalar (3.1.1) tenglamani qanoatlantirganida, (x', y') nuqta (3.1.1) tenglamaning yechimi bo'lsa, u holda (3.1.4) akslantirish (3.1.1) tenglamaning simmetriya tenglamasi deyiladi.

(3.1.2) almashtirishni $T(\theta)$ orqali , va (3.1.3)- almashtirishni $S(\theta)$ orqali belgilab olamiz. Barcha teskarilanuvchi chiziqli almashtirishlar to'plami ko'paytirish amaliga nisbatan gruppa hosil qiladi. Agar deteriminanti no'l bo'lmagan ikkita $T_1 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$ va $T_2 = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$ matritsalar uchun ko'paytma quyidagicha aniqlansa

$T_1 \cdot T_2 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, bu holda Gruppaning shartlari bajariladi:

(1) assotsiativlik: $(T_1 \cdot T_2) \cdot T_3 = T_1 \cdot (T_2 \cdot T_3)$:

(2) Birlik element mavjudligi: $T \cdot \mathrm{E} = \mathrm{E} \cdot T$, bu yerda $\mathrm{E} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$;

(3) Teskari element mavjudligi: $T \cdot T^{-1} = T^{-1} \cdot T = \mathrm{E}$.

Xususan, (3.1.1) aylanani o'ziga o'tkazuvchi elementlar to'plami gruppa hosil qiladi. Unga aylananing almashtirishlar gruppasi deyiladi. $T(\theta) = T(\theta')$ yoki $S(\theta) = S(\theta')$ bo'lishi uchun $\theta = \theta' + 2\pi n$ bo'lishi zarur va yetarli. Bu yerda n-butun son. Bu holda gruppa shartlari quyidagi ko'rinishda bo'ladi

$$\begin{aligned} &T(\theta_1) \cdot T(\theta_2) = T(\theta_1 + \theta_2). \\ &T(\theta_1) \cdot S(\theta_2) = S(\theta_1) \cdot T(-\theta_2) = S(\theta_1 + \theta_2) \\ &S(\theta_1) \cdot S(\theta_2) = T(\theta_1 - \theta_2). \end{aligned} \tag{3.1.5}$$

Agar almashtirishlarning o'zlarini qaramasdan ularga mos keluvchi (3.1.5) qoidalarni qarasak, aylana simmetriyalari abstrakt gruppa shartlari yordamida tavsiflanadi.

Faqat $T(\theta)$ almashtirishni qaraydigan bo'lsak. Agar $\theta = 0$ bo'lsa, u holda $T(0) = \mathrm{E}$ bo'lib, θ o'zgarganda, biz quyidagi almashtirishni ko'rishimiz mumkin

$$\begin{pmatrix} x(\theta) \\ y(\theta) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}. \tag{3.1.6}$$

bu jarayonda, (3.1.1) algebraik tenglamaning (x, y) yechimi aylana bo'ylab uzluksiz ko'chib yuradi.

(3.1.6) tenglikni θ bo'yicha differensiallaydigan bo'lsak, quyidagiga ega bo'lamiz

$$\frac{d}{d\theta}\begin{pmatrix} x(\theta) \\ y(\theta) \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}. \tag{3.1.7}$$

(3.1.6) almashtirish (3.1.7) tenglama va quyidagi boshlang'ich shartlar bilan birgalikda bir qiymatli aniqlanadi

$$\begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{3.1.8}$$

Ushbu

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

matritsa aylanishning infinitezimial generatori bo'ladi. Bu fikr quyida tushuntiriladi. Quyidagi munosabat o'rinli bo'lishini tekshirish qiyin emas

$$T(\theta) = e^{\theta\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}.$$

Bu tenglik o'ng tomonini θ ning kichik qiymatlarida darajali qatorga yoysak, quyidagi assimptotikaga ega bo'lamiz

$$T(\theta) = 1 + \theta\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + O(\theta^2). \tag{3.1.9}$$

Umumiy holda, agar θ-parametrga bog'liq bo'lgan $R(\theta)$ -almashtirish, quyidagi

$$R(\theta_1 + \theta_2) = R(\theta_1)R(\theta_2),$$

$$R(\theta) = 1 + \theta X + O(\theta^2)$$

tengliklarni qanoatlantirsa, u holda X birparametrli $R(\theta) = e^{\theta X}$ almashtirshning infinitezimial generatori bo'ladi. Agar $R(\theta)$ almashtirish f ga ta'sir ko'rsatsa, uni quyidagicha yozamiz

$$f^R(\theta) = R(\theta)f$$

va

$$\frac{df^R}{d\theta} = \frac{d}{d\theta}R(\theta)f = Xf^R(\theta). \tag{3.1.10}$$

Biz asosan funksiyalarning almashtirishlarini qaraymiz (jumladan infinitezimial almashtirishlarini ham). Masalan, ikki o'zgaruvchili $f(x,y)$ funksiya uchun quyidagi differensial tenglamani ko'rib chiqamiz

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-r^2\right) f(x,y) = 0. \qquad (3.1.11)$$

Bu yerda r miqdor (x,y) ga bog'liq bo'lmagan konstanta. (3.1.1) algebraik tenglama yechimlarini ikki o'lchamli (x,y) tekislikdan qidiramiz. (3.1.11) differensial tenglamaning yechimi $f(x,y)$ ni cheksiz o'lchamli ikki o'zgaruvchili vektor fazodan izlaymiz. (x,y) tekislikni burish ushbu

$$(T(\theta)\,\mathrm{g})(x,y) = g(x(-\theta),y(-\theta))$$

formula bilan ifodalanadigan $\mathrm{g} \to T(\theta)\,\mathrm{g}$ ta'sirni keltirib chiqaradi.

Ushbu $f^T(x,y,\theta) = (T(\theta) f)(x,y)$ almashtirishni qaraydigan bo'lsak, quyidagi infinitezimial almashtirishga ega bo'lamiz

$$\frac{\partial}{\partial\theta} f^T(x,y,\theta) = \left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right) f^T(x,y,\theta).$$

Shuning uchun $x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}$ operator almashtirishning infinitezimial generatori bo'ladi.

Agar f funksiya (3.1.11) tenglamaning yechimi bo'lsa, u holda $T(\theta) f$ ham shu tenglmaning yechim bo'ladi. Shu bilan birga (3.1.11) tenglama parallel ko'chirishga nisbatan ham invariant $(x,y) \to (x+a,y+b)$. Funksiyalar fazosida parallel ko'chirish tenglamasi quyidagi formula bilan beriladi:

$$f(x+a,y+b) = e^{a\frac{\partial}{\partial x}-b\frac{\partial}{\partial x}} f(x,y). \qquad (3.1.12)$$

Bu yerda $\frac{\partial}{\partial x}$ va $\frac{\partial}{\partial y}$ hosilalar siljishning infinitezimial generatori bo'ladi.

(3.1.12) tenglikdagi eksponentani qatorga yoyadigan bo'lsak, f funksiyaning (x,y) nuqta atrofidagi Teylor yoyilmasiga ega bo'lamiz.

Aytaylik X va Y chiziqli differensial operatorlar ushbu $e^{\theta X}$ va $e^{\theta Y}$ gruppa operatorlarining infentezimial generatorlari bo’lsin. Ularning ko’paytmasini qaraymiz. A va B operatorlarning kommutatori deb quyidagi ayirmaga aytiladi

$$A, B \ = AB - BA.$$

Bevosita hisoblashlar yordamida quyidagiga ega bo’lamiz

$$e^{\theta X} e^{\theta Y} = e^{\theta X + \theta Y + \frac{1}{2} \theta^2 \ X, Y \ + \frac{1}{12} \theta^3 \ X - Y[X, Y] \ + \dots} .$$

Bu formulaning o’ng tomonidagi eksponentada “uch nuqta” θ bo’yicha yuqori tartibga ega bo’lgan qo’shiluvchilarni bildiradi. Bu formuladagi ko’paytmadan tashqari barcha yig’indini [·,·] kommutatordan foydalangan holda ifodalashni ko’rsatish mumkin. Agar $X, Y \ = 0$ tenglik, X va Y operatorlar kommutativligini bildiradi, bu holda $e^{\theta X} e^{\theta Y} = e^{\theta(X+Y)}$ bo’ladi. Bu holda $e^{\theta X} e^{\theta Y}$ ikki almashtirishning kompozitsiasi $X + Y$ generatorga mos keluvchi $e^{\theta(X+Y)}$ almshtirish bilan ustma-ust tushadi. Umumiy holda $e^{\theta X} e^{\theta Y}$ va $e^{\theta(X+Y)}$ almashtirishlar ustma-ust tushmasligi mumkin, ammo ularning ayirmasi infinitezimial generatorlarning kommutatori orqali ifodalanadi.

g vektor fazoning ixtiyoriy ikki elementi $X, Y \in g$ uchun ularning $[X, Y] \in g$ kommutatori aniqlangan bo’lsa va quydagi munosabatlar bajarilsa:

$$\begin{aligned} &(1) \qquad X, Y \ = -[Y, X], \\ &(2) \qquad X, Y \ , Z \ + \ Y, Z \ , X \ + [\ Z, X \ , Y], \qquad (3.1.13) \\ &(3) \qquad \alpha X + \beta Y, Z \ = \alpha \ X, Z \ + \beta [Y, Z], \end{aligned}$$

g vektor fazoga Li algebrasi deyiladi. Agar yaqinlashish inobatga olinmasa, g Li algebrasining barcha quyidagi almashtirishlari

$$G = \{e^X : X \in g\}$$

gruppa tashkil qiladi.

3.1-§ Korteveg- de Friz tenglamasi va Toda zanjirining simmetriyalari

Yuqorida ko'rsatilganidek aylanishning infinitezimial generatorini qo'llashning ikki xil usuli mavjud: ikki o'lchamili fazoda

$$\begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix}$$

operator orqali, cheksiz o'lchamli fazoda esa $x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$ operator orqali.

Nochiziqli infinitezimial almashtirishlarni ham qarash mumkin. Ikki o'zgaruvchili $u(x,t)$ funksiya uchun quyidagi differensial tenglamani qaraymiz:

$$\frac{\partial u}{\partial t} = u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3}. \qquad (3.2.1)$$

Bu tenglamaga Korteveg-de Friz tenglamasi deyiladi. (3.2.1) tenglamaning $u\frac{\partial u}{\partial x}$ va $\frac{\partial^3 u}{\partial x^3}$ hadlari oldidagi koeffitsientlari birga teng qilib tanlangan, ammo ular nolmas o'zgarmas bo'lishi ham mumkin. Bu o'zgarmaslar $t,\ x$ va u larni almashtirish yordamida yo'qotiladi.

KdF tenglamasi u funksiyaning quyidagi

$$K\ u\ = u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} \qquad (3.2.2)$$

operator bilan beriladigan infinitezimial almashtirishini tavsiflaydi.

Umuman olganda $\frac{\partial u}{\partial t} = K(u)$ ko'rinishidagi tenglama evolyutsion tenglama deyiladi. $K(u)$-operatorning tuzulishiga bog'liq ravishda tenglama chiziqli yoki nochiziqli deyiladi. Agar $K(u)$-chiziqli operator bo'lsa, u holda $u \to K(u)$ almashtirish infinitezimial generator bo'ladi.

Bundan boshlab $K(u)$-hatto nochiziqli bo'lgan holda ham infinitesimial generator deb hisoblaymiz. Evolyutsion tenglamani funksiyaning infinitezimial almashtirishi sifatida talqin qilamiz va bu almashtirishlar orasidan KdF tenglamsining simmetriasini qidiramiz. Masalani quyidagicha qo'yamiz: KdF tenglamasi

$$\frac{\partial u}{\partial t} = K\ u \qquad 3.2.3$$

quyidagicha simmetriyaga egami

$$\frac{\partial u}{\partial s} = K \ u \ . \qquad (3.2.4)$$

(3.2.3) tenglama (3.2.4) simmetriyaga ega degani nimani bildirishini aniqlashtiramiz. Buning uchun uch o’zgaruvchili $u(x,t,s)$ funksiyani qaraymiz. Kelgusida foydalanishda qulaylik uchun hosilalarni quyidagicha belgilaymiz:

$$\frac{\partial u}{\partial t} = u_t, \qquad \frac{\partial^3 u}{\partial x^3} = u_{xxx} = u_{3x} \ldots$$

u funksiyaning va uning $u_x, u_{2x}, u_{3x} \ldots$ hosilalarining polinomi x ga nisbatan u funksiyaning differensial polinomi deyiladi. Masalan (3.2.2) polinom u funksiya-ning differensial polinomi bo’ladi.

$K(u)$ ifoda u funksiyaning differensial polinomi bo’lsin. (3.2.4)ni s vaqtga mos keluvchi evolyutsion tenglama sifatida qaraymiz va u $u(x,t,s=0)$ boshlang’ich shartda yechimga ega bo’lsin. Boshqacha aytganda $s=0$ vaqtda $u(x,t,s=0)$ ikki o’zgaruvchili funksiya (3.2.4) yechib quyidagiga ega bo’lamiz $u(x,t,s=\Delta s)$, Δs vaqtda. (3.2.4) tengalma KdF tenglamasining simmetriyasi bo’lishini ayta olamiz, bundan, agar $u \ x,t,s=0 \quad s=0$ vaqtda (3.2.3) tenglamaning yechimi bo’lsa u holda $u(x,t,s)$ ham s vaqtda tenglamaning yechimi bo’ladi.

t va s o’zgaruvchilarni teng kuchli deb qarab qo’yilgan savolimizni quyidagicha o’zgartirishimiz mumkin: t va s-ikkisi ham vaqtga bog’liq bo’lgan erkli o’zgaruvchilar va bizga quyidagi funksiya berilgan $u(x,t=0,s=0)$, $t=0$ va $s=0$. Bu holda $u(x,\Delta t,\Delta s)$ funksiyani $\Delta t, \Delta s$ vaqtda aniqlashnig ikki xil usuli mavjud:

$$\begin{array}{ccc} u \ x,t=\Delta t,s=0 & \xrightarrow{A_2} & u \ x,t=\Delta t,s=\Delta s \\ A_1 \uparrow & & B_2 \uparrow \\ u \ x,t=0,s=0 & \xrightarrow{B_1} & u \ x,t=0,s=\Delta s \end{array} \qquad (3.2.5)$$

Bu diagrammada yuqoriga yo’nalgan strelka (3.2.3) tenglamaning yechimini bildiradi, o’ngga yo’nalgani (3.2.4) tenglamaning yechimini bildiradi. Agar A_1, A_2 va B_1, B_2 strelkalar kompozitsiyasi bir xil natija bersa u holda (3.2.4) tenglama (3.2.3) tenglamaning simmetriyasi bo’ladi. (3.1.2) da $\Delta t, \Delta s$ larni juda kichik deb qarab limitga o'tadigan bo'lsak, $A_1 \circ A_2 = B_1 \circ B_2$ tenglik bajarilishi uchun, quyidagi tenglik o’rinli bo’lishi zarur

$$\frac{\partial}{\partial s} K \quad u \quad = \frac{\partial}{\partial t} K \quad u \tag{3.2.6}$$

(3.2.6) tenglik $K(u)$ ni ixtiyoriy tanlaganda ham bajariladimi? Masalan $K \quad u \quad = u^2$, kabi tanlasak, u holda (3.2.6) tenglikning chap qismi quyidagicha

$$(uu_x + u_{3x})_s = u^2 u_x + u(u^2)_x + (u^2)_{3x} = 3u^2 u_x + 6u_x u_{xx} + 2uu_{3x}$$

o’ng qismi

$$(u^2)_t = 2u^2 u_x + 2uu_{3x}$$

(3.2.6) tenglamaga quyidagi $0 = u^2 u_x + 6u_x u_{xx}$ qo’shimcha shart kiritiladi.

Bundan ko`rinadiki $K \quad u$ ixtiyoriy tanlanganda (3.2.3) va (3.2.4) tenglamalar bir biriga zid va (3.2.4) tenglama (3.2.3) tenglamaning simmetriyasi bo’lmaydi.

Agar (3.2.2) dagi u funksiyaning darajasini 2, u_x hosilaning- darajasi 3, u_{xx} hosilaning- darajasini 4 bo’lsa, u holda tenglikni o’ng qismi darajasi 5 va bir jinsli bo’ladi. Simmetriyani qaraganda almashtirishning infinitesimial generatorini bir jinsli differensial polinom deb qaraymiz.

Darajasi 7 ga teng bo’lgan bir jinsli differensial polinomning umumiy ko’rinishi quyidagicha bo’ladi:

$$C_1 u^2 u_x + C_2 uu_{3x} + C_3 u_x u_{xx} + C_4 u_{5x} \tag{3.2.7}$$

(3.2.4) tenglamadagi $K \quad u$ o’rniga (3.2.7) qo’yib hisoblasak $C_1 = 1$ bo’ladi. Hisoblashlardan keyin (3.2.3) va (3.2.4) tenglamalar mosligidan C_i- lar bir qiymatli aniqlanadi:

$$\frac{\partial u}{\partial s} = u^2 u_x + 2uu_{3x} + 4u_x u_{xx} + \frac{6}{5} u_{5x}. \tag{3.2.8}$$

Endi $K \quad u$ operator nochiziqli diskret operator bo’lganda ham uning simmetriyasi mavjudligini o’rganamiz. Bu masalani o’rganish muhim ahamiyatga ega. Masalan bunday operator sifatida Toda zanjiri turidagi ushbu nochiziqli diskret tenglamani qaraymiz:

$$\frac{da_n}{dt} = a_n = a_n \ \ a_{n+1}^2 - a_{n-1}^2 \ \ + a_n \ \ b_{n+1}^2 - b_n^2 \ \ + a_n(b_n - b_{n+1}) \qquad (3.2.9)$$

$$\frac{db_n}{dt} = b_n = 2a_n^2 \ \ b_{n+1} + b_n \ \ - 2a_{n-1}^2 \ \ b_n + b_{n-1} \ \ + 2(a_{n-1}^2 - a_n^2) \qquad (3.2.10)$$

(3.2.9)-(3.2.10) Toda zanjiri turidagi tenglamaning simmetriyaga ega yoki ega emasligi (3.2.9)-(3.2.10) tenglamaga qo’yilgan Koshi masalasini yechishda muhim ahamiyatga ega. Shu maqsadda (3.2.9)-(3.2.10) tenglamaning simmetriyalarini qidiramiz. Simmetriya tarifiga ko`ra ushbu

$$\frac{da_n}{d\tau} = \frac{da_{n\tau}}{dt} \qquad (3.2.11)$$

$$\frac{db_n}{d\tau} = \frac{db_{n\tau}}{dt} \qquad (3.2.12)$$

tenglikni qanoatlantiruvchi quyidagi

$$\frac{da_n}{d\tau} = f \ \ a_{n-1}, a_n, a_{n+1}, b_{n-1}, b_n, b_{n+1} \ \ , \qquad (3.2.13)$$

$$\frac{db_n}{d\tau} = g \ \ a_{n-1}, a_n, a_{n+1}, b_{n-1}, b_n, b_{n+1} \qquad (3.2.14)$$

ko`rinishga ega bo’ladigan qandaydir ikkinchi bir nochiziqli tenglamani mavjudligini ko’rsatish zarur, bunda $a_n = a_n \ \ t, \tau \ \ $, $b_n = b_n(t, \tau)$. Shu maqsadda biz f va g sifatida mos ravishda ushbu $a_n \ \ b_n - b_{n+1}$ va $2 \ \ a_{n-1}^2 - a_n^2$ ifodalarni olamiz va (3.2.11) va (3.2.12) tengliklarni bajarilishini tekshirib ko’ramiz. Bu holda (3.2.13) va (3.2.14) lar quyidagicha yoziladi:

$$\frac{da_n}{d\tau} = a_n \ \ b_n - b_{n+1} \qquad 3.2.15$$

$$\frac{db_n}{d\tau} = 2 \ \ a_{n-1}^2 - a_n^2 \qquad (3.2.16)$$

(3.2.9), (3.2.10) va (3.2.15), (3.2.16) tenglamalar uchun (3.2.11)- (3.2.12) tengliklarning bajarilishini to’g’ridan to’g’ri hisoblashlar orqali tekshirib ko’ramiz. Qulaylik uchun ushbu

$$\frac{da_n}{d\tau} = a_{n,\tau}$$

ko’rinishidagi belgilashni kiritamiz va (3.2.15) dan t bo’yicha differensialini hisoblaymiz:

$$\frac{d}{dt}\ a_{n,\tau} = \frac{d}{dt}\ a_n(b_n - b_{n+1}) = a_n\ (b_n - b_{n+1}) + a_n\ (b_n - b_{n+1}) .$$

Oxirgi tenglikdagi a_n, b_n lar o’rniga (3.2.9) va (3.2.10) ifodalarni olib kelib qo’yamiz va quyidagi hisoblashlarni amalga oshiramiz:

$$a_n\ (a_{n+1}^2 - a_{n-1}^2) + a_n\ (b_{n+1}^2 - b_n^2) + a_n\ (b_n - b_{n+1})\ (b_n - b_{n+1}) +$$

$$+a_n\ (2a_n^2\ (b_{n+1} + b_n) - 2a_{n-1}^2\ (b_n + b_{n-1}) + 2\ (a_{n-1}^2 - a_n^2)) -$$

$$-a_n\ (2a_{n+1}^2\ (b_{n+2} + b_{n+1}) - 2a_n^2\ (b_{n+1} + b_n) + 2(a_n^2 - a_{n+1}^2)) =$$

$$= a_n(a_{n+1}^2 b_n - a_{n-1}^2 b_n + b_n b_{n+1}^2 - b_n^3 + b_n^2 - b_n b_{n+1} -$$

$$-a_{n+1}^2 b_{n+1} + a_{n-1}^2 b_{n+1} - b_{n+1}^3 + b_n^2 b_{n+1} - b_n b_{n+1} + b_{n+1}^2$$

$$+2a_n^2 b_{n+1} + 2a_n^2 b_n - 2a_{n-1}^2 b_n - 2a_{n-1}^2 b_{n-1} + 2a_{n-1}^2 - 2a_n^2 -$$

$$-2a_{n+1}^2 b_{n+2} - 2a_{n+1}^2 b_{n+1} + 2a_n^2 b_{n+1} + 2a_n^2 b_n - 2a_n^2 + 2a_{n+1}^2) =$$

$$= a_n(a_{n+1}^2 b_n - 3a_{n-1}^2 b_n + b_n b_{n+1}^2 - b_n^3 + b_n^2 - 2b_n b_{n+1} -$$

$$-3a_{n+1}^2 b_{n+1} + a_{n-1}^2 b_{n+1} - b_{n+1}^3 + b_n^2 b_{n+1} + b_{n+1}^2$$

$$+4a_n^2 b_{n+1} + 4a_n^2 b_n - 2a_{n-1}^2 b_{n-1} + 2a_{n-1}^2 - 4a_n^2 -$$

$$-2a_{n+1}^2 b_{n+2} + 2a_{n+1}^2).$$

Demak

$$\frac{d}{dt}\ a_{n,\tau} = a_n(a_{n+1}^2 b_n - 3a_{n-1}^2 b_n - 3a_{n+1}^2 b_{n+1} + a_{n-1}^2 b_{n+1} - 2a_{n+1}^2 b_{n+2} -$$

$$-2a_{n-1}^2 b_{n-1} + b_n b_{n+1}^2 - b_n^3 + b_n^2 - b_{n+1}^3 + b_n^2 b_{n+1} + b_{n+1}^2 + 2b_n b_{n+1} +$$

$$+4a_n^2 b_{n+1} + 4a_n^2 b_n + 2a_{n-1}^2 + 2a_{n+1}^2 - 4a_n^2). \qquad (3.2.17)$$

Endi (3.2.9) dan τ bo'yicha differensialini hisoblaymiz:

$$\frac{da_n}{d\tau} = \frac{d}{d\tau}\ a_n\ a_{n+1}^2 - a_{n-1}^2\ + a_n\ b_{n+1}^2 - b_n^2\ + a_n(b_n - b_{n+1})\ =$$

$$= a_{n,\tau}\ a_{n+1}^2 - a_{n-1}^2\ + a_n\ 2a_{n+1,\tau}a_{n+1} - 2a_{n-1,\tau}a_{n-1}\ +$$

$$+a_{n,\tau}\ b_{n+1}^2 - b_n^2\ + a_n\ 2b_{n+1,\tau}b_{n+1} - 2b_{n,\tau}b_n\ +$$

$$+a_{n,\tau}\ b_n - b_{n+1}\ + a_n\ b_{n,\tau} - b_{n+1,\tau}\ .$$

Oxirgi tenglikdagi $a_{n,\tau}$, $b_{n,\tau}$ lar o'rniga (3.2.15) va (3.2.16) ifodalarni olib kelib qo'yib quyidagi hisoblashlarni amalga oshiramiz:

$$a_n\ b_n - b_{n+1}\ \ a_{n+1}^2 - a_{n-1}^2\ +$$

$$+a_n\ 2a_{n+1}\ b_{n+1} - b_{n+2}\ a_{n+1} - 2a_{n-1}(b_{n-1} - b_n)a_{n-1}\ +$$

$$+a_n(b_n - b_{n+1})\ b_{n+1}^2 - b_n^2\ + a_n\ 4\ a_n^2 - a_{n+1}^2\ b_{n+1} - 4\ a_{n-1}^2 - a_n^2\ b_n\ +$$

$$+a_n(b_n - b_{n+1})\ b_n - b_{n+1}\ + a_n\ 2\ a_{n-1}^2 - a_n^2\ - 2\ a_n^2 - a_{n+1}^2\ \ =$$

$$= a_n(a_{n+1}^2 b_n - a_{n-1}^2 b_n - a_{n+1}^2 b_{n+1} + a_{n-1}^2 b_{n+1}) +$$

$$+a_n\ 2a_{n+1}^2 b_{n+1} - 2a_{n+1}^2 b_{n+2}\ - 2a_{n-1}^2 b_{n-1} + 2a_{n-1}^2 b_n\ +$$

$$+a_n(b_n b_{n+1}^2 - b_n^3 + b_n^2 - b_{n+1}^3 + b_n^2 b_{n+1} + b_{n+1}^2 - 2b_n b_{n+1}) +$$

$$+a_n\ 4a_n^2 b_{n+1} - 4a_{n+1}^2 b_{n+1} - 4a_{n-1}^2 b_n + 4a_n^2 b_n\ +$$

$$+a_n\ 2a_{n-1}^2 + 2a_{n+1}^2 - 4a_n^2\ =$$

$$= a_n(a_{n+1}^2 b_n - 3a_{n-1}^2 b_n - 3a_{n+1}^2 b_{n+1} + a_{n-1}^2 b_{n+1} +$$

$$-2a_{n+1}^2 b_{n+2}\ - 2a_{n-1}^2 b_{n-1} + b_n b_{n+1}^2 - b_n^3 + b_n^2 - b_{n+1}^3 +$$

$$+b_n^2 b_{n+1} + b_{n+1}^2 - 2b_n b_{n+1} + 4a_n^2 b_{n+1} + 4a_n^2 b_n + 2a_{n-1}^2 + 2a_{n+1}^2 - 4a_n^2).$$

Bundan

$$\frac{da_n}{d\tau} = a_n(a_{n+1}^2 b_n - 3a_{n-1}^2 b_n - 3a_{n+1}^2 b_{n+1} + a_{n-1}^2 b_{n+1} - 2a_{n+1}^2 b_{n+2} -$$

$$-2a_{n-1}^2 b_{n-1} + b_n b_{n+1}^2 - b_n^3 + b_n^2 - b_{n+1}^3 + b_n^2 b_{n+1} + b_{n+1}^2 + 2b_n b_{n+1} +$$

$$+4a_n^2 b_{n+1} + 4a_n^2 b_n + 2a_{n-1}^2 + 2a_{n+1}^2 - 4a_n^2). \qquad (3.2.18)$$

ekanligi kelib chiqadi. Demak (3.2.17) va (3.2.18) lar aynan bir xil ifodalar ekan. Bundan (3.2.11) tenglik o'rinli ekanligini ko'rishimiz mumkin.

Endi (3.2.10) va (3.2.16) lar uchun (3.2.12) tenglik o'rinli bo'lishini tekshiramiz. Qulaylik uchun (3.2.16) dagi

$$\frac{db_n}{d\tau} = b_{n,\tau}$$

ko'rinishidagi belgilashni kiritamiz va (3.2.16) dan t bo'yicha differensialini hisoblaymiz:

$$\frac{d}{dt}\ \ b_{n\tau}\ \ = \frac{d}{dt}\ \ 2\ \ a_{n-1}^2 - a_n^2\ \ \ = 2\ \ 2a_{n-1}a_{n-1} - 2a_n a_n$$

yuqorida keltirilgan tenglikdagi a_n o'rniga (3.2.15) ifodani olib kelib qo'yib quyidagi hisoblashlarni amalga oshiramiz.

$$= 4((a_{n-1}\ \ a_n^2 - a_{n-2}^2\ \ + a_{n-1}\ \ b_n^2 - b_{n-1}^2\ \ + a_{n-1}(b_{n-1} - b_n))a_{n-1} -$$

$$-(a_n\ \ a_{n+1}^2 - a_{n-1}^2\ \ + a_n\ \ b_{n+1}^2 - b_n^2\ \ + a_n(b_n - b_{n+1}))a_n) =$$

$$= 4(a_{n-1}^2 a_n^2 - a_{n-1}^2 a_{n-2}^2 + a_{n-1}^2 b_n^2 - a_{n-1}^2 b_{n-1}^2 + a_{n-1}^2 b_{n-1} - a_{n-1}^2 b_n -$$

$$-a_n^2 a_{n+1}^2 + a_n^2 a_{n-1}^2 - a_n^2 b_{n+1}^2 + a_n^2 b_n^2 - a_n^2 b_n + a_n^2 b_{n+1}) =$$

$$= 4(2a_{n-1}^2 a_n^2 - a_{n-1}^2 a_{n-2}^2 + a_{n-1}^2 b_n^2 - a_{n-1}^2 b_{n-1}^2 + a_{n-1}^2 b_{n-1} -$$

$$-a_{n-1}^2 b_n - a_n^2 a_{n+1}^2 - a_n^2 b_{n+1}^2 + a_n^2 b_n^2 - a_n^2 b_n + a_n^2 b_{n+1})$$

Demak

$$\frac{d}{dt}\ \ b_{n\tau}\ \ = 4(2a_{n-1}^2 a_n^2 - a_{n-1}^2 a_{n-2}^2 + a_{n-1}^2 b_n^2 - a_{n-1}^2 b_{n-1}^2 +$$

$$+a_{n-1}^2 b_{n-1} - a_{n-1}^2 b_n - a_n^2 a_{n+1}^2 - a_n^2 b_{n+1}^2 + a_n^2 b_n^2 - a_n^2 b_n + a_n^2 b_{n+1})$$

Endi (3.2.10) dan τ bo'yicha differensialini hisoblaymiz:

$$\frac{d}{d\tau}\ b_n\ = \frac{d}{d\tau}\ 2a_n^2\ b_{n+1} + b_n\ - 2a_{n-1}^2\ b_n + b_{n-1}\ + 2\ a_{n-1}^2 - a_n^2\ =$$

$$= 4a_n a_{n,\tau}\ b_{n+1} + b_n\ + 2a_n^2\ b_{n+1,\tau} + b_{n,\tau}\ -$$

$$-4a_{n-1}a_{n-1,\tau}\ b_n + b_{n-1}\ - 2a_{n-1}^2\ b_{n,\tau} + b_{n-1,\tau}\ +$$

$$+4a_{n-1}a_{n-1,\tau} - 4a_n a_{n,\tau} =$$

yuqorida keltirilgan tenglikdagi $a_{n,\tau}$, $b_{n,\tau}$ lar o'rniga (3.2.15) va (3.2.16) ifodalarni olib kelib qo'yib quyidagi hisoblashlarni amalga oshiramiz.

$$= 4a_n a_n\ b_n - b_{n+1}\ \ b_{n+1} + b_n\ +$$

$$+2a_n^2\ 2\ a_n^2 - a_{n+1}^2\ + 2\ a_{n-1}^2 - a_n^2\ \ -$$

$$-4a_{n-1}a_{n-1}\ b_{n-1} - b_n\ \ b_n + b_{n-1}\ -$$

$$-2a_{n-1}^2\ 2\ a_{n-1}^2 - a_n^2\ + 2\ a_{n-2}^2 - a_{n-1}^2\ \ +$$

$$+4a_{n-1}a_{n-1}(b_{n-1} - b_n) - 4a_n a_n(b_n - b_{n+1}) =$$

$$= 4a_n^2 b_n^2 - 4a_n^2 b_{n+1}^2 - 4a_n^2 a_{n+1}^2 + 8a_{n-1}^2 a_n^2 +$$

$$+4a_{n-1}^2 b_{n-1}^2 - 4a_{n-1}^2 b_n^2 + 4a_{n-1}^2 a_{n-2}^2 +$$

$$+4a_{n-1}^2 b_{n-1} - 4a_{n-1}^2 b_n - 4a_n^2 b_n + 4a_n^2 b_{n+1}$$

Bundan

$$\frac{d}{d\tau}\ b_n\ = 4a_n^2 b_n^2 - 4a_n^2 b_{n+1}^2 - 4a_n^2 a_{n+1}^2 + 8a_{n-1}^2 a_n^2 +$$

$$+4a_{n-1}^2 b_{n-1}^2 - 4a_{n-1}^2 b_n^2 + 4a_{n-1}^2 a_{n-2}^2 +$$

$$+4a_{n-1}^2 b_{n-1} - 4a_{n-1}^2 b_n - 4a_n^2 b_n + 4a_n^2 b_{n+1}$$

$$\frac{d}{dt}\ b_{n\tau}\ = 4a_n^2 b_n^2 - 8a_{n-1}^2 a_n^2 - 4a_{n-1}^2 a_{n-2}^2 + 4a_{n-1}^2 b_n^2 - 4a_{n-1}^2 b_{n-1}^2 +$$

$$+4a_{n-1}^2 b_{n-1} - 4a_{n-1}^2 b_n - 4a_n^2 a_{n+1}^2 - 4a_n^2 b_{n+1}^2 + -4a_n^2 b_n + 4a_n^2 b_{n+1}$$

olingan natijalarni solishtirsak (3.2.12) tenglik o'rinli ekanligini ko'rishimiz mumkin.

Bulardan ko'rinadiki (3.2.9) va (3.2.10) tenglama simmetriyaga ega ekan va unga simmetrik bo'lgan tenglama klassik Toda tenglamasidan iborat ekan. Bundan shunday xulosa qililsh mumkinki (3.2.9) va (3.2.10) tenglamaga qo'yilgan Koshi masalasi global yechimga ega bo'ladi. Bu yechimning aniq ko'rinishi bir zonali xolda II-bo'lim oxirgi paragrafida aniq misol yordamida ko'rsatilgan.

II bob bo‘yicha xulosalar

Korteveg- de Friz tenglamasi va Toda zanjiri uchun simmetriya masalalari o’rganilgan. Xususan II-davriy Toda tenglamasining simmetriyasi bo’ladigan tenglama aynan Toda tenglamasi ekanligi ko’rsatilgan. Shu bilan birgalikda simmetriyaga ega bo’lgan nochiziqli tenglamalarni integrallash mumkinligi keltirilgan.

XULOSA

Ushbu ish moslangan manbali II-tur Toda tenglamasini teskari masalalar usulidan foydalanib, integrallashga bag‘ishlangan bo‘lib, quyidagi natijalar olindi:

1. Diskret Xill tenglamasi uchun qo‘yilgan to‘g‘ri va teskari masalalar o‘rganildi.

2. Moslangan manbali II-tur Toda tenglamasi keltirib chiqarildi.

3. Spektral parametrlarning o‘zgarish dinamikasi o‘rganildi.

4. Moslangan manbali II-tur Toda tenglamasi $m=2$ holida integrallandi.

5. Moslangan manbali II-tur Toda tenglamasini integrallash algoritmi keltirib chiqarildi.

6. KdF tenglamasining simmetriya masalasi o’rganildi.

7. Toda zanjiri turidagi nochiziqli diskret tenglama uchun simmetriya masalalari o’rganildi.

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