O`zbekiston Respublikasi Oliy va o`rta
maxsus ta`lim vazirligi

Namangan
muhandislik–qurilish instituti

Oliy matematika kafedrasi

Oily matematika fanidan misol va masalalar yechish uchun uslubiy qo’llanma

II kurs talabalari uchun

NAMANGAN - 2017

Oliy matematika kafedrasi
uslubiy seminarida ko`rib chiqilib,
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1-sonli majlis bayoni 2017 y

Namangan muhandislik-
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Kirish

Ushbu masalalar to'plami oliy texnika o'quv yurti o'qituvchi va talabalari oliy matematika fanidan amaliy mashg`ulot darslarida foydalanish uchun mo`ljallangan. Unda keltirilgan mavzular amaldagi ishchi dasturiga mos bo'lib, 2-semestrda foydalanish mumkin.

To'plam oliy matematikaning aniq integral, ko'p o'zgaruvchili funksiyalar, karrali integrallar, differensial tenglamalar, matematik fizika va egri chiziqli integrallarni o`z ichiga qamrab olgan.

To'plamni yozishda hozirga qadar rus va o`zbek tillarida chop etilgan adabiyotlardan foydalanildi, hamda mualliflar to`plagan tajribaga asoslanib oddiydan murakkabga ketma-ketligini saqlashga harakat qilindi. Kitob o`quv dasturiga moslab yozilgan bo`lib, har bir mavzuga kerakli formulalar keltirilib, na`muna tarzida masalalarning yechilishi keltirilgan. Mustaqil ishlash va uy vazifasi uchun masalalar va ularning javoblari keltirilgan

Mualliflar to'plamni ilmiy va uslubiy jixatdan yaxshilash uchun bildiriladigan takliflarni mamnuniyat bilan qabul qiladilar.

Mualliflar.

Aniq integral.

Aniq integralning xossalari   Nyuton - Leybnis formulasi.

1-misol. $\int_0^1 x^2 dx$; integralni ta`rif yordamida hisoblang.

Yechish: $f(x)=x^2$. $a=0$, $b=1$; $[0;1]$ kesmani n ta bo`lakka bo`lamiz.

$$\Delta x_k=(b-a)/n=1/n,\quad \xi_k=x_k \qquad x_0=0,\ x_1=\frac{1}{n};\ x_2=\frac{2}{n};\cdots;\ x_{n-1}=\frac{n-1}{n};\ x_n=\frac{n}{n}=1$$

$$f(\varsigma_1)=\left(\frac{1}{n}\right)^2;\ f(\varsigma_2)=\left(\frac{2}{n}\right)^2;\ldots,\ f(\varsigma_n)=\left(\frac{n}{n}\right)^2;\ f(\varsigma_k)\cdot\Delta x_k=\left(\frac{k}{n}\right)^2\cdot\frac{1}{n} \quad \text{u holda}$$

$$\int_0^1 x^2dx=\lim_{n\to\infty}\frac{1^2+2^2+3^2+\cdots+n^2}{n^2}=\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}=\lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}=\frac{1}{3}$$

2-misol. $\int_{-3}^{2}(2x+3)dx$ integralni hisoblang.

Yechish:

$$\int_{-3}^{2}(2x+3)dx=\int_{-3}^{2}2xdx+\int_{-3}^{2}3dx=x^2\Big|_{-3}^{2}+3x\Big|_{-3}^{2}=(2^2-(-3)^2)+(3\cdot 2-3\cdot(-3))=-5+15=10$$

3-misol. $\int_{\pi/6}^{\pi/4}\frac{4dx}{\cos^2 x}$ integralni hisoblang

Yechish: $$\int_{\pi/6}^{\pi/4}\frac{4dx}{\cos^2 x}=4\int_{\pi/6}^{\pi/4}\frac{dx}{\cos^2 x}=4\cdot tgx\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}=4\cdot\left[tg\frac{\pi}{4}-tg\frac{\pi}{6}\right]=4\cdot\left(1-\frac{\sqrt{3}}{3}\right)$$

Darsda yechish uchun misollar.
Quyidagi integrallarni hisoblang.

98. $\int_1^3 x^3dx$, j. 20

99. $\int_1^4 \sqrt{x}dx$, j. $\frac{14}{3}$

100. $\int_1^2\left(x^2+\frac{1}{x^4}\right)dx$, j. $\frac{63}{24}$

101. $\int_0^{\pi/4}\sin 4xdx$, j. $\frac{1}{2}$

102. $\int_0^3 e^{\frac{x}{3}}dx$, j. 3(e-1)

103. $\int_a^{a\sqrt{3}}\frac{dx}{a^2+x^2}$, j. $\frac{\pi}{12a}$

104. $\int 3x^2dx$, j .26

105. $\int_1^4\frac{x+1}{\sqrt{x}}dx$, j. $6\frac{2}{3}$

106. $\int_{-1}^{7}\frac{dx}{\sqrt{3x+4}}$, j. $\frac{8}{3}$

107. $\int_0^1\frac{dz}{(2z+1)^3}$, j. $\frac{2}{9}$

108. $\int_0^1 x\sqrt{1+x}dx$, j. $\frac{4\left(\sqrt{2}+1\right)}{15}$

109. $\int_{\pi/8}^{\pi/4}ctg^2xdx$, j. $\frac{1}{2}-\frac{\pi}{8}$

110. $\int_0^{\pi}\cos\frac{x}{2}\cos\frac{3x}{2}dx$ j.0

Mustaqil uy vazifasi uchun misollar

111. $\int_0^3 \left(3^{1-x} + \left(\frac{1}{3}\right)^{2x-1}\right) dx$, j . $\frac{1066}{243\ln 3}$

112. $\int_{-2}^{2} \frac{x^3 + x^2 + x + 1}{x^2 + 1} dx$,j. 4

113. $\int_1^c \frac{x^2 + 1}{2x^2} dx$, j . $\frac{c^2 - 1}{2c}$

114. $\int_{-3}^{2} (5x + 2) dx$, j . $-2\frac{1}{2}$

115. $\int_1^3 (2x^2 + x + 1) dx$, j . $23\frac{1}{3}$

Aniq integralni o`zgaruvchini almashtirib integrallash

1-misol. $\int_0^1 \frac{x^2 dx}{(x+1)^2}$ integralni hisoblang.

Yechish:

$$\int_0^1 \frac{x^2 dx}{(x-1)^2} = \left| \begin{array}{l} t = x+1,\ x = 1;\ t = 2 \\ x = t-1,\ x = 0;\ t = 1 \\ dx = dt \end{array} \right| = \int_1^2 \frac{(t-1)^2}{t^2} dt = \int_1^2 \frac{t^2 - 2t + 1}{t^2} dt = \int_1^2 dt - 2\int_1^2 \frac{dt}{t} + \int_1^2 \frac{dt}{t^2} = t\Big|_1^2 - 2\ln t\Big|_1^2 - \frac{1}{t}\Big|_1^2 =$$

$$= 2 - 1 - 2\ln 2 - \frac{1}{2} + 1 = \frac{3}{2} + 2\ln 2$$

2-misol. $\int_0^{\pi/2} \cos^2 x \cdot \sin x dx$ integralni hisoblang.

Yechish: $$\int_0^{\pi/2} \cos^2 x \cdot \sin x dx = \left| \begin{array}{l} t = \cos x, dt = -\sin x dx \\ x = 0, \quad t = 1. \\ x = \frac{\pi}{2}; \quad t = 0 \end{array} \right| = -\int_1^0 t^2 dt = \frac{1}{3} t^3 \Big|_0^1 = \frac{1}{3}$$

Darsda yechish uchun misollar.
Quyidagi integrallarni hisoblang.

116. $\int_1^8 \frac{xdx}{\sqrt{3x-1}}$, j.8

117. $\int_0^1 \frac{x^2 dx}{\sqrt{4 - x^2}}$, j. $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$

118. $\int_0^{\sqrt{a}} x^2 \sqrt{a^2 - x^2} dx$, j. $\frac{\pi a^2}{16}$

119. $\int_0^{\pi/2} x^2 \sqrt{9 - x^2} dx$, j. $\frac{81}{8}\pi$

120. $\int_1^2 \frac{dx}{(3x-4)^5}$, j. $1\frac{3}{12}$

121. $\int_3^8 \frac{xdx}{\sqrt{x+1}}$, j. $\frac{32}{3}$

122. $\int_{\frac{\sqrt{2}}{2}}^{1} \frac{\sqrt{1-x^2} dx}{x^2}$, j. $1 - \frac{\pi}{4}$

123. $\int_1^2 \frac{dx}{2x-1}$, j. $\frac{1}{2}\ln 3$

124. $\int_0^{\frac{\pi}{2}} \frac{cosxdx}{1+\sin^2 x}$, j. $\frac{\pi}{4}$

125. $\int_{\sqrt{3}}^{\sqrt{8}} x\sqrt{1+x^2}dx$, j. $\frac{19}{3}$

126. $\int_{-1}^{0} \frac{x^2 dx}{1-4x^3}$,

127. $\int_0^{\pi} \sin\left(\frac{\pi}{6}-2x\right)dx$,

128. $\int_0^{0.5} e^{\sin \pi x} \cos \pi x dx$,

Mustaqil uy vazifasi uchun misollar

129. $\int_1^3 x^3\sqrt{x^2-1}dx$, j. $\frac{464\sqrt{2}}{15}$

130. $\int_{-\frac{\pi}{12}}^{0} \sin\left(\frac{\pi}{4}-3x\right)dx$, j. $\frac{\sqrt{2}}{6}$

131. $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}dx$, j. $e-\sqrt{e}$

132. $\int_{25}^{196} \frac{dx}{x-4\sqrt{x}}$, j. $2\ln 10$

133. $\int_0^1 \frac{xdx}{1+x^4}$, j. $\frac{\pi}{8}$

134. $\int_4^{25} \frac{dx}{\sqrt{x}-1}$, j. $6+4\ln 2$

135. $\int_0^1 e^{x+e^x}dx$, j. $e^e - e$

136. $\int_1^3 \frac{\sqrt{x}}{x+1}dx$, j. $2\sqrt{3}-2-\frac{\pi}{6}$

Aniq integralni bo`laklab integrallash

Bo`laklab integrallash formulasi. $\int_a^b udv = (u\cdot v)\Big|_a^b - \int_a^b vdu$

1-misol. $\int_e^{e^2} x\ln xdx$ integralni hisoblang.

$$\int_e^{e^2} x\ln xdx = \left| \begin{matrix} u=\ln x, du=\frac{1}{x}dx \\ dv=xdx, v=\frac{x^2}{2} \end{matrix} \right| = \frac{x^2}{2}\ln x\Big|_e^{e^2} - \int_e^{e^2}\frac{x^2}{2}\cdot\frac{dx}{x} = e^4-\frac{e^2}{2}-\frac{1}{2}\frac{x^2}{2}\Big|_e^{e^2} = e^4-\frac{e^2}{2}-\frac{e^4}{4}+\frac{e^2}{4} =$$

$$=\frac{3}{4}e^4-\frac{e^2}{4}=\frac{e^2}{4}\left(3e^2-1\right)$$

2-misol $\int_0^1 xe^x dx$ integralni hisoblang.

Yechish: $\int_0^1 xe^x dx = \begin{vmatrix} u = x, du = dx \\ dv = e^x dx, v = e^x \end{vmatrix} = xe^x \Big|_0^1 - \int_0^1 e^x dx = e - e^x \Big|_0^1 = e - e + 1 = 1$

3-misol $\int_{-a}^{a} x\cos\frac{x}{a}dx$ integralni hisoblang.

Yechish:

$$\int_{-a}^{a} x\cos\frac{x}{a}dx = \begin{vmatrix} u = x, du = dx \\ dv = \cos\frac{x}{a}dx, v = a\sin\frac{x}{a} \end{vmatrix} = x \cdot a \cdot \sin\frac{x}{a}\Big|_{-a}^{a} - a\int_{-a}^{a}\sin\frac{x}{a}dx = a^2\sin 1 - (-a)a\sin(-1) -$$

$$- a \cdot a\left(\cos\frac{x}{a}\right)\Big|_{-a}^{a} = a^2\sin 1 - a^2\sin 1 + a^2\cos 1 + a^2\cos(-1) = 0$$

4 misol. $I = \int_0^{\pi/2} e^x \sin x dx$ integralni hisoblang.

Yechish:

$$I = \int_0^{\pi/2} e^x \sin x dx = \begin{vmatrix} u = e^x, dv = \sin x dx \\ du = e^x dx, v = -\cos x \end{vmatrix} = -e^x\cos x\Big|_0^{\pi/2} + \int_0^{\pi/2} e^x\cos x dx = \begin{vmatrix} u = e^x, dv = \cos x dx \\ du = e^x dx, v = \sin x \end{vmatrix} =$$

$$= -e^x\cos x\Big|_0^{\pi/2} + e^x\sin x\Big|_0^{\pi/2} - \int_0^{\pi/2} e^x\sin x dx = 1 + e^{\pi/2} - \int_0^{\pi/2} e^x \sin x dx$$

$$I = 1 + e^{\pi/2} - I$$

$$I = \frac{1}{2}(1 + e^{\pi/2})$$

Darsda yechish uchun misollar.

Aniq integrallarni bo`laklab integrallash yordamida hisoblang.

137. $\int_0^1 arctgx dx$, j. $\frac{\pi}{4} - \frac{1}{2}\ln 2$

138. $\int_1^4 \frac{\ln x}{\sqrt{x}}dx$, j. $4(\ln 4 - 1)$

139. $\int_0^1 xe^{3x}dx$, j. $\frac{2e^3 + 1}{9}$

140. $\int_0^1 \ln(x+1)dx$, j. $2\ln 2 - 1$

141. $\int_{-1}^1 xarctgx dx$, j. $\frac{\pi}{2} - 1$

142. $\int_1^e x^2 \ln x dx$, j. $\frac{2e^3 + 1}{9}$

143. $\int_0^{\pi/2} e^x \cos x dx$, j. $\frac{\left(e^{\frac{\pi}{2}} - 1\right)}{2}$

144. $\int_0^1 x\sin x dx$, j. π

145. $\int_1^e \sqrt[4]{x}\ln x dx$

146. $\int_1^e \sqrt[4]{x}\ln x dx$, j. $\frac{4\left(\sqrt[4]{e^5} + 4\right)}{25}$

147. $\int_{-1}^0 (2x+3)e^{-x}dx$, j. $3e - 5$

148. $\int_0^{\frac{\pi}{2}} (x-1)\cos x dx$, j. $\frac{\pi}{2} - 2$

149. $\int_0^1 xarctgxdx$, j. $\frac{\pi-2}{4}$

Mustaqil uy vazifasi uchun misollar

150. $\int_1^e \cos\ln xdx$, j. $\frac{e^{\frac{\pi}{2}}-1}{2}$

151. $\int_1^e x\ln^2 xdx$, j. $\frac{1}{4}\left(e^2-1\right)$

152. $\int_0^{\pi/2} x\cos xdx$, j. $\frac{\pi}{2}-1$

153. $\int_0^{\pi/2} e^{2x}\cos xdx$, j. $\frac{\pi^2-8}{32}$

154. $\int_1^e \ln^2 xdx$, j. $e-2$

155. $\int_{-1}^0 \arccos xdx$, j. $\pi-1$

Aniq integral yordamida yuzalarni hisoblash.

1-misol. $y=4x-x^2, y=0$ chiziqlar bilan chegaralangan soha yuzini toping.

Yechish: $y=4x-x^2$ va $y=0$ tenglamalarni birgalikda yechib, parabolani $0X$ o`qi bilan kesishish nuqtasini topamiz.
O(0;0) va M(4;0) nuqtalarda kesishadi.

$$S=\int_0^4 (4x-x^2)dx=\left[2x^2-\frac{1}{3}x^3\right]\Big|_0^4=\frac{32}{3} \quad \text{(kv.bir)}$$

Darsda yechish uchun misollar.

Quyidagi chiziqlar bilan chegaralangan sohani yuzini toping.

156. $y=-x^2, x+y+2=0$, j. $4{,}5$

157. $x=12\cos t+5\sin t, y=5\cos t-12\sin t$, j. $169\cdot\pi$

158. $y=16/x^2, y=17-x^2$ (I - CHorak) , j. 18

159. $x=a\cos^3 t, y=a\sin^3 t$ (astroida) , j. $\frac{3\pi a^2}{8}$

160. $y^2=4x^3, y=2x^2$, j. $\frac{2}{15}$

161. $xy=20, x^2+y^2=41$ (I - CHorak) , j. $\frac{41}{2}\arcsin\frac{9}{41}+20\ln\left(0{,}8\right)$

162. $y=\sin x, y=\cos x, x=0$, j. $\sqrt{2}-1$

Mustaqil uy vazifasi uchun misollar

Quyidagi chiziqlar bilan chegaralangan sohani yuzini toping

163. $y=0.25x^2, y=3x-0.5x^2$, j. 8

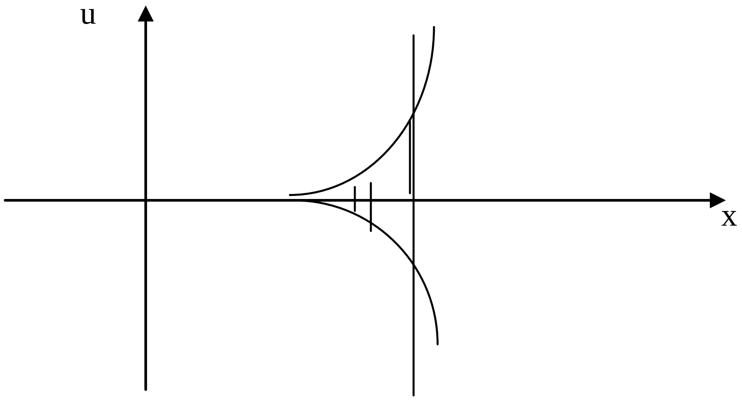
164. $xy=4\sqrt{2}, x^2-6x+y^2=0, y=0, x=4$, j. $\frac{9\pi}{4}-\sqrt{2}+4\sqrt{2}\ln 2-\frac{9}{2}\arcsin\left(\frac{1}{3}\right)$

165. $y=\frac{x^2}{2}, \quad x=1, \quad x=3$ parabola va to`g`ri chiziqlar va OX o`qi bilan chegaralangan soha yuzini hisoblang. , j. $\frac{13}{3}$

166. $x = 2 - y - y^2$ egri chiziq va ordinatalar o`qi bilan chegaralangan figuraning yuzini hisoblang. , j. $\frac{9}{2}$

Aniq integral yordamida jism hajmini hisoblang

1-masala $y^2 = (x-1)^3$ va $x = 2$ chiziqlar bilan chegaralangan figurani OX o`qi atrofida aylantirishdan hosil bo`lgan jismning hajmini hisoblang.
YEchish



$$V = \pi \int_1^2 y^2 dx = \pi \int_1^2 (x-1)^3 dx = \frac{1}{4}\pi (x-1)^4 \Big|_1^2 = \frac{1}{4}\pi \text{ (kub bir.)}$$

2-masala. $y = Sin\, x$ sinusoidaning yarim to`lqini OX o`qining $[0, \pi]$ kesmasi bilan chegaralangan figuraning OY o`qi atrofida aylanishidan hosil bo`lgan jismning hajmini hisoblang.
YEchish

$$V = 2\pi \int_0^\pi xy\, dx = 2\pi \int_0^\pi x Sin\, x\, dx = \begin{vmatrix} u = x\ , & du = dx \\ dv = Sin\, x\ , & v = -Cos\, x \end{vmatrix} = 2\pi \left(-x Cos\, x \Big|_0^\pi + \int_0^\pi Cos\, x dx \right) =$$

$$= 2\pi \left(-\pi \cos \pi + Sin\, x \Big|_0^\pi \right) = 2\pi^2 \text{ (куб бир.)}$$

3-masala Y=x^2 va y^2=8x parabolalar bilan chegaralangan figurani 0u o`qi atrofida aylantirishdan hosil bo`lgan jism xajmini hisoblang.

Yechish: y=x^2 va y^2=8x tenglamalarni birgalikda yechib kesishish nuqtalarini topamiz. X=0 va x=2

$$V_y = 2\pi \int_a^b x(y_2 - y_1) dx = 2\pi \int_0^2 x\left(\sqrt{8x} - x^2\right) dx = 2\pi \int_0^2 \left(\sqrt{8} x^{\frac{3}{2}} - x^3 \right) dx = 2\pi \left(\sqrt{8} \frac{2}{5} x^{\frac{5}{2}} - \frac{x^4}{4} \right) \Bigg|_0^2 =$$

$$= 2\pi \left(\frac{2\sqrt{8 \cdot 32}}{5} - 4 \right) = 2\pi \left(\frac{2 \cdot 16}{5} - 4 \right) = 2\pi \frac{12}{5} = \frac{24\pi}{5} \quad \text{kub birlik}$$

Darsda yechish uchun misollar.

Quyidagi chiziqlar bilan chegaralangan figuralarni koordinata o`qlari atrofida aylantirishidan hosil bo`lgan jismlarning hajmini toping.

167. $xy=9$, $y=3$, $y=9$ $0y$ o`q atrofida , ж. 18π

168. $y=10-x^2$, $y=x^2+2$ $0y$ o`q atrofida, ж. $0{,}16\pi$

169. $y=4-x^2$, $2x+y-4=0$ $0x$ o`q atrofida, ж. $6{,}4\pi$

Mustaqil uy vazifasi uchun misollar

Quyidagi chiziqlar bilan chegaralangan figuralarni koordinata o`qlari atrofida aylantirishidan hosil bo`lgan jismlarning hajmini toping.

175. $y=\cos\left(x-\dfrac{\pi}{3}\right)$, $x=0$, $y=0$, $(x>0)$ $0x$ o`q atrofida , j. $\dfrac{\pi}{4}\left(\dfrac{5\pi}{3}+\dfrac{\sqrt{3}}{2}\right)$

176. $x^2-y^2=4$, $y=2$ $0y$ o`q atrofida , j. $\dfrac{32\pi}{3}$

177. $x=\sqrt{y}$ $y=1$ $y=4$ $0y$ o`q atrofida , j. $\dfrac{15\pi}{2}$

178. $y=3x^2$ parabola $x=0$ $x=2$ $0x$ o`q atrofida , j. $57{,}6\pi$

Yoy uzunligi va sirt yuzalarni hisoblash

1-masala $y^2=x^3$ yarim kubik parabolaning koordinatalar boshidan A(4;8) nuqtagacha bo`lgan yoyi uzunligini toping.

Yechish: $y^2=x^3,\ \ y=x^{\frac{3}{2}},\ \ y'=\dfrac{3}{2}x^{\frac{1}{2}}$

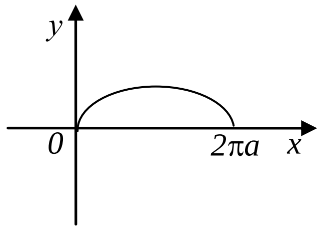
$$L=\int_0^4\sqrt{1+\left(\frac{3}{2}x^{\frac{1}{2}}\right)^2}\,dx=\int_0^4\sqrt{1+\frac{9}{4}x}\,dx=\frac{4}{9}\cdot\frac{2}{3}\left(1+\frac{9}{4}x\right)^{\frac{3}{2}}\Bigg|_0^4=\frac{8}{27}\left(10\sqrt{10}-1\right)\text{ (uzunlik birligi)}$$

2-masala X=a(t-sint), y=a(1-cost) sikloidaning bir arki uzunligini toping.

Echish: $x'=a(1-\cos t)$; $y'=a\sin t$

$$L=\int_0^{2\pi}\sqrt{a^2(1-\cos t)^2+a^2\sin^2 t}\,dt=a\int_0^{2\pi}\sqrt{2-2\cos t}\,dt=$$

$$=2a\int_0^{2\pi}\sin\frac{t}{2}dt=-4a\cos\frac{t}{2}\Bigg|_0^{2\pi}=8a\ \text{(uzunlik birlik)}$$

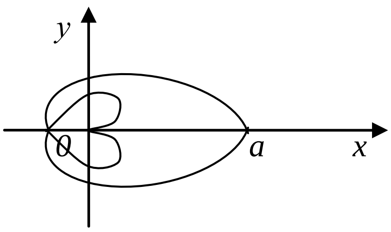


3- masala $r=a\sin^4\dfrac{\varphi}{4}$ yopik egri chiziq uzunligini toping.

Yechish: Berilgan funksiya juft funksiya. SHu sababli berilgan egri chiziq qutb o`kiga nisbatan simmetrik.

Nuqta butun egri chiziqni (burchak 0 dan 2π gacha o`zgarganda chizadi.

$r'=a\sin^3\dfrac{\varphi}{4}\cdot\cos\dfrac{\varphi}{4}$,



demak,

$$\frac{L}{2}=\int_0^{2\pi}\sqrt{a^2\sin^8\frac{\varphi}{4}+a^2\sin^6\frac{\varphi}{4}\cdot\cos^2\frac{\varphi}{4}}d\varphi=a\int_0^{2\pi}\sin^3\frac{\varphi}{4}d\varphi=-4a\int_0^{2\pi}\sin^2\frac{\varphi}{4}\cdot d\left(\cos\frac{\varphi}{4}\right)=$$

$$=-4a\int_0^{2\pi}\left(1-\cos^2\frac{\varphi}{4}\right)d\left(\cos\frac{\varphi}{4}\right)=-4a\left(\cos\frac{\varphi}{4}-\frac{\cos^3\frac{\varphi}{4}}{3}\right)\Bigg|_0^{2\pi}==-4a\left(1-\frac{1}{3}\right)=\frac{8}{3}a$$

demak, $L=\frac{16}{3}a$ (uzunlik birlik)

Darsda yechish uchun misollar.

Quyidagi egri chiziqlar yoylari uzunliklarini hisoblang.

181. $y=\ln(\sin x)$, $\frac{\pi}{3}\le x\le\frac{\pi}{2}$ j. $\frac{1}{2}\ln 3$

182. $x=\frac{1}{3}t^3-t,\quad y=t^2+2,\quad 0\le t\le 3$ j. $\sqrt{2}(\pi-1)$

183. $x=e^t\cos t.,\quad y=e^t\sin t\qquad t=0\qquad t=\ln\pi$ j. 12

Egri chiziq yoyining 0x o`qi atrofida aylanishdan hosil bo`lgan jism sirti yuzini hisoblang.

184. $y=x^3$, $0\le x\le\frac{1}{2}$ j. $\frac{61\pi}{1728}$

185. $x=t-\sin t$, $y=1-\cos t$ (bir arkasi) j. $\frac{64\pi}{3}$

186. $y=\frac{64}{x^2+16},\quad x^2=8y$ $0x$ o`q atrofida j. $16\pi(5\pi+\frac{8}{5})$

Mustaqil uy vazifasi uchun misollar

Egri chiziq yoyining 0x o`qi atrofida aylanishdan hosil bo`lgan jism sirti yuzini hisoblang.

187. $y^2=x$, $x^2=y$ $0y$ o`q atrofida j. $0{,}3\pi$

188. $y=\frac{x^2}{2}$, $y=\frac{x^3}{8}$ j. $\frac{4\pi}{35}$

Xosmas integrallarni hisoblash

$\int_a^{+\infty}f(x)dx\qquad\int_{-\infty}^{b}f(x)dx\qquad\int_{-\infty}^{+\infty}f(x)dx$ - ko`rinishidagi integrallar chegarasi cheksiz xosmas integrallar deyiladi.

$$\int_a^{+\infty}f(x)dx=\lim_{b\to+\infty}\int_a^b f(x)dx,\qquad\int_{-\infty}^{b}f(x)dx=\lim_{a\to-\infty}\int_a^b f(x)dx$$

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^{c} f(x)dx + \int_{c}^{+\infty} f(x)dx$$

ko`rinishida hisoblanadi.
Agar limit mavjud va chekli bo`lsa xosmas integral yaqinlashuvchi, aks xolda uzoqlashuvchi deyiladi.

1-misol. $\int_{a}^{+\infty} \cos xdx$ xosmas integralni hisoblang.

Yechish: $\int_{a}^{+\infty} \cos xdx = \lim_{b\to+\infty} \int_{0}^{b} \cos xdx = \lim_{x\to+\infty} \sin x /_{0}^{b} = \lim_{x\to+\infty} (\sin b - \sin 0) = \lim_{b\to+\infty} \sin b$

bunday limit mavjud emas . Demak, xosmas integral uzoklashuvchi.

2-misol. $\int_{-\infty}^{-1} \frac{dx}{x^2}$ xosmas integralni hisoblang.

Yechish: $\lim_{a\to-\infty} \int_{a}^{-1} \frac{dx}{x^2} = \lim_{a\to-\infty} \left[-\frac{1}{x} \right] \Big|_{a}^{-1} = \lim_{a\to-\infty} \left(1 + \frac{1}{a} \right) = 1$

limit chekli, demak, xosmas integral yaqinlashuvchi.

3-misol $\int_{-\infty}^{+\infty} \frac{dx}{1+x^2}$ xosmas integralni hisoblang.

Yechish: Integral ostidagi funksiya juft bo`lganligi sababli

$$\int_{-\infty}^{+\infty} \frac{dx}{1+x^2} = 2\int_{0}^{+\infty} \frac{dx}{1+x^2}$$

$$\int_{0}^{+\infty} \frac{dx}{1+x^2} = \lim_{b\to+\infty} \int_{0}^{b} \frac{dx}{1+x^2} = \lim_{b\to+\infty} arctgx \Big|_{0}^{b} = \lim_{b\to+\infty} arctgb = \frac{\pi}{2}$$

Demak,

$$\int_{-\infty}^{+\infty} \frac{dx}{1-x^2} = 2 \cdot \frac{\pi}{2} = \pi$$

Xosmas integral chekli limitga ega. Demak, integral yaqinlashuvchi.

4-misol. $\int_{0}^{+\infty} xe^{-x^2} dx$ xosmas integralni hisoblang.

Yechish:

$$\int_{0}^{+\infty} xe^{-x^2} dx = \lim_{b\to+\infty} \int_{0}^{b} xe^{-x^2} dx = \lim_{b\to+\infty} \left[-\frac{1}{2} e^{-x^2} \right]_{0}^{b} = \lim_{b\to+\infty} \left(\frac{1}{2} - \frac{e^{-b^2}}{2} \right) = \frac{1}{2}$$

Limit chekli, demak, xosmas integral yaqinlashuvchi

Darsda yechish uchun misollar

Xosmas integrallarni hisoblang va yaqinlashuvchilikka tekshiring.

278. $\int_{0}^{+\infty} \frac{ardtgx}{1+x^2} dx$ j: $\frac{256}{15}$

280. $\int_{-\infty}^{0} \frac{dx}{4+x^2}$ j: yaqinlashuvchi

279. $\int_{0}^{2} \frac{x^5}{4-x^2} dx$ j: $+\infty$

281. $\int_{0}^{1} \frac{dx}{\sqrt{x(1-x)}}$ j: $\frac{\pi}{6}$

282. $\int_{-1}^{1}\frac{dx}{x^2}$ j: $\frac{\pi}{6}$ 283. $\int_{0}^{1} x\lim^2 xdx$ j: π

Mustaqil uy vazifasi uchun misollar

284. $\int_{1}^{+\infty}\left(1-\cos\frac{2}{x}\right)dx$ j: yaqinlashuvchi 286. $\int_{1}^{+\infty}\frac{xarctgx}{\sqrt{1+x^2}}$ j: uzoqlashuvchi

285. $\int_{0}^{1}\frac{dx}{e^{\sqrt[3]{x}}-1}$ j: uzoqlashuvchi 287. $\int_{0}^{1}\frac{dx}{tgx-x}$

O`zgaruvchilari ajralgan va ajraladigan tenglamalar

Ushbu M(x)dx+N(y)dy=0 ko`rinishdagi tenglamaga o`zgaruvchilari ajralgan differensial tenglama deyiladi. Uning o`ziga xos tomoni shundaki, dx oldida faqat x ga bog`liq ko`paytuvchi, dy oldida esa faqat y ga bog`liq ko`paytuvchi turadi. Bu tenglamaning yechimi uni hadma-had integrallash yo`li bilan aniqlanadi:

$$\int M(x)dx+\int N(y)dy=C$$

Differensial tenglamaning oshkormas holda ifodalangan yechimi bu tenglamaning integrali deyiladi. Integrallash doimiysi C ni yechim uchun qulay ko`rinishda tanlash mumkin.

1- misol: tgxdx-ctgydy=0 tenglamaning umumiy yechimini toping.

Yechish: Bu yerda o`zgaruvchilari ajralgan tenglamaga egamiz. Uni hadma-had integrallaymiz:

$\int tgxdx-\int ctgydy=C$ yoki $-\ln|\cos x|-\ln|\sin y|=-\ln\overline{C}$

Bu yerda integrallash doimiysi C ni $-\ln\overline{C}$, ya`ni C= - $\ln\overline{C}$ orqali belgilash qulaydir, bundan

$\ln\sin y\cdot\cos x=\ln\overline{C}$ yoki $\sin y\cdot\cos x=\overline{C}$ umumiy integralni topamiz.

Ta`rif.

$$y' = f_1(x)f_2(y) \qquad (1)$$

ko`rinishdagi tenglamalar o`zgaruvchilari ajraladigan differensial tenglamalar deb ataladi, bu yerda $f_1(x)$ va $f_2(y)$ uzluksiz funksiyalar. (1) tenglamani yechish uchun unda o`zgaruvchilarni ajratish kerak. U holda (1) tenglama

$$\frac{dy}{f_2(y)}=f_1(x)dx \qquad (2)$$

ko`rinishga keladi. Bu tenglamada x o`zgaruvchi faqat o`ng tomonda, y o`zgaruvchisi esa chap tomonda ishtirok etyapti, ya`ni o`zgaruvchilar ajratildi. (2) tenglikni har ikki tomonini integrallab,

$$\int\frac{dy}{f_2(y)}=\int f_1(x)dx+C$$

ekanligini hosil qilamiz, bu yerda C ixtiyoriy o`zgarmas.

2-misol. y' =y/x tenglamani yeching.

Yechish: Berilgan tenglama (2) ko`rinishdagi tenglama, bu yerda $f_1(x)=1/x$ va $f_2(y)=y$. O`zgaruvchilarni ajratib, $\frac{dy}{y}=\frac{dx}{x}$ tenglamani hosil qilamiz. Uni integrallab $\int\frac{dy}{y}=\int\frac{dx}{x}+\ln C$, C>0 yoki lny+lnx=lnC va bu tenglikni potensirlab, y=Cx umumiy yechimni topamiz.

Faraz qilaylik, y=Cx umumiy yechimdan $x_0=1$, $y_0=2$ boshlangich shartlarni qanoatlantiruvchi xususiy yechim topish talab qilinyapti. Bu qiymatlarni y=C·x гa **x** вa **y** larni o`rniga qo`yib, 2=C·1 yoki C= 2 ni topamiz. Demak, xususiy yechim y=2x ekan.

<u>Quyidagi tenglamalarni yeching</u>

1. $x(y^2-4)dx+ydy=0$ J: $y^2-4=Ce^{-x^2}$
2. $y'\cos x=y/\ln y$, $y(0)=1$ J: $\frac{1}{2}\ln 2y\ln tg(\frac{x}{2}+\frac{\pi}{4})$
3. $y'=tgx\cdot tgy$. J: $\sin y\cos x=C$
4. $(1+x^2)dy+ydx=0$, $y(1)=1$. J: $y=e^{\frac{\pi}{4}arctgx}$
5. $\ln\cos y\,dx+x\,tgy\,dy=0$. J: $y=\arccos e^{cx}$
6. $\frac{yy'}{x}+e^y=0$, $y(1)=0$ J: $2e^{-y}(y+1)=x^2+1$
7. $y/y'=\ln y$, $y(2)=1$ J: $2(x-2)=\ln^2 y$
8. $y'+\sin(x+y)=\sin(x-y)$ J: $2\sin x+\ln\left|tg\frac{x}{2}\right|=C$
9. $x\sqrt{1+y^2}dx+y\sqrt{1+x^2}dy=0$ J: $\sqrt{1+x^2}+\sqrt{+y^2}=C$
10. $y'=2^{x-y}$, $y(-3)=-5$ J: $2^x-2^y=\frac{3}{32}$
11. $y'=sh(x+y)+sh(x-y)$ J: $y=\ln tg(chx+C)$
12. $x(y^6+1)dx+y^2(x^4+1)dy$, $y(0)=1$ J: $arctgx^2+2arctgy^3=\frac{\pi}{2}$

Bir jinsli va bir jinsliga keltiriladigan differensial tenglamalar Birinchi tartibli bir jinsli differensial tenglamalar

**1-Ta`rif.** f(x,y) funksiya x va y o`zgaruvchilarga nisbatan n- o`lchovli bir jinsli funksiya deb ataladi, agarda ixtiyoriy λ uchun

$$f(\lambda x,\lambda y)=\lambda^n f(x,y)$$

ayniyat o`rinli bo`lsa.

**2-Ta`rif**. Agarda f(x, y) funsiya x va y ga nisbatan 0- o`lchovli bir jinsli funksiya bo`lsa birinchi tartibli

$$\frac{dy}{dx}=f(x,y) \qquad (3)$$

differensial tenglama x va y ga nisbatan bir jinsli differensial tenglama deb ataladi.

Bir jinsli differensial tenglamani yechish. Faraz qilaylik, (3) bir jinsli differensial tenglama berilgan bo`lsin, u holda shartga ko`ra $f(\lambda x,\lambda y)=\lambda^0 f(x,y)$. Bu ayniyatda $\lambda=\frac{1}{x}$ deb olsak, $f(x,y)=f(1,\frac{y}{x})$ ni hosil qilamiz. Bu holda (3) tenglama quyidagi ko`rinishga keladi:

$$\frac{dy}{dx}=f(1,\frac{y}{x}) \qquad (4)$$

(4) da $u=\frac{y}{x}$, $y=u\cdot x$ almashtirish bajaramiz.

U holda $\frac{dy}{dx}=u+\frac{du}{dx}\cdot x$ ni hosil qilamiz. Hosilaning bu ifodasini (4) ga qo`yib,

$u+\frac{du}{dx}\cdot x=f(1,u)$ yoki $\frac{du}{f(1,u)-u}=\frac{dx}{x}$ tenglikni hosil qilamiz. Bu esa o`zgaruvchilari ajralgan differensial tenglamadir. Integrallab quyidagini topamiz:

$$\int\frac{du}{f(1,u)-u}=\int\frac{dx}{x}+\ln C\ ,\ \int\frac{du}{f(1,u)-u}=\ln|Cx|\ .$$

Integrallarni topgandan so`ng u o`rniga $\frac{y}{x}$ ni qo`yib, berilgan tenglamaning integralini $y=y(x,c)$ ko`rinishida topamiz.

3-misol. $\frac{dy}{dx}=\frac{xy}{x^2-y^2}$ tenglamani yeching.

Yechish: Tenglamaning o`ng tomonidagi funksiya 0-o`lchovli bir jinsli funksiya bo`lgani uchun tenglama bir jinsli differensial tenglama, shuning uchun $\frac{y}{x}=u$ almashtirishni bajaramiz. U holda y=ux, $\frac{dy}{dx}=u+x\cdot\frac{du}{dx}$. Bularni tenglamaga qo`yib $u+x\cdot\frac{du}{dx}=\frac{u}{1-u^2}$ yoki $x\cdot\frac{du}{dx}=\frac{u^3}{1-u^2}$ va o`zgaruvchilarni ajratib, $\frac{(1-u^2)du}{u^3}=\frac{dx}{x}$, ya`ni $(\frac{1}{u^3}-\frac{1}{u})du=\frac{dx}{x}$ tenglamaga kelamiz. Integrallash natijasida $-\frac{1}{2u^2}-\ln|u|=\ln|x|+\ln|C|$ yoki $-\frac{1}{2u^2}=\ln|uxc|$ munosabatlarni hosil qilamiz. Oxirgi tenglikda **u** o`rniga $\frac{y}{x}$ ni qo`yib, $-\frac{x^2}{2y^2}=\ln|cx|$ tenglamaning umumiy integralini topamiz. Ko`rinib turibdiki, y ni **x** orqali elementar funksiyalar yordamida ifodalab bo`lmaydi. Biroq **x** ni y orqali ifodalash mumkin: $x=y\sqrt{-2\ln|Cy|}$

4-misol. $\frac{dy}{dx}=\frac{x+y-3}{x-y-1}$ tenglamani yeching.

Yechish: Tenglamani bir jinsli tenglamaga aylantirish uchun $x=x_1+h$, $y=y_1+k$ almashtirishni bajaramiz. U holda tenglama $\frac{dy_1}{dx_1}=\frac{x_1+y_1+h+k-3}{x_1-y_1+h-k-1}$, h+k-3=0, h-k-1=0 tenglamalar sistemasini yechib h=2, k=1 ekanligini topamiz. Natijada bir jinsli $\frac{dy_1}{dx_1}=\frac{x_1+y_1}{x_1-y_1}$ tenglmani hosil qilamiz. $\frac{y_1}{x_1}=u$ almashtirishni bajarsak, u holda $y_1=ux_1$, $\frac{dy_1}{dx_1}=u+x_1\cdot\frac{du}{dx_1}$, $u+x_1\cdot\frac{du}{dx_1}=\frac{1+u}{1-u}$ bo`ladi va natijada $x_1\cdot\frac{du}{dx_1}=\frac{1+u^2}{1-u}$ o`zgaruvchilari ajraladigan tenglamaga ega bo`lamiz. O`zgaruvchilarni ajratamiz: $\frac{1-u}{1+u^2}du=\frac{dx_1}{x_1}$ integrallab $arctgu-\frac{1}{2}\ln(1+u^2)=\ln|x_1|+\ln|C|$, $arctgu=\ln\left|Cx_1\sqrt{(1+u^2)}\right|$ yoki $Cx_1\sqrt{1+u^2}=e^{arctgu}$ ekanligini topamiz.u o`rniga $\frac{y_1}{x_1}$ ifodani qo`yib, $C\sqrt{x_1^2+y_1^2}=e^{arctg\frac{y_1}{x_1}}$ ekanligini, va nihoyat, x va y o`zgaruvchilarga o`tib $C\sqrt{(x-2)^2+(y-1)^2}=e^{arctg\frac{y-1}{x-2}}$ natijani hosil qilamiz.

Quyidagi tenglamalarni yeching.

13. $(x^2+2xy)dx+xydy=0$ J: $\ln|x+y|+\dfrac{x}{x+y}=C$

14. $y'=\dfrac{y}{x}+\sin\dfrac{y}{x},\ \ y(1)=\dfrac{\pi}{2}$ J: $y=2xarctgx$

15. $xy'\sin(\dfrac{y}{x})+x=y\sin(\dfrac{y}{x})$ J: $Cx=e^{\cos\frac{y}{x}}$

16. $xy+y^2=(2x^2+xy)\cdot y'$ J: $y^2=Cxe\dfrac{-y}{x}$

17. $xyy'=y^2+2x^2$ J: $y^2=4x^2\ln Cx$

18.18. $y`=(\dfrac{y}{x})+\cos(\dfrac{y}{x})$ J: 1+sin(y/x)=Cxcos(y/x).

19. $(x^2+y^2)dx-xydy=0$ J: $y^2=x^2\ln Cx^2$.

20. $(x+y+2)dx+(2x+2y-1)dy=0$ J: x+2y+5ln$|x+y-3|$=C.

21. $(2x+y+1)dx+(x+2y-1)dy=0$ J: $x^2+y^2+xy+x-y=C_1, C_1=C^2-1$.

22. $2(x+y)dy+(3x+3y^{-1})dx=0, y(0)=2$ J: 3x+2y-4+2ln$|x+y-1|$=0.

11. Chiziqli differensial tenglamalar. Bernulli tenglamasi

1. Chiziqli differensial tenglamalar

**Ta`rif.** Noma`lum funksiya va uning hosilasiga nisbatan chiziqli bo`lgan tenglamaga chiziqli differensial tenglama deyiladi. Bunday tenglama

$$\frac{dy}{dx}+P(x)\cdot y=Q(x) \tag{11}$$

ko`rinishga ega bo`ladi, bu yerda P(x) va Q(x) berilgan uzluksiz funksiyalar.

$$y=e^{-\int Pdx}\left[\int Q(x)e^{\int Pdx}dx+C\right]$$

(11) ning umumiy yechimi bo`ladi.

1-misol. $\dfrac{dy}{dx}-\dfrac{2}{x+1}\cdot y=(x+1)^3$ tenglamani yeching.

Yechish: y=u v deb olsak, u holda $\dfrac{dy}{dx}=u\dfrac{dv}{dx}+v\dfrac{du}{dx}$

$\dfrac{dy}{dx}$ ifodasini berilgan tenglamaga qo`ysak,

$$u\frac{dv}{dx}+\frac{du}{dx}v-\frac{2}{x+1}uv=(x+1)^3$$

yoki

$$u(\frac{dv}{dx}-\frac{2}{x+1}v)+\frac{du}{dx}v=(x+1)^3. \tag{15}$$

v funksiyani aniqlash uchun $\dfrac{dv}{dx}-\dfrac{2}{x+1}v=0$ yoki $\dfrac{dv}{v}=\dfrac{2dx}{x+1}$ tenglamani hosil qilamiz. Bu yerdan ln| v|= 2ln|x+1| yoki $v=(x+1)^2$.

v ni ifodasini (15) tenglikka qo`yib, u ni aniqlash uchun $(x+1)^2 \frac{du}{dx} = (x+1)^3$ yoki $\frac{du}{dx} = x+1$ tenglamani hosil qilamiz, bu yerdan $u = \frac{(x+1)^2}{2} + C$. Demak, berilgan tenglamaning umumiy yechimi $y = \frac{(x+1)^4}{2} + C(x+1)^2$ bo`lar ekan.

Bernulli tenglamasi

Ta`rif.

$$\frac{dy}{dx} + P(x)\cdot y = Q(x)\cdot y^n, \ n \geq 2.$$

ko`rinishdagi tenglama Bernulli tenglamasi deb ataladi, bu yerda P(x) va Q(x) berilgan uzluksiz funksiyalar, n≠0;1.

Tenglamaning barcha hadlarini y^n ga bo`lamiz

$$y^{-n}\frac{dy}{dx} + P(x)\cdot y^{-n+1} = Q(x) \qquad (16)$$

va $z=y^{-n+1}$ almashtirishni bajaramiz, u holda

$$\frac{dz}{dx} = (-n+1)\cdot y^{-n}\frac{dy}{dx}.$$

Topilgan qiymatni (16) tenglamaga qo`yib, $\frac{dz}{dx} + (-n+1)P\cdot z = (-n+1)\cdot Q$ chiziqli tenglamani hosil qilamiz. CHiziqli tenglamaning umumiy integralini topgandan so`ng, z o`rniga $y^{-n+1}$ ni qo`yib, Bernulli tenglamasining umumiy integralini hosil qilamiz.

2–misol. Ushbu

$$\frac{dy}{dx} + xy = x^3\cdot y^3$$

tenglamani yeching.

Yechish: Tenglamaning barcha hadlarini u^3 ga bo`lamiz

$$y^{-3}\frac{dy}{dx} + xy^{-2} = x^3. \qquad (17)$$

va $z=\mathbf{y}^{-2}$ almashtirishni bajaramiz, u holda $\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$. Bu qiymatlarni (17) ga qo`yib

$$\frac{dz}{dx} - 2xz = -2x^3 \qquad (18)$$

chiziqli tenglamani hosil qilamiz. Uning umumiy integralini topamiz:

$$z=u\cdot\vartheta, \ \frac{dz}{dx} = u\frac{d\vartheta}{dx} + \vartheta\frac{du}{dx}.$$

Bu ifodalarni (18) tenglamaga qo`yamiz: $u\frac{d\vartheta}{dx} + \vartheta\frac{du}{dx} - 2xu\vartheta = -2x^3$ yoki $u(\frac{d\vartheta}{dx} - 2x\vartheta) + \vartheta\frac{du}{dx} = -2x^3$ qavs ichidagi ifodani nolga tenglab, $\frac{d\vartheta}{dx} - 2x\vartheta = 0$, $\frac{d\vartheta}{\vartheta} = 2xdx$, $\ln|\vartheta| = x^2$, $\vartheta = e^{x^2}$ ekanligini topamiz. u ni aniklash uchun

$$e^{x^2}\cdot\frac{du}{dx} = -2x^3$$

tenglamaga ega bo`lamiz. O`zgaruvchilarni ajratib

$$du = -2e^{-x^2}x^3dx, \ u = -2\int e^{-x^2}x^3dx + C$$

ekanligini topamiz. Oxirgi integralni bo`laklab

$$u = x^2e^{-x^2} + e^{-x^2} + C, \;\; z = u\cdot\vartheta = x^2 + 1 + Ce^{x^2}$$

ifodalarni topamiz. Demak, berilgan tenglamaning umumiy integrali $y^{-2} = x^2 + 1 + Ce^{x^2}$ yoki $y = \dfrac{1}{\sqrt{x^2 + 1 + Ce^{x^2}}}$ bo`lar ekan.

Quyidagi tenglamalarni yeching.

25. **y′cos²x+y=tgx, y(0)=0** **J:** $y=tgx-1+e^{-tgx}$.

26. $y' + \dfrac{2y}{x} = 3x^2y^{4/3}$ J**:** $y^{-1/3}=Cx^{2/3}-(3/7)x^{3}$.

27. $y'-ythx=ch^2x$ J**:** y=chx(shx+C).

28. $y' - \dfrac{y}{x-1} = \dfrac{y^2}{x-1}$ J**:** y=(x-1)(C-x).

29. $y' + \dfrac{xy}{1-x^2} = \arcsin x + x$ J**:** y= $\sqrt{1-x^2}\,[\frac{1}{2}(\arcsin\, x)^2 - \sqrt{1-x^2} + C]$.

30. $4xy'+3y=-e^x\cdot x^4y^5$ J**:** $y^{-4}=x^3(e^x+C)$.

31. $xy'-y=x^2\cos x$ J**:** y=x(sinx+C).

32. $y' + \dfrac{3x^2y}{x^3+1} = y^2(x^3+1)\sin x,$ y(0)=1 J**:** $y=\sec x/(x^3+1)$.

33. $y' + 2xy = xe^{-x^2}$ J**:** y= $e^{-x^2}(x^2/2 + C)$.

34. $ydx+(x+x^2y^2)dy=0$ J**:** x=1/[y(y+C)].

35. y′cosx+y=1-sinx J**:** cosx(x+C)/(1+sinx).

36. $y' + \dfrac{y}{x} = x^2y^4$ J**:** y= $\dfrac{1}{x\sqrt[3]{3\ln(C/x)}}$.

37. $(x^2\ln y-x)y'=y$ J: x= $\dfrac{1}{\ln y + 1 - Cy}$.

To`la differensialli tenglama. Integrallovchi ko`paytuvchi to`la differensialli tenglama

**Ta`rif.** Agar M(x,y)dx+N(x,y)dy=0 ko`rinishdagi tenglamaning chap qismi biror u(x,y) funksiyaning to`liq differensiali, ya`ni

$$du=M(x,y)dx+N(x,y)dy \qquad (19)$$

bo`lsa, u holda bunday tenglama to`liq differensialli tenglama deyiladi.
(19) tenglama to`liq differensialli tenglama bo`lishi uchun

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

shart bajarilishi kerak.

3–misol. $(3x^2+6xy^2)dx+(6x^2y+4y^3)dy=0$ tenglamaning umumiy yechimini toping.
Yechish: Bu yerda $M(x,y)=3x^2+6xy^2$, $N(x,y)=6x^2y+4y^3$.

$\frac{\partial N}{\partial y}=12xy, \quad \frac{\partial N}{\partial x}=12xy,$ yani $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}.$ $\frac{\partial u}{\partial x}=M(x,y)$ bo`lganligi sababli $\frac{\partial u}{\partial x}=3x^2+6xy^2.$

Bu tenglikni x bo`yicha integrallaymiz: $u=x^3+3x^2y^2+\varphi(y)$. Bundan $\frac{\partial u}{\partial y}=6x^2y+\varphi'(y).$

$\frac{\partial u}{\partial y}=N(x,y)$ ekanligini hisobga olsak,

$\varphi'(u)=6x^2y+4y^3-6x^2y$ yoki $\varphi'(u)=4y^3$. Bundan

$\varphi(y)=y^4+C$.

Demak, $u=x^3+3x^2y^2+y^4+C$

Yoki $x^3+3x^2y^2+y^4=C$.

1. Integrallovchi ko`paytuvchi

Agar $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$ bo`lsa, u holda ba`zi bir shartlar bajarilganda, shunday μ(x,u) funksiyani topish mumkinki, μMdx+μNdy=du bo'ladi. Bu μ(x,y) funksiya integrallovchi ko`paytuvchi deyiladi.

Quyidagi hollarda integrallovchi ko`paytuvchini topish oson:

1) $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=\Phi(x)$ bo`lganda, $\ln\mu=\int\Phi(x)dx$ bo'ladi.

2) $\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}=\Phi_1(y)$ bo`lganda, $\ln\mu=\int\Phi_1(y)dy$ bo'ladi.

4-misol. $(y+xy^2)dx-xdy=0$ tenglamani yeching.

Yechish: Bu yerda $M=y+xy^2$, $N=-x$, $\frac{\partial M}{\partial y}=1+2xy, \quad \frac{\partial N}{\partial x}=-1, \quad \frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$.

Demak, tenglamaning chap tomoni biror funksiyaning to`la differensiali emas. Bu tenglamaning faqat u ga bog`liq bo`lgan integrallovchi ko`paytuvchisi bormi degan masalani qaraymiz.

$$\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}=\frac{-1-1-2xy}{y+xy^2}=-\frac{2}{y},$$

bundan

$$\ln\mu=-2\ln y, \text{ ya'ni } \mu=\frac{1}{y^2}.$$

Berilgan tenglamaning μ ga ko`paytirganda keyin

$$\left(\frac{1}{y}+x\right)dx-\frac{x}{y^2}dy=0$$

tenglama hosil bo`ladi. Bu to`la differensialli tenglamadir, chunki

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}=-\frac{1}{y^2}.$$

Tenglamani yechib

$$\frac{x}{y}+\frac{x^2}{2}+C=0 \quad \text{yoki} \quad y=-\frac{2x}{x^2+2C}$$

umumiy integralni topamiz.

Quyidagi differensial tenglamalarning chap tomonlari to`liq differensialdan iborat ekanligi tekshirilsin va tenglamalar yechilsin

38. $(e^x+y+\sin y)dx+(e^y+x+x\cos y)dy=0$ J: $e^x+xy+x\sin y+e^y=C$.

39. $(x+y-1)dx+(e^y+x)dy=0$ J. $e^y+\frac{1}{2}x^2+xy-x=C$, $C=C_1+1$.

40. $(x\cos y-y\sin y)dy+(x\sin y+y\cos y)dx=0$ J: $e^x(x\sin y+y\cos y-\sin y)=C$.

41. $2xydx+(x^2-y^2)dy=0$ J: $3x^2y-y^3=C$.

42. $(2-9xy^2)xdx+(4y^2-6x^3)ydy=0$ J: $x^2-3x^3y^2+y^4=C$.

43. $\frac{y}{x}dx+(y^3+\ln x)dy=0$ J: $4y\ln x+y^4=C$.

44. $(10xy-8y+1)dx+(5x^2-8x+3)dy=0$ J: $5x^2y-8xy+x+3y=C$.

45. $3x^2(1+\ln y)dx=(2y-\frac{x^3}{y})dy$ J: $x^3+x^3\ln y-y^2=C$.

46. $2x\cos^2 ydx+(2y-x^2\sin^2 y)dy=0$ J: $x^2\cos^2 y+y^2=C$.

Quyidagi differensial tenglamalarning integrallovchi ko`paytuvchilari topilsin va tenglamalar yechilsin

47. $(x^2-y)dx+xdy=0$ J: $\mu=1/x^2$; $x+y/x=C$.

48. $y^2dx+(yx-1)dy=0$ J: $\mu=1/y$; $xy-\ln y=0$.

49. $(x^2+y^2+x)dx+ydy=0$ J: $2x+\ln(x^2+y^2)=C$.

50. $xy^2(xy'+y)=1$ J: $2x^3y^3-3x^2=C$.

51. $(x^2+3\ln y)ydx=xdy$ J: $x^2+\ln y=Cx^3$; $x=0$.

52. $2x\,\mathrm{tg}\,ydx+(x^2-2\sin y)dy=0$ J: $\mu=\cos y$; $x^2\sin y+\frac{1}{2}\cos 2y=C$.

53. $(e^{2x}-y^2)dx+ydy=0$ J: $\mu=e^{-2x}$; $y^2=(C-2x)e^{2x}$.

54. $(1+3x^2\sin y)dx-x\,\mathrm{ctg}\,ydy=0$ J: $\mu=1/\sin y$; $x/\sin y+x^3=C$.

55. $(\sin x+e^y)dx+\cos xdy=0$ J: $\mu=e^{-y}$; $e^{-y}\cos x=C+x$.

12. **Tartibini pasaytirish mumkin bo`lgan tenglamalar**

$$y^{(n)}=f(x)$$

Bunday ko`rinishdagi tenglamani n marta ketma-ket integrallash natijasida umumiy yechimi topiladi:

$$y^{(n)}=f(x),$$

$$y^{(n-1)}=\int f(x)dx+c_1=f_1(x)+c_1,$$

$$y^{(n-2)}=\int[f_1(x)+c_1]dx+c_2=f_2(x)+c_1x+c_2$$

………

$$y = f_n(x) + \frac{c_1}{(n-1)!}x^{n-1} + \frac{c_2}{(n-2)!}x^{n-2} + \ldots + c_{n-1}x + c_n, \qquad (22)$$

bu yerda $f_n(x) = \int\int\ldots\int f(x)dx^n$. (22) ni quyidagicha ham yozish mumkin:

$$y = f_n(x) + c_1x^{n-1} + c_2x^{n-2} + \ldots + c_{n-1}x + c_n$$

1-misol. $y''' = \sin x$ tenglamaning umumiy yechimi topilsin.

Yechish: $y''' = \frac{dy''}{dx}$ ekanligini e`tiborga olib, berilgan tenglamani $\frac{dy''}{dx} = \sin x$ yoki $dy'' = \sin x dx$ ko`rinishda yozish mumkin. Ketma-ket integrallab quyidagiga ega bo`lamiz:

$$y'' = \int \sin x dx + c_1 = -\cos x + c_1,$$

$$y' = \int(-\cos x + c_1)dx + c_2 = -\sin x + c_1x + c_2,$$

$$y = \int(-\sin x + c_1x + c_2)dx + c_3 = \cos x + \frac{1}{2}c_1x^2 + c_2x + c_3.$$

Demak, $y = \cos x + cx^2 + c_2x + c_3, \quad c = \frac{1}{2}c_1.$

Izlangan umumiy yechimga ega bo`ldik.

Quyidagi tenglamalarni yeching

1. $y^{IV} = \cos^2 x, \; y(0) = \frac{1}{32}, \; y'(0) = 0, \; y''(0) = \frac{1}{8}, \; y'''(0) = 0.$

2. $y''' = x\sin x, \; y(0) = 0, \; y'(0) = 0, \; y''(0) = 2.$

3. $y''' \sin^4 x = \sin 2x.$

4. $y'' = 2\sin x\cos^2 x - \sin^3 x$

5. $y''' = xe^{-x}, \; y(0) = 0, \; y'(0) = 2, \; y''(0) = 2.$

6. $y''' = \frac{6}{x^3}, \; y(1) = 2, \; y'(1) = 1, \; y''(1) = 1.$

7. $y'' = 4\cos 2x, \; y(0) = 0, \; y'(0) = 0.$

8. $y'' = \frac{1}{1+x^2}.$

9. $y'' = \frac{1}{\cos^2 x}, \; y\left(\frac{\pi}{4}\right) = \ln\sqrt{2}, \; y'\left(\frac{\pi}{4}\right) = 1.$

10. $y''' = x^{-2}.$ 11. $y^{IV} = \cos x.$

12. $y'' = \frac{1}{sin^2 x}$. 13. $y'' = xe^x$, $y(0) = 1$, $y'(0) = 2$.

14. $y'' = sin\, 2x$, $y(0) = 6$, $y'(0) = 0$.

Noma`lum funksiya oshkor holda qatnashmagan tenglamalar

$$F(x, y^{(k)}, y^{(k+1)} \ ..., y^{(n)}) = 0 \qquad (23)$$

tenglamada u funksiya oshkor holda katnashmagan.
Bu tenglamada

$$y^{(k)} = p(x)$$

almashtirishni bajarib, uni

$$F(x, p, p',,,, p^{n-k}) = 0$$

ko`rinishga keltiriladi. SHunday qilib, (23) tenglamani tartibi k birlikka pasayadi.

2-misol. $xy'' = y'\, ln\left(\frac{y'}{x}\right)$ tenglamaning umumiy yechimi topilsin.

Yechish: Bu tenglamada y funksiya oshkor holda katnashmagani uchun $y' = p(x)$ almashtirishni bajaramiz.
Bu holda, $y'' = p'$ o`rinli bo`ladi. Bularni tenglamaga qo`ysak,

$$x \cdot p' = p\, ln\frac{p}{x} \quad \text{yoki} \quad p' = \frac{p}{x} ln\frac{p}{x}.$$

Hosil bo`lgan tenglama birinchi tartibli bir jinsli tenglama bo`lganidan $\frac{p}{x} = t$ yoki $p = x \cdot t$ almashtirishni bajarsak, $p' = t + xt'$ ga ega bo`lamiz. Buni e`tiborga olib, tenglamani $t + xt' = t\, ln\, t$ yoki $xt' = t(ln\, t - 1)$ ko`rinishda yozish mumkin. O`zgaruvchilarni ajratsak,

$$\frac{dt}{t(ln\, t - 1)} = \frac{dx}{x}$$

tenglamaga ega bo`lamiz. Integrallash natijasida

$$ln(ln\, t - 1) = ln\, x + ln\, c_1 \text{ ёки } ln\, t - 1 = c_1 x,$$

bundan esa $t = e^{c_1 x + 1}$ kelib chiqadi. $t = \frac{p}{x}$ ekanini e`tiborga olsak,

$$p = xe^{c_1 x + 1}$$

hosil bo`ladi. $p(x) = y'$ dan $y' = xe^{c_1 x + 1}$ tenglik hosil bo`ladi.
Bundan esa izlangan umumiy yechim

$$y = \int xe^{c_1 x + 1} dx = \frac{1}{c_1} xe^{c_1 x + 1} - \frac{1}{c_1^2} e^{c_1 x + 1} + c_2$$

ko`rinishda hosil bo`ladi.

3-misol. $y'''(x - 1) - y'' = 0$ tenglamaning

$y(2)=2, \quad y'(2)=1, \quad y''(2)=1$ shartlarni qanoatlantiruvchi yechimi topilsin.

Yechish: $y''=p(x)$ va $y'''=p'$ almashtirish bajarsak, dastlabki tenglama $p'(x-1)=p$ yoki $\frac{dp}{p}=\frac{dx}{(x-1)}$ ko`rinishga keladi. Integrallash natijasida $ln\, p=ln(x-1)+ln\, c_1$ yoki $p=c_1(x-1)$ yechim hosil bo`ladi. Dastlabki belgilashni e`tiborga olib, $y''=c_1(x-1)$ natijaga ega bo`lamiz. Bu esa tartibi pasayadigan tenglamadan iborat. Ketma-ket integrallab:

$$y'=\int c_1(x-1)dx+c_2=\frac{1}{2}c_1x^2-c_1x+c_2,$$

$$y=\int\left(\frac{1}{2}c_1x^2-c_1x+c_2\right)dx+c_3=\frac{c_1}{6}x^3-\frac{c_1}{2}x^2+c_2x+c_3$$

umumiy yechimni hosil qilamiz. CHetki shartlarni e`tiborga olib

$y''(2)=1$ dan $1=c_1(2-1)$ yoki $c_1=1,$

$y'(2)=1$ dan $1=\frac{1}{2}\cdot 4-2+c_2$ yoki $c_2=1,$

$y(2)=2$ dan $2=\frac{8}{6}-\frac{4}{2}+2+c_3$ yoki $c_3=\frac{2}{3}$

natijalarni hosil qilamiz. Bundan esa xususiy yechimni

$$y=\frac{1}{6}x^3-\frac{1}{2}x^2+x+\frac{2}{3}$$

topamiz.

Quyidagi tenglamalarni yeching

15. $x^3y''+x^2y'=1$
16. $y''+y'tgx=sin\,2x$
17. $y''x\,ln\,x=y'$
18. $xy''-y'=e^x\cdot x^2$
19. $y''+2xy'^2=0$
20. $(1-x^2)y''-xy'=2$
21. $2xy'''\cdot y''=y''^2-a^2$
22. $(1+x^2)y''+1+y'^2=0$
23. $x^2y''=y'^2$
24. $y''(e^x+1)+y'=0$
25. $(1+x^2)y''+2xy'=x^3$
26. $y''tgx=y'+1$
27. $xy''+y'+x=0$
28. $y''-\frac{1}{x-1}y'=x(x-1), y(2)=1, y'(2)=-1$
29. $xy''=y'+x\,sin\frac{y'}{x}$
30. $(1-x^2)y''+xy'=2$

Argument oshkor holda qatnashmagan tenglama

$$F\left(y, y', y'', \ldots y^{(n)}\right) = 0$$

tenglamada erkli o`zgaruvchi x oshkor holda ishtirok etmaydi.
Bu tenglama

$$y' = p(y) \qquad (24)$$

almashtirish bilan tartibini bittaga pasaytirib yechiladi.

(24) almashtirishda: $y'' = p'(y) \cdot y' = p \cdot p'$,

va $y''' = p\left[p \cdot p'' + p'^2\right]$,...

o`rniga qo`yishlar bajariladi.

4-misol. $1 + y'^2 = y \cdot y''$ tenglamaning umumiy yechimini toping.

Yechish: $y' = p(y)$ va $y'' = pp'$ almashtirishlarni bajarsak, dastlabki tenglama

$1 + p^2 = y \cdot p \cdot p'$ ko`rinishga keladi, bu esa birinchi tartibli o`zgaruvchilari ajraladigan tenglamadir.

O`zgaruvchilarni ajratib, $\dfrac{pdp}{1 + p^2} = \dfrac{dy}{y}$ tenglamani hosil qilamiz.

Tenglikni integrallab, quyidagiga ega bo`lamiz:

$\dfrac{1}{2} ln\left|1 + p^2\right| = ln\, y + ln\, c_1$ yoki $1 + p^2 = c_1^2 y^2$, $p = \pm\sqrt{c_1^2 y^2 - 1}$. Dastlabki o`zgaruvchi y ga qaytib $y' = \pm\sqrt{c_1^2 y^2 - 1}$ yoki $\dfrac{dy}{\sqrt{c_1^2 y^2 - 1}} = \pm dx$ natijaga ega bo`lamiz.

Tenglikni integrallab $\dfrac{1}{c_1} ln\left(c_1 y + \sqrt{c_1^2 y^2 - 1}\right) = \pm(x + c_2)$ yoki

$y = \dfrac{1}{2c_1}\left(e^{\pm(x + c_2)c_1} + e^{\pm(x + c_2)c_1}\right) = \dfrac{1}{c_1} ch c_1 (x + c_2)$ izlangan umumiy yechimni hosil qilamiz.

Quyidagi tenglamalarni yeching

31. $y \cdot y'' + y'^2 = 0$
32. $y'' + 2y(y')^3 = 0$
33. $y'' tgy = 2y'^2$
34. $y''(2y + 3) - 2y'^2 = 0$
35. $y(1 - ln\, y)y'' + (1 + ln\, y)y'^2 = 0$
36. $y''(1 + y) = y'^2 + y'$
37. $yy'' + y = y'^2$
38. $y'^2 + 2yy'' = 0$
39. $yy'' - y'^2 = 0$, $y(0) = 1, \quad y'(0) = 2$

40.Egrilik radiusining oy o`qdagi proeksiyasi o`zgarmas a bo`lib $ox$ o`q bilan esa koordinata boshida kesishuvchi egri chiziq tenglamasini tuzing.

Ж: x $\sqrt{p}$ =lnp+C, y= $\sqrt{p}$ (4-lnp-C); y=0 .

41. Suyuqlikka tashlangan m massali jism o`z og`irligi tufayli cho`ka boshladi. Agar suyuqlik qarshiligi jism tezligiga proporsional bo`lsa, harakat qonunini toping.

J: x=C(p-1)-2+2p+1, y=Cp^2(p-1)-2+p^2; y=0; y=x-2

42. $2y\,y'' = (y')^2$ J: xp2=p+C, y=2+2Cp-1-lnp.

43. $y''y^3 = 1$ J: y= y=Cx-lnC; y=lnx+1 .

44. $2y\,y'' = 1 + y'^2$ J: Cx-C2; maxsus integral y= $\frac{x^2}{4}$

45. $y \cdot y'' = y'^2 + y^2\, ln\, y$ J: y=Cx-a $\sqrt{1+c^2}$; maxsus integral x^2+y^2=a^2

46. $y'' = y' / \sqrt{y}$ J: y=Cx+ $\frac{1}{2c^2}$; maxsus integral y=1,5x $^{\frac{2}{3}}$

14.O`zgarmas koeffisientli chiziqli bir jinsli tenglama

1-misol. $y'' - 7y' + 6y = 0$ tenglamaning umumiy yechimi topilsin.

Yechish: $k^2 - 7k + 6 = 0$ xarakteristik tenglamani tuzib, $k_1 = 1$ va $k_2 = 6$ ildizlarga ega bo`lamiz, bularga esa $e^x$ va $e^{6x}$ xususiy yechimlar, mos keladi. Bu yechimlar chiziqli erkli bo`lganidan, umumiy yechim (33) formulaga asosan quyidagi ko`rinishda yoziladi:

$$y = c_1 e^x + c_2 e^{6x}$$

2-misol. $y^{IV} - 13y'' + 36y = 0$ tenglamaning umumiy yechimi topilsin.

Yechish: Xarakteristik tenglama
$k^4 - 13k^2 + 36 = 0$ ko`rinishda bo`lib, uning ildizlari $k_{1,2} = \pm 3,\ k_{3,4} = \pm 2$. Bunga mos $e^{-3x}, e^{3x}, e^{-2x}, e^{2x}$ funksiyalar chiziqli erkli bo`lganligidan, umumiy yechim (33) formulaga asosan

$$y = c_1 e^{-3x} + c_2 e^{3x} + c_3 e^{-2x} + c_4 e^{2k}.$$

Quyidagi tenglamalarning umumiy yechimlari topilsin

1. $y'' - 4y' + 3y = 0$
2. $y'' - 4y' + 4y = 0$
3. $y'' - 4y' + 13y = 0$
4. $y'' - 4y = 0$
5. $y'' + 4y = 0$
6. $y'' + 4y' = 0$
7. $y'' - y' - 2y = 0$
8. $y'' + 25y = 0$
9. $y'' - y' = 0$
10. $y'' + 4y' + 4y = 0$

11. $y^{IV} - 2y''' + y'' = 0$

12. $y^{IV} + a^4 y = 0$

13. $y^{IV} + 5y'' + 4y = 0$

14. $y'' - 3y' + 2y = 0$

Quyidagi tenglamalarning boshlang`ich yoki chetki shartlarni qanoatlantiruvchi yechimi topilsin

15. $y'' + 5y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = -6$

16. $y'' - 10y' + 25y = 0, \quad y(0) = 0, \quad y'(0) = 1$

17. $y'' - 2y' + 10y = 0, \quad y\left(\frac{\pi}{6}\right) = 0, \quad y'\left(\frac{\pi}{6}\right) = e^{\frac{\pi}{6}}$

18. $y'' + 3y' = 0, \quad y(0) = 1, \quad y'(0) = 2$

19. $y'' + 9y = 0, \quad y(0) = 0, \quad y\left(\frac{\pi}{4}\right) = 1$

20. $y'' + y = 0, \quad y(0) = 1, \quad y'\left(\frac{\pi}{3}\right) = 0$

21. $9y'' + y = 0, \quad y\left(\frac{3\pi}{2}\right) = 2, \quad y'\left(\frac{3\pi}{2}\right) = 0.$

22. $y'' - y = 0, \quad y(0) = 2, \quad y'(0) = 4.$

23. $y'' + 2y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 1.$

15. O'zgarmas koeffisientli chiziqli bir jinsli bo'lmagan differensial tenglamalar

1-misol. $y'' + y' - 2y = \cos x - 3\sin x$ tenglamaning $y(0) = 1, \quad y'(0) = 2$ boshlang`ich shartlarni qanoatlantiruvchi yechimi topilsin.

Yechish: Xarakteristik tenglama $k^2 + k - 2 = 0$, uning yechimlari esa $k_1 = -2, \quad k_2 = 1$ bo`lgani uchun bir jinsli tenglamaning umumiy yechimi

$$y = c_1 e^{-2x} + c_2 e^x$$

ko`rinishda bo`ladi. $f(x) = e^{0x}(\cos x - 3\sin x)$, ya`ni $\alpha = 0, \ \beta = 1$ bo`lgani uchun xususiy yechimni (43) formulaga asosan

$$U(x) = A\cos x + B\sin x$$

ko`rinishda izlaymiz. $U(x)$ ni tenglamaga qo`ysak:

$-A\cos x - B\sin x - A\sin x + B\cos x - 2A\cos x - 2B\sin x = \cos x - 3\sin x$ yoki $(B - 3A)\cos x - (3B + A)\sin x = \cos x - 3\sin x$.

Mos koeffisientlarni tenglab, quyidagi sistemaga ega bo`lamiz:

$$\begin{cases} B - 3A = 1, \\ 3B + A = 3, \end{cases} \quad \text{yoki} \quad A = 0, \ B = 1$$

Bundan esa umumiy yechim $y = c_1 e^{-2x} + c_2 e^x + \sin x$ ko`rinishda ekanligini topamiz. $c_1$ va $c_2$ koeffisientlarni topish uchun boshlang`ich shartlardan foydalanib, quyidagi sistemani hosil qilamiz: $\begin{cases} c_1 e^0 + c_2 e^0 + \sin 0 = 1, \\ -2c_1 e^0 + c_2 e^0 + \cos 0 = 2, \end{cases}$ yoki $c_1 = 0,\ c_2 = 1$. Demak, $y = e^x + \sin x$ izlangan yechim bo`ladi.

3-misol. $y'' + y = xe^x + 2e^{-x}$ tenglamaning umumiy yechimini toping.

Yechish: Xarakteristik tenglama $k^2 + 1 = 0$, uning ildizlari esa $k_{1,2} = \pm i$ bo`ladi. SHuning uchun bir jinsli tenglamaning umumiy yechimi

$$y = c_1 \cos x + c_2 \sin x$$

ko`rinishda bo`ladi. $f(x) = f_1(x) + f_2(x) = xe^x + 2e^{-x}$ bo`lgani uchun $\alpha_1 = 1,\ \alpha_2 = -1,\ \beta_1 = \beta_2 = 0,\ p_1(x) = x$ demak, xususiy yechimni $U(x) = U_1(x) + U_2(x) = (Ax + B)e^x + Ce^{-x}$ ko`rinishda izlaymiz. Tegishli hosilalarni hisoblab tenglamaga qo`ysak:

$$2Ae^x + (Ax + B)e^x + Ce^{-x} + (Ax + B)e^x + Ce^{-x} = xe^x + 2e^{-x},$$

$$(2Ax + 2A + 2B)e^x + 2Ce^{-x} = (1x + 0)e^x + 2e^{-x}.$$

Noma`lum koeffisientlarni aniqlash uchun quyidagi sistemani hosil qilamiz:

$$\begin{cases} 2A = 1, \\ 2A + 2B = 0, \\ C = 1. \end{cases} \quad \text{yoki} \quad A = \frac{1}{2},\ B = -\frac{1}{2},\ C = 1.$$

Demak, dastlabki tenglamaning umumiy yechimi quyidagicha bo`ladi

$$y = c_1 \cos x + c_2 \sin x + \frac{1}{2}(x - 1)e^x + e^{-x}.$$

Quyidagi tenglamalarni yeching

1. $y'' - 2y' + y = e^{2x}$
2. $y'' - 4y = 8x^3$
3. $y'' + 3y' + 2y = \sin 2x + 2\cos 2x$
4. $y'' + y = x + 2e^x$
5. $y'' + 3y' = 9x$
6. $y'' + 4y' + 5y = 5x^2 - 32x + 5$
7. $y'' - 3y' + 2y = e^x$
8. $y'' + 5y' + 6y = e^{-x} + e^{-2x}$

9. $y''' + y'' = 6x + e^{-x}$

10. $y'' + y' - 2y = 6x^2$

Differentsial tenglamalar sistemasi.

1-misol**.** Sistemaning yechimini toping.

$$\begin{cases} \dfrac{dx}{dt} = x - 5y; \\ \dfrac{dy}{dt} = 2x - y. \end{cases}$$

Yechish. Berilgan sistemaning xarakteristik tenglamasi

$$\begin{vmatrix} 1-r & -5 \\ 2 & -1-r \end{vmatrix} = r^2 + 9 = 0$$

ko'rinishda bo'lib, u $r_{1,2} = \pm 3i$ ildizlarga ega.

sistemaga ega bo'lamiz. $r_1 = 3i$ uchun

$$\begin{cases} (1-3i)\lambda_1 - 5\mu_1 = 0, \\ 2\lambda_1 - (1+3i)\mu_1 = 0. \end{cases}$$

$\lambda_1 = 5$ deb, $\mu_1 = 1 - 3i$ ni topamiz. U holda

$$x_1 = 5e^{3it}, \qquad y_1 = (1-3i)e^{3it}$$

xususiy yechimlarni topamiz. $r_2 = -3i$ ni qo'yib, $\lambda_2 = 5,$ $\mu_2 = 1 + 3i$ larni topamiz.

U holda xususiy yechimlar

$$x_2 = 5e^{-3it}, \qquad y_1 = (1+3i)e^{-3it}$$

ko'rinishda bo'ladi.

Yangi fundamental yechimlar sistemasiga o'tamiz:

$$\left.\begin{array}{ll} \overline{x_1} = \dfrac{x_1 + x_2}{2}, & \overline{x_2} = \dfrac{x_1 - x_2}{2}, \\ \overline{y_1} = \dfrac{y_1 + y_2}{2}, & \overline{y_1} = \dfrac{y_1 - y_2}{2}, \end{array}\right\}$$

Bundan Eyler formulasi $e^{\pm \alpha i t} = \cos \alpha t \pm i \sin \alpha t$ dan foydalanib

$$\overline{x_1} = 5\cos 3t, \quad \overline{x_2} = 5\sin 3t,$$

$$\overline{y_1} = \cos 3t + 3\sin 3t, \quad \overline{y_2} = \sin 3t - 3\cos 3t$$

larni topamiz.

U holda berilgan sistemaning umumiy yechimi quyidagi ko'rinishda bo'ladi:

$$x(t) = 5C_1 \cos 3t + 5C_2 \sin 3t;$$

$$y(t) = C_1(\cos 3t + 3\sin 3t) + C_2(\sin 3t - 3\cos 3t).$$

2-misol**.** Sistemaning yechimini toping.

$$\begin{cases} \frac{dx}{dt} = 2x + y, \\ \frac{dy}{dt} = 4y - x. \end{cases}$$

Yechish**.** Sistemaning xarakteristik tenglamasi

$$\begin{vmatrix} 2-r & 1 \\ -1 & 4-r \end{vmatrix} = r^2 - 6r + 9 = 0$$

$r_1 = r_2 = 3$ ildizga ega. Sistemaning yechimini

$$\begin{cases} x = (\lambda_1 + \mu_1 t)e^{3t}, \\ y = (\lambda_2 + \mu_2 t)e^{3t} \end{cases}$$

ko'rinishda izlash kerak. $3(\lambda_1 + \mu_1 t) + \mu_1 = 2(\lambda_1 + \mu_1 t) + (\lambda_2 + \mu_2 t)$ tenglikka ega bo'lamiz. Chap va o'ng tomondagi bir xil darajali t ning koeffitsientlarini tenglashtirib

$$\begin{cases} 3\lambda_1 + \mu_1 = 2\lambda_1 + \lambda_2, \\ 3\mu_1 = 2\mu_1 + \mu_2 \end{cases}$$

sistemani hosil qilamiz. Bundan

$$\begin{cases} \lambda_2 = \lambda_1 + \mu_1, \\ \mu_2 = \mu_1 \end{cases}$$

ni topamiz. λ_1 va μ_1 sonlarni ixtiyoriy parametr deb olishimiz mumkin. $\lambda_1 = C_1$ va $\mu_2 = C_2$ deb belgilasak, (55) sistemaning umumiy yechimi

$$\begin{cases} x = (C_1 + C_2 t)e^{3t}, \\ y = (C_1 + C_2 + C_2 t)e^{3t} \end{cases}$$

ko'rinishda bo'ladi.

Darsda yechish uchun misollar

Differentsial tenglamalar sistemasini yeching.

288. $\begin{cases} \frac{dx}{dt} = -9y \\ \frac{dy}{dt} = x \end{cases}$ 289. $\begin{cases} \frac{dx}{dt} = y + t \\ \frac{dy}{dt} = x - t \end{cases}$ 290. $\begin{cases} \frac{dx}{dt} = x - 4y \\ \frac{dy}{dt} = x + y \end{cases}$ 291. $\begin{cases} \frac{dx}{dt} = -3x - y \\ \frac{dy}{dt} = x - y \end{cases}$

292. $\begin{cases} \frac{dx}{dt} = x + 5y \\ \frac{dy}{dt} = -x - 3y \end{cases}$ $x(0) = -2,\ y(0) = 1$ 293. $\begin{cases} \frac{dx}{dt} = 8y - x \\ \frac{dy}{dt} = x + y \end{cases}$

294. $\begin{cases} \frac{dx}{dt} = 4x - 3y \\ \frac{dy}{dt} = 3x + 4y \end{cases}$ 295. $\begin{cases} \frac{dx}{dt} = 2x + y \\ \frac{dy}{dt} = x - 3y \end{cases}$ $x(0) = 0,\ y(0) = 0$

296. $\begin{cases} \dfrac{dx}{dt} = x + y \\ \dfrac{dy}{dt} = 4y - 2x \end{cases}$ $x(0) = 0 \quad y(0) = -1$

Mustaqil uy vazifasi uchun misollar

Differentsial tenglamalar sistemasini yeching.

297. $\begin{cases} y' = 7y + 3z \\ z' = 6y + 4z \end{cases}$ 298. $\begin{cases} y' = -7y + z \\ z' = -2y - 5z \end{cases}$ 299. $\begin{cases} y' = 5y - z \\ z' = y + 3z \end{cases}$

Sonli qatorlar. Qatorlar yig'indisini topish.

1-misol. $\sum_{n=0}^{\infty} \frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots + \frac{1}{2^n} + \ldots\ldots$ qatorni tekshiring va yig'indisini toping.

Yechish:Bu qator birinchi hadi $a_1 = 1$ va maxraji $q = \frac{1}{2}$ bo'lgan cheksiz kamayuvchi geometrik progressiyadir. Geometrik progressiyani dastlabki n ta hadining yig'indisi

$A_n = \dfrac{a_1(1-q^n)}{1-q}$ formulaga ko'ra $A_n = \dfrac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}} = 2\left[1-\left(\frac{1}{2}\right)^n\right]$

Bundan

$$A = \lim_{n\to\infty} A_n = \lim_{n\to\infty} 2\left[1-\left(\frac{1}{2}\right)^n\right] = 2$$

Demak, qator yaqinlashuvchi bo'lib, uning yig'indisi 2 ga teng, yahni:

$$\sum_{n=0}^{\infty} \frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{2^2} + \ldots\ldots\ldots + \frac{1}{2^2} + \ldots\ldots = 2$$

2-misol**.** $\sum_{n=1}^{\infty} \frac{4}{4n^2-1} = \frac{4}{3} + \frac{4}{15} + \frac{4}{35} + \ldots\ldots\ldots + \frac{4}{4n^2-1} + \ldots\ldots$

qatorning A_n qismiy yig'indisini va A yig'indisini toping.

Yechish. qatorning umumiy hadini

$$a_n = \frac{4}{4n^2-1} = \frac{4}{(2n-1)(2n+1)} = \frac{2}{2n-1} - \frac{2}{2n+1} \qquad \forall n \in N$$

sodda kasrlar ayirmasi ko'rinishda yozish mumkin ekanligini ehtiborga olib, berilgan qatorni

$$\left(2-\frac{2}{3}\right)+\left(\frac{2}{3}-\frac{2}{5}\right)+\frac{2}{5}-\ldots\ldots\ldots-\frac{2}{2n-1}+\left(\frac{2}{2n-1}-\frac{2}{2n+1}\right)+\ldots\ldots$$

shaklda yozish mumkin. qavslarni ochib, soddalashtirganimizdan so'ng, bu qatorning dastlabki n ta hadining qismiy yig'indisi quyidagicha bo'ladi:

$A_n = 2-\frac{2}{2n+1}$ Demak, $A=\lim\limits_{n\to\infty} A_n = \lim\limits_{n\to\infty}\left(2-\frac{2}{2n+1}\right)=2$

Yahni, qator yaqinlashadi va uning yig'indisi 2 ga teng.

Darsda yechish uchun misollar

Berilgan qatorlar uchun yaqinlashishning zaruriy sharti bajariladimi?

36. $\frac{2}{3}+\frac{4}{9}+\frac{6}{27}+\frac{8}{81}+\ldots;$

37. $2+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\ldots+\frac{n+1}{n}+\ldots;$

38. $\frac{2}{1}+\frac{5}{8}+\frac{10}{27}+\frac{17}{64}+\ldots+\frac{n^2+1}{n^3}+\ldots;$

39. $\sum\limits_{n=1}^{\infty}(n^2+9)\cdot \arcsin\frac{1}{n^2+5}$

40 . $\sum\limits_{n=1}^{\infty}\left(\frac{n-1}{n+1}\right)^n$

Berilgan qatorlarni yaqinlashuvchi ekanini ko'rsating va yig'indisini toping.

41. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{n\cdot(n+1)}+\ldots;$

42. $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots+\frac{1}{(2n-1)\cdot(2n+1)}+\ldots;$

43. $\sum\limits_{n=1}^{\infty}\frac{2n+1}{n^2(1+n)^2}$ 44. $\sum\limits_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}$

Mustaqil uy vazifasi uchun misollar

Berilgan qatorlar uchun yaqinlashishning zaruriy sharti bajariladimi?

45. $\frac{1}{1\cdot 3\cdot 5}+\frac{1}{3\cdot 5\cdot 7}+\frac{1}{5\cdot 7\cdot 9}+\ldots;$ 46. $\sum\limits_{n=1}^{\infty}\frac{1}{(2n+1)(2n+3)(2n+5)}$

47. $\sum\limits_{n=1}^{\infty}\frac{n}{(2n-1)^2\cdot(2n+1)^2}$

Berilgan qatorlarni yaqinlashuvchi ekanini ko'rsating va yig'indisini toping.

48. $\sum_{n=1}^{\infty}\frac{1}{(2n+1)(2n+3)(2n+5)}$ 49. $\sum_{n=1}^{\infty}\frac{n}{(2n-1)^2\cdot(2n+1)^2}$

50. $\sum_{n=1}^{\infty}\frac{1}{(3n+1)\cdot(3n-2)}$

Musbat xadli qatorlar yaqinlashishining taqqoslash va Dalamber alomati

Agar $\sum_{n=1}^{\infty}a_n=a_1+a_2+\ldots\ldots+a_n+\ldots\ldots$

qatorning barcha hadlari manfiy bo'lmasa, yahni $a_n\geq 0,\quad n\in N$ bo'lsa, uni musbat hadli qator deb ataymiz.

1-misol. Ushbu $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\ldots\ldots+\frac{1}{n(n+1)}+$ qatorning yaqinlashishi tekshirilsin.

Yechish. Har qanday n uchun berilgan qatorning hadlari $\frac{4}{1\cdot 3}+\frac{4}{3\cdot 5}+\frac{4}{5\cdot 7}+\ldots\ldots+\frac{4}{(2n-1)(2n+1)}+\ldots\ldots$ qatorning mos hadlaridan kichik, yahni $\frac{1}{n(n+1)}<\frac{4}{(2n-1)(2n+1)},\qquad \forall n\in N$

Ammo, keyingi qator yaqinlashadi Demak berilgan qator ham yaqinlashadi.

Dalamber alomati. Agar musbat hadli qator uchun $U_n=\frac{a_{n+1}}{a_n}$ nisbat $n\to\infty$ da l (chekli) limitga ega bo'lsa, yahni

$\lim_{n\to\infty}U_n=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=l$ bo'lsa, u holda: $l<1$ bo'lganda, qator yaqinlashuvchi; $l>1$ bo'lganda, qator uzoqlashuvchi bo'ladi.

Agar $l=1$ bo'lsa, qatorning yaqinlashishi yoki uzoqlashishi haqidagi savolga, boshqa alomatlar yordamida javob topish mumkin.

2-misol. Ushbu

$$\sum_{n=1}^{\infty}\frac{2n-1}{\left(\sqrt{2}\right)^n}=\frac{1}{\sqrt{2}}+\frac{3}{2}+\frac{5}{2\sqrt{2}}+\ldots\ldots+\frac{2n-1}{\left(\sqrt{2}\right)^n}+\ldots\ldots$$

qatorning yaqinlashishi tekshirilsin.

Yechish. Bunda $Q_n=\frac{2n-1}{\left(\sqrt{2}\right)^n},\qquad Q_{n+1}=\frac{2n+1}{\left(\sqrt{2}\right)^{n+1}}$ shu sababli

$$\frac{Q_{n+1}}{Q_n}=\frac{2n+1}{\left(\sqrt{2}\right)^{n+1}}:\frac{2n-1}{\left(\sqrt{2}\right)^n}=\frac{2n+1}{\sqrt{2}(2n-1)}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{2n+1}{\sqrt{2}(2n-1)}=\frac{1}{\sqrt{2}}\lim_{n\to\infty}\frac{2n+1}{2n-1}=\frac{1}{\sqrt{2}}<1$$

Demak, qator yaqinlashadi.

3-misol. Ushbu

$$\sum_{n=1}^{\infty}\frac{3^n n!}{n^n}=3+\frac{3^2\cdot 2!}{2^2}+\frac{3^3\cdot 3!}{3^3}+\frac{3^4\cdot 4!}{4^4}+\ldots\ldots+\frac{3^n n!}{n^n}+\ldots$$

qatorning yaqinlashishi tekshirilsin.

$$a_n=\frac{3^n\cdot n!}{\mathrm{n}^{\mathrm{n}}},\qquad a_{n+1}=\frac{3^{(n+1)}(n+1)!}{(n+1)^{n+1}},\qquad \frac{a_{n+1}}{a_n}=\frac{3\cdot n^n}{(n+1)^n},$$

Yechish.

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{3\cdot n^n}{(n+1)^n}=3\lim_{n\to\infty}\frac{1}{\left(1-\frac{1}{n}\right)^n}=\frac{3}{e}>1,\qquad (e\approx 2{,}71)$$

Demak, qator uzoqlashadi, uning umumiy hadi $a_n\to\infty$

Eslatma. Dalamber alomati faqat $\lim\limits_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)$ mavjud va birdan farqli bo'lgan holda berilgan musbat hadli qator yaqinlashadimi yoki yo'qmi degan savolga javob beradi.

Darsda yechish uchun misollar

Berilgan qatorlarni taqqoslash alomatlaridan foydalanib, yaqinlashishi tekshirilsin:

51. $\frac{1}{1\cdot 2}+\frac{1}{3\cdot 2^3}+\frac{1}{5\cdot 2^5}+\ldots+\frac{1}{(2n-1)2^{2n-1}}+\ldots$

52. $\frac{1}{2\cdot 5}+\frac{1}{3\cdot 6}+\frac{1}{4\cdot 7}+\ldots+\frac{1}{(n+1)(n+4)}+\ldots$

53. $\frac{2}{3}+\frac{3}{8}+\frac{4}{15}+\frac{5}{24}\ldots+\frac{n+1}{(n+2)n}+\ldots$

54. $\frac{1}{1+5}+\frac{1}{1+5^2}+\frac{1}{1+5^3}+\ldots+\frac{1}{1+5^n}+\ldots$

55. $\sin\frac{\pi}{2}+\sin\frac{\pi}{4}+\sin\frac{\pi}{8}+\ldots+\sin\frac{\pi}{2^n}+\ldots$

Berilgan qatorlarni Dalamber alomatidan foydalanib yaqinlashishi tekshirilsin:

56. $tg\frac{\pi}{4}+2tg\frac{\pi}{8}+3tg\frac{\pi}{16}+\ldots+n\cdot tg\frac{\pi}{2^{n+1}}+\ldots$

57. $\frac{2}{1}+\frac{2\cdot 5}{1\cdot 5}+\frac{2\cdot 5\cdot 8}{1\cdot 5\cdot 9}+\ldots+\frac{2\cdot 5\cdot 8\ldots(3n-1)}{1\cdot 5\cdot 9\ldots(4n-3)}+\ldots;$

58. $sin\frac{\pi}{2}+4\,sin\frac{\pi}{4}+9\,sin\frac{\pi}{8}+\ldots+n^2\cdot sin\frac{\pi}{2^n}+\ldots$

59. $\frac{1}{3}+\frac{1\cdot 3}{3\cdot 6}+\frac{1\cdot 3\cdot 5}{27\cdot 6}+\ldots+\frac{1\cdot 3\cdot 5\ldots(2n-1)}{3^n\cdot n!}+\ldots;$

60. $\frac{1}{\sqrt{3}}+\frac{3}{(\sqrt{3})^2}+\frac{5}{(\sqrt{3})^3}+\ldots+\frac{2n-1}{(\sqrt{3})^n}+\ldots;$

61. $\sum_{n=1}^{\infty}\frac{3^n}{(2n+1)\cdot 2^n}$

62. $\sum_{n=1}^{\infty}\frac{(2n+1)!}{(3n+4)\cdot 3^n}$

Mustaqil uy vazifasi uchun misollar

Berilgan qatorlarni taqqoslash alomatlaridan foydalanib, yaqinlashishi tekshirilsin:

63. $\frac{1}{2}+\frac{1}{5}+\frac{1}{8}+\frac{1}{11}+\ldots+\frac{1}{3n-1}+\ldots$

64. $\frac{1}{\ln 2}+\frac{1}{\ln 3}+\frac{1}{\ln 4}+\frac{1}{\ln 5}\ldots+\frac{1}{\ln(n+1)}+\ldots$

Berilgan qatorlarni Dalamber alomatidan foydalanib yaqinlashishi tekshirilsin:

65. $\sum_{n=1}^{\infty}\frac{3^n}{n\cdot 2^n}$ 66. $\sum_{n=1}^{\infty}\frac{1}{(5n-4)\cdot(4n-1)}$ 67. $\sum_{n=1}^{\infty}\frac{n!}{5^n}$

68. $\sum_{n=3}^{\infty}\frac{7^{3n}}{(2n-5)!}$ 69. $\sum_{n=1}^{\infty}\frac{2n-1}{2^n}$

Musbat xadli qatorlar yaqinlashishining Koshi va integral alomati.

Koshi teoremasi. Agar musbat hadli qator uchun

$U_n=\sqrt[n]{a_n}$ miqdor $n\to\infty$ da l chekli limitga ega bo'lsa, yahni

$$\lim_{n\to\infty}U_n=\lim_{n\to\infty}\sqrt[n]{a_n}=l$$

bo'lsa, u holda:

1) $l<1$ bo'lgan holda qator yaqinlashuvchi ;
2) $l>1$ bo'lgan holda qator uzoqlashuvchi bo'ladi.

Agar $\sqrt[n]{a_n}<1$ bo'lib, $\lim_{n\to\infty}\sqrt[n]{a_n}=1$ bo'lsa, shubhali hol deyiladi, chunki bu holda qator yo yaqinlashuvchi, yo uzoqlashuvchi bo'lib, masalani aniqlash uchun qo'shimcha tekshirish kerak bo'ladi.

1-misol. Ushbu $\sum_{n=1}^{\infty}\left(\frac{n+1}{2n-1}\right)^n=2+1+\frac{4}{5}+\frac{5}{7}+\ldots\ldots+\left(\frac{n+1}{2n-1}\right)^n+\ldots\ldots$

qatorning yaqinlashishi tekshirilsin.

Yechish. Koshi alomatidan foydalanamiz:

$$\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\left(\frac{n+1}{2n-1}\right)^n}=\lim_{n\to\infty}\frac{n+1}{2n-1}=\frac{1}{2}<1$$

Demak, berilgan qator yaqinlashuvchi bo'ladi.

2-misol. $\lim_{n\to\infty}\frac{a^n}{n!}=0$ ekanligini isbotlang.

Yechish. Bu yerda $f(n)=(a^n/n!)$ $f(n+1)=\left[a^{n+1}/(n+1)!\right]$.

$\sum_{n=1}^{\infty}\frac{a^n}{n!}$ qatorni tuzamiz va Dalamber alomati yordamida yaqinlashishini tekshiramiz: $\lim_{n\to\infty}\frac{f(n+1)}{f(n)}=\lim_{n\to\infty}\frac{a^{n+1}n!}{(n+1)!\,a^n}=\lim_{n\to\infty}\frac{a}{n+1}=0<1$

qator yaqinlashuvchi. Demak,

$\lim_{n\to\infty}(a^n/n!)=0$.

Bundan $\lim_{n\to\infty}\sqrt[n]{a_n}=0<1$, yahni Koshi alomatiga ko'ra $\sum_{n=1}^{\infty}a_n$ qator yaqinlashuvchi . Demak, $\lim_{n\to\infty}\frac{n^{n^2}}{[(3n)!]^n}=0$

qator yaqinlashishining integral alomati

Ushbu $\sum_{n=1}^{\infty}a_n=a_1+a_2+a_3+\ldots\ldots+a_n+$ qatorning hadlari musbat, lekin o'suvchi bo'lmasin, yahni

$a_1\geq a_2\geq a_3\geq\ldots\ldots\geq a_n\geq\ldots\ldots>0$ deb faraz qalaylik.

Agar $x\geq 1$ lar uchun aniqlangan $f(x)$ funktsiya uzluksiz, musbat va monoton kamayuvchi bo'lib,

$f(1)=a_1,\quad f(2)=a_2,\ \ldots\ldots\ldots,f(n)=a_n,\ldots\ldots$

bo'lsa, qatorning yaqinlashishi uchun xosmas integralning

$\int_1^{\infty}f(x)dx$ yaqinlashuvchi bo'lishi zarur va yetarli.

3-misol. Ushbu $\sum_{n=1}^{\infty}\frac{1}{n^m}=\frac{1}{1^m}+\frac{1}{2^m}+\frac{1}{3^m}+\ldots\ldots\ldots+\frac{1}{n^m}+$

qatorning yaqinlashishi tekshirilsin.

Yechish. $f(x)=\frac{1}{x^m}$ deb olamiz Bu funktsiya teoremaning hamma shartlarini qanoatlantiradi. quyidagi integralni qaraymiz:

$$\int_1^N \frac{dx}{x^m} = \begin{cases} m \neq 1 \quad \text{булганда} \quad \left. \frac{x^{1-m}}{1-m} \right|_1^N = -\frac{1-N^{1-m}}{1-m}, \\ m = 1 \quad \text{булганда} \quad \ln x \left.\right|_1^N = \ln N \end{cases}$$

$N \to \infty$ da $m > 1$ bo'lsa $\int_1^\infty \frac{dx}{x^m} = \frac{1}{m-1}$

Yahni, xosmas integral chekli shuning uchun berilgan qator yaqinlashadi;

$m \leq 1$ bo'lsa, $\int_1^\infty \frac{dx}{x^m} = \infty$ bo'ladi. Yahni integral cheksiz. qator uzoqlashadi.

Darsda yechish uchun misollar

Koshi alomatiga asosan quyidagi qatorlarning yaqinlashishi tekshirilsin:

70. $\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \ldots + \left(\frac{n}{2n+1}\right)^n + \ldots$

71. $\arcsin 1 + \arcsin^2 \frac{1}{2} + \arcsin^3 \frac{1}{3} + \ldots + \arcsin^n \frac{1}{n} + \ldots$

72. $\frac{1}{\ln 2} + \frac{1}{\ln^2 3} + \frac{1}{\ln^3 4} + \frac{1}{\ln^4 5} + \ldots + \frac{1}{\ln^n(n+1)} + \ldots$

73. $arctg 1 + arctg^2 \frac{1}{2} + arctg^3 \frac{1}{3} + \ldots + arctg^n \frac{1}{n} + \ldots$

74. $\frac{1}{2} + \left(\frac{2}{5}\right)^3 + \left(\frac{3}{8}\right)^5 + \ldots + \left(\frac{n}{3n-1}\right)^{2n-1} + \ldots;$

75. $\sum_{n=1}^{\infty} 3^{n+1} \cdot \left(\frac{n+2}{n+3}\right)^{n^2};$

Integral alomati bilan quyidagi qatorlarning yaqinlashishi tekshirilsin:

76. $\frac{1}{2\ln^2 2} + \frac{1}{3\ln^2 3} + \frac{1}{4\ln^2 4} + \ldots;$

77. $\frac{e^{-\sqrt{1}}}{\sqrt{1}} + \frac{e^{-\sqrt{2}}}{\sqrt{2}} + \frac{e^{-\sqrt{3}}}{\sqrt{3}} + \ldots;$

78. $\sum_{n=1}^{\infty} \frac{2n-3}{n(n+1)};$

79. $\sum_{n=2}^{\infty} \frac{n}{\sqrt{n^2-1}};$

80. $\sum_{n=1}^{\infty} \left(\frac{1+n}{1+n^2}\right)^2$

81. $\sum_{n=1}^{\infty} \frac{n}{1+n^2}$

82. $\sum_{n=3}^{\infty} \frac{1}{n^4-9}$

Mustaqil uy vazifasi uchun misollar

Koshi alomatiga asosan quyidagi qatorlarning yaqinlashishi tekshirilsin:

83. $arctg1 + arctg^2 \frac{1}{2} + arctg^3 \frac{1}{3} + \ldots + arctg^n \frac{1}{n} + \ldots$

84. $\frac{1}{2} + \left(\frac{2}{5}\right)^3 + \left(\frac{3}{8}\right)^5 + \ldots + \left(\frac{n}{3n-1}\right)^{2n-1} + \ldots;$

85. $\sum_{n=1}^{\infty} \left(\frac{n-1}{n+1}\right)^{n^2+4n+5}$; 86. $\sum_{n=1}^{\infty} 3^{-n} \cdot \left(\frac{n+1}{n}\right)^{n^2}$; 87. $\sum_{n=1}^{\infty} \left(\frac{an}{n+2}\right)^n$; $(a > 0)$

Integral alomati bilan quyidagi qatorlarning yaqinlashishi tekshirilsin:

88. $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1};$

89. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(n+1)};$

90. $\sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{(2n-3)^2}};$

91. $\sum_{n=1}^{\infty} \frac{1}{(n+1) \cdot ln(n+1)};$

92. $\sum_{n=2}^{\infty} \frac{1}{n \cdot ln^3(n)};$

Ishorasi navbatlashuvchi qatorlar. Leybnis teoremasi.

Ishoralari navbatlashuvchi qatorlar deb, hadlari navbat bilan musbat va manfiy ishoraga ega bo’lgan qatorlarga aytiladi.

$a_1 - a_2 + a_3 - a_4 + \ldots\ldots - (-1)^{n-1} a_n + \ldots\ldots$

kabi yozish qulaydir (bu yerda hamma $a_n > 0$ deb hisoblanadi).

Leybnits teoremasi. Agar ishoralari navbatlashuvchi qatorning hadlari absolyut qiymatlari bo’yicha monoton kamayuvchi bo’lsa

$a_1 > a_2 > a_3 > \ldots\ldots > a_n > \ldots\ldots$

va $n \to \infty$ da qatorning n - hadi a_n nolga intilsa, yahni $\lim_{n\to\infty} a_n = 0$ bo’lsa, u holda qator yaqinlashuvchi bo’ladi va uning yig’indisi musbat bo’lib birinchi haddan katta bo’lmaydi.

Agar ishorasi almashinuvchi qator yaqinlashsa, lekin uni hadlarining absolyut qiymatlaridan tuzilgan qator uzoqlashsa, u holda berilgan ishorasi almashinuvchi qator shartli yaqinlashuvchi qator deb ataladi.

1-misol. Leybnits alomatidan foydalanib, ushbu qatorning

$$\frac{2}{4} - \frac{3}{7} + \frac{4}{12} - \frac{5}{19} + \ldots\ldots\ldots + (-1)^{n-1} \frac{n+1}{n^2+3}$$

yaqinlashishini tekshiring.

Yechish. Berilgan qatorning hadlari absolyut qiymati bo’yicha monoton kamayadi :

$$\frac{2}{4} > \frac{3}{7} > \frac{4}{12} > \frac{5}{19} > \ldots\ldots$$

va uning umumiy hadi esa $n \to \infty$ da nolga intiladi, yahni :

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+1}{n^2+3} = 0.$$

Demak, qator Leybnits alomatiga ko'ra yaqinlashadi.

2 - misol. Ushbu $1{,}1 - 1{,}01 + 1{,}001 - 1{,}0001 + \ldots$
qatorning yaqinlashishi tekshirilsin.

Ye<u>chish.</u> qatorning hadlari absolyut qiymatlari bo'yicha kamayuvchi:
$1{,}1 > 1{,}01 > 1{,}001 > 1{,}0001 > \ldots\ldots,,$

Lekin, $n \to \infty$ da nolga intilmaydi:

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(1 + \frac{1}{10^n}\right) = 1 \neq 0$$

Demak, qator uzoqlashadi chunki Leybnits teoremasining ikkinchi sharti (yahni qator yaqinlashishining zaruriy sharti) bajarilmadi.

Darsda yechish uchun misollar

Berilgan qatorlarni yaqinlashishi aniqlansin:

93. $1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \frac{1}{9^2} - \ldots + \frac{(-1)^{n+1}}{(2n-1)^2}\ldots;$

94. $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots + \frac{(-1)^{n+1}}{2n-1}\ldots;$

95. $1 - \frac{1}{2} + \frac{1}{5} - \frac{1}{10} + \frac{1}{17} - \ldots + (-1)^n \frac{1}{n^2+1} + \ldots;$

96. $1 - \frac{2}{7} + \frac{3}{13} - \ldots + (-1)^{n-1} \frac{n}{6n-5} + \ldots;$

97. $\sqrt{\frac{1}{101}} - \sqrt{\frac{1}{201}} + \sqrt{\frac{1}{301}} - \sqrt{\frac{1}{401}} + \ldots + (-1)^{n-1}\sqrt{\frac{1}{100n+1}} + \ldots$

98. $1 - \frac{1}{2} + \frac{1}{5} - \frac{1}{10} + \frac{1}{17} - \ldots + (-1)^n \frac{1}{n^2+1} + \ldots;$

99. $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} \cdots\cdots (-1)^{n-1} \frac{1}{\sqrt{n}} + \cdots$

100. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots + (-1)^{n-1}\frac{1}{n} + \ldots;$

Mustaqil uy vazifasi uchun misollar

Berilgan qatorlarni yaqinlashishi aniqlansin:

101. $\frac{\ln 2}{2}-\frac{\ln 3}{3}+\frac{\ln 4}{4}-\frac{\ln 5}{5}-\ldots+(-1)^n\frac{\ln\ n}{n}+\ldots;$

102. $1-\frac{2!}{2^2}+\frac{3!}{3^3}-\frac{4!}{4^4}+\frac{5!}{5^5}-\ldots+(-1)^n\frac{n!}{n^n}+\ldots;$

103. $1-\frac{1}{3^2}+\frac{1}{5^2}-\frac{1}{7^2}+\frac{1}{9^2}-\ldots+\frac{(-1)^{n+1}}{(2n-1)^2}\ldots;.$

104. $0{,}1-\frac{0{,}1^2}{2}+\frac{0{,}1^3}{3}-\frac{0{,}1^4}{4}+\frac{0{,}1^5}{5}-\ldots+\frac{(-1)^{n+1}0{,}1^n}{n}+\ldots;$

Funktsional qatorlar

Tahrif. Hadlari o'zgaruvchi x ning biror X sohada aniqlangan funktsiyalaridan iborat bo'lgan

$$\sum_{n=1}^{\infty}U_n(x)=U_1(x)+U_2(x)+\ldots\ldots+U_n(x)+\ldots\ldots \qquad (1)$$

qator funktsional qator deyiladi.

x ga aniq son qiymatlar berib, cheksiz ko'p turli sonli qatorlarni hosil qilamiz. Bular yaqinlashuvchi va uzoqlashuvchi bo'lishi mumkin.

x ning (1) funktsional qator yaqinlashadigan qiymatlar to'plami, shu qatorning yaqinlashish sohasi deyiladi.

Tahrif. Agar har qanday $\varepsilon>0$ uchun x ga bog'liq bo'lmagan shunday nomer N mavjud bo'lsaki, $n\succ N$ bo'lganda X sohaning hamma x lari uchun bir vaqtda

$$|S_n(x)-S(x)|<\varepsilon \quad \text{yoki} \quad |r_n(x)|<\varepsilon \qquad (2)$$

tengsizlik o'rinli bo'lsa, u holda, qator X sohada tekis yaqinlashuvchi deyiladi.

Agar X sohada yaqinlashuvchi bo'lgan (1) funktsional qator aytilgan xossaga ega bo'lmasa, u holda bu qatorga notekis yaqinlashuvchi qator deyiladi.

1-misol.

$$\sum_{n=1}^{\infty}x^{n-1}=1+x+x^2+\ldots\ldots+x^n \quad \text{funktsional qatorni qaraymiz.}$$

Yechish. Bu qator x ning $(-1;\ 1)$ intervaldagi hamma qiymatlarida, yahni x ning $|x|<1$ shartni qanoatlantiruvchi hamma qiymatlarida yaqinlashadi va yig'indisi $\frac{1}{1-x}$ ga teng, chunki bu maxraji x ga teng bo'lgan kamayuvchi geometrik progressiyadir. SHunday qilib berilgan qator $(-1;\ 1)$ intervalda

$$\frac{1}{1-x}=1+x+x^2+x^3+\ldots\ldots+x^n+\ldots\ldots$$

qatorning yig'indisi bo'lgan $S(x)=\frac{1}{1-x}$ funktsiyani aniqlaydi.

2-misol. Ushbu $\sum_{n=1}^{\infty}\frac{1}{(n+x)(n+x+1)}$

funktsional qatorning yaqinlashish sohasi va yig'indisini toping.

Ye<u>chish.</u> $S_n(x)=\frac{1}{(n+x)(n+x+1)}$ funktsiya $x=-n$ va $x=-(n+1)$ nuqtalarda aniqlanmagan (bu nuqtalar funktsiyani uzulish nuqtalari), shuning uchun biz qatorni $x\neq -n \quad (n=1,\ 2,\ 3,\ldots\ldots)$ nuqtalarda tekshiramiz.

$$S_n(x)=\frac{1}{n+x}-\frac{1}{n+x+1}$$

ekanligini ehtiborga olib,qatorning n -qismiy yig'indisini quyidagi ko'rinishda yozamiz:

$$S_n(x)=\frac{1}{(1+x)(2+x)}+\frac{1}{(2+x)(3+x)}+\ldots\ldots\frac{1}{(n+x)(n+x+1)}=$$

$$=\left(\frac{1}{1+x}-\frac{1}{2+x}\right)+\left(\frac{1}{2+x}-\frac{1}{3+x}\right)+\left(\frac{1}{3+x}-\frac{1}{4+x}\right)+\ldots\ldots+$$

$$+\left(\frac{1}{n-1+x}-\frac{1}{n+x}\right)+\left(\frac{1}{n+x}-\frac{1}{n+x+1}\right)=\frac{1}{1+x}-\frac{1}{n+x+1}$$

Demak, $\lim\limits_{n\to\infty}S_n(x)$ mavjud va

$$S(x)=\lim_{n\to\infty}S_n(x)=\lim_{n\to\infty}\left(\frac{1}{1+x}-\frac{1}{n+x+1}\right)=\frac{1}{1+x}\ ,$$

yahni, qator x ning $x\neq -n \quad (n=1,\ 2,\ 3,\ldots)$ shartni qanoatlantiruvchi barcha qiymatlarida yaqinlashadi va yig'indisi $S(x)=\frac{1}{1+x}$ funktsiyadan iborat bo'ladi.

Darsda yechish uchun misollar

105. Ushbu $\frac{4-x}{7x+2}+\frac{1}{3}\left(\frac{4-x}{7x+2}\right)^2+\frac{1}{5}\left(\frac{4-x}{7x+2}\right)^3+\ldots;$ funktsional qatorning $x=0$ va $x=1$ nuqtalarda yaqinlashishi tekshirilsin.

106. Ushbu $\frac{1}{1+x^2}+\frac{1}{1+x^4}+\frac{1}{1+x^6}+..;$ funktsional qatorning yaqinlashishi tekshirilsin va yaqinlashish sohasi topilsin.

107. Ushbu $\sum_{n=1}^{\infty}\frac{(-1)^n n}{x^4+n^2}$; funktsional qatorning $(-\infty,\infty)$ oraliqda tekis yaqinlashishi ko'rsatilsin.

108. Ushbu $x+\frac{x^2}{2}+\frac{x^3}{4}+\frac{x^4}{8}+\ldots$, funktsional qatorning $(-2;2)$ intervalda notekis yaqinlashishi isbot qilinsin.

Berilgan funktsional qatorlarning yaqinlashish intervalini toping.

109. $x+\frac{x^2}{\sqrt{2}}+\frac{x^3}{\sqrt{3}}+\frac{x^4}{\sqrt{4}}+\frac{x^5}{\sqrt{5}}+\ldots+\frac{x^n}{\sqrt{n}}+\ldots$

110. $\frac{x}{1+x}+\frac{x^2}{1+x^4}+\frac{x^3}{1+x^6}+\frac{x^4}{1+x^8}+\frac{x^5}{1+x^{10}}+\ldots+\frac{x^n}{1+x^{2n}}+\ldots$

111. $\frac{1}{1+x}+\frac{1}{1+x^2}+\frac{1}{1+x^3}+\frac{1}{1+x^4}+\ldots+\frac{1}{1+x^n}+\ldots$

112. $\frac{\cos x}{e^x}+\frac{\cos 2x}{e^{2x}}+\frac{\cos 3x}{e^{3x}}+\frac{\cos 4x}{e^{4x}}+\frac{\cos 5x}{e^{5x}}+\ldots+\frac{\cos nx}{e^{nx}}+..$

113. $\frac{\sin x}{1}+\frac{\sin 2x}{2^2}+\frac{\sin 3x}{3^2}+\frac{\sin 4x}{4^2}+\frac{\sin 5x}{5^2}+\ldots+\frac{\sin nx}{n^2}+..$

114. $e^{-x}+e^{-4x}+e^{-9x}+e^{-16x}+\ldots+e^{-n^2x}+\ldots$

Mustaqil uy vazifasi uchun misollar

Berilgan funktsional qatorlarning yaqinlashish intervalini toping.

115. $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots+(-1)^{n+1}\frac{x^{2n-1}}{(2n-1)!}+\ldots,$

116. $(x+8)+\frac{(x+8)^2}{2^2}+\frac{(x+8)^3}{3^2}+\frac{(x+8)^4}{4^2}+\ldots+\frac{(x+8)^n}{n^2}+\ldots$

117. $\ln x+\ln^2 x+\ln^3 x+\ln^4 x+\ldots+\ln^n x+\ldots$

118. $x+x^4+x^9+x^{16}+\ldots+x^{n^2}+\ldots$

119. $tg\,x+\frac{tg^2 x}{2^2}+\frac{tg^3 x}{3^2}+\frac{tg^4 x}{4^2}+\frac{tg^5 x}{5^2}+\ldots+\frac{tg^n x}{n^2}+\ldots$

Darajali qatorlar. Yaqinlashish intervali.

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_nx^n + \ldots \quad (1)$$

ko'rinishdagi funktsional qatorga darajali qator deyiladi. Bunda a_i $(i=\overline{0,\infty})$ lar o'zgarmas sonlar bo'lib, darajali qatorning koeffitsientlari deyiladi.

x argumentga tayin bir qiymat berish natijasida (1) darajali qator sonli qatorga aylanadi va bu sonli qator yaqinlashuvchi yoki uzoqlashuvchi bo'lishi mumkin. (1) darajali qator yaqinlashadigan barcha $x=\bar{x}$ nuqtalardan tashkil topgan $X=\{\bar{x}\}$ to'plamga darajali qatorning yaqinlashish sohasi deyiladi.

1-teorema. (Abelg' teoremasi) Agar (1) darajali qator 0 dan farqli $x=\bar{x}$ qiymat uchun yaqinlashuvchi bo'lsa, u holda bu qator $|x|<|\bar{x}|$ tengsizlikni qanoatlantiruvchi istalgan x uchun absolyut yaqinlashuvchi bo'ladi; agar qator $x=\bar{x}$ qiymat uchun uzoqlashuvchi bo'lsa, u holda bu qator $|x|>|\bar{x}|$ tengsizlikni qanoatlantiruvchi istalgan x da uzoqlashuvchi bo'ladi.

$\bar{x}=R$ desak, bu R songa darajali qatorning yaqinlashish radiusi deyiladi.

$(-R;\ R)$ interval esa yaqinlashish intervali deyiladi.

Darajali qatorning yaqinlashish radiusi R ni aniqlash uchun Dalamber belgisidan foydalanib, quyidagiga ega bo'lamiz :

$$R=\lim_{n\to\infty}\frac{|a_n|}{|a_{n+1}|}$$

Darajali qatorning yaqinlashish radiusi R ni aniqlash uchun shunga o'xshash Koshi alomatidan ham foydalansak, u holda

$$R=\frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{a_n}}$$

1-misol. Ushbu

$$\sum_{n=1}^{\infty}\frac{3^n x^n}{2^n\sqrt{n}}=\frac{3x}{2}+\frac{3^2x^2}{2^2\sqrt{2}}+\frac{3^3x^3}{2^3\sqrt{3}}+\ldots$$

darajali qatorning yaqinlashish intervali topilsin.

Yechish. Dalamber belgisini tatbiq etamiz:

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\left|\frac{3^n\cdot 2^{n+1}\sqrt{n+1}}{3^{n+1}2^n\sqrt{n}}\right|=\lim_{n\to\infty}\left|\frac{2}{3}\sqrt{\frac{n+1}{n}}\right|=\frac{2}{3}\lim_{n\to\infty}\sqrt{\frac{1}{1+\frac{1}{n}}}=\frac{2}{3}$$

Demak, $R=\frac{2}{3}$ bo'ladi. Bundan $(-\frac{2}{3};\frac{2}{3})$ interval berilgan qator uchun yaqinlashish intervali bo'ladi.

Endi intervalning chetki nuqtalarida (yahni $x=-\frac{2}{3}$ va $x=\frac{2}{3}$ da) berilgan qatorning yaqinlashishini, tekshiramiz.
Buning uchun har bir chegarada yakka-yakka holda tekshiramiz.

a) $x=-\frac{2}{3}$ bo'lganda $\sum_{n=1}^{\infty}\frac{3^n\left(-\frac{2}{3}\right)^n}{2^n\sqrt{n}}=1+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}-\ldots$ ishoralari navbatlashuvchi sonli qator hosil bo'ladi. qator uchun Leybnits teoremasining ikkala sharti ham bajariladi, yahni

1) $1>\frac{1}{\sqrt{2}}>\frac{1}{\sqrt{3}}>\frac{1}{\sqrt{4}}>\ldots;$

2) $\lim\limits_{n\to\infty}\frac{1}{\sqrt{n}}=0$

va demak, berilgan qator $x=-\frac{2}{3}$ nuqtada yaqinlashuvchi. b) $x=\frac{2}{3}$ bo'lganda

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}=1+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}-\ldots \qquad (5.8)$$

musbat hadli sonli qator hosil bo'ladi. (5.8) qator uzoqlashuvchi.
Demak, berilgan darajali qator uchun yaqinlashish intervali $\left[-\frac{2}{3};\frac{2}{3}\right[$ yoki $-\frac{2}{3}\le x<\frac{2}{3}$ bo'ladi.

2-misol. $$x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots+\frac{x^n}{n!} \qquad (5.9)$$

qatorning yaqinlashish intervali topilsin.
Yechish. Berilgan qatorga Dalamber belgisini tatbiq etsak,

$$\lim_{n\to\infty}\left|\frac{x^{n+1}n!}{x^n(n+1)!}\right|=|x|\lim_{n\to\infty}\frac{1}{n+1}=0<1$$

limitning x ga bog'liq emasligi va birdan kichikligi tufayli qator x ning barcha qiymatlarida yaqinlashadi, yahni qatorning yaqinlashish radiusi $R=\infty$ bo'lib, yaqinlashish intervali $]-\infty,+\infty[$ dan ibrat.

Darsda yechish uchun misollar

Berilgan darajali qatorlarning yaqinlashish intervali aniqlansin va intervalining chegaralarida ham qatorni yaqinlashishi tekshirilsin:

120. $1+\frac{1}{3\cdot 2}x+\frac{1}{3^2\cdot 3}x^2+\frac{1}{3^2\cdot 4}x^3+\ldots\frac{1}{3^n\cdot(n+1)}x^n+;$

121. $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-\ldots+(-1)^n\frac{x^n}{n}+\ldots$

122. $x+\frac{x^2}{20}+\frac{x^3}{300}+\frac{x^4}{4000}+\ldots+\frac{x^n}{n\cdot 10^{n-1}}+\ldots$

123. $1+2x^2+4x^4+\ldots+2^{n-1}x^{2(n-1)}+\ldots$

124. $x+4x^2+27x^3+256x^4\ldots+(n\cdot x)^n+\ldots$

125. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{(2n)!}$;

126. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{(n+1)\cdot 7^{n-1}}$;

127. $\sum_{n=1}^{\infty}\frac{x^n n}{(n+1)2^n}$;

128. $\sum_{n=1}^{\infty}\left(\left(\frac{n+1}{n}\right)^n x\right)^n$;

129. $\sum_{n=1}^{\infty}\frac{(x-2)^n n}{(2n-1)2^n}$;

130. $\sum_{n=1}^{\infty}\frac{x^n}{(n+1)^2 e^{n-1}}$;

Mustaqil uy vazifasi uchun misollar

Berilgan darajali qatorlarning yaqinlashish intervali aniqlansin va intervalining chegaralarida ham qatorni yaqinlashishi tekshirilsin:

131. $x-\frac{x^3}{3\cdot 3!}+\frac{x^5}{5\cdot 5!}-\frac{x^7}{7\cdot 7!}\ldots+\frac{x^{2n-1}}{(2n-1)\cdot(2n-1)!}+\ldots$

132. $1+x+2x^2+6x^3+24x^n\ldots+n!\cdot x^n+\ldots$

133. $\sum_{n=1}^{\infty}\frac{2^n\cdot n!}{(2n)!}x^{2n}$

134. $x+\frac{x^2}{2^2}+\frac{x^3}{3^2}+\frac{x^4}{4^2}+\frac{x^5}{5^2}+\ldots+\frac{x^n}{n^2}+\ldots$

Ikki o`lchovli integral va uning yordamida yuza hisoblash.

$$S=\lim_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\sum\sum\Delta x\Delta y=\int\int_{(S)}dxdy=\int_a^b dx\int_{y_1(x)}^{y_2(x)}dy$$

Agar, (S) soha $h\le y\le \ell$, $x_1(y)\le x\le x_2(y)$ tengsizliklar bilan aniqlangan bo`lsa u holda $S=\int\int_{(S)}dxdy=\int_h^{\ell}dy\int_{x_1(y)}^{x_2(y)}dx$

1-misol: $\int_{-2}^{1}\left(\int_x^{2-x^2}dy\right)dx$ ni hisoblang.

Yechish: $\int_{-2}^{1}\left(\int_x^{2-x^2}dy\right)dx=\int_{-2}^{1}(2-x^2-x)dx=\left[2x-\frac{x^3}{3}-\frac{x^2}{2}\right]\Bigg|_{-2}^{1}=\frac{9}{2}$

2-misol: $x=4y-y^2$ va $x+y=6$ chiziqlar bilan chegaralangan figuraning yuzi hisoblansin.

Yechish: $x=4y-y^2$ va $x+y=6$ chiziqlarni kesishish nuqtasini topamiz.

$\begin{cases}x=4y-y^2\\ x+y=6\end{cases}$ ni yechib A(4;2) va V(3;3) nuqtalarni topamiz.

Natijada:

$$S=\int_D dxdy=\int_2^3 dy\int_{6-y}^{4y-y^2}dx=\int_2^3 x\Big|_{6-y}^{4y-y^2}dy=\int_2^3(-y^2+5y-6)dy=\left[-\frac{1}{3}y^3+\frac{5}{2}y^2-6y\right]_2^3=\frac{1}{6}$$

$$\cdot\int_0^{\frac{\pi}{6}}\left(\frac{4}{3}\cos^2\theta-1\right)d\theta=\int_0^{\frac{\pi}{6}}\left(\frac{2}{3}+\frac{2}{3}\cos 2\theta-1\right)d\theta$$

Darsda yechish uchun misollar

347. Hisoblang. $\int_0^{2\pi} cos^2 x dx \int_0^a y dy$ 348. Hisoblang. $\int_1^3 dx \int_{x^3}^{x} (x-y) dy$

349. $\iint_D y \ln x dx dy$ D soha quyidagi chiziqlar bilan chegaralangan.

$xy=1, \quad y=\sqrt{x} \quad x=2$ J: $5(2\ln 2-1)/8$

Quyida berilgan chiziqlar bilan chegaralangan
figuralarning yuzini hisoblang

350. $x=y^2-2y \quad x+y=0 \quad J: \frac{1}{6}$ 352. $y=2-x \quad y^2=4x+4 \quad J: \frac{64}{3}$

351. $y^2=4x-x^2 \quad y^2=2x \quad J: 2\pi-16/3$ 353. $3x^2=25x \quad 5x^2=9y \quad J: 5$

Mustaqil uy vazifasi uchun misollar

Ikki o`lchovli integralni hisoblang

354. $\iint_D xy dx dy \qquad (0\le x\le 1, \quad 0\le y\le 2)$

355. $\iint_D e^{x+y} dx dy \qquad (0\le x\le 1, \quad 0\le y\le 1)$

Quyida berilgan chiziqlar bilan chegaralangan
figuralarning yuzini hisoblang

356. $y^2+2y-3x+1=0$
$3x-3y-7=0 \qquad J: 125/18$

357. $y=4x-x^2 \quad y=2x^2-5x \quad J: 27/2$

Egri chiziqli integrallar

Aniq integral tushunchasini integrallash sohasi tekislikda yotuvchi qandaydir egri chiziqni bir qismi bo’lgan hol uchun umumlashtiramiz.

Bunday turdagi integrallar egri chiziqli integrallar deb ataladi.

Egri chiziqli integrallar ikki turda bo’ladi: birinchi va ikkinchi tur egri chiziqli integrallar. Tushunishga qulaylik tug’dirish maqsadida tahrifni tekislikda yotgan egri chiziq uchun beramiz.

$$\sum_{i=1}^{n} f(M_i^*)\Delta l_i$$

yig’indini tuzamiz, bu yerda Δl_i $M_{i-1}M_i$ yoy uzunligi, M_i^* $M_{i-1}M_i$ yoyning ixtiyoriy nuqtasi.

Tahrif. Agar integral yig’indi $\Delta l_i \to 0$ da J limitga ega bo’lsa, u holda bu limit AV egri chiziq bo’yicha f(x,y) funktsiyadan olingan I-tur egri chiziqli integral deb ataladi va quyidagicha belgilanadi.

$$J=\int_{AB} f(M) dl$$

$$J = \int_{AB} f(x, y)dl$$

I-tur egri chiziqli integralni aniq integralga keltirish mumkin. AV egri chiziqda parametr sifatida A nuqtadan boshlanadigan yoy uzunligi l ni olsak, egri chiziqli $x = x(l), y = y(l)$ parametrik ko'rinishini hosil qilamiz.

U holda I-tur egri chiziqli integral

$$\int_{AB} f(xy)dl = \int_0^l f(x(l), y(l))dl$$

AV egri chiziq $x = \varphi(t),\ y = \psi(t)$ parametrik tenglamalar bilan berilgan bo'lsin. U holda I-tur egri chiziqli integral quyidagi formula bo'yicha hisoblanadi.

$$\int_{AB} f(x, y)dl = \int_\alpha^\beta f[\varphi(t), \psi(t)]\sqrt{(\varphi'(t))^2 + (\psi'(t))^2}\,dt \qquad (1)$$

Agar AV egri chiziq $y = y(x)$, tenglama bilan berilgan bo'lsa $(a \le x \le b)$, u holda I-tur egri chiziqli integral

$$\int_{AB} f(x, y)dl = \int_a^b f(x, y(x))\sqrt{1 + [y'(x)]^2}\,dx \qquad (2)$$

ko'rinishni oladi.

II - tur egri chiziqli integral

$$\int_{AB} P(x, y)dx + \int_{AB} Q(x, y)dy$$

yig'indi ikkinchi tur egri chiziqli integral deb ataladi va

$$\int_{AB} P(x, y)dx + Q(x, y)dy$$

ko'rinishda belgilanadi.

Faraz qilaylik, AV egri chiziq $x = \varphi(t), \quad y = \varphi(t), \quad \alpha \le t \le \beta$ parametrik tenglamalar bilan berilgan bo'lsin, bu yerda $\varphi(t)$ *ва* $\psi(t)$ uzluksiz $\varphi'(t)$ *ва* $\psi'(t)$ hosilalarga ega bo'lgan funktsiyalar.

U holda quyidagi formulalar o'rinli.

$$\int_{AB} P(x, y)dx = \int_\alpha^\beta P[\varphi(t), \psi(t)]\varphi(t)$$

$$\int_{AB} Q(x, y)dx = \int_\alpha^\beta Q[\varphi(t), \psi(t)]\varphi(t)$$

$$\int_{AB} P(x, y)dx + Q(x, y)dy = \int_\alpha^\beta \{P[\varphi(t), \psi(t)]\varphi(t) + Q[\varphi(t), \psi(t)]\psi(t)\}dt \qquad (3)$$

1-misol. $\int_c (x^2+y^2)dl$ egri chiziqli integralni $x=a\cos t,\quad y=a\sin t \quad o\le t\le 2\pi$

parametrik formulalar bilan berilgan S aylana bo'yicha hisoblang.
Echish: Integralni (1) formula bo'yicha hisoblaymiz.
$\varphi'(t)=-a\sin t,\quad \psi'(t)=\cos t$ bo'lganligi uchun

$$\int_c (x^2+y^2)dl=\int_o^{2\pi}(a^2cos^2t+a^2\sin^2 t)\sqrt{a^2\sin^2 t+a^2cos^2t}\,dt=$$

$$\int_o^{2\pi}a^2(cos^2t+\sin^2 t)\sqrt{a^2(\sin^2 t+a^2cos^2t)}\,dt=\int_o^{2\pi}a^2\cdot a\,dt=2\pi a^3.$$

2-misol. $\int_c (x-y)de$ egri chiziqli integralni hisoblang, bu yerda s to'g'ri chiziqning

A(0;0) nuqtadan V (4;3) nuqtagacha bo'lgan qismi:
Echish: Avval A(0;0) va V(4;3) nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini tuzib olamiz.

$A(x_1y_1)$ *ва* $B(x_2y_2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini topish

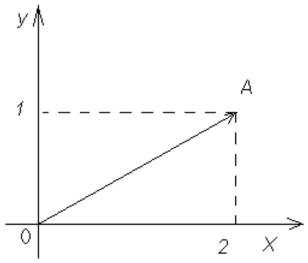
$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ formulasiga asosan $\frac{x-0}{4-0}=\frac{y-0}{3-0}$ bu yerdan $y=\frac{3}{4}x \qquad y'=\frac{3}{4}$

bo'lgani uchun (2) formulaga asosan

$$\int_c (x-y)dl=\int_0^4 (x-\frac{3}{4}x)\sqrt{1+\frac{9}{16}}\,dx=\frac{5}{16}\int_0^4 xdx=\frac{5}{32}x^2\Big|_0^4=\frac{5}{2}.$$

3-misol. $\int_L 2xydx-x^2dy$ egri chiziqli integral hisoblang, bu yerda L O(0,0) va A(2,1)

nuqtalarni birlashtiruvchi to'g'ri chiziq kesmasi.
Echish.



OA kesmani ko'ramiz. Integrallash yo'li tenglamasini tuzamiz. Buning uchun ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasiga $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ ga ko'ra OA to'g'ri chiziq tenglamasini tuzamiz.

$$OA:\ \frac{x-0}{2-0}=\frac{y-0}{1-0}\ \Rightarrow\ y=\frac{1}{2}x,\quad 0\le x\le 2 \quad \text{formulaga asosan}$$

$$\int_L 2xydx-x^2dy=\int_0^2 2x\frac{1}{2}xdx-x^2(\frac{1}{2}x)'dx=\int(x^2-\frac{1}{2}x^2)dx=\frac{1}{2}\int_0^2 x^2dx=\frac{1}{2}\cdot\frac{x^3}{3}\Big|_0^2=\frac{4}{3}.$$

Mustaqil yechish uchun misollar

quyidagi egri chiziqli integrallar hisoblansin:

26 $\int\limits_L (x^2+y^2)dx + ydy$ egri chiziqli integralni hisoblang. Bu yerda L $y=e^x$ egri chiziqning A(0;1) nuqtalardan V(1;e) bo'lgan yoyini hisoblang.

27. $\int y^2dx + 2xy\,dy$ *интеграл* $x=a\cos t_1$ $y=a\sin t$ aylana bo'yicha.

28. $\int ydx - xdy$ *интеграл* $x=\cos t_1$ $y=a\sin t$ ellips yoyi bo'yicha.

29. $\int\left(\frac{x}{x^2+y^2}dx - \frac{y}{x^2+y^2}dy\right)$ integral markazi koordinatalar boshida bo'lgan aylana bo'yicha.

30. $\int\frac{ydx+xdy}{x^2y^2}$ integral $y=x$ to'g'ri chiziqning $x=1$ dan $x=2$ gacha kesmasi bo'yicha.

31. $\int yzdx + xzdy + xydz$ integral t 0 dan 2π gacha o'zgarganida

$x=a\cos t$ $y=a\sin t$, $z=rt$ chizig'ining yoyi bo'yicha.

32. $\int xdy - y\,dx$ *интеграл* $x=a\cos^3 t_1$ $y=a\sin^3 t$ astroidaning yoyi bo'yicha. .

33. $\int xdy - yxd$ *интеграл* $x=\frac{3at}{1+t^3}$; $y=\frac{3at^2}{1+t^3}$ Dekart yaprog'ining sirtmog'i bo'yicha.

34. $\int xdy - ydx$ *интеграл* $x=a(t-\sin t)$, $y=a(1-\cos t)$ $(0\le t\le 2\pi)$ egri chiziq bo'yicha.

35. $\int\limits_{AB}(x^2-y^2)dx + xydy$ bu yerda AV A(1;1) V(3;4) nuqtalardan o'tuvchi to'g'ri chiziq.

Ikki o`lchovli integral va uning yordamida yuza hisoblash.

$$S=\lim_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\sum\sum\Delta x\Delta y=\iint\limits_{(S)}dxdy=\int_a^b dx\int_{y_1(x)}^{y_2(x)}dy$$

Agar, (S) soha $h\le y\le \ell$, $x_1(y)\le x\le x_2(y)$ tengsizliklar bilan aniqlangan bo`lsa u holda

$$S=\iint\limits_{(S)}dxdy=\int_h^{\ell}dy\int_{x_1(y)}^{x_2(y)}dx.$$

1-misol: $\int_{-2}^{1}\left(\int_{x}^{2-x^2} dy\right)dx$ ni hisoblang.

Yechish: $\int_{-2}^{1}\left(\int_{x}^{2-x^2} dy\right)dx = \int_{-2}^{1}(2-x^2-x)dx = \left[2x-\frac{x^3}{3}-\frac{x^2}{2}\right]\Bigg|_{-2}^{1} = \frac{9}{2}$

2-misol: $x = 4y - y^2$ va $x + y = 6$ chiziqlar bilan chegaralangan figuraning yuzi hisoblansin.
Yechish: $x = 4y - y^2$ va $x + y = 6$ chiziqlarni kesishish nuqtasini topamiz.

$\begin{cases} x = 4y - y^2 \\ x + y = 6 \end{cases}$ ni yechib A(4;2) va V(3;3) nuqtalarni topamiz. Natijada:

$$S = \int_D dxdy = \int_2^3 dy \int_{6-y}^{4y-y^2} dx =$$

$$\int_2^3 x\Bigg|_{6-y}^{4y-y^2} dy = \int_2^3(-y^2+5y-6)dy = \left[-\frac{1}{3}y^3+\frac{5}{2}y^2-6y\right]_2^3 = \frac{1}{6}$$

Darsda yechish uchun misollar

1. Hisoblang. $\int_0^{2\pi} cos^2 xdx \int_0^a ydy$
2. Hisoblang. $\int_1^3 dx \int_{x^3}^{x}(x-y)dy$
3. $\iint_D y\ln xdxdy$ D soha quyidagi chiziqlar bilan chegaralangan.

$xy = 1,\quad y = \sqrt{x}\quad x = 2$ J: $5(2\ln 2 - 1)/8$

Quyida berilgan chiziqlar bilan chegaralangan
figuralarning yuzini hisoblang

4. $x = y^2 - 2y \qquad x + y = 0 \qquad J: \frac{1}{6}$
5. $y = 2 - x \qquad y^2 = 4x + 4 \qquad J: \frac{64}{3}$
6. $y^2 = 4x - x^2 \qquad y^2 = 2x \qquad J: 2\pi - 16/3$
7. $3x^2 = 25x \quad 5x^2 = 9y \qquad J: 5$

Mustaqil uy vazifasi uchun misollar
Ikki o`lchovli integralni hisoblang

1. $\iint_D xydxdy \qquad (0 \le x \le 1, \quad 0 \le y \le 2)$

2. $\iint_D e^{x+y}dxdy \qquad (0 \le x \le 1, \quad 0 \le y \le 1)$

Quyida berilgan chiziqlar bilan chegaralangan
figuralarning yuzini hisoblang

3. $y^2 + 2y - 3x + 1 = 0$
$3x - 3y - 7 = 0 \qquad J:125/18$

4. $y = 4x - x^2 \quad y = 2x^2 - 5x \qquad J:27/2$

5. Egri chiziqli integrallarni hisoblash.

I-tur egri chiziqli integral quyidagi formula bo`yicha hisoblanadi.

$$\int_{AB} f(x,y)dl = \int_\alpha^\beta f[\varphi(t),\psi(t)]\sqrt{(\varphi'(t))^2 + (\psi'(t))^2}\,dt \quad (1)$$

Agar AB egri chiziq $y = y(x)$, tenglama bilan berilgan bo`lsa $(a \le x \le b)$, u holda I-tur egri chiziqli integral

$$\int_{AB} f(x,y)dl = \int_a^b f(x,y(x))\sqrt{1+[y'(x)]^2}\,dx \qquad (2)$$

II – TUR EGRI CHIZIQLI INTEGRAL.

$$\int_{AB} P(x,y)dx + \int_{AB} Q(x,y)dy$$

$$\int_{AB} P(x_1y)dx = \int_\alpha^\beta p[\varphi(t),\psi(t)]\varphi(t) \qquad \int_{AB} Q(x_1y)dx = \int_\alpha^\beta Q[\varphi(t),\psi(t)]\varphi(t)$$

$$\int_{AB} P(x_1y)dx + Q(x_1y)dy = \int_\alpha^\beta \{P[\varphi(t),\psi(t)]\varphi(t) + Q[\varphi(t),\psi(t)]\psi(t)\}dt$$

1-misol. $\int_c (x^2+y^2)dl$ egri chiziqli integralni $x = a\cos t, \quad y = a\sin t \quad o \le t \le 2\pi$ parametrik formulalar bilan berilgan S aylana bo`yicha hisoblang.

Yechish: Integralni (1) formula bo`yicha hisoblaymiz . $\varphi'(t) = -a\sin t \quad \psi'(t) = \cos t$ bo`lganligi uchun

$$\int_c (x^2+y^2)dl = \int_o^{2\pi} (a^2cos^2t + a^2\sin^2 t)\sqrt{a^2\sin^2 t + a^2cos^2t}\,dt =$$

$$\int_o^{2\pi} a^2(cos^2t + \sin^2 t)\sqrt{a^2(\sin^2 t + a^2cos^2t)}\,dt = \int_o a^2 \cdot a\,dt = 2\pi a^3$$

2-misol. $\int\limits_c (x-y)de$ egri chiziqli integralni hisoblang, bu yerda s to`g`ri chiziqning A(0;0) nuqtadan B (4;3) nuqtagacha bo`lgan qismi:

Yechish: Avval A(0;0) va V(4;3) nuqtalardan o`tuvchi to`g`ri chiziq tenglamasini tuzib olamiz.

$A(x_1 y_1)$ *ва* $B(x_2 y_2)$ nuqtalardan o`tuvchi to`g`ri chiziq tenglamasini topish

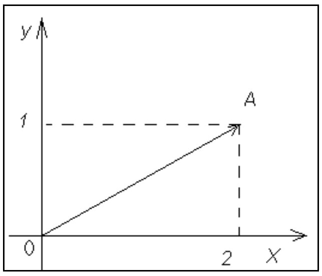
$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ formulasiga asosan $\frac{x-0}{4-0}=\frac{y-0}{3-0}$ bundan $y=\frac{3}{4}x$ $y'=\frac{3}{4}$.

(2) formulaga asosan

$$\int\limits_c (x-y)dl=\int\limits_0^4 (x-\frac{3}{4}x)\sqrt{1+\frac{9}{16}}dx=\frac{5}{16}\int\limits_0^4 xdx=\frac{5}{32}x^2\Big|_0^4=\frac{5}{2}$$ ni topamiz.

3-misol. $\int\limits_L 2xydx-x^2dy$ egri chiziqli integralni hisoblang, bu yerda L O(0,0) va A(2,1) nuqtalarni birlashtiruvchi to`g`ri chiziq kesmasi

Yechish:



OA kesmani ko`ramiz. Integrallash yo`li tenglamasini tuzamiz. Buning uchun ikki nuqtadan o`tuvchi to`g`ri chiziq tenglamasi $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ ga ko`ra OA to`g`ri chiziq tenglamasini tuzamiz.

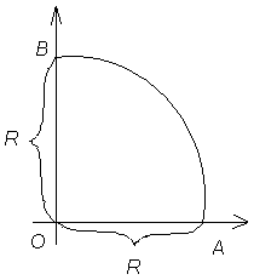
$$OA:\ \frac{x-0}{2-0}=\frac{y-0}{1-0}\ \Rightarrow\ y=\frac{1}{2}x,\quad 0\le x\le 2$$ formulaga asosan

$$\int\limits_L 2xydx-x^2dy=\int\limits_0^2 2x\frac{1}{2}xdx-x^2(\frac{1}{2}x)'dx=\int(x^2-\frac{1}{2}x^2)dx=\frac{1}{2}\int\limits_0^2 x^2dx=\frac{1}{2}\cdot\frac{x^3}{3}\Big|_0^2=\frac{4}{3}$$

4-misol. $\int\limits_L (x+y)dx+(x-y)dy$ egri chiziqli integralni hisoblang, bu yerda L-aylananing t o`sib borish yo`nalishida olingan $x=R\cos t,\ \ y=R\sin t,\ \ 0\le t\le\frac{\pi}{2}$ yoyi.

Yechish: Aylananing yoyini ko`ramiz

$t=0$ da $\begin{cases}x=R\\ y=0\end{cases}$ $\quad t=\frac{\pi}{2}$ da $\begin{cases}x=0\\ y=R\end{cases}$



$$dx = (R\cos t)'dt = -R\sin t dt,$$
$$dy = (R\ \sin t)'dt = R\ \cos t dt.$$

formulaga asosan

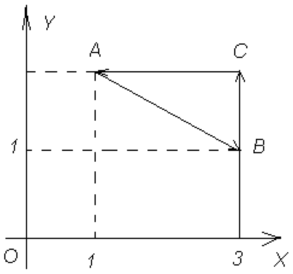
$$\int\limits_L (x+y)dx + (x-y)dy = \int\limits_0^{\frac{\pi}{2}} (R\cos t + R\sin t)(-R\sin t)dt + (R\cos t - R\sin t)R\cos t dt =$$

$$= R^2 \int\limits_0^{\frac{\pi}{2}} (-\sin t \cos t - \sin^2 t + \cos^2 t - \sin t \cdot \cos t)dt = R^2 \int\limits_0^{\frac{\pi}{2}} (\cos 2t - \sin 2t)dt =$$

$$= R^2 (\frac{1}{2}\sin 2t + \frac{1}{2}\cos 2t)\Bigg|_0^{\frac{\pi}{2}} = R^2 (\frac{1}{2}\sin\pi + \frac{1}{2}\cos\pi - \frac{1}{2}\sin 0 - \frac{1}{2}\cos 0) = -R^2$$

5-misol.

$\int\limits_L 2xydy - 3ydx$ egri chiziqli integralni hisoblang, bu yerda *L A(1;2) B(3;1), C(3;2)* uchlarga ega bo`lgan uchburchak konturi. Konturni aylanib o`tish yo`nalishi, soat ko`rsatkichi yo`nalishiga qarama-qarshi bo`lgan yo`nalish.



Yechish: ABC uchburchakni qurib olamiz. Uchburchak konturi uch qismdan iborat bo`lgani uchun integral uchta integral yig`indisiga teng bo`ladi.

$$\int\limits_L 2xydy - 3ydx = \int\limits_{AB} 2xdy - 3ydx + \int\limits_{BC} 2xdy - 3ydx + \int\limits_{CA} 2xdy - 3ydx$$

Konturning har bir qismi tenglamasini ikki berilgan nuqtalardan o`tuvchi to`g`ri chiziq tenglamasidan foydalanib tuzib olamiz.

$$AB: \ \frac{x-1}{3-1} = \frac{y-2}{1-2} \Rightarrow y = -\frac{x}{2} + \frac{5}{2} \qquad (dy = -\frac{1}{2}dx, 1 \le x \le 3)$$

$$BC: \frac{x-3}{3-3} = \frac{y-1}{2-1} \Rightarrow x = 3 \ \ (dx = 0,\ 1 \le y \le 3)$$

$$CA:\ \frac{x-3}{1-3}=\frac{y-2}{2-2}\Rightarrow y=2\ \ (dy=0,\ 1\le x\le 3)$$

Endi har bir integralni alohida hisoblab chiqamiz.

$$\int\limits_{AB}2xdy-3ydx=\int\limits_{1}^{3}2x(-\frac{1}{2}x)dx-3(-\frac{x}{2}+\frac{5}{2})dx=\int\limits_{1}^{3}(-x+\frac{3}{2}x-\frac{15}{2})dx=$$

$$=\int\limits_{1}^{3}(\frac{1}{2}x-\frac{15}{2})dx==(\frac{x^2}{4}-\frac{15}{2}x)\Big|_{1}^{3}=-13;$$

$$\int\limits_{BC}2xdy-3ydx=\int\limits_{1}^{2}6dy-3y0=6y\Big|_{1}^{2}=12-6=6$$

$$\int\limits_{CA}2xdy-3ydx=\int\limits_{3}^{1}2x0-3\cdot 2dx=-6x\Big|_{3}^{1}=12.$$

Demak, $\int\limits_{L}2xdy-3ydx=-13+6+12=5$

Darsda yechish uchun misollar

Quyidagi egri chiziqli integrallar hisoblansin:

1. $\int\limits_L (x^2+y^2)dx + ydy$ egri chiziqli integralni hisoblang. Bu yerda L $y=e^x$ egri chiziqning A(0;1) nuqtadan V(1;e) nuqtagacha bo\`lgan yoyini hisoblang.

J: $e^2-\frac{2}{3}$

2. $\int y^2dx+2xy\,dy$ *интеграл* $x=a\cos t_1$ $y=a\sin t$ aylana bo\`yicha.

J.O

3. $\int ydx - xdy$ *интеграл* $x=\cos t_1$ $y=a\sin t$ ellips yoyi bo\`yicha.

J.-$2\pi ab$

4. $\int\left(\frac{x}{x^2+y^2}dx-\frac{y}{x^2+y^2}dy\right)$ integral markazi koordinatalar boshida bo\`lgan aylana bo\`yicha. J.0

5. $\int\frac{ydx+xdy}{x^2y^2}$ integral $y=x$ to\`g\`ri chiziqning $x=1$ dan $x=2$ gacha kesmasi bo\`yicha. J.$\ln 2$.

6. $\int yzdx+xzdy+xydz$ integral t= 0 dan 2π gacha o\`zgarganida $x=a\cos t$ $y=a\sin t$, $z=rt$ chizig\`ining yoyi bo\`yicha. J.0.

Mustaqil uy vazifasi uchun misollar

1. $\int xdy-y\,dx$ integral $x=a\cos^3 t$ $y=a\sin^3 t$ astroidaning yoyi bo\`yicha. *ж* $\frac{3}{4}\pi a^2$ (astroidaning ikkilangan yuzi).

2. $\int xdy-yxd$ integral $x=\frac{3at}{1+t^3}$; $y=\frac{3at^2}{1+t^3}$ Dekart yaprog\`ining sirtmog\`i bo\`yicha. J. $3a^2$ (ko\`rsatilgan sirtmoq bilan chegaralangan sohaning ikkilangan yuzi).

3. $\int xdy - ydx$ integral $x=a(t-\sin t)$, $y=a(1-\cos t)$ $(0\le t\le 2\pi)$ egri chiziq bo\`yicha. J. $-6\pi a^2$ (sikloidaning bitta arki va Ox o\`q bilan chegaralangan sohaning ikkilangan yuzi).

4. $\int\limits_{AB}(x^2-y^2)dx+xydy$ bu yerda AB A(1;1) B(3;4) nuqtalardan o\`tuvchi to\`g\`ri chiziq. J: $\frac{67}{6}$

Asosiy adabiyotlar

1. Claudio Canuto, Anita Tabacco. Mathematical Analysis I. Sprinder-Verlag Italia, Milan 2008.
2. Claudio Canuto, Anita Tabacco. Mathematical Analysis II. Sprinder-Verlag Italia, Milan2010.
3. Erving Kreyszig, Herbert Kreyszig, Edward Normuton. Advanced engineering Mathematics. New York, 2011.
4. H. Latipov va boshqalar. Oliy matemetika. T. «Aloqachi» -2005.
5. R.E.Danko. Oliy matematika misol va masalalarda. «O'zbekiston faylasuflari milliy jamiyati» 2007
6. A.Y. Normanov. Analitik geometriya. T. 2008
7. Soatov Yo. U.. Oliy matematika .1 tom, T.O'qituvchi, 1992.
8. Soatov Yo. U. Oliy matematika .III tom T.O'qituvchi, 1995.
9. Xurramov Sh.R. Oliy matematika. Misollar. Nazorat topshiriqlari. l-qism. T.: Fan va texnologiyalar, 2015

Qo'shimcha adabiyotlar:

1. Gerd Baumann. Mathematics for Engineers I. Munchen, 2010.
2. Lay, David C. Linear algebra and its applications. 2012.
3. N.Ya. Vilinkin, Ye.S. Kuniskaya, A.M. Mordkovix "Matematicheskiy analiz" integralnoe ischisleniya. Moskva 1979.
4. A. Sa'dullaev, H. Mansurov, G. Xudoyberganov va boshq. "Matematik analiz kursidan misol va masalalar to'plami" Toshkent, O'zbekiston nashriyoti 1993.
5. N.SH. Kremer. "Visshiy matematika dlya ekonomistov" Moskva 2004.
6. http://www.mcce.ru, http://lib.mexmat.ru
7. http:// www.a-geometry.narod.ru